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THE OPTIMIZATION OF DYNAMIC PROPERTIES OF PLANAR LINKAGES BY INTERNAL MASS REDISTRIBUTION

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Abstract

The literature of the passive balancing of planar mechanisms was reviewed. A numerical optimization method was considered to be the most promising technique available and was thus chosen for implementation. An interactive, graphically-oriented computer program for balancing planar mechanisms was written. The slider crank, four bar, and Watt's six bar linkages were balanced using the program with good results. The numerical optimization method, as implemented in this work, performed well. Comments concerning the use of the program were made and areas for improvement and for further work were noted.

Chapter 1
Introduction

All machines are composed of combinations of moving masses. The accelerations of these moving masses cause the development of inertia forces. Typically, because of the difficulty of analysis, the effect of these inertia forces is not taken into account in many aspects of the machine's design, particularly in the early stages.

The usual strategy for designing a linkage mechanism has three stages [39]. In the first stage the mechanism type, geometric parameters (e.g. link lengths), and input motion(s) are chosen so as to satisfy all the necessary kinematic requirements upon the mechanism (e.g. output motion(s)). In the second stage, with heavy reliance upon the designer's experience, the size and shape of the machine elements are designed so as to embody the geometric parameters chosen in the first stage and also satisfy their strength and material requirements. The kinematics and force calculations performed during the first two stages are made assuming the machine base and all the machine elements are rigid and the speed of the driving motor is constant.

However, the inertia forces cause the machine to perform in ways the designer has not taken into account. The inertia forces act as inputs to the base of the machine, causing a forced vibration, which in turn causes the base of the machine to deflect. The other members of the machine also suffer deformation. The inertia forces feed back into the driving motor, causing speed fluctuations. These effects in turn affect the kinematic behavior (e.g. output motion(s)) of the machine, so that the output(s) may be changed as to no longer provide the required motion that the machine was designed to perform.

The third and final stage in the usual strategy of design attempts to overcome the effects of the inaccurate assumptions made in the earlier stages by "adjusting" the design so as to achieve satisfactory performance. This often occurs after a prototype of the mechanism has been built and is being tested. The usual "adjustments" made to remedy a mechanism suffering from the afore stated problems have been:

1. to slow down the mechanism (thus reducing the magnitude of the inertia forces).

2. to add a flywheel or to increase the size of the flywheel if one exists (to smooth variations in the input speed).

3. to begin a slow iterative procedure of altering the various parameters of the mechanism (usually through a series of prototypes), attempting to improve the mechanism's dynamic performance and allow operation at the speeds desired.

If the addition of a flywheel (or adding a larger flywheel) works, then this solution is generally acceptible, since the cost of the modification is relatively small. However, the other alternatives are neither direct nor attractive. To slow down the mechanism means to cut down the productivity of the manufacturing process, possibly to the point where it is no longer profitable. An interative, trial and error redesign of the mechanism can be extremely expensive, and there is no guarantee of obtaining the desired results.

An alternate way to anticipate these kinds of problems is to consider the effects of the inertia forces on the machine's behavior earlier in the design process and attempt to minimize their net effects by balancing the mechanism.

The balancing of a mechanism can take two forms: active or passive. In active balancing additional moving masses are added to the mechanism in such a way as to counteract the forces produced in the unbalanced mechanism.

Passive balancing seeks to redistribute the internal mass of the members of the mechanism so as to diminish or eliminate the unwanted inertia forces. At the current time, passive balancing efforts usually involve the design of counterweights to be added to the existing members, as the synthesis of actual mechanism members is a great deal more difficult to perform. Passive balancing usually costs less than active balancing and has the advantage of not adding any additional moving parts to the machine.

The use of balancing to "adjust" the design in order to optimize in some sense the dynamic performance of the mechanism generally falls into the third stage of the design, with the useful exception that it can be done in advance of the construction of a prototype mechanism. Hence, balancing can avoid or reduce the "adjusting" effort involved in the performance testing/evaluation process.

The goal of this work is to develop an interactive, computer-aided-engineering program to perform the passive balancing of a linkage or mechanism through counterweight synthesis. Its eventual use will be by machine designers at the AMP corporation of Harrisburg, Pennsylvania, in conjunction with their existing machine-design software. The program should have the following characteristics:

1. It should be able to work with a wide variety of linkages and mechanisms.

2. It must not require an undo understanding of the analytical and kinematic aspects of a machine in order to be used, i.e., it must be usable by an average engineer after some limited training.

3. It must allow the user flexibility in terms of achieving the effects he desires.

A review of the current literature in the field of passive balancing will be presented first to show the state-of-the-art in this field. After this, a description of the program developed will be given, followed by a number of problems to demonstrate the capabilities of the program and a conclusion.

Chapter 2
Literature Review

The following is a review of the literature in the field of passive balancing. It is not intended to be a complete survey of the literature in the field of balancing, but rather this review will concern itself mainly with the papers of passive balancing which present topics in the methodology of balancing, as opposed to papers presenting applications of previously advanced balancing techniques. Furthermore, this review will be limited to the more modern trends in this field. The list of references, however, contains not only the papers cited in this review, but a number of additional papers in the field of balancing which treat the subject matter not addressed here.

It should be pointed out that this review is an abstraction and update of the review done by Lowen, Tepper, and Berkof [23]. Their work contains additional information on this topic and a more extensive list of references.

First, several terms must be defined. The shaking force in some coordinate direction is defined as the vector sum of all of the bearing reactions on the frame of a mechanism in that coordinate direction. For example, referring to Figure 4-1, the shaking force in the x-direction (or the X-shaking force) would be $F_X = -R_{AX}$. A mechanism is generally said to be fully force-balanced if the shaking forces in the specified coordinate direction(s) are identically zero. A mechanism is generally said to be partially force-balanced if the shaking forces in the specified coordinate direction(s) have been reduced from those of the unbalanced mechanism.

The shaking moment is defined as the sum of the moments caused by the bearing reactions of a mechanism on its base, taken relative to some point.

Sometimes the input torque or force is included, sometimes it is not, depending on the physical configuration of the device.

The modern developments in the field of passive balancing began in the late 1960's with the publication of the paper by Berkof and Lowen [4]. In this paper they presented the technique for fully force-balancing planar linkages known as the Method of Linearly Independent Vectors. This technique makes the center of mass of the linkage stationary, so that the sum of the external forces in any coordinate direction on the moving members of the linkage must be zero. This contition is accomplished by writing the equation for the center of mass of the linkage and then substituting into it the loop equation(s) [24, p.233-35] for the linkage. The resulting equation will have several time-dependent terms, each multiplied by another term which is a function only of the mass properties of the links. By setting each of the terms which multiply the time-dependent terms equal to zero, the center of mass is made stationary, and a number of conditions on the inertial properties of the links are determined.

Tepper and Lowen [29] generalized the Method of Linearly Independent Vectors for single degree-of-freedom linkages. Furthermore, they showed that, in general, in order for it to be possible to fully force-balance a linkage, a path to ground through revolute joints only must exist for each link, or (equivalently in a kinematic sense), that each loop of the linkage may contain at most one prismatic joint. Such a linkage can be balanced by the "apparent" minimum of $n/2$ counterweights for an n-linked mechanism.

Walker and Oldham [37], following an earlier paper by Smith et al. [28], presented a theory of balancing based on an alternate interpretation of the work of Berkof and Lowen. This paper presented equations which allow one to write directly the necessary and sufficient conditions on the counterweights to force-balance a mechanism, instead of extracting them from the kinematic equations of motion. Extending their work, Walker and Oldham [38] developed a formula for determining the minimum number of counterweights needed to fully force-balance a multi-degree-of-freedom linkage and, in addition, established a means of identifying which links in a linkage should have the counterweights attached to them.

While all the work above concentrates on full force-balancing, the workers in this field also turned their attention to the effect of full balancing upon the other characteristics of a linkage, i.e. upon the shaking moment and the input torque. It was found that the shaking moment and input torque of the fully force-balanced linkage often became worse than those of the unbalanced linkage.

Berkof and Lowen first examined these effects in [5] and [21] for the four-bar linkage. They presented the theory in [5] which used a least-squares optimization technique to develop the conditions upon the mass properties of the links in the mechanism needed to minimize the shaking moment of a fully force-balanced four-bar linkage. In [21] they applied their conditions to a four-bar linkage of standard configuration, i.e. one in which the geometries of the links are defined by non-dimensional ratios, such as the ratios of the lengths of the coupler and follower to the length of the crank and the width of the center sections of the links to the length of the crank. They generated graphs and tables in terms of selected ratios, and showed how to use them to design a fully force-balanced linkage with minimum shaking moment. However, for certain values of the parameters, no solution was possible. Furthermore, it was noted that in attempting to apply their general moment-balance criteria, the counterweights needed could be unrealistic due to their size and/or location.

Tepper and Lowen [30] studied the mathematical nature of the root-mean-square shaking moment function for unbalanced planar linkages and showed that the root-mean-square shaking moment is constant when taken with respect to all points along the concentric isomomental ellipses in the mechanism plane. Further, the root-mean-square shaking moment is minimum when taken with respect to the point at the center of the family of isomomental ellipses. They showed that if the shaking moment of the fully force-balanced linkage with respect to this point is lower than that of the unbalanced linkage, then the shaking moment of the fully force-balanced linkage will be lower than that of the unbalanced linkage when taken with respect to all points in the mechanism plane. This provides an easy criteria for judging the effect of full force-balancing upon the shaking moment for a particular linkage.

Carson and Stephens [7] continued the work of Berkof and Lowen for the problem of the shaking moment balance of the four-bar linkage (of standard configuration, again because of the difficulty of solution), examining the feasible ranges of the design parameters developed by Berkof and Lowen in [21].

Other work to minimize the shaking moment and input torque has been performed. Wiederrich and Roth [41] showed how to perform a full force-balance and a full or partial moment-balance of a four-bar linkage by working directly with the expressions for the linear and angular momentum. Berkof [6] discussed the least-square minimization of the input torque of a force-balanced four-bar linkage, basing his work on the expression of the input torque as a function of the first derivative of the kinetic energy. Haines [17] treated the problem of optimizing the root-mean-square shaking moment and/or input torque of a fully force-balanced four-bar linkage, subject to physical constraints on the parameters (e.g. constraints on the size and location of counterweights as determined by the mechanism envelope). Elliot and Tesar [13], making use of the linear qualities of Berkof and Lowen's technique, developed a generalized methodology which allows for full force-balancing and moment balancing with full force-balancing about points in the mechanism plane which do not coincide with the center of a fixed revolute joint. Their technique also allows for the satisfaction of user-specified non-zero values of the dynamic properties of the mechanism, including input torque, shaking force, shaking moment, kinetic energy, and combinations thereof, at specified points in the motion of the mechanism. They noted, however, that the counterweights so obtained were sometimes unrealisitic (e.g. negative masses were required).

All the above techniques have been spawned by the pioneering work of Berkof and Lowen, and begin with full shaking force balancing and develop from this premise. Other techniques and methods have been applied, however.

For instance, Triacamo and Lowen [32], building on a graphical technique developed earlier by Gheronimus [14], developed a method for balancing four-bar linkages which, given a prescribed maximum shaking force to be realized by the counterweighted linkage, allows for a substantial flexibility in the choice of the location and size of counterweights. Lowen, Tepper, and Berkof [23] describe the work of Urba [34, 35, 36], who in [34] used "best-fit" coefficients of a truncated Fourier series to describe the angular accelerations of the links of a four-bar mechanism. She then used this description to determine the best input link counterweight to perform partial force-balancing of four-bar linkages in [35], and to balance actively four-bar linkages in [36].

Non-linear programming has also been applied to this problem. Lowen, Tepper, and Berkof [23] describe the work of Dresig and Schoenfeld [11, 12], who applied several numerical optimization techniques (Powell's method, Monte Carlo techniques, and the Gauss-Seidel method), with several forms of the object function (Chebyshev-type and root-mean-square type), to the balancing problem. Walker and Haines [39] synthesized counterweights for a Watt's six-bar linkage using numerical optimization of an object function that considered the combined effect of shaking force, shaking moment, input torque, and bearing forces. This work was followed by an experimental study of the results of counterweighting upon the six-bar linkage [40]. Tricamo and Lowen [33], extending the method developed in a companion paper [32], used a generalization of their graphical technique combined with numerical optimization to minimize the combined effects of bearing forces, input torque, and shaking moment while synthesizing counterweights which produce a prescribed maximum shaking force. Lee and Cheng [20] examined the combined balancing of shaking force, shaking moment, and input torque of a four-bar linkage using heuristic optimization. In this study they discretized the solution space into a set of finite points and then used a random-walk type of optimization technique for solution. Rao [25] examined the combined shaking force, shaking moment, and input torque of a four-bar linkage using multiobjective function optimization techniques.

In summary, there are two major trends in passive balancing. The first is that work which considers full force-balancing first (either by the Method of Linearly Independent Vectors or one of its derivatives or related methods) and then attempts to optimize the remaining dynamic properties of the mechanism.

A considerable amount of analytical work has been done with this approach, and the solutions and their limitations are well known. Criterion exist for judging whether or not the linkage can be fully force-balanced; and if it can, the techniques for performing the full force-balance are relatively simple. Optimizing the other properties is not as direct, but it can be accomplished. However, these techniques have only been applied to mechanisms containing lower-order pairs (i.e. revolute and prismatic joints) and not mechanisms which contain higher order kinematic pairs (e.g. cams), and thus the number of mechanisms to which it can be applied is limited. Furthermore, the insistance upon full force-balancing as a prerequisite for further balancing is questionable, because of the constraints it imposes upon the designer's choice of what he wishes to achieve by balancing.

The second major trend is the use of numerical optimization. Almost any criterion of success can be used with numerical optimization, but it can be difficult to obtain reliable results, and often a great deal of guesswork and experience is required. It can also be very expensive in terms of computer time. However, this procedure allows great flexibility in terms of what the designer chooses to optimize and how he chooses to perform the analysis. He can attempt to optimize virtually any of all of the dynamic properties of any mechanism for which a description of the mechanism's kinematic behavior exists.

There are other techniques, such as those of Tricamo and Lowen [32, 33] and Urba [34, 35, 36], whose work does not fall into the above stated categories. They were mentioned because the uniqueness of their methods may have promise in providing a base for further analytical work, and, at some time in the future, may yield techniques with more general applicability.

After reviewing the literature, given the objectives of this investigation, the most viable solution technique was deemed to be a numerical optimization method. One reason for not choosing the methods of the first category is that their insistance upon full force-balancing was considered to restrain unnecessarily the designer's choices in balancing. Futhermore, they are applicable only to mechanisms having lower order pairs, which further limits severely the range of mechanisms to which the balancing program would be applicable. The techniques other than non-linear optimization mentioned above which have the capability of overcoming these defects, such as the work of Urba and the methods developed by Elliot and Tesar, have not been been refined to the point where they are generally applicable. Thus, the logical choice is that of numerical optimization.

The optimization method is the only method available at this time which can treat a wide variety of mechanisms and allows for designer flexibility. It may not be attractive in the sense of embodying elegant mathematics, as it is a brute-force technique which draws heavily upon the ever-increasing speed of computers to make it attractive. At some future time, analytical techniques may be developed that will be suitably general and flexible to meet the objectives set forth in the Introduction; but as for now, such analytical techniques do not exist and non-linear optimization is the best technique available.

Chapter 3
Description of Balancing Program

An interactive program for the balancing of mechanisms was written to satisfy the following goals:

1. It should be able to work with a wide variety of linkages and mechanisms.

2. It must not require an undo understanding of the analytical and kinematic aspects of a machine in order to be used, i.e., it must be usable by an average engineer after some limited training.

3. It must allow the user flexibility in terms of achieving the effects he desires.

The following is a description of the program's use, salient features, and overall organization. In Figure 3-1 is shown an outline of the operational procedure of the balancing program. The details and tasks indicated in the figure are discussed in the material that follows.

Program Preparation

In order to understand the function and use of the balancing program, it is necessary to remember where, in the usual strategy of design (as described in the Introduction), the balancing program is used by the designer and to what end. The designer should have completed both the first design stage (type synthesis of the mechanism and synthesis of geometric parameters and input motion necessary to meet the kinematic requirements of the mechanism) and the second design stage (designing the size and shapes of the mechanical elements so as to embody the geometric parameters chosen in the first stage and to meet static strength and materials requirements). The balancing program is then used to design counterweights for the links so as to improve the dynamic

1. **Program Preparation**

   a. Design mechanism to produce output motion; design trial geometry for links.

   b. Decide what dynamic properties should be modified, and to what extent any one property should be optimized at the expense of other properties, as well as any quantitive limits or requirements on any dynamic property.

   c. Design the object function to be used.

2. **Program Initialization**

   a. Enter kinematic data of mechanism, mass properties of trial geometry,

   b. Enter state of mass properties of counterweights (i.e. identifying each mass property as constant or variable), bounds on variables, and initial guess for counterweights.

   c. Enter object function weights, penalty function parameters.

3. **Program Operation** -- Loop until user quits

   a. Enter number of simplex steps to be taken, go, view plots.

   b. Reinitialize simplex, enter number of steps to be taken, go, view plots.

   c. Edit current value of mass properties, state of mass properties, bounds on mass properties, object function weights, penalty function parameters.

Figure 3-1: Outline of Balancing Program Operation properties of the mechanism. Thus the balancing program is used in the third stage of the usual strategy of design, with the important exception that the "adjustments" occur while the mechanism is still on the drawing board.

Before initializing the balancing program, the designer must contemplate how sensitive the machine will be with regard to the shaking force, shaking moment, and input torque that the mechanism being designed will produce. He must come to an understanding of what kind of shaking force, etc. the machine can tolerate, or will require, to guarantee smooth operation. An understanding of what sacrifices can be made in the optimization of certain properties relative to other properites is also required, as it may not be possible to obtain exactly what the designer wishes. Hence, to get the "best" solution, he may have to modify his goals after he has a better idea of what he can achieve. The word "best" is in quotes because the decision of what constitutes "best" is extremely subjective and is based to a great extent on intuition and experience.

The designer must next design an object function which will provide a measure of the quantites to be minimized in the balancing program. He may feel it necessary to examine only one dynamic property, or he may feel it necessary to include a relatively large number of properties in the object function. This process must be done with some care and foresight, as otherwise getting the desired overall results is virtually impossible.

The general form of the object function, E, consists of a "root-mean-square-like" integral of the weighted sum of the squares of the dynamic properties of interest, I, plus penalty function terms, D and F.

$$E = I + D + F$$

The integral forms the major part of the object function; the penalty functions are used to limit the ranges of the variables and to penalize certain situations the designer deems unacceptible, i.e. when the force at a bearing exceeds the safe design limit.

The integral portion of the object function, I, is this:

$$I = \int_{cycle} \sqrt{\sum_{i=1}^{n} \alpha_i \left(\frac{P_i}{V_i}\right)^2}\, d\theta,$$

where

1. P_i is the i-th physical term (physical terms are the dynamic properties of interest, i.e. the shaking force in the X-direction, the input torque, etc).

2. V_i is the maximum absolute value of the i-th physical term over the input cycle for the uncounterweighted mechanism.

3. α_i are the weighting factors for the sum of squares of the physical terms, reflecting the amount the i-th physical term should be optimized relative to the other physical terms. The larger the α_i , the greater the importance of the term.

4. n is the number of physical terms in the summation.

5. "Cycle" refers to the full cycle of the input motion of the mechanism.

The penalty function on the range of variables, D, is of the form

$$D = \sum_{i=1}^{m} (G(x - l_i) + G(u_i - x)),$$

and the penalty function for the prescribed design limits on the joint forces is

$$F = \sum_{i=1}^{l} G(F_i - D_i),$$

where

$$G(q) = \begin{cases} H \times q & \text{if } q < -\epsilon \\ 0 & \text{if } |q| \leq \epsilon \\ R/q & \text{if } q > \epsilon \end{cases} .$$

Here

1. m is the number of variables.

2. G is the actual penalty for a violation of a bound by an amount q (G is not calculated for a variable or joint force if that bound or design limit is not being enforced).

3. u_i is the upper bound on the i-th variable.

4. l_i is the lower bound on the i-th variable.

5. H is a large number (e.g. 10^{10}).

6. R is the sharpness factor, which is set and modified by the user.

7. ϵ is a small number (e.g. 10^{-10}).

8. l is the number of joints.

9. F_i is the force on the i-th joint.

10. D_i is the design limit for the force on the i-th joint.

In practice, the integral is evaluated at a finite number of points around the input cycle of the mechanism, and thus when something is said to be evaluated over the input cycle of the mechanism, it should be taken to mean over the finite set of points in the input cycle of the mechanism used to evaluate the integral.

The penalty function "sharpness" factor determines how "sharply" the penalty function is applied as a variable approaches a bound. The larger the number, the more severely excursions toward the boundary (from inside the allowable region) are penalized. However, because of magnitude of penalization of excursions beyond the bound, the larger the sharpness factor, the more smooth the transition going from inside the bound to outside the bound. This property is important, because numerical optimization methods will frequently "stall" against a bound in space which is too sharp. Therefore, the user should begin with a relatively large value of the sharpness parameter (about 10 is usually good). Then, when the solution appears to be converging (especially to a point near a boundary), the sharpness parameter should be decreased and the search should continue. Each time the solution converges, the sharpness parameter should be decreased until the user is satisfied with the solution. There are no hard and fast rules for using a sharpness parameter such as this; experience is the best guide, and fortunately exact choices are rarely needed to get a solution.

Program Initialization

The initialization of the balancing program consists of inputting the results obtained from the first two stages of design, namely:

1. The type of mechanism and all necessary geometric parameters (e.g. link lengths, cam profiles).
2. The input motion.
3. The external forces and torques.
4. The mass properties. For a planar linkage these are the mass of the link, polar radius and polar angle of the center of mass of the link relative to some reference joint (as dictated by the kinematic analysis routines), and the centroidal inertia, all specified in a frame located on that member, of each of the moving members of the uncounterweighted mechanism.
5. The simplex optimizer step sizes.

The designer must specify the state of each of the mass properties of the counterweights for each member of the mechanism. The state of a mass property refers to whether that mass property will be considered by the program as a variable or a constant. If the designer chooses that mass property to be variable, he must then specify whether or not the absolute value of that mass property will be used. If he chooses not to use the absolute value of that mass property, he must then decide if he wishes to enforce a lower bound on that mass property (i.e. to keep the optimizer from choosing a value below that of the lower bound). The designer must also decide, whether or not he has chosen to use the absolute value, if he wishes to enforce an upper bound on that variable (i.e. to keep the optimizer from choosing a value above that of the upper bound). The state of any or all of the mass properties can be changed at any time in the operation of the program.

The designer must also provide an initial guess for the mass properties of the counterweights. This initial guess will be treated as the current best estimate of the optimum mass properties of the counterweights for the mechanism. The optimizer will then attempt to find continually better estimates for the optimum mass properties of the counterweights.

The designer must enter the object function and the object function weights (which express the relative importance of the terms of the object function) and the penalty function sharpness parameter. These can be changed by the user at any time during the operation of the program.

Finally, the designer must enter the simplex optimizer step sizes. These are used when the optimizer initializes or reinitializes the simplex. A step size must be given for each variable. The value of the step size is approximately that for which, holding all the other varibles constant, a Taylor series centered at the initial guess, carried to the first-order term, would be accurate. Fortunately, these values need only be approximate, although it is always best to make them larger rather than smaller.

Program Operation

The operational mode of the program is a cycle or loop. At any time during the operation of the program, there exists a current best guess for the optimum set of counterweights for the mechanism being balanced. As the program goes through the loop, the goal of the program is to improve the current best estimate. It must be kept in mind that, as the program is operating, it always deals with this current best guess.

The loop begins with the user telling the program how many simplex steps he wishes to take. The program takes that number of steps, pauses, and then draws a series of plots. These plots provide information about about the recent progress of the opimizer and the degree to which the current best estimate has modifed the dynamic properties of the mechanism. This information forms the basis from which the designer will choose the course of action which will be taken next.

The first display is a plot of the normalized average object function value of the simplex versus the step number. This information gives the designer idea of what kind of progress the numerical optimization routine is making.

Next the program draws a plot of the square root of the weighted sum of squares of the terms of the object function (the integrand of the main term of the object function) versus input variable (e.g. crank angle). Following this, the program sequentially displays the plots of each of the physical terms used in defining the object function versus the input variable. On these plots, two lines are drawn, one for that quantity for the uncounterweighted linkage and one for the linkage with the current best estimate of the counterweights. This allows the designer to see how the dynamic properties have been modified from those of the uncounterweighted linkage.

From these plots, the user must decide what to do next.

If he feels that the optimizer is making good progress towards the kind of solution that he wants, he should go to the top of the loop and continue.

If he feels that the optimizer has just about reached a minimum and wants to be sure, he can perturb the current best estimate (by changing the value of a variable slightly) and go to the top of the loop. After the optimizer has taken some more steps and he sees the series of plots, he should look to see whether or not the optimizer has begun to make any significant progress toward a new set of values. If it has, the numerical optimizer most likely had become stalled in a "corner" of the surface of the object function, and disturbing the point helped to get the optimizer going again. If it has not, the optimizer has probably reached a local minimum, and the designer should output the results and quit the loop, either to try to find another solution or to make use of the results he has.

He may choose to edit the current best-guess mass properties of the counterweights and restart the process. This is simply equivalent to choosing another initial guess.

If one or more of the variables on which he put no bounds are getting into regions where he does not wish them to be, he may choose to add or to modify the bounds on some or all the variables.

If the properties are not being modified as he deems appropriate, he may choose to change the weights of the object function.

If he feels the optimizer is converging to a solution which has variables near their limits, he may wish to "sharpen" the penalty function by reducing the value of the sharpness parameter.

Finally, he may simply quit.

Program Generalities

In order to understand the structure of the program, it is necessary to describe some of the complexities of numerical optimization. There are two main problems associated with multi-dimensional, non-linear optimization. One is to determine when the optimizer has converged on a minimum. The second is to try to design an object function which accurately incorporates the desires of the designer.

The balancing program deals with these difficulties in a unique way. Unlike most optimizers, the user controls the progress of the optimizer from the keyboard. The graphics portion of the program provides the visual feedback link to the user from which the designer makes decisions and directs the progress of the program.

The balancing program addresses the first difficulty by asking the user to decide if convergence has occured based upon information it presents to the user, namely the plot of the normalized value of the object function versus the iteration counter of the optimizer. "Normalized" in this case means that the current object function value is divided by the value of the object function for the uncounterweighted linkage. Thus, if the line on the plot dips below unity, the object function value of the counterweighted linkage is less than that of the uncounterweighted linkage, and at least some small success has been achieved. The shape of the line serves to indicate if and how well the process is converging. If the line seems to be tending toward an asymptote as the iteration number increases, it would seem that the process is "stalling". The optimizer may either have converged or have hit a region where progress may be slow for a while. In such a case, if the user simply perturbs the point (moving it by a small amount), thereby reinitializing the simplex, he can better discern what has happened. If the optimizer resumes making good progress in minimizing the function, the optimizer had previously reached a region of slow progress. If the optimizer makes little progress but tends back toward the first point, the optimizer has probably converged.

The balancing program tries to address the second difficulty: "What parameter(s) shall I minimize?" by treating the object function as a necessary evil. The program regularly plots the dynamic properties the designer has chosen, such as the shaking force in each direction, the shaking moment, and the input torque, with the property for the counterweighted linkage plotted on the same graph as the property for the uncountweighted linkage. The designer then can see how each of the dynamic properties is being modified. If one property is getting worse rather than better, he can increase the weight on that property to try to cause the optimizer to seek solutions which minimize that property more specifically. In this way, the designer can vary the weights of object function in order to make the object function better reflect his interests.

This program is unique in the sense that, the user is required to use his engineering judgment and physical insight to help guide the program. The use of engineering judgment and physical insight are of course desirable in all facets of design, but this program attempts to encourage the designer to use them and become actively involved in the operation of the program. This is in direct constrast to simply treating the program as a black box into which one dumps data and from which one gets the answer, and, hence, develops no great understanding of either the problem being solved or how the solution was obtained. The degree to which this technique is successful could yield important insight into how to best design the algorithms to be used for all computer-aided-engineering software.

It should be pointed out that this program is the first attempt at a continuously evolving program. The software will continue to evolve with updated versions as users point out deficiencies and features which are seldom if ever useful, and make suggestions for new features. One area of probable redesign is the object function; it is doubtless that the current form will be supplemented by additional forms that will better reflect the users' design decisions.

Chapter 4
Examples

4.1 Slider-Crank Linkage

The first linkage to be investigated using the balancing program will be the slider-crank. Because of its relatively simple kinematic relationships, it is possible to obtain approximate analytical solutions for the shaking forces and shaking moment [24, p.530-537]. Furthermore, because of its use in the internal combustion engine, the slider-crank has been examined in great detail and the basics of balancing it are well known and establish procedures.

The linkage data for this example are contained in Appendix A, as are all data of for the various sets of counterweights. The crank speed was 60.0 radians per second. A schematic drawing of the linkage is given in Figure 4-1.

The balancing solution sought will attempt to find the counterweights for the crank and connecting rod which best balance the linkage. However, since this linkage exists only on paper, no physical design restrictions or considerations exist. Therefore, several of solutions will be obtained, using the balancing program, simply to show the range of solutions possible.

The integral portion of the object function to be used is:

$$I = \int_0^{2\pi} \sqrt{\alpha_1\left(\frac{F_X}{V_X}\right)^2 + \alpha_2\left(\frac{F_Y}{V_Y}\right)^2 + \alpha_3\left(\frac{M_A}{V_M}\right)^2}\ d\theta,$$

where

1. F_X , F_Y , and M_A are the shaking force in the X-direction, the shaking force in the Y-direction, and the shaking moment about the crankpin (A), respectively.

2. V_X , V_Y , and V_M are the maximum absolute values over the cycle

of input motion of F_X , F_Y , and M_A , respectively, for the uncounterweighted linkage, hereafter called divisors.

3. α_1 , α_2 , and α_3 are the weights for F_X , F_Y , and M_A , respectively.

4. θ is the crank angle (input).

For reference, a standard balancing recipe was used, as prescribed in a machine design textbook [24]. This solution is approximate, being obtained from a truncated series solution for the shaking forces and shaking moment, and considers only the first harmonic components of the shaking force and shaking moment.

In the discussion which follows, all references to the value of a dynamic property refer to the percent change of the root-mean-square (RMS) value of that dynamic property from that of the unbalanced linkage. The percent changes in the root-mean-square values of the X-shaking force (F_X), the Y-shaking force (F_Y), and the shaking moment about the crankpin (M_A) are given in Table 4-1 for the various counterweight sets to be examined, along with the weights used to obtain the counterweight sets and the divisors. Plots of F_X , F_Y , and M_A are given in Figures 4-2, 4-3, and 4-4, respectively.

The classical solution reduces F_X , increases F_Y , and zeroes the shaking moment.

The solutions under the columns counterweight sets #1 and #2 were obtained using the balancing program. Counterweight set #1, for which the weights α_1 , α_2 , and α_3 were chosen as specified in Table 4-1, reduces F_X by an amount slightly greater than does the classical solution, increases F_Y by an amount about 13% less than the amount the classical solution does, and reduces M_A by 94.0%, as opposed to a full balance of it, as the classical solution does.

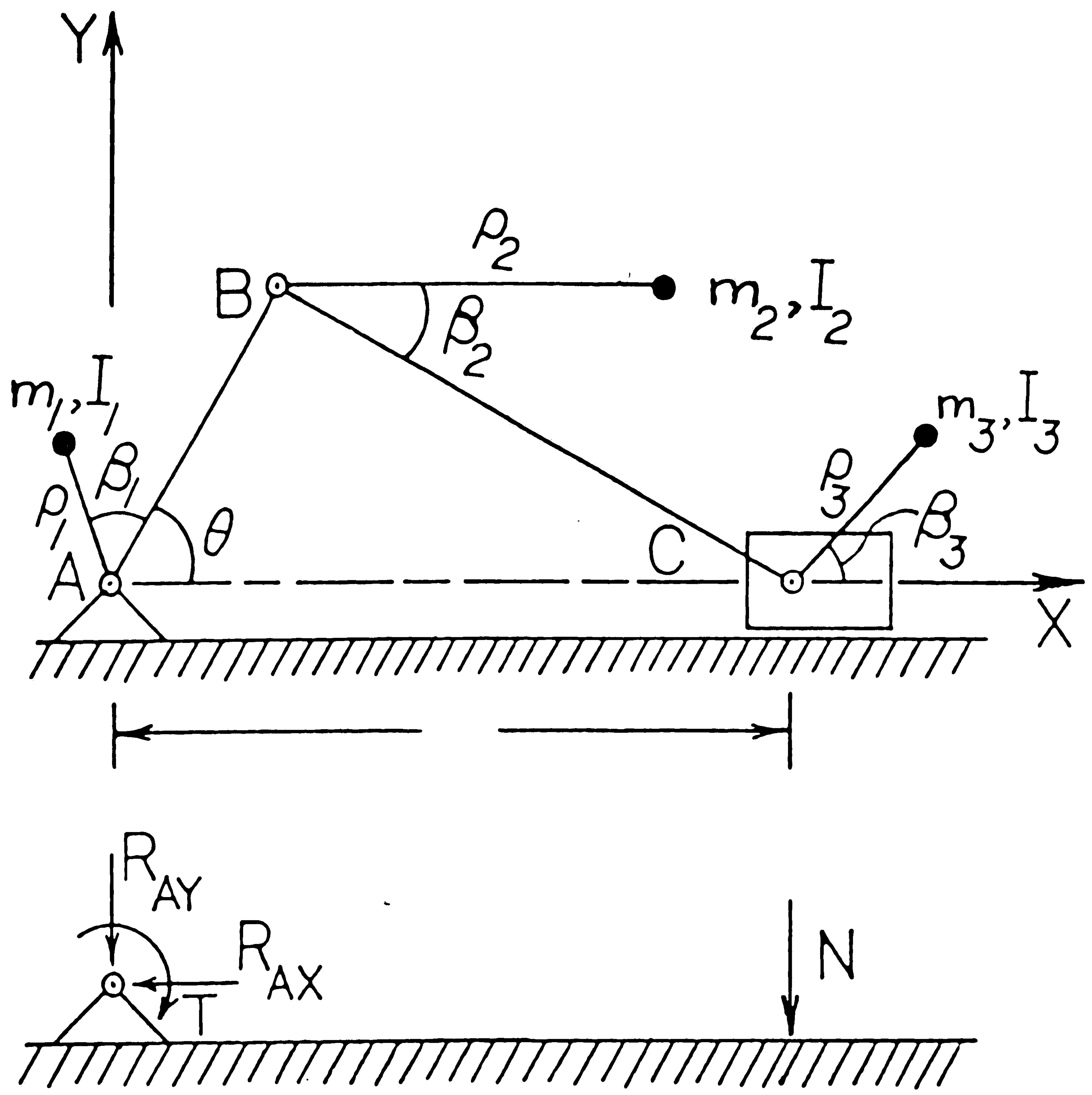
Table 4-1: %RMS Changes, Weights, and Divisors for Slider-Crank

Term	Classical Solution	Counterweight Set #1	Counterweight Set #2
F_X	-50.1%	-53.8%	-41.7%
F_Y	+53.9%	+40.9%	-18.3%
M_A	-100.0%	-94.0%	-93.2%
α_1	-----	10.4011	10.4011
α_2	-----	0.407923	0.815846
α_3	-----	0.0001	0.0001
V_X	-----	1133.80N	1133.80N
V_Y	-----	224.536N	224.536N
V_M	-----	3.51558N·m	3.51558N·m

Thus, the counterweight set #1 yields results similar to those produced by the counterweights chosen by the classical solution, except that it trades off the zeroing of M_A for a greater reduction in F_X and for a smaller increase in F_Y. In most cases, since the magnitude of the shaking moment is relatively small compared to the magnitudes of the shaking forces, this solution would probably be more useful.

Counterweight set #2 reduces all three dynamic properties. The reduction in the X-shaking force is about 9.6% less than that of classical solution. The Y-shaking force is decreased by 18.3%, as opposed the 53.9% increase that the classical solution causes. The reduction in M_A is about 7% less than that of the classical solution. This counterweight set is probably the best of the three for most situations, since it leads to a good reduction in F_X without causing a marked increase in F_Y, with M_A still being reduced markedly. For purposes of comparison, α_g for counterweight set #2 was twice the value used in finding counterweight set #1.

It can be seen that the balancing program has generated solutions which are comparable to the classical solution. The solutions presented are but two of an seemingly infinite number of solutions which could be found; they were chosen to demonstrate the variety of solutions which can be obtained. For these solutions, it was not necessary to enforce bounds on any of the variables to prevent them from becoming obviously unreasonable. For a real mechanism, constraints might exist which cause the counterweight sets found by the balancing program to be unfeasible.



$$F_X = -R_{AX}$$

$$F_Y = -R_{AY} - N$$

$$M_A = -s \circ N - T$$

Figure 4-1: Slider-Crank Linkage

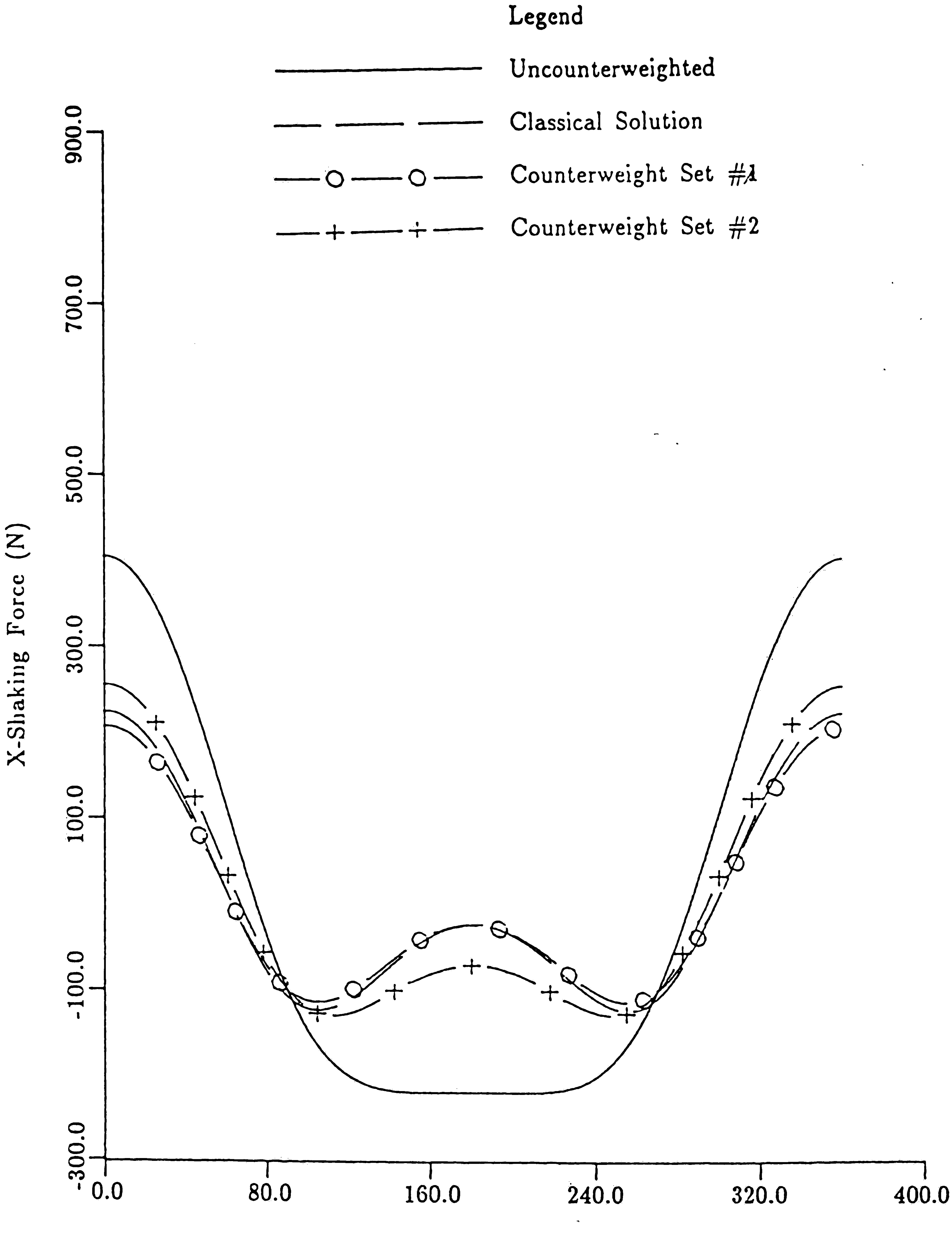


Figure 4-2: X-Shaking Force vs. θ for Slider-Crank

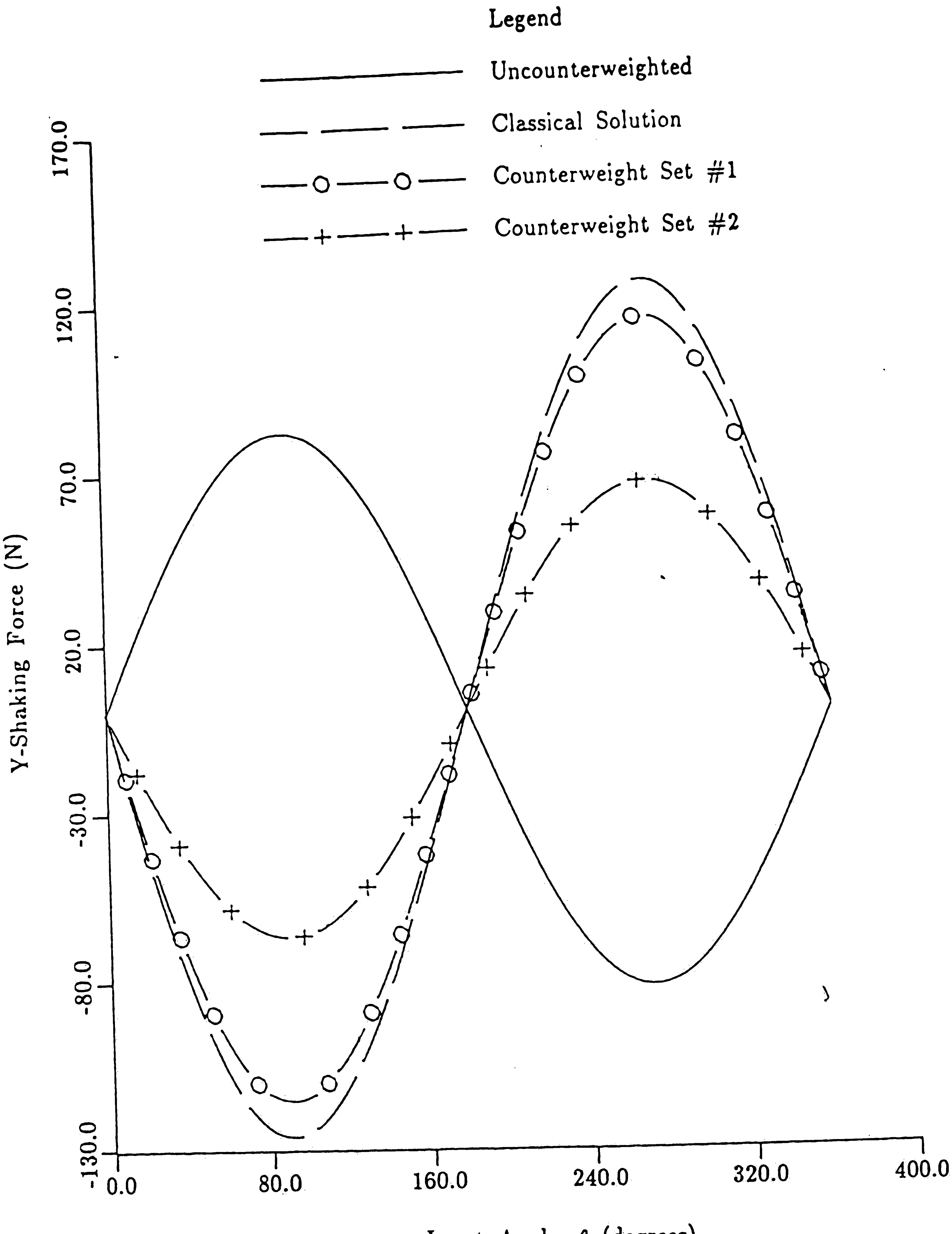


Figure 4-3: Y-Shaking Force vs. θ for Slider-Crank

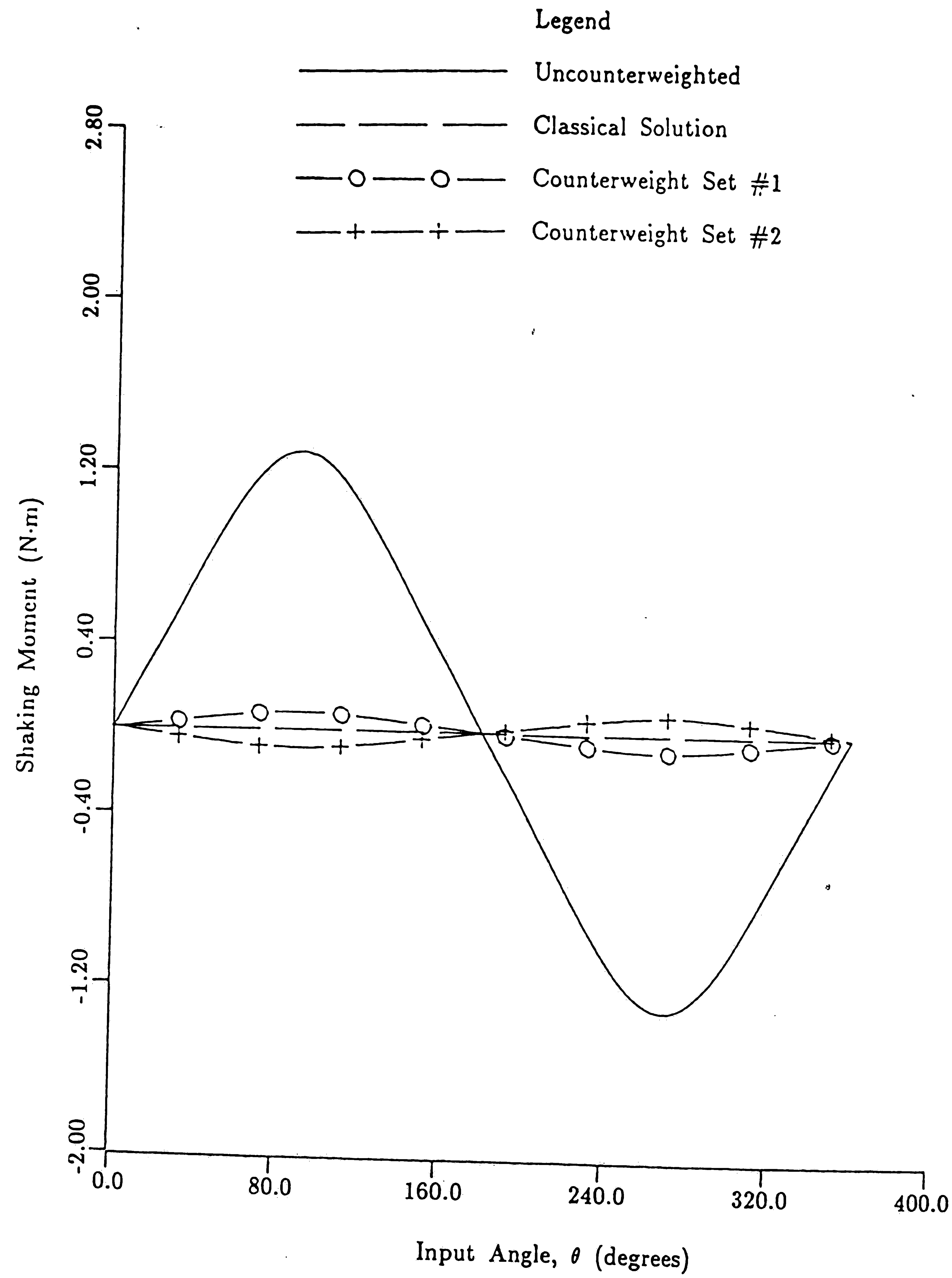


Figure 4-4: Shaking Moment about A vs. θ for Slider-Crank

4.2 Four-Bar Linkage

The second linkage to be examined will be the four-bar. Data for the example linkage and the counterweight sets are in Appendix B. The crank speed was 150.0 radians per second. A schematic drawing of the linkage is given in Figure 4-5.

The combined effects of the shaking forces, the shaking moment, and the input torque will be considered. The integral portion of the object function to be used is:

$$I = \int_0^{2\pi} \sqrt{\alpha_1\left(\frac{F_X}{V_X}\right)^2 + \alpha_2\left(\frac{F_Y}{V_Y}\right)^2 + \alpha_3\left(\frac{M_A}{V_M}\right)^2 + \alpha_4\left(\frac{T}{V_T}\right)^2}\; d\theta,$$

where

1. F_X , F_Y , M_A , T are the shaking force in the X-direction, the shaking force in the Y-direction, the shaking moment about the crankpin (A), and the input torque, respectively.

2. V_X , V_Y , V_M , and V_T are the maximum absolute values over the cycle of input motion of the F_X , F_Y , M_A , and T, respectively, for the uncounterweighted linkage, hereafter called divisors.

3. α_1 , α_2 , α_3 , and α_4 are the weights for F_X , F_Y , and M_A , and T respectively.

4. θ is the crank angle (input).

For purposes of comparison, the Method of Linearly Independent Vectors with least-squares optimization of the shaking moment [5] has been applied to produce a set of counterweights for the example linkage. This technique first determines conditions which zero the shaking forces in the X and Y direction and then minimizes the shaking moment.

In the following discussion, any references to the value of a dynamic property will refer to the percent change of the RMS value of that property from that of the unbalanced linkage. Table 4-2 contains the percent changes of the X shaking force (F_X), the Y-shaking force (F_Y), the shaking moment about the crankpin (M_A), and the input torque (T) for the various counterweight sets, as well as the weights used to obtain the counterweight sets and the divisors. Plots of F_X , F_Y , M_A , and T are given in Figures 4-6, 4-7, 4-8, and 4-9, respectively.

Table 4-2: %RMS Changes, Weights, and Divisors for Four-Bar

Term	Berkof and Lowen	Counter-weight Set #1	Counter-weight Set #2	Counter-weight Set #3
F_X	-100.0%	-73.0%	-51.4%	+48.1%
F_Y	-100.0%	-74.9%	-79.4%	+64.8%
M_A	+127.0	+7.2%	-42.7%	-14.8%
T	+21.1%	-39.4%	+15.5%	-19.5%
α_1	-----	0.025	0.250	0.005
α_2	-----	0.025	0.250	0.005
α_3	-----	0.750	1.000	1.000
α_4	-----	1.500	0.250	1.000
V_X	-----	820.546N	820.546N	820.546N
V_Y	-----	688.508N	688.508N	688.508N
V_M	-----	65.2159N·m	65.2159N·m	65.2159N·m
V_T	-----	12.5423N·m	12.5423N·m	12.5423N·m

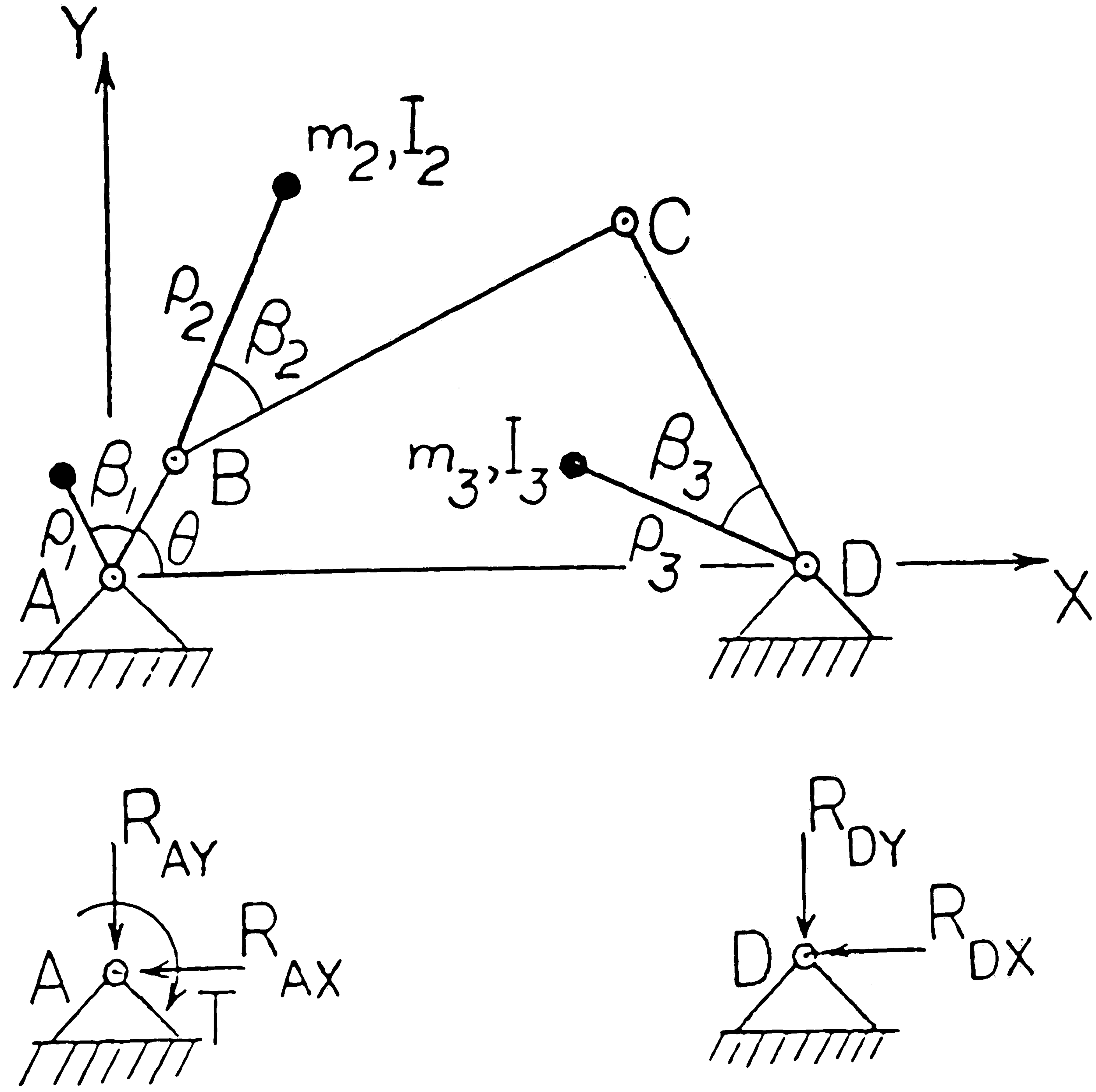
The balancing program was used to find three solutions. The first counterweight set was found with T more heavily weighted than F_X , F_Y , and M_A. Significant reductions in F_X , F_Y , and T are obtained. The shaking moment is increased, however.

Counterweight set #2 was found with M_A weighted more heavily than T, F_X , and F_Y . Good reductions in F_X , F_Y , and M_A are found. T is increased, however.

For comparison, it is seen that the counterweight set found by the method of Berkof and Lowen zeroed both F_X and F_Y (i.e. a full force-balance), while significant increases in M_A and T are induced. Both counterweight sets #1 and #2 offer good reductions in F_X and F_Y , while neither increase either M_A and T as severely as the counterweight set obtained with the method of Berkof and Lowen. In addition, both sets offer a decrease in either M_A or T.

This demonstrates how the insistance upon full force-balancing before balancing other dynamic properties can limit the range of solutions that the designer may obtain. It is most likely that, in actual practice, either of counterweights sets #1 or #2, obtained by the balancing program, would be more useful, as the increases in the shaking moment and input torque of the counterweight set due to Berkof and Lowen's solution would make it unattractive in most situations.

Counterweight set #3 offers reductions in both M_A and T, at the expense of increases in F_X and F_Y . It was found to be very difficult to find a counterweight set which offered improvements in both M_A and T. This is thought to be due to the nature of the kinematics of the mechanism. This example serves to illustrate that it may not always be possible to find exactly the kind of solution the designer desires.



$$F_X = -R_{AX} - R_{DX}$$

$$F_Y = -R_{AY} - R_{DY}$$

$$M_A = -\overline{AD} \cdot R_{DY}$$

Figure 4-5: Four-Bar Linkage

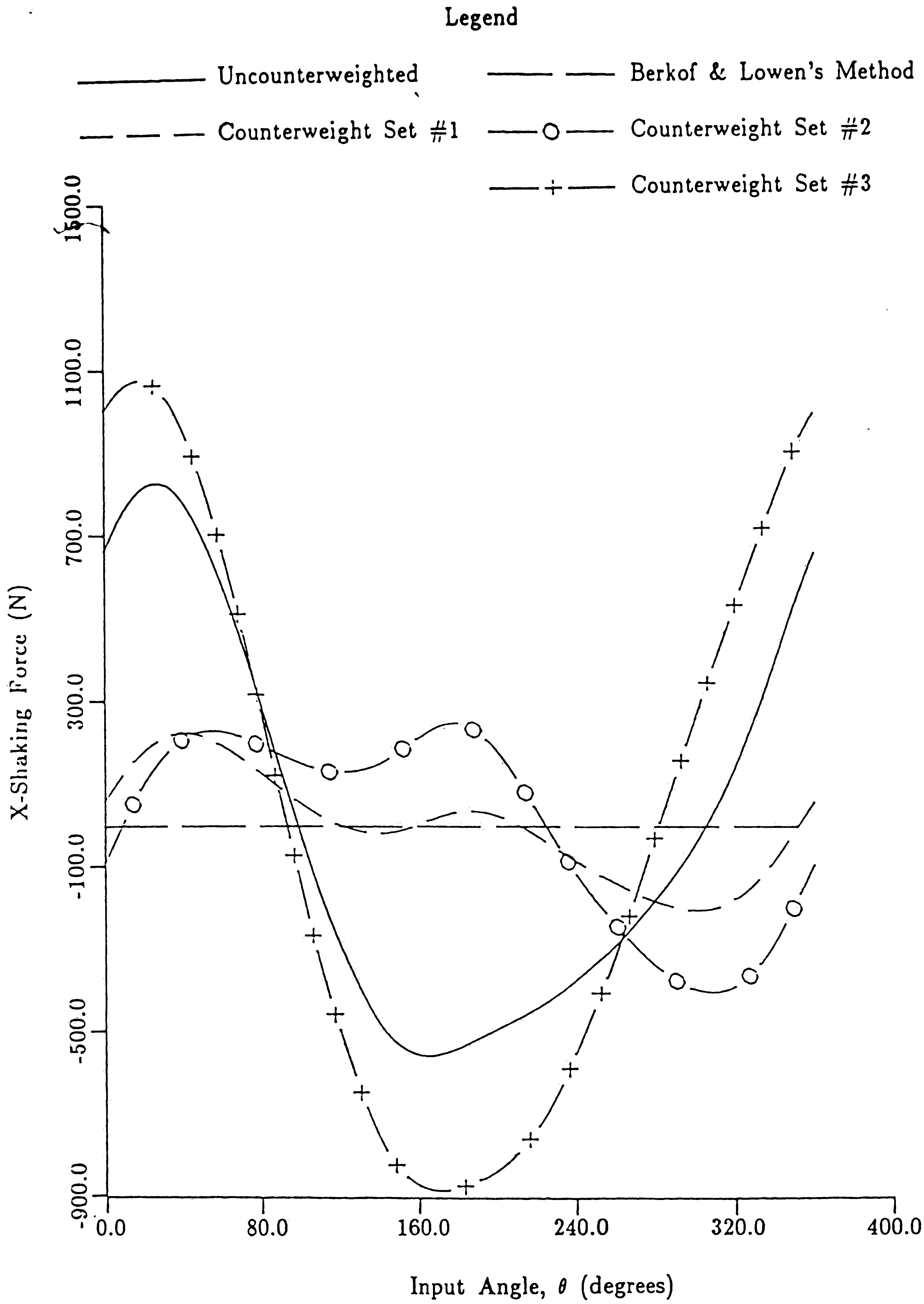


Figure 4-6: X-Shaking Force vs. θ for Four-Bar

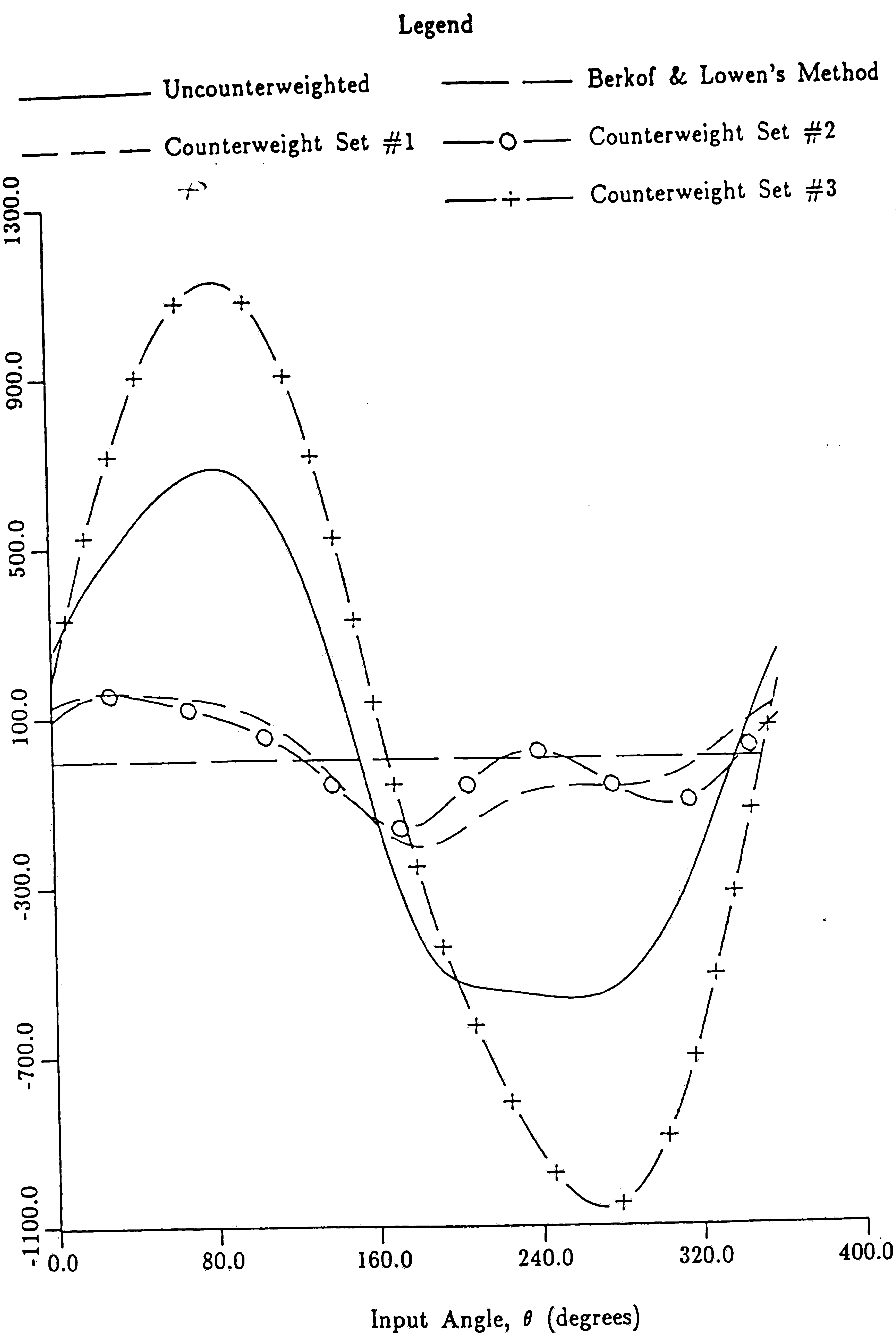


Figure 4-7: Y-Shaking Force vs. θ for Four-Bar

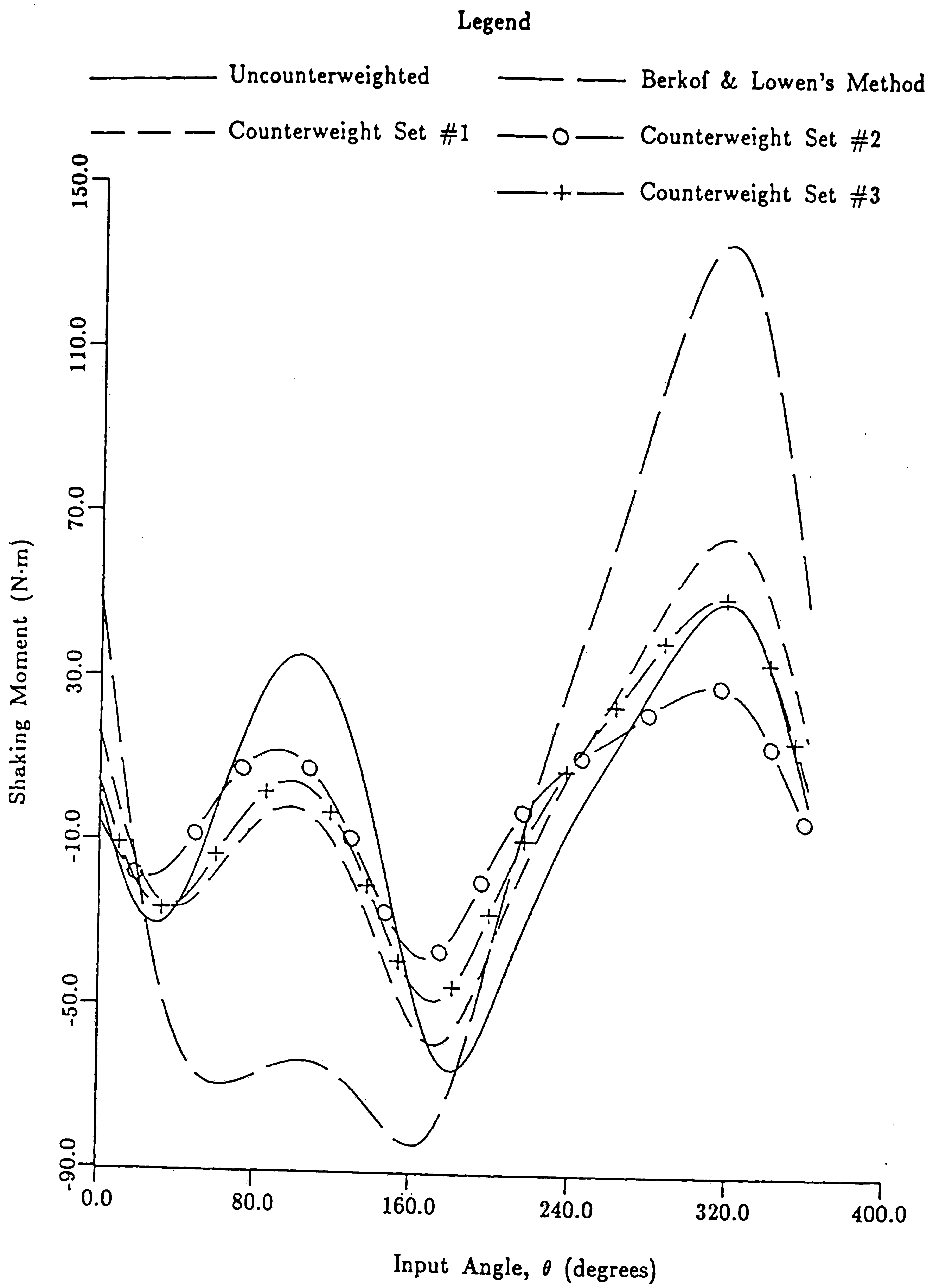


Figure 4-8: Shaking Moment about A vs. θ for Four-Bar

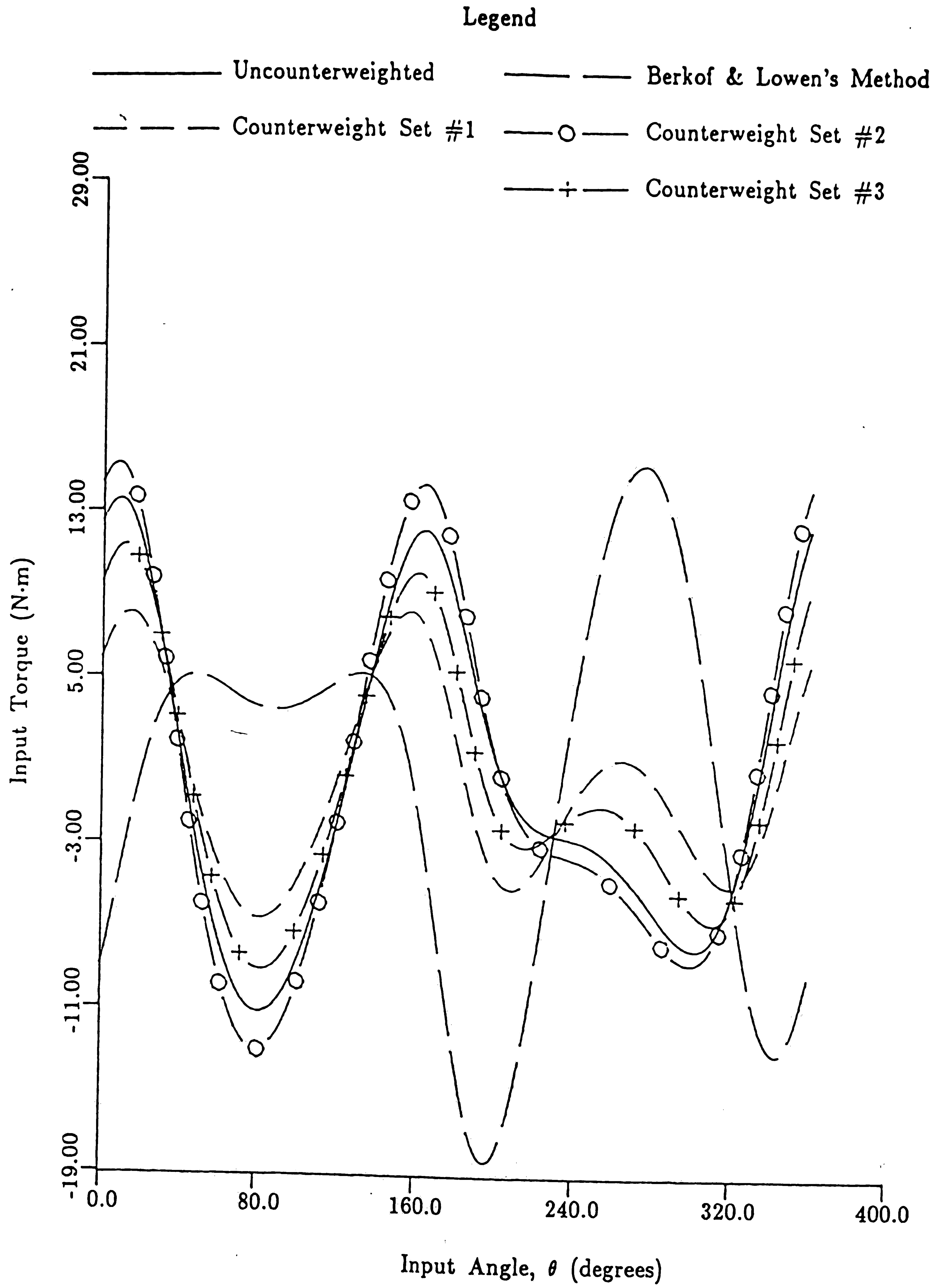


Figure 4-9: Input Torque vs. θ for Four-Bar

4.3 Watt's Six-Bar Linkage

The third and final linkage to be examined will be a Watt's six-bar linkage, as shown in Figure 4-10. Data for the linkage and for the counterweight sets are in Appendix C. The crank speed was 62.83 radians per second.

This linkage has five moving links, all of which may be counterweighted, and thus it is significantly more complex than either of the two linkages previously examined. Furthermore, bounds on variables will be used, as well as design limits on joint forces, adding to the complexity of the problem.

As with the four bar linkage, the combined effects of the shaking forces, the shaking moment, and the input torque will be considered. The integral portion of the object function to be used is:

$$I = \int_0^{2\pi} \sqrt{\alpha_1\left(\frac{F_X}{V_X}\right)^2 + \alpha_2\left(\frac{F_Y}{V_Y}\right)^2 + \alpha_3\left(\frac{M_A}{V_M}\right)^2 + \alpha_4\left(\frac{T}{V_T}\right)^2}\; d\theta,$$

where

1. F_X , F_Y , M_A , T are the shaking force in the X-direction, the shaking force in the Y-direction, the shaking moment about the crankpin (A), and the input torque, respectively;

2. V_X , V_Y , V_M , and V_T are the maximum over the cycle of input motion of the F_X , F_Y , M_A , and T, respectively.

3. α_1 , α_2 , α_3 , and α_4 are the weights for F_X , F_Y , and M_A , and T respectively;

4. θ is the crank angle (input).

For comparison, two counterweight sets given by Walker and Haines have been included, referred to as Walker and Haines set F and set G. These sets were obtained for slightly different considerations (with gravity acting in the Y direction) than those of this investigation (gravity acting in the Z direction), but the effect is rather small compared to the inertial forces and thus these counterweight sets still provide useful comparisons. Set F is an example of an input torque balance, which balances the input torque at the expense of the other dynamic properties. Set G is a full force-balance example.

As before, any references to the value of a dynamic property will refer to the percent change of the RMS value of that property from that of the unbalanced linkage. Table 4-3 contains the percent changes of the X-shaking force (F_X), the Y-shaking force (F_Y), the shaking moment about the crankpin (M_A), and the input torque (T) for the various counterweight sets, as well as weights used in obtaining the counterweight sets and the divisors. Plots of F_X , F_Y , M_A , and T are given in Figures 4-11 through 4-18.

In finding counterweight sets #1 and #2, F_X and F_Y were weighted more heavily than M_A and T , and both counterweight sets reduce F_X and F_Y more than M_A and T. The weights on M_A and T used in finding set #2 were higher than those used in finding set #1; hence set #2 reduces F_X and F_Y less than set #1 while increasing M_A and T less. Neither sets achieve the reductions in F_X and F_Y that Walker and Haines' full force-balance (set G) does, but both counterweight sets produce much lower increases in M_A and T than set G. Because of this, it is more likely that these counterweight sets would be more useful for most machine design situations.

In finding counterweight set #3, M_A and T were weighted more heavily than F_X and F_Y . This set produces reductions in all the dynamic properties, which is unusual, with particularly good reductions in F_Y and T. Since counterweight set #3 reduces T only 8.2% less than Walker and Haines' input torque balance (set F), without increasing the other dynamic properties, it is

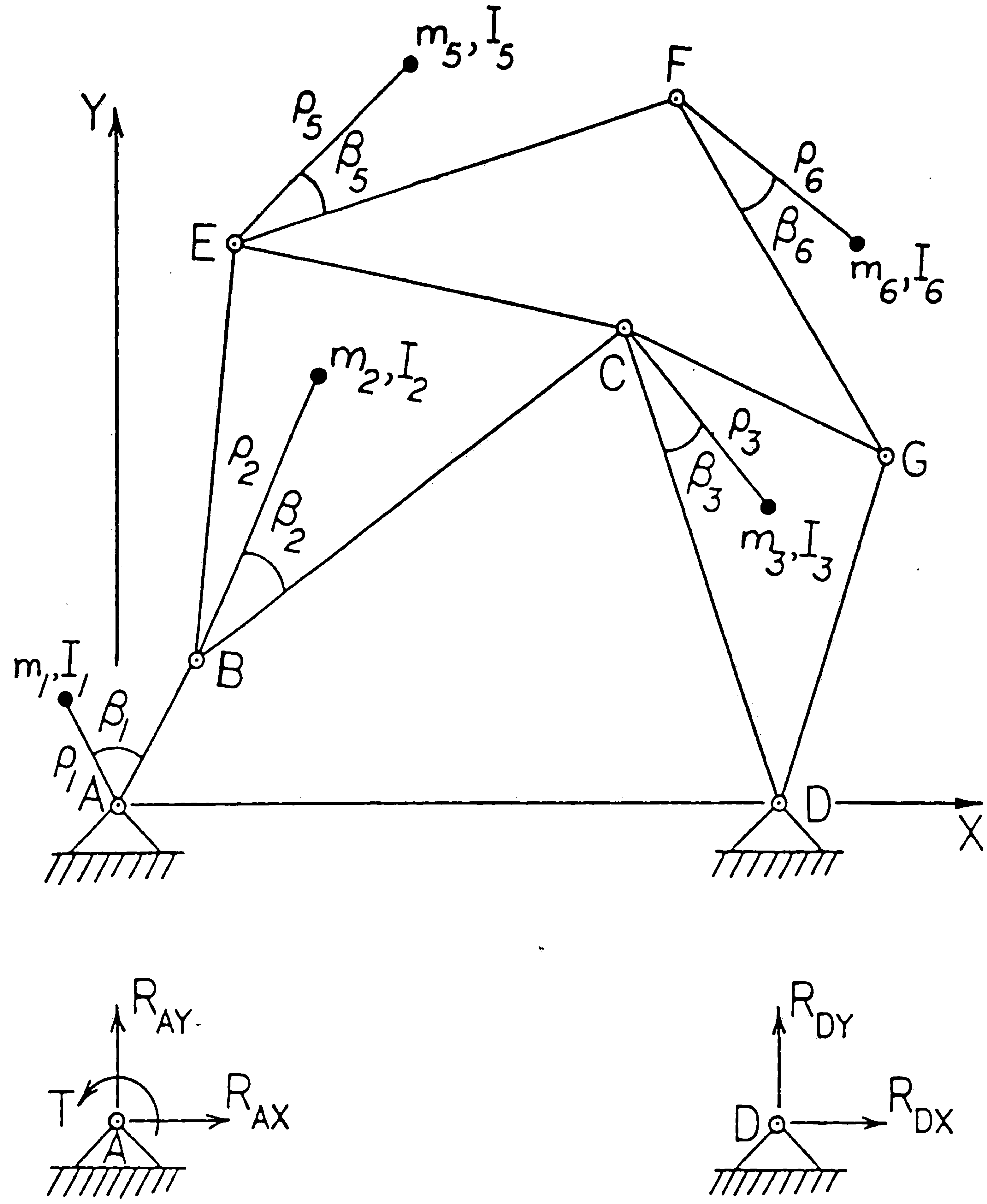
Table 4-3: %RMS Changes, Weights, and Divisors for Watt's Six-Bar

Term	Walker & Haines' [F]	Walker & Haines' [G]	Counter-weight Set #1	Counter-weight Set #2	Counter-weight Set #3
F_X	+48.7%	-99.8%	-72.1%	-73.1%	-13.6%
F_Y	+59.9%	-99.7%	-83.3%	-55.9%	-69.9%
M_A	+33.2%	+165.6%	+43.0%	+28.8%	-4.5%
T	-54.1%	+59.8%	+7.1%	+17.8%	-45.9%
α_1	-----	-----	1.000	1.000	0.100
α_2	-----	-----	1.000	1.000	0.100
α_3	-----	-----	0.200	0.500	1.000
α_4	-----	-----	0.200	0.500	1.000
V_X	-----	-----	643.695N	643.695N	643.695N
V_Y	-----	-----	346.389N	346.389N	346.389N
V_M	-----	-----	395.096N·m	395.096N·m	395.096N·m
V_T	-----	-----	23.4071N·m	23.4071N·m	23.4071N·m

unquestionably a better design choice in most situations.

The bounds applied to variables in this example were applied to the polar angles of the variables, specifically, the bounds on the polar angles were placed at zero and at 2π. This was done only for the sake of testing, as the nature of the trigonometric functions makes this unnecessary. As will be discussed in the Conclusion, the presence of the bounds made operation somewhat more difficult.

Also present in this example were bounds on the joints forces at joints A, B, C, D, E, and F. The three counterweight sets all satisfied the requirements on the joint forces. The bounds on the maximum joint forces, as well as the maximum joint forces found for each counterweight set, are tabulated in Appendix C.



$$F_X = R_{AX} + R_{DX}$$

$$F_Y = R_{AY} + R_{DY}$$

$$M_A = \overline{AD} \circ R_{DY}$$

Figure 4-10: Watt's Six-Bar Linkage

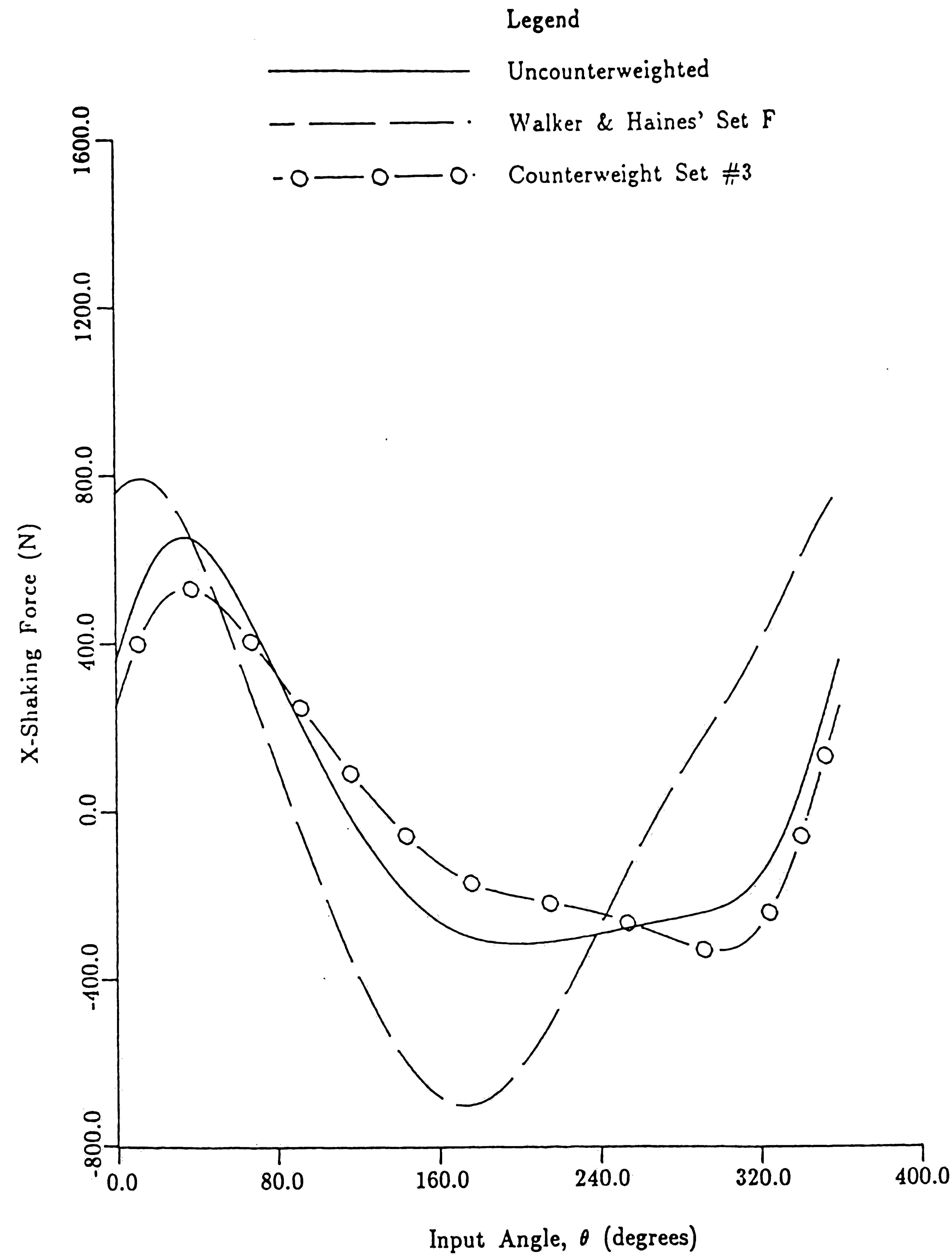


Figure 4-11: X-Shaking Force vs. θ for Six-Bar, Plot #1

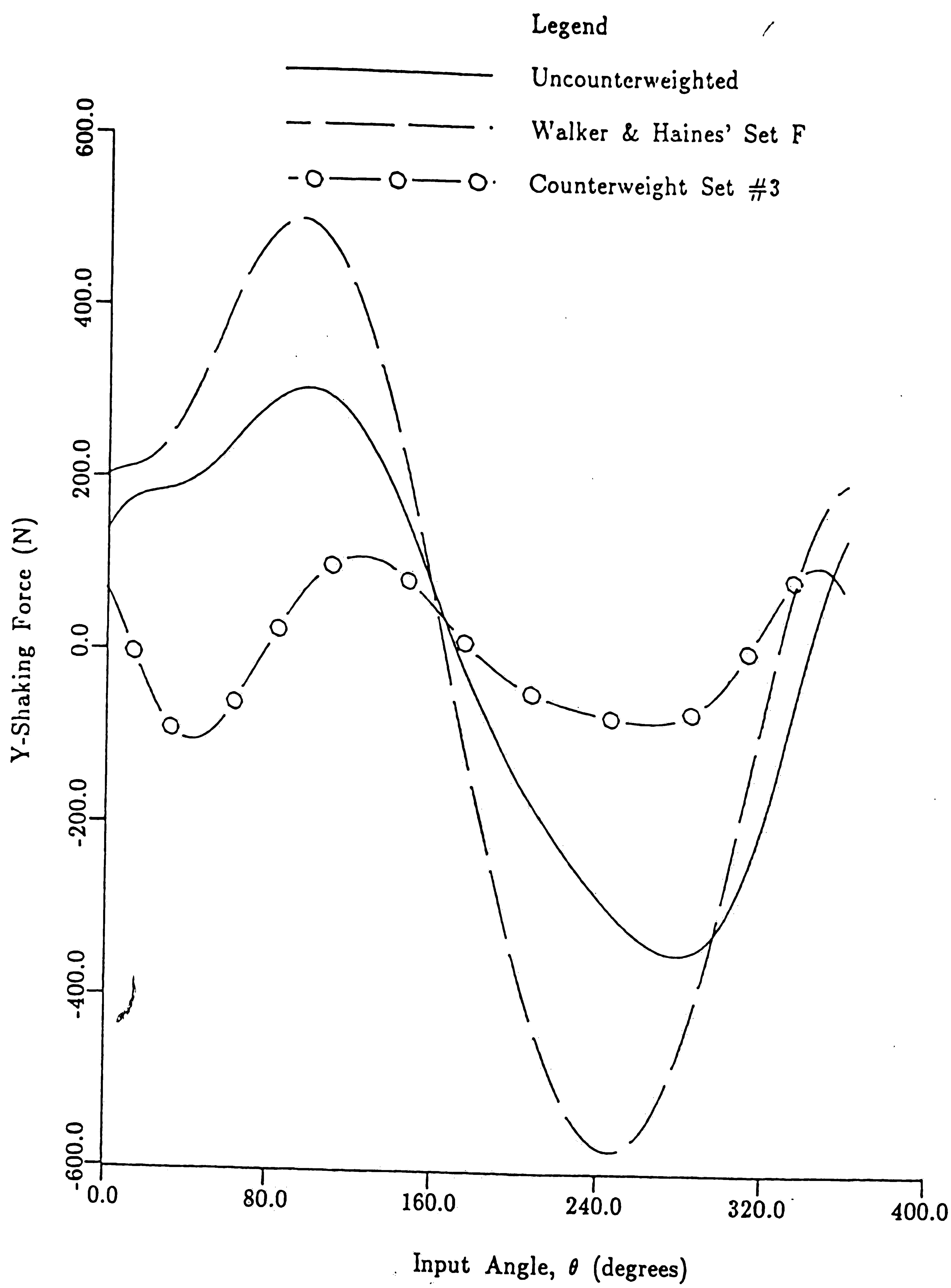


Figure 4-12: Y-Shaking Force vs. θ for Six-Bar, Plot #1

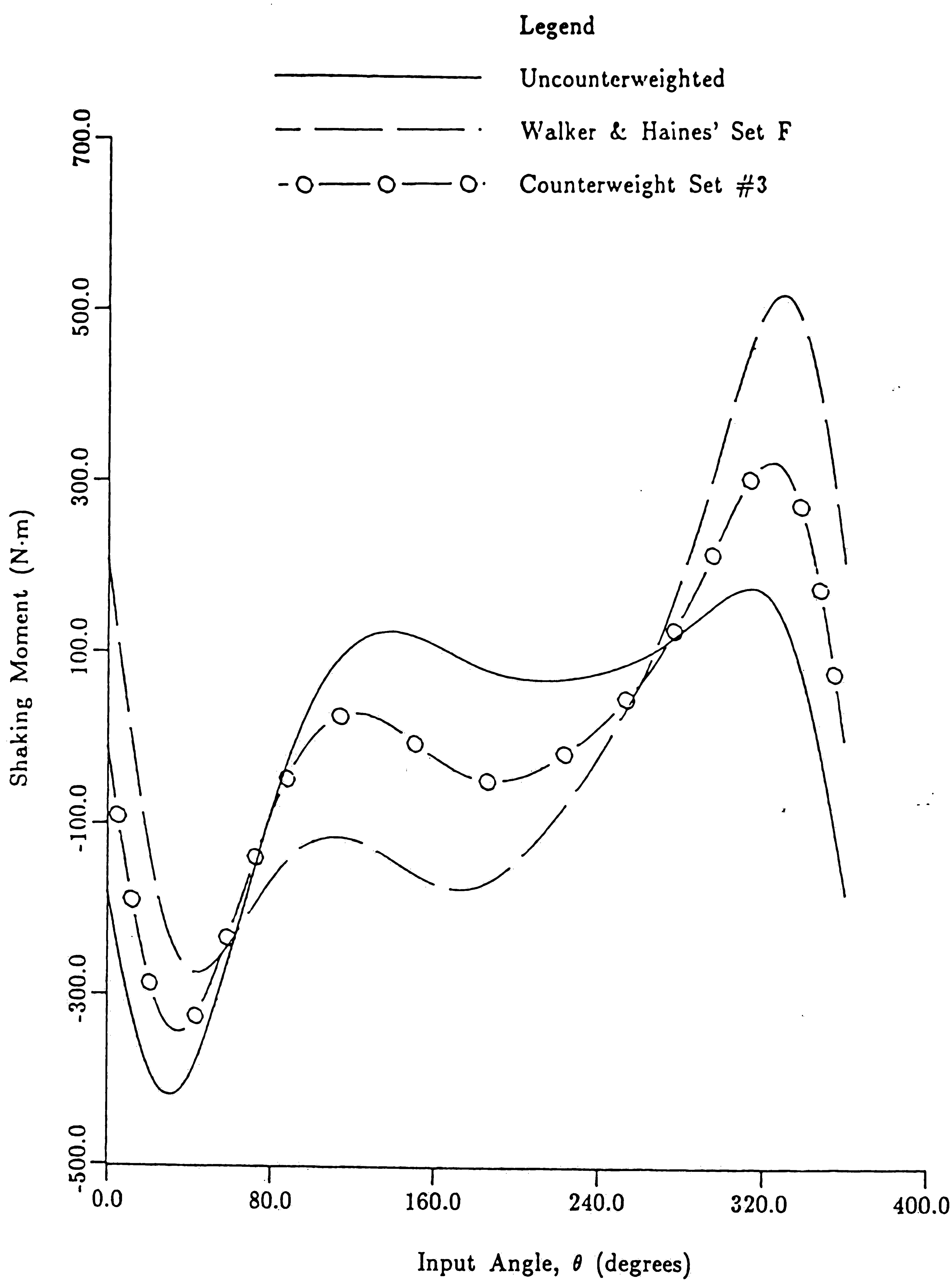


Figure 4-13: Shaking Moment about A vs. θ for Six-Bar, Plot #1

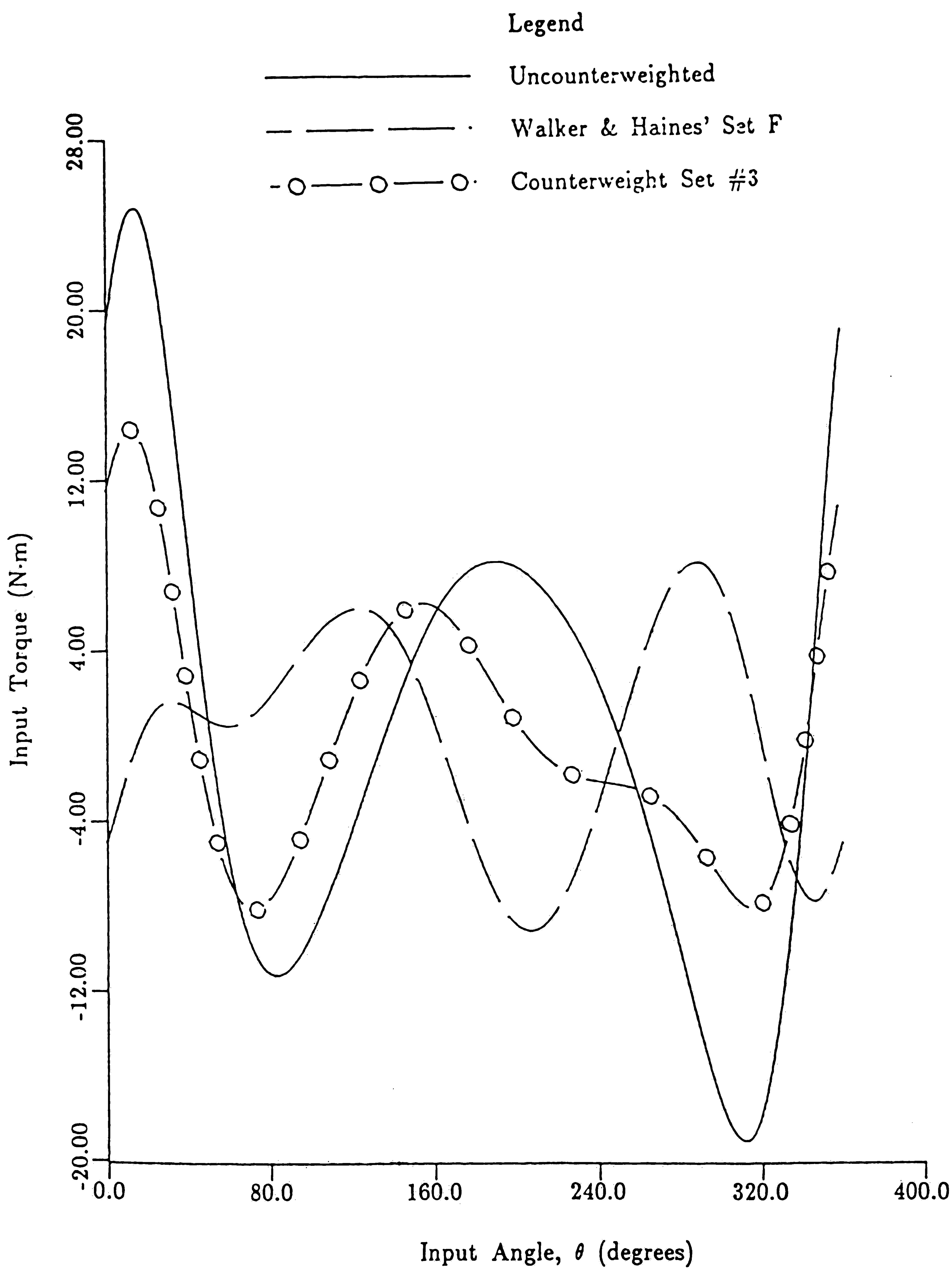


Figure 4-14: Input Torque vs. θ for Six-Bar, Plot #1

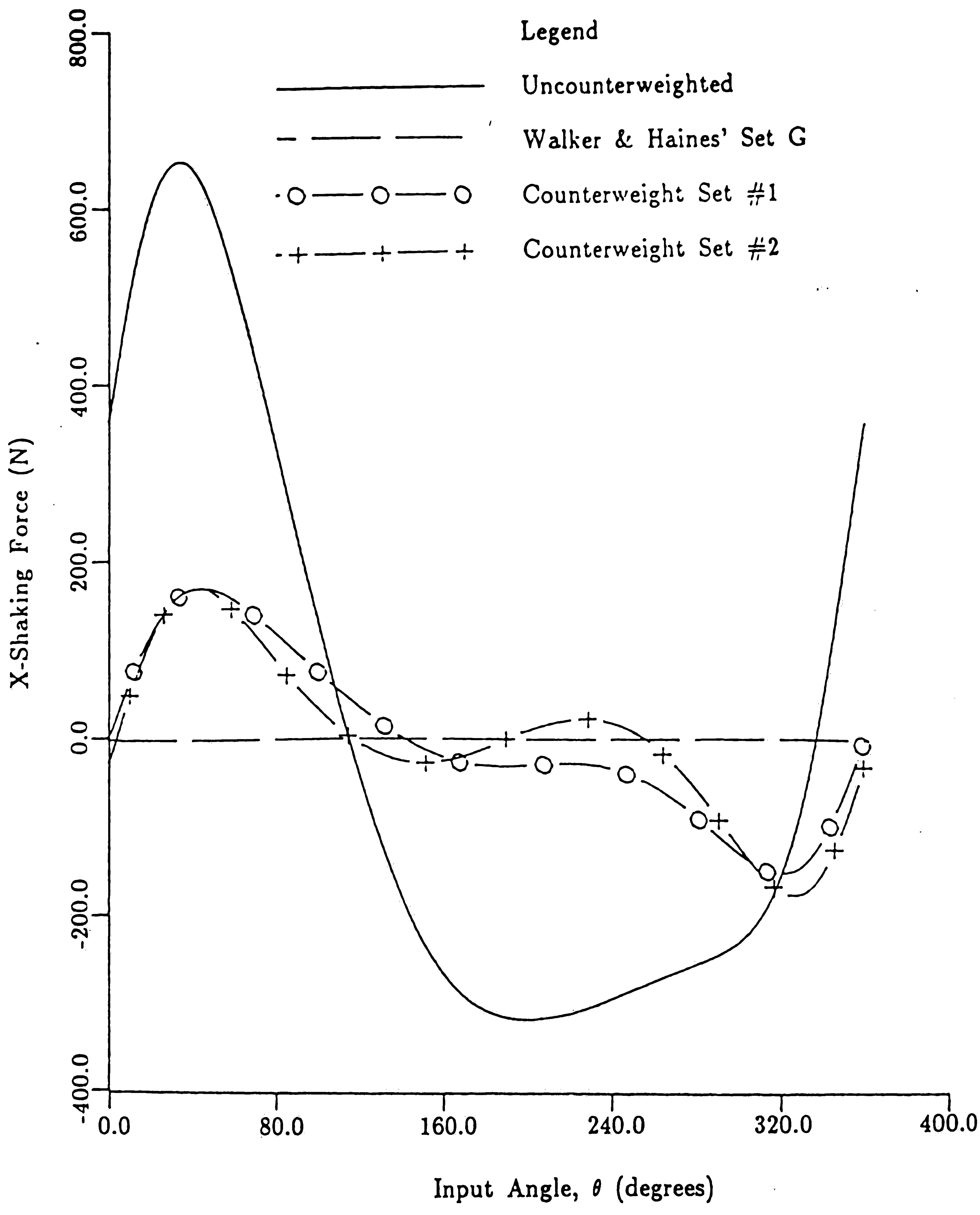


Figure 4-15: X-Shaking Force vs. θ for Six-Bar, Plot #2

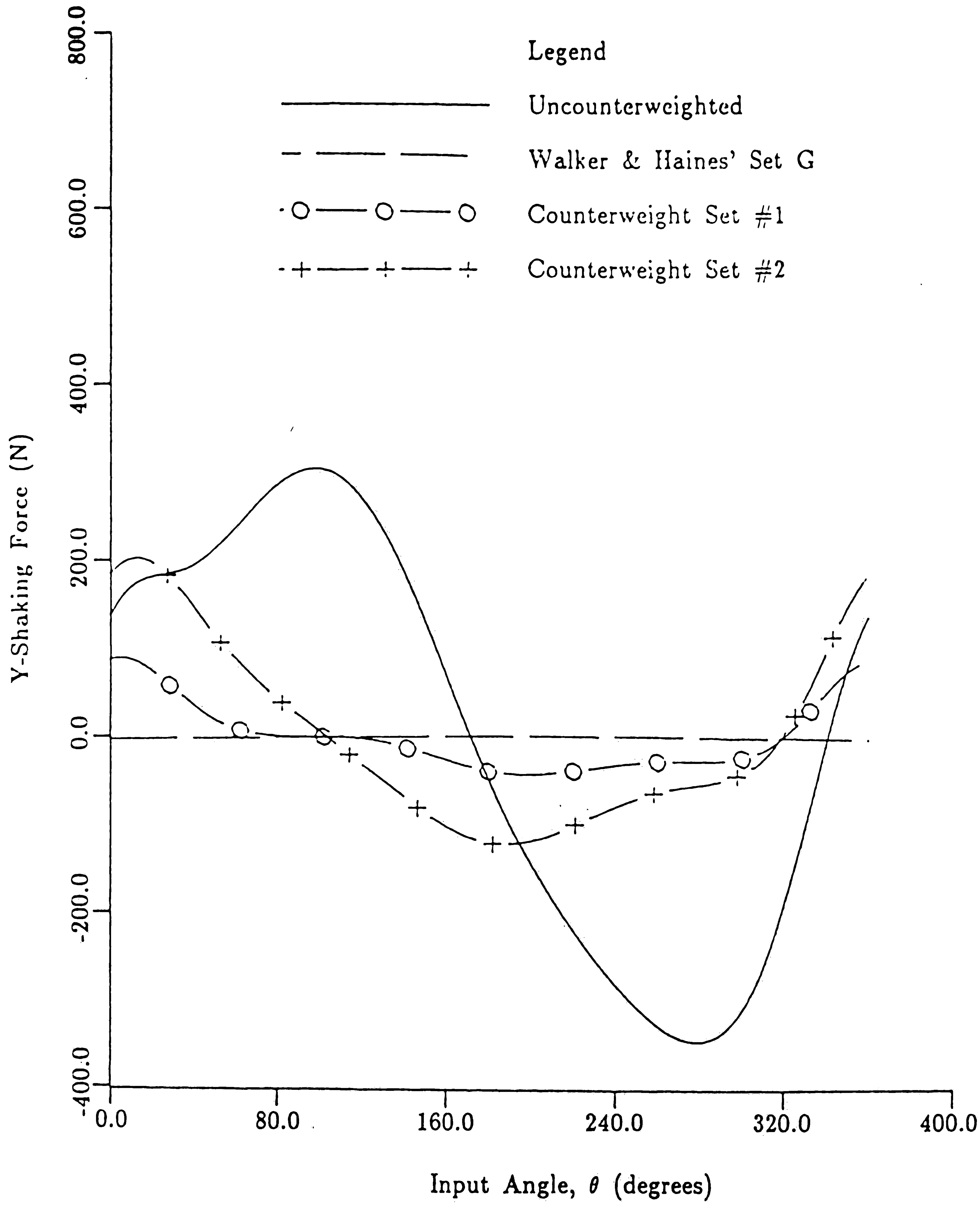


Figure 4-16: Y-Shaking Force vs. θ for Six-Bar, Plot #2

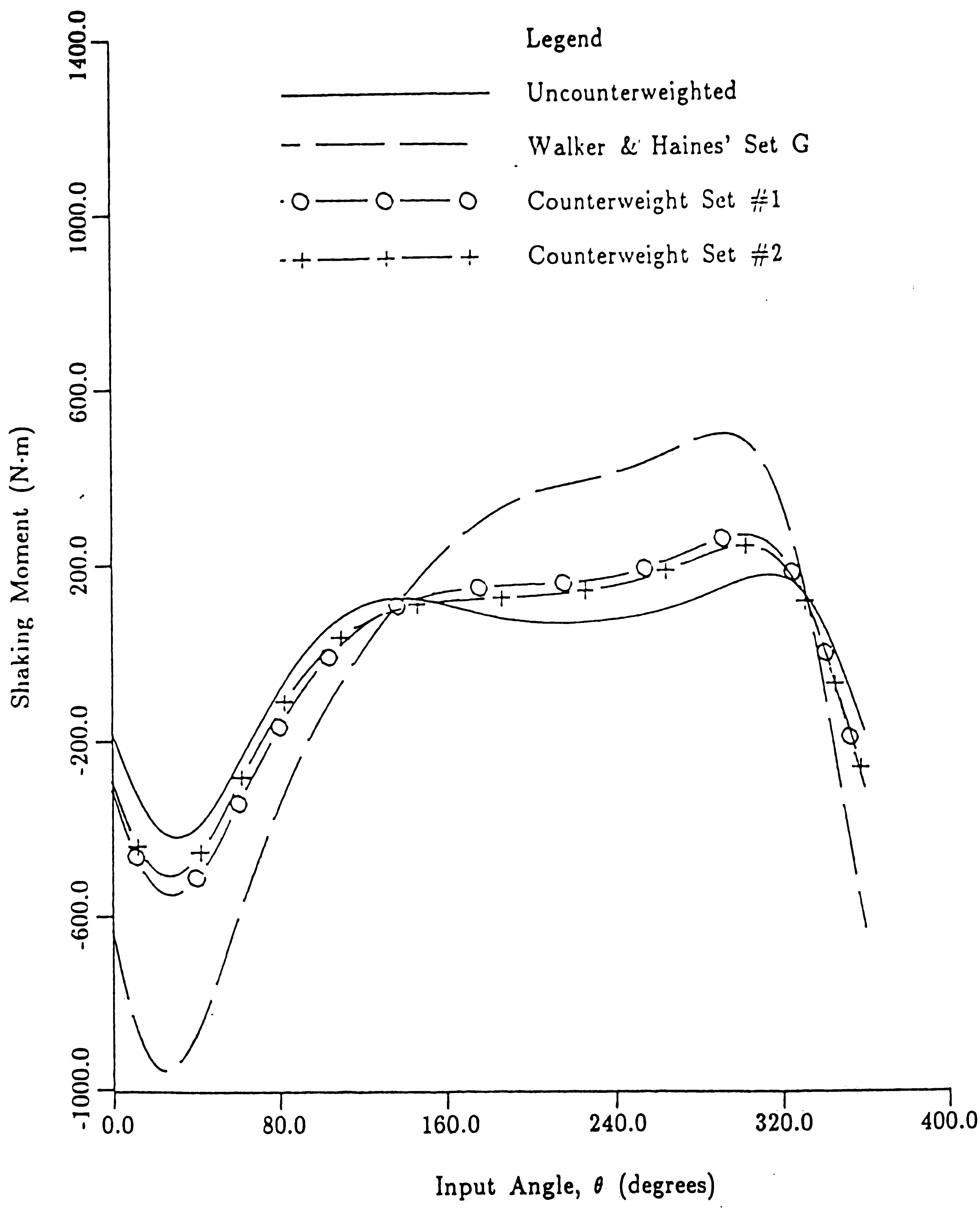


Figure 4-17: Shaking Moment about A vs. θ for Six-Bar, Plot #2

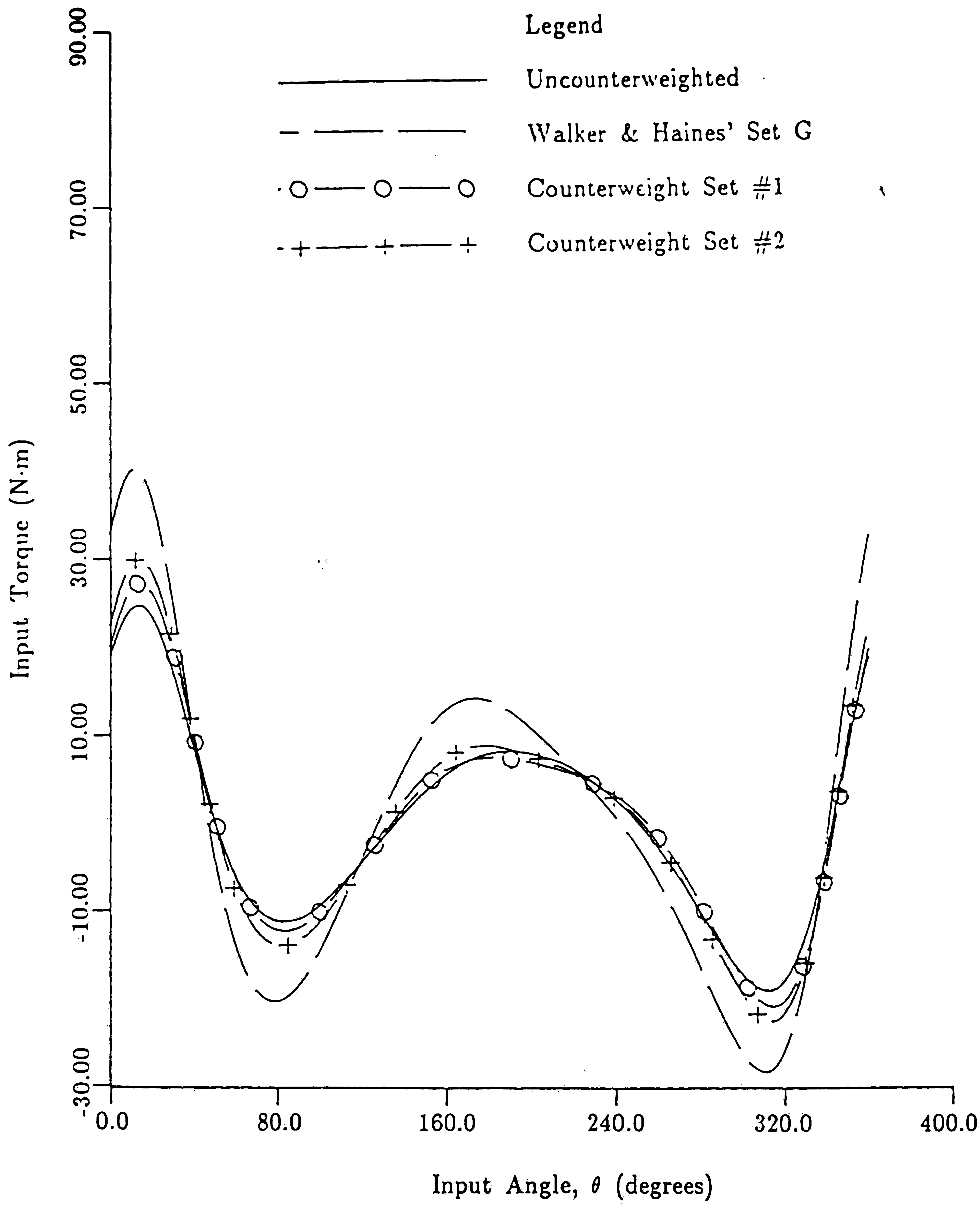


Figure 4-18: Input Torque vs. θ for Six-Bar, Plot #2

Chapter 5
Concluding Remarks

As the examples treated in the previous chapter indicate, the balancing program is capable of producing useful solutions. However, there are several issues concerning its use which need to be pointed out.

First, some observations about the object function are in order. One of the difficulties in designing an appropriate object function involves what the user thinks an object function should describe versus what results it may actually produce. To demonstrate, consider a four-bar parallelogram linkage with an ideal massless coupler, Figure 5-1, operating at a constant speed ω_c, with no external forces or torques exerted on the system. The massless coupler does not make this example unrealistic, since a real coupler whose mass center lies on its line of centers may be reduced (using a kinematically equivalent link [24]) to a coupler with point masses at either end and a massless hoop inertia (located in the midsection of the link). The point masses may then be lumped together with those on the crank (AB) and follower (CD), resulting in the same system as will be treated here, since the assumption of constant input velocity makes no use of the value of the hoop inertia of the kinematically equivalent link.

In addition, it will be assumed that the inertia of the crank and follower are sufficient to avoid any difficulties at the singular points of the linkage. Intuitively one realizes that the shaking force in both directions, the shaking moment about the crankpin, and the input torque can all be made identically zero by adding counterweights such that the mass centers of the crank and follower are located at their points of rotation. However, if one were to attempt to balance this linkage and include only the shaking force (in either

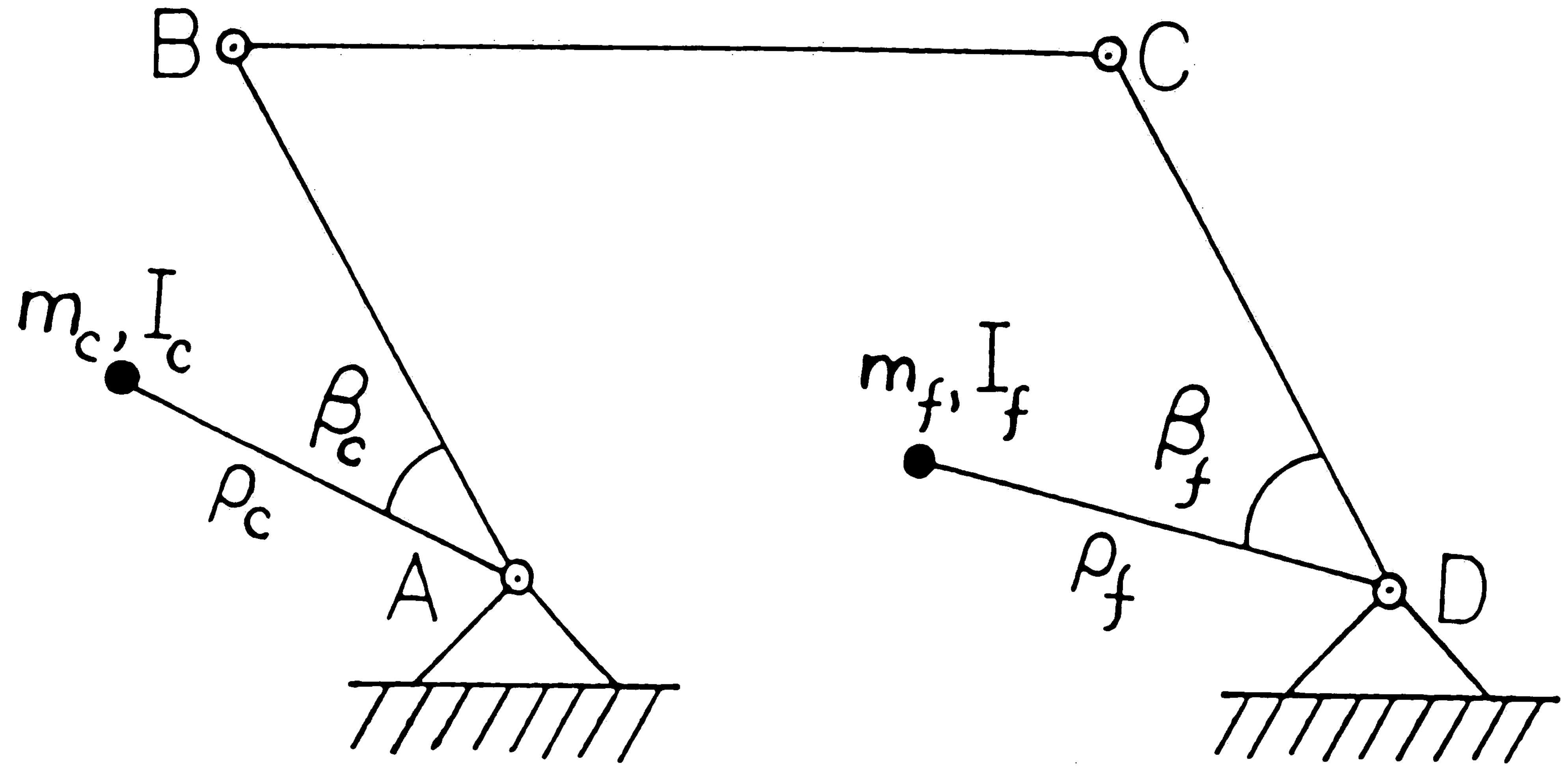


Figure 5-1: Four-Bar Parallelogram Linkage

direction or both) in the object function, one would probably not get that as a solution.

Summing the net force in each coodinate direction, one obtains

$$F_X = -(m_c \cdot r_c \cdot \sin(\theta + \beta_c) + m_f \cdot r_f \cdot \sin(\theta + \beta_f)) \cdot \omega^2$$

and

$$F_Y = -(m_c \cdot r_c \cdot \cos(\theta + \beta_c) + m_f \cdot r_f \cdot \cos(\theta + \beta_f)) \cdot \omega^2.$$

From this, it can be seen that

$$F_X = F_Y = 0$$

can be realized by using counterweights on the crank and follower which are such as to cause the net mass properties of the crank and follower to satisfy

$$\beta_f = \beta_c + \pi,$$

and

$$m_c \cdot r_c = m_f \cdot r_f .$$

Therefore, if one used the program to balance this linkage and defined an object function that minimized only the shaking forces, one could get either the solution originally expected (which make the polar radius of mass centers of counterweighted crank and follower zero), or the solution satisifying the conditions above. In practice, it is most likely that one would not get the solution originally expected, as it is but one solution which satisfies the condition above.

However, if the shaking moment was also included in the object function definition, only the intuitive solution would result. In this example, the above condition would not zero out the shaking moment and therefore would not minimize the object function.

This simple example demonstrates how the user can be "fooled" by his object function definition. This is an extremely simple linkage, and an alert kinematician would have grasped the peculiarities arising from the geometry immediately. However, similar situations can occur with other linkages, and the designer must beware of such.

A second problem in the area of the object function concerns the penalty functions. Knowing when and how to modify the penalty function sharpness factor, for instance, requires experience and judgment, along with some good fortune. The designer must be aware of the effects of the penalty function on the object function definition, as they can cause results which might otherwise mislead him. For instance, sometimes the graph of the object function versus the input will show that the object function is uniformly lower across the input motion for the counterweighted mechanism than for the uncounterweighted mechanism. This should mean that a good set of counterweights has been found. If, however, one of the variables is near its bound, the object function value may be so inflated that the plot of the normalized object function value versus the simplex step number may show that no success has yet been achieved, i.e. that the line has not yet dipped below unity. If this is so, reducing the value of the sharpness parameter will result in a rapid reduction in the values of the object function. This will show the progress made and allow further progress. If the designer does not take the presence of the penalty function into account, situations such as this will appear to be non-sensical, causing him to wonder what is wrong with the program.

A third problem is the difficulty of providing initial guesses for the counterweights which will lead to an optimal solution. To help deal with this problem, a program option is given which allows the designer to generate a limited number of solutions randomly and store those with the least object function values for future use in the program.

Designers must also recognize the limitations of the state of undestanding of the optimization technique. Because of the uncertainties of non-linear optimization, there is no way to guarantee that one has the global minimum. The best that can be hoped for is that one will find a good, or at least a better, solution, In any case the time required to balance most linkages is small enough that it will be worthwhile to attempt the balancing of the linkage. In many cases the designer will be able to make a distinct improvement.

A number of capabilities of the program have not been examined. First, the program allows any mechanism to be balanced, but so far only linkages have been treated. This is due to the lack of suitable routines to handle the more difficult kinematics of mechanism elements such as cams.

Furthermore, while this program was designed primarily to perform passive balancing, the design of the program has been such as to also allow the solution of the active balancing problem. By examining the mechanism to be balanced and its active balancing mechanism as one mechanism, making the mass properties of the components of the active balancing mechanism identically zero, and restricting the program so as to find counterweights only for the active balancing mechanism, the program can be used to determine these counterweights. This result will provide the optimal mass properties for the members of the active balancing mechanism. This has not yet been attempted.

Another area in which no work has been done is in the area of balancing mechanisms which already exist. By measuring the necessary force components as functions of the input variables directly, it is not necessary to know the mass properties of the mechanism already built. By using superposition techniques, the balancing program can be used to find the counterweights for the mechanism. This too will ultimately be attempted.

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Appendix A
Data for Slider-Crank Linkage

Table A-1:

LINK	LINK SUFFIX	MASS (kg)	CENT. INERTIA (kg m^2)	RADIAL OFFSET ρ (m)	ANGULAR OFFSET (DEG.)	LINK LENGTHS β (x10^{-2} m)
AB	1	0.300	0.000200	0.030	0.0	AB 5.00
BC	2	0.460	0.000900	0.050	0.0	BC 12.50
CD	3			1.110		

Table A-2: Data for Counterweight Sets

LINK	MASS (kg)	CENTROIDAL INERTIA (kg m^2)	RADIAL OFFSET (m)	ANGULAR OFFSET (degrees)
Classical Solution [24]				
AB	1.268	0.0	0.050	180.0
BC	0.220	0.001684	0.625	0.0
CD	0.0	0.0	0.0	0.0
Counterweight Set #1				
AB	0.731	0.0	0.075	180.0
BC	1.790 10^{-6}	7.754 10^{-4}	0.054	0.0
CD	0.0	0.0	0.0	0.0
Counterweight Set #2				
AB	0.619	0.0	0.067	180.0
BC	1.073 10^{-7}	8.809 10^{-4}	0.078	0.0
CD	0.0	0.0	0.0	0.0

Appendix B
Data for Four-Bar Linkage

Table B-1: Geometric Data for Four-Bar Linkage

LINK	LINK SUFFIX	MASS (kg)	CENT. INERTIA (kg m^2)	RADIAL OFFSET ρ (m)	ANGULAR OFFSET β (DEG.)	LINK LENGTHS (x10^{-2} m)
AB	1	0.300	0.000400	0.030	0.0	AB 5.00
BC	2	0.450	0.006653	0.125	15.0	BC 20.00
CD	3	0.690	0.001700	0.025	10.0	CD 15.00
AD	4					AD 27.50

Table B-2: Data for Counterweight Sets

LINK	MASS (kg)	CENTROIDAL INERTIA (x10^{-3} kg m^2)	RADIAL OFFSET (m)	ANGULAR OFFSET (degrees)
Berkof and Lowen [5, 21]				
AB	1.582	-----	0.057	180.0
BC	1.160	1.363	0.048	195.0
CD	0.528	0.282	0.033	190.0
Counterweight Set #1				
AB	0.791	-----	0.075	182.0
BC	0.692	0.004	0.027	195.8
CD	0.667	0.002	0.021	153.8
Counterweight Set #2				
AB	0.574	-----	0.085	189.0
BC	0.489	0.049	5.11 10^{-5}	4.5
CD	0.789	0.014	0.039	137.2
Counterweight Set #3				
AB	0.540	-----	4.65 10^{-3}	329.5
BC	0.368	0.055	0.028	183.9
CD	0.795	0.007	0.025	146.1

Appendix C
Data for Watt's Six-Bar Linkage

Table C-1: Geometric Data for Watt's Six-Bar Linkage

LINK	LINK SUFFIX	MASS (kg)	CENT. INERTIA (kg m^2)	RADIAL OFFSET ρ (m)	ANGULAR OFFSET β (DEG.)	LINK LENGTHS (x10^{-2} m)
AB	1	0.377	-	0.031	0.0	AB 6.00
BCE	2	0.740	0.000373	0.126	12.7	BE 15.00 EC 15.00 CB 20.00
CDG	3	0.540	0.000243	0.116	7.8	CG 11.00 GD 13.00 DC 18.00
AD	4					AD 25.00
EF	5	0.644	0.000405	0.080	11.6	EF 17.50
FG	6	0.427	0.001443	0.114	7.6	FG 15.00

Table C-2: Peak Joint Forces

Joint [Design Limit]	Walker & Haines' [F]	Walker & Haines' [G]	Counter-weight Set #1	Counter-weight Set #2	Counter-weight Set #3
A [6000N]	964.90881N	1142.6113N	662.77368N	743.35937N	473.81470N
B [1250N]	1067.6130N	1205.9109N	826.58874N	913.71960N	822.78955N
C [1250N]	566.51465N	765.80005N	414.84216N	478.78998N	325.17322N
D [1090N]	634.64563N	1144.1528N	560.72327N	522.91736N	399.85669N
E [1250N]	273.96356N	295.44366N	338.52319N	343.66296N	290.15686N
F [1250N]	44.066154N	105.25644N	153.56085N	154.59659N	55.183647N

Table C-3: Data for Counterweight Sets

LINK	MASS (kg)	CENTROIDAL INERTIA ($x10^{-3}$ kg m^2)	RADIAL OFFSET (m)	ANGULAR OFFSET (degrees)
Walker and Haines' Set F				
AB	0.963	-----	0.044	152.2
BCE	1.515	0.98	0.147	238.5
CDG	-----	-----	-----	-----
EF	-----	-----	-----	-----
FG	-----	-----	-----	-----
Walker and Haines' Set G				
AB	0.963	-----	0.044	152.2
BCE	-----	-----	-----	-----
CDG	3.35	11.38	0.262	357.4
EF	-----	-----	-----	-----
FG	0.38	0.15	0.305	1.2
Counterweight Set #1				
AB	1.498	-----	0.046	166.5
BCE	0.040	1.224	$8.05\ 10^{-4}$	93.4
CDG	3.607	0.198	0.215	359.56
EF	0.010	0.967	0.024	53.4
FG	0.341	1.963	0.325	320.0
Counterweight Set #2				
AB	0.926	-----	0.086	161.8
BCE	0.067	1.161	0.030	66.6
CDG	3.480	8.281	0.208	356.3
EF	0.024	0.501	0.048	39.2
FG	0.366	0.256	0.328	318.4
Counterweight Set #3				
AB	1.105	-----	0.092	196.0
BCE	0.754	0.912	0.162	265.6
CDG	0.001	0.841	0.099	6.4
EF	0.012	0.247	0.021	156.5
FG	0.015	0.899	0.011	355.2

Biography

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