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Commentary on the 1968 Edition of the Specification for the Design of Cold-formed Steel Structural Members

George Winter

COMMENTARY ON THE 1968 EDITION OF THE SPECIFICATION FOR THE DESIGN OF COLD-FORMED STEEL STRUCTURAL MEMBERS

by
GEORGE WINTER

Cold-Formed Steel Design Manual — Part V

AMERICAN IRON AND STEEL INSTITUTE
150 EAST FORTY-SECOND STREET
NEW YORK, NEW YORK 10017

American Iron and
Steel Institute.
COLD-FORMED STEEL
DESIGN MANUAL

COMMENTARY

ON THE

1968 EDITION OF THE

SPECIFICATION FOR THE DESIGN OF COLD-FORMED STEEL STRUCTURAL MEMBERS

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LESLIE A. BARRON
VICE PRESIDENT

May, 1970

Enclosed is a copy of -

COMMENTARY ON THE 1968 EDITION
OF THE SPECIFICATION FOR THE DESIGN OF
COLD-FORMED STEEL STRUCTURAL MEMBERS

By
GEORGE WINTER, PhD

This Commentary contains a comprehensive discussion of the 1968 Edition of American Iron and Steel Institute's "Specification for the Design of Cold-Formed Steel Structural Members". As in previous editions, it includes background material on the general characteristics of cold-formed steel structures, justification for the various provisions of the Specification, and documentation of the research results upon which these provisions are based. Particular emphasis is given to the new provisions for the design of compression members subject to torsional-flexural behavior, and for the utilization of the cold work of forming.

Additional information on cold-formed steel design, illustrative examples, and charts and tables are in preparation and will be distributed by the Institute as soon as available.

The enclosed errata sheet pertains to the first printing of the 1968 Edition of the "Specification for the Design of Cold-Formed Steel Structural Members". If you did not receive this edition, single copies are available without charge upon request to the Engineering Division, American Iron and Steel Institute.

LESLIE A. BARRON

Enclosures

AMERICAN IRON AND STEEL INSTITUTE
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Errata Sheet

Specification for the Design of Cold-Formed Steel Structural Members, 1968 Edition, 1st Printing

Pg. viii Add the following definition after r_y:

r_I Radius of gyration of I-section about the axis perpendicular to the direction in which buckling would occur for the given conditions of end support and intermediate bracing, if any, in. 4.3

Pg. 8 Add the subscript to w_f in the definitions following Table 2.3.5

Pg. 8 Definition of w_f, second line should read, "...webs of box-..."

Pg. 12 Detriment is misspelled in the second footnote

Pg. 14 In the third line of Section 3.4.3, add a left parenthesis to the second term of the expression, i.e.,

$(f_v/F_v)^2$

Pg. 15 Sixth line should read, "(a) Beams having single unreinforced webs:"

Pg. 18 In Section 3.6.1.2, the last line of the first paragraph should read "...Section 3.6.1.1..."

Pg. 19 The fifth term defined should be t_i

Pg. 21 In Section 3.7.2 (ii), the second term of the interaction equation should read

$$\frac{C_{TF}\,\sigma_{b1}}{\sigma_{bT}\left(1-\frac{\sigma_{TF}}{\sigma_e}\right)}$$

Pg. 27 In the third line, S_{max} should be s_{max}

Pg. 27 Vertical is misspelled in the definition of the term g

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PREFACE

The use of sheet and strip steel for structural purposes does not represent a new development. The employment of cold-formed steel structural members, such as roof deck, floor and wall panels, and structural sections was begun several decades ago. Development on a large scale, however, was hampered by the absence of an appropriate design specification. Such a special specification, it became evident, was desirable not only because the performance of cold-formed members under load differs in several significant respects from that of hot-rolled steel construction, but more important perhaps, the forms, shapes, means of connection, etc., which have developed in cold-formed construction differ in so many respects from those of heavy steel structures that design specifications written for the latter cannot possibly cover the former satisfactorily.

Realizing this situation, the Committee on Building Research and Technology of American Iron and Steel Institute in 1939 instituted a research undertaking at Cornell University for the purpose of developing factual information on which to base a design specification for this type of construction. Research projects have been carried out continuously since 1939. Based on research results and on rapidly accumulating practical experience the first edition of the Specification for the Design of Light Gage Steel Structural Members was published by American Iron and Steel Institute in 1946. Since 1949, a Design Manual containing important supplementary material for use in design has also been published. Over the years, the Specification and the Manual have been revised and enlarged to reflect technological developments and research results. In order to more completely describe their scope of application, the present titles of both the Specification and the Manual refer to the Design of Cold-Formed Steel Structural Members.

While most of the findings of the research project at Cornell University and other relevant material have been published through normal channels, a need was felt as early as 1947 for a systematic discussion of the background of the Specification. At first, this information was supplied in the form of a correlation of the Cornell research results with the Specification. Subsequently, the Committee decided to publish a systematic discussion of the behavior under load of cold-formed structures and of the background and justification of the various provisions of the Specification, so that designers, building officials, and others could gain a clearer understanding of this type of construction. Dr. George Winter, the director of the research undertaking at Cornell University continuously since 1939, was asked to draft an appropriate Commentary. This Commentary was first published in 1958.

Specifically, it was the purpose of that Commentary

(a) to offer to the interested structural engineer a brief but coherent presentation of the characteristics and performance of thin-walled steel structures in his accustomed language rather than in that of the specialized research investigator;

(b) to furnish to teacher and student background material for a study of cold-formed steel design methods;

(c) to provide a record of the reasoning behind, and justification for the various provisions of the Specification;

(d) to provide, by cross-referencing of the various provisions with the published supporting research data, as complete a research documentation as is possible.

It was hoped that in this manner the Commentary would be useful to the practicing engineer who uses the Manual and Specification, to those who for various reasons are interested in the background and basis of the various provisions and methods in these documents, and to those who will be responsible for future revisions and editions of the Specification and Manual. The wide and favorable reception of the Commentary has since justified these hopes. To cite but one instance, in recent years material on cold-formed construction has been included in several college texts and engineering handbooks, stimulated largely by the information presented in the Commentary.

As on previous occasions, the present new edition of the Commentary became necessary to reflect and provide the background for the changes and additions in the 1968 edition of the Specification and the corresponding revisions of the Design Manual. A major expansion of the Specification has been the inclusion of design provisions for compression members subject to torsional-flexural behavior. An analytical procedure for utilization of strengthening caused by the cold work of forming has also been added. Many other provisions have been improved or expanded in detail. In addition, the Specification has been correlated as much as possible with the American Institute of Steel Construction Specification for the Design, Fabrication and Erection of Structural Steel for Buildings. Hence, the most significant difference between the AISC and the AISI Specifications is that the AISC Specification covers hot-rolled shapes and built-up members, while the AISI Specification deals with members which are cold-formed to shape from flat steel.

While this Commentary undertakes to summarize the chief research results on which the Specification is based, many important details had to be omitted. The reader who wishes to have more complete information, or who may have questions which are not answered by the abbreviated presentation of the Commentary, should refer to the original research publications to which reference is made throughout.

TECHNICAL COMMITTEE ON STRUCTURAL
RESEARCH AND DESIGN SPECIFICATIONS
American Iron and Steel Institute

March 1970

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NOTATION

NOTE: Symbols not listed here are the same as those defined in the Specification for the Design of Cold-Formed Steel Structural Members.

Symbol	*Definition*	*Section*
A_{eff}	Effective area of a compression member	D.2(c)
A_f	Area of compression flange, in.^{2}	C.1
d	Total deflection of a beam-column, in.	F.2
d_b	Bending deflection, in.	F.2
E_t	Tangent modulus of elasticity, ksi	B.1, D.1
e	Edge distance, in.	G.3
F_{at}	Allowable stress for pure torsional buckling of point-symmetrical shapes, ksi	D.2(e)
F_{b2}	Permissible outer fiber stress for laterally unbraced compression flanges, ksi	E.3
F_{cr}	Buckling stress for unstiffened element with largest w/t, ksi	D.2(c)
F_{ult}	Collapse stress of longitudinally compressed thin tube	C.6
f_1	Actual compression stress at junction of flange and lip, ksi	C.4
f_2	Actual compression stress at free edge of lip, ksi	C.4
f_{max}	Stress in compression element, at supported edge, ksi	C.1, C.2(a), C.2(c)
I	Moment of inertia, in.^{4}	C.2(b), C.3(c)
I_{yt}	The moment of inertia of the tension portion of a section about its gravity axis parallel to the web, in.^{4}	E.1(a)
k	Edge support coefficient for plate buckling	C.1, C.4, C.7(a), C.7(b), G.4
M	Bending moment, kip-in.	F.2
M_{all}	Allowable bending moment, kip-in.	C.2(c)
M_b	Maximum bending moment due to transverse loads only, kip-in.	F.2
M_{ult}	Bending moment at failure, kip-in.	C.2(c)
n	Safety factor	F.2
P_{cr}	Elastic buckling load for "equivalent column," kips	E.3
P_e	Euler buckling load	F.3
P_{TFO}	Elastic, torsional-flexural buckling load for a concentrically loaded column with singly-symmetrical cross-section, kips	D.2(d)
P_{tm}	Flexural buckling load by tangent modulus theory, kips	D.1
P_x	Flexural buckling load for bending about the x-axis only, kips	D.2(d)

Symbol	*Definition*	*Section*
P_{ult}	Failure load of a short compression member	D.2(c)
P_{ult}	Total load on connection at failure, kips	G.3(a)
P_y	Axial load at which failure would occur by simple yielding due to compression plus bending	F.3
P_{yield}	Yield load of short compact compression member, kips	D.1
P_{ϕ}	Torsional buckling load, kips	D.2(d)
Q	Applied load, kips	E.2(a), E.2(c)
S	Section modulus, in.^{3}	C.2(b), C.2(c), F.3
S_{Fy}	Section modulus for maximum stress equal to yield point, in.^{3}	C.2(c)
T	Factor in design procedure for laterally unbraced compression flanges	E.3
T_o	Factor in design procedure for laterally unbraced compression flanges	E.3
σ_b	Bending stress, ksi	F.2
σ_c	Compression stress, ksi	F.2
σ_{cr}	Lateral buckling stress, ksi	E.1(a), E.1(c)
σ_{cr}	Elastic buckling stress for plates, ksi	C.1, C.7(a)
σ_f	Failure stress, ksi	F.2
σ_{fb}	Simple bending failure stress, ksi	F.2
σ_{fc}	Column failure stress, ksi	F.2
σ_{ey}	Flexural buckling stress about the y-axis, ksi	F.2
σ_{max}	Maximum elastic stress at any section of a member, ksi	F.2
σ_{net}	Average stress on net section at failure, ksi	G.3(a)
σ_{tm}	Tangent modulus buckling stress, ksi	D.1
σ_u	Ultimate tensile strength, ksi	G.3(a)
μ	Poisson's ratio	C.1, E.1(a)

A. INTRODUCTION

Cold-formed steel construction takes its name from the fact that members are cold-formed, in rolls or brakes, from flat steel, generally not thicker than ½ and as thin as about 0.0149 in.

Cold-formed members, as distinct from heavier, hot-rolled sections, are used essentially in three situations: (1) where moderate loads and spans render the thicker, hot-rolled shapes uneconomical, (2) where, regardless of thickness, members are wanted of cross-sectional configurations which cannot economically be produced by hot-rolling or by welding of flat plates, and (3) where it is desired that load-carrying members also provide useful surfaces, such as in floor and wall panels, roof decks, and the like. Accordingly, one can broadly divide cold-formed members into individual structural sections on the one hand, and panels and decks on the other.

Cold-formed structural sections often have outlines generally similar to those of hot-rolled shapes. However, the peculiarities of fabrication, of usage and of strengthwise optimum shape usually dictate variation from the customary sections (I's, channels, angles, etc.). Thus, provision is often made for nailability by shaping the member to provide a nailing slot; flanges are often furnished with stiffening lips at the edges to guard against local buckling and thereby to improve the strength-weight ratio; while I-shapes can be hot-rolled in one piece, they can be conveniently made of sheet or strip steel by welding together two or more cold-formed pieces (such as two channels spot-welded back to back); and special shapes not used in hot-rolled construction are often favorable for reasons of fabrication and strength, such as hat-shaped sections.

Cold-formed components are also employed as parts of members which may also contain other components of a different kind. A case in point is an open web joist with cold-formed especially shaped chords, but with web members consisting of hot-rolled bars. The main considerations which determine these structural sections are economy of material (i.e., favorable strength-weight ratio), ease of mass production, versatility, and provision for effective and simple connection in the structure.

In contrast to individual structural sections, whose main and almost only function is that of carrying load, the structural strength of panels and decks is only one of several desired characteristics and functions. To take floor or roof panels as an example, apart from developing the necessary strength for carrying the vertical floor load, it has been shown by many full scale tests that, if adequately connected to each other and to the supporting beams, they develop very considerable strength as shear diaphragms to resist force in their own planes. They are, therefore, widely used in this manner to resist and transmit horizontal forces from wind, earthquake, or similar actions (Ref. A.1). In addition, these panels also supply the flat surface on which to apply the flooring or roofing proper or to pour concrete fill; moreover, in many cases they provide space, in the cells, to locate electrical and other conduits; frequently they are acoustically conditioned to permit them to act as sound absorption materials, thereby improving the acoustics of the space of which they form the ceiling; provision is often made for lighting recessed in the panels; and, finally, good nesting in packaging, to minimize bulk and thereby shipping costs, is often important. Panels are shaped to meet, in varying degrees as required by the particular application, several or all of these and similar requirements. Optimum strength, then, is desired only in a conditional sense, i.e., insofar as it is compatible with the various other enumerated features. In consequence of their specific usage, the shapes of the many current types of panels and decks are entirely different from any used in hot-rolled construction.

It will be clear from this brief discussion that hot-rolled and cold-formed steel structural members actually supplement each other. In some structures cold-formed members constitute the entire framing, primary and secondary. In others the main structural framing is of heavy members hot rolled or built up from flat plates and shapes, whereas secondary members (such as joists), and load-resisting surfaces (such as floors, roofs, and curtain walls) are cold-formed.

In contrast to hot-rolling, the cold-forming processes (Refs. A.2, A.3) coupled with automatic welding, permit an almost infinite variety of shapes to be produced. A considerable number of shapes, as well as their usage, are described and illustrated in Ref. A.4. This freedom to produce a great variety of shapes has the consequence that a design specification or code, in order to be useful in this field, must enable the designer to compute the properties and performance of practically any conceivable shape of cold-formed structural member, regardless of whether or not that particular shape was in actual use at the time when the specification was written. It is this requirement for versatility, in addition to the inherent structural peculiarities of thin-walled members, which dictates the specific character of the *American Iron and Steel Institute Design Specification* and *Manual*.

In addition to versatility of shape, the methods of production cause other differences between hot-rolled members or members fabricated from flat plates and shapes, and cold-formed members. In the former, residual cooling stresses from hot-rolling or welding significantly influence behavior, particularly of compression members or components. Such cooling stresses are absent in cold-formed members. But these, in turn, are subjected during the forming processes to selective strain-hardening. This strain-hardening affects response to load in a manner quite different from that of cooling stresses in, say, hot-rolled members. These differences are reflected in the applicable design specifications.

Much of the research on which the *Manual* is based, has been carried out on specimens made from relatively thin sheet or strip steel, in the thickness range of 0.03 to 0.10 in. Specimens in this range either were available or were easier to produce than heavy members would have been. Member behavior depends only on material properties and on dimensional ratios, not on absolute dimensions. Two areas where absolute thickness could have been suspected to influence behavior are those of strain-hardening (increase in yield strength) due to cold-forming and of bolted connections. Because of this possibility, research on cold-forming effects covered a thickness range from 0.06 to 0.16 in. with a few specimens as thick as ¼ in., and tests on bolted connections covered thicknesses from 0.036 to 0.19 in. No influence of thickness was apparent in this wide range. One can state that the design methods of the *Manual* can apply to members of any thickness capable of being cold-formed.

B. MATERIALS, SAFETY FACTORS, BASIC DESIGN STRESSES

1. MATERIAL

Several grades of structural quality carbon and high strength low alloy sheet and strip steel (without and with zinc coating) are standardized by the American Society for Testing and Materials. These standards are listed in *Section 1.2* of the *Specification*. They cover a considerable range of strength properties the most important of which is the yield point. The yield points covered by the various ASTM grades of structural quality steel are included in *Table 3.1, Section 3.1* of the *Specification* and range from 25 to 50 ksi. The table also includes yield points up to 65 ksi, reflecting the fact that *Section 1.2* permits the use of steels other than ASTM grades of sheet and strip. Sheet and strip steels with yield points lower than 33 ksi and plate steels lower than 36 ksi are rarely used for structural purposes. 1.2*

A second important property is ultimate tensile strength. Specified tensile strengths range from about 1.8 times the yield point for low yield strength steels to about 1.3 times the yield point for high yield steels. (For certain special applications which require only relatively mild cold-forming, e.g. corrugated sheet, steels with yield points exceeding 80 ksi are used, such as ASTM A446, Grade E. These steels have very low tensile to yield strength ratios.)

The third structurally important property of steels is ductility, which is the ability of a metal to undergo sizeable permanent deformations prior to fracture. Ductility is generally measured by the permanent elongation of a tensile specimen after fracture. For sheet and strip steels specified minimum elongations in a 2 in. gage length range from about 15 to 27 percent, and for plates and bars, in an 8 in. gage length, from 14 to 21 percent (except for Grade E type steels of limited application, which show much lower elongations). It is not established that steels with elongations smaller than these, when used in moderate thicknesses and not subject to severe impact, are less suitable structurally. In fact, research underway at this writing (1969) seems to indicate that amounts of ductility considerably less than the 14 to 15 percent lower limit found in most ASTM specifications are amply adequate to ensure satisfactory static structural performance of members and connections of moderate thickness.

Section 1.2 does not list ASTM grades of steel plates or bars suitable for cold-formed construction. In this respect the ASTM grades listed in the Specification of the American Institute of Steel Construction (Ref. C.6) represent steels which, in thicknesses suitable for cold-forming, are appropriate for this type of construction. 1.2

In addition to the steels covered by ASTM Specification, other steels are in use for structural purposes. These are permitted under *Section 1.2* of the *AISI Specification* which reads, in part:

*Marginal notes indicate section of *Specification* under discussion.

"The above listing does not exclude the use of steel up to and including one-half inch in thickness ordered or produced to other than the listed specifications provided such steel conforms to the chemical and mechanical requirements of one of the listed specifications or other published specification which establishes its properties and suitability, and provided it is subjected by either the producer or the purchaser to analyses, tests and other controls to the extent and in the manner prescribed by one of the listed specifications."

The strength of steel structural members depends primarily on the yield point but also on the shape of the initial portion of the stress-strain diagram, chiefly in cases where local or overall buckling determines this strength. Cold-formed structural members, in common with hot-rolled steel shapes, exhibit one of the two types of stress-strain diagrams shown on Fig. B.1. Steels of type (a) of Fig. B.1 are known as sharp yielding, those of type (b) as gradual yielding. For the former the yield point is defined by the level at which the stress-strain diagram becomes horizontal. For the latter there is, in general, no such horizontal portion and specifications define the yield point or strength by a stipulated offset or a stipulated total elongation.

The strength of members which fail by buckling depends not only on the yield point and on Young's modulus E (i.e., the slope of the initial straight portion of the stress-strain curve) but also on the "tangent modulus" E_t, i.e., the slope of the stress-strain curve at the stress at which buckling occurs. It is seen

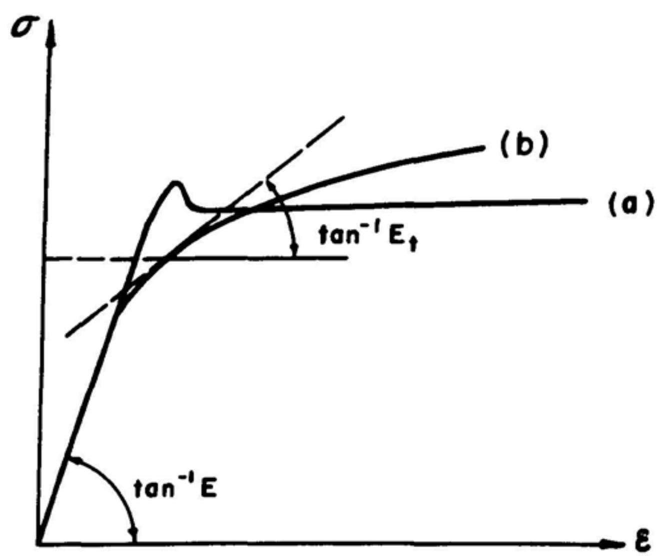


Fig. B.1

from Fig. B.1 that in this respect sharp yielding steels often result in larger buckling strength than gradual yielding steels. Indeed, for the former $E = E_t$ right up to the yield point, whereas in the latter, once the proportional limit is exceeded, i.e., once the stress-strain curve begins to deviate from the straight line, the tangent modulus E_t becomes progressively smaller than Young's modulus E. This affects the buckling resistance adversely. To account for this eventuality, the various buckling provisions in the Specification are written for gradual yielding steels, whose proportional limit is not lower than about 70 percent of the specified minimum yield point.

In contrast to the yield point and the shape of the initial portion of the stress-strain diagram, the ultimate tensile strength has little effect on static member strength. However, the strength of certain types of connections and of some other details depends not only on the yield point, but on the tensile strength as well.

Another quality which is often essential to satisfactory structural performance is weldability (as determined by the chemistry of the steel). It is the combination of these various properties (yield point, tensile strength, ductility, weldability, etc.) which, for purposes of *Section 1.2* of the *Specification* deter- 1.2
mines the "suitability" of a given steel for use in cold-formed construction.

The *AISI Specification* and *Manual* apply to carbon and low alloy steels, but not to non-ferrous metals or to many highly alloyed steels, such as the austenitic stainless steels. This is so because the structural performance of metal members depends not only on their strength properties (yield point, tensile strength, etc.) but also on the modulus of elasticity and on the shape of the stress-strain curve. These affect particularly the buckling characteristics, whether local or general, of the member; and since various forms of buckling play a more important part in the dimensioning of thin-walled than of more stocky members, attempts to adapt design procedures developed for one metal, such as mild structural steel, to some other metal by mere substitution of corresponding properties are particularly inappropriate in this field.

Extensive recent research into the structural performance of stainless steel members (Refs. B.1, B.2) has led to publication by the American Iron and Steel Institute of a separate specification for the design of stainless steel structures (Ref. B.3).

2. UTILIZATION OF COLD WORK

It has long been known that any cold work, such as cold stretching, bending, etc., affects the mechanical properties of steel. Generally, such operations produce strain-hardening, that is, they increase the yield point and to a lesser degree the tensile strength, and they decrease the ductility as measured by elongation in a tensile test. Cold work of one sort or another occurs in all cold-forming operations, such as roll forming or forming in press brakes. In this respect the properties of the steel in the member as formed are, to various degrees, different from those of the steel prior to forming.

The 1962 edition of the *Specification,* for the first time, permitted basing allowable design stresses on the raised yield strength of the steel in the formed member. This more economical procedure was restricted to certain parts of the *Specification*. Also, since little was known quantitatively about the effects of cold work on steel properties, it was stipulated that for any particular shape the as-formed steel properties had to be proved by test before they could be utilized in design determinations. Since that time a large amount of research has been carried out on the details and quantitative aspects of the effects of cold-forming, the major parts of which are summarized in Refs. B.4 through B.6. Based on these findings, the present edition of the *Specification* contains more detailed and more liberal provisions for utilizing the strengthening effects of cold work in design.

Depending on shape and manufacturing process, the type and amount to which steel is cold-worked in the cold-forming process varies widely. When sections are produced in press brakes, the flat portions of the shape, such as a channel, are generally not cold-worked at all. The strain-hardening is entirely concentrated in the corners and its effect is the greater the sharper the curvature. In members produced by cold-rolling the largest amount of cold work, likewise, is concentrated in the corners. However, the flat portions of a section (henceforth called "flats") usually also receive varying amounts of cold work from two sources: for one, the pressures exerted by the rolls produce a certain amount of permanent deformation, i.e. cold work; for another, a portion of the shape which may be flat in its final configuration, frequently is bent first in one direction and then in the other in the various stages of the rolling process, each of these producing definite strain-hardening effects. Tubes undergo cold work of still another character. The flat material is gradually bent to the circular shape of round tubes which results in reasonably uniform plastic strains throughout the section and is also subjected to a certain amount of transverse squeezing in the forming process. Square and rectangular tubes are formed to shape from round tubes; this involves additional sharp cold work in the corners, a milder amount in the flats which are produced by reverse bending of the pertinent portions of the round tube and further transverse squeezing throughout the forming process. Two things are clear from this description. Since in most shapes the various portions of the cross-section experience different amounts of cold work, the as-formed properties of the steel will not be uniform throughout the cross-section. Also, the amount of cold work and its effects depends not only on the geometry of the final shape but on the entire forming history.

In the simplest kind of cold work, uniform cold stretching in one direction, the following effects are observed (Ref. B.4): Increasing amounts of cold stretching progressively increase the tension yield point when the material, subsequent to stretching, is stressed in the same direction as the prior stretching. The ultimate tensile strength is also increased but by smaller percentages. On the other hand, when the material is compressed in the direction of prior cold stretching, the yield point is raised to a much lesser amount or, for some steels, may not be raised at all (Bauschinger effect). If subsequent compression stresses act in the direction perpendicular to the prior cold stretching, the yield point is raised considerably, whereas for tension in that same transverse direction the increase is much smaller or, in some cases, absent (inverse Bauschinger effect). The amount of strain-hardening is approximately proportional to that of prior cold stretching. For any given amount of cold stretching, strain-hardening is greater for steels with the larger ratios of virgin ultimate to virgin yield strength, F_u/F_y. For non-stabilized steels (e.g. rimmed or semi-killed steels) aging subsequent to cold stretching results in a marked increase in proportional limit, yield point and ultimate strength, in tension as well as compression, and transversely as well as longitudinally.

The various parts of a cold-formed shape undergo cold work much more complex than simple cold stretching, but the effects can be understood on the basis of the simple behavior just described.

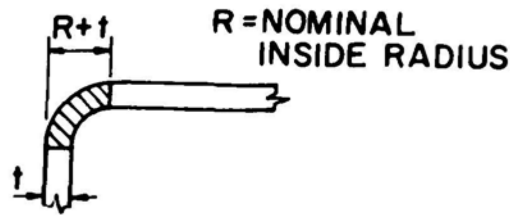
When a *corner* is formed, by any of the forming processes, the outer layers are permanently stretched circumferentially and compressed radially, while the inner layers are permanently compressed circumferentially and stretched radially. All these deformations occur transverse to the axial direction of the member, i.e. transverse to the stresses which act when the member is later used in a structure as a column, beam, or otherwise. In this situation it is easy to show, and has been verified by test, (Ref. B.5) that there is no significant Bauschinger effect in corners. That is, when tested longitudinally, it is found that the yield point of formed corners is substantially the same in tension and in compression.

Just as for simple cold stretching, it was found for corners that the amount of strain-hardening increases with the degree of cold work, i.e. with the magnitude of permanent strain. From geometry it is easily seen that the permanent strain is proportional to the ratio of inside corner radius to thickness of material, R/t. This is illustrated in Fig. B.2 which shows the initial portions of stress-strain diagrams of corners formed from two different steels, one of which is killed (stabilized, Fig. B.2(b)) and the other is semi-killed (aging, Fig. B.2(c)). In each case tension and compression stress-strain curves are shown for the virgin steel before forming, and for two corners of sharply differing R/t ratios. It will be observed that the corner curves for compression and tension are quite close to each other (no significant Bauschinger effect), that the curves for the smaller R/t ratios lie substantially above those for the larger ratios, and that the semi-killed steel Fig. B.2(c) exhibits a much higher proportional limit and sharper yielding than the killed steel. From theoretical consideration and from testing well over a hundred corners made from different types of steel, of different thicknesses, and with varying R/t ratios (Ref. B.5), it was found that the yield point of a formed corner can be calculated from

$$F_{yc} = B_c F_y / (R/t)^m \tag{B.1}$$

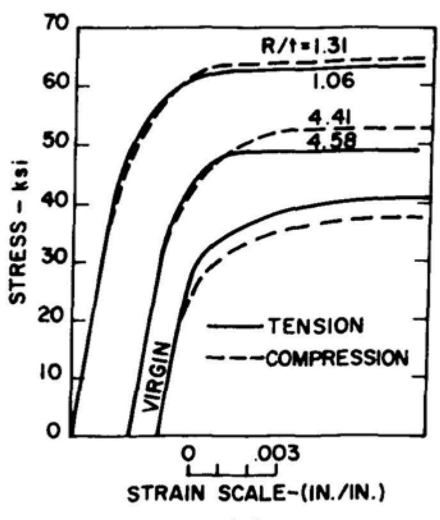
where B_c and m depend on the ratio of ultimate to yield strength of the virgin material as follows: $B_c = 3.69\ (F_u/F_y) - 0.819\ (F_u/F_y)^2 - 1.79$ and $m = 0.192\ (F_u/F_y) - 0.068$. It is this formula which is given in *Section 3.1.1.1(a) for* 3.1.1.1(a) calculating corner yield strength. It was found to apply with the same accuracy to killed as to semi-killed or rimmed steel.

No such equation can be given for calculating the yield point of the *flats* in a cold-formed section. This is so because, as was pointed out, the degree of



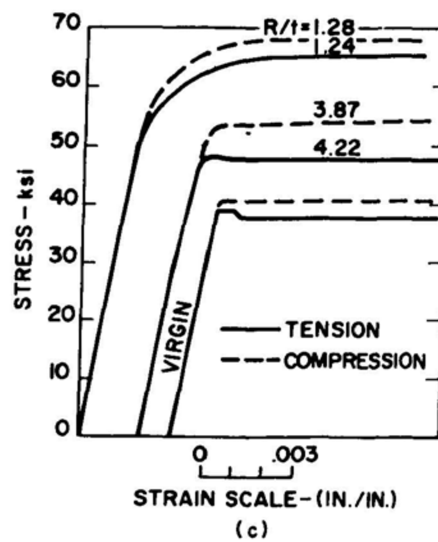
Cross-Sectional Dimensions of Specimens

Fig. B.2(a)



Typical Stress-Strain Curves for Cold-Rolled Killed Corners

Fig. B.2(b)



Typical Stress-Strain Curves for Hot-Rolled Semi-Killed Corners

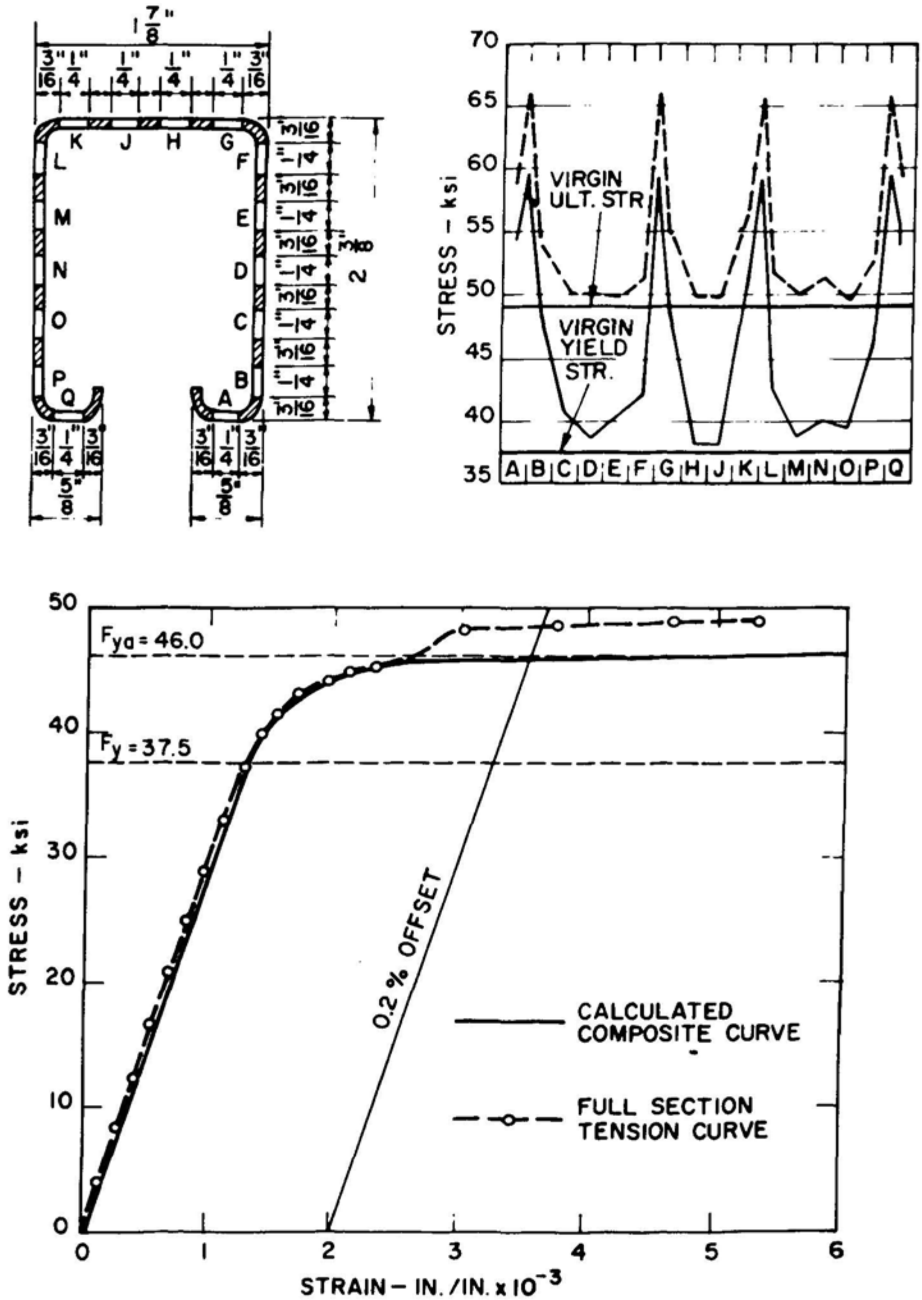
Fig. B.2(c)

strain-hardening of the *flats* depends on the entire forming history ranging from almost no cold work in pressbraked sections to a large amount of it in roll formed square and rectangular tubes and other shapes such as some joist chords.

Figs. B.3 and B.4 show the distribution of yield and ultimate strengths throughout the various portions of two roll-formed shapes as determined by coupon tests cut from these shapes (see Ref. B.6). Fig. B.3 refers to a relatively thin 16 gage section with large amounts of flat material, Fig. B.4 to a relatively stocky 9 gage shape where much of the material is in curved portions. It is seen that in the former the yield point and tensile strength of the flats have been raised very little above their virgin values, while in the latter all of the material, regardless of location, shows very considerable strain-hardening. Because the properties of the flats in the formed section are not predictable in the same manner as those of the corners, *Section 3.1.1.1* of the *Specification* 3.1.1.1
provides that the properties of flats shall either be determined by test or, in the absence of such tests, shall be assumed equal to the virgin properties. It can be seen that for the shape of Fig. B.3, where most of the cold work effects are concentrated in the corners, assuming the flats to have the same properties as the virgin material is a reasonable approximation. On the other hand, this same assumption when made for the shape of Fig. B.4, would greatly underestimate the strength of the flat portions and, thus, would fail to exploit fully the beneficial effects of cold-forming. These effects can be fully utilized if the properties of the flats are determined by the test procedure stipulated in *Section 6.3.2* of the *Specification.* 6.3.2

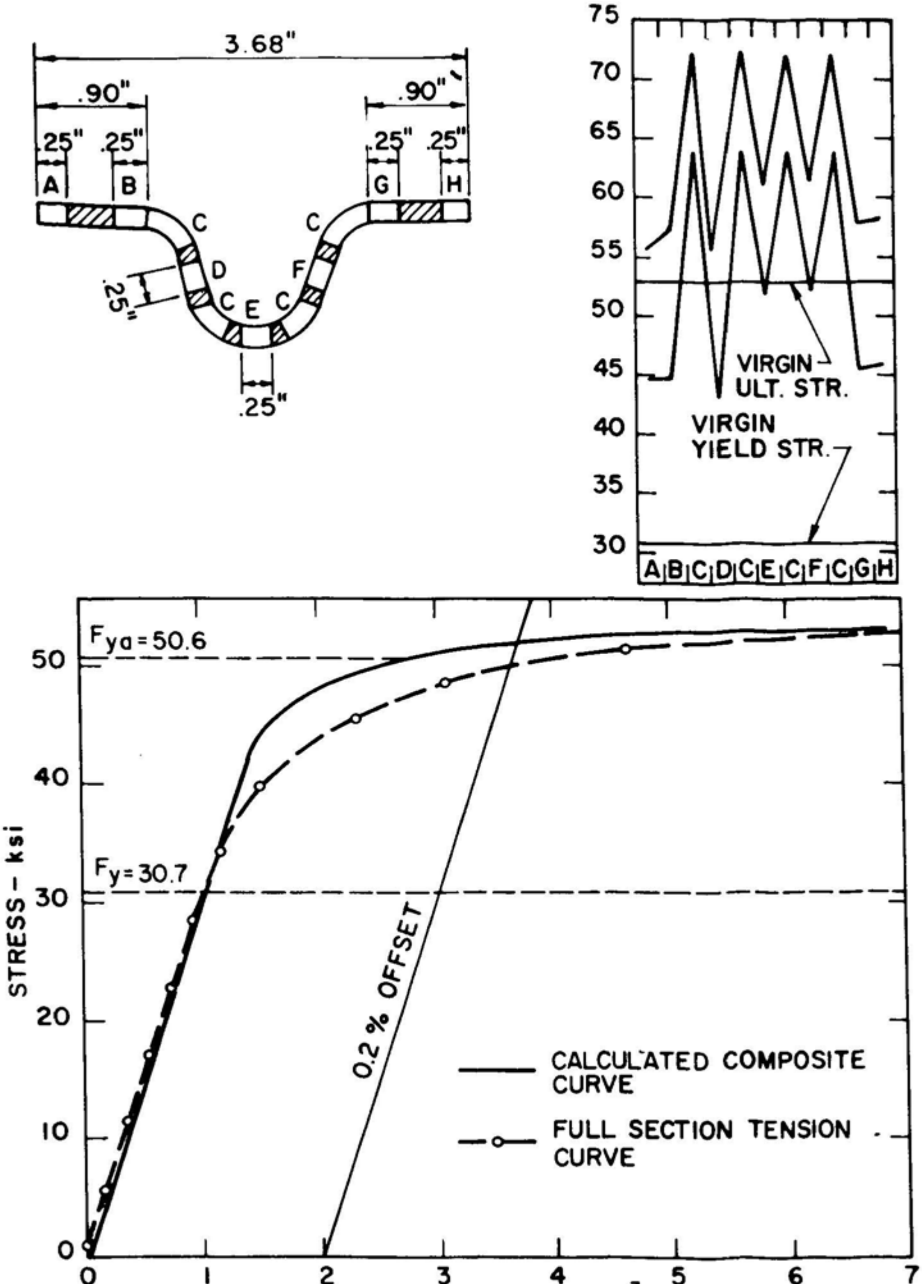
This procedure consists in cutting tensile coupons at least from the middle of each flat and subjecting it to a standard tensile test. As is seen from Fig. B.3, a more realistic and economical determination is obtained if additional coupons are taken from locations about midway between the centerline of a flat and its tangent point with the adjacent corner. The yield points so determined must be multiplied by the ratio of the specified minimum yield point to the actual virgin yield point. This necessitates making tensile tests on coupons taken from virgin material of the same coil of which the tested shape has been formed. This is necessary because as-formed strength of flats is about proportional to the virgin strength of the material. Many coils will have actual strengths significantly higher than the specified minimum. Hence, basing strength calculations on the unadjusted test values of the flats would overestimate the actual available strength of similar sections made of a coil whose strength happens barely to exceed the specified minimum.

What the designer needs, in the end, are the *average properties of the entire formed section* and, particularly, the average tensile yield point. For this purpose the *Specification* distinguishes between compact and non-compact shapes. The former are defined in *Section 3.1.1.1(a)* as compression members 3.1.1.1(a)
with $Q = 1$ or flexural members whose compression flanges have $Q = 1$. For such members, which are not subject to local buckling, full advantage can be taken of cold work effects. The *Specification* provides three alternative methods for determining full section properties of such members: either by tensile tests carried out on short lengths of the full sections, according to the methods



Tensile Stress-Strain Characteristics of Roll-Formed Hot-Rolled Semi-Killed Section

Fig. B.3



Tensile Stress-Strain Characteristics of a Roll-Formed Hot-Rolled Semi-Killed Joist Chord

Fig. B.4

Section 6.3.1(a); or by full section compression tests according to *Section*
6.3.1(b) *6.3.1(b)*; or by calculating a weighted averaged yield point F_{ya} as follows:

$$F_{ya} = C\,F_{yc} + (1\text{-}C)\,F_{yf} \qquad \text{(B.2)}$$

Here F_{yc} is the yield point of the corners as calculated from Eq. B.1, F_{yf} is the weighted average tensile yield point of the flats determined as previously described, and C is the ratio of the total corner area to the total area of the full section.

Figs. B.3 and B.4 each show the composite stress-strain curve calculated by weighted averaging as in Eq. B.2, and also show the curve obtained by full section tension tests. It can be seen that the weighted averages agree with the full section test results very satisfactorily. The figures also illustrate the overall benefit of the cold work of forming, which is seen to be substantial for both shapes, but more so for the stockier shape of Fig. B.4 than for Fig. B.3 where the strain-hardening is chiefly concentrated in the corners.

For sections for which $Q < 1$, a more conservative approach for utilizing
3.1.1.1(b) cold work effects is provided in *Section 3.1.1.1(b)*. This is so because, for such sections, it is possible that the raised strength obtained particularly in the corners cannot be fully mobilized because of premature buckling of the flats. Correspondingly, this section in essence stipulates that the corner strengthening effects shall be neglected in such shapes. It provides that the full section yield point shall be: either the weighted average yield point of the flats as determined by tests; or, in the absence of such tests; the specified minimum yield point of ASTM steels; or, for non-ASTM steels, the yield point as verified
6.3.3 by the procedures of *Section 6.3.3*. Subsequent research may permit liberalization of this conservative approach to the effects of cold work in shapes with $Q < 1$. Preliminary data seem to indicate that while the excess corner strength may indeed be incapable of full mobilization, neglecting it completely may be more conservative than necessary.

3. SAFETY FACTORS

The safety factor may be stated as being the ratio of the specified design strength to the specified design load. Except for the simplest cases the computation of the actual ultimate strength of a structure is not a simple matter. Therefore, without entering into a discussion of the intrinsic meaning of a safety factor, for the purposes of this Commentary its conventional definition will be adopted, which can be stated thus: the safety factor is the ratio of stress at incipient failure to the calculated stress at design load. In some cases, such as for columns, beam-columns, etc., it is the ratio of the calculated load at incipient failure to the design load.

In steel structures, for the most simple cases, such as tension, bending, simple compression without buckling, etc., it is assumed that failure is beginning to occur when the maximum stress computed by simple, accepted procedures, becomes equal to the yield point. (For some types of hot-rolled construction, plastic design methods recognize higher failure loads than those causing incipient yielding. For the applicability of plastic design to cold-formed

construction, see H.3 of this Commentary.) For these simple cases, the safety factor as conventionally defined is simply the ratio of the yield point to the design stress. The *AISI Specification* is based on a safety factor of 1.67 = 1/0.6, this being the ratio of the yield point F_y to the basic design stress F
(Section 3.1). In some special cases, such as in the design of some types of con- 3.1
nections, higher safety factors are incorporated in the design provisions. These safety factors are practically identical with those employed in the American Institute of Steel Construction Specification, Ref. C.6.

In conformity with all American structural design specifications, the *Specification* permits a 25 percent reduction in the nominal factor of safety for members or assemblies stressed by *wind* or *earthquake forces,* or by the simultaneous action of dead and live load (if additive) plus wind or earthquake
forces. Thus *Sections 3.1.2.1* and *3.1.2.2* of the *Specification* permit a 33⅓ per- 3.1.2.1
cent increase in all allowable stresses for the two described situations, provided, 3.1.2.2
however, that the dimensions of the members determined in this manner be no less than those required to carry the appropriate combinations of dead plus live load without wind or earthquake, at the applicable unincreased allowable stress. A similar increase in allowable stress is provided for roofs which may be subjected to an accumulation of water due to storms, known as ponding.

Special safety factors used in portions of the *Specification* are discussed where appropriate in this Commentary.

4. BASIC DESIGN STRESSES

Under essentially static loading as it occurs in buildings, failure of steel structural members is initiated by yielding except in those cases where some form of buckling occurs at stresses below the yield point. Accordingly, the term "basic stress" *(Section 3.1)* applies to those situations where members fail by yielding. Special reduced design stresses are provided in various parts of the *Specification* for those frequent cases where the strength of a member is governed by buckling rather than by yielding.

In conformity with the stipulated safety factor, *Section 3.1* specifies that 3.1
the basic design stress in tension or bending shall be equal to

$$F = 0.6\,F_y$$

that is, the specified minimum yield point of the particular steel divided by the safety factor. Numerical values for F are given for some of the yield point values which are stipulated in the various ASTM Standards listed in *Section 1.2*. These values are obtained by rounding off to the nearest ksi the applicable yield point multiplied by 0.6. The list does not imply that other yield point values are not equally admissible. All this refers to that common situation where design stresses are based on the specified minimum yield point for the steel before forming.

When advantage is taken of the strength increase which can be obtained by cold working (see B.2 above), then the basic design stresses are based on the full section yield point.

Section 3.4.1 specifies the maximum design stress in shear as $F_v = 0.4\,F_y$. The accepted von Mises yield theory indicates that yielding in shear occurs at a stress equal to 0.577 of the yield stress in tension, F_y. Consequently, the safety factor against incipient yielding in shear is $0.577/0.4 = 1.44$, as compared to the basic safety factor of 1.67. This apparent reduction from the basic safety factor is justified by long-standing use and by the minor consequences of incipient yielding in shear, compared with those associated with tension and compression yielding (Ref. C.6).

No investigation of the effect of cold work on the yield stress in shear is known to the writer. Because maximum shear stresses occur in the central portions of webs of flexural members, while the greatest effects of cold work are concentrated at and near the flange corners, it is suggested that F_v be based on the virgin yield point rather than the full section yield point F_{ya}.

When stresses are caused in whole or in part by wind or earthquake forces, *Sections 3.1.2.1* and *3.1.2.2* of the *Specification* provide the customary 33⅓ percent increase in allowable design stresses, in agreement with what has been said in more detail in B.3, above.

In the *Tables of Section Properties, Part IV* of *Manual,* data are given for two specific values of the basic stress, usually for $F_b = 20$ ksi (corresponding to $F_y = 33$ ksi) and $F_b = 30$ ksi (corresponding to $F_y = 50$ ksi, the largest value likely to be used under normal circumstances in building construction). As indicated in the *Manual,* appropriate properties for steels with basic stresses other than these two values are found with sufficient accuracy by direct interpolation or extrapolation.

C. LOCAL BUCKLING OF THIN ELEMENTS

1. GENERAL

In heavy steel construction the chief forms of buckling that are considered in design are column buckling (which governs the allowable stress P/A depending on the slenderness L/r) and lateral buckling of unbraced beams (which governs the allowable bending stress depending, in the AISC Specification, on the parameters Ld/A_f or L/r). Local buckling of the various plate-shaped components of which heavy structural sections consist needs rarely to be considered because these plates are usually so stocky, i.e., have such small width-thickness ratios, that they will not buckle at stresses below the yield point. There are exceptions to this situation, such as thin webs of plate girders. In contrast, in cold-formed construction, the individual components of the sections are frequently so thin, i.e., their flat-width ratios, w/t, are so large, that they will buckle at stresses below the yield point if subjected to compression, shear, bending, or bearing. It is necessary, therefore, to design such members so that, at design load, adequate safety exists against failure by local buckling. In this respect the situation is similar to that in aircraft construction where, likewise, thin-walled members are used extensively and where local buckling constitutes one of the chief design criteria.

It is well known that a concentrically loaded, elastic column will buckle at the Euler critical stress

$$\sigma_e = \frac{\pi^2 E}{(KL/r)^2} \tag{C.1}$$

where K is a coefficient which depends on the manner of end support. It is equal to 1 if both ends are hinged, ½ if both ends are fixed, 2 if one end is fixed and the other unsupported, etc.

If a thin plate, such as the top flanges of the two beams of Fig. C.1 is longitudinally compressed it will buckle and distort in a wavelike manner as shown on that figure. Under ideal conditions this will occur at a stress determined by an equation which is very similar to the Euler formula for columns, namely

$$\sigma_{cr} = k \frac{\pi^2 E}{(1-\mu^2)\,(w/r)^2}$$

where the term involving Poisson's ratio, μ, comes from the fact that a plate extends in two dimensions, in contrast to a column. The radius of gyration, r, of a plate of thickness, t, is $r = t/\sqrt{12}$. If this is substituted in the above equation, one gets the critical plate buckling stress in the usual form (see e.g., p. 320 of Ref. C.1)

$$\sigma_{cr} = k \frac{\pi^2 E}{12(1-\mu^2)(w/t)^2} \tag{C.2}$$

As in the case of columns, the factor k depends on the manner in which the plate is supported, chiefly along the longitudinal edges parallel to the compression stress. In the case of the flange of Fig. C.1(a) where one edge is supported by a thin web while the other, outer edge is unsupported, k is about equal to 0.5; for the case of Fig. C.2(b), where both longitudinal edges are supported or stiffened by thin webs, k is, conservatively, equal to about 4.

In column design a safe design stress P/A is obtained by dividing the buckling stress of Eq. C.1 (or some modification thereof) by an appropriate safety factor. One might think, then, that in order to obtain safe working stresses for compressed plate elements, such as the top flanges of the beams of Fig. C.1, one would similarly divide the buckling stresses of Eq. C.2 by a safety factor. While this is the proper procedure for some kinds of plates it is very wasteful for others because these latter plates are able to resist without failure much larger stresses than are computed from Eq. C.2. To understand the reason for such different behavior it is necessary to visualize physically the manner in which a plate buckles.

Imagine for simplicity a square plate uniformly compressed in one direction, with the unloaded edges simply supported. Since it is difficult to visualize the performance of such two-dimensional elements, the plate will be replaced by a model which is shown on Fig. C.2(a). It consists of a grid of longitudinal and transverse bars in which the material of the actual plate is thought to be concentrated. Since the plate is uniformly compressed, each of the longitudinal struts represents a column loaded by P/5, if P is the total load on the plate. As the load is gradually increased the compression stress in each of these struts will reach the critical buckling value (Eq. C.1) and all five struts will tend to buckle simultaneously. If these struts were simple columns, unsupported except at the ends, they would simultaneously collapse through unrestrainedly increasing lateral deflection. It is evident that this cannot occur in the grid model of the plate. Indeed, as soon as the longitudinal struts start deflecting at their buckling stress, the transverse bars which are connected to them must stretch like ties in order to accommodate the imposed deflection. Like any structural material they resist stretch and, thereby, have a restraining effect on the deflections of the longitudinal struts.

The tension forces in the horizontal bars of the grid model correspond to the so-called membrane stresses in a real plate. These stresses, just as in the grid model, come into play as soon as the compression stresses begin to cause buckling waves. They consist mostly of transverse tension, but also of some shear stresses, and they counteract increasing wave deflections, i.e. they tend to stabilize the plate against further buckling under the applied increasing longitudinal compression. Hence, the resulting behavior of the model is as follows: (a) there is no collapse by unrestrained deflection, as in unsupported columns, and (b) the various struts will deflect unequal amounts, those nearest the supported edges being held almost straight by the ties, those nearest the center being able to deflect most.

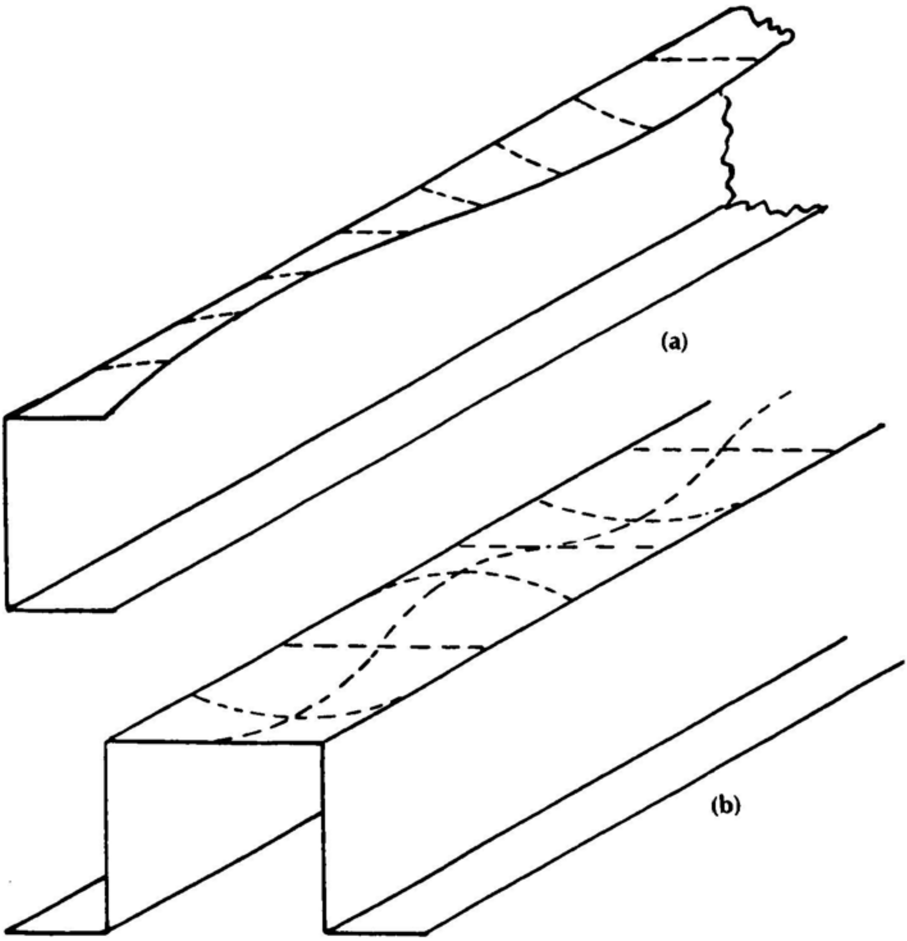


Fig. C.1

In consequence of (a) the model, or the plate which it represents, will not collapse and fail when its buckling stress (Eq. C.2) is reached; in contrast to columns it will merely develop slight deflections but will continue to carry increasing load. This is known as the post-buckling strength of plates. In consequence of (b) the struts (strips of the plate) closest to the center, which deflect most, "get away from the load," and hardly participate in carrying any further load increases. These center strips may in fact, even transfer part of their pre-

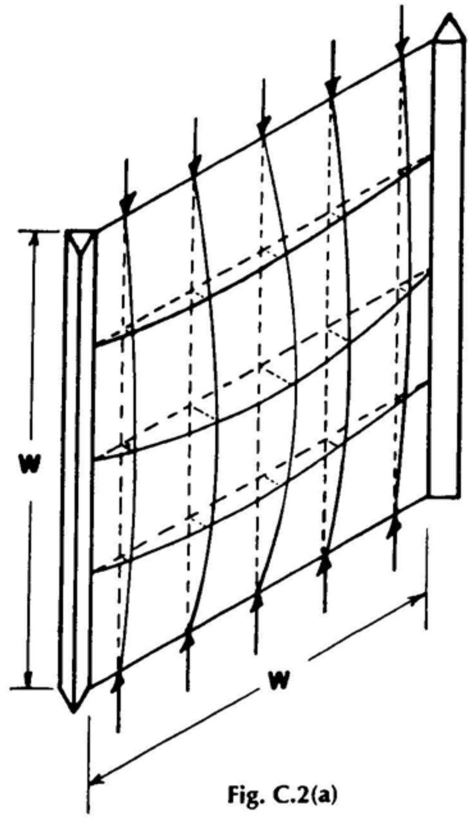


Fig. C.2(a)

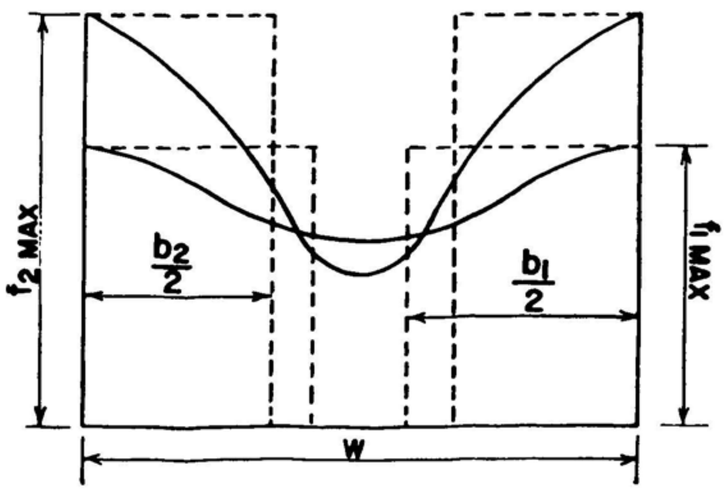


Fig. C.2(b)

buckling load to their neighbors. In contrast the struts (or strips) closest to the edges, held straight by the ties, continue to resist increasing load with hardly any increasing deflection. For the plate this means that the hitherto uniformly distributed compression stress re-distributes itself in a manner shown on Fig. C.2(b), the stresses being largest at the edges and smallest in the center. With further increase in load this non-uniformity increases further, as also shown on Fig. C.2(b). The plate fails, i.e., refuses to carry any further load increases, only when the most highly stressed strips, near the supported edges, begin to yield, i.e., when the compression stress f_{max} reaches the yield point F_y.

This post-buckling strength of plates was discovered experimentally in 1928, and an approximate theory of it was first given by Th. v. Karman in 1932. (See pp. 478-9 of Ref. C.1). It has been used in aircraft design ever since. A graphic illustration of the phenomenon of post-buckling strength will be found in the series of photographs on Fig. 7 of Ref. A.2.

The model of Fig. C.2(a) is representative of the behavior of a compression element supported along both longitudinal edges, as the flange in Fig. C.1(b). In fact, such elements buckle into approximately square waves as shown on that latter figure, and the grid can be regarded as a model of any one such wave. In contrast, if a model were to be made for the top flange of Fig. C.1(a) it would consist of a grid in which each tie would be supported only at one end, but would be free at the outer edge. It is immediately evident that such ties will have little restraining influence on the buckling deflections of the compression struts of the grid. This means that compression plates longitudinally supported along only one edge exhibit much smaller membrane stresses and, therefore, develop buckling waves of considerable magnitude almost immediately upon reaching their critical buckling stress and will show less post-buckling strength than those supported along both edges. This difference in the behavior of the two types of compression plates is fully borne out by tests (Ref. C.2). It is for this reason that different design procedures applying to each of them are necessary.

Correspondingly, *Section 2.2* defines a stiffened compression element as a 2.2
portion of a cross-section stiffened along both longitudinal edges (such as in Fig. C.1(b)); an unstiffened element as one stiffened along only one of the two longitudinal edges (such as in Fig. C.1(a)); and a multiple-stiffened element as one having one or more intermediate stiffeners between the edges (for examples see *Chart 2.3.1(A)* of *Manual)*. The buckling and post-buckling strength of each of these is determined by their degree of thinness, which is expressed by the ratio of flat width of the compression element to its thickness, designed as the flat-width ratio, w/t.

2. STIFFENED COMPRESSION ELEMENTS

(a) Effective Width

It was pointed out that Fig. C.2(b) represents the state of stress in a stiffened compression element when buckling (slight, and usually hardly perceptible waving) has taken place, and that failure is initiated when the maximum edge stress reaches the yield point. It would be awkward in design to take explicit account of this non-uniform stress distribution. This difficulty is obviated by employing the well known device of the "effective design width," which is illustrated in Fig. C.2(b). The total compression force in the element, say the flange of Fig. C.1(b), is equal to the area under the stress distribution curve times the thickness of the element. The same total force is obtained if the actual element with its non-uniform distribution is replaced by one of reduced, effective width, b, and with constant stress of magnitude f_{max}. The two elements will be equivalent if the effective width has been so chosen that the area under the actual stress distribution curve is equal to the two rectangular areas f_{max} b/2 shown in dashed lines on Fig. C.2(b). In this manner the central portion of stiffened compression elements is thought of as removed, and the element of actual width, w, is replaced by one of effective width, b, *(Section 2.2(e))*. Fig. C.2(b) also shows that the effective width decreases with increasing edge stress f_{max}. Corresponding effective cross-sections are shown on *Chart 2.3.1(A)* of the *Manual*.

In order to determine the effective width, some 150 tests have been carried out at Cornell University, on sections with stiffened compression elements whose w/t ratios ranged from 14.3 to 440. The majority of these are reported in Refs. C.2 to C.5 (those not reported were tests made on proprietary sections, and not intended for publication). From these tests the following formula was derived (See Appendix of Ref. C.2a and Refs. C.3, 4, 5):

$$\frac{b}{t} = 1.9\sqrt{\frac{E}{f_{max}}}\left(1 - \frac{0.475}{w/t}\sqrt{\frac{E}{f_{max}}}\right) \tag{C.3}$$

This equation is merely an experimental modification of that originally proposed by v. Karman (See Ref. C.3), which has long and successfully been used in aircraft design. It was found that failure loads and deflections at service loads of thin-walled beams are safely and conservatively predicted on the basis of Eq. C.3. Also, out-of-plane distortions immediately preceding failure were found to be small, of the order of 0.2 percent to 1 percent of the width even in very thin compression flanges with w/t up to about 250.

Provisions of the *Specification* have been based on Eq. C.3 for over twenty years, with uniform success and, in many cases, with somewhat excessive conservatism. The latter is due to the fact that, because of the novelty of the concept of post-buckling strength back in 1946, Eq. C.3 had been selected to represent close to a lower bound rather than the average of test results. Two decades of successful use which confirmed the described conservatism, now justifies a very slight liberalization of Eq. C.3, consisting in changing the constant in the last term from 0.475 to 0.415. Hence, the new equation (which is still
2.3.1.1 close to a lower bound) on which the present *Section 2.3.1.1* of the *Specification* is based, reads:

$$\frac{b}{t} = 1.9\sqrt{\frac{E}{f_{max}}}\left(1 - \frac{0.415}{w/t}\sqrt{\frac{E}{f_{max}}}\right) \qquad C.4$$

It will be found that effective widths calculated by Eqs. C.3 and C.4 differ from each other by at most about 10 percent and in the vast majority of cases by half or less of this amount. It should be observed, in addition, that if the effective widths of the compression elements of a member are changed by a certain amount, the change of calculated strength or deflection for the entire member is only a fraction of this amount.

The situation is illustrated in Fig. C.3 which, for $F_y = 33$ ksi, shows curves representing the 1962 effective width provisions based on Eq. C.3 and the 1968 provisions based on Eq. C.4. These provisions are obtained if, in Eq. C.4, E = 29,500 ksi is substituted which results in the formula "for deflection determination" in *Section 2.3.1.1.* It is seen that the effective width b (or the ratio b/t) depends on the maximum edge stress f_{max} (simply denoted by f in the *Specification)* and on the flat width ratio w/t. *Charts 2.3.1.1(C) and 2.3.1.1(D)* of the *Manual* show this relationship. One sees that for any given stress there is a definite value of w/t below which the element is fully effective, i.e. w = b. This particular transition value, designated by $(w/t)_{lim}$, is found from the appropriate formula in *Section 2.3.1.1*. Similar equations "for load determination" are obtained from Eq. C.4 by introducing the safety factor as explained in (c), below, and corresponding curves are shown in *Charts 2.3.1.1(A) and 2.3.1.1(B)* of the *Manual*. 2.3.1.1

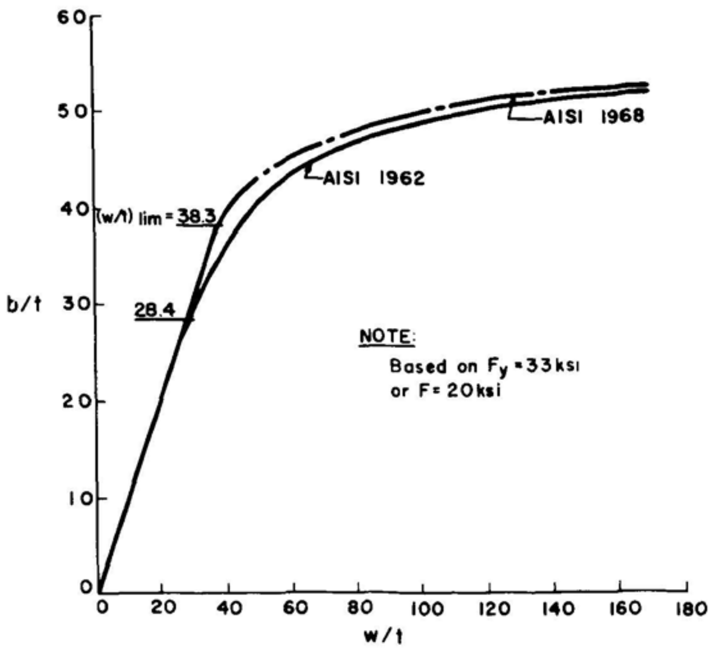


Fig. C.3

It is seen from Fig. C.3 that the two curves are of the same nature and numerically quite close to each other. One of the chief practical advantages of the new version, more important than the slight gain in effective width, is this: The pre-1968 version resulted in $(w/t)_{lim} = 28.4$ and required reduced widths to be calculated and used for any w/t larger than this limit. At the same time, the width reduction was so gradual that even at $w/t = 40$ the effective width ratio was $b/t = 36.6$, a reduction of less than 10 percent for a 40 percent increase in flat-width ratio. Hence, shapes with compression flanges in the range of $w/t = 28.4$ to 40 had to be tediously calculated for a reduced effective width, though the resulting section properties of the members remained almost identical with those for the full section. The new provisions raise $(w/t)_{lim}$ to 38.3 (for this particular stress) and at that value the new curve, as can be seen, takes off at an angle from the straight-line which represents full effectiveness $(b/t = w/t)$. This means that the region where effective width calculations are required but produce only very slight and practically negligible effects is greatly reduced, saving wasted effort for the designer.

Another feature of the new provisions is that they have much reduced the differences between effective width provisions in the *AISI Specification* and in the AISC Specification (Ref. C.6). The 1963 AISC Specification contained effective width provisions which did not depend on the actual flat-width ratio w/t, and which were markedly conservative in some ranges and markedly unconservative in others when compared with the *AISI* provisions, differences amounting to up to 20 percent. In the 1969 AISC Specification the form of the provisions is the same as in the *AISI Specification* and, while numerically the two will still not be entirely identical, the differences which result in part from differences in section geometry, will become practically insignificant, amounting to a maximum of about 9 percent in rare cases.

Special, very slightly more liberal provisions, identical in both the *AISI* and the AISC *Specification*, are made for square and rectangular manufactured tubes. These are strictly standardized, closed shapes (in contrast to the great variety of specialized and mostly open shapes which can and are being cold-formed by individual manufacturers), produced by special rolling processes (see B.2, herein) of great regularity and control.

(b) Variable Section Properties

When a member containing a stiffened compression element, is subjected to load, what happens according to Eq. C.4 is this: When the stress on the element of given w/t (say, 70) is gradually increased there is at first, at low stresses, no buckling (waving) and consequently no reduction in effectiveness $(b = w)$. When a definite stress is reached which can be computed from the formula for $(w/t)_{lim}$ (or read from *Table 2.3.1.1* or *Charts 2.3.1.1)* the effective width begins to be less than the actual width. For $w/t = 70$. *Chart 2.3.1.1 (C)* shows this stress to be approximately 10 ksi. As the stress is further increased, the effective width decreases (see Fig. C.2(b)). For $w/t = 70$, for example, at a stress of 30 ksi, the effective width has decreased to 48.5t as can be read from *Chart 2.3.1.1(C)*.

It follows that the effective area, say, of the compression flange of a beam decreases as the load increases. In consequence of this process the neutral axis moves toward the tension flange and the effective properties of the cross section, such as A, I, and S, decrease with increasing load. This process is shown schematically (with corresponding actual and equivalent stress distributions) in the top part of Fig. C.4, which is identical with Fig. 6 of Ref. C.4. The bottom part of that figure shows the measured position of the neutral axis of two typical tests. The neutral axis is seen to be located somewhat below the centroidal axis even at relatively low loads, and to descend as the load is increased; also, the axis for the beam with the larger w/t is seen to lie below that for the other beam since the larger the w/t the larger the loss in efficiency, or the smaller the b/w.

The fact that effective section properties change with stress or load has to be considered in design, as explicity specified in *Section 2.3* of the *Specification*. This is one of the reasons why in the *Tables of Section Properties* of the *Manual* a number of properties are given for two basic stresses, usually $F_b = 20$ ksi and 30 ksi. As indicated in the *Manual* knowing a given property at two sufficiently different stress levels, it is usually accurate enough to obtain the same property at some other stress level by interpolation or extrapolation (the latter within reasonable limits). For shapes for which no tables are available in the *Manual* (i.e., for the preponderant majority of members in actual use) a similar procedure is advisable of computing the properties at two or more stresses, and using interpolation for additional information.

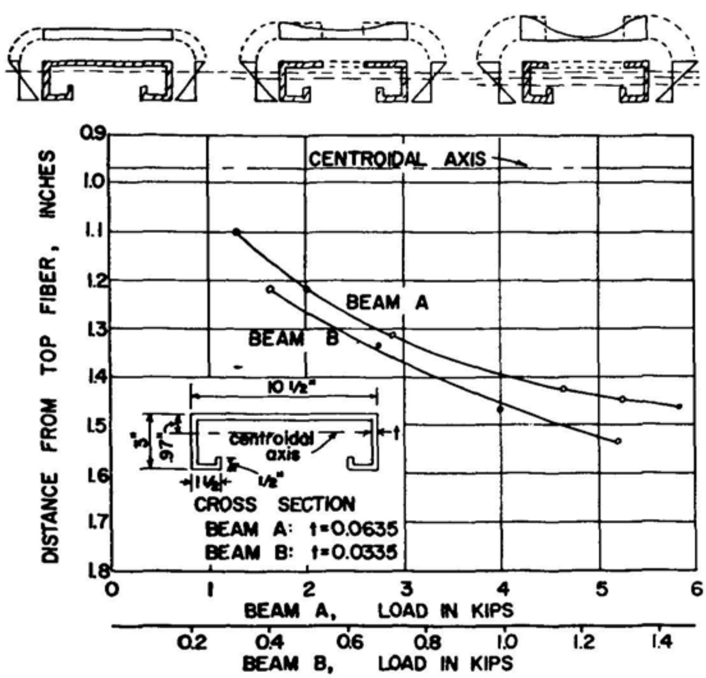


Fig. C.4

(c) Formulas for Load and for Deflection Determination

It has been pointed out that stiffened compression elements fail when the edge stress, f_{max}, which is equal to the stress on the effective area (see Fig. C.2(b)), reaches the yield point. In order to compute the failure moment, M_{ult} of a beam which fails in compression yielding one would, therefore, have to compute its section modulus S for a stress equal to the failure stress, i.e., the yield point, and multiply it by the yield point, so that

$$M_{ult} = S_{F_y} \times F_y = S_{1.67F} \times 1.67\,F$$

Then the allowable moment is

$$M_{all} = M_{ult}/1.67 = S_{1.67F} \times F$$

It is likely to cause confusion to ask the designer to determine the effective width and the section modulus for one stress, 1.67F, and then to multiply that modulus by another stress, F, to obtain an allowable bending moment. In order
2.3.1.1 to obviate this confusing necessity, *Section 2.3.1.1* contains a special formula for effective width for computing allowable moments and loads. This is obtained from the original formula (the one for deflection determination in *Section 2.3.1.1)* by substituting 1.67f for f. The formula for load determination, consequently, is adjusted in such a manner that the designer, when he substitutes his design stress, actually determines the effective width for 1.67 times the design stress, as is necessary in order to compute the correct section properties for determining load capacity. Accordingly, *Charts 2.3.1.1(A)* and *2.3.1.1(B)* actually give b/t ratios for unit stresses 1.67 times those shown. On the other hand, the computation of deflection should be based upon the width which is effective under the stress caused by the actual applied load. Hence, the deflection formulas and *Charts 2.3.1.1(C)* and *2.3.1.1(D)* give the b/t ratios for the actual stresses.

A special situation arises when dimensioning members for wind or earthquake forces alone, or, for combinations of such forces with dead and live loads. It was indicated in B.3, above, that for this situation the *Specification* provides a 25 percent reduction in nominal safety factor and that, correspond-
3.1.2 ingly, in *Section 3.1.2* it permits a 33⅓ percent increase in allowable stress. When calculating carrying capacities of members with variable section properties, i.e. members which incorporate stiffened compression elements, the following applies:

Basically, the section properties regardless of the type of loading, are to be calculated for the stresses at incipient failure. As was just explained, this is done by using the ordinary allowable stresses (which are based on a safety factor of 1.67) in connection with the effective width formulas "for load determination" which are compensated for that same safety factor. When dealing with wind or earthquake, the allowable stresses are increased, but they are to be referred to the same section properties defined above. To
2.3.1.1 achieve this, *Section 2.3.1.1* provides that the effective width shall be deter-

mined for 0.75 times the stress caused by wind or earthquake or by the applicable combination of live and dead load plus wind or earthquake. To illustrate, suppose one wants to determine the wind load capacity of a member made of a steel with $F_y = 33$ ksi. The allowable stress, according to *Section* 3.1.2, is $1.33 \times 20 = 26.7$ ksi. According to *Section 2.3.1.1* the appropriate property, say the section modulus, is to be determined for a stress $0.75 \times 26.7 = 20$ ksi. This section modulus, times the allowable wind stress of 26.7 ksi, gives the maximum permissible wind moment in the member.

3. STIFFENERS AND MULTIPLE-STIFFENED COMPRESSION ELEMENTS

To be effectively stiffened, a compression element can be supported along both longitudinal edges by webs, such as in the hat, box, or U-sections of *Charts 2.3.1* of the *Manual*. In this case, if the webs are properly designed (see *Section 3.4 and 3.5* of the *Specification,* discussed in C.7, herein), they provide adequate stiffening for the compression elements by preventing their longitudinal edges from out-of-plane distortion. On the other hand, in many cases only one longitudinal edge is stiffened by a web, while support of the other is provided by a special edge stiffener. In most cases the special edge stiffener takes the form of a simple lip, such as in the channel and I-sections of *Charts 2.3.1* of the *Manual*. Not infrequently, other shapes are used for edge stiffeners, such as the hook joint shown in *the Manual,* in connection with *Example 2*.

The structural efficiency of a stiffened element always exceeds that of an unstiffened element with the same w/t by a sizeable margin, except for low w/t not exceeding $63.3/\sqrt{F_y}$. However, when stiffened elements of large w/t are used, inspection of *Charts 2.3.1.1* will show that the material is not employed economically inasmuch as an increasing proportion of the width of the compression element becomes ineffective. Thus, for w/t of the order of 100 only about one-half of the width is effective, and the fraction becomes even smaller for larger w/t. On the other hand, in many applications of cold-formed construction, such as the entire field of building panels and decks, maximum coverage is desired and, therefore, large flat-width ratios are called for. In such cases, structural economy can be improved by providing additional "intermediate" stiffeners between the main stiffeners along the edges, i.e. between webs or between a web and an edge stiffener. Such intermediate stiffeners provide optimum stiffening if they do not participate in the wave-like distortion of the compression element. In that case they break up the wave-pattern so that the two strips to each side of the intermediate stiffener distort substantially independently of each other, each in a pattern similar to that shown for a simple, stiffened element in Fig. C.1(b). Compression elements furnished with such intermediate stiffeners are designated as "multiple-stiffened elements." Two examples are shown in *Chart 2.3.1(A)* of the *Manual*.

In designing stiffened elements with edge stiffeners, and multiple-stiffened elements, information is needed (a) on the properties required of edge stiffeners and of intermediate stiffeners in order that they provide adequate support, and (b) on the manner in which the effective widths of such compression elements, the effective stiffener areas, and the resulting cross-sectional properties of the member are to be computed.

(a) Edge Stiffeners

It is evident that in order for an edge stiffener to provide the necessary support for the compression element, it must possess sufficient rigidity. Otherwise it might buckle perpendicular to the plane of the element which it is supposed to stiffen, in the general manner of a compression strut. It was noticed in several early tests of lipped, double-channel I-sections that premature failure had occurred because edge stiffeners were inadequate. To determine the required minimum stiffness theoretical determinations (unpublished), somewhat similar to those on pp. 367 to 370 of Ref. C.7, were made. This analysis gave the necessary dimensions to make the critical buckling stress of an edge-stiffened flange equal to that of the identical flange but stiffened by webs along
2.3.2.1 both edges. *Section 2.3.2.1* represents a simple but close fit to those findings.

The analysis as such deals only with critical buckling stresses of the type of Eq. C.2; no attempt is made to include post-buckling strength, theoretical treatments of which would become prohibitively involved. It has been established experimentally, however, that the stiffener dimensions obtained from the theoretical analysis (and, thereby, from *Section 2.3.2.1)* are satisfactory to develop the full effective width of edge-stiffened compression elements. In particular, the lips of the 20 types of beam specimens of Table 2 of Ref. C.2 had been designed to these requirements. The satisfactory performance of these members is evident from that table, while in previous tests with dimensionally deficient stiffeners unsatisfactory results had been obtained. For stiffeners which do not satisfy *Section 2.3.2.1*, see (b), below.

(b) Intermediate Stiffeners

In regard to the necessary rigidity of intermediate stiffeners, the following reasoning had been verified by tests: An edge stiffener, whose rigidity is stipu-
2.3.2.1 lated in *Section 2.3.2.1*, is required to stiffen only one compression element. In contrast, an intermediate stiffener must stiffen two such elements, one to either side of the stiffener. It seems reasonable to expect, then, that the required minimum rigidity of an intermediate stiffener will be twice that of an edge stiffener. To obtain pertinent information on this question, and on the overall performance of multiple-stiffened elements, sixteen tests have been carried out on beams of inverted U-shape with multiple-stiffened top flanges. The w/t of the sub-elements ranged up to about 160, a sub-element being a flat portion between two stiffeners at least one of which is an intermediate stiffener; for exact definition see *Section 2.2(c)*. Duplicates of these same specimens, but without intermediate stiffeners, had been tested previously, facilitating an accurate assessment of the effect of intermediate stiffening.

To check the assumption that the required rigidity of an intermediate stiffener is twice that of an edge stiffener for the same w/t the stiffeners on half of the 16 beams were given moments of inertia exactly twice those of *Section 2.3.2.1*, whereas those of the other half were given four times that amount (i.e. eight times those of *Section 2.3.2.1)*. The test results showed that no improvement in stiffening effect was obtained through the heavier stiffeners, indicating that the lighter stiffeners were sufficient to produce optimum effect. (On the other hand, proprietary panel tests carried out by a major panel producer showed a significant loss of stiffening action if stiffeners are used with moments of inertia appreciably less than twice those of *Section 2.3.2.1.)*

Accordingly, *Section 2.3.2.2* specifies that the minimum moment of inertia of intermediate stiffeners shall be twice that of edge stiffeners as specified in *Section 2.3.2.1*. **2.3.2.2**

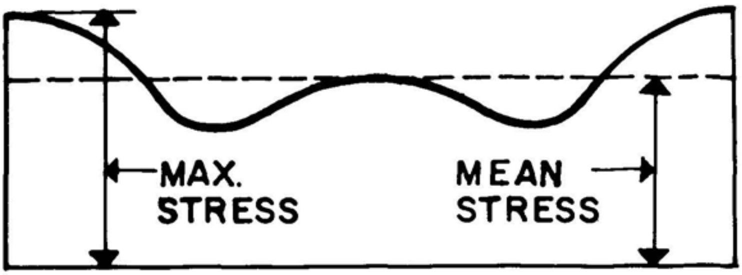
The question "when is a stiffener a stiffener and not a web" is not always easy to decide. From the preceding discussion it appears that in clarifying this question one has to distinguish between flexural members (beams, floor and roof panels, etc.) and compression members. In flexural members a stiffener is an appropriately shaped portion which is entirely or almost entirely subject to compression i.e. is located entirely or almost entirely on the compression side of the neutral axis. It is for such portions that the theoretical analysis has been carried out and it is for such portions that it has been verified by tests. If a substantial part of a stiffener, such as a rib in a panel, reaches beyond the neutral axis into the tension zone, evidently a much more stable situation is obtained. No research information defining this situation is known to the writer and, therefore, recourse should be had to *Section 6, Tests*. Purely as a subjective opinion the writer is inclined to say that if at least one-third of the total area of a stiffener is located on the tension side of the neutral axis, he would regard its stiffening effect equivalent to the full stiffening provided by a web, rather than the lesser stiffening provided by a stiffener as per *Section 2.3.2*. In compression members, on the other hand a distinction evidently cannot be based on presence or absence of tension stresses. Here the writer is inclined to designate a component as a web which provides full stiffening only if its depth is equal to the full depth of the section, i.e. one which reaches all the way from one flange to the other.

Another question equally difficult to decide is that of assessing the effect of a stiffening rib or groove with deficient properties, i.e., with depth or moment of inertia smaller than stipulated in *Section 2.3.2.1*. Analytical information of considerable complexity could be worked out for this situation, and would need confirmation by test and probably excessively cumbersome formulation in specification terms. The scattered and incidental test information on this question seems to indicate that ,though substandard stiffeners do have a stiffening effect, it is very much smaller than for stiffeners which satisfy *Section 2.3.2.1*. It would be well to avoid such stiffeners, but if they are used for some non-structural reasons, again their effect must be assessed by means of tests according to *Section 6*.

(c) Effective Width and Effective Stiffener Area

The tests on members with intermediate stiffeners showed that the effective width of a sub-element is less than that of an ordinary stiffened element of the same w/t, particularly for w/t exceeding about 60. This can be explained in the following manner:

In any flanged beam the normal stresses in the flanges are the result of shear stresses between web and flange. The web, as it were, originates the normal stresses by means of the shear it transfers to the flange. The more remote portions of the flange obtain their normal stress through shear from those closer to the web, and so on. In this sense there is a difference between webs and intermediate stiffeners in that the latter is not a shear-resisting element and, therefore, does not "originate" normal stresses through shear. On the contrary, any normal stress in the stiffener must have been transferred to it from the web or webs through the intervening flange portions. As long as the sub-element between web and stiffener is flat or only very slightly buckled (i.e., with low w/t) this shear proceeds unhampered. In this case, then, the stress at the stiffener is equal to that at the web and the sub-element is as effective as a regular stiffened element of the same w/t.



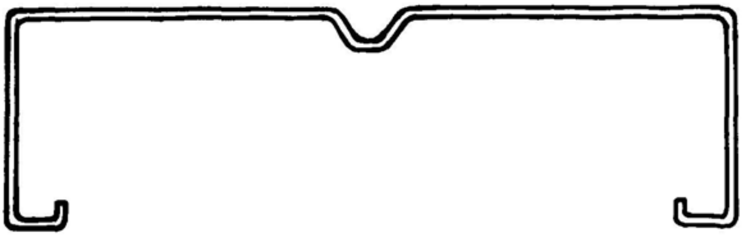


Fig. C.5

However, tests indicate that for larger w/t the slight buckling waves of the sub-element interfere with complete shear transfer and create a "shear lag" (somewhat similar to that reflected in *Section 2.3.5* and discussed elsewhere in this Commentary). Consequently, the stress-distribution in a multiple-stiffened element, when the w/t of the sub-elements exceed about 60, can be thought of as represented in Fig. C.5. That is, since the edge stress of a sub-element is less at the stiffener than at the edge, its effective width is less than that of the corresponding stiffened element (with same w/t). Also, the efficiency of the stiffener itself is reduced by this lower stress which fact is best accounted for by assigning a reduced, effective area to the stiffener.

The quantitative formulation, from the test results, of the situation just described qualitatively was originally given in terms of special effective width formulas especially applicable to multiple-stiffened elements. It can be shown that the simple reduction formula of *Section 2.3.1.2* gives results which are practically identical with the explicit formulas derived from the test results.

Consequently, the effective widths of sub-elements are identical with those obtained from *Section 2.3.1.1* provided w/t is less than 60. For larger w/t the effective widths of *Section 2.3.1.1* are reduced according to the simple formula of *Section 2.3.1.2*. Also, in view of the reduced efficiency of intermediate stiffeners just described, their effective area for determining properties of sections of which they are part, is to be determined from the simple formulas for A_{ef} also given in *Section 2.3.1.2*.

What has been said so far in regard to effects and behavior of intermediate stiffeners, holds identically for edge stiffeners. In fact, an edge stiffener can be regarded as one-half of an intermediate stiffener, as was discussed in the preceding section. Correspondingly, if the shape of Fig. C.5 were cut in two along the centerline of the intermediate stiffener, each half would become an edge stiffener, but this would not change the shear flow and other characteristics in each half of the original member. For this reason the same provisions in regard to effective width and effective stiffener area which have just been discussed in regard to the effects of intermediate stiffeners, also hold for situations where an edge stiffener is employed.

It should be noted that the reduction in efficiency provided by *Section 2.3.1.2* does not substantially detract from the very considerable gain in structural economy obtained by intermediate stiffeners. For instance, if a stiffened element has $w/t = 180$, with $F = 20$ ksi (for load determination) its efficiency b/w is only 29 percent. If one intermediate stiffener is provided at the center line, the w/t of each of the two sub-elements generally will be less than half depending on shape of stiffener (see *Manual, Charts 2.3.1*). Assuming this ratio to be 85, from *Section 2.3.1.2* the efficiency b_e/w is found to be 53 percent, a considerable improvement. For an element with $w/t = 120$ stiffened to result in two sub-elements with $w/t = 55$ each, the respective efficiencies are 42 percent and 77 percent. These two examples show the sizable effect of intermediate stiffening.

Provisions *(a), (b), and (c) of Section 2.3.2.2* reflect the fact previously discussed, that intermediate stiffeners, due to shear lag across slightly waved sub-elements are not as effective as complete webs would be. Consequently, if a number of stiffeners were placed between webs at such distances that the resulting sub-elements have w/t of considerable magnitude, there would be a rapidly cumulative loss of effectiveness with increasing distance from the web. Provisions *(a)* and *(b)* in essence provide that if w/t of the sub-elements exceeds $(w/t)_{lim}$ *(Section 2.3.1.1)*, i.e., if they are in the slightly buckled state so that shear transfer is interfered with, only such intermediate stiffeners which are adjacent to a web shall be regarded as effective. Contrariwise, if stiffeners are so closely spaced that the sub-elements show no tendency to slight buckling (i.e., $w/t < (w/t)_{lim}$) the entire intermediately stiffened element, including stiffeners, will be fully effective. That is what provision *(c)* specifies. The limiting condition of the latter case is a corrugated sheet in which sub-elements have disappeared, as it were, and the entire element consists of closely spaced stiffeners. Provision *(c)* also specifies for such closely stiffened elements an effective thickness t_s for computing, when needed, the flat-width ratio of the entire element (including stiffeners). It is easily checked that this t_s is the 2.3.2.2

thickness of a solid plate having the same moment of inertia as the actual, closely stiffened element. It should be emphasized that this equivalent thickness t_e is to be used only for determining an equivalent flat-width ratio w/t_e for the purpose of calculating the effective width of such a compression element with closely spaced stiffeners. That is, this flat-width ratio w/t_e is to be used instead of w/t in *Section 2.3.1.1* in order to determine b/t_e and, thereby, the effective width b of the entire, closely stiffened element. Once b is determined, section properties such as A, I, etc. are, of course, calculated using the actual steel thickness t.

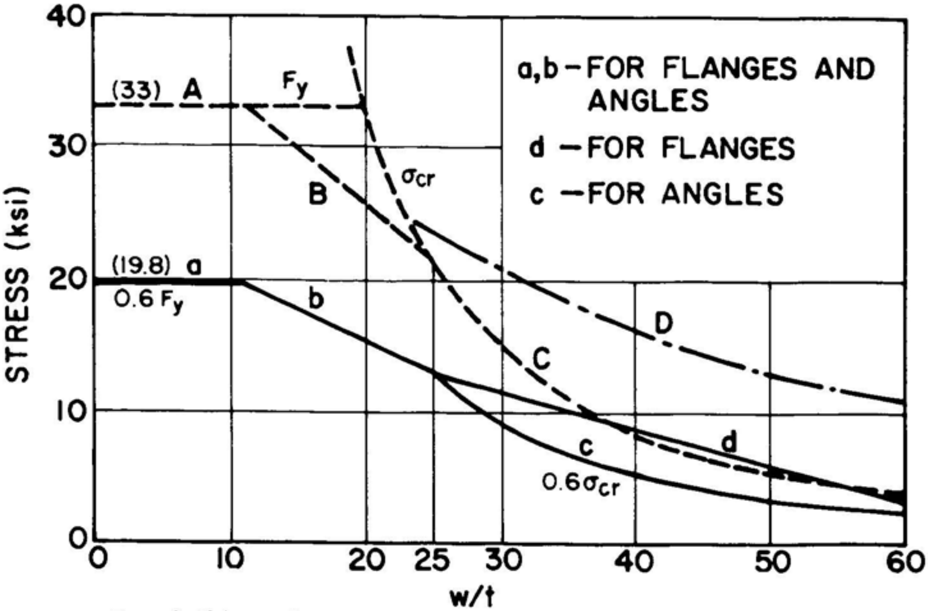
4. UNSTIFFENED COMPRESSION ELEMENTS

It has been pointed out under C.1 General, that unstiffened compression elements can be thought of as represented by the model of Fig. C.2(a), except that ties are held along one edge only. In consequence, it was pointed out, their restraining influence is weaker and, correspondingly, unstiffened elements develop considerable deformation immediately upon reaching their buckling stress and show less post-buckling strength than stiffened elements. Actually, the model of Fig. C.2(a) is incomplete. Since plates resist not only normal strains but also shear strains, the model should be completed by introducing diagonals into the rectangular panels formed by the struts and the ties. These represent membrane shear stresses, which also contribute to post-buckling strength.

The experimental evidence which has led to the allowable stresses on un-
3.2 stiffened compression elements, given in *Section 3.2* of the *Specification* or *Chart 3.2*, of the *Design Manual*, is presented in detail in Ref. C.2 and the Appendix of Ref. C.2(a). The substance of these provisions is best visualized by means of Fig. C.6, which is substantially identical with Fig. 8 of Ref. A.2.

It has been pointed out that the critical buckling stress of unstiffened elements is given by Eq. C.2 with, conservatively, $k = 0.5$. This critical stress, as a function of w/t is shown by the curved, dashed line C. (For ideal hinge support along the stiffened edge one would have $k = 0.425$; it is realized that in some types of cross-sections with relatively stiff webs k can assume values in excess of 0.5. However, to combine safety with simplicity, no variation of the restraint coefficient k has been introduced in the *Specification,* particularly since the cited test evidence did not seem to support values significantly in excess of 0.5.)

If steel were always sharp yielding (see Fig. B.1) and if compression elements were ideally plane, the horizontal line A drawn at the yield point would set an upper limit to the buckling stress. That is, for a steel with yield point of 33 ksi (for which Fig. C.6 is drawn), elements with w/t in excess of 20 would fail by buckling at stresses below the yield point; elements with w/t smaller than 20 would fail by simple yielding at 33 ksi. (A similar reasoning holds, and a corresponding figure can be drawn, for any other yield point.) It is well known that such ideal conditions do not exist and that, in consequence, compression plates of moderate w/t buckle below the value given by Eq. C.2. (See e.g., Fig. 9.48, p. 410 of Ref. C.7.) As was pointed out under B.1, above, many of the customary sheet and strip steels tend toward gradual yielding (see Fig. B.1) and, in addition, the cold-forming process itself tends to set up residual



Permissible Design Stress for Unstiffened Compression Elements for F_y = 33 ksi

Fig. C.6

stresses which also lower the proportional limit. Both these influences tend to lower actual inelastic buckling stresses for moderate w/t below their theoretical elastic value of Eq. C.2. On the basis of the experimental evidence of Fig. 14 of Ref. C.2, line B, of Fig. C.6, has been drawn as representing those stresses at which sudden and pronounced inelastic buckling occurred in the tests. Such buckling did not result in immediate complete failure of the member, particularly for w/t exceeding about 20; however, the "kinks" caused by buckling were so sharp that any existing additional strength was considered useless in view of excessive distortion. The general expression of line B is similar to that of Eq. 10 of Ref. C.2. In that reference the limit up to which failure would occur by yielding rather than by buckling (intersection of lines A and B in Fig. C.6) had been set at w/t = 12 and the end point of line B at w/t = 30.

At that time (1946) yield strengths in excess of about 33 ksi were not yet contemplated for use, and the experimental work was carried out on steels of about that or of only slightly higher strength. Since then, and particularly in recent years, even higher strength steels of structural quality have become available both for conventional and for cold-formed construction. *Section 3.2* in the present edition of the *Specification* has been adjusted to permit the safe design of unstiffened elements for steels of any yield point from 33 ksi up to about 100 ksi. This has been done on the basis of theoretical considerations and of additional tests on members with unstiffened elements formed of a special steel with F_y = 83 ksi and intentionally low ductility. The results of these tests are reproduced and discussed later herein. A footnote in *Section 3.2* takes care of the rare case when steels with yield point lower than 33 ksi are used.

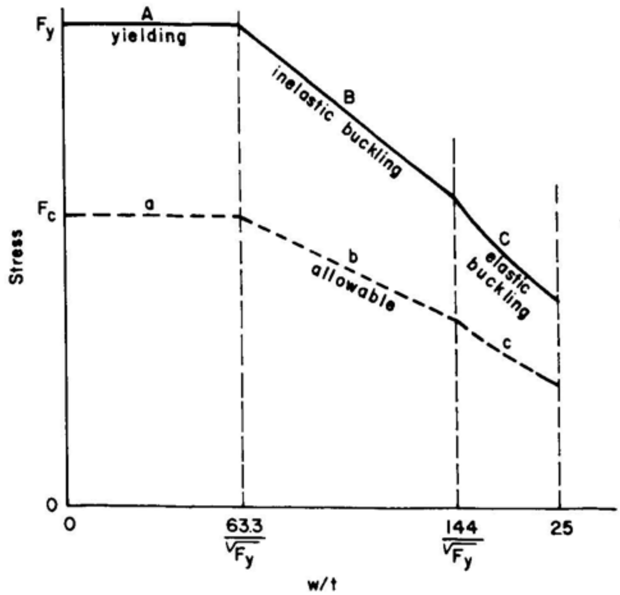
This extension to higher strength steels, and the revisions connected with it, have permitted the provisions on unstiffened compression elements of this and of the AISC Specification (Ref. C.6) to be made substantially similar. Contradictions which existed in previous editions of the two specifications have been eliminated so that essentially identical answers are now obtained for similar situations by both specifications, due account being taken of essential differences in geometry of hot-rolled vs. cold-formed shapes.

To extend the design provisions to steels of medium and high yield points, the limit up to which failure occurs by yielding rather than buckling, has been made dependent on F_y; that is, this limit is specified as $(w/t)_{lim,1} = 63.3/\sqrt{F_y}$. For $F_y = 33$ ksi this gives a limit $w/t = 11$ instead of 12 as in Ref. C.2. It is further assumed conservatively that the proportional limit is about 65 percent of the yield point. This determines the end point of the straight line B for inelastic buckling, at $(w/t)_{lim,2} = 144/\sqrt{F_y}$. Again, for $F_y = 33$ ksi this gives a limit of 25 as compared with 30 in Ref. C.2. For steels with higher yield point there is a third region, from $(w/t)_{lim,2}$ to $w/t = 25$ where elastic buckling according to Eq. C.2 is assumed to constitute the limit of structural usefulness. This situation is shown on Fig. C.7, where for higher strength steels the straight-line representing inelastic buckling is again designated as B and the curve according to Eq. C.2 which represents elastic buckling as C.

In order to arrive at allowable stresses for this range of w/t from 0 to 25, the ordinates of the lines A, B, and C evidently must be divided by the safety factor of 1.67. This results in the allowable stresses given by provisions (a), (b),
3.2 and (c) in *Section 3.2* of the *Specification,* which are also shown on Fig. C.7. To aid in visualizing these provisions, Fig. C.8 shows graphs of allowable stresses on unstiffened elements up to $w/t = 25$ for four different yield points. Also, for $F_y = 33$ ksi and 50 ksi the provisions of the 1962 edition of the *Specification* are shown in dashed lines. It is seen that for the yield point range envisaged in the 1962 edition, the new values are very close to those of 1962.

To verify the provisions in the range of very high strength steels, special tests have been made, as mentioned before. These were similar to those for low strength steels in Ref. C.2, i.e. they consisted in testing short compression members made of two channels connected back to back. The w/t varied from 7 to 20 and the yield point, as mentioned, was 83 ksi. The results are shown on Fig. C.9 which also shows the allowable stresses according to the new *Section 3.2,* multiplied by the safety factor of 1.67, i.e. the predicted failure stresses, as it were. It is seen that in the region of inelastic buckling the specification provision, line B, is verified closely and somewhat conservatively. In the region of elastic buckling, curve C, considerable conservatism is evident, chiefly because of the presence of post-buckling strength (see below).

Flanges with w/t larger than about 25 behave substantially the same, regardless of yield point because elastic buckling occurs at stresses sizably below the yield point and is independent of it. At about the theoretical buckling stress, Eq. C.2 with $k = 0.5$, they distort quite gradually and return to their original shape upon unloading. Also, such flanges show sizable post-buckling



Unstiffened Elements Failure Stresses and Allowable Stresses for O < w/t < 25

Fig. C.7

strength. All this is so because the buckling stress is considerably below the yield point (see curve C of Fig. C.6) so that sizable waving can occur without permanent set being caused by the additional stresses due to distortion. In this range the post-buckling strength of unstiffened flanges is significant. Detailed information is contained in Refs. C.2, C.2(a) and, particularly, Eqs. 12 and 14 of Ref. C.8. From this information, curve D has been drawn conservatively to show that compression stress at which an unstiffened flange fails in the post-buckling range. It is seen that for values beyond w/t = 25, the post-buckling strength given by Curve D is considerably larger than the elastic buckling stress given by Curve C. Consequently, in this case, in order to prevent major distortions from occurring at service loads it is sufficient to insure that the design stress exceed the theoretical buckling stress by at most a small margin. The post-buckling strength is then sufficient to provide adequate safety against actual collapse. For this reason, in the range of w/t from 25 to 60, the straight line d has been chosen as representing satisfactorily the allowable stress on which to base design. It starts at w/t = 25 with a stress equal to 1/1.67 of the critical buckling stress (to provide adequate safety against pronounced and permanent buckles) and is so located that in the region of w/t = 40 to 60 the allowable stress is practically identical with the theoretical buckling stress (to prevent sizable distortion at design load, safety being provided by post-buckling strength). Line d in Fig. C.6 represents the formula designated by "for all other sections" in *Section 3.2(d)*. From the information presented in Table 4 and on p. 55 of Ref. C.2(a) it is seen that for the tested specimens with flanges w/t in the range 3.2(d)

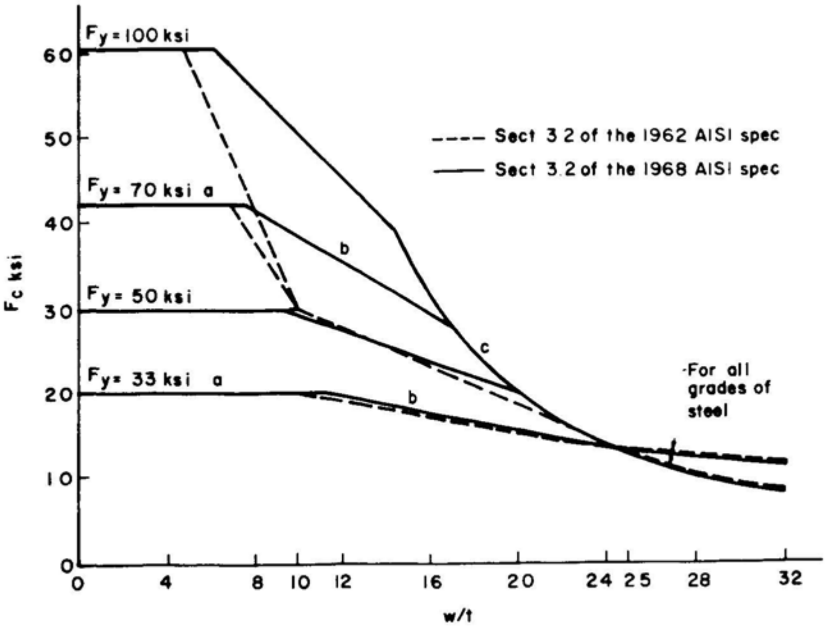
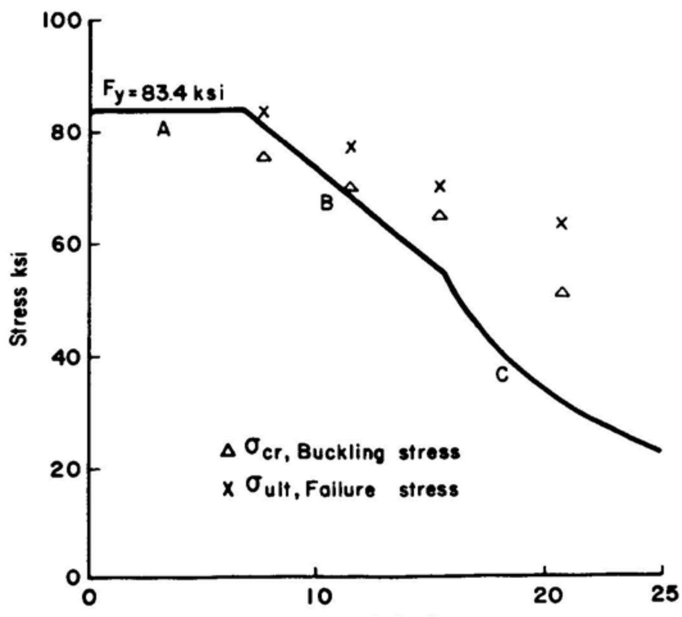


Fig. C.8



Buckling of High Strength Steel Unstiffened Compression Elements

Fig. C.9

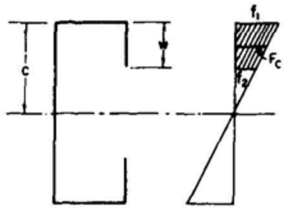
of w/t from 25 to 60, safety factors against collapse ranged from 2.1 to about 4 for beams and from 1.85 to more than 3 for studs, the higher values applying to the larger w/t ratios. First barely noticeable flange distortions occurred for the large w/t ratios at stresses equal to at least 0.7 times F_c as given in *Section 3.2(d)*, while for the smaller w/t ratios (25 to 35) they occurred at stresses 1.3 to 1.6 times F_c.

While a limited amount of post-buckling strength is available in unstiffened elements, which has been made use of in the provision just cited for the range from about w/t = 25 to 60, there is a type of cross-section composed entirely of unstiffened elements which shows little or no post-buckling strength. This is the angle section when used for compression struts. (Cruciform sections have the same characteristic but have no application in cold-formed construction.) This is so because, when an equal-leg, thin angle reaches the buckling stress of the two equal, component plates, both of them buckle in the same direction; this results in a twisting distortion of the angle as a whole, leading to early collapse. (See e.g., Ref. C.7, Fig. 9.7, p. 363.) Consequently, for a safe design of such angles it is necessary that the design stress not exceed the critical buckling stress divided by the safety factor, since little or no reserve strength is available beyond the buckling stress. The corresponding curve is that designated by c in Fig. C.6; it corresponds to the stipulation "for angle struts" in *Section 3.2(d)*.

It should be noted that for unstiffened elements the allowable stress decreases very rapidly with increasing w/t ratios, beyond $w/t = 63.3/\sqrt{F_y}$. Consequently, in designing shapes for load carrying purposes, the use of unstiffened elements with w/t substantially exceeding $63.3/\sqrt{F_y}$, will usually be found entirely uneconomical. Design stresses up to w/t = 60 are provided nevertheless in the *Specification*, this was done because in cold-formed construction the shape of members is often dictated by other than structural considerations. In such cases it may be desirable to be able to compute the carrying capacity of a member which incorporates unstiffened elements with large w/t ratios, even though from a purely structural standpoint such a member may be uneconomical.

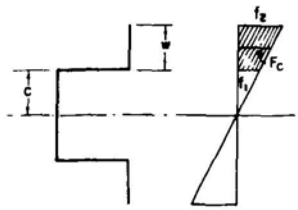
This entire discussion applies to unstiffened elements in which the compression stress before buckling is constant throughout the width w. This will be so in the majority of cases; that is, in concentrically loaded compression members or in flexural members where the unstiffened element is parallel to the neutral axis. There are situations, however, where this is not so. Two of these are illustrated on Fig. C.10, where flexural members are shown with lips turned in or out. These lips represent unstiffened elements disposed perpendicular to the neutral axis. It is seen that the compression stress on these elements is not of constant magnitude but varies in proportion to the distance from the neutral axis.

An exact determination of the buckling conditions of such elements is of a high degree of complexity, since they depend not only on the ratio of f_1 to f_2 but also on the location of the stiffened edge in relation to the stress distribution. Evidently, if that edge is stiffened which is subject to the maximum stress (Fig. C.10(a)), a more stable situation obtains than when the opposite is true (Fig. C.10(b)). For purposes of design it is sufficiently accurate to assume, how-



$$f_1 = F_c \cdot \frac{C}{C - w/2}$$

(a)



$$f_1 = F_c \frac{C}{C + w/2}$$

(b)

Fig. C.10

ever, that two dimensionally identical unstiffened plates, one compressed uniformly and the other non-uniformly, will buckle at the same total critical compression force. Correspondingly, the allowable stresses can then be determined from the requirement that the total permissible compression force in the variably stressed element shall be the same as in the dimensionally identical, uniformly stressed element when designed according to *Section 3.2*. It is clear from Fig. C.10 that this requirement is satisfied when the average stress on the variably stressed element is equal to F_c as stipulated in *Section 3.2*, i.e.,

$$(f_1 + f_2)/2 = F_c$$

In order to satisfy this requirement the stress f_1 in the adjoining, stiffened element must be limited appropriately. The suggested procedure can, therefore, be described as follows:

(i) From the w/t of the unstiffened element determine the allowable compression stress F_c according to *Section 3.2*. For the variably stressed element, this is the allowable stress at the center line of the element, i.e., distant w/2 from either edge.

(ii) Determine the corresponding maximum allowable stress f_1 on the contiguous stiffened element from the fact that stress varies proportionately to distance from the neutral axis. (For the case of elements disposed perpendicular to the axis, this is shown on Fig. C.10.) Evidently, the stress on the contiguous, stiffened element is also limited by F; that is, the smaller of the two values, f_1 or F, governs.

5. CALCULATION OF SECTION PROPERTIES OF BEAMS

It has been pointed out in C.2(b) above, that the effective properties of sections containing stiffened compression elements vary with load. This is so because the effective width changes with stress. It is for this reason that different properties are used for determining allowable loads on the one hand, and for calculating deflections under actual service loads on the other, as has been explained in C.2(c). Deflection requirements very often govern the design of floor and roof members (panels and decks) and utilization of load computation section properties for these cases would be unnecessarily restrictive and uneconomical.

It has been noted that, for load calculation, the effective width of the compression flange of flexural members is calculated for the basic design stress F. This is true in many cases but there are important exceptions. One concerns the case where the compression flange consists of unstiffened as well as stiffened elements. Such are, for instance, the flanges of the lipped channels of Fig. C.10; while the horizontal portions are stiffened, the lips are unstiffened elements. When the w/t of the unstiffened lips exceeds $63.3/\sqrt{F_y}$, the design stress on the stiffened flange elements must be modified according to *Section 3.2* of the 3.2
Specification as discussed in C.4 of the Commentary. The effective width of the stiffened flanges must be determined according to the computed maximum allowable stress (F_c) or basic design stress ($F = 0.6\ F_y$), whichever is smaller.

(In *Section 2.3.2.1* the use of simple lips as edge stiffeners is restricted to 2.3.2.1
elements with w/t not exceeding 60. Table 2.3.2.1(B) shows that for w/t = 60 and the most frequent yield point range, the minimum required d/t is 10.9. For customary corner radii, this results in a w/t of the lip of about 8 to 9. Hence, if it were attempted to stiffen by simple lips, compression elements with w/t significantly exceeding 60, lips with w/t exceeding $63.3/\sqrt{F_y}$ would be required. This would necessitate a reduction of the allowable compression stress below F to prevent premature buckling of the stiffening lip. This is one reason why simple lips are restricted to elements with w/t not exceeding 60.)

Another situation in which the effective width of compression flanges is computed for a stress less than F is the following: If the distance from the compression fiber to the neutral axis is equal to or greater than that to the tension fiber, the compression stress is equal to or greater than the tension stress; in this case the compression stress governs and the effective width is computed for the stress, F (*See Example 4 of Manual).* Contrariwise, if the neutral axis is closer to the compression flange, the tension stress is the greater, governs, and must not exceed F. In this case the effective width of the compression flange is computed for the smaller stress which occurs in that flange when the stress in the tension flange is F. That compression stress can be computed only if the location of the neutral axis is known; but that location, in turn, depends on the as yet unknown effective width of the compression flange. In this case, therefore, section properties are best computed by successive approximation, as illustrated in *Example No. 7* of the *Manual*. (It is possible to determine the location of the neutral axis by setting up explicit, usually quadratic equations, instead of using successive approximations. Except for special situations, the successive approximation procedure as illustrated in the Example will generally be found simpler and faster.)

Finally, when it is desired to compute deflections under design load it is the bending moment rather than the stress which is known. The effective width must be computed for that compression stress which is caused by the known moment, but that stress cannot be computed unless the section modulus, and hence the effective width corresponding to that as yet unknown compression stress, is determined. In this case, too, a small number of successive approximations leads to the desired result, as is illustrated in *Example 7* of the *Manual.*

6. CYLINDRICAL TUBES IN COMPRESSION OR BENDING

The principal structural application of thin-wall tubes is for compression members in view of their favorable ratio of radius of gyration to area, and in view of the fact that their radius of gyration is the same in all directions. Like other thin-wall compression members, tubes must be designed to provide adequate safety not only against column buckling but also against local buckling. It is well known that the classical theory of local buckling of longitudinally compressed cylinders (Ref. C.7, p. 457) overestimates the actual buckling strength, often by 200 percent and more. It is also known, from theoretical investigations by v. Karman and others, that inevitable imperfections of shape and of axiality of load reduce the actual strength of compressed tubes radically below their theoretical value. In view of this it seemed advisable to rely largely on test results for developing adequate design provisions to safeguard against local buckling.

A systematic evaluation of test evidence obtained by a number of investigators was given by Plantema (Ref. C.9). Important additional tests not included in Ref. C.9 are found in Ref. C.10. These have been checked against the evaluation of Ref. C.9, and it was found that Plantema's graphical representation (see below) also fits these additional tests conservatively. In consequence, *Section*
3.8 *3.8* of the *Specification* is based on the information of Ref. C.9 in the following manner:

Plantema found from tests on longitudinally compressed thin tubes of mild steel possessing a definite yield point, that the ratio of collapse stress to yield point F_{ult}/F_y, depends on the parameter (E/F_y) (t/D) in the manner shown on Fig. C.11 (t = wall thickness, D = mean diameter of tube). Line 1 corresponds to collapse stresses below the proportional limit, line 2 to collapse stresses between proportional limit and yield point (the approximate proportional limit being on the average 83 percent of the yield point, at pt. B of Fig. C.11), line 3 to that range where collapse occurs at the yield point. In other words, in the range of line 3 local buckling does not occur before yielding, and no reduction of allowable stress below that permitted on a solid section is necessary. In regions 2 and 1 collapse by local buckling occurs before the yield point is reached; if tubes thin enough to fall into that range are used, their allowable stresses would have to be reduced to safeguard against local buckling. It is seen that pt. A delimits the range of tubes which do not collapse by local buckling and that for this point (E/F_y) $(t/D) = 8$. Substituting $E = 29{,}500$ ksi one finds

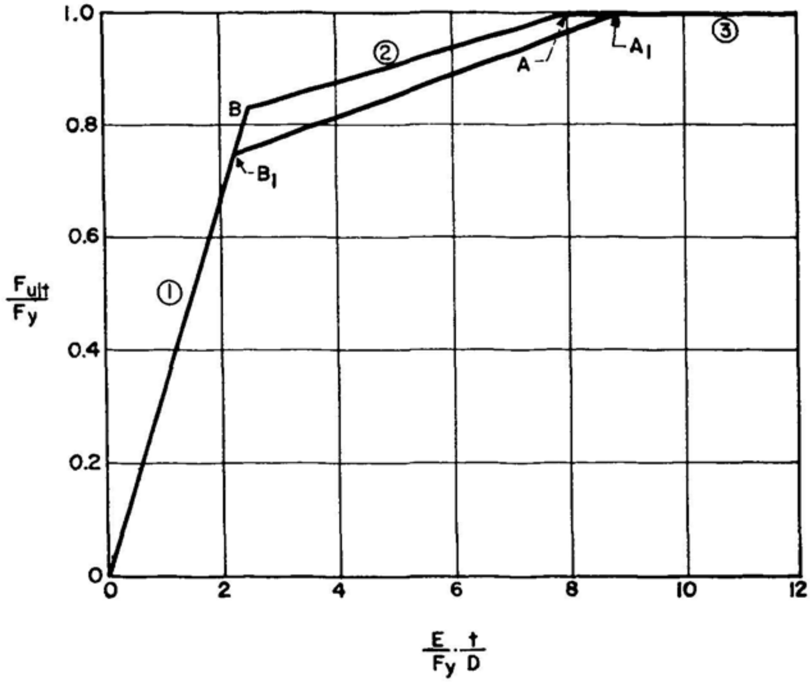


Fig. C.11

that tubes with D/t less than $3{,}700/F_y$ are safe from failure caused by local buckling.

Section 3.8 of the *Specification* is based on line A_1B_1 of Fig. C.11 which is seen to be conservatively chosen relative to the Plantema-Wilson-Newmark test evidence. That is, instead of a proportional limit of $0.83F_y$ (point B) the more conservative and consistent value $0.75F_y$ (point B_1) has been chosen. Further, again as a matter of some conservatism, the maximum D/t ratio below which local buckling need not be considered, has been lowered from $3{,}700/F_y$ (point A) to $3{,}300/F_y$ (point A_1). Correspondingly, when D/t is smaller than this value, *Section 3.8* permits the full allowable stress $F = 0.6\ F_y$ to be used (unless other provisions, such as *Section 3.6* for column buckling, require a lower stress). For D/t ratios between those of points A_1 and B_1, i.e. between $3{,}300/F_y$ and $13{,}000/F_y$, the maximum allowable compression stress is smaller than F and is given by the ordinates of line A_1B_1 divided by the safety factor, 1.67. No provision is made for tubular members with D/t larger than $13{,}000/F_y$. It is seen that the buckling stress of such extremely thin tubes would be governed by the steeply inclined line 1. That is, such tubes would require a sharply reduced and thereby mostly uneconomical design stress, and would also be very sensitive to geometric and other imperfections. 3.8

There are situations where it is appropriate to use cylindrical tubular shapes for flexural members. Since the tendency toward local buckling in the compression half of a flexural tubular member is essentially the same as in a tubular compression member, the same limitations on wall thickness and allowable stresses apply in both situations.

7. WEBS OF BEAMS

In regard to webs of beams the designer of cold-formed steel construction is faced with somewhat different problems than he is in heavy, hot-rolled construction. In the latter, webs with h/t in excess of 70 are usually furnished with stiffeners to avoid reduction of allowable stress. Such webs occur only in fabricated sections (plate girders) since, for hot-rolled sections h/t does not exceed about 60. Moreover, in plate girders bearing stiffeners are frequently provided at reaction and load points. The problem, therefore, in hot-rolled construction is primarily that of correct stiffener design. In contrast, in cold-formed construction h/t ratios exceeding 70 are frequent. At the same time the fabrication process (production in forming rolls or press or bending brakes) generally makes it economically impracticable to employ stiffeners, except under unusual conditions. Consequently, the problem here is primarily that of so limiting the various allowable web stresses that adequate stability is obtained without the use of stiffeners.

(a) Shear

The elastic stress at which a web, considered as simply supported along both flanges, buckles when subject to shear only is given by Eq. C.2, herein, with $k = 5.35$ (see Ref. C.7). Below the proportional limit, with $E = 29{,}500$ ksi, this gives $\sigma_{cr} = 142{,}000/(h/t)^2$. If the elastic critical shear buckling stress computed in this manner is larger than the proportional limit of the material in shear, the actual shear buckling stress is smaller than this elastic critical stress because, above the proportional limit, the effective modulus is smaller than Young's modulus, E. If h/t is so small that local buckling in shear will not occur, then failure will occur by simple yielding at a shear stress of about 0.577 F_y (see B.4, Basic Design Stresses, herein).

3.4.1 The provisions of *Section 3.4.1* are obtained directly from this basic information by applying a safety factor ranging from 1.44 to 1.67 to the shear stresses which, depending on h/t, produce yielding, inelastic buckling, or elastic buckling. In particular, for elastic buckling in the range of large h/t values, an allowable stress of $85{,}200/(h/t)^2$ is obtained by multiplying by 0.6 the above indicated elastic buckling stress of $142{,}000/(h/t)^2$. This stress is based on $E = 29{,}500$ ksi. The corresponding provision in the AISC Specification (Ref. C.6) for elastic shear buckling of webs without stiffeners is derived entirely identically, but a modulus $E = 29{,}000$ ksi is used which gives $F_v = 83{,}200/(h/t)^2$. For the sake of uniformity between the two specifications, the slightly lower AISC value has been adopted, which makes *Section 3.4.1(b)* identical with the corresponding AISC provision.

Section 3.4.1(a) for inelastic buckling in the intermediate h/t range is also identical with the corresponding AISC provision.

Previous editions of the *Specification* provided a safety factor of 2.22 against elastic shear buckling, 1.65 against yielding in shear, and a gradual transition between the two values for inelastic shear buckling. Because of the presence of some post-buckling strength in the range of elastic shear buckling, and the proven use, both by test and in practice, of a uniform safety factor of 1.67 in the AISC shear provisions, the use of a special safety factor for shear buckling was discontinued in the present edition. Such a step was already anticipated in the Commentary to the 1962 edition, III, 7, b.

(b) Bending

Webs of beams can buckle not only in shear, but also due to the compression stresses caused by bending. The corresponding theoretical critical buckling stress is given on p. 377 of Ref. C.7. It is identical with Eq. C.2, herein, with $k = 23.9$. For steel this results in $\sigma_{cr} = 640{,}000/(h/t)^2$. However, just as in the case of stiffened compression elements, it is well known that webs in bending do not fail at these theoretical buckling stresses, but develop sizable post-buckling strength, accompanied by slight waving (see e.g. Ref. C.11).

For this reason, and in accord with current practice in plate girder design, particularly in bridges (see Ref. C.12), only a small factor of 1.23 has been applied to the above expression to obtain the formula for the maximum allowable bending stress in webs of *Section 3.4.2,* namely, $F_{bw} = 520{,}000/(h/t)^2$. 3.4.2
This small safety factor is sufficient to prevent development of wave-like web distortion at design loads; the necessary strength reserve is provided by the post-buckling strength.

It should be added that when unstiffened webs are subject to bending only, such as in the region of maximum moment in beams, *Section 3.4.2* need be checked for high strength steels only, but not for steels having yield points of less than 40 ksi. In fact, substitution of the maximum allowable h/t ratio, 150 (see *Section 2.3.4)* in the formula of *Section 3.4.2* gives $F_{bw} = 23$ ksi, which corresponds to a yield point of 40.6 ksi. On the other hand, for high strength steel (e.g., $F = 30$ ksi corresponding to $F_y = 50$ ksi) the allowable bending stress in webs with high h/t ratios must be reduced in accordance with *Section 3.4.2.*

(c) Combined Bending and Shear

In cantilevers, at supports of continuous beams, and in other situations, high bending moments combine with large shear forces and webs must be safeguarded against buckling due to this combination. The simultaneous action of bending and shear stresses produces buckling at lower unit stresses than when one is present without the other. Eq. 762, p. 407 of Ref. C.1 permits one to compute pairs of shear and bending stresses which, when acting simultaneously, will result in web buckling. The corresponding formula in *Section 3.4.3* is 3.4.3
identical with the quoted equation of Ref. C.1, except that it is given in terms of allowable stresses rather than stresses which produce buckling; that is, it contains the necessary safety factors. This provision, $(f_{bw}/F_{bw})^2 + (f_v/F_v)^2 = 1$, is known as an "interaction formula" since it permits one to determine the effect of one type of stress on the allowable value of another type of stress. The well known formula for simultaneous bending and compression, $(f_a/F_a) + (f_b/F_b) = 1$ which is used in various forms in many design codes, including the *AISI Specification,* is another example of such an interaction formula.

It should be noted that *Section 3.4.3* provides safety specifically against elastic instability, that is, against elastic buckling of webs under simultaneous shear and bending. It is not intended to supply safety against yielding (rather than buckling) in bending and shear. This has to be checked separately. That is if, in a given case, the criterion of *Section 3.4.3* is satisfied, one still has to make sure, individually, that the actual bending stress does not exceed the basic design stress F and that the actual shear stress f_v does not exceed 0.4 F_y *(Section 3.4)*. This is necessary in those cases, and they are very frequent, where $F_{bw} >$ 0.6 F_y and/or $F_v > 0.4$ F_y. This situation, that is safety against yielding, is checked individually rather than by an interaction formula because the maximum shear stress and the maximum bending stress occur in different locations, the former at the neutral axis and the latter at the web-flange junction. For this reason there is no significant interaction of bending and shear stresses as far as initiation of web yielding is concerned.

(d) Bearing (Web Crippling)

Concentrated loads or reactions of beams, applied over short lengths, produce a high local intensity of load which can cripple unstiffened thin webs. This is why, in plate girder construction and sometimes also in deep hot-rolled girders, bearing stiffeners are provided at points of concentrated reactions or loads. Ways have been found to incorporate the forming of such bearing stiffeners for end reactions in the mass production process of some types of long-span cold-formed shapes. However, the preponderant majority of cold-formed flexural members continues to be produced with plane, unstiffened webs which must, therefore, be checked against web crippling at reactions and, occasionally, at load points. A theoretical analysis of this phenomenon is extremely complex since it involves a combination of non-uniform stress distribution (the stresses radiating out from the loaded length into the adjacent portions of the web), elastic and plastic instability due to stresses so distributed, and local yielding in the immediate region of load application. The complexity is aggravated by the bending produced by eccentric application of the load caused by the curved transition from web to bearing flange. In view of this analytical complexity, reliance has to be placed almost exclusively on experimental evidence. For this reason a total of 290 web crippling tests have been carried out and the provi-
3.5 sions of *Section 3.5* of the *Specification* and *Charts 3.5 of the Design Manual* based upon the results of those tests. (A theoretical investigation which takes account of at least some of the enumerated influences has been published by one of the writer's collaborators—Ref. C.13. It shows reasonable agreement with the general trend of the experimental evidence.)

Two types of specimens have been investigated: (a) beams the configuration of which virtually prevents rotation of the web out of its plane at the bearing length, and (b) beams where such rotation is not only possible but, to some degree, is actually promoted by the very configuration of the member. Fig. C.12(a) shows one type of section of category (a), including the kind of distortion which obtains on web crippling; the fact that both flanges bear symmetrically counteracts rotation and provides considerable fixity to the web along its transition to the flange. Fig. C.12(b) shows one type of section of category (b); here the one-sided flange permits a rotation which lifts the tip of the flange off the seat, and this rotation is in fact accentuated by the eccentricity of the web with regard to the point of application of the bearing force at the end of the transition radius.

136 tests were carried out on two types of I-sections providing the high

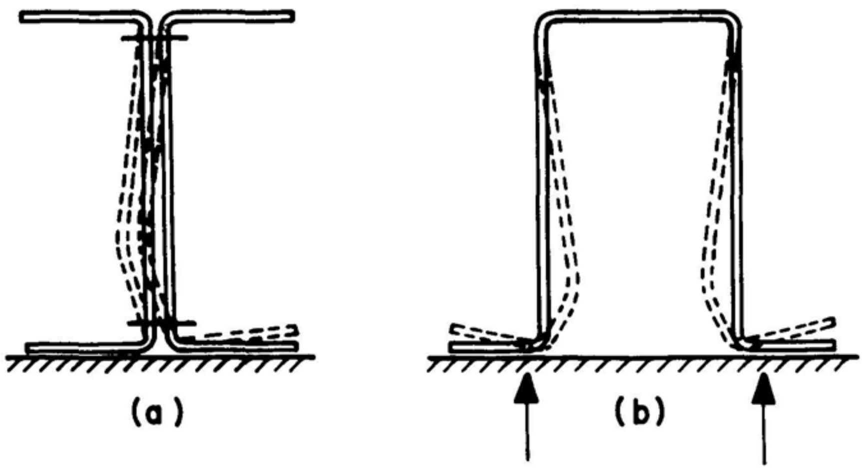


Fig. C.12

degree of fixity of Fig. C.12(a) and are reported in Ref. C.14. These covered h/t ratios from 30 to 175 and N/t ratios from 7 to 77 (h = depth of section, N = length of bearing). The provisions of *Section 3.5(b)* are identical with Eqs. 3 and 4, p. 18 and p. 19 of Ref. C.14, divided by a safety factor of 2.2. The latter factor was chosen (a) in view of significant scattering of the test results and, more important, (b) because the tested specimens represent probably the optimum amount of web restraint likely to be met in practice. 3.5(b)

154 additional tests have been performed, 128 of them on specimens of the type shown on Fig. C.12(b), and 26 on specimens of the same type but inverted. The latter position provides somewhat more web restraint than the former. The h/t ratios ranged from 49 to 200, and the N/t ratios from 12 to 40. As is evident from Fig. C.12(b), the lateral distance of the reaction or load from the center-line of the web is likely to be a significant factor since it is this eccentric location which produces bending of the web. For this reason the ratio of R/t was also varied in these tests, R being the inside corner radius.

In contrast to the previous tests with high degree of restraint, it was found that for these specimens the h/t ratio affected the crippling strength significantly. In consequence, this strength was found to depend on four variables: N/t, h/t, R/t, and F_y. The simplest expressions that could be developed to represent these test results with reasonable accuracy are incorporated in *Section 3.5(a)*. The explicit formulas for P_{max} are written for the most frequent situation, i.e., R/t = 1. For R/t values other than one, correction factors are given separately.

As is evident from Fig. C.12, the bend radius which governs web crippling refers to that bend which is in bearing. In terms of Fig. C.12, if the upper and lower bend radii were different, the radius of the bottom bends should be used
3.5(a) in *Section 3.5(a)* when checking for bearing at the supports, as shown. If, somewhere along the beam, a concentrated force were applied to the top flange, the radius of the upper bends would have to be used at that location. In the 154 tests which have been referred to, R/t ratios ranged from about 1 to about 3, which covers most of the customary range. For unusually large R/t ratios the formula of *Section 3.5(a)* can be used up to $R/t = 4$, but can not be relied upon beyond this value. In case a larger radius is used, web crippling strength must be ascertained by test (*see Section 3.5(a)(3)*).

The formulas of *Section 3.5(a)* have been derived from the 128 tests with the weakest degree of web restraint (see Fig. C.12(b). The 26 tests on the inverted sections showed larger web strength, but still considerably less than those obtained for the high degree of fixity of Fig. C.12(a). Since the 128 tests on which *Section 3.5(a)* is based represent the lowest degree of web restraint likely to be found in practice, a lower safety factor than adopted for *Section 3.5(b)* seemed warranted. Consequently, the formulas of *Section 3.5(a)* incorporate a safety factor of 1.85.

The provisions of *Section 3.5* apply only to webs with h/t ratios less than
2.3.4 150. This is so because *Section 2.3.4* requires that unstiffened webs have h/t ratios not exceeding 150, and that webs with larger h/t ratios must be furnished with adequate stiffeners to transmit reactions and concentrated loads, if any (see H.2 of this Commentary). Since *Section 3.5* specifically applies to unstiffened webs, the limitation to $h/t = 150$ follows automatically from *Section 2.3.4*.

It is noted that the equations in *Section 3.5* in the present edition of the *Specification* are wholly identical with those in the 1962 edition. The apparent differences in these equations are entirely due to the fact that these formulas are now written in terms of F_y rather than F and that the former emphasis on steels with $F_y = 33$ ksi has been eliminated.

D. COMPRESSION MEMBERS

1. GENERAL

The carrying capacity of concentrically compressed members, such as columns, studs, etc., may be limited by any of the following factors:

1. Simple yielding, which will fail a short, compact member when it reaches its yield load, $P_{yield} = F_y A$.

2. Flexural buckling, when a slender member of doubly symmetrical section, or one which is not susceptible to, or is braced against twisting, fails by flexural buckling about its axis of least resistance, at a load

$$P_{tm} = \sigma_{tm} A$$

where the buckling stress

$$\sigma_{tm} = \pi^2 E_t/(KL/r)^2 \qquad \text{(D.1)}$$

The tangent modulus E_t, as previously defined in B.1, Materials, is the slope of the stress-strain curve at the level of the buckling stress σ_{tm}; it is equal to Young's modulus E in the lower, straight-line portion of the stress-strain diagram. The effective length KL depends on the conditions of restraint at the two ends of the member.

3. Local buckling, which can fail a thin-walled member when its individual flat compression elements collapse in the manner discussed in C.1 to C.4, above.

4. Torsional-flexural buckling, by simultaneous twisting and bending, which can occur in members whose shear center and centroid do not coincide and which are torsionally weak (thin open sections, in contrast to closed, thick-walled, or solid shapes).

Some shapes which are not susceptible to torsional-flexural buckling are: all closed sections, such as square, rectangular or round tubes or other closed shapes made by welding together two or more pieces (e.g. a hollow section produced by welding two C-shapes toe-to-toe); all solid shapes; also, when concentrically loaded, all thin-walled open shapes whose shear center and centroid coincide, such as doubly-symmetrical I-shapes and point-symmetrical Z-shapes. Many other thin-walled open shapes are subject to torsional-flexural buckling, but whether they will so buckle depends on their specific dimensions; among these are channel-, C-, hat- and plain or lipped angle sections, I-sections with unequal flanges, and others. In all these sections, shear center and centroid do not coincide so that the axial load applied at the centroid does not pass through the shear center. It is this fact which causes the tendency to torsional-flexural buckling. Ways for determining whether a given member of such shape and of given dimensions will in fact buckle torsional-flexurally or simply flexurally are discussed in (d), below; graphical design aids for this purpose are provided in Part IV of the *Manual*.

5. Torsional buckling, by twist without bending, which can occur in certain open, thin-walled short members in which shear center and centroid coincide (e.g. I- or Z-sections); this mode is rarely important for realistic dimensions.

Allowable stresses for compression members must safeguard against any and all of these occurrences, singly or in combination. Because of the described variety of ways in which compression members can fail, the determination of allowable stresses on them is necessarily of some complexity. This accounts for
3.6 the relatively elaborate provisions given in *Section 3.6* of the *Specification;* their practical use is greatly facilitated by a variety of design aids given in Parts II and IV of the *Manual.*

2. ALLOWABLE STRESSES IN AXIAL COMPRESSION

(a) Safety Factor

The allowable stresses in axial compression incorporate a safety factor of $23/12 = 1.92$, which is about 15 percent larger than the basic safety factor of 1.67 used in most parts of the *Specification*. This increase compensates for the greater sensitivity of compression members to accidental imperfections of shape or accidental load eccentricities, when compared to tension members or beams.

For hot-rolled construction, the AISC (Ref. C.6) permits a variable safety factor equal to $23/12 = 1.92$ for slender members, but decreasing to 1.67 when L/r becomes zero. For the more compact shapes and frequently more precise end connections in hot-rolled construction this reduced safety factor for stocky columns appears justified because of their smaller sensitivity to end eccentricities and their ability to sustain larger than yield point stresses because of strain-hardening. Most cold-formed sections, in contrast, are more difficult to connect with precision, are thinner and therefore more sensitive to local crippling because of imperfect end connections, and will not develop strain-hardening because of prior local buckling. For these reasons, in the *Specification,* the safety factor throughout *Section 3.6* is kept at the more conservative constant value of 1.92 for members of any slenderness KL/r. An exception is made in
3.6.1.1(b) *Section 3.6.1.1(b)* where the AISC's sliding safety factor is adopted for flexural buckling of relatively stocky sections whose response is more akin to those in hot-rolled construction.

(b) Flexural Buckling

The stress P/A at which flexural buckling occurs is given by Eq. D.1 and is seen to depend on the tangent modulus E_t. Most members in cold-formed as in hot-rolled construction show a gradual-yielding stress-strain curve (curve b in Fig. B.1). The details of the shape of the stress-strain curve of a given cold-formed member depends on that of the steel before forming and on the effects of cold work in the forming process (see B.2, herein), just as in hot-rolled members it depends chiefly on the the residual cooling stresses. It is evidently impossible to take explicit account of these random variations. For this reason, in hot-rolled as in cold-formed construction for columns of small or moderate slenderness Eq. D.1 is conservatively approximated by

$$\sigma_{tm} = F_y - \left(\frac{F_y^2}{4\pi^2 E} \right) (KL/r)^2 \tag{D.2}$$

Recent test confirmation is given in Ref. B.6, pp. 463-4. It is seen that for very short columns, when KL/r approaches zero, σ_{tm} approaches F_y, i.e. failure obtains by simple yielding. On the other hand, for columns of large slenderness σ_{tm} falls into the straight-line portion of the stress-strain diagram; hence, E_t becomes E and σ_{tm} becomes equal to the Euler stress

$$\sigma_e = \pi^2 E/(KL/r)^2 \tag{D.3}$$

Allowable design stresses, F_{a1}, are then obtained by dividing by the safety factor, n, the stress σ_{tm} from Eq. D.2 for low and moderate KL/r ratios, and the stress σ_e from Eq. D.3 for large KL/ r values.

Correspondingly, the allowable unit stresses for axially loaded columns not subject to torsional-flexural buckling in *Section 3.6.1.1* of the *Specification* are obtained from the following two formulas:

$$F_{a1} = F_y/n - \left(\frac{F_y^2}{4n\pi^2 E} \right) (KL/r)^2 \tag{D.4}$$

for small and medium values of KL/r, and

$$F_{a1} = (\pi^2 E/n)\,(KL/r)^2 \tag{D.5}$$

for large values of KL/r.

The limiting value of KL/r below which D.4 and above which D.5 holds, is obtained by equating the right sides of these two equations and solving for KL/r. This gives

$$(KL/r)_{lim} = \pi\sqrt{2E/F_y} \tag{D.6}$$

It will be found that if in Eqs. D.4 to 6 the values $n = 23/12 = 1.92$ and $E =$ 29,500 ksi are substituted, then the formulas of *Section 3.6.1.1* of the *Specification* are obtained for members in which local buckling need not be considered (i.e., for $Q = 1$, see below). For the specific case of $F_y = 33$ ksi the corresponding curves are shown on Fig. D.1.

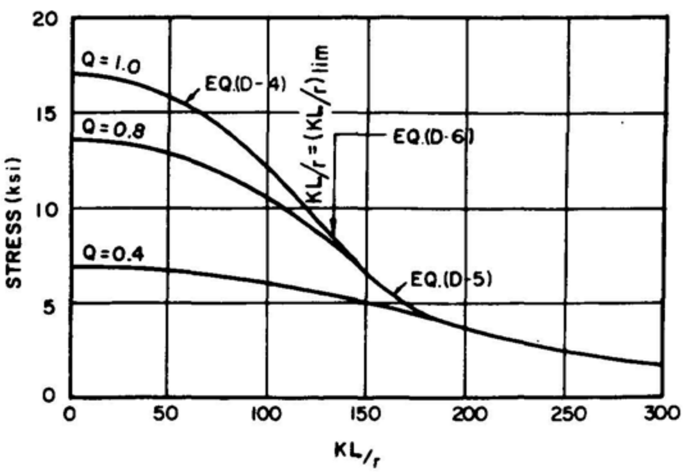
Effective Length Factor K

The effective length factor K accounts for the influence of restraint against rotation and translation at the ends of a column on its carrying capacity. For the simplest case, a column with both ends hinged and braced against lateral translation, buckling occurs in a single half-wave and the effective length KL, being the length of this half-wave, is equal to the actual physical length of the column (Fig. D.2); correspondingly, for this case, $K = 1$. This situation is approached if a given compression member is part of a structure which is braced in such a manner that no lateral translation (sidesway) of one end of the column relative to the other can occur. This is so for compression members in trusses, or for columns or studs in a structure with x-bracing, diaphragm bracing, shear-wall construction or any other provision which prevents horizontal displacement of the upper relative to the lower column ends. In these situations it is safe and only slightly, if at all, conservative to take $K = 1$.

If translation is prevented and abutting members (including foundations) at one or both ends of the member are rigidly connected to the column in a manner which provides substantial restraint against rotation, K-values smaller than 1 (one) are sometimes justified. Representative values are, for end conditions approaching the ideal (hinged or fixed) conditions:

End A	End B	Theoretical K	Recommended K
hinged	hinged	1.0	1.0
hinged	fixed	0.7	0.8
fixed	fixed	0.5	0.65

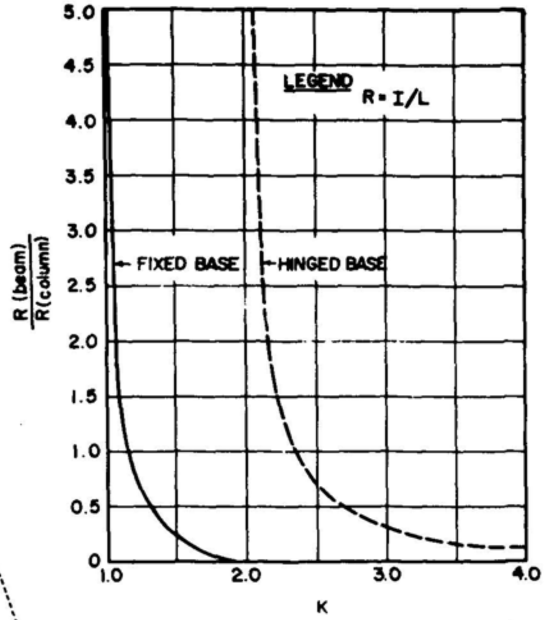
The last column gives values recommended by the Column Research Council (Ref. D.1) which take account of the fact that complete fixity against rotation is never attained. In trusses the intersection of members provides rotational restraint to the compression members at service loads. However, as the collapse load is approached, the member stresses approach the yield point which greatly reduces the restraint they can provide. For this reason K = 1 in all trusses, regardless of whether they are welded, bolted, or otherwise connected.



Column Design Curves, F_y = 33 ksi

Fig. D.1

On the other hand, when no lateral bracing against sidesway is present, such as in the portal frame of Fig. D.3, the structure depends on its own bending stiffness for lateral stability. In this case, when failure occurs by buckling of the columns, it invariably takes place by the sidesway motion shown. This occurs at a lower load than the columns would be able to carry if they where braced against sidesway and the figure shows that the half-wave length into which the columns buckle is longer than the actual column length. Hence, in this case K is larger than 1 (one) and its value can be read from the graph of Fig. D.4, (Ref. D.5). Since column bases are rarely either actually hinged or completely fixed, K-values between the two curves should be estimated depending on actual base fixity.



Effective Length Factor K in Laterally Unbraced Portal Frames
Fig. D.4

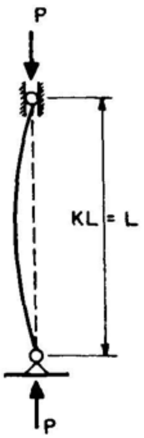


Fig. D.2

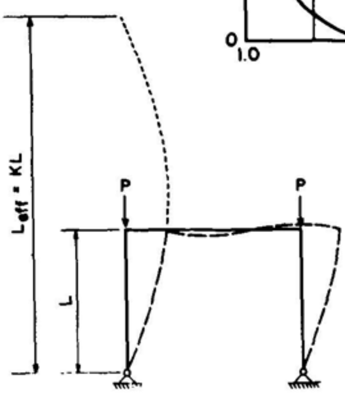


Fig. D.3

Fig. D.4 can also serve as a guide for estimating K for other simple situations. For multi-bay and/or multi-story frames, which are rare in cold-formed construction, simple alignment charts for determining K are given in Ref. D.1, in addition to other useful information on K-values. Reference is also made to the Commentary on the AISC Specification, Ref. C.6.

Whether or not sidesway is effectively prevented is evident by simple inspection in most cases. The bracing system must be sufficiently strong and rigid to counteract sidesway effectively, but the required strength to resist the lateral force exerted from the column upon the bracing system is generally only a few percent of the axial column load. For ambiguous cases, Ref. D.6 gives a reasonably simple method for determining the required bracing characteristics.

(c) Effect of Local Buckling on Column Strength

The effect which local buckling of thin-walled compression members can have in reducing column strength is expressed, in *Section 3.6.1*, by a "form 3.6.1

factor" Q. The meaning of the form factor Q is easily understood as follows:

A very short, compact concentrically loaded compression member ($L/r \rightarrow 0$) fails through simple yielding rather than buckling, at the yield stress F_y. This is correctly reflected in Eq. D.2 from which, for such short pieces, the ultimate failure stress is

$$(P/A)_{ult} = F_y \tag{D.7}$$

A similarly short piece of thin-wall compression member may however fail through local buckling at a stress smaller than the yield point. Hence, for such a member

$$(P/A)_{ult} = QF_y \tag{D.8}$$

where Q is a factor, smaller than one (1), which represents the weakening influence of local buckling. Evidently, Q depends on the form or shape of the thin-walled section and, for this reason, is known as a form factor.

From what has been said in C.2, above, it is clear that a short compression member which consists entirely of stiffened elements (e.g., a closed, rectangular tube) fails under a load,

$$P_{ult} = A_{eff}F_y$$

where A_{eff} is the sum of the effective areas of all the stiffened compression elements, computed for F_y (or for F if the formulas "for load determination" of *Section 2.3.1.1* are used; see C.2(c) above). Dividing both sides by the unreduced area A, one has

$$(P/A)_{ult} = (A_{eff}/A)F_y$$

from which, by comparison with Equation D.8, one sees that for such members

$$Q_a = A_{eff}/A \tag{D.9}$$

In contrast, if a short member consists entirely of unstiffened elements (e.g., an angle section), from what has been said in C.4, above, it is clear that it will fail by local buckling at a load

$$P_{ult} = F_{cr}A$$

where $F_{cr} = 1.67\ F_c$ is the stress at which the unstiffened element with the largest *w/t ratio buckles* (F_c *being the allowable stress on that element, Section 3.2,* and 1.67 being the safety factor). Consequently

$$(P/A)_{ult} = F_{cr} = (F_{cr}/F_y)F_y = \quad (1.67F_c/1.67F_b)F_y = (F_c/F_b)F_y$$

From this, by comparison with Equation D.8, it is seen that for such members

$$Q_s = F_c/F_b \tag{D.10}$$

Finally, if a member consists of both stiffened and unstiffened elements (e.g., a Z-section) its useful limit will be reached when its weakest unstiffened element buckles at the stress F_{cr} (i.e., 1.67 F_c). At this stress the effective area A_{eff} will consist of the unreduced area of all unstiffened elements plus the reduced (effective) area of all stiffened elements; the latter is to be computed for that stress at which such buckling occurs, i.e., for F_{cr} (or for F_c if the formula or chart "for load determination" is used). Consequently, for such mixed sections the ultimate load is

$$P_{ult} = F_{cr}A_{eff}$$

From this

$$(P/A)_{ult} = (A_{eff}/A)\ (F_{cr}/F_y)\ F_y = (A_{eff}/A)\ (F_c/F_b)\ F_y$$

Comparison with Eqs. D.8 to D.10 shows that for this case

$$Q = (A_{eff}/A)\ (F_c/F_b) = Q_a Q_s \qquad \text{(D.11)}$$

This discussion furnishes the reasons for the determination of Q for these three cases, as prescribed in *Section 3.6.1.1(a)*.

From Equations D.7 and D.8 it is seen that for short members ($L/r \rightarrow 0$) the simple equation for calculating the ultimate load due to yielding,

$$P_{ult} = A\ F_y$$

can also be made to apply to failure by local buckling, merely by replacing F_y by QF_y. In completely the same manner, in order to compute the failure load (or the ultimate stress $n(P/A)$ in Equation D.4) for thin-wall members of ordinary length ($L/r > 0$), it is merely necessary to replace F_y by QF_y in the corresponding equation. The same, then, holds true for determining allowable stresses. Hence, in order that Equation D.4 apply also to thin-wall members, it is merely necessary that F_y be replaced by QF_y.

It is in this manner that the first equation in *Section 3.6.1.1* has been obtained.

It will be noticed that of the two general equations for allowable stresses, Equations D.4 and D.5, only the former (for the lower range of KL/r) contains F_y and that, correspondingly, in *Section 3.6.1.1* the form factor Q appears only in the equations pertaining to that slenderness range. This can be understood from the fact that for large slendernesses, when Equation D.3 applies, the stresses at which the column buckles are so low that they will not cause any local buckling before ordinary column buckling has taken place (see also Refs. C.4, D.2).

The described method furnishes design formulas which provide adequate safety against the combinations of column and local buckling which can occur in thin-wall construction. There are cases where the actual strength of a compression member, by test, will be found to exceed that reflected in this method. This occurs, particularly, for sections which consist chiefly of stiffened elements (possibly including unstiffened elements with w/t not much exceeding 10), but which incorporate one or two unstiffened elements with large w/t. In that case the stress F_c, valid for the entire section, is governed by that small unstable portion of the section which consists of these unstiffened elements. If such a column is loaded to failure it will be found that these particular unstiffened elements develop rapidly increasing buckling waves at loads which are satisfactorily predicted by the above method. The column continues to resist increasing loads, however, since the major portion of its area consists of elements which are much more stable than those which have buckled. The method of

Section 3.6.1.1 does not account for the excess strength of these sections because it is intended to provide adequate safety not only against actual collapse but also against prohibitively large local distortions, even though these may not result in immediate collapse. In general, members of such shape, which incorporate one or two unstiffened elements with large w/t do not represent good, i.e., economical design. They can occur if a member is used primarily for other purposes (such as facing of a wall corner), but is also called upon to resist some small loads. In such cases prevention of distortion under load is an important consideration, and is adequately provided for by *Section 3.6.1.1*.

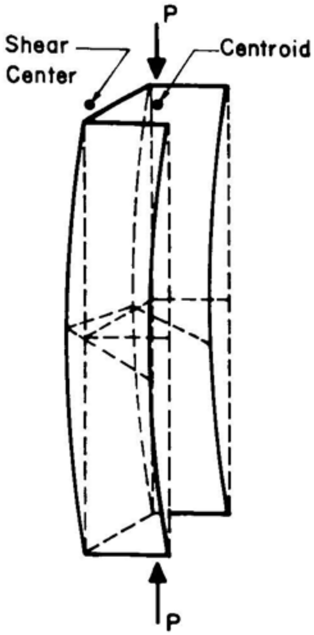
Design Charts

The explicit equations of *Section 3.6.1.1* are easily plotted in a practical form. Thus, the upper curve in Figure D.1 shows, for steel having a minimum yield point of 33 ksi, Eqs. D.4 and D.5 which are identical with those of *Section 3.6.1.1* for $Q = 1$ (no local buckling). On the same figure two other curves are shown for $Q = 0.8$ and 0.4. It is seen that the influence of local buckling on column strength is very pronounced for relatively small KL/r, but decreases rapidly for high values of KL/r. *Chart 3.6.1.1(B)* of the *Manual* gives a family of such curves, for values of QF_y from 6 to 62 from which, with sufficient accuracy, the allowable stress F_{a1} can be read for any combination of KL/r and QF_y.

(d) Torsional-Flexural Buckling

Centrally loaded columns can buckle by bending in one of the principal planes; or by twisting about the shear center; or by simultaneous bending and twisting. As was indicated previously herein (see D.1) members can, but need not, buckle torsional-flexurally if they are thin-walled, of open section, and if their shear center and centroid do not coincide (e.g. channels, C- and hat-sections, angles, I-sections with unequal flanges). This type of buckling, for a channel stud with both ends prevented from moving, is shown on Fig. D.5. Such torsional-flexural buckling can occur at loads well below those which would cause simple flexural buckling.

It should be strongly emphasized that one needs to design for torsional-flexural buckling only when it is physically possible for such buckling to occur. This means that if a member is so connected to other parts of the structure such as wall sheathing that it can only bend but cannot twist, it needs to be designed for flexural buckling only. This may hold for the entire member or for individual parts. For instance, a channel member in a wall or the chord of a roof truss is easily connected to girts or purlins in a manner which prevents twisting at these connection points. In this case torsional-flexural buckling needs to be checked only for the sub-lengths between such connections. Likewise, a doubly-symmetrical compression member can be made up by connecting two spaced channels at intervals by batten plates. In this case each channel constitutes an "intermittently fastened singly-symmetric component of a built-up shape" in the sense of *Section 3.6.1.2* and *3.6.1.3*. Here the entire member, being doubly-
3.6.1.2 symmetrical, is not subject to torsional-flexural buckling so that this mode
3.6.1.3 needs to be checked only for the individual component channels between batten connections.



Torsional-Flexural Buckling of a Channel in Axial Compression
Fig. D.5

Refs. D.3 and D.4 show that the elastic, concentric torsional-flexural buckling load of a column of singly-symmetrical shape (such as those enumerated above) can be found from the equation

$$P_{TFO} = \frac{1}{2\beta} \left[(P_x + P_\phi) - \sqrt{(P_x + P_\phi)^2 - 4\beta P_x P_\phi}\right] \tag{D.12}$$

If both sides of this equation are divided by the cross-sectional area A, one obtains the equation for the elastic, torsional-flexural buckling stress σ_{TFO} given in *Section 3.6.1.2(a)* of the *Specification:* 3.6.1.2(a)

$$\sigma_{TFO} = \frac{1}{2\beta} \left[(\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4\beta\sigma_{ex}\sigma_t}\right] \tag{D.13}$$

For this equation, as in all provisions which deal with torsional-flexural buckling, the x-axis is the axis of symmetry; $\sigma_{ex} = \pi^2 E/(KL/r_x)^2$ is the flexural Euler buckling stress about the x-axis (see Eq. D.3) and β and σ_t are torsional quantities which are defined in *Section 3.6.1.2(a)*. It is worth noting that the torsional-flexural buckling stress σ_{TFO} is always lower than the Euler stress σ_{ex} for flexural buckling about the symmetry axis. Hence, for these singly-symmetrical sections flexural buckling can only occur, if at all, about the y-axis which is the principal axis perpendicular to the axis of symmetry.

If buckling occurs elastically, i.e., if σ_{TFO} is in the straight-line portion of the stress-strain curve, then the allowable stress is simply obtained by dividing the buckling stress by the safety factor 23/12, i.e.

$$F_{a2} = 12\sigma_{TFO}/23 = 0.522\,\sigma_{TFO} \qquad \text{(D.14)}$$

3.6.1.2 as given in *Section 3.6.1.2*. If buckling occurs at higher stresses in the curved, inelastic portion of the stress-strain curve, then theory and tests given in Ref. D.4 have shown that in this range the same type of parabolic expression (as exemplified by the curve Q = 1 in Fig. D.1) can be used for torsional-flexural as for flexural buckling. Using the same safety factor, 1.92, this gives

$$F_{a2} = 0.522\,F_y - \frac{F_y^2}{7.67\,\sigma_{TFO}} \qquad \text{(D.15)}$$

also as given in *Section 3.6.1.2*.

3.6.1.2(a) *Inspection of Section 3.6.1.2(a) will show that in order to calculate β and σ_t* for use in Eq. D.13 it is necessary to determine x_o = distance between shear center and centroid, J = St. Venant torsion constant, and C_w = warping constant, in addition to several other, more familiar cross-sectional properties. Because of these complexities, the calculation of the torsional-flexural buckling stress cannot be made as simple as that for flexural buckling. However, a variety of design aids, given in *Parts II* and *IV* of the *Manual,* considerably simplify these calculations at least for the most common cold-formed shapes.

For one thing, any singly-symmetrical shape can buckle either flexurally about the y-axis or torsional-flexurally, depending on its detailed dimensions. For instance, a channel stud with narrow flanges and wide web will generally buckle flexurally about the y-axis (axis paralled to web); in contrast a channel stud with wide flanges and a narrow web will generally fail in torsional-flexural buckling. One can determine the mode which governs by calculating F_{a1} for flexural buckling from *Section 3.6.1.1* and F_{a2} for torsional-flexural buckling from *Section 3.6.1.2;* the smaller of the two will govern design. This cumbersome method is avoided by using the diagrams in *Part IV*. These were developed for common shapes in Refs. D.3 and D.4; they permit one to determine which of the two buckling modes governs, depending on simple combinations of the cross-sectional dimensions and the length of the member. If the pertinent diagram indicates that the member buckles flexurally about the y-axis, then *Section 3.6.1.1* governs and the entire complex calculation by *Section 3.6.1.2* can be omitted.

On the other hand, if torsional-flexural buckling is indicated, the supplementary information and design aids in *Parts II* and *IV* of the *Manual* facilitate and expedite the necessary calculations according to *Section 3.6.1.2*.

For *nonsymmetric open shapes* the analysis for torsional-flexural buckling becomes extremely tedious unless its need is sufficiently frequent to warrant computerization. For one thing, instead of the quadratic equations D.12 and D.13, cubic equations have to be solved. For another, the calculation of the required section properties, particularly C_w, becomes quite complex. The

3.6.1.2 method of calculation is outlined in *Part II* of the *Manual*. *Section 3.6.1.2* of the *Specification* provides that calculation according to this section shall be used or tests according to *Section 6* shall be made when dealing with nonsymmetrical open shapes.

All that has been said so far refers to members subject to torsional-flexural buckling, but made up of elements whose w/t ratios are small enough so that no local buckling will occur. In terms of the *Specification* this means that what has been said refers to members with Q = 1 (see *Section 3.6.1.1* of the *Specification,* and D.2(b) in this Commentary). For shapes which are sufficiently thin, i.e. with w/t ratios sufficiently large, local buckling can combine with torsional-flexural buckling similar to the combination of local with flexural buckling which was discussed in D.2(c).

An accurate design method which would take account of this combination would be entirely too complex, apart from the fact that adequate research information to date is not available. *Section 3.6.1.3* of the *Specification* prescribes a method of dealing with simultaneous local and torsional-flexural buckling which is known to be conservative, possibly excessively so. The method is entirely analogous to that described in D.2(c) for flexural buckling and thus needs no further explanation. For the case at hand it is definitely conservative because local and torsional-flexural buckling are basically interrelated and, therefore, accounting for the former separately by means of Q means that portions of both effects are accounted for twice. For instance, in a short, thin-walled, equal-leg angle strut, local and torsional-flexural buckling are almost identical; local buckling will produce in each leg a single buckling half-wave from end to end, but since both waves will occur in the same direction (maintaining the right angle at the joint) the entire angle, by the same token, will twist. Hence, in this particular case, local and torsional buckling become almost indistinguishable and accounting for the first by Q and for the second by Eqs. D.14 and D.15, as stipulated in *Section 3.6.1.3,* almost means accounting for the same effect twice. For shapes other than equal-leg angles the overlap is less pronounced, but it is present. 3.6.1.3

Thus, the fact that *Section 3.6.1.3* is known to be conservative is the reason for stipulating that, as an alternate to the Q-method, the combined buckling strength may be ascertained by test.

From the viewpoint of design economy, the use of shapes with Q substantially less than one will be found highly uneconomical. This is so because using shapes which are subject to torsional-flexural buckling by itself results in poorer economy than using shapes which will fail only flexurally. The added weakening produced by local buckling will further reduce economy. It is suggested, therefore, that in connection with torsional-flexural buckling the use of sections with Q less than, say, 0.8 be avoided by substituting other, more favorable shapes. Within this range of Q from 0.8 to 1.0, on the other hand, the conservatism built into *Section 3.6.1.3* is of limited economic consequence, if any.

(e) Torsional Buckling

It has been pointed out under D.1 above, that purely torsional buckling, i.e. failure by sudden twist without concurrent bending, is also possible for certain thin-walled open shapes. These are all point-symmetrical shapes (in which shear center and centroid coincide), such as doubly-symmetrical I-shapes, anti-symmetrical Z-shapes, and such unusual sections as cruciforms, swastikas, and the like. Under concentric load, torsional buckling of such shapes very rarely governs design. This is so because most such members of realistic slenderness will buckle flexurally or by a combination of flexural and local buckling, at loads smaller than those which would produce torsional buckling. This is why the *Specification* makes no provision for checking torsional buckling.

However, for relatively short members of this type, carefully dimensioned to minimize local buckling, such torsional buckling cannot be completely ruled out. If such buckling is elastic, it occurs at the stress σ_t defined in *Section 3.6.1.2(a)*. From this, allowable stresses can be derived in the same manner as
3.6.1.2 was done for torsional-flexural buckling in *Section 3.6.1.2*. This leads to the following allowable stresses F_{at} for purely torsional buckling of point-symmetrical shapes:

$$\sigma_t > \tfrac{1}{2} F_y: \qquad F_{at} = 0.522\, F_y - F_y^2/(7.67\sigma_t) \tag{D.16}$$

$$\sigma_t \leq \tfrac{1}{2} F_y: \qquad F_{at} = 0.522\sigma_t \tag{D.17}$$

where F_y and σ_t are defined in *Sections 3.6.1.1 and 3.6.1.2*.

Of course, it must also be ascertained that such sections do not buckle flexurally, i.e. their average axial stress P/A, likewise, shall not exceed F_{a1} as specified in *Section 3.6.1.1*. In fact, as was indicated, the latter requirement will almost always be found to govern.

3. WALL STUDS

Cold-formed steel studs in walls or load-carrying partitions are often employed in a manner foreign to heavy steel framing, but which has been used consistently in timber framing of residential and other light construction. Such studs are faced on both sides by a variety of wall materials such as fiber board, pulp board, plywood, gypsum board, etc. While it is the main function of such wall sheathing to constitute the actual outer and inner wall surfaces and to provide the necessary insulation, they also serve as bracing for the wall studs. The latter, usually of simple or modified I- or channel-shape with webs placed perpendicular to the wall surface, would buckle about their minor axes, i.e., in the direction of the wall, at prohibitively low loads. They are prevented from doing so by the lateral restraint against deflection in the direction of the wall provided by the wall sheathing. If this lateral support is correctly designed, such studs, if loaded to destruction, will fail by buckling out of the wall; since this buckling, then, occurs about the major axis, the corresponding buckling load obviously represents the highest load which the stud can reach. The wall sheathing, therefore, contributes to the structural economy by substantially increasing the usable strength of the studs.

5.1 *Section 5.1* formulates the necessary requirements in order to assure that the wall sheathing provides the lateral support necessary for the described optimum functioning of the studs. The provisions of *Section 5.1* are almost entirely based on Ref. D.7, which utilizes the result of 102 tests on studs (mostly with lateral bracing), of 24 tests on a variety of wall materials, and of detailed theoretical analysis, to arrive at appropriate design requirements.

In order that collateral wall material furnish the necessary support to the studs to which it is attached, the assembly (studs, wall sheathing, and connections or attachments between the two) must satisfy three requirements: (1) The spacing between attachments (screws, nails, clips, etc.) must be close enough to prevent the stud from buckling in the direction of the wall between attachments. (2) The wall material must be rigid enough to minimize deflection of the studs in the direction of the wall which, if excessive, could lead to failure in one of two ways; (a) the entire stud could buckle in the direction of the wall in a manner which would carry the wall material with it, and (b) it could fail simply by being overstressed in bending due to excessive lateral deflection. (3) The strength of the connection between wall material and stud must be sufficient to develop a lateral force capable of resisting the buckling tendency of the stud without failure of the attachment proper, by tearing, loosening, or otherwise.

The first of these conditions is satisfied by the second requirement of provision (b) of *Section 5.1*. This stipulates that the slenderness ratio a/r_2 for minor-axis buckling between attachments (i.e., in the direction of the wall) shall not exceed one-half of the slenderness ratio L/r_1 for major-axis buckling, i.e., out of the wall. This means that with proper functioning of attachments, buckling out of the wall will always occur at a load considerably below that which would cause the stud to buckle laterally between attachments. Even in the unlikely case that an attachment were defective to a degree which would make it completely inoperative, the buckling load would still be the same for both directions (i.e., $a/r_2 = L/r_1$).

In regard to requirement (2), the rigidity of the wall material plus attachments is expressed as its modulus of elastic support, k_w, i.e., the ratio of the applied force to the stretch produced by it in the sheathing-attachment assembly. The method for determining the actual value of k_w for any given assembly (sheathing, means of connection, and stud) is given in *Part II* of the *Manual*.

Section 5.1(c) specifies the minimum modulus k_w which must be furnished by the collateral material in order to satisfy requirement (2), above, i.e., to prevent excessive "give" of the stud in the direction of the wall. This requirement, in the form of the equation in *Section 5.1(c)*, is identical with Equation 15 of Ref. D.7. The latter defines the minimum rigidity (or modulus k_w) which is required to prevent the lateral buckling of a stud which is loaded by $P = AF_y$, i.e., is stressed right up to the yield point of the steel. On the other hand, the maximum load permitted on a stud by *Section 3.6.1* is $P = AF_{a1}$. It is seen from Eq. D.4, that even for very short studs (i.e., $L/r \rightarrow 0$) $F_{a1} = P/A$ can not exceed $F_y/1.92 = 0.522\, F_y$. *Section 5.1(c)*, then, specifies the modulus required to safeguard the stud from lateral buckling under a load at least equal to 1.92 times the design load; in other words, it contains a safety factor of 1.92 for short studs, and an increasingly larger factor for larger L/r ratios. These relatively large factors are justified because the specified value of k_w is for an ideally straight and concentrically loaded stud. It is easily shown (Ref. D.6) that if studs are initially crooked (as is practically inevitable, at least to some degree), the required value of k_w exceeds the one which is necessary under "ideal" conditions, the more so the larger the initial deviation from straightness. The sliding safety factor incorporated in *Section 5.1(c),* which increases with increasing L/r, takes account of this situation.

It is seen from *Section 5.1(c)* that the required modulus of support k_w is directly proportional to the spacing of attachments, a. As a rule, the value of a will be selected on the basis of the second provision of *Section 5.1(b),* (i.e., a $\leq$ L r_2/ 2 r_1) and the next one will determine whether the actual test value of k_w exceeds the minimum required for that particular value of a in *Section 5.1(c).* For most normal combinations of materials and dimensions, this will be the case. However, should the actual magnitude of k_w fall below the required value, it is then necessary to reduce the spacing a accordingly. The spacing which is required in this case is that given in the first of the two requirements of *Section 5.1(b).* This formula for a_{max}, evidently, is nothing but the formula for k_w in *Section 5.1(c),* solved to give a for a given k_w. In this manner requirements (1), for spacing, and (2), for rigidity, above, are seen to be to some degree interdependent.

It remains to satisfy requirement (3), above, to the effect that the strength of the attachment of wall material to the stud must be sufficient to permit the stud to develop its maximum load carrying capacity. This is achieved by means of *Provision (d)* of *Section 5.1.*

Theory indicates that an ideal (straight, concentric) stud which is elastically supported at intermediate points (such as by wall attachments) will not exert any force on these attachments until it reaches its buckling load. In contrast, analysis and test indicate that intermediately supported "real," i.e., imperfect studs (crooked, eccentric) do exert pressure on their supports, increasingly so as the load on the stud is increased. Accordingly, design requirements must be based on a reasonable amount of assumed imperfection. The formula for the required minimum strength of attachment, P_{min}, in *Section 5.1(d)* is based on Equation 17 of Ref. D.7. That equation, in turn, expresses the strength of support required for a stud which has an initial crookedness and/or load eccentricity. In the *Specification,* a crookedness tolerance of stud length/480 has been assumed.

It will be noted in *Section 5.1(d)* that the value of eccentricity assumed there is equal to stud length/240, rather than stud length/480. By this means a safety factor of two (2) is incorporated in the formula for P_{min}, if it is assumed that initial crookedness is the only imperfection which affects the eccentricity. However, this assumption is not always justified. In fact, load eccentricity affects the required value of P_{min} in much the same way as initial crookedness. In cases where imperfections happen to be so arranged that the load eccentricity is in the same direction as the initial crookedness, the effects of these two influences are additive in regard to the required strength of attachment. To take care of this possibility, the safety factor has been increased over the value of two (2) indicated above. Comparison of the formula for P_{min} in *Section 5.1(d)* with Equation 17 of Ref. D.7 shows that this has been accomplished by omitting the factor 2 (two) under the radical in the denominator. In consequence, when the first term in the denominator far exceeds the second, (as is almost always the case), an additional safety factor of $\sqrt{2} = 1.41$ has been incorporated. In the relatively rare cases where the first term is not very much larger than the second, the additional factor so incorporated is even larger. It is seen, then, that the requirement for P_{min} contains an overall safety factor (including the effect of accidental eccentricity) of at least 2 x 1.41 = 2.82. This factor is somewhat higher than the overall safety factor of 1.92 in the column formulas (see D.1(a) above) in order to account for the fact that connections of two unlike materials, such as achieved by the attachments under consideration, are likely to contain some element of uncertainty not present in the design of a single individual member, such as a column.

4. I- OR BOX-SHAPED COMPRESSION MEMBERS MADE BY CONNECTING TWO CHANNELS

The only two-flanged shapes which can be cold-formed from a single sheet without welding are Channels or Zee's, without or with lips. Except for light loads, I-shaped sections are often preferable for compression members. In cold-formed construction these can be produced by connecting two channels back to back.

For two connected channels to function as a single compression member it is necessary to make the longitudinal spacing between connections (e.g. spot welds) close enough to prevent the component channels from buckling individually about their own axes parallel to the web at a load smaller than that at which the entire compression member would buckle. This requirement is similar to the first of the requirements discussed for wall studs in D.3. Just as in that case, in order to satisfy this requirement *Section 4.3(a)* stipulates that 4.3(a)
the slenderness ratio of the individual channel between welds or other connectors, s_{max}/r_{cy} be not larger than one-half of the pertinent slenderness ratio L/r_I of the entire compression member.

The new edition of the *Specification* is more specific than previous editions in defining what this pertinent slenderness ratio is, by specifying more closely the definition of r_I. This is "the radius of gyration of the I-section about the axis perpendicular to the direction in which buckling would occur for the given conditions of end support and intermediate bracing, if any." For a free-standing I-section stud, for instance, buckling would undoubtedly occur about the minor axis, i.e., the axis parallel to the web; hence, in the case r_I is the radius of gyration relative to this axis. However, if the same stud is part of a wall and so designed that the wall sheathing prevents buckling about the minor axis, then, if the stud were loaded to failure, it would buckle perpendicular to the wall. Such buckling would then occur about the major axis of the I-section, and r_I is to be taken about that axis.

The literal application of this provision implies that the stud is specifically designed for one given application, and that the connection spacing is correspondingly determined. For standard, mass-produced shapes this will frequently not be the case. In this situation a conservative approach must be taken. That is, r_I must be the radius of gyration about the minor axis. Furthermore, since the spacing s_{max} also depends on the length of the compression member, and since this length will not be known for such standardized shapes, the only way is to anticipate conservatively the shortest length for which the given shape is likely to be used, and to take this length for L in *Section 4.3(a)*.

Compression members can also be made by connecting two channels tip-to-tip to form a box shape. Lipped channels facilitate fabrication of such shapes by welding. Although the *Specification* does not explicity say so, it is clear that *Section 4.3(a)* also applies to this case without change, provided r_1 is defined as the larger of the two radii of gyration of the box-shaped section.

E. FLEXURAL MEMBERS

1. LATERAL BUCKLING

(a) I-Shaped Beams

If an equal-flanged I-beam of length L is laterally unsupported (unbraced), it may fail in lateral, torsional-flexural buckling. The type of deformation which occurs, twisting and simultaneous transverse bending, is shown on Fig. E.8(a) in *Section E.3,* below. *Section 3.3* contains provisions to safeguard against such failures. In the elastic range, the maximum fiber stress at which such buckling occurs (Ref. E.1) can be written as 3.3

$$\sigma_{cr} = \frac{E\pi^2}{2(L/d)^2}\sqrt{\left(\frac{I_y}{2I_x}\right)^2 + \left(\frac{J\,I_y}{2(1+\mu)\,I_x{}^2}\right)\left(\frac{L}{\pi d}\right)^2} \qquad \text{(E.1)}$$

In this equation d is the depth, and J the torsional constant of the section; μ is Poisson's ratio, and the other terms have their usual meaning. It has been shown (Ref. E.2) that this same equation is a reasonable and generally conservative approximation for most unusual kinds of loadings. (It should be noted that Eq. E.1 is merely a simple transformation of the long established equation for the critical moment of an I-beam in pure bending; see e.g., Refs C.7, D.1.) A permissible design stress could, therefore, be obtained by dividing the buckling stress of Eq. E.1 by a safety factor. However, this formula is generally regarded as too unwieldly for routine design use. For this reason various approximate simpler formulas have long been in use for laterally unbraced beams. For example, for hot-rolled construction the first term under the radical is often negligible as compared with the second term. For this situation, corresponding design provisions are then based on eliminating that term.

Conversely, it is shown in Ref. E.1 that for cold-formed, thin-walled sections of ordinary dimensions the first term under the square root in Eq. E.1 usually considerably exceeds the second. This first term expresses the portion of the lateral strength due to the lateral bending rigidity of the beam (i.e., bending about the axis through the web).

In previous editions of the *Specification,* greatly simplified design provisions have been derived from Eq. E.1 by omitting the second term under the square root and introducing further approximations which simplified the final formula at the price of sacrificing some accuracy and economy. This feature of lateral buckling of beams has been re-studied and re-formulated in both the *AISI* and the AISC (Ref. C.6) *Specifications,* particularly in regard to I-shaped beams with unequal flanges. Such beams occur with increasing frequency in both types of construction.

The writer has analyzed beams with unequal flanges (Ref. E.3) and derived an equation for the buckling stress of such beams which (Eq. 4.32 of Ref. D.1) can be written as

$$\sigma_{cr} = \frac{\pi^2 Ed}{2 S_{xc} L^2} \left(I_{yc} - I_{yt} + I_y \sqrt{1 + \frac{4 G J L^2}{\pi^2 I_y E d^2}} \right) \tag{E.2}$$

where S_{xc} = section modulus relative to compression fiber; I_{yc} and I_{yt} = moments of inertia, about center line of web, of compression and tension portion of section, respectively; G = shear modulus of elasticity. It is easily verified that for equal-flange I-beams, where $I_{yc} = I_{yt} = I_y/2$, Eq. E.2 reduces to Eq. E.1. As has been explained, for thin-walled beams the second term under the square root, which represents the contribution of the St. Venant torsion stiffness, can be conservatively neglected without much loss in economy. Then, considering that $I_y = I_{yc} + I_{yt}$, a conservative simplification of Eq. E.2 is

$$\sigma_{cr} = \frac{\pi^2 E d I_{yc}}{L^2 S_{xc}} \tag{E.3}$$

In this form the equation applies, strictly, to the case of uniform bending (M = constant throughout the span). It is quite accurate for other loadings such as uniform load or equal quarter-point loads, but becomes excessively conservative for the case of unequal end moments, particularly if they are opposite in direction. This can be rectified by multiplying the right-hand side by a factor C_b which depends on the ratio of the end moments (see Eq. 4.13 of Ref. D.1) and which is the same as used in the AISC Specification. If this is done, and a safety factor of 1.67 = 1/0.6 applied, one obtains the basic allowable stress
3.3(a) formula in *Section 3.3(a):*

$$F_b = 0.6 \pi^2 E C_b \frac{d I_{yc}}{L^2 S_{xc}} \tag{E.4}$$

As explained, this formula applies to unsymmetrical I-sections as well as symmetrical I and channel sections and hence is an improvement over the 1962 edition which only applied to the symmetrical cases. It may be noted that for plate girders consisting of a web plate and two unequal flange plates, such as they are universally used in hot-rolled steel construction, the term dI_{yc}/S_{xc} is very closely equal to the square of the radius of gyration about the web axis of the compression flange plus the adjacent one-third of the compression part of the web. The AISC Specification (Ref. C.6) for lateral buckling is written in terms of this particular radius of gyration. However, for cold-formed I-shapes, with rounded corners, lips on one or two flanges, and other possible variations of shape, this approximation is not always satisfactory, which accounts for the use of dI_{yc}/S_{xc} in the *AISI Specification.*

Just as in the case of compression members (see *Section D.1(a)* of this Commentary), Eq. E.2 as well as the simplified Eq. E.3 derived from it, is valid only as long as σ_{cr} is below the proportional limit. If this is not the case, the value of the effective modulus is smaller than the elastic modulus E, the more so the shorter the beam (see B.1 of this Commentary). Correspondingly, in this inelastic range the buckling stress becomes a progressively smaller fraction of σ_{cr} the smaller the slenderness. Just as in the case of compression members, the relatively wide range over which the shape of the stress-strain curve can vary makes it impossible to account for this influence rigorously (see *D.1(a)*).

To reflect this situation, *Section 3.3(a)*, similarly to *Section 3.6*, specifies a certain limiting value $L^2S_{xc}/dI_{yc} = 1.8\ \pi^2EC_b/F_y$ below which the allowable stress is to be computed from the first of the two formulas for F_b in *Section 3.3*. This is seen to be similar in form to the corresponding inelastic equation for compression members, i.e. the first of two formulas in *Section 3.6.1.1*, which was explained in terms of Eq. D.2, herein. When the stress given by the first formula in *Section 3.3(a)* becomes larger than the stress $F = 0.6F_y$ (which occurs at $L^2S_{xc}/dI_{yc} = 0.36\pi^2EC_b/F_y$), the basic design stress F, of course, must be used in design. The entire situation is pictured in Fig. E.1. It is seen that in the low range of slenderness, i.e. for beams with relatively small unbraced lengths, any weakening effect of possible lateral buckling is so small as to be negligible, particularly because the design provisions have been derived by approximations which are on the conservative side, as has been discussed. These beams, then, are designed for the unreduced stress F. Only when the slenderness of the unbraced length exceeds the indicated values, reduced allowable stresses must be used.

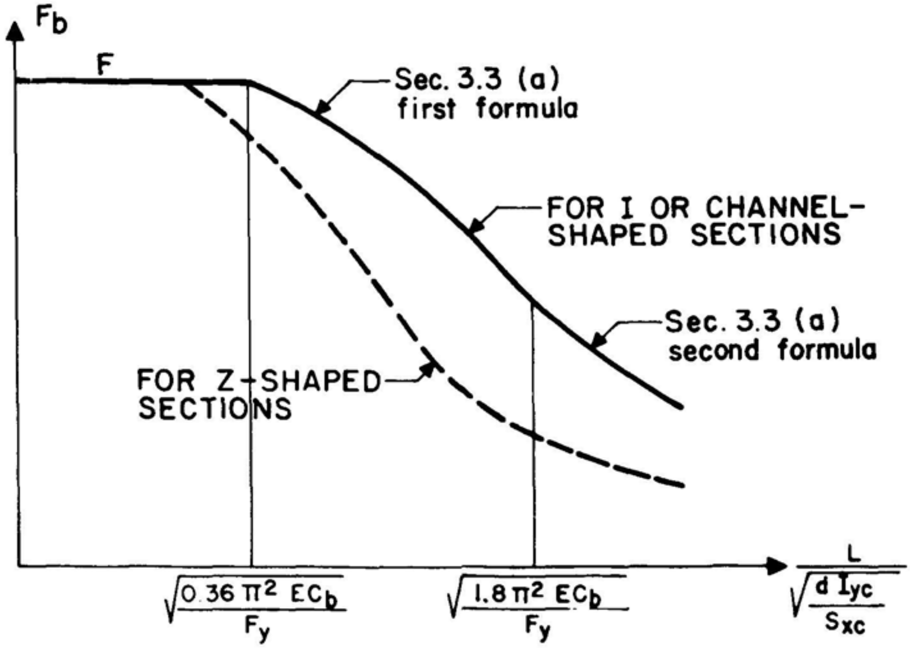


Fig. E.1

(b) Channel- and Z-Shaped Beams

Channels and Z-beams, when loaded in the plane of the web, twist and deflect laterally on account of their asymmetry, unless appropriately braced. Provisions governing the spacing of such braces are given in *Section 5.2* of the *Specification*. In general, braces located according to these provisions will be at sufficiently close intervals to prevent lateral buckling of the beams between braces. It is possible, however, that beams of unusually large span/width ratios would be liable to buckling between braces. Therefore, even though brace
5.2 location will usually be governed by *Section 5.2*, it is necessary to set an upper limit for distance between bracing, or to reduce the stress correspondingly so
5.2.3 as to prevent possible buckling between braces. This is indicated in *Section 5.2.3*
3.3 *and is achieved by means of those provisions of Section 3.3 which apply to* channels and Z-shapes.

It has been shown by H. N. Hill (Ref. E.4) that Equation E.1 applies to channel beams without change, as a very satisfactory approximation. For this
3.3(a) reason, in *Section 3.3(a)* the same formula is listed for channels as for I-shapes. From the same paper *(Ref. E.4)* it can be shown that if a channel and a Z-beam have the same slenderness ratio, the Z-beam will buckle at a lower stress, the amount of difference varying, depending on details of shape. In view of the
3.3(b) fact that *Section 5.2* rather than *Section 3.3(b)* will usually govern bracing of Z-beams (see above), and also in view of the fact that the tendency of a Z-beam to deflect slightly even between braces lowers its buckling strength, no special elaborate formulas seemed warranted, but a rather conservative approach seemed indicated. For this reason *Section 3.3(b)* specifies the allowable stress of a Z-beam as one half of that of a channel or I-beam of the same slenderness ratio *when* L^2S_{xc}/dI_{yc} exceeds $0.9\pi^2EC_b/F_y$, *with corresponding transitional* values for smaller slenderness ratios.

The resulting relations are also graphed in Fig. E.1 and *Charts 3.3 of the Design Manual.*

(c) Box- and Hat-Shaped Beams

3.3 It will be noted that the requirements of *Section 3.3* are specifically restricted to single-web beams. This is so because Equation E.2 from which these requirements have been derived, applies only to single-web sections. However, two-web sections, such as box, hat, or U-shapes, are incomparably more stable laterally than single-web sections (of the same depth/width ratio). In situations where lateral stability is essential, such two-web sections are, therefore, decidedly preferable.

5.3 In previous editions of the *Specification, Section 5.3,* stipulated that closed *box-type sections can be used as beams with length/width-ratios up to 75* without any stress reduction for lateral buckling. Even though the latter is not explicitly stated in *Section 5.3*, this is the intent of that section and is made clear by the parenthetical phrase in *Section 3.3* which specifically excludes box-shaped members from the restrictions of that section. The justification for this treatment of box-shaped members can be found in Refs. D.1 and E.2. In particular, Figure 4 of Ref. E.2 shows that even for a box-beam of unusually unfavorable dimensions (extremely large depth/width ratio, see Figure 3 of Ref. E.2) the failure stress is practically unaffected by lateral buckling up to L/b as high as 100.

This information was developed for steels of moderate yield points, of the order of 33 ksi. With steels of significantly greater strength having come into use both for hot-rolled and for cold-formed construction it became necessary to adjust this provision for these high strength steels. Correspondingly, the new editions of both the *AISI* and AISC *Specifications* contain identical provisions, as stated in *Section 5.3,* to the effect that for closed, box-shaped beams bent about the major axis, the laterally unsupported length shall not exceed $2{,}500/F_y$ times the distance between webs. For $F_y = 33$ ksi, this works out to 75.8 times the distance between webs, practically identical with the previous provisions, but for higher-strength steel the unbraced length must now be correspondingly reduced.

No simple information on hat sections has been developed to date. This is the reason why the Specification does not contain any provisions on unbraced hat sections, even though such sections are particularly favorable when used without intermediate bracing. Hat sections used as beams are more stable against lateral buckling when the closed side of the hat is in compression. Let the y-axis be the axis of symmetry, and let bending be applied about the x-axis. Then the following can be said, conservatively, about using unbraced hat sections: (a) For any hat section the I_y of which is equal to or exceeds I_x, no stress reduction for lateral buckling is necessary, no matter what the length/width ratio. This is so because, regardless of shape, only beams bent about the "strong axis" show any tendency for lateral buckling; this tendency can be described as a desire of the beam to flip over into its weak position. Evidently, if $I_y \geq I_x$, there is no such tendency. Inspection will show that the majority of the hat-sections tabulated in the *Manual* fall into that category. (b) For hat sections where the reverse is true ($I_y < I_x$), it is a safe procedure to determine the allowable stress from the formula

$$F_b = \frac{151{,}900}{(L/r_y)^2}$$

Comparison with *Section 3.6* shows that this is the formula for slender columns. In applying it to hat section beams, r_y is the radius of gyration about the vertical axis of that portion of the hat section which is in compression. This procedure is justified and conservative because the lateral stability of any beam is greater than the buckling strength which its compression portion would have if it were separated from the tension portion and loaded as a column. This is so because this portion, being in tension, tends to stay straight and thus has a stabilizing influence on the compression portion. (For an illustration see Figure 4 of Ref. E.2).

If desired, more accurate allowable stresses for any beam of singly symmetrical section loaded in the plane of symmetry can be obtained from:

$$\sigma_{cr} > 0.545\, F_y: \qquad F_b = 2F_y/3 - F_y^2/(5.4\sigma_{cr})$$

$$\sigma_{cr} \leq 0.545\, F_y: \qquad F_b = 0.6\sigma_{cr}$$

where σ_{cr} stands for σ_{bT} or σ_{bC}, as applicable, to be determined as specified in *Section 3.7.2*. (Evidently, F_b also cannot exceed the applicable allowable stress of *Section 3.1* or *3.2*.) These stipulations provide the same basic safety factor of 1.67 against lateral buckling as does *Section 3.3*.

2. CHANNEL AND Z-BEAMS

Among hot-rolled sections, I-shapes are most favorable for use as beams because a large portion of the material is located in the flanges, at the maximum distance from the axis. In cold-formed construction the only two-flange shapes which can be formed of one single sheet (without welding or other connecting) are the channel, the Z-shape, and the hat. Of these, the hat-shape has the advantage of symmetry about the vertical axis and of great lateral stability; its use is correspondingly increasing, but is hampered occasionally in view of the presence of two separate webs which pose problems of access, connection, etc.

Channels and Z-shapes continue to be widely used. Neither of them is symmetrical about a vertical plane. Since, in most applications, loads are applied in the plane of the web, lack of symmetry about the plane calls for special measures to forestall structurally undesirable performance (lateral deflection, twisting, etc.). The Specification contains provisions for the required bracing if applied to both tension and compression flanges.

(a) Connecting Two Channels to Form an I-Beam

There are various ways of connecting two or more cold-formed shapes to produce an I-section. One of these is by spot-welding an angle to each flange of a channel. Another is to connect two channels back to back by two rows of spot-welds (or other connectors) located as closely as possible to top and bottom flange. Provisions for the correct proportioning of the connecting welds for
4.3(b) such shapes are given in *Section 4.3(b)* of the Specification.

In view of lack of symmetry or anti-symmetry about a vertical plane, the so-called shear center of a channel is neither coincident with the centroid (as it is in symmetrical or anti-symmetrical shapes) nor is it located in the plane of the web. The shear center is that point in the plane of a beam section through

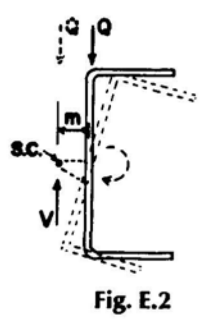


Fig. E.2

which a transverse load must act in order to produce bending without twisting. In a channel this point S.C. is located a distance m back of the midplane of the web, as shown in Figure E.2. The distance m for channels with and without flange lips is given in *Section 4.3(b)*. The internal shear force passes through this point. Consequently, if the external load Q were applied at the same point (such as by means of the dotted bracket in Fig. E.2) the two forces would be in line and simple bending would result. Since loads in most cases actually act in the plane of the web, each such load produces a twisting moment Qm. Unless these torques are balanced by some externally applied counter-torques, undesirable twisting will result.

If two channels are joined to form an I-beam, as shown in Fig. E.3(a), each of them is in the situation shown in Figure E.2 and tends to rotate in the sense indicated by the arrow on that figure. The channels, then, tend through rotation to separate along the top, but this tendency is counteracted by the forces in the connections joining them. These forces T_s, constitute an opposing couple; they are shown in Figure E.3(b) which represents a short portion of the right channel, of length equal to the connection spacing s. This portion, delimited by dotted lines in Figure E.3(a), contains a single pair of connections, and Q is the total force acting on that piece of one channel, i.e., half the total beam load over the length s. From the equality of moments

$$Qm = T_s g \quad \text{so that} \quad T_s = Q(m/g)$$

It is seen that the connection force T_s depends on the load acting in the particular connection interval s. If q is the intensity of load on the beam at the location of the particular connection, the load on one channel is $Q = qs/2$. Substituting this in the above equation, one has the maximum permissible connection spacing

$$s_{max} = \frac{2gT_s}{m\,q}$$

which is the formula of *Section 4.3(b)*. 4.3(b)

It is seen that the required connection strength depends on the local intensity of load on the beam at that connection. Generally, beams designed for "uniform load" actually are usually subjected to more or less uneven loads, such as from furniture, occupants, etc. It is, therefore, specified that for "uniformly loaded beams" the local load intensity q shall be taken as three times the uniform design load. "Concentrated" loads or reactions P are actually distributed over some bearing length N; if N is larger than the connection spacing s, then the local intensity is obviously P/N. If, on the other hand, the bearing length is smaller than the weld spacing, then the pair of connections nearest to the load or reaction must resist the entire torque (P/2)m, so that $T_s = Pm/2g$. This is how the appropriate connection strength T_s is specified in *Section 4.3* for this case.

The above requirements are adequate to insure the necessary strength of the connections. However, if for relatively light loading the spacing s_{max} assumes relatively large values, the strong twisting tendency may cause the two channels to distort excessively between connections, by separation along the top flange. For the case of channels placed individually and braced against each other, it is

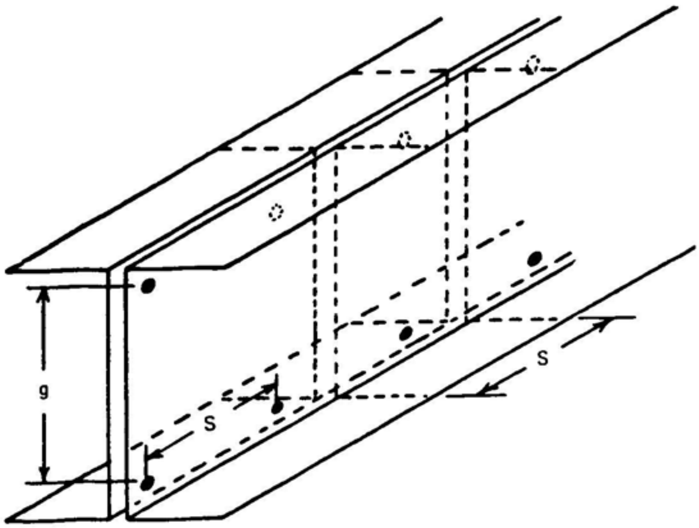


Fig. E.3 (a)

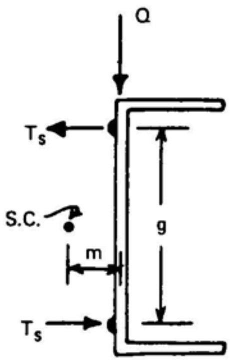


Fig. E.3 (b)

shown in E.2(b), below, that a maximum connection spacing of L/4 is adequate to safeguard against such deformation. In channels connected back to back, continuous contact along the bottom flange further counteracts such twist; for this reason a larger spacing, such as L/3 would be adequate. However, in conformity with the general approach described for compression members in D.3 and D.4, above, it was assumed that an occasional connection may be defective to the extent of being entirely inoperative. In this case a maximum spacing $s_{max} = L/6$ would still constitute an adequate safeguard, and this is how the limit $s_{max} = L/6$ was arrived at in *Section 4.3(b).*

(b) Bracing of Single-Channel Beams

If channels are used singly as beams, rather than being paired to form I-sections, they must evidently be braced at intervals so as to prevent them from rotating in the manner indicated in Figure E.2. Figure E.4, for simplicity, shows two channels braced at intervals against each other. The situation is evidently much the same as in the composite I-section of Figure E.3(a), except that the role of the connections is now played by the braces. The difference is that the two channels are not in contact, and that the spacing of braces is generally considerably larger than the connection spacing. In consequence, each channel may actually rotate very slightly between braces, and this will cause some additional stresses which superpose on the usual, simple bending stresses. Bracing must be so arranged that: (a) these additional stresses are small enough so that they will not reduce the carrying capacity of the channel (as compared to what it would be in the continuously braced condition); (b) rotations must be kept small enough to be unobjectionable (e.g., in regard to connecting other portions of the structure to the channels), of the order of 1 to 2 degrees.

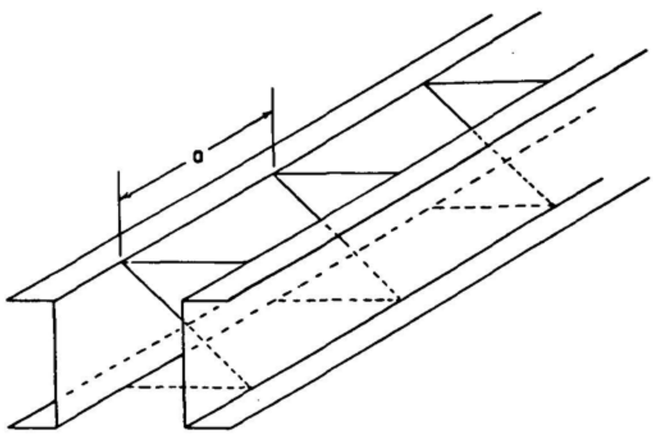


Fig. E.4

In order to develop information on which to base appropriate bracing provisions, seven different channel shapes have been tested. Each of these was tested with full, continuous bracing; without any bracing; and with intermediate bracing at two different spacings. In addition to this experimental work, an approximate method of analysis was developed and checked against the test results. A condensed account of this work is given in Ref. E.5. It is indicated in that reference that the above requirements are satisfied for most distributions of beam load if between supports not less than three equidistant braces are

placed (i.e., at quarter-points of the span, or closer). The exception is the case where a large part of the total load of the beam is concentrated over a short portion of the span; in this case an additional brace must be placed at such a
5.2.1 load. Correspondingly, *Section 5.2.1* provides that the distance between braces shall not be greater than one-quarter of the span; it also defines the conditions under which an additional brace must be placed at a load concentration.

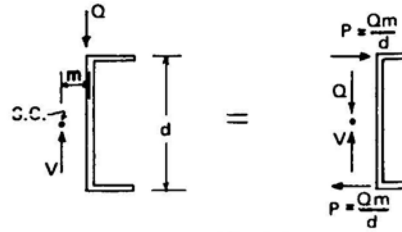


Fig. E.5

For such braces to be effective it is not only necessary that their spacing be appropriately limited; in addition, their strength must suffice to provide the force required to prevent the channel from rotating. It is, therefore, necessary also to determine the forces which will act in braces, such as those forces shown in Figure E.5. These forces are found if one considers that the action of a load applied in the plane of the web (which causes a torque Qm) is equivalent to that same load when applied at the shear center (where it causes no torque) plus two forces $P = Qm/d$ which, together, produce the same torque Qm. As is sketched in Figure E.6, and shown in some detail in Ref. E.5, each half of the channel can then be regarded as a continuous beam loaded by the horizontal forces f and supported at the brace points. The horizontal brace force is then, simply, the appropriate reaction of this continuous beam. The provisions of

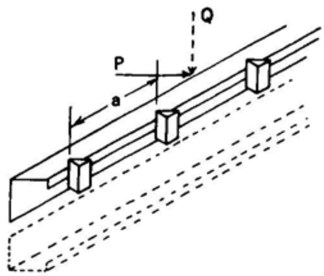


Fig. E.6

5.2.2 *Section 5.2.2* represent a simple and conservative approximation for determining these reactions, which are equal to the force P_L which the brace is required to resist at each flange.

(c) Bracing of Z-Beams

Most Z-sections are anti-symmetrical about the vertical and horizontal centroidal axes, i.e. they are point-symmetrical. In view of this, the centroid and the shear center coincide and are located at the midpoint of the web. A load applied in the plane of the web has, then, no lever arm about the shear center ($m = 0$) and does not tend to produce the kind of rotation a similar load would produce on a channel. However, in Z-sections the principal axes are oblique to the web (Figure E.7). A load applied in the plane of the web, resolved in the direction of the two axes, produces deflections in each of them. By projecting these deflections onto the horizontal and vertical planes it is found that a Z-beam loaded vertically in the plane of the web deflects not only vertically but also horizontally. If such deflection is permitted to occur then the loads, moving sideways with the beam, are no longer in the same plane with the reactions at the ends. In consequence, the loads produce a twisting moment about the line connecting the reactions. In this manner it is seen that a Z-beam, unbraced between ends and loaded in the plane of the web, deflects laterally and also twists. Not only are these deformations likely to interfere with a proper functioning of the beam, but the additional stresses caused by them produce failure at a load considerably lower than when the same beam is used fully braced.

In order to develop information on which to base appropriate bracing provisions, 19 tests have been carried out on three different Z-shapes, unbraced as well as with variously spaced intermediate braces. In addition, an approximate method of analysis has been developed and checked against the test results. An account of this is given in Ref. E.6. Briefly, it is shown there

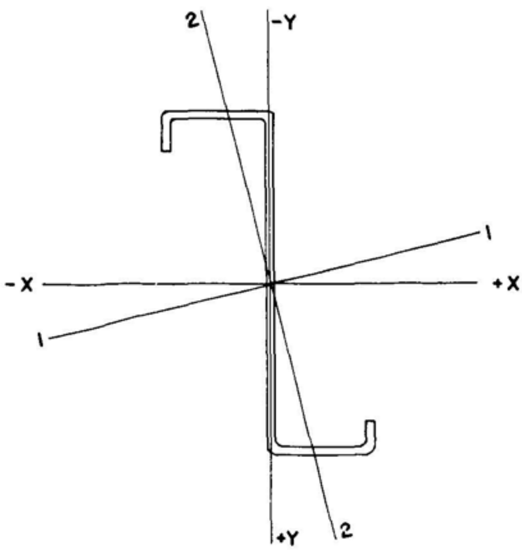


Fig. E.7

that intermittently braced Z-beams can be analyzed in much the same way as intermittently braced channels. It is merely necessary, at the point of each actual vertical load Q, to apply a fictitious horizontal load $P = Q(I_{xy}/I_y)$. One can then compute the vertical and horizontal deflections, and the corresponding stresses, in conventional ways by utilizing the convenient axes x and y (rather than 1 and 2, Figure E.7), except that certain modified section properties have to be used.

In this manner it has been shown that as to location of braces the same provisions which apply to channels are also adequate for Z-beams. Likewise, the forces in the braces are again obtained as the reactions of continuous beams horizontally loaded by fictitious loads P. It is in this manner that the
5.2 provisions applicable to bracing of Z-shaped beams in *Section 5.2* have been arrived at.

The following general observations may be appropriate: Since Z-shapes and channels are the simplest two-flange sections which can be produced by cold-forming, one is naturally inclined to use them as beams under vertical load. However, in view of their lack of symmetry, such beams require special measures to prevent tipping at the supports, as well as relatively heavy bracing to counteract lateral deflection and twisting in the span. Their use is indicated chiefly where continuous bracing exists, such as when they are incorporated in a rigid floor system, so that special intermittent bracing may be required during erection only. It is for this erection condition that *Section 5.2* may be chiefly useful. For conditions other than these, serious consideration should be given to hat sections. These have the same advantages as channel and Z-sections (two-flange section produced by simple cold-forming) but none of their disadvantages, and are, in fact, in some respects superior to I-sections (see E.1(c) above).

3. LATERALLY UNBRACED COMPRESSION FLANGES

The type of lateral buckling discussed in E.1 above, occurs in the manner shown in Fig. E.8(a). It consists of lateral deflection and rotation of the entire cross-section and occurs in thin open sections loaded so as to cause bending about the major axis and free to move in a direction parallel to that axis. Appropriate design provisions for this type of performance have been discussed. In thin-wall construction, however, another type of lateral buckling is possible and is of considerable practical interest. This type is illustrated in Fig. E.8(b). For the case shown there, the U-shaped beam is bent about the minor (horizontal) axis; this eliminates any tendency for the entire beam to buckle laterally. However, the two top flanges, being in compression, tend to behave like columns and to buckle individually as shown, unless lateral restraint prevents such motion. As indicated in Fig. E.8(b), if such buckling occurs, it is accompanied by distortion of the entire cross-section. That is, since the tension flange remains straight and does not displace laterally, the compression flanges can buckle only by causing the webs and, thereby, also the bottom flange to bend out-of-plane as shown.

Safety against this type of buckling must be provided in a considerable variety of practical situations, such as: hat sections when used in such a manner that the brims are in compression and are not restrained laterally; sheet-stiffener combinations loaded in bending in such a manner that the sheet is in tension and the unrestrained flanges of the stiffeners in compression; etc. In these and similar cases the only feature which prevents the compression flange from buckling laterally are the webs which connect them to the laterally stable tension flanges so that lateral buckling can occur only by means of the shape distortion illustrated in Fig. E.8(b). Whether or not safety against such

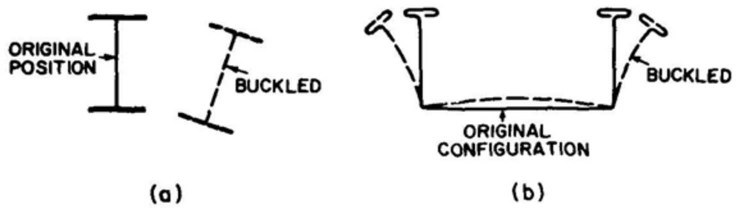


Fig. E.8

buckling is adequate depends, therefore, on whether or not the rigidity of the rest of the sections (webs, bottom flange) is sufficient to restrain the compression flanges from buckling.

A method for estimating the allowable compression stresses which will provide such safety is given in *Section 3* of *Part II, Supplementary Information* of the *Manual*. The method presented in that section is not a part of the *Specification*, that is, it is not mandatory. This procedure is based on a considerable simplification of the complex analysis of this type of elastic instability. The results have been checked against more than a hundred tests on beams with seven different configurations and ranging in thickness from 12 to 20 gage. Details of the analysis, the development of the design method, and a summary of test results are given in Ref. E.7.

In substance, the compression flange plus a part of the compression portion of the web act like a column on an elastic foundation. The elastic foundation which counteracts the tendency of this "equivalent column" to buckle laterally is provided by the out-of-plane bending stiffness of the web and bottom flange. A unit length of these connected elements can be regarded as a rigid frame which acts as a spring in furnishing elastic support to the compression flange. Step 1 of the design procedure defines the dimensions of the "equivalent column" and Step 3 indicates the way of calculating the spring constant of the elastic support.

If the top flange, when buckling, would only bend laterally, the above information would suffice to analyze it by well established theory as a column on an elastic foundation. However, as is seen from Fig. E.8(b), as it bends the flange also twists. The weakening influence of this torsional action is incorporated in the factors T and T_o in Steps 4 to 6 which give an approximate method for calculating the critical load P_{cr} of the "equivalent column." Step 7 defines that slenderness ratio which a simple Euler column would have in order to possess the same critical stress P_{cr}/A as the top flange under consideration. Knowing this slenderness ratio, one utilizes Section 3.6 of the Specification to 3.6
find the pertinent allowable compression stress F_a of the equivalent column, Step 8. This is the stress which is permissible at the centroid of that column. For dimensioning a flexural member, however, one wants to utilize the com-

pression stress at the outer fiber rather than that at the centroid of the compression portion. On the basis that stresses are proportional to distances from the neutral axis, Step 9, therefore, permits one to calculate the permissible outer fiber stress F_{b2} from the previously determined value of F_a.

For details, reference is made to Ref. E.7 and to *Example No. 17* of the *Manual*.

F. COMBINED COMPRESSION AND BENDING

1. General

Loadings which result in combined compression and bending may consist either of longitudinal forces applied eccentrically; or of concentric longitudinal forces acting simultaneously either with end moments or with transverse loads somewhere along the member; or of any combination of these. Thin-walled members can respond to such loading in a complex variety of ways, depending on their shape, slenderness, direction, line of action and relative magnitude of the various loadings, conditions of bracing, etc. With few exceptions an exact stress analysis would be too time-consuming for design use. This is why approximate design methods, in themselves fairly complex, have been developed for most of the more frequent situations, based on thorough comparison with rigorous theory and in most cases well verified by tests. There are a few situations, not as rare as one might wish, for which practicable design methods do not exist. It is advisable to avoid such situations where possible (e.g. by selecting appropriate shapes, providing suitable bracing, etc.); if this is not possible, recourse must be had to capacity determination by test, according to *Section* 6 of the *Specification*. 6

For ease of orientation, the various cases will first be listed and described in general terms, just as has been done for compression members in D.1, herein. Thereafter, the individual specification provisions for these various situations will be explained and documented. For brevity, members in combined compression plus bending will be designated as beam-columns.

Torsionally stable shapes, such as closed rectangular tubes, when *bending* acts *about the minor axis,* deflect in the plane of applied bending and fail on the concave side in the region of maximum moment by yielding or local buckling. When *bent about the major axis* they can fail in the same manner (mostly for large eccentricities); however, they can also fail by simple flexural buckling about the minor axis (i.e. normal to the plane of applied bending), if the load at which such buckling occurs is smaller than that which causes failure by yielding or local buckling (mostly for small eccentricities).

Doubly-symmetric open shapes, such as I-shapes, when *bending* acts *about the minor* axis likewise simply fail flexurally on the concave side in the region of maximum moment by yielding or local buckling. When *bent about the major axis* they may fail flexurally in the same manner; however, they can also collapse in lateral, *torsional-flexural buckling* in a manner similar to that discussed for purely flexural members in E.1, herein. (It will be remembered that concentrically loaded, doubly-symmetrical compression members do not buckle torsional-flexurally because the concentrically applied load coincides with the shear center. See D.2.(d), herein. When eccentrically loaded, the load no longer passes through the shear center, and this makes torsional-flexural buckling possible.)

Singly-symmetric open shapes, such as channels, angles, C-, or hat-, shapes, etc. hereafter shall have their axis of symmetry designated as the x-axis, as is
3.7.2 done in *Section 3.7.2*. When *bending is applied in the plane of symmetry* (i.e. about the y-axis) they, too, can simply fail flexurally on the concave side in the region of maximum moment by yielding or local buckling. Alternatively, they, also, can collapse in torsional-flexural buckling, particularly if the eccentric load is applied on the open side of such shapes as channels or hats. When *bending is applied in any plane other than that of symmetry*, singly-symmetric beam-columns will continuously bend in both directions and also twist as the load is increased, for the same reasons which were explained for beams of such shape in E.2 herein. Satisfactory ways for calculating the behavior of such members are not available. In the special case *when bent about the x-axis* (symmetry axis), twisting should be prevented by suitable bracing similarly as discussed in E.2(b) herein; in this case failure can occur only flexurally by yielding or local buckling.

Unsymmetrical open shapes, when used as beam-columns, in general will continuously twist and bend in a plane inclined to that of applied bending, as load is increased. Again, satisfactory ways of calculating the behavior of such members are not available. When, for non-structural reasons, their use cannot be avoided, their load capacity must be ascertained by test according to *Section* 6 of the *Specification*.

2. Torsionally Stable and Doubly Symmetrical Open Shapes.

In modern American design specifications, the tool for dealing with combined stress situations is the *interaction equation*. Its nature can be understood as follows: The maximum elastic stress σ_{max} at any section of a member, caused by an axial force P and a simultaneous bending moment M, is obtained from

$$P/A + M/S = \sigma_{max} \quad \text{or} \quad \frac{P/A}{\sigma_{max}} + \frac{M/S}{\sigma_{max}} = 1 \qquad \text{(F.1)}$$

Let $P/A = \sigma_c$, the compression stress, $M/S = \sigma_b$, the bending stress, and σ_f that stress which causes incipient failure, e.g. by yielding or local buckling. Then, from Eq. F.1, the condition that a member, at the particular section, is on the point of failing, is

$$\sigma_c + \sigma_b = \sigma_f \quad \text{or} \quad \sigma_c/\sigma_f + \sigma_b/\sigma_f = 1 \qquad \text{(F.2)}$$

It is seen that for simple compression Eq. F.2 becomes $\sigma_c = \sigma_f$ and for simple bending $\sigma_b = \sigma_f$. Eq. F.2 can be generalized by recognizing that the stress causing failure in simple compression, σ_{fc} (e.g. the stress at which the member, acting as a column, will buckle) need not be the same as the stress causing failure of the same member in simple bending, σ_{fb} (e.g. the yield point). Now, since the first term of Eq. F.2 refers to the compression component, and the second to the bending component of the combined stress situation, the equation can be generalized to the form in which it is used in design codes;

$$\sigma_c/\sigma_{fc} + \sigma_b/\sigma_{fb} = 1 \tag{F.3}$$

The equation is evidently correct for the two extreme situations of either $\sigma_c = 0$ (bending only) or $\sigma_b = 0$ (compression only). It has been shown that for the entire range of combined stress, i.e. for any ratio of M/P, Eq. F.3, with suitable modifications where needed, is a reliable approximation, not only in the elastic but also in the inelastic range. (For a more extensive discussion of this interaction equation, see Ref. F.1, particularly the writer's Ch. 4.)

Equation F.3 defines the state of incipient failure. In design, in order to calculate combinations of compression stresses $f_a = P/A$ and bending stresses $f_b = M/S$ which will insure the necessary margin of safety against failure, the failure stresses in the denominators of Eq. F.3 must be replaced by the respective allowable stresses F_a for compression without bending, and F_b for bending without compression. This result in the design interaction equation

$$f_a/F_a + f_b/F_b \leq 1 \tag{F.4}$$

which is used throughout *Section 3.7* of the *Specification*. It defines permissible 3.7
combinations of simultaneous compression stresses f_a and bending stresses f_b such that in combined stress the same safety factors are maintained as in other parts of the *Specification*. It is essential that the appropriate allowable stresses be substituted for F_a and F_b depending on the case at hand.

Eqs. F.3 and F.4 recently have been further generalized to apply to *bi-axial bending*, as follows:

$$f_a/F_a + f_{bx}/F_{bx} + f_{by}/F_{by} \leq 1 \tag{F.5}$$

The quantities have the same significance as in Eq. F.4, except that the subscripts x and y apply to bending about these principal axes. The equation is used in this form in the latest edition of the AISC Specification and for uniformity is also incorporated in the *AISI Specification*. The equation is known (Ref. D.1, p. 164) to be reasonable and conservative as long as the bi-axial behavior is chiefly flexural, e.g. for box-shaped members and most cases of doubly symmetrical I-shapes *Section 3.7.1* of the *Specification*. At this time there is no 3.7.1
known evidence that the equation applies to the extremely complex situation of bi-axial bending of singly symmetrical or unsymmetrical shapes, *(Sections 3.7.2, 3.7.3* and *3.7.4* of the *Specification)*. 3.7.2

For Eq. F.4 to be correct, it is necessary that one substitute the maximum 3.7.3
value of M which occurs anywhere along the member under the action of the 3.7.4
simultaneous external loading, i.e. the axial force, transverse loads, and end moments if any. In most cases this moment is larger than that which would occur in the same member if the axial force were absent. For example, in a uniformly loaded simple beam the maximum moment is $M_b = wL^2/8$, and is associated with a deflection $d_b = (5/384)(wL^4/EI)$. If an axial force P is additionally

applied to this member, the deflection increases to a value closely equal to $d = d_b(1 - (P/A)/\sigma_e)$ where σ_e is given in Eq. D.3. The force P applied concentrically at the ends now has a lever arm d with respect to the centroid of the mid-span section of the beam, causing an additional moment $P \times d$. Consequently, at that section, the moment is

$$M = wL^2/8 + P \times d$$

This is the maximum moment anywhere along the beam and must be used in Eq. F.4 and those derived from it. The situation is similar for other types of loading so that, in general, the moment

$$M = M_b + P \times d$$

must be used in the interaction equation, M_b being the beam moment which would occur in the member if P were absent.

It is seen that M is larger than M_b, by an amount which depends on the axial force, but also on the particular type of loading, since the deflection d_b will be different for different load distributions. It has been shown (see e.g. Refs. F.1, D.1, F.2) that the maximum moment in the beam-column (i.e. the member subject to simultaneous compression and bending loads) can be calculated with satisfactory accuracy from

$$M = M_b C_m/(1 - (P/A)/\sigma_e)$$

Here $C_m/(1 - (P/A)/\sigma_e)$ is known as a modification factor, that is, it determines the amount by which the moment M_b caused by flexural loading is modified by the presence of the axial force P. (For the meaning of σ_e see Eq. D.3.) C_m is a coefficient which depends on type and distribution of the flexural loading. For most cases it can be taken sufficiently accurately and slightly conservatively as 0.85, except for the situations where other values are specified in
3.7 *Section 3.7*. (See also Ref. C.6.)

For the particular case of compression plus unequal end moments, it has been shown (Ref. D.1) that $C_m = 0.6 + 0.4\ (M_1/M_2)$ but not less than 0.4, as specified in *Section 3.7*.

The bending stresses f_b which are caused in the absence of P by the beam moment M_b alone, are evidently modified in the same ratio as the moment itself. Correspondingly, to compute the correct flexural stress for use in Eq. F.4, f_b must be multiplied by the same modification factor which was shown to apply to M_b. If this is done and the safety factor $n = 1.92$ correctly introduced in the modification factor, Eq. F.4 takes the form,

$$f_a/F_a + C_m\, f_b/[1 - f_a/(\sigma_e/n)]\ F_b \leq 1 \qquad \text{(F.6)}$$

It is this equation which is the basis for that specified for combined compression
3.7.1 and bending; the first of the two expressions in *Section 3.7.1*, with σ_e/n designated as F'_e.

Evidently the moment M in the beam-column exceeds the moment M_b by the amount $P \times d$ only in those places along the member where deflections d are possible. This is not the case at supports or other places where deflection in the plane of applied bending is prevented. Also, since $C_m \leq 1$, it is possible for the entire modification factor $C_m/(1 - f_a/F'_e)$ to be smaller than 1 (one), particularly for stocky members. This can result in situations where the permissible bending stress f_b in the presence of axial load could turn out to be larger than permitted when bending alone is present. To safeguard against this possibility of local overstressing at braced points or when the modification factor is smaller than one, *Section 3.7.1* prescribes a further check of the form 3.7.1

$$f_a/F_{ao} + f_b/F_b \leq 1 \qquad \text{(F.7)}$$

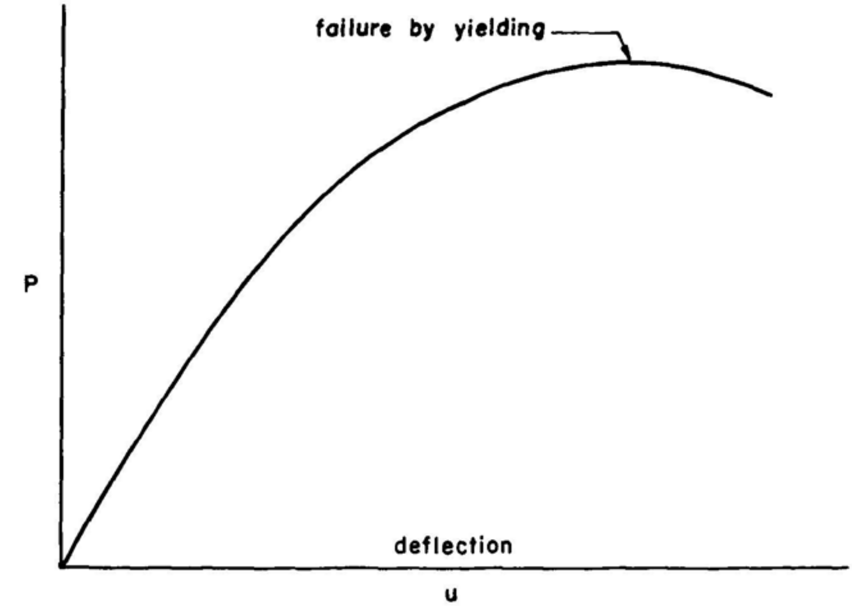
Finally, when the axial force is so small that its effects are insignificant, ($f_a/F_a \leq 0.15$) *Section 3.7.1* stipulates that the modification factor shall be dispensed with so that Eq. F.4 applies without modification. Not only is this a conveniently simpler provision but it, too, prevents situations where, when $C_m = 0.85$ and $f_a << F'_e$, the main equation which includes the modification factor could result in $f_b > F_b$.

In all this it is essential that the correct value be substituted for F_b, which is to be determined according to *Section 3.1, 3.2,* or *3.3,* as the case may be. Thus, for members bent about the minor axis or which are torsionally very rigid (such as box-beams) *Section 3.3* need not be checked since lateral buckling cannot occur. Otherwise, if lateral buckling is possible, F_b must be determined according to *Section 3.3* and also according to *Section 3.1* or *3.2,* as applicable. If the section contains unstiffened compression flanges, then in order to safeguard these flanges against local buckling, F_b must be determined according to *Section 3.2*. If the section contains no unstiffened flanges, then *Section 3.1* applies.

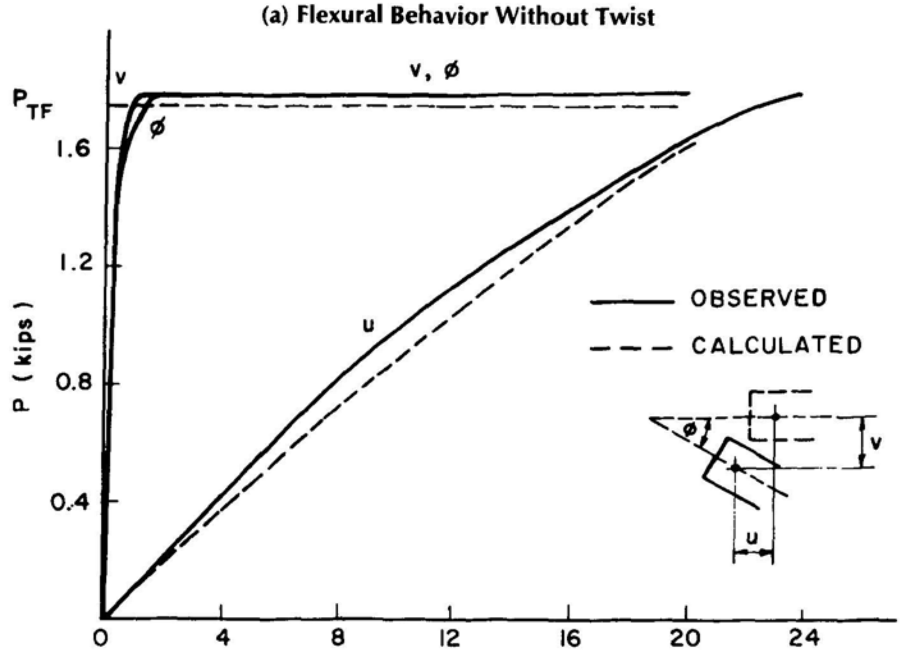
For simplicity, this discussion was presented for uniaxial bending. It was pointed out that for the situation covered by *Section 3.7.1* (doubly-symmetric shapes and those that will not show significant twisting), Eq. F.4 for uniaxial bending has been generalized to Eq. F.5 for bi-axial bending plus compression. Thus, *Section 3.7.1* in its entirety is formulated for the general case of bi-axial 3.7.1
bending; naturally, when one deals with a case of uni-axial bending, the pertinent second or third term of the respective interaction equation is simply omitted.

3. Singly-Symmetrical Open Shapes

As has been discussed in F.1, above, singly-symmetrical open shapes such as channels, hats, etc., when bending is applied in the plane of symmetry (i.e. about the y-axis) can fail in one of two ways: (a) As loads and deflections increase gradually, simple bending in the plane of symmetry will finally cause yielding or local buckling to occur at the location of maximum moment. (b) Alternatively, purely flexural bending in the plane of symmetry will again proceed gradually as loads are increased, but at some definite load the member will suddenly buckle by twisting and simultaneous transverse deflection. The buckling mode

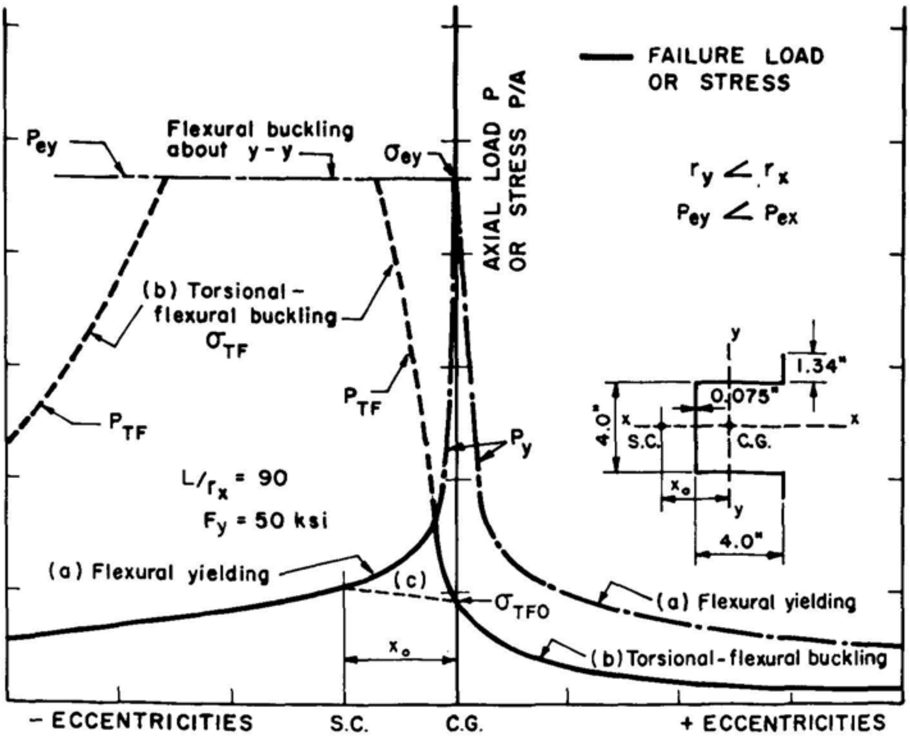


(a) Flexural Behavior Without Twist



(b) Torsional—Flexural Buckling

Channel Member in Eccentric Compression
Fig. F.1



Strength of Eccentrically Compressed Hat Section

Fig. F.2

is similar to that on Fig. D.5, except that now the load is applied eccentrically in the plane of symmetry. The two types of behavior are shown on Fig. F.1 which illustrates in (a) the continuous and gradual approach to failure, and in (b) the sudden torsional-flexural buckling. Fig. F.1(b) is a direct reproduction of one of the numerous tests carried out at Cornell University (Refs. F.3, F.4.)

Whether, and at what load or stress, behavior (a) or (b) obtains depend in each case on shape, dimensions and length of the member and the eccentricity of load. Fig. F.2 from Ref. F.3 illustrates this. The solid curve shows how the failure load or stress changes as the eccentricity is varied over the range $\pm 4x_o$ (x_o = distance between centroid, C.G. and shear center, S.C.). If the load is concentric, this hat section member buckles torsional-flexurally at the stress σ_{TFO} which is seen to be much lower than that stress σ_{ey} at which it would buckle flexurally about the y-axis if it were concentrically loaded and braced to prevent twisting. Also given are two curves showing, for each eccentricity, (a) the load or stress at which failure would occur by simple yielding due to compression plus bending, and (b) the load or stress at which failure would occur by torsional-flexural buckling. Which failure will actually take place at any given eccentricity depends on which curve falls below the other at that particular eccentricity. Thus, it is seen that for all eccentricities on the open side of the section torsional-flexural buckling occurs because curve (b) falls below curve (a). Conversely, beyond the shear center on the closed side of the section, simple flexural failure occurs because curve (a) falls below curve (b). Between shear center and centroid one mode changes to the other; also, in this range, small changes in eccentricity cause large changes in theoretical failure load, with a sharp peak located close to the centroid.

This behavior is typical of all singly-symmetrical sections, with minor variations for T- and unsymmetrical I-sections, but the relative locations of the curves can vary considerably. In any event, if torsional-flexural buckling occurs at all, it will do so for eccentricities on the open side of the section, but not for eccentricities on the closed side beyond the shear center. (Only for T's and unsymmerical I's can torsional-flexural buckling also occur in that range of negative eccentricities. This is discussed later herein.) Since it is not possible without calculation to predict which curve lies below the other, particularly for eccentricities on the open side, one always must calculate (a) the load or stress at which flexural failure, and (b) the different load or stress at which torsional-flexural buckling will occur; the lower of the two, evidently, governs design.

3.7.2 This situation is reflected in the design provisions of *Section 3.7.2* of the *Specification. Sub-section (i)* checks whether safety is provided against simple flexural failure (curves (a) of Fig. F.2). It is seen that this provision is identical in form with *Section 3.7.1*. The only difference is that *Section 3.7.2(i)* is formulated to apply only to flexural failure without lateral buckling.

Sub-section (ii), in terms of Fig. F.2, applies to curve (b) to the right of the centroid C.G. It permits one to determine that elastic stress σ_{TF} at which torsional-flexural buckling occurs. One does this by solving the third equation in *Section 3.7.2(ii)* for σ_{TF}. This is seen to be an interaction equation of the same form as Eq. F.6, except that it refers to failure stresses rather than allowable stresses, i.e. it does not contain any safety factors. The term C_{TF} is the same as C_m in *Section 3.7.1*, except that no lower limit is specified, whereas C_m has a lower limit of 0.4. It is shown in Ref. F.4, on the basis of large numbers of comparative calculations for a variety of shapes and, for each shape, for a considerable range of eccentricities and slendernesses, that for unequal end eccentricities the use of C_{TF} results in a satisfactorily close and conservative approximation of the exact value of σ_{TF}. Once σ_{TF} is calculated, allowable stresses F_a based on a safety factor of 23/12, are obtained from the first two expressions in *Section 3.7.2(ii)*.

Sub-section (iii) of 3.7.2 applies to the range of eccentricities between the centroid C.G. and the shear center S.C. As was discussed above and as is seen from Fig. F.2, it is in this range that the failure mode changes from torsional-flexural buckling to simple flexural failure. Also, in this same range small changes in eccentricity result in large changes in failure load or stress, with a sharp peak as shown. Hence, any small inaccuracy in eccentricity, because of imperfections and the like, could result in decidedly unconservative designs. To avoid this situation it was decided to cut off the sharp peak in this eccentricity range, and to base allowable stresses on the straight cut-off line (c) of Fig. F.2. The formula in *sub-section* (iii) represents this straight-line, in terms of allowable (rather than failure) stresses. This sub-section need be used only if, for concentric loading, the member would fail in torsional-flexural buckling rather than in flexural buckling about the y-axis, i.e. if $F_{a1} > F_{a2}$. In terms of Fig. F.2, if the concentric flexural buckling stress σ_{ey} were lower than the concentric torsional-flexural stress σ_{TFO}, then no torsional-flexural buckling could occur for any eccentricity between S.C. and C.G. and, hence, only *Section 3.7.2(i)* would govern.

To summarize: for singly-symmetrical open shapes (not including I- and T-sections), torsional-flexural buckling need not be checked for eccentricities on the side of the shear center opposite from the centroid, i.e. on the closed side of the section. In this case only *Section 3.7.2(i)* need be used. If the eccentricity is on the same side of the shear center as the centroid, it is not possible by inspection to determine whether simple flexure or torsional-flexural buckling will govern. Hence, one needs to determine the allowable stress both by *Section 3.7.2(i)* and either *3.7.2(ii)* or *3.7.2(iii)* (depending on eccentricity), and take 3.7.2(i)
the lower value. 3.7.2(ii)

For I-sections, with unequal flanges and for T-sections the situation is dif- 3.7.3(iii)
ferent inasmuch as it is possible for torsional-flexural buckling to occur also for eccentricities on the side of the shear center opposite to that of the centroid. This is immediately evident if one recalls that for symmetrical I-sections loaded in eccentric compression, failure can be either flexural or torsional-flexural (see F.1 and F.2, above). The symmetrical I-section is one special case, and the T-section another, of the general case of an I-section with unequal flanges. All these shapes can, for appropriate dimensions and slendernesses, fail torsional-flexurally for eccentricities on either side of the shear center.

Section 3.7.2(iv)b, therefore, specifically applies to such sections when the 3.7.2(iv)b
load is applied on the side of the shear center opposite from the centroid. The provision is very similar in form to *Section 3.7.2(ii)*. That is, one first has to determine σ_{TF} from the third equation, which is seen to be of the usual interaction type. Once σ_{TF} is calculated, one determines the allowable stress from the first or second equation for F_a, as applicable.

When the load is applied between centroid and shear center, the same kind of transition straight-line (c) in Fig. F.2 is employed for these sections. The corresponding provision, *Section 3.7.2(iv)a* is identical in form to *Section 3.7.2(iii)* 3.7.2(iv)a
which holds for the same eccentricity range, but for sections other than I and T. The difference is that for I- and T-sections, when the load is applied at the shear center ($e = x_O$) it is possible for either flexural or torsional-flexural failure to occur. Hence, to determine the correct left end-point of line (c) in Fig. F.2 for such shapes, i.e. the stress F_{ac}, one has to calculate it for both flexure (i) and torsional flexure (iv,b) and take the lower of the two values.

Complete analytical documentation, supported by test results, for all these procedures is given in Refs. F.3 and F.4. Design aids which greatly simplify the use of these somewhat complicated provisions are presented in the Manual.

G. CONNECTIONS

1. GENERAL

A considerable variety of means of connection finds application in cold-formed construction. Without any claim for completeness, these may be listed as follows:

(a) Welds, which may be subdivided into resistance welds, mostly for shop fabrication, and fusion welds, mostly for erection welding.

(b) Bolts, which may be subdivided into unfinished bolts without special control on bolt tension, and high-strength bolts with or without controlled, high bolt tension.

(c) Rivets. While hot rivets have little application in cold-formed construction, cold rivets find considerable use, particularly in special forms, such as blind rivets (for application from one side only), tubular rivets (to increase bearing area), high shear rivets, explosive rivets, and others. Most of these are proprietary products.

(d) Screws, mostly self-tapping screws of a considerable variety of shapes.

(e) Special devices, among which may be mentioned: (i) metal stitching, achieved by tools which are special developments of the common office stapler, and (ii) connecting by upsetting, by means of special clinching tools which draw the sheets into interlocking projections.

The Specification contains provisions only for welded and for bolted connections. Classes (c), (d), and (e), above, mostly refer to a variety of proprietary devices in regard to which information on strength of connections must be obtained from manufacturers or from tests carried out by or for the prospective user. In regard to riveting and, to a lesser extent, screwing, the data given in the Specification in regard to bolting can be used as a general guide, except for the shear strength of the rivet or screw which depends on the material and shape of the connector and may be quite different from that of a bolt of equal diameter.

2. WELDING

(a) Resistance Welds

Spot welding in its normal form as well as by projection welding is probably the most important means of shop connecting in cold-formed steel fabrication. *Section 4.2.2* gives allowable design values per spot, depending exclusively 4.2.2
on the thickness of the thinnest connected sheet. This is so because the American Welding Society's Recommended Practice for Resistance Welding, on which *Section 4.2.2* is exclusively based, contains definite recommendations on electrode diameter, current, etc., depending on sheet thickness. The use of the design values of that Section, which are based on a safety factor of about 2.5, is therefore justified only if the quoted Recommended Practices are strictly followed.

(b) Fusion Welds

Fusion welding is used for connecting cold-formed steel members to each other as well as connecting such members to heavy, hot-rolled steel framing (such as floor panels to beams and girders of the steel frame). It is used in fillet welds, butt welds (rather rarely), and in plug welds.

4.2.1 The provisions for allowable stresses in fusion welds in *Section 4.2.1* have been expanded in the present edition of the *Specification*. In shear, previous editions were intended to cover only moderate strength steels and corresponding electrodes, a situation for which a single allowable shear stress of 13.6 ksi was provided. Since then, higher strength steels have come into increasing use, and correspondingly higher strength electrodes have been introduced. Accordingly, the new edition of the *Specification* provides for three strength ranges of steel and three grades of electrodes, E60, E70 and E80. The allowable shear stress for lower strengths of steel welded with E60 electrodes has remained unchanged, 13.6 ksi. Appropriately higher strength steels welded with E70 electrodes permit a higher shear stress of 15.8 ksi. This value is the same as that given in the 1963 edition of the AISC Specification. Finally, for steels with yield points exceeding 50 ksi and welded with E80 electrodes, a shear stress of 17.7 ksi is permitted.

A total of 151 tests on welded connections for which under present provisions an allowable shear stress of 13.6 ksi would apply, have been made at Cornell University, more as a limited check on the applicability of the same value to cold-formed as to heavier hot-rolled steel. It was found that with this allowable stress, safety factors of no less than 2.7 obtained, indicating more than adequate safety in the tested range. In the 1969 edition of the AISC Specification (Ref. C.6), a substantial increase in allowable shear stresses on welds is provided. Whether this increase can safely be extended to the lighter sheet and strip gages has not been established at this writing. Extensive research is now underway on the strength of those types of fusion welds specific to much cold-formed construction in the thinner materials (mostly fillet, puddle, and plug welds). Until the results of this research become available, the present edition continues the conservative allowable stresses previously in use in both specifications rather than adopting the increases now in the AISC Specification without proof of their applicability to thin materials.

It is mentioned in *Section 4.2.1* that shear stresses are referred to "the throat" of the weld. The throat is a fictitious dimension, equal to 0.707t (t being the sheet thickness), the meaning of which is shown in Figure G.1. That is, in welding thin sheet the weld shape generally obtained is that shown in the figure, with the thickness of the weld actually exceeding that of the sheet. It is the intent of *Section 4.2.1* to disregard any material deposited beyond the dashed line in Figure G.1, and to calculate the throat thickness in the same manner as in heavy welded construction.

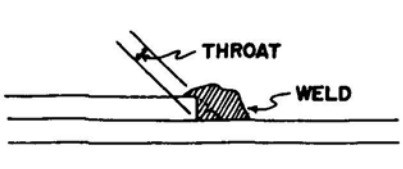


Fig. G.1

When plug welds are made with pre-punched holes, the length of the fillet weld for computing weld strength can correctly be assumed to be the perimeter of the hole.

Another type of connection is sometimes known as puddle weld. Basically, it is a plug weld except that no pre-punched holes are employed. Instead, a hole is burned into the upper sheet, which is then filled with a puddle of weld metal to fuse to the lower sheet or plate. This procedure requires special welding skill and experience, but has been used for a variety of connections, particularly for connecting panels or decks to supporting steel beams. In Ref. G.5 some such uses are illustrated and representative strength values are given for connections made with such welding.

3. BOLTING

(a) Standard-Strength Bolts, ASTM A307

The nature of cold-formed construction generally precludes the use of turned bolts in fitted (reamed) holes. The provisions of *Section 4.5* are, therefore, 4.5
written for unfinished bolts in oversize holes (usually 1/16 in. oversize for bolts of 1/2 in. diameter and larger, and 1/32 in. for smaller bolts). The provisions of that section are based on 574 tests on bolted connections reported in Ref. G.1, supported by the data from 602 tests in Ref. G.2.

The four provisions of *Section 4.5* safeguard against the four types of failure observed in these tests, generally with a safety factor of 2.2 or larger.

(i) For relatively small edge distances (in line of stress) failure occurs by shearing of the connected sheet along two parallel lines one bolt diameter apart (see Figure 3.I of Ref. G.1). This occurs at a shear stress of 0.7 F_y, i.e., at a total load $P_{ult} = 2 \times 0.7$ t e F_y, where e is the edge distance. Hence, e = $P_{ult}/1.4$ tF_y. It is specified in *Section 4.5.1* that the edge distance shall not be less than $P/0.6$ $F_y t$. Since $F_y = 1.67$ F, the safety factor is seen to be $1.4/0.6 = 2.33$.

(ii) For larger edge distances failure may occur by material piling up in front of the bolt and the bolt cutting through the sheet (see Figure 3.II of Ref. G.1) which was found to occur at a bearing stress equal to 4.8 F_y. *Section 4.5.3* per- 4.5.3
mits a bearing stress of 2.1 F_y. Considering again, that $F_y = 1.67$ F, it is seen that the safety factor in *Section 4.5.3* is $4.8/2.1 = 2.28$.

(iii) It is inevitable for any type of connection (with the possible exception of butt welds) to create stress concentrations in the connected parts. In bolted connections there are two causes for such stress concentration: (a) the presence of a hole or holes which is known to result in elastic stress concentration factors of about 2.5 to 3.0; (b) the fact that at the hole or holes a concentrated localized force is transmitted by the bolt to the sheet, plate, or other connected part. In those situations where progressive local yielding is capable of causing sufficient plastic stress redistribution to eliminate the stress concentration, tensile failure at a net section through a hole or holes occurs at a load equal to the net area times the tensile strength of the steel. If plastic stress redistribution is not capable of completely eliminating the stress concentration, the average stress on the net section, σ_{net}, at failure will be smaller than the tensile strength of the material, σ_u.

Recent tests at Cornell University (unpublished) have shown that plastic redistribution is capable of eliminating the stress concentration caused by the mere presence of a hole in a plate in tension, even in steels of considerably less ductility than stipulated in current ASTM specifications; that is, the average stress at failure on the net section through the hole or holes is simply equal to the steel tensile strength, $\sigma_{net} = \sigma_u$.

However, if the additional stress concentration caused by the local force transfer between bolt and sheet or plate becomes pronounced, it may cause net section tearing at average stresses below the steel tensile strength. The pertinent tests of Refs. G.1 and G.3 showed that such weakening of the net section results when bolts are widely spaced in the direction perpendicular to the transmitted force. From these tests the following equation was obtained:

$$\sigma_{net} = (0.1 + 3d/s)\, \sigma_u \leq \sigma_u \qquad \text{(G.1)}$$

where d = bolt diameter; s = spacing of bolts perpendicular to line of stress (for single bolts, s = blank width of sheet); σ_u = tensile strength of connected
4.5.2 steel. The provisions for stress on the net section, *Section 4.5.2,* in past editions of the *Specification* were based directly on Eq. G.1.

These tests were carried out on connections with a single bolt in line of stress (and with one or two bolts in a line perpendicular to that of stress). In this case the entire force in the single net section through the hole or holes, is transmitted at that section from the one sheet to the other by the bolt or bolts in that section. This makes for a very large additional stress concentration caused by the localized bolt forces. The situation is shown in Fig. G.2(a) where, in section a-a, the force in the net section is P and the force transmitted by the bolt in that section is also P.

This sharp stress concentration is much relieved when more than one bolt in line of stress is used. For instance, for three bolts in line of stress as in Fig. G.2(b), each bolt transfers one-third of the total force. Thus, in section a-a, while the total force in the section is P as before, the portion of that force which is transferred at that section and which causes stress concentration, is only P/3. Hence, if the sharp stress concentration in Fig. G.2(a) causes a reduced net section strength according to Eq. G.1, one should expect a much smaller weakening or none at all from the much milder stress concentration of Fig. G.2(b).

In order to explore this situation, additional bolted connection tests were carried out at Cornell University as follows:

8 single bolt tests entirely duplicating the earlier test procedure,
7 two-bolt-in-line-of-stress tests connecting elements of equal thickness,
4 two-bolt-in-line-of-stress tests connecting elements of unequal thickness,
4 three-bolt-in-line-of-stress tests (elements of equal thickness).

Fig. G.3 gives the results of those of these tests which resulted in failure by tearing at the net section.

It is clearly seen that, as expected, failure in the net section in the two-bolt tests occurred at a much higher stress σ_{net} than in single-bolt connections and, on the average, occurred at a somewhat higher stress yet in the three-bolt con-

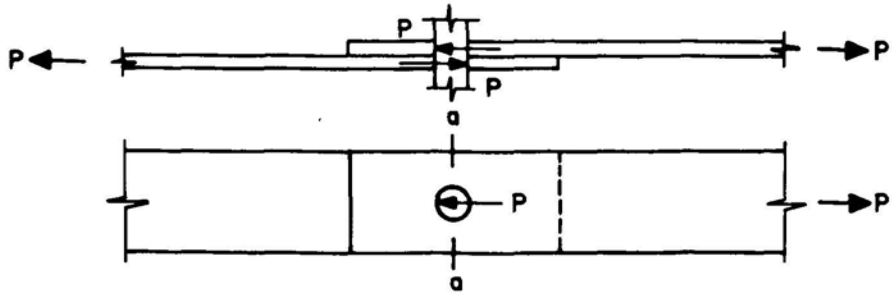


Fig. G.2(a)

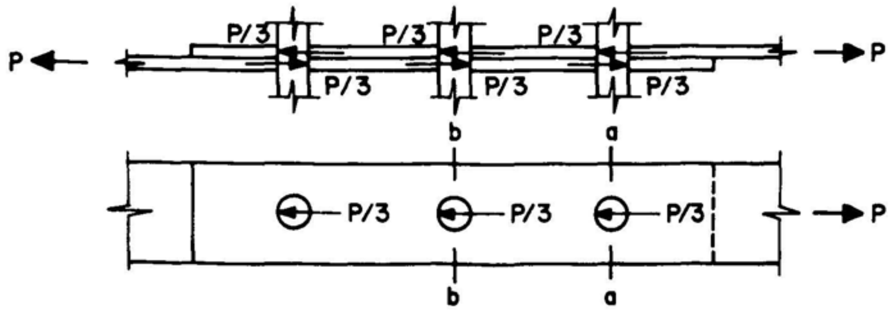
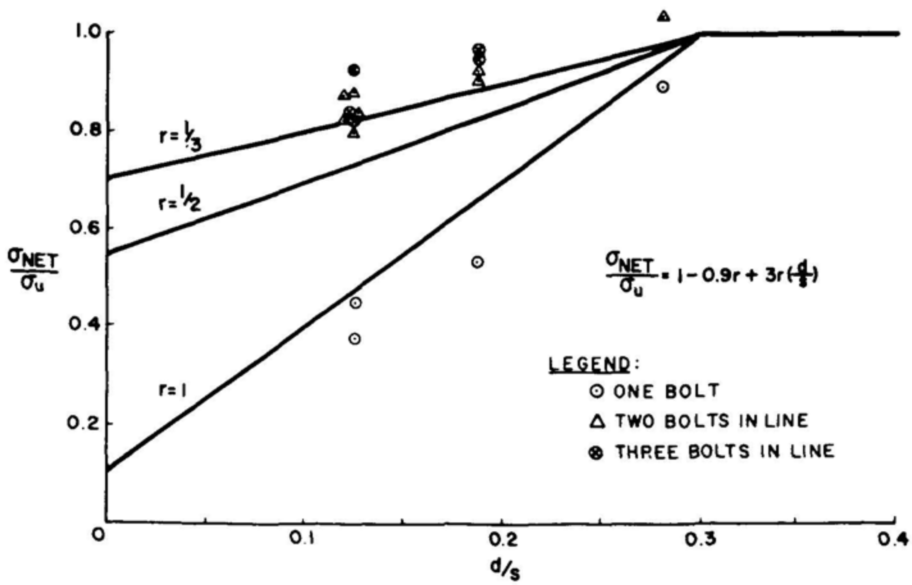


Fig. G.2(b)



Multiple-Bolt Connection Tests
Fig. G.3

nections. To represent this situation conservatively and in a manner consistent with the earlier findings on single-bolt connections, the following formula was developed:

$$\sigma_{net} = (1 - 0.9r + 3rd/s)\ \sigma_u \leq \sigma_u \tag{G.2}$$

where r = force transmitted by the bolt or bolts at the section considered, divided by the force in the member at that section. For instance, in Fig. G.2(a), r = 1 in section a-a because in that section the force in the member is P and the force transmitted by the bolt is also P. In contrast, in Fig. G.2(b), in section a-a the force transmitted by the bolt is P/3 while the force in the top member at that section is P; hence, at a-a the ratio r = 1/3. In section b-b the force transmitted is again P/3, while the force in either member at that section is 2P/3; hence at b-b the ratio r = 2/3. It is seen that for one-bolt-in-line, connections, with r = 1, Eq. G.2 reduces to Eq. G.1 which was the basis for the corresponding provisions in previous editions of the Specification.

The three inclined straight-lines on Fig. G.3 represent Eq. G.2 for one-bolt (r = 1), two bolt (r = 1/2) and three-bolt (r = 1/3) connections. It is seen that the single-bolt tests fall along, and somewhat below the line for r = 1. This amount of scatter is perfectly normal and was experienced in the large number of previous tests. It is the reason for using a safety factor of about 2.25 (see below) for net section failure as compared to the basic safety factor of 1.67. The two-bolt and three-bolt test results are seen to be well above the corresponding lines representing Eq. G.2. This conservatism was thought necessary primarily because of the limited number of tests on which Eq. G.2 is based. The tests made on connections of two elements of different thickness merely confirmed that it is appropriate and safe to base design directly on the smaller of the two thicknesses.

In bolted connections of thin material it is essential that a washer be placed under the nut, that the connection be firmly tightened, and preferably that another washer be placed under the head. Tests have shown that in connections without washers under the heads and which were only "fingertight," the head can dig into the connected element and, thereby, reduce the net section strength on the order of 20 percent.

In agreement with the new research evidence just described, the present edition of the *Specification*, in *Section 4.5.2,* liberalizes the allowable stress in the net section in accordance with Eq. G.2 for connections with more than one bolt in line of stress. There is no change if only one bolt is used in line of stress.

The pertinent tests of Refs. G.1 and G.3 show that the load at which tearing occurs, correlates better with the tensile strength than with the yield point of the steel. That is, tearing seems to begin when the stress at the point of concentration in the net section reaches the tensile strength of the material. For simplicity, the provision of *Section 4.5.2* is written in terms of the basic design stress $F = 0.6\ F_y$, i.e. basically in terms of the yield point rather than the tensile strength. The desired safety factor, as in (i) and (ii) above, is approximately 2.25. This factor evidently is assured for all steels in which the ratio of specified minimum tensile to specified minimum yield strength is at least $2.25 \times 0.6 = 1.35$

(in round numbers). Such is the case in all moderate-strength structural sheet and strip. However, for some of the higher strength structural sheet steels the ratio of specified tensile to yield strength may be smaller than 1.35. In such cases, to assure maintaining a safety factor of 2.25, the footnotes to *Section 4.5* provide that instead of F_y a smaller stress, namely the tensile strength divided by 1.35 shall then be used for the appropriate design determinations.

(iv) The tests of Ref. G.1 indicate that shear failure of the bolts occurs at a stress, conservatively equal to 0.6 times the tensile strength of the bolt material; this shear stress was computed on the root area of the thread. *Section 4.5.4* 4.5.4
specifies a flat value of 10 ksi for the allowable shear stresses which are identical with those of the AISC Specification (Ref. C.6).

In members which are designed utilizing the material properties in the as-formed condition, the dimensioning of bolted connections must be based on material properties and allowable stresses of the sheet or strip before forming, rather than on those for the as-formed condition.

(b) High-Strength Bolts, ASTM A325

High-strength bolts conforming to ASTM Specification A325 are employed in two types of connections: (1) ordinary connections in which, as with ASTM A307 bolts, slip into bearing at design loads is permissible, and (2) special connections in which, by prescribed torquing of the high-strength bolts, a high contact pressure is produced between connected parts. In this case, if the faying surfaces have an adequate coefficient of friction, the resulting friction force transmits the entire load in the connection, resulting in the fact that connected parts do not slip as loads are applied. Such no-slip connections have definite advantages where fatigue conditions prevail or where even small deformations are detrimental to the serviceability of the structure. The usual surface of hot-rolled steel, when clean, provides the friction necessary for this purpose.

To investigate the possible advantages of using high-strength bolts in cold-formed steel construction, 476 tests have been made on connections of this type, which are reported in Ref. G.3. Faying surfaces were of the three types ordinarily met in such construction, namely galvanized, painted, or bare steel not having undergone any special cleaning. It was found that: (a) with the torques prescribed by the Specification of the Research Council on Riveted and Bolted Structural Joints (Ref. G.4), non-slip connections could be achieved with such surfaces if shear stresses were kept at appropriately low values; (b) shear failure of the bolts occurred, conservatively, at a stress on the root section equal to the same fraction of 0.6 times the tensile strength of the bolts, as it did in unfinished bolts; (c) once slip into bearing had occurred, failure in the connected sheets would occur at the same loads as with A307 bolts.

In view of findings (c), above, the results previously discussed for A307
bolts under (i), (ii), and (iii), and the corresponding provisions of *Sections 4.5.1* 4.5.1
to *4.5.3* of the *Specification* apply without change to connections with high- 4.5.2
strength bolts. 4.5.3

The chief practical advantage of A325 bolts, then, lies in their higher shear strength which makes it possible to use a much smaller number of bolts in such connections where bolt shear governs. To reflect this higher shear strength, *Sec-*

tion 4.5.4 provides for high-strength bolts the same allowable shear stresses as are implied in the above-mentioned Specification of the Research Council (Ref. G.4) for thick-walled, hot-rolled construction. In order to reflect the difference between the effective shear areas depending on whether the shear plane passes through the threaded or the unthreaded portion of the bolt, that specification prescribes a smaller shear stress in the former than in the latter case, 15 ksi vs. 22 ksi, computed on the gross area.

As to connections in which slip into bearing is prevented, experience has shown that the need for preventing slip hardly ever arises in cold-formed construction and that the type of surfaces which would be needed for this purpose is difficult to realize in such construction. For this reason the present edition of the *Specification* no longer provides for such non-slip connections. Under exceptional circumstances, should prevention of slip be necessary, appropriate guidance can be obtained directly from the research results given in Ref. G.3.

4. SPACING OF CONNECTIONS IN COMPRESSION ELEMENTS

If compression elements are joined to other parts of the cross-section by intermittent connections, such as spot welds, these connections must be sufficiently closely spaced to develop the required strength of the connected element. For instance, if a hat section is converted into a box shape by spot-welding a flat plate to it, and if this member is used as a beam with the flat plate up, i.e., in compression, (see Figure G.4), then the welds along both lips of the hat must be spaced so as to make the flat plate act monolithically with the hat. If welds are appropriately placed, this flat plate will act as a "stiffened compression element" with width w equal to distance between rows of welds, and the section can be calculated accordingly.

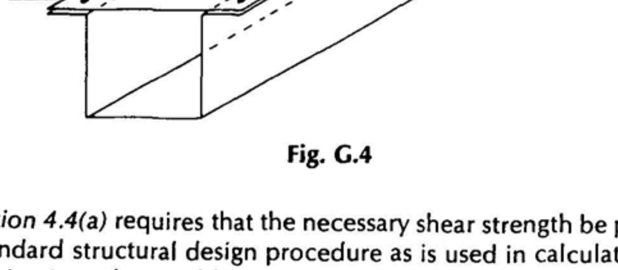


Fig. G.4

4.4(a) *Section 4.4(a)* requires that the necessary shear strength be provided by the same standard structural design procedure as is used in calculating flange connections in riveted or welded plate girders or similar structures; it needs no further comment.

4.4(b) *Section 4.4(b)* ensures that the part of the sheet between two adjacent welds will not buckle as a column at a stress below 1.67f, where f is the design stress of the connected compression element. Taking a strip of the compression plate between two welds of the described box section, for instance, it is seen that it could buckle away from the lips of the hat between welds (as shown in dashed lines on Figure G.4) if the weld spacing were too large. This strip, therefore, acts as a column of length equal to the clear distance between adjacent connections. In view of the kind of connection provided by the welds, end rotation of the "column" is practically prevented so that the strip acts as a "fixed-fixed" column the effective length of which is, theoretically, half the clear distance between connections. In order to account for the weakening influence of inelastic buckling, the provision is based on a more conservative assumption, that is, on an effective length equal to 0.6s. This is conservative in that (a) the coefficient is taken as 0.6 instead of 0.5, and (b) the length is taken as the center distance instead of the clear distance between connections (such as spot welds). On this basis the formula of *Section 4.4(b)* is obtained directly from the general Euler formula $\sigma_e = \pi^2 E/(KL/r)^2$ by substituting, as just explained, $\sigma_e = 1.67f$, $K = 0.6$, $L = s$, and $r = t/\sqrt{12}$, and solving for s. *Chart 4.4* of the *Design Manual* is a graphical presentation of this formula.

The provision is similar to that used for corresponding situations in aircraft construction. Even though no tests specifically aimed at verifying this provision have been made under the Cornell project, one of the major panel manufacturing firms in its development work has tested it extensively and has found it reliable.

Section 4.4(b) ensures satisfactorily close spacing to make a row of connections act as a line of stiffening for all situations, with the possible exception of relatively narrow unstiffened elements with w/t up to about 20. The allowable stresses for unstiffened elements *(Section 3.2)* are based on a buckling stress computed from a buckling coefficient of $k = 0.5$ (See D.1 and D.4, above). If an outstanding flange were ideally simply supported (hinged) at the web, it would have a buckling coefficient of 0.425 and would buckle in a half-wave equal to its full length (see Ref. C.1, p. 330, Table 26). The chosen coefficient of 0.5, therefore, corresponds to a slight rotational restraint of the unstiffened element along its supported edge and to a correspondingly smaller half-wavelength. Without detailed investigation the accuracy of which would be somewhat fictitious, this length can be assumed as being not less than 6w, judging from Table 9.2, p. 362 of Ref. C.7. In order for an intermittently connected line to act as one of continuous stiffening, at least two connections should be located within one half-wave. 4.4(b) 3.2

It is this consideration which has led to the provision of *Section 4.4(c)* which stipulates that for unstiffened elements connections should be made at distances not exceeding 3w. For large w/t ratios, stipulation (b) will automatically provide that this is so. Hence, provision (c) governs only for relatively narrow unstiffened elements. 4.4(c)

According to *Section 3.2* of the *Specification*, the limiting flat width for which the allowable stress is 0.6 F_y, i.e. below which failure occurs by yielding and above which it occurs by local buckling, is $w = 63.3t/\sqrt{F_y}$. Correspondingly, for this condition *Section 4.4(c)* stipulates a maximum permissible weld spacing equal to three times this amount, i.e. $s = 190t/\sqrt{F_y}$. If the flat width of the unstiffened element is larger than $63.3t/\sqrt{F_y}$ by a sufficient amount so that the allowable compression stress according to *Section 3.2* drops to $0.54\ F_y$ or less, a 20 percent increase in this maximum permissible weld spacing is provided, making it $s = 228t\sqrt{F_y}$.

H. MISCELLANEOUS

1. UNUSUALLY WIDE, STABLE BEAM FLANGES

Compression flanges of large w/t ratios tend to lose their stability through buckling; corresponding design provisions have been discussed in C, above. However, if flanges are unusually wide they may require special consideration even if there is no tendency to buckling, such as in tension flanges. Two matters need consideration for such elements: shear lag, which depends on the span-width ratio and is independent of the thickness, and curling which is independent of the span and does depend on the thickness.

(a) Shear Lag

In metal beams of the usual shapes, the normal stresses are induced in the flanges through shear stresses transferred from the web to the flange. These shear stresses produce shear strains in the flange which, for ordinary dimensions, have negligible effects. However, if flanges are unusually wide (relative to their length) these shear strains have the effect that the normal bending stresses in the flanges decrease with increasing distance from the web. This phenomenon is known as shear lag. It results in a non-uniform stress distribution across the width of the flange, similar to that in stiffened compression elements (see C.1, above), though for entirely different reasons. As in the latter case (see C.2(a), above), the simplest way of accounting for this stress variation in design is to replace the non-uniformly stressed flange of actual width w_f by one of reduced, effective width subject to uniform stress.

Theoretical analyses by various investigators have arrived at results which differ but little numerically (see p. 12 and pp. 124-5 of Ref. H.1). The provisions of *Section 2.3.5* are based on the analysis and supporting experimental evidence 2.3.5
obtained by detailed stress measurements on eleven beams, reported in Ref. H.2. In fact, the values of effective widths in Table 2.3.5 are taken directly from Curve A of Figure 4 of that reference.

It will be noted that according to *Section 2.3.5*, the use of a reduced width for stable, wide flanges is required only for concentrated load. For uniform load it is seen from Curve B of the quoted figure that the width reduction due to shear lag for any but unrealistically large width-span ratios is so small as to be practically negligible.

The phenomenon of shear lag is of considerable consequence in naval architecture and aircraft design; in cold-formed construction it is infrequent that beams are so wide as to require significant reductions according to *Section 2.3.5*.

(b) Flange Curling

In beams which have unusually wide and thin, but stable flanges (i.e., primarily tension flanges with large w/t ratios), there is a tendency for these flanges to curl under load. That is, the portions of these flanges most remote from the web (edges of I-beams, center portions of flanges of box or hat beams) tend to deflect toward the neutral axis. Deformations of this type have been observed in a number of tests at Cornell University.

An approximate, analytical treatment of this problem is given in the latter part of Ref. C.3. In *Section 2.3.3(d)* there is given a formula which permits one to compute the maximum admissible flange width w_f for a given amount of tolerable curling, c_f. This formula is obtained directly from Equation 11 of Ref. C.3, if that equation is solved for the flange width (called b in that reference).

2.3.3 It will be noted that *Section 2.3.3(d)* does not stipulate the amount of curling which can be regarded as tolerable, but merely suggests in the footnote that an amount equal to about 5 percent of the depth of the section is not excessive under usual conditions. It will be found that the cases are relatively rare in which curling becomes a significant factor in limiting flange width, except where for the sake of appearance it is essential to closely control out-of-plane distortions (e.g., when flat ceilings are to be formed of very wide, cellular panels).

2. LIMITATIONS ON FLAT-WIDTH RATIOS

2.3.3 *Sections 2.3.3(a) to (c)* and *Section 2.3.4* contain limitations on permissible
2.3.4 flat-width ratios of compression flanges and of webs of beams. As all such limitations, the exact values indicated in these sections are to some extent arbitrary. They do, however, reflect a body of experience and are intended to delimit practical ranges.

The limitation to a maximum w/t of 60 for compression elements stiffened by a simple lip has been discussed in the parenthetical paragraph in C.5, above. It is based on the fact that the stiffening lip itself is an unstiffened element. If its d/t exceeds about 10, this would call for stress reduction in the lip according to *Section 3.2*, and a corresponding reduction in the flange. However, for flanges with w/t significantly exceeding 60, lips with d/t less than 10 are inadequate ac-
2.3.2.1 cording to *Section 2.3.2.1*, so that w/t = 60 is a practical limit for lip-stiffened elements.

The limitation to w/t = 90 for flanges with edge stiffeners other than lips merely expresses the fact that still thinner flanges are quite flexible and liable to be damaged in transport, handling, and erection.

Much the same can be said for the limitation to w/t = 500 of web-stiffened compression elements. The Note specifically states that wider flanges are not unsafe, but that stiffened flanges exceeding w/t = 250 and unstiffened flanges exceeding w/t = 30 are likely to develop noticeable, though structurally harmless, distortions at design loads. In both cases the upper limit is set at twice that ratio at which first noticeable deformations are likely to appear, based on observation of such members under test. These upper limits, then, will generally keep such distortions to reasonable limits.

2.3.4 The limit h/t = 150 *(Section 2.3.4)* applies to webs typical for cold-formed construction. Such webs are generally unstiffened, in contrast to webs of plate girders which are always furnished with stiffeners at supports and concentrated loads, and often also with intermediate vertical or additional horizontal stiffeners. Also, in contrast to plate girder webs, the webs of cold-formed beams connect to the flanges through rounded corners; this makes it inevitable that loads and reactions are introduced into the web with some eccentricity which tends to distort the cross-sections for webs. Tests of cold-formed flexural members have been made with webs with h/t up to 175 in one series and up to 200 in another (see C.7(d), herein). For these extreme ratios it was found not only that the allowable web shear stresses and bearing values are so low as to make such members uneconomical, but also that they tend to cross-sectional distortion under load and in handling. For these reasons the limit $(h/t)_{max} = 150$ has long been maintained in the *Specification*. The present edition liberalizes this limit to $(h/t)_{max} = 200$ for those situations where adequate means are provided of transmitting concentrated loads and or reactions into the web. Such means can be separate (as in plate girders) or integrally cold-formed transverse stiffeners or, in the case of reactions, appropriate framing details which provide for transmission of reactions without causing web distortion.

3. APPLICATION OF PLASTIC DESIGN TO COLD-FORMED STRUCTURES

Plastic design is based on the proven proposition that a mild steel beam does not fail when the yield stress is reached in the outer fiber. It continues to function and gives way through excessive deformation only when yielding has practically reached the neutral axis from both sides, thus forming a "yield hinge." In continuous structures, yield hinges form successively and produce a redistribution of moments which generally permits a more economical design. Failure occurs only when enough hinges have formed to convert the structure (rigid frame, continuous beam, etc.) into a mechanism. This requires that the hinges undergo considerable rotations without local buckling of the flanges or webs, while the steel in practically the entire section is yielding. In order to ensure such behavior, w/t and h/t must be strictly limited to prevent premature local buckling (see Ref. H.3).

Most shapes now in use in cold-formed steel structures have w/t and h/t considerably in excess of the limits imposed by the requirements of plastic design. They are, therefore, not capable of developing plastic hinges satisfactorily and maintaining them throughout the required rotations without premature local buckling. It follows that plastic design methods are not applicable to cold-formed steel construction in its present form, unless such construction is surrounded with additional safeguards.

4. TESTS FOR SPECIAL CASES

Section 6 of the *Specification* covers (a) situations in which test results are needed to determine those mechanical properties on which design calculations shall be based, and (b) situations where "calculation of safe load carrying capacity or deflection cannot be made in accordance with the provisions (of the main body of) this Specification."

The first of these situations refers to the utilization of the strengthening effects of cold work in the calculation and dimensioning of members. Explicit methods for such utilization are incorporated for the first time in *Section 3.1.1.* 3.1.1
of the present edition of the *Specification,* and discussed in detail in *Section B.2* of this Commentary. It was pointed out there that as-formed mechanical properties, in particular the yield strength, can be determined either by full-section tests or by calculating the strength of the corners and computing the

weighted average of strength of corners and of flats. The strength of flats in turn, as was pointed out, can be taken as the virgin strength of the steel before forming, or can be determined by special tension tests on speciments cut out of
6.3.1 the flat portions of the formed section. *Section 6.3.1* in considerable detail spells out the types and methods of these tests, and their number as required for use in connection with *Section 3.1.1*. These provisions are self-explanatory. For details of some testing procedures which have been used for such purposes, but which in no way should be regarded as mandatory, reference is made to Refs. B.3, B.4 and B.5.

6.2 *Section 6.2* of the *Specification* makes provision for proof of structural adequacy by load tests. The intent of this section is clearly expressed by the word "special" in the title, and by the restriction (see *Section 6.1(a)*) to cases where . . ."calculation of safe load-carrying capacity or deflection cannot be made in accordance with the provisions of . . . this Specification."

It is evidently not the intent of this provision to substitute proof of structural adequacy by load test for design calculations according to the Specification. This is so because for structures of such shape and type that they can be calculated according to the *Specification*, the results of such calculations usually possess a greater degree of certainty than the results of load tests. This is easily illustrated by the following example: It is extremely unlikely that for a test structure for which the minimum yield point of steel of 33 ksi is specified, a steel with exactly 33 ksi yield point actually will be furnished. If the steel actually supplied has a 40 ksi yield point, the test load will generally be higher than if steel of minimum specified yield point had been used. However, in many cases the strength of cold-formed (and other) steel structures is not proportional to the yield point. It is impossible, therefore, to deduce by simple proportionality from the test results obtained on the higher strength steel what the load capacity would have been had a lower strength steel been used. However, since the structure in this example was specified to be made of steel having a minimum yield point of 33 ksi, the yield point of acceptable steels for structures built according to the tested sample can be as low as 33 ksi. In this case, then, the result of the load test will give quite *inadequate information on the minimum strength of the actual prototype* structure. Other similar examples could be added.

It is, therefore, clearly the intent of *Section 6* that structures should be designed according to the provisions of the *Specification*, without requiring load tests, in all cases where such design is possible. This is universally accepted good engineering practice and applies equally to any other design specification.

There are however, in cold-formed steel (as in other kinds of structures) perfectly acceptable and safe types of construction whose composition or configuration are not covered by provisions of the *Specification*. Their performance and adequacy therefore, cannot be demonstrated by reference to the *Specification*. To mention but one example: It has been pointed out in G.1, above, that apart from those methods of connection covered in the *Specification*, a number of other means of connecting are in use. The fact that these are not specifically covered in the *Specification* is not intended to exclude their use. However, since structures so connected cannot be calculated according to the *Specification* (at least as to strength of connections), tests according to *Section 6* are the only means of supplying proof of structural adequacy. Other similar examples could be added.

Provision (b) of *Section 6.2* prescribes that the structure under test load shall support without failure at least twice the live load plus one-and-one-half the dead load. For the usual ratios of live to dead load, the minimum carrying capacity so defined gives an overall safety factor somewhat larger than the basic value of 1.67 on which the body of the *Specification* is based. This is so because, within that body, carefully selected safety factors larger than 1.67 have been used in a number of instances where this appeared desirable. This is pointed out in various places in this Commentary. No such differentiation is possible in a load test. Accordingly, for such tests only a somewhat larger safety factor is likely to provide the same degree of overall safety which is stipulated throughout those parts of the *Specification* which relate to design. In addition, *Section 6.2(b)* also provides that no untoward local distortions shall occur at test loads equal to dead load plus one-and-one-half live load.

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