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Stiffness and Damping of Sands in Torsion Shear

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SYNOPSIS: A laboratory investigation was carried out into stiffness and damping of sands as sheared in a torsional shear apparatus. In the drained monotonic and cyclic loading tests, a particular care was taken of the small strain measurements in which the secant stiffness was measured over a wide range of shear strain from about $10^{-6}$ to $10^{-3}$. Despite the marked differences in the grain size and the sample preparation method among the sands, a fairly good coincidence of the secant stiffness was seen, in common, in the range of shear strain less than about $1 \times 10^{-5}$ between two types of test using the monotonic and cyclic loadings. However, the response was softer in the monotonic loading tests for the larger strains. It has also been pointed out that the damping when examined in relation to the normalized secant shear modulus was scarcely affected by the confining pressure, and that the values of damping were smaller than those so far available in the literature.

INTRODUCTION

In view of examining elastic properties of soils, it is of interest to know whether the secant stiffness at small strains is similar for the cases as sheared by means of dynamic, static cyclic and monotonic loadings. The stiffness and damping of isotropically consolidated sands as subjected to dynamic or static cyclic loadings has been intensively investigated in the laboratory using, for example, a resonant-column apparatus, a torsion shear apparatus and a triaxial apparatus (e.g., Hardin and Drnevich, 1972; Iwasaki et al., 1978; Kokusho, 1980 among others). However, the strain levels which may be investigated in a resonant-column apparatus are restricted to a range from $10^{-4}$ to $10^{-3}$. Also it is quite difficult to impose small number of cycles, say less than several hundreds, in a controlled manner. In torsional shear and triaxial devices these disadvantages can theoretically be overcome, if decent instrumentation is incorporated. However, the capability of the strain measurement is usually larger than $10^{-4}$.

Teachavorasinskun et al. (1991), using the torsional shear apparatus described in this paper, have shown that the secant stiffness observed for the range of shear strains less than $10^{-6}$ was similar between the static cyclic and monotonic loading tests. Similar results have also been obtained from the triaxial tests (Shibuya et al., 1990a and 1990b). It has also been pointed out that, using the measured values of initial stiffness and peak strength, the hyperbolic function proposed by Kondner (1953) overestimated the stiffness of Toyoura sand as monotonically sheared and it fitted the stress-strain relation well as cyclically sheared.

This paper presents the results of drained monotonic and static cyclic torsion shear tests performed on four kinds of anisotropically consolidated sands. In these tests, the shear strains were measured with an accuracy of the order of about $10^{-6}$ (0.0001%). The interpretation of the test results concentrated on the effects of the pattern of loadings, including the results of dynamic loading tests available in the literature, on the stiffness and damping spanning the wide range of shear strain.

TORSION SHEAR APPARATUS (TSA)

The TSA developed at IIS was used for the study. Drained simple shear tests involving no radial and circumferential strains were performed on specimens of saturated Toyoura sand. The detailed aspects of the TSA and the computer-based servo-control system have been given by Pradhan et al. (1988) and Teachavorasinskun et al. (1991). The specimens of other sands were sheared in a fashion of torsion shear which involved no control of horizontal strains. A relay system was incorporated to accurately measure a wide range of shear strain with an accuracy of about $1 \times 10^{-6}$ (Teachavorasinskun et al., 1991).

SAMPLE PREPARATION

The results of tests performed on four Japanese sands: i.e., Toyoura, Hamaoka, Sengenya and Onahama sands, are presented in this paper. The tests on Onahama sand were performed by Kong et al. (1988). The grain size distribution together with their physical properties are shown in Fig. 1 and Table 1, respectively.

A hollow cylindrical specimen having dimensions of 6cm i.d. x 10cm o.d. x 20 cm high was used for the tests performed on Toyoura, Hamaoka and Sengenya sands, whereas a shorter sample with the height of 10cm was prepared for the tests on Onahama sand. The sample preparation methods are shown in Table 2. Each sample preparation method and the low pressure levels that applied was in accordance with those employed in the model tests (as an example of Toyoura sand, see Tatsuoka et al., 1991). The wet-tamping method is described as the air-dried grains were tamped inside the hollow cylindrical moulds in five and eight layers, for Onahama and Hamaoka sands respectively. The water content as the sample prepared was 1.9% for Hamaoka sand and 0.5% or 1.9% for Onahama sand. The density of air-pluviated Toyoura sand specimens was controlled by having different, but fixed, free fall heights during the preparation. The specimens of Sengenya sand were prepared in four layers by spooning dry grains through water into the hollow cylindrical moulds. The density was controlled by applying vibration, in each layer, by means of tapping the side of the outer mould.

All the specimens were subjected to a suction of 0.05 kPa/cm², and the initial void ratio, e₀, was measured. Except for Onahama sand, the specimens were anisotropically consolidated to reach the prescribed value
which gives rise to approximately 40 conditions for the Hamaoka and Toyoura sand, the value of \( K \) was taken as 0.346 kgf/cm². The overconsolidated consolidation, the value of \( K \) of axial stress, \( \sigma_a \) during the anisotropic consolidation, the value of \( K = \frac{\sigma_a}{\sigma_r} \) (c.f., \( \sigma_r \): radial stress) was fixed at the values of 0.5 and 0.375 for Hamaoka and Sengenyama sand, respectively. In each test of Toyoura sand, the value of \( K \) was taken as 0.52 x e_{o,0} which gives rise to approximately 40 conditions for the sand (Okochi and Tatsuoka, 1984). The overconsolidated specimens of Toyoura sand were prepared by reducing both \( \sigma_r \) and \( \sigma_a \) from a common stress point at \( \sigma_a = 2 \) kgf/cm², by having the value of \( K \) equal to 0.52 x e_{o,0} x (OCR)^m x \( \Phi \). For unsaturated specimens of Hamaoka and Onahama sands, the confining pressure was applied by means of a partial vacuum.

The monotonic loading tests were carried out at the fixed, but different, value of \( \sigma_a \) using a constant shear strain of 0.01 %/min. In the static cyclic loading tests, the stage-cyclic loadings were applied, under a constant \( \sigma_a \), in a manner that the cyclic shear strain amplitude was increased in steps using a sinusoidal cyclic shear stress with a frequency of 0.1 Hz. Although the volume change of the vacuumed specimens was not directly measured, the current radii of the specimens were estimated based on the relationship between the stress ratio, \( \epsilon = \sigma_a / \sigma_r \), and the volumetric strain which was obtained from the other monotonic loading tests performed on a saturated specimen with the similar void ratio and the same initial stress conditions.

Table 1 The physical properties of testing materials

<table>
<thead>
<tr>
<th>Sands</th>
<th>Preparation methods</th>
<th>Specimens Consolidation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamaoka</td>
<td>Wet-tamping (8 layers)</td>
<td>wet Anisotropic</td>
</tr>
<tr>
<td>Toyoura</td>
<td>Air-pluviation</td>
<td>Saturated Anisotropic</td>
</tr>
<tr>
<td>Onahama</td>
<td>Wet-tamping (5 layers)</td>
<td>wet Anisotropic</td>
</tr>
<tr>
<td>Sengenyama</td>
<td>under-water vibration (4 layers)</td>
<td>Saturated Anisotropic</td>
</tr>
</tbody>
</table>

N.B: \( w \) water content as prepared.

A comparison of the stress-strain relations between the two specimens of Hamaoka, Toyoura and Sengenyama sands as subjected to the monotonic and cyclic shear is made in Fig. 3, in which the stress-strain relation of the monotonic loading test is compared to the relationship between the single amplitude shear stress, \( \Delta \tau = \Delta \sigma_a \), and the single amplitude shear strain, \( \Delta \varepsilon \), obtained from the cyclic loading test. In these tests, the specimens were sheared at the same axial stress of \( \sigma_a = 2 \) kgf/cm².
Despite that the specimen subjected to the monotonic shear was slightly denser, it was apparent that the response was softer in the specimen subject to the monotonic shear than in the cyclic shear. The tendency was more obvious at larger shear strains. Similar results have also been observed in the tests using dense Toyoura and Hamaoka sands (Teachavorasinskun et al., 1991).

The secant stiffness of the monotonic and cyclic loading tests, \( G_{\text{sec}} = \frac{\Delta \tau}{\Delta \gamma} \) and \( G_{\text{sec}} = \frac{d \tau}{d \gamma} \), is examined in relation to the corresponding shear strains using a logarithmic scale (Figs. 4 and 5). In Fig. 4, the solid line represents the results of cyclic loading tests, in which the relationship was obtained using a resonant-column and a torsional shear for \( d \tau / d \gamma \) less than and larger than \( 10^{-5} \), respectively (Iwasaki et al., 1978). The values of \( G_{\text{sec}} \) at shear strain corresponding to \( 10^{-5} \), which is denoted as \( G_{\text{sec}}^0 \), in the cyclic loading test were 1010 and 1340 kpg/cm² for the cases of loose and dense specimens tested with \( p' = 1 \) kpg/cm², respectively. Provided that the value of \( G_{\text{sec}}^0 \) increases in proportion to the power of 0.4 for \( p' \), the corrected value of \( G_{\text{sec}}^0 \) becomes pretty close to and slightly larger than \( G_{\text{sec}}^{0.4} \) of the monotonic loading tests in the cases of the loose and dense specimen, respectively (for \( G_{\text{sec}}^{0.4} \) values, see Fig.2).

In Fig. 5, the comparison is made in terms of the stiffness as subjected to the static cyclic shear and monotonic shear. In the case of tests on Sengenyama sand (Fig. 5c), the effect of difference in density is corrected by assuming that the secant shear modulus is proportional to the function of \((2.17+e)/1+e\) (Hardin and Richart, 1963). Note that, for all the tests, the secant stiffness at shear strain level at about \( 10^{-6} \) was almost the same for two specimens subjected to the monotonic and static cyclic shear. Furthermore, in the case of tests on Toyoura sand which include the results of dynamic loading test (Iwasaki et al., 1978), it may be seen that the secant shear moduli at shear strain level of \( 10^{-6} \) were close to each other among the monotonic, static cyclic and resonant column tests.

Another comparison of the stiffness between the monotonic torsional simple shear tests and the cyclic loading tests (the static cyclic loading tests plus the results by Iwasaki et al., 1978) is made, for the same void ratio and the mean principal stress, in Fig. 6. In this figure, the secant stiffness...
moduli of Toyoura sand are directly compared at various strain levels. Note that the secant moduli coincided with one to the other at shear strains less than $2 \times 10^5$ irrespective of the overconsolidation ratio up to four. However, the difference in stiffness between two types of tests was less significant for the overconsolidated specimens (Fig 6(b)).

The results of Hamaoka sand are shown in Fig.7, in which the secant moduli normalized by $(2.17-e)^2/(1+e)$ are plotted against $p'$. As can be seen in this figure, the values of $p'$ spanned between 0.027 and 0.54 kgf/cm$^2$ for the ranges of $e_0$ between 0.61 and 0.63. Like the other kind of sands, the unsaturated specimens exhibited similar trends in that the secant moduli were almost the same at $Y_{\infty}$ equal to $10^{-6}$ (i.e., $(G_{\infty}/G_{\infty}) = 1$) and that, at large strain levels, the stiffness in the cyclic loading tests was higher than that of the monotonic loading tests, irrespective of the mean stress.

**SHEAR STRAIN DEPENDENCY OF SECANT STIFFNESS AND DAMPING AS SUBJECTED TO CYCLIC SHEAR**

The results of static cyclic loading tests are herein examined. The hyperbolic stress-strain relationship can be conveniently denoted in the following form.

$$G_{\infty}/G_{\infty} = 1/(1+e_0/e)$$  \hspace{1cm} (1)

**Fig.6(a) Comparisons of secant shear moduli between the monotonic and resonant column tests (Toyoura sand) for normally consolidated specimens, (1) loose specimens (2) dense specimens.**

**Fig.5 A comparison of secant shear moduli between monotonic and cyclic loading tests, (a) Hamaoka sand, (b) Toyoura sand and (c) Sengenyama sand.**
Fig.6(b) Comparisons of secant shear moduli between the monotonic and resonant column tests (Toyoura sand) for over-consolidated specimens. (1) loose specimens (2) dense specimens.

The symbol of $\gamma_{0.5}$ refers to the shear strain at the value of $G_{s}/G_{max}$ equal to 0.5. Note that, if $\gamma_{0.5}$ is equal to the reference strain $\gamma_0$, the Eq.(1) coincides with the original hyperbolic equation, which is:

$$G_{s}/G_{max} = 1/[1+(\gamma/\gamma_0)]$$

The stress-strain relationship represented in Eq.(1) is convenient for practical use since it simply requires the value of $\gamma_{0.5}$ whereas the relation expressed in Eq.(2) needs two parameters.

Figures 8 and 9 show the relationship between the normalized secant shear modulus, $G_{s}/G_{max}$, and $\gamma/\gamma_0$ for Hamaoka and Onahama sands, respectively. In Fig.8, the results of monotonic loading tests are also shown for comparison. In these figures, the hyperbolic relationship is shown using a dashed curve. Note that (i) the measured relationship was stable for the specimens subjected to cyclic shear; i.e., it was scarcely affected by the kind of sands, the wide range of confining pressure, sample preparation method and the degree of saturation; (ii) the stress-strain relationship can be approximately modelled by using the hyperbolic function shown in Eq.(1). The values of $\gamma_{0.5}$ are plotted against $\gamma_0$ in Fig.10. It should be noted that the values of $\gamma_{0.5}$ increased as $\gamma_0$ increased (see the equations shown in Fig.10). In the case of cyclic loading tests, due to the difference in density, the values of $\gamma_{0.5}$ at similar $\gamma_0$ were larger in the case of Onahama sand than Toyoura and Hamaoka sands (i.e., $D_0=11\%$ and $35\%$ for Onahama sand, and $32\%$ for Toyoura and Hamaoka sands). The values of $\gamma_{0.5}$ were smaller in the monotonic loading tests than the cyclic loading tests for similar $\gamma_0$ (Fig.9). It has been reported that the hyperbolic stress-strain relationship using the measured $G_{max}$ and $\gamma_{0.5}$ (Eq.(2)) drastically overestimated the stiffness of Toyoura sand as subjected to monotonic shear.

The damping characteristics can be seen in Figs. 11 and 12, in which the coefficient of hysteresis damping, $h$, is plotted against $\gamma_0$ for Hamaoka and Onahama sands. In the case of Hamaoka sand (Fig.11), the effect of the confining pressure was obvious in that, in the region of $\gamma_0$ less than about 10−3, the value of $h$ decreased as the confining pressure increased, whereas the effect was not significant for the wet specimens of Onahama sand.
The relationship between 'h' and $G_{0.5}/(G_{0.5})_o$ is shown in Fig.13, in which the results of the four sands are separately presented. Figure 14 shows the average curves, together with some other experimental and theoretical relationships so far proposed. It should be noted that, when the damping is examined in relation to the stiffness, the relationship is scarcely affected by the confining pressure (Fig.13). The damping of sands proposed in the past, based on the theoretical and experimental investigations, seems to be larger than those measured in the present study. Most likely, this may be owing to the poor measurement of shear strain in the previous study. The difference in the value of 'h', say as much as twice, will lead to significant error in estimating seismic ground motion using seismic response analysis.

Fig.9 Relationship between the normalized secant shear modulus and $\gamma / \gamma_{\text{ref}}$ for specimens of Onahama sand ((a) $w=0.5$% (b) $w=1.8$%).

Fig.11 Coefficient of hysteretic damping versus shear strain (Hamaoka sand).

Fig.12 Coefficient of hysteretic damping versus shear strain (Onahama sand).

Fig.10 Relationship between $\gamma / \gamma_{\text{ref}}$ and $p'$ for specimens of Hamaoka, Onahama and Toyoura sands.
CONCLUSIONS

1. For a range of shear strain less than about $7 \times 10^{-6}$, the secant stiffness of the sands was scarcely affected by the type of dynamic, static cyclic and monotonic loadings. Hence the shear modulus in this small strain region is elastic.

2. In the cyclic loading tests, the relationship between $G_{aw}/(G_{aw})_o$ and $\gamma^0_0$ was scarcely affected by the kind of sands, sample preparation methods, degree of saturation and the confining pressure.

3. The relationship between $G_{aw}/(G_{aw})_o$ and the damping was also unaffected by the confining pressure.

4. The damping of sands proposed in the past appears to be unreasonably high owing to the inaccuracy of the shear strain measurements.

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Fig. 13 Relationship between the normalized shear modulus and the coefficient of hysteretic damping. (a) Hamaoka sand. (b) Toyoura and Sengenyama sands and (c) Onahama sand.

Fig. 14 Comparison of theoretical and experimental relationship between the normalized shear modulus and the coefficient of hysteretic damping.
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REFERENCES


Appendix: 'Relay' system used to measure shear strain of the hollow cylindrical specimen.

The shear strain, $\gamma_{rel}$, of both the monotonic and cyclic loading tests performed on Hamaoka, Toyoura and Senenyama sands were measured using a relay system (Fig.A-1). It incorporates two proximity transducers, with the capacities of a 4 mm and a 8 mm, together with the potentiometer. The resolution of the proximity transducers decreases in proportion to the capacity. The 'Relay' measurement is such that, as shear strain increases, the measurement is relayed in sequence of a 4 mm proximeter, a 8 mm proximeter and the potentiometer for the ranges of shear strain ($\gamma_{rel}$) corresponding respectively to less than 0.3%, less than about 1% and more than 1%. The resolution of a proximity transducer with a 4 mm capacity is 0.4 $\mu$m in the current system. This enables the shear strain to be measured by an accuracy of $10^{-6}$. Therefore, by having the 'Relay' system, it is possible to measure the shear strain for a wide range between $10^{-6}$ and $10^{-1}$.

Fig. A-1