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PRESSURE DROP IN HELICALLY COILED TUBES

by

Richard N. Meier

A THESIS

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L A Wenzel
Professor in Charge

Jan. 21, 1961
Date

Head of the Department

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NOMENCLATURE

A	Cross sectional area.
D_c	Diameter of coil
D_t	Inside diameter of tube (D_t/D_c curvature ratio)
f_c	Fanning friction factor for coiled tube
f_s	Fanning friction factor for straight tube (f_s/f_c friction of ratio)
G	Mass velocity
g_c	32. 12 gravitational constant
L	Length
N_{Re}	Reynolds number
ΔP	Pressure drop
W	Mass flow rate
ρ	Density
μ	Viscosity

ABSTRACT

Pressure drop in three helically coiled tubes with a tube to coil diameter ratio of 0.048", 0.100", and 0.125" respectively was investigated at Reynolds Numbers from 70 to 70,000 using nitrogen and helium. Thin walled 3/32" copper tubing was finned before coiling to avoid the usual flattening effects.

The friction factor for straight tube was found to be lower than that for the corresponding coiled tube, the ratio of f_s/f_c approaching one at low Reynolds Numbers (Figure 4). The transition to turbulent flow occurred at Reynolds Numbers of 6000 to 10,000, after which the ratio f_s/f_c may be considered independent of Reynolds Number for design purposes.

For estimation of pressure drop in coils,

$$f_s/f_c = \ln\left[\ln\left(\frac{12.0}{N_{Re}^{0.2}}\right)\right] - 0.13 \ln (D_t/D_c)$$

$$N_{Re} < 2100$$

where $f_s = 16/N_{Re}$. In the turbulent region above Reynold's Numbers of 10,000

$$f_s/f_c = 1.08 - 4.17 \, (D_t/D_c)$$

Incorrect

$N_{Re} > 10,000$

$D_t/D_c > .03$

$$f_s = 0.0014 + \overset{0.125}{\cancel{0.090}} \left(\frac{\mu}{DG}\right)^{\overset{0.32}{\cancel{0.27}}}$$

For estimating pressure drop in coils, Figure 6 and Figure 7 provide design charts showing the effect of curvature on the friction factor ratio.

INTRODUCTION

Coils of tube are chiefly used in connection with heating and refrigeration in order to transfer heat from one fluid to another. Since both the pressure required to obtain the necessary circulation and the heat transfer at that circulation rate are dependent upon the resistance to flow, it is important to have design charts or equations for estimating pressure drop in coils with various degrees of curvature.

While the relationships for pressure drop in straight tube have been thoroughly investigated and proven, the flow of a fluid in curved pipe follows a more complex path, even in the laminar region, and obtaining theoretical or empirical relationships is difficult. Secondary flow is imposed on the normal flow profile and the flow pattern becomes a three dimensional consideration. Furthermore, the degree of curvature is an additional variable to be considered in the final analysis.

These complexities have prevented integration of the equations of motion and have limited both the quantity and scope of the theoretical and experimental investigations.

Past research has been mainly concentrated in the turbulent region; the derived relationships, however, are inconsistent with one another and usually correlated only the data of the particular investigation. Little work has been done in the laminar region, and no investigator has considered both regions simultaneously.

The purpose of the research reported here was to investigate both regions of flow in coils with different degrees of curvature, and to develop a design correlation for each region of flow, consistent with both the data obtained and that of other experimentors.

THEORETICAL BACKGROUND

The theoretical background is quite limited because of the difficulty in solving the equations of motion for expressions describing the flow behavior.

For a small degree of curvature, Dean (4) made certain assumptions as to the relative magnitude of the fluid velocities and, for a given pressure gradient in the laminar region of flow, obtained an expression for the ratio of velocities in a curved pipe to those in a straight pipe of equal cross section. He further developed an expression for the velocity at any point relative to the center-line velocity. Both of these solutions were experimentally verified by White (21), however, the limitation that only a small degree of curvature be considered makes the expressions of little value.

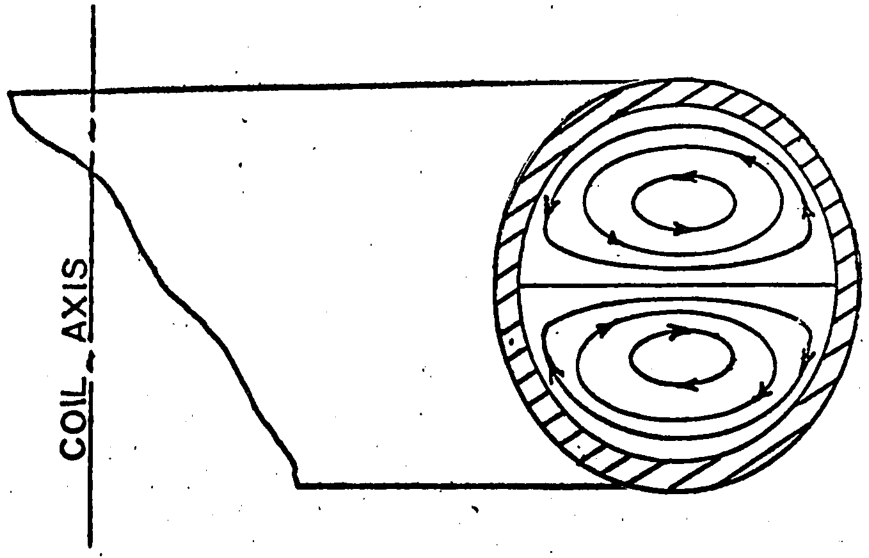
In the turbulent region above Reynolds Numbers of 150,000, Weske (20) derived a relation for the net pressure drop in curved ducts as a function of Reynolds Number and diameter ratio by integrating the equations of motion of the shedding layer, somewhat analogous to the 'buffer' layer in straight tube. For more details, see reference. He reports that experimental results support the assumptions he made in the development, however, quantitative information is not available. A few others (1, 16) have developed theoretical equations, but experimental results fail to verify their validity.

EXPERIMENTAL BACKGROUND

The first significant results of experimental work on fluid flow inside coiled tube was reported by Eustice (8), who experimented with the injection of dye into water flowing in a glass U-tube. It was observed that fluid particles in the center of the pipe traveled to the outside wall as a result of the centrifugal force of the water rounding the bend. Once at the wall, the fluid split into two streams, each returning along one of the sides of the tube, thus introducing a circular motion of two longitudinal vortexes. The same was latter observed by Taylor (19), i.e. the fluid formed two helixes, symmetrically disposed with regard to the diameter through the coil axis and the tube (see Figure 1). Hawes (13) characterized the flow in coils by three stable regions.

Figure 1

Fluid Flow in Coiled Tube



Using a coil with a D_t/D_c ratio of 0.100, he found: (1) laminar flow below $N_{Re} = 30$; (2) Exceedingly stable double-helical type flow for $30 < N_{Re} < 8000$; (3) Turbulence above $N_{Re} = 8000$.

The work of White (21) further verified the indications of increased stability of flow through coiled tube. Together with the results of Taylor (19) and Adler (1), White's values for critical Reynold's Number are shown on Figure 2.

White made a study of the laminar flow of water through coils with curvature ratios D_t/D_c equal to 0.066, 0.020, and .0005. White correlated his data plus that of Grindley and Gibson (11) by plotting f_c/f_s vs. $N_{Re}\,(D_t/D_c)^{1/2}$ on log-log coordinates. The data fell on one curve and the relationship was given

$$f_s/f_c = 1 - \left[1 - \left(\frac{11.6}{K}\right)^{0.45}\right]^{2.22}$$

where $K = N_{Re}(D_t/D_c)^{1/2}$

to relate the friction factor ratios to the Reynolds Number and curvature ratio. White sets the limitation that $11.6 < K < 2000$ and suggests for $K < 11.6$, the friction factor for straight tubes be used.

White (22) later did work in the turbulent region also, and proposed the relation

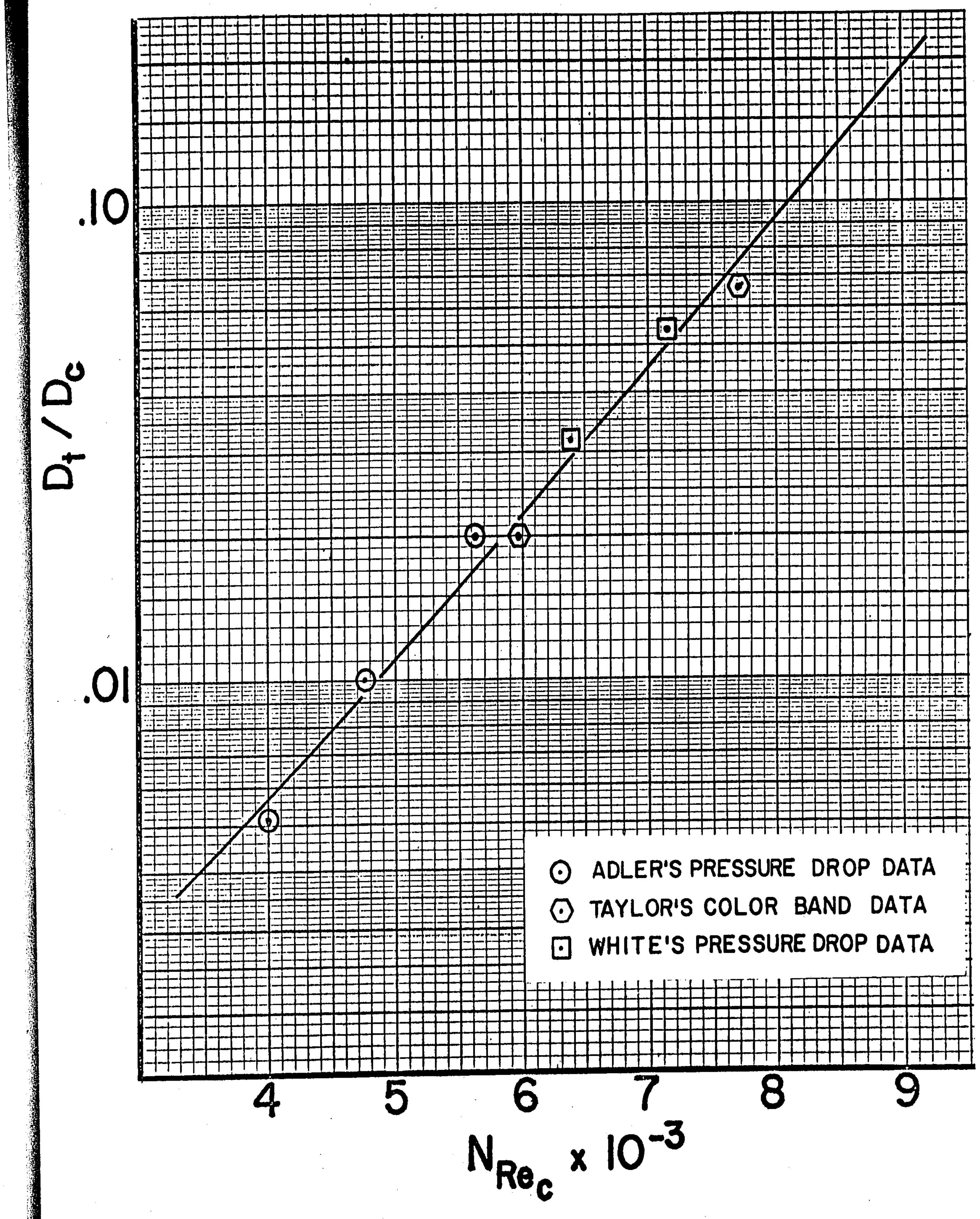


FIGURE 2 EFFECT OF CURVATURE RATIO ON CRITICAL REYNOLDS NUMBER

$$f_c = 0.08\ N_{Re}^{-1/4} + 0.012\ (D/D_c)^{1/2}.$$

To express f_c Lorentz (16) proposed a similar equation

$$f_c = 0.08\,N_{Re}^{-1/4} + 0.32\ (D/D_c)^{2}$$

Lapin (15) conducted pressure drop measurements at higher Reynolds Numbers for air flowing inside coils made of 0.495" I. D. tube and having curvature ratios of 0.03 and 0.135. His data was not correlated at the time of the experimentation and is presented now with that of the author.

Christensen obtained data for the flow of three fluids (air, toluene, and water)(3) through three helical coils having curvature ratios of 0.049, 0.092, and 0.136 at Reynolds Numbers from 3 to 150,000. For his .049 and .092 coils, he proposed the correlation

$$f_c = 0.08\ Re^{-1/4} + 0.34\ (D_t/D_c)^{1.65}\ \left(\frac{DG}{\mu}\right)^{-0.10}$$

He proposed another correlation for coils having curvature ratios greater than 0.092, since his smallest coil did not agree with his other two, however, the author believes the data for this particular tube to be in error since no other data, including that of the author, indicate that the friction factor is not a function of the curvature ratio at larger values of D_t/D_c.

Finally, it should be mentioned that several comprehensive studies [Richter (18), Adler (1), Itō (14)] have been made using a curved pipe of one turn or less.

Table 1 is a summary of the experimental investigation of pressure drop in coiled tubing to include the work of the author.

TABLE 1

SUMMARY OF EXPERIMENTAL WORK FOR PRESSURE DROP IN COILED TUBES

Diameter of Pipe or Tube (inches)	Average Diameter of Coil (inches)	Curvature Ratio (D_t/D_c)	Range of Flow Reynolds Number	Fluids Used	Experimentor	Comparison With Results of this Investigation
0.125*	14.40*	0.009	25 to 1,400	Air	Grindley and Gibson	In agreement.
0.250*	12.50*	0.020	16 to 13,000	Water	White	In agreement with author at low Reynolds Numbers.
0.495	16.75	0.030	3,000 to 50,000	Air	Lapin	In agreement.
0.081	1.69	0.048	60 to 66,000	Nitrogen Helium	Meier	In agreement in laminar region.
.....	.....	0.049	3,000 to 100,000	Air Toluene Water	Christensen	In agreement
0.406*	6.15*	0.066	0.06 to 41,000	Oil Water	White	Not in agreement with White's later work in the turbulent region - not considered.
.....	.....	0.092	6,000 to 100,000	Air Toluene Water	Christensen	In agreement.

*converted from cm.
(continued)

TABLE 1 (cont'd)

SUMMARY OF EXPERIMENTAL WORK FOR PRESSURE DROP IN COILED TUBES

Diameter of Pipe or Tube (inches)	Average Diameter of Coil (inches)	Curvature Ratio (D_t/D_c)	Range of Flow Reynolds Numbers	Fluids Used	Experimentor	Comparison with Results of this Investigation
0.081	0.806	0.100	60 to 76,000	Nitrogen Helium	Meier	In agreement.
0.081	0.648	0.125	60 to 72,000	Nitrogen Helium	Meier	In agreement.
0.495	3.68	0.135	3,000 to 50,000	Air	Lapin	In agreement.
.....	.....	0.136	4,000 to 100,000	Air Toluene Water	Christensen	Data for this coil not in agreement with Christensen's other data - not considered.
.....	.....	.....	Turbulent region	.....	White	Original data unobtainable at time of this work.

TEST PROGRAM

The test program was planned using one diameter of tubing for three coils of varying curvature ratio. The ratios were chosen so as to best supplement or verify existing data.

Only one tube diameter was used since the primary aim of the investigation was to study the effect of curvature ratio on pressure drop by varying the diameter of the coil. It was therefore desirable to eliminate any other sources of possible variation in the data.

The particular tube diameter used was chosen for several reasons. It is much smaller than that of other investigators and therefore strengthens the validity of any correlation obtained by incorporating their data in its formulation. It was geometrically convenient, since a large variation in curvature ratio could be obtained without necessitating the use of large and unwieldy test sections. Finally, it was particularly suitable for finning, a process by which flattening of the tube during coiling was kept to a minimum.

Both the laminar and turbulent region of flow were investigated since the data for each coil could be compared with that of investigators of both regions.

Compressed helium and nitrogen were chosen as test fluids since they were readily available, convenient to use, and suitable for the Reynolds numbers to be investigated.

TEST APPARATUS

The apparatus can be seen in Figs. 3a,b,c. The flow diagram is shown in Fig. 3d. The gas from a compressed gas cylinder was throttled through a Hoke Regulator and entered a 51" straight calming section where the inlet temperature was measured. Located at the end of the calming section and the beginning of the test section was the inlet pressure tap. This tap was connected to an 18" Heiss 500 psi Pressure Gauge, subdivided in increments of 0.5 psi, and then to one side of a pressure drop manifold. To the other side of the manifold was connected the outlet pressure tap, located immediately after the test section. The manifold consisted of a 46" glass water manometer, a 36" mercury manometer, and a 0 to 100 psi Barton differential pressure gauge, subdivided in 1 pound increments. The outlet temperature of the gas was then measured, after which it passed through a Brooks rotameter and was vented.

When making a test section, the copper tubing was annealed and then finned. The finning was not soldered to the tube, but was held there by mechanical bond. This flexible circular support prevented flattening during coiling of the tube. To check this, a cross-section of finned tube from coil 0.125 examined under a microscope showed no signs of crimping from the finning and appeared to be in round.

After coiling a test section, a brass sleeve with a 3/32" hole was soldered to each of the ends. Through this a .030" hole was drilled into the tube, particular care being taken that the tube remained in round and that rough edges were smoothed. A modified Swagelok tee was then carefully soft soldered to the sleeve taking care that no solder block the hole. The calming tube was then butted up against the test section tube inside the sleeve, and the end of the sleeve soft soldered to the calming tube. Finally, connections were made with regular Swagelok fittings on 1/4" tubing.

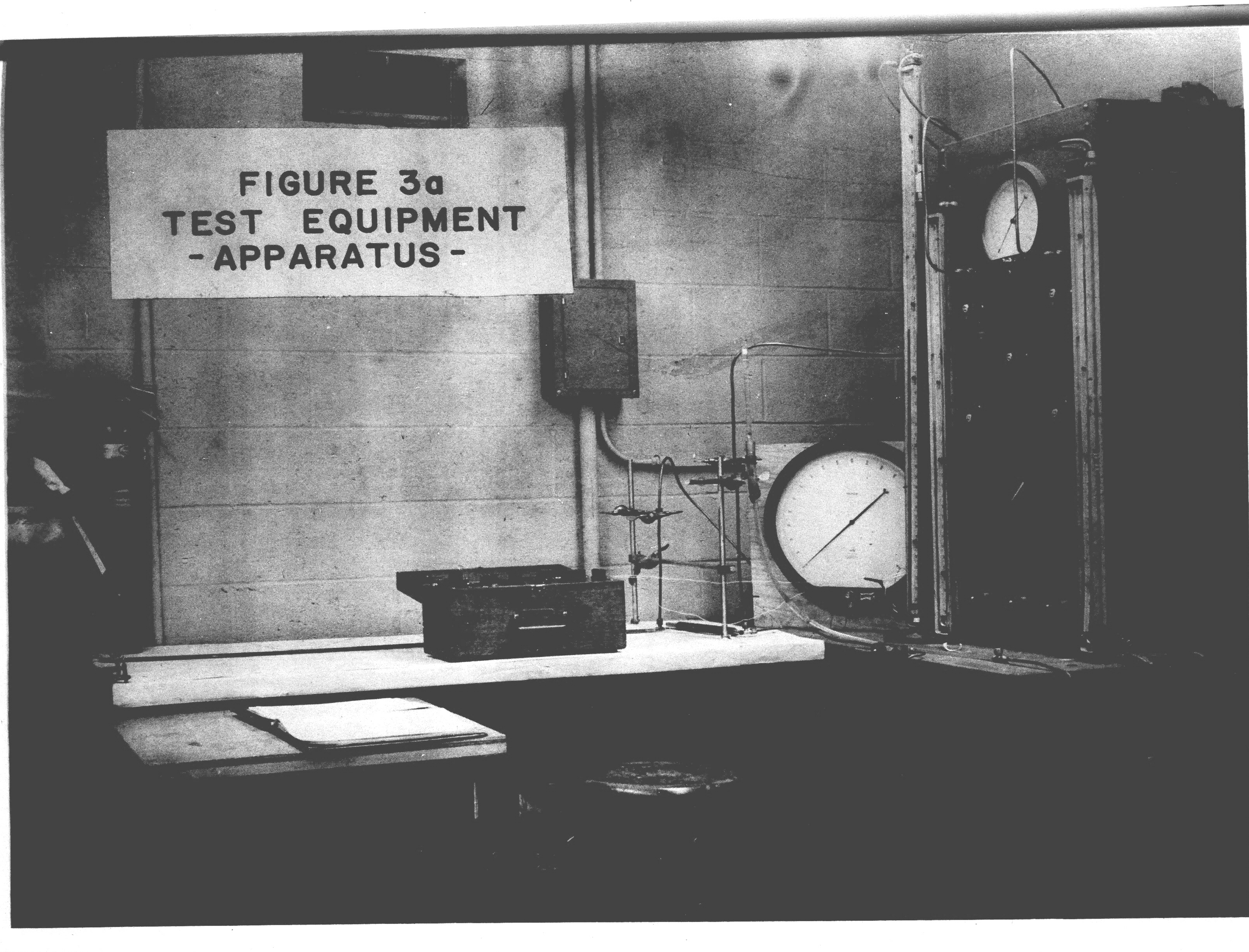


FIGURE 3a
TEST EQUIPMENT
-APPARATUS-

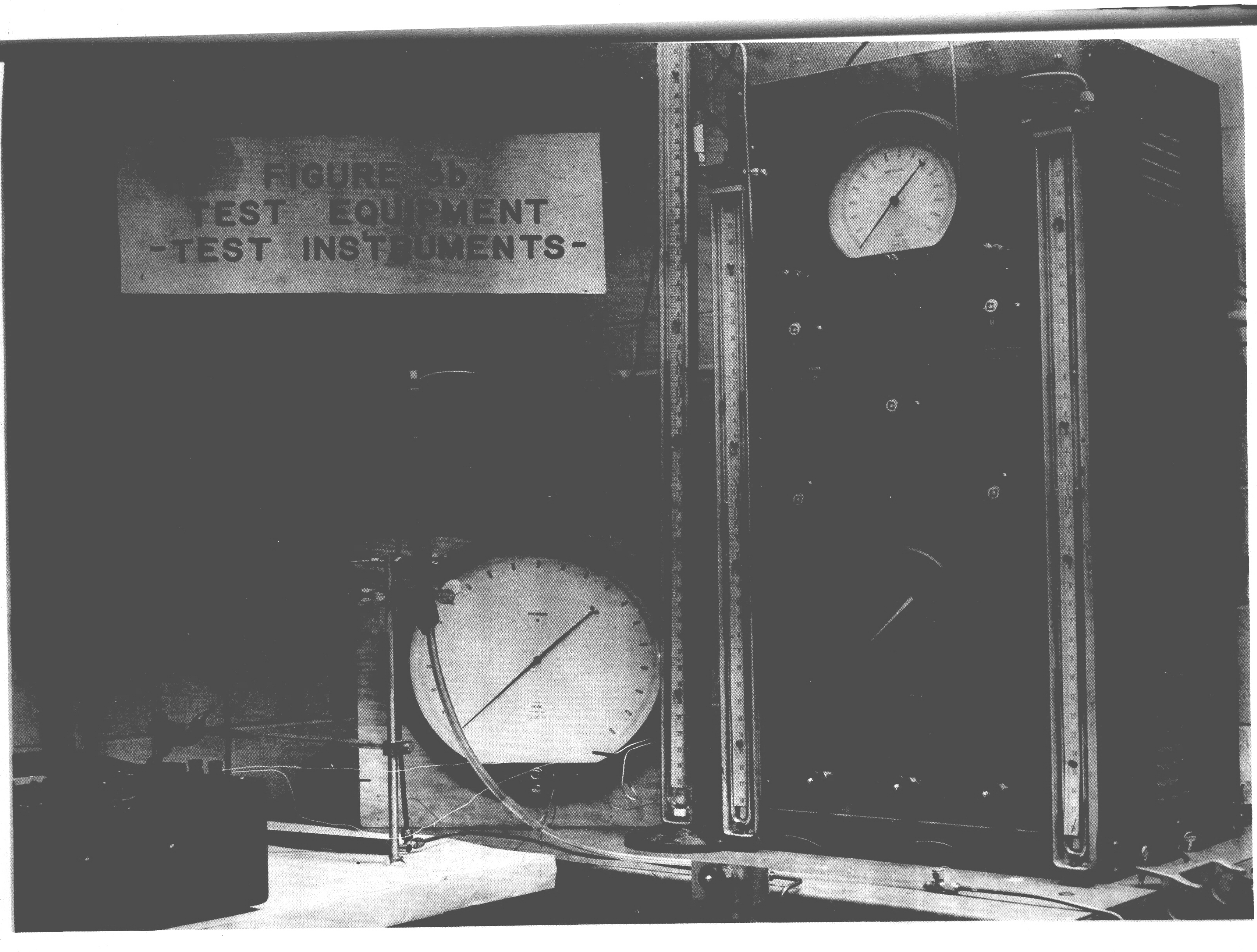


FIGURE 3b
TEST EQUIPMENT
-TEST INSTRUMENTS-

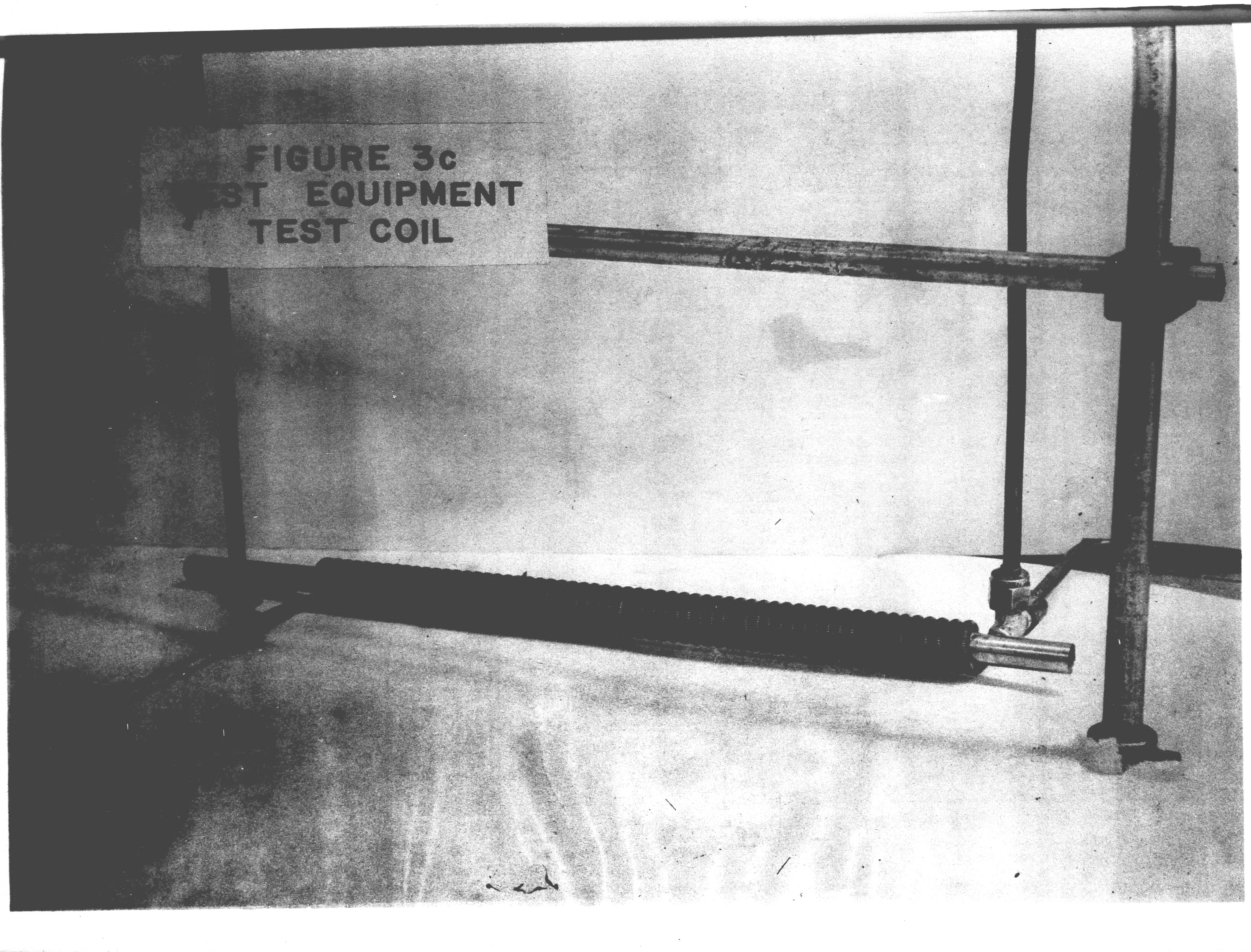


FIGURE 3c
TEST EQUIPMENT
TEST COIL

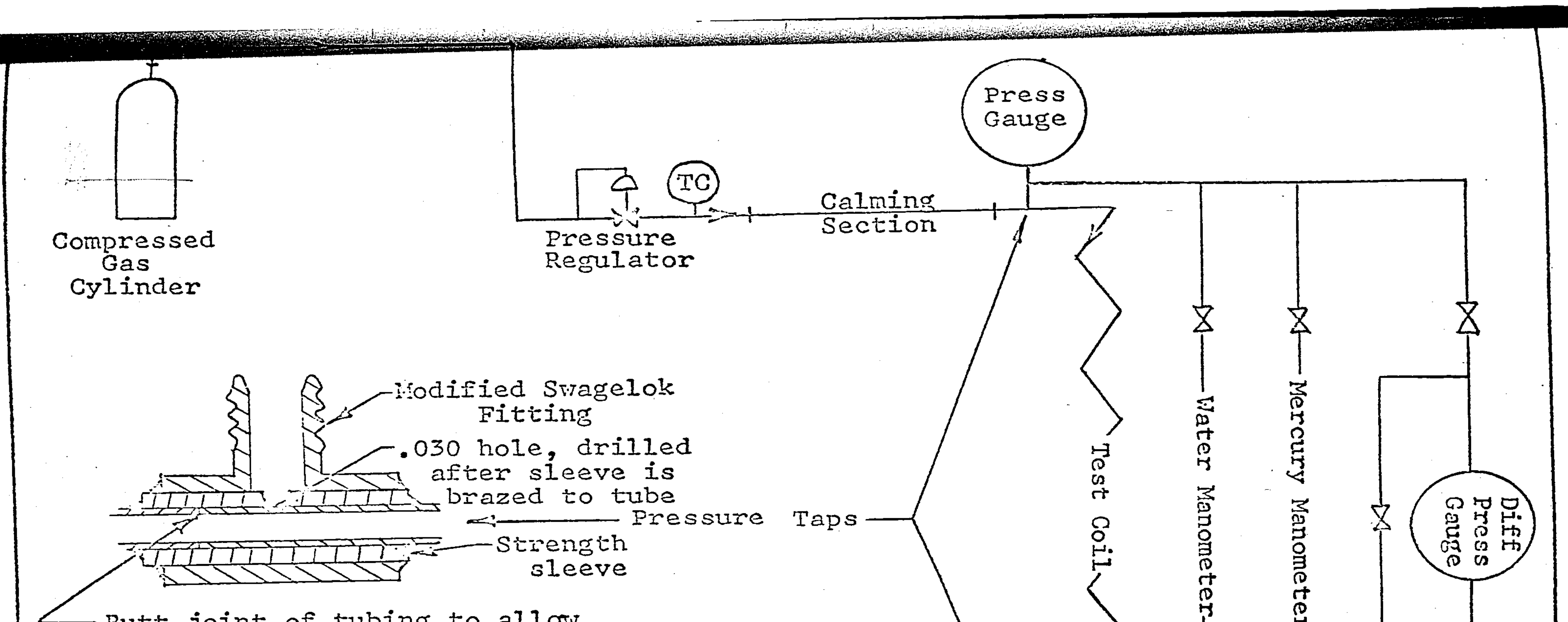


FIGURE 3d
TEST EQUIPMENT
FLOW DIAGRAM

TEST PROCEDURE

The gas valve was opened and the pressure regulator adjusted to the desired inlet pressure. With the throttling valve closed, the manometer was checked for zero. The gas was then throttled through the test section at different flow rates. After steady state conditions were attained, the inlet and outlet temperatures, the inlet pressure, the pressure drop, and the flow rate were recorded.

CALCULATION OF RESULTS

The exact diameter of the tubing was determined by testing a typical length of straight tubing in the laminar region of flow and using the data obtained together with the formula

$$f_s = 16/N_{Re}$$

to calculate the true diameter. This was necessary since the allowable tolerances of the die used in drawing a lot of tubing is significant with regard to the total diameter. The exact diameter determined in this manner was found to be 0.081" as compared to the nominal diameter of 0.078". This data is shown in Table 2 & Fig. 11.

The arithmetic average of the temperature and pressure between the inlet and outlet to the test section was used in determining the average density and viscosity of the fluid.

The flowrates were determined from a rotameter conversion chart which had been made for each gas from the data obtained by using a wet test meter.

A sample set of calculations is given in the appendix.

DISCUSSION OF RESULTS

The tabular results are summarized in Tables 2 thru 7 in the appendix. These tabular results are shown graphically in Fig. 4 & 5.

As expected, f_c at a given Reynolds number is considerably above that for a corresponding straight tube, the difference being greater at the larger curvature ratios. With Reynolds number as a parameter, a plot (Fig. 8) of the ratio of these friction factors, f_s/f_c, vs the curvature ratio D_t/D_c represents straight lines with a slope of -0.3. Replotting the original data points as $(D_t/D_c)^{0.3/2.303}(e^{f_s/f_c})$ vs N_{Re} on semi-logarithmic paper (Fig. 9), the data fall on a straight line with one exception. The data for coil 0.125 between a Reynolds number of 70 and 350 are not correlated. Upon rechecking the original data, it was noticed that the flowrates for these particular ten points were taken on a rotameter which, on the following run , was found to be cracked at the base. Since it was not possible to correct the data, it was shown separately in Fig. 10.

A cross-plot of the points from the curve in Fig. 9 resulted in the design chart for pressure drop in the laminar region of flow (Fig. 6).

As indicated in the f_c vs N_{Re} plot (Fig. 4), this graph shows that the coiling of straight tube does not alter the friction factor at low Reynolds numbers and small curvature ratios, the upper allowable limit of Reynolds number decreasing with increasing curvature ratio.

Once the influence of curvature takes effect, the difference in f_s and f_c increases steadily with increasing Reynolds number. The effect of increasing curvature ratio, however, is greatest at initial bending and becomes less as D_t/D_c values become larger. An abrupt change in trend takes place at Reynolds numbers close to 10,000. This is in agreement with other experimentors (1) (19) (21), who suggest that in this region the onset of turbulence takes place. For coils 0.048, 0.100, and 0.125, Fig. 4 indicates critical Reynolds numbers of approximately 7000, 8000, and 8200, respectively. This may be compared with Fig. 2. The data, then, are in agreement with the findings of Hawes (13): at low Reynolds numbers f_s/f_c approaches one and laminar flow is present; at Reynolds numbers in the range of 10 to 10,000, f_s/f_c decreases and an exceedingly stable double-helical type of flow (Fig. 1) is present; at still higher Reynolds numbers, turbulent flow is present.

In the turbulent region, a plot of f_s/f_c vs D_t/D_c is essentially a straight line approaching $f_s/f_c = 1$ asymptotically. At higher values of

D_t/D_c, the points for Reynolds numbers of 10,000 and 50,000 fall on oneanother, indicating little variation of f_s/f_c with N_{Re}. At lower values of curvature ratio, there is some difference in these points. This could be attributed to the fact that any error in the slopes of Fig. 4 & 5 would become increasingly noticeable with decreasing D_t/D_c values since f_c approaches f_s. Since the highest Reynolds number investigated was about 70,000, it is difficult to derive a correlation showing the effect of Reynolds number on f_s/f_c. The straight line through the points in Fig. 7, however, should represent values of f_s/f_c which, when used together with the Koo equation for pressure drop in straight tube, will represent friction loss in coils quite satisfactorily for design purposes.

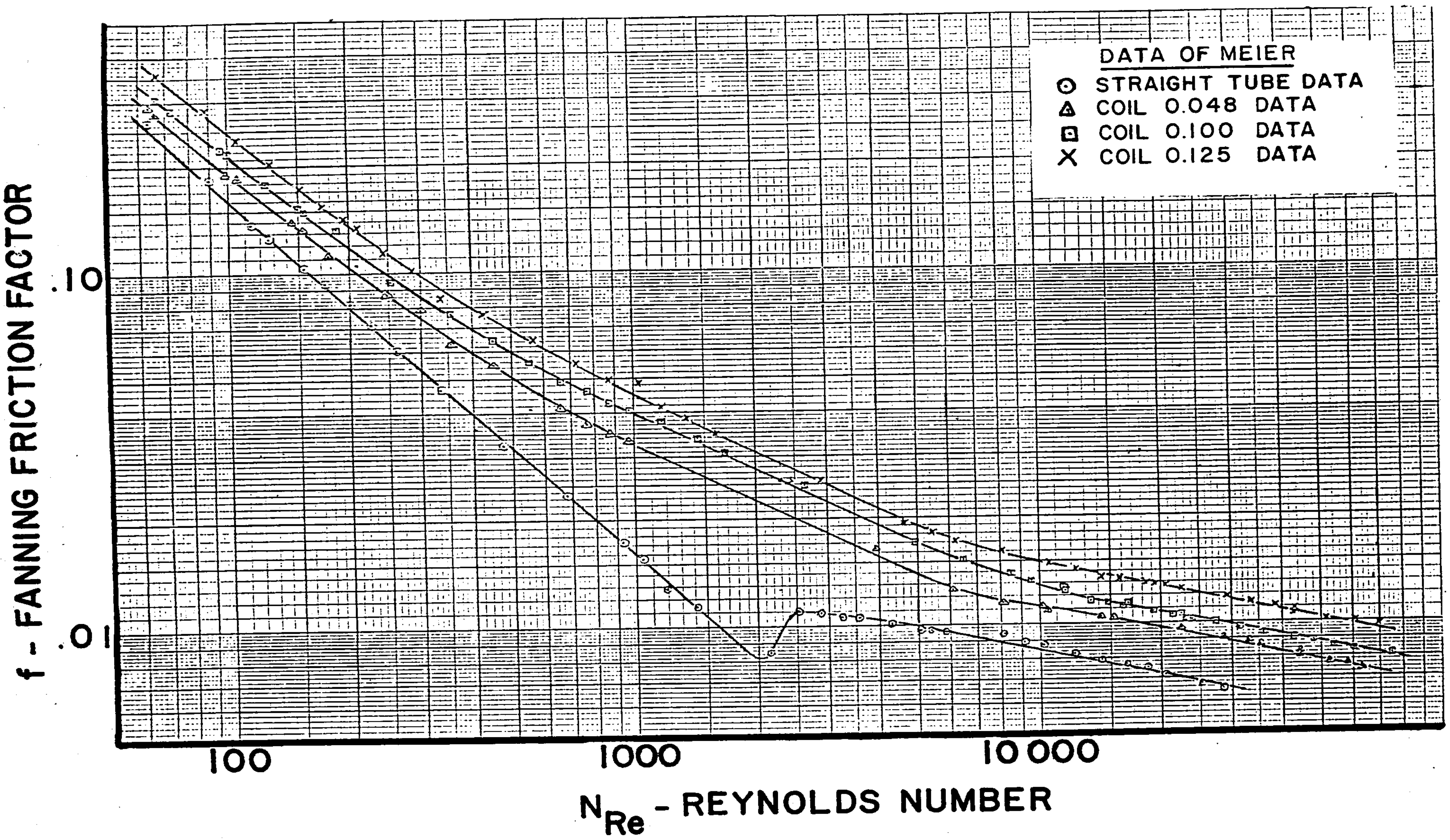


FIGURE 4 EFFECT OF REYNOLDS NUMBER ON FRICTION FACTOR

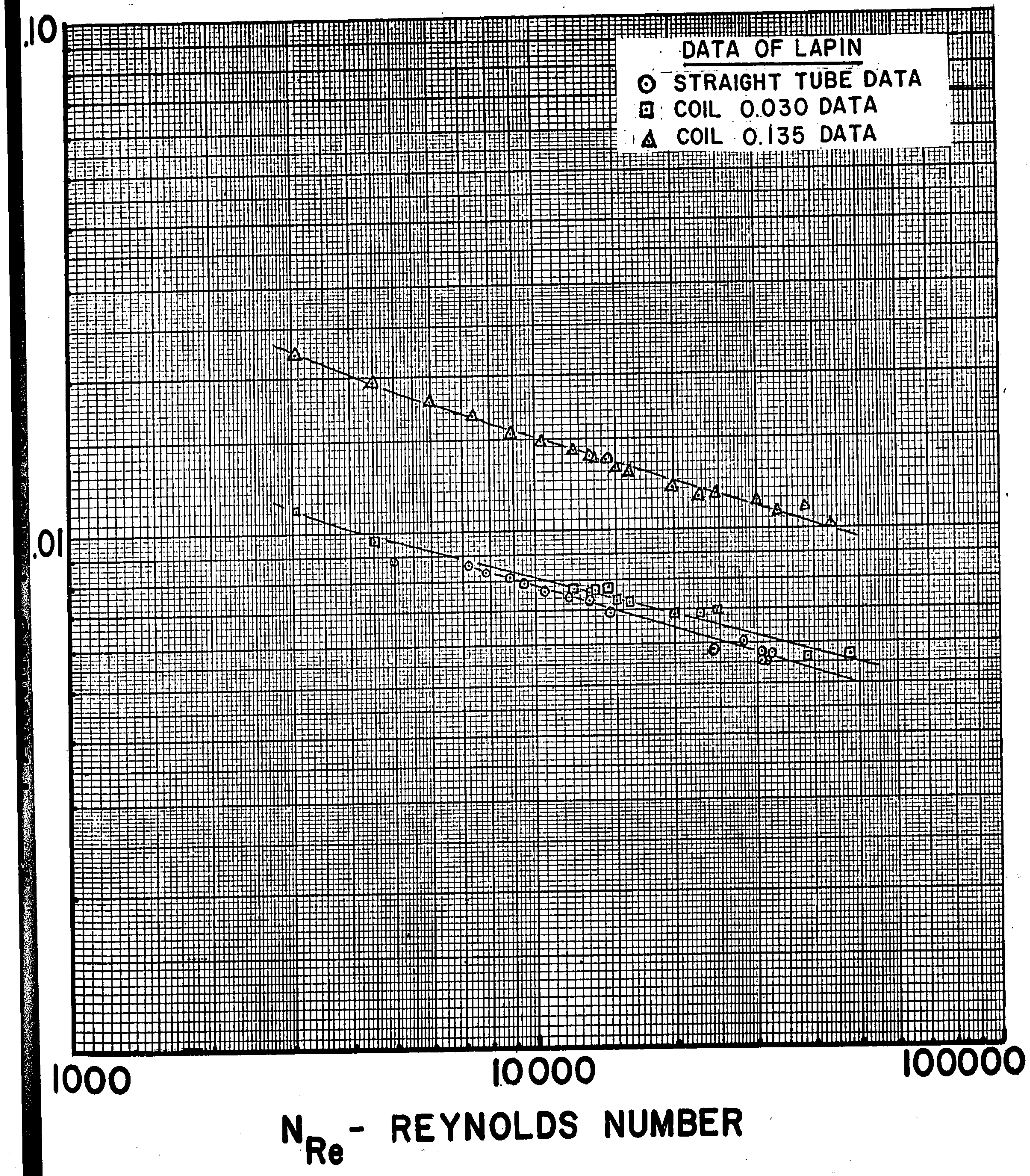


FIGURE 5

FFECT OF REYNOLDS NUMBER ON FRICTION FACTOR

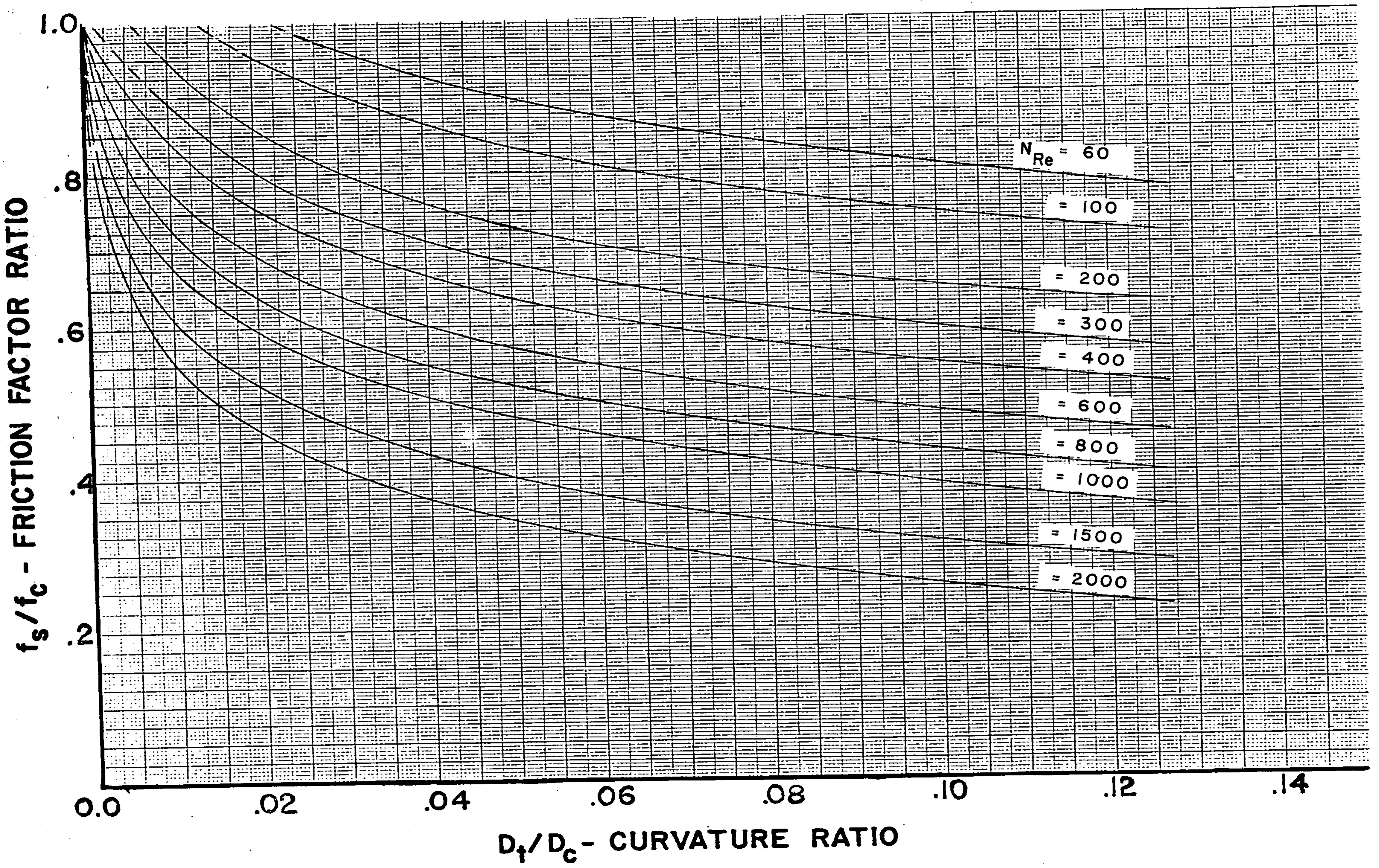


FIGURE 6 EFFECT OF CURVATURE RATIO ON FRICTION FACTOR - LAMINAR REGION

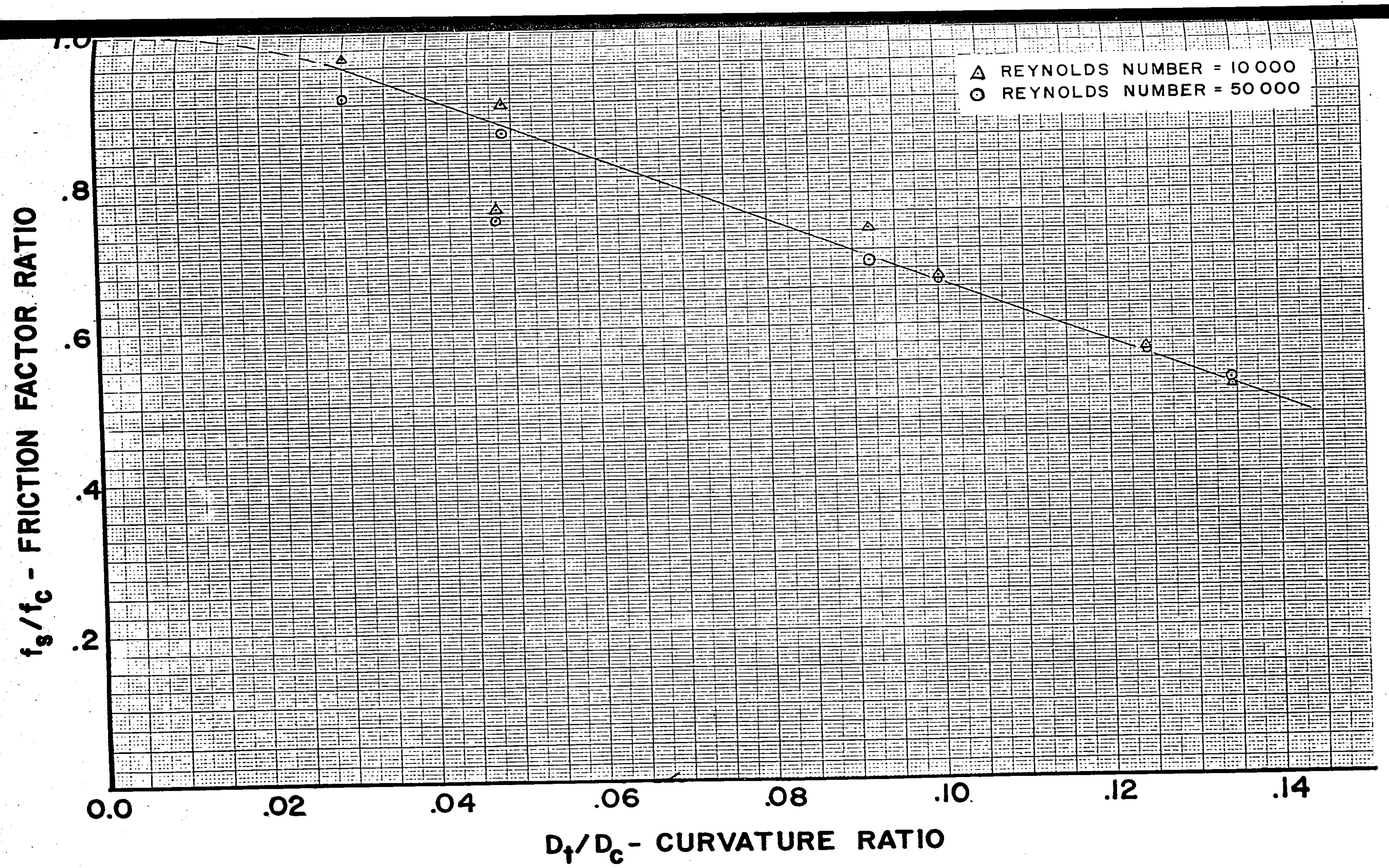


FIGURE 7 EFFECT OF CURVATURE RATIO ON FRICTION FACTOR - TURBULENT REGION

CONCLUSIONS

The friction factor in smooth helically coiled tubes is higher than that for a corresponding straight tube. The effect on the friction factor for a given increase in curvature is greatest at initial bending, becoming less at higher D_t/D_c values. At very low Reynolds numbers, the ratio f_s/f_c approaches a value of one. The Reynolds number below which $f_s/f_c = 1$ is determined by the curvature ratio and varies inversely with this curvature ratio.

The onset of turbulence in coiled tube occurs at much higher Reynolds numbers than in corresponding straight tube, thereby indicating the exsistance of a more stable type of flow. In the truly turbulent region, the ratio f_s/f_c may be considered independent of N_{Re} without serious error.

For design purposes, the curves in Fig. 6 may be used to estimate pressure drop in coiled tube in the laminar region of flow. In the turbulent region, Fig. 7 together with the Koo equation for flow in straight tubes may be used.

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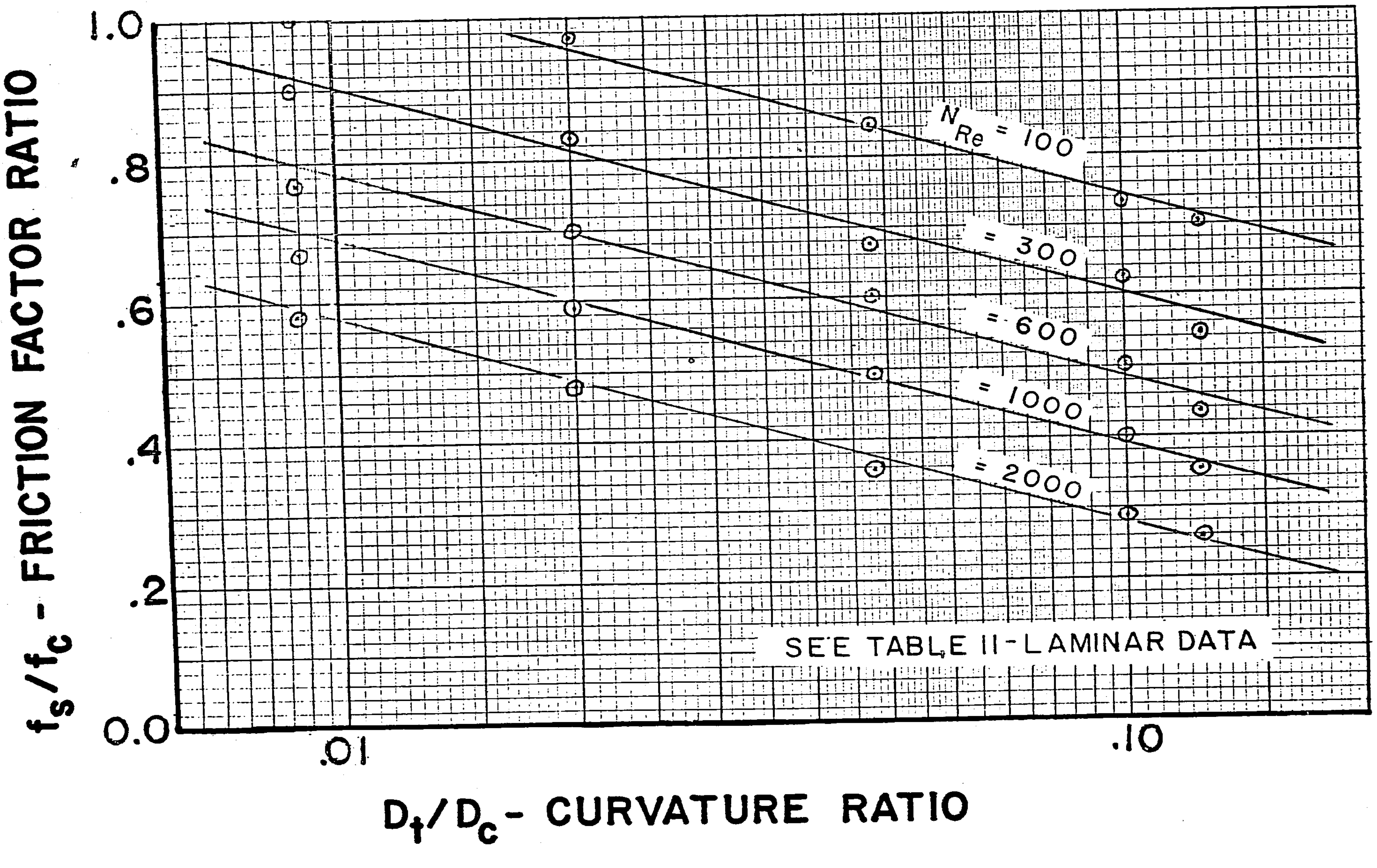

FIGURE 8 EFFECT OF CURVATURE RATIO ON FRICTION FACTOR RATIO

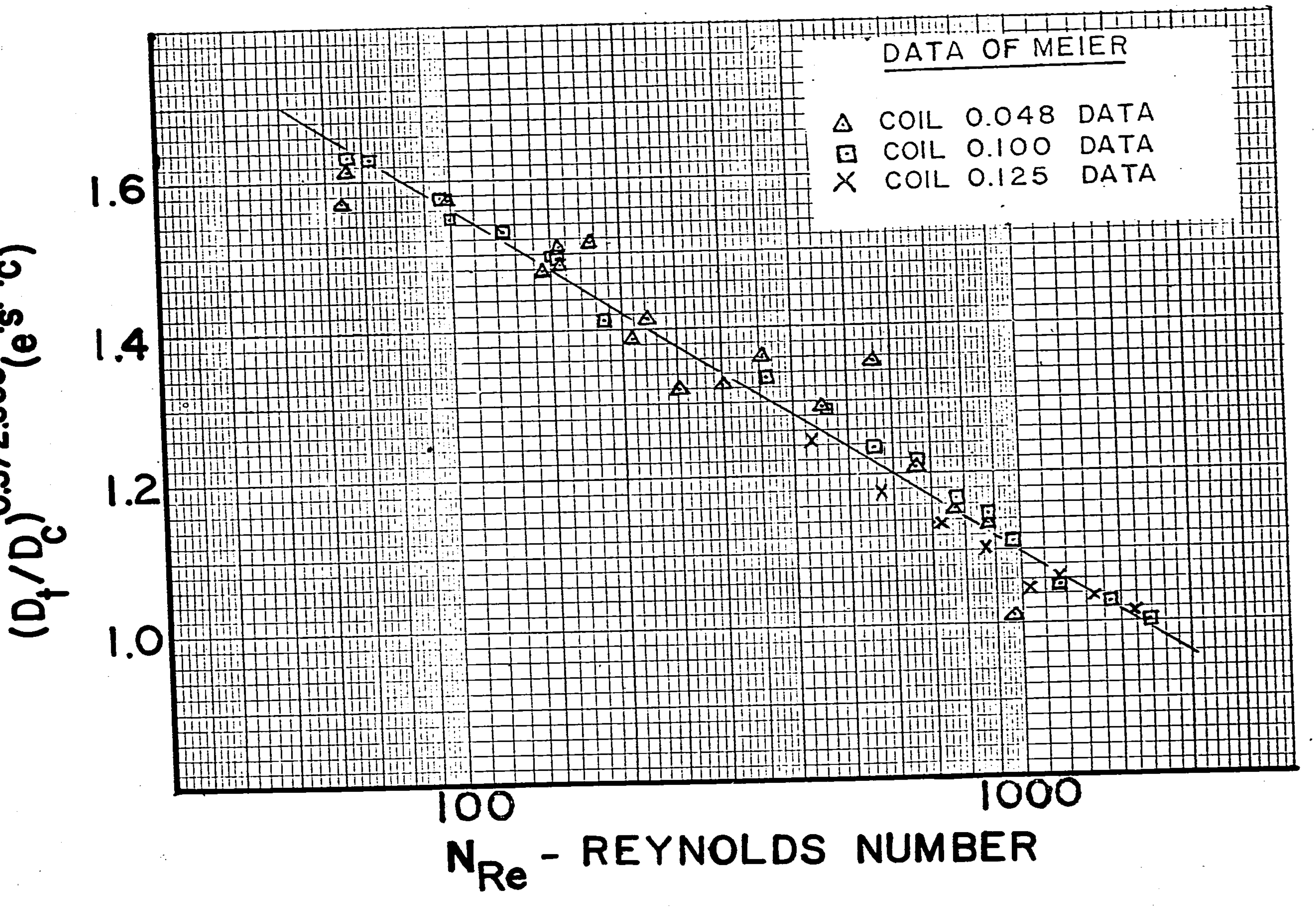

FIGURE 9 CORRELATION FOR PRESSURE DROP IN COILED TUBE - LAMINAR REGION

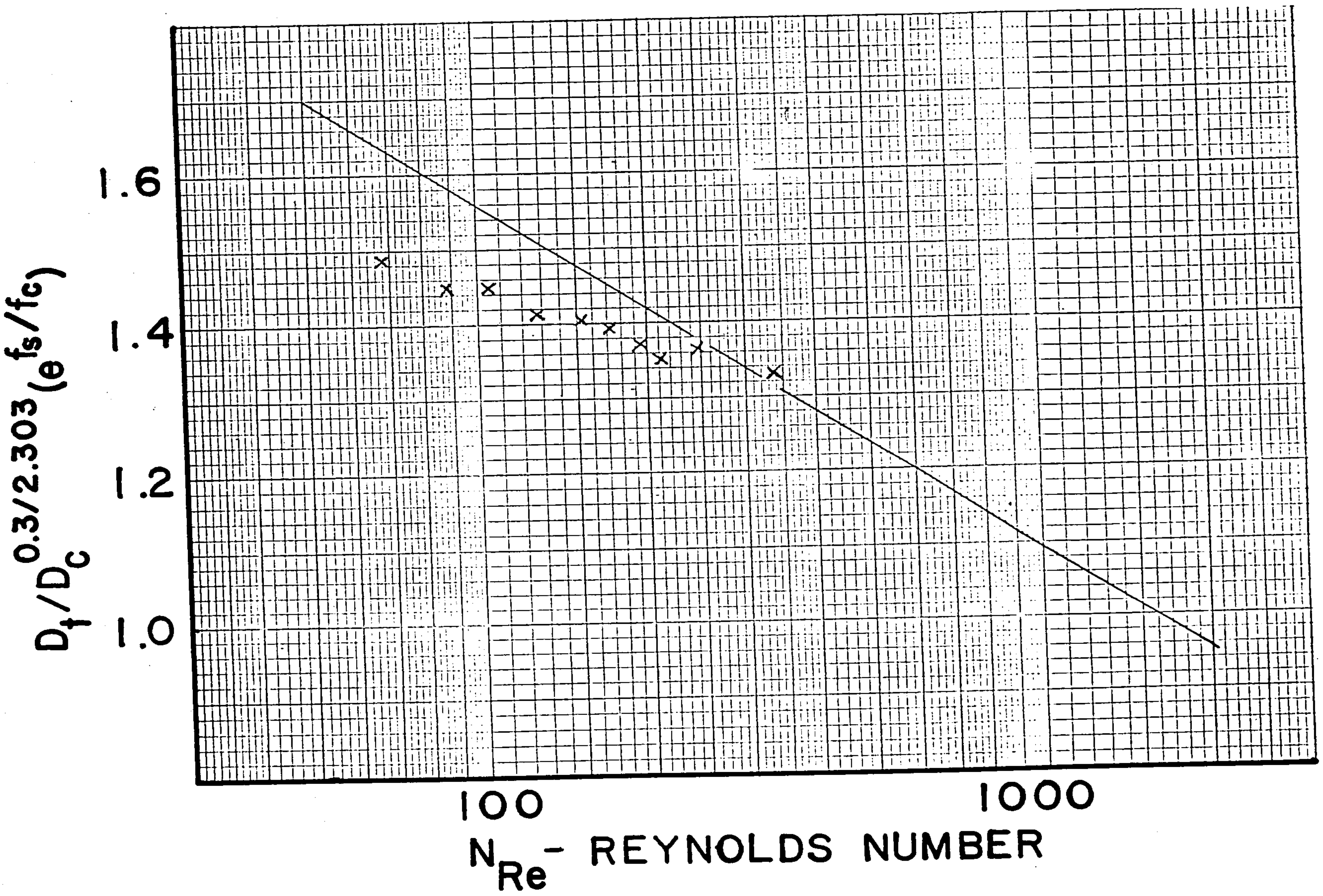

FIGURE 10 SUSPECTED DATA POINTS

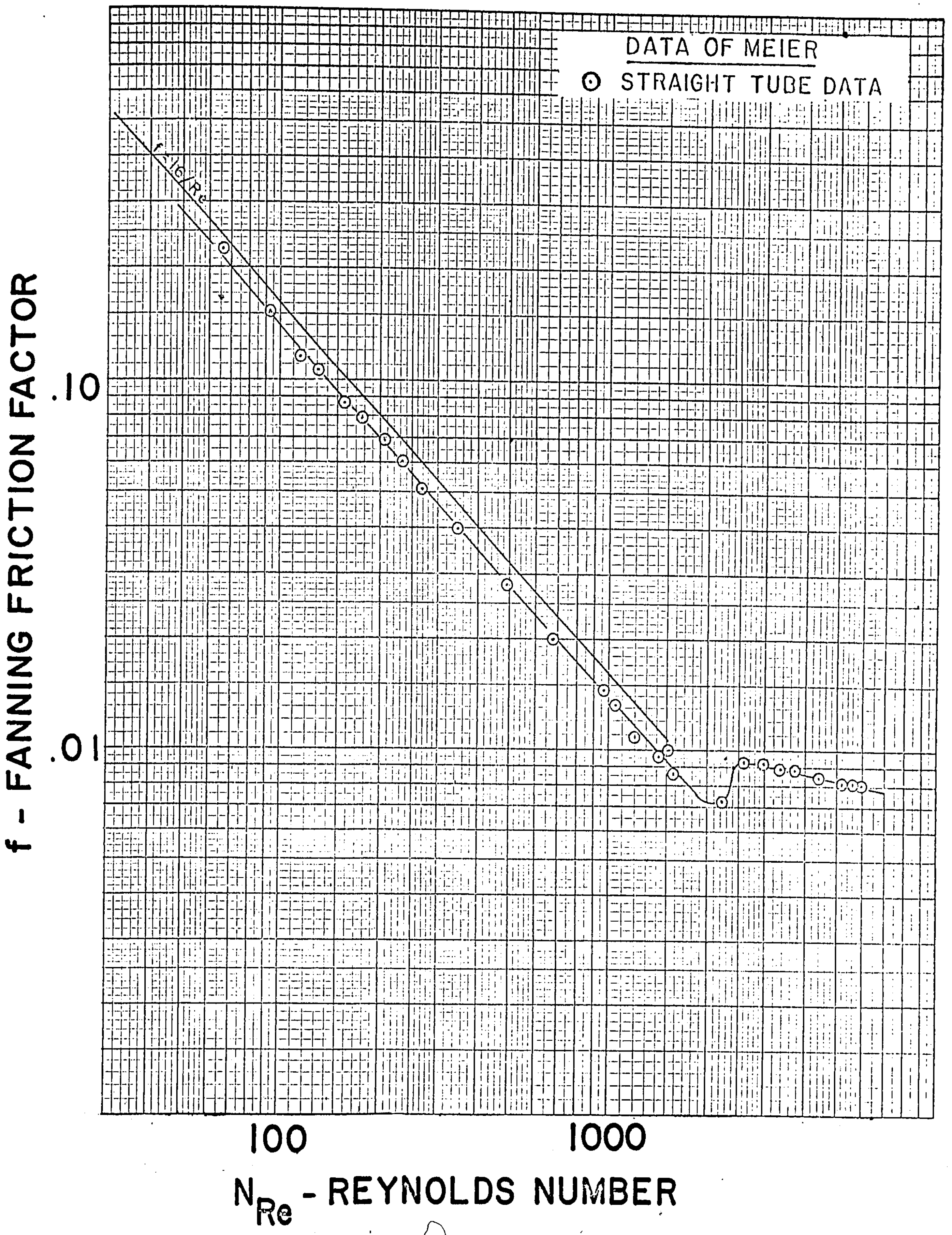

FIGURE 11 EFFECT OF REYNOLDS NUMBER ON FRICTION FACTOR

APPENDIX

TABLE 2
DATA - STRAIGHT TUBE

Tubing	0.093" O.D. x 0.078" I.D. (nominal) 0.081" I.D. (actual)
Length Between Pressure Taps	123.8"
Average Fluid Temperature	74°F

Point Number	Notebook * Page No. & Section	Fluid Used	Inlet Pressure (PSIA)	Pressure Drop	Flow (l/min.)	Reynolds Number (N_{Re})	Fanning Friction Factor (f)	Adjusted Reynolds Number (N_{Re})	Corrected Fanning Friction Factor (f)
1	83-1	He	33.54	11.63 H_2O	2.60	233	0.0604	225	0.0716
2	"	"	33.7	8.88	2.00	179	0.0787	173	0.0933
3	"	"	33.9	6.40	1.46	131	0.107	127	0.129
4	"	"	34.0	4.57	1.04	93	0.152	89.9	0.185
5	"	"	34.1	3.40	0.74	66	0.223	63.8	0.264
6	"	"	33.9	5.59	1.31	117	0.116	113	0.138
7	"	"	33.8	7.77	1.78	159	0.0870	154	0.103
8	"	"	33.7	10.45	2.31	207	0.0691	200	0.0819
9	"	"	33.5	13.13	3.01	269	0.0512	260	0.0607
11	"	"	33.7	1.76 Hg	5.50	493	0.0276	476	0.0327
12	"	"	33.1	3.65	10.75	964	0.0146	932	0.0173
13	"	"	32.8	4.57	13.80	1238	0.0109	1197	0.0129
14	"	"	32.2	6.58	18.30	1641	0.00863	1586	0.0102
15	"	"	32.7	5.68	16.25	1457	0.00964	1408	0.0114
16	"	"	33.5	4.04	11.95	1072	0.0132	1036	0.0156
17	"	"	34.0	2.58	7.85	704	0.0200	680	0.0237
18	"	"	34.5	1.20	3.85	345	0.0396	333	0.0469
24	"	N_2	69.5	11.4	14.75	10500	0.00739	10140	0.00816
25	"	"	67.5	21.67	20.7	14700	0.00667	14208	0.00791
26	"	"	64.5	40.52	27.15	19300	0.00635	18653	0.00753
28	"	"	118.1	31.98	36.54	26000	0.00669	25129	0.00675
29	"	"	116.2	41.20	41.30	29400	0.00553	28415	0.00656
31	"	"	91.4	6.92	13.3	9450	0.00756	9133	0.00896
32	"	"	90.0	11.74	17.7	12600	0.00698	12178	0.00828
33	"	"	88.2	20.52	23.8	16900	0.00645	16334	0.00765
34	"	"	85.8	31.87	29.7	21100	0.00599	20400	0.00710
35	"	"	61.0	46.95	7.47	5222	0.00816	5047	0.00967
36	"	"	61.3	34.80	6.37	4441	0.00848	4292	0.0101
37	"	"	61.5	25.33	5.35	3716	0.00886	3592	0.0105
38	"	"	61.7	16.85	4.32	2984	0.00921	2884	0.0109
39	"	"	62.1	7.25	3.32	2224	0.00718	2149	0.0851
40	"	"	62.0	4.82	2.35	1528	0.0101	1477	0.0120
42	"	"	61.7	12.74	3.81	2594	0.00924	2507	0.0110
43	"	"	61.5	20.51	4.84	3361	0.00877	3248	0.0104
44	"	"	77.8	48.52	8.65	6054	0.00806	5851	0.00956
45	"	"	78.1	41.75	8.03	5600	0.00816	5412	0.00967

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TABLE 3
DATA - COIL 0.048

Tubing	0.093" O.D. x 0.078" I.D. (nominal) 0.081" I.D. (actual)
Average Fluid Temperature	73°F
Mandrel	1.543"
O.D. Finned Tube	0.181"
Length Between Pressure Taps	325.5"
Diameter Tube/Diameter Coil	.0478"
Number of Coils	59

Point Number	Notebook* Page No. & Section	Fluid Used	Inlet Pressure (PSIA)	Pressure Drop	Flow	Reynolds Number (N_{Re})	Fanning Friction Factor (f)	$\frac{f_s}{f_c}$	$(D/D_c)^{.13}_{e}\ f_s/f_c$
2	79-1	N_2	91.7	6.34 HG	.205 SCFM	3972	0.0164	-	-
3	"	"	90.7	11.98	.316	6118	0.0126	-	-
4	"	"	90.4	19.52	.417	8080	0.0116	-	-
5	"	"	89.1	32.85	.540	10438	0.0109	-	-
6	"	"	89.5	15.5 PSI	.526	10148	0.0112	-	-
7	"	"	87.2	32.2	.731	14111	0.0104	-	-
8	"	"	87.0	39.1	.784	15174	0.0104	-	-
12	"	"	199.0	29.1	1.17	22400	0.0966	-	-
13	"	"	195.9	48.2	1.49	28705	0.0907	-	-
14	"	"	193.0	76.8	1.83	35278	0.0867	-	-
15	"	"	376.0	30.0	1.74	33151	0.0874	-	-
16	"	"	371.1	53.3	2.34	44556	0.0822	-	-
17	"	"	370.5	52.2	2.34	44556	0.0801	-	-
18	"	"	367.5	72.1	2.75	52384	0.0760	-	-
19	"	"	365.2	91.2	3.06	58280	0.0762	-	-
20	"	"	398.5	95.7	3.32	63209	0.0735	-	-
1	80-1	He	37.2	4.57 HG	4.1 l/min.	357	0.0634	.707	1.362
2	"	"	36.9	8.59	6.4	556	0.0412	.697	1.353
3	"	"	36.6	13.9	8.8	764	0.0384	.544	1.160
4	"	"	36.3	20.8	11.2	973	0.0334	.491	1.101
5	"	"	36.6	16.7	10.0	870	0.0350	.523	1.136
6	"	"	37.0	10.8	7.6	660	0.0414	.585	1.209
7	"	"	37.5	6.38	5.2	451	0.0547	.647	1.287
11	80-2	"	44.4	7.4 H_2O	0.74	64.3	0.293	.846	1.570
12	"	"	44.4	18.7	1.67	145	0.140	.786	1.478
13	"	"	44.1	36.5	2.90	252	0.0944	.672	1.320
15	"	"	44.4	12.7	7.22	106	0.185	.816	1.523
16	"	"	43.9	44.7	3.45	300	0.0788	.676	1.326
17	"	"	43.9	33.4	2.85	247	0.0871	.743	1.416
18	"	"	44.1	21.8	2.05	178	0.111	.809	1.512
25	"	"	63.4	5.34	8.8	209	0.106	.722	1.388
26	"	"	63.2	10.3	13.6	153	0.132	.792	1.488
27	"	"	62.8	14.0	16.6	98.6	0.191	.849	7.575
28	"	"	62.5	18.2	19.5	65.7	0.279	.873	1.612

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TABLE 4
DATA - COIL 0.100

Tubing	0.093" O.D. x 0.078" I.D. (nominal) 0.081" I.D. (actual)
Average Fluid Temperature	73°F
Mandrel	0.625"
O.D. Finned Tube	0.181"
Length Between Pressure Taps	194.0"
Number of Coils	79

Point Number	Notebook* Page No. & Section	Fluid Used	Inlet Pressure (PSIA)	Pressure Drop	Flow	Reynolds Number (N_{Re})	Fanning Friction Factor (f)	$\frac{f_s}{f_c}$	$(D/D_c)^{.13}_{e}\ f_s/f_c$
1	78-1	N_2	116.7	1.78 Hg	3.17 l/min	2630	0.0251	-	-
2	"	"	116.0	6.61	9.15	6490	0.0151	-	-
3	"	"	115.5	10.63	12.0	8500	0.0139	-	-
4	"	"	114.5	18.36	16.5	11700	0.0123	-	-
5	"	"	113.5	28.56	21.05	14900	0.0115	-	-
6	"	"	112.9	36.05	23.5	16700	0.0113	-	-
8	"	"	65.5	7.78	6.97	4940	0.0170	-	-
10	"	"	63.5	24.80	13.48	9560	0.0132	-	-
11	"	"	62.7	35.95	16.07	11400	0.0127	-	-
12	78-2	"	118.3	12.2 PSIA	19.7	13400	0.0117	-	-
14	"	"	112.5	36.7	33.0	22500	0.0106	-	-
15	"	"	114.5	25.0	0.99 SCFM	19100	0.0108	-	-
16	"	"	112.0	38.3	1.19	23000	0.0101	-	-
17	"	"	109.0	62.2	1.41	27300	0.0100	-	-
19	78-3	"	211.5	9.3	23.8 l/min	16200	0.0111	-	-
20	"	"	210.0	15.7	33.5	21500	0.0106	-	-
21	"	"	209.2	19.0	1.24 SCFM	24000	0.0101	-	-
22	"	"	206.2	32.0	1.61	31100	0.00964	-	-
23	"	"	203.8	43.2	1.86	36000	0.00934	-	-
24	"	"	200.0	65.4	2.24	43100	0.00901	-	-
25	78-4	"	300.4	49.2	2.52	48100	0.00888	-	-
26	"	"	294.9	64.0	2.85	54400	0.00858	-	-
27	"	"	309.0	75.2	3.16	60300	0.00839	-	-
29	"	"	286.7	92.1	3.98	75100	0.00811	-	-
30	78-5	He	33.7	33.7 H_2O	2.97 l/min	2858	0.101	-	-
31	"	"	33.8	23.0	2.15	187	0.133	.643	1.408
32	"	"	33.9	6.59	0.76	66	0.309	.790	1.632
33	"	"	33.9	10.12	1.12	100	0.218	.734	1.542
34	"	"	33.9	17.83	1.79	155	0.149	.693	1.482
35	"	"	57.1	8.49	0.82	72	0.283	.785	1.624
36	"	"	56.7	11.97	1.10	96	0.221	.754	1.573
37	"	"	56.3	16.20	1.42	124	0.179	.721	1.525
38	"	"	55.9	20.70	1.73	150	0.152	.701	1.495
28	79-3	"	36.8	3.3 Hg	4.10	357	0.0762	.588	1.334
29	"	"	36.5	6.08	6.40	556	0.0560	.513	1.237
30	"	"	36.3	9.8	8.80	764	0.0461	.453	1.166
31	"	"	35.9	14.5	11.20	973	0.0405	.425	1.111
32	"	"	36.1	11.77	10.00	870	0.0422	.434	1.143
33	"	"	36.5	7.63	7.60	660	0.0490	.494	1.214
34	"	"	36.9	4.51	5.20	451	0.0642	.551	1.286
26	"	"	63.2	10.26 H_2O	13.6 l/min	1180	0.0383	.353	1.053
27	"	"	62.8	14.04	16.6	1440	0.0384	.324	1.024
28	"	"	62.5	18.2	19.5	1690	0.0313	.302	1.002

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DATA - COIL 0.125

Tubing	0.093" O.D. x 0.078" I.D. (nominal) 0.081" I.D. (actual)
Average Fluid Temperature	72°F
Mandrel	0.467"
O.D. Finned Tube	0.181"
Length Between Pressure Taps	123.0"
Number of Coils	59

Point Number	Notebook* Page No. & Section	Fluid Used	Inlet Pressure (PSIA)	Pressure Drop	Flow (l/min)	Reynolds Number (N_{Re})	Fanning Friction Factor (f)	$\frac{f_s}{f_c}$	$(D/D_c)_e^{.13} f_s/f_c$
2	82-1	N_2	89.5	1.97 Hg	4.20	2910	0.0255	-	-
3	"	"	89.1	3.90	6.77	4690	0.0192	-	-
4	"	"	89.0	5.04	7.90	5460	0.0181	-	-
5	"	"	88.7	6.19	9.00	6220	0.0171	-	-
6	"	"	88.1	10.23	11.80	8160	0.0161	-	-
7	"	"	87.4	16.16	15.30	10580	0.0148	-	-
8	"	"	86.6	21.23	17.70	12250	0.0141	-	-
9	"	"	85.8	28.66	20.70	14300	0.0134	-	-
10	"	"	85.0	36.35	23.0	15900	0.0133	-	-
11	"	"	109.0	17.90 PSI	26.8	18450	0.0130	-	-
12	"	"	107.3	22.7	29.9	20600	0.0128	-	-
13	"	"	105.5	28.4	33.0	22700	0.0124	-	-
14	"	"	106.5	20.0	28.1	19350	0.0129	-	-
16	"	"	98.8	58.8	42.4	29200	0.0118	-	-
17	"	"	165.7	35.4	49.3	33700	0.0113	-	-
18	"	"	162.5	48.2	56.2	38450	0.0111	-	-
19	"	"	158.7	66.2	63.3	43300	0.0109	-	-
21	"	"	157.3	63.9	63.5	43300	0.0105	-	-
22	"	"	305.0	40.0	76.8	51600	0.0101	-	-
23	"	"	300.5	56.8	90.1	60600	0.0101	-	-
24	"	"	295.2	78.0	103.5	69600	0.00983	-	-
25	"	He	33.7	18.52 H_2O	2.42	210	0.133	.572	1.350
26	"	"	33.7	14.45	1.98	171	0.155	.603	1.393
27	"	"	33.8	10.28	1.47	128	0.203	.616	1.410
28	"	"	33.8	6.96	1.02	88	0.285	.638	1.442
29	"	"	33.8	8.28	1.22	105	0.237	.643	1.449
30	"	"	33.9	5.02	0.77	66.6	0.360	.667	1.484
31	"	"	33.8	12.39	1.75	152	0.172	.612	1.404
32	"	"	33.6	16.90	2.25	194	0.141	.584	1.366
33	"	"	33.6	21.40	2.82	244	0.113	.580	1.360
34	"	"	33.5	31.25	3.90	337	0.0855	.554	1.326
35	"	"	33.4	45.45	4.95	428	0.0765	.488	1.241
36	"	"	33.9	7.05 Hg	8.35	722	0.0557	.397	1.138
37	"	"	33.3	12.32	11.95	1034	0.0488	.3155	1.044
38	"	"	33.9	19.16	15.70	1350	0.0382	.309	1.028
39	"	"	33.8	25.68	18.60	1600	0.0346	.289	1.017
41	"	"	33.8	14.70	13.66	1180	0.0409	.330	1.060
42	"	"	34.3	9.32	10.1	870	0.0492	.367	1.100
43	"	"	34.8	4.90	6.6	570	0.0645	.434	1.175

* Air Products Research Notebook No. 142

TABLE 6
LAPIN DATA - COIL 0.030

Tubing	0.0413 Ft.
Mean Coil Diameter	1.395 Ft.
Length Between Pressure Taps	19.72 Ft.
Diameter Tube/Diameter Coil	0.0295"
Number of Coils	4.5

Point Number	Notebook* Page No. & Section	Fluid Used	Outlet Pressure (Hg)	Pressure Drop	Average Temperature °C	Flow (SCFM)	Reynolds Number (N_{Re})	Fanning Friction Factor Before Coiling (Straight)
1	25-1	Air	2.45	3.30" Hg	19.0	11.21	32,480	.00562
2	"	"	2.30	3.10"	19.0	10.87	31,470	.00558
3	"	"	2.30	3.10"	19.0	10.64	30,800	.00583
4	"	"	2.05	2.70"	19.0	9.71	28,110	.00601
5	"	"	1.60	2.05"	19.0	8.44	24,440	.00591
6	"	"	1.00	9.40" H_2O	18.0	4.59	14,450	.00690
7	"	"	0.95	8.10"	20.0	4.16	13,100	.00720
8	"	"	0.75	6.80"	19.5	3.75	11,810	.00732
9	"	"	0.75	5.60"	19.2	3.34	16,510	.00765
10	"	"	0.65	4.70"	19.0	3.01	9,480	.00787
11	"	"	0.70	4.20"	19.0	2.80	8,810	.00813
12	"	"	0.60	3.40"	18.5	2.46	7,740	.00848
13	"	"	0.50	2.95"	18.0	2.28	7,180	.00855
14	"	"	0.45	1.50"	18.5	1.58	4,970	.00898
								After Coiling
4	27,28-1	Air	1.3	7.8" H_2O	22.5	3.83	12,050	.00763
5	"	"	1.0	5.3"	22.5	3.28	10,330	.00768
6	"	"	1.5	8.4"	22.5	4.20	13,220	.00757
7	"	"	2.6	9.8"	22.5	4.57	14,390	.00774
8	"	"	0.6	1.3"	22.5	1.43	4,500	.00961
9	"	"	0.4	0.7"	22.5	0.97	3,050	.0111
10	"	"	1.8	10.2"	26.0	4.65	13,460	.00760
11	"	"	2.0	12.0"	24.0	5.17	14,980	.00728
12	"	"	2.2	13.5"	24.0	5.52	15,990	.00724
13	"	"	2.6	16.1"	24.0	5.58	16,170	......
14	"	"	2.9	19.3"	23.7	6.90	19,980	.00681
15	"	"	3.7	24.2"	23.5	7.82	22,650	.00684
16	"	"	4.5	28.5"	23.3	8.51	24,650	.00699
30	"	"	9.2	4.1"	23.0	13.34	38,640	.00564
34	"	"	13.1	5.5"	22.0	16.67	48,280	.00570

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TABLE 7
LAPIN DATA - COIL 0.135

Tubing	.0413 Ft.
Mean Coil Diameter	0.306 Ft.
Length Between Pressure Taps	14.42 Ft.
Diameter Tube/Diameter Coil	0.135
Number of Coils	15

Point Number	Notebook* Page No. & Section	Fluid Used	Outlet Pressure (Hg)	Pressure Drop	Average Temperature °C	Flow (SCFM)	Reynolds Number (N_{Re})	Fanning Friction Factor Before Coiling (Straight)
1	25-1	Air	11.10	2.70" Hg	19.0	11.21	32,480	.00573
2	"	"	10.60	2.50"	19.0	10.87	31,470	.00557
3	"	"	10.25	2.50"	19.0	10.64	30,800	.00577
4	"	"	9.00	2.25"	19.0	9.71	28,110	.00603
5	"	"	6.95	1.75"	19.0	8.44	24,440	.00583
6	"	"	2.65	9.00" H_2O	18.0	4.59	14,450	.00690
7	"	"	2.50	7.80"	20.0	4.16	13,100	.00730
8	"	"	1.95	6.60"	19.5	3.75	11,800	.00740
9	"	"	1.70	5.40"	19.2	3.34	10,510	.00760
10	"	"	1.45	4.60"	19.0	3.01	9,480	.00790
11	"	"	1.40	4.10"	19.0	2.80	8,810	.00810
12	"	"	1.10	3.25"	18.5	2.46	7.750	.00830
13	"	"	1.05	2.90"	18.0	2.28	7.180	.00860
14	"	"	0.70	1.45"	18.5	1.58	4,980	.00880
								After Coiling
1	27,28-1	Air	1.1	3.0" H_2O	22.5	1.89	5,950	.0180
2	"	"	1.5	4.3"	22.5	2.36	7,430	.0167
3	"	"	1.8	5.6"	22.5	2.82	8,880	.0154
4	"	"	2.9	9.2"	22.5	3.83	12,060	.0143
5	"	"	2.3	7.2"	22.5	3.28	10,330	.0149
6	"	"	3.4	10.6"	22.5	4.20	13,220	.0139
7	"	"	3.9	12.2"	22.5	4.57	14,390	.0137
8	"	"	0.8	1.9"	22.5	1.43	4,500	.0196
9	"	"	0.6	1.0"	22.5	0.97	3,050	.0222
10	"	"	4.0	12.7"	26.0	4.65	13,460	.0138
11	"	"	4.6	14.7"	24.0	5.17	14,980	.0132
12	"	"	5.1	16.2"	24.0	5.52	15,980	.0130
14	"	"	7.2	22.3"	23.7	6.90	19,980	.0121
15	"	"	9.0	26.5"	23.5	7.82	22,650	.0116
16	"	"	10.5	29.9"	23.3	8.51	24,650	.0117
18	"	"	14.7	38.9"	23.1	10.35	29,980	.0114
28	"	"	16.7	3.2" Hg	23.0	11.50	33,310	.0108
30	"	"	21.2	3.9"	23.0	13.34	38,640	.0110
32	"	"	25.2	4.4"	23.0	15.18	43,960	.0101

*Air Products Research Notebook No. 25

TABLE 8

CROSS PLOT DATA

N_{Re}	f_s/f_e								
	Grindley & Gibson .009	White .02	Lapin .030	Meier .048	Christensen .049	Christensen .092	Meier .100	Meier .125	Lapin .135
100	1.000	0.971	.....	.842	.....	.....	.734	.704	.....
300	0.900	0.833	.....	.676	.....	.....	.625	.544	.....
600	0.769	0.700	.....	.601	.....	.....	.504	.435	.....
1,000	0.667	0.591	.....	.492	.....	.....	.400	.356	.....
2,000	0.580	0.481	.....	.357	.....	.....	.291	.259	.....
10,000	.....	.....	0.963	.759	.899	.729	.664	.567	0.513
20,000	.....	.....	0.944	.742	.887	.714	.667	.567	0.518
35,000	.....	.....	0.917	.739	.867	.693	.661	.566	0.519
50,000	.....	.....	0.909	.744	.859	.685	.659	.563	0.521
70,000	.....	.....	.....	.742	.849	.669	.659	.565	.....

SAMPLE CALCULATION

Coil 0.100
Tube I.D. = 0.081"
Length of tubing = 123" = 10.25'
Flow rate = 2.37 std liters / minute nitrogen
Av. temperature in coil = 72° F.
Inlet pressure to coil = 89.7 psia
Pressure drop thru coil = 11.0" H_2O = 0.390 psi = 56.0 psf
Av. pressure = 89.7 - (0.390/2) = 89.5 psia
Density @ 89.5 psia & 72° F. = 0.445 lb/ft^3
Viscosity @ 89.5 psia & 72° F. = 11.67×10^{-6} lb/ft-sec
Cross-sectional area = $(6.75 \times 10^{-3})^2 \times 0.785 = 3.56 \times 10^{-5}\ ft^2$

$$G = 2.37\ l/min \times 0.0353\ ft^3/l \times 0.073\ lb/ft^3 \times 1/60\ min/sec \times$$
$$10^5/3.56\ ft^{-2} = 2.86\ lb/ft^2\text{-sec}$$

$$G^2 = 8.15\ lb^2/ft^4\text{-sec}^2$$

$$N_{Re} = DG/\mu = \frac{.00675\ ft \times (2.86\ lb/ft^2\text{-sec})}{11.67 \times 10^{-6}\ lb/ft\text{-sec}} = 1660$$

$$f_c = \Delta P D g_c \rho / 2LG^2 = \frac{56.0\ lb/ft^2 \times .00675\ ft \times 32.2\ lb_m\text{-}ft/lb_f\text{-sec}^2 \times .445}{2 \times 10.25\ ft \times 8.15\ lb^2/ft^4\text{-sec}^2 \qquad ft^3/lb}$$
$$= 0.032$$

$$f_s/f_c = \frac{16/N_{Re}}{f_c} = 0.301$$

True Diameter Determination

$$\Delta P = \left[4fL/D\right]\left[(W^2 \times 16/\pi^2 D^4\ 2g_c\rho\right]$$

In laminar flow $f = KN_{Re}$ where K = 16 $f = K\mu\pi D/4W$
From straight tube data (Fig 11) K = 14 (using nominal diameter)
Nominal tube diameter = 0.078"

$$\Delta P = 4(K\mu\pi D/4W)(L/D)\left[(W^2 \times 16/\pi^2 D^4\ 2g_c\rho\right] = K(\text{Constants})(1/D^4)$$

$$\frac{\Delta P = K(\text{Constants})(1/D^4)}{\Delta P' = K'(\text{Constants}')(1/D'^4)}$$

where D' refers to the nominal diameter
and D refers to the true diameter
$\Delta P = \Delta P'$, Constants = Constants',
$D/D' = \sqrt[4]{K/K'}$

$$D = 0.078"\ \sqrt[4]{16/14} = 0.081"$$