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A tool to study dynamic phenomenon of a four bar linkage and its application in balance program verification /

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A TOOL TO STUDY DYNAMIC PHENOMENON OF

A FOUR BAR LINKAGE AND ITS APPLICATION IN

BALANCE PROGRAM VERIFICATION

BY

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Abstract

This project has provided a vibration response chassis which is a powered four bar linkage that can be used to perform machine design simulations in dynamic balancing, cyclical load application and other phenomenon associated with machine dynamics. The machine was instrumented with a programmable controller to control an electric brake on the output of the linkage.

The vibration response chassis was first used to verify the accuracy of the Mechanical Linkage Balancing Program. The program generates counterweight information to optimally reduce the shaking forces of a variety of mechanical devices. To check the program's ability to balance a four bar linkage the mechanical properties of the four bar linkage in the vibration response chassis were entered in the program and counterweights were made according to the program output. To measure the relative change in shaking forces, the frame of the vibration response chassis was mounted to vibrate in one degree of freedom and the acceleration of the frame due to the shaking forces was measured. The maximum acceleration of the frame without counterweights was reduced by 52.2% when the specified counterweights were installed. The acceleration responses with the counterweights in various positions indicate the computer generated locations were

optimal.

The mechanism was then used to study the affect of an applied cyclical load on the shaking forces of the linkage. The applied load to the oscillating output of the linkage had no effect on shaking forces at high speed cycle speeds (325 RPM). The frame accelerations increased 32% without counterweights and 63% with counterweights in place when the load was applied to the linkage while running at low speed (195 RPM). At low speeds the applied load caused speed variations in the machine cycle which increased the shaking forces. The brake was not able to cause speed variations in the linkage at high speed because of the higher rotational inertia of the mechanical system.

Introduction

The purpose of this project is to provide a flexible tool to carry out machine design simulation experiments and use the tool to verify the results of the mechanical linkage balancing program. The four bar linkage was chosen as the simulation mechanism because of its wide application in design. The project required the design of a motorized four bar linkage which could be instrumented to measure a variety of phenomena characteristic of dynamic motion. The machine was initially configured to measure shaking forces caused by the inherent unbalance of the linkage. An electric hysteresis brake was mounted on the oscillating output shaft of the linkage to simulate loading conditions. The brake was used to study the effects of cyclical loading on the shaking forces of the linkage. Other simulations which could be conducted on the machine will be reviewed at the end of this report.

The mechanical linkage balance program assists the machine designer by providing counterbalance weights and positions to optimally reduce the shaking forces of any single input cam mechanism, slider crank linkage or four bar linkage. This project verified the program's ability to balance a four bar linkage. An experiment was designed to measure the difference in shaking forces generated by the linkage without counter weights and then with the specified counter weights in place. This was done by mounting the frame of the four bar linkage such that it could move, or vibrate in one degree of freedom. The acceleration of the frame due to the shaking forces could then be measured to determine the relative unbalance.

Description of the Mechanism

The mechanical device for this project had to provide the flexibility to measure many parameters of dynamic motion accurately. A motor driven four bar linkage mounted in a rigid frame was selected as the best device to carry out the machine simulations because of the various balance, friction and dynamic elasticity it demonstrates. It is also used widely in design as a common way to convert rotational motion into oscillating motion.

The supporting frame is made of 2 x 6 inch steel channel with 5/16" plate welded on the ends to insure rigidity. The crank and follower are each formed by two 3/4 inch by 2 inch steel plates while the connecting rod is more slender in design. The connecting rod is easily replaceable so connecting rods of different materials and design can easily be made and installed. A split journal bearing is used on the ends of the connecting rod. Self

aligning pillow block ball bearings are used on the input and output shafts. Threaded shafts in the pivot ends of the crank and follower allow easy placement and change of counterbalance weights. (see figure 1)

The basic dimensions of the initial linkage are as follows.

a = 15 inches
b = 2 inches
c = 15.125 inches
d = 4 inches

One complete revolution of the input shaft results in a 60.76 degree oscillation of the output shaft. The Newton-Raphson method was used to calculate the output shaft position at various input shaft positions. A BASIC program to perform the position calculations and its output appear in appendix A.

Gearbelts are used to transfer mechanical power from the drive motor to input shaft and from the output shaft to brake. Gearbelts provide smooth power with zero slip at low tension. This minimizes the cantilever forces on the drive shafts.

The drive motor is a Woods, 1/3 horsepower, AC, synchronous model. The E-Trol SCR controller permits infinite speed adjustment between zero and 2400 RPM in either direction. Analog speed and load meters are also part of the system and provide instant speed and load readings. See figure 2 for drive system layout. The drive gearbelt pulleys were configured to provide a three to one speed reduction from the motor to the input shaft for a top input shaft speed of 800 RPM.

An electric brake is mounted to the end of the frame and is driven by gearbelt from the oscillating output shaft. A Deltran hysteresis brake was selected to simulate the loading of the machine. Hysteresis brakes use the magnetic field generated by an energized coil to retard motion. A permanent magnet rotor spins inside the coil field in the same manner as an electric motor, only the applied voltage produces a stopping torque.

The hysteresis brake is good for experimental work for a number of reasons. The torque generated is always constant for a given applied voltage. Conventional brake loads generally vary with speed and run time due to changes in the coefficient of friction of the brake surfaces with temperature. The Deltran hysteresis brake applies a constant torque over the entire speed range and is not affected by run time. The load is infinitely variable and linear with applied voltage, and the load is applied instantaneously which is important for this application as the on/off cycle time is as low as a tenth of a second. The Deltran HB-420 applies 120 ounce inches at 24 volts DC.

Programmable Controller

External instrumentation and devices such as the electric brake can be controlled with the programmable controller. The controller takes position information off the input or output shafts through an incremental encoder and, according to the user written program, operates outputs to perform functions such as turn on and off relays or trigger an oscilloscope. The programmable encoder used in this machine is the Encoder Products Company 7252 Motion Controller equipped with the main processor and two counter boards.

The 7252 Motion Controller is primarily designed to control the position, velocity or torque of D.C. motors. The servo system operates as a closed loop system receiving its feedback from an incremental shaft encoder with two channels in quadrature. However the system is modular in design so that it can be customized to perform most any control application. The main system processor occupies one of six available slots in the 7252 housing. The remaining slots can be used for servo control boards or any other control board. Optional boards include digital and analog I/O's, high speed counters, digital displays, preset thumbwheels and serial communication boards.

The programming language for the 7252 controller is

Intel's MCS BASIC-52 which was specifically designed for control systems. There are specialized commands for each board which allows sophisticated control routines with a minimum of programming skills. The system has 8K of RAM to accommodate user defined programs.

The processor can be programmed with any RS-232C ASCII terminal. The terminal plugs into the port on the front of the processor. A personal computer with a terminal emulation package can also be used if it is set for an 8 bit structure, no parity, no auto line feed, full duplex and upper case. A standard 25 pin RS-232C connector is used between the terminal and processor although only pins two, three and seven are used for communication.

There are two methods of storing an inputed program. For program development and short term storage the RAM is maintained when the power is turned off by a board mounted battery. To enable the battery back up jumper H1 must be moved to the right most two pins as viewed from the board's front panel. The battery will maintain the program and internal clock for about six months and therefore should be disconnected if the unit is not used for an extended period. Once a "production" program has been developed, it can be burned into permanent EPROM or EEPROM on the processor board by simply using the PROG

command. The EPROM is mounted in quick release sockets for easy exchange.

Physical connections of encoders and other input and output devices is made through the back panel. A row of 36 terminals is available for each board. There are also connections for line power and a user power supply which provides twelve and five volt DC power for external device operation.

System Configuration for Brake Operation

The electric brake is mounted on the output shaft of the vibration response chassis to simulate machine loading of an oscillating output. The motion controller can be programmed to accurately turn on and off the brake at any point in the machine cycle. The following is a description of the configuration and program which controls the brake application.

-Controller Configuration

To switch the brake on and off, two high speed counter boards and the main processor are used. One counter board turns the brake on and the other turns it off. The counter boards can accept bi-directional counts up to 250 KHz from the encoder. With a 720 pulse per revolution encoder, such as the Encoder Products model 715, the counter can keep up with speeds up to 20,800 RPM. Each counter also has a preset register feature which accepts a preset count. When the count from the encoder reaches the preset value, an auxiliary output on the counter board activates. This is useful for time critical events and can provide either a latched or pulsed (100ms) output. A status word can be read to determine if the count is less than, greater than, or equal to the preset value.

In this application both counters were set to provide a latched auxiliary output. This is done by setting the left microswitch of the two switch set on the counter board to the closed position. This setting will cause the output to go on when the preset is reached and stay on until it is reset by a command from the user defined program. The counter boards were also set to accept an interrupt signal from the main processor board. The interrupt signal allows instant counter and output reset from an external signal and is accepted by the counter board only when the second microswitch of the two switch set is in the open position.

As stated earlier, one counter board is used to turn on the brake and the other is used to turn it off. To accomplish this the switching transistor in the "turn off" board was rewired to operate in a normally closed manner. This means the auxiliary output will be on until the preset count is reached and then turn off once the

count equals the preset. The auxiliary outputs on the counter board can switch up to 30 VDC at 250 milliamps. Since the brake may draw more than the switching capacity, a relay is used to switch the brake. The power from a 9 volt battery is switched in series through the two counter boards and powers the relay coil. A diode and 47 f capacitor are required across the relay coil to arrest spikes and noise which tend to disrupt the microprocessor. (fig 3)

-Encoder Configuration

To control the brake, the motion controller must know the exact position of the input shaft. This is done by counting the pulses from the encoder which is attached to the input shaft. The encoder consists of a shaft on which a disc with clear and opaque segments around its circumference is mounted. An optical switch goes on and off as the clear and opaque segments pass between the switch and internal light source. The number of segments on the disc determines the number of pulses per revolution and thus the accuracy of the encoder.

The encoder used in this application is an Encoder Products Model 715-1 accu-coder. This model generates 720 pulses per revolution. The encoder is wired to the controller as shown in figure 4.

The encoder is mounted to the side of the chassis frame and is attached to the input shaft with a W. M. Berg bellows shaft coupler. This type of shaft coupler permits both axial and angular misalignment with no backlash and therefore no loss in resolution. A clockwise rotation of the encoder shaft produces an increasing count and a counter clockwise rotation decreases the programmer count.

To operate the brake both counter boards must have the same count and therefore the count signal to both boards must come from one encoder. To accomplish this the encoder is wired to one board and then jumpers run to the second counter board as shown in figure 3.

Programming the Controller

In addition to the common BASIC programming statements, MCS BASIC-52 has been enhanced by using a group of ROM resident subroutines called EXTENDED BASIC ROUTINES (EBR's). These routines allow simple programming of the special functions associated with the various optional boards. There are four such routines which interact with the counter board. The routines use a PUSH statement to load the parameters into a stack, and a CALL statement to transfer control to the EBR. Any parameter returned from the EBR can be assigned to a variable with a POP statement.

The four counter board EBR's are outlined below.

For more detail see chapters four and five of the 7252 Motion Controller Instruction Manual.

-The following statements would obtain the actual count from a counter board and assign the value to variable A.

```
PUSH BD#, 10
CALL 4000H
POP A
```

Where BD# is the slot where the counter board is located.

-To load a value into a counter board's preset register, only two statements are required.

```
PUSH BD#, N, 11
CALL 4000H
```

Where N is the counter value to be loaded.

-The following statements send operating instructions to the counter board. Such instructions tell the counter to:

Start - enables the counter

Stop - disables counter

Clear - sets counter registers to zero

Load - loads value of preset to counter register

The statements to send the instructions are:

```
PUSH BN#, CN, 12
CALL 4000H
```

Where CN is the command number which defines which instructions are to be sent to the counter board according to the following table.

Table 1. Counter Board Operating Instruction Table

CN	Start	Stop	Clear	Load	Reset
2		X		X	X
3	X			X	X
4		X	X		X
5	X		X		X
6		X			X
7	X				X
10		X		X	
11	X			X	
12		X	X		
13	X		X		
14		X			
15	X				

-The following statements call the fourth EBR which reads the status register of the counter board in binary, converts it to decimal and assigns the value to variable A.

```
PUSH BN#, 13
CALL 4000H
POP A
```

Where A is the decimal equivalent of a four digit binary number in which the bits represent the following.

X(1) X(2) X(3) X(4)

(1) Latched high when count reaches preset, must be reset

(2) Low when count = preset

(3) Low when count > preset

(4) Low when count < preset

These routines can be used with common basic

statements to control any rotating equipment.

Another important feature of the 7252 Motion Controller is the interrupt enable. This feature allows the control of the program to instantly shift to any designated line upon instruction from the program or an external trigger. In this particular application the interrupt was used to reset the counters to zero at the start of each shaft revolution. This is accomplished with the a limit switch which is tripped once per revolution by a set screw on the input shaft. The switch is wired to apply 5 VDC across backpanel connections A2(+) and A1(-) on the main processor board. This causes the program control to shift to a section of the program which resets the counters to zero. Each time the interrupt is used it must be reset with the RETI command.

-Sample program to run brake

The following is a simple program which will turn on and off the electric brake at any point in the machine cycle. The program first requests the user to input the desired brake on and off positions. The input can be between zero and 720 as the encoder produces 720 counts per revolution. The starting point of the count can be set by adjusting the position of the set screw which trips the interrupt switch.

```
10 ONEX 200
    REM: Shift control to line 200 upon interrupt signal
20 INPUT "INPUT BRAKE ON POSITION",N
    REM: Assigns ON value to variable N
30 INPUT "INPUT BRAKE OFF POSITION",F
    REM: Assigns OFF value to variable F
40 PUSH 1,5,12
    REM: Starts, clears and resets counter board #1
50 CALL 4000H
    REM: Go to counter command routine
60 PUSH 2,5,12
    REM: Starts, clears and resets counter board #2
70 CALL 4000H
    REM: Go to counter command routine
80 PUSH 1,N,11
    REM: Assigns N to preset of counter board #1
90 CALL 4000H
    REM: Go to preset counter routine
100 PUSH 2,F,11
    REM: Assigns F to preset of counter board #2
110 CALL 4000H
    REM: Go to preset counter routine
120 PUSH 1,10
    REM: Read count on board #1
130 CALL 4000H
    REM: Go to read counter routine
140 POP A
    REM: Assigns count to variable A
150 PUSH 2,10
    REM: Read count on board #2
160 CALL 4000H
    REM: Go to read counter routine
170 POP B
    REM: Assigns count to variable B
180 PRINT A,"          ",B
    REM: Prints A and B
190 GOTO 120
    REM: Read next count
200 PUSH 1,5,12
    REM: Start, clear, reset counter #1
210 CALL 4000H
    REM: Go to counter command routine
220 PUSH 2,5,12
    REM: Start, clear, reset counter #2
230 CALL 4000H
    REM: Go to counter command routine
240 RETI
    REM: Resets interrupt
250 GOTO 80
```


Balancing Verification

-Method of Testing

The first experiment performed on the vibration response chassis was to verify the Mechanical Linkage Balancing Program. The program provides counter balance masses and locations to optimally balance a variety of mechanical devices. To verify the program's ability to reduce shaking forces in the vibration response chassis, the frame was mounted so it could vibrate in one degree of freedom. The acceleration of the machine frame due to the shaking forces caused by the machine imbalance, could then be measured without and with the specified counterweights in place.

The motion of the chassis was restricted to one degree of freedom to simplify the instrumentation and insure accuracy. With a single degree of motion, a single accelerometer could be used to measure the entire motion of the frame. The signal from the accelerometer could then be sent to an oscilloscope to determine maximum acceleration of the frame.

The chassis was mounted to pivot at the base of the motor end of the frame and was supported by industrial machine mounts at the other end. The mounts acted as springs which flexed in response to the shaking forces of the machine. The accelerometer was mounted on the frame such that it was in line with the motion of the frame (i.e. perpendicular to a line from the pivot point). See figure 5.

To insure the frame absorbed all the shaking forces generated, the chassis frame was mounted on a 600 lbf iron table with a 300 lbf steel plate placed across the legs. This weight kept the chassis from lifting the table but a small amount of wobble was noted when the machine was run at certain speeds.

To obtain a usable signal from the accelerometer, the signal is first amplified and then put through a low pass filter. The filter smooths the signal by eliminating the high frequency mechanical and electrical noise. The signal then goes to the oscilloscope, which traces the voltage output over time.

The machine mounts used for the balancing experiments were Tech Products model number 52062. The mounts are rated for 40 lbf and four were used in parallel to support the free end of the frame. With these mounts in place the natural frequency of the motion was 31.1 Hz. (see graph 1) Since operation of the machine near any multiple of the resonance frequency would tend to distort the results, a speed was selected that was not a multiple of the natural resonance. It was found that 325 RPM provided enough vibration amplitude to

generate a clear signal from the accelerometer and did not allow the frame to vibrate at its natural frequency. All runs were done at 325 rpm. The proper speed was assured with a stroboscope.

A plot of the frame acceleration was taken first without counterweights and then with the specified weights in place. The acceleration was then recorded with the counter weights in other positions to insure the specified location was optimal. If the positions were incorrect, the shaking forces and corresponding accelerations would continue to decrease as the true optimal positions were approached.

-Balance Program Inputs and Outputs

The mechanical linkage balancing program requires three properties of each rigid member of the linkage (i.e. crank, connecting rod, follower) to generate optimal counterweight size and location. The properties are mass, location of mass center, and rotational moment of inertia about the mass center. These values are calculated in appendix B and listed below.

Table 2. Mechanical Properties of Linkage

Rigid Member	Mass (lbm)	Center of Mass Location (in)	Rotational Moment of Inertia ($lbm*in^2$)
Crank	.028620	.212305	.022946
Follower	.020633	1.18742	.083118
Con Rod	.010312	7.56250	.432258

For the crank and follower, the center of mass location is measured from the centerline of the respective shafts. The center of mass location for the connecting rod is measured from the center of a pivot hole.

The machine configuration and properties were entered into the program and the following results were generated as the optimal locations for the given counterweights.

Table 3. Mechanical Linkage Balancing Program Output

	Total Counterweight Mass	Distance from Shaft Center to Mass Center
Crank	.0030141 lbm	1.3806 in
Follower	.0104272 lbm	3.7673 in

-Results and Interpretation

Graphs three through 14 are traces of the output voltage from the accelerometer versus time. All traces were graphed with the instrumentation at the following settings.

Table 4. Instrumentation Settings for Acceleration Measurement

Accelerometer Amplifier	100X
Signal Filter, Low Pass	50 Hz
Oscilloscope, scan time	.050 sec/div.
Oscilloscope, amplitude scale	.050 volts/div.

The vertical scale shows major divisions as defined by the oscilloscope; each division equivalent to .04883 g's. The time scale shows ten divisions of time which is equal to 2.71 revolutions of the crank. As shown on graph 2 there is background drift in the signal. To reduce the uncertainty caused by this drift, the maximum change in acceleration during one cycle was used as the imbalance measure.

-Error Analysis

To determine the uncertainty of the acceleration change measurements, a plot was made of the accelerometer output when the machine was at rest. (see graph 2) The noise and drift are from floor vibrations and electrical ground noise. The maximum uncertainty the noise may have generated in any low point to high point measurement is determined by the maximum change in amplitude of the background noise over the time it takes the acceleration curve to go from a minimum to a maximum. This time interval for 325 RPM is .0219 seconds from graph 3. The maximum amplitude change over this time interval on the background noise plot is .00626 g's. Therefore the maximum error in any peak to peak acceleration measurement is .00626 g's.

All trials were done with the counterweights specified by the computer. Positions are in inches and measured from the center of the respective shafts.

Table 5. Balance Verification Results

Graph Number	Crank C-weight Position	Follower C-weight Position	Max. Change in G's
3	None	None	.1541
4	Optimal	Optimal	.07357
5	None	Optimal	.09333
6	Optimal	None	.1455
7	2.00	Optimal	.08687
8	2.50	Optimal	.08942
9	3.50	Optimal	.09036
10	4.50	Optimal	.1091
11	Optimal	2.83	.08994
12	Optimal	3.15	.08232
13	Optimal	3.45	.07832
14	Optimal	4.10	.08291

The accelerations caused by the shaking forces of the mechanism were reduced by 52.2% with the addition of the optimally placed counterweights. It is clear from the data that the counterweight positions generated by the balancing program are optimal. Graphs 15 and 16 show the measured acceleration of the frame with the counterweights in various positions. For each graph the opposite weight was placed in the optimal position (i.e. for the follower graph, the crank counterweights were set at 1.38 inches). The optimal position for the crank

counterweights was as close to the shaft center as the machine configuration would permit so it was impossible to obtain data with the weights closer to the shaft.

Graphs nine and ten show that the follower counterweights reduced the machine imbalance far more than the crank counterweights. This is because the follower is more out of balance than the crank as indicated by the center of mass calculations. The follower shaft will also generate a larger moment about the pivot than the crank for a given force because it is farther from the pivot point.

Effect of Cyclical Load Application on Machine Balance

This experiment investigated the effect, on machine balance, of applying a cyclical load to the output shaft of the vibration response chassis. The machine frame accelerations were measured and interpreted in the same manner as for the balance verification part of this project.

The machine was first run at 325 RPM with the optimal counterweights in place and its acceleration response was recorded. The electric brake was then programed to apply a load through 180 degrees of the out stroke motion and the acceleration response was again recorded. This process was repeated without any counterweights in place.

The machine was then run at 195 RPM and the above combination of counter weights and loads were applied. To obtain a usable response from the frame at this speed, the four 52062 springs mounted in parallel were replaced with two pairs of softer 52061 springs mounted in series, which reduced the spring rate by more than 50%.

The instrumentation settings used for the balance verification part of this project were used again for the data taken at 325 RPM. However the oscilloscope time scale was changed to .1 second/division for the data recorded at 195 RPM.

The results of this experiment are shown below.

Table 6. Brake Application Results

Graph Number	Counterweight Positions	Brake Status	Max. Change in G's	Speed RPM
17	None	On	.1246	325
18	None	Off	.1336	325
19	Optimal	On	.05513	325
20	Optimal	Off	.05551	325
21	None	On	.2292	195
22	None	Off	.1731	195
23	Optimal	On	.1238	195
24	Optimal	Off	.07640	195

At 325 RPM the application of the brake had no significant effect on the frame acceleration with and without the counterweights in place. The acceleration response of the frame is reduced from the balance verification portion of this project because the original output shaft pulley was replaced with a larger pulley to

provide a greater braking torque. There was a measurable increase in frame acceleration when the cyclical load was applied to the output shaft at 195 RPM in both the unbalanced and optimally balanced conditions. The frame accelerations increased 32% without counterbalance weights in place, and 62% with optimal counterbalance weights in place.

While the linkage cycled at 325 RPM, the brake was unable to overcome the rotational inertia of the motor and linkage, so there was no change in speed of the linkage through the cycle. The linkage therefore generated the same shaking forces and resulting frame accelerations with and without the applied load. The brake was able to affect the rotational speed of the linkage at the lower speed (195 RPM). These variations in speed through the rotational cycle generated the increased shaking forces. The optimally balanced linkage was effected more because any changes to the system would tend to increase the imbalance whereas this may not be the case with the unbalanced linkage.

These results indicate predictable shaking forces can only be assured when the inertia or power of the linkage maintains a constant speed through the cycle. If an applied cyclical load causes variation in velocity through the cycle, these variations should be accounted for when using computer aided design tools such as the Mechanical Linkage Balancing Program.

Future Applications

The vibration response chassis was designed with the flexibility to study many aspects of machine dynamics. In addition to machine balancing and applied load simulation, the machine can be used to study elastodynamic phenomena. The deflection of machine parts in motion becomes an important aspect of machine design as speeds and required accuracies increase.

-A Method to Measure Dynamic Deflection

The following is a possible approach to actually measure the deflection of linkage in motion. The connecting rod of the machine can easily be replaced with a more slender design of a less rigid material such as aluminum. With a such a connecting rod, the crank and follower members could be considered rigid and all of the deflection in the linkage could be assumed to be in the connecting rod.

To actually determine the amount of flex in the linkage, the angular position of the output shaft with respect to the input shaft would be measured at various points in its oscillatory motion while the machine is cycled at a very slow speed. This would provide a position profile of the output shaft without any flex in

the linkage. The machine would then be run at a speed where the connecting rod could be expected to flex due to inertial forces. The positions of the output shaft could then be measured and the difference between the positions at slow speed and positions at high speed would indicate the amount of flex in the connecting rod.

A deflection of .020in. in the connecting rod corresponds to only a .3 degree change in the position of the output shaft. To measure the output angle to this accuracy while the machine is running will require a rotary variable differential transformer (RVDT). This device is similar to the more common linear variable differential transformer only the RVDT outputs a voltage proportional to its angular position rather than its linear position. The resolution of the position measurement is determined by the method used to measure the output voltage. To obtain the output shaft angles at the predetermined input shaft positions the motion controller would be programmed to send an electrical pulse when the input shaft reached the desired position. This pulse would trigger the oscilloscope which would read and store the output voltage of the RVDT at the desired instant. This procedure may require additional refinement as actual deflection predictions become available and the RVDT is mounted on the machine.

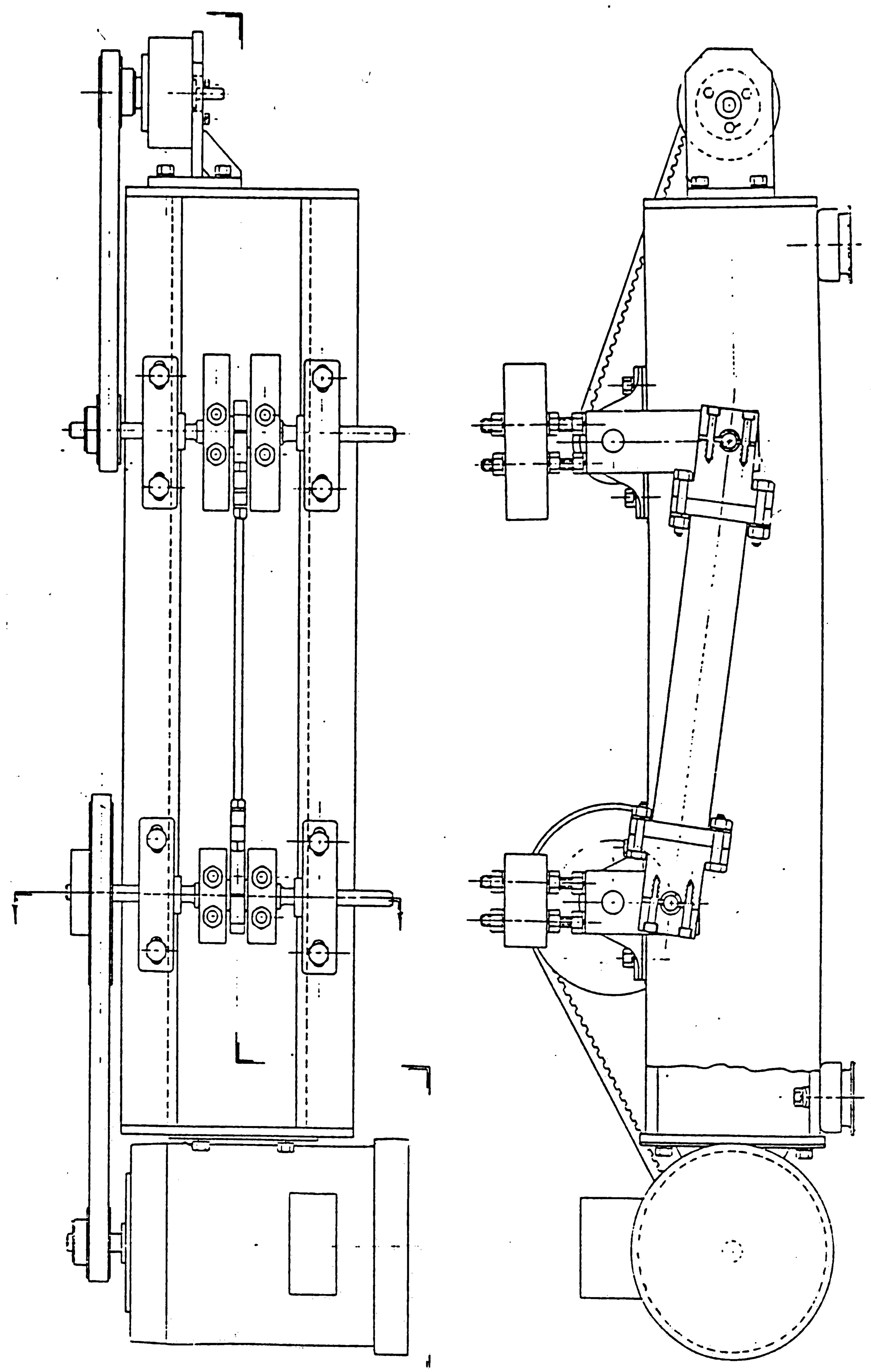
-A Method to Measure Frictional Losses

The frequency response chassis can also easily be used to measure frictional losses in a mechanical linkage. The method would require the generation of a current versus torque curve for the drive motor and the instrumentation needed to measure the current draw of the motor while it runs on the machine. With such a configuration, the amount of torque being applied to overcome friction could be calculated from the measured current draw of the motor. The frictional characteristics of various materials could be examined by simply replacing the bronze bushings used in the crank-connecting rod joint and connecting rod-follower joint with the desired material. The ball bearings used on the crank and follower shafts are also simple to replace with other types of bearings.

There are other areas where the vibration response chassis may provide experimental information. The variation in rotational velocity due to applied loads through the cycle can be measured and the affect of various flywheels on the velocity variation could be analyzed. The machine can also be used to study motion associated with slider crank mechanisms by replacing the follower assembly with a sliding surface. Similar balance, friction and elasticity investigations could

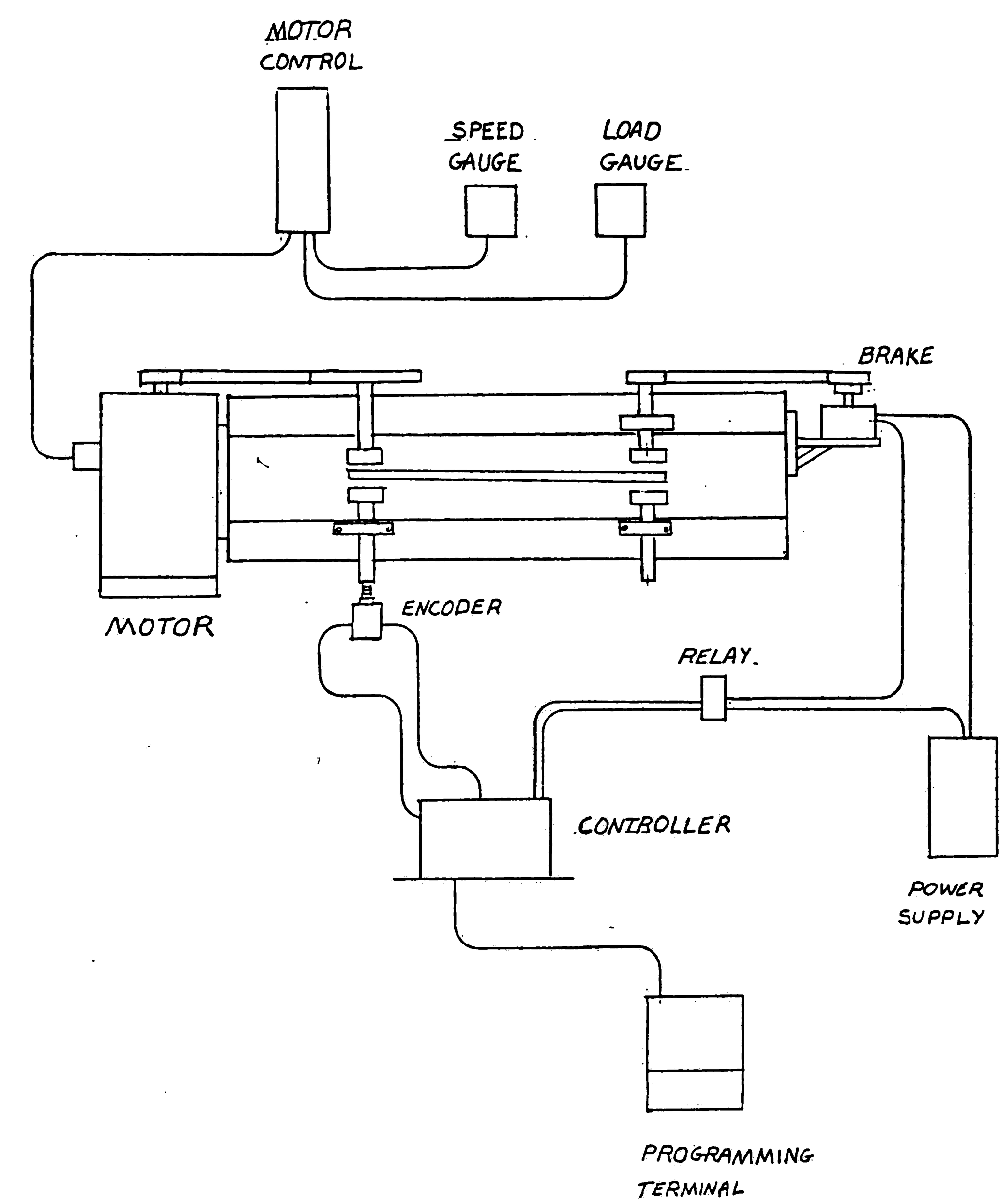
then be performed.

This project has shown that the mechanical linkage balance program can optimally position counterweights to reduce the shaking forces of a four bar linkage. It has also shown that applied cyclical loads to the output shaft of a four bar linkage which cause variation in speed through the machine cycle increases shaking forces. The vibration response chassis is a useful tool in the study of machine dynamics with potential applications in elastodynamics, friction loss and rotational inertia studies.



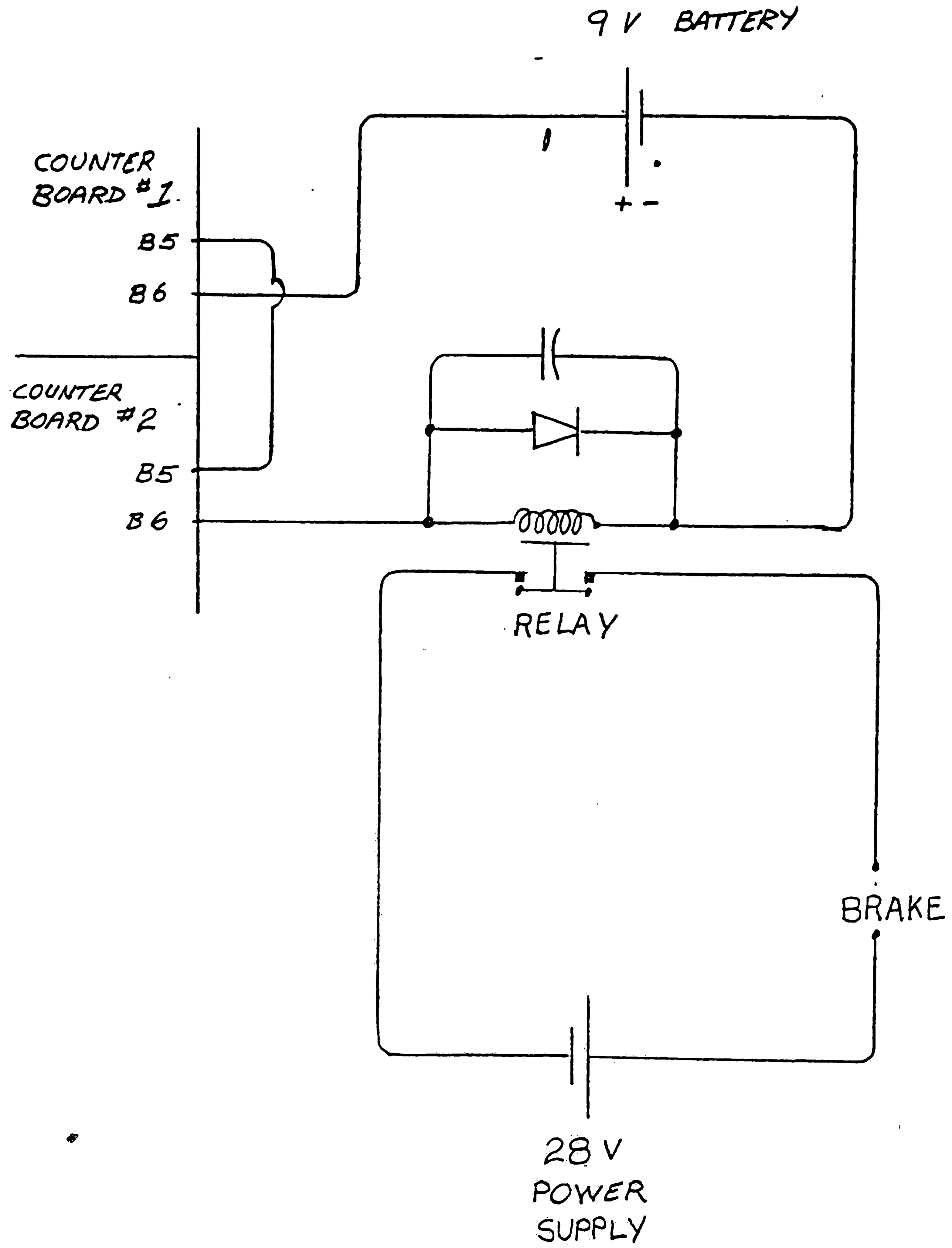
Vibration Response Chassis

Figure 1



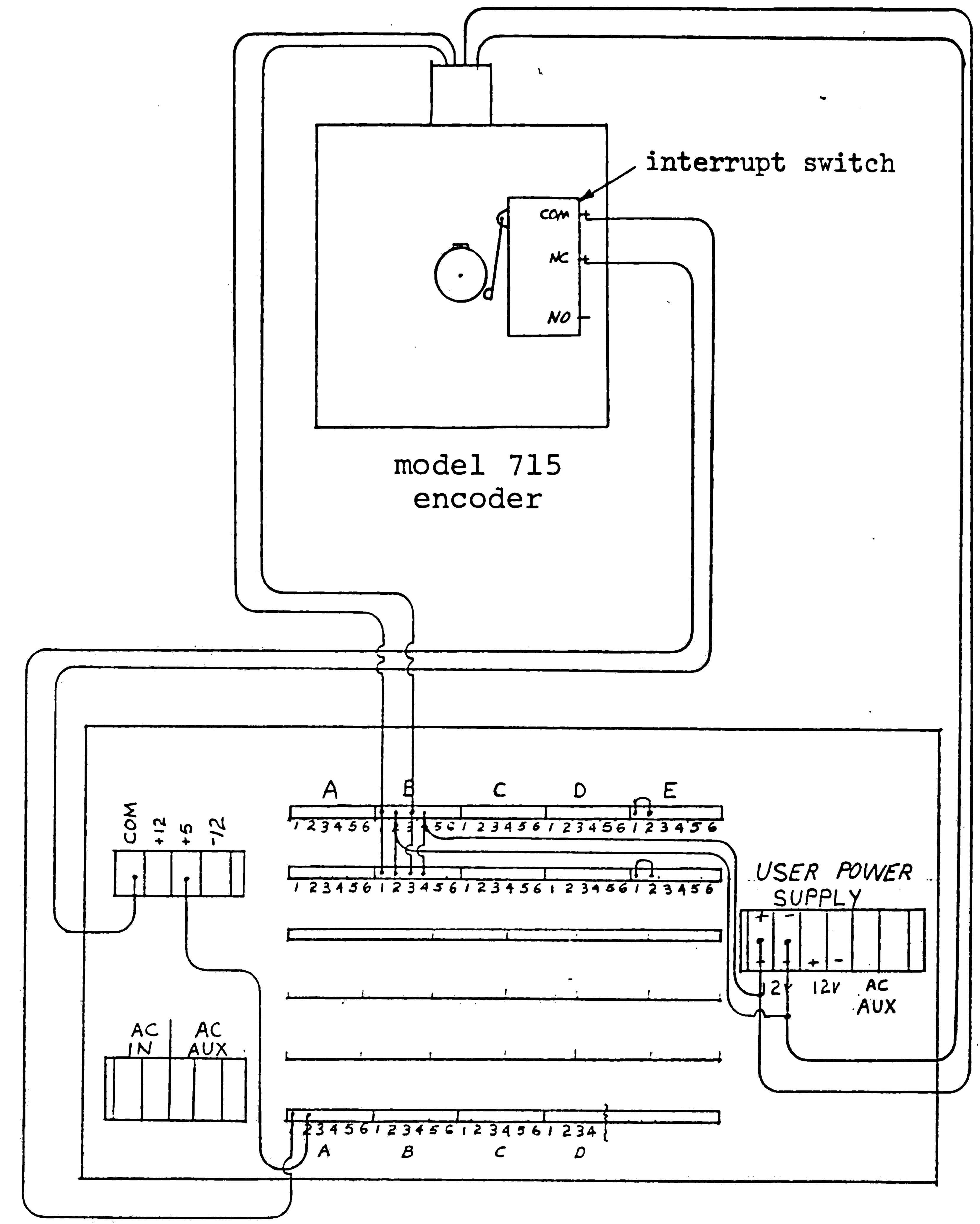
Vibration Response Chassis Control System

Figure 2



Brake Control Circuit

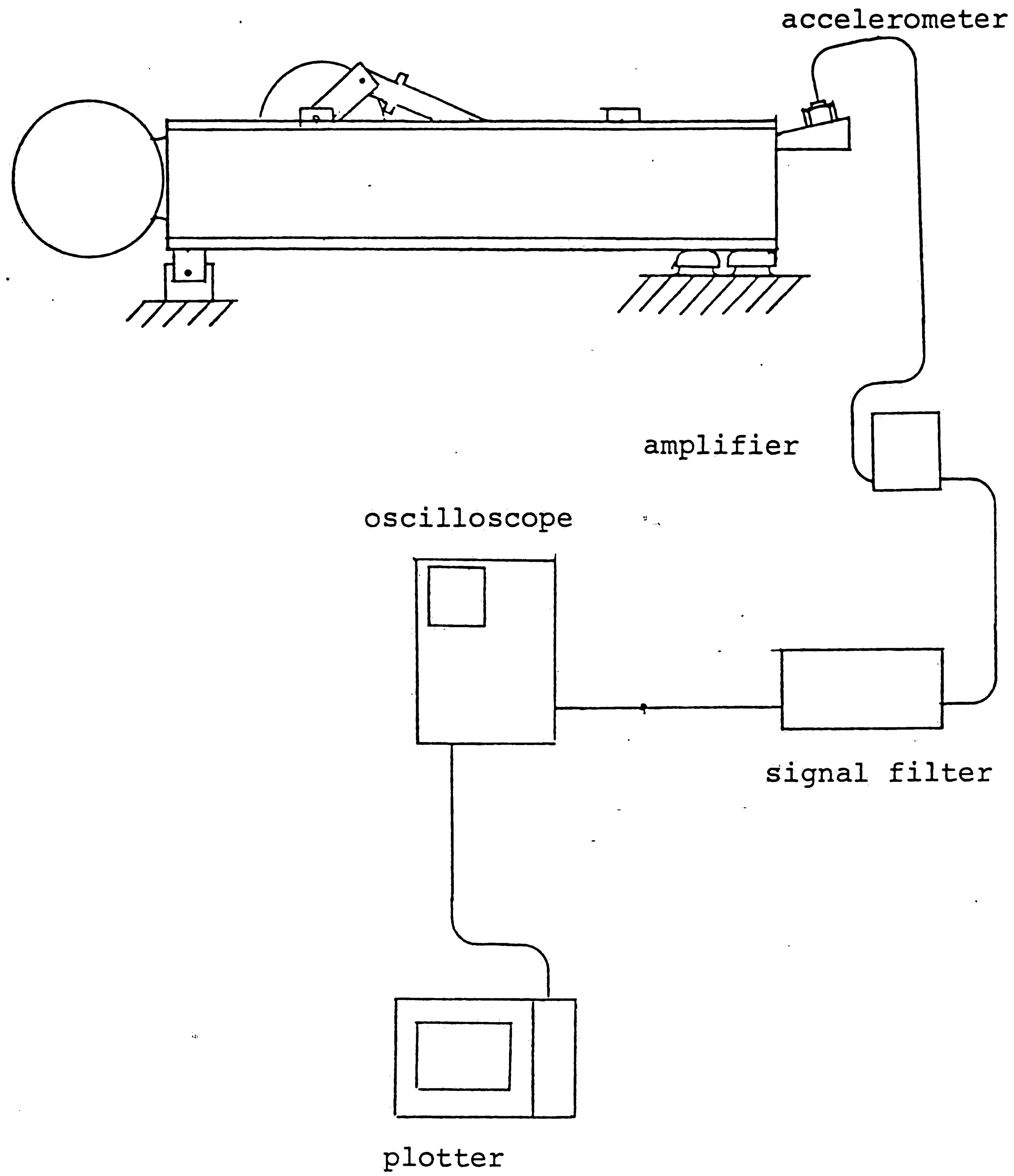
Figure 3



motion controller back panel

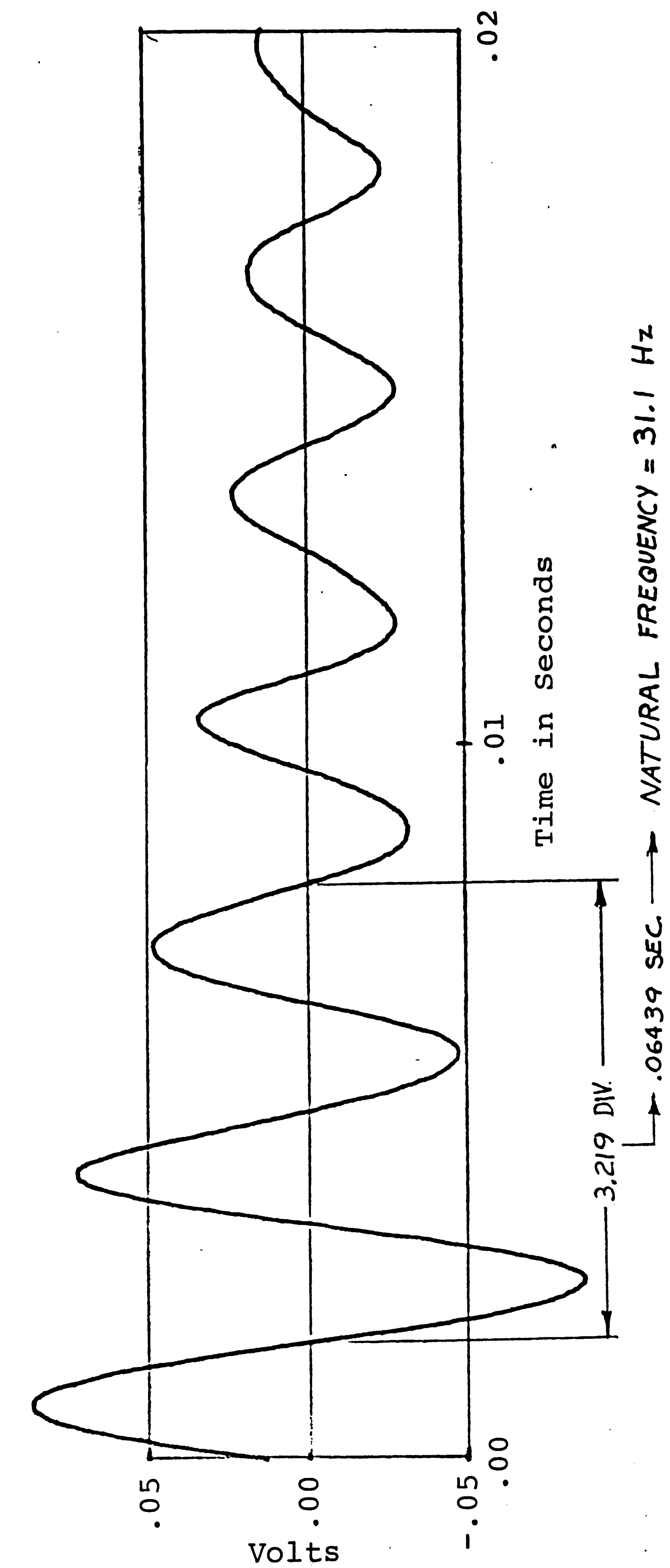
Encoder Wiring

Figure 4



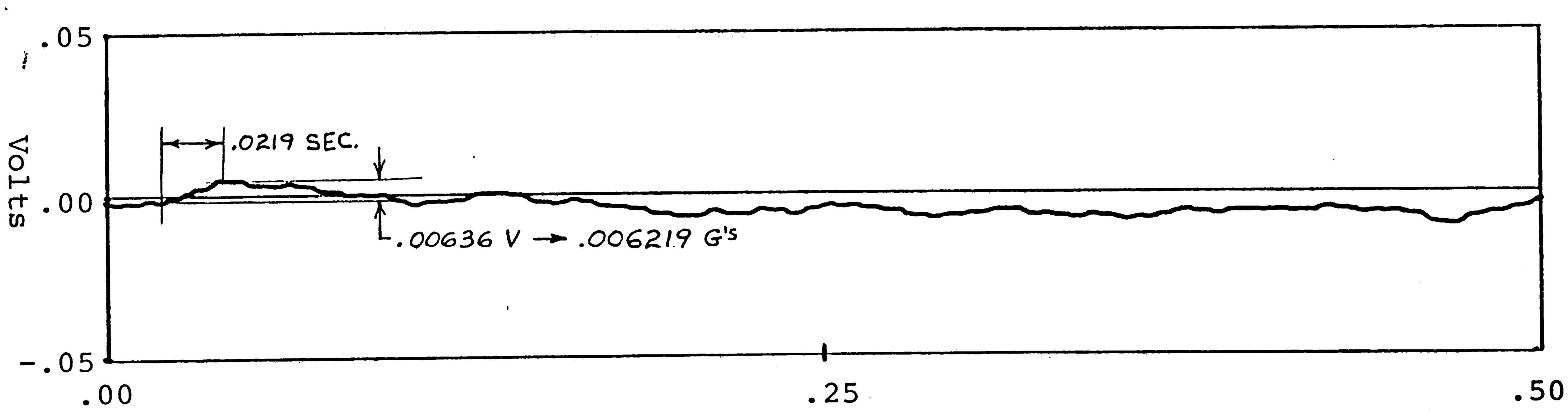
Equipment Configuration for Frame Acceleration Measurement

Figure 5



Natural Response of the Vibration Response Chassis

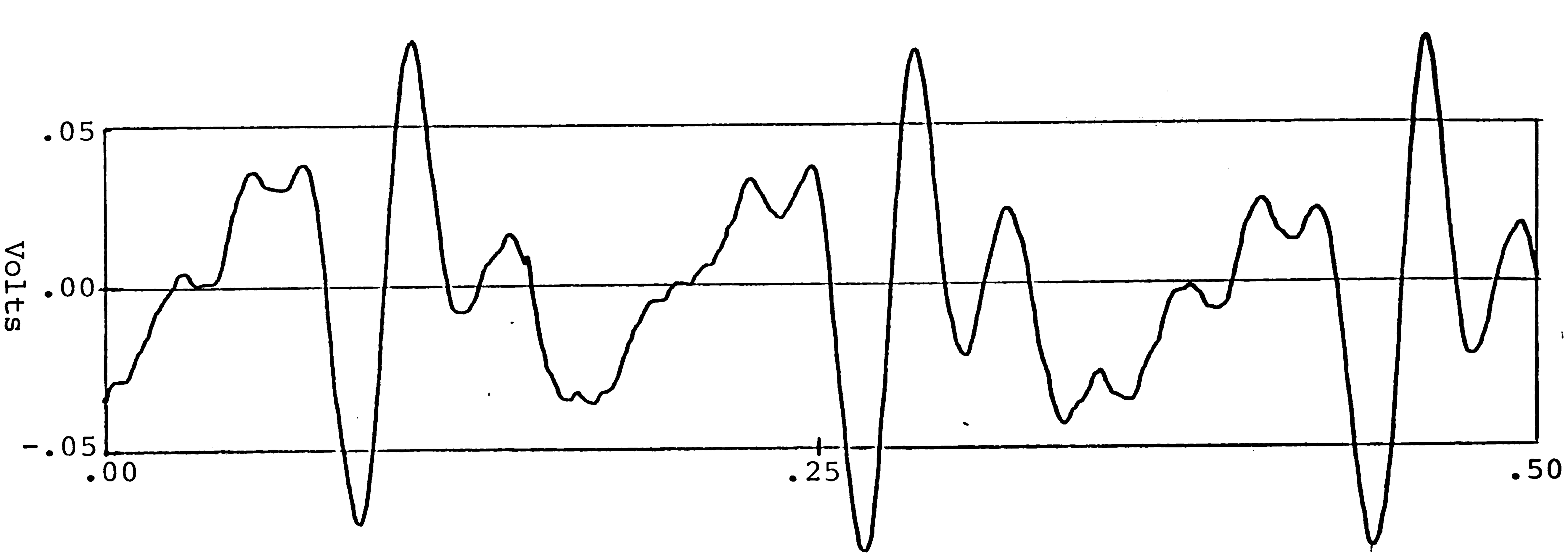
Graph 1



Time in Seconds

Frame Acceleration With
Linkage at Rest

Graph 2



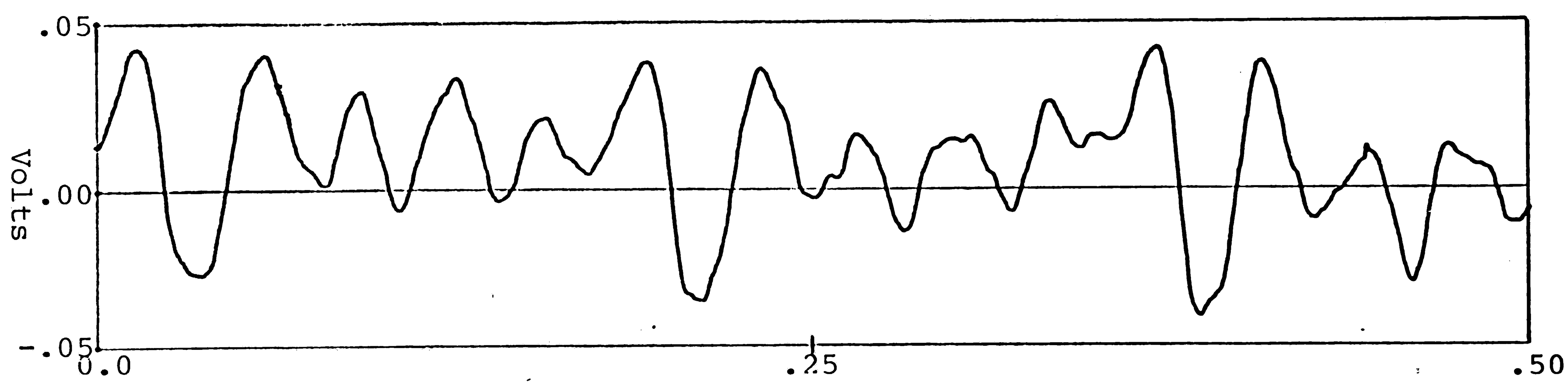
Time in Seconds
325 RPM

Counterweight Positions
Crank-None Follower-None

Frame Acceleration

Graph 3

38



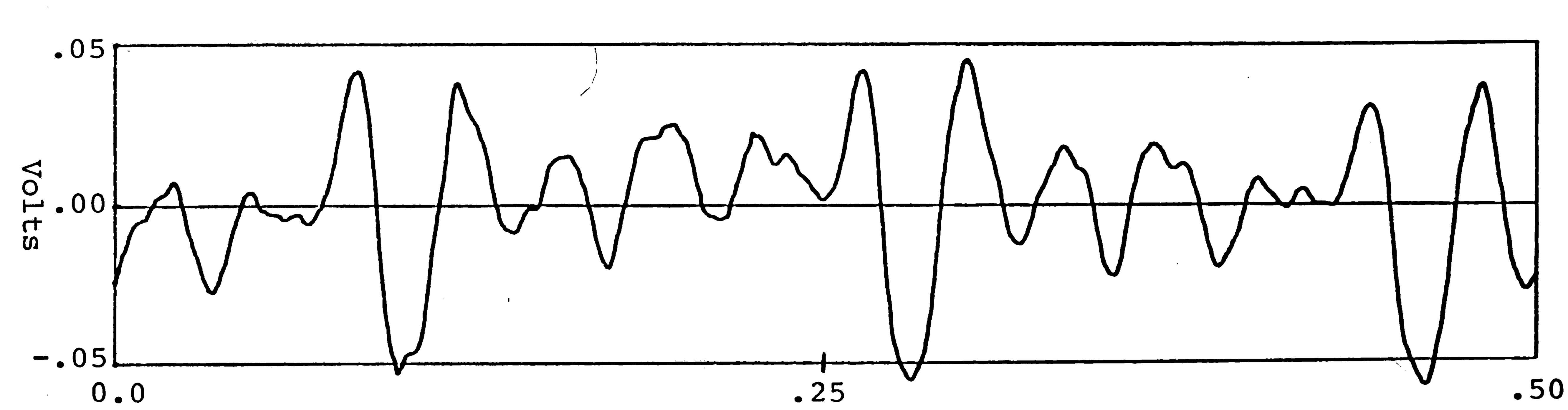
Time in Seconds
325 RPM

Counterweight Positions
Crank-Optimal Follower-Optimal

Frame Acceleration

Graph 4

39

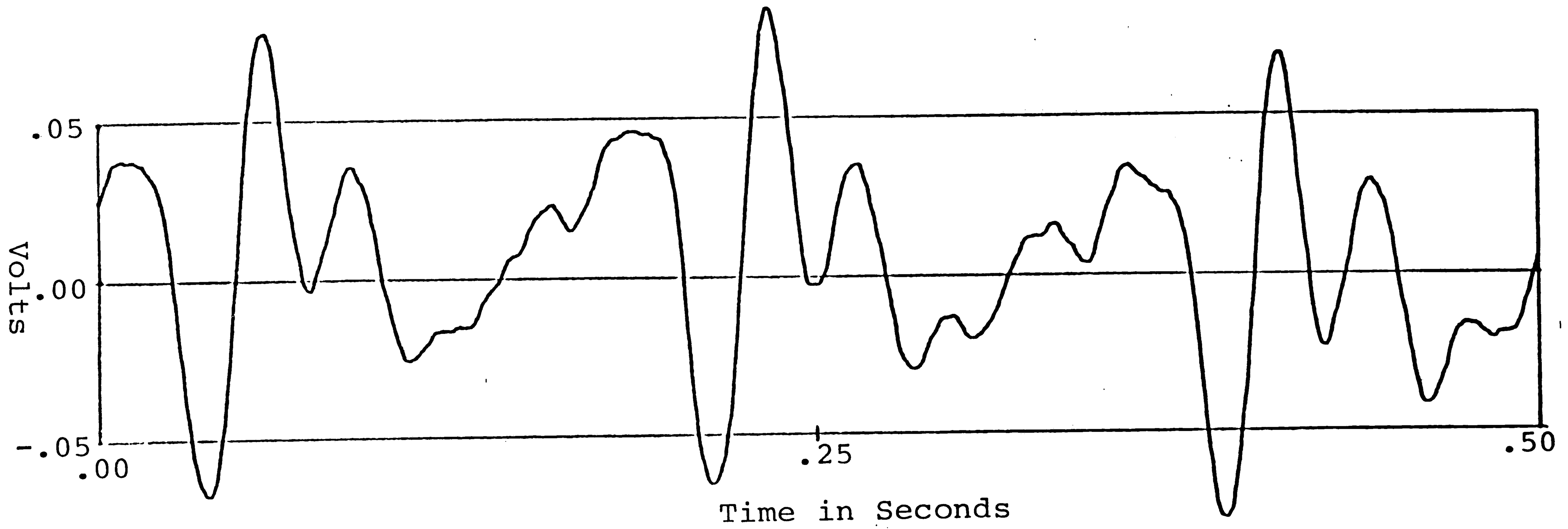


Time in Seconds
325 RPM

Counterweight Positions
Crank-None Follower-Optimal

Frame Acceleration

Graph 5

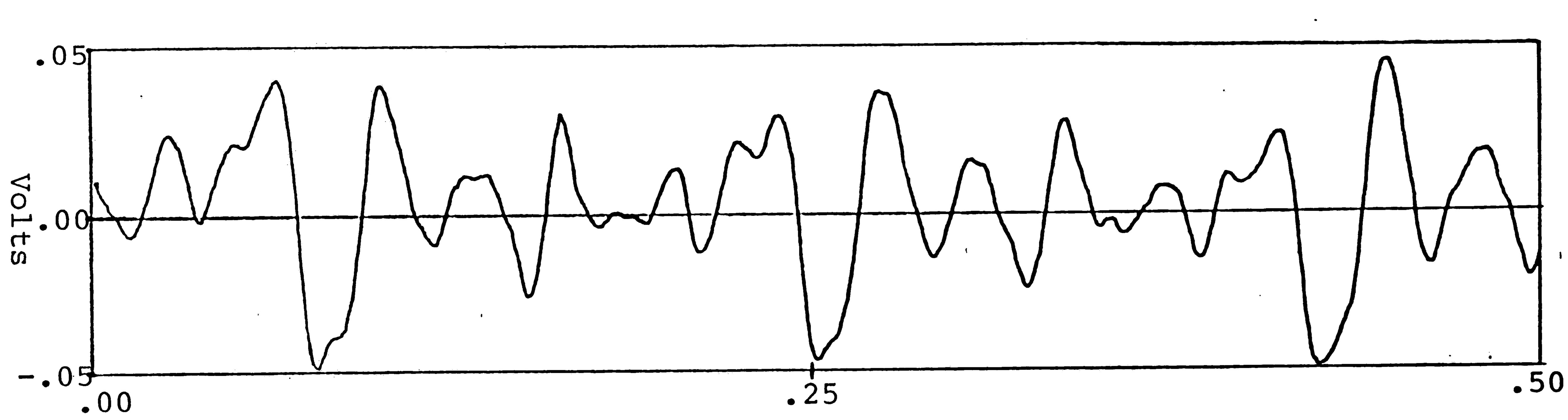


Time in Seconds
325 RPM

Counterweight Positions
Crank-Optimal Follower-None

Frame Acceleration

Graph 6

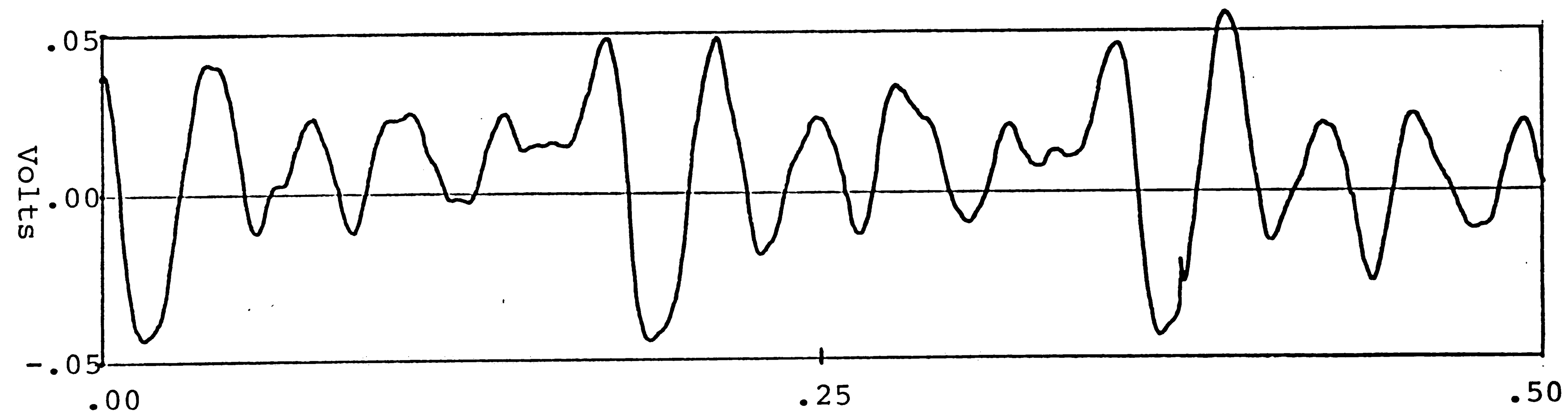


Time in Seconds
325 RPM

Counterweight Positions
Crank-2.00in Follower-Optimal

Frame Acceleration

Graph 7

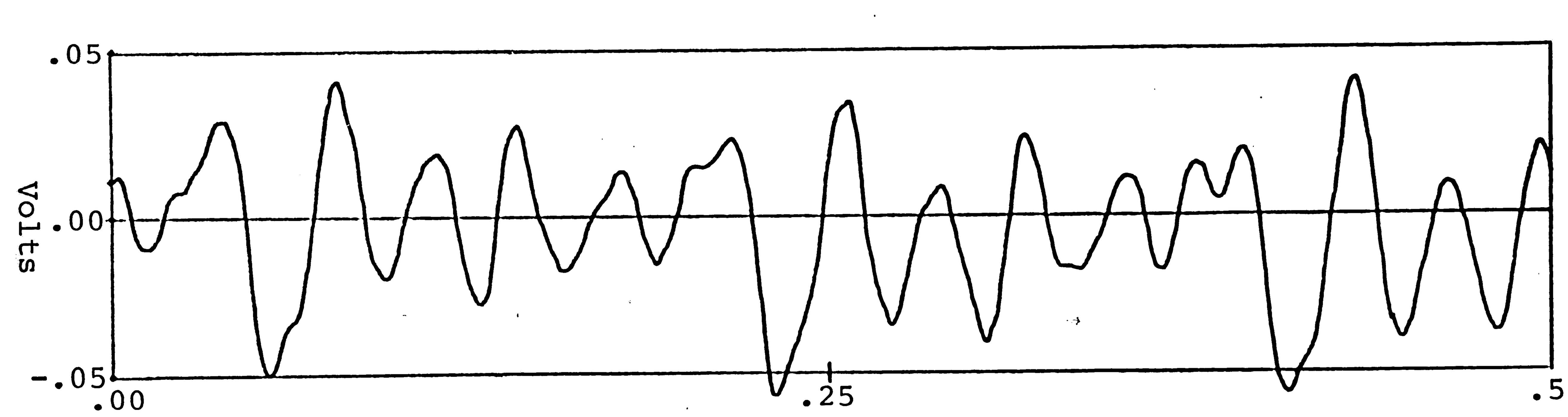


Time in Seconds
325 RPM

Counterweight Positions
Crank-2.50in Follower-Optimal

Frame Acceleration

Graph 8

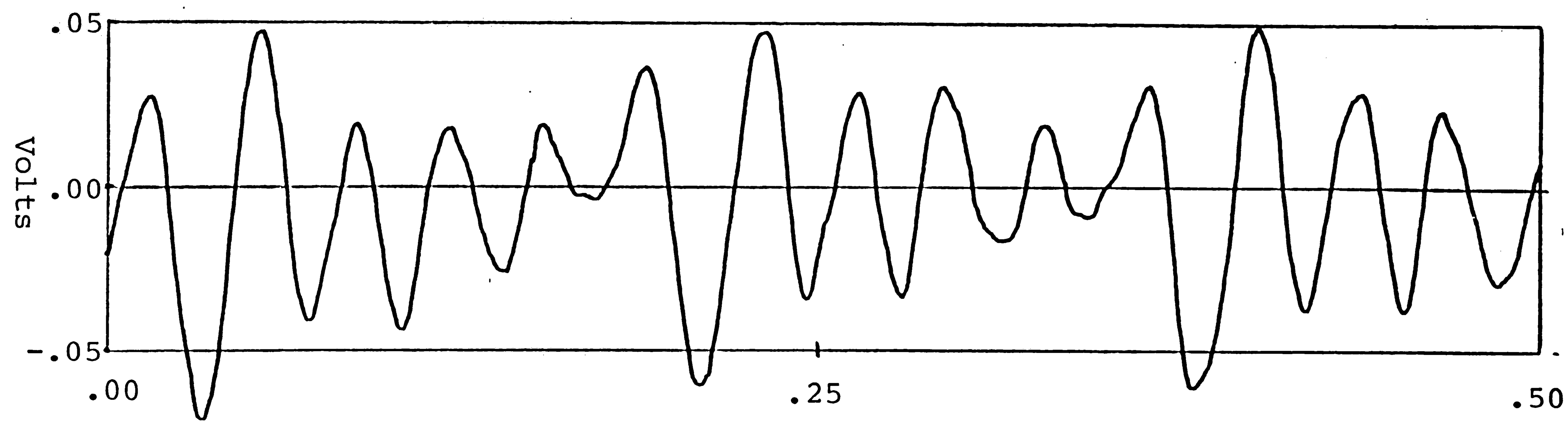


Time in Seconds
325 RPM

Counterweight Positions
Crank-3.50in Follower-Optimal

Frame Acceleration

Graph 9

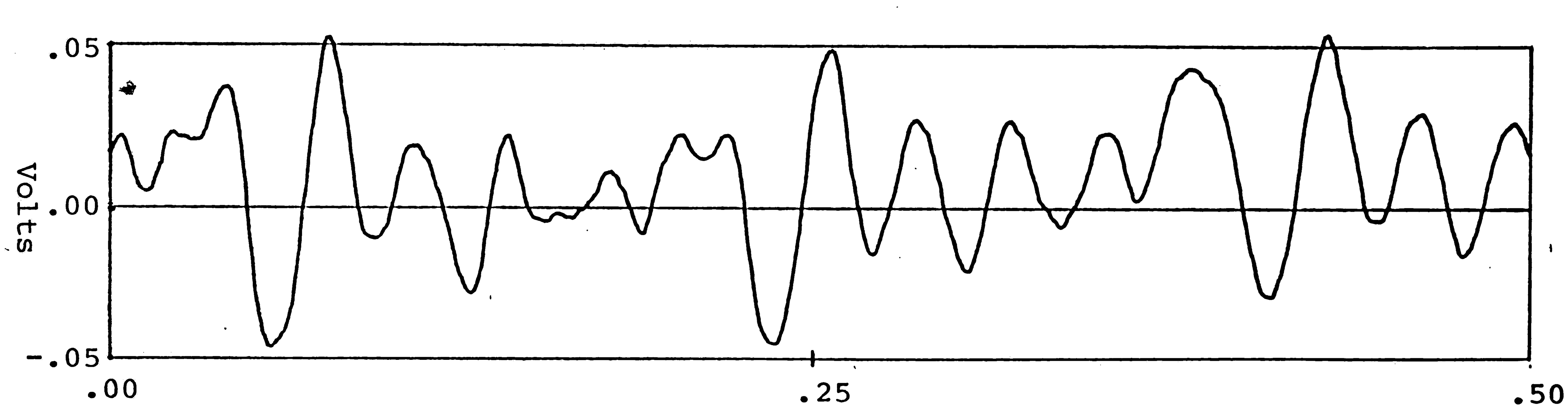


Time in Seconds
325 RPM

Counterweight Positions
Crank-4.50in Follower-Optimal

Frame Acceleration

Graph 10

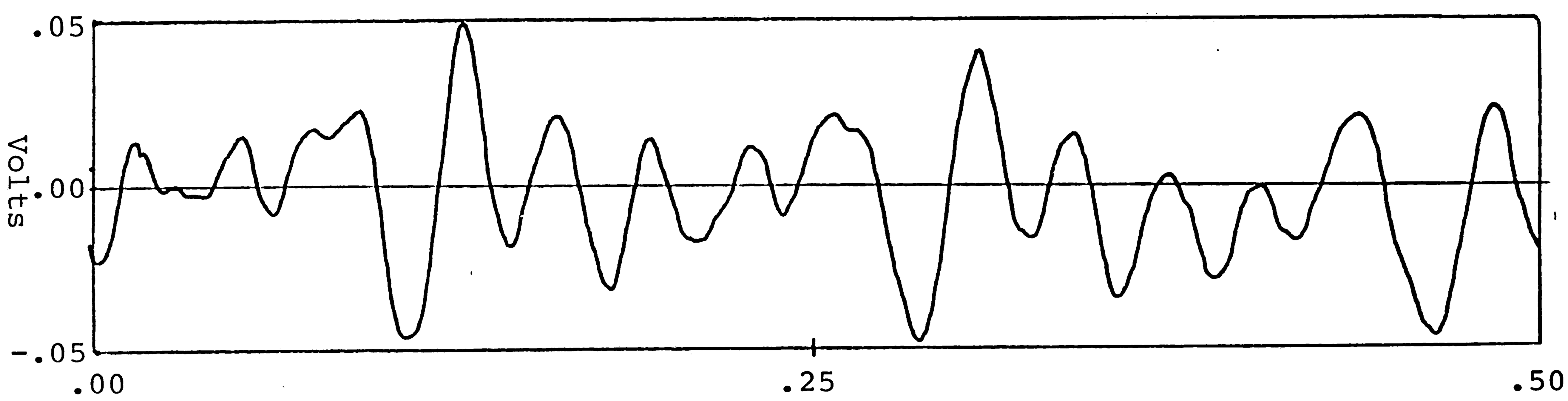


Time in Seconds
325 RPM

Counterweight Positions
Crank-Optimal Follower-2.83in

Frame Acceleration

Graph 11

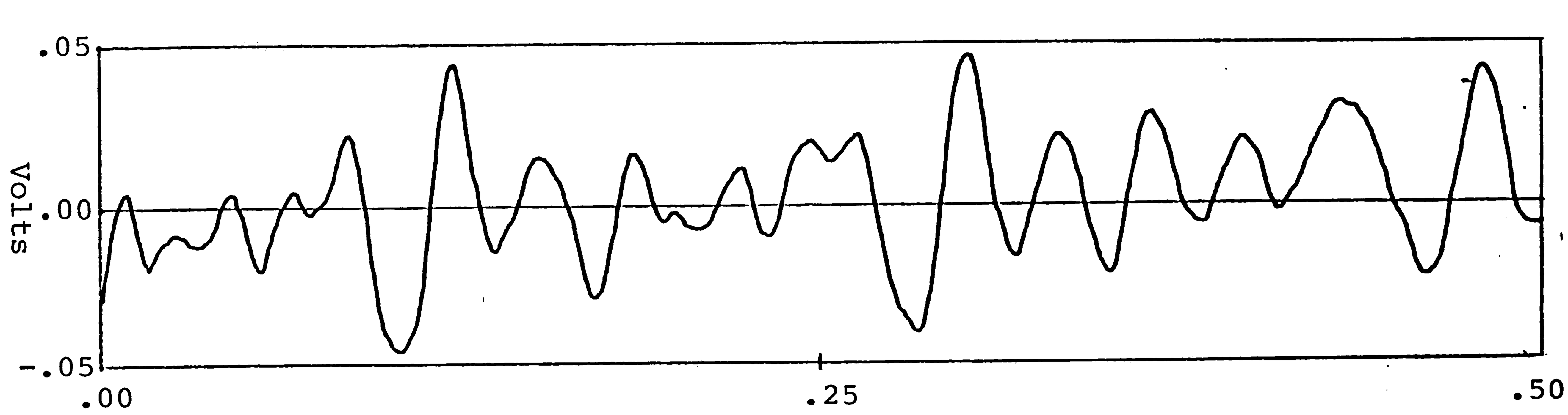


Time in Seconds
325 RPM

Counterweight Positions
Crank-Optimal Follower-3.15in

Frame Acceleration

Graph 12

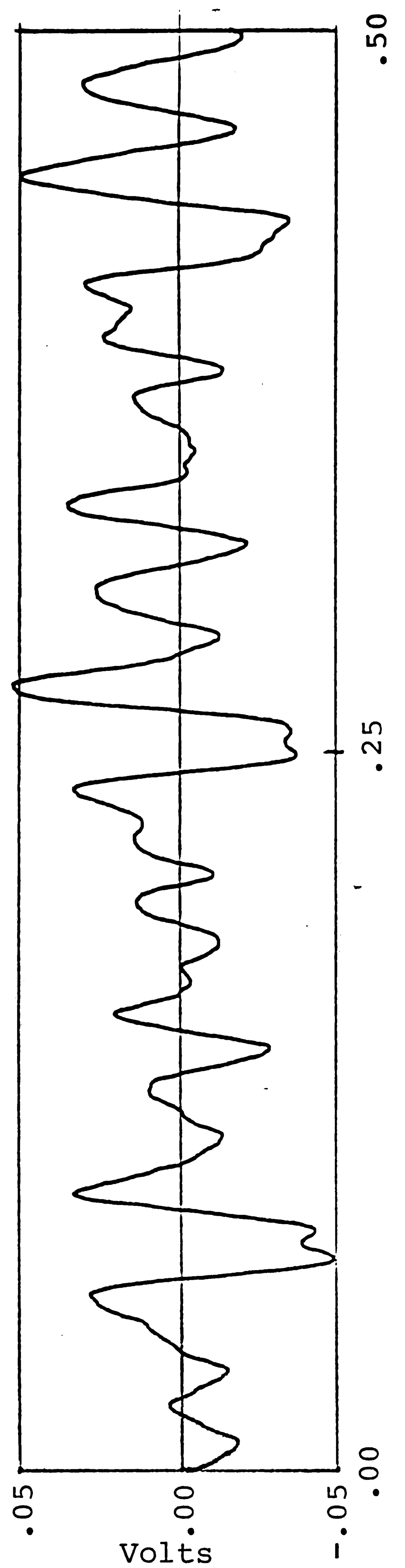


Time in Seconds
325 RPM

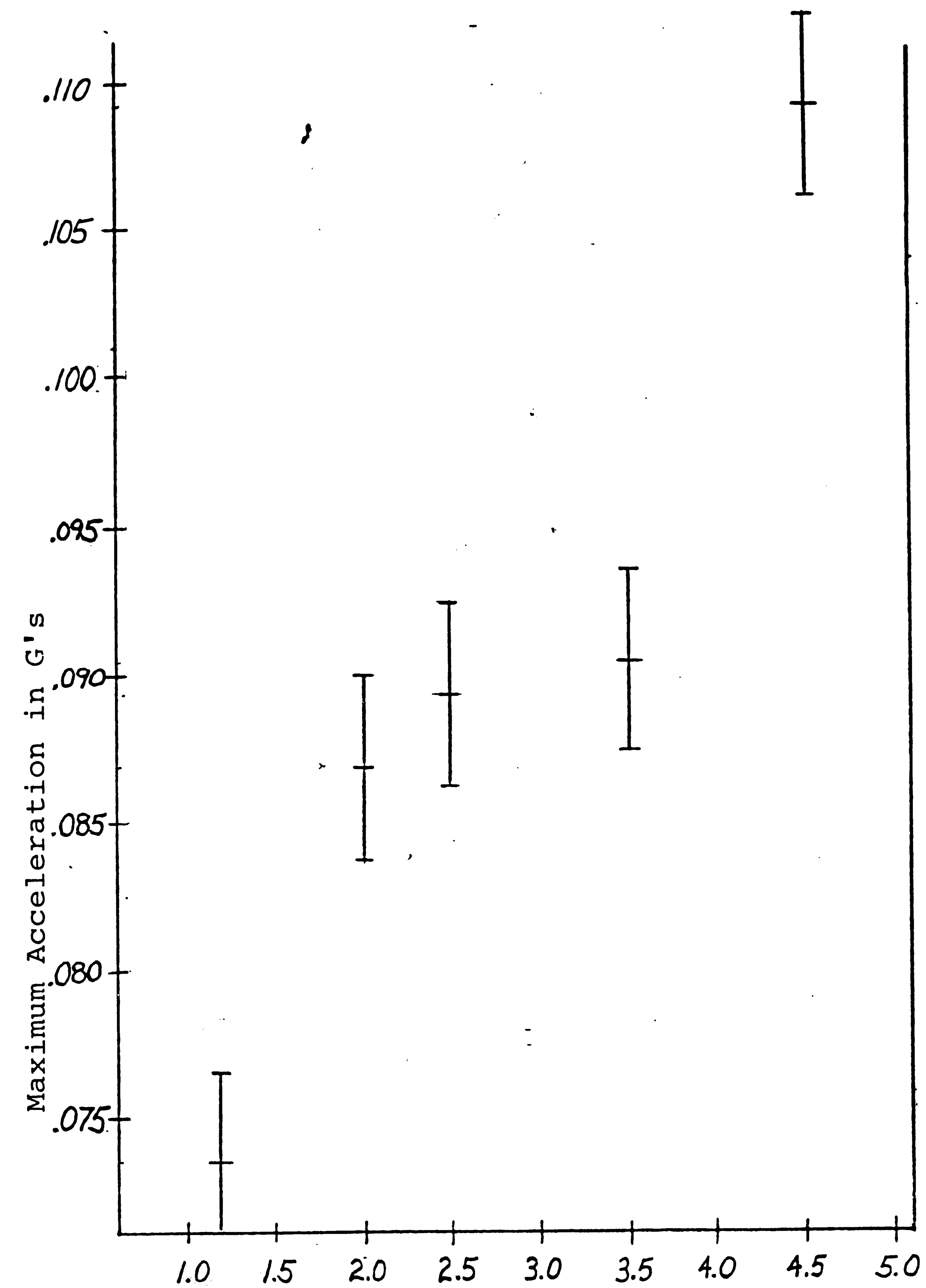
Counterweight Positions
Crank-Optimal Follower-3.45in

Frame Acceleration

Graph 13

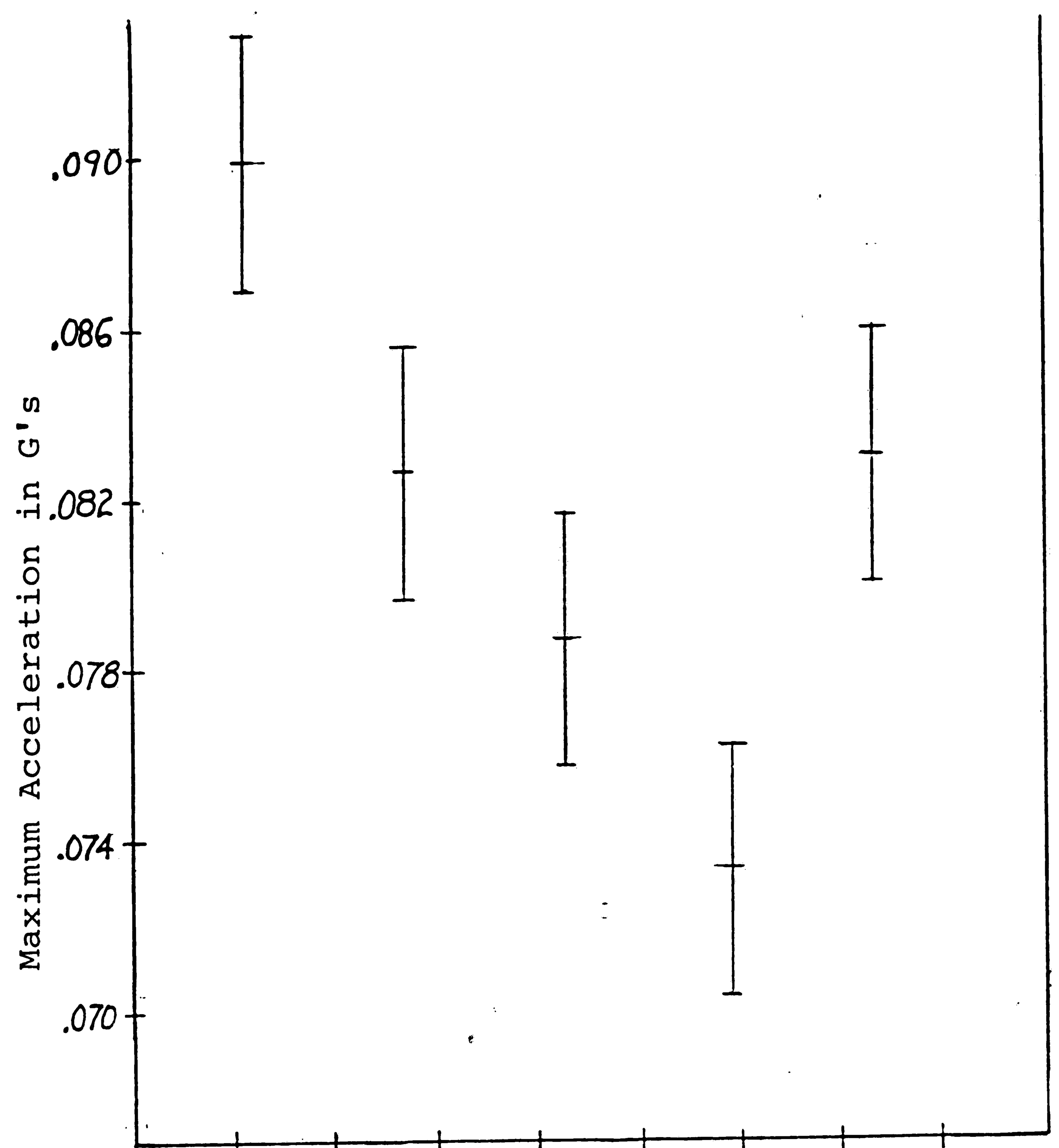


Time in Seconds
325 RPM
Counterweight Positions
Crank-Optimal Follower-4.10in
Frame Acceleration
Graph 14

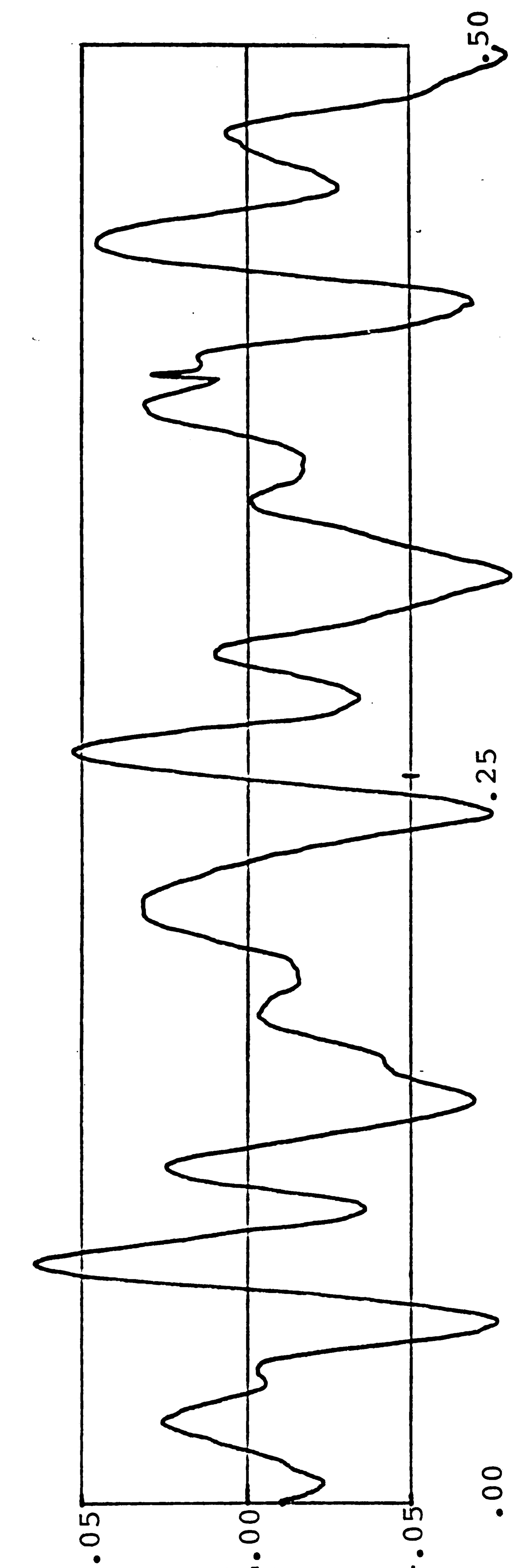


Crank Counterweight Distance
in Inches

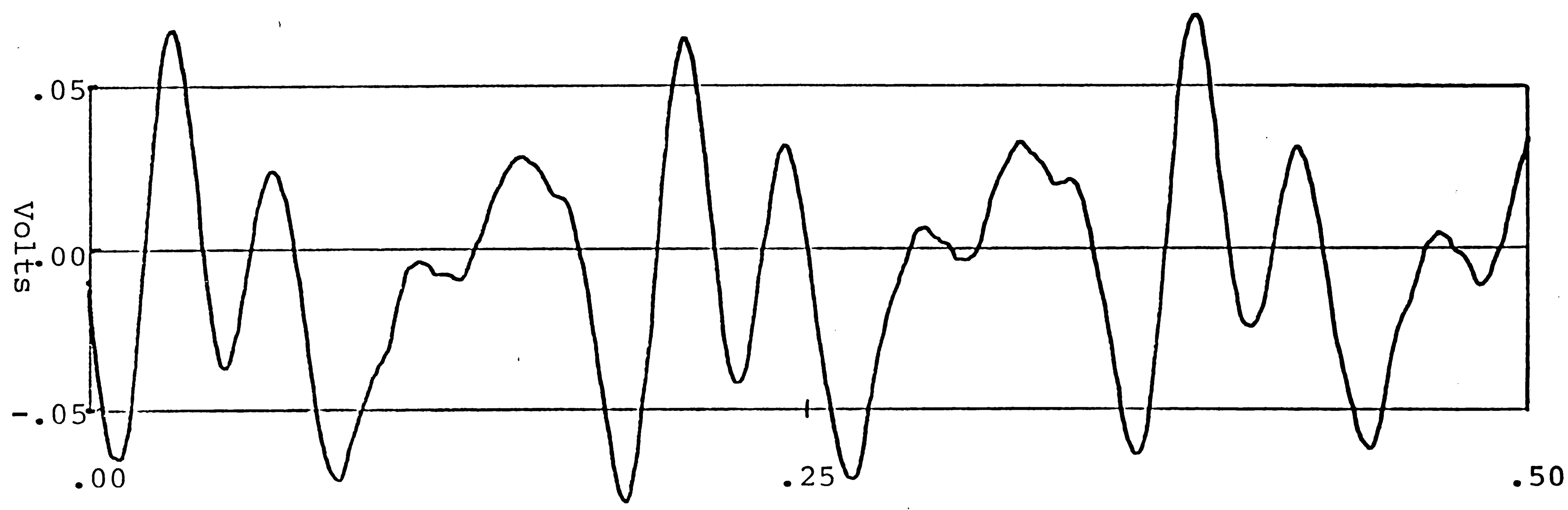
Frame Acceleration at Various Crank
Counterweight Locations
Graph 15



Frame Acceleration at Various Follower
Counterweight Locations
Graph 16



Time in Seconds
325 RPM
Load Applied No Counterweights
Frame Acceleration
Graph 17

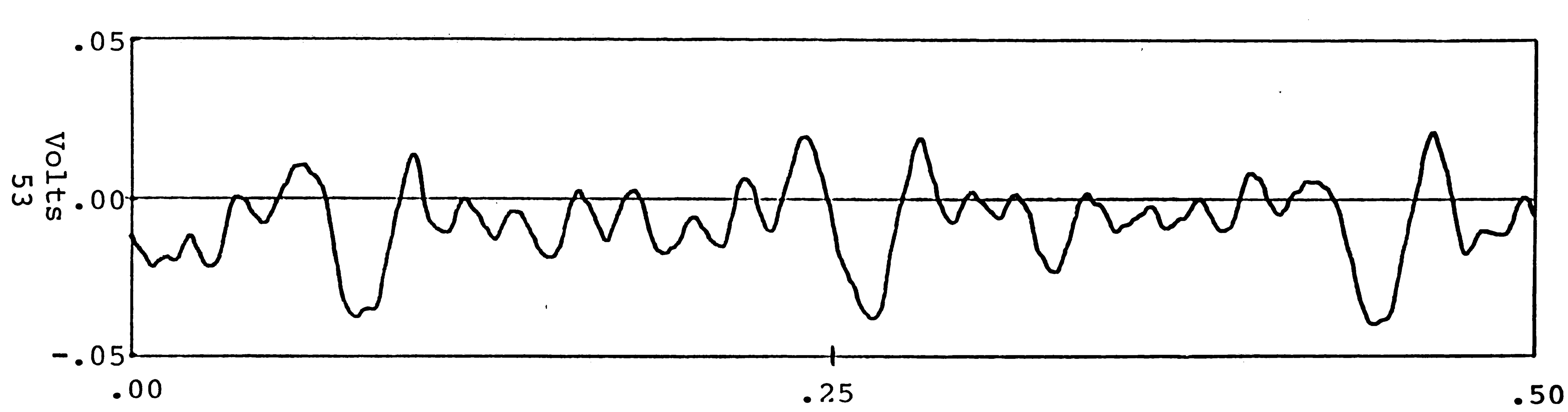


Time in Seconds

No Load Applied 325 RPM No Counterweights

Frame Acceleration

Graph 18

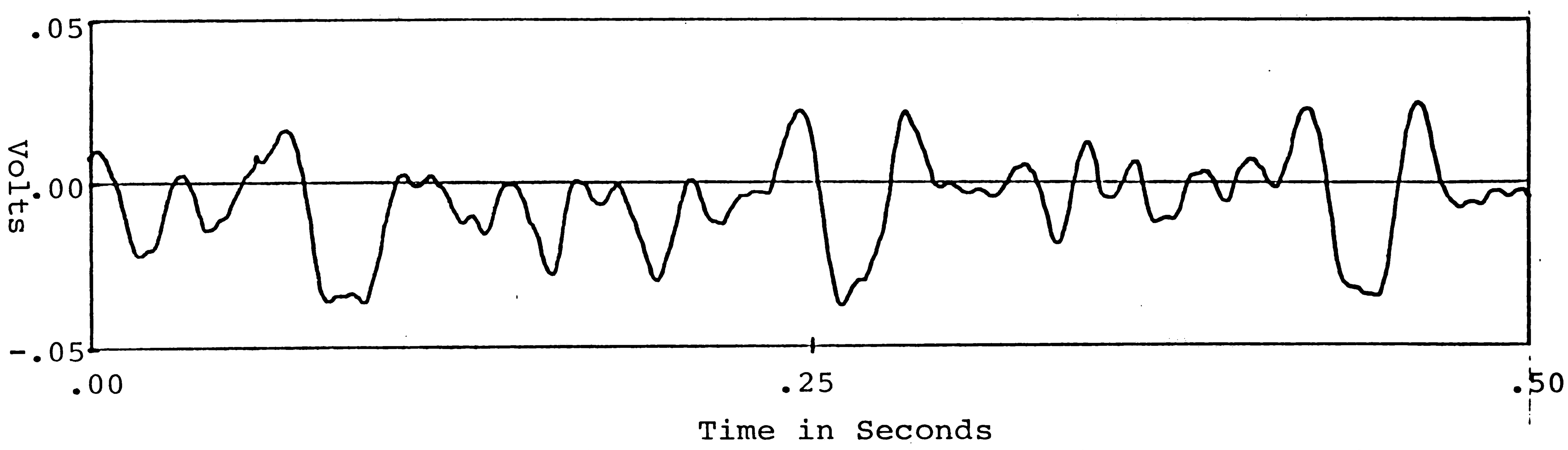


Time in Seconds

Load Applied 325 RPM Optimal Counterweights

Frame Acceleration

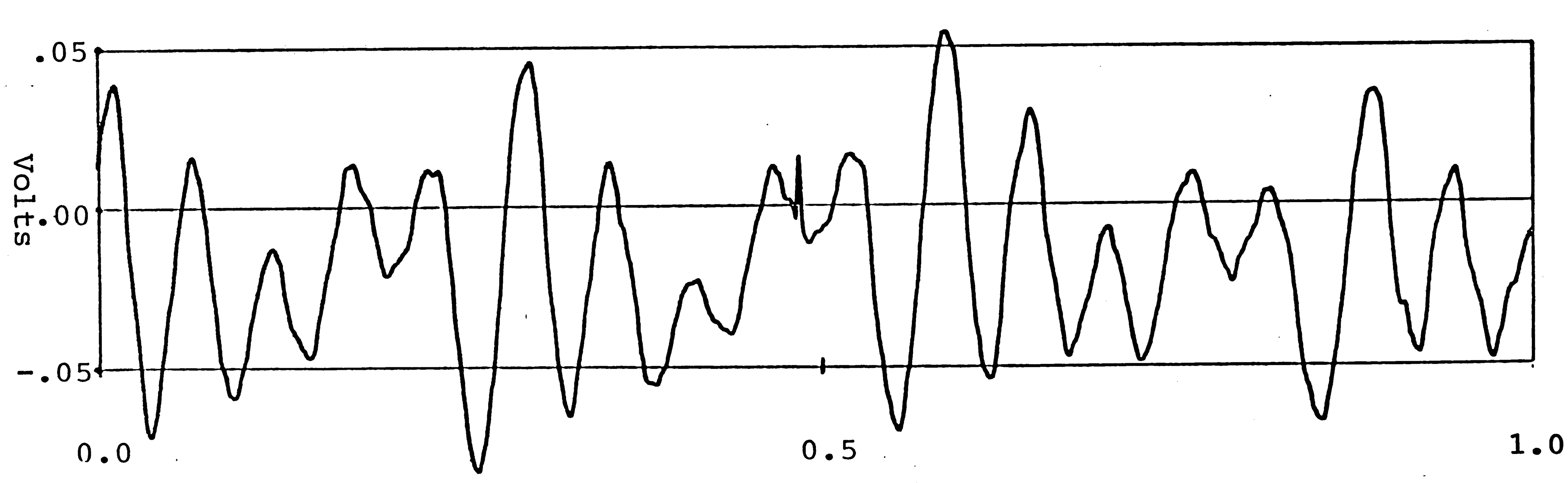
Graph 19



No Load Applied 325 RPM Optimal Counterweights

Frame Acceleration

Graph 20

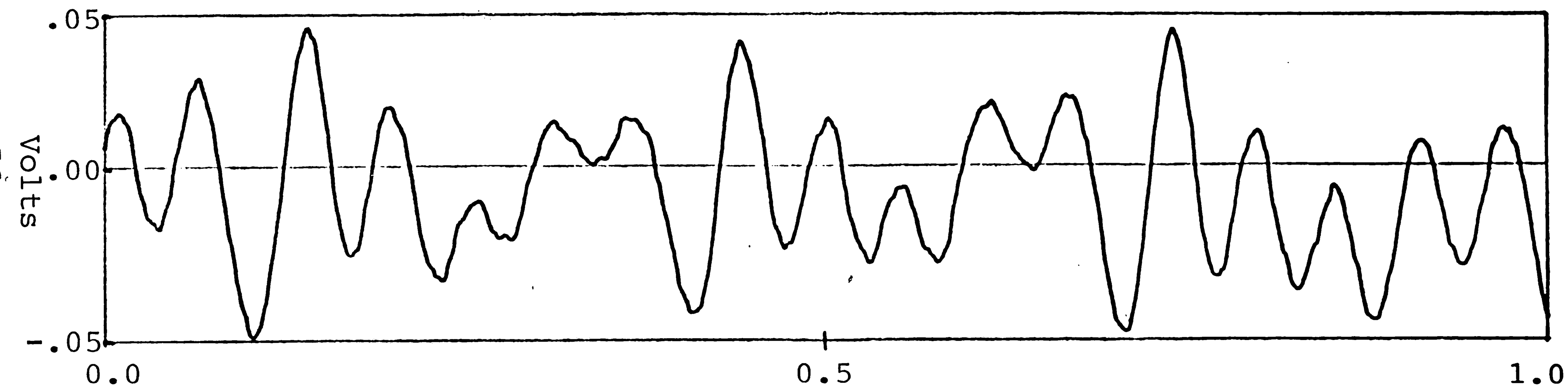


195 RPM

Load Applied No Counterweights

Frame Acceleration

Graph 21

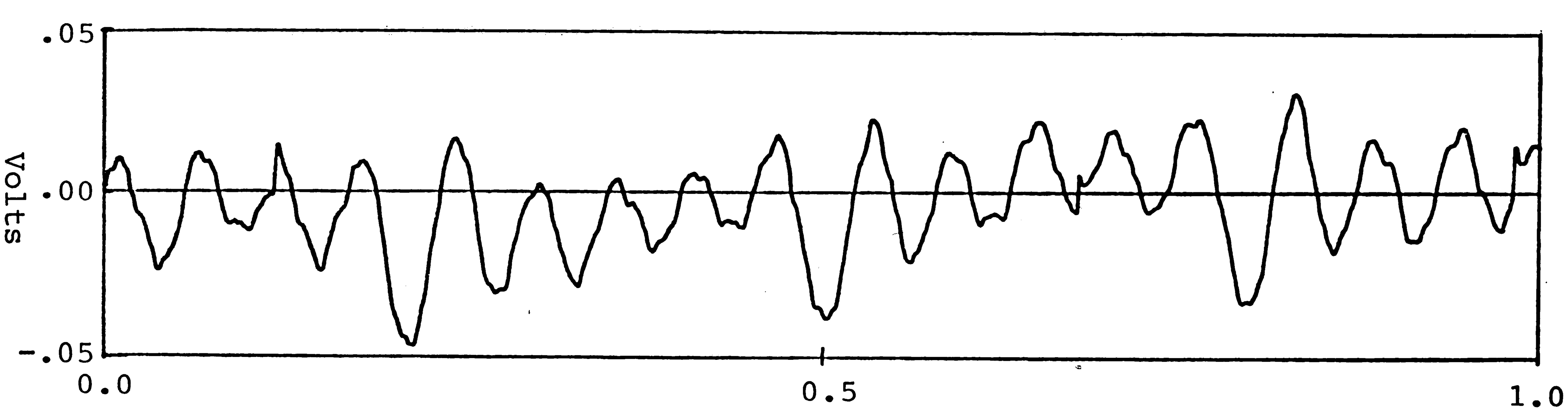


Time in Seconds
195 RPM

No Load Applied No Counterweights

Frame Acceleration

Graph 22

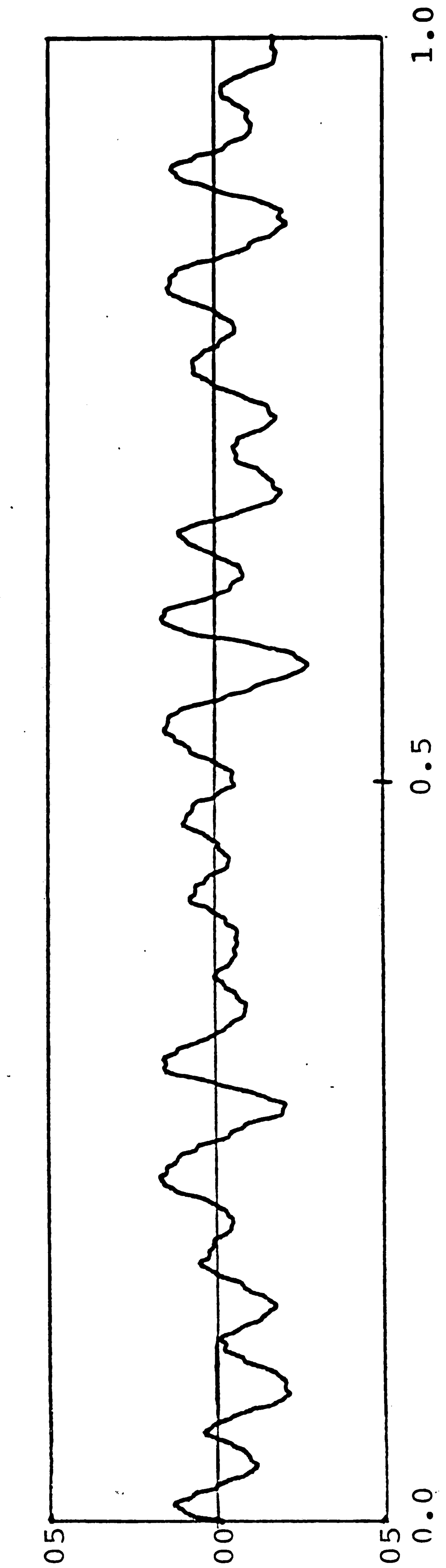


Time in Seconds
195 RPM

Load Applied Optimal Counterweights

Frame Acceleration

Graph 23



Volts

Time in Seconds

195 RPM

No Load Applied Optimal Counterweights

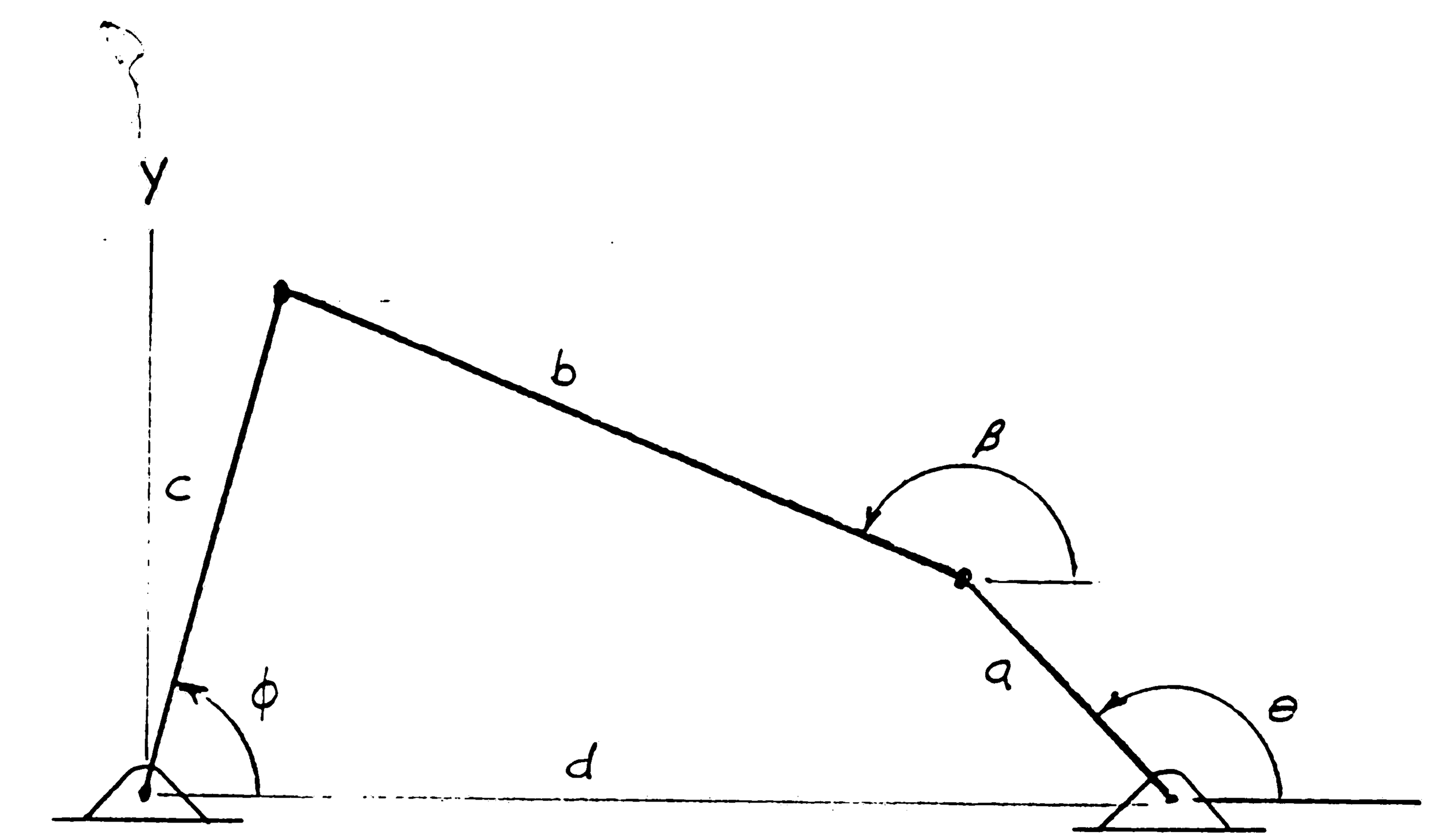
Frame Acceleration

Graph 24

Appendix A

Output Shaft Position Calculation Program

This program uses the Newton-Raphson method to find the roots of the equations of position of any four bar linkage. The link lengths and angles are defined as follows.



The sum of the x components and the y components yield the equations.

$$b(\cos\beta) - c(\cos\phi) + d + a(\cos\theta) = 0$$

$$b(\sin\beta) - c(\sin\phi) + a(\sin\theta) = 0$$

Solving these equations for $b(\cos\beta)$ and $b(\sin\beta)$, then squaring both sides and summing the result yields.

$$b^2 = c^2 + d^2 + a^2 - 2dc(\cos\phi) - 2ca(\cos\phi)(\cos\theta)$$
$$- 2ca(\sin\phi)(\sin\theta) + 2da(\cos\theta)$$

Dividing both sides by 2ac and letting

$R_1 = d/c \qquad R_2 = d/a \qquad R_3 = (d^2 + a^2 - b^2 + c^2)/2ca$

the above equation can be rewritten as

$$R_1(\cos\theta) - R_2(\cos\phi) + R_3 - \cos(\theta - \phi) = 0$$

This is known as Freudenstein's equation and can be used to find the relation between the input and output angles of any four bar linkage. It can not be solved for for a given value of so a program was written to solve for interatively using the Newton-Raphson method. The following program solves for output positions at five degree increments of input but could be easily changed to solve for any increment.

Variable Definitions:

A	length of input crank a, inches
B	length of connecting rod b, inches
C	length of follower c, inches
D	length of fixed link d, inches
DELTH	increment of input angle , dergrees
TH	value of input angle , degrees
THMX	maximum value of input angle , degrees
R1, R2, R3	constants calculated from link lengths
PHI	value of output angle, degrees
PHI1	improved value of output angle, degrees
FOFPH	Freudenstein's equation
DFOFPH	derivative of Freudenstein's respect to
EPSI	accuracy check, radians

```
10 REM   FOUR BAR LINKAGE POSITIONS
20 PRINT "FOUR BAR LINKAGE POSITIONS"
30 READ A,B,C,D
50 PRINT "POSITIONS FOR A=";A;" B=";B;" C=";C;" D=";D
70 READ TH,THMAX,PHI,DELTH,EPSI
80 R1=D/C
90 R2=D/A
100 R3=(D*D+A*A-B*B+C*C)/(2*C*A)
110 TH=TH*1.745329E-2
```



```
120 PHI=PHI*1.745329E-2
130 FOFPH=R1*COS(TH)-R2*COS(PHI)+R3-COS(TH-PHI)
140 DFOFPH=R2*SIN(PHI)-SIN(TH-PHI)
150 PHI1=PHI-FOFPH/DFOFPH
160 IF ABS(PHI1-PHI)>EPSI THEN GOTO 180
170 IF ABS(PHI1-PHI)<=EPSI THEN GOTO 200
180 PHI=PHI1
190 GOTO 130
200 TH=CINT(TH/1.745329E-2)
210 PHI1=PHI1/1.745329E-2)
220 PRINT "          ";TH;"          ";PHI1
230 TH=TH+DELTH
240 IF (TH-THMAX)>0 THEN GOTO 280
250 IF (TH-THMAX)<=0 THEN GOTO 260
260 PHI=PHI1
270 GOTO 110
280 END
290 DATA 2,15.125,4,15
300 DATA 0,360,50,5,.00001
```

Output: Modified to fit on page

FOUR BAR LINKAGE POSITION
POSITIONS FOR A=2 B=15.125 C=4 D=15

TH	PHI1	TH	PHI1
5	56.44124	185	114.1718
10	57.28826	190	113.0060
15	58.32881	200	109.8424
20	59.55316	205	107.8973
25	60.95411	210	105.7516
30	62.52011	215	103.4387
35	64.23924	220	100.9922
40	66.09867	225	98.44452
45	68.08500	230	95.82644
50	70.18447	235	93.16609
55	72.38317	240	90.48899
60	74.66717	245	87.81773
65	77.02255	250	85.17233
70	79.43550	255	82.57016
75	81.89228	260	80.02646
80	84.37917	265	77.55441
85	86.88237	270	75.16565
90	89.38794	275	72.87044
95	91.88159	280	70.67804
100	94.34863	285	68.59693
105	96.77374	290	66.63507
110	99.14084	295	64.80012
115	101.4329	300	63.09954
120	103.6320	305	61.54077

125	105.7187	310	60.13131
130	107.6725	315	58.87868
135	109.4719	320	57.79046
140	111.0939	325	56.87412
145	112.5150	330	56.13693
150	113.7115	335	55.58572
155	114.6598	340	55.22660
160	115.3382	345	55.58572
165	115.7274	350	55.10394
170	115.8120	355	55.34648
175	115.5821	360	55.79277
180	115.0342		

Appendix B

Mass Property Calculations

The machine balance program requires the calculation of the mass, center of mass location and rotational moment of inertia about its mass center of each rigid member of the linkage (i.e. crank assembly, connecting rod assembly, follower assembly).

General equations for the three mechanical parameters are first developed for the crank and follower since they only differ in one dimension.

Density of steel = .0007332lbm/in^3

Acceleration of gravity = 386 in/sec^2

Mass of Crank and Follower

a = length of members between centers

b = total length of member

Mass = ρ[2 x 2.0in x .75in x b + (π/4) x .5^{2}in x .75in + 4((π/4(1.5in^2 x .125in) + (π/4) x .75^{2}in x 10.75in - 8 x

(π/4) x .125^{2}in x .5in + 4 x (π/4) x .375^{2}in x 3.5in + 4 x (π/4) x (5/8)^{2}in x .4in] + mass of pulley

Equation for center of mass for crank and follower with counter weight shafts in place. The baseline for the calculation is the centerline of the shaft center.

Rcm = (π/mass) x [3.0^{2}in x b(b/2 - 1in) + a(π/4) x .5^{2}in x .75in + a(π/4) x 2 x 1.5^{2}in x .125in - a(π/4) x 4 x .125^{2}in x .5in - 1.2in x 4 x (π/4) x .625^{2}in x .4in - 2.75in x 4 x (π/4) x .375^{2}in x 3.5in]

simplified

Rcm = [3.0in^2 x b(b/2 - 1) + a(.56451in^3) - 4.84124in]/[3b + 7.7681 + pulley weight/ρ]

A general equation for the rotational moment of inertia for the crank and follower about the mass center is calculated below. Listed are the rotational moment of inertias for each component of the crank/follower then the transfer function to move it to the center of mass of the assembly.

Main Shaft: Icm + md^2 = .5(π/4) x .75^{2}in x 10.75in(.75in/2)^2 + (π/4) x .75^{2}in x 10.75in x Rcm^2

Thrust washers: Icm + md^2 = 4 x .5(π/4) x 1.5^{2}in x .125in x .75^{2}in + 2(π/4) x 1.5^{2}in x .125in x Rcm^2

$+ 2(\pi/4) \times 1.5^2 in \times .125in(a - Rcm)^2$

Sides: $Icm + md^2 = (1/12)3^2 in \times b(b^2 + 2^2 in) + 3^2 in \times b(b/2 - 1 - Rcm)^2$

Rod Journal: $Icm + md^2 = .5(\pi/4) \times .5^2 in \times .75in \times .25^2 in + (\pi/4) \times .5^2 in \times .75in(a - Rcm)^2$

Pin Holes in Sides: $Icm + md^2 = -4[.25(\pi/4) \times .125^2 in \times .5in(.125/2)^2 + 1/12 \times (\pi/4) \times .125^2 in \times .5in \times .5^2 in] - 4(\pi/4).125^2 in \times .5in(Rcm^2 + .75^2 in) - 4(\pi/4).125^2 in \times .5in((a-Rcm)^2 + .75^2 in)$

Counter Weight Shafts: $Icm + md^2 = 4[.25(\pi/4).375^2 in \times 3.5in(.375/2)^2 + (1/12)(\pi/4).375^2 in \times 3.5in \times 3.5^2 in] + .375^2 in\ 3.5in((2.75in + Rcm)^2 + (5/8)^2 in)$

Jam Nuts: $Icm + md^2 = 4[.25(\pi/4).625^2 in \times .4in(.625/2)^2 in + (1/12)(\pi/4).625^2 in \times .4in(.4/2)^2] + 4(\pi/4).625^2 in \times .4in((Rcm + 1.2in)^2 + (5/8)^2 in)$

Large Pulley (used only for crank calculation): $Icm + md^2 = .5(5.453lbf/386in/sec^2) \times .3^2 in + (5.453lbf/386in/sec^2) \times Rcm^2$

Small Pulley (used only for follower calculation): $.5(.6719lbf/386in/s^2) \times 1^2 in + (.6719lbf/386in/s^2) \times Rcm^2$

Simplifying and summing the components yields

$Icm/\rho = 2.192176 + 5.190991 \times Rcm^2 + .589048(a - Rcm)^2 + .25(b^2 + 4) + 3b(b/2 - 1 - Rcm)^2 - .0245436(Rcm^2 + .5625) - .0245437((a - Rcm)^2 + .5625) + 1.546252((2.75 + Rcm)^2 + .39062) + .490874((Rcm + 1.2)^2 + .39062)$

for crank only add: $.04238 + .014127Rcm^2$

for follower only add: $.00087034 + .0017407Rcm^2$

Crank Calculations: a = 2in   b = 4in

Mass = .02862lbm

Crank Center of Mass = .212304in

Rotational Moment of Inertia about Center of Mass

$Icm = .229467lbf*in^2$

Follower Calculation: a = 4in   b = 6in

Mass = .0206328lbf

Center of Mass from shaft centerline = 1.18742in

Rotational Moment of Inertia about Center of Mass

$Icm = .0831175lbf*in^2$

Connecting Rod Calculations:

Mass = $(2 \times 2.5in \times 2in \times .5in) - 2(\pi/4).5^2 in \times .5in + 2 \times 3.5in \times 1in \times .5in + 10.13in \times 2in \times .25in + 8(\pi/4).5625^2 in \times .35in$

Mass = .0103115lbf

Center of Mass from shaft centerline = 7.5625in (by symetry)

Rotational Moment of Inertia about Center of Mass =

$Icm = .432257lbf*in^2$

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Biography

Daniel R. Brooks was born in Suffern, New York, on July 14, 1960, of Robert and Jane Brooks. He obtained a Bachelor of Science Degree in Mechanical Engineering from the Rochester Institute of Technology in May 1983. Professional experience includes employment with the Delco Products Division of General Motors as a Manufacturing Engineering working in the area of automated assembly of automotive actuators.