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The Impact of Algebra and Trigonometry to Calculus Performance

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ABSTRACT: The study aims to determine the impact of Algebra and Trigonometry to the students' performance in Calculus. To achieve this objective, the researcher considered the following: (1) identification of the major topics in Differential Calculus and Integral Calculus with the respective applications of Algebra and Trigonometry; (2) computation of the coefficients of correlation to determine the relationship of the students' performance in the Pre-Calculus and Calculus courses; and (3) documentation of the significant findings of related investigations conducted in different countries. One hundred (100) engineering students participated in the study where final grades in Algebra, Trigonometry, Differential Calculus and Integral Calculus were elicited from them. The results of the study confirmed what the researchers of previous studies assumed that the deficiencies in Algebra and Trigonometry skills continue to impact adversely to Calculus students. The low rating of the students in Calculus was seen to be attributed to the poor performance in the pre-requisite subjects. As documented from related investigations, it is further concluded that students generally lack both conceptual and procedural understanding of Calculus because of the observed deficiencies in their mathematical content knowledge in the Pre-Calculus courses.

Keywords: Algebra, Calculus, Differential Calculus, Integral Calculus, Pre-Calculus, Trigonometry

INTRODUCTION

The students' deficiencies in terms of readiness in learning Pre-Calculus and Calculus has been observed over the past few decades in several countries. An inadequate comprehension of the concepts and high failure rate among the students has been a matter of great concern of various countries till now (Bokhari, Samman, & Yushau, 2006). Many institutions have given the impression to the mathematics entry point as a decisive factor to control in their efforts to improve student performance, retention and diversity in the students' chosen discipline (Ennis, Sullivan, Louie, & Knight, 2013). Previous studies revealed that only a small percentage of students taking Pre-Calculus even begin a Calculus course (Gordon, 1994; as cited in Jarrett, 2000).

Pre-Calculus mathematics courses are often the critical filter to many college majors. These introductory Calculus courses are sometimes the cause of change of major for college students (Whitely, 1987; as cited in Jarrett, 2000). Many of these students appear to be motivated for and

capable of Pre-Calculus success often do not do well; thus, results in changes of the major and loss of potential in the mathematics related careers (Kauffmann et al., 2011). Studies show that success in one of these Pre-Calculus courses does not insure that a student is successful in subsequent courses (Jarrett, 2000).

Calculus is regarded as the backbone of the undergraduate students in programs such as mathematics, sciences, engineering and technology. It initiates a solid process of learning among the students in different dimensions, forces them to utilize all faculties of their brain and above all, reshapes their mind to retain the material in this process (Bokhari et al., 2006). Unfortunately, the course often turns deterrents in the pursuance of higher education. Anecdotal information and experience indicate that a substantial proportion of the college students fail to meet the minimum grade criteria in the subject (Mwavita, 1994; Seymour & Hewitt, 1997; Jarrett, 2000; Bokhari et al., 2006; Islam & Al-Ghassani, 2015). Some of these students ultimately have to leave the university because of their repeated poor performance in Calculus (Islam & Al-Ghassani, 2015).

Because of the importance of the Pre-Calculus and Calculus courses in a wide range of disciplines in the undergraduate courses, the researcher opted to conduct an investigation in order to determine the impact of the Pre-Calculus courses to the Calculus performance of the students in college.

METHODOLOGY

The study used the descriptive method of research. It deals with the gathering of facts or information pertaining to the given conditions or situations for the purpose of description and inclusion of proper analysis and interpretation. To examine the impact of the Pre-Calculus courses to the Calculus performance of the students in college, the researcher considered the following methods: (1) identification of the major topics in Differential Calculus and Integral Calculus with the respective applications of Algebra and Trigonometry; (2) computation of the coefficients of correlation to determine the relationship of the students' performance in the Pre-Calculus and Calculus courses; and (3) documentation of the significant findings of related investigations conducted in different countries. The Pre-Calculus courses



considered in this study are Algebra and Trigonometry; while the Calculus subjects are Differential Calculus and Integral Calculus.

In the identification of specific applications of the Pre-Calculus subjects to the topics contained in Calculus, the researcher presented in tabular form the following: (1) the topics included in the Differential Calculus with the applications of Algebra; (2) the topics included in the Integral Calculus with the applications of Algebra; (3) the topics included in the Differential Calculus with the applications of Trigonometry; and (4) the topics included in the Integral Calculus with the applications of Trigonometry.

To describe quantitatively the impact of Algebra and Trigonometry to Calculus, the researcher elicited from one hundred (100) engineering students their grades in the Pre-Calculus subjects (Algebra and Trigonometry) and Calculus subjects (Differential Calculus and Integral Calculus). Algebra and Trigonometry are the mathematical foundation subjects of the first year students prior to their advancement to Differential Calculus and subsequently to Integral Calculus.

The coefficients of correlation using the Pearson's product moment of correlation (r) were computed and analyzed together with the corresponding p -values to be able to conclude whether the relationship is significant or not. To note, the correlation coefficient does not only describe the relationship between variables; it also allows one to make prediction from one variable to another (Jackson, 2009). The correlation between variables indicates that when one variable is present at a certain level, the other also tends to be present at a certain level. Also employed in the analysis of data

is the coefficient of determination (r^2), a measure of the proportion of the variation in one variable that is accounted for by another variable, calculated by squaring the correlation coefficient (Jackson, 2009).

Finally, the research findings of previous investigations focusing on the impact of the Pre-Calculus to Calculus performance of the students were documented to provide insights as regards to students' learning of Calculus in the global setting.

RESULTS AND DISCUSSION

The Impact of Algebra to Calculus

For better understanding of the impact of Algebra to Calculus, the researcher identified some of the major topics in Calculus with the corresponding significant applications of Algebra. In the following table, the applications of Algebra to Differential Calculus are presented. The applications stated in the second column play significant contribution to be able to solve the problems in the topics identified in Differential Calculus. For example, in the first topic, critical points of a curve can only be located upon equating the first derivative of the given function to zero and solving for the values of unknown variable. Moreover, the points of inflection can only be determined upon equating the second derivative of the given function to zero and subsequently solve the unknown variable. In most cases, functions are in the higher degree form that require the technique of factoring, completing the square, or with the use of quadratic formula to be able to get the critical numbers. The other applications of Algebra to Differential Calculus are presented in the succeeding numbers in Table 1.

Table 1: Topics in Differential Calculus with the corresponding applications of Algebra

	Topics in Differential Calculus	Applications of Algebra
1	Locating critical points and points of inflection of a curve	Finding the values of unknown variables by factoring, completing the square or with the use of the quadratic formula
2	Using Newton's method to find the roots of polynomial equations	Employing the concept of zeroes of the equation
3	Determining the angle of intersection of a pair of curves	Solving systems of linear and/or quadratic equations to identify the points of intersection
4	Obtaining the Maclaurin's series of an infinite series	Computing the values of the function by the concept of composition
5	Calculating approximate values of radical numbers using differentials	Applying the laws of radicals

In the subsequent table, the applications of Algebra to Integral Calculus are indicated. In the first topic, whenever the integrand is in the form of rational function, especially of higher degree, the only way to calculate the integral is to resolve first the given rational function into partial fractions. Moreover, if the given fraction is in improper form (the degree of the polynomial in the numerator is either equal of greater than the degree of the polynomial in the denominator), then, the long method of division of polynomials, as learned from Algebra,



should first be applied before resolving into partial fractions. All other significant applications of Algebra to Integral Calculus are shown in the succeeding numbers in Table 2.

Table 2: Topics in Integral Calculus with the corresponding applications of Algebra

	Topics in Integral Calculus	Applications of Algebra
1	Integrating rational function	Resolving proper or improper fraction into partial fractions
2	Employing algebraic substitution in integration	Applying the laws of exponents
3	Using reciprocal substitution in integration	Simplifying complex fractions
4	Finding the area and volume of a solid of revolution	Solving systems of linear and/or quadratic equations to identify the limits
5	Testing for convergence of infinite series using integral test	Determining the general term of a series

To be able to measure the impact of Algebra to Calculus, the final grades of the student-participants in the subjects were elicited. To note, the final grades of the students in the mathematics subjects were based on the following: students' grades in the midterm examination, final examination, quizzes, problem sets or exercises, recitation, and attendance. Table 3 shows that the correlation of the students' grades in Algebra and Differential Calculus is 0.6667 with $p < 0.00001$. A p-value of less than 0.05 indicates that the correlation is significant. Similarly, the correlation of the students' grades in Algebra and Integral Calculus was computed. The r-value of 0.5399 with $p < 0.00001$ is indicative of significant correlation of the students' grades in the said subjects.

Table 3: Correlations of students' grades in Algebra and Calculus

Performance in mathematics	r-value	p-value	Conclusion
Algebra and Differential Calculus	0.6667	< 0.00001	significant at $p < 0.05$
Algebra and Integral Calculus	0.5399	< 0.00001	significant at $p < 0.05$

For a wider understanding of the impact of Algebra to Calculus, the researcher made a documentation of the significant findings of some related studies conducted in different countries. Presented in the subsequent discussions are the results of the investigations conducted by different researchers in the following countries: Australia, Japan, United States of America, Singapore, and South Africa.

The students' responses to Calculus word problems involving rates of change were analyzed from a group of students in Australia. Investigators found in their study that students' poor understanding of variables and symbols seemed to be a major source of difficulties in Calculus. Detailed analysis of the researchers revealed students' errors, in which variables are treated as symbols to be manipulated rather than as quantities to be related. The investigators surmised that the students had developed an "abstract-apart" concept of a variable rather than the "abstract-general" concept that is needed for the successful study of Calculus (White & Mitchelmore, 1996).

The students' conceptual understanding of Calculus and their ability to use Algebra to solve traditional Calculus problems were examined from the groups of students in Japan and United States of America. The findings indicated that students' difficulties in Calculus stem from their difficulties in dealing with functions in Algebra. It was further pointed out from the study that while many of the students could carry out the procedures required to find and

use derivatives in sketching graphs of functions, almost all the students had a poor understanding of functions, which had led to misconceptions in solving problems, especially in the application of differentiation and in integration (Judson & Nishimori, 2005; as cited in Yee & Lam, 2008).

The students' errors in integration of rational functions were studied from a group of students in Singapore. Investigators concluded that errors were generally classified in the study as those due to the students' weak algebraic concepts and their lack of understanding of the concept of integration. Detailed findings of the researchers revealed that some students had approached integration with a poor background in Algebra which could have caused them misread the integration or misinterpret symbols and carry out algebraic work such as indices and long division of polynomials incorrectly (Yee & Lam, 2008).

The effect of deficiencies in basic Algebra and Pre-Calculus to Calculus class was determined to a group of students in the United States of America. Findings revealed that students can generally grasp the new concepts that are being taught, but inadequate skills in Algebra and Pre-Calculus hinder them from correctly completing computational problems. Based on the data analysis, the authors were able to determine that factoring and working with rational functions are the two main skills with which students struggle. Investigators concluded further that students can



apply correctly the quotient rule for finding the derivative, but they commit mistakes such as illegal cancellation of terms when they try to simplify the expression (Agustin & Agustin, 2009).

Students' errors and misconceptions in Calculus were identified from a group of students in South Africa. The investigators outlined the nature of students' mathematical errors and misconceptions. Their analysis showed that most of the errors and misconceptions were due to knowledge gaps in basic Algebra. Accordingly, students' performance in Calculus is undermined by weak basic algebraic skills of factorization, handling operations in directed numbers, solving equations and poor understanding of indices (Luneta & Makonye, 2010).

Students' errors when calculating the derivatives of trigonometric functions were analyzed from another group of students in South Africa. It was found that some errors displayed by students were errors of interpretation and linear extrapolation. The researcher concluded that interpretation errors occurred when students wrongly interpreted a

concept due to over-generalization of the existing schema; while linear extrapolation errors occurred when students over-generalized the property $f(a + b) = f(a) + f(b)$, which applies only when f is a linear function, to the form $f(a * b) = f(a) * f(b)$, where f is any function and $*$ is any operation (Siyepo, 2015).

The Impact of Trigonometry to Calculus

To be able to recognize the impact of Trigonometry to Calculus, the researcher identified some of the major topics in Calculus with the respective significant applications of Trigonometry in the following table. In the first topic, whenever polar curves are given, the equations are usually expected to be in trigonometric form. If the problem requires the measure of the angle of intersection of the given polar curves, then, the initial step is to find the points of intersection of the curves. In doing so, the points of intersection can be determined by solving the unknown variable in the given trigonometric equations. The other applications of Trigonometry to Differential Calculus are presented in the succeeding numbers in Table 4.

Table 4: Topics in Differential Calculus with the corresponding applications of Trigonometry

	Topics in Differential Calculus	Applications of Trigonometry
1	Getting the angle of intersection of polar curves	Determining the values of unknown in trigonometric equations
2	Obtaining solutions to problems on time rates involving triangles	Employing trigonometric relations and identities
3	Solving maxima and minima problems with application of right triangles	Applying the concept of angle of elevation/depression or solutions of right triangles
4	Finding the circle of curvature of curves in polar form	Simplifying trigonometric equations
5	Measuring the angle between the radius vector and tangent	Calculating the angle of the function tangent

Indicated in the following table are the applications of Trigonometry to Integral Calculus. On the first topic, to be able to determine the integrals of trigonometric functions, the knowledge of Trigonometry is required. In most cases, integrand in trigonometric form must be simplified first or expressed to its equivalent form by applying trigonometric identities prior to integration. All other significant applications of Trigonometry to Integral Calculus are shown in the succeeding numbers in Table 5.

Table 5: Topics in Integral Calculus with the corresponding applications of Trigonometry

	Topics in Integral Calculus	Applications of Trigonometry
1	Determining integrals of trigonometric functions	Simplifying integrals with the use of trigonometric identities
2	Using trigonometric substitution in integration	Employing Pythagorean relations of trigonometric functions
3	Translating trigonometric integral to its equivalent expression	Applying the product to sum identities of trigonometric functions
4	Finding integrals by half-angle substitution	Making use of half-angle formula of tangent
5	Calculating the area of polar curves	Evaluating trigonometric expressions

In order to measure the impact of Trigonometry to Calculus, the final grades of the student-participants in the subjects were elicited. As mentioned earlier, the final grades of the students in the mathematics subjects are



based on the following: students' grades in the midterm examination, final examination, quizzes, problem sets or exercises, recitation, and attendance. The correlation of the students' grades in Trigonometry and Differential Calculus as shown in Table 6 is 0.2698 with a corresponding p-value of 0.006635. A p-value of less than 0.05 indicates that the correlation is significant. Similarly, the correlation of the students' grades in Trigonometry and Integral Calculus was computed. The r-value of 0.5732 with $p < 0.00001$ is indicative of significant correlation of the students' grades in the said subjects.

Table 6: Correlations of students' grades in Trigonometry and Calculus

Performance in mathematics	r-value	p-value	Conclusion
Trigonometry and Differential Calculus	0.2698	0.006635	significant at $p < 0.05$
Trigonometry and Integral Calculus	0.5732	< 0.00001	significant at $p < 0.05$

For a deeper understanding of the impact of Trigonometry in Calculus, some related studies conducted in different countries were documented by the researcher. Presented in the succeeding discussions are the results of the investigations conducted by different researchers in the following countries: United States of America, Singapore, Turkey, Nigeria and South Africa.

The efficacy of a conceptual change model to amend distributive law misconceptions was investigated in a classroom setting with a group of Calculus students in the United States of America. It was observed that students mistakenly apply the distributive property to functions or operations that are not distributive. According to the researchers, often such mistakes indicate an underlying misconception rather than just a thoughtless error. This misconception, as they analyzed, is compelling and acts as a barrier to learn the correct understanding of certain functions and correct ways to manipulate non-distributive functions or operations. It was further concluded that this misconception impacts learning the logarithmic, exponential, and trigonometric functions; and this also leads to difficulty comprehending a fundamental expression used frequently in Calculus (Skane & Graeber, 1993).

Difficulties in solving integration problems in Calculus were analyzed from a group of students in Singapore. It was found that some technical errors relate to the lack of mathematical content knowledge in Trigonometry. The errors of this nature, according to the investigator, were due to students' lack of knowledge in trigonometric identities; this occurred in question where, in order to be able to integrate, students had to change the trigonometric expression to an equivalent trigonometric identity. It was further observed in the study that only a small number of students remembered the required formula needed; the other

errors were due to the students' lack of knowledge in solving trigonometric equations (Kiat, 2005).

The gap on students' understanding of real numbers and trigonometric relations was explored to a group of Calculus students in Turkey. In this study, the two levels were defined for a measure of students' knowledge; the first one is radian concept and the second one is a trigonometric function concept. It has been realized in the investigation that students' skills for usage of radian concept have not been developed. The findings have shown that the students perform unsatisfactorily in the operation with real numbers of trigonometric expressions. It was further concluded that there are two different types of error that the freshman Calculus students made; the first one is the lack of knowledge and the second one is the manipulative (Orhun, 2010).

The algebraic pre-requisite skills of students as incorporated into the applied Calculus optimization problem solution were assessed from a group of students in Nigeria. It was concluded from the study that an accumulation of structural and procedural errors help in worsening the solution processes of non-algebraic equations (trigonometric). Most common among operational breakdown, according to the investigation, is inability to find the sum of two trigonometric fractions as well as clearing the resultant fraction. Moreover, it was observed that students find hard to identify, recall, and use the appropriate trigonometric identity, in addition to recognizing quadratic structures express in trigonometric terms in order to get its solution (Usman, 2012).

As earlier mentioned, errors when calculating the derivatives of trigonometric functions were analyzed from a group of students in South Africa. It was also found from that study that some errors displayed by students were procedural. Procedural errors occurred when students failed to carry out manipulations or algorithms, even if concepts were



understood. The findings revealed that the participants were not familiar with basic operational signs such as addition, subtraction, multiplication and division of trigonometric functions. The participants, as concluded by the researcher, demonstrated poor ability to simplify once they had completed differentiation (Siyepo, 2015).

CONCLUSIONS

It is apparent from the research findings that the students' knowledge of Algebra has significant impact to their performance in Calculus. This can be described quantitatively by the coefficients of determination (r^2) which disclosed that 44.44 percent and 29.15 percent of the variation of the students' performance in Differential and Integral Calculus, respectively, are accounted for by their performance in Algebra.

It is likewise evident from the results of the investigation that the students' understanding of Trigonometry has also relevant impact to their performance in Calculus. This can be depicted numerically by the coefficients of determination which revealed that 7.28 percent and 32.85 percent of the variation of the students' performance in Differential and Integral Calculus, respectively, are accounted for by their performance in Trigonometry.

The results of this study confirmed what the other researchers suspected that the deficiencies in Algebra and Trigonometry skills continue to impact adversely to Calculus students. Thus, the low rating of the students in Calculus can be attributed to the poor performance in the pre-requisite subjects. As documented from related investigations earlier mentioned, it is further concluded that students generally lack both conceptual and procedural understanding of Calculus because of the observed deficiency in their mathematical content knowledge in Pre-Calculus courses.

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RECOMMENDATIONS

The results of this study encourage students to pay close attention in learning the basic knowledge of mathematics, like Algebra and Trigonometry, for this serve as their foundation to advance to the succeeding mathematics courses like Differential and Integral Calculus. If this will be neglected, students are likely to become progressively more confused and in the long run they may not survive in their higher mathematics courses.

It is hoped that these findings will prompt mathematics teachers to re-examine their instructional strategies and practices in teaching Pre-Calculus and Calculus courses. Teachers may take into consideration that their students may learn best by being exposed to problem-based curricula that allow them to explore the mathematical content in a way that develops both their students' conceptual and procedural understanding of mathematics. A continued emphasis on and reinforcement of such students' understanding and skills, may not only translate to a higher chance of success in Differential and Integral Calculus, but also will enhance the learning environment of their students' to advance mathematics courses.

The outcome of this study may provide insights to school administrators to give attention not only to overall high school grades of the students, but also to high school mathematics grades, particularly for students' admission into mathematics, sciences, engineering and technology based programs. It may be considered to revisit the existing retention program and take into account the necessity of providing academic support programs for students who were admitted in the disciplines earlier mentioned but later found to have low performance after taking the basic mathematics courses.

Lastly, this study hopefully could spur further research into students' errors and difficulties in advance mathematics courses.



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