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Use of the Analytic Hierarchy Process for Industrial Site Selection

by

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Abstract

Most site selection algorithms accept only objective, quantifiable input, and use cost as the only decision criterion. This is not representative of the manner in which most site selection decisions are made, in which the goal of minimizing cost is tempered by a need for non-quantifiable measures of suitability. These non-quantifiable measures may range from the skills of the local workforce to the personal dispositions of managers toward individual sites. To date, only the Brown-Gibson Algorithm has been widely recognized as a means to select sites based on both quantifiable and non-quantifiable input. This algorithm and some of its weaknesses are explored.

A general method of combining quantitative and non-quantitative input to general decisions was developed in the late 1970's and has been applied to a variety of problems. The method is the Analytic Hierarchy Process (AHP), and here its suitability to site selection is explored. A step-by-step guide to using AHP for site selection is developed, along with techniques useful in adapting facility location problems to AHP analysis. Detailed examples using AHP for general selection as well as site selection problems are provided.

AHP is found to offer better solutions to the site selection problem, overcoming several of the difficulties of the Brown-Gibson algorithm. However, it requires more difficult computations. Because of the importance of the location decision in most cases, the use of AHP is recommended.

Chapter 1
The Site Selection Problem

1.1 Introduction to Facilities Location

Site selection is a type of facility location problem concerned with selecting locations for production and support facilities. Facility location problems are a wide-ranging class of problems concerned with locating one or more **objects** somewhere in a given **area** to serve some **purpose**. In these problems, the placement of the objects is done to satisfy some notion of **optimality**. Over the spectrum of facility location problems, objects and areas vary widely. **Objects** could include such things as naval bases, warehouses, fire stations, work-in-process (WIP) storage within a factory, or pieces of individual equipment. The **areas** corresponding to these objects could be the entire earth for the naval base, a distribution zone for the warehouse, part of a town for the fire station, and a factory (or part of a factory) for the WIP storage and factory equipment.

The **purpose** served by an object depends greatly upon the object itself. Generally, an object's purpose is to provide or consume goods or services. For example, warehouses provide goods to users and consume them from producers. Production facilities consume raw materials and provide processed goods. WIP storage facilities provide the service of storing work in process.

The definition of **optimality** which is used in a facilities location problem will usually affect greatly the resulting choice of location. Some of the criteria often optimized are financial measures, such as total long-term cost (minimized), return on investment (maximized), initial investment (minimized), revenue (maximized) and profit (maximized). Other quantifiable yet non-financial measures may also be chosen to be optimized. An example of a quantifiable non-financial measure would be delivery time of supplies, which would be important for production plants which use perishable supplies. Finally, some measures are not easily quantifiable and very subjective in nature. These include community attitudes toward a new facility, educational opportunities near a site, the Occupational Safety and Health Administration (OSHA) regulations and other regulations governing the placement of production equipment, and even aesthetics.

1.2 Site Selection: The Facilities Location Problem Applied to Production Sites, Warehouses and Offices

Site selection is a problem encountered by firms when they need to expand or relocate any of their facilities. Such facilities may include production facilities, warehouses, field offices and sales offices. Site selection generally does not include the problem of locating specific pieces of machinery in a production facility, although facility location algorithms can certainly be used in addressing such problems. There are a number of reasons for this distinction. First, the scales of the problem are so different that individual techniques differ in their application. When locating equipment within a factory, the exact locations of and routes between stations requiring transportation services are known. When locating the factory itself, such information is often not known and is subject to change. Second, the operating cost of a machine rarely changes as a function of its location within a factory, whereas operating costs may vary greatly as a function of location in site selection problems. Third, equipment location within a factory rarely involves the degree of commitment and upheaval that facility location does. Finally and most importantly, equipment location relies almost entirely on "hard" numbers directly associated with cost, whereas site selection decisions often rely at least partly on subjective factors not easily expressed in monetary terms. These subjective factors can include factors which are absolutely necessary but not quantifiable, such as whether industrially or commercially zoned sites exist in a certain area, to those which are ill-defined in nature, such as the personal impressions of management toward a certain site.

Each company will have its own procedure for selecting sites. Such procedures may range from the rigidly structured to unwritten, informal practices. Regardless, they all accomplish the same general steps. These general steps are:

1. The need for a facility is recognized.
2. A general location is identified.
3. Selection criteria are identified
4. Candidate sites are selected.
5. A final site is selected.

The first two steps will sometimes be done together. This often occurs when sales offices or warehouses are to be located, as these facilities tend to be closely linked to geographic locations. Thus, the original need for the facility may be identified as "a sales office west of the Rockies" or, on a different scale, "a warehouse in the north end of town." Some companies use the same criteria for site selection as they use for other business decisions, and thus never explicitly perform the third step. Such companies typically judge all decisions by one form of financial return, such as a payback period or rate or return. The fourth step may be done in one step if the general location is small, or in several passes if the general location is large. If done in several passes, the first pass might identify regional locations and the second pass could identify specific sites within a chosen region. The final step is usually made by a committee of the firm's management, who use both quantitative input and personal biases to select one final site.

While all procedures include these five basic steps, many firms' procedures will include additional steps. If operating licenses are required or a firm's production has an environmental impact, then the site selection procedure may include community surveys, environmental impact studies, public hearings, and other steps specific to the facility's needs.

1.3 Classification

Site selection problems can generally be classified into four types of problems, each with its own recognized methods of solution. The classes are defined by the types of decision factors they contain. Any site selection problem will contain factors in one of these four categories:

- Quantitative Factors Defined over a Single Domain
- Quantitative Factors Defined over Multiple Domains
- Only Non-Quantitative Factors
- Quantitative and Non-Quantitative Factors

Quantitative factors are those factors which can be quantified in standard, recognized units. They would include utility costs, delivery times, and distances to other facilities. **Non-quantitative factors** are those which cannot be quantified in units which are directly usable in the site selection problem. Examples of such factors include a site's aesthetics, the quality of local education, and the availability of labor in the area. Note that while statistics can be compiled for some of these factors, they are not in units which are meaningful for the site selection problem. A **domain** is the dimension over which a factor is defined. Domains include money, time, distance, customer counts, and others. Note that domains and units are different. Conversions can be made between units defined in common domains, such as dollars and deutschmarks, or miles and meters. This allows quantities defined in different units to be compared directly against each other. No direct conversion can be made between different domains, such as money and distance.

1.3.1 Quantitative Factors Defined over a Single Domain

This class of problems is the most straight-forward and simplest to solve. Because all factors are quantitative, no surveys and no estimations are needed. Because only one domain is involved, all factors may be added directly to one another. The solution is to simply calculate the costs of each candidate site and choose the cheapest. An example of a problem of this class is the location of a "lights-out" production facility where only operating costs are to be considered. Because all costs are monetary, only a single domain is involved. Note, however, that if distances or delivery times are to be factors, or any considerations to personnel are to be included, then the problem falls outside of this class.

1.3.2 Quantitative Factors Defined over Multiple Domains

A problem consisting only of quantitative factors defined over multiple domains would include only factors which could be measured, but which cannot be directly translated into common units. This would include site selection locations with monetary factors as well as time and distance factors. All are quantitative, but there is no direct way to compare, for example, the cost of utilities with the distance to the nearest rail line, or either of those factors with the time required to reach a hospital.

Such problems can be approached in two different ways. The first is to develop some kind of conversion factor among the various factors. This is best accomplished by translating all non-monetary factors into the monetary domain, because money is the only domain where indifference can be assumed. **Indifference** means that a dollar spent on one item has the same value to the buyer as a dollar spent on another. This is not the case with time or distance, as a minute required to reach a highway may have different value than a minute required to reach a hospital. Differentiation can even be made within a single factor, as the distance to a large customer may be much more important than the distance to a small one. The problem facing the analyst, then, is to convert these times and distances to dollar values. This is not an obvious conversion, yet managers implicitly assign dollar values to non-monetary things during the normal course of their jobs. Thus, it is possible although strong management talent is probably needed to do a credible analysis.

The second approach to problems with quantitative factors defined over multiple domains is to approach the problem as one with both quantitative and non-quantitative factors. In the analysis, the monetary factors would be treated as quantitative and the non-monetary ones treated as qualitative. The algorithms would be simplified somewhat, as comparisons between sites for individual factors would not rely on time-consuming pairwise comparisons, but instead on values assigned directly from the quantitative factors. Comparisons are still necessary, however, between the various factors independent of sites. Note that this has the effect of using the non-monetary quantitative measurements and implicitly developing conversions between the various non-monetary factors via the pairwise comparisons.

1.3.3 Only Non-Quantitative Factors

This type of problem seldom occurs in site selection, as there is almost always some cost or revenue which is a function of location. If such a problem was encountered, however, it could be solved in a number of ways. One strategy is to use either of the algorithms recommended for solving problems with both quantitative and non-quantitative factors (see Section 1.3.4). In the case of the Brown-Gibson Algorithm, the Objective Factor Decision Weight (*X*, defined in Section 2.4.7) is zero, since there are no objective factors to weight. In the case of AHP, no changes in the algorithm are necessary because the algorithm does not distinguish between quantitative and non-quantitative factors.

1.3.4 Quantitative and Non-Quantitative Factors

A problem with quantitative and non-quantitative factors is the most complex of the four classes, and can also be considered as a general case of which the other two classes are instances. A site selection problem in this category would include both quantitative factors, such as costs, and qualitative factors, such as the desirability of the area for the employees who will have to live there.

Traditionally, the qualitative factors have had no place in formal analysis. Cost alone drove the location decision, and when qualitative factors were included they would be used as an initial screen to exclude candidate sites. The decision from among the remaining sites would be cost-based and not include the relative merits of the sites with regard to qualitative factors. If any qualitative factors were considered, they would likely be limited to the personal biases of the managers involved with the selection.

Recent industrial trends since the 1970's have made traditional cost analysis an incomplete basis for site selection problems. As more and more companies demand Just-In-Time (JIT) relationships with their suppliers, firms are ensuring that new—and sometimes existing—facilities are located near their customer base. Increased technological demands are forcing companies to congregate in regional centers. Competitors in single industries have always been located near each other, since many are started by employees of older companies in the industry. However, the rapid pace of technological advances today almost *requires* companies in high technology fields to be clustered in regional centers. This is because strong competitors need word of advances in leading-edge research very quickly, and being located in the same community as other researchers facilitates this. Also, a general shortage of properly trained personnel in high-technology areas makes jobs plentiful and greatly increases the cost of luring top people to remote locations.

Methods of selecting sites based on both quantitative and non-quantitative factors have received relatively little attention in existing literature. Within the realm of site selection, nearly all algorithms use only monetary factors, or else are solutions to the transportation problem and focus on minimizing time, distances or costs. If non-quantitative factors are to be included in these algorithms, they must be expressed in terms of quantitative factors such as money, time and distance.

One notable exception is the Brown-Gibson algorithm [6], which allows both quantitative and non-quantitative factors to be included in an analytical procedure. The algorithm selects the best site from a group of candidate sites. It was introduced in 1972, yet is endorsed unchanged from its original form in very recent texts [6] [26].

The problem of combining quantifiable and non-quantifiable factors in a single decision transcends site selection. It is primarily in other fields that advancements in combining quantifiable and non-quantifiable factors have occurred. The most widely used and highly endorsed technique developed in recent years for solving such problems is the Analytic Hierarchy Process (AHP), developed by Dr. Thomas L. Saaty. This technique has been used to address problems as diverse as analyzing political candidacies and selecting transportation systems for a Third-world country.

1.4 Related Problems

To apply AHP to the site selection problem, it is helpful to understand its application to other problems, and to highlight similarities between site selection and other problems. Two problems to which AHP has successfully been applied are the selection of Research and Development projects when resources prohibit pursuing all candidate projects, and the evaluation of employee performance.

1.4.1 Managerial Selection of Research and Development Projects

Like site selection, the selection of research and development projects involves both objective and subjective input. Objective factors include pro forma cost and revenue data associated with each project, as well as estimated completion times. Subjective factors include the effect of a project's selection on the organization's knowledge base, whether the expertise gained from a project will be useful in future projects, and whether the project uses technology which will distinguish the organization from its competitors. The factor of "effect on the organization's knowledge base" may have several components, such as how much the organization's breadth of knowledge will be increased by using new technologies, and whether the project is challenging enough to keep employees from leaving the organization.

The problem of selecting research and development projects is analogous to the site selection in that critical quantitative data includes costs and estimated completion times, and that non-quantitative data is important to the decision. Unlike most instances of the site selection problem, several projects may be included in the solution whereas only one site is generally desired in site selection. AHP has been successfully applied to this problem by Matthew J. Liberatore [16].

1.4.2 Employee Performance Evaluation

Evaluation of employee performance includes factors which range from the purely quantitative to the purely subjective. Quantitative measurements include measurements of production output and error rates. Subjective factors might include how well the employee works with his coworkers. Such factors must be combined into a single scale to effectively evaluate employees for purposes of adjusting salaries and charting futures within the organization. The problem of evaluating employees is a problem with quantitative and non-quantitative factors, and can be approached with some of the same techniques used to approach the site selection problem. Usually, no selection is intended as a result of the evaluation, rather it is only to compare employees to their own potential and their coworkers. This is analogous to developing a rating for an individual candidate site based on quantitative and non-quantitative factors, but not selecting a final site. This problem has been addressed both with AHP and in studies before the advent of AHP [15].

1.5 Organization of this Paper

The use of AHP for site selection problems is presented in five major sections. The first section has introduced the problem of site selection. The next chapter, Chapter 2, will present existing methods of analysis. Generic site selection procedures for the various classes of problems are presented, including the Brown-Gibson algorithm. Some of the Brown-Gibson algorithm's problems are discussed at the end of the chapter. Chapter 3 introduces the Analytic Hierarchy Process, and includes an example of the process applied to a consumer's selection of an automobile. Some concepts from graph theory are presented, and used to present a way of simplifying the selection process. Chapter 4 applies AHP to the problem of site selection, and discusses various issues which are specific to the use of AHP for site selection. An example site selection problem is solved using both AHP and Brown-Gibson, and the results are compared. Conclusions and a summary are presented in Chapter 5.

There are four appendices. Many of the subjective factors which might be included in any specific site selection problem are discussed in Appendix A. Similarly, objective factors are presented in Appendix B. In Appendix C, an expression for the weights of factors in AHP is derived so that AHP analysis is equivalent to standard economic analysis. This equivalency is shown in a site selection example which uses only monetary factors; it is solved using both AHP and standard economic analysis, and the results are equal. Appendix D presents various methods of financial justification and applies each of them to an example.

Chapter 2
Generic Site Selection Procedures

2.1 Generic Site Selection Procedure Using Quantitative Factors Over One Domain

When quantitative factors defined over only one domain are used, the selection of a site becomes a relatively straightforward task. An example in which quantitative factors span only one domain would be a site selection based only on financial factors. The single domain would be dollars, and the procedure used to select the site could be any of the financial justification methods normally used by management in making capital decisions. Such methods include Present-Worth Amount (PW), Annual Equivalent Amount (AE), Future-Worth Amount (FW), Rate of Return and Payback Period [25]. An additional basis of comparison worth noting is the Project Balance, which identifies the cost or profit of a project if it is terminated before its planned maturity. Because industrial plants often require large cash outlays over several financial periods, they are often subjected to unplanned changes in funding levels. The Project Balance concept identifies the net cost or benefit to the firm if the project (ie., the facility's construction) is frozen before it is finished.

The reason that such methods can be used is that the single domain inherently allows factors to be ranked and weighted simply by comparing their magnitude. The assumption is that a dollar is worth a dollar, regardless of where it is spent. This is the Indifference Assumption, previously mentioned in Section 1.3.2. The example presented in this chapter to illustrate the various bases for economic comparison shows a site selection based only on quantitative factors over a single domain.

If the single domain is not financial, it is quite possible that the indifference assumption may not be valid. Such a case might be the location of an ambulance station, where the domain is response time. The response time to certain locations, such as nursing homes and schools, may be more important than the average response time to single-family residences. In this case, the indifference assumption is invalid and a way of weighting the relative importance of the different response times must be found. This problem then becomes of the quantitative factor, multiple-domain type. Appendix D discusses the various financial justification methods in detail, and each method is used to analyze an example.

2.2 Generic Site Selection Procedure Using Quantitative Factors Over Multiple Domains

When decision factors, as described in Section 1.3 are defined over different domains, a way must be found to equitably combine the domains. This can only be accomplished if a suitable conversion method exists between the domains. For example, if one factor is defined in dollars and another, involving direct labor content, is defined in hours, the direct labor content can be easily converted to dollars by multiplying the hours by the expected direct labor rate, and adding any overhead. A less easily handled example of quantitative factors defined over multiple domains would be when one factor is defined in dollars and a second factor, defined in hours, involves the time needed to receive emergency spare parts. The conversion of resupply hours to dollars is not self-evident like the conversion of direct labor hours to dollars.

Generally, analysis is made easier and more believable when conversions between various domains can be made in standard and widely-accepted ways. Such conversions include direct labor hours into dollars, machine and utility usage into dollars, conversions among various national currencies, and conversions between labor hours of different pay grades. When conversions cannot be easily made, an algorithm like the Brown-Gibson procedure or the modified procedure presented in this thesis must be used.

2.3 Generic Site Selection Procedures Using Only Non-Quantitative Factors

It is very rare that a site selection problem would encompass only non-quantitative factors, but a brief discussion of techniques for solving such problems is presented here for completeness. These techniques are borrowed from other fields where decisions based solely on non-quantitative factors are common. Such fields include marketing and non-functional design aspects of product design. Two of the various methods available to combine non-quantitative factors are used more commonly than the rest. They are the Jury of Executive Opinion and the Delphi Technique, and both are survey methods which require a panel whose opinions are combined into specific recommendations.

2.3.1 Jury of Executive Opinion

The simplest and most commonly used method is the Jury of Executive Opinion [13, p. 284]. This method entails surveying company executives to determine their preferences, then using this information to select a solution. In the case of a site selection problem, the highest level of executives knowledgeable about the specific site selection dilemma would be asked their preferences among the candidate sites. These would then be either averaged to produce an ordered list, or presented to the group or to an ultimate decision maker as raw data, from which a single choice would be made.

2.3.2 Delphi Technique

Delphi has been used in a wide variety of applications, and would certainly be a valid method to solve a site selection problem based on non-quantitative factors. It is also very useful in addressing the subjective factor side of a site selection problem based on both quantitative and non-quantitative factors and could be easily incorporated into the Brown-Gibson algorithm. In the Delphi technique, a panel of experts is employed—often at considerable expense—and given an extensive questionnaire relating to the problem at hand. They answer the questions and provide additional comments. A central coordinator then compiles their answers and circulates the results, with all the comments, back to the panel. The committee's membership and all of their input is kept anonymous in order to avoid domination by strong personalities or reputations and keep the problem at hand as the basis of all discussion. A consensus emerges through this process, although several iterations may be required.

Either the Jury of Executive Opinion or the Delphi Technique could be appropriate to a site selection problem based on only subjective factors. As noted, however, such problems are extremely rare.

2.4 Site Selection Procedure Using Quantitative and Non-Quantitative Factors: The Brown-Gibson Algorithm

The Brown-Gibson algorithm was first published in 1972 by Phillip A. Brown and David F. Gibson [6]. The algorithm offered a way to combine objective and subjective factors. It was intended to fill a gap left by other location algorithms, which used either lists of subjective factors or mathematical models of objective factors. Before the Brown-Gibson algorithm, no existing algorithms used both subjective and objective factors as input to individual decisions. Gibson isolated the procedure of the Brown-Gibson algorithm into the ten discrete steps shown in Figure 2-1.

The Brown-Gibson Site Selection Procedure

1. Define critical factors, objective factors, subjective factors
2. Evaluate critical factor measures
3. Evaluate objective factor measures
4. Determine subjective factor weights
5. Determine site weights
6. Evaluate subjective factor measures
7. Determine objective factor decision weight
8. Calculate location measures
9. Perform sensitivity analysis
10. Select facility site

Figure 2-1: The Brown-Gibson Site Selection Procedure [10]

Various references provide detailed examples of how the Brown-Gibson algorithm can be applied to site selection problems [6] [10] [26]. The following sections describe the Brown-Gibson Algorithm in detail, without examples.

2.4.1 Define Critical Factors, Objective Factors, Subjective Factors

The first step in using the Brown-Gibson procedure is to select which factors will be used for the particular problem being considered, and to group the factors into three categories. Essential to the understanding of the algorithm is an understanding of the three classes of factors which comprise input to the site selection algorithm:

- Critical factors

  Critical factors are those which must be present at a site for that site to be considered. They can be judged on a binary scale: either present or absent. Examples include access to rail lines if the facility to be located is a warehouse requiring rail shipment, an outlet to the sea if the facility is a shipyard, or proper licenses if the facility requires licenses to operate.

- Objective factors

  Objective factors are those which can be measured. The Brown-Gibson procedure allows objective factors to include only financial factors, such as prevailing labor wages, acquisition cost of the site and state and local taxes. Non-financial objective factors such as transportation times and distances must be considered as subjective factors when using the algorithm.

- Subjective factors

  Subjective factors are those which cannot be readily measured in dollars. They include the availability of transportation and education, expectations of changes in objective factors, and judgements of organized labor activity. Appendix A discusses many possible subjective factors.

2.4.2 Evaluate Critical Factor Measures

All critical factors must be evaluated for all sites, and any site not having all critical factors present is eliminated from further consideration.

2.4.3 Evaluate Objective Factor Measures (OFM_i)

An Objective Factor Measure is developed for each site from the sum of all objective factors associated with the site. The objective factors must be expressed in terms of dollars, and as a cost (as opposed to a revenue). The user is left to determine exactly how to express in consistent units those cash flows which occur over a period of time, such as payments made over many years. The algorithm assumes that a standard engineering economy method of developing a consistent time frame for costs will be used. The algorithm's expression for the Objective Factor Measure is

$$OFM_i = [OFC_i \times \sum_i (1/OFC_i)]^{-1}$$

where

OFC_i is the sum of all objective factors (costs) for site i

OFM_i is the objective factor measure for site i [10]

2.4.4 Determine Subjective Factor Weights (SFW_i)

Subjective factor weights are independent of the candidate sites. To determine the relative importance of the identified subjective factors, the factors are evaluated against each other in exhaustive pairwise comparisons (see Section 3.1 for a discussion of pairwise comparisons). The method of comparison uses a binary scale, assigning 1 to the more preferable of any two factors and a zero to the less preferable. If they are of equal preference, each is a assigned a 1. The factor's Subjective Factor Weight is then calculated as the ratio of its total of ones to the total number of ones assigned to all subjective factors:

$$SFW_i = \frac{\sum_i 1}{\sum_{all\ i} 1}$$

where SFW_i is the subjective factor weight of the ith subjective factor.

2.4.5 Determine Site Weights (SW_{ik})

For each subjective factor, sites are evaluated in exhaustive pairwise comparisons to determine the relative desirability of the sites with respect to the particular subjective factor. The comparisons are conducted in exactly the same way as the subjective factor comparisons in the previous step, with ones and zeroes assigned as the result of each comparison. The site weights are determined in the same way that subjective factor weights are determined in the previous step. At the conclusion of this step, each site will have one site weight for each subjective factor.

2.4.6 Evaluate Subjective Factor Measures (SFM_i)

Each site will have one Subjective Factor Measure, which is the sum of each factor's Subjective Factor Weight and the Site Weight for the particular site for that factor.

$$SFM_i = \sum_k (SFW_k \times SW_{ik})$$

where

SFM_i	is the Subjective Factor Measure of site i
SFW_k	is the Subjective Factor Weight of factor k
SW_{ik}	is the Site Weight of site i with respect to factor k.

2.4.7 Determine Objective Factor Decision Weight (X)

The Objective Factor Decision Weight is "the relative importance of objective factors to the location decision [10]." At this point in the procedure, dimensionless indices exist for the total objective factor measure (OFM_i) and the total subjective factor measure (SFM_i) of each site. The Objective Factor Decision Weight will be used to combine these two measures into a single measure of the site's worthiness. By definition, the Objective Factor Decision Weight must be a number between zero and unity, and the difference between unity and the weight will be the weight of the subjective factor measure.

Thus, an Objective Factor Decision Weight of 0.5 would have the effect of granting subjective and objective factors equal importance. A higher number would put more emphasis on the objective factors. This weight should be determined by those responsible for the decision, and reflect company policy, past data, and current management sentiments. Its value is usually determined by a committee [10].

2.4.8 Calculate Location Measures (LM_i)

This step simply combines the Subjective Factor and Objective Factor Measures according to the Objective Factor Decision Weight calculated in the previous step:

$$LM_i = X \times OFM_i + (1 - X) \times SFM_i$$

where

LM_i	is the Location Measure of site i
X	is the Objective Factor Decision Weight determined in the above step
OFM_i	is the Objective Factor Measure of site i
SFM_i	is the Subjective Factor Measure of site i

The site with the highest Location Measure is the most preferable for the specified Objective Factor Decision Weight (X). That is the site which should be chosen, if the results of the procedure are to be used.

2.4.9 Perform sensitivity analysis

Because the selection of the Objective Factor Decision Weight (X) has such an important effect on the final Location Measure, and is so subjective in its nature of selection, Brown and Gibson recommend performing sensitivity analysis of the Location Measures with respect to changes in X. It is an easy matter to graph results of this analysis because Location Measure is a linear function of X. The result of the sensitivity analysis can be more confidence in the algorithm's resulting champion if it is discovered that that site is most preferable over a wide range of Objective Factor Decision Weights. It is also noted that sensitivity analyses can be performed on the location measures for changes in other inputs to the algorithm, such as changes in labor costs or production requirements. The results of these changes, however, will not be as easily determined as changes in the Objective Factor Decision Weight.

2.4.10 Select facility site

The final step is to select a site for the facility. The Location Measures of the sites give a sense of the magnitude of the differences among the sites as well as the order of preference of the sites. The site with the highest Location Measure should be pursued as the first choice, but if the difference between its Location Measure and the Location Measures of other sites is small, those other sites may ultimately be chosen instead of the highest ranking site.

2.5 Weaknesses and Practical Difficulties of the Brown-Gibson Algorithm

There are a number of weaknesses in the Brown-Gibson algorithm which keep it from being a useful tool in general site selection problems. These weaknesses range from difficult-to-interpret results to practical difficulties in using the algorithm for some types of problems.

2.5.1 Interpretation of Results

Chief among the weaknesses of the algorithm is the interpretation of its results. The algorithm produces a weight for each site, all of which sum to unity. One of the weights will usually be greater than all of the others, indicating that the corresponding site is the most preferable. In practice, however, the numbers are all close to each other and it is difficult to determine *how much* better one site is than another. Thus, when the algorithm results are brought to the people who must select the ultimate site, the relative preferability of one site to another is not obvious. Furthermore, since the numbers are often very close, it may be more correct to decline to rate one site over another, due to the judgement errors inherent in obtaining the input to the algorithm.

2.5.2 Finding a Meaningful Objective Factor Decision Weight (*X*)

Much rigorous calculation is done to determine the Objective Factor Measure (*OFM*) and Subjective Factor Measure (*SFM*) of a site. The two are then combined, however, by using an Objective Factor Decision Weight (*X*) which is of paramount importance to the ultimate weights yet has no analytic basis whatsoever. The number is determined by the management responsible for selecting the new site, based on its own practices and preferences; the Brown-Gibson algorithm gives no advice for selecting a meaningful value of X. Due to the difficulty in selecting a meaningful weight, sensitivity analysis is suggested as the next step after determining a value of X.

In practice, X can be determined by the results rather than contributing to them. Because the problem is linear, with only Objective and Subjective Factor Measures, a graph of the sensitivity analysis can be made and the values of X at which results change can be identified. The question to the analyst then becomes, for instance, "Is my Objective Factor Decision Weight less than 0.73?" rather than "What is my Objective Factor Decision Weight?" This greatly simplifies part of the difficulty with the Objective Factor Decision Weight, as it is easier to provide a yes or no answer than a real number. However, a problem could have many intersections of the site lines, leading to many more binary questions.

If the analyst wants to know the order of sites below the top-rated site, then more binary questions will need to be answered if the site lines intersect below the site line segment which defines the top rated site at any given value of X. Answers to the binary questions are critical, as the preference order of sites is by definition different on each side of an intersection of site lines.

All of these practical questions in assigning a value to X are eclipsed by the difficulty in understanding the real meaning of X. By assigning a relative weight to Objective and Subjective Factor Measures, the algorithm assumes each to be a monolithic factor, rather than the combination of many factors, great and small. Each can contain very important factors as input, such as the capital required to build or prepare the site or the site's suitability to its intended purpose. Each Factor Measure can also contain relatively minor input, such as the cost differential between suppliers to different sites of low-cost items or perhaps the difference in climate between sites if climate is judged to be of low importance. There will be both major and minor components in both the Objective and Subjective Factor Measures, thus clouding the meaning of an Objective Factor Decision Weight. The composite nature of the Factor Measures usually makes the selection of a valid Objective Factor Decision Weight based upon the problem's components an impossible task.

2.5.3 Amount of Required Pairwise Comparisons

The Brown-Gibson procedure is awkward to use on problems with either a large number of candidate sites or a large number of subjective factors because the required number of pairwise comparisons becomes unwieldy. This is a practical rather than a theoretical limitation. Because it is important that such comparisons are uniform and unbiased, they are usually made by a committee. If there are a large number of comparisons to be made, it is not reasonable that a committee will give similar consideration to comparisons at the end of a very long meeting than to those at the beginning. Consider a site selection problem with ten subjective factors and six candidate sites. Calculation of the Subjective Factor Weights will require $(10^2 + 10)/2 = 45$ comparisons (see Section 3.1). For each of the ten subjective factors, calculation of the Site Weights will require $(6^2 + 6)/2 = 15$ comparisons. Thus, the entire evaluation requires $45 + 10\times15 = 195$ pairwise comparisons. This is likely to be too much work to be done responsibly at one sitting, and multiple sessions are likely to have different biases. It is not impossible to get good results from the algorithm on problems of this size, but the path to such results is likely to be very long and exhausting.

Chapter 3
The Analytic Hierarchy Process

3.1 Background

Much research has addressed the problem of measuring significant factors defined over different domains. One recent and popular method is the "analytic hierarchy process," (AHP) which was developed in the early 1970's by Dr. Thomas L. Saaty [18]. AHP has been applied to a wide variety of problems characterized by either inputs defined over multiple domains, and inputs defined over non-objective domains. Some applications include transportation planning in third world countries, deciding faculty tenure, political candidacies, energy rationing, the future of higher education in the United States, and the selection of research projects from a selection which is wider than available resources [16] [19] [21] [22] [23].

The process is roughly based on the perceived method of operation of the human mind. As Saaty explains, when the brain is confronted with many input factors to a complex decision,

> it aggregates them into groups, according to whether they share certain properties. Our model of this brain function allows a repetition of this process, in that we consider these groups, or rather their identifying common properties, as the elements of a new level in the system. These elements may, in turn, be grouped according to another set of properties, generating the elements of yet another "higher," level, until we reach a single "top" element which can often be identified as the goal of our decision-making process.
> [18, p. x]

Thus, AHP decomposes problems into a hierarchical tree of component priority problems. In the theory's purest form, each component problem is addressed by an exhaustive series of pairwise comparisons. A pairwise comparison is a comparison of two individual factors of a problem, the result of which is to rank them equally important, or to rank one as superior to the other. AHP's pairwise comparisons are more sophisticated than those used in the Brown-Gibson procedure because a magnitude is assigned to every judgement of superiority. The method used to produce rankings from an exhaustive series of pairwise comparison also differs between the two methods. An exhaustive series of pairwise comparisons consists of all possible pairings of alternatives. For n factors, this amounts to $(n^2-n)/2$ comparisons. When the number of factors is too large to make $(n^2-n)/2$ comparisons feasible, a simplifying method exists; it is discussed in Section 3.3.3.

3.2 A Detailed Description of the Analytic Hierarchy Process

The Analytic Hierarchy Process involves three basic steps:

1. State the problem as a hierarchical structure
2. Compare the branches at each branch node in the hierarchy
3. Combine pairwise preferences into a ranking of alternatives

3.2.1 Stating problems as hierarchies

As in the quest to solve most engineering problems, proper problem statement can make the difference between an easily solved problem and a very difficult problem. AHP requires that problems be reduced to hierarchies. This is done by decomposing the goal of the problem into the components which define it. These components may be subdivided into subcomponents, but the bottom level of the hierarchy will be the various options which can be taken to achieve the original goal. Thus, the general hierarchy is as shown in Figure 3-1.

Goal
|
Characteristics of the Goal
|
Actions which can be Taken to Meet the Goal

Figure 3-1: General Hierarchy of an AHP Representation

3.2.2 Comparing the Branches at the Nodes in the Hierarchy

With the three-tiered hierarchy, there are two basic questions which must be answered in order to develop a preference among the alternatives. Between the bottom two levels of the hierarchy, one must ask "How would each action contribute to each particular characteristic of the goal?" Between the top two levels of the hierarchy, the question is "What is the relative importance of each characteristic to the goal?"

A brief example can give meaning to these questions. In a site selection problem, the goal at the top of the hierarchy may be to have a plant which best meets the needs of the corporation. The second level of the hierarchy would consist of the corporation's needs. The third level would be the various sites available to the corporation. Assume, for example, that the corporation has just two needs: profitability and long-term capacity. Also assume that only two sites are being considered: Site A and Site B. The questions which would need to be answered are

- What would be Site A's contribution to Profitability?
- What would be Site B's contribution to Profitability?
- What would be Site A's contribution to Long-term Capacity?
- What would be Site B's contribution to Long-term Capacity?
- What is the relative importance of Profitability to the Needs of the Corporation?
- What is the relative importance of Long-term Capacity to the Needs of the Corporation?

Sometimes these questions are easily answered. Traditional cost and transportation studies can produce reasonably accurate figures for the profitability of potential plant sites, and allow the analyst to rank the various sites with respect to profitability. Other questions are not so easily answered, and AHP's way of dealing with these questions is to mandate pairwise comparisons. This avoids the problem of quantifying a particular site with respect to an unquantifiable attribute. Again, a brief example can explain the use of pairwise comparisons:

Suppose that one of the attributes desired in a site was community support for the new facility, and that four sites are being considered. Some community support can be quantified in monetary terms, but overall community support takes on many forms and is not quantifiable in every respect. Rather than ask that the four sites be immediately ranked in order of decreasing community support, AHP instead asks that each site be compared against the others with respect to community support. These pairwise comparisons would then be aggregated into a ranking of the sites with respect to community support.

When comparing two branches with respect to the characteristic at a branching node (such as two sites with respect to community support), the two branches can either be judged equal or one can be judged superior to the other. To perform a pairwise comparison, AHP works with ratios of importance. If two branches are equal, the ratio will be unity. If one is superior, a measure of superiority is needed. Saaty defines ratios between 1 and 9, where 1 indicates no preference between the branches and 9 indicates that one is far superior to the other. As a measure of *inferiority*, the reciprocals of these numbers are used. Saaty's description of the numerical rankings is presented in Figure 3-2. The scale recommended by Saaty stops at nine so that all rankings are within one order of magnitude. The reason for this is that if one factor is an order of magnitude greater than another, the problem is poorly defined. Factors less than an order of magnitude as great as the problem's most important factors should not be included in the analysis.

Scale of Relative Importance		
Intensity of Relative Importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Moderate importance of one over another	Experience and judgment slightly favor one activity over another.
5	Essential or strong importance	Experience and judgment strongly favor one activity over another
7	Demonstrated importance	An activity is strongly favored and its dominance is demonstrated in practice.
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation.
2, 4, 6, 8	Intermediate values between the two adjacent judgments	When compromise is needed.
Reciprocals of above nonzero numbers		If an activity has one of the above numbers (e.g. 3) compared with a second activity, then the second activity has the reciprocal value (i.e., 1/3 when compared to the first).

Figure 3-2: Scale of Relative Importance [20, p. 27]

A series of pairwise comparisons should be made so that each alternative at a node has been compared with every other alternative. As discussed in Section 3.1, this requires $(n^2-n)/2$ comparisons. The results of these comparisons are normally expressed in a matrix $\overline{\mathbf{A}}$, defined as

$$\begin{matrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{matrix}$$

This matrix has several distinct characteristics. First, all elements on the principal diagonal equal unity because every branch is equal to itself. Second, every element a_{ji} is equal to $1/a_{ij}$. This is obvious from the definition of the components of $\overline{\mathbf{A}}$, for if branch i is a_{ij} times as preferable as branch j, then branch j must be $1/a_{ij}$ times as preferable as branch i. This second preference $(1/a_{ij})$ is the definition of a_{ji}. Finally, all elements of $\overline{\mathbf{A}}$ must be positive, because only positive preferences are defined.

Having defined the matrix $\overline{\mathbf{A}}$, it is now possible to see how the pairwise comparisons can be combined into a ranking of the branches at the node. This is best explained by working backwards from the ranking of each comparison to the idea of the weights of the individual factors which, divided into each other, form the matrix [18] [20] [17].

Suppose that a ranking, or weight, exists for each of the branches at a given node. If the ideal pairwise comparisons are made, then for each element of the matrix $\overline{\mathbf{A}}$, $a_{ij} = w_i/w_j$, where w_i is the weight of branch i and w_j is the weight of branch j. Thus, an equivalent representation of $\overline{\mathbf{A}}$ is

$$\begin{matrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{matrix}$$

Assume for the moment that all comparisons are ideal—that is, all a_{ij} are exact ratios of the weights of factors i and j. Consider the weight of a given factor i, w_i. For a problem with n factors, it can be shown that the product of n

and w_i is vector of all factor weights $\vec{w}$, and the i^{th} row of the $\overline{\mathbf{A}}$ matrix:

$$n \times w_i = w_i + w_i + \cdots + w_i$$

$$n \times w_i = \frac{w_1}{w_1} \times w_i + \frac{w_2}{w_2} \times w_i + \cdots + \frac{w_n}{w_n} \times w_i$$

$$n \times w_i = a_{i1} \times w_1 + a_{i2} \times w_2 + \cdots + a_{in} \times w_n$$

$$n \times w_i = [a_{i1} \;\; a_{i2} \;\; \cdots \;\; a_{in}] \times \vec{w}$$

When the weights of all factors are viewed this way, the resulting set of equations can be written as

$$n \times \vec{w} = \overline{\mathbf{A}} \times \vec{w} \tag{3.1}$$

In matrix theory, if there exist a number n and and vector $\vec{w}$ such that equation (3.1) holds, then $\vec{w}$ is an **eigenvector** of matrix $\overline{\mathbf{A}}$ which corresponds to **eigenvalue** n. Thus, by finding the right eigenvector of a matrix $\overline{\mathbf{A}}$, one can find the priorities expressed by $\overline{\mathbf{A}}$. Up to this point, however, it has been assumed that the judgements used to form $\overline{\mathbf{A}}$ are ideal. When subjective judgements are used to form $\overline{\mathbf{A}}$, then each element a_{ij} may not equal exactly the ratio of weights w_i/w_j. This means that equation (3.1) is no longer valid. Saaty states, however, that this formulation can still be used even if $\overline{\mathbf{A}}$ deviates from a matrix of ideal weight ratios because of two facts of matrix theory [18, p. 51]. The first fact is that if all diagonal elements of a matrix are unity (ie., $a_{ii} = 1$), then for an $n \times n$ matrix, the sum of all of the matrix's eigenvalues equals n. Thus, when judgements are ideal and equation (3.1) holds, all eigenvalues except n must equal zero. Thus, n is the largest eigenvalue of $\overline{\mathbf{A}}$. The second fact is that if the elements a_{ij} of a positive reciprocal matrix change by small amounts, then the eigenvalues of the matrix also change by small amounts. $\overline{\mathbf{A}}$ is by definition a positive reciprocal matrix, so by combining these two rules one can deduce that small deviations in elements of $\overline{\mathbf{A}}$ from ideal ratios of weights will keep the largest eigenvalue close to n and the remaining eigenvectors close to zero. Thus, the components of the eigenvector corresponding to the largest eigenvalue will be close to the true priority vector. The elements of the eigenvector represent the priorities of the individual branches at any one node, and are the basis of comparison among the branches.

3.2.3 Ranking Alternatives

The alternatives are normally the terminating nodes at the bottom of the hierarchy. These nodes, and all others in the tree, receive a score based upon the preferences identified at the branch nodes higher in the hierarchy. These preferences are equal to the components of the eigenvector corresponding to the largest eigenvalue of the preference matrix.

The procedure is best understood by example. Consider the following simplified problem:

Problem:

A person wishes to buy an automobile. He has narrowed his choices down to three different models, from which he must choose one. He has decided to base his decision on the car's aesthetics, warranty and performance rating.

Solution:

This problem could be modeled as a three-tiered hierarchy. The top level would be the object of the entire decision, "satisfaction with his purchase." The next level would consist of the criteria on which the decision is to be based: Aesthetics, Warranty and Performance Rating. The final level would be the available choices: Chevrolet, Dodge and Ford. The hierarchy is shown in Figure 3-3.

Analysis begins by comparing the factors on the second level against each other. Using Saaty's suggested scale of 1 to 9, the consumer first compares Aesthetics and Performance Rating. He has great trust in automobile magazines

Satisfaction with his Purchase

Aesthetics
Warranty
Performance Rating

Chevrolet
Dodge
Ford

Figure 3-3: Hierarchy for Automobile Selection Example

and decides that their opinions are much more important than his attraction to the style of the car, and thus he assigns Performance Rating a 5 to 1 priority over Aesthetics. Next, Warranty and Performance Rating are compared. The consumer hopes that sound design and construction, as rated by the Performance Rating, will make the warranty irrelevant. He assigns Performance Rating a 3 to 1 advantage over Warranty. Finally, Aesthetics and Warranty are compared. The style-conscious consumer believes Aesthetics to be much more important than the warranty and thus assigns Aesthetics a 5 to 1 advantage over Warranty. The resulting matrix of comparisons for this level is as shown in Figure 3-4.

Top Level Hierarchy			
	Aesthetics	Warranty	Perfromance Rating
Aesthetics	-1-	5	1/5
Warranty	1/5	-1-	1/3
Performance Rating	5	3	-1-

Figure 3-4: Relative Weights of Factors for Selecting an Automobile

Next, a similar comparison is made for each car model in the next tier of the hierarchy, using only one of the criteria in the second level as the sole basis of judgement. Assume the consumer's own tastes lead him to assign weight ratios for the car models' aesthetics as shown in Figure 3-5.

Aesthetics			
	Chevrolet	Dodge	Ford
Chevrolet	-1-	1/3	1/5
Dodge	3	-1-	1
Ford	5	1	-1-

Figure 3-5: Relative Weights of Aesthetics among Automobile Choices

Comparisons for Warranty probably cannot be made on the basis of comparing the length of time for which the car is covered, since warranties typically cover different components of the car for different time periods and levels of service. After careful examination of the warranty plans, the consumer compares all pairs of automobiles and develops the relative weights shown in Figure 3-6. Finally, consultation with automobile publications yields comparisons for performance ratings as shown in Figure 3-7.

Warranty			
	Chevrolet	Dodge	Ford
Chevrolet	-1-	1/3	3
Dodge	3	-1-	5
Ford	1/3	1/5	-1-

Figure 3-6: Relative Weights of Warranties among Automobile Choices

Performance Rating			
	Chevrolet	Dodge	Ford
Chevrolet	-1-	1	1/3
Dodge	1	-1-	1/3
Ford	3	3	-1-

Figure 3-7: Relative Weights of Car Performance among Automobile Choices

The above comparisons complete the information needed to conduct AHP analysis to determine the consumer's best choice. The next step in the AHP analysis is to find the eigenvector associated with the largest eigenvalue for each of the comparison matrices. This can be a difficult calculation, especially for large comparison matrices. Saaty recommends several approximation methods, and the best of them has been made simple by technology now available in hand-held scientific calculators. That method is to approximate each component of the eigenvector by the nth root of the product of the n elements in each row of $\overline{\mathbf{A}}$. Symbolically,

$$e_i = \left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}$$

Because the components of the eigenvector represent priorities, or weights, they can be scaled. It is convenient to have them sum to unity, so normalize them to do so.[1] When scaled to sum to unity, the components can easily be interpreted as percentage weights. Figure 3-8 shows the actual eigenvector components of the priority vector of each matrix ($\|e_i\|$), as well as the approximation ($\|(\prod a_{ij})^{\frac{1}{3}}\|$).

To compute each model's overall rating, multiply its score for each attribute by that attribute's contribution to the overall objective. A uniform way of expressing this is as the product of a matrix $\overline{\mathbf{E}}$ and a vector $\vec{w}$, where each column of $\overline{\mathbf{E}}$ is the priority vector of one of the attributes and $\vec{w}$ is the overall priority vector of the attributes. Both $\overline{\mathbf{E}}$ and $\vec{w}$ are formed from the normalized priorities already calculated. In the example,

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[1] The symbol $\|x\|$ is used to denote components of a vector "normalized" so that their *scalar* sum is unity. Note that this differs from the usual mathematical definition, in which components of a vector are normalized so that their *vector* sum is unity.

Overall Priority of Each Factor						
	Aesthetics	Warranty	Performance Rating	$(\prod a_{ij})^{\frac{1}{3}}$	$\|(\prod a_{ij})^{\frac{1}{3}}\|$	$\|e_i\|$
Aesthetics	1	5	0.2	1	0.25829	0.25828
Warranty	0.2	1	0.33333	0.40548	0.10473	0.10473
Performance Rating	5	3	1	2.46621	0.63699	0.63699

Aesthetic Weights						
	Chevrolet	Dodge	Ford	$(\prod a_{ij})^{\frac{1}{3}}$	$\|(\prod a_{ij})^{\frac{1}{3}}\|$	$\|e_i\|$
Chevrolet	1	0.33333	0.2	0.40548	0.11397	0.11397
Dodge	3	1	1	1.44225	0.40539	0.40539
Ford	5	1	1	1.70998	0.48064	0.48064

Warranty Weights						
	Chevrolet	Dodge	Ford	$(\prod a_{ij})^{\frac{1}{3}}$	$\|(\prod a_{ij})^{\frac{1}{3}}\|$	$\|e_i\|$
Chevrolet	1	0.33333	3	1	0.25829	0.25855
Dodge	3	1	5	2.46621	0.63699	0.64415
Ford	0.33333	0.2	1	0.40548	0.10473	0.09730

Performance Weights						
	Chevrolet	Dodge	Ford	$(\prod a_{ij})^{\frac{1}{3}}$	$\|(\prod a_{ij})^{\frac{1}{3}}\|$	$\|e_i\|$
Chevrolet	1	1	0.33333	0.69336	0.2	0.2
Dodge	1	1	0.33333	0.69336	0.2	0.2
Ford	3	3	1	2.08008	0.6	0.6

Figure 3-8: Priority Vectors of Each Comparison Matrix

$$\overline{\mathbf{E}} = \begin{matrix} 0.11397 & 0.25855 & 0.2 \\ 0.40539 & 0.64415 & 0.2 \\ 0.0.48064 & 0.09730 & 0.6 \end{matrix} \qquad \vec{w} = \begin{matrix} 0.25828 \\ 0.10473 \\ 0.63699 \end{matrix}$$

The vector resulting from the multiplication of $\overline{\mathbf{E}} \times \vec{w}$ is

$$\begin{matrix} 0.18391 \\ 0.29956 \\ 0.48078 \end{matrix}$$

Normalization yields

$$\begin{matrix} 0.19073 \\ 0.31067 \\ 0.49860 \end{matrix}$$

These correspond to the weights of the Chevrolet, Dodge and Ford models respectively. Based upon this analysis, the Ford model is the car which most fulfills this particular consumer's needs. The next best choice would be the Dodge, but its score of 0.31067 is only 62.3% of the Ford's score. If the two scores were closer, the consumer might reasonably set aside AHP's recommendations and select the Dodge. To do so anyway would suggest a lack of confidence in the judgements used in the AHP analysis. Sometimes such a lack of confidence is justified. Another reason for not selecting the top-rated option is if the problem itself changes. If, for instance, the Ford model's price is double that of the Dodge model, the consumer may well select the Dodge. In this case, price is an important decision factor and should have been included in the original analysis. It can still be included with a minimum amount of extra work: pairwise comparisons with respect to price must be performed for all of the automobiles and the priority vector of the resulting matrix calculated, and pairwise comparisons between price and the other attributes must be performed and a new top-level priority vector calculated. Then, the new values must be multiplied in another $\bar{\mathbf{E}} \times \vec{w}$ calculation.

3.2.4 Consistency

AHP assumes that the pairwise comparisons will result in judgements which are close to the ratios of the factors' actual weights. Saaty defines a consistency ratio which can be used for a comparison matrix to judge whether this condition is met [18]. To determine this Consistency Ratio (*CR*), a Consistency Index (*CI*) must first be calculated. Recall from Section 3.2.2 that the largest eigenvalue of a completely consistent comparison matrix is equal to the order of the matrix. (ie., for a comparison matrix of n factors, the largest eigenvalue λ_{max} is equal to n.) The Consistency Index is defined as

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

To form the Consistency Ratio, the Consistency Index is divided by a Random Index (*RI*):

$$CR = \frac{CI}{RI}$$

The Random Index (*RI*)is an average Consistency Index for random reciprocal matrices consisting of numbers from 1 through 9 (the AHP comparison ratios) and their inverses. This Random Index was found to be a function of the matrix order, n, and Saaty gives values of the Random Index for matrices of orders from 1 to 15. These values are reprinted in Figure 3-9. A perfectly consistent matrix will have a Consistency Ratio of zero. Generally, a Consistency Ratio of 0.10 or less is considered acceptable.

One obstacle to calculating the Consistency Ratio is that λ_{max} is not usually known. It is the difficulty of calculating this eigenvalue and its associated eigenvector which prompted the approximation technique described in

Random Index (*RI*)[2]

n	*RI*	n	*RI*	n	*RI*
1	0	6	1.24	11	1.51
2	0	7	1.32	12	1.48
3	0.58	8	1.41	13	1.56
4	0.90	9	1.45	14	1.57
5	1.12	10	1.49	15	1.59

Figure 3-9: Values for the Random Index (*RI*) [18, p. 21]

Section 3.2.3. Similarly, there is an approximation technique to estimate the value of λ_{max}. That technique is to multiply the comparison matrix by the weight vector which was calculated from it, which produces a vector result. Divide each component of this vector by the corresponding component of the weight vector, which produces a set of n values. The average of these n values is an approximation of λ_{max}. Symbolically, for comparison matrix $\overline{\mathbf{A}}$ and weight vector $\vec{w}$, form a new vector $\vec{v}$ defined by

$$\vec{v} = \overline{\mathbf{A}} \times \vec{w}$$

λ_{max} is then approximated by

$$\lambda_{max} = \frac{v_1 / w_1 + v_2 / w_2 + \cdots + v_n / w_n}{n}$$

$$\lambda_{max} = \frac{\sum_{i=1}^{n} v_i / w_i}{n}$$

As an example of this calculation, consider the automobile selection example in in Section 3.2.3. The Consistency Ratio (*CR*) of each matrix can be calculated and then used to judge the matrix's consistency. Consider the top-level com-

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[2] The Random Index (*RI*) should increase as the order of the matrix increases. The drop in the value of *RI* between $n = 11$ and $n = 12$ is due to a change in the sample size used to determine *RI*, which is an average value. For matrices up to size $n = 11$, a sample size of 500 was used. For matrices from sizes $n = 12$ to $n = 15$, a sample size of 100 was used [18].

parison matrix, shown in Figure 3-4. Multiplying this matrix by the calculated weights (shown in Figure 3-8) yields the vector

$$\begin{matrix} 0.90933 \\ 0.36872 \\ 2.24258 \end{matrix}$$

Dividing each of these components by the corresponding calculated weight gives the values of

$$\begin{matrix} 3.52071 \\ 3.52063 \\ 3.52059 \end{matrix}$$

The average of these values is $\lambda_{max} = 3.52064$. The Consistency Index (CI) is calculated as

$$CI = \frac{3.52064 - 3}{3 - 1}$$

$$CI = 0.26032$$

For a matrix of order 3, the Random Index (RI) is 0.58. Dividing 0.26032 by 0.58 yields a Consistency Ratio (CR) of 0.44883. Because this number is much greater than 0.10, it indicates that something is very wrong in the original comparison matrix. Indeed, upon reexamination of the matrix, a flaw is discovered: the Performance Rating is five times as important as Aesthetics, which are five times as important as the Warranty. Thus, the Performance Rating should be around 25 times as important as the Warranty. Instead, it was rated as only three times as important—less than the ratio between Performance Rating and Aesthetics. This calls for reevaluation of the original judgements; the new data are shown in Figure 3-10. The ratio between Performance Rating and Aesthetics, and between Aesthetics and Warranty, have been reduced from 5 to 3. The ratio between Performance Rating and Warranty has been increased from 3 to 5. The weights of the factors are then recalculated. That the new weights are equal to the old weights is a coincidence, but related to the small size of this matrix: normally, a revision would not entail changing all of the comparison values. Because the weights remain unchanged, the end results also remain unchanged. The Consistency Ratio (*CR*) of the new matrix is a much more acceptable 0.03320. The Consistency Ratio of the other comparison matrices in the example are 0.02505 for Aesthetics, 0.03697 for Warranty, and 0.0 for Performance Ratio, which is perfectly consistent.

Revised Overall Priority of Each Factor						
	Aesthetics	Warranty	Performance Rating	$(\prod a_{ij})^{\frac{1}{3}}$	$\|(\prod a_{ij})^{\frac{1}{3}}\|$	$\|e_i\|$
Aesthetics	1	3	0.33333	1	0.25829	0.25828
Warranty	0.33333	1	0.2	0.40548	0.10473	0.10473
Performance Rating	3	5	1	2.46621	0.63699	0.63699

Figure 3-10: Revised Overall Comparison Matrix and Priority Vector

If the Consistency Ratio is greater than 0.10, then the pairwise comparisons should be reevaluated. There are several ways to go about this reevaluation. The best, and most time-consuming, is to consider the actual judgements themselves to see if any are in error. Another, analytic, way is to compare each element of the comparison matrix with the ratio of the weights calculated for its component factors. That is, compare each a_{ij} with the ratio w_i / wj, where w_i and w_j were calculated from the comparison matrix. Replace the a_{ij} which most poorly matches its corresponding w_i / w_j with the calculated ratio of weights. Which element is most "poorly matched" is a matter determined by heuristics; two good heuristics are to evaluate the quality of the match by the absolute difference between a_{ij} and w_i / w_j, and by the ratio of w to w_i / w_j.

Saaty describes other heuristics as well [18]. There is inherent danger in revising judgements in this way, as answers are distorted somewhat by the use of inputs (the weight ratios) which are known to be based on faulty judgements. Caution must be exercised when revising judgments in this way.

3.3 Reduction of the Amount of Pairwise Comparisons

Problems with many comparison matrices or with large comparison matrices (large values of n) will require so many comparisons that strict use of AHP will be infeasible or, at best, very time consuming. For these cases, an analyst may wish to use a simplifying technique which reduces the amount of pairwise comparisons. The price of this shortcut is that the independence of information in the analysis deteriorates.

3.3.1 Some Concepts from Graph Theory

It is helpful to look at the $\overline{\mathbf{A}}$ matrix as a graph in order to understand how the process of creating it can be shortened. This requires some elementary understanding of the field of mathematics known as graph theory. The various terms used in graph theory all have rigorous mathematical definitions, but more practical descriptions will be developed here for use in better understanding AHP and the site selection problem.[3]

A **graph** is a collection of **vertices** (points) which are joined by **edges** (lines). An edge may connect only two vertices, and not all vertices need to share edges with each other. For the purpose of AHP, consider only non-trivial graphs which contain at least one edge and two vertices, and where no edge connects a single vertex with itself. A **path** is a series of edges which are connected

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[3] There are many excellent texts on the topic of graph theory. The author particularly recommends [3], [4], [8] and [27], from which much of this presentation originates.

to one another, end to end. If a path exists between all points in a graph, the graph is said to be **connected**. Figure 3-11*a* contains a graph which is not connected; Figure 3-11*b* contains a connected graph. In Figure 3-11*b*, one path between vertices A and D is the edges which go from A to B to D. Other paths between A and D are A-C-B-D and A-C-D.

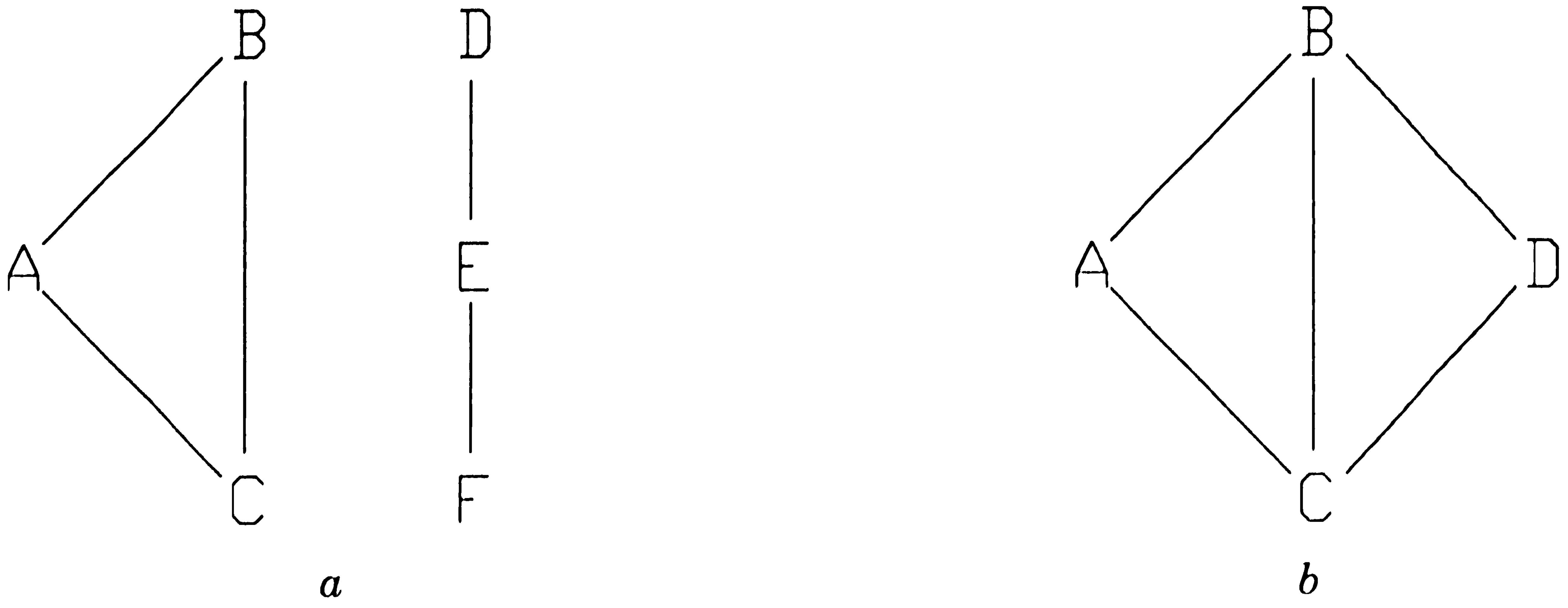

Figure 3-11: *a*: a disconnected graph
b: a connected graph

If it is possible in a graph to have a path between a vertex and itself, without retracing any edge in the path, then that path is called a **circuit**. There are at least two paths between any two vertices on a circuit. The path A-B-D-C in Figure 3-11*b* is a circuit. A connected graph with no circuits is called a **tree**. Of the various properties of trees, the most important one with respect to AHP is that there is only one path between any two distinct vertices of a tree.

Another notion of graph theory which must be understood in order to simplify AHP is that of subgraphs. A **subgraph** is a graph which is a subset of another graph. The subgraph contains some or all of the parent graph's vertices and some or all of its edges, but has no vertices or edges which are not in the parent graph. If the subgraph contains all of the vertices in the parent graph, it is said to be a **spanning subgraph**.

It is possible, given a connected graph with circuits, to remove edges which comprise circuits until no circuits are left. Because only edges, and not vertices, were removed, the resulting graph is a spanning subgraph of the original. Because it has no circuits, it is also a tree. Such a subgraph is called a **spanning tree**. A spanning tree of a graph contains all of the vertices of the parent graph, but has a distinct path between any two vertices. These characteristics of a spanning tree allow it to be used as an approximation of the parent graph in certain circumstances, and AHP meets these circumstances.

3.3.2 The Comparison Matrix as a Graph

The $\overline{\mathbf{A}}$ matrix can be viewed as a graph. The factors which are used in the pairwise comparisons become the vertices of the graph. The values of the elements of $\overline{\mathbf{A}}$, a_{ij} are assigned to the edges connecting vertices i and j. Because the bottom triangle of the matrix is the reciprocal of the top triangle, only one half of the matrix needs to be included in the graph. Otherwise, there would be two edges between each pair of vertices (a_{ij} and a_{ji}). While this would still form a valid graph, it introduces unneeded complexity. Also, connections between each vertex and itself are not shown. These connections represent the diagonal a_{ii} elements, which are defined as unity. Figure 3-12 shows a sample $\overline{\mathbf{A}}$ matrix and its associated graph. Ordinarily, edges of a graph would not have values assigned to them. Here, the values are not an inherent part of the graph but rather labels used to refer to the edges.

For a comparison matrix with n rows and columns, each vertex will have $n-1$ edges connected to it. The total number of edges will be

$$\text{number of edges} = \frac{n \text{ vertices } \times (n-1) \text{ edges/vertex}}{2}$$

The two in the denominator is to compensate for double counting each edge by

$$\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{array}$$

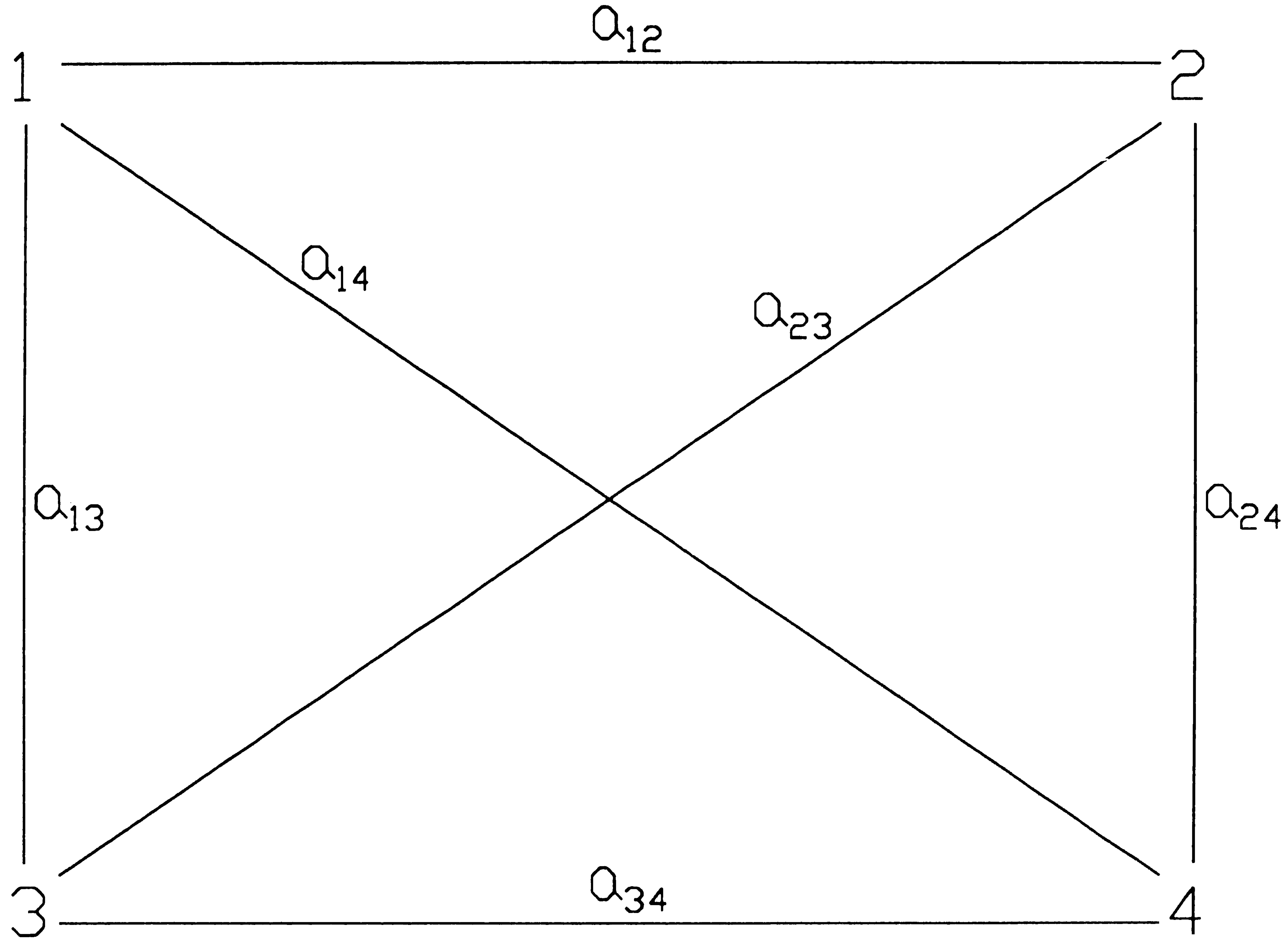

Figure 3-12: An $\overline{\mathbf{A}}$ matrix and its associated graph

counting it at each of its vertices. Note that the resulting expression is the same as the expression developed in Section 3.1 for the number of pairwise comparisons which must be made at a node with n factors. Each edge of the graph matches with one pairwise comparison.

3.3.3 The Minimum Comparison Matrix: A Spanning Tree

If the ratios in the $\overline{\mathbf{A}}$ matrix are ideal—that is, if they are perfectly correct ratios of the weights of the various decision factors (see Section 3.1)—then for any three factors i, j and k, $a_{ik} = a_{ij} \times a_{jk}$. Thus, if any two of a_{ij}, a_{ik} and a_{kj} are known, the third can be calculated. If either a_{ij} or a_{jk} are unknown, it too can be calculated from two other elements. By extension, any element of $\overline{\mathbf{A}}$ can be calculated from any number of other elements of $\overline{\mathbf{A}}$, if those elements can form a chain of the type

$$a_{bz} = a_{bc} \times a_{cd} \times a_{de} \times \cdots \times a_{wx} \times a_{xy} \times a_{yz}$$

Substituting a product of other elements of $\overline{\mathbf{A}}$ for a particular a_{ij} will yield the true value of a_{ij} only if all of the elements used in the substitution are ideal. To verify if the elements are ideal, it is necessary to perform the pairwise comparison of i and j to find a_{ij}. Doing so, however, defeats the purpose of the original substitution, which was done to reduce the number of necessary comparisons. Still, a substitution can be done if the analyst has confidence in the judgements which will form the substitution. If the analyst finds some comparisons particularly difficult, examining a substitution may serve as a valuable second opinion or sanity check.

Spanning trees can be used to reduce the number of pairwise comparisons, but the results obtained from such use will tend to be less accurate than if exhaustive pairwise comparisons are made. The reason for this degradation of accuracy is that for each eliminated comparison, a calculated ratio is substituted for the result of an actual comparison. The calculated ratio is a function of other comparisons in the matrix, so a faulty judgement in any of those comparisons has a greater influence on the overall results than if it were used only once in the matrix.

As an example, consider three factors in a comparison matrix: Quality of Local Roads, Delivery Truck Downtime, and Missed Shipments. It may be relatively easy to compare Quality of Local Roads to Delivery Truck Downtime. Similarly, a relationship may exist between Delivery Truck Downtime and Missed Shipments. The relationship between Quality of Local Roads and Missed Shipments, however, may be very nebulous. In this case, the analyst may wish to employ the concept of a spanning tree and define this comparison ratio as

$$\frac{\text{Local Roads}}{\text{Missed Shipments}} = \frac{\text{Local Roads}}{\text{Truck Downtime}} \times \frac{\text{Truck Downtime}}{\text{Missed Shipments}}$$

or

$$a_{\text{LR, MS}} = a_{\text{LR, TD}} \times a_{\text{TD, MS}}$$

where

LR	is the Quality of Local Roads
MS	is the measure of Missed Shipments
TD	is the Delivery Truck Downtime

If this expression is substituted for an acutal pairwise comparison of Quality of Local Roads and Missed Shipments, the number of pairwise comparisons among these three factors decreases from three to two.

Chapter 4
Site Selection Using the Analytic Hierarchy Process

With few modifications, AHP can be used for site selection in the same way it was used for automobile selection in Section 3.2.3. Individual sites would be used in place of automobile models, and factors such as building cost, labor availability and utility costs would be used in place of aesthetics, warranty and performance rating.

4.1 Monetary Objective Factors

Use of monetary objective factors (ie., costs and revenues) raises several issues which can affect the process of analysis as well as its outcome. Among these issues are whether monetary factors can be combined with each other or kept separate, and how monetary factors should be brought to a common basis of comparison. There are many monetary objective factors likely to be encountered in a site selection problem. Appendix B lists and discusses some of the most common ones.

4.1.1 Aggregation vs. Detailing of Monetary Factors

If conducted using the formulation in Appendix C, AHP analysis is equivalent to economic analysis. There are two extremes in analyzing the monetary objective factors in a site selection problem. At one extreme, all such factors can be grouped together into a single figure and that figure can be used as one factor in the top level of the AHP analysis. At the opposite extreme, finely detailed cost components can each be considered as factors in the selection decision. Either method, or any hybrid combination of them, can be used; their results are identical. If more than one monetary factor is used, they will need to be combined using AHP; if only one is used, it is determined by the standard economic formulation. Both will yield the same results but, as Appendix C.2 shows, standard economic analysis is much easier to execute than AHP. Also, using a single monetary factor based on economic analysis has obvious appeal to managers who are concerned about the ultimate impact of the location decision on their bottom lines.

The only real difference between the methods is in the comparison of monetary factors with non-monetary factors; implementation of AHP differs slightly between the case of a single monetary factor and multiple monetary factors.

4.1.1.1 Aggregation of Monetary Factors

Consider the case of an AHP analysis where all monetary factors are aggregated into a Present-Worth value for each site. The pairwise comparisons between the single monetary factor and the non-monetary factors take the form of assigning monetary value to each non-monetary factor. This would seem to make AHP unnecessary, because a standard economic analysis could then be conducted among factors which are all expressed in monetary terms. However, AHP is still useful because the other comparisons of the non-monetary factors among themselves will temper the comparisons made with the monetary factor. Thus, if comparisons made with the monetary factor are in error, the other comparisons will result in final priority weights which are close to the true weights. The analyst must find a comfortable trade-off between the ease of using economic analysis instead of AHP for the monetary factors and the confidence gained in using AHP based on numerous factors instead of only a few factors.

4.1.1.2 Use of Finely Detailed Monetary Factors

There are disadvantages in using many finely-detailed monetary factors. Obviously, the number of required pairwise comparisons will increase as more factors are added. The comparisons between monetary factors are strictly numerical and can thus be easily automated. However, the greater problem is in comparing monetary factors with non-monetary factors. This is a problem for two reasons. First, there will be many monetary versus non-monetary comparisons and the evaluation committee could quickly tire of performing them. Second, some of the comparisons will be between completely unrelated factors. For instance, the committee may be asked to weigh the cost of plastic components against the quality of elementary schools. Although managers are routinely called upon to make value judgements between alternatives, comparing many such diverse factors is likely to produce almost random results and thus introduce error into the analysis.

The implementation of AHP also differs if multiple monetary factors are used. If monetary factors are not aggregated, the **inverse** of costs must be used when ranking sites with respect to monetary factors which are costs. This is because AHP assumes weights to be measures of desirability. For revenue factors, the ratio between the merit of two sites is the ratio of the monetary quantities: a site with a $200,000 revenue is twice as good as a site with a $100,000 revenue. For cost factors, the opposite is true: a site with an associated $200,000 cost is half as good as a site that only costs $100,000. Note that inverses should be used only when ranking sites with respect to individual cost factors. When comparing factors among themselves, the magnitude of the costs (ie., their absolute value) should be used to judge the factor's relevance against other monetary factors. An expression for a number representative of a monetary factor is derived in Appendix C.

4.1.2 Bringing Monetary Factors to a Common Basis of Comparison

Monetary factors are likely to be expressed in a variety of ways. A labor cost will likely be defined as dollars per employee-hour or perhaps per employee-year. Site costs will be in absolute dollars, electricity costs will be in cents per kilowatt-hour. The challenge in comparing these costs is to express all costs in a like manner. A number of ways of converting future cash flows to present values and present values to future cash flows are discussed in Section 2.1 and Appendix D. Of those methods, only Present-Worth (*PW*), Annual Equivalent Amount (*AE*) and Future Worth (*FW*) should be considered for manipulation of financial factors in AHP analysis. The other methods (Rate of Return, Payback Period and Project Balance) do not yield monetary units but rather interest rates and time periods. These bases of comparison are often used for judging one project against another, but cannot be used to judge components of a single project in AHP. This is because of the difficulty of assigning revenues to individual cost components. Without revenues, all costs have infinite payback periods and no rate of return.

The three valid methods of converting monetary factors to common bases of comparison are equivalent, as shown in Appendix D.3. Thus, any of them can be used. To avoid unnecessary recalculations, it is recommended that all factors be converted to one of these bases even if it seems unnatural for any individual comparison. For example, if fifteen monetary factors are defined as annual cash flows and three are defined as initial lump sum costs, the three lump sum costs should be converted to annual cash flows and those numbers used for all comparisons.

Conversion of monetary factors defined in units of money per unit of some item or service requires knowing how much of the item or service will be used.

This quantity may vary from site to site. For example, not only might electricity rates differ between two locations, but the amount of electricity expected to be consumed may differ as well.

4.1.3 Depreciation of Capital Expenses and Cash Flows

When calculating the Present-Worth of cash flows (or the Annual Equivalent or Future Worth of lump sums), an interest rate is required. The selection of a proper interest rate is important if the analysis is to be worthwhile. Most companies will have a specified internal rate of return, or a hurdle rate which is normally used in such calculations. Often, these rates are inflated. If the rate is too high, future cash flows are given too low a weight and results are skewed toward those projects with the fastest return rather than the greatest return. Industrial site relocations tend to be expensive undertakings with impacts far into the future. Where a company is participating in a trend, such as migration of certain industries to the sunbelt, the advantage of relocation may not be felt for several years until after the relocation. During that time, the company is establishing itself in its new location, developing and improving relations with local suppliers and distributors, and otherwise moving along the learning curve of its new facility after disrupting operations [1]. It is important that a realistic interest rate be used, or either the wrong site will be selected or any relocation at all will appear—perhaps wrongly—unprofitable. One researcher suggests that, to the surprise of many managers, a rate of eight percent reflects the true cost of capital to most companies over a period of many years [14].

A second parameter of cash flows is the time period over which flows are to be considered. Again, because relocations tend to have significant long-term effects, lengthy periods should be considered. This can entail considerable risk, as many companies in modern business climates cannot even say with certainty what business they will be in after five or ten years, much less what the parameters of that business will be. Monetary factors should be included for as many periods as the analyst can confidently predict. If the precision of predictions are doubtful, approximated figures and educated guesses are preferable to simply ignoring future flows.[4]

4.2 An Example of AHP Used for Site Selection

The following example is a site selection problem involving eight factors and six sites. It is representative, but on a smaller scale, of typical industrial relocation problems. The factors are a mix of critical, subjective, objective non-monetary and objective monetary factors. Solutions using both AHP and Brown-Gibson will be presented.

Problem:

A company is in need of greater production capacity than can be provided by its single plant in California. It has decided to open a second plant, and to reduce its distribution costs it has decided to consider only locations which are east of the Mississippi River. Two hundred thousand square feet of floor space is needed, which includes space for the manufacturing floor, offices, aisle space, and support functions including a first aid station. The company believes that an accurate measure of its expected return on investment is 10%. The plant is expected to have a service life of 20 years. The company is considering six different locations for the new plant, and a number of factors have been identified for inclusion in the site selection procedure:

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[4] Kaplan [14] explains convincingly that zero is an arbitrary number, and there is no particular reason to favor it over other numbers for assignment to flows which are not known precisely.

- Cost of Buying or Leasing the Facility (Facility Cost)

  This is a quantitative, monetary factor which includes all costs associated with purchasing a facility or obtaining a long-term lease, as well as the costs to be incurred to make the site suitable for production.

- Business Climate

  This is a subjective factor which is included in the analysis at the request of the sales division. The company traditionally has exceptionally strong sales near its plant, and thus does not want to find itself in an economic backwater. The company's products are mostly commercial, and it is important that the new facility be located in a thriving commercial area.

- Distance to Airport

  There needs to be an airport nearby which is capable of handling jet aircraft and which has cargo storage capacity. This is because the company provides overnight delivery of rush orders to customers throughout the country, as well as overnight delivery of replacement parts to its field service offices. Additionally, executives of customer companies are brought to the site from around the country when large orders are being pursued. In addition to producing products, the plant is also to be a showcase.

- Distance to Hospital

  For safety purposes, the site should be near a hospital because toxins are used in its production processes. The company's insurance agent requires that a hospital be within a ten-minute ambulance ride, or else that a doctor be stationed inside the plant. If a doctor is stationed inside the plant, a much larger and more elaborate first-aid station is required than would otherwise be needed.

- Availability of Electricity
- Cost of Electricity

  The company's manufacturing is heavily automated, which has resulted in low direct labor costs but very large utility bills. Currently, the company averages $100,000 of electricity per month. The company's management has decided that the new location must be served by a power company which currently operates at less then 90% of its power plant capacity. The cost of electricity is expected to vary among the location choices and is large enough to merit inclusion in the decision process. The price at the company's California location is 6 cents per kilowatt-hr.

- Cost of Living

The company intends to transfer a number of managers and engineers to the new location, and does not want their life-styles to suffer as a result of moving to a more expensive area. Nor does the company want to give salary increases to support the move. Most of all, it wants to dissuade valued employees from leaving the company in favor of staying in their present homes. The Consumer Price Index, compiled by the U.S. Labor Department's Bureau of Labor Statistics, will be used as the basis for this quantitative, nonmonetary factor.

- Quality of Life

The company recognizes that, despite even a large reduction in the cost of living, many employees will not relocate if the new location does not have facilities for recreation, education, and entertainment which they have come to enjoy in their present location. No strict limits have been placed on this factor, but management has decided that the overall quality of life in the new location should weigh in the location decision.

The company has contracted a commercial real estate broker who has identified six candidate sites in the eastern United States. These are:

1. **Andover, Massachusetts.** A renovated textile plant, originally built in the 1850's, can be rented for $4.50/sq. ft. per year. It is an aesthetically attractive plant, and is located seven minutes by car from a major hospital. The nearest suitable airport is 40 miles away, and can be reached in about an hour and fifteen minutes. The local economy is very healthy, and primarily technology- and finance-based.
2. **Scranton, Pennsylvania.** A lot is available for a long-term lease, and the owner will build a factory to suit the renter's needs. The lease requires a $2 million initial payment and an annual rent of $4/sq. ft. of floor space once the renter occupies the site. The economy is stable, with only very growth. The nearest jet airport is 30 miles away (about a 40 minute drive). A suitable hospital is 30 minutes away from the site.
3. **Atlanta, Georgia.** A warehouse would be bought for $5.5 million. A hospital is five minutes from the site, and the city's major airport is twenty minutes from the site. No site renovations or preparations are needed.
4. **Memphis, Tennessee.** An existing warehouse would be rented for $3/sq. ft. per year. The State of Tennessee would pay for up to $2.5 million for equipment and any necessary site preparation. A hospital is eight minutes from the site, and the airport is a 35 minute drive from the site.

5. **Staten Island, New York.** An existing factory is available in a long-term lease for $5/sq. ft. per month. No renovations would be required. A hospital is 8 minutes from the site and each of New York's airports are within a 30 minute drive; the nearest is about 10 minutes away.
6. **Baltimore, Maryland.** A warehouse would be bought for $3 million. The site is five minutes from a hospital, and twenty-five minutes from the airport.

4.2.1 AHP Solution

First, critical factors must be judged. Only two critical factors have been specified: first, the plant must be in the eastern United States; second, there must be sufficient electricity supply. All of the plants meet the geographical requirement, but a survey of regional power companies shows that the New York area power companies operate at above the acceptable capacity limit, and indeed the city has even had "brown-outs" in recent years. For this reason, the New York site is eliminated from further consideration.

Next, the hierarchy to be used for the location decision is developed. The incentive offered by the State of Tennessee has caused the company to add another factor, Equipment. Originally, it was thought that the cost of the plant's equipment would be $12 million, regardless of the site selected. The offer from Tennessee, however, has made equipment costs a function of location. The other states were approached for incentives, but because the plant is very automated, it employs few people and no other incentives were offered. The decision hierarchy is shown in Figure 4-1.

As a first step, rank the sites with respect to each individual factor. (Alternatively, the factors could be judged among themselves before ranking sites with respect to individual factors.) The first factor is Facility Cost. The cash flows associated with the Facility Cost of each site are calculated using an

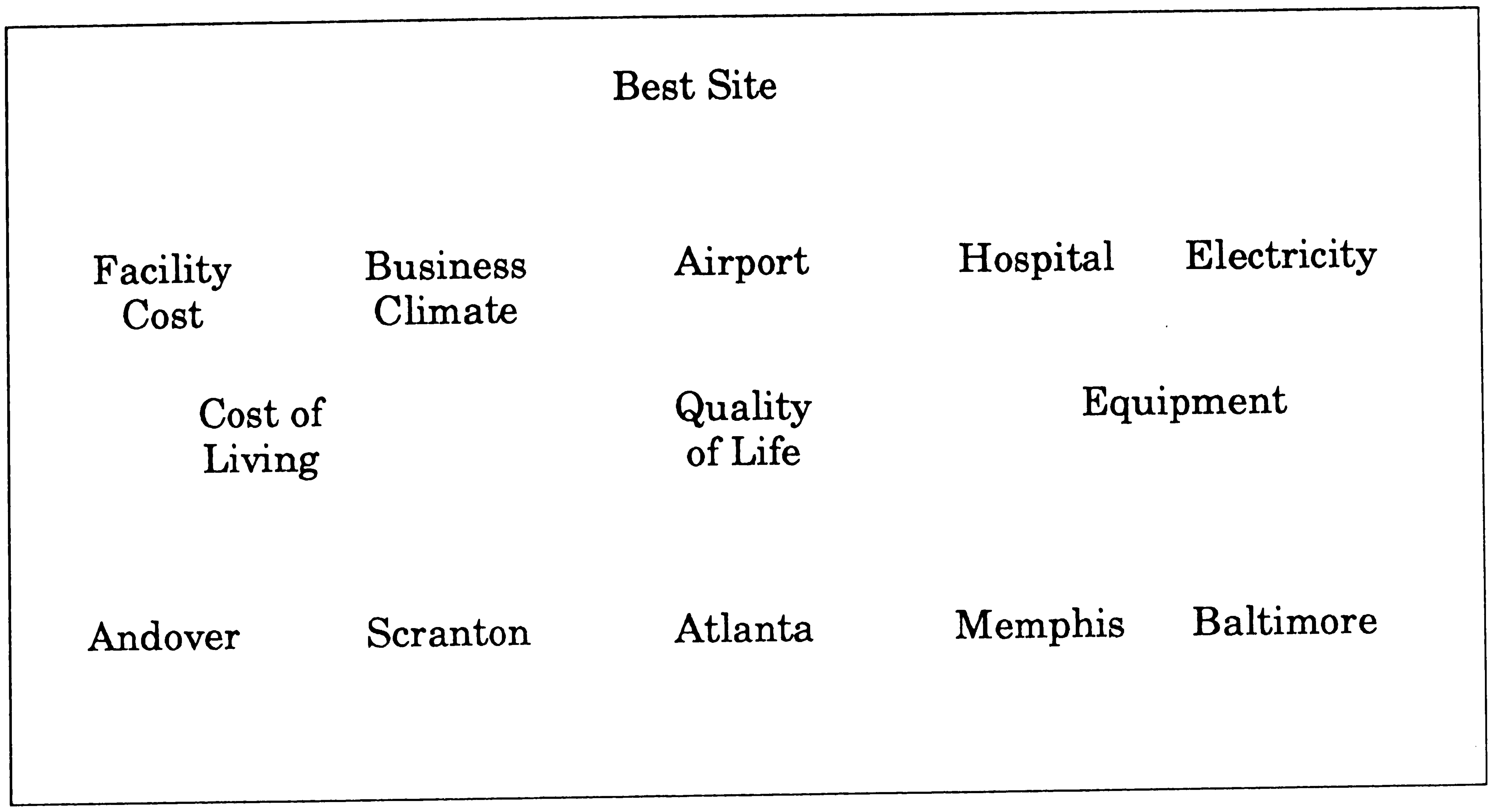

Figure 4-1: The AHP Hierarchy for the Site Selection Example

interest rate of 10% and are shown in Figure 4-2.

Weight of Sites with Respect to Facility Cost						
Site	Initial Cost ($)	Annual Cost ($ per ft^2)	Total Annual Cost ($)	PW ($)	$\frac{1}{PW} \times 10^{-7}$	$\|\frac{1}{PW}\|$
1	0	4.50	900,000	7,662,207	1.30511	0.13367
2	2,000,000	4.00	800,000	6,810,851	1.34964	0.13823
3	5,500,000	0	0	5,500,000	1.81818	0.18621
4	0	3.00	600,000	5,108,138	1.95766	0.20050
6	3,000,000	0	0	3,000,000	3.33333	0.34139

Figure 4-2: Site Weights with Respect to Facility Cost

A committee from the sales organization was asked to perform pairwise comparisons of the sites with respect to business climate. Because local sales were the prime motivation for including this factor in the analysis, the company's management felt that the sales organization was best suited to make the evaluations. Their pairwise comparisons are shown in Figure 4-3, as are the eigenvector approximations of the judgements. The Consistency Ratio for this matrix is $CR = 0.023622$, which indicates that the matrix is of acceptable consistency.

Weight of Sites with Respect to Business Climate						
	Site 1	Site 2	Site 3	Site 4	Site 6	$(\prod a_{ij})^{\frac{1}{5}}$	$\|(\prod a_{ij})^{\frac{1}{5}}\|$
1	-1-	5	2	5	3	2.72407	0.42766
2	0.2	-1-	0.33333	2	0.33333	0.53649	0.08423
3	0.5	3	-1-	5	1	1.49628	0.23491
4	0.2	0.5	0.2	-1-	0.33333	0.36710	0.05763
6	0.33333	3	1	3	-1-	1.24573	0.19557

Figure 4-3: Site Weights with Respect to Business Climate

To weigh sites with respect to airport distances, the inverse of the driving time is used. The inverse is used because, like costs, smaller measures are better than larger measures. Figure 4-4 shows the weight of each site with respect to Airport Location.

Weight of Sites with Respect to Airport Location			
Site	Driving Time	$\frac{1}{\text{Driving Time}}$	$\|\frac{1}{\text{Driving Time}}\|$
1	75	0.01333	0.08498
2	40	0.025	0.15933
3	20	0.05	0.31867
4	35	0.02857	0.18210
6	25	0.04	0.25493

Figure 4-4: Site Weights with Respect to Airport Location

The company needs either a hospital within ten minutes of the plant, or to have a doctor on site at the plant. Upon further discussion, the company's management has concluded that, if a hospital is within the required ten minute ride, the time required to reach the hospital is not a factor. Thus, a one minute ride is as meaningful to them as a nine minute ride. All sites except Scranton (Site 2) meets the ten-minute requirement, so this factor can be eliminated from the AHP analysis.

For the lone site which requires an in-house doctor, it is estimated that the doctor will cost $200,000 per year. This is the cost of the doctor's salary and benefits, more elaborate medical equipment than would normally be found in the plant's first-aid station, and additional floor space needed to house the medical office. Annualized at 10%, this sum has a Present-Worth of $1,072,713. This amount will be considered as part of the equipment cost of added to the equipment cost for Site 2 only.

The next factor to be considered is the cost of electricity. In its California plant, the company's electricity usage averages $100,000 per month, and electricity is bought from the local power company for 6 cents per kilowatt hour. Thus, the plant's average usage is $\$100{,}000 / \$0.006$ per kW-h $= 1{,}666{,}667$ kW-h per month. Electricity used for production is not expected to change at the new location, since the production equipment is to be a duplicate set of the existing equipment in California. The other major use of electricity would be for heat, at those sites which have electric heating and air conditioning. This is a utility cost and is not being considered in this example.

The local power companies in each of the five locations have provided the company with costs per kilowatt-hour. These costs are a function of many factors, including amount of electricity used and the plant's specific location within the power company's service area. These rates, and the factor's weight calculations, are shown in Figure 4-5.

To evaluate the Cost of Living factor, the Consumer Price Index (CPI) is used. Data is available for most of the cities being considered; where it is not,

Weight of Sites with Respect to Electricity Cost					
Site	Cost ($ per kw-h)	Total Cost per Month ($)	PW ($)	$\frac{1}{PW} \times 10^{-7}$	$\lVert\frac{1}{PW}\rVert$
1	0.06	100,000	10,216,276	0.978830	0.17677
2	0.055	91,667	9,364,954	1.06781	0.19284
3	0.04	66,667	6,810,885	1.46824	0.26516
4	0.05	83,333	8,513,530	1.17460	0.21213
6	0.062	103,333	11,796,781	0.847689	0.15309

Figure 4-5: Site Weights with Respect to Electricity Cost

data for a nearby city of similar size is used.[5] Data for the Cost of Living Factor is shown in Figure 4-6.

Weight of Sites with Respect to Cost of Living		
Site	CPI	‖CPI‖
1	319.3	0.19955
2	308.1	0.19255
3	331.7	0.20730
4	314.2	0.19636
6	326.8	0.20424

Figure 4-6: Site Weights with Respect to Cost of Living

To evaluate the Quality of Life, the scoring of Rand McNally's Places Rated Almanac [5] was used. This is an index in which low scores are preferable, so the inverse of the score will be used to evaluate each site. Pairwise comparisons are unnecessary because firm numbers exist for each site and all judgements would be consistent, thus producing the same weights obtained

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[5] The specific index used is the CPI-W, which is revised for "urban wage earners and clerical workers [28]." This index is intended to represent the average working population. A larger-based index, the CPI-U, also includes unemployed and retirees with low or no pensions. Data for Memphis was not available, so data for St. Louis, MO, was substituted. All data is for either April, 1986 or May, 1986 (not all cities are evaluated every month). For all cities, the CPI was defined to be 100 in the base year of 1967.

by simply using raw scores. Figure 4-7 shows the weights assigned to each site for this factor.

Weight of Sites with Respect to Quality of Life			
Site	Score	$\frac{1}{\text{Score}} \times 10^{-3}$	$\lVert\frac{1}{\text{Score}}\rVert$
1	751	1.33156	0.25710
2	1,151	0.86881	0.16775
3	923	1.08342	0.20919
4	1,216	0.82237	0.15879
6	932	1.07296	0.20717

Figure 4-7: Site Weights with Respect to Quality of Life

The final remaining factor to be evaluated is the Equipment factor. The expected price for new equipment at any of the locations was \$12 million, but the State of Tennessee's offer to pay up to \$2.5 million of that cost if Site 4 (Memphis) is chosen has made the cost of equipment a site-dependent factor. The usual analysis of cost factors for Equipment is shown in Figure 4-8.

Weight of Sites with Respect to Cost of Equipment			
Site	Cost ($Millions)	$\frac{1}{\text{Cost}}$	$\lVert\frac{1}{\text{Cost}}\rVert$
1	12	0.083333	0.19
2	12	0.083333	0.19
3	12	0.083333	0.19
4	9.5	0.10526	0.24
6	12	0.083333	0.19

Figure 4-8: Site Weights with Respect to Equipment Cost

Having weighted all sites for all factors, the only remaining evaluation to be performed is to weight all factors against each other. This will be accomplished by pairwise comparisons. For all monetary factors, the expression for representative costs derived in Appendix C, Equation (C.3), will be evaluated. For comparisons of monetary factors against monetary factors, the ratio of these numbers will be used. All other comparisons will be determined subjectively by the evaluation committee.

From Appendix C, Equation (C.3) is evaluated for each of the objective factors. Equation (C.3) is

$$W_j = \frac{\sum_{k=1}^{n} C_{kj}}{\sum_{l=1}^{m} \sum_{k=1}^{n} C_{kl}}$$

Figure 4-9 shows the results of these calculations. The ratio between these numbers will be the pairwise comparison ratios between these factors.

Weights of Objective Factors			
Factor	$\sum_{k=1}^{n} C_{kj}$	$\sum_{l=1}^{m} \sum_{k=1}^{n} C_{kl}$	W_j
Facility Cost	28,081,196	132,283,622	0.21228
Electricity Cost	46,702,426	132,283,622	0.35305
Equipment Cost	57,500,000	132,283,622	0.43467

Figure 4-9: Weights of Objective Factors

Next, the pairwise comparisons are performed. All comparisons except those between monetary factors and other monetary factors are subjective in nature and are made by the site selection committee. The comparisons between monetary factors are the appropriate ratios of the weights calculated in Figure 4-9. The comparison matrix and the resulting factor weights are shown in Figure 4-10. For this matrix, the Consistency Ratio is $CR = 0.06179$, which indicates that the judgements are acceptable.

Now the weight of each site can be evaluated. These calculations are shown in Figure 4-11.

Comparison Matrix of All Factors

	Facility Cost	Business Climate	Airport Location	Electricity Cost	Cost of Living	Quality of Life	Equipment Cost
Facility Cost	-1-	0.33333	5	0.60128	5	2	0.48837
Business Climate	3	-1-	5	1	7	3	3
Airport Location	0.2	0.2	-1-	0.11111	0.5	0.14286	0.11111
Electricity Cost	1.66312	1	9	-1-	9	5	0.81222
Cost of Living	0.2	0.14286	2	0.11111	-1-	0.5	0.11111
Quality of Life	0.5	0.33333	7	0.2	2	-1-	0.2
Equipment Cost	2.04763	0.33333	9	1.23120	9	5	-1-

Weights of All Factors

Factor	$(\prod a_{ij})^{\frac{1}{7}}$	$\|(\prod a_{ij})^{\frac{1}{7}}\|$
Facility Cost	1.25466	0.12620
Business Climate	2.66110	0.26767
Airport Location	0.23116	0.02325
Electricity Cost	2.46122	0.24756
Cost of Living	0.32120	0.03231
Quality of Life	0.71263	0.07168
Equipment Cost	2.29986	0.23133

Figure 4-10: Weights of All Factors

AHP analysis shows that Site 1 (Andover, Massachusetts) is the best site for this particular location problem, and will most closely meet the needs the company has defined. A good second choice would be Site 3 (Atlanta, Georgia). The company may eventually decide on this site as a result of factors not in-

Overall Weights of Sites	
Site	Weight
1	$0.12620 \times 0.13367 + 0.26767 \times 0.42766 + 0.02325 \times 0.08498 +$ $0.24756 \times 0.17677 + 0.03231 \times 0.19955 + 0.07168 \times 0.25710 +$ 0.23133×0.19 $= 0.24591$
2	$0.12620 \times 0.13823 + 0.26767 \times 0.08423 + 0.02325 \times 0.15933 +$ $0.24756 \times 0.19284 + 0.03231 \times 0.19255 + 0.07168 \times 0.16775 +$ 0.23133×0.19 $= 0.15363$
3	$0.12620 \times 0.18621 + 0.26767 \times 0.23491 + 0.02325 \times 0.31867 +$ $0.24756 \times 0.26516 + 0.03231 \times 0.20730 + 0.07168 \times 0.20919 +$ 0.23133×0.19 $= 0.22508$
4	$0.12620 \times 0.20050 + 0.26767 \times 0.05763 + 0.02325 \times 0.18210 +$ $0.24756 \times 0.21213 + 0.03231 \times 0.19636 + 0.07168 \times 0.15879 +$ 0.23133×0.24 $= 0.17072$
6	$0.12620 \times 0.34139 + 0.26767 \times 0.19557 + 0.02325 \times 0.25493 +$ $0.24756 \times 0.15309 + 0.03231 \times 0.20424 + 0.07168 \times 0.20717 +$ 0.23133×0.19 $= 0.20466$

Figure 4-11: Overall Site Weights

cluded in the analysis, including (but not limited to) the personal preferences of the executive who is ultimately responsible for the decision. The other sites are poorer matches, with Sites 2 and 4 (Scranton, Pennsylvania and Memphis, Tennessee) being particularly bad for the needs of this company. Worst of all would be Site 5 (Staten Island, New York), which was eliminated early in the analysis because it did not have both necessary critical factors.

4.2.2 Brown-Gibson Solution

The same example can be analyzed with the Brown-Gibson Algorithm. The Objective Factor Measures are calculated in Figure 4-12. Subjective Factor Weights are calculated in Figure 4-13. The Site Weights for each subjective factor are calculated in Figure 4-14. To determine superiority for Brown-Gibson analysis, the matrices developed for AHP analysis were consulted and the factor with the greater weight was marked as superior. The numerous pairwise comparisons required among the five sites to determine the Site Weights for each factor are not shown. In the Quality of Life comparisons, scores between Sites 3 and 6 were nearly equal (923 and 932, respectively) and a tie was declared for those two sites. The Subjective Factor Weights and the Site Weights are combined in Figure 4-15 to produce Subjective Factor Measures for each site.

Calculation of Objective Factor Measures						
Site	Facility Cost ($)	Electricity Cost ($)	Equipment Cost ($)	Total Cost ($)	$\frac{1}{\text{Total Cost}} \times 10^{-8}$	$\left\|\frac{1}{\text{Total Cost}}\right\| = OFM_i$
1	7,662,207	10,216,276	12,000,000	29,878,483	3.34689	0.17554
2	6,810,851	9,364,954	12,000,000	28,175,805	3.54914	0.18615
3	5,500,000	6,810,855	12,000,000	24,310,885	4.11338	0.21574
4	5,108,138	8,513,530	9,500,000	23,121,668	4.32494	0.22684
6	3,000,000	11,796,781	12,000,000	26,796,781	3.73179	0.19573

Figure 4-12: Calculation of Objective Factor Measures

Calculation of Subjective Factor Weights								
Factor	B-A	B-C	B-Q	A-C	A-Q	C-Q	$\sum 1$	$\lVert\sum 1\rVert$
Business Climate (B)	1	1	1				3	0.5
Airport Location (A)	0			0	0		0	0
Cost of Living (C)		0		1		0	1	0.16667
Quality of Life (Q)			0		1	1	2	0.33333

Figure 4-13: Calculation of Subjective Factor Weights

Calculation of Site Weights for Business Climate Factor		
Site	$\sum 1$	$\lVert\sum 1\rVert = SW_{i,k}$
1	4	0.36364
2	1	0.09091
3	3	0.27273
4	0	0
5	3	0.27273

Calculation of Site Weights for Cost of Living Factor		
Site	$\sum 1$	$\lVert\sum 1\rVert = SW_{i,k}$
1	1	0.1
2	5	0.5
3	0	0
4	3	0.3
5	1	0.1

Calculation of Site Weights for Airport Location Factor		
Site	$\sum 1$	$\lVert\sum 1\rVert = SW_{i,k}$
1	0	0
2	1	0.1
3	4	0.4
4	2	0.2
5	3	0.3

Calculation of Site Weights for Quality of Life Factor		
Site	$\sum 1$	$\lVert\sum 1\rVert = SW_{i,k}$
1	4	0.36364
2	1	0.09091
3	3	0.27273
4	0	0
5	3	0.27273

Figure 4-14: Calculation of Site Weights

Calculation of Subjective Factor Measures	
Site	Subjective Factor Measure
1	$0.5 \times 0.36364 + 0 + 0.16667 \times 0.1 + 0.33333 \times 0.36364 = 0.31970$
2	$0.5 \times 0.09091 + 0 + 0.16667 \times 0.5 + 0.33333 \times 0.09091 = 0.15909$
3	$0.5 \times 0.27273 + 0 + 0.16667 \times 0 + 0.33333 \times 0.27273 = 0.22727$
4	$0.5 \times 0 + 0 + 0.16667 \times 0.3 + 0.33333 \times 0 = 0.05000$
6	$0.5 \times 0.27273 + 0 + 0.16667 \times 0.1 + 0.33333 \times 0.27273 = 0.24394$

Figure 4-15: Calculation of Subjective Factor Measures

Having obtained both the Objective Factor Measure and the Subjective Factor Measure of each site, a plot can now be made to see the site recommendations as a function of X, the Objective Factor Decision Weight. This is shown in Figure 4-16. It is clear from this figure that the most highly recommended site can be one of three candidates, depending on the value of X which is selected. For much of X's range, the top choice is Site 1 (Andover, Massachusetts), which was the top choice in the AHP analysis. However, all sites except for Site 2 occupy the second spot for some value of X and there are nine separate possible rankings depending upon the value of X. None of these rankings is the same as the ranking calculated using AHP. These are listed in Figure 4-17. It is not possible to tell from the information already derived in this example what an appropriate value of X is, since most objective factors were greatly inferior to the subjective factors, but one of them, Business Climate, was more important than any subjective factor. Referring to the weights of all seven factors as calculated in the AHP solution (see Figure 4-10), the top ranked factor (Business Climate) and the bottom three (Quality of Life, Cost of Living and Airport Location) were all subjective. Objective factors occupied three intermediate rankings (spots two three and four). It cannot be said that, as a group, one set was superior or inferior to another. Most likely, a value somewhere above $X = 0.5$ and below $X = 0.9$ would be chosen, leaving six of the nine rankings eligible. It is not significant that the first and last choices are reversed at the extreme values of *X*, although this further illustrates the Brown-Gibson method's dependence on this hard-to-determine parameter.

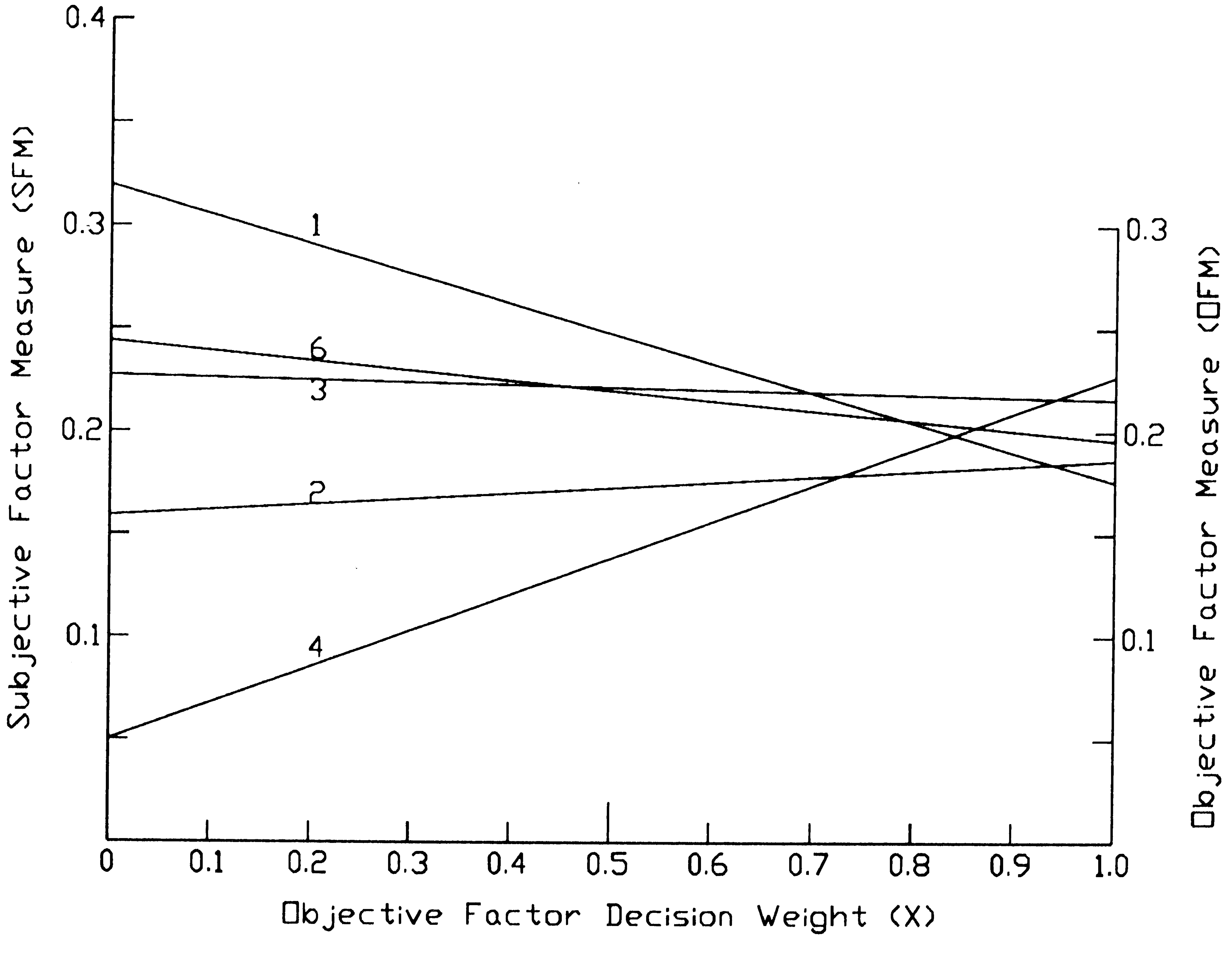

Figure 4-16: Site Rankings as a Function of *X*

Final Rankings of Sites									
AHP	$X <$ 0.454	0.454 $<X<$ 0.696	0.696 $<X<$ 0.728	0.728 $<X<$ 0.789	0.789 $<X<$ 0.840	0.840 $<X<$ 0.861	0.861 $<X<$ 0.938	0.938 $<X<$ 0.941	$X >$ 0.941
1	1	1	3	3	3	3	3	3	4
3	6	3	1	1	6	6	4	4	3
6	3	6	6	6	1	4	6	6	6
4	2	2	2	4	4	1	1	2	2
2	4	4	4	2	2	2	2	1	1

Figure 4-17: Calculation of Subjective Factor Measures

Chapter 5
Conclusions

The original motivation for this exploration of AHP's applicability to the site selection was to overcome three difficulties in using the Brown-Gibson procedure. Those difficulties are

1. The procedure is time consuming, requiring many pairwise comparisons.
2. The differences in final scores among sites may not accurately reflect the differences in the values of the sites themselves.
3. Assignment of the Objective Factor Decision Weight (X) is difficult, especially considering the very great impact it has on the final solution.

AHP contributes significantly toward the reduction and elimination of the second and third difficulties, at the expense of the first. AHP is more difficult and more time consuming than Brown-Gibson, although it has the significant advantage that much more of the work can be computerized than in the Brown-Gibson Algorithm. Most of the work required by AHP is numerical calculation; there are many cases where pairwise comparisons can be skipped because their immediate objective, branch weights, can be calculated in other ways. This was shown in the the site selection example in Chapter 4, where among all individual factors only the Business Climate factor actually required pairwise comparisons. However, AHP requires more complex calculations than the Brown-Gibson algorithm, and some of the simplifications which avoided sets of pairwise comparisons in AHP could also be applied to Brown-Gibson.

The second difficulty with the Brown-Gibson algorithm is in the interpretation of its results. AHP offers improved results, due to a better comparison scheme. In Brown-Gibson, any pairwise comparison results in one of three assignments: superiority (1-0), equality (1-1), or inferiority (0-1). AHP offers seventeen different assignments, including eight shades of superiority and inferiority (the integers 2 through 9 and their reciprocals). Thus, the difference between one site being slightly better than another versus being greatly better than another is not lost in the (1-0) selection choice. AHP's use of eigenvector components is more theoretically sound than the Brown-Gibson algorithm's weighted average method, which relies on the nebulous Objective Factor Decision Weight (X).

The greatest advantage of AHP comes from the equal treatment given to all factors, whether subjective or objective. Whereas the Brown-Gibson divides factors into two sets, all factors are compared against each other in AHP. The division of factors into subjective and objective sets in Brown-Gibson cannot be justified by the characteristics of the site selection problem. If segregation of factors must be done, it would perhaps be more reasonable to divide factors on some other basis, such as wheter they are generally "more important" or "less important" factors. Regardless of the basis for segregation, segregation itself brings an additional difficulty. Division into two sets requires that a weighting factor be assigned, and as shown in the example in Chapter 4, a value for this factor can be difficult to assign. Unfortunately, the effect of this weighting factor on the results is great, as shown by the nine different possible site rankings depending on the value of X.

AHP offers a more accurate solution to the site selection problem than Brown-Gibson, and the solution relies less heavily on any given input to the solution than Brown-Gibson's reliance on the Objective Factor Decision Weight. The price for the advantages of this algorithm is increase computational difficulty. Because plant location decisions usually are very important, involving much money and affecting the company far into the future, they usually take a relatively long time to occur—often over a year. This certainly allows the time, and provides the motivation, for using AHP instead of Brown-Gibson.

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Appendix A
Some Subjective Factors Likely to be Used in Site Selection

Each site selection problem will require its own set of subjective factors. The following is a discussion of some of the more likely subjective factors to be found in most site selection problems. Individual problems, however, are likely to have some specific subjective needs not listed here. The more general factors discussed here include:

- Transportation Availability
- Employee Transportation
- Climate
- Weather Emergencies
- Industrial Sites
- Utility Availabilities
- Aesthetics
- Educational Facilities
- Union Activity
- Competition
- Community Predisposition
- Proximity to Customers
- Labor Availability
- Housing
- Recreational Facilities
- Future Growth
- Community Services
- Proximity to Suppliers
- Local Labor Practices
- Ability to Combine Site's Functions
- Proximity to Existing Facility
- Cost of Living
- Complementary Industries

A.1 Transportation Availability

This factor is likely to be relevant in problems of any scale. It is intended to reflect the availability of shipping sources and routes for incoming materials and equipment as well as finished goods. Included in this factor is the site's proximity to highways, ports, rail lines and airports; as well as the conditions of these facilities. Any special costs in using these facilities (such as tolls required to reach the site from the main interstate system) would also be considered in this factor. Also note any restrictions, such as bridge weight limits or tunnel cargo rules, which would restrict the transport of supplies, finished product, or the moving of machinery to the plant.

A.2 Employee Transportation

This factor allows such things as proximity to subways and bus lines to be judged, as well as rush-hour traffic levels near the plant and the size of the road network around the plant. Placement of a plant with many employees on a site served by only one or two two-lane roads is likely to detract from the site's attractiveness and may necessitate such programs as flexible reporting time and company-sponsored car- and van-pools to relieve traffic problems around the plant. Truck deliveries might also need to be scheduled around heavy traffic times, either voluntarily or by local ordinance.

A.3 Climate

Climate would be a relevant factor only when candidate sites are geographically separated by long distances. Differences in temperatures can affect the heating and cooling systems needed for the plant, and the costs associated with these systems. In many high technology applications, the manufacturing processes themselves can be affected by the temperature and humidity of the plant, which could perhaps require special air filtration systems. This factor would include only the effects of average, predictable, seasonal climate changes.

A.4 Weather Emergencies

Separate from consideration of average climate changes are the weather emergencies which inevitably happen in many parts of the world. Heavy snow storms, floods, tornadoes and hurricanes often result in downtime for plants. In emergencies, workers often remain in their homes (or leave work early), transportation becomes unavailable or interrupted, power may be cut, or in the worst case the plant may be damaged. The vulnerability of each potential site to such disasters is accounted for in this factor.

A.5 Industrial Sites

This factor is relevant when individual candidate sites have already been determined. Components of this factor include whether a site is flat, is cleared of brush or existing facilities needing destruction, and is zoned appropriately (if at all). It also includes the general suitability of existing buildings to their proposed new purpose, although the specific costs of renovating them should be accounted for as an objective factor. The subjective factor involved here is the opinion of how suitable, after necessary renovations, the individual sites are for their new purposes. It is a subjective judgement of the desirability of different sites, all technically feasible, with regards to industrial operation.

A.6 Utility Availabilities

Some sites may have water, sewer and electricity lines in place while others may require installation of these utilities. In some communities, utility systems are at full capacity and unable to accept new customers. In other places, the local water supply may be unsuitable for use in the plant's processes. Some sites may even require bottled water for drinking. Such constraints would be included in this factor. As with the "Industrial Sites" category, the direct cost of rectifying the shortcomings of any site should be considered as an objective factor. The subjectivity involved is estimating the risk involved in installing or changing utilities. Areas of risk are that deadlines might not be met, costs might be exceeded, and some planned systems might prove technically infeasible. Many areas of the United States have strained water systems, and have written laws requiring new users to be queued while awaiting water availability. A slow-moving queue could result in a delay of the facility's opening, and the risk of such a queue would be a subjective factor in this category.

Of course, the cost of utilities is also an important factor, but is generally included as an objective factor. Cost can, however, also be included as a subjective factor if there are reasons to expect an abnormal change in them. For instance, if a growing area has little excess power capacity, it is likely that a new power plant may be built during the life of the plant. This could have a large effect on the cost of electricity.

A.7 Aesthetics

The aesthetics of a plant setting can make a difference in how employees perceive their value, and their plant's value, to a company. More importantly, aesthetics can affect how customers perceive the value of the plant and its products to the company. As an example, the author's own experience includes a company which moved from an old, functional plant to a new one a few miles away primarily to influence customers' opinions. In this case, the company's business was such that customers often visited the plant before placing orders. The company found that many customers questioned the parent company's dedication to the business unit and its products, because from the plant's appearance it seemed that not much money was being put into the business unit. As a result, sales were slipping, and it was discovered that competitors were even encouraging potential customers to visit the plant for comparison with their own!

A.8 Educational Facilities

Proximity to colleges and technical schools is increasingly seen as an advantage in industries which require an increasingly sophisticated work force. The success of North Carolina's Research Triangle and Massachusetts' Route 128 region attest to this, as do other areas where educational and industrial parties have formed partnerships. While recent initiatives such as the National Technical University have sought to use satellite technology to bring education into remote plants, they have not completely duplicated all of the benefits of physical proximity to schools. Such benefits include the interaction between students from different companies, the availability of professors for individual assistance, group visits to other facilities, university consulting services, and exposure to experts in areas not directly related to the plant's primary business. In addition to the use of outside educational facilities by their employees for individual study, many companies rely upon local schools to develop in-house training programs and for specialized consulting help. In addition, a strong technical institution can, by being a neutral forum, foster the kind of interaction between individual companies which results in successful joint ventures. Educational facilities also usually sponsor seminar series which employees can attend to keep their knowledge up to date. Beyond the enrichment of present employees, colleges and universities can provide a steady stream of graduates skilled in computer and other technologies as the facility's needs grow.

Finally, an often overlooked benefit of being near a university is the research conducted with government and other funds. Such sponsorship is contributing more and more to joint research programs with industry and sometimes funds entire projects [11].

A.9 Union Activity

The relative strength of unionized labor in a geographic area can determine whether a plant will be unionized or not. A unionized plant in a company whose other sites are not unionized, or the reverse, can be burdensome to a company. Additionally, even though a "new reality" of cooperation is touted between labor and management, the absence of organized labor is still preferred by most management. Many companies not weak enough to gain union concessions but not strong enough to make great profits have responded to their predicament in recent years by moving to "right to work" states, leaving amidst charges by labor of "union busting." "Right to work" laws do not prohibit unions, but rather weaken their financial base by allowing workers at a unionized plant to not be members of the union, and not requiring non-members to support the union financially [7].

A.10 Competition

The presence of competition in close proximity to a plant can be a mixed blessing. While it encourages tight profit margins and makes sales more difficult to close, it also provides greater access to the trends of the plant's industrial segment and more up to date information about the actions of specific competitors. Moreover, in some industries, a certain "credibility" is established or maintained by locating in an area where the rest of the industry is centered. Examples of this include California's San Jose area ("Silicon Valley") for computeer chip manufacturers and New York City for stock brokers. Proximity of competition also has an affect on personnel policies and retention. Nearby competition provides a ready source of expert new personnel, although it also provides attractive alternatives for the firm's own personnel. The result is that in areas with many competitors in a single industry, firms often have lucrative benefit packages: salaries are very competitive and firms are almost obligated to pay prevailing wages to attract employees; but to keep employees, companies try to create extremely comfortable atmospheres with, perhaps, child care, longer vacation periods, and flexible reporting hours. Often, policies whose direct cost may be small (such as flexible reporting hours) can make a work environment which is difficult to leave.

A.11 Community Predisposition

The community's predisposition to a plant is more likely to be an issue if the plant uses or produces dangerous materials or something not accepted by all members of society (such as cosmetics products involving experiments on animals). Additionally, community objections can be expected if a plant would use enormous local resources such as water or electricity. If a plant crosses none of these boundaries, it is likely to be welcomed in most communities or, at worst, greeted ambivalently. Those plants which could face opposition—such as nuclear weapons plants or chemical plants—may strongly wish to avoid locating in areas where they are likely to face challenges at every step on the path to becoming a fully operational, licensed plant. Additionally, if there are any questions of environmental damage posed by the plant's operation, it may be unwise to locate the plant in a community which has traditionally or recently been the scene of environmental mishaps or legal challenges.

In areas that have experienced much recent growth, residents may want to regulate and in some instances freeze the development of the area. This could limit the new facility's freedom in expanding or even place restrictions on its initial layout, capacity and appearance.

A.12 Proximity to Customers

Many firms seek to locate facilities near their customers. This saves transportation cost (an objective factor) but has other benefits as well. For makers of industrial products, it is easier to work on joint projects together and to coordinate the communications which must take place with the customer when custom-designed products are to be produced. It is also easier to coordinate just-in-time deliveries. In some cases, close proximity may be required for just-in-time deliveries to be feasible. Consider an automobile plant which knows its final production schedule just hours before color-dependent components, such as seats, are needed in its assembly operation. A seat plant or at least a seat warehouse must be located close to the customer's plant for deliveries to be timely. Additionally, longer delivery distances increase the chance for delays due to inclement weather or other problems.

Manufacturers of non-commercial products may also choose to locate near major customer areas in order to increase their good will with their customers. Consumers like to buy locally-made goods because it helps the local economy, they feel a connection with the company, they know where to turn if they encounter problems with the product, and in many cases they feel the product is fresher. While freshness is of obvious importance for food products, non-consumable products produced locally can be perceived to have spent less time in warehouses and to be the latest models available.

Finally, locating a facility in a particular foreign country can be essential to gaining or maintaining a presence in that country's market. Many countries have trade barriers such as quotas and tariffs which can be avoided by producing in that country.

A.13 Labor Availability

Traditionally, manufacturers have asked whether a site had enough labor nearby to fulfill their employment needs. While this is still a valid concern, advancing technology and decreasing labor content have caused manufacturers to ask whether there is enough labor of the right type. Various regions of the United States and the world have high concentrations of skilled labor in various areas, such as machining, electronics design and assembly, and others. Aside from traditional skills, certain types of manufacturing may require specific levels of education, which tend to vary across different regions.

Gross statistics are available from many sources (usually governmental) about regional unemployment rates, including seasonal variations. Generally, areas with high unemployment are receptive to new plants and offices, and state governments will often provide incentives for firms to locate in these areas. Firms with seasonal production schedules may wish to locate in areas whose seasonal high employment period is different from their own.

Finally, if the firm has non-standard labor practices, it may wish to judge local labor's receptivity to such practices. For example, firms requiring 12-hour shifts for three or four days per week, or firms using "Japanese" styles of worker self-management, may wish to locate in areas where these practices are not unheard of, in order to lessen the shock to new employees and reduce the training necessary for the firm to provide.

A.14 Housing

All employees need a place to live. The larger a new plant's employment relative to the existing population, the greater will be its impact on local housing. If the presence of a new facility will bring many new people into an area, it may significantly increase the price of housing in the area. This increase can filter to salary requirements of employees. Beyond mere housing cost, local community services such as hospitals, schools and even the transportation network may become overloaded by the population increase. This could lead to new public building projects and increased taxes.

Earlier in this century, some companies avoided these problems by building "company towns." The firm would build housing for its employees and either retain ownership of the dwellings or sell them to employees at reduced prices. This leverage over employees usually resulted in employees staying with the company for their entire working lives.

Often, the equivalent of company towns are built when a facility must be located where no towns exist. Such facilities include mines and oil fields. In these cases, towns are developed where virtually the entire economy relies on one facility, or dormitories are built for workers to stay in while commuting to their homes during weekends and extended off-periods. Off-shore oil platforms with residence facilities are an example of such dormitories.

A.15 Recreational Facilities

The abundance and type of recreation available near a site greatly impacts the quality of life of the employees of the site. "Recreation" can be defined very locally as well as regionally. A local definition might involve the availability of areas for lunchtime runners and the existence of industrial-park softball leagues. Regional definitions, which usually have more impact, involve the availability within a reasonable drive of skiing, sailing, beaches, parks, hunting, fishing, shopping, sports teams, theaters, resorts, and cultural activities. Good recreational facilities will make the site more attractive to employees being asked to relocate there, as well as helping the company's interpersonal atmosphere flourish when employee's similar outside interests are stimulated.

A.16 Future Growth

If future growth is anticipated for the area around a candidate site, several positive effects are possible for the facility. Local markets for their products are likely to expand as industry and population grows. The labor supply will likely expand or at least remain steady. Community services and recreation will grow to meet the needs of the larger population. This will involve increased public cost, but the tax base will expand also, thus decreasing the individual facility's portion of the tax base.

A.17 Community Services

Community services include fire, police and ambulance services; snow removal; schools; road maintenance and construction; and others. The specific services and levels of service usually vary from community to community, even within a region. Often, local governments wishing to attract industries will enhance specific services to woo specific facilities. Such enhancements might include maintenance of private plant roads.

Regions or individual states may have specific programs geared to help attract, develop and maintain industry. These range from matching-fund and research-oriented programs such as Pennsylvania's Ben Franklin Partnership Program, to regional business-to-business colloquia intended to encourage local business to support other local businesses.

A.18 Proximity to Suppliers

Manufacturers wishing to reduce the costs of maintaining large inventories of raw materials may wish to implement just-in-time delivery of their supplies. This requires timely deliveries from suppliers, which is best accomplished if suppliers are nearby for two significant reasons: nearby suppliers can more accurately schedule deliveries because there are fewer uncontrollable variables in their delivery process; and transport time is less, allowing the facility to order supplies later in his production cycle and thus allowing the facility more flexibility.

Reduced transport time also often translates to reduced transport cost, and this savings can be passed on to the facility in the form of lower material cost.

Finally, being near suppliers may alleviate technical difficulties. For example, Bethlehem Steel, in Bethlehem, PA, receives oxygen gas via a pipeline from a nearby industrial gas supplier, Airco Industrial Gases. The pipeline runs approximately 1 mile from the gas plant, under a river, into the steel mill. Such a flow of gas would be impossible if the two plants were far apart. Other businesses with perishable supplies, such as dairies and fresh food processing plants, need to be near their suppliers in order to have quality ingredients in their products.

A.19 Local Labor Practices

Standard labor practices and policies differ between various parts of the United States and various countries in the rest of the world. There may be regional holidays and vacation periods which could affect the plant's operation or its interaction with other plants in the parent company. For example, a large proportion of private companies in Massachusetts close for Patriot's Day, a state holiday, in April of each year. This holiday is not celebrated in other states. In France, many businesses shut down in August for a vacation period. Following these customs could cause difficulties in the plant's supply to or consumption of other plants' manufacturing. Not following these customs could put the plant at odds with local customs and employee expectations.

"Flex time" is becoming common in many areas of the country. This is a policy which allows employees to set their own hours, within certain predefined limits. (Often, the rules take the form of specifying a starting time of between certain morning hours, with a finishing time eight hours after the employee chose to begin his or her day.) As the number of working couples continues to increase, this becomes an important policy in the eyes of employees who must deliver or retrieve their children from school or day care in conjunction with their spouse. If a firm locates in a region where flex time is very common, it should be prepared to seriously consider incorporating this practice in its operations.

Some practices are considered standard in some locales and valuable benefits in others. In remote plants, free parking is assumed as a right, but in a city location, it can have value to employees who would otherwise have to pay for public transportation or pay expensive private parking fees. Child care or full or partial reimbursement for it is another benefit which is becoming common among urban and metropolitan employers. In rural areas, this might be considered extraordinary.

When international locations are considered, many labor practices emerge. Outside of the United States and Japan, annual vacation periods are typically much longer than is common inside the United States. For long-term employees, vacations can amount to several months per year; in India, even beginning engineers typically receive one month vacation, as opposed to two weeks in the United States. Wage practices also vary; in some countries an employee's marital status and number of dependence contribute to his or her salary reviews.

A.20 Ability to Combine Site's Functions

Often, it is convenient to combine two separate functions in one site. Such combinations may include a sales office with a service center, a factory with a warehouse or distribution center, or one of many other combinations. In problems outside of traditional manufacturing, it may make sense to combine such functions as ambulance and fire stations, since such services are often needed together.

Combining a manufacturing site with a warehouse or distribution center allows tighter control of inventory, reduces the amount of inventory in transit at any given time, and allows quicker response by the manufacturing organization to changes in the product demand mix. Combining a factory with a service center facilitates better communication of field problems from the service organization back to manufacturing, so that recurrent problems can be more quickly investigated and corrected. It also makes communication of work-arounds and solutions from manufacturing to the field easier.

Combining a sales office with a field support center helps keep local contacts updated for both organizations, as they can be kept in one place. More importantly, field service personnel—who often visit customers more frequently than sales personnel—can alert sales to changes in their customers' operations and personnel, and to new opportunities caused by changes in their customers' business.

A.21 Proximity to Existing Facility

This factor is usually included in site selection decisions. If a new site is intended to replace the existing site, it is likely that this factor will be given so much importance that it will be treated as a critical factor and used to eliminate sites early in the evaluation process. One survey of companies which opened plants between 1970 and 1980 found that 30% of the companies which conducted location searches never searched beyond the borders of a single state [24]. There could be many reasons for such a limited search, including other subjective factors such as proximity to suppliers, customers or natural resources. One likely reason, however, is proximity to an existing facility. This factor is important for many reasons, including:

- Valued employees whom the company would like to transfer to the new site are not left behind because they do not want to relocate.
- Relocation cost of employees is saved. Usually, only management and professional staff is transferred, but such transfers can be very expensive, involving thousands of dollars. Money is spent to cover some or all of the cost of selling employees' current homes and buying their new homes, moving and storing their possessions, and paying for their living expenses and transportation during the move. Data for 1985 estimated that transfers cost an average of $48,000 per employee and had risen three-fold over the past six years [11].
- Some of the most valued employees may refuse to relocate. Personal attachments to local culture, communities and friends may outweigh devotion to the employer. Also of increasing importance as the nation's number of dual-income households grow is the job opportunities in a new location for a working spouse. According to one research group, national statistics indicate that less than a quarter of employees are willing to relocate [11].
- The company has first-hand knowledge of business, labor, real es-

tate and other conditions in communities where they are already located, and faces less risk of miscalculating new conditions.

- Less training cost is necessary for workers in the new location, as most will transfer from the old site. If the new site is far, much of the new work force will need to be recruited from outside the company.
- Less time and money is spent in travelling between sites during the construction and start-up of the new site.

A.22 Cost of Living

The economics for employees living near the site will be different at different site locations. One measure of this is the Consumer Price Index, which is published by the Department of Labor's Bureau of Labor Statistics. This is a very broad measure which may or may not be appropriate for an individual company. Generally, the Cost of Living factor reflects the differences in the cost of items bought by most employees who will be located at the new site.

A.23 Complementary Industries

It is usually beneficial to locate near industries which are complementary to one's own. Such industries are often suppliers and customers to each other, or have common customers. The advantages of locating near customers and suppliers have already been explored (see Appendices A.12 and A.18). If two companies have common customers, customers will often arrange to visit one when visiting the other, and one company will often recommend another to fulfill a customer need which it cannot fill itself (providing that the two companies do not compete with each other).

Another advantage is that complementary industries often require the same manufacturing skills, ensuring that there will be a ready supply of expertise in the area. (This can also be harmful if the company fears its own workforce will be raided.) Examples of complementary industries include auto parts, auto tires, and automobile assembly; packaged foods and containers; and paper and office supplies.

Appendix B
Some Objective Factors Likely to be Used in Site Selection

Some costs associated with starting or relocating a facility are independent of the site selected, while others very much depend upon the site. This second group is presented here. Some or all of these may be applicable for a specific site selection problem, and specific problems may also include other factors not mentioned here. The objective monetary factors discussed here include:

- Labor Costs
- Utility Costs
- Inventory Costs
- Supply Costs
- Taxes and Tax Incentives
- Freight, Packing and Insurance Costs for Initial Move
- Extra Cost of Satisfying Demand During Initial Move
- Site Acquisition Costs
- Site Renovation and Preparation Costs
- Cost of Obtaining Licenses
- Landscaping and Seasonal Care
- Industrial Park Fees
- Security

B.1 Labor Costs

Savings in Labor Costs have traditionally been a prime cause of industrial relocations. Many companies with manually-intensive operations roam the world in search of cheap labor, and have moved factories between Puerto Rico, Singapore, Mexico and other third-world countries. Within the United States, labor cost is cited as a chief reason for the migration of many industries from

Northern states to the Sun Belt. Right-to-Work laws (see Appendix A.9) are usually given the most credit for the shift, but high wages tend to go together with unionization. When considering the quantitative effects of site selection on the cost of labor, it is essential that the types of labor available at a candidate site (see Appendix A.13) not be overlooked.

Federal and local sources can be used to find the prevailing wages at each site for the various types of labor needed. Good local sources include chambers of commerce; at the federal level, the Bureau of Labor Statistics can provide data. The quantitative cost of labor is the sum of wages and benefits which must be paid if the site is selected. Either rough numbers, such as a total plant population multiplied by a general wage, can be used, or a detailed assessment of labor costs can be made. Naturally, a detailed assessment will be more accurate, and should be made if at all possible.

B.2 Utility Costs

Utility costs include the cost of electricity, water, natural gas and sewerage. Any of these may not apply to a specific problem, especially if the plant intends to produce its own power or does not need one of the utilities. The cost of utilities is significant in most site selection problems; one survey showed manufacturers ranked energy costs a close second to wages as the most critical factor in selecting sites [2]. A good estimate of utility costs can be made by measuring the quantity of the utility used (Kilowatt-hours of electricity, gallons of water, etc.) and calculating costs for a site using local rates. These figures, however, may require adjustment because of changes in consumption cause by new technology used in the new site, or by changes in consumption required by conditions at the new site. An example of the latter of these conditions is increased power usage for refrigeration in warmer climates. Beyond the cost of utilities used directly for production, more utilities may be consumed simply by the facility itself. Plants in colder climates will have greater heating bills; an older building will likely be more poorly insulated than a newer one. Examining the history of utility usage for specific buildings (unless the new facility is being built "green field") can yield valuable data, as can assistance from local utility companies.

B.3 Inventory Costs

Location far from particular suppliers may result in changes of inventory policy to reduce the risk of running out of supplies. Increasing an inventory reorder point or a safety stock level will increase the cost associated with the increased inventory. This is particularly true if the company pays for the supplies from the time the supplier ships them, as there will be more supplies in the "inventory pipeline" at any given time. Similarly, moving closer to particular suppliers may decrease inventory costs.

B.4 Supply Costs

Some materials used in production can be expected to vary by location. These variations are due mainly to transportation costs of raw materials faced by local suppliers, as well as their local production costs. The alternative of using distant suppliers (perhaps those which served a relocating plant's old site) will increase transportation costs. An analysis of supply alternatives should be conducted to determine which costs will be included in the location analysis.

B.5 Taxes and Tax Incentives

State taxes vary from state to state, and local taxes vary from locality to locality. Common taxes include property taxes, income taxes, inventory taxes, sales taxes and use taxes. Any or all of these may have to be paid by the company. Other taxes will be levied on employees, and may thus influence the wages they command. Personal taxes may also contribute to an individual employee's decision to accept relocation; the value of retaining or losing such employees is a subjective factor in the company's view.

A positive aspect of taxes is tax incentives. Every state offers reduced, eliminated or deferred taxation to companies which invest in a way that the state desires. In some cases, this may mean relocating to an economically depressed area in the state; in other cases, this may mean the hiring of minorities and the use of minority-controlled suppliers. An excellent and impartial reference to the taxes and incentives of every state in the United States is the *Directory of Incentives for Business Investment and Development in the United States: A State-By-State Guide* [9].

B.6 Freight, Packing and Insurance Costs for Initial Move

If equipment is being transferred to the new facility, there will certainly be a cost associated with moving the equipment. This may be a simple charge from a moving company, which includes labor for packing and unpacking the equipment, or it may be very complicated. Large pieces of equipment may necessitate bringing workers from the equipment's manufacturer to disassemble it and then reassemble it in the new location. In the worst case, walls or windows may need to be removed to move the equipment. Moving large computers usually requires the manufacturer's service personnel to secure disk drives and other system components for shipment. Insurance for items in transit can usually be purchased through the moving company, although inquiries to the relocating firm's own insurance company should be made to see if separate insurance is redundant.

B.7 Extra Cost of Satisfying Demand During Initial Move

If production is to be moved from an existing site to a new site without disrupting shipments, extra cost is likely to be incurred. An inventory will likely be built up in advance of the move to handle demand while production is stopped. This will result in inventory costs and perhaps overtime production costs as well if the facility is already operating at full capacity. For example, if production is expected to be disrupted for one week due to a move, the company may want to have one and one half or two weeks of extra inventory on hand at the beginning of the move. If production is already at full capacity, the plant's management may decide to add 25% overtime capacity. (Adding workers for a second shift for two weeks is not likely to be feasible.) Thus, it would take six to eight weeks of overtime to build up the excess inventory. Additionally, temporary storage space (and transportation to it) may need to be rented to hold the excess inventory.

The possibility of purchasing a second set of production equipment should be explored. When the extra cost of producing to cover the off-line period is added to the cost of moving the equipment, it may be cheaper to buy new equipment for the new site and sell or even discard the existing equipment. The advantages of newer technology can perhaps sway this decision if the costs are close.

B.8 Site Acquisition Costs

The site's acquisition cost is the cost of buying or leasing the land on which the facility is to be built, or the cost of buying or leasing an existing facility. This factor should include the purchase price or rent, as well as any other fees to be incurred in the purchase if the site is selected. These other fees could range from a realtor's fee to delinquent back taxes owed on the property, to settlement of any other claims on the property. A sale agreement will normally specify exactly what is to be paid by the purchaser. If the site is being purchased from a governmental body, the sale may involve special obligations whose cost should be considered part of the cost of acquiring the site.

If the site is in an industrial park, there may be other details to be considered with the cost of the site. One is parking, for spaces in such parks are often allotted to individual tenants. The relocating company will want to ensure that its lease includes adequate parking spots for employees and visitors. Adding parking spots to the lease will likely involve extra cost.

B.9 Site Renovation and Preparation Costs

Rarely is a site in "move in" condition. Usually, some alterations are necessary to accommodate the material handling and storage systems of a manufacturer or the particular production equipment that is to be used. Such modifications can range from major changes, such as the installation of freight elevators, widening of doorways, removal and construction of walls, structural reinforcement of the building, and construction of a water tower, to less drastic changes such as modifications of restroom facilities for handicapped use, installation of industrial gas storage tanks if required for production, and constructing signs. Some of these costs will often be included in the Site Acquisition Cost.

B.10 Cost of Obtaining Licenses

Depending on what the facility is to produce, special licenses or permits may be required. These are likely to have costs, in the form of fees as well as services required to obtain the licenses. For example, an environmental impact study may be required to obtain a building permit. Both the study and the permit will likely cost money. The local chamber of commerce can usually be very helpful in alerting new businesses to what licenses are required, but a general rule is to consult all governing bodies for a comprehensive list. Governing bodies would include boroughs, cities, counties and states. Individual agencies of the federal government may also need to be consulted, depending on what is to be produced (ie., the Nuclear Regulator Commission for atomic energy plants, the Department of Defense for plants making military items, etc.). Other governing bodies would include water commissions and districts, which may be comprised of several counties and thus not under the jurisdiction of any individual member, or even any individual state if the district lies in a multi-state area.

B.11 Landscaping and Seasonal Care

Even if a company rents its facility, it is usually responsible for landscaping and seasonal care. Landscaping may entail only the occasional cleaning of parking lots, or could involve the maintenance of expansive lawns and gardens which are often found at executive sites. In northern climates, seasonal care includes removal of snow from parking lots. The roof of the building may also need snow removal if the roof is flat and snowfalls are heavy. If the site is in an industrial park, the park will usually provide these services in return for a fee required of all residents.

B.12 Industrial Park Fees

If the site is in an industrial park, a fee beyond the rent will probably be required. This fee will be used to maintain common areas in the park, such as parking lots and lawns, as well as common services such as exterior lights, signs, and perhaps a security patrol.

B.13 Security

Security needs are a function of the location of the site, the products produced there, and the facilities used to produce them. Plants producing products which can be easily transported and disposed will usually need tighter security than those producing large, specialty items which cannot be easily stolen and which require continuing service from the manufacturer. If the plant has much expensive general-purpose equipment such as computers, security needs will increase. Increased security is also appropriate if products are stored in open areas outside of the plant. The cost of security can include alarm systems, patrols, card-access systems or similar restrictive systems, a patrol car, surveillance cameras and a central security station. Usually a security consultant is hired to determine a plant's security needs; the cost of the consultant is also a component in this factor.

Appendix C
Use of AHP for Only Monetary Factors

In this appendix, the expression for monetary factor weights is derived, in which equivalence of AHP and economic analysis (for purely monetary problems) is maintained, and an example using only monetary factors is provided.

C.1 Derivation of Monetary Factor Weights

This derivation proceeds from equating the definitions of factor weight, as defined in AHP and the standard economic justification methods such as Present-Worth, Annual Equivalent, and Future-Worth. Definitions used are as follows:

C_{ij}	the value at site i of factor j. Revenues are positive, costs are negative.
W_j	the AHP weight (priority) of factor j
S_i	the normalized weight of site i
n	the total number of sites
m	the total number of monetary factors

C.1.1 Formulation

For any site i, consider the site's weight as defined in the AHP formulation. The site's raw, or unnormalized, weight is the sum of the product of each factor's relative weight multiplied by the site's weight with respect to that factor,

$$W_1 \times w_{i1} + W_2 \times w_{i2} + \cdots + W_m \times w_{im}$$

This sum is then normalized by the sum of the AHP weights of all of the sites, so that for any site i,

$$S_i = \frac{W_1 \times w_{i1} + W_2 \times w_{i2} + \cdots + W_m \times w_{im}}{\sum_{k=1}^{n} (W_1 \times w_{k1} + W_2 \times w_{k2} + \cdots + W_m \times w_{km})}$$

Next consider the weight of site i as defined in the economic formulation. The raw (unnormalized) value of the site is the sum of all cash flows associated with the site:[6]

$$C_{i1} + C_{i2} + \cdots + C_{im}$$

This sum is then normalized by the sum of the economic value of all of the sites. Thus, for site i, the economic formulation of its weight is

$$S_i = \frac{\sum_{j=1}^{m} C_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{m} C_{kj}}$$

The goal of this exercise is to solve for all W_j, which are the priorities of the monetary factors. Setting the two expressions for S_i equal, a set of m simultaneous equations is obtained, with m unknown variables:

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[6] This simple sum represents the Present-Worth if the various cost components are the Present-Worths of the components, the Annual Equivalent if the various components are Annual Equivalents, or the Future-Worth if the various components are Future-Worths.

$$S_1 = \frac{W_1 \times w_{11} + W_2 \times w_{12} + \cdots + W_m \times w_{1m}}{\sum_{k=1}^{n} (W_1 \times w_{k1} + W_2 \times w_{k2} + \cdots + W_m \times w_{km})} = \frac{\sum_{j=1}^{m} C_{1j}}{\sum_{k=1}^{n} \sum_{j=1}^{m} C_{kj}}$$

$$S_2 = \frac{W_1 \times w_{21} + W_2 \times w_{22} + \cdots + W_m \times w_{2m}}{\sum_{k=1}^{n} (W_1 \times w_{k1} + W_2 \times w_{k2} + \cdots + W_m \times w_{km})} = \frac{\sum_{j=1}^{m} C_{2j}}{\sum_{k=1}^{n} \sum_{j=1}^{m} C_{kj}}$$

$$\cdots \qquad \cdots \qquad \cdots \qquad \cdots \qquad \cdots$$

$$S_n = \frac{W_1 \times w_{n1} + W_2 \times w_{n2} + \cdots + W_m \times w_{nm}}{\sum_{k=1}^{n} (W_1 \times w_{k1} + W_2 \times w_{k2} + \cdots + W_m \times w_{km})} = \frac{\sum_{j=1}^{m} C_{nj}}{\sum_{k=1}^{n} \sum_{j=1}^{m} C_{kj}}$$

This set of equations is linearly dependent, and thus its m equations cannot be used alone to solve for its m unknowns. Section C.1.3 shows that the equations are indeed linearly dependent. An additional condition must be introduced to allow solution; this final condition will be a scaling factor, which will be imposed once a general solution has been obtained. As a final task in formulating the solution, write the simultaneous equations in a generalized form for any site i:

$$\frac{W_1 \times w_{i1} + W_2 \times w_{i2} + \cdots + W_m \times w_{im}}{\sum_{k=1}^{n} (W_1 \times w_{k1} + W_2 \times w_{k2} + \cdots + W_m \times w_{km})} = \frac{\sum_{j=1}^{m} C_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{m} C_{kj}}$$

$$\frac{\sum_{j=1}^{m} W_j \times w_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{m} W_j \times w_{kj}} = \frac{\sum_{j=1}^{m} C_{nj}}{\sum_{k=1}^{n} \sum_{j=1}^{m} C_{kj}} \qquad \text{(C.1)}$$

C.1.2 Solution of Simultaneous Equations for W_j

Any w_{ij} is defined as

$$w_{ij} = \frac{C_{ij}}{\sum_{k=1}^{n} C_{kj}}$$

Thus, each of the simultaneous equations becomes, for site i,

$$\frac{\sum_{j=1}^{m} W_j \times \frac{C_{ij}}{\sum_{k=1}^{n} C_{kj}}}{\sum_{k=1}^{n} \sum_{j=1}^{m} W_j \times \frac{C_{kj}}{\sum_{l=1}^{n} C_{lj}}} = \frac{\sum_{j=1}^{m} C_{nj}}{\sum_{k=1}^{n} \sum_{j=1}^{m} C_{kj}}$$

Assume the denominators of each side of each of the equations are equal:

$$\sum_{k=1}^{n} \sum_{j=1}^{m} W_j \times \frac{C_{kj}}{\sum_{l=1}^{n} C_{lj}} = \sum_{k=1}^{n} \sum_{j=1}^{m} C_{kj} \tag{C.2}$$

This assumption will later be shown to be valid when a solution is found. When the denominators are assumed equal, the simultaneous equations now become, for each i,

$$\sum_{j=1}^{m} (W_j \times \frac{C_{ij}}{\sum_{k=1}^{m} C_{kj}}) = \sum_{j=1}^{m} C_{ij}$$

The left-hand side of the equation can be rewritten so that C_{ij} has a coefficient which is independent of i and equal for any j in all of the equations:

$$\sum_{j=1}^{m} [(\frac{W_j}{\sum_{j=1}^{m} C_{kj}} \times C_{ij}] = \sum_{j=1}^{m} C_{ij}$$

An obvious solution to the equations is to force the coefficient of C_{ij} on the left-hand side to equal unity:

$$\frac{W_j}{\sum_{k=1}^{n} C_{kj}} = 1$$

This yields a solution for each W_j:

$$W_j = \sum_{k=1}^{n} C_{kj}$$

Substituting this expression in Equation (C.2) shows that the assumption of making the denominators equal is valid, and thus the original equations are satisfied. The final step in the solution is to normalize the values of W_j. Because the equations are linearly dependent, the solution derived for W_j is just one of many possible solutions. In the normal AHP process, weights at any node (ie., any set of W_j) are normalized so that their sum is unity. For the W_j derived in this case, normalization produces

$$W_j = \frac{\sum_{k=1}^{n} C_{kj}}{\sum_{l=1}^{m} \sum_{k=1}^{n} C_{kl}} \tag{C.3}$$

Verify that this is a valid solution by substituting it into the generalized form of the original simultaneous equations (Equation (C.1)):

$$\frac{\sum_{j=1}^{m} \frac{\sum_{k=1}^{n} C_{kj}}{\sum_{l=1}^{m} \sum_{k=1}^{n} C_{kl}} \times \frac{C_{ij}}{\sum_{k=1}^{n} C_{kj}}}{\sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\sum_{k=1}^{n} C_{kj}}{\sum_{l=1}^{m} \sum_{k=1}^{n} C_{kl}} \times \frac{C_{ij}}{\sum_{k=1}^{n} C_{kj}}} = \frac{\sum_{j=1}^{m} C_{ij}}{\sum_{i=1}^{n} \sum_{k=1}^{m} C_{ik}}$$

The summations $\sum_{k=1}^{n} C_{kj}$ cancel in both the numerator and denominator of the left-hand side, producing

$$\frac{\dfrac{\sum_{j=1}^{m} C_{ij}}{\sum_{l=1}^{m}\sum_{k=1}^{n} C_{kl}}}{\dfrac{\sum_{i=1}^{n}\sum_{j=1}^{m} C_{ij}}{\sum_{l=1}^{m}\sum_{k=1}^{n} C_{kl}}} = \frac{\sum_{j=1}^{m} C_{ij}}{\sum_{i=1}^{n}\sum_{k=1}^{m} C_{ik}}$$

The denominators on the left-hand side cancel, leaving

$$\frac{\sum_{j=1}^{m} C_{ij}}{\sum_{i=1}^{n}\sum_{k=1}^{m} C_{ik}} = \frac{\sum_{j=1}^{m} C_{ij}}{\sum_{i=1}^{n}\sum_{k=1}^{m} C_{ik}}$$

Thus, the original assumption was valid, and on the supposition that AHP and economic weights can be made equal, an expression for the AHP priorities of monetary factors in terms of the factors' values at the sites has been derived.

C.1.3 Linear Dependence of the Simultaneous Equations

It is necessary to show that the equations are indeed not independent. This can be accomplished by adding the equations, resulting in the single equation

$$\frac{W_1 \times \sum_{k=1}^{n} w_{k1} + W_2 \times \sum_{k=1}^{n} w_{k2} + \cdots + W_m \times \sum_{k=1}^{n} w_{km}}{\sum_{k=1}^{n} (W_1 \times w_{k1} + W_2 \times w_{k2} + \cdots + W_m \times w_{km})} = \frac{\sum_{k=1}^{n}\sum_{j=1}^{m} C_{kj}}{\sum_{k=1}^{n}\sum_{j=1}^{m} C_{kj}}$$

The right-hand side of this equation is equal to unity; the denominator of the left-hand side can be expressed as a set of individual sums.

$$\frac{W_1 \times \sum_{k=1}^{n} w_{k1} + W_2 \times \sum_{k=1}^{n} w_{k2} + \cdots + W_m \times \sum_{k=1}^{n} w_{km}}{W_1 \times \sum_{k=1}^{n} w_{k1} + W_2 \times \sum_{k=1}^{n} w_{k2} + \cdots + W_m \times \sum_{k=1}^{n} w_{km}} = 1$$

The resulting left-hand side reduces to unity, and thus the sum of the equations is universally true and is independent of the equations themselves. This means that any member of the set of equations can be derived if the other equations are known, and thus they are not independent.

C.2 An Example Using Only Monetary Factors

Assume that there are three candidate sites and five monetary factors. The five monetary factors include revenues associated with two products, A and B; sales of these products are expected to be a function of the site selected. The other monetary factors are costs: labor, transportation and material. Only that material whose cost is affected by location is included in the material factor. All cash flows associated with the monetary factors are expressed as Present-Worth (PW) values. Data for each site is shown in Figure C-1.

Site	Revenue A	Revenue B	Labor	Transport	Material
1	100	150	-50	-25	-100
2	110	135	-45	-50	-90
3	100	200	-75	-20	-100

Figure C-1: Data for Exclusively Monetary Factor Example

C.2.1 Economic Analysis of Example

The economic analysis weight of each site is simply the sum of that site's cash flows divided by the sum of all sites' cash flows:

$$S_i = \frac{\sum_{j=1}^{m} C_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{m} C_{kj}}$$

Thus, the weight of each site is:

$$\text{Site 1:}\quad \frac{100 + 150 - 50 - 25 - 100}{240} = \frac{75}{240} = 0.3125$$

$$\text{Site 2:}\quad \frac{110 + 135 - 45 - 50 - 90}{240} = \frac{60}{240} = 0.2500$$

$$\text{Site 3:}\quad \frac{100 + 200 - 75 - 20 - 100}{240} = \frac{105}{240} = 0.4375$$

C.2.2 AHP Analysis of Example

To conduct an AHP analysis of the example, it is first necessary to define the hierarchy. Figure C-1 shows the hierarchy to be used in the analysis.

Best Site

Revenue A Revenue B Labor Transport Material

Site 1 Site 2 Site 3

Figure C-2: The AHP Hierarchy for Exclusively Monetary Factor Example

Determine the weight of each site with respect to each factor. These weights are defined by

$$w_{ij} = \frac{C_{ij}}{\sum_{k=1}^{n} C_{kj}}$$

These computations are shown in Figure C-3.

Site	Revenue A	Revenue B	Labor	Transport	Material
1	$\frac{100}{310} = 0.3226$	$\frac{150}{485} = 0.3093$	$\frac{-50}{-170} = 0.2941$	$\frac{-25}{-95} = 0.2632$	$\frac{-100}{-290} = 0.3448$
2	$\frac{110}{310} = 0.3548$	$\frac{135}{485} = 0.2784$	$\frac{-45}{-170} = 0.2647$	$\frac{-50}{-95} = 0.5263$	$\frac{-90}{-290} = 0.3103$
3	$\frac{100}{310} = 0.3226$	$\frac{200}{485} = 0.4124$	$\frac{-75}{-170} = 0.4412$	$\frac{-20}{-95} = 0.2105$	$\frac{-100}{-290} = 0.3448$

Figure C-3: Weights of Sites with Respect to Each Factor

The priority of each factor with respect to each other factor must also be determined. As previously derived,

$$W_j = \frac{\sum_{k=1}^{n} C_{kj}}{\sum_{l=1}^{m} \sum_{k=1}^{n} C_{kl}}$$

These computations are shown in Figure C-4.

Factor	Weight
Revenue A	$\frac{310}{240} = 1.2916$
Revenue B	$\frac{485}{240} = 2.0208$
Labor	$\frac{-170}{240} = -0.7083$
Transport	$\frac{-95}{240} = -0.3958$
Material	$\frac{-290}{240} = -1.2083$

Figure C-4: Weights of Sites with Respect to Each Factor

Now that the priorities at each node are known, the rankings of the sites can be determined. Figure C-5 shows these calculations, and the results are indeed equal to the results of the economic analysis.

Site	Rank
1	$1.2916 \times 0.3226 + 2.0208 \times 0.3093 + -0.7083 \times 0.2941 +$ $-0.3958 \times 0.2632 + -1.2083 \times 0.3448$ $= 0.3126$
2	$1.2916 \times 0.3548 + 2.0208 \times 0.2784 + -0.7083 \times 0.2647 +$ $-0.3958 \times 0.5263 + -1.2083 \times 0.3103$ $= 0.2501$
3	$1.2916 \times 0.3226 + 2.0208 \times 0.4124 + -0.7083 \times 0.4412 +$ $-0.3958 \times 0.2105 + -1.2083 \times 0.3448$ $= 0.4376$

Figure C-5: Ranking of Sites According to AHP Analysis

Appendix D
Financial Justification Methods

In this appendix, six financial justification methods are presented. These methods are Present-Worth Amount (PW), Annual Equivalent Amount (AE), Future-Worth Amount (FW), Rate of Return, Payback Period and Project Balance. Following the introduction of these is a single example which is analyzed using each of the methods.

D.1 Present-Worth Amount (PW)

The Present-Worth Amount is the value of all future cash flows associated with the project, discounted to the present value. The costs and revenues of the project from each year of its projected life are discounted by an interest rate i, such that the costs and revenues of each future year t years from the present is discounted by a factor of $1/(1+i)^t$. Thus, the net present worth of a project is the sum of these discounted flows over the anticipated life of the project:

$$PW = \sum_{t=0}^{n} F_t \times (1+i)^{-t}$$

where F_t is the cash flow of year t, defined as total revenues minus total costs during year t.

The Present-Worth method has the advantage of combining the financial flows over the life of the project into a single number for use in decision making. It is, however, very dependent on the choice of interest rate i and the expected life of the project n. Problems arise if an interest rate is selected to match anticipated inflation or bank interest rates, but the economy performs in an unexpected manner making the interest rate unrealistic. Also, guessing the cash flows many years into the future can be very difficult, especially when many companies today cannot say with certainty what businesses they will be in after ten or twenty years.

If the Present-Worth of a facility at each candidate site can be calculated, it can be used as a basis for comparison among the sites. This requires that all input factors either be components of a financial cash flow, or easily convertible to components of a financial cash flow.

D.2 Annual Equivalent Amount (AE)

The Annual Equivalent Amount of a cash flow is the flow which can be expected to occur in an "average" year of the project's life. Computationally, it is very similar to the Present-Worth Amount, and one can be derived from the other. Its advantage is it expresses costs and revenues as events over time, which is how they usually occur, rather than as one lump sum. An interest rate is used to discount future cash flows when calculating the single annual figure. The Annual Equivalent Amount is often preferred if costs or revenues occur cyclically. This would be the case for plants with large recurring expenses, although such expenses are important for facilities location only if their amount is a function of the plant's location. A major disadvantage of this method is that annual cash flows on a project may never be anywhere near the average figure calculated. Typically, a facility location project will encounter large costs before any revenues, and then will gain (hopefully) significant profits once most of the construction and production start-up work is completed.

The Annual Equivalent Amount of a project is defined as

$$AE = PW \times (A/P,i,n)$$

Note that because $(A/P,i,n)$ is a constant, both the Annual Equivalent Amount and the Present-Worth Amount are equivalent bases for comparison of projects.[7] A project with *x* times the Present-Worth of another will have *x* times the Annual Equivalent Amount as well. Continuing with the definition of the Annual Equivalent Amount, one can substitute the definition derived for PW and the definition of *(A/P,i,n)* to obtain the more fundamental definition of the Annual Equivalent Amount as

$$AE = \{\sum_{t=0}^{n} F_t \times (1+i)^{-t}\} \times \{\frac{i(1+i)^n}{(1+i)^n - 1}\}$$

[25, p. 143]

In a site selection problem, the annual equivalent amount of operating at various sites would be the basis of comparison among the sites. This method might be preferred for computational ease if the particular problem has many components of operating cost known on an annual or any other time basis. In this case, the initial and other non-recurring costs associated with the plant would be annualized by multiplying them by the factor *(A/P,i,n)*. The sum of these annualized figures would be the annual equivalent amount for comparison.

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[7] This paper has adopted the notation for engineering economy factors used in [12] and [25]. *(w/x,y,z)* means "the value of w, given values of x, y and z." Symbols used are

A	The annualized value of a cash amount
F	The future value of a cash amount
P	The present value of a cash amount
i	The applicable interest rate
n	The applicable number of time periods (usually years)

D.3 Future-Worth Amount (FW)

Still another variation of the Present-Worth Amount method is the Future-Worth Amount. This is the sum of cash flows expressed as a value in some future year. For the value in year n, it is defined as

$$FW = \sum_{t=0}^{n} F_t \times (F/P,i,n\text{-}t)$$

This is mathematically equivalent, however, to the product of the project's Present-Worth Amount and a constant:

$$FW = PW \times (F/P,i,n)$$

[25, p. 148]

Present-Worth, Annual Equivalent and Future Worth are all equivalent bases of comparison. Different firms prefer one over the others for various reasons, including ease of computation and the existence of systems already using one of the methods.

D.4 Rate of Return

The Rate of Return is defined as the interest rate which causes the Present-Worth Amount of a project to be zero. The Rate of Return is also commonly referred to as the Return on Investment (ROI). It is interpreted as being the interest rate which the project will provide to the money invested in it. The Rate of Return is often compared with current external investment interest rates, which are used as a benchmark to screen internal spending proposals from further consideration. The reason for this is that money would be better invested in a bank at a known, guaranteed interest rate rather than in a project expected to provide less of a return. Usually, companies specify minimum rates of return which are even higher than those reasonably available external to the firm. These strict requirements have been criticized for placing too much emphasis on the financial returns of projects, at the expense of their longer-term strategic importance to the company [14].

The Rate of Return is the interest rate i which causes the Present-Worth of a cash flow to be zero. It can be defined as i, such that

$$0 = \sum_{t=0}^{n} F_t (1 + i)^{-t}$$

This can be expressed as an nth degree polynomial in i, whose real, positive root will be the Rate of Return. Rather than solve such an equation, an approximate Rate of Return is interpolated from standard $(P/F,i,n)$ tables.

As a basis of comparison, the Rate of Return is desirable because it requires no assumption of future interest rates. In this method, an interest rate is the result of calculations rather an an input factor to them. This reduces the uncertainty of calculations to the magnitude and the timing of future cash flows.

In a site selection problem, the rates of return would be the basis of comparison. The site with the highest rate of return would be the most preferable, and the ratios among the rates would be interpreted as the ratios among the sites' desirability.

D.5 Payback Period

The Payback Period is simply the time required to recoup the initial cost associated with a project. It does not consider the time value of money, meaning it uses an interest rate of zero. Because it ignores the time value of money, the Payback Period is sometimes referred to as the "Undiscounted Payback Period," to distinguish it from the "Discounted Payback Period," which would account for the time value of money. The Discounted Payback Period may be found from the Project Balance, which is described in Appendix D.6. The mathematical definition of Payback Period is the smallest value of n such that

$$0 \geq \sum_{t=0}^{n} F_t$$

[25, p. 159]

Often, the inequality of this definition is satisfied rather than the equality, and linear interpolation is used to arrive at a non-integer number. The Payback Period is very easy to calculate but has two serious shortcomings: it ignores the time value of money and it ignores all cash flows after the initial cost is recouped. The latter weakness has the effect of ignoring true net profit—money received beyond that which was spent. Thus, profitable projects with longer Payback Periods can appear less desirable than break-even projects with shorter Payback Periods. Generally, projects with shorter Payback Periods are preferred because they are perceived to involve less risk of unrecouped spending. In a facility location problem, the sites with the shortest payback periods would be considered more preferable, but because of its serious shortcomings it is recommended that this method be avoided in favor of one of the other methods presented here.

D.6 Project Balance

The Project Balance method differs from those already described in Appendices D.1 through D.5 in that cash flows are presented as a function of time for each project, rather than as a single index. A non-zero interest rate may be used, and the method shows the net cost or income from a project as a function of time. This allows management to see the cost of prematurely stopping a project, which is often considered when periodic decisions of continued funding are required.

For any single project, the Project Balance is a function of time since the project began. This function is defined recursively:

$$PB_0 = F_0$$

$$PB_t = PB_{t-1} \times (1 + i) + F_t$$

[25, p. 163]

The Project Balance at any future time T can be found from

$$PB_T = \sum_{t=0}^{T} F_t \times (1 + i)^{T-1}$$

[25, p. 164]

Expressing the resulting function PB_t in a graph provides a simpler way of understanding the cash flow associated with a project. Such a graph is shown in Figure D-1. From it, the discounted payback period T_0 may be found. This is the time when the project's net worth accumulates to zero, but differs from the undiscounted payback period in that a non-zero interest rate is used. The general advantages of the Project Balance method over the single-index methods are

- A discounted payback period is provided.
- The maximum possible loss is provided.
- The rate of profit accumulation is provided.

All of these are evident from a graphical presentation of the Project Balance function.

D.7 An Example of the Various Bases of Comparison

Problem:

Site A and Site B are two candidate sites being considered for a new plant. The total costs and revenues predicted for each site are shown in Figure D-2. Assume an interest rate of 15%, compounded annually. Determine which site is economically preferable using each of the criteria discussed in this chapter.

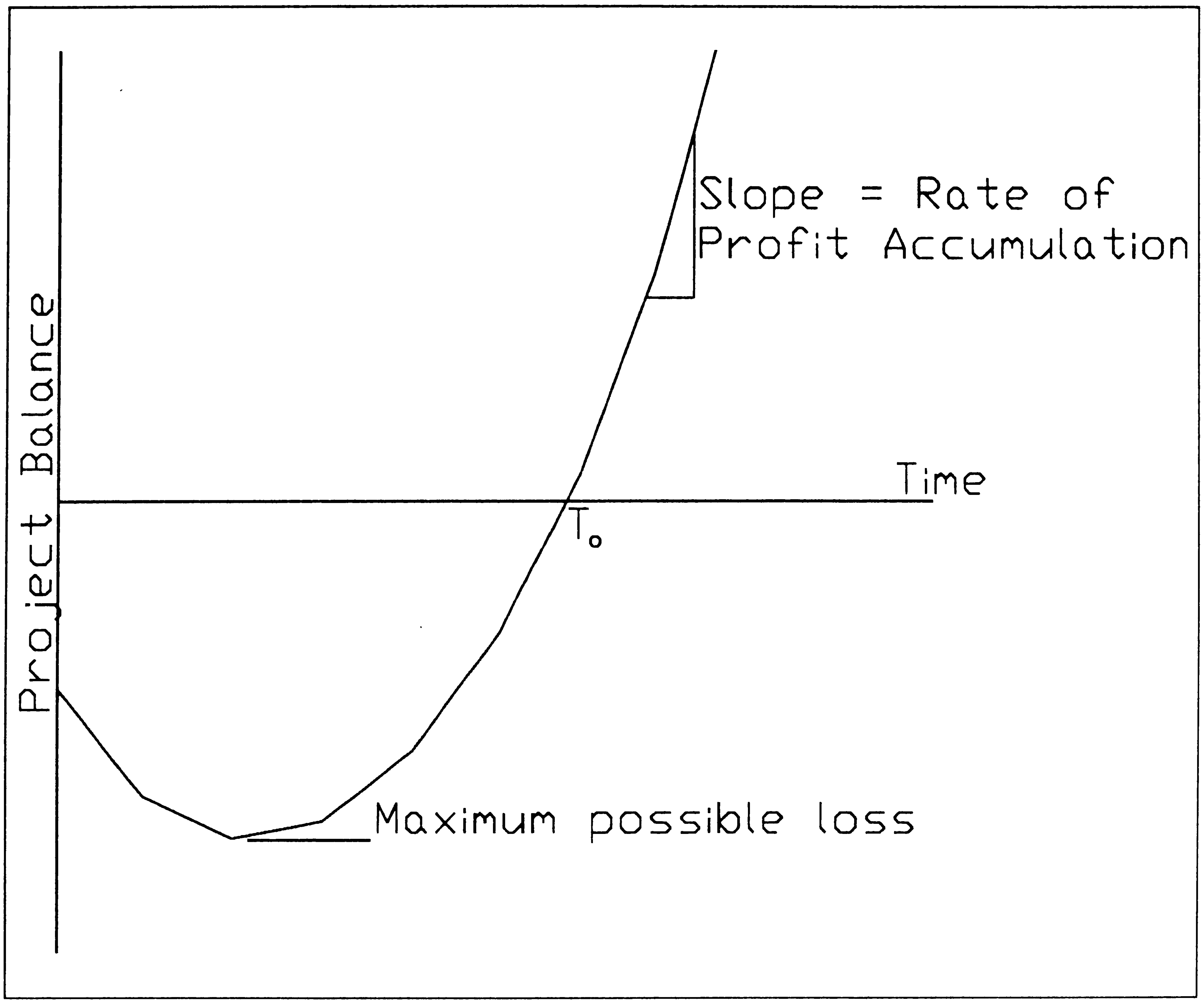

Figure D-1: The Project Balance function, PB_t. Time T_0 is the discounted payback period.

Estimated Costs and Revenues, by Site				
Year	Site A Cost	Site A Revenue	Site B Cost	Site B Revenue
0	10,000	1,000	11,000	0
1	8,000	4,000	8,000	3,000
2	1,000	6,000	500	6,000
3	1,000	11,000	500	12,000
4	2,000	12,000	1,000	13,000
5	2,000	15,000	1,000	17,000

Figure D-2: Initial Cost and Revenue Data for an Example Site Selection Based on Quantitative Factors Defined Over One Domain

Solutions:

Regardless of which method is used, the first step it to compute the net revenue in each year by subtracting that year's costs from its revenues. The results of these calculations are shown in Figure D-3.

Site A			
Year	Cost	Revenue	Net Revenue
0	10,000	1,000	(9,000)
1	8,000	4,000	(4,000)
2	1,000	6,000	5,000
3	1,000	11,000	10,000
4	2,000	12,000	10,000
5	2,000	15,000	13,000

Site B			
Year	Cost	Revenue	Net Revenue
0	11,000	0	(11,000)[8]
1	8,000	3,000	(5,000)
2	500	6,000	5,500
3	500	12,000	11,500
4	1,000	13,000	12,000
5	1,000	17,000	16,000

Figure D-3: Net Revenues Associated with each Site in Example

The various methods of determining preference can now be employed, using the Net Revenue of each site for each of the years as the initial data for analysis.

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[8] The accounting notation of indicating negative amounts by enclosing them in parentheses is adopted for use in this example.

D.7.0.1 Present-Worth Amount (PW)

The Present-Worth Amount is defined as

$$PW = \sum_{t=0}^{n} F_t \times (1 + i^{-t})$$

For Site A:

$$PW = -9{,}000 + \frac{-4{,}000}{1.15} + \frac{5{,}000}{1.15^2} + \frac{10{,}000}{1.15^3} + \frac{10{,}000}{1.15^4} + \frac{13{,}000}{1.15^5}$$

$$PW = 10{,}058.45$$

For Site B:

$$PW = -10{,}000 + \frac{-5{,}000}{1.15} + \frac{5{,}500}{1.15^2} + \frac{11{,}500}{1.15^3} + \frac{12{,}000}{1.15^4} + \frac{16{,}000}{1.15^5}$$

$$PW = 11{,}188.27$$

Site B has a greater Present-Worth Amount than site A and is thus preferable to it.

D.7.0.2 Annual Equivalent Amount (AE)

Calculate the Annual Equivalent Amount from the Present-Worth Amount:

$$AE = PW \times (A/P,i,n)$$

The value of $(A/P,i,n)$ is independent of the site:

$$(A/P,i,n) = \frac{i \times (1 - i)^n}{(1 + i)^n - 1}$$

$$(A/P,i,n) = 0.298316$$

For Site A:

$$AE = 10{,}058.45 \times 0.298316$$

$$AE = 3{,}000.60$$

For Site B:

$$AE = 11{,}188.27 \times 0.298316$$

$$AE = 3{,}337.63$$

Again, site B is economically preferable to site A.

D.7.0.3 Future-Worth Amount (FW)

Like the Annual Equivalent Amount, calculate the Future-Worth Amount from the Present-Worth Amount:

$$FW = PW \times (F/P,i,n)$$

The value of $(F/P,i,n)$ is independent of site:

$$(F/P,i,n) = (1 + i)^n$$

$$(F/P,0.15,5) = (1 + 0.15)^5$$

$$(F/P,0.15,5) = 2.01136$$

For Site A:

$$FW = 10{,}058.45 \times 2.01136$$

$$FW = 20{,}231.16$$

For Site B:

$$FW = 11{,}188.27 \times 2.01136$$

$$FW = 22{,}503.64$$

Again, site B is economically preferable because of its higher Future Worth. Note that the ratio of Future Worths of B to A equals the ratio of Annual Equivalent Worths of B to A as well as the ratio of Present Worths of B to A. These ratios will always be equal, because Present-Worth, Annual Equivalent Worth and Future Worth are all defined to be proportional to each other.

D.7.0.4 Rate of Return

Find the Rate of Return by calculating the Present Worth for various interest rates, then interpolating to find a sufficiently accurate rate such that the Present Worth is zero. Because each site had a positive Present Worth at the stated interest rate of 15%, use a figure higher than 15% to begin the search for the Rate of Return.

For Site A:

30%: $PW = 2{,}435.99$

35%: $PW = 754.80$

36%: $PW = 454.77$

37%: $PW = 165.60$

38%: $PW = -113.23$

The Rate of Return for site A lies between 37% and 38%, slightly closer to 38%.

For Site B:

35%: $PW = 169.25$

36%: $PW = -184.43$

The Rate of Return for site B lies about midway between 35% and 36%. Because site A has a higher rate of return, it is economically preferable to site B. Note that this contradicts the results of the previous methods of analysis. This is primarily because Site B's high net revenues are far in the future and thus heavily discounted by the compounded interest rate.

D.7.0.5 Payback Period

The undiscounted Payback Period is easily found by summing the net revenues of the current and all previous time periods, and identifying when that sum reaches zero. The accumulated net revenues associated with each site are shown in Figure D-4. Both sites reach an undiscounted break-even point between years 2 and 3. Linear interpolation shows that site A reaches its break-even at 2.8 years and site B breaks even at 2.91 years. Because its Payback Period is shorter, site A is economically preferable to site B.

Site A				
Year	Cost	Revenue	Net Revenue	Total Net Revenue
0	10,000	1,000	(9,000)	(9,000)
1	8,000	4,000	(4,000)	(13,000)
2	1,000	6,000	5,000	(8,000)
3	1,000	11,000	10,000	2,000
4	2,000	12,000	10,000	12,000
5	2,000	15,000	13,000	25,000

Site B				
Year	Cost	Revenue	Net Revenue	Total Net Revenue
0	11,000	0	(11,000)	(11,000)
1	8,000	3,000	(5,000)	(16,000)
2	500	6,000	5,500	(10,500)
3	500	12,000	11,500	1,000
4	1,000	13,000	12,000	13,000
5	1,000	17,000	16,000	29,000

Figure D-4: Total Net Revenues Associated with Each Site in Example

D.7.0.6 Project Balance

The Project Balance is the discounted version of the total net revenue. Use of the formulas

$PB_0 = F_0$ and

$PB_t = PB_{t-1} \times (1 + i) + F_t$

yields the data shown in Figure D-5. Both sites have a discounted payback period that lies between three and four years. Linear interpolation shows site A's payback period to be 3.34 years and site B's payback period to be 3.49 years. Using this as a basis of comparison, site A is the preferable site.

The five methods of comparison in this example show that either Site A or Site B can be justified. The methods used all have a firm financial foundation, and all are currently used by management of leading corporations. Which methods are superior to the others is a matter of debate, although often several

Site A				
Year	Cost	Revenue	Net Revenue	Project Balance
0	10,000	1,000	(9,000)	(9,000)
1	8,000	4,000	(4,000)	(14,350)
2	1,000	6,000	5,000	(11,502.50)
3	1,000	11,000	10,000	(3,227.88)
4	2,000	12,000	10,000	6,287.94
5	2,000	15,000	13,000	20,231.14

Site B				
Year	Cost	Revenue	Net Revenue	Project Balance
0	11,000	0	(11,000)	(11,000)
1	8,000	3,000	(5,000)	(17,650)
2	500	6,000	5,500	(14,797.50)
3	500	12,000	11,500	(5,517.13)
4	1,000	13,000	12,000	5,655.31
5	1,000	17,000	16,000	22,503.60

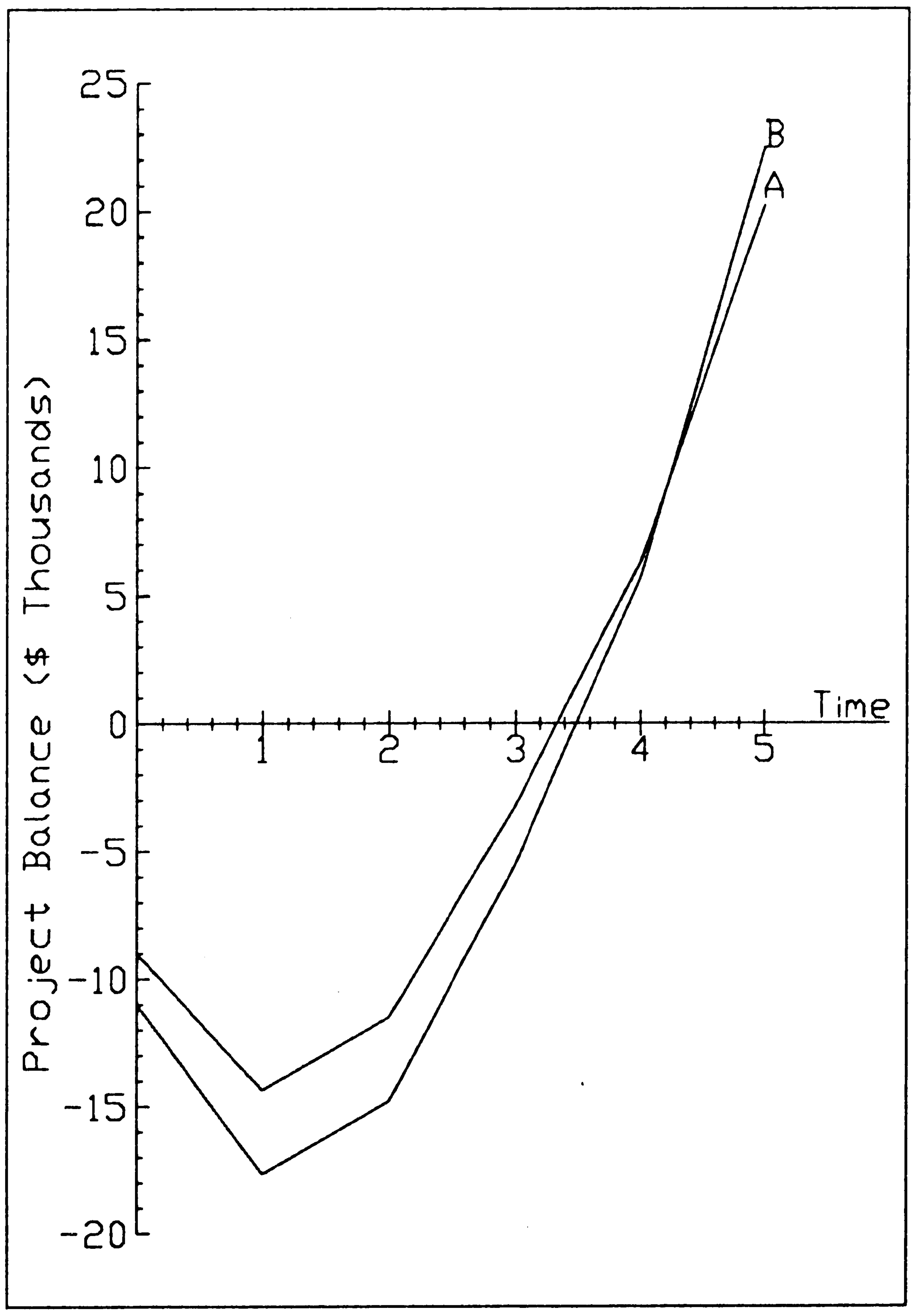

Figure D-5: Project Balance Associated with Sites in Example methods will be used to reenforce the justification of any particular project.

VITA

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