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Random growth in two dimensional hexagonal mesh

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Random Growth In Two Dimensional Hexagonal Mesh

By

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1.0: Abstract.

The characteristics of simple random growth in two dimensional hexagonal mesh are studied and analyzed. A numerical solution of four growth models each emulating processes arising in the growth of an epidemic and mechanical fracture are presented. The development of computer simulation programs for each model, provided a graphical and statistical results that describes the mesh's response to the growth processes chosen. The epidemic processes which is represented by two percolation models, illustrates the fact that the use of a hexagonal element mesh does not effect or alter the deterministic behavior expected. The failure of the two mechanical growth models in providing a comprehensive conclusion, necessitate additional evaluation of each model definition and its given parameters. The validity of using a hexagonal mesh to study the mechanical failure growth models remains in question.

1.1: Introduction.

Random growth models have been used to describe a large variety of naturally and artificially spreading phenomena in nature. Applications for these models include processes arising in the growth of an epidemic,[1,2,3] and mechanical fracture.[5] In recent years, the development and availability of high-speed computers have attracted increased interest in the development of computer simulation techniques that are used to study the growth process in these models. Prior to the presentation of the techniques and their results, a description of the random growth models and their relationship to the problems that they emulated would help the reader in understanding the basic concept of random growth.

One of the major scientific goals is to obtain a solution for complex engineering problems at minimal cost in resources and equipment. Examples of these problems in the topics outlined above can be found in the study of random growth behavior of an infected cell in a body, the spread of an epidemic in nature, and the growth of a fracture in an object. A person knowledgeable in these topics may not find it difficult to define the governing equations and boundary conditions for these problems, but the analytical solution can be complicated and in some cases it is not achievable. Several alternatives are available to overcome these challenges. One possibility is to reduce the complexity of the problem by making simplified assumptions that ignore the difficulties presented and reduce the problem to one that could be handled. The reliability of this method is not desirable in all cases, and sometimes this procedure leads to inaccuracies and wrong conclusions. A more viable alternative is to retain the complexities of the problem and attempt to find an approximate numerical solution and analysis. The numerical analysis method begins by modeling the problem with a region of space in which a particular phenomenon is occurring. This region possesses all the unknowns that are dictated by the problem definition and conditions. To reduce the problem to one of finite number of unknowns, the region is divided into elements. These elements contain specific nodes or nodal points at which the approximating function are defined in terms of random variables at each node.[4] This region is defined as a mesh or network within a given boundary that contains uniform elements. The mesh definition and structure varies for each random growth model.

To develop a better understanding of these models, computer simulation techniques are used to study the growth phenomena in a suitably defined mesh. The results obtained from these analyses reflect the global behavior of the problem that is emulated by these conditions.[3] To establish a solid foundation for the numerical solution approach, it is important to fully understand and evaluate the simple random growth in a given mesh without considering the complexities dictated by the specified requirements and mesh construction.

The intent of this paper is to present a computer analysis of several random growth models imposed on a two dimensional hexagonal mesh. The structure of a two dimensional hexagonal mesh is described in section 1.2. The mesh structure provides a simple node layout for growth analysis. The simplicity of the node layout yields an efficient computer program that requires minimal computing time to generate data bases. Since large data bases can be generated economically, they can be used to study model behavior.

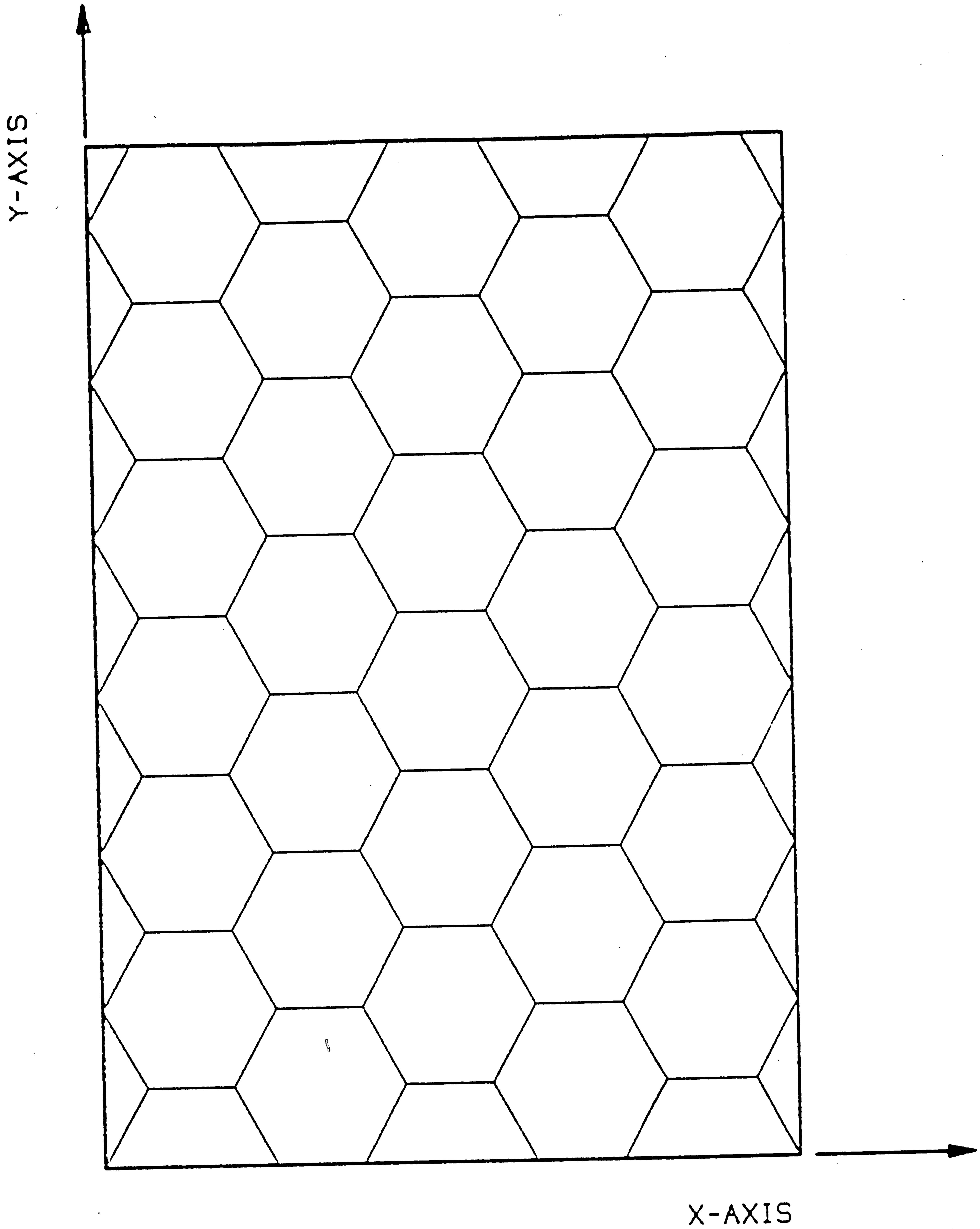
1.2: Mesh Definition and Structure

A mesh is constructed from individual elements contained within a known boundary. The shape of these elements is uniform within a given mesh. The most common shapes of elements used in simulation techniques are square, triangular, and hexagonal. An element is constructed from nodes positioned along its perimeter. These nodes are considered as sites where the probability of status change could occur at a given time interval. The passage or link for random growth between nodes is restricted to be along the perimeter of the element only. These links can be described as bonds between nodes such that these bonds will conform to the geometric shape of the defined element. Both nodes and bonds are shared by neighboring elements.

Each mesh is given a set of conditions that models the object behavior. These conditions are then applied to nodes contained within the mesh. The bond effect on mesh response to random growth is dependent on the given conditions. In some models the bonds are defined as having no influence on node status at any given step.[1,2] However, more detailed models necessitate added complexities of bond specifications,[5,6] which do have a direct impact on the growth process.

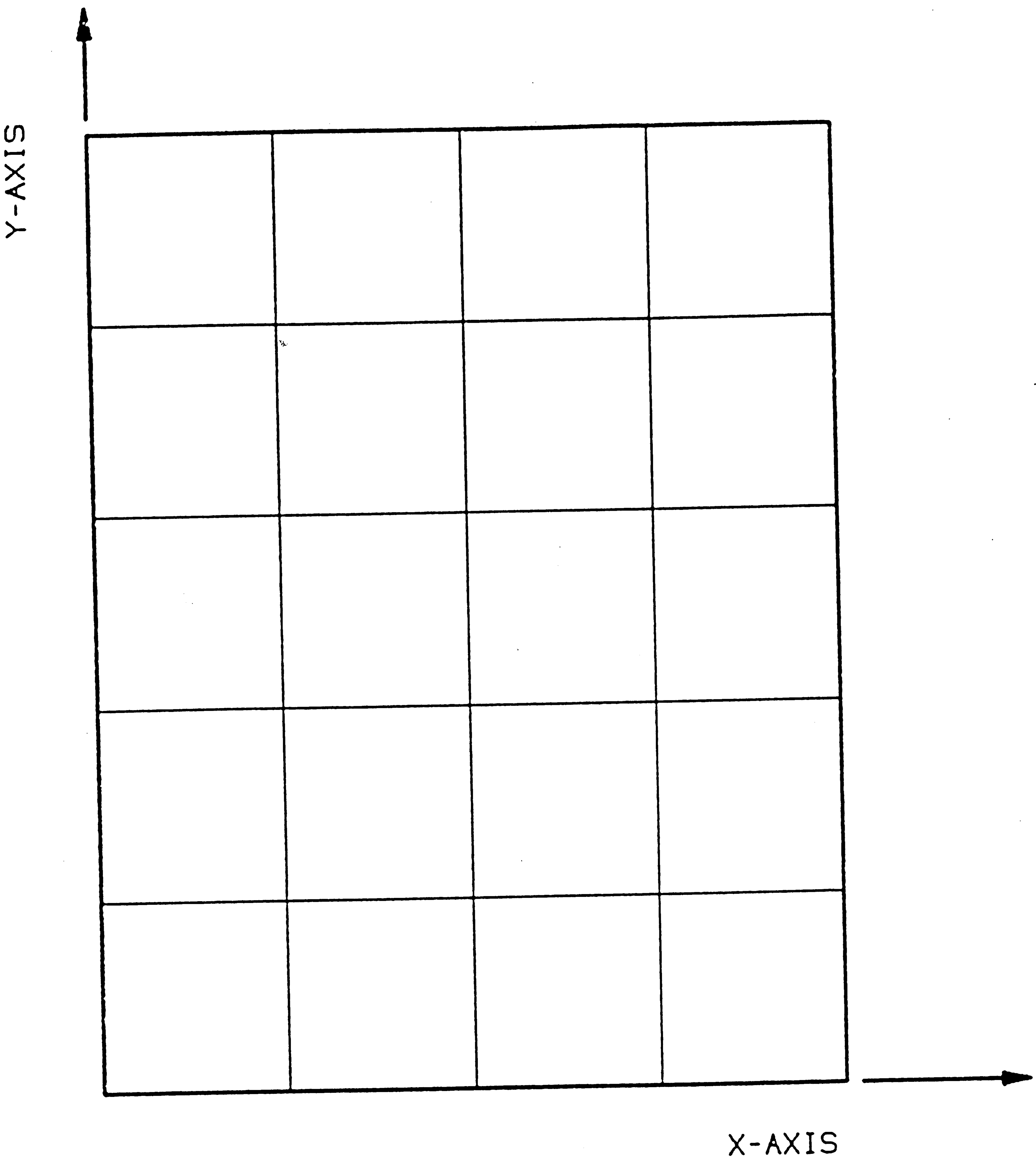
The relative size or scale of a particular mesh, and elements contained within, is considered irrelevant to the size of the physical object or the natural phenomena that the model represents. A mesh could be viewed on a microscopic or on an infinitely large scale. For example, the same mesh structure used to describe an epidemic growth process for an infected cell in a body could be used to describe the outbreak of measles in a population, or the spread of a forest fire.[3] It is true that each of these growth models require a different set of conditions and their actual size varies, however the basic approach to the numerical solution remains the same.

The choice of element type in a mesh is a matter of engineering judgment based on accumulated experience.[4] In some cases it is possible to choose the elements in a way that leads to an exact representation, but this occurs only in special cases. For example, a model that emulates a support structure that consist of several trusses which possess a specific uniform geometric shape. The most logical element shape should be that of the actual physical geometric shape. The reasoning behind choosing a hexagonal mesh to that of a square or triangular, is related to its geometric structure. The geometric structure of a hexagonal mesh shown in Figure 1, is such that a typical node is surrounded by three neighbors, with an exception of nodes along the mesh boundary. In the case of square or triangular mesh the node layout increases in complexity. For example, a non-boundary node in a square mesh shown in Figure 2, is surrounded by four neighbors. A typical non-boundary node in a triangular mesh shown in Figure 3, is surrounded by six neighbors. Since the relative scale of an element is irrelevant to the mesh size, the added complexity to the node layout may not be necessary.



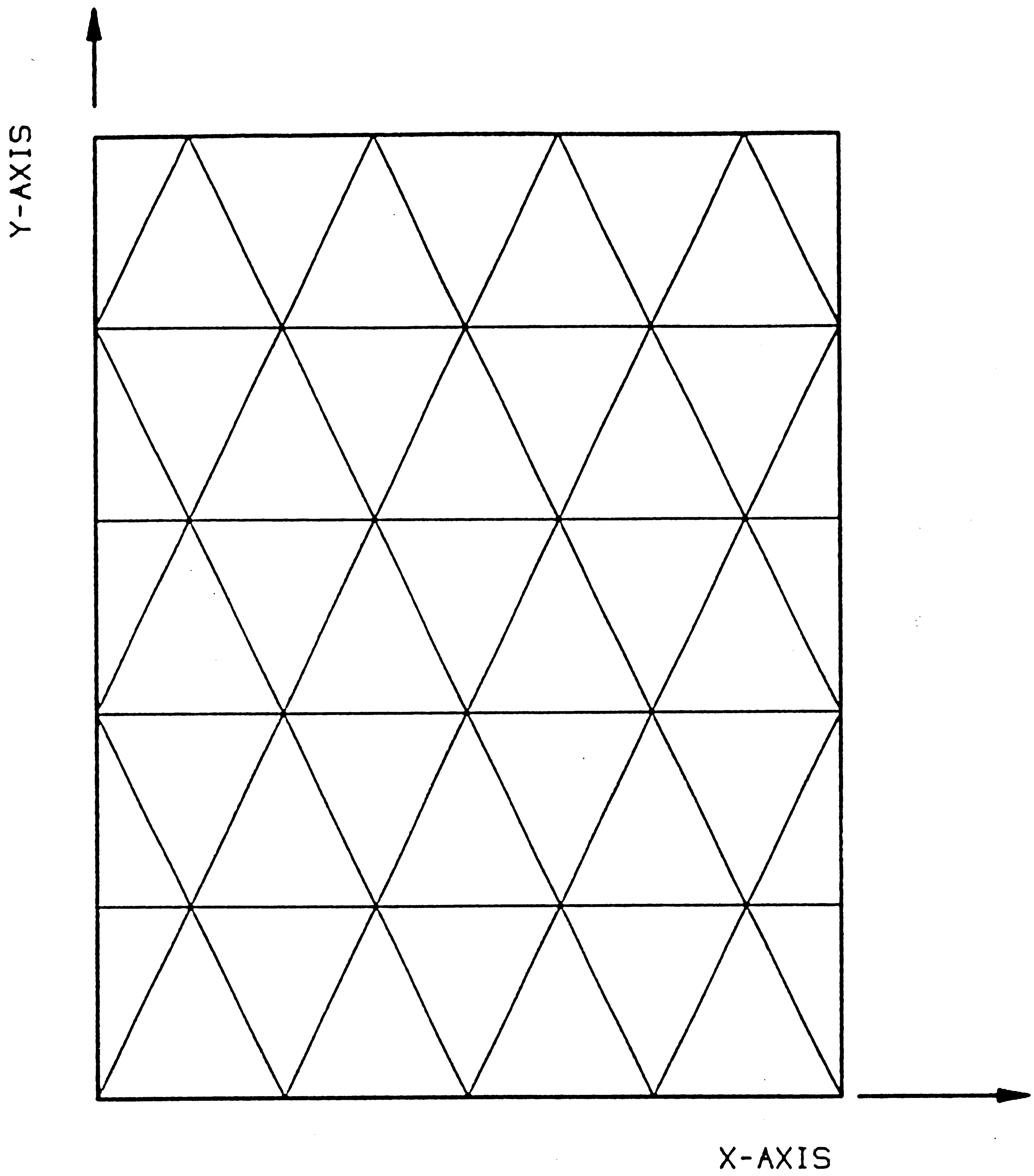
HEXAGONAL MESH

FIGURE 1



SQUARE MESH

FIGURE 2



TRIANGULAR MESH

FIGURE 3

1.3: Model Definition.

The computer simulation techniques and programs are based on two growth models with variation of each. The models depicted are the percolation growth model, and the mechanical failure growth model.

The percolation growth model is chosen to represent a simple random growth process in a suitably defined hexagonal mesh. This mesh exhibits the same structure as defined previously in section 1.2. An analogy of the word percolation is one based on visualizing the mesh as a fluid flow network. Within this network are bonds and nodes acting as pipes and valves. The probability of node/valve failure is considered as a random variable which designates its status. If a node/valve fails it is considered open, otherwise it remain closed until such time where the failure occurs. When a node fails the fluid is permitted to freely flow through the connecting bonds/pipes. If the nodal probability of failure increases to a point where the entire network is saturated, the mesh is considered as reaching its critical probability of failure. This analogy is used to describe the random growth in the two models presented in section 1.3.1. In substituting an epidemic process for the fluid flow process, the model will then emulate the nodes as cells and the bonds as a path in which the epidemic is spread throughout the network.

At the end of the process the growth is represented by a cluster of failed nodes. The size and shape of the cluster is dependent on the probability of nodal failure. Various growth behavior are studied by altering the definition of the nodal probability of failure.

The mechanical failure growth models differ from the percolation models by their definition of the growth process relative to the mesh nodes and bonds. This is achieved by using the hexagonal mesh construction previously defined and altering the definition of the nodes relative to the corresponding bonds. The nodes are considered as sites where the growth is allowed to propagates along the connecting bonds based on the given probability of nodal failure. For example, if the model emulates the study of a fracture growth in an object that is subjected to an external force, the mesh is viewed on a microscopic scale. The location of each node is arbitrary selected by the type of elements contained within . The mechanical failure growth models described in section 1.3.2 are chosen to emulate simple problems without the added complexities. An example of these models lies in the need to analyze the fracture propagation that originate from a void in a mechanical shaft, or a structural member that is subjected to various loading conditions.

1.3.1: Percolation growth model.

The percolation growth model is based on an example given by Richardson.[2] This model emulates the spread of infection in a body. The infection is originated from a single cell where an infected cell is represented by a "1" status, otherwise it is considered healthy and represented by a "0". The Gp model is described as being a honeycomb (hexagonal element) mesh Z, of n dimensional space having a node at the geometric center, such that Z has central symmetry. The initial conditions at time t=0, is such that the node at center is "1", and all other nodes are "0". Time is assumed to progress in discrete jumps (t=0,1,2,...). Assume that if a node is "1" at time t, it remains "1" at all future times. If a node is "0" at time t, and all of its neighbors are "0" at that time, then it remains "0" at time t+1. However, if a node is "0" at time t, and at least one of its neighbors is "1" at that time, then it becomes "1" at time t+1 with probability p and remains "0" with probability of 1-p. The probability p is an independent random variable, such that p belongs to [0,1].

To reduce the model complexity, additional restrictions and/or assumptions are added to the basic definition. These restrictions are related to the overall shape of the mesh, orientation of elements contained within, condition of nodes along mesh boundary, and the effect of bonds on the growth process. It is assumed that the overall shape of a mesh is a square with m x m dimensions, that contains hexagonal elements such that one of the three bonds formed at the triple-junction node is parallel to the X-axis (Figure 1). The growth or passage between two adjacent nodes located along the boundary is not permitted. The reasoning behind this assumption is to eliminate the possibility of having what could be described as a loop back growth. Therefore, the passage to and from a node along the boundary is via neighboring nodes that are not located along the boundary. The bonds are assumed to have no effect on the percolation growth process. They are used as a conduit between neighboring nodes. The growth progression through a bond is considered complete for all times if and only if the two defining nodes obtain the same status "1" at any given time.

To obtain a variation to the Gp model, consider a Gq model such that all the conditions, restrictions, and assumptions defined for the Gp model remain the same except for the probability of growth. If a node is "0" at time t and at least one of its neighbors is "1" at that time, then it becomes "1" at time t+1 with probability q and remains "0" with probability of 1-q, where q=(1/i), and i is defined as the number of nodes with "0" status at time t, such that each have at least one of their neighbors with "1" status. The nodes that define the value of i, are located around the cluster. For example, at time t=0 the cluster consist of a single failed node at mesh center. The single node is surrounded by three nodes in which the growth could spread. The value of i at this time is equal to three, and the value of p is equal to 1/3. This process continues until the growth is stopped.

The two percolation growth models Gp and Gq, emulates two different epidemic processes. This is achieved by altering the probability of failure for each. The Two models should provide sufficient details to study the mesh response to each growth. In the Gp model, p is defined as a random variable with its value remaining constant for each sample. The value of q in the Gq model is a dependent random variable, where q is inversely proportionate to the number of nodes surrounding the cluster at each time step. The results and analysis of these models found in chapters 3 and 4, will illustrate the basic difference between the two growth processes and their effect on the mesh.

1.3.2: Mechanical failure growth model.

The mechanical failure growth model is derived from the general failure of an object under a tensile force. Two models are developed to define this behavior, the Mp model and the Mpc model. Both models share basic definitions and mesh construction. Each has a two dimensional hexagonal mesh Z of m x m dimensions that is subjected to a tensile force along the boundaries parallel to the X-axis. The node at the center, which provides mesh Z with central symmetry is assumed to be "1" at time t=0, and all other nodes are "0" at the same time. This indicates that the node at center has failed and all other nodes are normal.

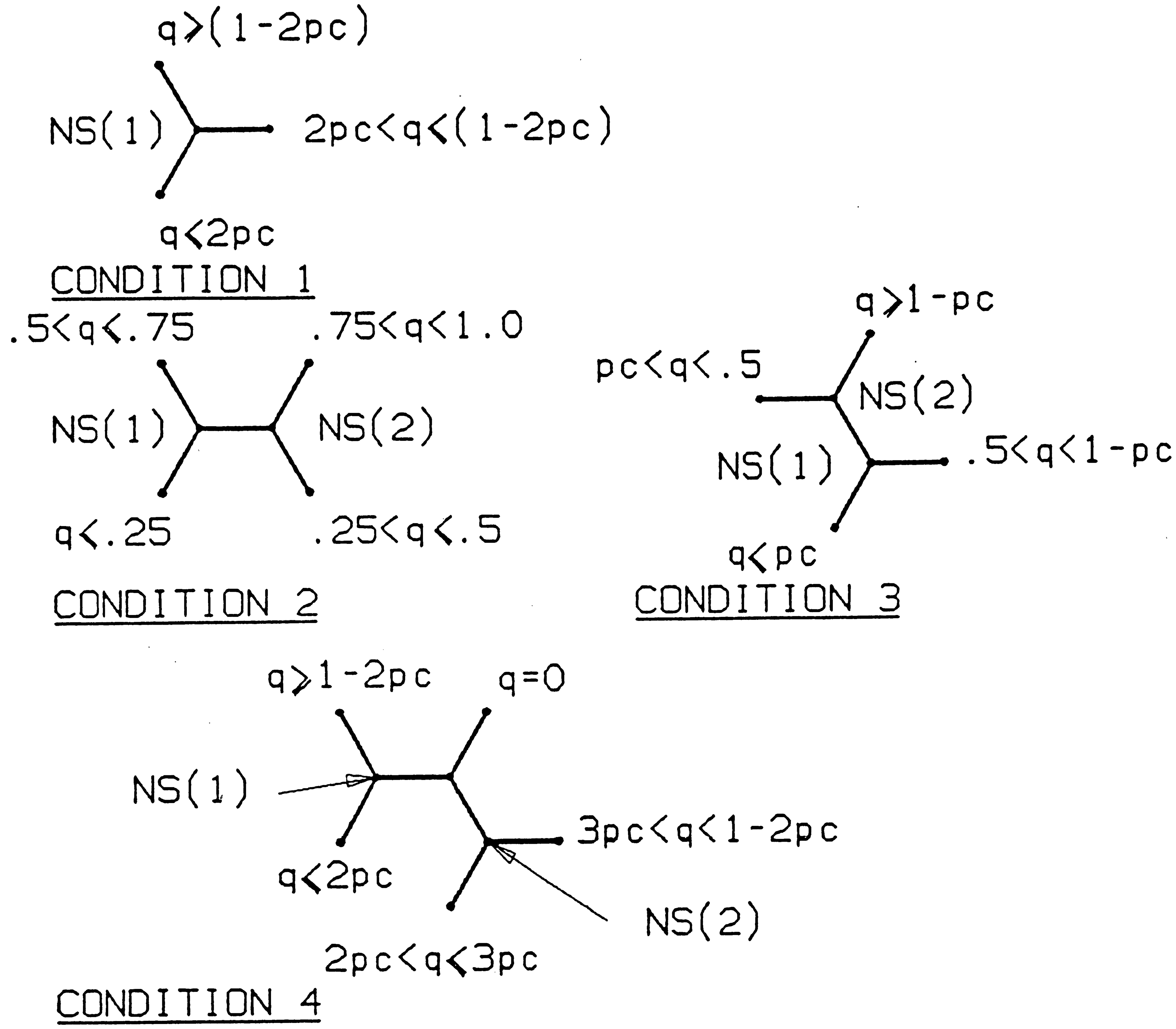
The conditions for growth between two adjacent nodes along the mesh boundary and element orientation remains the same as in the Gp model. Additionally, the growth progression within mesh Z is conducted along the leading nodes of the fracture path, no secondary fractures are allowed to branch off this growth profile.

The Mp model is used to emulate the fracture growth in an object such that the probability of fracture growth is conducted along the weakest node surrounding its leading nodes. To simplify this model, it is assumed that the nodal bonds have no effect on the growth process. They are considered as being the defining shape of the fracture profile. In order to avoid loop back growth, the fracture progression can not be repeated along a failed bond, where a failed bond is categorized as having its defining nodes achieving a failed status at time t, which will remain as such for all future time. A bond is considered normal if at least one of its defining nodes is "0" at any given time.

The probability of fracture growth is defined such that as time progresses in discrete jumps, the probability of node failure is considered only at the neighboring nodes surrounding the fracture of two leading nodes. It should be noted that at time t=1, the fracture starts at a single triple junction node as defined by the initial conditions at time t=0. The probability of fracture growth at time t is defined as a random variable p, such that p=max(p1,p2,...,pk), where pk's are random variables indicating the probability of failure of each neighboring node surrounding the fracture two leading nodes.

The Mpc model varies from the Mp by the bond orientation relative to the X and Y-axis. The probability of fracture propagation through the mesh is affected by the number of horizontal bonds that surrounds its leading nodes. Instead of allowing the fracture to seek the weakest link in the network, the fracture is probabilistically trained to seek the shortest route to the boundaries that are perpendicular to the direction of the force applied. This is achieved by defining the probability of fracture growth at time t as a random variable q, such that q belongs to [0,1]. As time progresses in discrete jumps, the probability of nodal failure of each node surrounding the fracture leading nodes is defined as a function of pc which belongs to [0,1]. The value pc is equal to 1/c, such that c is a constant. Figure 4, illustrates the various conditions that apply during the nodal simulation. It should be noted that condition 1, is applicable at time t=1 only. The fracture failure originates at a single node failure. The remaining conditions in this figure occur at any given time t greater than 1. To eliminate having invalid inequalities shown in Figure 4, additional restrictions are applied to the value of c, such that c is greater than 6.

The results and analysis for these models are illustrated in chapters 3 and 4, respectively. The effect of the growth behavior on the hexagonal mesh are presented and analyzed.



pc=1/c, where c is a constant, such that c>2.
q= probability of fracture growth at time t.

NOTE:

- NS(1), and NS(2) for conditions 2,3, and 4 indicates the fracture location at time t-2 and t-1.

Mpc MODEL NODAL CONDITIONS

FIGURE 4

2.0: Existing models.

A variety of compatible models exist in the fields of percolation and mechanical failure topics. Each of these models are targeted to a specific application that dictates a set of conditions used to obtain final results in the study of random growth behavior in an applicable mesh.

For unspecified reasons, the majority of referenced authors selected the use of a square or triangular mesh structure for their computer simulation analysis, and in some cases referred to possible results that could be obtained by using a hexagonal mesh.

The following sections in this chapter will highlight these compatible models, and the specific results that are related to the models given in chapter one. It should be noted that this outline is not a comprehensive study of these models. For material omitted, the reader should refer to the sources listed in the reference table.

2.1: Richardson's growth model.

Richardson's growth model (Gp) described in section 1.2.1, was introduced by Richardson in his paper titled "RANDOM GROWTH IN A TESSELLATION".[2] Richardson applied the Gp model on a square tessellation for various probability values (p), with different number of time discrete intervals. The basic results obtained are directly related to the overall shape of the random growth profile. In his conclusion, Richardson stated that as p approaches 1 the growth profile edges become smooth with a diamond-like shape. Conversely, as p decreases the roughness of the growth profile shape increases, and become circular.

The proof for the model behavior was later expanded upon by Durrett.[3,8] Durrett's use of the same model did not provide any indication of the effect of substituting a hexagonal mesh for the square tessellation. His analysis did confirm the growth profile as stated by Richardson.

Meakin,[7] in similar analysis hinted that if similar procedures and conditions are applied to a large hexagonal mesh, the growth distortion could take on a hexagonal shape as p approaches 1. Similarly as p decreases the roughness of the growth profile shape increases, and becomes circular. It should be noted that the results obtained by Meakin are based on a similar model to that of Richardson. The deviation is induced to the conditions applied to the nodal interaction with the basic growth cluster. In addition, the mesh size used is relatively large compared to that used in the previous analysis.

2.2: Hooke-type spring model.

A variation of the classical Hooke-type spring model was used by Beale and Srolovitz.[9] In their study, they evaluated the elastic fracture growth in random material such as minerals and ceramics.

The model construction is based on a two dimensional triangular network of springs. Initial conditions and elastic parameters govern the status of each spring at various steps. The status of each spring is defined as being present (normal) or absent (failed) from the system. The mesh is subjected to a uniform external strain applied in the X- direction, which continues until no connected path exist across the sample. The boundary conditions are maintained in the horizontal direction, while the top and bottom surfaces are free.

In their conclusion, the mesh breakdown process was described as having two steps of failure. The first step causes a number of springs to fail that forms a critical defect crack which results in a system with zero elastic modules. Hence, under a large strain a fracture forms across the entire sample resulting in the system failure.

Of particular interest from the results obtained in this model, is the system or mesh behavior during the second step of breakdown. The fracture propagation conditions are similar to the mechanical failure growth model given in section 1.3.2. These similarities are reflected by the fracture growth. The fracture is originated from a given area (single nodal failure verses critical crack failure). Additionally, the basic assumption that the system in the second step of failure has no elastic properties also apply to the Mp and Mpc models. The intent of this comparison is to evaluate the results of the Mp and the Mpc models relative to that of the models used by Beale and Srolovitz.

2.3: Elastic percolation model.

The three models used by Sahimi and Goddard,[6] are based on a random network of two dimensional triangular Hooke-type springs. Each having a special case in which both the spring constants and the critical strain are stochastic quantities.

In their analysis, they stated that a single crack is formed which propagates throughout the mesh, hence splitting it into two pieces. Additionally, they stated that the crack tip or leading edges, seek the easiest path through the mesh or network. Their observation regarding side branches occurrence to the fracture is assumed as having no statistical value.

Similarly, the results and observations made about the crack growth in these models are similar to that of the parameters given to the Mp and Mpc models. The intent is to compare the results of the Mp and Mpc models to that of Sahimi and Goddard. This evaluation should illustrate the effect of the fracture growth propagation in a hexagonal mesh model.

3.0: Computer simulation development.

The development and execution of the computer programs, is carried out on a compatible "IBM" PC-XT ! computer. The computer hardware is equipped with an Intel 8086 main processor, 640 K-bytes of main memory , and PC-DOS 3.10 as an operating system.

WATFOR-77 !! fortran compiler and editor are used for writing, compiling and executing the program source code.

The computer hardware computing capabilities are determined by the size of its memory and its central processing unit (CPU) type. These limitation affected the size of the program output data. Hence, in order to decrease the time required to execute the simulation program for each sample, the mesh overall size is limited to a specific number of elements within it. After several experiments, it was observed that a reasonable mesh size should contain no more than about 3,530 nodes. This equates to about 42 by 42 hexagonal elements.

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In the previous chapter, two assumptions were stated regarding the mesh size and symmetry. The mesh could be viewed on a microscopic or an infinitely large scale. In some models the mesh does not reflect the actual physical size of the region in which the growth phenomena is occurring. The data and analysis obtained from the simulation, is used to statistically estimate the global behavior of the entire space. For example, if the model emulates the spread of infection in a body, the mesh and its contents are not a true representation of the body. The mesh and its nodes are but a small sample in which the random growth is studied. Therefore, the limitation imposed by the number of elements allowed in a given mesh should not effect the output data that describe the growth behavior in the given model.

3.1: Computer simulation program.

The computer program developed for the analysis of the random growth models is formatted into several modules or subroutines, each addressing the general and specific requirements given in the model definition. The basic objective is targeted to the user interface. A user friendly program provides an easier interface and flexibility. By minimizing the time required for program input and setup, which in turn reduces the time required by the user to generate large numbers of output data, is viewed as being essential to the model analysis.

Some of the general features and flexibility of the program are common to the input data required, mesh nodal matrix, graphical and statistical output. For example, the program used to simulate the Gp model requires minimal input. The user has to input the number of nodes along the X-axis and the Y-axis (Figure 5), the number of time steps or intervals, the number of samples desired for this specific run, and finally in this example only the nodal probability value p (Section 1.3.1).

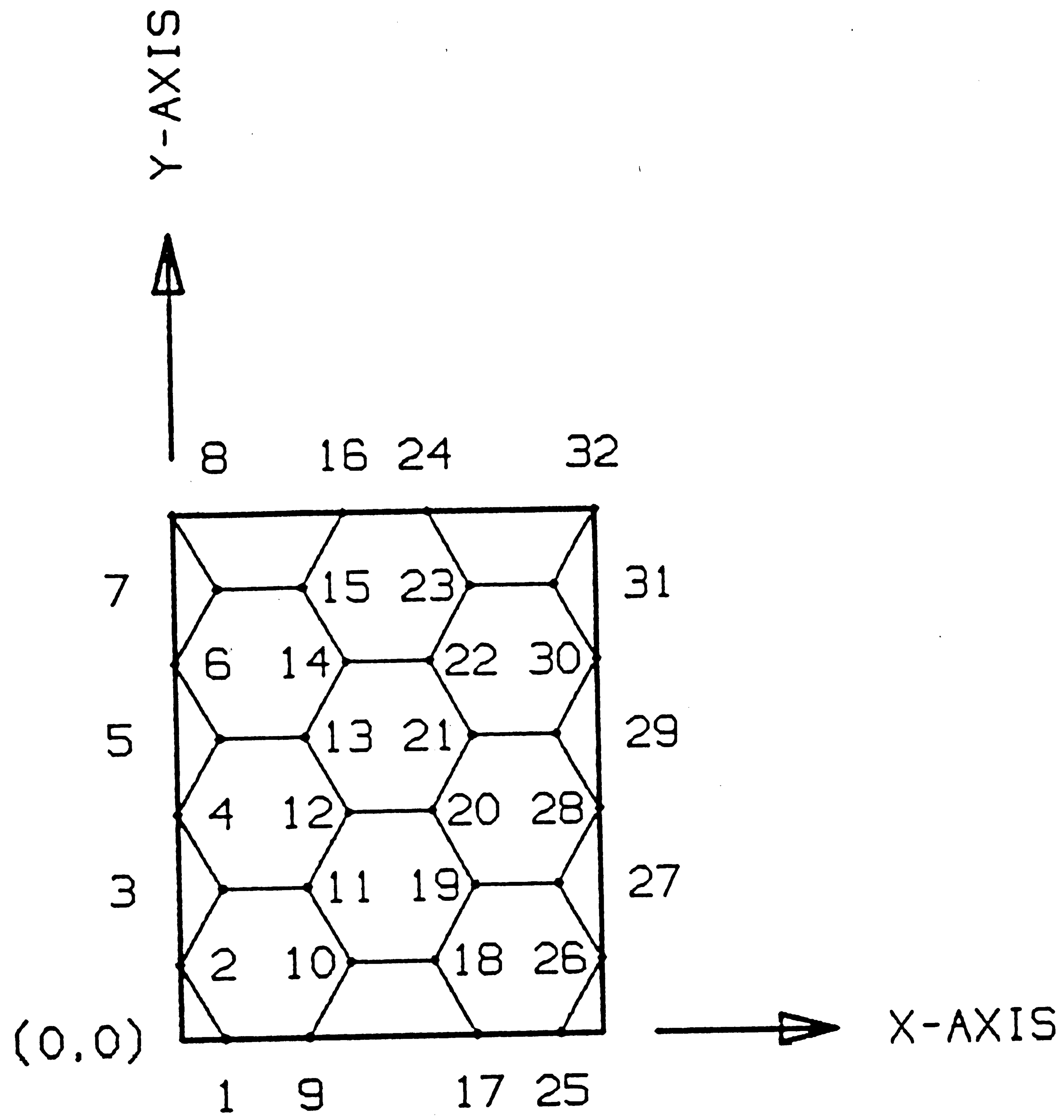
The construction of the nodal matrix is carried out by the computer program. This matrix contains vital nodal information regarding each node identification number, node position relative to its surrounding neighbors, node boundary condition if applicable, and node status at each time interval. During the execution of the program, the information contained in the nodal matrix is stored, retrieved, and updated at every time step.

To minimize the storage space allocation for the nodal matrix, hence increase the computer program efficiency, a nodal identification procedure was developed. The specific geometric properties of the hexagonal element mesh were used to define the conditions of the nodes to each node in question at the required time interval.

Figure 5 illustrates the input required for a typical mesh, the mesh orientation relative to the axis, and each node numbering sequence, which is stored in the nodal matrix NODE. The size of matrix NODE is NN by 5, where NN is the total number of nodes contained within the mesh, such that NN=(NX)(NY)(2).

Figure 6 shows the various conditions and values assigned to the nodal matrix. The status of each node is defined by the value of NODE(NNi,4), and NODE(NNi,5). When NODE(NNi,5) equals one, node NNi has failed, and when NODE(NNi,5) equals zero, node NNi is normal. It should be noted that the value of NODE(NNi,4) and NODE(NNi,5) are equal. NODE(NNi,4) is used internally to the program as a counter for various steps taken during each time interval.

The graphical output generated is relatively simple and self explanatory. Several selected examples are

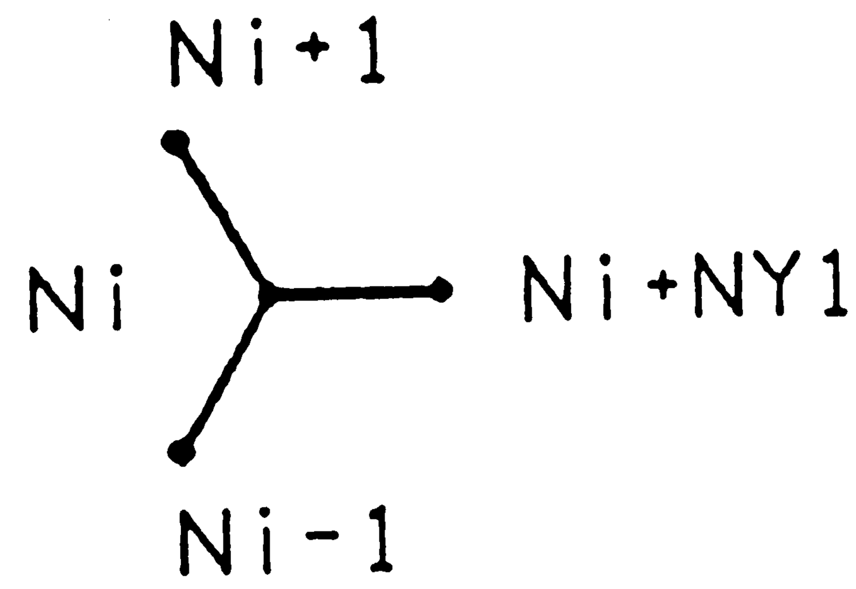


INPUTS:

Number of nodes along the X-axis: NX=4
Number of nodes along the Y-axis: NY=4
Number of discrete time intervales: NTIME=A
Number of samples to be evaluated : NRUN=B
Where, A and B are constants.

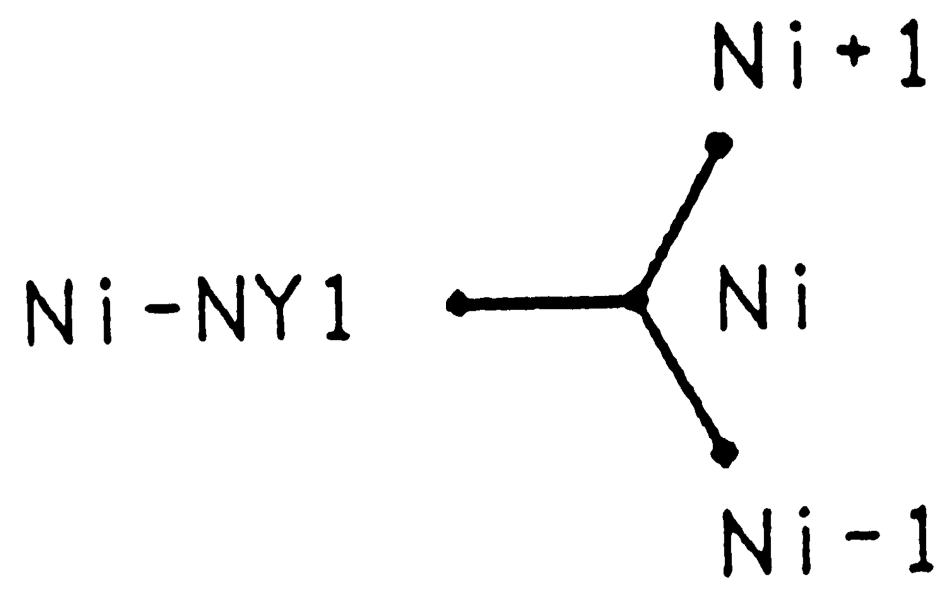
MESH CONSTRUCTION

FIGURE 5



NODE(NN,1)=Ni
NODE(NN,2)=1

FIGURE 3a



NODE(NN,1)=Ni
NODE(NN,2)=0

FIGURE 3b

Where, NY1=NY*2

If NODE(NN,2)=-1 This implies that the node is located along mesh boundary.

Evaluation of NODE(NN,3) is required such that NODE(NN,3)=C.

If C=1: Node Ni is along the X-axis boundary.

If C=2: Node Ni is along the boundaries parallel to the Y-axis.

If C=-1: Node Ni is along the boundary parallel to the X-axis.

NODAL MATRIX DEFINITION

FIGURE 6

shown in the following sections of this chapter. The use of the final stored values of NODE(NNi,5) which is either a zero or one, indicates the status of node NNi. The program will generate and store the graphical output of each sample with a set of one's and zero's defining the random growth profile.

To further reduce the output data files, and limit the memory size required by the program, some of the statistical data used during and/or at the final steps of the program are not printed. This data is found in the source code under debug routine headings. At the discretion of the user these subroutine could be activated and printed for additional statistical information.

Typical to all of the computer simulation programs, the type of the random number generator used has a direct impact on the results obtained. In this project, two random number generators are used.[10,11] Each random number generator subroutine provides a set of random numbers having a uniform distribution. It should be noted that regardless of which subroutine is used, extensive evaluation and testing is required to insure the validity of these numbers. A hint to the reader regarding this issue could be summarized by stating that the mean value of a set of uniformly distributed random numbers ranging from zero to one, should be one half, with a standard deviation equal to 0.2887.

The random number generator produces a sequence of numbers between zero and unity. To initialize this sequence a seed is required. If this seed remains constant throughout the simulation program, some of the samples will yield identical results. Therefore, in order to generate large numbers of different samples, the random generator seed should be randomly selected and utilized by the program for each run. Hence, an argument to reset the seed is incorporated in the main computer simulation program.

The computer program used to simulate the random growth profile for the various models, requires alteration and modifications to the specific modules that address unique conditions that applies to each model. Hence, each model has it's individual program, however the basic structure of each remains the same. A helpful hint to the reader is related to the management of these files. The naming convention for each file should reflect the model name, and its revision level. Implementing this procedure would avoid misplacement or misuse of old files.

3.1.1: Other computer programs.

Two computer programs are used to analyze the statistical output for each model. The first program will fit an equation to a set of data. The least squares method is used for the curve fitting program. This program is written in Basic and is compatible with the hardware listed in section 3.0. The program fits a straight line, an exponential curve, and a power curve to a given set of X, Y coordinate points. Coordinate points are imported from the statistical output data files generated by the main simulation program. To help determine which equation best fits the data, the following information is printed for each equation:

Coefficient of determination (1.0 is a perfect fit).

Coefficient of correlation (1.0 is a perfect fit).

Standard error of estimate (0.0 is a perfect fit).

After the first step has been completed, one or more of the three equations are chosen to calculate the Y intercept for a specific value of X.

Upon completion of the curve fitting program, the data associated with the equation chosen that best fit the modeled data is again imported to a plotting program. This program is a standard software package called LOTUS, which is commercially available.

LOTUS is a copyright of Lotus Development Corporation. For operation and use of this software package, the reader should refer to the documentation that is provided with this package.

3.2: Growth profile and results.

The results obtained from the computer simulation programs describing each model growth profile, are presented in the following sections of this chapter. The presentation covers the standard procedure and steps taken throughout the development of each model. In addition, selected graphical and statistical data obtained from the simulation programs, are presented.

It should be stated that during the initial stages of the project development, considerable amount of time was consumed in understanding of the fundamental characteristics and behavior of each model. The various observation made based on these experiments, necessitated additional simulation time.

The model analysis includes a description of some of the observations, however, the elementary notes are omitted for added clarity.

3.2.1: Gp model.

The computer simulation results obtained for the Gp model are derived from two sets of data, each contain nine different runs having ten mesh samples each.

The nodal probability value p for each run is incremented by one tenth (0.10), starting with p equal to 0.10, and ending with p equal to 0.90.

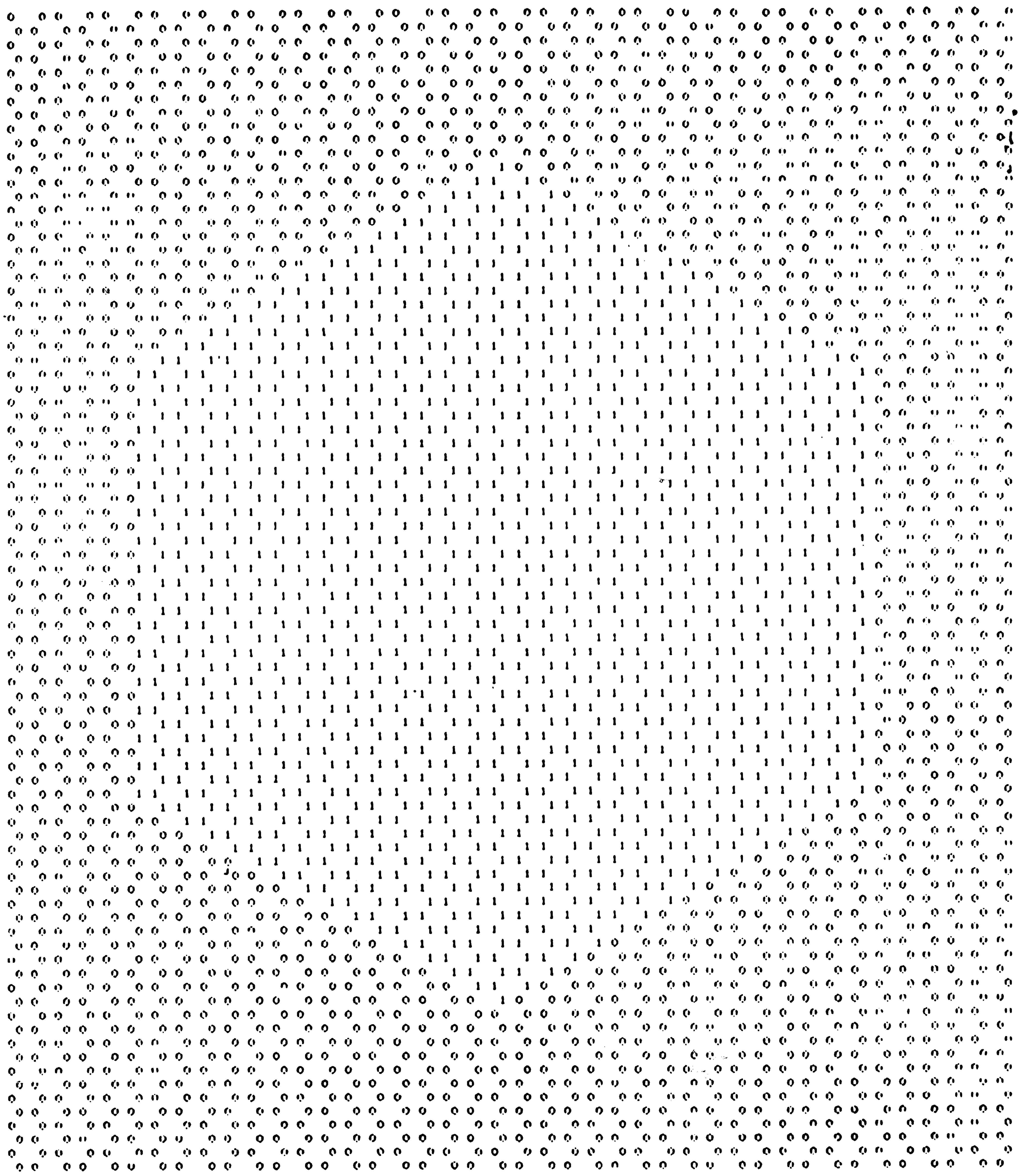
The difference between the two sets is in the mesh size. For the first set the mesh size is relatively small. The mesh is constructed from 20 by 20 elements, which equates to 800 nodes. However, for the second set the mesh contained 42 by 42 elements, which equates to 3,528 nodes. From experimentation, the value of the time discrete parameter (NTIME) is determined as being equal to 10 for the first set, and 30 for the second. The duration of each time step is not measured in terms of actual CPU time.

The growth profile shape generated from both sets is identical. Therefore, the presentation is focused on the second set in order to avoid repetitions.

The shape of the growth profile obtained, is observed at three different ranges of the nodal probability p. The three ranges are at the upper, middle, and lower values of p, which belongs to the interval [0,1]. It is noted that a shape transformation occurs between each range. The shape of the growth profile at the upper value of p, such that p ranges between 0.80 and 0.90, is clearly defined by Figure 7 and 8. These figures show that the growth profile take on a hexagonal shape, at p equal to 0.80 and 0.90 respectively. This hexagonal shape is symmetrical around the mesh initial point of failure that is defined in the model initial conditions.

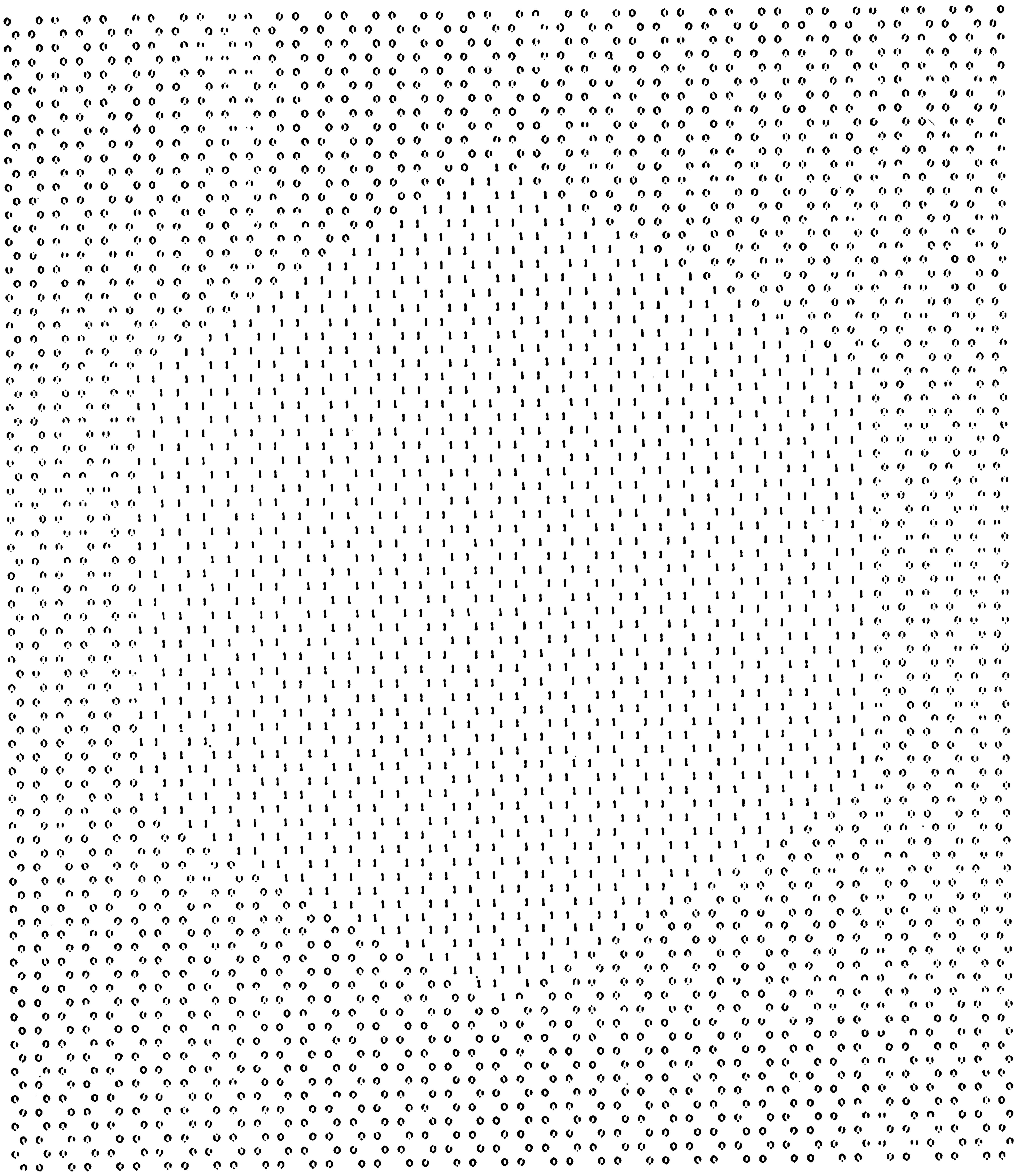
As p decreases and enters into the second range, the growth profile shape increases in roughness. Figure 9, shows a typical growth profile for p equal to 0.70. It should be noted that the shape is not yet circular, but instead a transformation occurs between the nodal probability values of 0.70 and 0.80.

The transformation of the growth shape profile continues for values of p ranging between 0.40 and 0.60. Figure 10 shows the growth profile shape for p equal to 0.50 which indicates that the shape become more circular. Further evaluation of this figure indicates that the growth shape is not exactly circular. Several normal nodes are contained within a circle drawn around the growth contour. This circle has its center at the initial point of failure (fixed at mesh center), and a radius r encompassing the outer edges of the cluster. The indication that the growth profile losses its symmetrical behavior, continues into the lower range of the nodal probability (p ranges between 0.10 and 0.20).



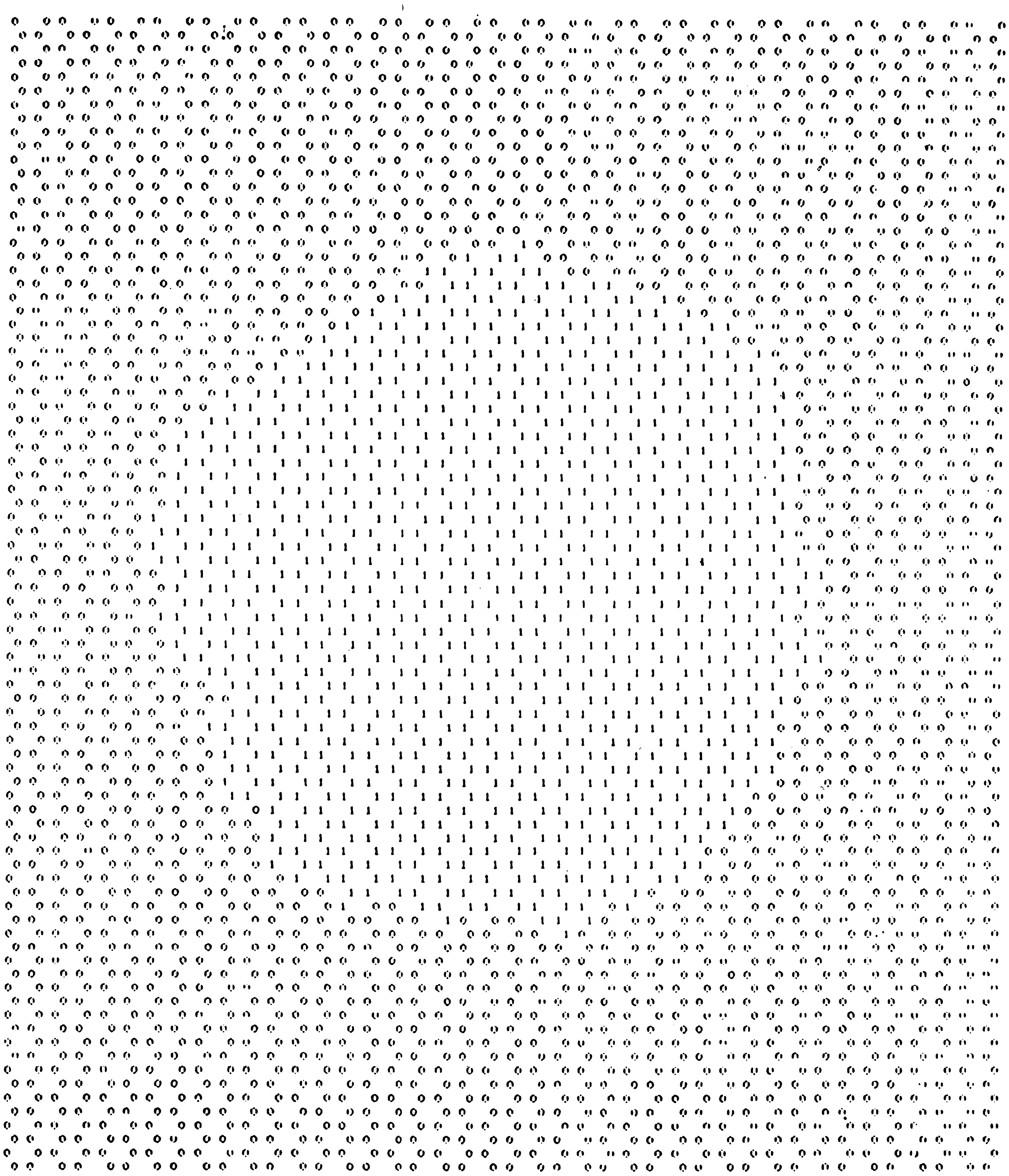
Gp model graphical output, with p=0.80.

FIGURE 7



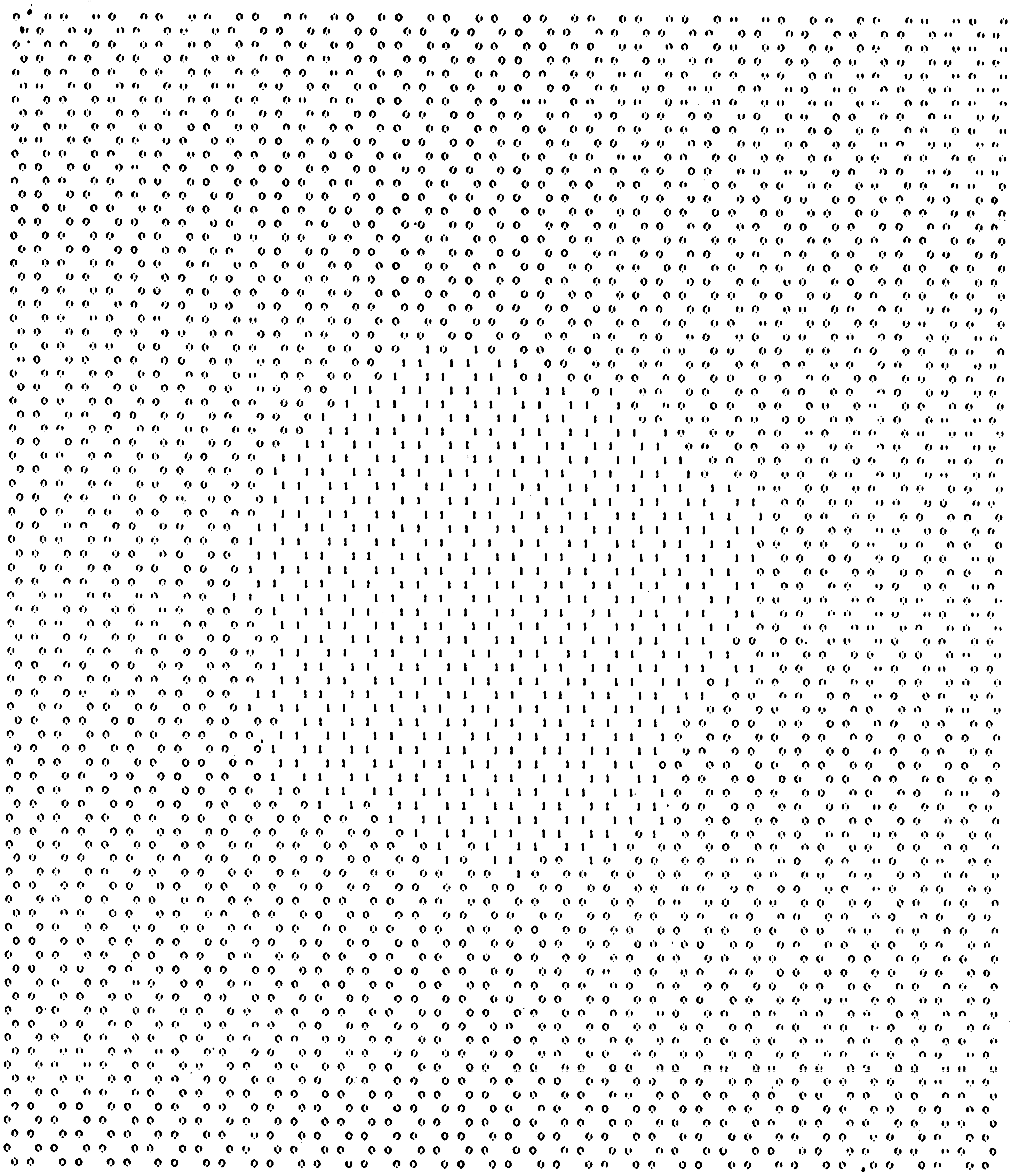
Gp model graphical output, with p=0.90.

FIGURE 8



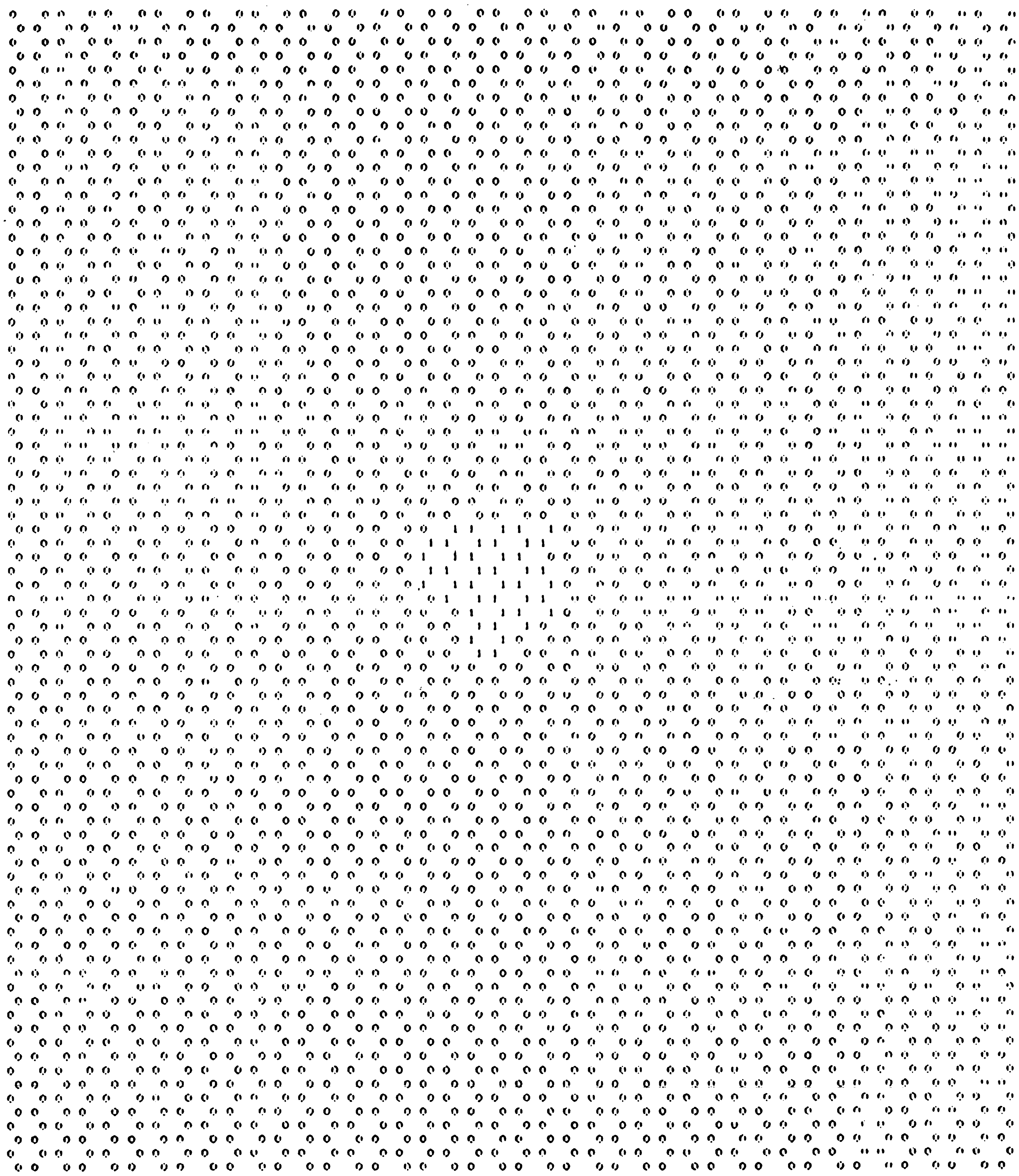
Gp model graphical output, with p=0.70.

<u>FIGURE 9</u>



Gp model graphical output, with p=0.50.

<u>FIGURE 10</u>



Gp model graphical output, with p=0.10.

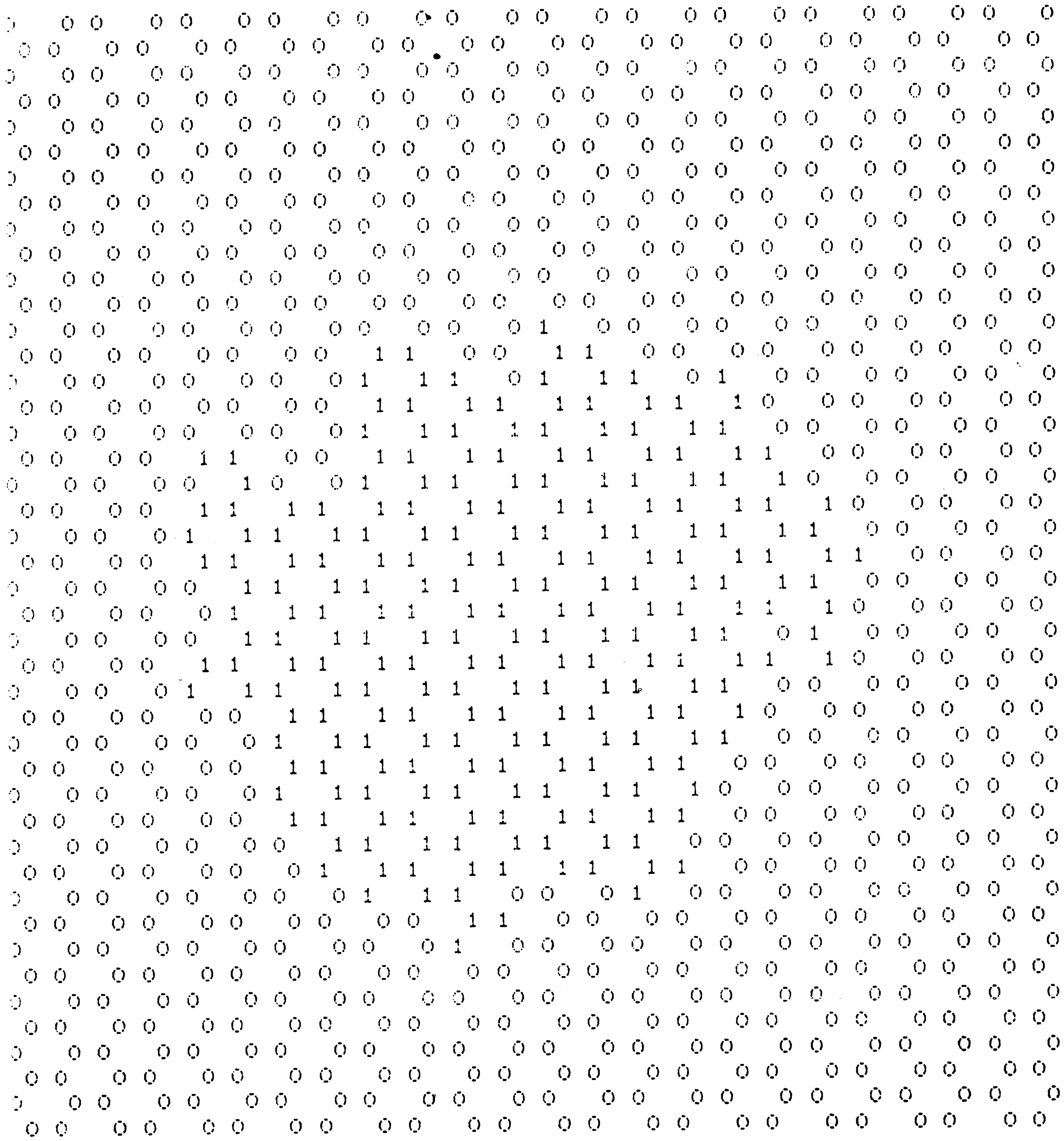
FIGURE 11

Figure 11 shows such results for p equal to 0.10. It is obvious that the growth profile in Figure 11 for p equal to 0.10, at NTIME equals to 30 is relatively small. An average of about 37 failed nodes are contained within the growth contour which is contributed to the low probability of nodal failure. Therefore, additional data is required to further study this behavior for the same value of p at different time interval value. Additional simulation was carried out at NTIME values equal to 40, 45, 49, and 50.

The results from the simulation at the various values of NTIME did not yield any change to the growth profile shape. Hence no clear observation could be drawn regarding this behavior, except for the fact that as NTIME increases, the nodal growth rate increases and eventually encompasses the entire mesh (at NTIME equal 50). This observation was made for all values of p. Further analysis of this model are presented in next chapter.

3.2.2: Gq model.

The graphical output data obtained from the simulation for the percolation model Gq yield the observation that the random growth profile has no

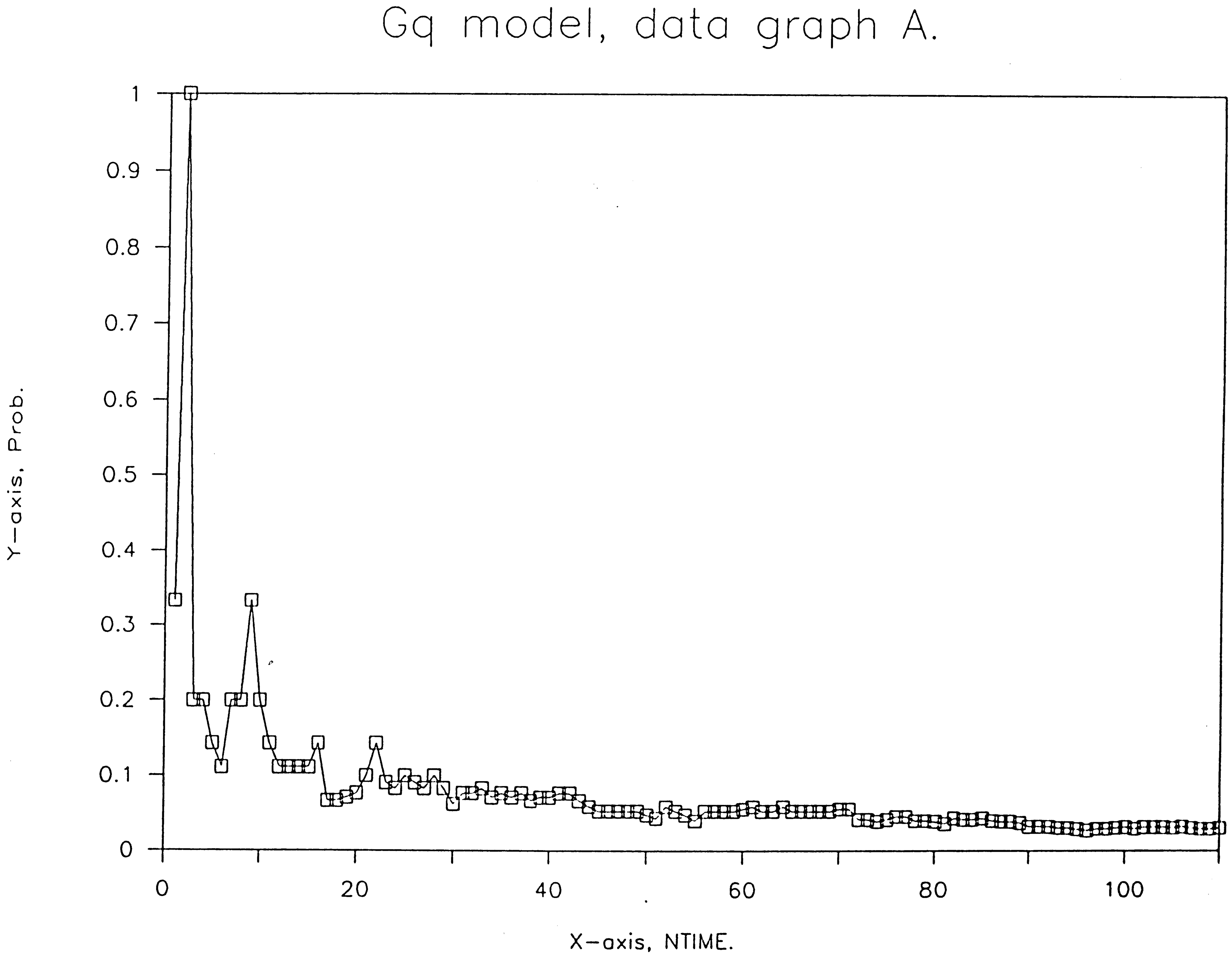


Gq model graphical output.

FIGURE 12

specific geometric shape. The occurrence of sporadic branching and irregular shape as shown in Figure 12 indicates that the growth profile is not symmetrical around the initial point of failure.

Further evaluation of the statistical output data indicates that the growth process weakens as time progress. Using the plotting program previously described in section 3.1.1, a plot of the nodal probability of failure verses time indicate that the value of q (q=1/n) decreases as time progresses. Figures 13, illustrates a selected sample data. The curve fitting programs indicate that the statistical output data is best fitted by an equation of a power curve such that q(t)=A*t^(B). Figures 14, represent a plot of the estimated equation for the selected sample. This behavior indicates that as time progresses, the number of failed nodes increases and the probability of failure decreases. It is estimated that the value of A is equal to 0.541, and the value of B is equal to -0.595.



Gq model, data graph A.
Y-axis, Prob.
X-axis, NTIME.
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
20
40
60
80
100

FIGURE 13

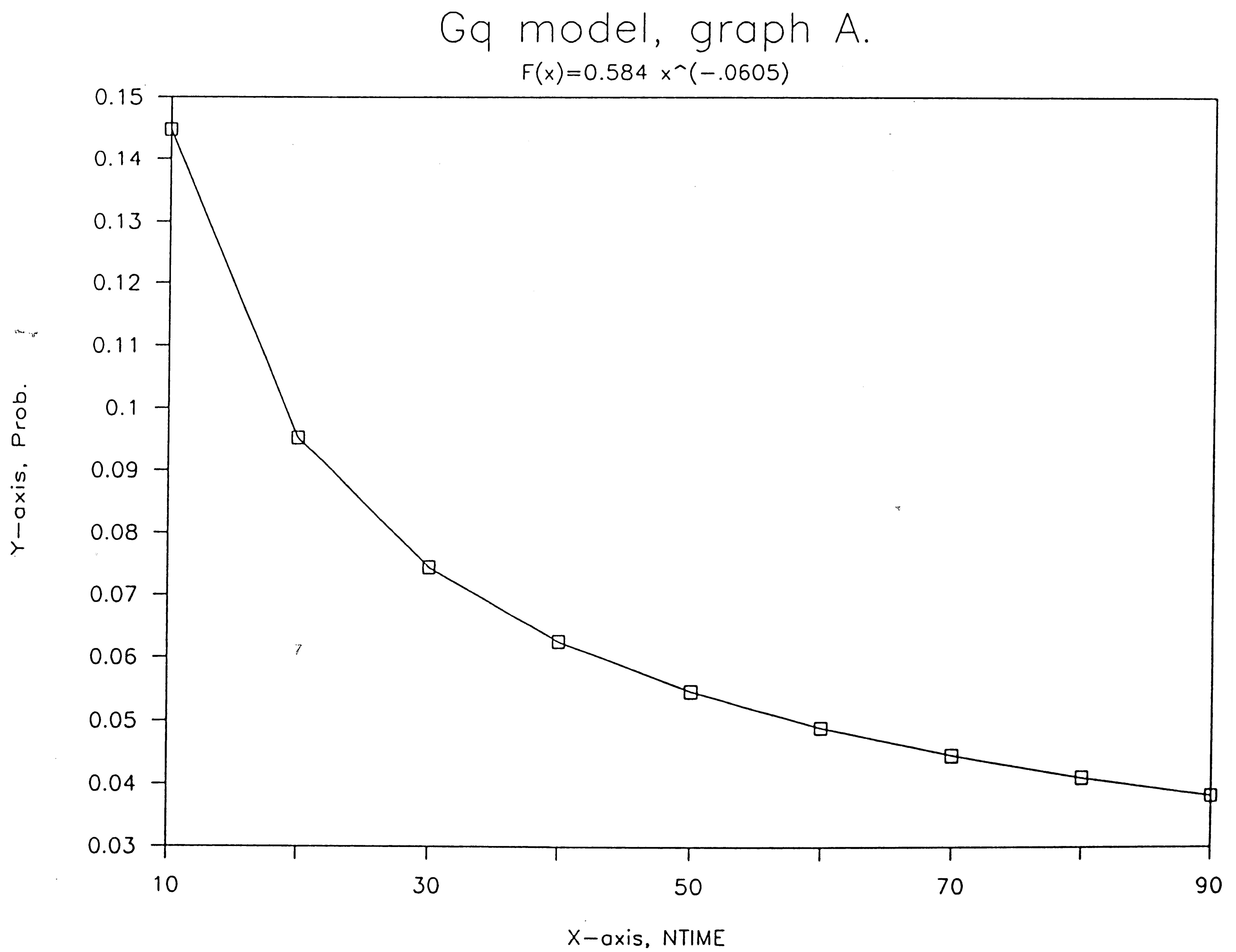

Gq model, graph A.
F(x)=0.584 x^(-.0605)
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
Y-axis, Prob.
10
30
50
70
90
X-axis, NTIME

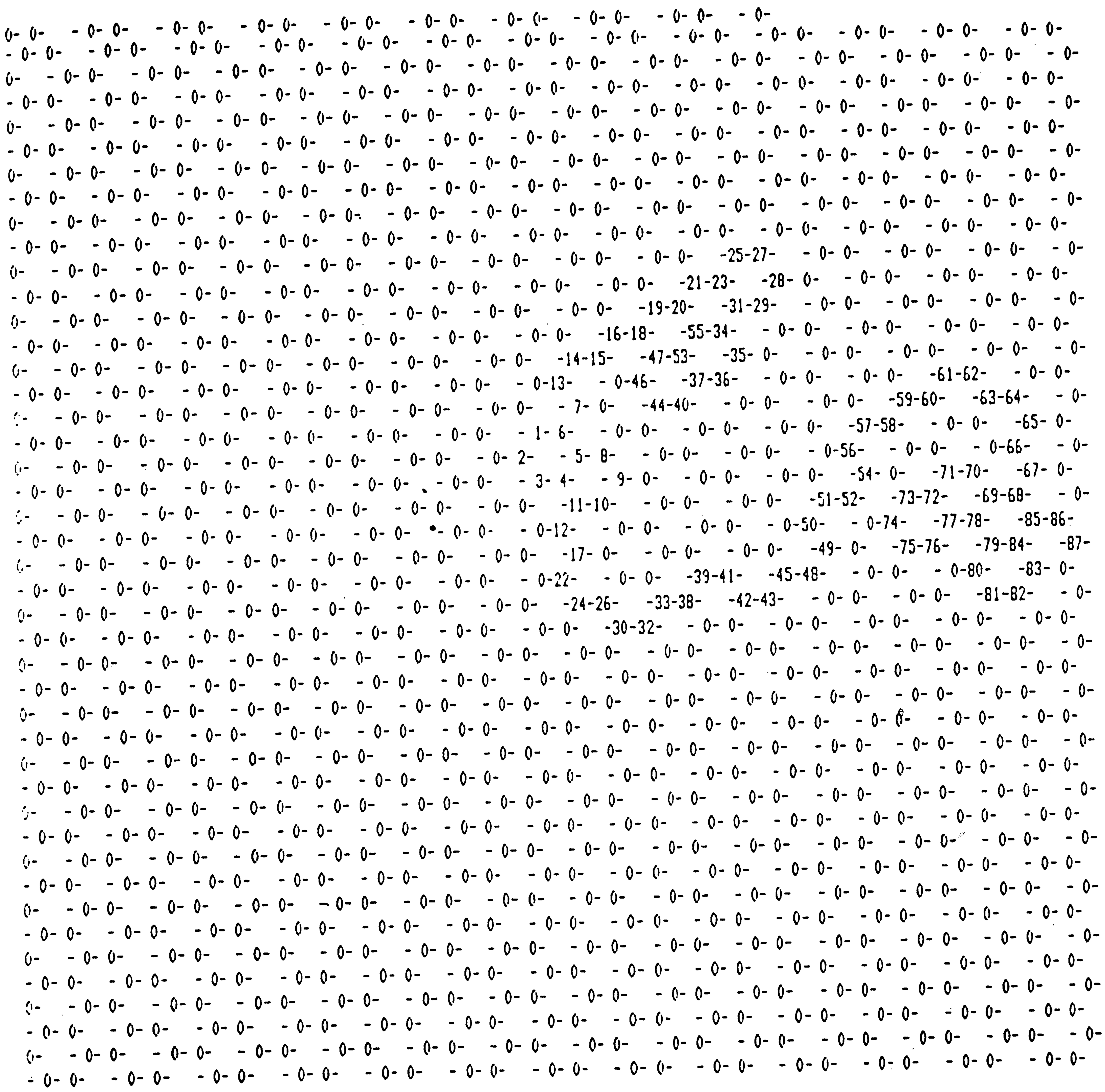
FIGURE 14

3.2.3: Mp model.

During the initial development of the simulation program for the Mp model an evaluation of the graphical and statistical results did not provide a clear indication of how the fracture growth formation and propagation occurs throughout the mesh, hence alteration to the graphical and statistical output were deemed necessary.

This alteration is related to the sequence of nodal failure at a given time, where each failed node is numbered in order of its failure at the same time interval. Similarly, the node number and its order of failure at the same time including the probability of failure are added to the statistical data output.

The results obtained from the graphical output indicates that the fracture growth propagation throughout the mesh along its leading nodes does not follow a consistent path or a deterministic behavior. The crack tendency to loop back on its previous path and the occurrence of artificial growth prevents the mesh from failing. Figure 15, illustrate a typical graphical output which clearly shows this behavior. Therefore, no conclusive results could be drawn from the computer simulation program graphical output data.



Mp model, graphical output.

FIGURE 15

An evaluation to the statistical output data indicates that their is no correlation between the value of p and the sequence of failure at the same time step. Figure 16 illustrates a plot of the sample data presented in Appendix A. This figure clearly indicates that the data are badly scattered. The results obtained by applying the curve-fitting program to the sample data, indicates that the correlation coefficient value is less than 0.04. For example, the equation of a straight line obtained, is F(t)=0.777+(-0.004*t), with a correlation coefficient equal to 0.068. The correlation coefficient for an exponential curve is 0.083. Similarly the correlation coefficient for a power curve is .047. The above curves are illustrated in Figure 17.

The lack of comprehensive results from the graphical and statistical output data necessitate further evaluation of the probability of fracture growth p. The Mp model as defined earlier in section 1.3.2 emulates a fracture growth in a hexagonal mesh. This growth is allowed to be conducted along the weakest link in the network. To satisfy these requirements, p is defined to be equal to the maximum of (p1,p2,...,pk). The pk's are independent random variables that each equates to the probability of failure of the neighboring nodes in question at the desired time step.

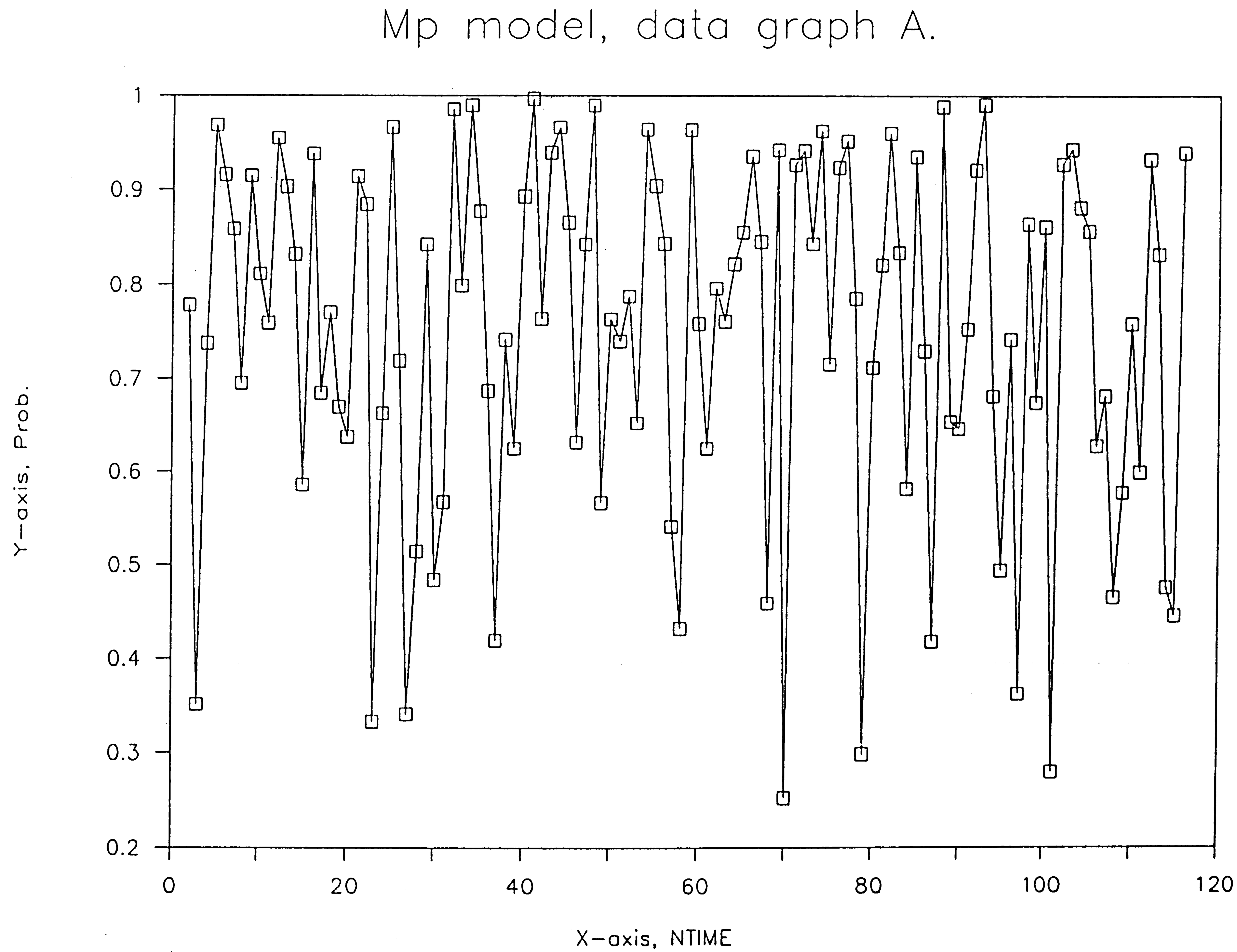
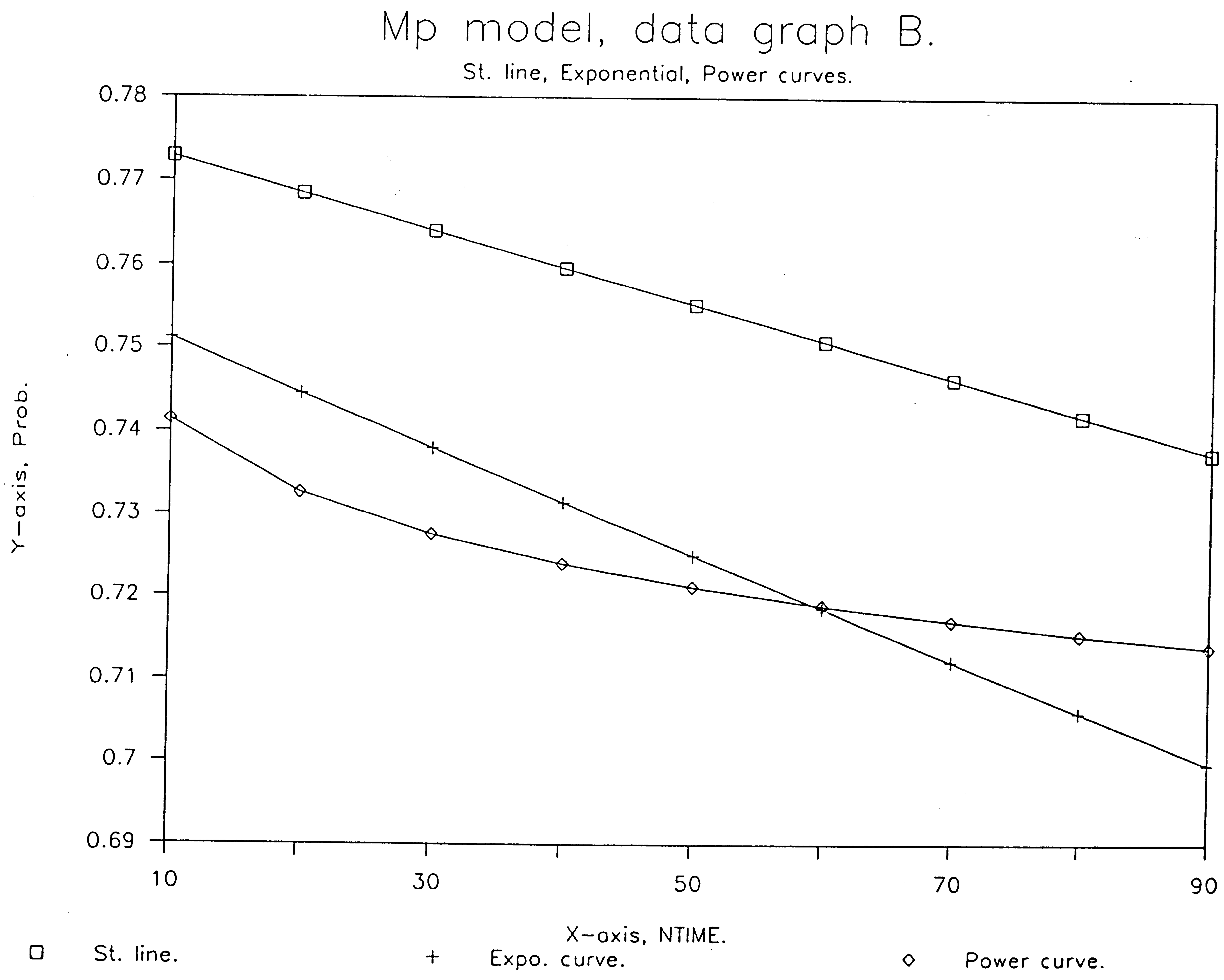

Mp model, data graph A.
Y–axis, Prob.
X–axis, NTIME
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
20
40
60
80
100
120

FIGURE 16



Mp model, data graph B.
St. line, Exponential, Power curves.
0.78
0.77
0.76
0.75
0.74
0.73
0.72
0.71
0.7
0.69
Y-axis, Prob.
10
30
50
70
90
X-axis, NTIME.
St. line.
Expo. curve.
Power curve.

FIGURE 17

Given these conditions the fracture is allowed to propagates freely through the mesh along its weakest nodes. This "free flow" behavior which prevented the mesh from breaking is reflected by the data.

Further analysis of this model are presented in chapter 4.

3.2.4: Mpc model.

The computer simulation carried out for the Mpc model at various values of c (section 1.3.2), indicates that at the lower values of c (pc inversely proportional to c) the growth propagation through the mesh has the tendency to loop back on its previous path, which prevents the mesh from failing. The sporadic occurrence of artificial growth is also observed. It should be noted that these conditions are present regardless the size of the given mesh.

Figure 18, shows a typical graphical output indicating that one fracture edge reached the boundary, with the other looping back on its previous path.

By incriminating the value of c (in increments of 50), the results obtained from the graphical output

```
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 4- 3-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    -14-13-    - 8- 7-    - 1- 2-    - 0- 0-    - 0- 0-    - 0- 0-
0-    -18-17-    -10- 9-    - 0- 0-    - 5- 6-    - 0- 0-    - 0- 0-    - 0-
-22-21-    - 0- 0-    - 0- 0-    - 0- 0-    -11-12-    -25-24-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    -15-16-    -23- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    -19-20-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-
0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0-
- 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-    - 0- 0-
```

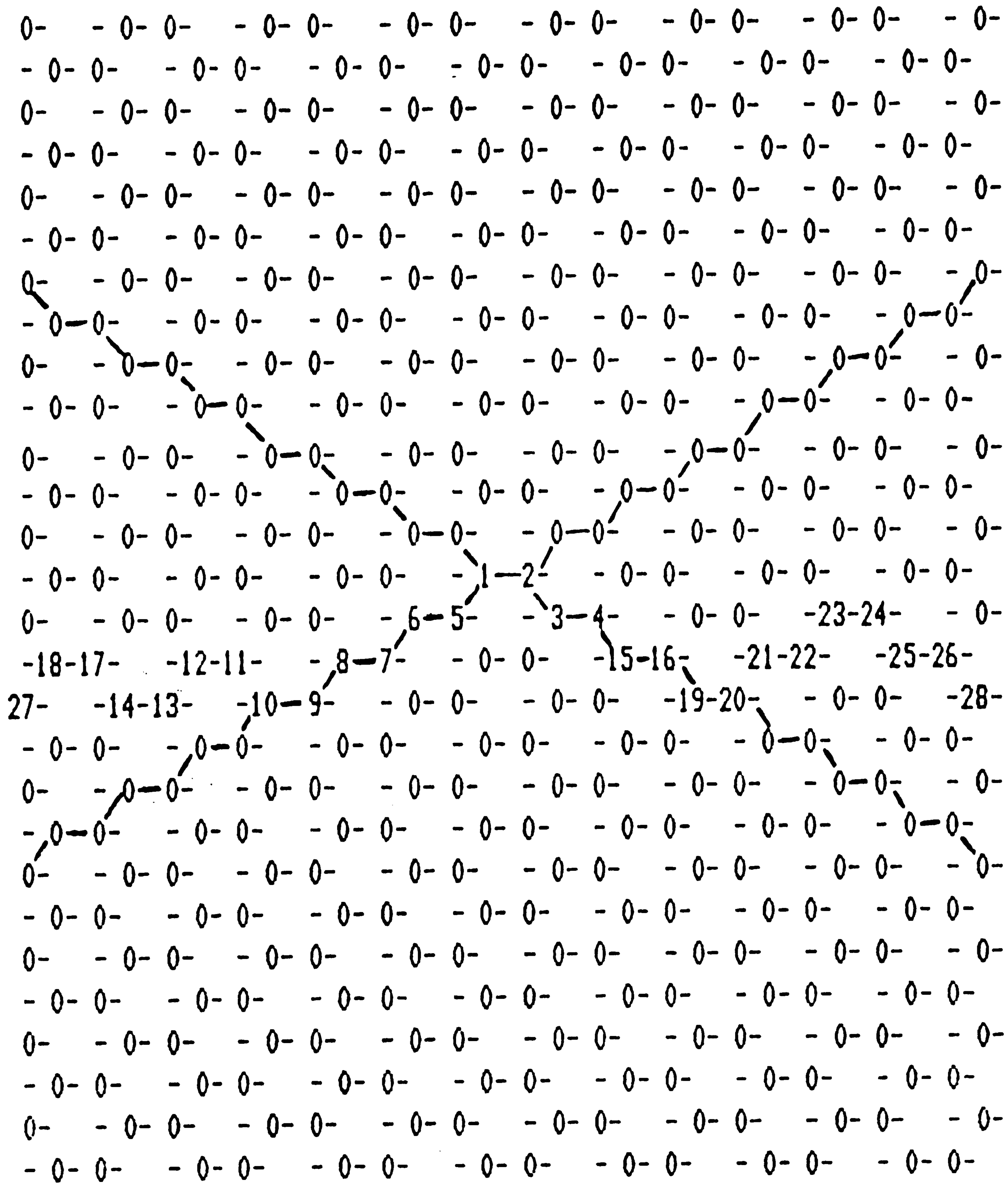
Mpc model, graphical output (loop back)

FIGURE 18

indicates that the occurrence of artificial growth and loop back condition decreases accordingly. At higher values of c (c ranges between 500 to 800) the probability of fracture growth along the horizontal bonds is increased accordingly (section 1.3.2). The growth profile propagation through the mesh becomes more apparent that is, the mesh failure is achieved for every sample.

The fact that pc decreases as c increases, which decreases the probability of failure along the nodes diagonal to the fracture leading nodes, enhances the probability of failure along the nodes parallel to the X-axis. After evaluating several of the graphical data output, it is observed that there exists an envelope in which the fracture growth occurs. This envelope is defined by the initial point of failure , and propagates through the mesh along four lines, to form two triangular envelopes. The defining edges of this envelope are shown in figure 19.

A plot of the fracture probability of growth q verses time t, obtained from the statistical output data are similar to the previous plot presented for the Mpc model. The data is again badly scattered, and no correlation exists between the two values. These plots are omitted to avoid repetition.



Mpc model,graphical output (failed sample).

<u>FIGURE 19</u>

4.0: Random growth model analysis.

4.1: Percolation growth model.

The growth profile obtained from the percolation growth models Gp, and Gq, varies in shape and statistical characteristics. It should not be surprising that the results are different. The fundamental difference between the two models is induced by the definition of each model nodal probability of failure p and q, at a given time t. In the Gp model, p is constant at all times, where for the Gq model, q is a variable that is dependent on the number of failed nodes i surrounding the growth cluster at the same time t, such that $q=1/i$.

The analysis is focused on the common behavior of the mesh response to these conditions which is dependent on its critical probability of failure. The final growth profile shape and its characteristic are also dependent on the closeness of the conditions given, to that of the mesh critical probability.

It is observed that the mesh response is different, based upon achieving and /or exceeding its critical probability, to that of the opposite. The mesh critical probability could be defined as a function of nodal probability of failure at time t, number of failed nodes at the same time, and the number of time discrete jumps t.

The results obtained from the Gp model compared to the results obtained from the existing models, indicates that the growth shape profile for the hexagonal mesh is indeed similar to that of a square mesh. The deterministic shape of the growth profile at higher values of p, with its symmetrical properties, lead to the observation that the rate of nodal breakdown exceed the mesh critical probability of failure. Surpassing this threshold, the mesh breakdown is apparent from the figures presented.

The opposite apply at the lower value of p, where the growth process rate is slowed due to the low probability of nodal failure, which in turn reduces the number of failed nodes at any given time. Thus reducing the number of failed nodes at the end of the time discrete interval (NTIME). To further illustrate this behavior, an examination of the statistical output of the Gq model indicates that the growth propagation rate is slowed as q decreases. This requires an increase of the value of the time discrete intervals NTIME. For example, the value of NTIME used in the Gq model is extremely larger than that used for the Gp model. The fact that for lower values of p the number of failed nodes at the end of each run amounts to few nodes, indicates that the correlation between the number of nodes that have at least one of their neighbors with a failed status verses time, is dependent on the nodal probability of failure.

The difficulties in measuring the mesh critical probability of failure is imposed by the lack of statistical output data generated from the model computer simulation program. The expectation of having a set or deterministic value for the mesh critical probability did not necessitate the need for this data. Hence, no quantitative analysis could be included to obtain the relationship between the parameters that defines the mesh critical probability. The mesh's critical probability of failure is extremely important to the analysis of these models.

4.2: Mechanical failure growth model analysis.

The definition of the mechanical failure growth model stated in section 1.3.2, is based on the general failure of a mechanical object under a tensile force. It is assumed that this force is applied along the mesh boundaries that are parallel to the X-axis. The typical or classical mesh response to these conditions is expected to result in the mesh failure such that a fracture will originate from the single point of failure and will propagate through the mesh along a path that leads to the boundaries perpendicular to that at which the force is applied, in this case the Y-axis.

The result obtained from the Mp model clearly indicates that the parameters and conditions applied to this model does not simulate the conditions of an object under a tensile force. The repeated occurrence of artificial failure, and the constant tendency of the fracture to loop back on its previous path, indicates that the nodes surrounding the fracture leading nodes are independent to each others status, and to that of the fracture. In the definition of the Mp model it is assumed that the nodal bonds have no effect on the growth process, and the probability of fracture growth at time t (larger than zero) is defined as p, such that p=max(p1,p2,...pk) (section 1.3.2). The assumption that these conditions simulate the physical object is false. These conditions provide an environment where the nodes in question at time t are independent of the status of the fracture length, direction, the number of failed nodes at the same time, and the direction of the force applied. This evaluation explain the occurrence of artificial failure. An artificial failure is considered when the mesh fails due to a fracture that takes a path parallel to that of the force applied.

Even though the Mp model failed in providing a meaningful graphical and statistical results that simulate the intended conditions imposed, the result obtained provides a foundation for the Mpc model development.

The technical difficulties and their solution encountered during the development of the computer program are incorporated in the Mpc model. For example, additional debug routines are added to the computer source code to allow additional statistical output which showed detailed analysis at each time step. Furthermore, additional steps are taken to minimize the time required by the user to modify the value of the main parameter c. This allowed the generation of large amount of data at various values, which intern illustrated the model failure growth propagation.

Regardless of the failure or success of these models in showing the final fracture propagation and profile, the analysis should be focused on the validity of choosing a hexagonal mesh to emulate the mechanical failure growth model, and evaluation of the basic model definition and assumptions. The basic difference between a hexagonal mesh and a triangular mesh is in the node orientation relative to the axes. For example, the fracture leading nodes for a hexagonal mesh at any given time are surrounded by at least two nodes, such that the relative orientation of these nodes changes , from one node on the horizontal path, with the other on a diagonal path, to both nodes on a diagonal path. In a triangular mesh the fracture leading nodes are surrounded by five nodes, such that at any given time their exists at least one node along a horizontal path.

Therefore, the probability of the fracture to propagate along the shortest route to the boundaries is enhanced by having at least one horizontal bond surrounding the leading node at any given time step.

The occurrence of fracture loop back condition in the Mp model, and specifically at the lower values of c In the Mpc model is surprising. Such a poor performance shown by the Mpc model could be contributed to its definition and assumption, however in the Mpc model, the nodal probability of failure of the horizontal nodes is higher than that of a diagonally positioned nodes. Hence, it could be stated that the hexagonal mesh structure induce such a behavior.

5.0: Conclusion.

The definition, results, and analysis of the random growth models depicted for the computer simulation programs illustrates the expected and the simulated behavior of a hexagonal mesh response and failure.

The mesh response to the random growth process as outlined by the Gp, and Gq models in the percolation growth study, clearly indicates that the use of a hexagonal element mesh does not effect or alter the deterministic behavior expected. The mesh structure provides a simple node layout, which results in an efficient computer program. Hence, large amount of graphical and statistical data could be generated economically. The deficiency noted in the computer program in providing additional statistical data to enable further evaluation of the behavior of the mesh critical probability could be easily resolved by adding several statements in the program source code.

The mesh response for the mechanical failure models is not as clearly defined. The fact that the statistical and graphical results shows the unpredictable mesh response to these conditions clearly indicates that the use of a hexagonal mesh contributes to this behavior. Hence, caution should be taken in selecting the use of this mesh structure verses that of a triangular element mesh. However, if it is necessary to use a hexagonal mesh for the computer simulation, care must be taken to address the difficulties encountered. This could be achieved by altering the model definition to reflect the true behavior of the growth phenomenon that they describe. For example, in the definition of the Mp model it is assumed that the bonds have no effect on the fracture propagation, the fracture is allowed to travel freely along the weakest link in the mesh. In comparison to the true physical object, the horizontal bonds relative orientation to the direction of the force applied are weaker than the diagonal bonds. The Mpc model which adheres to these assumptions showed similar symptoms at the lower values of the constant c, however the fracture growth at the higher values of c did provide the expected results. In the following section, several suggestions are recommended for future development which attempt to eliminate this undesirable response.

5.1: Future development.

The analysis and results of the percolation and mechanical growth models presented in previous chapters, necessitate the need for further development and enhancements.

Future development of the percolation models should begin by modifying the computer simulation program. These modifications are targeted to include pertinent statistical output data that pertain to the mesh critical probability of failure. The correlation between the parameters that describe the mesh's response to the modeled growth, will determine the mesh critical probability of failure. The mesh critical probability of failure is defined as a function of the nodal probability of failure at time t, number of failed nodes at the same time, and the number of time discrete jumps t. It should be noted that the value of the nodal probability (p), defined in the Gp model is constant for each program run, where each run contain several mesh samples. Therefore, the focus should be concentrated on the correlation between the number of failed nodes, and the value of the time parameter. It is recommended to follow the steps presented in the preceding chapters. For example, The computer programs outlined in chapter 3, provide a good foundation for manipulating the data obtained from the main computer simulation program. The ease in transferring the data from one program to another have proven valuable. These programs will reduce the user interface time that is required for data analysis.

Another challenging objective is related to the two computer programs presented in section 3.1.1. The development is targeted to incorporating the curve fitting program into the LOTUS software package. This task will require a solid knowledge of the LOTUS software and its capabilities. It is highly recommended that prior to experimenting with other growth models, the user should compare the accuracy of the results obtained from the modified programs to that of the models presented in this paper. Upon finalization of the computer programs the user could experiment with various growth models. Several growth models are presented by Richardson,[2] and Durrett.[3] The complexity of these model varies for each. It is recommended to begin by choosing simple models during the initial phase of the development.

The failure of the mechanical growth models in providing meaningful graphical and statistical results that describe the mesh's response to the conditions given, necessitate a comprehensive evaluation of the basic assumption that govern the growth processes. It is recommended to introduce several new restrictions regarding the bond effect on the probability of fracture growth. The bonds should be treated as members connecting each node. For example, each bond is treated as a simple supported beam. The amount of deflection in each beam is determined by its relative orientation to the axis, the direction of the force applied on the mesh, and the number of failed nodes at the same time. It is difficult to judge if the above model enhancements will yield to a definite conclusion. The validity of using a hexagonal mesh construction to study the mechanical growth failure remains uncertain.

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Appendix A

Mp model statistical sample data.

NTIME	Probability p
2	0.7781
3	0.3516
4	0.7377
5	0.9688
6	0.9159
7	0.8591
8	0.6951
9	0.9145
10	0.8113
11	0.7586
12	0.9548
13	0.9032
14	0.8322
15	0.5856
16	0.9379
17	0.6849
18	0.7699
19	0.6700
20	0.6374
21	0.9143
22	0.8850
23	0.3332
24	0.6628
25	0.9668
26	0.7188
27	0.3412
28	0.5143
29	0.8427
30	0.4842
31	0.5672
32	0.9857
33	0.7987
34	0.9895
35	0.8777
36	0.6867
37	0.4188
38	0.7413
39	0.6243
40	0.8931
41	0.9965
42	0.7633
43	0.9394
44	0.9664
45	0.8656
46	0.6311
47	0.8427
48	0.9895
49	0.5665
50	0.7629
51	0.7394

52	0.7869
53	0.6520
54	0.9640
55	0.9042
56	0.8437
57	0.5408
58	0.4315
59	0.9635
60	0.7576
61	0.6242
62	0.7957
63	0.7606
64	0.8216
65	0.8551
66	0.9351
67	0.8453
68	0.4586
69	0.9421
70	0.2524
71	0.9262
72	0.9411
73	0.8432
74	0.9622
75	0.7150
76	0.9234
77	0.9515
78	0.7845
79	0.2986
80	0.7114
81	0.8205
82	0.9599
83	0.8339
84	0.5813
85	0.9349
86	0.7290
87	0.4177
88	0.9885
89	0.6534
90	0.6458
91	0.7519
92	0.9208
93	0.9904
94	0.6808
95	0.4934
96	0.7410
97	0.3623
98	0.8643
99	0.6741
100	0.8610
101	0.2794
102	0.9270
103	0.9428
104	0.8814
105	0.8565

106	0.6271
107	0.6812
108	0.4648
109	0.5772
110	0.7579
111	0.5988
112	0.9320
113	0.8317
114	0.4756
115	0.4449
116	0.9393

Vita

Edmond H. Atiyeh was born on May 22, 1959 in Beirut, Lebanon. He is the son of Hanna I. Atiyeh and the late Salma Azar. On September 23, 1989 he was married to his wife Andrea.

He completed his high school education in Lebanon, and immigrated to the United States Of America in August, 1967. He later became a citizen in September, 1987. He attended Lafayette College in Easton, Pennsylvania and graduated in May, 1980 with a Bachelor of Science in Mechanical Engineering.

He accepted a position as a product engineer at Altech Industries in Allentown, Pennsylvania. In February, 1982 he accepted a position as a mechanical engineer at FMC Corporation Material Handling Systems Division in Chalfont, Pennsylvania. He continue to pursue his career at FMC as a senior design engineer and a project manager for the Automatic Guided Vehicle Systems.