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Effects Of Radiation And Viscous Dissipation On Mhd Boundary Layer Flow Due To An Exponentially Moving Stretching Sheet In Porous Medium

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Abstract: Aim of the paper is to investigate radiation effects on the MHD flow over an exponentially moving stretching sheet placed in a porous medium. A variable magnetic field is applied normal to the sheet. Similarity transformation is used to convert the governing nonlinear partial differential equations into a system of ordinary differential equations which are solved numerically using forth order Runge- Kutta integration scheme with shooting iteration technique. The effects of physical parameters on the dimensionless velocity and temperature profiles are depicted graphically and analyzed in details. Finally numerical values of physical quantities, such as the local skin friction coefficient and the local Nusselt number are presented in the tabular form.

Keywords: Stretching sheet, Porous medium, Boundary layer flow, Gebhart number.

1. Introduction

The study of boundary layer flow over a stretching surface is important as it occurs in industry, for example materials manufactured by extrusion, glass fiber and paper production, cooling of metallic plates in a cooling bath etc. In these cases the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching. By drawing such strip in an electrically conducting fluid subject to magnetic field or through porous media, the rate of cooling can be controlled and final product of desired characteristic might be achieved. Gebhart [1] introduced effects of viscous dissipation in natural convection. Carne [2] obtained flow past a stretching plate. Gupta [3] studied heat and mass transfer on a stretching sheet with suction or blowing. Chen & Char [4] reported heat transfer of a continuous stretching surface with suction or blowing. Karimi et al. [5] discussed a numerical modeling for natural convection heat transfer in porous media with generated internal heat sources. Sharma [6] discussed free convection effects on the flow of an ordinary viscous fluid past and infinite vertical porous plate with constant suction and constant heat flux. Pal & Shivakumara [7] studied mixed convection heat transfer from vertical plate embedded in a sparsely packed porous medium. Sharma & Singh [8] obtained numerical solution of transient MHD free convection flow of an incompressible viscous fluid along an inclined plate with ohmic dissipation. Ishak [9] obtained MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. Kameswaran et al. [10] discussed on radiation effects on hydromagnetic Newtonian liquid flow due to an exponential stretching sheet. Sharma [11] studied effects of viscous dissipation and heat source on unsteady boundary layer flow and heat transfer past a stretching surface embedded in a porous medium using element free Galerkin method.

Aim of the paper is to investigate effects of radiation and viscous dissipation on steady flow of a viscous incompressible electrically conducting fluid over an exponentially moving stretching sheet in porous medium.

2. Mathematical model

Consider the two dimensional steady boundary layer flow and heat transfer through an incompressible viscous electrically conducting fluid past a semi infinite exponentially stretching sheet embedded in porous medium. The origin of the system is located at the slit from which the sheet is drawn. The x-axis is taken along the continuous stretching surface and points in the direction of motion. The y–axis is perpendicular of the plate. The sheet velocity is assumed to vary as an exponential function of distance x from the slit. The temperature away from the fluid is assumed to be T_∞. The sheet ambient temperature is also assumed to exponential function of distance from the slit. A variable magnetic field of strength $B(x)$ is applied normal to the sheet. The governing continuity, momentum and heat transfer equations are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u-\frac{1}{K_p(x)}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_P}\frac{\partial q_r}{\partial y}+\frac{\sigma B^2(x)}{\rho C_P}u^2+\frac{\upsilon}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where u and v are velocity components in the x, y-directions, respectively $\upsilon(=\mu/\rho)$ is the kinematic viscosity, μ is the coefficient of viscosity, ρ is the density , σ is the electrical conductivity of the fluid, T is the fluid temperature , $\alpha(=\kappa/\rho C_P)$ is the thermal diffusivity, κ is the thermal conductivity C_P is the specific heat at constant pressure and q_r is the radiation heat flux.

The corresponding boundary conditions are

$$u(x,0)=u_w(x)=Ue^{x/L}, v(x,0)=0,$$
$$T(x,0)=T_w=T_\infty+T_s e^{2x/L}, \quad u(x,\infty)=0, T(x,\infty)=T_\infty \tag{4}$$

Here subscripts w,∞ refer to the surface and ambient conditions respectively. U is the characteristic velocity, T_s is the static temperature and L is characteristic length.

3. Method of solution

To facilitate a similarity solution, the magnetic field $B(x)$ and permeability of the porous medium $K_P(x)$ are assumed to be of the form

$$B(x)=B_0 e^{x/2L}, K_P(x)=\rho e^{-x\backslash L} \tag{5}$$

where B_0 is a constant. It is also assumed that fluid is weakly electrically conducting so that the induced magnetic field is negligible. Following Rosseland's approximation, the radioactive heat flux q_r is modeled as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{6}$$

Where σ^* is the Stefan-Boltzman constant, k^* is the mean absorption coefficient. Assuming that the temperature difference within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature $T^4 \equiv 4T_\infty^3 T - 3T_\infty^4$, we have

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}. \tag{7}$$

Introducing the following dimensionless variables, parameters and similarity variable

$$u=Ue^{x/L}f', v=-\left(\frac{\upsilon U}{2L}\right)^{1/2}[f+\eta f']e^{x/2L}, T=T_\infty+T_s e^{2x/L}\theta, \beta=\frac{2L}{\rho U_0},$$
$$\eta=\left(\frac{U}{2\upsilon L}\right)^{1/2} y e^{x/2L}, M=\frac{2\sigma B_0^2 L}{\rho U}, K=\frac{4\sigma^* T_\infty^3}{3k^* k}, \Pr=\frac{\rho\upsilon C_P}{\kappa}, Gb=\frac{U^2}{C_P T_s} \tag{8}$$

Where η is the similarity variable, $f(\eta)$ is dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature and the prime indicates differentiation with respect to η.

Using equations (7) and (9) into the equations (1) to (3), we get

$$f'''+ff''-2(f')^2-(M+\beta)f'=0 \tag{9}$$

$$\left(1+\frac{4}{3}K\right)\theta''+\Pr(f\theta'-4f'\theta)+Gb\Pr\left\{M(f')^2+(f'')^2\right\}=0 \tag{10}$$

Where prime denotes differentiation with respect to η and M is the magnetic parameter, β is the local porosity parameter, K is the radiation parameter , $\Pr$ is the Prandtl number and Gb is the Gebhart number.

The boundary conditions are reduced to

$$f(0)=0, f'(0)=1, \theta(0)=0, f'(\infty)\to 0, \theta(\infty)\to 0 \tag{11}$$

4. Skin friction coefficient

The shearing stress at the surface of the wall τ_w is given by

$$\tau_w=-\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}=-\frac{\mu U_0}{L}\sqrt{\frac{\mathrm{Re}}{2}}e^{3x/2L}f''(0) \tag{12}$$

Where $\mathrm{Re}=\frac{LU}{\upsilon}$ is the Reynolds number.

The skin friction coefficient is defined as

$$C_f=\frac{2\tau_w}{\rho U^2}$$
$$\Rightarrow \frac{C_f \rho U}{\mu\sqrt{2\,\mathrm{Re}}}e^{-3x/2L}=-f''(0) \tag{13}$$

5. Heat transfer coefficient

The rate of heat transfer at the surface is given by

$$q_w=-\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}=-\frac{\kappa(T_w-T_\infty)}{L}\sqrt{\frac{\mathrm{Re}}{2}}e^{x/2L}\theta'(0) \tag{14}$$

The Nusselt number is defined as

$$Nu_x=\frac{x}{\kappa}\frac{q_w}{(T_w-T_\infty)} \tag{15}$$

$$\Rightarrow \frac{Nu_x e^{-x/2L}}{(x/L)\sqrt{\mathrm{Re}/2}} = -\theta'(0)$$

Equations (9) and (10) with boundary conditions (11) are solved numerically using Runge- Kutta fourth order integration scheme with shooting iteration technique. Numerical values of skin friction coefficient and the Nusselt number are derived and presented through tables. The effects of physical parameters on the velocity and temperature profiles are shown in figures.

6. Results and discussion

It is observed from the figure (1) that the velocity profiles decrease due to increase in the magnetic parameter as the Lorentz force opposes the flows which decelerate the flow. Figure (2) depicts that fluid velocity profiles decrease due to increase in porosity parameter which agrees with natural phenomena. It is noted from figures (3) and (5) that fluid temperature increases rapidly due to a slight increase in radiation parameter or Gebhart number respectively. Figures (4) and (6) depict that fluid temperature increases a bit due to increase in magnetic parameter or porosity parameter respectively.

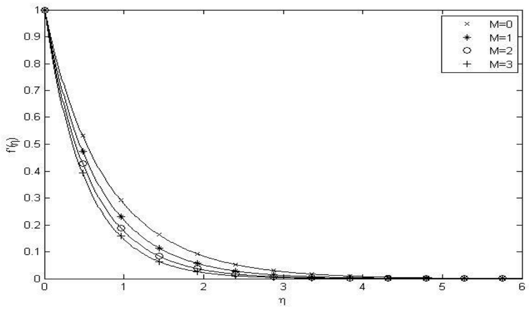

Figure 1. Velocity profiles versus η when $\mathrm{Pr}=7, K=0.5, Gb=0.2, \beta=1.$

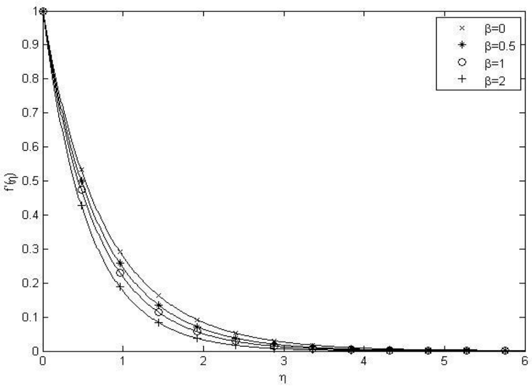

Figure 2. Velocity profiles versus η when $\mathrm{Pr}=7, K=0.5, Gb=0.2, M=1.$

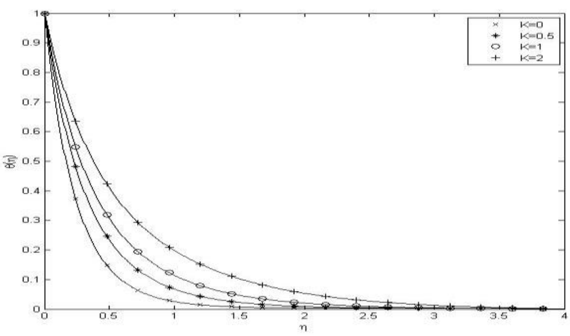

Figure 3. Temperature profiles versus η when $\mathrm{Pr}=7, M=1, Gb=0.2, \beta=1.$

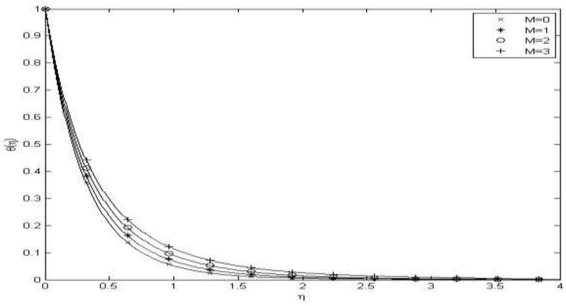

Figure 4. Temperature profiles versus η when $\Pr = 7, K = 0.5, Gb = 0.2, \beta = 1.$

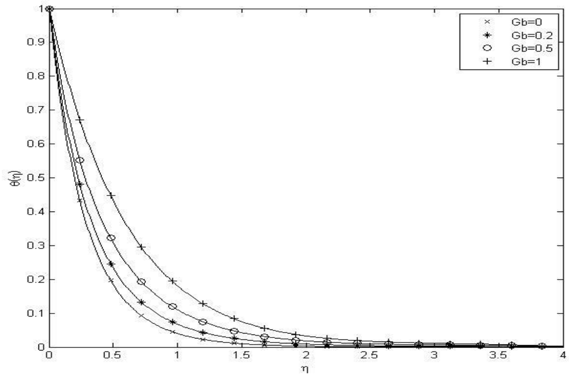

Figure 5. Temperature profiles versus η when $\Pr = 7, K = 0.5, M = 1, \beta = 1.$

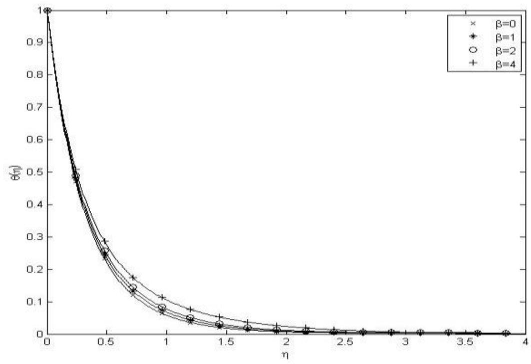

Figure 6. Temperature profiles versus η when $\Pr = 7, K = 0.5, Gb = 0.2, M = 1.$

Table1. Comparison of the Skin friction coefficient $-f''(0)$ at the surface for various values of physical parameters.

M	β	**Kameswaran et al [2012]**	**Present study**
0	0	1.281809	1.282110
1	0	1.629178	1.629180
2	0	1.912620	1.912620
3	0	2.158736	2.158736
1	1	--	1.91260
1	2	--	2.15873

1	4	--	2.58113

Table2. Numerical values of Nusselt number $-\theta'(0)$ at the surface for various values of physical parameters.

M	K	Gb	β	$-\theta'(0)$
1	0.5	0.2	1	3.77101
2	0.5	0.2	1	3.56030
3	0.5	0.2	1	3.23341
1	1.0	0.2	1	3.11931
1	2.0	0.2	1	2.39550
1	0.5	1	1	1.92221
1	0.5	2	1	0.39130
1	0.5	0.2	2	3.72656
1	0.5	0.2	4	3.63242

The values of skin friction coefficient in the absence of some physical parameters (i.e. $\Pr = 0$, $K = 0$, $Gb = 0$) are shown in Table 1. It is observed from the table 1 that skin friction coefficient increases with the increase of magnetic field or porosity parameter. Some results reported by Kameswaran et al are also included in this table. It is seen that the agreement between the previously published results with the present one is very good. We can conclude that this method works efficiently for the present problem. It is noted from the table 2 that the Nusselt number decreases with the increase of magnetic parameter, radiation parameter, Gebhart number or porosity parameter, respectively.

7. Conclusions

In this paper we have studied the two dimensional steady boundary layer flow and heat transfer through an incompressible viscous electrically conducting fluid past a semi infinite exponentially stretching sheet embedded in porous medium. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. The results for the prescribed the skin friction coefficient and Nusselt number at the plate surface are presented and discussed. We can conclude that

1. Fluid velocity profiles decrease due to increase in the magnetic parameter.

2. Fluid velocity profiles decrease due to increase in porosity parameter.

3. Fluid temperature increases due to increase in radiation parameter, magnetic parameter, Gebhart number or porosity parameter respectively.

4. Skin friction coefficient increases with the increase of magnetic field or porosity parameter.

5. Nusselt number decreases with the increase of magnetic parameter, radiation parameter, Gebhart number or porosity parameter, respectively.

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