Distortion in Perspective Projection

Robert P. Kelso Sr.
Professor (retired) Louisiana Tech University, kelso@coes.latech.edu

Abstract

The paper presents a unique approach in associating perspective projection with the image beheld by the eye and demonstrates that all graphical and photographic perspective projections must contain distortion when compared to the image beheld by the eye.

INTRODUCTION

This paper defines distortion in perspective projection as the difference between normal vision and the emulation of normal vision by graphics or photography.

Section (1.0) first analyzes the distortion of perspective projection produced on a plane-of-projection and demonstrates that such distortion is due to the lack of perpendicularity among the projectors to the plane-of-projection. The paper further offers an observation on problems associated with viewing perspective pictures.

Section (2.0) then investigates an alternate surface, the sphere, which permits all projectors toward its center to be perpendicular to a surface of projection.

Section (3.0) demonstrates that a perspective projection onto a spherical surface of projection is analogous to light rays into the eye, which rays are similarly perpendicular to the spherical-shaped retina, and reproduces thereon and thereby the image beheld by the eye.

Section (4.0) concludes that there is no remedy available to correct for the intrinsic distortion in perspective projection because it is impossible to rectify onto a planar surface, without distortion, an image on a sphere.

1.0 DISTORTION ON THE PLANE-OF-PROJECTION

Fig. 1 and Fig. 2 demonstrate a) that distortion is due to the lack of perpendicularity among the projectors to the plane-of-projection and acknowledges b) that only a single projector of sight, namely, the axis of the cone of vision, from the object to the eye may be orthogonal to a plane-of-projection and c), concludes therefore, that all projectors other than b) result in the distortion of the image. This is also true of the camera: only a single ray of light, namely, the principal optical axis, may be orthogonal to the film and, therefore, all other rays of light result in the distortion of the photograph.

Fig. 1 identifies the elements of Fig. 2. As an aid to the reader's intuition the illustrations use a human form as an analog for the Station Point (Focal Point). The object projected is a base molding for a building column.

Fig. 2 compares a projection onto a plane from one direction with a projection onto the same plane of the same object from a different direction—thereby creating different angles among the projectors to the plane-of-projection. Pivoting of the object eliminates differing viewing directions as a factor which might cause a difference in the images on the Projection Plane. The difference between the two images, as seen in the front view, is apparent.

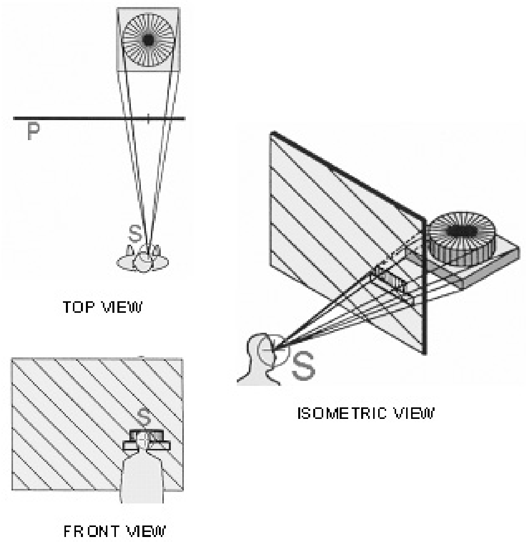

FIGURE 1: *Orthographic Top Front and Ilsometric Views*

Fig. 1 is an orthographic top-front, and isometric view of a pictorial analog of a standard graphical/photographic perspective projection. P designates the planar surface-of-projection. In photography, the projection plane (film) is located to the rear of the Station Point, S. This placement does not alter the geometric principles or the characteristics of the developed image. Note that an eye at S views both the object and the image of the object as if the projection plane did not exist. That is to say, to the eye, from the Station Point, the image of the object on the plane-of-projection is indistinguishable from the object itself.

Fig. 2 demonstrates the basis of perspective distortion on the projection plane.

The object in Fig. 1 is pivoted by an arbitrary amount about the Station Point, S, so that at all times the object appears unchanged to the eye. This causes the new image to appear the same to the eye as does the original image, namely, indistinguishable from the object itself. But the Front View in Fig. 2 reveals the new image to be clearly different from the original image. Therefore, because the only difference in the two projections is the angles they form with the plane-of-projection, this paper concludes that the two images are different because of the differences in the angles which the respective projectors form with the projection plane.

This logic extends to Fig. 1: the lines of sight from the object in its original position, at an angle other than perpendicular to the Projection Plane, produces a distortion of the image. This essentially encompasses all of the lines of sight in the projection other than the central axis of vision, which is to say all the lines of projection other than the axis of the cone of vision. Therefore, the paper concludes that any perspective projection of any image onto any plane-of-projection is necessarily distorted.

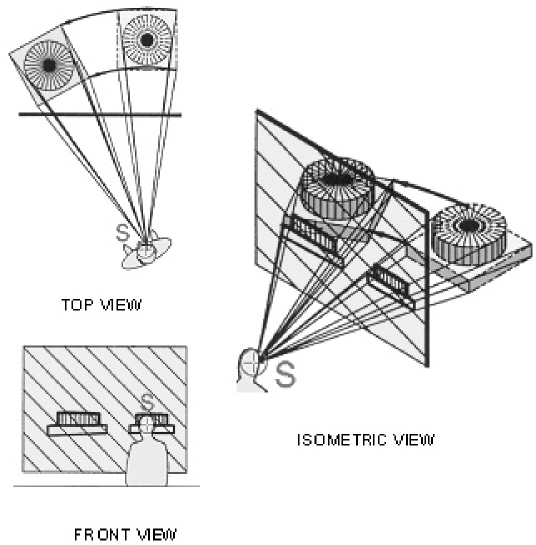

***FIGURE 2**: Pivoting the Model*

[Note: An exception to the phenomenon of distortion in graphics/photography would seem to be the case in which spatially parallel straight lines in nature are parallel to the plane-of-projection: they project as parallel line images onto the plane-of-projection. This is true on a plane-of projection, but because—as will be demonstrated—the retina is spherical in shape, straight lines that are parallel to a plane-of-projection cannot be parallel to a spherical surface, e.g., the retina. In other words, the exception applies to planar projection but not to vision.]

From an analysis of the above, it follows that an inspecting eye of a standard perspective image may avoid the distortion on The Plane-of-Projection by viewing the image from the exact position of its related Station Point/Focal Point. The likelihood of a casual viewer looking from precisely that point—the point at which the plane-of-projection distortion disappears—is vanishingly small. Therefore, an inspecting eye itself is challenged with an intrinsic distortion different from the intrinsic distortion on the plane-of-projection: the amount of the eyeball's chance distance from the related Station Point.

2.0 SPHERICAL SURFACE OF THE PROJECTION

The eye distorts reality as evidenced by spatially-parallel straight lines appearing to intersect—in the manner of receding straight railroad tracks. The author takes the concept a step further by asserting that an observer standing between the rails will see the rails as simultaneously intersecting in both (opposite) directions if peripheral vision permitted – in the manner of a Chameleon with lateral eye locations in which separate images from both eyes are simultaneously merged into one by the brain—just as the human brain merges the images from two front-facing eyes. How is this possible? After all, the geometry of classical foreshortening (convergent) theory applies to only a single direction of the central axis of the cone of vision. The author argues that it is possible because the retina of the eye is spherical in shape.

The retina, as a part of the spherical orb of the eyeball, is a sector of a sphere. Therefore, the image projected through the lens to the retina to the brain is the consequence of a projection onto a spherical (sector) surface-of-projection rather than onto a planar surface-of-projection, as is the case in both graphics and photography.

To accommodate this phenomenon the paper proposes a spherical model as a surface of projection rather than the standard planar model.

Figs. 3 - 9 present a spherical surface-of-projection as an analog of the retina, i.e., a 'projection sphere' rather than a 'projection plane'. The illustrations demonstrate that the step-by-step projection of spatially parallel lines onto the retina-analog sphere-of-projection mimics human eyesight.

The center of the sphere is analogous to the Focal Point of the retina. The resulting intersections of the projected lines as they appear upon the "picture sphere" resolve the enigma of spatially parallel lines appearing to intersect—in "both directions".

A brick wall serves as a model for spatially parallel lines.

[Artistic Note: The initial illustrations include strategically placed great circles to enhance the illusion of a sphere.]

Fig. 3 features a brick wall with parallel horizontal and vertical edges. The wall serves as an object in nature an image of which is to be projected onto the retina-analog spherical surface (P). By steps in the following illustrations, the edges are projected toward the center (S) of the "picture sphere", namely, toward the analog of the Focal Point of the retina. The spherical surface-of-projection is analogous to the spherical (sector) shape of the retina.

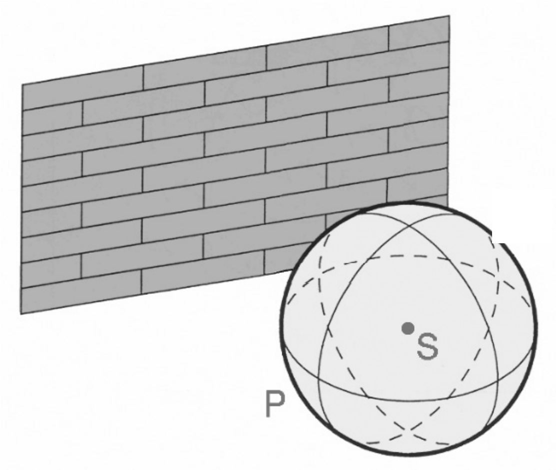

***FIGURE 3**: Sphericial Surface of Projection and Object Wall*

Fig. 4 shows the perspective projection of the horizontal top line of the wall to the Focal Point of the retina-analog,. [In life, the Focal Point of the retina is slightly in front of the retina—from where the image becomes inverted before contacting the retina and its subsequent transmission to the brain.] The intersecting projectors define a plane that intersects the sphere-of-projection in a great circle (G). [Note: For clarity, selected "artistic" great circles are occasionally omitted.]

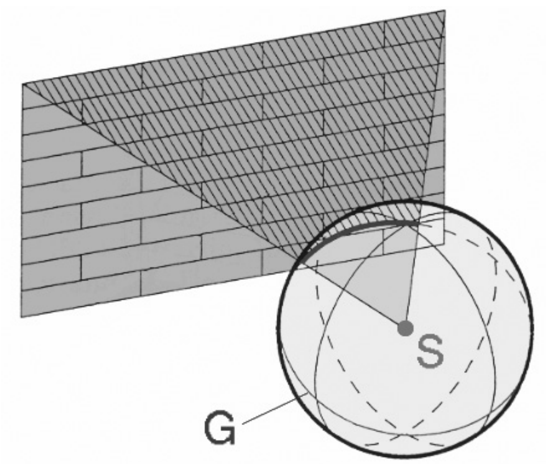

***FIGURE 4**: Projection of the Top Line of the Wall*

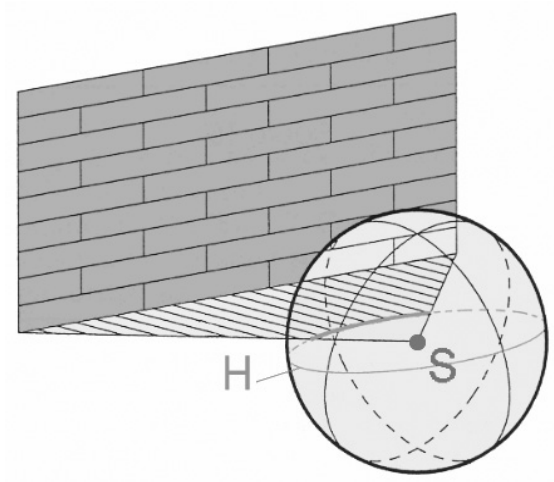

***FIGURE 5**: Projection of the Bottom Line of the Wall*

Fig. 5 similarly shows that the projection of the horizontal bottom line of the wall creates the great circle (H).

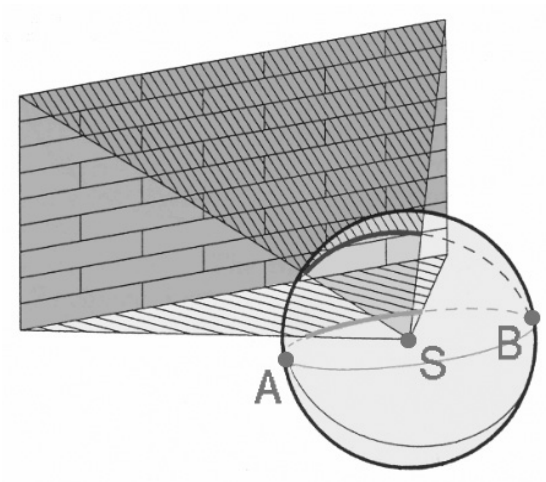

***FIGURE 6**: Intersection of Great Circles of the Wall*

Fig. 6 shows that the two great circles G and H intersect at points A and B. (Take note that A and B do not lie on the outer circle of the illustration, here artistically forming the sphere in the illustration.) A viewer at S ("Station Point"/ Focal Point) looking in the direction of A sees all horizontal lines of the wall as appearing to converge at A; the same is true of point B. The points of intersection, A and B, are analogous to the two points of intersection observed as one peers from between the rails down a railroad track simultaneously "in both directions".

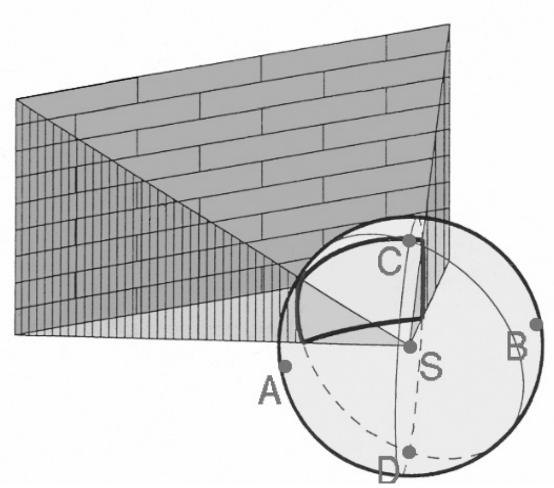

FIGURE 7: Projection of the Vertical Sides of the Wall

Fig. 7 shows a similar determination of the two vanishing points, C and D, for the spatially vertical lines of the wall. The perspective image now has four vanishing points, A, B, C, and D. Two of the vanishing points appear at the "east-west" poles and two at the north-south poles. By logical extension, a perspective image of a single plane may have an unlimited number of vanishing points as defined by an unlimited number of parallel straight lines thereon and their spatial directions. Note if the vertical lines of the wall represented the vertical corners of a tall building, a viewer at S looking upwards towards C will observe the vertical corners of the building as converging toward that point.

Fig. 8 shows that the horizon line, Z, is determined by a plane defined by the four vanishing points A, B, C, and D and is observed as a straight line from S, as are all great circles. Note that the horizon line of the wall is determined by vanishing points; the horizon line does not determine the vanishing points. A view from S towards any point along the perimeter Z perceives that point as located on the horizon line of the vertical wall. The horizon line would appear as horizontal if the wall were horizontal—and be more familiar to graphicians.

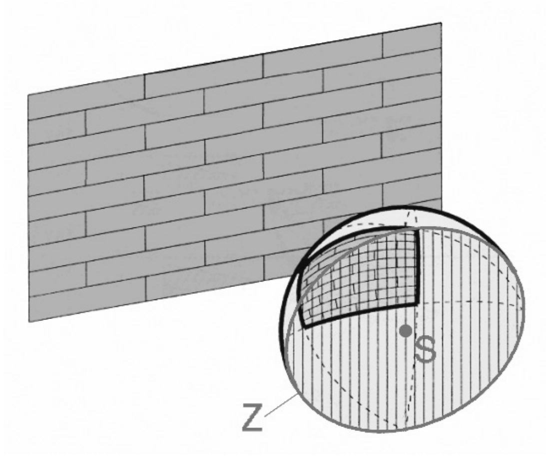

FIGURE 8: Hemisphere of Projection

CONCLUSION

From the above the paper concludes that if, in graphical/photographic perspective projection, it were possible to project an image onto a spherical surface and then to perfectly rectify the sphere into a planar surface the result would be a perspective projection image without distortion. However, this is not possible because it is impossible, geometrically, to perfectly rectify a sphere, e.g., the globe, into a plane, as especially cartography confirms. Therefore, all standard, perspective projection images are necessarily distorted.