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Effect of viscosity on residence time distributions in short unagitated tubular vessels

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EFFECT OF VISCOSITY ON RESIDENCE-TIME DISTRIBUTIONS IN SHORT UNAGITATED TUBULAR VESSELS

by

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EFFECT OF VISCOSITY

ON

RESIDENCE-TIME DISTRIBUTIONS

IN

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______________________ Date

______________________ Professor in Charge

______________________ Head of the Department

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SUMMARY

Residence-time distributions have been studied for a number of years. Distributions have been obtained for a number of fluids (1,2,8) at various viscosities and Reynold's Numbers flowing through short, unagitated, tubular vessels. Attempts (1) have been made to correlate the parameters associated with these distributions with parameters associated with vessel geometry and fluid properties. These correlations are restricted to water as the flowing media.

This report attempted to apply these same correlations to viscous solutions with little success. Other correlations were attempted using the "free jet" theory, i.e. considering the vessel entrance to be a free jet, as a base but these were unsuccessful due in part to the scatter of the available experimental data. It was found that the free jet theory does not hold even for small length to diameter ratios as had been previously suggested. (2,8)

The Flow Model concept (6,7) was investigated and the parameters associated with the model were found to be primarily dependent on the vessel geometry rather than fluid properties. It is felt that fluid properties have a direct bearing on the diffusion process but that these properties do not influence the flow pattern to a great extent. Thus the vessel geometry becomes the dominant characteristic in determining the flow model parameters. Little can be learned with respect to the actual flow pattern within the vessel by such bulk effluent measurements and future work should continue to measure the effect of viscosity on point concentrations within the vessel.

2. INTRODUCTION

2.1 Subject

This report discusses "residence time" distributions, i.e. the distribution of times required for the passage of fluid elements through an unagitated, tubular vessel. Knowledge of these distributions is important for the design and operation of continuous flow reactors and similar process equipment.

2.2 Purpose

Residence time distributions have been obtained by previous investigators using experimental concentration versus time measurements. These distributions have been presented and their general chacteristics have been quantitatively discussed. Attempts have been made to correlate the data obtained with dimensionless parameters which account for fluid properties and vessel geometry. These parameters were found to be insufficient for correlation of viscous solutions. The purpose of this report is to discuss these residence time distributions and the parameters characterizing them with regard to the dimensionless groups characterizing the fluid flow and the vessel geometry for both water and more viscous sugar solutions. It is also to account for the discrepancy found between correlations presented for water data and the experimental data obtained with viscous sugar solutions through the use of flow models and fluid mechanics.

3. BACKGROUND

3.1 Residence-Time Distributions

The "residence-time" is simply the time required for a fluid element to pass through a vessel. If the paths of molecules are traced while they travel through the vessels the molecules will be seen to travel various paths before emerging from the vessel. Each of these paths require a different time of passage or "residence time." The residence times for each individual molecule may be summed to give a distribution of residence times versus the number of molecules having this particular residence time. This is the Residence-Time Distribution.

This distribution can be obtained analytically for only a very few cases. Thus residence-time distributions are usually determined experimentally. Danckwerts (4) has suggested a number of experimental techniques for determining residence time distributions. These methods consist of introducing a tracer into the stream flowing into the vessel. The tracer concentration is measured in the effluent stream as a function of time to give the required distribution.

The method of introducing the tracer differs between the techniques. The tracer may be injected continuosly at a constant rate starting at a time zero or the tracer may be injected in slugs, or pulses, that is, all the tracer may be introduced at once, essentially instantaneously. These two methods of tracer introduction yield the distribution curves shown in

Figures 3.1A and 3.1B. These curves illustrate the tracer concentration in the effluent stream versus time for both of these methods of tracer introduction.

Curves of this nature have been obtained experimentally at Lehigh University by injecting a dispersion of carbon black dissolved in the solution to be used. (Water or sugar solutions) Adler (1) has obtained residence-time distribution curves by injecting slugs of tracer into a stream of fluid flowing into a short, unagitated tubular vessel. A schematic diagram of such a vessel is shown in Figure 3.1C. Vessels are considered to be "short" if the length to diameter ratio is low. Since almost all the work considered had a length to diameter ratio less than 10, we will assume we are dealing with short vessels.

Adler's work was largely restricted to injections of tracer into water streams although he made a few experiments with viscous streams. This latter work indicated that the Reynold's Number should account for any change in viscosity. Sabnis (8) extended this work by using the same techniques as Adler but confining his work to viscous solutions of 10%, 20%, 30% and 40% sugar. His work indicated that the Reynold's Number was insufficient to characterise the effect of viscosity on the residence-time distribution. Thus residence-time distribution curves have been obtained for a number of flow cases by the methods outlined by Danckwerts, but the data does not fit any general pattern.

FIGURE 3.1A

TYPICAL TRACER CONCENTRATION - TIME DIAGRAMS

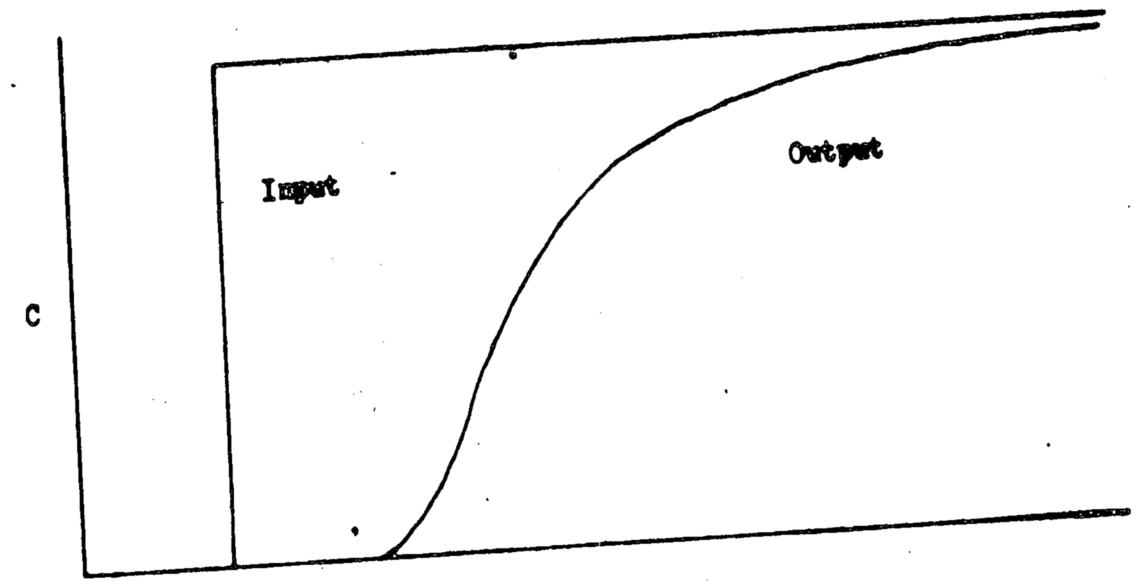
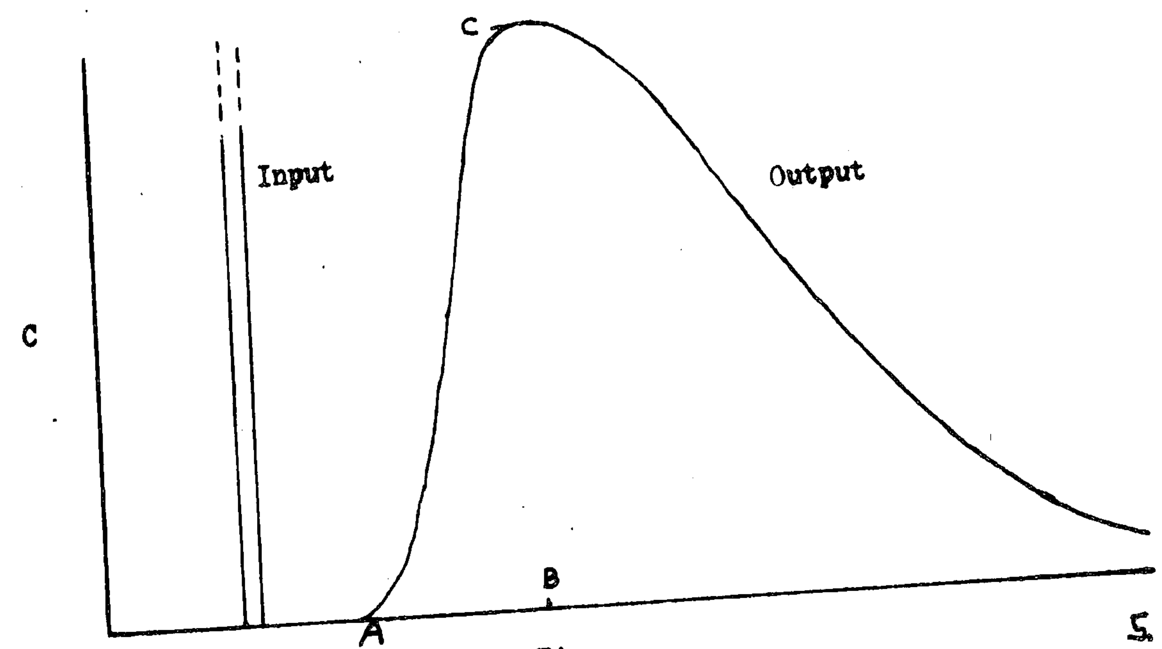


FIGURE 3.1 B



SCHEMATIC OF SHORT, UNAGITATED VESSEL

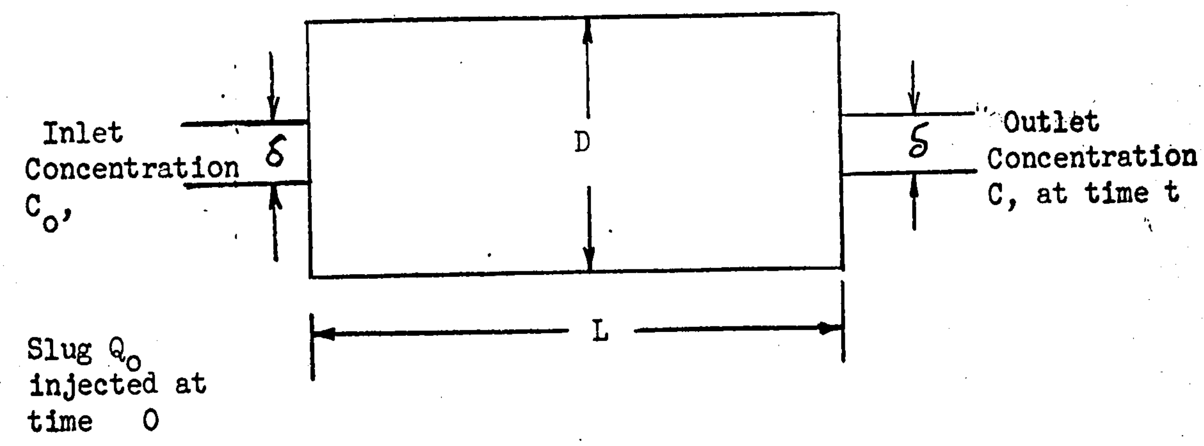


FIGURE 3.1C

3.2 Representation and Characterization of Residence-Time Distributions

Residence-time distributions have usually been presented in a graphical form with dimensionless coordinates to generalize the results. These distributions can be characterized by these dimensionless coordinates and by variables associated with these coordinates. It is usual (for the continuous input case) to plot the concentration in the effluent stream as a dimensionless concentration ratio, C/C_O. This represents the ratio of the output concentration, C to the input concentration, C_O. Since the input concentration is constant, the variations in the effluent concentration as a function of time is obtained. The abscissa of such a plot is the dimensionless parameter, vt/V where v is the constant volumetric rate of flow, V the total volume of the vessel, and t is the time elapsed since the tracer was first introduced.

If the tracer is introduced in pulses a different ordinate is used. It is very similar to the previous ratio C/C_O but contains the parameters, CV/Q_O. In this case the concentration is multiplied by the volume divided by the quantity of tracer introduced by the pulse, Q_O. As in the case of the continuous introduction of tracer the dimensionless concentration parameter is plotted versus the dimensionless time parameter, vt/V. This is sometimes referred to as the reduced time. Both of these residence-time distribution curves are related and the dimensionless parameters associated with each of these curves are likewise related.

There are four general parameters which are associated with the pulse response curves. These parameters are the start time, the peak time, the peak concentration, and the slope of the exponentially decaying portion of the residence-time distribution curve or the "K" value. The start time is simply the time necessary for the first detectable portion of tracer to reach the monitoring section in the effluent tube. This parameter is characteristic for both continuous and pulse tracer injections. The peak time is the time required for the maximum concentration to emerge from the vessel under consideration. Thus on Figure 3.1B the peak time would be the time at which the dimensionless concentration parameter is at its maximum value, point B. The third parameter which is characteristic of pulse tracer injection response is the peak concentration. This is the concentration of tracer in the effluent stream at the peak time. On figure 3.1B this is the value of the dimensionless parameter at point C. The last variable which is used to characterize the residence-time distribution curve is the so called "K" value. This is the slope of the decaying portion of the residence-time distribution curve. This value is then characteristic of the decay of tracer concentration in the fluid remaining in the vessel.

Each of these dimensionless parameters has been obtained from experimental data by Adler (1,2) and Sabnis (8) and tabulated. Correlations have been attempted to relate these four dimensionless parameters to parameters characteristic of the system under consideration.

3.3 Analytical Solutions For Residence-Time Distributions

Analytical solutions for the residence-time distribution yielding the concentration or the dimensionless concentration parameter as a function of time or dimensionless time have been presented for a number of the less complex systems. Cases for which analytical solutions are available include perfect mixing in the flow vessel, laminar flow within the vessel with negligible diffusion, turbulent flow through packed beds, and extremely long vessels operating in either laminar or turbulent flow.

Adler (1,2) has presented the analytical solution for a perfectly mixed vessel into which an instanteous injection of tracer Q_o is made. By writing a simple differential equation the following dimensionless equation for concentration as a function of time can be obtained.

$$\frac{CV}{Q_o} = e^{-vt/V} \qquad 3.3(1)$$

This equation yields the residence-time distribution for a perfectly mixed vessel. Similarly the solution for the case of laminar flow with negligible diffusion has been presented by Taylor (10) using the length of the vessel and the time-velocity distribution. He gives the solution in the form of the concentration as a function of the other variables;

$$C = \frac{Q_o}{\pi r^2 v(0)t} \qquad 3.3(2)$$

where v(0)t is the time dependent centerline velocity and r is the radius of the tube. Expressions using the universal diffusion solution (Fick's equation) have been presented and discussed by a number of researchers. Adler (1,2) presents a complete derivation of an equation based on Fick's equation. His equation, in terms of dimensionless groups is,

$$\frac{CV}{Q_0} = \frac{1}{2\sqrt{\pi\left(\frac{\nu t}{V}\right)\left(\frac{D}{\alpha L}\right)}} \; e^{-\frac{\left\{1-\frac{\nu t}{V}\right\}^2}{4\left(\frac{\nu t}{V}\right)\left(\frac{D}{\alpha L}\right)}} \qquad 3.3\ (3)$$

The dimensionless concentration parameter and therefore the residence-time distribution are thus related to the velocity of the fluid flowing and the diffusivity as well as the dimensionless time parameter.

Sabnis (8) has presented the analytical solution for the residence-time distribution considering axial dispersion of particles in flow through a packed bed. This analytical solution considers the bed to consist of a series of randomly arrayed void cells each with a holding or residence-time. The solution generated yields the concentration as an error function of the dispersion coefficient and time.

Each of these solutions is valid only for a particular case and cannot be used for our general case which consists of steady flow through a short, unagitated, tubular vessel. These solutions may, however, be used as guidelines for consideration of the mixing process within the vessel and the flow pattern in general

4. CORRELATION OF EXPERIMENTAL DATA

4.1 Theoretical Discussion of Flow Parameters

The Navier-Stokes equation may be used as a basis for the theoretical foundation of the dimensionless parameters associated with the flow pattern through short, unagitated, tubular vessels. For a cylindrical system the Navier-Stokes equation may be written as;

$$\rho\left[\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_z}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}\right] = -\frac{\partial P}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right] + g\rho_z$$

where V_z is the velocity in the flow direction (z),

V_θ the velocity in the angular direction (θ), and

V_r the velocity in the radial direction (r).

The shear stresses are denoted by the τ symbol with the subscripts showing the direction and tangency of the shear vectors.

The above equation can be shown to be characterized by four dimensionless groups. This has been demonstrated in previous reports of work on the subject of residence-time distributions. (4,8) The method by which this is accomplished consists of making each term in the above equation domensionless by multiplying by the appropriate length and velocity terms. By suitable rearrangement the following dimensionless groups are obtained; Vz/V_O, vt/V, z/L, Vr/V_O, r/δ, and $P-P_O/P_z-P_O$. (P is the pressure and the subscript denotes the location of the pressure measurement.) The next step involves the mathematical solution of the resulting differential equations. Finally the four dimensionless parameters $\frac{\delta v_0 \rho}{\mu}$, N_{Re}, L/d, d/δ, and $P_z-P_O/\rho v_0^2$ are obtained. These are of great importance in using the results from work in one system to predict results in another system. Adler (1) states that the last dimensionless group, the Euler Number, may be approximately resolved by a consideration of the physical properties which contribute to the pressure drop. From this consideration he showed that the Euler Number is approximately a function of the other three dimensionless parameters. This leaves just three parameters L/d, d/δ, and N_{Re} to characterize the fluid flow in the system and to relate one system to another system. It is important to point out the physical significance of each of these three groups. The Reynolds Number, N_{Re} represents the ratio of the inertia forces to the viscous forces, the parameter L/d is the ratio of the length of the vessel to its diameter, and the ratio d/δ is the ratio of the diameter of the vessel to the diameter of the inlet tube.

4.2 Previous Correlation Attempts - The Free Jet Theory

Adler (1,2) has made an attempt to correlate the peak-times obtained by experimental measurements in water systems with the "free-jet" theory. The free jet theory describes the centerline velocity of a fluid emerging from a jet into an unconfined area. Basicly he assumed the free jet theory to be a reasonable approximation of the flow pattern within the vessel if the vessel was short. The peak time would be given by the centerline velocity of the inlet jet. Using the experimental data and some empirical modifcations he obtained fair correlation for very short vessels with water as the flowing media.

The procedure he used to develop the free jet correlation is paraphrased below.

Schlichting (9) gives the following equation for the mean center-line velocity of a turbulent, three dimensional, unconfined jet;

$$U = \frac{3}{8\pi} \cdot \frac{J}{E_o} \cdot \frac{1}{z} \qquad 4.2(1)$$

where $U = U(z)$ is the mean center-line velocity. J is the kinematic momentum of the jet defined by Schlichting as:

$$J = 2\pi \int_0^\infty u^2 r \, dr \qquad 4.2(2)$$

and z is the distance from the jet face. In 4.2(2) u is the velocity of the fluid. The kinematic momentum is considered constant with respect to z. E_o is the eddy viscosity which Schlichting gives as;

$$E_o = 0.0256 b_{\frac{1}{2}} U \qquad 4.2(3)$$

This was obtained from experimental measurement rather than theoretical development. The term $b_{\frac{1}{2}}$ is the radius of the jet at the point where the velocity is $\frac{1}{2}U$. ($\frac{1}{2}$ the center-line velocity.)

Next he evaluates the kinematic momentum given in equation 4.2(2) by assuming a flat velocity profile so that,

$$J = a \, \bar{u}^2 \qquad 4.2(4)$$

where a is the cross-sectional area of the inlet port and $\bar{u}$ is the average fluid velocity. The equation for the eddy viscosity is evaluated at the inlet port by setting the term $b_{\frac{1}{2}}$ equal to k, where k is a constant between $\frac{1}{2}$ and 1. Since a flat velocity profile has been assumed U is equal to $\bar{u}$

so that

$$E_o = 0.0256k\delta\bar{u} \qquad 4.25(5)$$

Equation 3.2(1) is modified to provide a finite velocity at $z = 0$. The equation is integrated assuming constant J and E_o. This yields,

$$\frac{LE_o}{2Jt} + \frac{bLE_o}{Jt} = \frac{3}{8\pi} \qquad 4.2(6)$$

where b was the constant added to z to give the finite velocity at $z = 0$. ($b + z$ is really the distance where the velocity is U) The values for J and E_o can be substituted into the above equation and the resulting equation can be rearranged in dimensionless groups. The equation resulting is;

$$\frac{(L/d)}{\left(\frac{d}{\delta}\right)\left(\frac{vt}{V}\right)_p} = \frac{3}{(16)(0.0256k)\left[1 + 2\left(\frac{b}{L}\right)\right]} \qquad 4.2(7)$$

where b is still a constant which needs to be evaluated. Substituting into modified equation 4.2(1) with $z = 0$ we get;

$$\bar{u} = 3J/8E_o b \qquad 4.2(8)$$

This can be rearranged and solved for b substituting the previous equations for J and E_o. This yields;

$$b = \frac{3.66\delta}{k} \qquad 4.2(9)$$

where δ is the inlet port diameter and k is the previously defined constant. Substituting this result into equation 4.2(7) and solving for the peak time $(vt/V)_p$ yields;

$$\left(\frac{vt}{V}\right)_p = \frac{(L/d)}{\left(\frac{d}{\delta}\right)}\left[\frac{k}{7.32} + \frac{1}{\left(\frac{L}{d}\right)\left(\frac{d}{\delta}\right)}\right] \qquad 4.2(10)$$

which is the correlation of the peak time with the free jet theory. This is all that the free jet theory will reveal but it is noted that the Reynold's Number was not included as a parameter in the above equation. A glance at the data which Adler obtained shows a weak dependence on the Reynold's Number and on the ratio d/δ. What Adler did next was to say that "k", the previously defined constant, was not really constant at all but a function of the ratio d/δ. Using the data at a Reynold's Number of 4,000 he obtained an equation which describes the dependence of k on the ratio of d/δ. This was purely empirical. The equation he obtained was further modified by the inclusion of a term to account for the Reynold's Number dependence. Thus the equation becomes:

$$\left(\frac{vt}{V}\right)_p = \frac{\left(\frac{L}{d}\right)}{\left(\frac{d}{\delta}\right)}\left[0.135\left(\frac{d}{\delta}\right)^{0.2} + \frac{1}{\left(\frac{L}{d}\right)\left(\frac{d}{\delta}\right)}\right]\psi N_{Re}^{n} \qquad 4.2(11)$$

where ψ and n are constant to be determined by plotting the data. Plotting on log-log paper yielded Adler the following equation for the peak time as a function of the variables a associated with the free jet theory.

$$\frac{\left(\frac{vt}{V}\right)\left(\frac{d}{\delta}\right)}{\left(\frac{L}{d}\right)\left[0.135\left(\frac{d}{\delta}\right)^{0.2} + \frac{1}{\left(\frac{L}{d}\right)\left(\frac{d}{\delta}\right)}\right]} = 0.347\left(N_{Re}\right)^{0.13} \qquad 4.2(12)$$

At the conclusion of his presentation Adler states that since the equation was based on sound theoretical principles with only modest empirical correlation the equation should be valid at L/D ratios other than 2. (The ratio at which the empirical correlation was made.) This is especially true at ratios less than 2 where better agreement with the free jet theory was expected. Thus an equation for the correlation of the peak times of residence-time distributions has been presented which is based on theory and which should be valid at L/d ratios less than 2 regardless of the nature of the fluid flowing. No further attempts were made at correlation of the data and the data which Sabnis (8) later took with viscous solution was not used to prove or disprove the correlation presented by Adler.

4.3 Modification of The Free-Jet Theory For Viscous Solutions

Naturally the first attempts which were made by the author to generalize the results of previous work for all flow systems were centered around the correlation which Adler had presented using the free jet theory. Actually what Adler was doing when he empirically correlated the data he had obtained was finding an equation for his constant k which fit the data. If one rearranges the equations, the equation for the dependence of k on the other parameter can be obtained.

$$k = 2.54\,(N_{Re})^{0.13}\left[0.135\left(\frac{d}{\delta}\right)^{0.2}\right] + \frac{2.32}{\left(\frac{d}{\delta}\right)\left(\frac{L}{d}\right)}\left[0.347\,(N_{Re})^{0.13} - 1\right] \qquad 4.3(1)$$

Since this rather complex equation which correlated Adler's data did not correlate the data obtained by Sabnis for peak times (See Figure 4.3) the above equation for the constant does not provide the relationship desired for the general case.

If the free jet theory previously presented is again considered the actual k value may be obtained. If k is substituted for $b_{\frac{1}{2}}$, the radius at which the velocity is ½ of the center-line velocity, k may be obtained from consideration of fluid mechanics. We know that (2) the following relationships are true;

For turbulent flow

$$\frac{V_z}{V_z(MAX)} = \left[1 - \frac{r}{R}\right]^{1/7} \qquad 4.3(2)$$

For laminar flow

$$\frac{V_z}{V_z(MAX)} = 1 - \left(\frac{r}{R}\right)^2 \qquad 4.3(3)$$

Substituting $r = k\delta$ and $R = \delta$ with $U_{\frac{1}{2}}/U = \frac{1}{2}$ the following values are obtained;

For turbulent Flow $k \cong 1$

For laminar Flow $k \cong 0.707$

Since the type of flow is directly proportional to the Reynold's Number;

$$k = f(N_{Re}) \text{ with } 0.707 < k < 1.0.$$

Using the viscosity data obtained by Sabnis the values of k which would satisfy the theoretical expression for peak times obtained by Adler were obtained. They are presented in Figures 4.3(B) and 4.3(C) for $L/d = 2$ and $L/d = 5$ respectively. It is interesting to note that they vary from 1.2 to 3.1 for L/d ratios of 2 and from 1.5 to 9.3 for L/d ratios of 5. This confirms the fact that the free jet theory is not valid at high L/d ratios but it also shows that even at low L/d ratios ($L/d = 2$) the theory does not conform to the values obtained from fluid mechanics. Thus it seems that the free jet theory is not really a valid correlation even at low L/d ratios as the experimental k values are all greater than the 1.0 maximum obtained from fluid mechanics.

4.4 Correlation Results

For the flow cases in which the Reynold's number is less than 2,000 in the inlet pipe, a fairly good representation for the peak time data can be obtained as a function of the viscosity for L/d ratios of 2.0 and 5.0. Once the Reynold's Number rises to about 3,000 a wide scattering of data points is observed and it does not seem possible to correlate these points with

FIGURE 4.3 (A)
Peak Time vs. N_{Re}

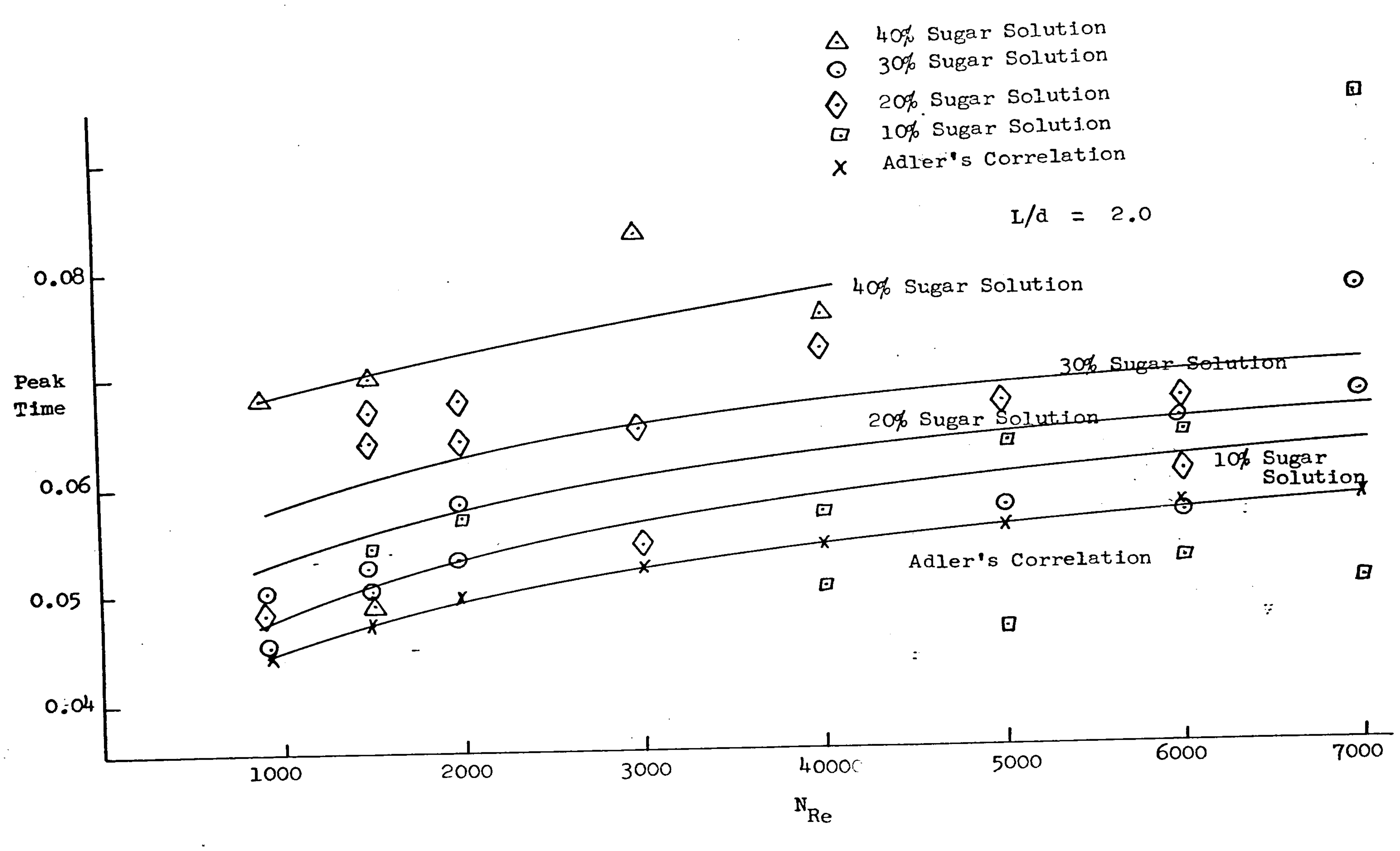
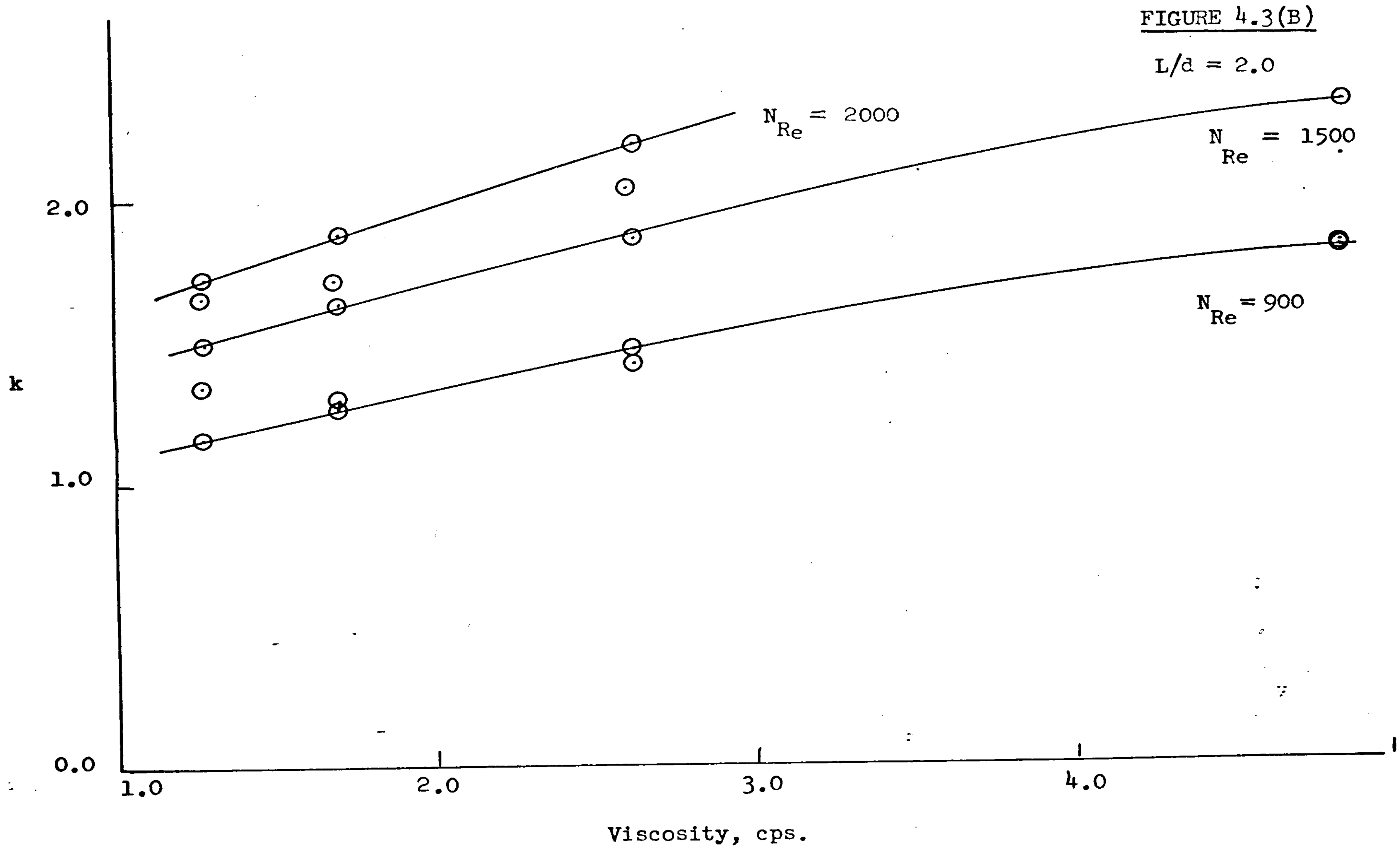


FIGURE 4.3(B)



L/d = 2.0
N_{Re} = 2000
N_{Re} = 1500
N_{Re} = 900
k
2.0
1.0
0.0
1.0
2.0
3.0
4.0
Viscosity, cps.

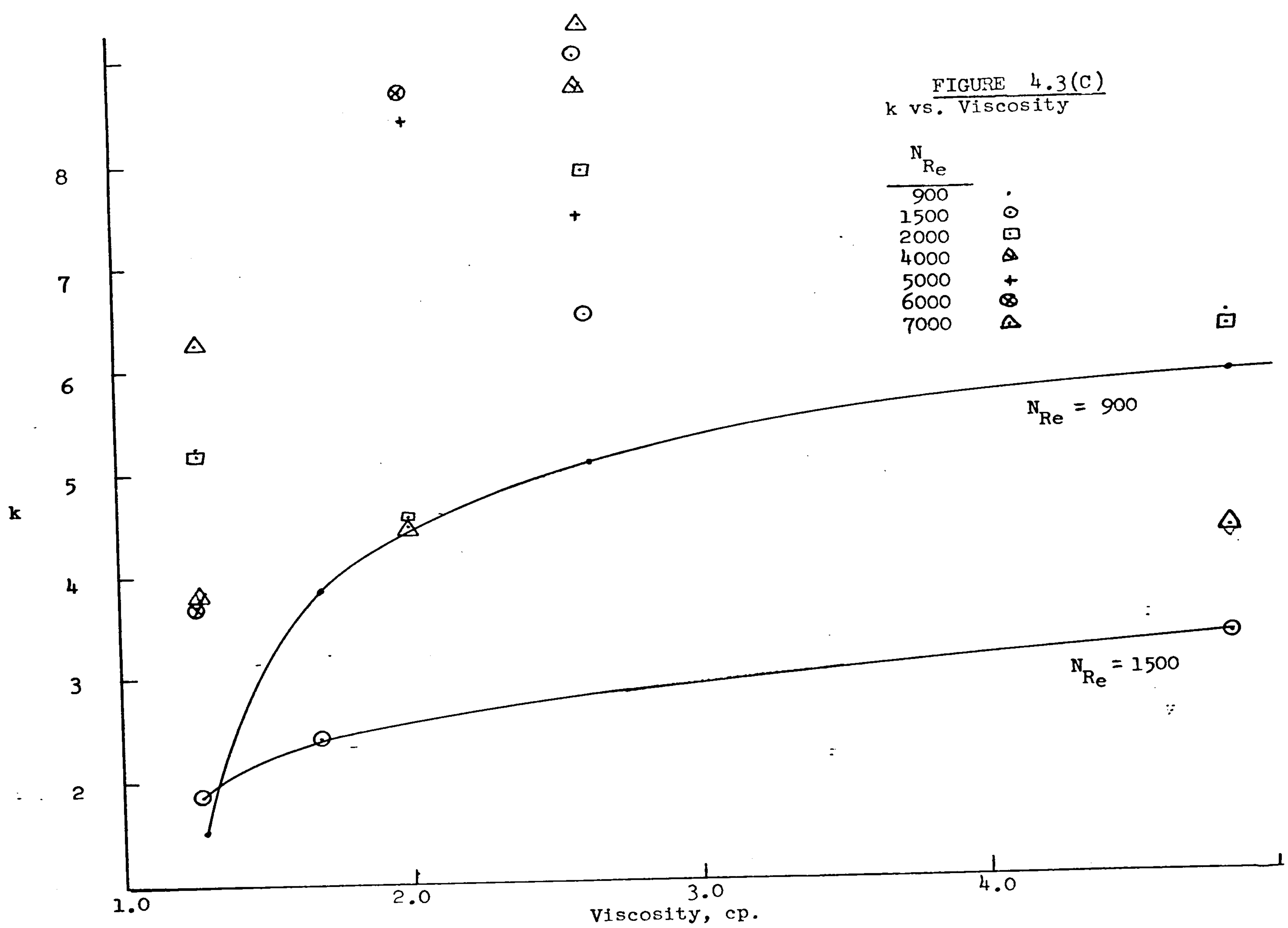


FIGURE 4.3(C)
k vs. Viscosity
N_{Re}
900
1500
2000
4000
5000
6000
7000
N_{Re} = 900
N_{Re} = 1500
k
8
7
6
5
4
3
2
1.0
2.0
3.0
4.0
Viscosity, cp.

a theoretical or empirical equation. This is especially true at higher L/d ratios.

Investigating the effect of the viscosity on the constant k showed that the constant is only slightly effected by the viscosity at low L/d ratios (L/d = 2.0) but it is clearly an exponentially increasing function of the viscosity at higher L/d ratios. Similarly the constant k is not effected by the Reynolds Number at low L/d ratios but the dependence of k on the Reynolds Number is great at higher L/d ratios. It does seem possible that the free jet theory would hold at L/d ratios lower than the ones at which the data were taken. The investigations have pointed in this direction but the free jet is not useful for experimental data taken for short, tubular vessels. Some attempts were made to include the Reynolds Number and the viscopsity into semi-empirical equations in the hope that a useful, if not theoretical, correlation couldbe obtained. One attempt was fairly successful in the low Reynolds Number region. This equation gave the k value as a function of the Reynolds Number and viscosity. (The Reynolds Number was again insufficient to characterize the conditions.) This equation was not very general and could not be applied to any other conditions. Equations which took into account all of the parameters were unmanageable and did not lead to further understanding of the physical situation. The conclusion of the author's work with the free jet theory and other empirical correlations are that the peak time, peak concentration, and "K" values cannot be correlated by an equation. The data obtained by measuring the concentration at the outlet of the vessel can be used to obtain the residence-time distribution but this distribution cannot be characterized or correlated by theory or by empirical manipulation of the variables concerned.

5. THE FLUID - FLOW MODEL

Previous investigators have proposed the use of models to describe the actual flow conditions within the vessel in order to correlate the behavior of the residence-time distributions obtained experimentally. These have included a "Dead" volume model which pictured the vessel as divided into separate sections. One of these sections was the "live" volume through which the fluid was flowing. The second section was a "dead" volume which, while not stagnant, was circulating very slowly when compared to the live volume. The idea of a dead volume and the fact that the decaying portion of the residence-time distribution curve was exponential led to the suggestion of a model for the flow pattern within the vessel. (1,2)

With a steady stream flowing through the vessel, the vessel has a certain live and a certain dead volume. The injection of a tracer pulse into the inlet stream caused the dye concentration in the live volume to increase immediately. Since turbulence and eddy effects are present in the vessel, some of the dye travels into the dead volume. After the main dye pulse has past by this section, the dye which has been circulating in the dead volume begins to diffuse back into the main stream. If this possible model is correct the initial section of the residence-time distribution curve would be steeper than a curve given by the universal diffusion theory. Likewise the decaying portion of the curve would be shallower. Previous experimental work

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concluded that the residence-time distributions obtained did differ from the universal diffusion theory has had been predicted. This was evidence that the idea of a dead volume was sound but it did not give any quantitative information on the relative amounts of live and dead volumes.

Another model was proposed by Sabnis(8) which was very similar. He proposed that some of the fluid within the vessel be considered as perfectly mixed while another portion of the fluid is considered to bypass the totally mixed section. This portion is channeled directly to the outlet section. Again this provides a model will help to understand the actual flow behavior but it does not provide a model which will explain the physical flow situation within the vessel.

5.1 General Description of Flow Models

Levenspiel (6,7) has proposed a method for obtaining a model to fit a particular flow system. If a fluid flows through a vessel in plug flow or completely mixed flow, the performance of the vessel can be determined by a straight forward method. If the flow deviates from these two patterns it is more difficult to predict the actual behavior. The best method, according to Levenspiel, is to represent the real vessel by a flow model and determine the parameters of the flow model. This flow model can then be used to predict the performance of other similar vessels. For cases which deviate greatly from plug flow situations, Levenspiel showed that the best method for predicting the vessel performance was to consider the vessel as composed of interconnected flow regions with various modes of flow possible between and around these regions. Models which are based on these interconnected regions are called "mixed models". They have been used in this report to investigate the characteristics of the residence-time distribution curves already available.

The mixed models were considered to contain the following regions. (Not all the regions need be present in a particular case.)

(1) Plug Flow Regions

(2) Backmix Flow Regions (regions where fluid is completely mixed and uniform in composition.)

(3) Dispersed Plug Flow Regions (regions where a diffusion like process is superimposed on plug flow.)

(4) Deadwater Regions (regions where fluid is completely stagnant.)

In addition to these type regions there are various possible methods of flow betweem regions. These include;

(1) Bypass Flow (a portion of the fluid bypasses the vessel or a particular flow region.)

(2) Recycle Flow (a portion of the fluid leaving the vessel or leaving a flow region is recirculated and returned to mix with fresh fluid.)

(3) Cross Flow (interchange of fluid occurs between two streams flowing in parallel through different flow regions.)

In the above definition for deadwater regions, the fluid was considered to be stagnant/nonmoving. This is not physically possible in the actual flow vessel so that a "practical" definition for stagnant is necessary. Usually material which has been in the vessel twice the mean residence-time is considered to be stagnant. The problem in using these flow models to determine the relative volumes of the various regions and various types of flow is to fit the experimental results to such a model. Levenspiel (7) has given methods for determining quantitatively the various volumes of the vessel under consideration. In general, the following methods are used to determine the volume fractions of each type of flow.

(1) Deadwater Regions: A reasonable cut-off point, such as $vt/V = 2$, is selected and the mean of the ditribution curve up to that point is obtained. If there were no deadwater regions the mean of the distribution would be unity. The difference between the unity and the mean of the residence-time distribution curve is the volume fraction of the dead-water region;

$$V_D/V = 1 - \theta_c \qquad 5.1(1)$$

(2) Pypass Flow: For bypass flow to exist the residence-time distribution curve must have a high initial concentration followed by a sharp drop to the normal distribution. This high initial concentration corresponds to the initial fluid which bypasses the mixing area.

(3) Plug Flow Regions: The amount of the volume which is engaged in plug flow can easily be obtained by the time required for the first trace of the injected tracer to emerge

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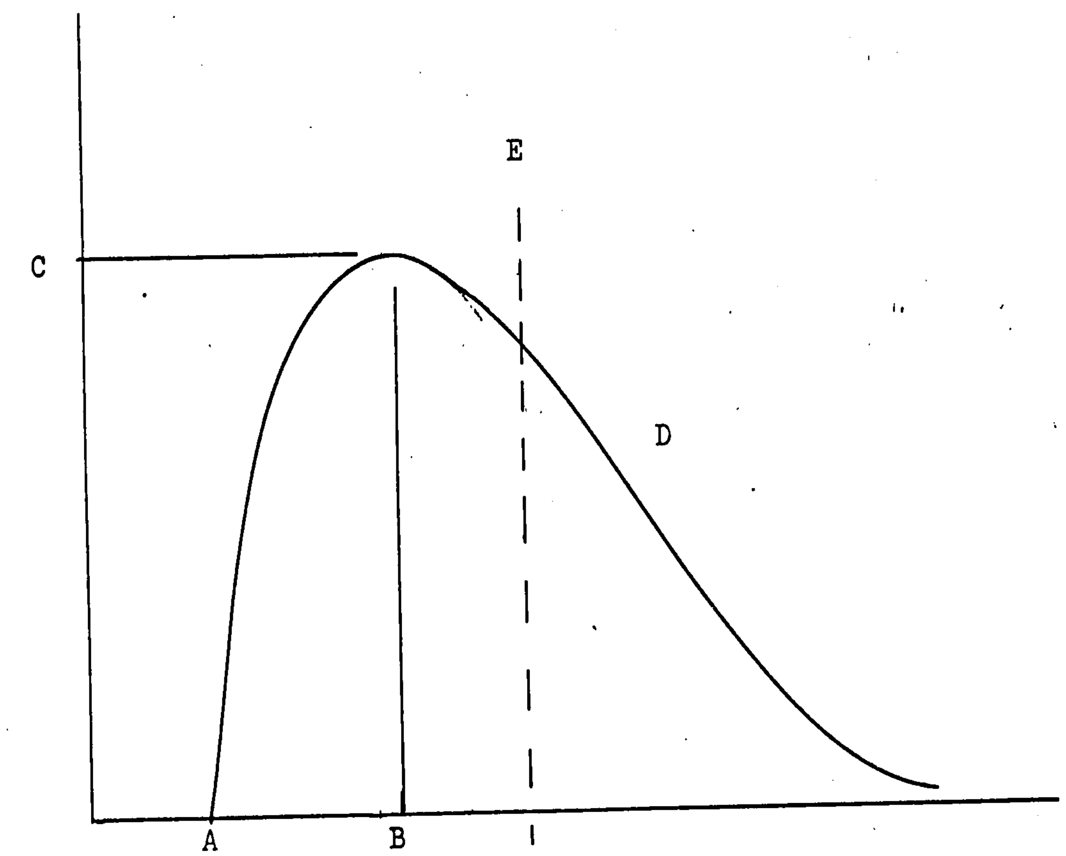
FIGURE 5.1(A)

TYPICAL RESIDENCE - TIME DISTRIBUTION CURVE

AND PARAMETERS



Dimensionless Concentration versus Dimensionless Time from the vessel.

(4) Backmix Flow: The fraction of the vessel volume that may be considered as totally mixed can be obtained from the slope of the exponentially decaying portion of the residence-time distribution curve.

Figure 5.1(A) shows a typical distribution curve and the parameters associated with the curve. From this curve we are able to get the peak time, peak concentration, "K" value, and start time. We can also obtain the mean of the residence-time distribution and the deviation from this mean.

TABLE 5.1A

Parameters of Residence-Time Distribution Shown in Figure 5.1(A)

A.	Start Time	0.248
B.	Peak Time	0.616
C.	Peak Concentration	0.963
D.	"K" Value	1.23
E.	Mean of Distribution	0.77

From considerations of the distribution curve the following quantities may be determined; (Assuming the flow model presented in Figure 5.1(A))

$$\frac{V_D}{V} = \frac{\text{Volume of Deadwater}}{\text{Total Volume}} = 1 - \theta_c = 0.23$$

$$\frac{V_B}{V} = \frac{\text{Volume of Backmix}}{\text{Total Volume}} = 1/K = 0.813$$

$$\frac{V_P}{V} \simeq \frac{\text{Volume of Plug Flow}}{\text{Total Volume}} = \text{Start Time} = 0.248$$

A sketch of the flow model we are considering can be seen in Figure 5.1 (B). It is noted that the summation of the volume fractions in the above table is greater than unity. This is from the vessel.

(4) Backmix Flow: The fraction of the vessel volume that may be considered as totally mixed can be obtained from the slope of the exponentially decaying portion of the residence-time distribution curve.

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TABLE 5.1

Parameters of Residence-Time Distribution Shown in Figure 5.1(A)

1.	Start Time	0.[illegible]48
2.	Peak Time	0.616
3.	Peak Concentration	0.959
4.	"K" Value	1.[illegible]3
5.	Mean of Distribution	0.77

From considerations of the distribution curve the following quantities may be determined: (Assuming the flow model presented in Figure 5.1(B))

$$\frac{V_D}{V} = \frac{\text{Volume of Deadwater}}{\text{Total Volume}} = 1 - 2c = 0.[illegible]3$$

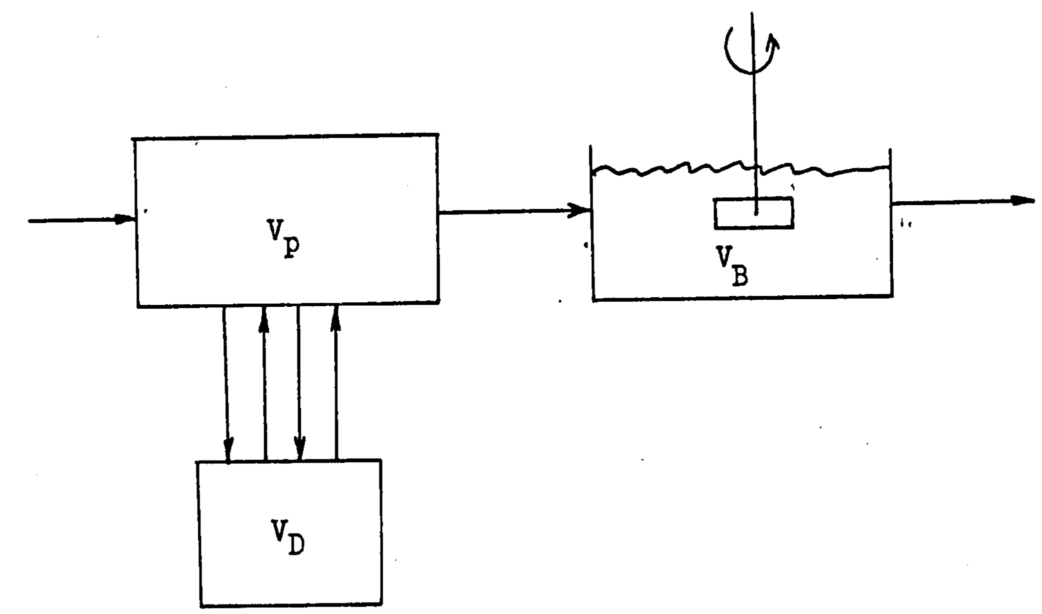
$$\frac{V_B}{V} = \frac{\text{Volume of Backmix}}{\text{Total Volume}} = 1/K = 0.[illegible]13$$

$$\frac{V_P}{V} = \frac{\text{Volume of Plug Flow}}{\text{Total Volume}} = \text{Start Time} = 0.248$$

A sketch of the flow model we are considering can be seen in Figure 5.1 (B). It is noted that the summation of the volume fractions in the above table is greater than unity. This is

FIGURE 5.1(B)

FLOW MODEL SCHEMATIC



probably due to the experimental errors in the residence-time distribution curves. In order to get these fractions on a sound basis they may be normalized to give a total of unity. This method was used on the experimental data and a quantitative picture of the flow within the vessel as obtained. Thus the dead volume and the plug flow volume are each about 20% of the total volume.

Normalized Parameters

V_D/V	=	0.18
V_B/V	=	0.63
V_p/V	=	0.19

5.2 Application of Model Concept to Residence-Time Distributions Obtained Experimentally

In order to apply the model concept developed in the previous section it is necessary to first assume a model to fit the experimental data. This is not very difficult as the general shape of the curve eliminates bypass flow. Thus we are left with a model which in general contains plug flow, backmix flow, and dead volumes. There must be transport between these volumes.

The procedure outlined in the past section was used to correlate the experimental data obtained by Adler and Sabnis. The mean of the distribution was taken and the ratio V_D/V, the dead volume region, was obtained by subtracting this mean from unity. The value of the backmix fraction was found as the reciprocal of the slope of the exponentially decaying portion of the residence-time distribution curve, "K". The start time became the ratio of the plug flow volume to the total volume.

This "raw" data was taken and normalized so that the volume fractions would sum to unity. The results are presented in the following tables. Table 5.2(A) is for an L/d ratio of 5.0 while Table 5.2(B) tabulates the values for an L/d ratio of 2.0.

5.3 Flow Model Results

An examination of these volume fractions points to a few interesting characteristics of the flow model and the data considered.

(1) At higher L/d ratios (5) the effects of viscosity and Reynold's Number are not apparent in the calculated volume fractions. This may be because they are hidden by the wide scatter of experimental data or it may be that they do not really exist. At isolated parameters (i.e. a Reynold's Number of 5000) a definite trend with respect to viscossity is noted but changing the parameter does not show any substantiating trend.

(2) At lower L/d ratios (2) the scatter is still apparent and only at a Reynold's Number of 2000 is a trend apparent.

(3) The wide scatter of data makes it difficult to statistically analyze the data. A perusal of the data in Table 5.2(A) and Table 5.2(B) shows that data within a viscosity range (regardless of Reynold's Number) and

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TABLE 5.2 (A)

VOLUME FRACTIONS FOR FLOW MODEL

L/d = 5 d/δ = 10

% Sugar	N_{Re}	$\frac{V_D}{V}$	$\frac{V_P}{V}$	$\frac{V_B}{V}$	Sum	Normalized Parameters $\frac{V_D}{V}$	$\frac{V_P}{V}$	$\frac{V_B}{V}$
40	900	0.28	0.17	0.8	1.25	0.23	0.13	0.64
30*		0.69	0.09	0.14	0.92	0.75	0.09	0.16
20*		0.70	0.11	0.06	0.86	0.81	0.13	0.06
10		0.67	0.07	0.35	1.09	0.61	0.07	0.32
Water		0.27	0.06	0.93	1.26	0.21	0.05	0.74
40	2000	0.16	0.17	1.27	1.59	0.09	0.12	0.79
30		0.35	0.17	0.40	0.92	0.38	0.19	0.43
20		0.20	0.15	0.91	1.26	0.16	0.11	0.73
10		0.24	0.14	1.13	1.50	0.16	0.09	0.75
Water		0.26	0.07	0.83	1.15	0.22	0.06	0.72
40	4000	0.28	0.16	0.97	1.40	0.20	0.11	0.69
30		0.20	0.12	1.37	1.69	0.12	0.07	0.81
20		0.26	0.13	0.79	1.18	0.22	0.10	0.67
10		x	x	x	x	x	x	x
Water		0.25	0.06	0.80	1.14	0.23	0.06	0.73
40	5000	0.18	0.14	1.32	1.63	0.11	0.09	0.81
30		0.18	0.19	1.00	1.37	0.13	0.14	0.73
20		0.24	0.15	0.78	1.19	0.20	0.13	0.66
10		0.35	0.10	0.38	0.83	0.42	0.12	0.46
Water		x	x	x	x	x	x	x
40	6000	x	x	x	x	x	x	x
30		0.25	0.14	1.20	1.59	0.16	0.09	0.76
20		0.23	0.11	1.20	1.54	0.15	0.07	0.78
10		0.23	0.21	0.24	0.68	0.34	0.31	0.35
Water		x	x	x	x	x	x	x

Notation:

* These runs are not typical of experimental residence-time distributions and may be subject to experimental error.

x No data was taken with these particular parameters.

TABLE 5.2(B)

VOLUME FRACTIONS FOR FLOW MODEL

% Sugar	N_{Re}	$\frac{V_D}{V}$	$\frac{V_P}{V}$	$\frac{V_B}{V}$	Sum	Normalized Parameters $\frac{V_D}{V}$	$\frac{V_P}{V}$	$\frac{V_B}{V}$
40	900	0.47	0.03	0.85	1.36	0.34	0.03	0.63
30		0.63	0.04	0.94	1.61	0.39	0.03	0.59
20		0.69	0.04	0.78	1.51	0.46	0.02	0.52
10		0.79	0.03	0.39	1.20	0.67	0.01	0.32
Water		0.37	0.03	1.53	1.93	0.20	0.01	0.79
40	2000	x	x	x	x	x	x	x
30		0.0	0.05	1.87	1.92	0.0	0.03	0.97
20		0.09	0.04	2.43	2.56	0.04	0.02	0.94
10		0.03	0.04	2.05	2.39	0.13	0.02	0.85
Water		0.48	0.04	1.17	1.68	0.29	0.03	0.68
40	4000	0.10	0.07	0.83	0.99	0.10	0.07	0.83
30		x	x	x	x	x	x	x
20		0.09	0.04	2.25	2.38	0.04	0.02	0.94
10		x	x	x	x	x	x	x
Water		0.48	0.35	1.22	2.05	0.23	0.18	0.59

Notation:

x No data taken with these particular parameters.

$$\frac{L}{d} = 2 \qquad \frac{d}{\delta} = 10$$

within a Reynolds Number range (regardless of viscosity) the data fall within 0.1. This is about the accuracy of the techniques used to obtain the distributions and the volume fractions from these distributions. Thus we can average the volume fractions obtained from the experimental data to obtain the volume fractions as a function of vessel geometry.

If the averaging technique is used as visualized in point (3) above, the results shown in Table 5.3 are obtained.

TABLE 5.3

AVERAGE VOLUME FRACTIONS

$\frac{L}{d}$ = 2 and 5

	$\frac{V_D}{V}$	$\frac{V_B}{V}$	$\frac{V_P}{V}$
L/d = 2	0.21	0.69	0.10
L/d = 5	0.24	0.72	0.03

The above table shows that the length to diameter ratio has an effect on the backmix and plug flow volumes but has little or no effect on the dead volume. As the length to diameter ratio is increased, the amount of plug flow decreases and mixing becomes more general. This seems to be in line with our intuitive thinking about the actual flow behavior. It is surprising that the flow model does not tell us more about the effects of fluid parameters on the residence-time distribution parameters.

6. PRESENT STATE OF KNOWLEDGE OF RESIDENCE-TIME DISTRIBUTIONS

Even though a great deal of data has been taken in the form of residence-time distributions, we still do not know too much about the actual flow patterns within the vessel. Knowledge is also lacking in how these flow patterns effect the various parameters associated with the residence-time distributions. Each of these parameters from the residence-time distribution have a direct counterpart in the fluid flow model concept.

Start Time	V_P/V	-	Plug Flow Volume Fraction
"K" Value	V/V_B	=	Backmix flow Volume Fraction

The peak time and peak concentration are related to the deviation of the mean from unity although this relationship is not as straightforward as the others.

Direct Correlation between parameters associated with the residence-time distributdon and the flowing fluid properties have not been obtained for the general case. It is possible to obtain such a correlation for each restricted case but this is not meaningful for a different vessel or a fluid with different properties.

It seems theoretical and empirical correlations of residence-time distribution parameters cannot be obtained with any validity. The best hope for continuing work lies in the use of the model concept which shows the volume fractions within the vessel to be functions of vessel geometry rather than fluid properties as the first correlations had assumed. Using models of the flow system as described previously, we can obtain some knowledge of the effect of geometry on the residence-time distribution for short, unagitated vessels. The effects of Reynold's Number and viscosity still are not understood. Tabulated data which gives exit concentration as a function of time does not give enough information to derirve a correlation. The data which is presently being taken at various points within the vessel may help to show the actual flow patterns and the actual effects of vsicosity and fluid flow parameters.

There are a few general effects which have been found to contribute to the overall behavior of the effluent concentration. We know that the fluid initially travels through the vessel in a main "live volume" stream. There is a certain stagnant (dead volume) area. At the interfacial area between the live and dead volumes there is a shear set up. The dead volume is circulating slowly and there is a shear force at the vessel wall along with the interfacial shear. When the tracer is injected into the main stream, it flows through the live volume. At the interface with the dead volume a concentration gradient is set up. This concentration gradient naturally causes diffusion from the main stream into the dead volume. The concentration of tracer increases in the dead volume and the tracer enters the dead volume circulation system. After the tracer pulse has passed the dead volume interface, a concentration gradient is set up in the opposite direction. The tracor will flow back from the dead volume into the main stream which is now free of tracer. In order to do this, however, the circulation path within the dead-water region must bring the tracer back to the interface. Since the tracer is spread along the circulation path, it will arrive at the interface at different times. Thus the concentration gradient at the interface will change and the diffusion process will be slowed down. Since the circulation pattern spreads the tracer out within the dead volume, it will take a longer time for the tracer to be transported out of the dead volume than it took to transport the tracer into this dead volume. This explains the fast rise in the distribution curve and the slower decay.

The above picture is qualitative. The model concept does give us a more quantitative picture of the volumes and interfaces involved. No further quantitative information seems obtainable by measuring the concentration of tracer in the effluent stream.

It is not possible to describe or correlate the residence-time distribution parameters for short, unagitated tubular vessels solely by the use of effluent concentration measurements. Within the vessel it is possible to obtain concentration measurements for viscous solutions similar to those taken for water systems. In this way changes in flow patterns, interfacial areas, and volumes can be seen as a function of viscosity if such a function does indeed exist. Differences between Reynold's Numbers and viscosities do not explain differences in distribution parameters. Perhaps these could be accounted for by using the model concept and measuring the changes in interfacial area and volumes. Measurements on concentration of tracer within the vessel at various points would show such changes.

7. APPENDIX

7.1 NOMENCLATURE

c - concentration

C - concentration of tracer in outflowing stream

C_o - inlet concentration or concentration of tracer in inflowing stream

∂ - partial differential operator

d - diameter of tubular vessel

D - dispersion coefficient

k - constant

K - decay constant

L - length of vessel

N_{Re} - Reynolds number

P - pressure

Q - quantity of tracer injected instanteously into inlet stream

r - radial distance

R - radius of vessel

t - time

u - velocity in Z (longitudinal) direction

v - volumeteric rate of flow

V - volume of vessel

V_B - volume of backmix flow

V_D - volume of deadwater

V_P - volume of plug flow

x - distance in axial direction of vessel

δ - inlet or outlet port diameter

μ - viscosity

ρ - specific gravity

τ - shear stress

7.2 LITERATURE CITATIONS

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7.4 LOCATION OF ORIGINAL DATA

All data used in this research work will be found in the following reports:

1. Adler, R.J. Residence-Time Distributions of Short Tubular Vessels, Ph. D. Thesis, Lehigh University (1959)
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