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The algebraic solution of highly indeterminate plane rigid framed rectilinear structures /

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The Algebraic Solution of Highly Indeterminate Plane Rigid Framed Rectilinear Structures

by

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Abstract

A study was made of the feasibility of applying a computer program system for symbolic mathematical operations to obtain closed-form solutions to large scale structural systems. The computer program system accepts problems defined in algebraic form and provides solutions also in algebraic form, using combinations of the same symbolic quantities used to define the problem. Numerical substitution into the algebraic expressions for solutions results in direct calculation of results.

It was desired to obtain closed form solutions for the internal forces and external displacements of typical multi-bay, multi-storey plane rigid building frames. An algebraic model and programs were set up for analysis of plane rigid frames by the direct stiffness method. Trials at applying the model to the general frame form proposed showed that the method requires excessive computer central memory storage. Only by reducing the problem to a one-storey by one-bay frame was it possible to obtain the desired solution on the computer equipment available.

Closed form solutions for the model deflections and member stress resultants were successfully obtained. Parameter studies were performed by substituting ranges of numerical values into the final algebraic expressions, while holding the remaining parameters constant. An extensive set of curves was prepared from the parameter studies, presenting the results in the form of graphs of direct magnitudes and of percentage changes in magnitude.

The minor importance of cross-sectional area properties, suggests that a multi-storey frame model based on rotational degrees of freedom, and a single sway degree of freedom per level would yield meaningful results. This would

considerably reduce the size of the system stiffness matrix requiring solution.

The algebraic deflection and stress resultant equations for the structure provide a basis to conduct an immediate parametric analysis of the structure, using explicit closed form expressions.

One benefit of a symbolic algebraic solution, would be to provide a computer-based 'expert system' with as many 'up-front' constraints as possible, which has the immediate effect of choking the feasible solution domain to a much smaller set, not to mention the merits of the use of a precise structural model.

Chapter 1
Introduction

1.1 Problem Statement

The investigation of the behavior of rectilinear plane rigid framed structures, as are found in single and multi-storey building frames, is conducted using algebraic methods rather than numerical methods. The final solutions for all structural displacements and internal member stress resultants are obtained as algebraic expressions containing combinations of the symbolic quantities and parameters used to describe the properties of the structure. Since it is desired to apply the method to structures too large to solve by hand, a computer package for algebraic mathematical processing, called REDUCE, is employed.

1.2 Objective

The objective is to formulate a set of general equations that describe the interrelation of structural, geometric properties and loads to the deflections and stress resultants, in an indeterminate rectilinear elastic planar frame system. The original need that inspired this approach, was the desire to formulate closed form algebraic solutions that would permit a reliability analysis to be performed for a general frame structure, without the need to resort to Monte Carlo numerical integration techniques.

1.3 Method

The method of structural analysis used, corresponds to the Bernoulli-Navier beam models that are commonly used in many linear elastic numerical finite element packages. The algebraic model is linear with respect to the load and joint deflection terms, and not necessarily linear in its geometric and member section terms. The behavior of the whole structure is characterized by the nodal behavior of the model, which serves to define the complete deflection, rotation, moment and shear diagrams for each member, with the requirement of deflection and rotational continuity at each joint.

The test case structure that was extensively analyzed in this thesis, was a one-level by one-bay rigid frame, with fully rigid support conditions at the base. This general model consisted of six degrees of freedom, and three degrees of indeterminacy, and is discussed at length in the next chapter.

For the purpose of a coherent presentation of the general equations, that are presented in Appendix D, in graphical form; numerical values are inserted into the general solutions so that a study can be made of the effects of varying a few selected parameters.

1.4 Applications of Algebraic Solutions

Once the correct algebraic relationships can be established between loading and the attainment of limit states, then a set of design rules may be formulated in algebraic terms, involving loading, geometry, material, section and limiting value factors, based on some rational design policy.

Another benefit of expressing the structure's behavior algebraically is that it is then possible to directly integrate all stochastic parameters of the system, and thereby assess the distribution of the structure's load capacity, assuming a linear model is appropriate in approximating the behavior of the system.

A full rigid plastic analysis could be carried out based on the formulation of released sections in the structure (ie. hinges and slip planes), however such a problem becomes combinatorially large as the number of release sites increases.

Chapter 2
Definition of the Model's Parameters

2.1 Overview of the System Model

The behavior of the structure is fully defined in the deflection (stiffness) domain, using assembled 'first order' member stiffness properties, the particulars of which are presented in Appendices B and C. The problem is formulated in the familiar direct stiffness method using a basic element of the form pictured in Figure 1. The element stiffness is defined in the form, and is discussed in more detail in Appendix B.

$\{\mathbf{s}\}=\{\mathbf{P},\mathbf{V},\mathbf{M}\}=[\mathbf{k}]\cdot\{\mathbf{u},\mathbf{v},\theta\}=[\mathbf{k}]\cdot\{\mathbf{u}\}$

The local member stiffness is converted to global member stiffness through equilibrium, and direction cosine matrix transforms, giving members end forces in global coordinates in terms of structure node displacements in global coordinates, which is covered more fully in Appendix C. By compatibility the member displacements and structure displacements at any node are identical. By equilibrium, the sum of all member end forces at a node equals the external applied loads at the loads. This allows the total structural stiffness matrix to be stacked in the form:

$\{\mathbf{w}\}=[\mathbf{K}]\cdot\{\delta\}$

This formulation produces a set of equations expressing the load as linear functions of all deflection terms. The property of linearity eases the solution of these equations, so that the deflection terms may be uniquely expressed in terms of the applied loads, or vice versa, or some hybrid set. The functional relationship between geometric and structural properties on one hand and load and deflection terms on the other is not linear. Therefore an arbitrary solution of one set of these terms with respect to another is non trivial.

The initial problem is to express the deflection terms with respect to a set of equations containing purely load, geometric and structural parameters. Thus in a linear model the assembled global stiffness matrix need only be inverted to achieve this goal.

Once the deflections are known in terms of the load and structural parameters, the member end stress resultants can also be formulated in terms of the load and structural parameters.

The structural interaction between load and deflected geometry is constructed in terms of the assembly of the effect of the stiffness of each member. The assembly is based on the requirement that the total resultant end stresses in all members framing into a common node are in equilibrium with all applied node forces. The other assembly requirement is that deflections of all member end points connected at a common node are of equal magnitude and direction, a requirement of compatibility.

Briefly the model of the structure is formulated using the following criteria:

- The structure is characterized in two dimensions.
- A 'cantilever' type member element is used.
- The axial behavior arises from a uniform strain profile.
- The flexural behavior arises from a pure linear strain profile.
- The stress-strain material behavior is linear elastic.
- The members frame into each other at right angles.
- The members form closed rectangular cells.

- Intersecting members form monolithic joints.
- Loads are applied at joints, or are uniform over the member.
- Modelling nodes are only at member joints.

Przemienski [1968] was the source of all elemental formulations used in this model, of which only the first-order linear model was carried out to completion in this thesis.

The simplest case, the one-level by one-bay rigid frame shown in Figure 2, was analyzed first in order to confirm the correctness of the solution method, the veracity of the program that was written, and the correct functioning of the REDUCE package. Limitations in storage capacity did not permit a larger system to be analyzed, and so the the remainder of this chapter outlines the specific model of the one-level by one-bay rigid frame. Appendix A expounds the more general model that was originally contemplated when this work was started.

2.2 System Parameters

2.2.1 Geometric Parameters

The one-level by one-bay rigid frame in this investigation is parameterized by two variables (γ,**L**), which have the following meanings:-

- γ - cell aspect ratio, beam span = γ**L**
- **L** - floor-to-floor height

The investigation imposes the uniformity requirement that all floors are of similar height. Since floors are often of similar function this constraint is practical. Since the investigation deals only with rectilinear frames, the bay lengths must remain of constant span for all levels in a given bay. The choice of the uniform length parameter 'L', as the floor-to-floor height was made in the light of the fact that for most multi-storey frames, this parameter is constant and controlled by architectural necessity.

The factor γ is also denoted in Roman typeface by G, and called the span-to-height ratio.

2.2.2 Structural Parameters

Each member is modelled as a prismatic section for the purposes of simplicity. This is not a restriction in the general REDUCE model that has been formulated and non-prismatic beams can be modelled using altered stiffness coefficients that can be derived numerically in terms of the section properties of some conveniently chosen standard section (A_0, I_0). In either case the structural parameters can still be compactly expressed in terms of area and moment of inertia.

In general, a more complex formulation could be derived directly from the cross-sectional shape function of the beam. In practice most beams are prismatic, or approximately prismatic.

These factors are denoted in Roman typeface by AB(i,j), IB(i,j) for beams and AC(i,j) and IC(i,j) for columns. Simplified notation has been used in the one-level by one-bay rigid frame problem, where AB,IB is used for beams, and AL,IL for the left-hand column and AR,IR is used for the right-hand column.

2.2.3 Load Parameters

The system is subjected to two load effects, gravity and lateral 'wind' loads. Gravity loads are modelled as uniformly distributed loads over an entire beam member, and produce four non-zero force and moment components in the system load vector. Lateral loads are applied assuming a linear increasing load effect up the side of the frame. These loads are modelled as lumped nodal sway forces, parameterized by the maximum load W_{max}, which occurs at the top level of the frame system.

- **W** - Uniform Load, Floor loading.
- **WD** - Wind load.

For the case of the one-level by one-bay rigid frame, these factors are denoted in Roman typeface by W for beam loads, and WD or P for lateral loads.

2.2.4 Deflection - Response Parameters

For the one-level by one-bay rigid frame the only deflection unknowns occur at the top level, at the beam-column corner points. Thus there are six deflection unknowns, denoted in vector form as the matrix DV(i,1), where:

- i=1 -> left-hand horizontal (X) sway deflection.
- i=2 -> left-hand vertical (Y) shortening deflection.
- i=3 -> left-hand joint rotation due to beam loads/side-sway
- i=4 -> right-hand horizontal (X) sway deflection.
- i=5 -> right-hand vertical (Y) shortening deflection.
- i=6 -> right-hand joint rotation due to beam loads/side-sway

The REDUCE programs that solve for the stress resultants, use DV(i,1) for i=7,8,9, as the zero deflection values of the fixed boundary conditions in this specific problem, for the sake of maintaining program generality.

2.2.5 Internal Stress Resultant Parameters

Stress resultant parameters are similar to the deflection parameters, in that they may be expressed in terms of the structural, geometric and load parameters. These factors, as shown in Figure 3, are denoted in Roman typeface by P_a, V_a, M_a, M_b and M_{max}. The M_b term is calculated directly from the V_a and M_a terms using moment equilibrium, and the M_{max} term is obtained after solving the deflected shape differential equation. (See references in Harrison [1980] and Boresi et al. [1978]) The formulation of the stress resultant transform matrix is given in Appendices B and C.

2.3 Boundary Conditions

The support conditions for the problem are confined to the base of the structure. The boundary conditions may be parameterized by direct stiffness coefficients (K_x,K_y,K_θ), where a fixed support corresponds to infinite stiffness for each coefficient. However the use of generalized boundary conditions expands the number of unknown deflections that must be solved for. It was not possible to include generalized boundary stiffness properties in the analysis due to limitations in the REDUCE system. (See Section 4.5 for further discussion)

Chapter 3
Specification in Dimensionless Parameters

The one-level by one-bay rigid frame system can be expressed in terms of dimensionless parameters. Any algebraic representation of a system, which is dimensionally consistent, is of a form that is essentially dimensionless. It is not until specific dimensionally consistent values are assign to each parameter that a general solution, becomes specific to a given set of units.

It can be often illustrative to express all parameters in a form relative to some chosen parameter, such that both are dimensionally consistent. The most common method is to divide a parameter through by some dimensionally consistent term, thereby producing a 'dimensionless' parameter, γ the span to height ratio is such a parameter.

The structural section properties were redefined in terms of the height L, and the relationship between dimensionless area and dimensionless inertia was expressed as another ratio Q, where $Q=A \cdot L^2/I$.

The loads as described already reflect a dimensionless form, in that the force unit is the simplest integral component of a load (compared to using mass length per time squared). The uniform loads are expressed in terms of a parameter specific to a given span, these parameters could be re-expressed in terms of WD_{max}, however this only serves to add another ratio parameter that is not meaningful, as lateral and gravity forces can act independently.

The reformulation on this basis did not lead to any simplifying terms appearing in the solution. The only result was the occurrence of 'Q' terms replacing area 'A' terms. This method would be illustrative if Q was made uniform for the whole structure. This would bring Q out as a constant of the system stiffness and flexibility matrices. Choosing Q to take a specific value would remove all Q factors, and simplify the system equations to involve only inertia 'I' terms, for each member. Thus member sizing could be ascertained using one design unknown per member. For the one-level by one-bay rigid frame these would be three unknown inertial terms, or really only two, the beam and column terms, in a practical design problem. Since the number of equations for sway and critical moments under various load conditions exceeds the number of design unknowns, the design will be controlled by the worst case values of these equations.

Chapter 4
The Algebraic Solution Method

4.1 Preamble

The primary numerical form in structural engineering work is the rational number. In algebraic terms, this form has its analog as a linear function divided by another linear function; such as two multivariate polynomials.

$$\frac{22}{3}, \quad \frac{a}{b}, \quad \frac{P(x_1,x_2,..)}{Q(x_1,x_2,..)}$$

The term 'rational' will be used as an adjective denoting the property of the the algebraic rational function form.

4.2 The REDUCE Algebraic Processor

REDUCE is a software package to do numerous mathematical tasks in a general algebraic manner. This package was available on the DEC-20 computer, and runs as a shell on a LISP interpreter. The reader should refer to Hearn [1985] for details about the use of this system.

Problems are stated in algebraic form and the results are outputted in algebraic form. The user has the option of requesting resulting algebraic expressions in the form of FORTRAN statements for use in later computer programs or in a display form suitable for printing with raised superscripts and lowered subscripts (depending on the printing device).

The processing required in this thesis work was the manipulation of simple rational forms, with integer powers. In particular all manipulations would be on matrices, which would only require simple multiplication and subtraction operations on these rational forms. REDUCE has features that make it much more flexible than required for this project, to the extent that its flexibility detracted from its capacity to represent the simple forms of this project in an efficient manner, in terms of storage space and computation time. The data structures used in REDUCE are general purpose LISP lists, which are not the most efficient manner of representing rational forms.

The methods and multiple programs reflected more the need to side-step storage limitation problems, than mirror the actual needs of a computation scheme for deriving an algebraic solution. These limitations also fettered the size of the structure that could be handled and meant that REDUCE was only able to handle a one-level by one-bay rigid frame. This was the price incurred in using a general purpose algebraic package, such as REDUCE.

4.3 The REDUCE Solution Scheme

Due to REDUCE storage limitations, it was necessary to divide the scheme of solution over several sub-programs. Four different catagories of files were created:

- PROGRAM - A file containing mainly REDUCE procedures and information management commands, designated in this text by capitals - SOLVE.
- INPUT FILE - A file containing equations that are constants of the system, designated in this text by bold lettering - **STRUC**.
- DATA FILE - A file generated by a REDUCE program, formatted for another program, designated in this text by italic capitals - *DELTA*.
- OUTPUT FILE - A file generated and formatted for the purposes of printing, designated in this text by underlined capitals - <u>OUTSR</u>.

The names and functions of the various files are outlined in Appendix C.

The solution scheme requires the setting up of two INPUT files,

- **STIF** - Contains the basic stiffness coefficients K_{11},..etc
- **STRUC** - Contains the size of the frame, bays and levels, and any information about similarity of section properties, span ratios and numeric constants

Appendix B contains the details of these input files.

4.3.1 Element Stiffness

The use of a file like **STIF** permits flexibility in producing formulations for alternate elemental stiffness matrices, based on shear strain energy and first order approximation to PΔ second order axial-moment interaction or non-prismatic members, using REDUCE as a means of algebraically integrating and deriving the stiffness coefficients, in which case, **STIF** would be a DATA file from another REDUCE program. This study utilized the common first order axial/flexural stiffness properties of a prismatic element.

STIF is called up specifically for use by the programs GENEL and MAKELK, that generate the 6x6 elemental stiffness matrix for a planar beam member and its corresponding stress resultant transformation matrix, which was trimmed to be 3x6 for the purposes of saving storage space in later stress resultant calculations. **STIF** is called up by all the other programs, but only to get the element title information for printing purposes, which was done to prevent confusion between output results from different analyses.

4.3.2 System Interconnectivity

The use of the file **STRUC** permits the specification of any number of potential frame types and formats. The use of the REDUCE LET statement in this file allows for parametric simplification, and condensation of any structural or load parameter (ie. **I**, **W**..etc) into an alternate algebraic or numeric form. This file also contains a title line for output identification purposes. **STRUC** is specifically used by the programs SOLVE, STRESS, MAXIM and similar analogs to these programs.

4.3.3 System Solution

The solution scheme, requires the running of GENEL, to create the elemental stiffness matrix and elemental stress resultant transform matrix in DATA files *ELSTIF* and *ELSTRS*. SOLVE is run, which examines **STRUC** and *ELSTIF* and generates the deflection vector solution in a DATA file *DELTA*. The stress resultants are formed by running STRESS, which takes *DELTA* and *ELSTRS*, and produces the OUTPUT file <u>OUTSR</u>, which contains the section axial, shear and moments at the member ends. The program MAXIM is similar to STRESS, but produces the middle span extremum of the bending moment due to the placement of uniformly distributed loads on a member, creating the OUTPUT file <u>MAXSR</u>. The split up of the tasks between STRESS and MAXIM was required due to storage problems, in fact MAXIM can only handle a single stress computation without bombing out due to storage limitations, for a one bay by one level frame. The programs INFLU and MAXINF produce the influence coefficients for the system, under all load conditions at the end and maximum middle span moment sections. The OUTPUT files are called <u>IFLSR</u> and <u>IFLMX</u> respectively. These programs formulate the stress due to a unit effect of each load condition (point or distributed) on the structure, and are not just an alternate form of the system flexibility matrix.

4.4 Solution Expressions as FORTRAN Statements

In order to facilitate a better understanding of the meaning of the closed form elastic solution of the one-level by one-bay rigid frame system, a REDUCE program PLOTS was devised to convert the deflection and stress resultant equations into FORTRAN statement form. Only the sway deflection and the base, column-beam corner, and maximum middle span beam moments were used in the later FORTRAN based analysis. The FORTRAN equations expressed these effects in terms of unit lateral and gravity loads seperately, and so were forms of the influence coefficients in a FORTRAN format.

A FORTRAN program was used to generate plots of each deflection and moment effect against variations in some other two parameters. The complexity of the algebraic solution made it imperative that a graphic form of representation be produced to aid in making credible observations about the behavior of a one-level by one-bay rigid frame.

4.5 The Limitations of REDUCE

The major problem in the use of REDUCE, was that it was unable to invert matrices of any size greater than 6x6, without encountering storage limits. REDUCE could not handle the inversion of the system stiffness matrix of a two bay by one level rigid frame, which was 9x9 in size and had zero coefficients. There are other compact methods to solve the simultaneous equations, which would have benefited from the symmetry and positive definiteness of the matrix. These methods were not used, as the subsequent computation of a deflection vector and the stress resultants were the bottlenecks in using REDUCE. This limitation required the splitting up of stress calculations into the programs STRESS and MAXIM.

The complexity of each term along the multi-program solution chain became larger and larger, which resulted in storage capacity limits being encountered early in the chain for large systems, or at later stages for smaller systems. The simplification and cancelation of terms that did occur, did not control the growth of this complexity to any diminishing degree. For instance the deflection vector represented a data entity comparable to the size of the system flexibility matrix and the applied load vector multiplied together with only marginal simplification of terms. Furthermore the symmetry and potential compactness of the flexibility matrix is lost upon multiplication. However as REDUCE does not try to store symmetric matrices compactly, this is not a consideration in this study. In any future development, it is strongly recommended to keep all terms as individual matrices, and multiply out to get specific coefficients only at the time they are needed, Nash [1979] outlines methods that could be more economic.

By the time it came to multiplying the elemental stress resultant transform matrix into the deflection vector the propagation of the size of stress terms was very large, in fact only three terms at a time could be formulated without exceeding REDUCE's storage limits. Thus even though the one-level by one-bay rigid frame system was the simplest indeterminate system possible, there were considerable problems in producing final solutions in terms of stresses and deflections.

Chapter 5
Program Verification

The programs used and the REDUCE system itself were verified as operating correctly in three ways,

- Dimension Reduction Analysis
- Numeric Case Study and Comparison against a Numerical Structural Analysis Program
- Simplified Reformulations

5.1 Dimension Reduction Analysis

This method reduces all parameters down to their lowest form as functions of measurement units. Dimensional analysis was used to provide a check on the consistency of the measurement units in the equations resulting from the REDUCE solution. For instance Young's Modulus E, is reduced as force over length squared and moment is force times length. For this problem the elemental dimensional forms are force and length, rather than mass, time and length. Substitution into all equations for system stiffness, flexibility, force and deflection vectors should yield simple rational forms reflecting the dimensional characteristics of each equation or coefficient. Rather than follow the combinations of units through the path of the solution by hand, this analysis was carried out by the REDUCE program SOLFLX, which produced the OUTPUT file AKAFLX. The first part of the file lists the general solution in algebraic form, the latter half gives the dimensionally reduced form, of the general solution, after substituting force and length factors for the primary parameters; E, I, A, ..etc.

The proof of the dimensional correctness is given in Appendix D, Section D.3.

5.2 Numeric Case Study

This method provided verification that the addition and multiplication of algebraic elements in REDUCE had taken place correctly, and had yielded equations that were correctly formed, as well as dimensionally consistent. STRUCTRD, a FORTRAN program used in a microcomputer environment (see Driscoll [1985]), was executed for a one-level by one-bay rigid frame with fixed numerical properties. The STRUCTRD output included all intermediate matrix steps and values. The REDUCE program SOL1, which substitutes numeric values for all algebraic system parameters, was executed and the numeric results were copied to the OUTPUT file AKAFLX.

The two output files were merged and abridged, and appear in Figure 5-0, where it is evident that there was precise agreement.

Figure 5-1: Partial Listing of STRUCTRD and SOL1 Output

In the following partial listing and mixing of the outputs from the FORTRAN program STRUCTRD and SOL1, the text in **bold**face is output from the REDUCE program SOL1, for a specific solution as given below.

% STRUCTURAL SPECIFICATIONS:
% ELEMENT TYPE IS ELASTIC FLEXURAL/AXIAL ELEMENT
% SYSTEM TYPE IS RIGID SUPPORTS, FULL 3 DOF PER NODE
% NUMBER OF BAYS = 1
% NUMBER OF LEVELS= 1
% NUMBER OF NODES = 2
% GLOBAL D.O.F. = 6

% ** SPECIFIC SOLUTION ******
E=30000
L=12
I=500
A=100
G=1

WD=5
W=1

```
O       THESIS - SPACE FRAME 1 BAY x 1 LEVEL
O       NUMBER OF MEMBERS=  3
O       NUMBER OF JOINTS =  4
O       ITYPE            =  3
        NUMBER OF MEMBER GEOMETRIES =      2
        NUMBER OF MATERIAL PROPERTY SETS =      1

O       NUMBER OF KNOWN LOAD COMPONENTS        6
        NUMBER OF UNKNOWN LOAD COMPONENTS      6
        NUMBER OF LOAD CASES                   1

        GEOMETRIC PROPERTIES OF MEMBERS
        NO  NAME             A/AO      IZ/IO      IY/IO      IP/IO
         1      COLUMN     100.000   500.000       .000       .000
         2      BEAM       100.000   500.000       .000       .000

        PHYSICAL PROPERTIES OF MATERIALS
        NO  NAME            E/EO        G/EO
         1       STEEL 30000.000        .000
O       PROPERTIES OF MEMBERS
O       JOINT NO      X/LO          Y/LO          Z/LO
O            1          .00           .00           .00
O            2        12.00           .00           .00
O            3          .00         12.00           .00
O            4        12.00         12.00           .00

        MEMBER    I     J   LENGTH/LO   SHAPE            MATERIAL              THETA3
          NO     END   END            (NO)  (NAME)     (NO)  (NAME)          (DEG)   (RAD)
           1      1     3     12.00    1     COLUMN     1       STEEL         .00    .0000
           2      2     4     12.00    1     COLUMN     1       STEEL         .00    .0000
           3      3     4     12.00    2     BEAM       1       STEEL         .00    .0000
O       EO=   .10000E+01    /  2       CA=    1.000
        LO=        1.000               CL=    1.000
        IO=        1.000    4          CD=    1.000
        AO=        1.000    2          CE=    1.000
                                       CF=    1.000
```

S EXCHANGE MATRIX
SIZE 12 ROWS BY 12 COLUMNS

	1	2	3	4	5	6
1	.10000E+08	-.62500E+06	.62500E+06	.25000E+07	.62500E+06	.00000
2	-.62500E+06	.35417E+06	.00000	-.62500E+06	-.10417E+06	.00000
3	.62500E+06	.00000	.35417E+06	.00000	.00000	-.25000E+
4	.25000E+07	-.62500E+06	.00000	.10000E+08	.62500E+06	.62500E+
5	.62500E+06	-.10417E+06	.00000	.62500E+06	.35417E+06	.00000
6	.00000	.00000	-.25000E+06	.62500E+06	.00000	.35417E+
7	.25000E+07	.00000	.62500E+06	.00000	.00000	.00000
8	.00000	-.25000E+06	.00000	.00000	.00000	.00000
9	-.62500E+06	.00000	-.10417E+06	.00000	.00000	.00000
10	.00000	.00000	.00000	.25000E+07	.00000	.62500E+
11	.00000	.00000	.00000	.00000	-.25000E+06	.00000
12	.00000	.00000	.00000	-.62500E+06	.00000	-.10417E+

S EXCHANGE MATRIX (CONTINUED)
SIZE 12 ROWS BY 12 COLUMNS

	7	8	9	10	11	12
1	.25000E+07	.00000	-.62500E+06	.00000	.00000	.00000
2	.00000	-.25000E+06	.00000	.00000	.00000	.00000
3	.62500E+06	.00000	-.10417E+06	.00000	.00000	.00000
4	.00000	.00000	.00000	.25000E+07	.00000	-.62500E+
5	.00000	.00000	.00000	.00000	-.25000E+06	.00000
6	.00000	.00000	.00000	.62500E+06	.00000	-.10417E+
7	.50000E+07	.00000	-.62500E+06	.00000	.00000	.00000
8	.00000	.25000E+06	.00000	.00000	.00000	.00000
9	-.62500E+06	.00000	.10417E+06	.00000	.00000	.00000
10	.00000	.00000	.00000	.50000E+07	.00000	-.62500E+
11	.00000	.00000	.00000	.00000	.25000E+06	.00000
12	.00000	.00000	.00000	-.62500E+06	.00000	.10417E+

***** AKA - GLOBAL STIFFNESS MATRIX *****

u_3	354166.66	0	625000	(-250000)	0	0
v_3	0	354166.66	625000	0	-104166.6	625000
γ_3	625000	625000	10000000	0	(-625000)	2500000
u_4	(-250000)	0	0	354166.66	0	625000
v_4	0	-104166.6	(-625000)	0	354166.66	(-625000)
γ_4	0	625000	2500000	625000	(-625000)	10000000

FLEXIBILITY MATRIX
SIZE 6 ROWS BY 6 COLUMNS

	1	2	3	4	5	6
1	.16657E-06	.25532E-06	-.63717E-06	.20661E-07	-.25532E-06	-.48623E-
2	.25532E-06	.37872E-05	-.15319E-05	.25532E-06	.21277E-06	-.15319E-
3	-.63717E-06	-.15319E-05	.90759E-05	-.48623E-06	.15319E-05	.72646E-
4	.20661E-07	.25532E-06	-.48623E-06	.16657E-06	-.25532E-06	-.63717E-
5	-.25532E-06	.21277E-06	.15319E-05	-.25532E-06	.37872E-05	.15319E-
6	-.48623E-06	-.15319E-05	.72646E-05	-.63717E-06	.15319E-05	.90759E-

***** FLEXIBILITY MATRIX *****

0.907587E-5	0.153191E-5	-0.63717E-6	0.726455E-5	-0.15319E-5	-0.48623E-6
0.153191E-5	0.378723E-5	-0.25531E-6	0.153191E-5	0.212765E-6	-0.25531E-6
-0.63717E-6	-0.25531E-6	0.166572E-6	-0.48623E-6	0.255319E-6	0.206610E-7
0.726455E-5	0.153191E-5	-0.48623E-6	0.907587E-5	-0.15319E-5	-0.63717E-6
-0.15319E-5	0.212765E-6	0.255319E-6	-0.15319E-5	0.378723E-5	0.255319E-6
-0.48623E-6	-0.25531E-6	0.206610E-7	-0.63717E-6	0.255319E-6	0.166572E-6

DISPLACEMENT UNITS: LINEAR DEFLS.(DX,DY,DZ)
ROTATIONS(THETAX,THETAY,THETAZ) RADIANS

DISPLACEMENT MATRIX
SIZE 12 ROWS BY 1 COLUMNS

	1
1	.00000

```
 2     .00000
 3     .00000
 4     .00000
 5     .00000
 6     .00000
 7     .38134E-04
 8    -.16340E-04
 9    -.41821E-05
10     .43568E-04
11    -.31660E-04
12    -.14349E-05
```

% ***** SOLUTION DV ******
DV(1,1) := 0.38134083E-4
DV(2,1) := - 0.16340425E-4
DV(3,1) := - 0.41820956E-5
DV(4,1) := 0.43568044E-4
DV(5,1) := - 0.31659574E-4
DV(6,1) := - 0.14349257E-5

```
      FORCE UNITS: LINEAR FORCES
                   MOMENTS    -

      FORCE      MATRIX
       SIZE   12 ROWS BY    1 COLUMNS
             1
 1    -1.3585
 2     4.0851
 3     13.379
 4    -3.6415
 5     7.9149
 6     23.643
 7     .00000
 8    -6.0000
 9    -12.000
10     5.0000
11    -6.0000
12     12.000
```

FG(1,1) := 0
FG(2,1) := (-6)
FG(3,1) := (-12)
FG(4,1) := 5
FG(5,1) := (-6)
FG(6,1) := 12

```
      STRESS RESULTANT UNITS: FORCES(AXIAL,SHEAR)
                              MOMENTS    -
```

BASIC ELEMENT STRESSES

```
    STRESSES FOR MEMBER    1  RUNNING FROM JOINT    1   TO    3
       STRESS     MATRIX
        SIZE    6 ROWS BY    1 COLUMNS
             1
 1     4.0851
 2     1.3585
 3     13.379
 4    -4.0851
 5    -1.3585
 6     2.9233
```

PA := 4.0851063
VA := 1.3584906
MA := 13.378562

MB := 2.9233245

```
    STRESSES FOR MEMBER    2  RUNNING FROM JOINT    2   TO    4

       STRESS     MATRIX
        SIZE    6 ROWS BY    1 COLUMNS
             1
 1     7.9149
 2     3.6415
```

PA := 7.9148935
VA := 3.6415093

```
3    23.643                    MA := 23.642713
4    -7.9149
5    -3.6415
6    20.055                    MB := 20.055399

   STRESSES FOR MEMBER   3  RUNNING FROM JOINT   3  TO   4

        STRESS    MATRIX
          SIZE   6 ROWS BY   1 COLUMNS
             1
1   -1.3585                    PA := - 1.3584902
2   -1.9149                    VA := - 1.9148935
3   -14.923                    MA := - 14.923324
4    1.3585
5    1.9149
6   -8.0554                    MB := - 8.0553990
                               MAX MOMENT MMAX = 23.267371
                                 MAX AT    XMAX = 4.0851064
```

5.3 Simplified Forms

This method is similar to the numeric case study, except that the problem is reduced to a form containing only a handful of parameters. The simplest form for this problem was to ignore all axial effects, set the span and height as equal, and set all moments of inertia as common valued. Thus all equation forms would involve only factors in E,I,L.

The REDUCE program called SIMPLE, makes use of the OUTPUT/DATA file AKAFLX and made a series of parameter simplifications of the general solution, for the cases where:-

- All columns have the same section properties
- All column and beam sections have the same section properties
- All sections are the same and the span and height are the same
- As above with axial effects ignored

These simplified analyses are listed in Appendix D, Section D.2.

The discussion of the plotted results also provides corroboration with the work of Kleinlogel [1981], who used simplified equations; see Section 7.2.

Chapter 6
Discussion of the Algebraic Results

6.1 Preamble

The frame was modelled as rigidly connected to the ground, in order to simplify the general 12x12 stiffness matrix for a frame with general boundary condition lumped stiffness terms, into a 6x6 matrix with zero boundary deflections.

6.2 The General Solution

REDUCE was unable to find one common factor in either the numerator or denominator of any coefficient in the system flexibility matrix, deflection or stress resultants. Thus the solution was always in the rational form of a multivariate polynomial numerator over a multivariate polynomial denominator.

The results were extracted from the OUTPUT and DATA files, and transcribed into a form suitable for printing. A text editor was used for this purpose in order to reduce the likelihood of typographic errors being introduced as the alterations were made. These equations are listed in Appendix D, Section D.1 and should be faithful copies of the true output, however the reader should use the actual OUTPUT file listings as the primary source for reference purposes. These equations represent the complete and most concise closed form solution to this small problem.

Chapter 7
Discussion of Plotted Results

7.1 Preamble

For the ease of presentation all plotted graphs appear at the end of the thesis, starting on page 47.

For the purpose of a coherent presentation of the general equations, that are presented in Appendix D, in graphical form; numerical values are inserted into the general solutions so that a study can be made of the effects of varying a few selected parameters. Extensive graphs have been composed that depict the deflection and stress resultant variation with respect to parameters reflecting geometry, sectional properties and loading. The ranges of structural values chosen were based on typical rolled steel section properties; gravity and lateral loads were considered separately. Geometric ratio, which had the most profound influence, was used as the main "x" ordinate for all graphs.

7.2 Other Algebraic Work

For the purposes of comparing the results of this work with prior work in the field of structural analysis, Kleinlogel [1981] was used as the primary reference. Kleinlogel's basic findings are presented in Figure 4.

Kleinlogel has derived for a rigid frame, with rigid supports, the following primary structural parameters:

$$k = \frac{I_b}{I_c \cdot \gamma}$$
$$N_1 = k + 2$$
$$N_2 = 6k + 1$$

re-expressed, these produce the three ratios

$$\frac{1}{k}=\frac{\gamma L}{I_b}.\frac{I_c}{L}$$

$$\frac{k}{N_2}=\frac{1}{6+1/k} \; approx \; \frac{1}{7}$$

$$\frac{3k+1}{N_2}=\frac{3+1/k}{6+1/k}$$

For uniformly distributed gravity loads only,

$$M_a=\frac{WL^2\gamma^2}{12N_1}$$

$$M_b=-2M_a$$

$$M_c=\frac{WL^2\gamma^2}{8}+M_b$$

For a lateral side load only,

$$M_a=\frac{PL}{2}.\frac{3k+1}{N_2}$$

$$M_b=\frac{PL}{2}.\frac{3k}{N_2}$$

7.3 Simplifications in Graphical Results

Infinite section area is approximated numerically by setting the area values to 10^{10}. This may be the root of slight numeric deviation of a few points in some of the sway plots. No effort was made to track down a numeric anomaly.

All plots are made with Young's Modulus set as 1.0, this has no effect on moment values, and the sway values need only be divided by E to get physically correct values of sway.

All loads are **unit** loads.

The following standard properties in SI units were employed and the values used are typical real world averages:

- L - Height = 3.5 meters
- I - Inertia = 0.001 meter^4

- A - Area = 0.02 $meter^2$ (10^{10} for infinity)

7.4 Method of Presentation

The results have been presented in six individual sections. Each section represents a specific choice of free and fixed parameters. Only two free parameters could be used so as to avoid clutter in the graphs. The span-to-height ratio was chosen as a free variable on the 'x' ordinate in all cases, so that the effect of the other free variable may be seen by scanning the contours for a specific span-to-height value. The graphs showing percentage change with respect to unity of the free parameter provide a means to gauge the effect of span-to-height variation with respect to the other free parameter.

The sway and the moments at critical sections are shown along the 'y' ordinate on each of the graphs. These sections are depicted in Figure 5, and a pictographic legend highlighting the effect versus the free parameters has been used to simplify references to the plotted results.

The reader should look at several graphs to appreciate the information content in them, before reading the discussions in the remaining sections of this chapter.

There are five families of graphs that are presented:

- span-to-height & Stiffness Ratio of Beam and Column
- span-to-height & Height for Fixed I,A
- Uniform Inertia in Beam and Col. & span-to-height
- Column Inertia & Beam Inertia (for span-to-height Ratios of 1,2,3,4,5)
- Column Area & Beam Area

7.5 Verification - Numerical Test Case

A check was made to ensure that the FORTRAN functions were producing correct results by setting the values of the structural properties to be the same as the values used in the REDUCE program verification. Figures 6 and 7 show that for G=1, the results match those computed by REDUCE.

7.6 Span-to-Height Ratio and Relative Stiffness Effects

7.6.1 Gravity Loading

The plot of the base moment shown in Figure 8 shows that the moment increases quadratically with the span-to-height ratio , this behavior agrees with the formulation in Kleinlogel, and simple structural mechanics expectations. Increasing the stiffness of the column inertia relative to the beam's inertia increases the base moment M_a. This is to be expected as a stiffer column will carry more of the moment distributed over the beam. Figure 9 shows the percentage change in the moment, and it illustrates that a change in relative stiffness has only a moderate influence on the base moment M_a, a matter of a few percent. The influence of span-to-height ratio on the relative stiffness is negligible for span-to-height ratios exceeding one, and diminishes as this ratio increases.

The plot of the maximum middle span moment shown in Figure 10 shows that the moment increases quadratically with the span-to-height ratio; this behavior agrees with the formulation in Kleinlogel, and simple structural mechanics expectations. Increasing the stiffness of the column inertia relative to the beam's inertia increases the maximum middle span moment M_c. This is to be expected as a stiffer column will carry more of the moment distributed over the beam. Figure 11 shows the percentage change in the moment, and it illustrates that a change in relative stiffness has a strong influence on the maximum middle span moment M_c, a matter of approximately 20-30% for an eight fold stiffness change.

The plot of the side sway shown in Figure 12 shows that the sway increases more than quadratically with the span-to-height ratio. Increasing the stiffness of the column inertia relative to the beam's inertia decreases the sway. This is to be expected as a stiffer column will bend less. Figure 13 shows the percentage change in the sway, and it illustrates that a change in relative stiffness has only a moderate influence on the magnitude of the sway, a matter of a few percent. The influence of span-to-height ratio on the sway reduction effects of relative stiffness is negligible for span-to-height ratios exceeding one, and diminishes as this ratio increases.

In all cases the increase in relative stiffness, has had a diminishing effect in reducing or increasing the moment or sway. The moment relations are symmetric, and the sway is anti-symmetric as expected.

7.6.2 Lateral Loading

The plot of the base moment depicted in Figure 14 shows that the moment increases hyperbolically with the span-to-height ratio; this behavior agrees with the formulation in Kleinlogel, with an upper bound of 1.75 (ie. L/2). Increasing the stiffness of the column inertia relative to the beam's inertia increases the base moment M_a. This is to be expected as a stiffer column will carry more of the moment distributed over the beam. The effect of this stiffness agrees with the 'k' factor used by Kleinlogel, where a span to depth ratio of a given value has the same effect as a similar relative column to beam stiffness value. Figure 15 shows the percentage change in the moment, and it illustrates that a change in relative stiffness has a great influence on the base moment M_a, a matter of a approximately 40% percent overall. The influence of span-to-height ratio on the relative stiffness is is appreciable for span-to-height ratios exceeding one, and stabilizes to produce large percentage effects as this ratio increases.

The plot of the column-beam corner moment depicted in Figure 16 shows that the moment decreases hyperbolically with the span-to-height ratio; this behavior agrees with the formulation in Kleinlogel, where the moment approaches an upper bound of zero, as would be expected in the case of a very flexible beam. Increasing the stiffness of the column inertia relative to the beam's inertia decreases the column-beam corner moment M_b. This is to be expected as a stiffer column will carry more of the moment distributed over the beam; this is also reflected in the Kleinlogel 'k' formulation. Figure 17 shows the percentage change in the moment, and it illustrates that a change in relative stiffness has a strong influence on the column-beam corner moment M_b, which increases over the range shown. The expected asymptotic behavior is only seen for the quadruple relative stiffness at the largest span-to-height ratio, where a clear 60% reduction in the beam moment is shown. Since these moments are of small magnitude, this is of minor consequence.

The plot of the side sway depicted in Figure 18 shows that the sway increases more or less linearly with the span-to-height ratio. Increasing the stiffness of the column inertia relative to the beam's inertia decreases the sway. The effect of relative stiffness does produce diminishing results in sway reduction. Figure 19 shows the percentage change in the sway, and it illustrates that a change in relative stiffness has only a great influence on the magnitude of the sway, a matter of a 25% percent, which is independent of the span-to-height ratio value. The influence of span-to-height ratio on the sway reduction effects of relative stiffness is negligible for span-to-height ratios exceeding one.

The moment relations are anti-symmetric, about the center line of the frame as would be expected.

7.7 Span-to-Height Ratio and the Effect of Height Variation

In this section the percentage change plots, showed no variation due to variation in span-to-height ratio, and so have not been included in the thesis body. However the variations due to height changes have been discussed.

7.7.1 Gravity Loading

The plot of the base moment shown in Figure 20 shows that the moment increases quadratically with the span-to-height ratio; this behavior agrees with the formulation in Kleinlogel, and simple structural mechanics expectations, where for example the moment for L=1 and γ=4 is the same as L=4 and γ=1 (see figure). Increasing the height increases the base moment M_a, which is exactly parabolically related to L. The percentage change in the moment (not shown), is purely a constant with respect to span-to-height ratio, for a given choice of L.

The plot of the middle span moment shown in Figure 21 shows that the moment increases quadratically with the span-to-height ratio. The observations made for the base moment M_a apply for this section's behavior, with the maximum positive moment increasing as column stiffness is diminished by length increase, or relative inertia decrease.

The plot of the side sway shown in Figure 22 shows that the sway increases quadratically with the span-to-height ratio. Increasing the height of the column dramatically increases the sway. This is clearer of the percentage change plot, where the effect of span-to-height ratio is constant, but the sway magnitude increases as the square of the increase in height. This is expected as the effect of a given distributed load on the beam is to apply a simple end moment to a column, which has parabolic deflection with respect to increasing length.

In all cases an increase in height, has had a parabolically increasing effect in increasing the moment or sway. The moment relations are symmetric, and the sway is anti-symmetric as expected.

7.7.2 Lateral Loading

The plot of the base moment depicted in Figure 23 shows that the moment increases hyperbolically with the span-to-height ratio; this behavior agrees with the formulation in Kleinlogel, with an upper bound of L/2. Each L plot has its own upper bound. The moment is linearly related to the height, doubling as the height doubles.

The plot of the column-beam corner moment depicted in Figure 24 shows that the moment decreases hyperbolically with the span-to-height ratio; this behavior agrees with the formulation in Kleinlogel, where the moment approaches an upper bound of 3/14.

The plot of the side sway depicted in Figure 25 shows that the sway is more or less constant with the span-to-height ratio, over the range of values shown. Only with a large height does the span-to-height ratio show an asymptotically diminishing sway effect. Furthermore the height has a cubic effect on the increase in the sway deflection, as is the case for a tip loaded cantilever.

The moment relations are anti-symmetric, about the center line of the frame as would be expected.

7.8 Uniform Inertia and Span-to-Height Ratio Variation

7.8.1 Lateral Loading

The plot of the side sway depicted in Figure 26 shows that the sway is hyperbolically decreasing with the uniform frame stiffness. There is slight reduction in sway, by the uniform increase in the section inertias of both the beams and columns. The difference in the sway magnitude for various span-to-height ratios is also asymptotic, so that span-to-height ratio has a lesser direct effect in controlling sway for heavier sections. Figure 27 shows that the percentage change in sway is fixed for a given span-to-height ratio, for all section inertias. It can be seen that the decrease in sway is approximately 15-20% for each doubling of the span-to-height ratio.

7.9 Column and Beam Inertia and Span-to-Height Variation

All plots were generated using integer values of span-to-height of 1,2,3,4,5. This section will highlight differences observed between I_c, I_b and γ.

7.9.1 Gravity Loading

The plots in Figures 28 to 32 show that the base moment was practically constant with respect to the column inertia, for beams of stiffnesses less than the column's. This trend and the percentage charts in Figures 33 to 37 show that the span-to-height ratio was of no consequence in this relationship. The span-to-height ratio directly increased the base moment, as discussed in Section 7.6. For higher beam stiffness the moment is more hyperbolic, reaching a asymptote more gradually. For high values of inertia, the moment variation between the highest and lowest stiffness for the beam was only 20% decreasing to 10% for a span-to-height ratio of 5. The percentage change is basically constant for most choices of column inertia.

The plots in Figures 38 to 42 show that the maximum middle span moment was practically constant with respect to the column inertia, for beams of stiffnesses less than the column's. The behavior is very similar to the base moment case. This trend and the percentage charts in Figures 43 to 47 show that the span-to-height ratio was of no consequence in this relationship. The span-to-height ratio directly increased the maximum middle span moment, as discussed in Section 7.6. For higher beam stiffness the moment is more hyperbolic, reaching a asymptote more gradually. For high values of inertia, the moment variation between the highest and lowest stiffness for the beam was 25% decreasing to 7% for a span-to-height ratio of 5. The percentage change is basically constant for most choices of column inertia, but in all cases diminishes as column inertia gets higher. A doubling of the beam stiffness, produced a more than proportional increase in the maximum middle span moment.

The plots in Figures 48 to 52 show that the beam closure was practically constant with respect to the column inertia, for beams of stiffnesses less than the column's. The behavior is very similar to the base moment case. This trend and the percentage charts in Figures 53 to 57 show that the span-to-height ratio was of little consequence in this behavior. For high values of inertia, the moment variation between the highest and lowest stiffness for the beam was 2% increasing to 10% for a span-to-height ratio of 5. The percentage change is basically constant for most choices of column inertia, but in all cases diminishes as column inertia gets higher.

7.9.2 Lateral Loading

The plots in Figures 58 to 62 show similar behavior to the gravity case. However the spread of base moment values is much more sensitive to the beam stiffness over the whole range of column stiffnesses used. Furthermore the behavior is below the asymptote and is hyperbolic in all cases. It is only for high span-to-height ratios that the moment relationship levels off to an upper bound value.

The plots in Figures 68 to 72 show that decreasing beam, increasing column stiffness and increasing span-to-height ratio drive the moment at the column-beam corner closer to its asymptotic upper bound of zero.

The plots in Figures 78 to 82 show that increasing beam, increasing column stiffness and decreasing span-to-height ratio drive the side sway closer to its asymptotic lower bound of zero. The percentage reduction in sway due to increasing column stiffness is marginal, however beam stiffness variation plays a major role, with a difference of 8% to 5% depending on span-to-height (which has marginal influence). With high column stiffnesses the effect of beam stiffness variation on sway was practically a constant.

7.10 Column Section Area and Beam Section Area Variation

All plots were made with a span-to-height ratio of two.

7.10.1 Gravity Loading

The plot in Figure 88 shows that the base moment was constant with respect to the column area. Variation with choice of beam area was slight, there is a 15% increase from the moment due to a beam of a quarter the section area of that of the column, to an infinite beam area.

The plot in Figure 89 shows that the maximum middle span moment was constant with respect to the column area. Variation with choice of beam area was slight, there is a near zero decrease from the moment due to a beam of a quarter the section area of that of the column, to an infinite beam area, so that neither column or beam area has any noteworthy effect on the variation of the maximum middle span moment.

The plot in Figure 90 shows that the beam closure was constant with respect to the column area. Variation with choice of beam area was moderate, there is a 37% decrease from the sway due to a beam of a quarter the section area of that of the column, to an infinite beam area.

7.10.2 Lateral Loading

The plot in Figure 91 shows that the base moment was practically constant (ie. close to asymptote) with respect to the column area, only increasing for very small column areas. Variation with choice of beam area was slight, there is an 8% decrease from the moment due to a beam of a quarter the section area of that of the column, to an infinite beam area. In terms of the absolute moment this reduction is slight.

The plot in Figure 92 shows that the column-beam corner moment was practically constant (ie. close to asymptote) with respect to the column area, only increasing for very small column areas. Variation with choice of beam area was slight, there is an 5% decrease from the moment due to a beam of a quarter the section area of that of the column, to an infinite beam area. In terms of the absolute moment this reduction is slight.

The plot in Figure 93 shows that the side sway was practically constant with respect to the column area. Variation with choice of beam area was moderate, there is a 10% decrease from the sway due to a beam of a quarter the section area of that of the column, to an infinite beam area. Furthermore the choice of using a beam with double the area of the column, halved the sway.

Chapter 8
Recommendations

8.1 Computability

The purpose of this work was to explore the potential of using computers to solve the algebra of structural analysis, and to apply this scheme of solution to large scale structural systems. As was explained in Section 4.5 it was not possible to do a large scale problem using REDUCE. However the solution was successful for a problem limited enough to fit the capacity of the computing available equipment.

The following recommendations have evolved from the experiences gained by the author as a result of the execution of this work, and are not specifically derived from any tangible results.

All matrix algebraic terms are best left in an expanded form, by deferring multiplication of matrices explicitly. This has a drawback in that multiple references to a specific element will require repeated recalculation, via row and column multiplication. However in a dawning age of parallel processing, repetitive matrix operations (including symbolic operations) would not be too time consuming. This approach reduces the amount of memory required to store the values of any combination of matrix terms, and can utilize the compactness of sparsity and symmetry that exists in discretized structural problems, which was discussed in Section 4.5.

As most engineering problems involve rational forms or adequately precise rational approximations, an engineering algebraic system could be devised that represented rationals in their most compact form, as discussed in Section 4.1.

This numerator-denominator form, would use storage more efficiently, and operations should execute faster.

The coefficients of an inverted matrix could be more compactly stored as a nested product, ie $f_{ij}=\alpha_n\cdot(\alpha_{n-1}\cdot(\ldots\alpha_0+\beta_1)..+\beta_{n-1})+\beta_n$

For symbolic problems, the force vector is symbolic, so that it is not necessary to find the explicit inverse matrix, rather a Choleski decomposition scheme would suffice, followed by back-substitution.

To cope with the generation of long rational polynomial forms in a large structure, a virtual memory system is desirable, so that the symbolic processor may store final results on a disk file as they are generated and so release core memory for ongoing operations.

Chapter 9
Conclusions

9.1 The One Level by One Bay Model

The one-level by one-bay rigid frame represented a simple, yet complicated algebraic problem. The results presented in Chapter 5, showed that algebraic processing, and specifically the REDUCE system can analyze these types of problems, and produces correct answers.

The general solution, presented in Appendix D, had no simplifying factors, and the the rational polynomial functions expressed its correct and most rudimentary form. The closed solution obtained was very complex and thus is of no benefit to designers using hand computation schemes to make design decisions. However the general solution could provide a direct model to another computer-based 'expert system', which could make design decisions.

The one-level by one-bay rigid frame problem could also have been handled by the flexibility method of analysis. This was not done, as in a general highly indeterminate problem the flexibility method is not the best.

The formulation of the general algebraic solution of the structure permits a precise stochastic analysis of the structure, using explicit algebraic integration techniques, rather than using numerical methods such as Monte Carlo sampling.

The generality of the method could be used to derive general solutions of higher order elements, including shear and $P\Delta$ first order approximation of second order stability analyses. The REDUCE programs developed in this thesis are directly applicable to these forms of analyses.

Of greater complexity is plastic analysis, which is in general a combinatorial problem, over a general set of load paths. The REDUCE programs developed, could be simply modified to perform a plastic analysis. However the complexity becomes truly enormous, as the structural terms in the stiffness matrix are altered from one stage to the next.

9.2 A Better General Model

The graphs of sway and moment with respect to variation of structural and geometric parameters, in Chapter 7, showed that the complex equations could assume more simplistic forms, within a constrained real-world domain of design values. This would provide a basis for formulating simplified equations to satisfy any need for hand computation.

The minor importance of cross-section area as a design parameter was demonstrated in Section 7.10, and suggests that a model based on rotational degrees of freedom, and a single sway degree of freedom per level would yield meaningful results as is detailed in Amirikian [1942]. This would considerably reduce the size of any system matrix requiring solution. On this basis it would be feasible to conduct analyses for a two-storey by one-bay problem, and a one-storey by up to four bay frame, so long as there were no more than six unknowns in the system, for the computer system that was available.

9.3 The Future Potential for the Algebraic Analysis of Systems

In the 1950's system solutions, using numerical matrix methods of solution were considered pioneering, even though the formalism of system representation in matrix form had been developed prior to that time. In the 1980's such numerical analysis is common place, and consumes millions of dollars of computer time, as finite element analyses of similar systems are solved time and time again. The research literature of this decade does not abound with any rule-of-thumb methods that may have been in vogue prior to the 1960's.

Artificial intelligence research in this decade, will produce computers that will be able to treat the algebraic solution process, as a more lightweight exercise in years to come, similar in extent to the numeric revolution of the 1960s.

The quest to derive complete closed-form discrete element solutions of continuum systems has many immediate applications.

Many researchers are devoting efforts to formulate 'rule-based' expert systems. The rules are usually heuristic (rule-of-thumb) in nature and are often formulated from grossly simplified models. As artificial intelligence technology evolves for creating better symbolic models, it will become worthwhile to develop closed-form and general algebraic solutions, of the form that was used to generate the results presented in Chapter 7. Furthermore a simplistic heuristic is more problem dependent, and as an aid to design could produce flawed designs. From a computability aspect, a simplistic rule could lead to a search for some 'optimal' design over an unwieldingly large domain.

The merit in a symbolic approach, is to provide a computer-based 'expert system' with as many 'up-front' constraints as possible, which has the immediate effect of choking the feasible solution domain to a much smaller set, while also providing the basis of a more precise and less problem-dependent model.

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Full Page Figures & Plots

Figure 1: The Primary Cantilever Element

Figure 2: The One Level by One Bay Rigid Frame

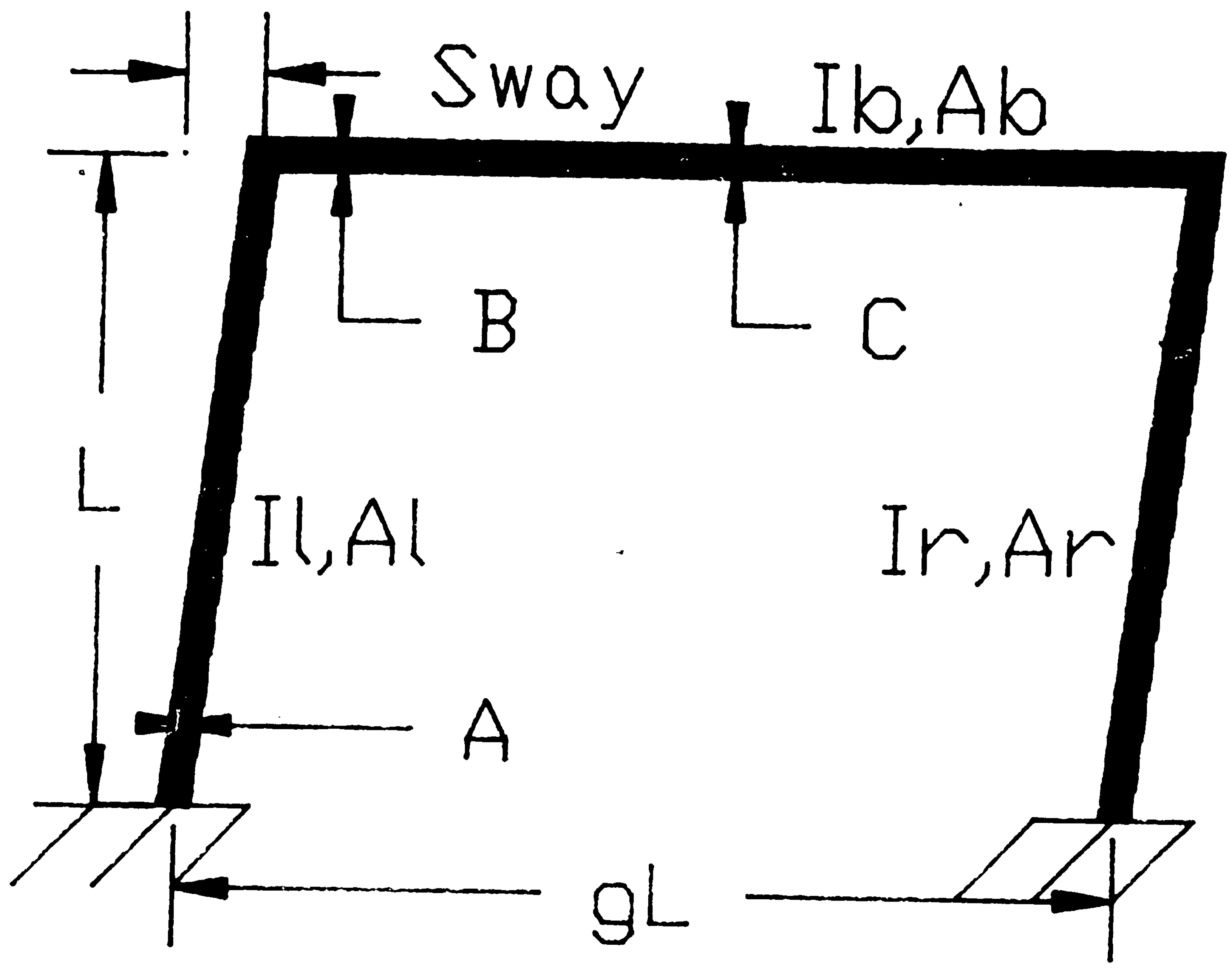
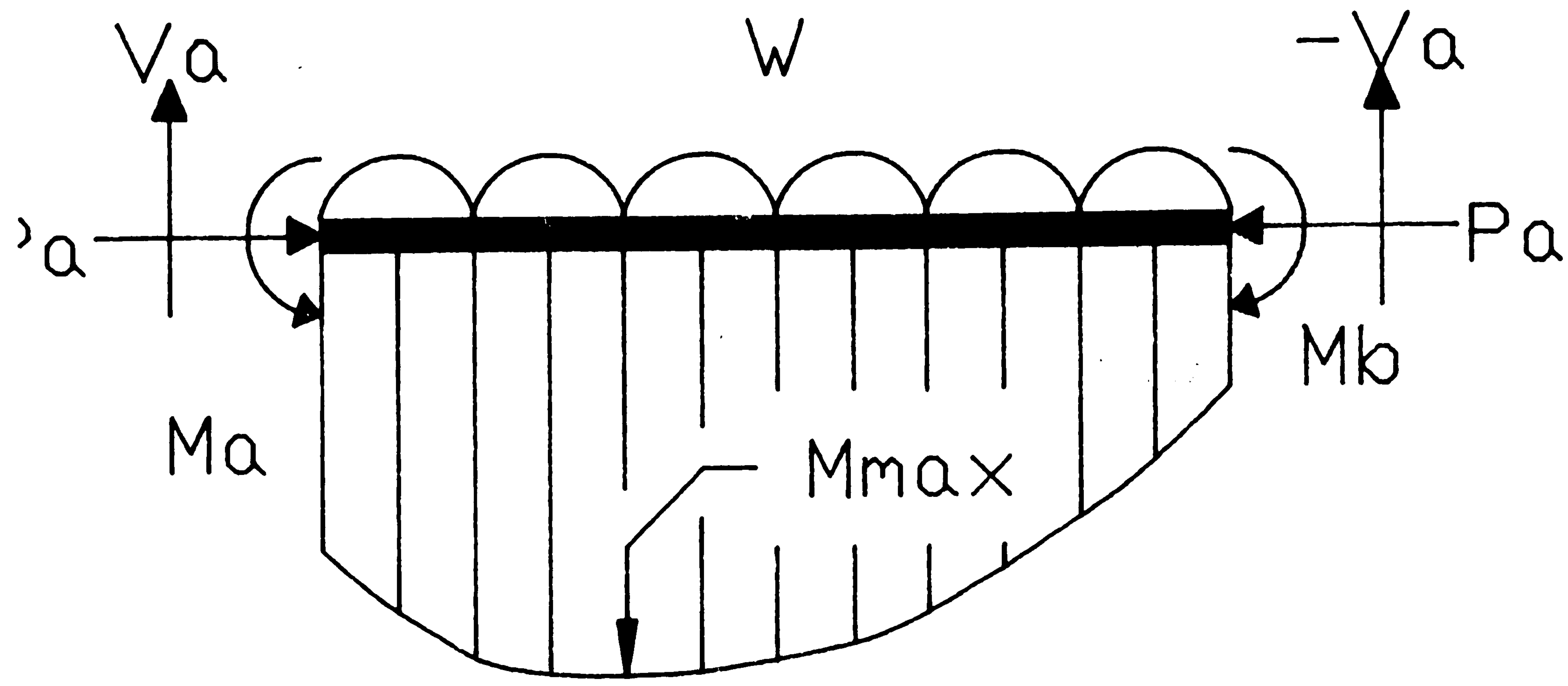
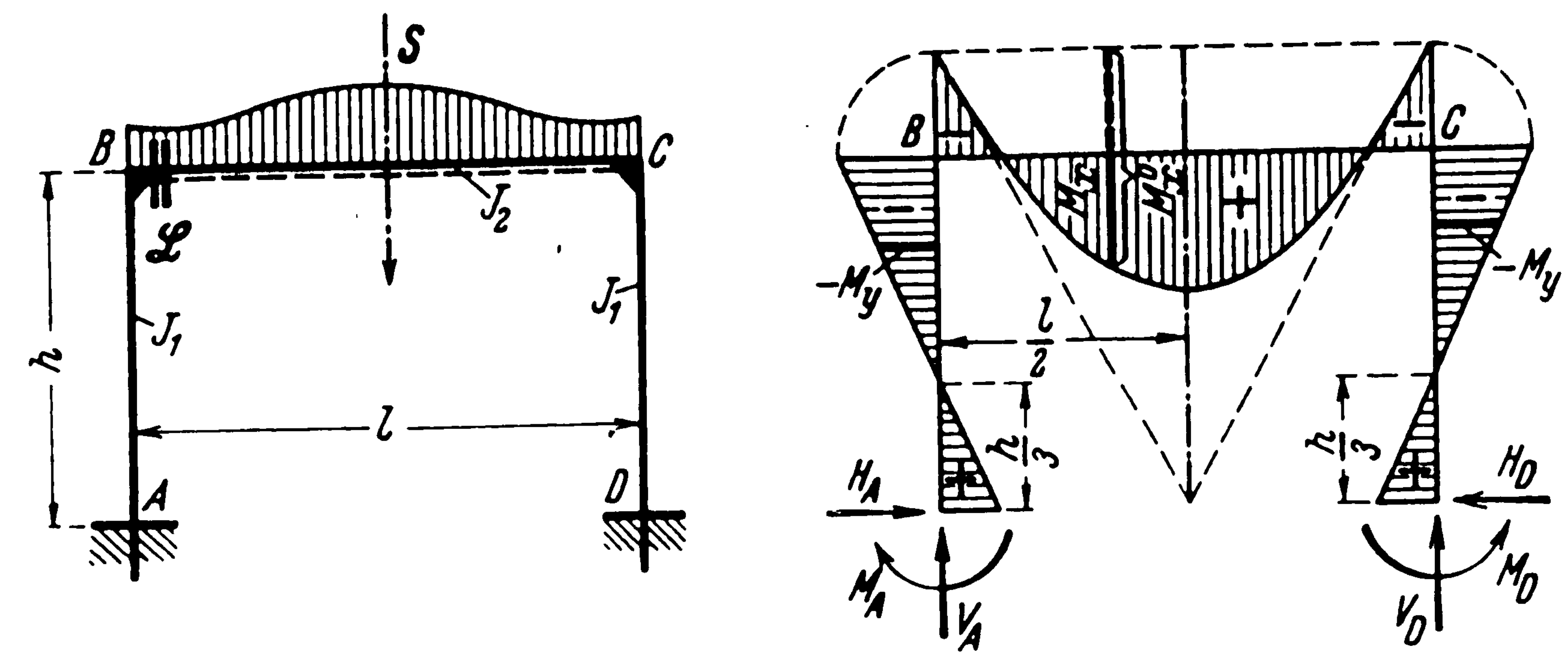


Figure 3: The Member Stress Resultants



Case 41/3: Girder loaded by any type of vertical load, acting symmetrically



$$M_A = M_D = +\frac{\mathfrak{L}}{3N_1};\qquad H_A = H_D = \frac{3M_A}{h}\qquad V_A = V_D = \frac{S}{2};$$

$$M_B = M_C = -2M_A\qquad M_x = M_x^\bullet + M_B\qquad M_y = M_A - H_A y.$$

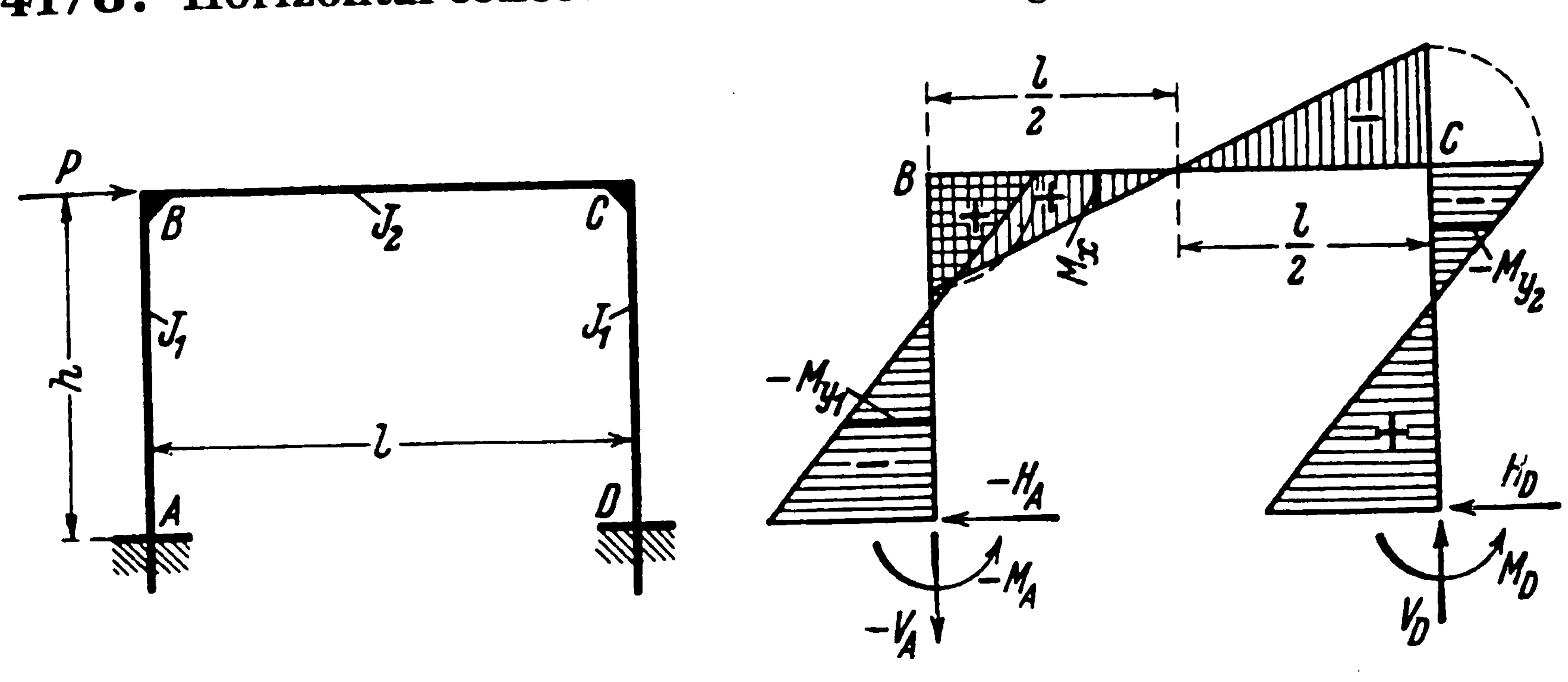
Special case 41/3a: Uniformly distributed load $S = ql$

$$M_A = M_D = +\frac{ql^2}{12N_1}\qquad V_A = V_D = \frac{ql}{2}\qquad \max M_x = \frac{ql^2}{8} + M_B.$$

All other formulas as above.

*For an antisymmetrical load ($\mathfrak{N} = -\mathfrak{L}$) $X_1 = 0$, $X_3 = \mathfrak{L}/N_2$; $M_D = M_C = -M_A = -M_B = \mathfrak{L}/N_2$ and $H_A = H_D = 0$.

Case 41/8: Horizontal concentrated load at the girder



$$\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = \mp\frac{Ph}{2}\cdot\frac{3k+1}{N_2}\qquad \left.\begin{matrix} M_B \\ M_C \end{matrix}\right\rangle = \pm\frac{Ph}{2}\cdot\frac{3k}{N_2};$$

$$H_D = -H_A = \frac{P}{2};\qquad V_D = -V_A = \frac{2M_B}{l};$$

$$M_{y1} = M_A + \frac{P}{2}y_1\qquad M_x = \frac{x' - x}{l}M_B\qquad M_{y2} = M_D - \frac{P}{2}y_2.$$

Figure 5: Critical Frame Sections and Sway Location

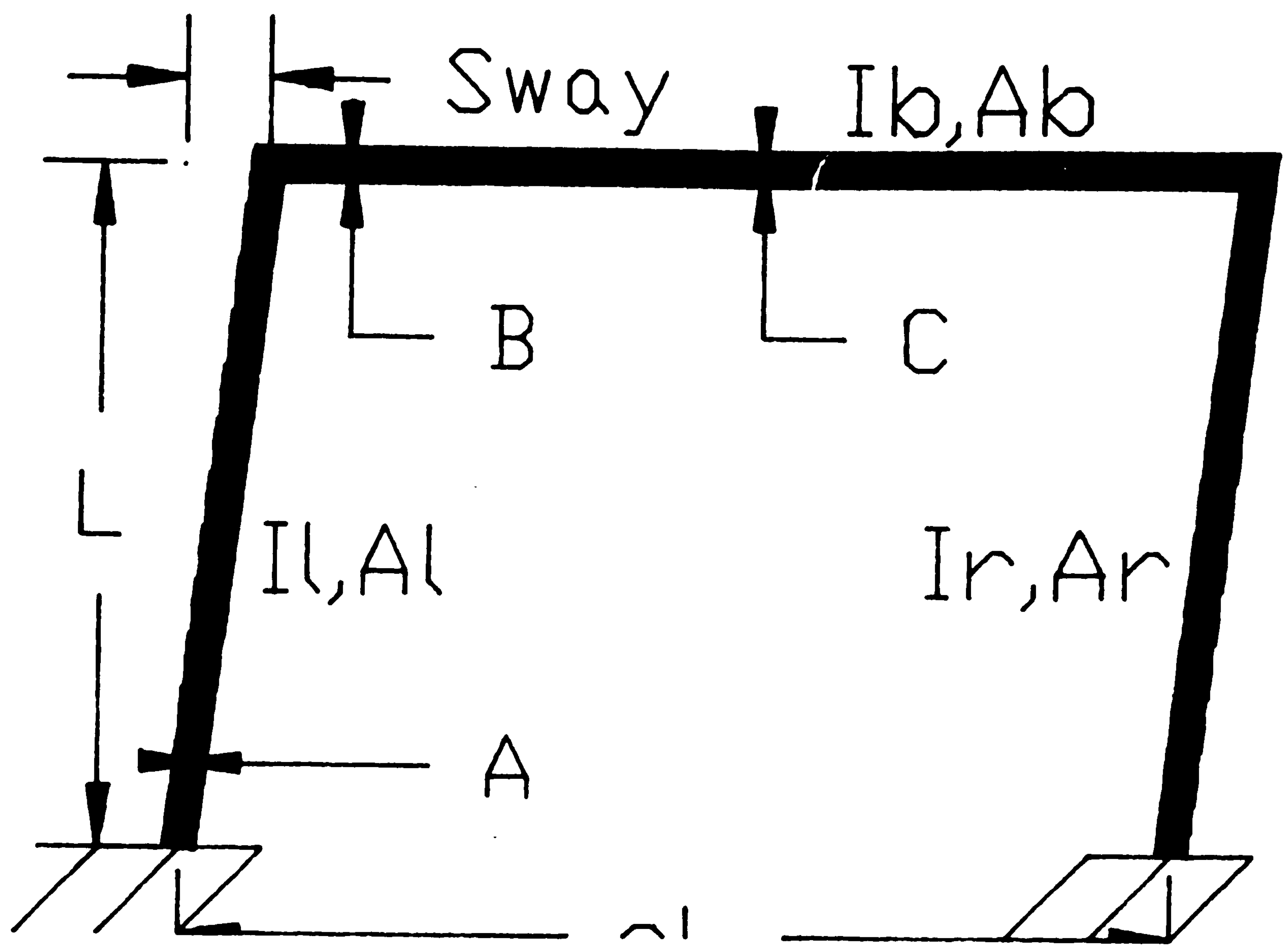


Figure 6: Moments at Critical Sections under Test Load

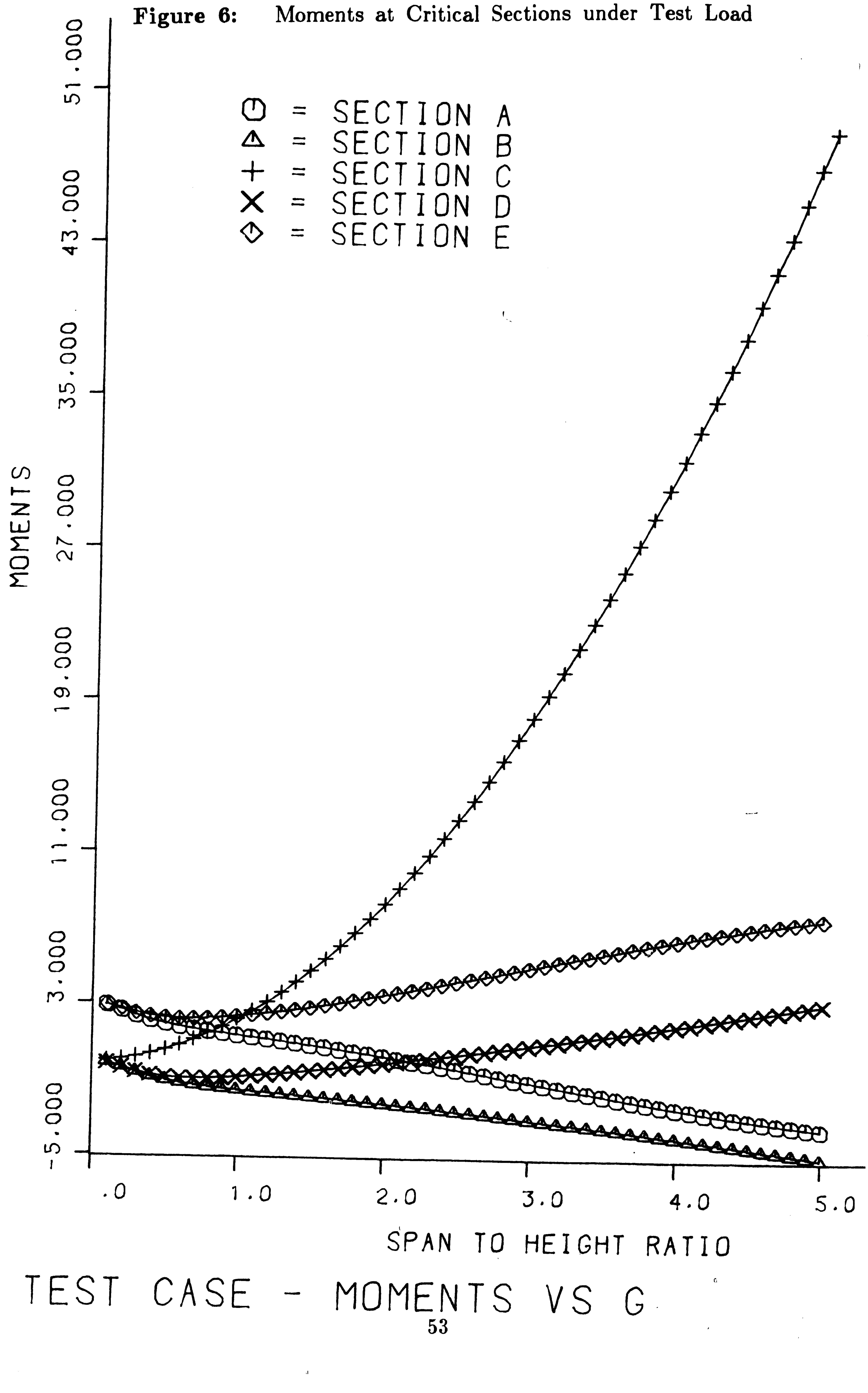


Figure 7: Side Sway under Test Load

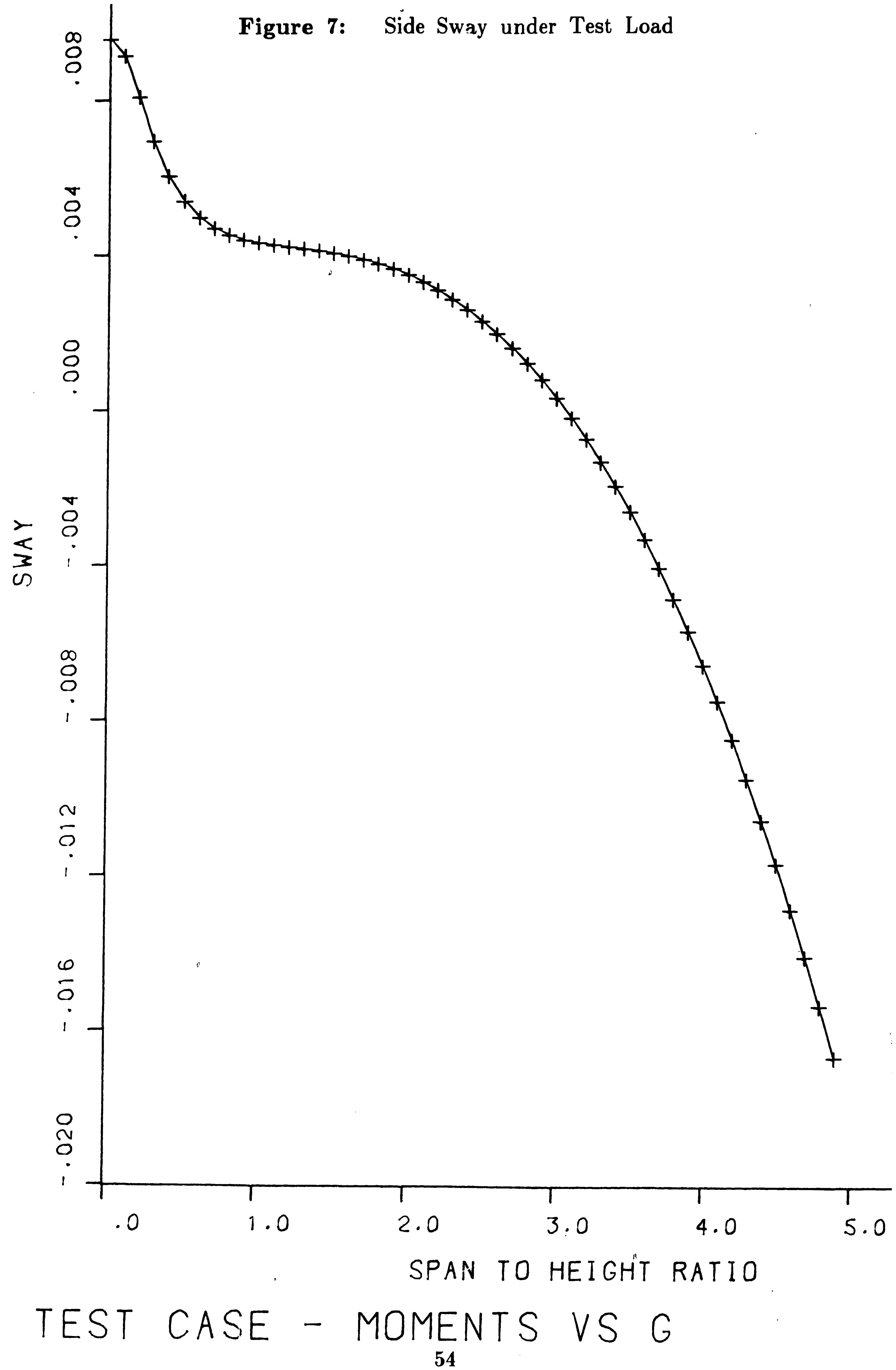


Figure 8: Base Mom. vs Span-Height & Col. Inertia under Grav. Load

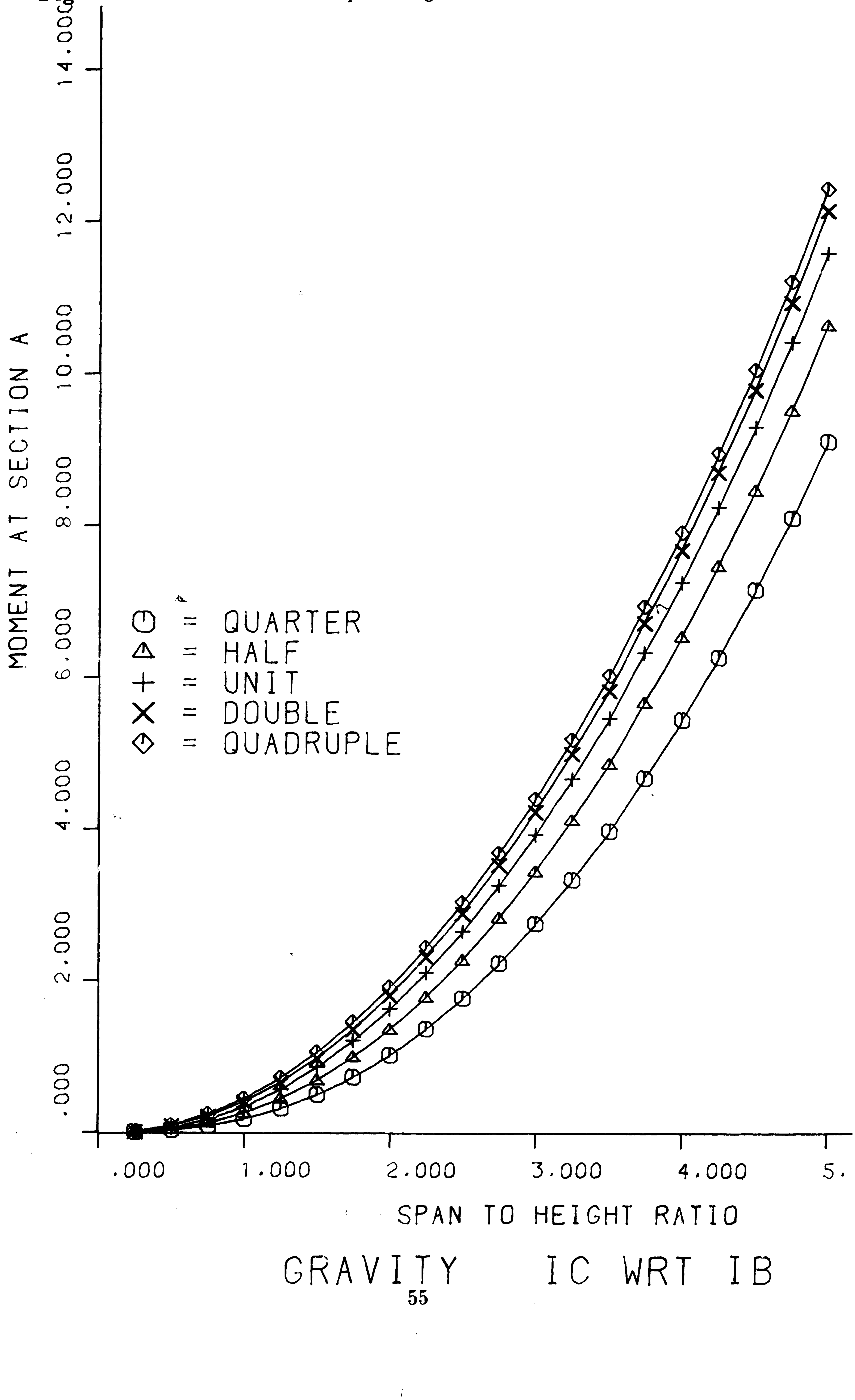


Figure 9: % Base Mom. vs Span-Height & Col. Inertia under Grav. Load

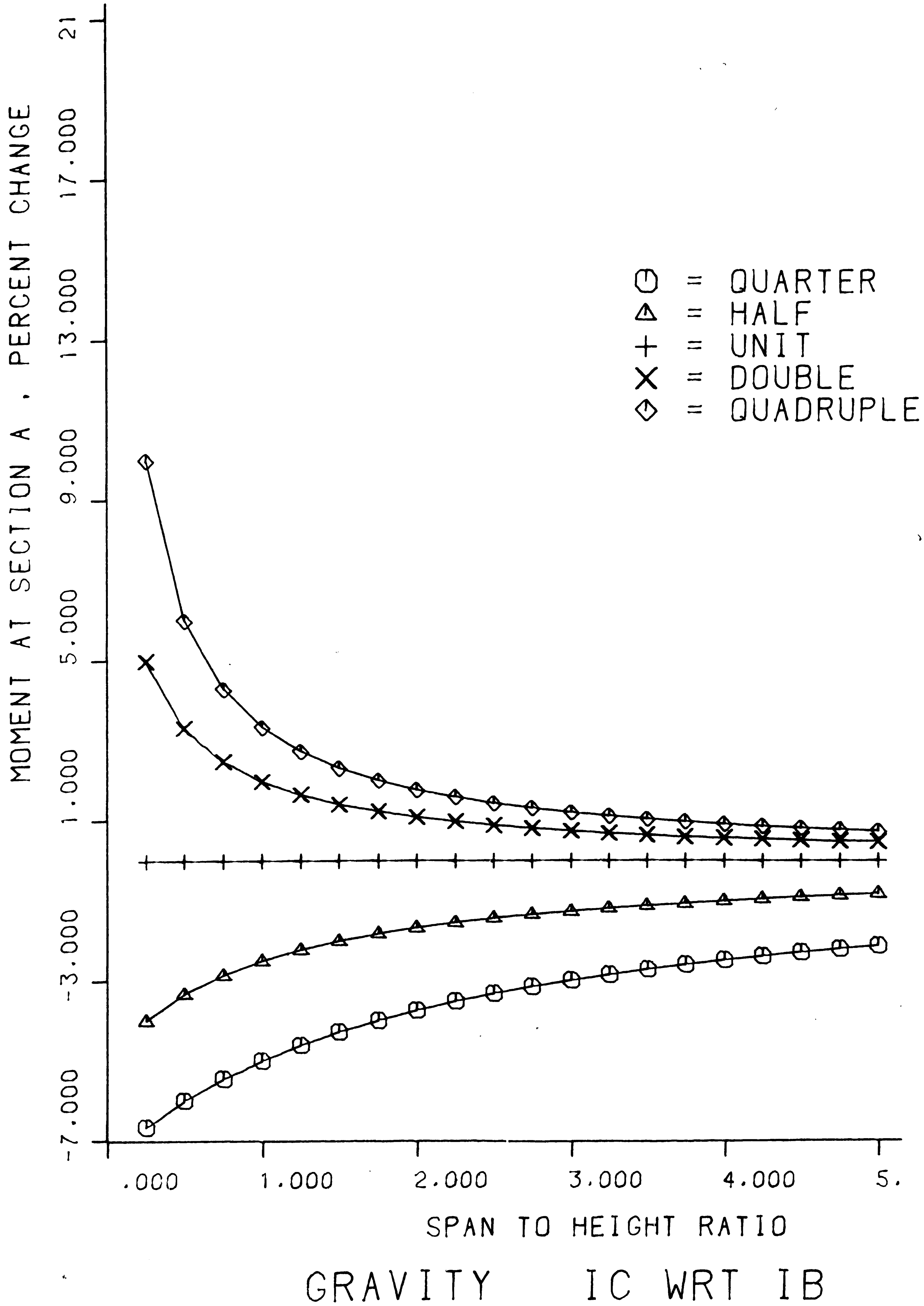


Figure 10: Midspan Mom. vs Span-Height & Col. Inertia under Grav. Load

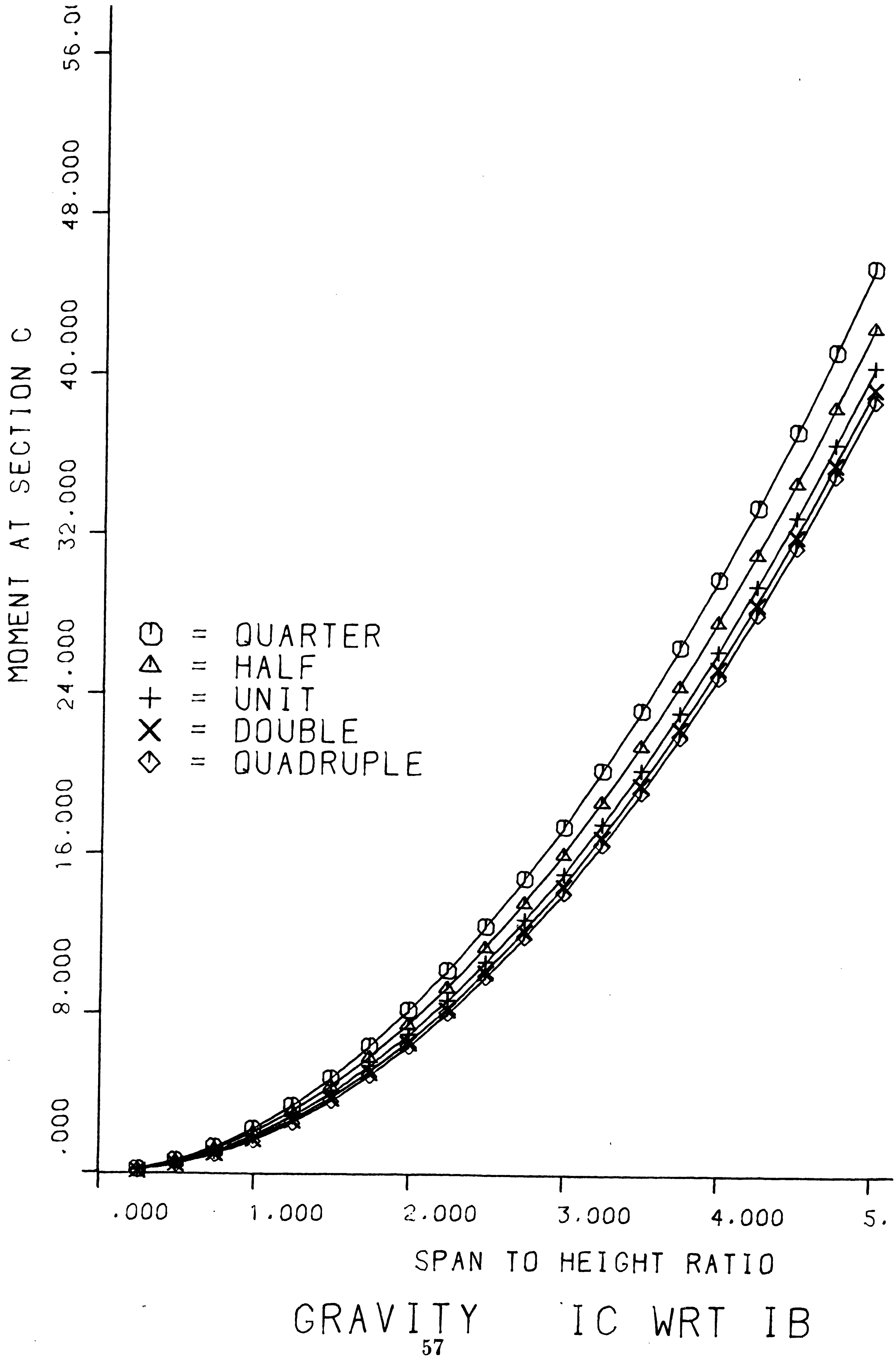


Figure 11: % Mid Mom. vs Span-Height & Col. Inertia under Grav. Load

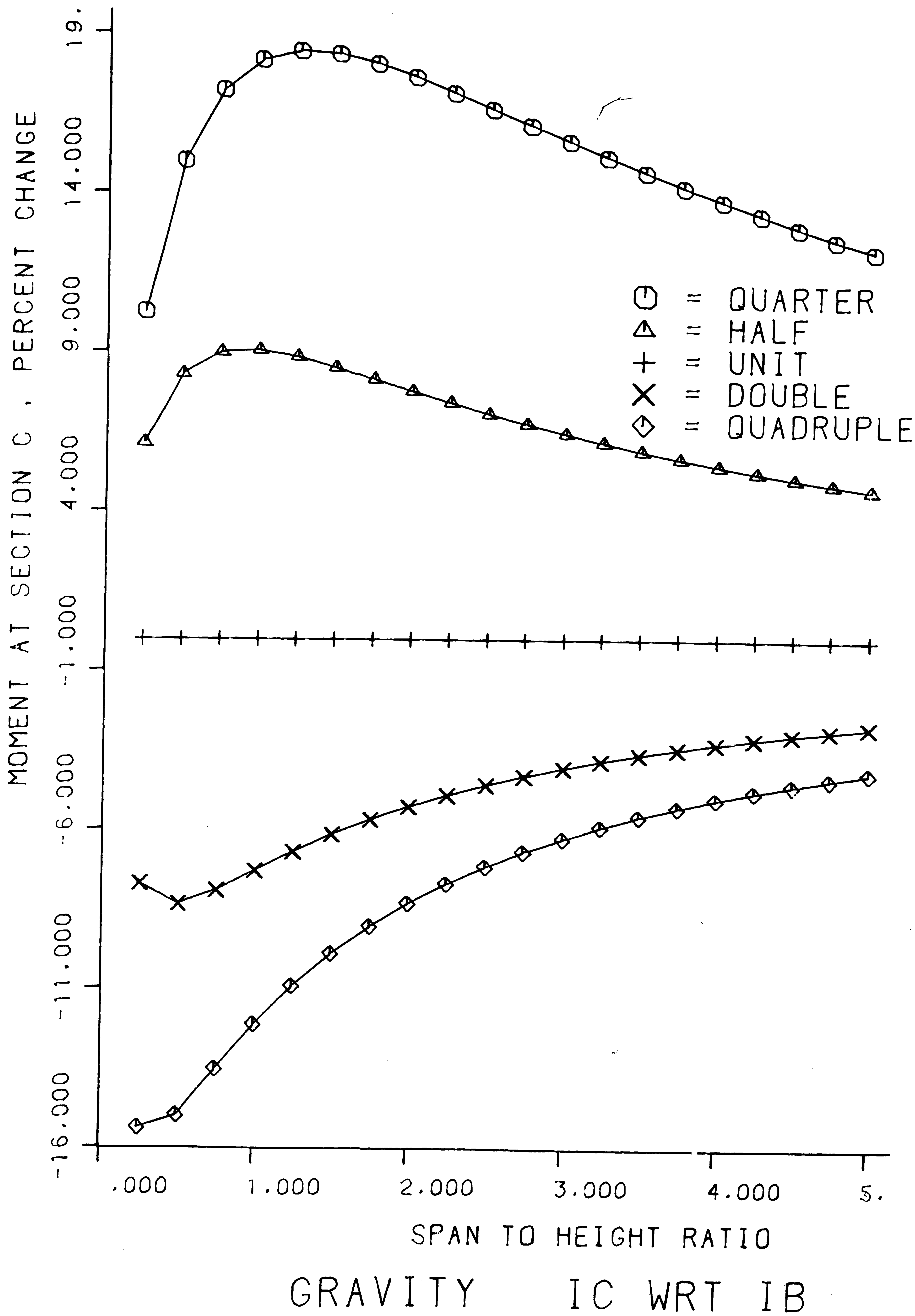


Figure 12: Sway vs Span-Height & Col. Inertia under Grav. Load

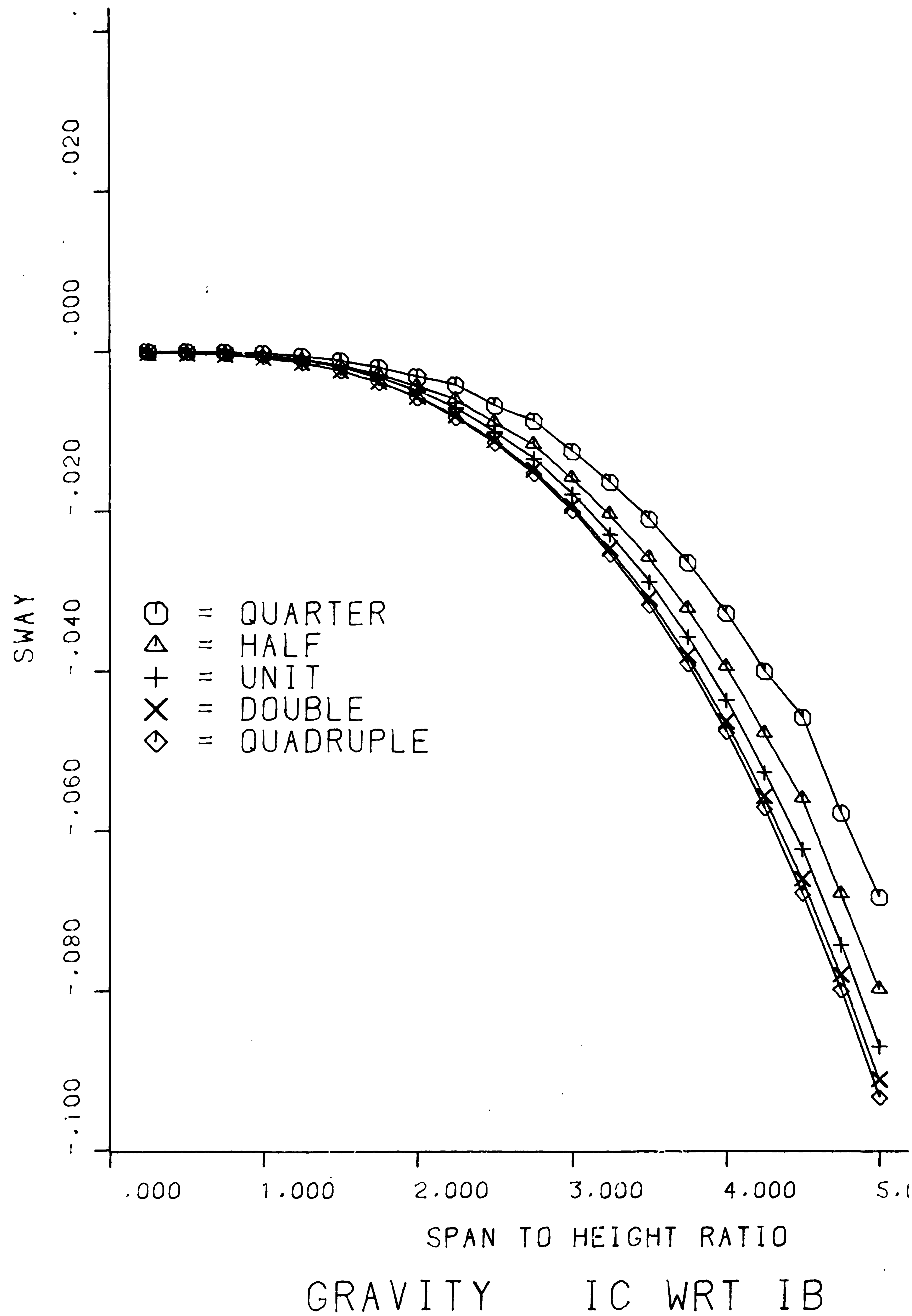


Figure 13: % Sway vs Span-Height & Col. Inertia under Grav. Load

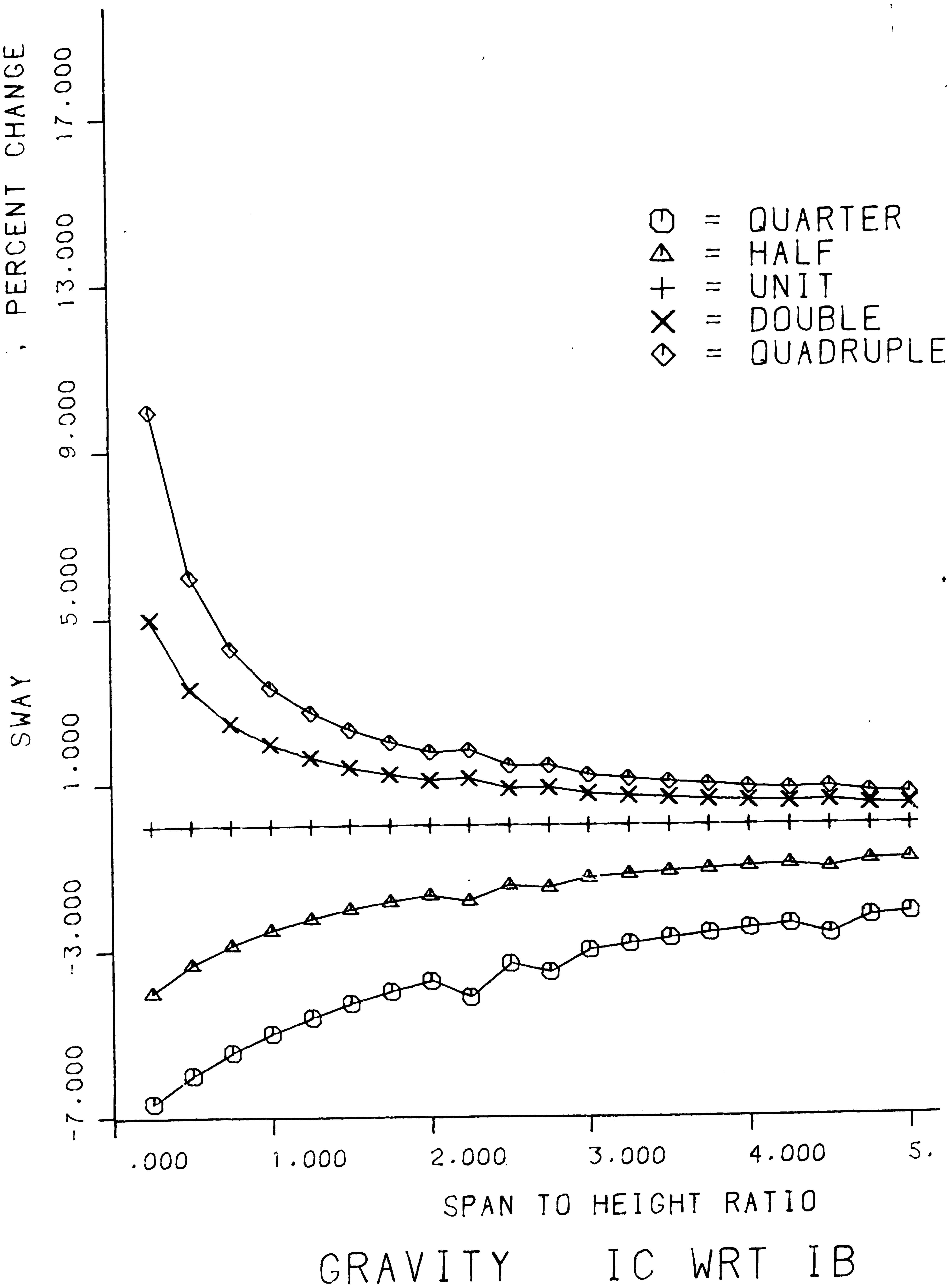


Figure 14: Base Mom. vs Span-Height & Col. Inertia under Lat. Load

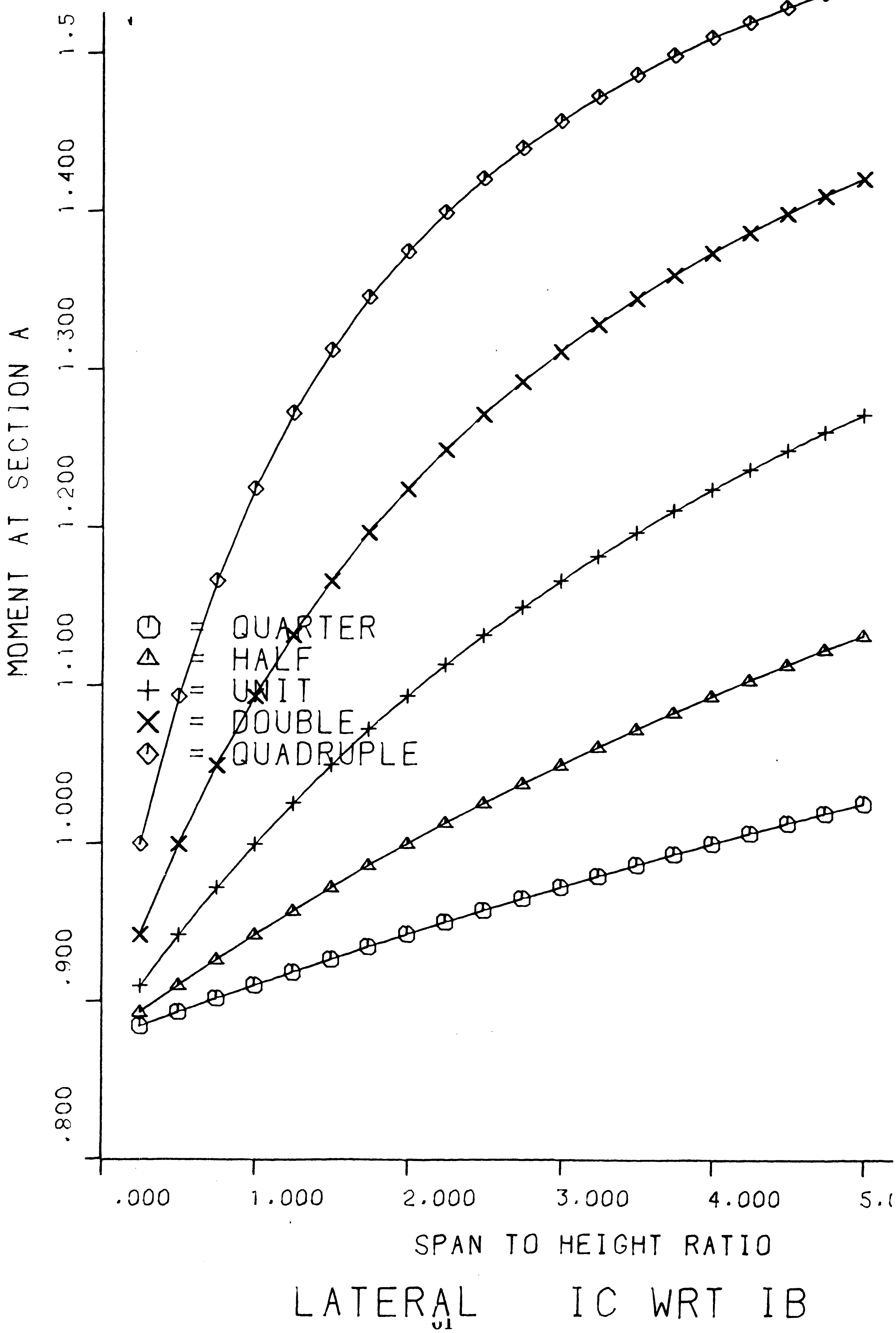


Figure 15: % Base Mom. vs Span-Height & Col. Inertia under Lat. Load

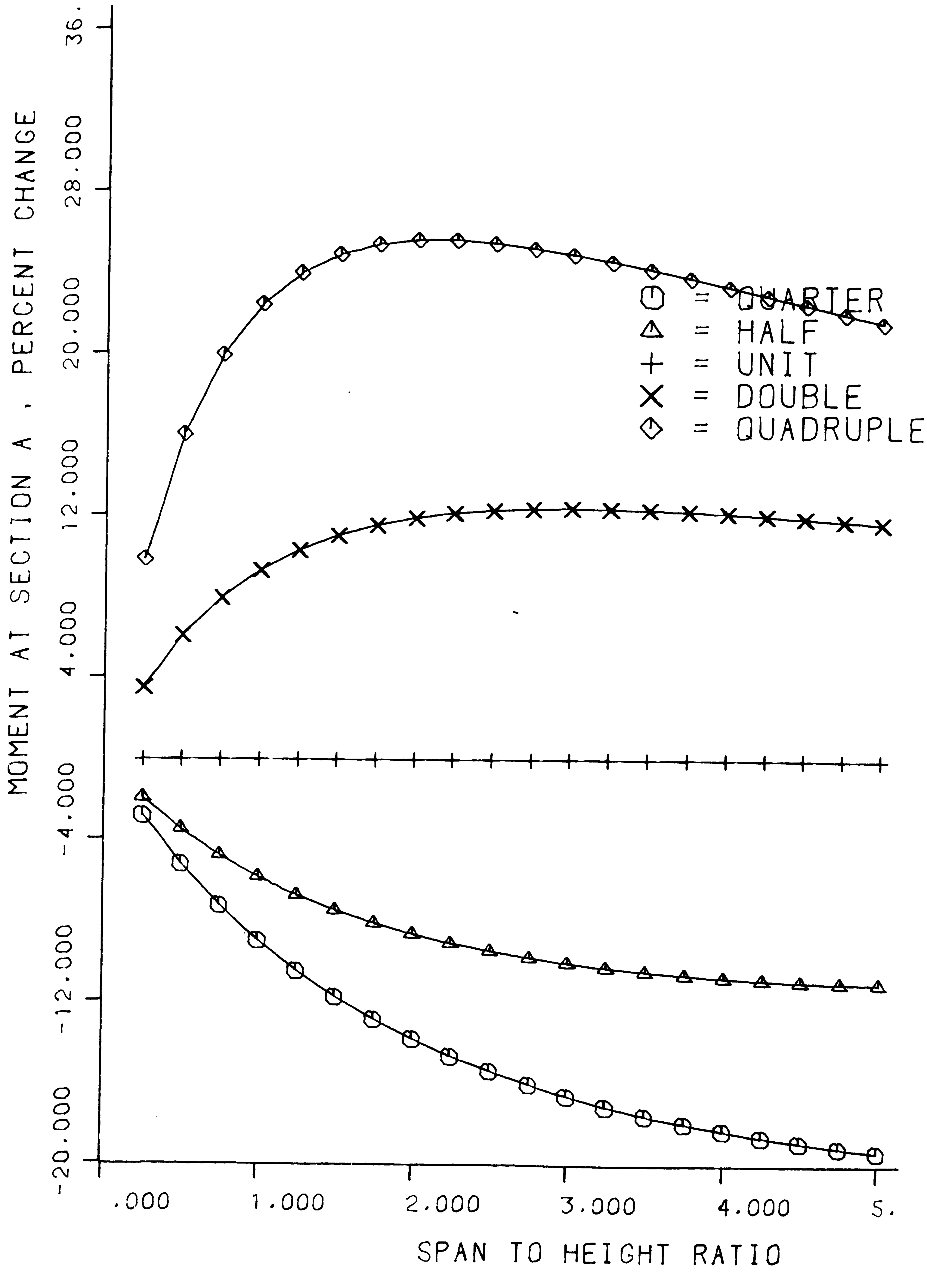


Figure 16: Corner Mom. vs Span-Height & Col. Inertia under Lat. Load

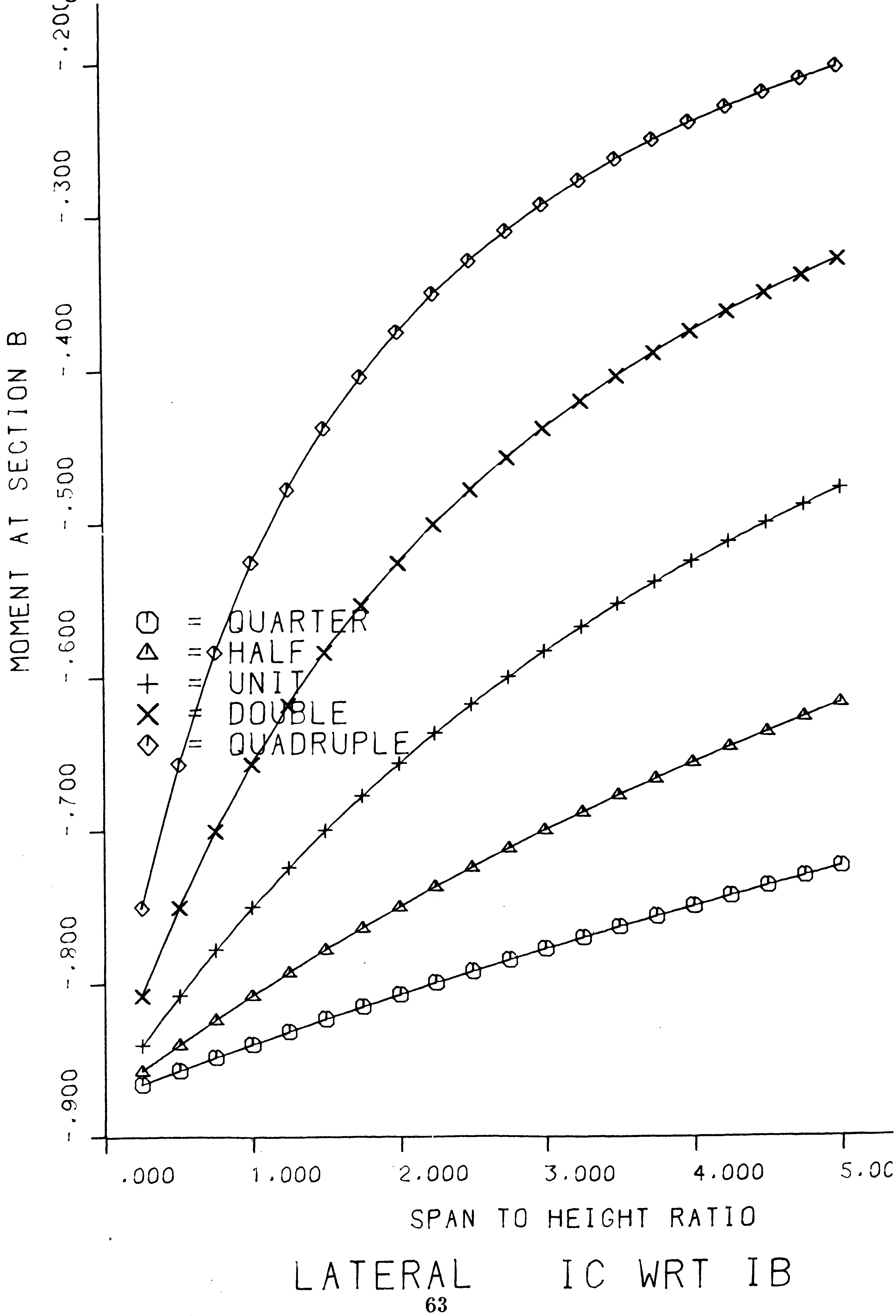


Figure 17: % Corner Mom. vs Span-Height & Col. Inertia under Lat. Load

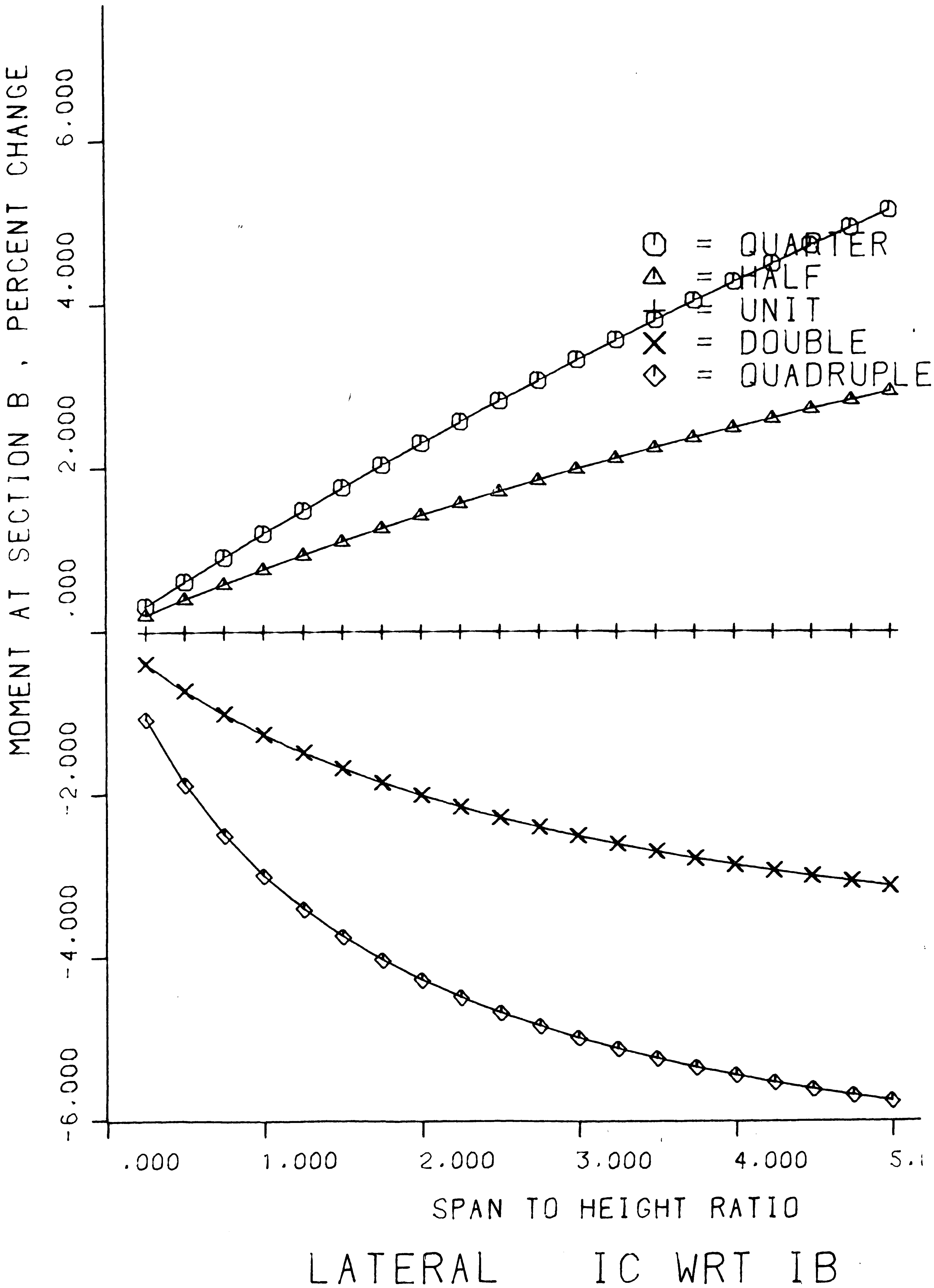


Figure 18: Sway vs Span-Height & Col. Inertia under Lat. Load

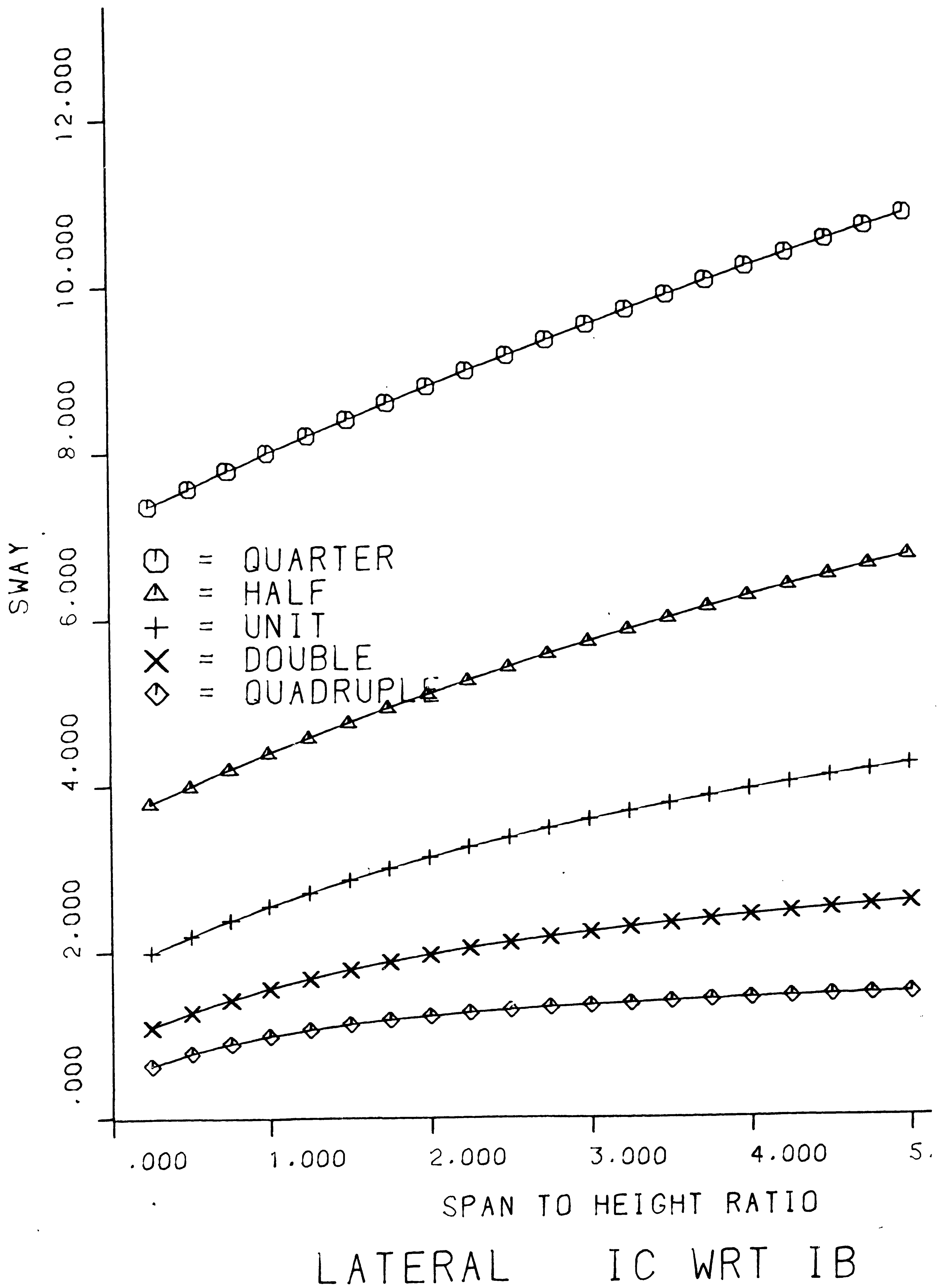


Figure 19: % Sway vs Span-Height & Col. Inertia under Lat. Load

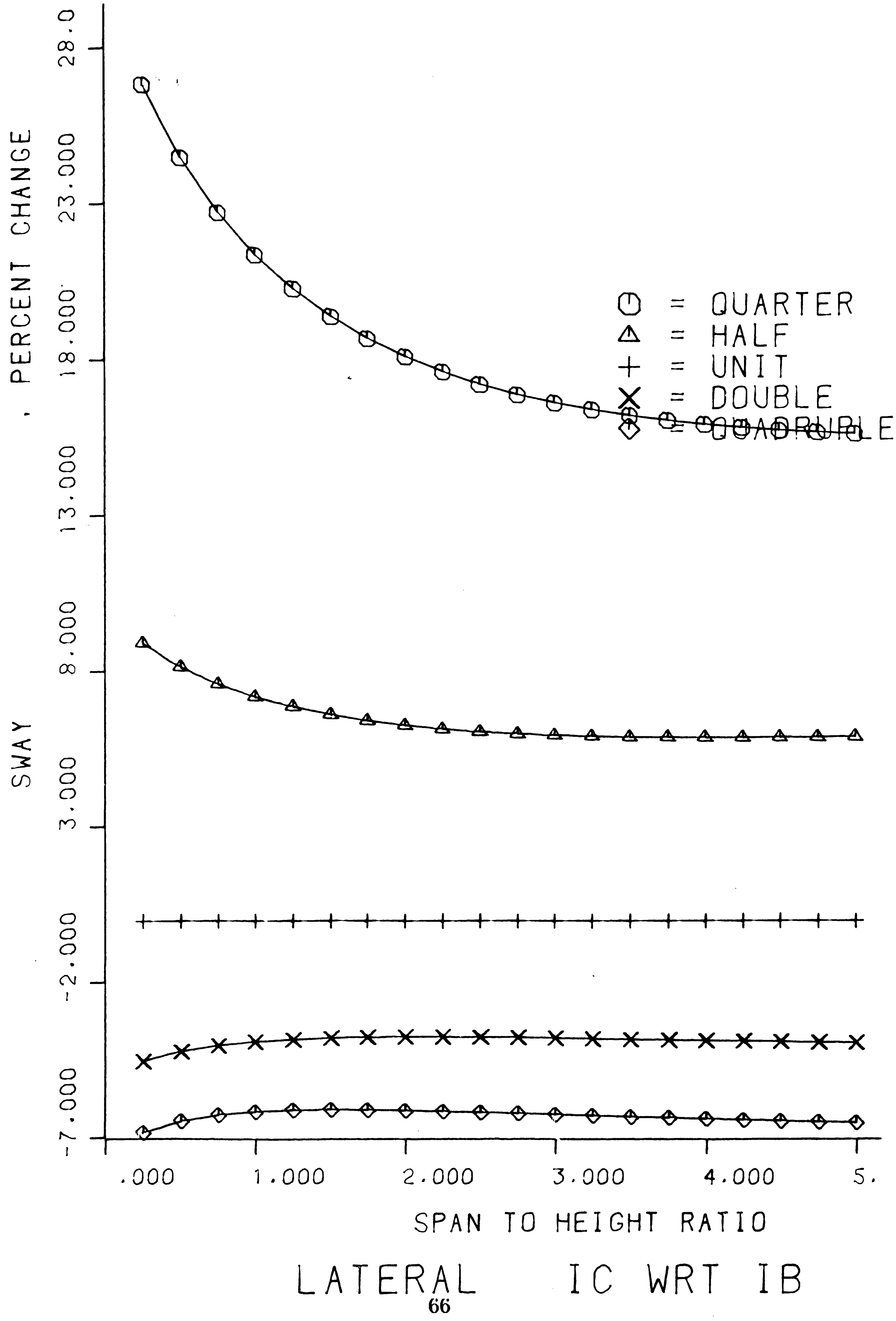


Figure 20: Base Mom. vs Span-Height & Height under Grav. Load

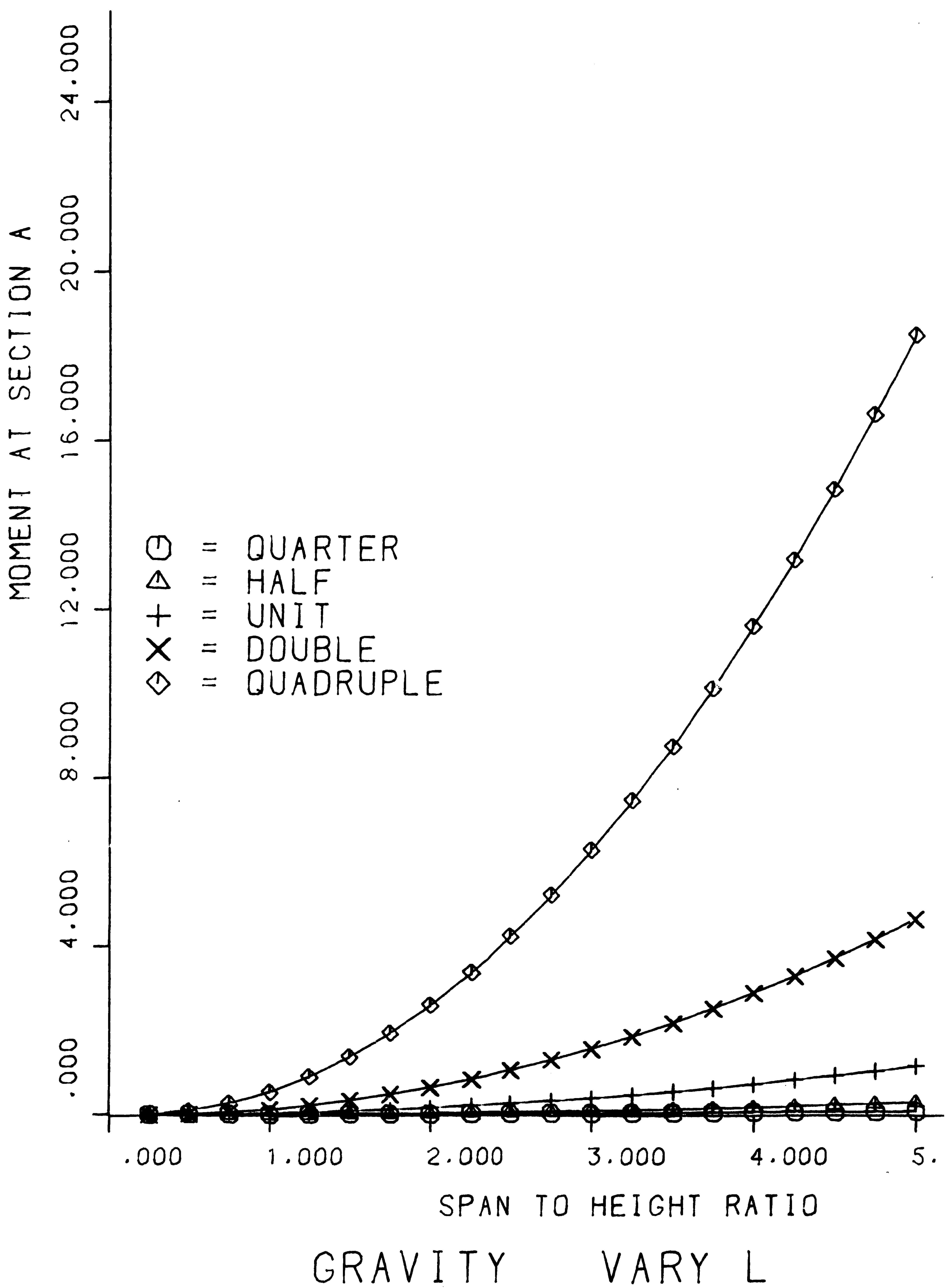


Figure 21: Midspan Mom. vs Span-Height & Height under Grav. Load

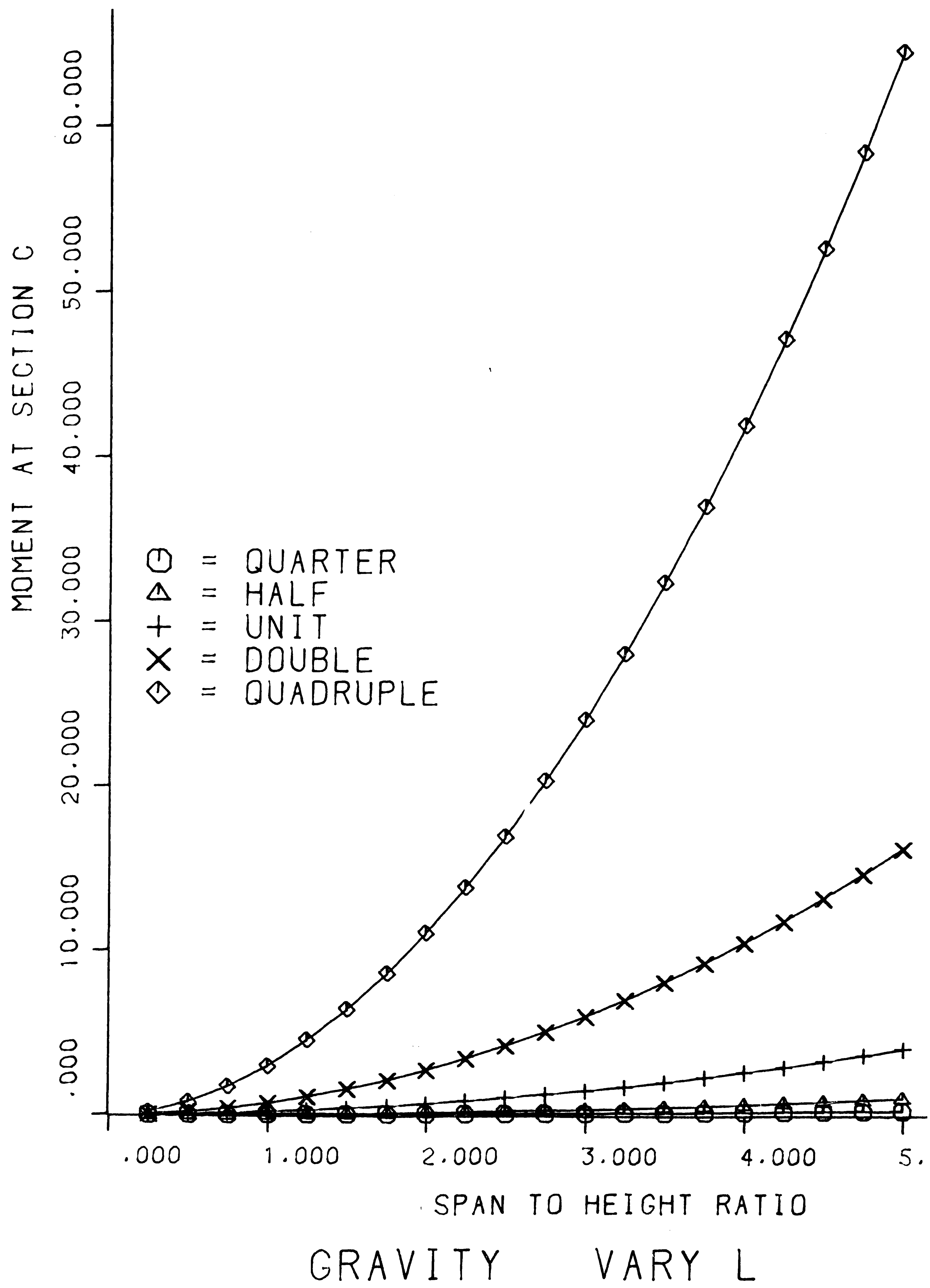


Figure 22: Sway vs Span-Height & Height under Grav. Load

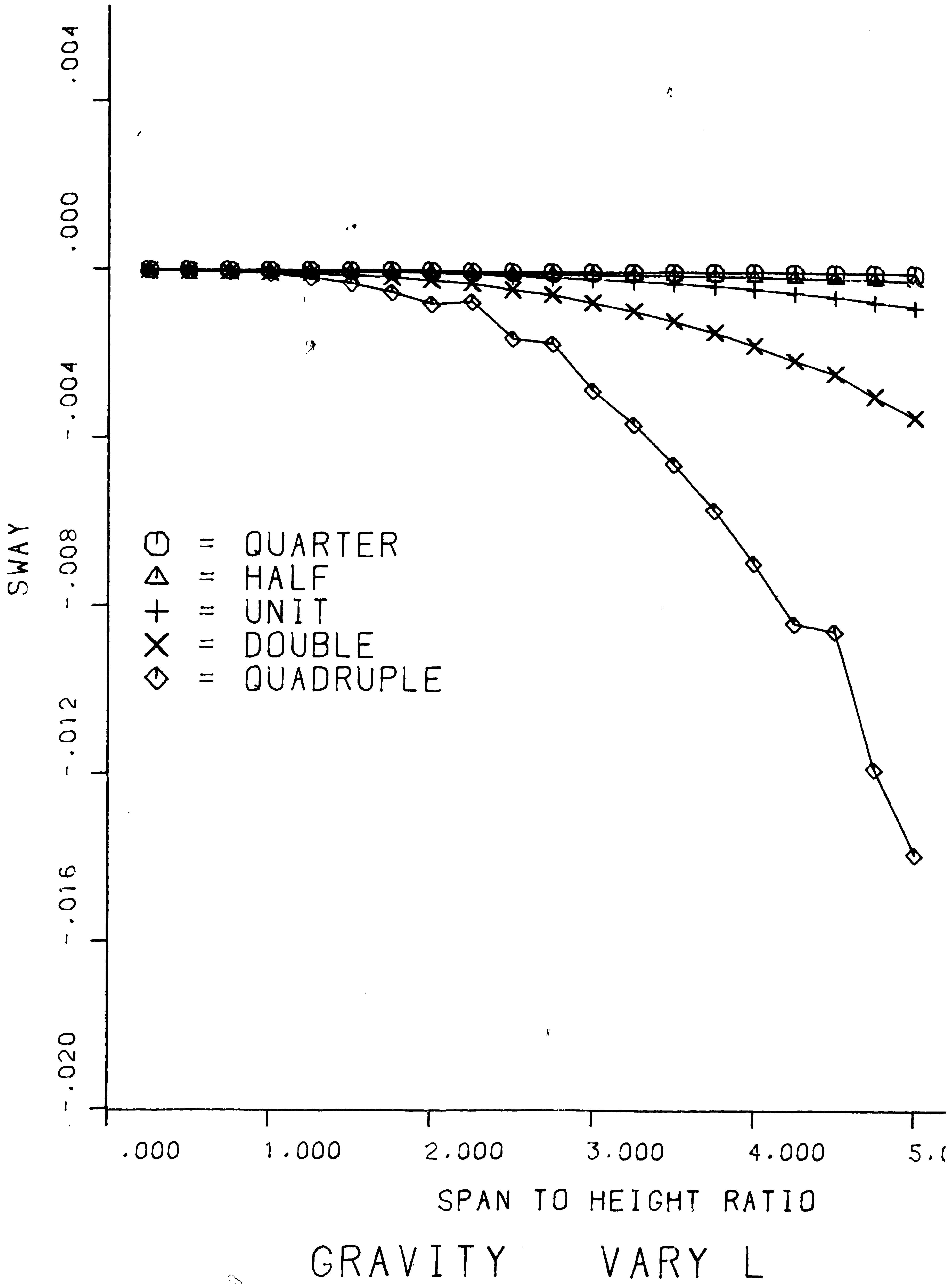


Figure 23: Base Mom. vs Span-Height & Height under Lat. Load

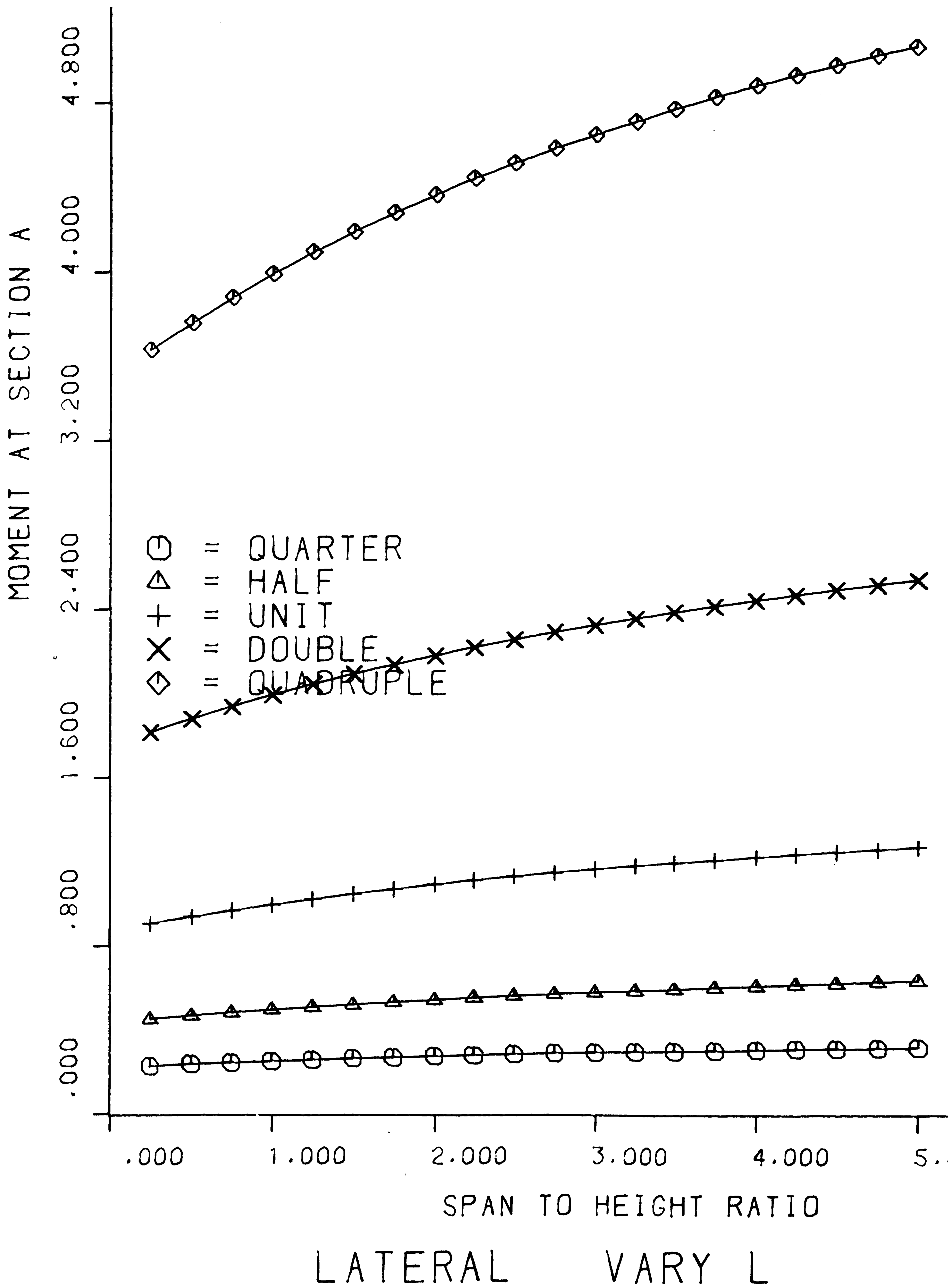


Figure 24: Corner Mom. vs Span-Height & Height under Lat. Load

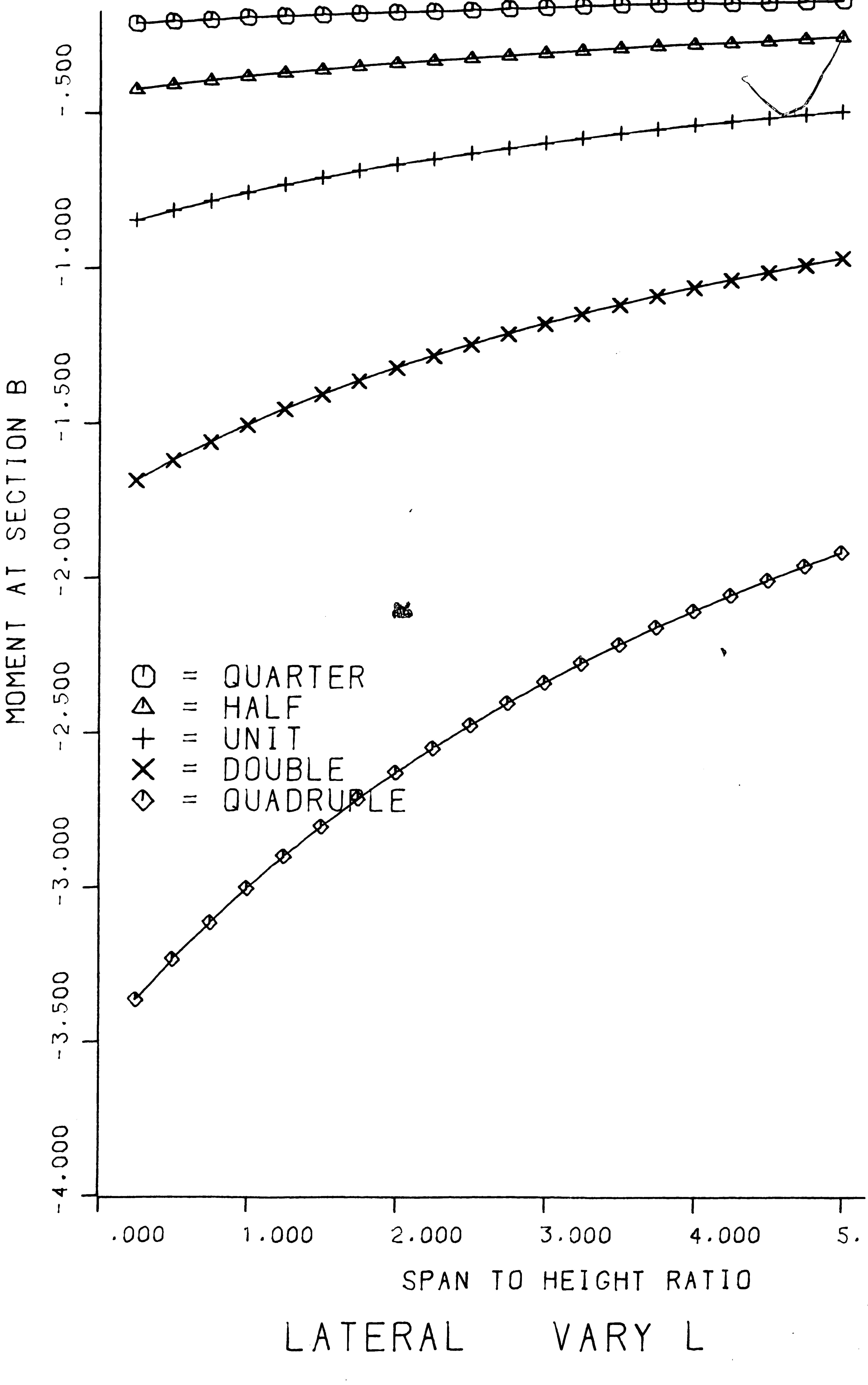


Figure 25: Sway vs Span-Height & Height under Lat. Load

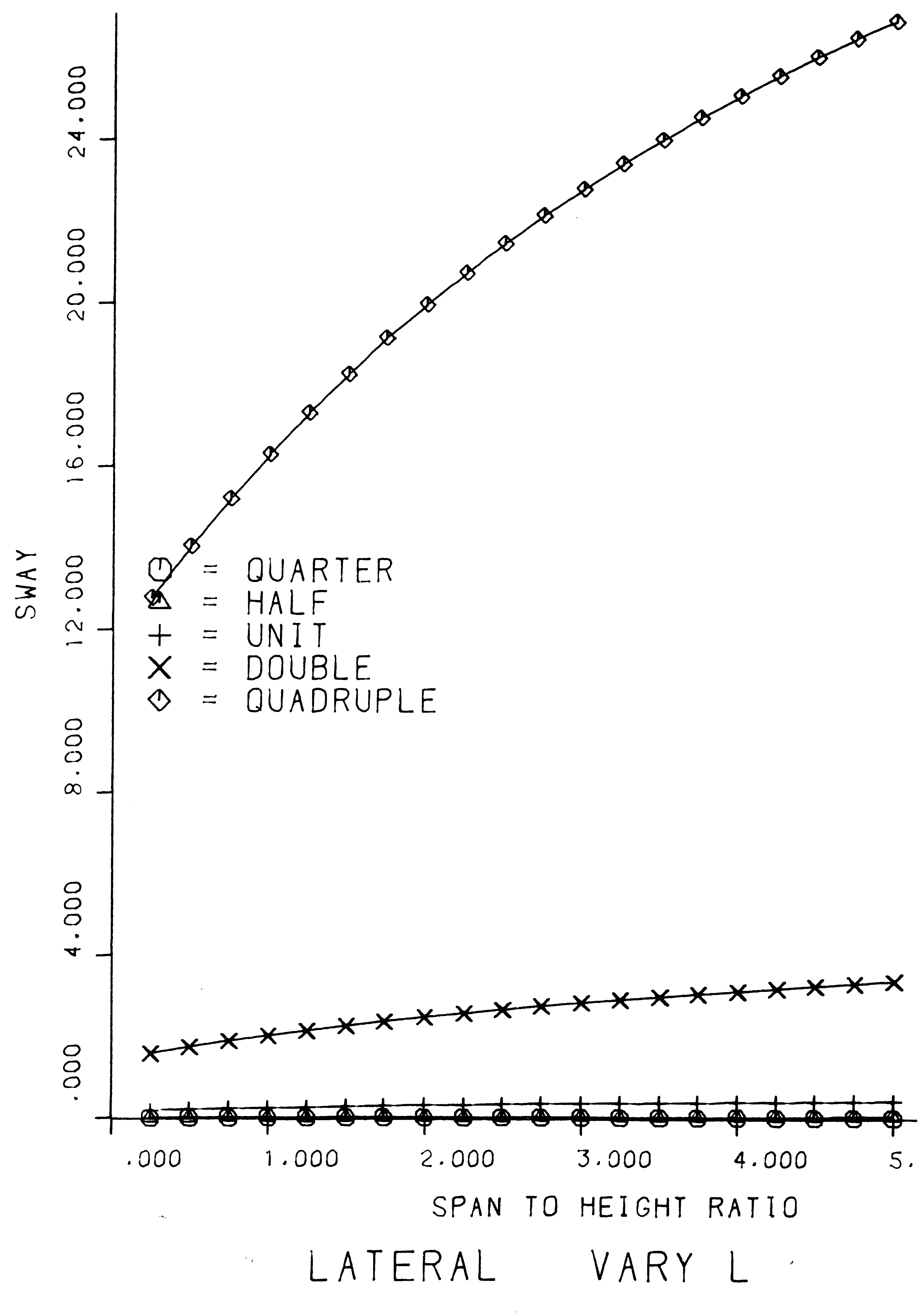


Figure 26: Sway vs Col. Inertia & Span-Height under Lat. Load

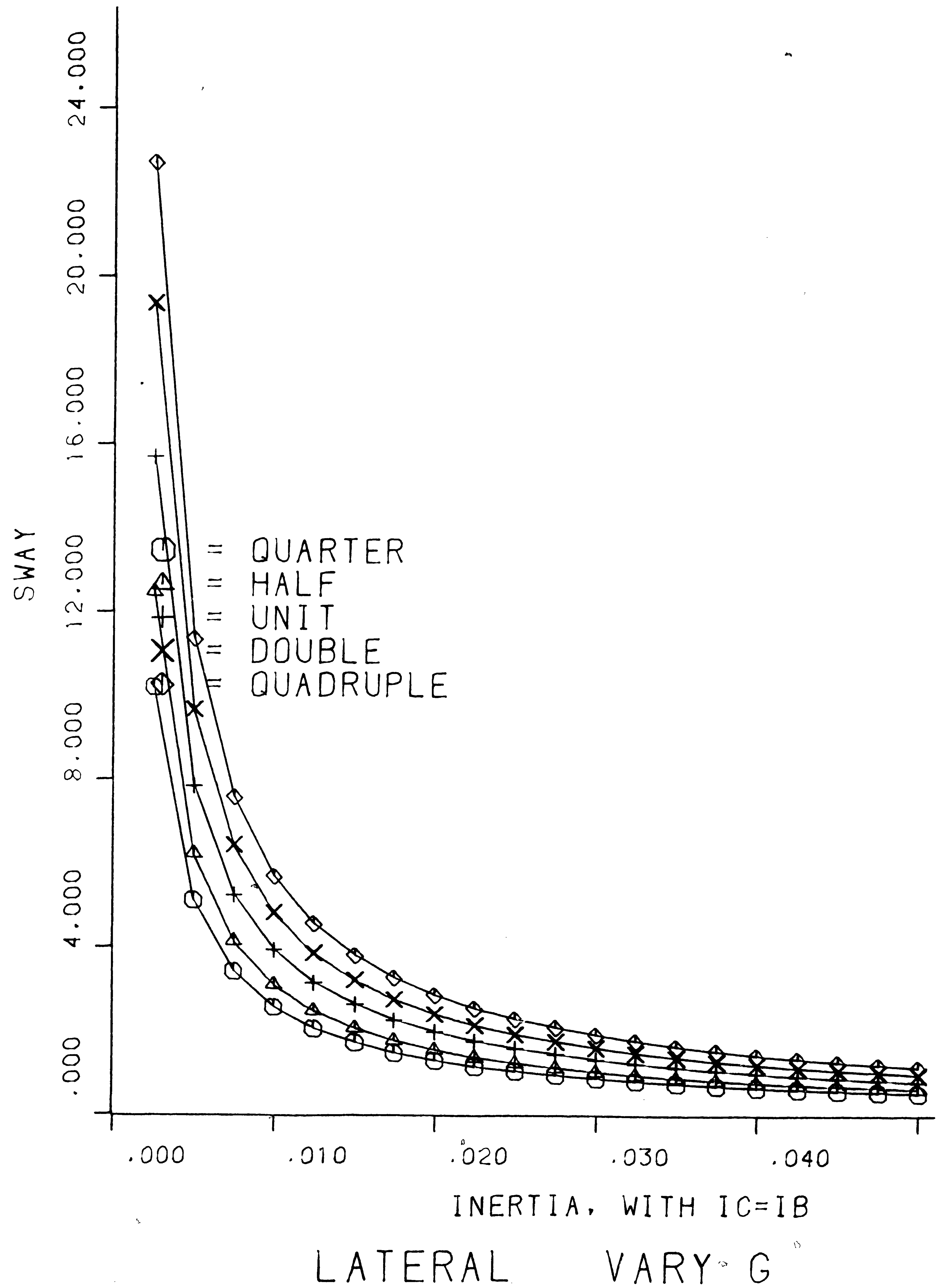


Figure 27: % Sway vs Col. Inertia & Span-Height under Lat. Load

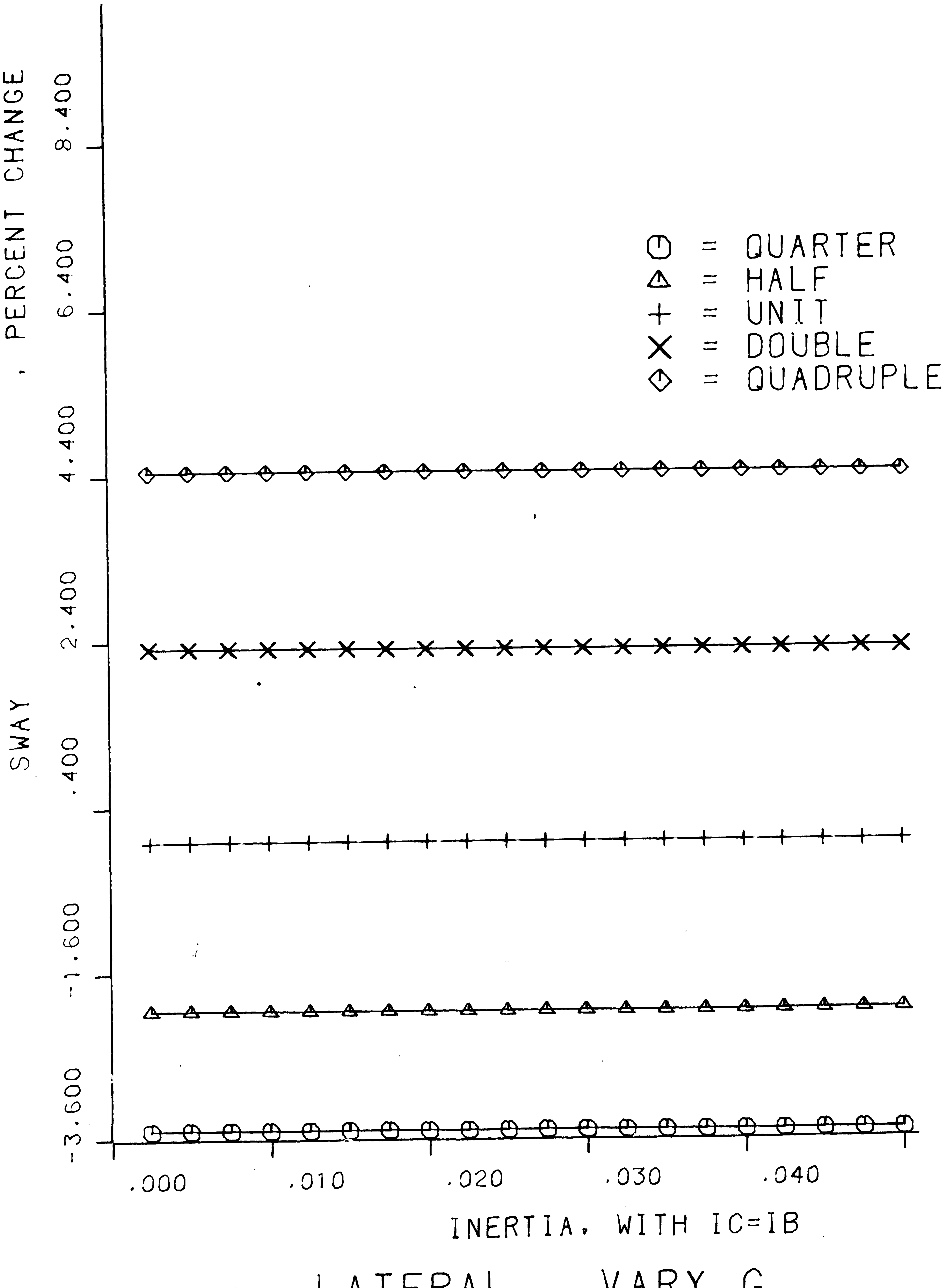


Figure 28: Base Mom. vs Col. Inertia & Beam Inertia, G=1, Grav. Load

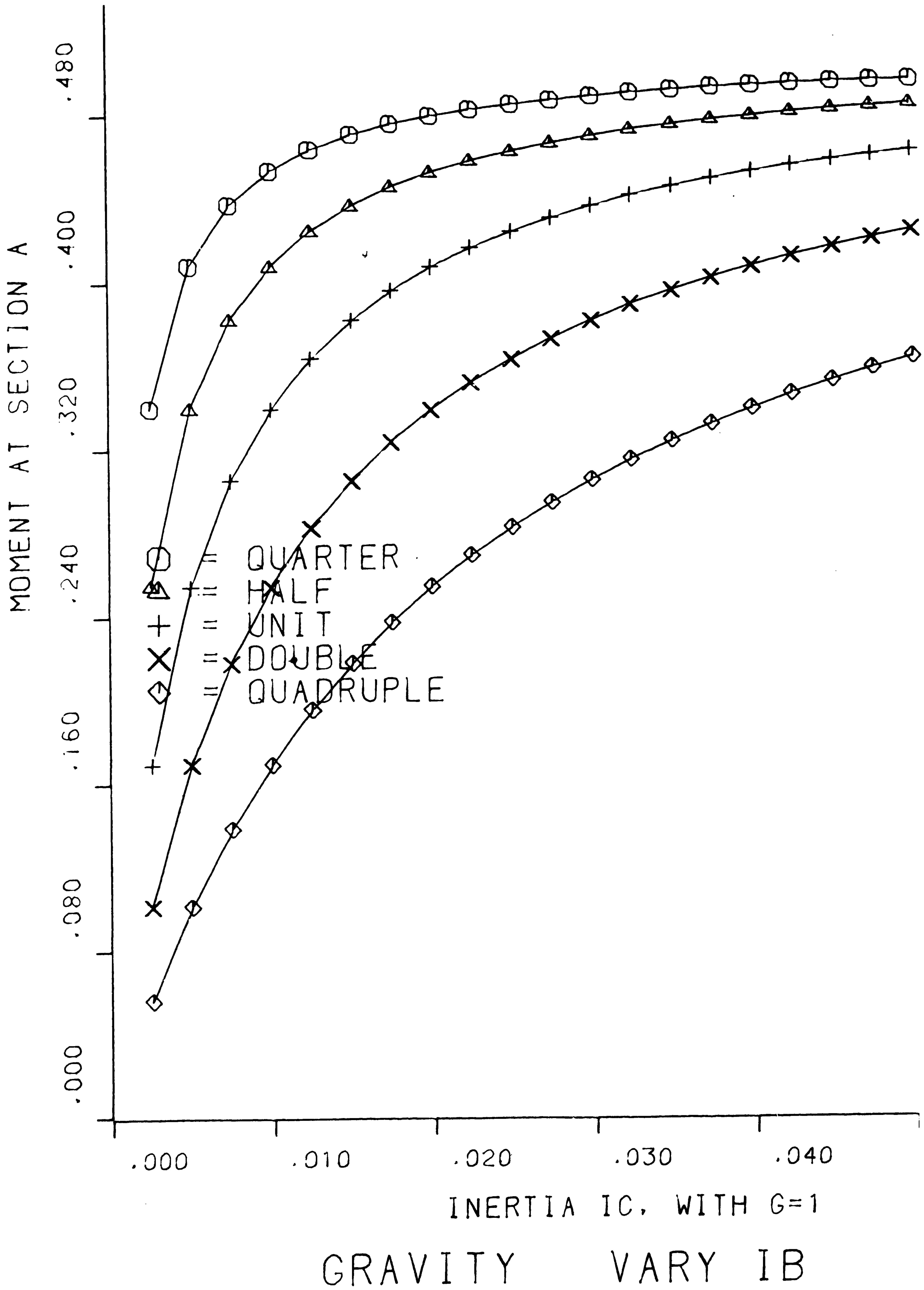


Figure 29: Base Mom. vs Col. Inertia & Beam Inertia, G=2, Grav. Load

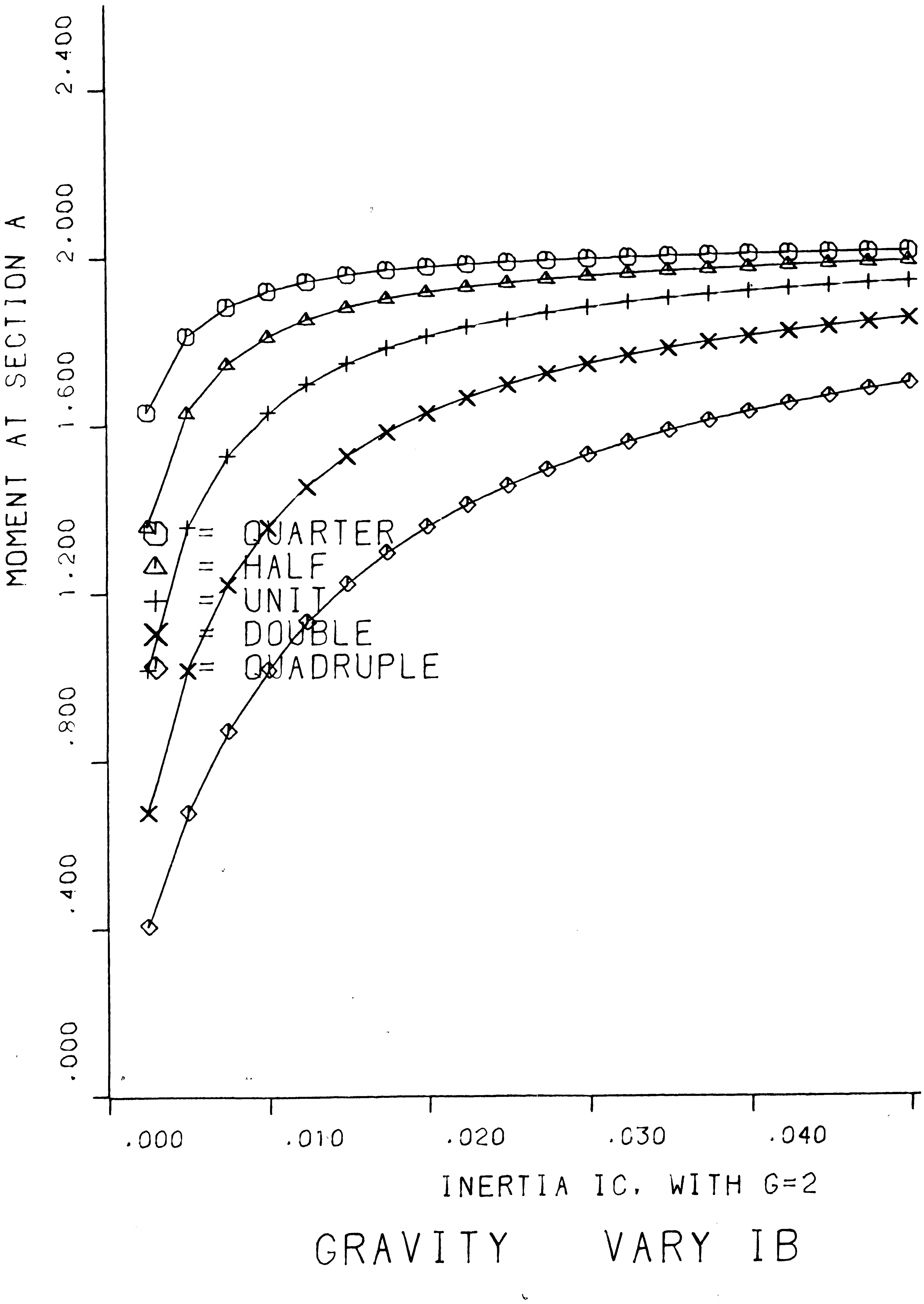


Figure 30: Base Mom. vs Col. Inertia & Beam Inertia, G=3, Grav. Load

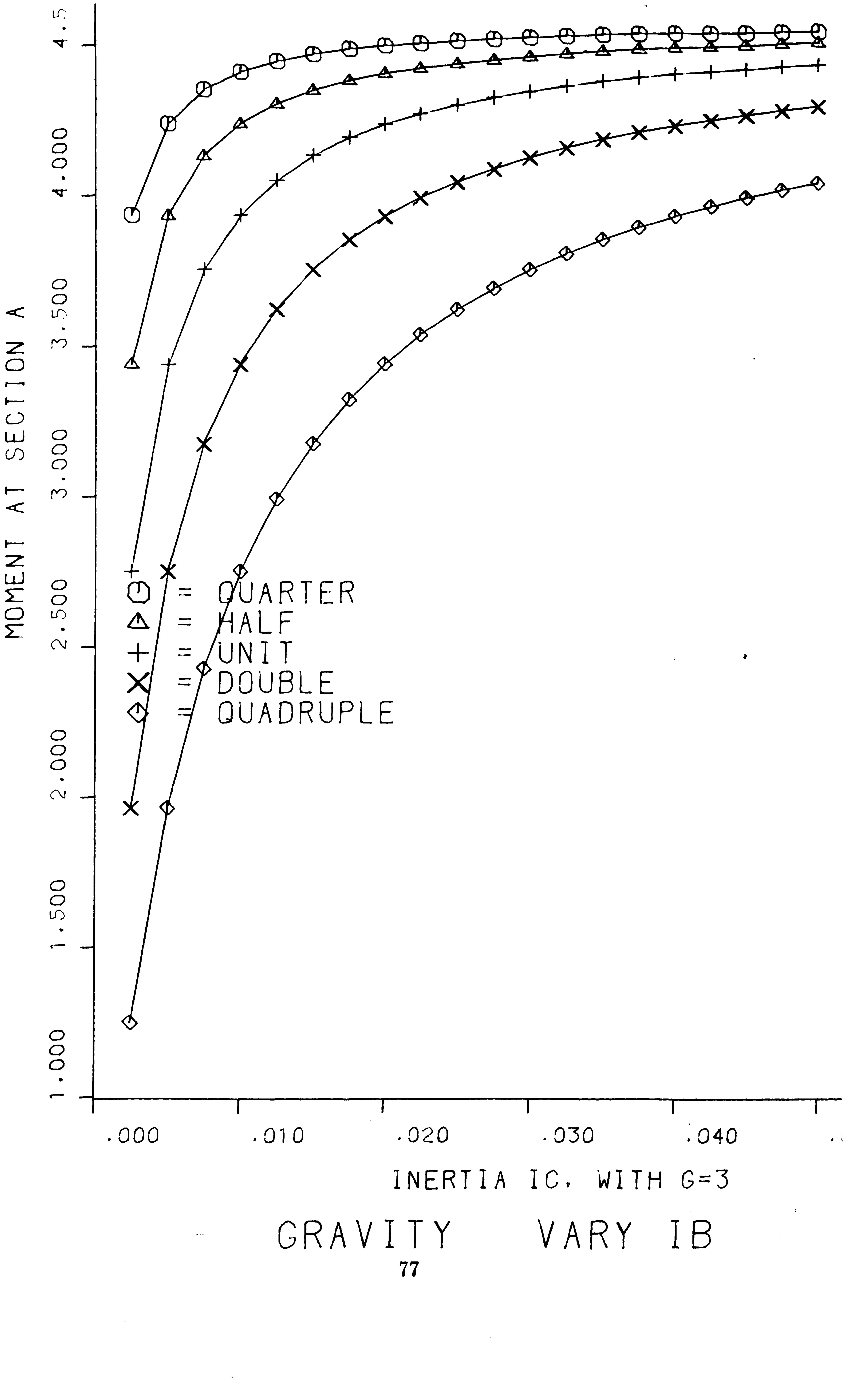


Figure 31: Base [illegible] & Beam Inertia, G=4, Grav. Load

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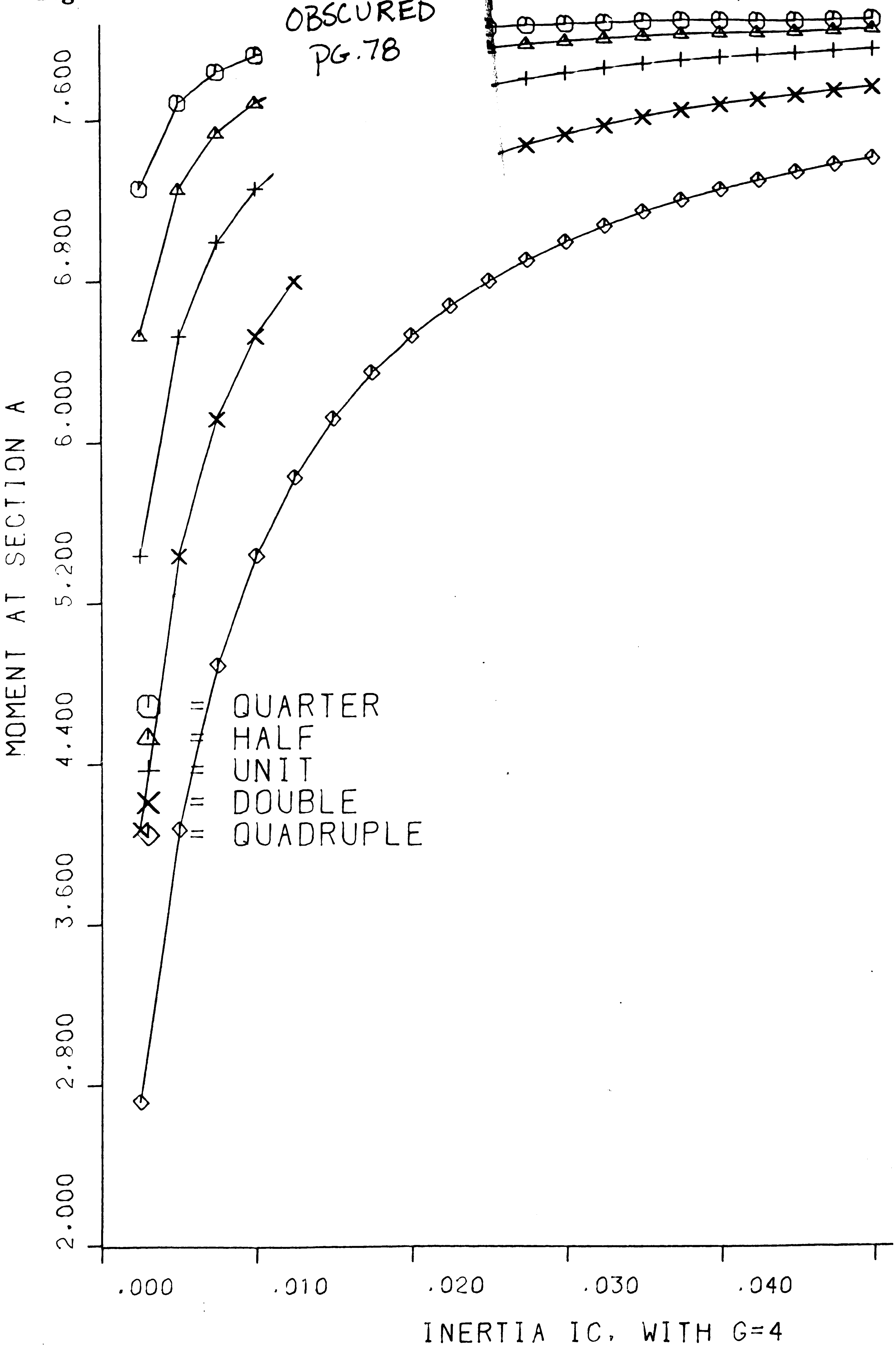

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Figure 32: Base Mom. vs Col. Inertia & Beam Inertia, G=5, Grav. Load

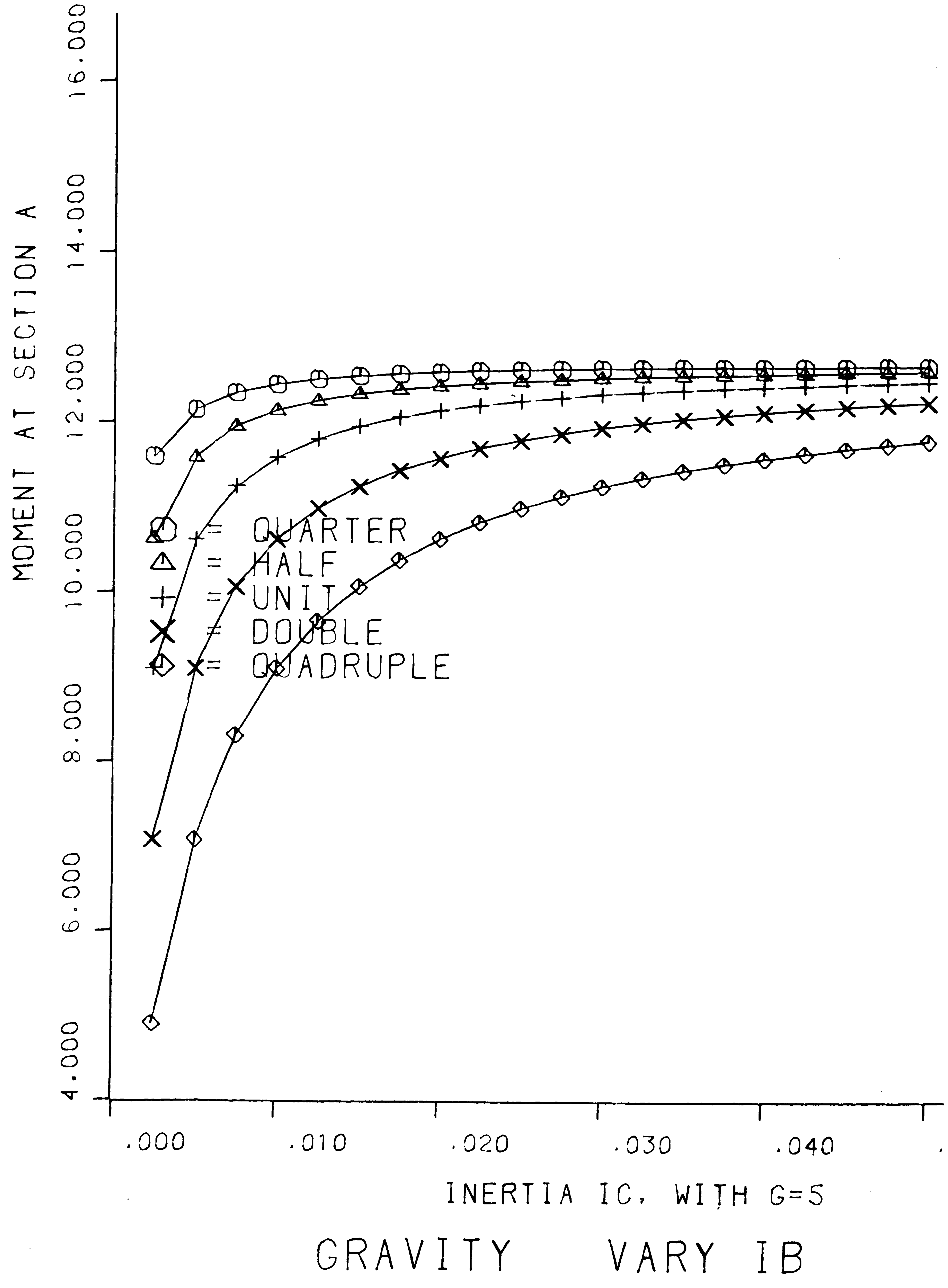

Figure 33: % Base Mom. vs Col. Inertia & Beam Inertia, G=1, Grav. Load

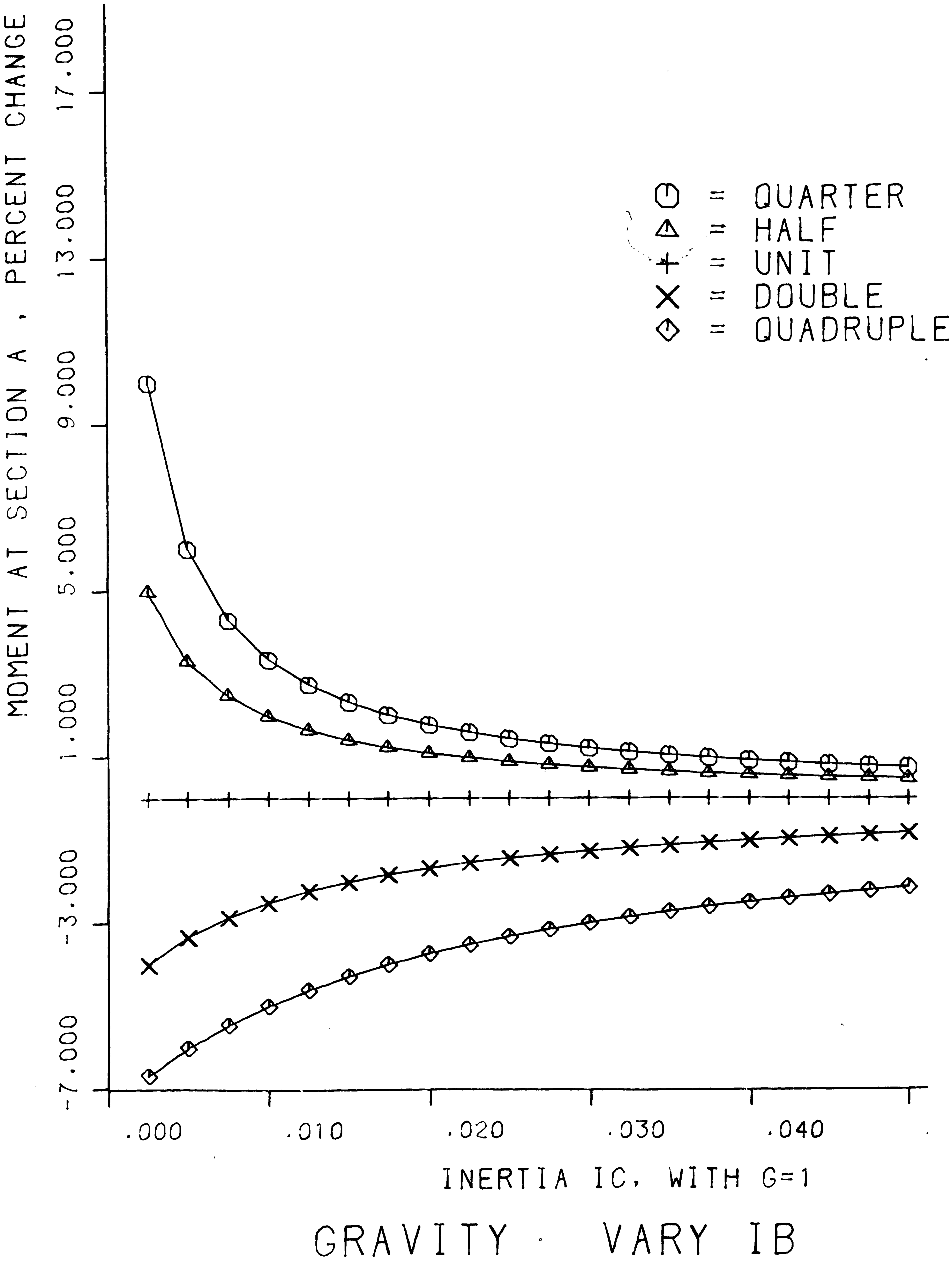

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Figure 34: % Base Mom. vs Col. Inertia & Beam Inertia, G=2, Grav. Load

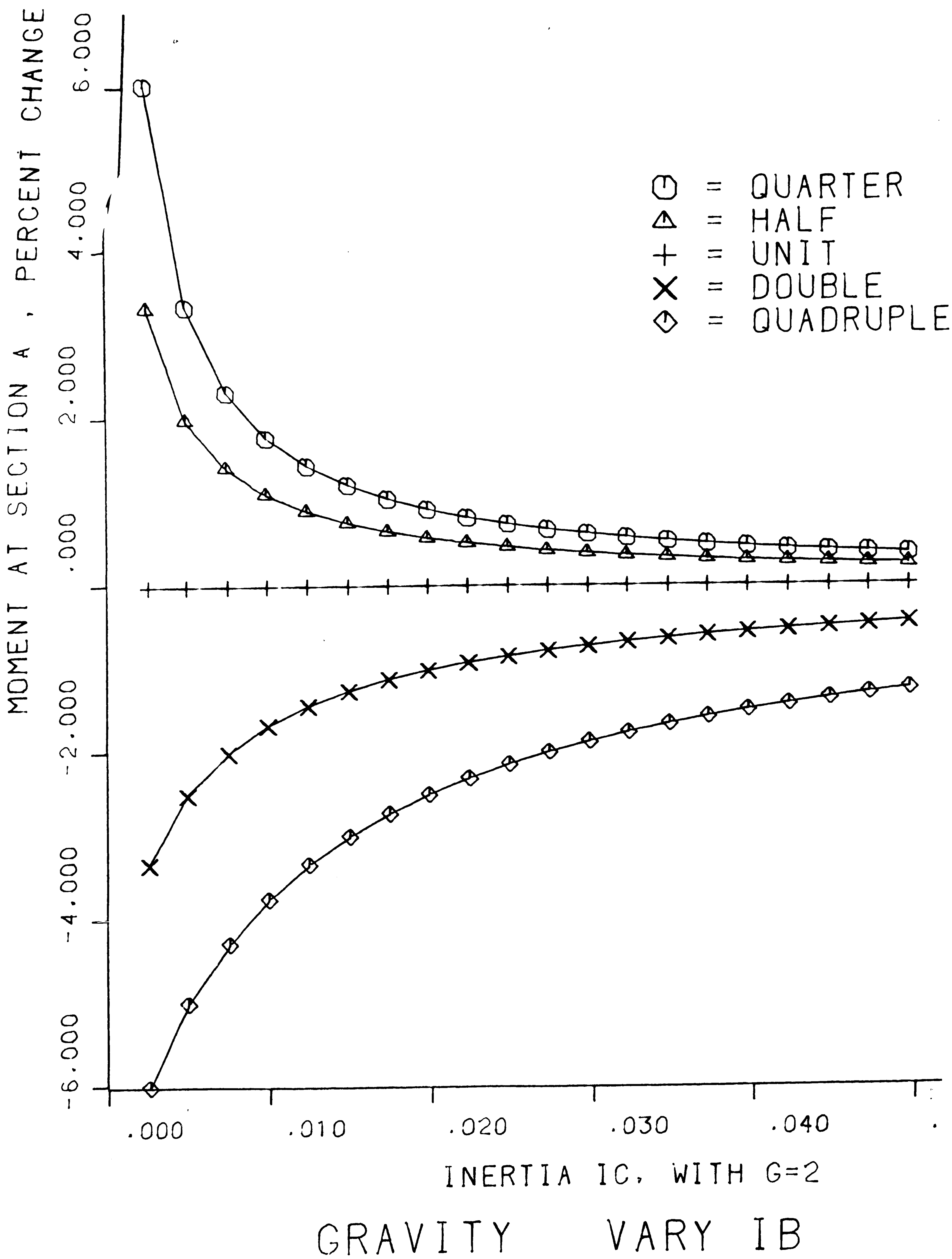


Figure 35: % Base Mom. vs Col. Inertia & Beam Inertia, G=3, Grav. Load

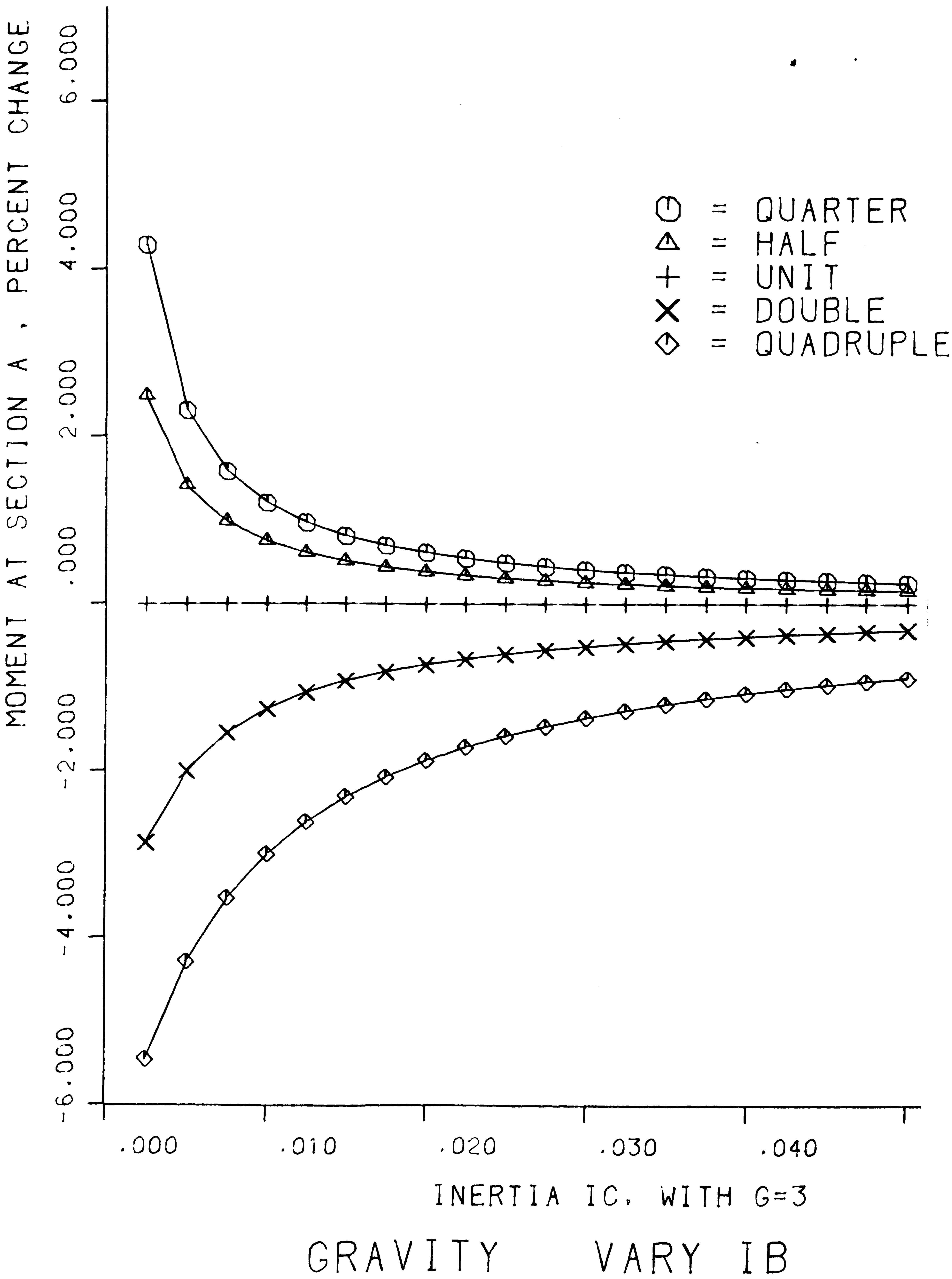


Figure 36: % Base Mom. vs Col. Inertia & Beam Inertia, G=4, Grav. Load

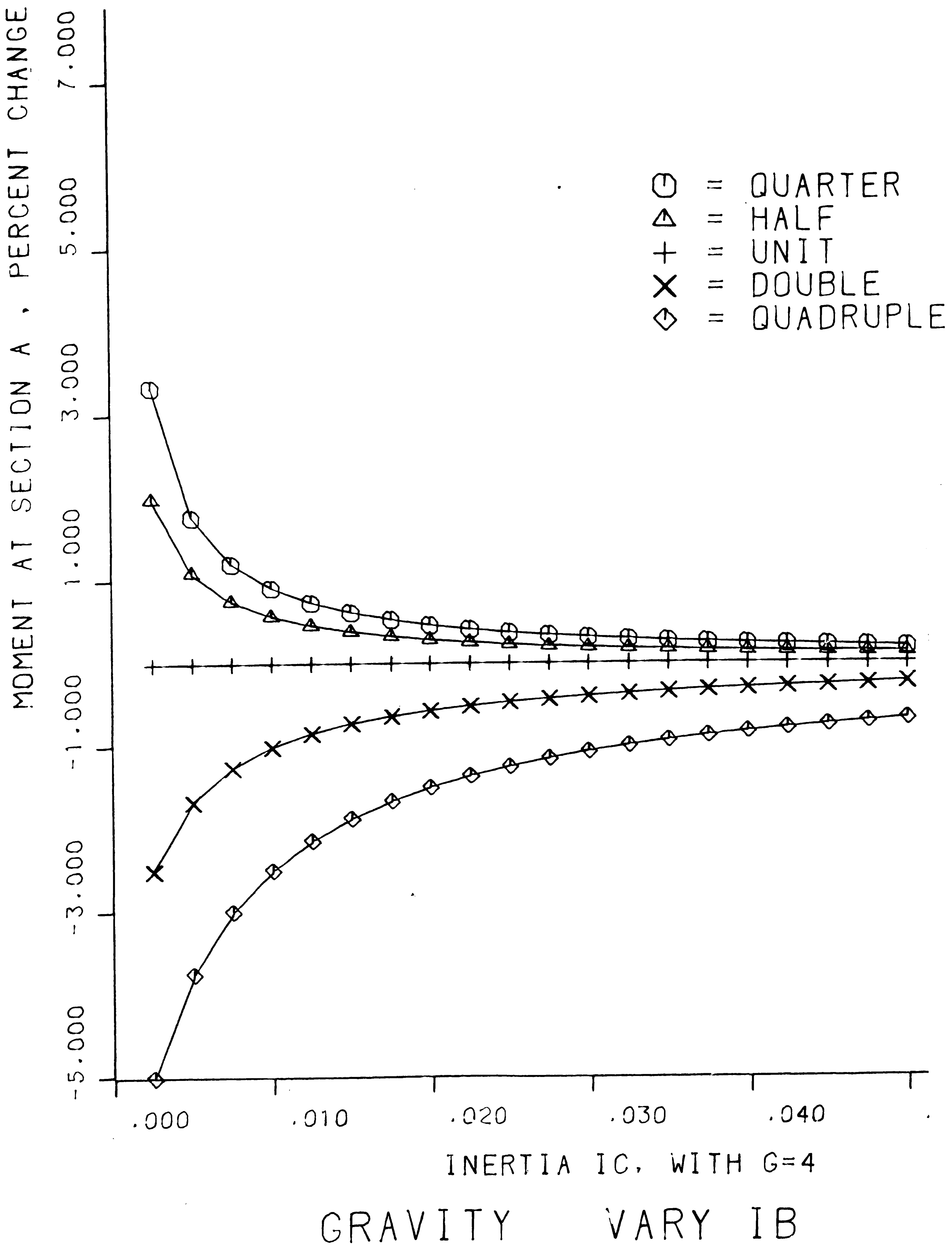


Figure 37: % Base Mom. vs Col. Inertia & Beam Inertia, G=5, Grav. Load

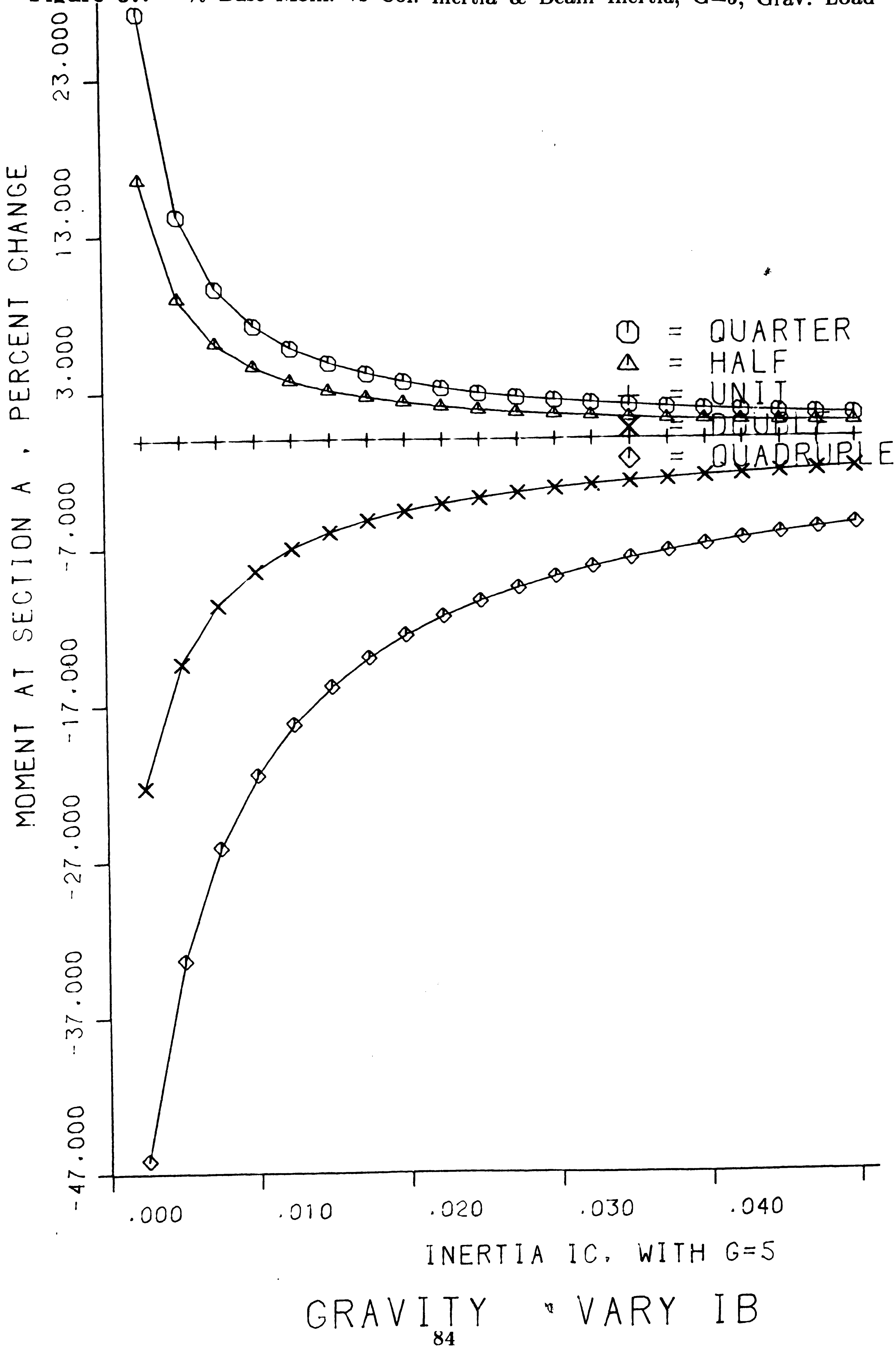


Figure 38: Midspan Mom. vs Col. Inertia & Beam Inertia, G=1, Grav. Load

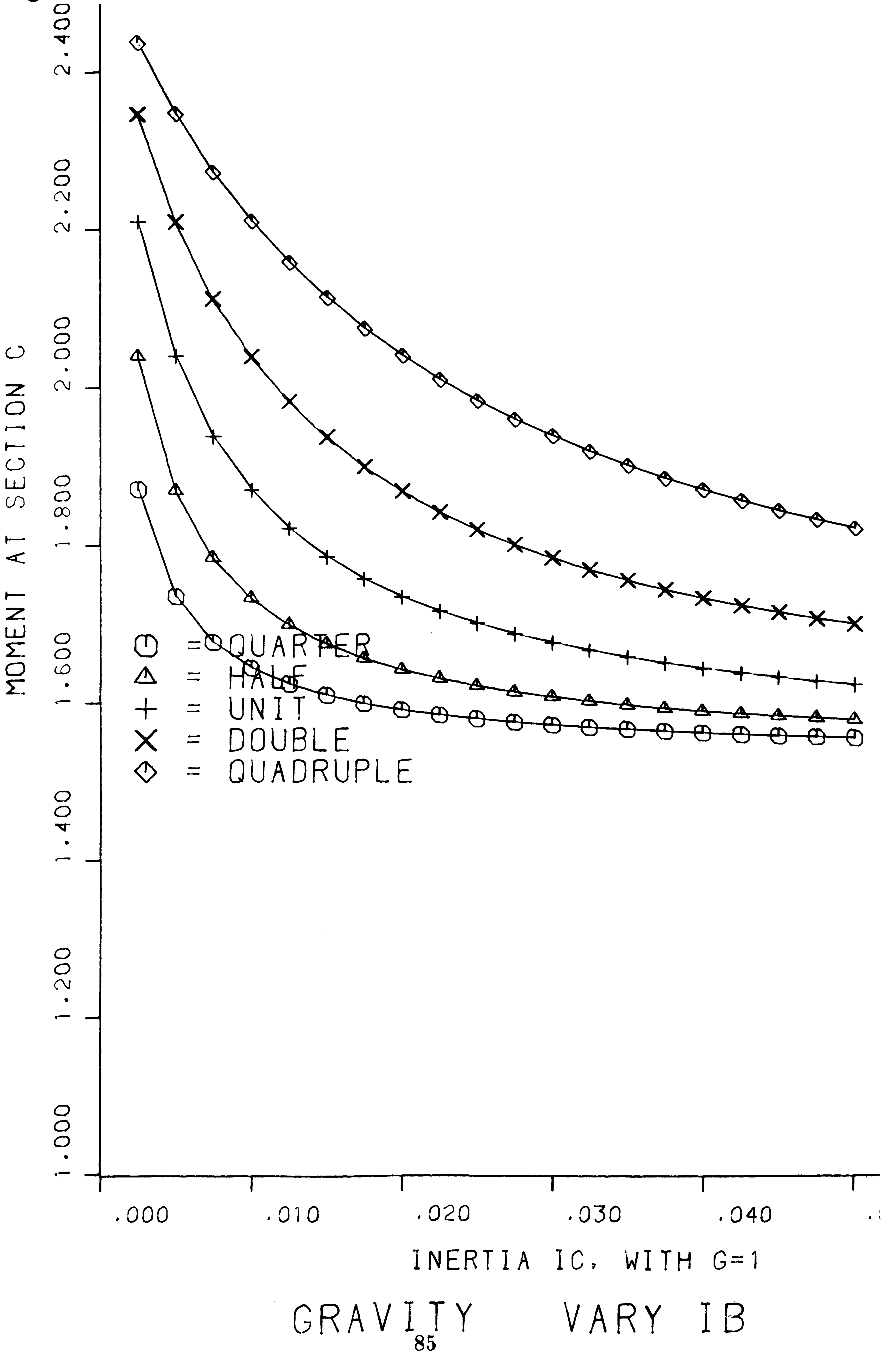


Figure 39: Midspan Mom. vs Col. Inertia & Beam Inertia, G=2, Grav. Load

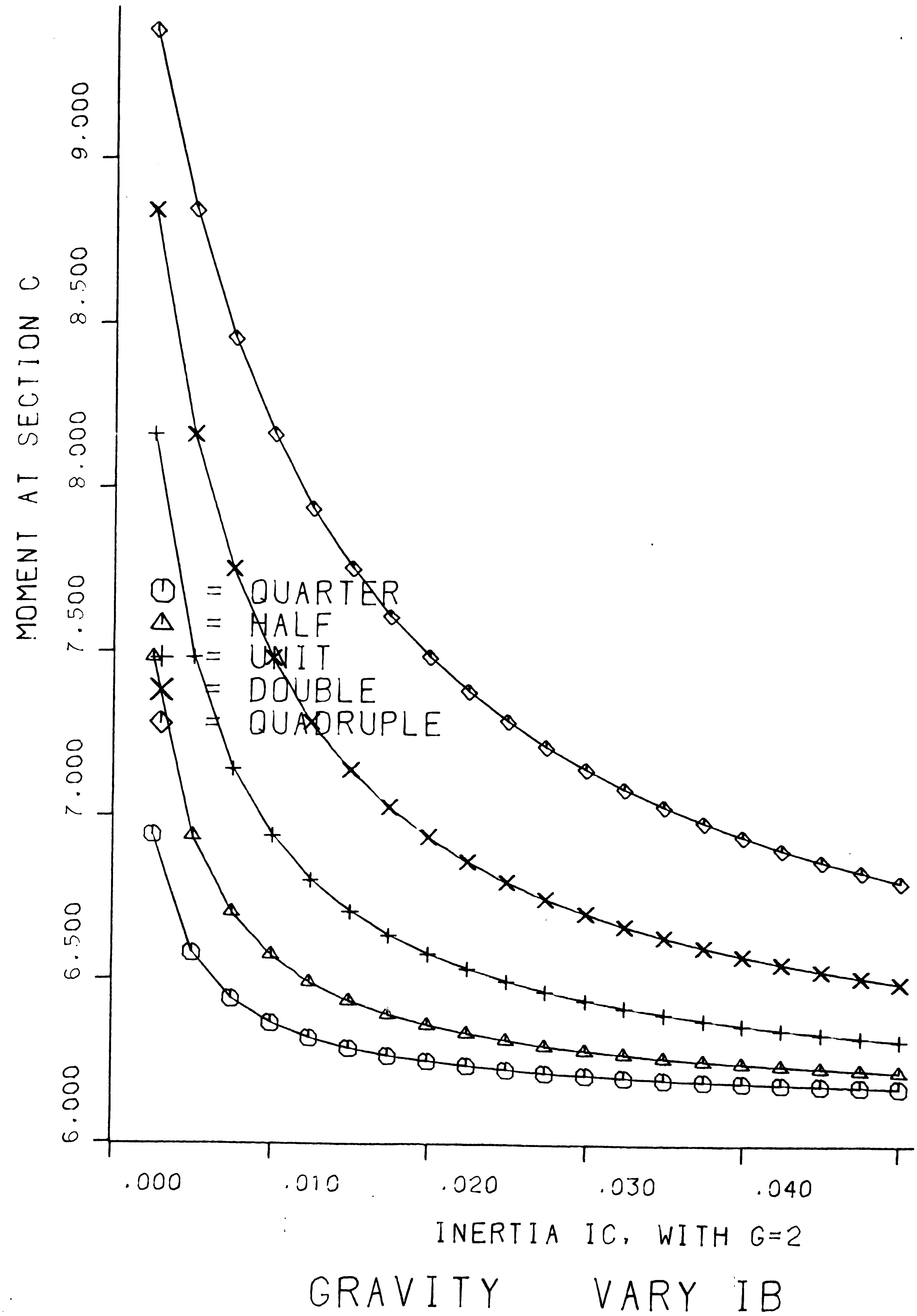


Figure 40: Midspan Mom. vs Col. Inertia & Beam Inertia, G=3, Grav. Load

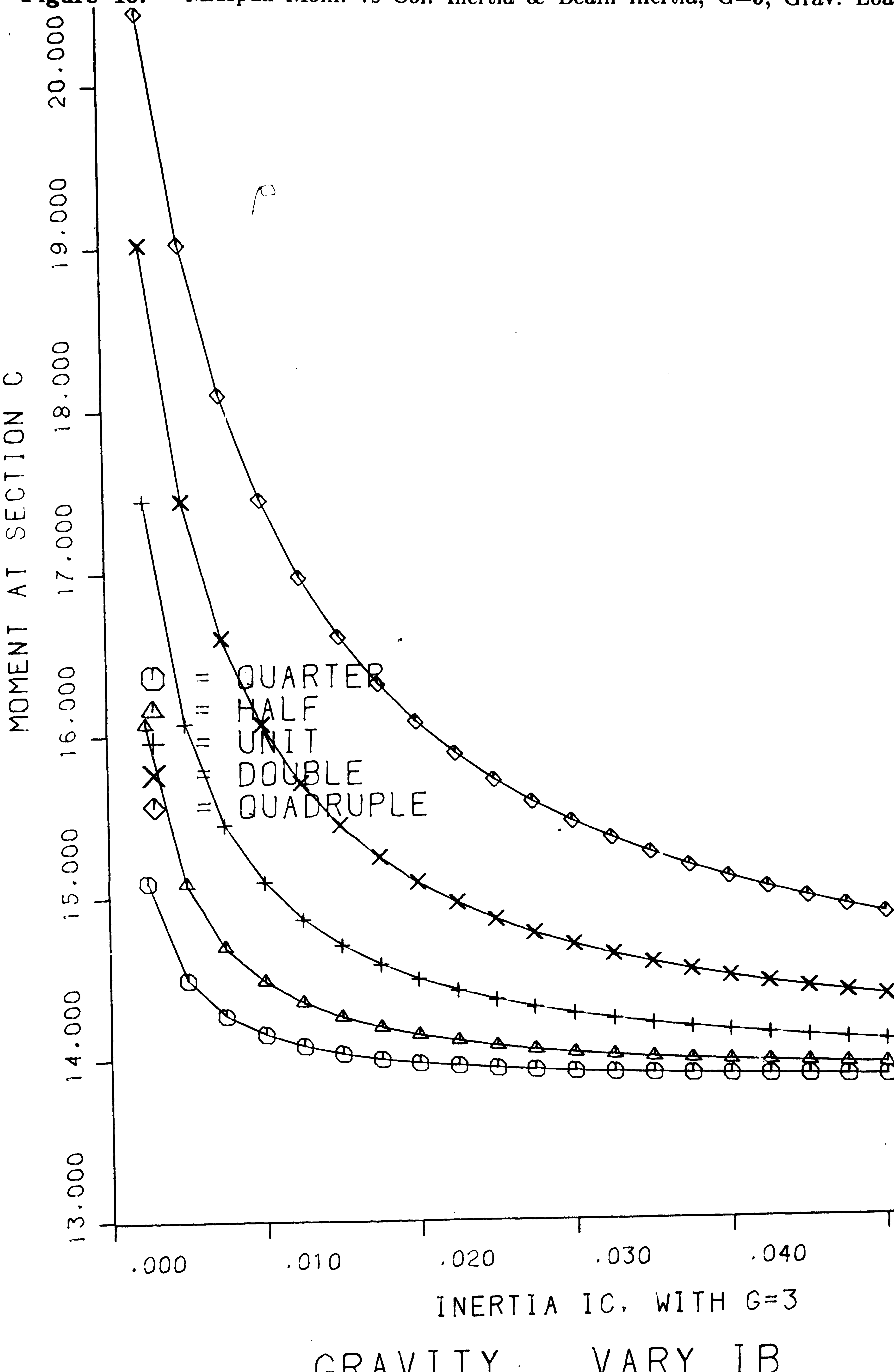


Figure 41: Midspan Mom. vs Col. Inertia & Beam Inertia, G=4, Grav. Load

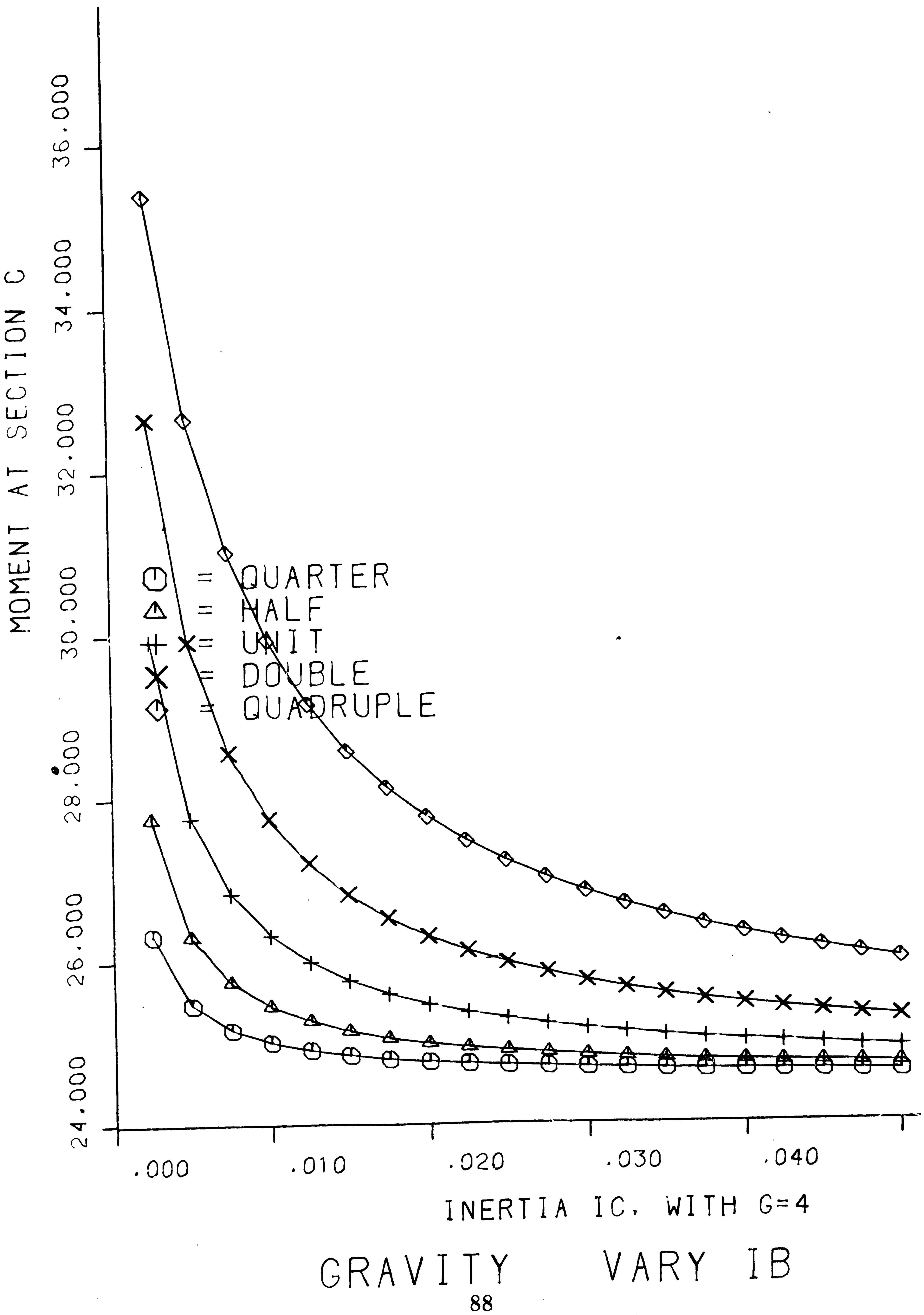


Figure 42: Midspan Mom. vs Col. Inertia & Beam Inertia, G=5, Grav. Load

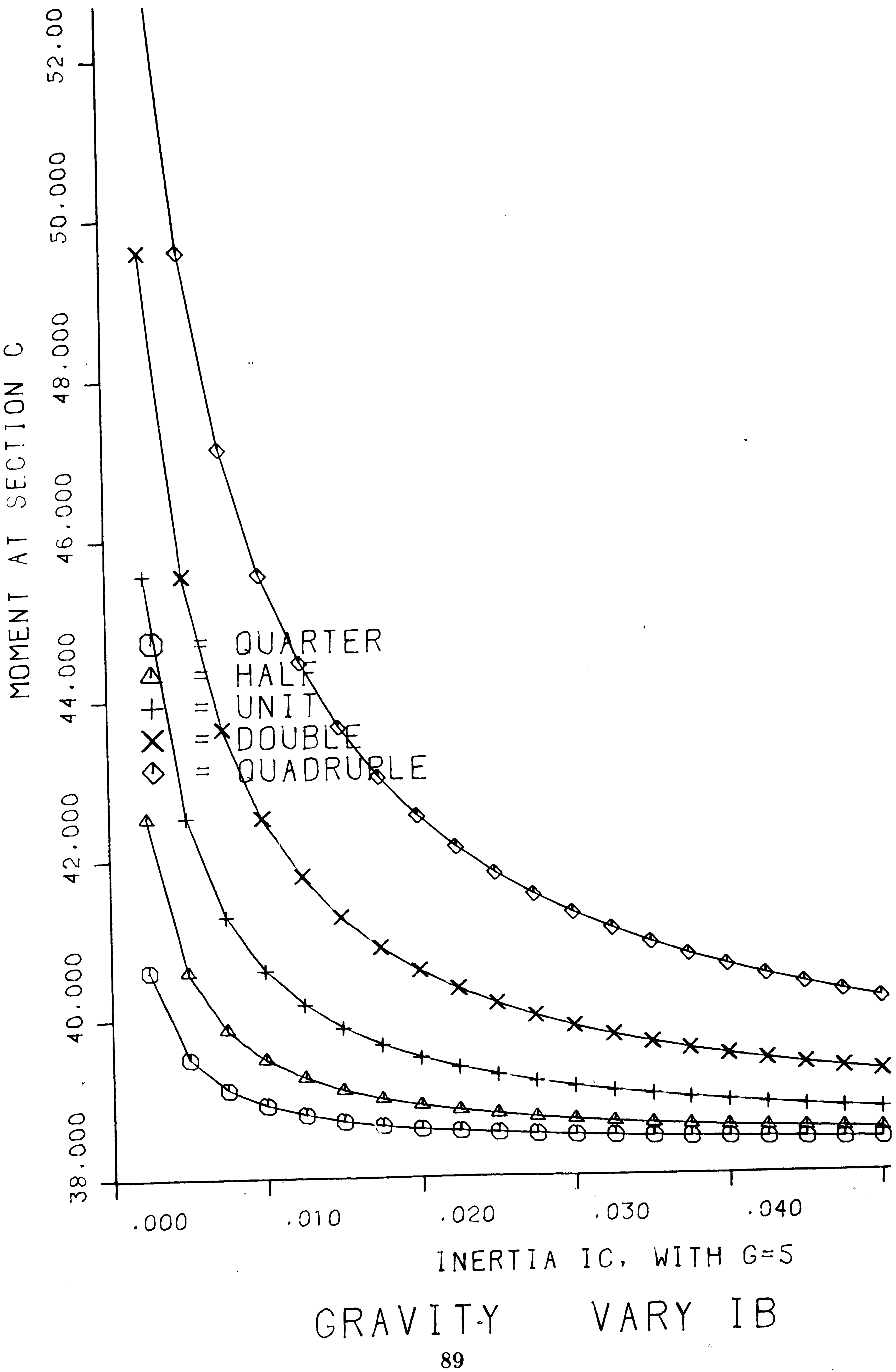


Figure 43: % Mid Mom. vs Col. Inertia & Beam Inertia, G=1, Grav. Load

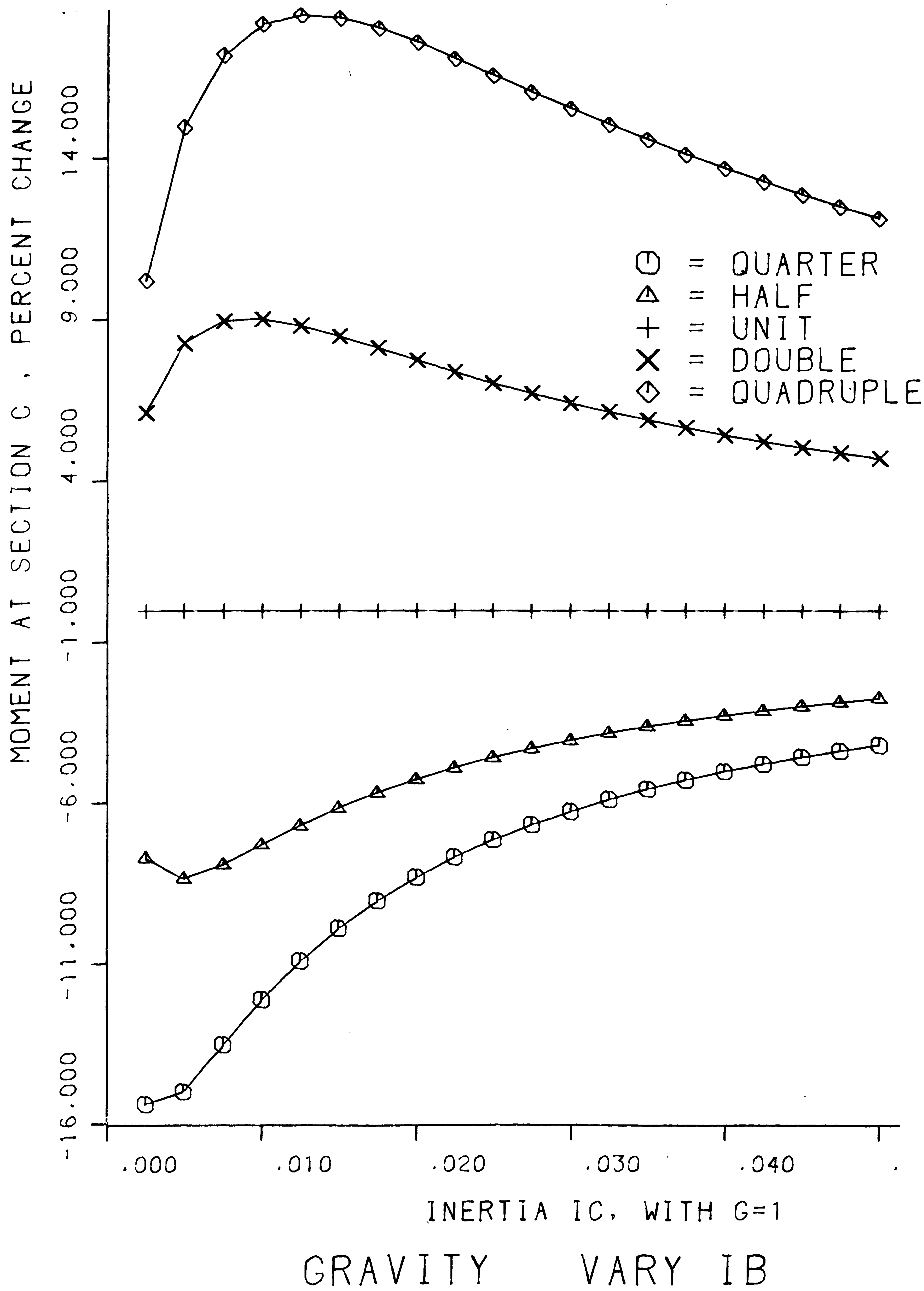


Figure 44: % Mid Mom. vs Col. Inertia & Beam Inertia, G=2, Grav. Load

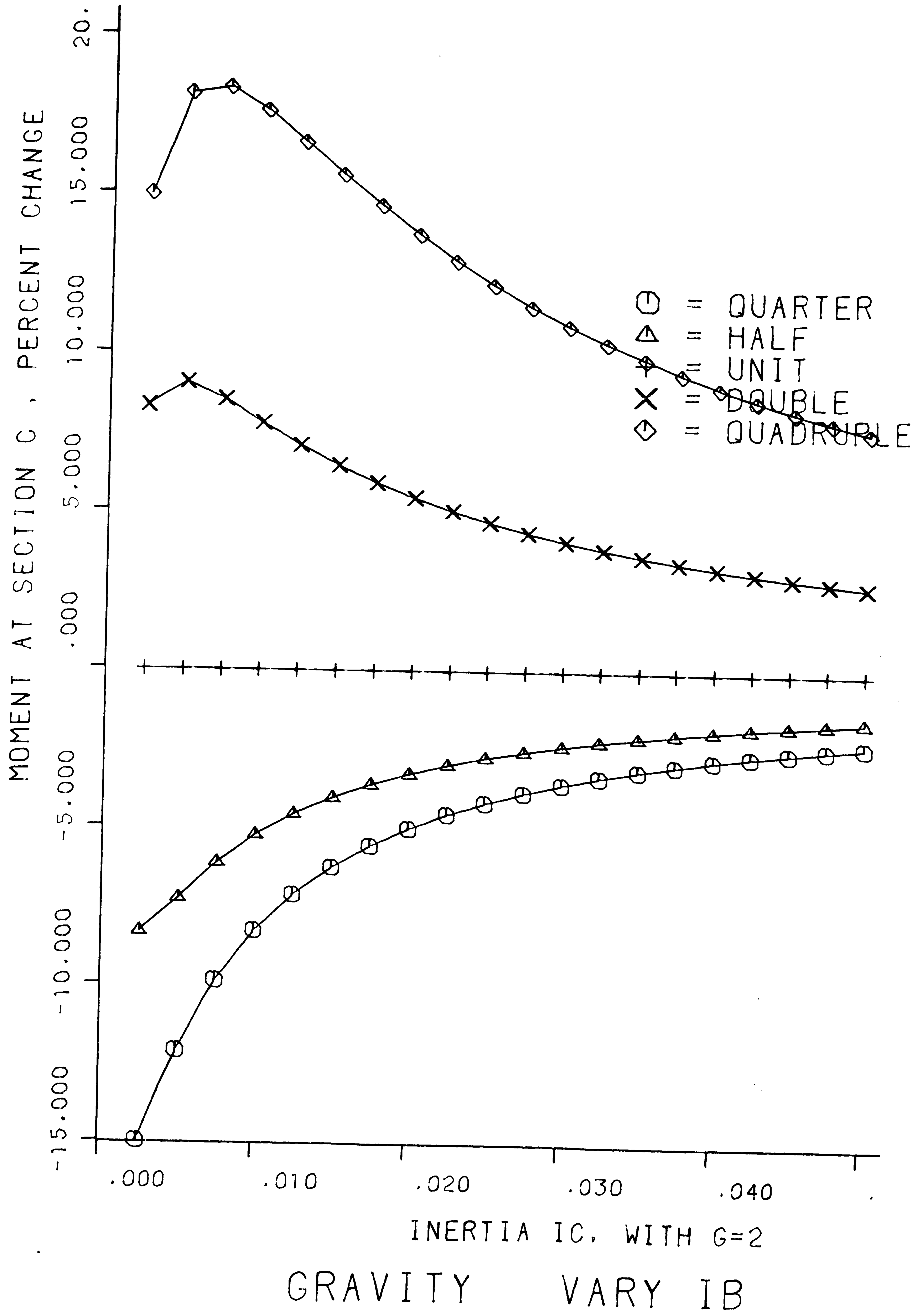


Figure 45: % Mid Mom. vs Col. Inertia & Beam Inertia, G=3, Grav. Load

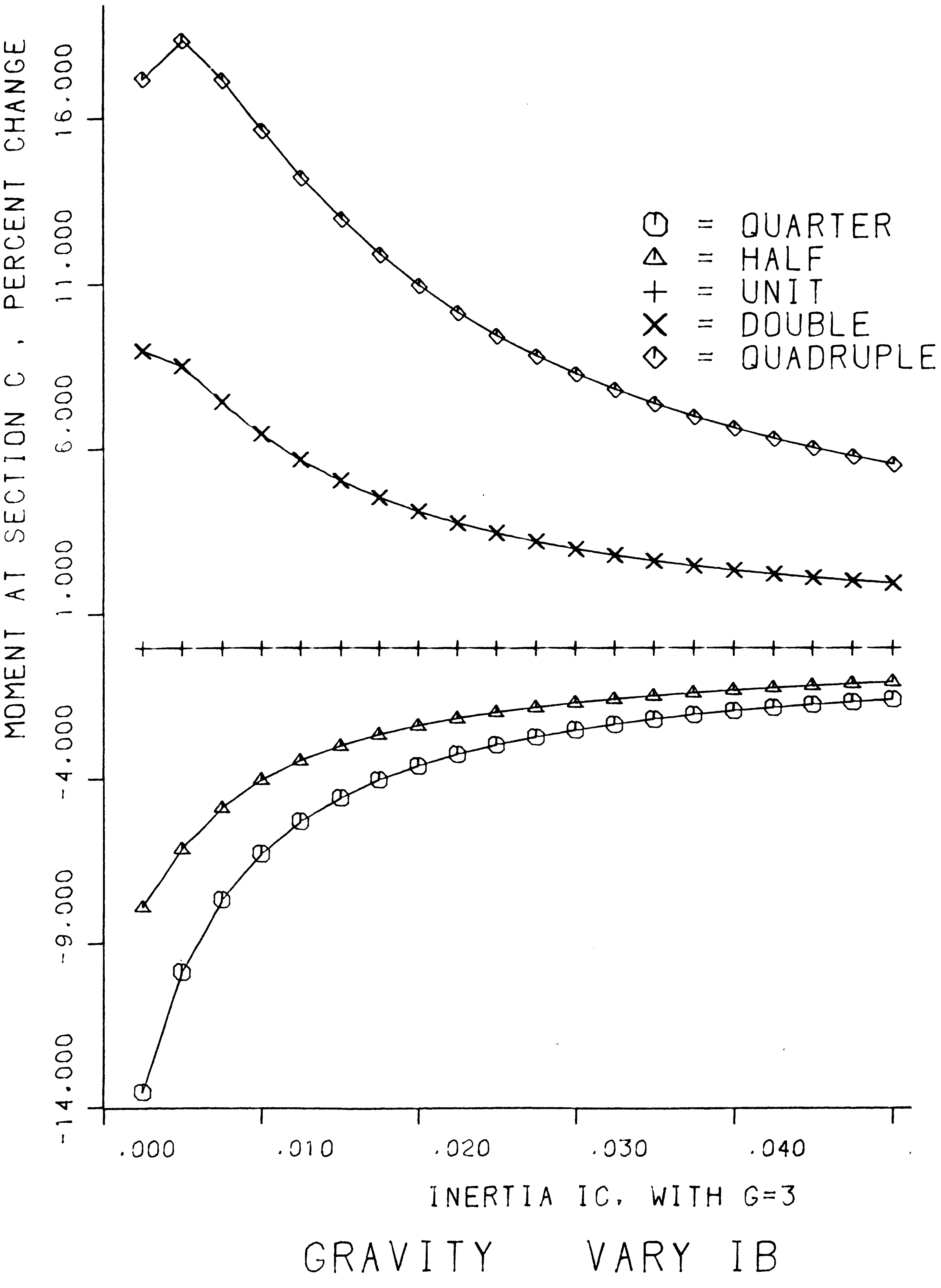


Figure 46: % Mid Mom. vs Col. Inertia & Beam Inertia, G=4, Grav. Load

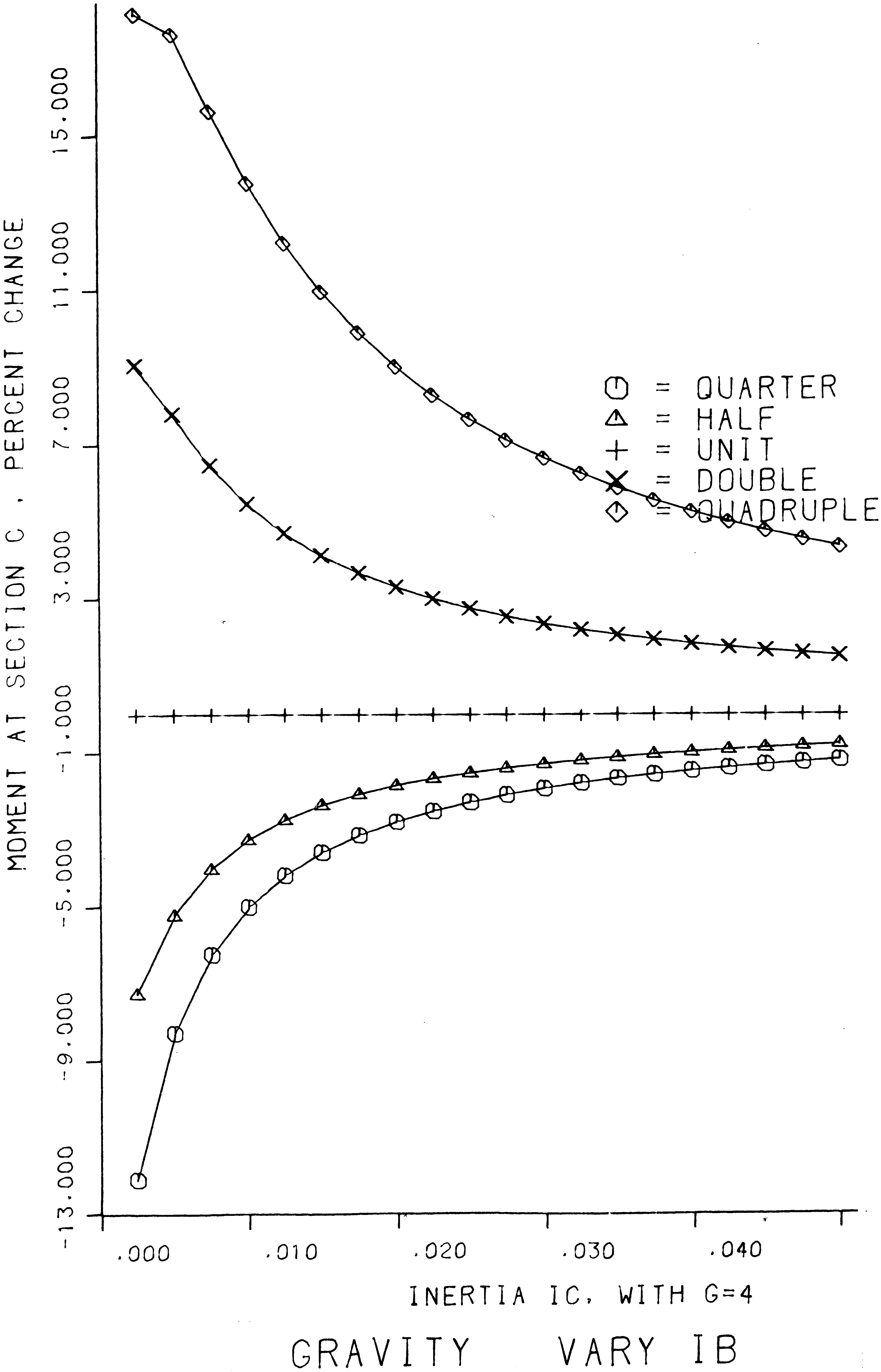


Figure 47: % Mid Mom. vs Col. Inertia & Beam Inertia, G=5, Grav. Load

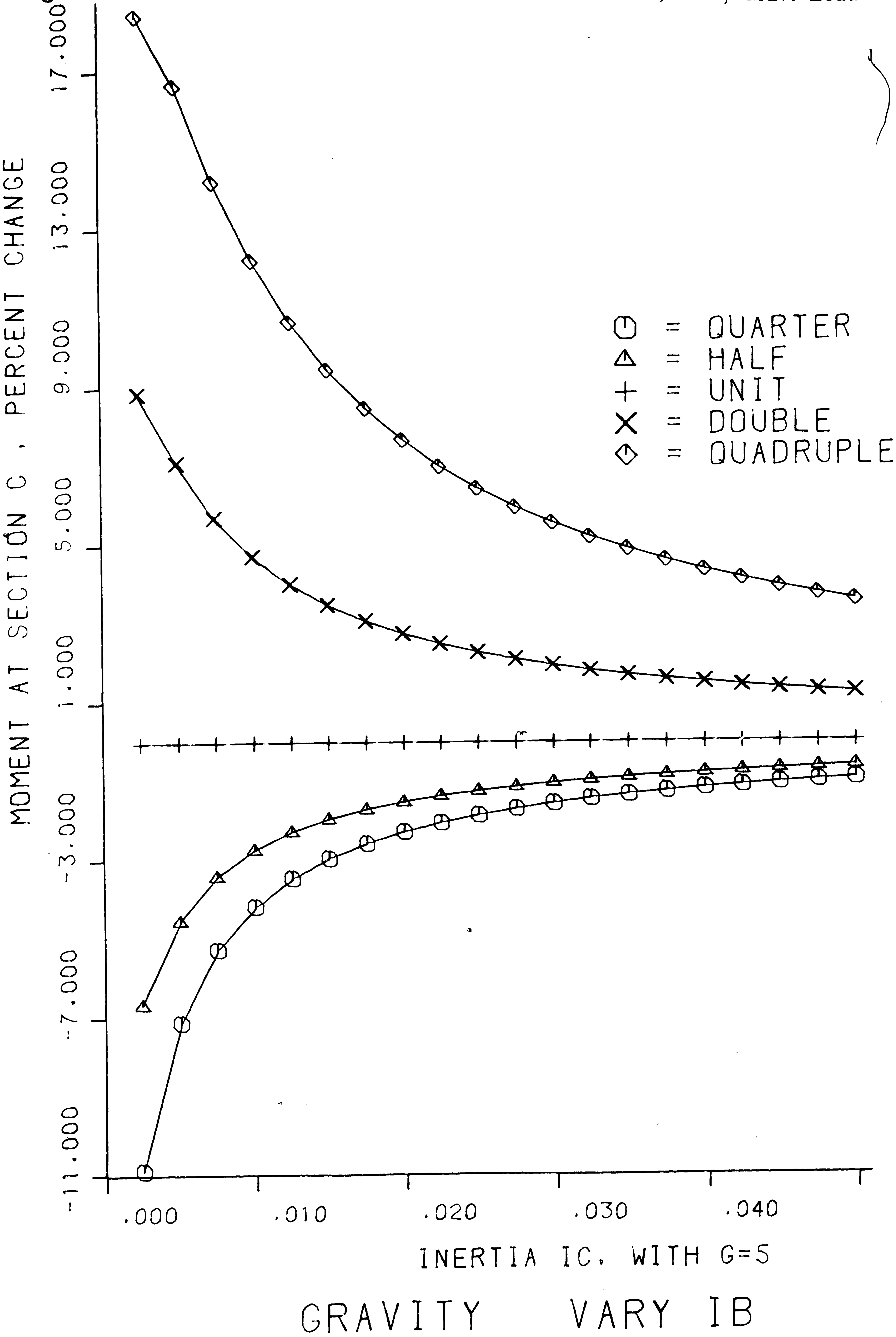


Figure 48: Sway vs Col. Inertia & Beam Inertia, G=1, Grav. Load

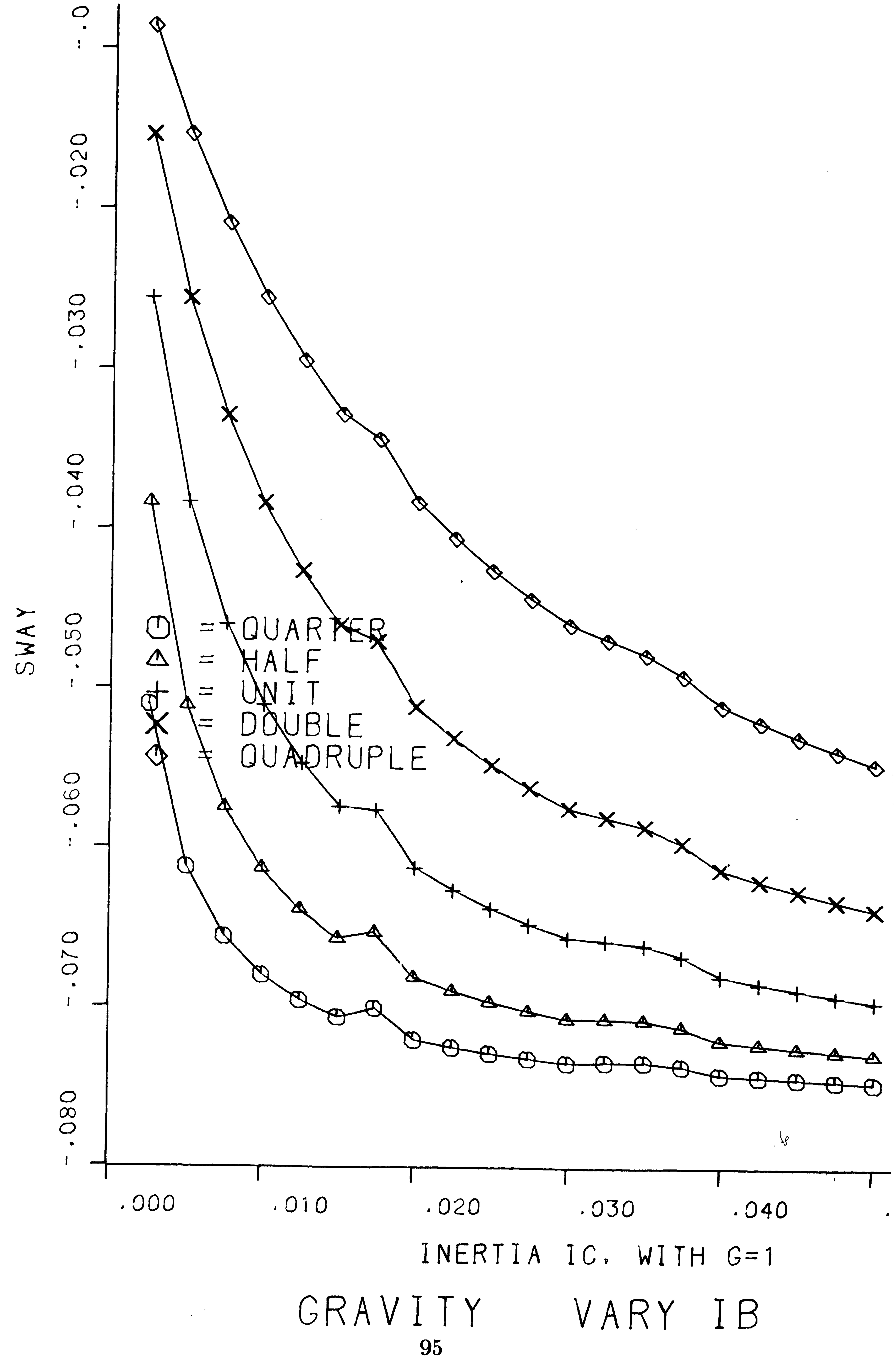


Figure 49: Sway vs Col. Inertia & Beam Inertia, G=2, Grav. Load

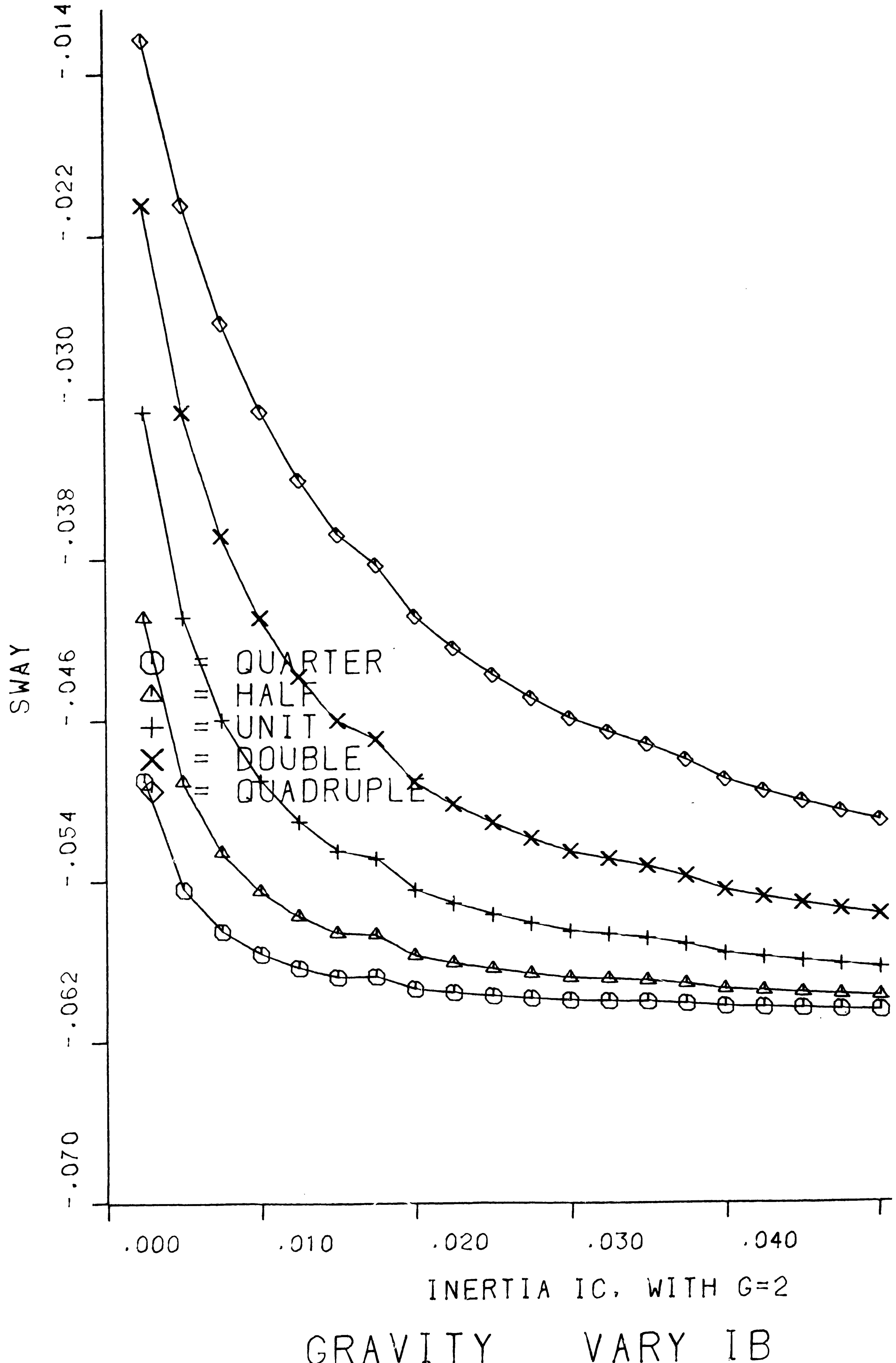


Figure 50: Sway vs Col. Inertia & Beam Inertia, G=3, Grav. Load

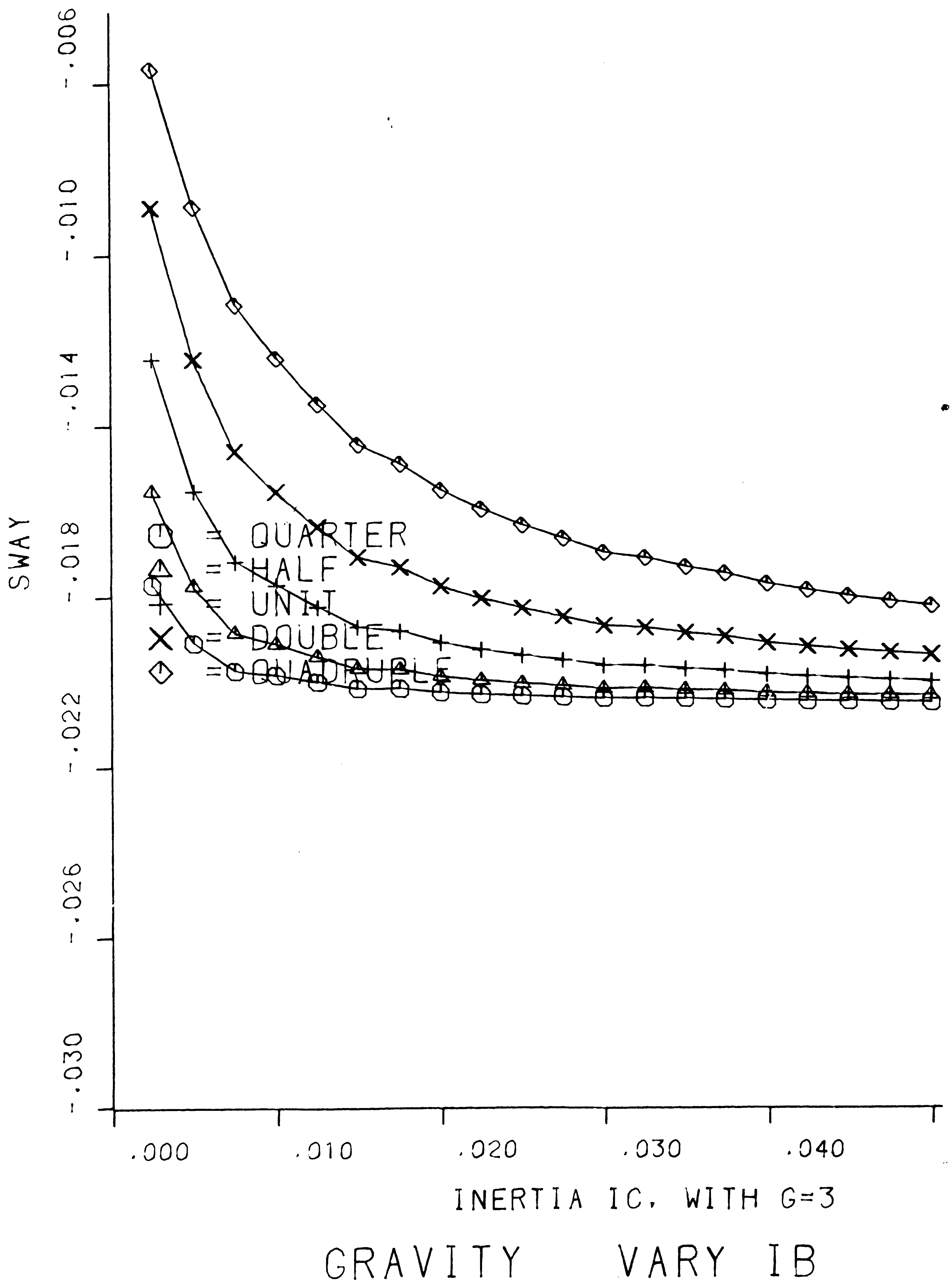


Figure 51: Sway vs Col. Inertia & Beam Inertia, G=4, Grav. Load

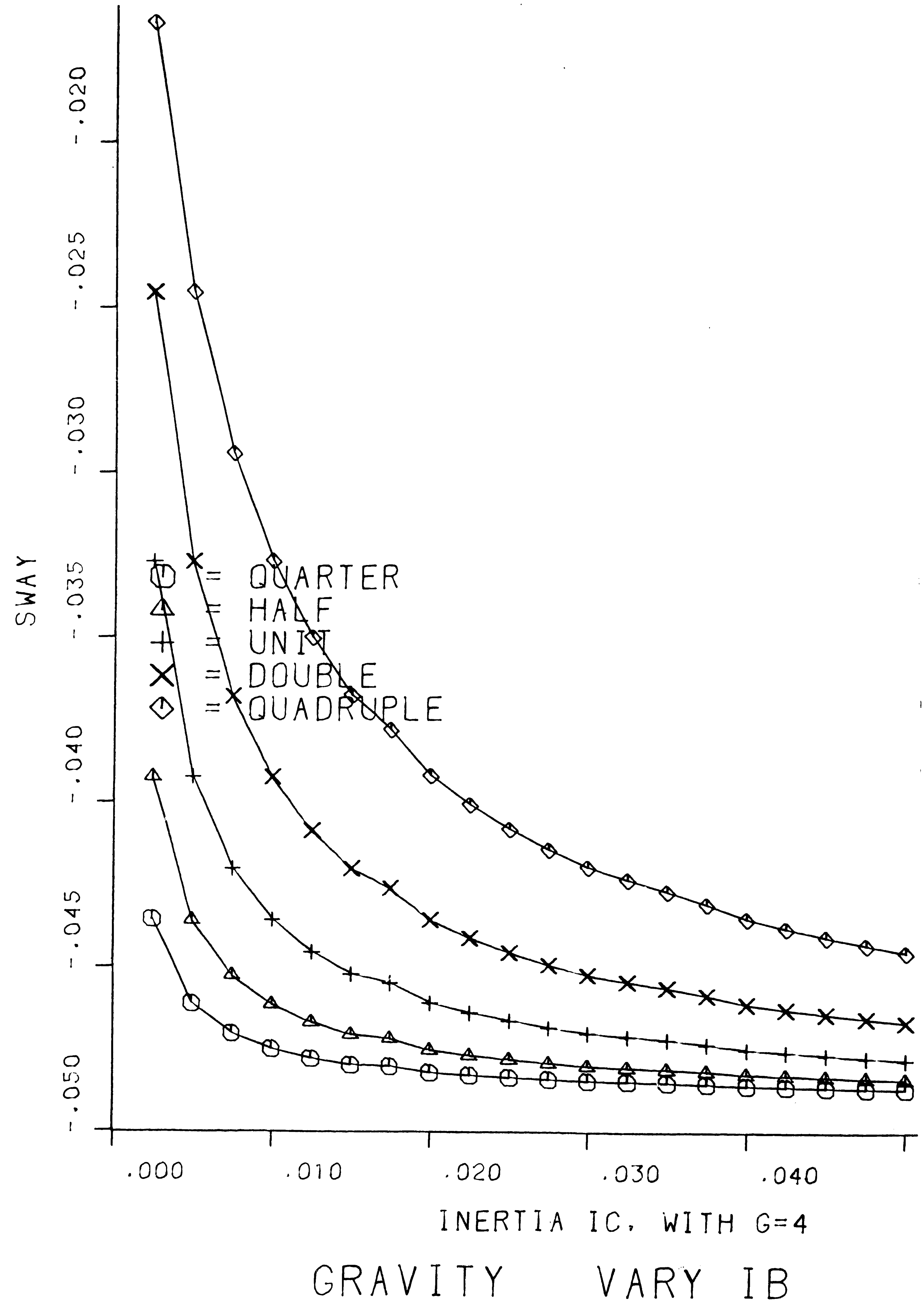


Figure 52: Sway vs Col. Inertia & Beam Inertia, G=5, Grav. Load

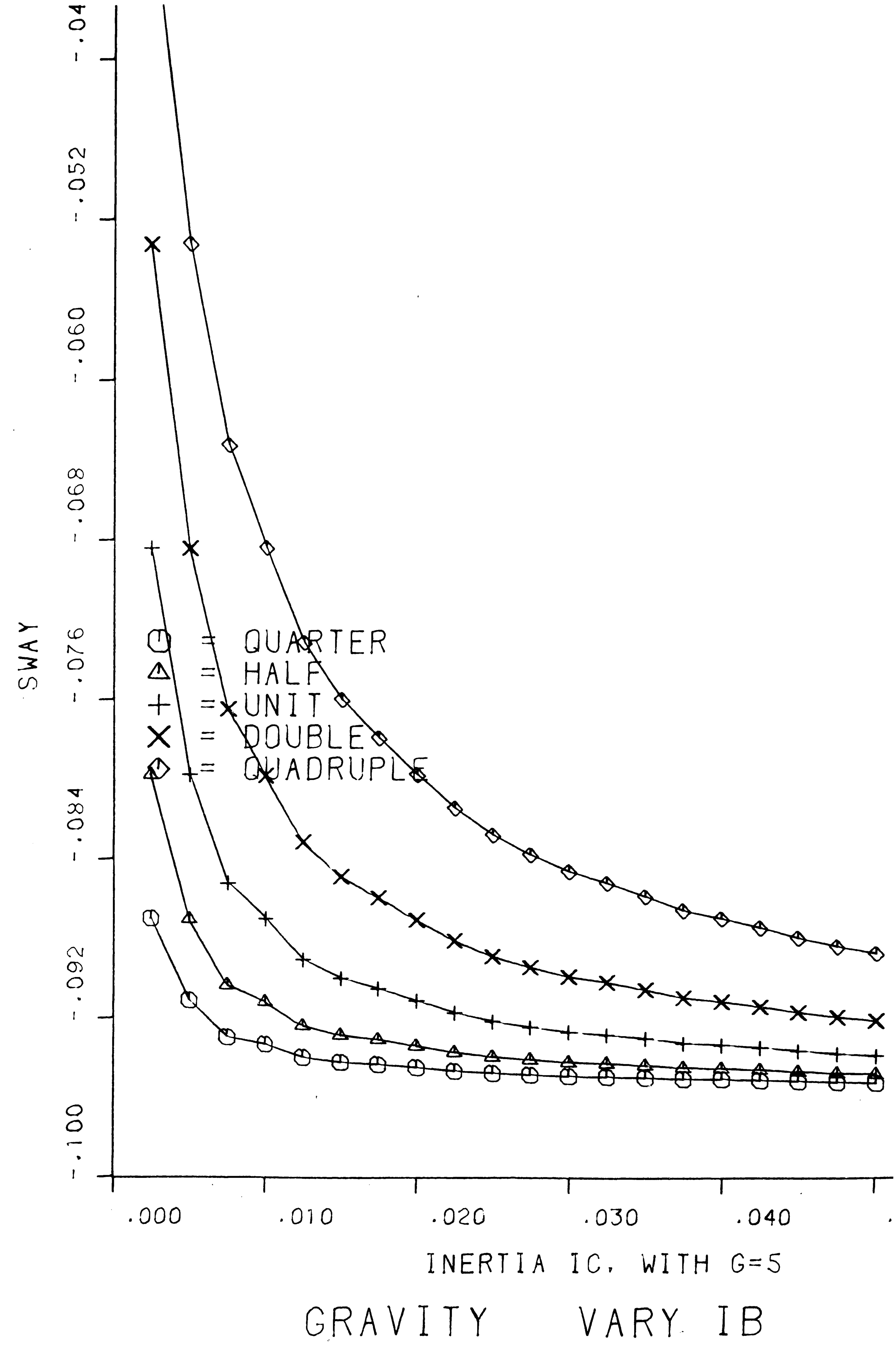


Figure 53: % Sway vs Col. Inertia & Beam Inertia, G=1, Grav. Load

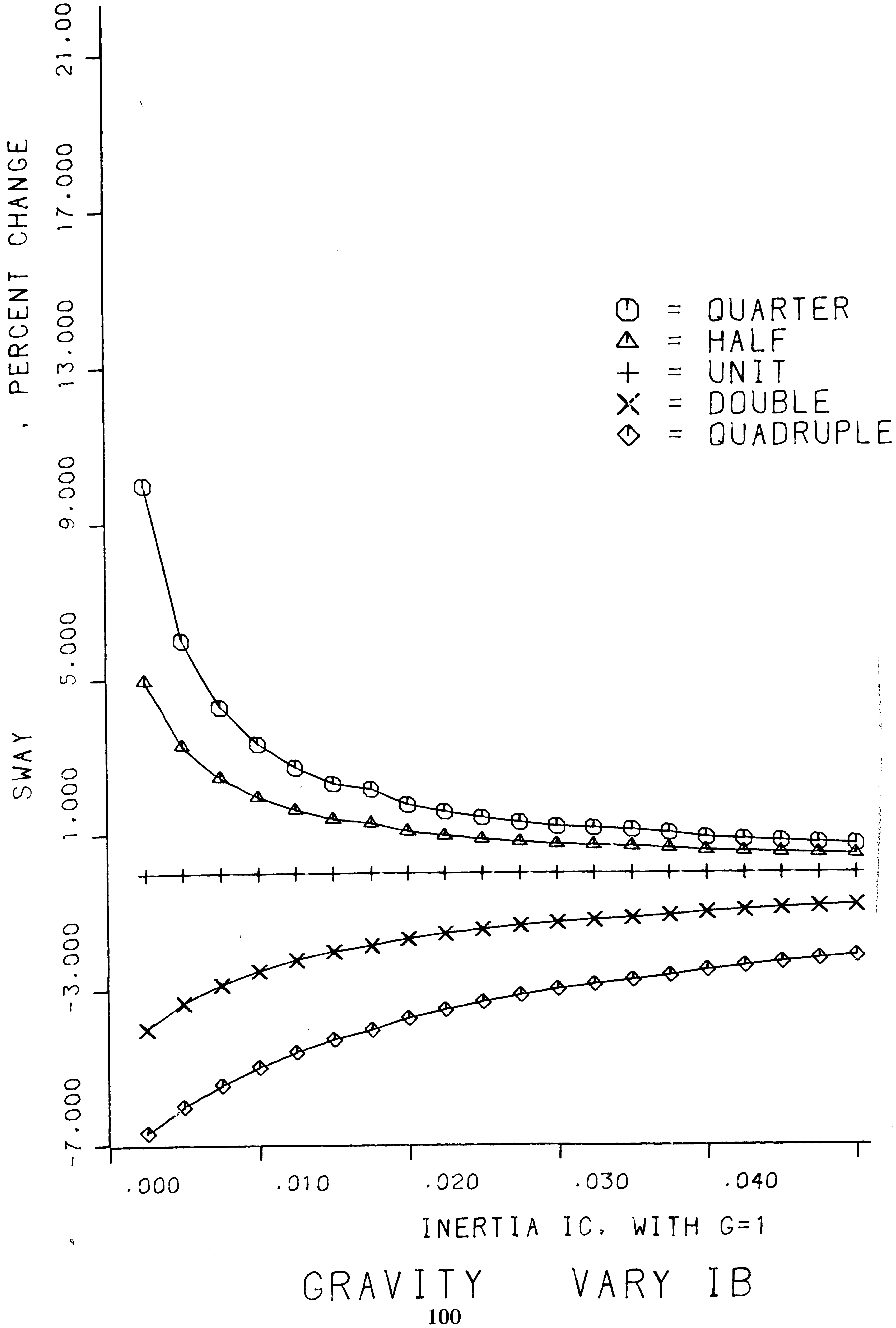


Figure 54: % Sway vs Col. Inertia & Beam Inertia, G=2, Grav. Load

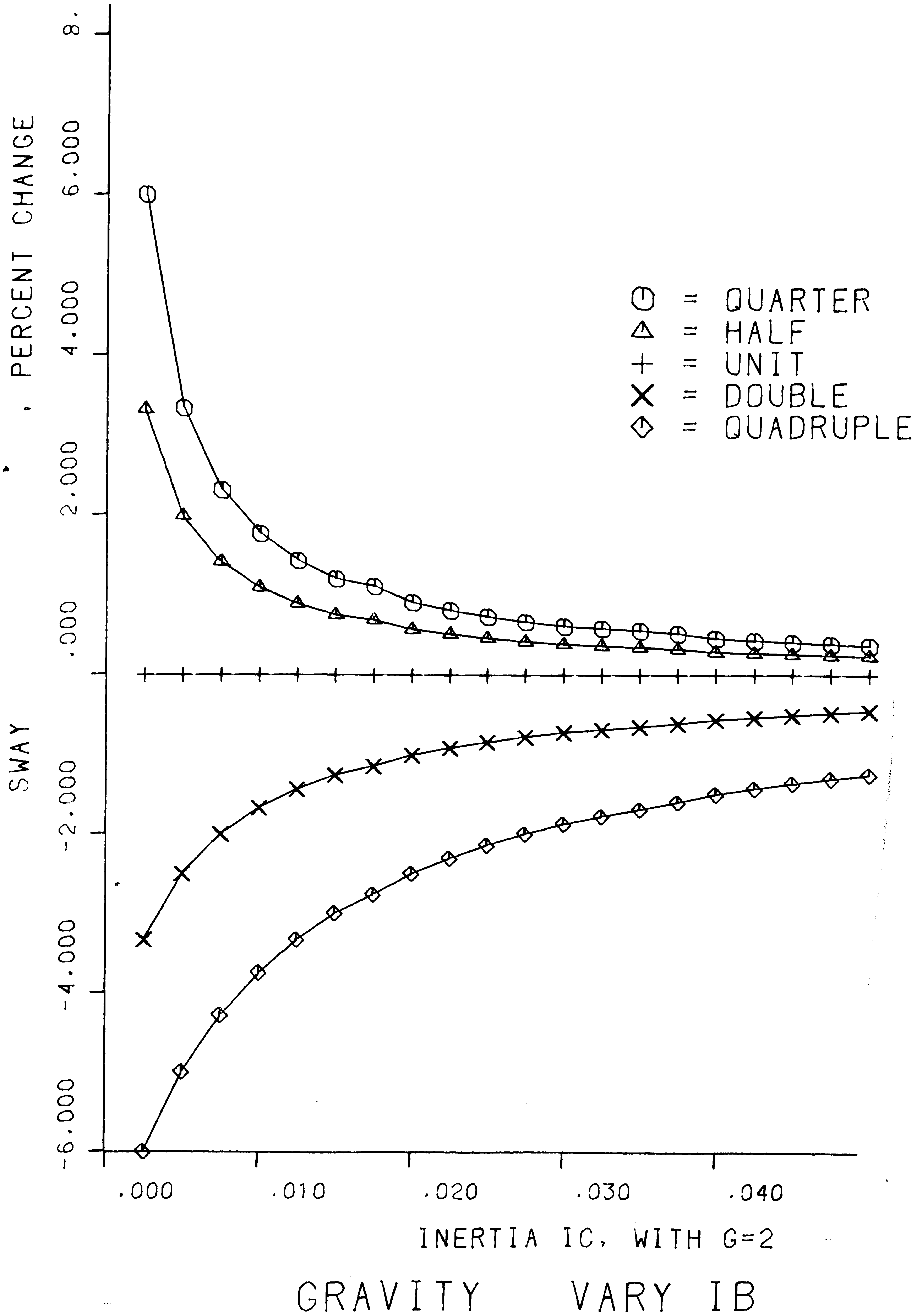


Figure 55: % Sway vs Col. Inertia & Beam Inertia, G=3, Grav. Load

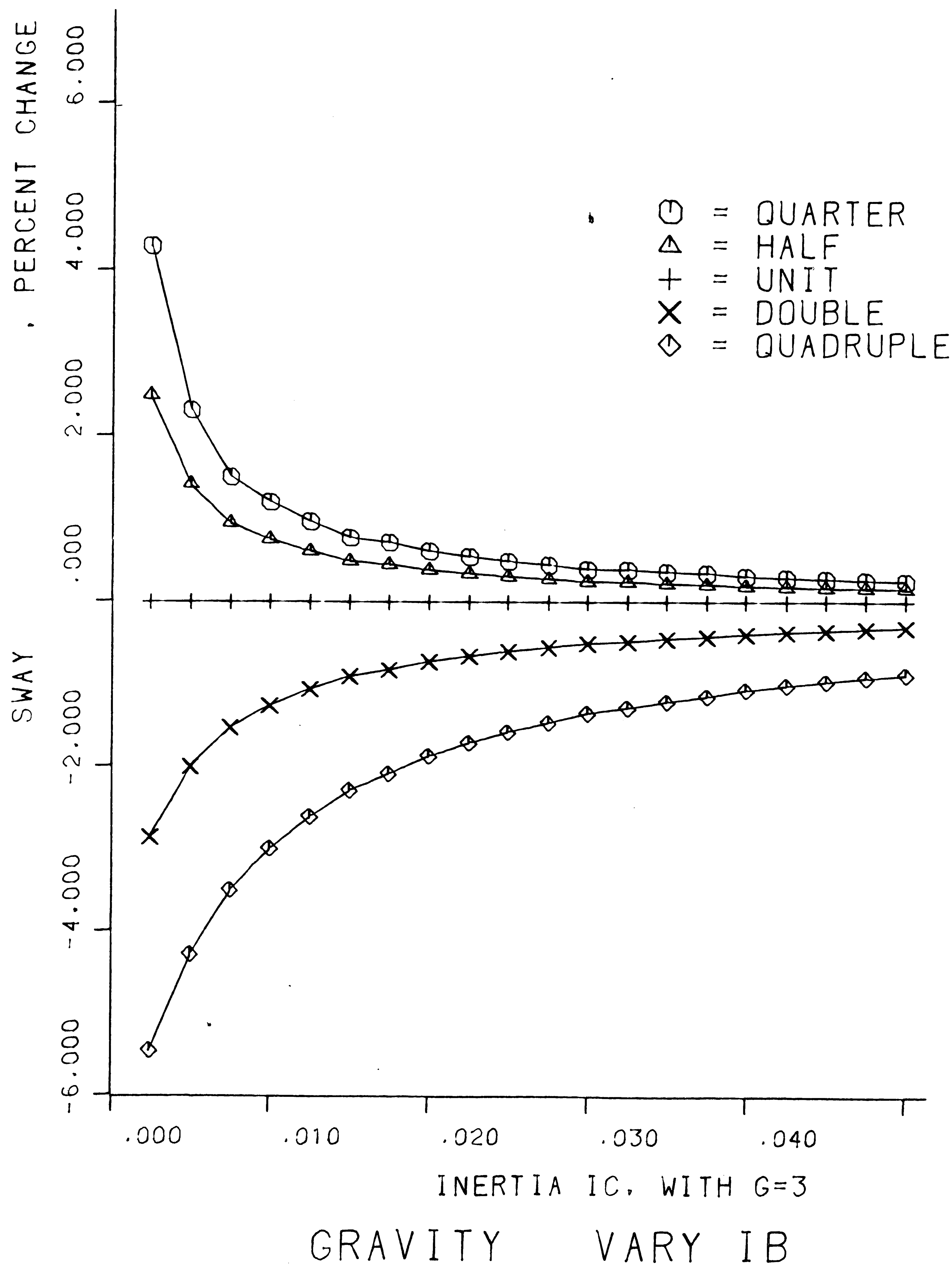


Figure 56: % Sway vs Col. Inertia & Beam Inertia, G=4, Grav. Load

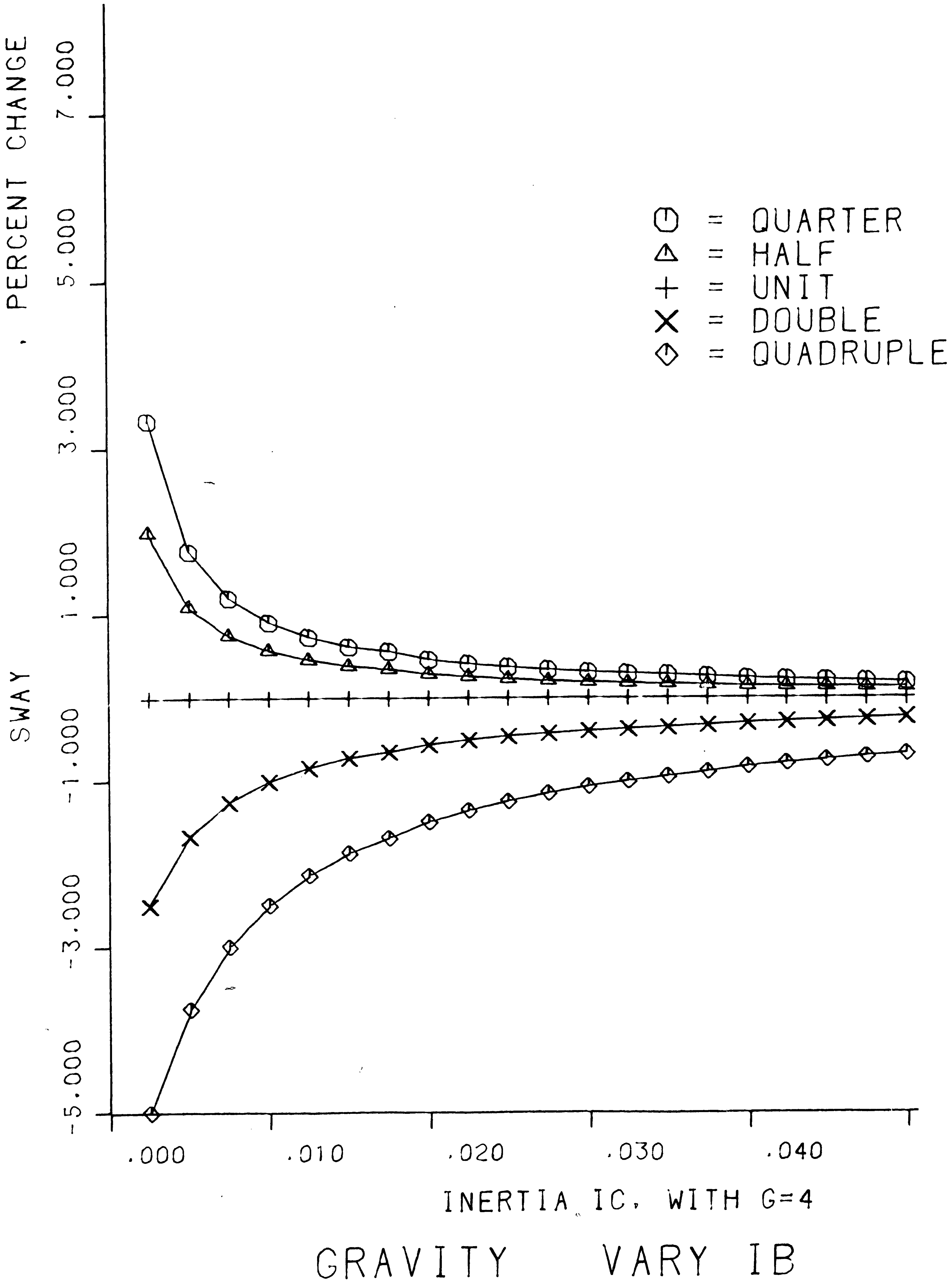


Figure 57: % Sway vs Col. Inertia & Beam Inertia, G=5, Grav. Load

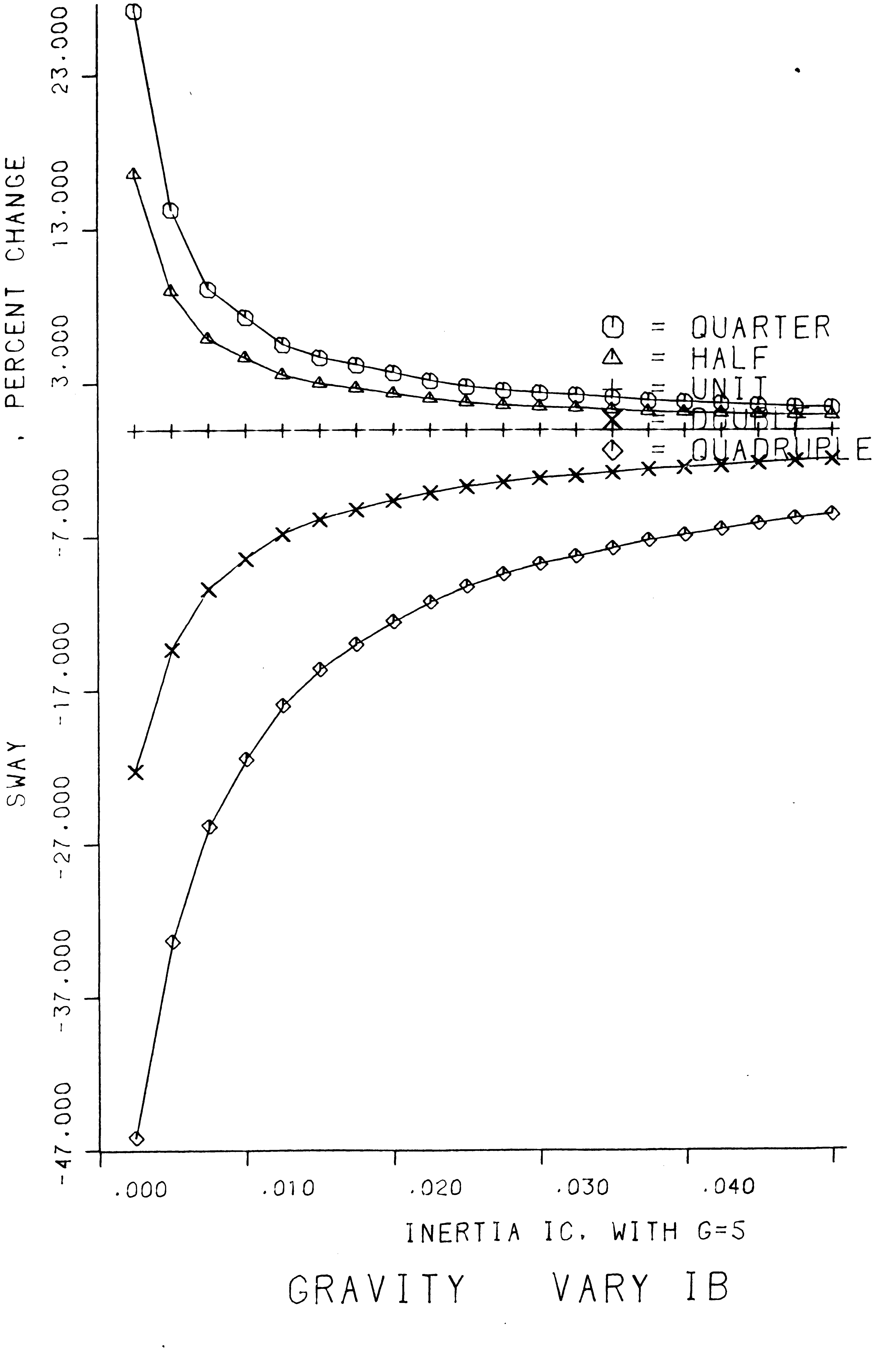


Figure 58: Base Mom. vs Col. Inertia & Beam Inertia, G=1, Lat. Load

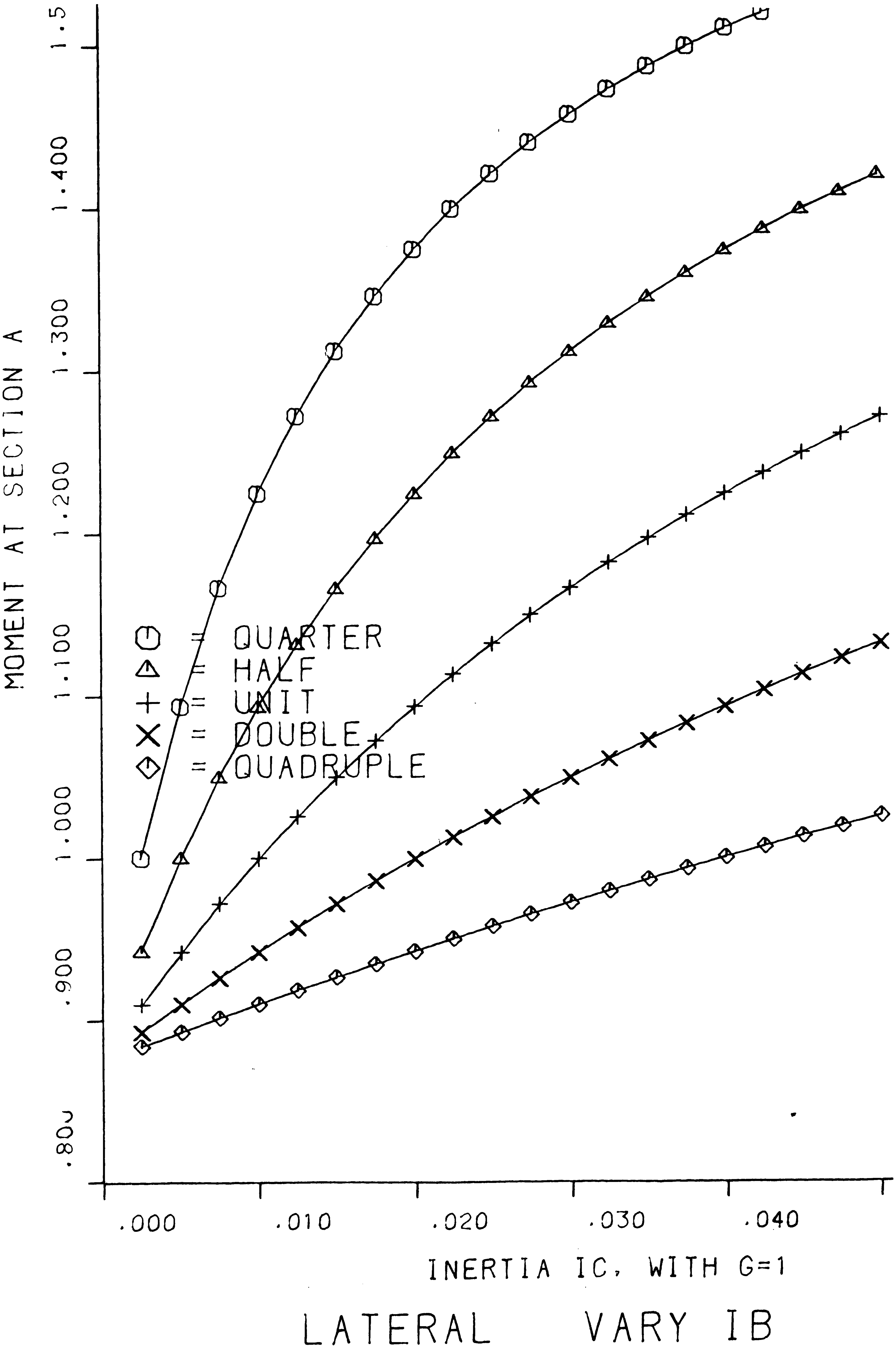


Figure 59: Base Mom. vs Col. Inertia & Beam Inertia, G=2, Lat. Load

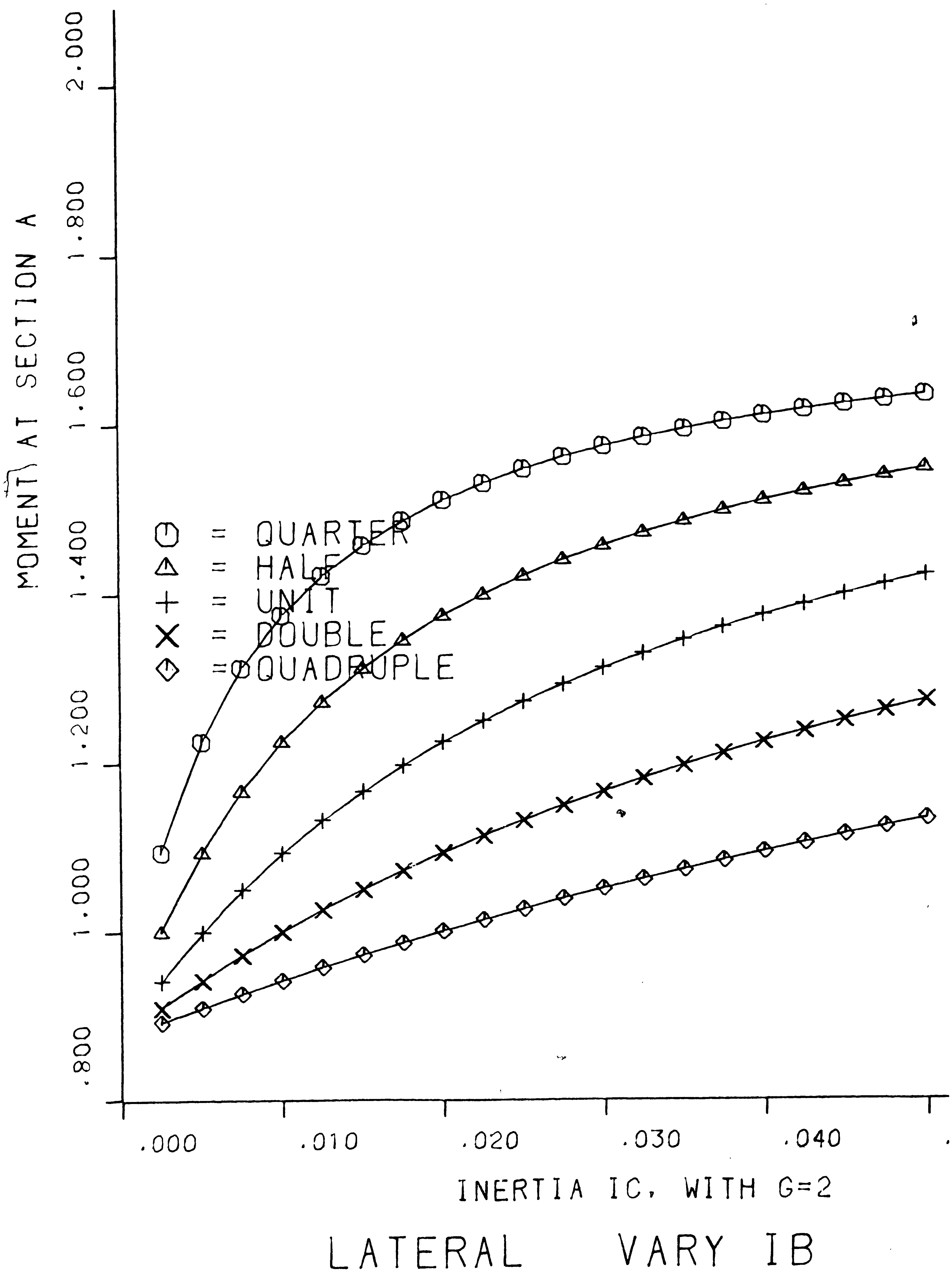


Figure 60: Base Mom. vs Col. Inertia & Beam Inertia, G=3, Lat. Load

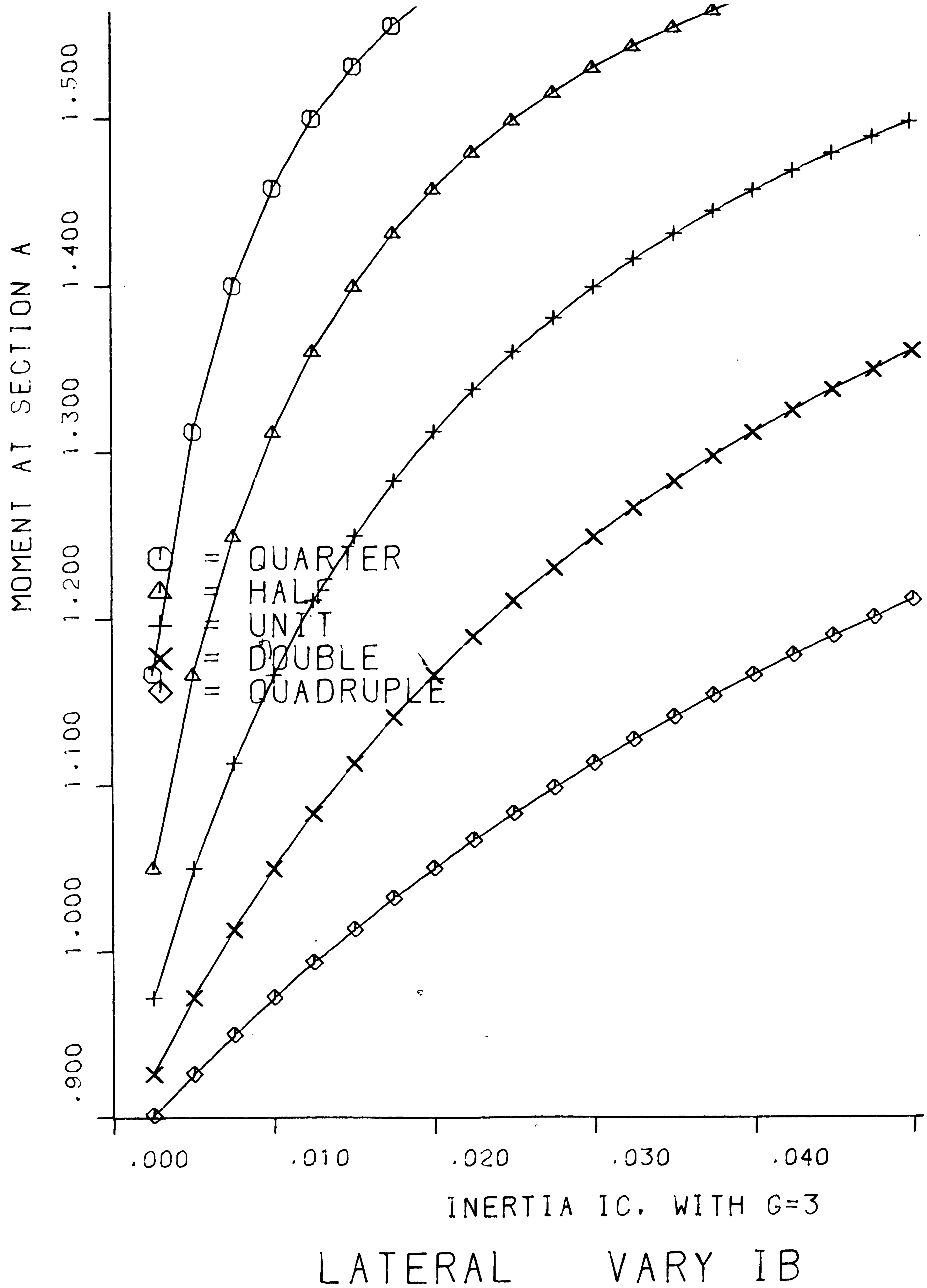


Figure 61: Base Mom. vs Col. Inertia & Beam Inertia, G=4, Lat. Load

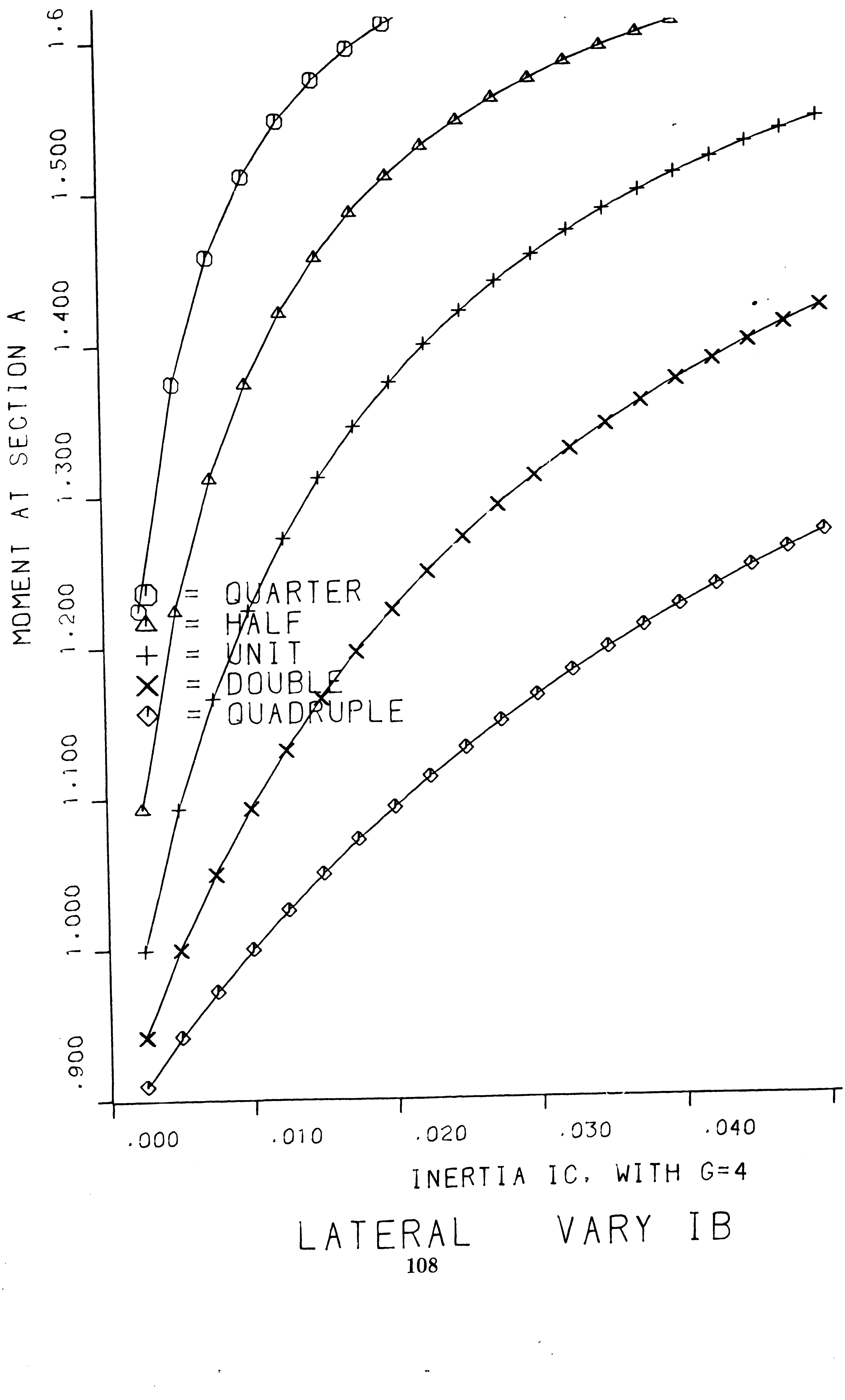


Figure 62: Base Mom. vs Col. Inertia & Beam Inertia, G=5, Lat. Load

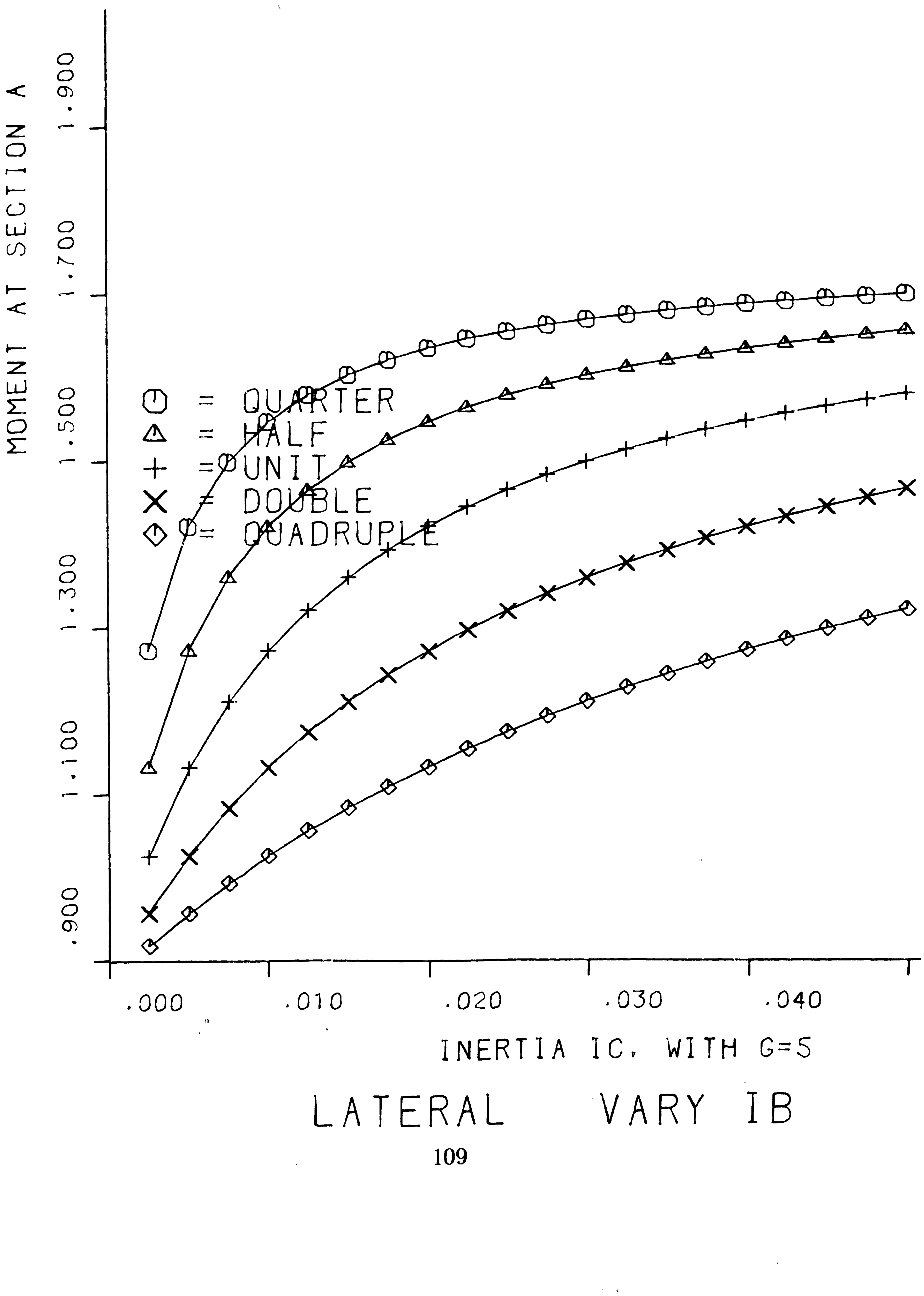


Figure 63: % Base Mom. vs Col. Inertia & Beam Inertia, G=1, Lat. Load

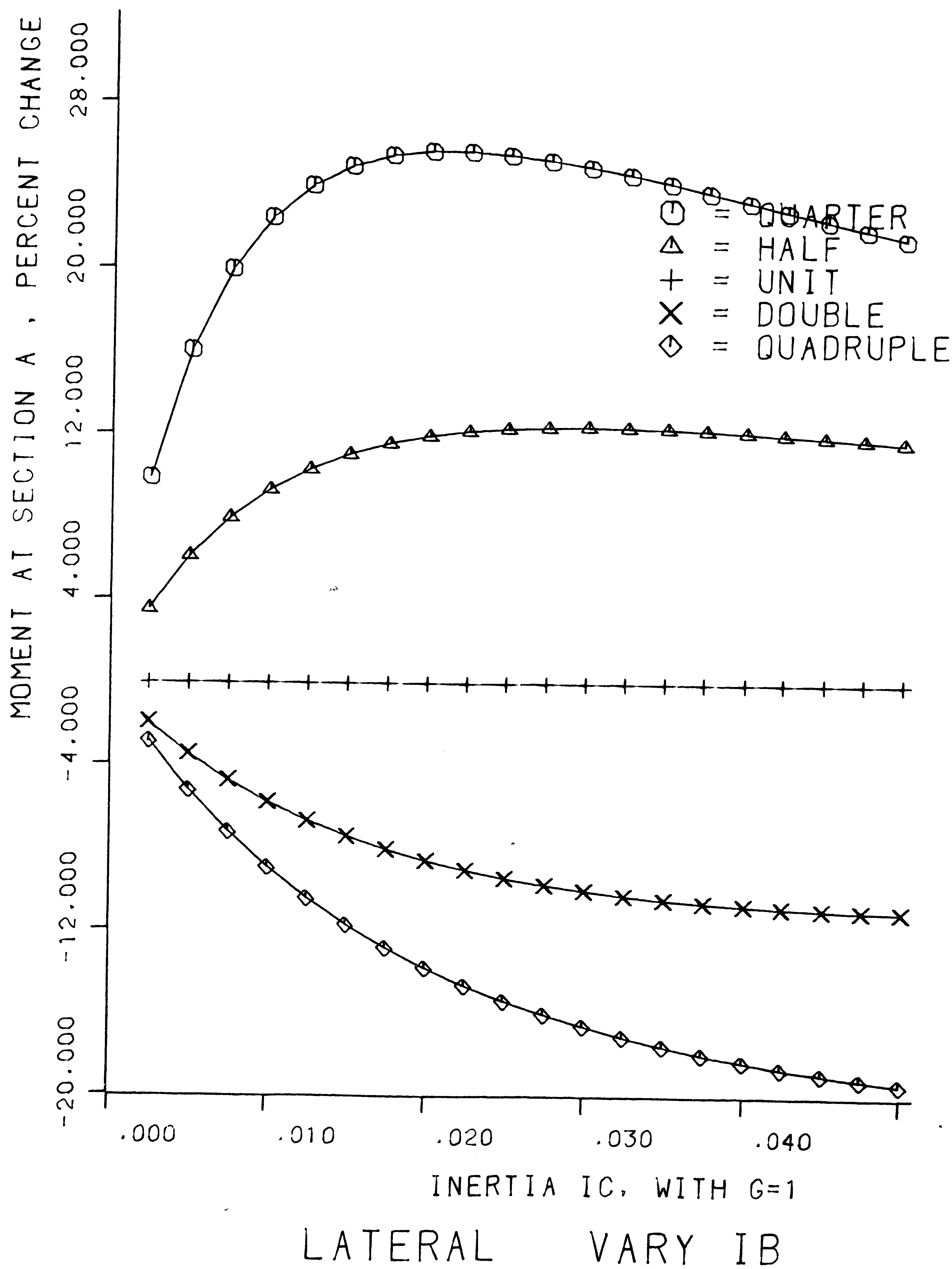


Figure 64: % Base Mom. vs Col. Inertia & Beam Inertia, G=2, Lat. Load

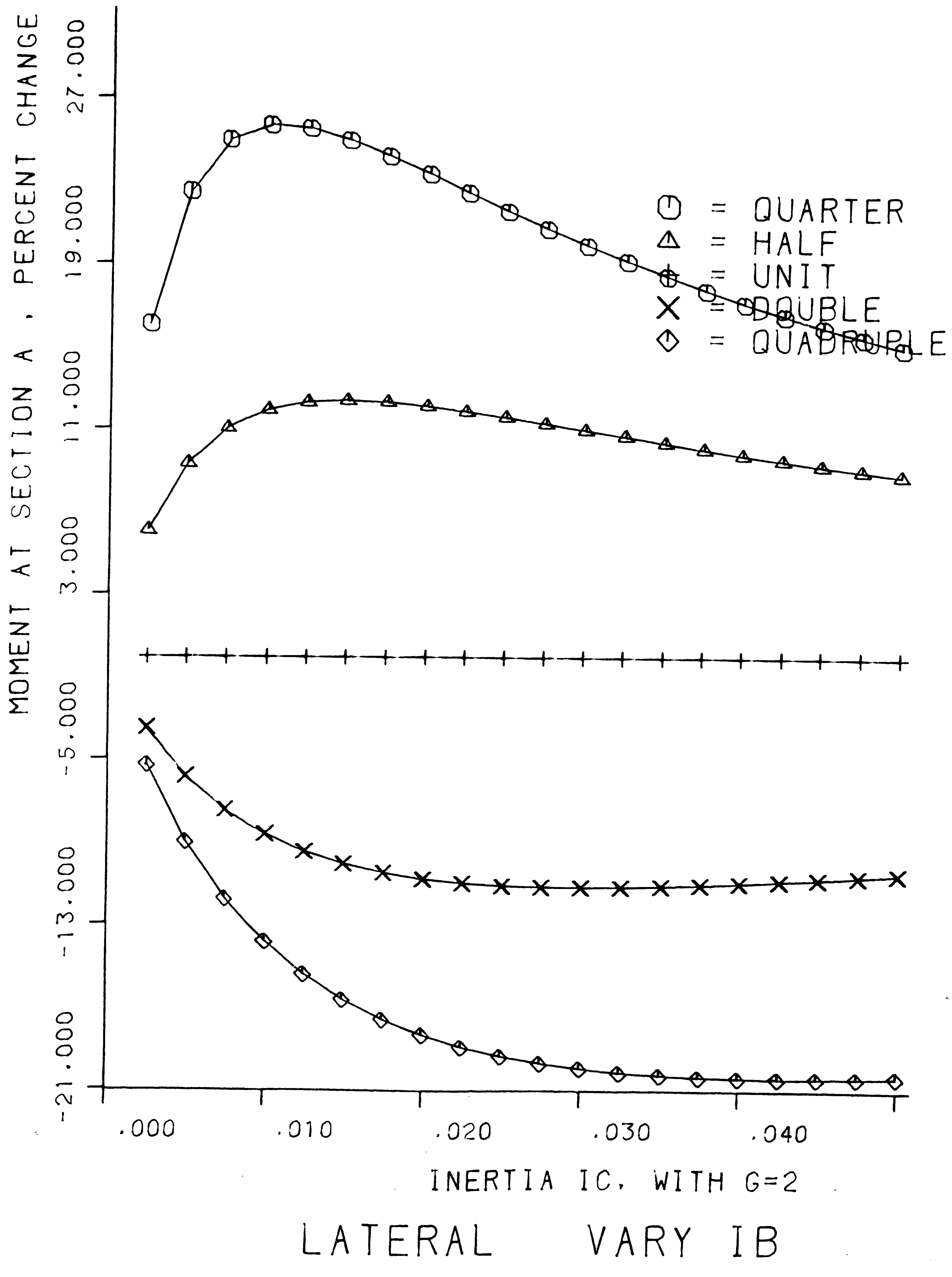


Figure 65: % Base Mom. vs Col. Inertia & Beam Inertia, G=3, Lat. Load

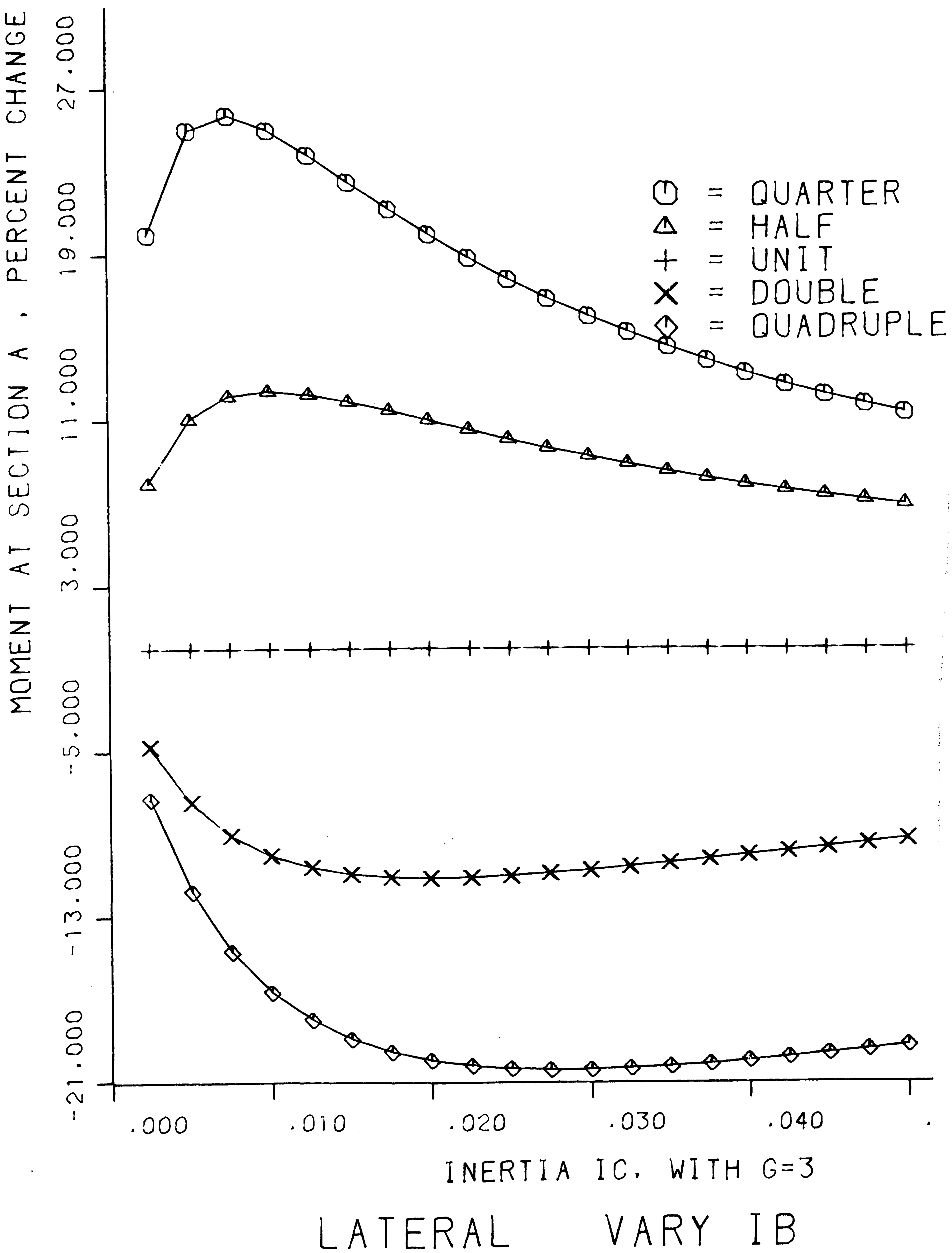


Figure 66: % Base Mom. vs Col. Inertia & Beam Inertia, G=4, Lat. Load

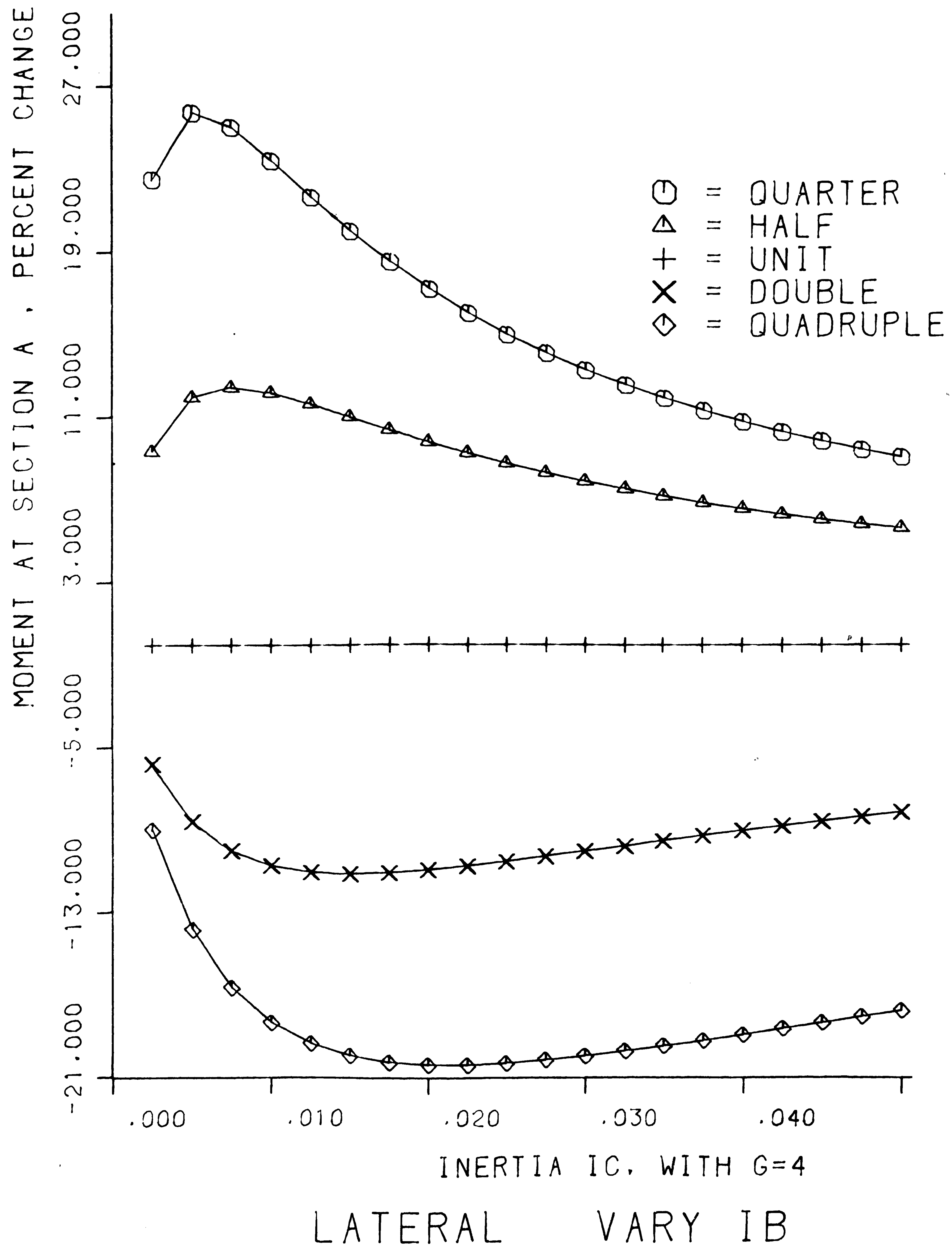


Figure 67: % Base Mom. vs Col. Inertia & Beam Inertia, G=5, Lat. Load

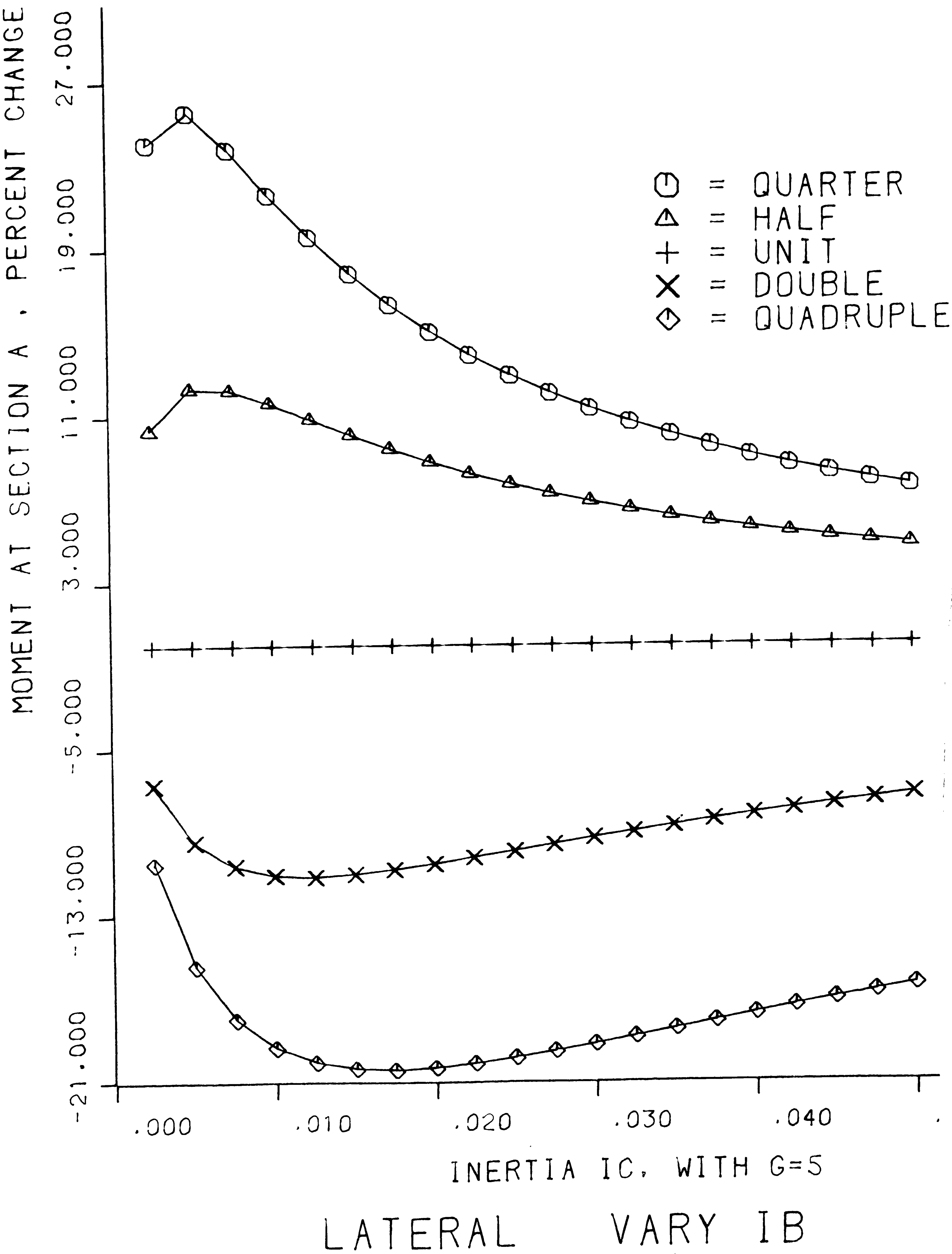


Figure 68: Corner Mom. vs Col. Inertia & Beam Inertia, G=1, Lat. Load

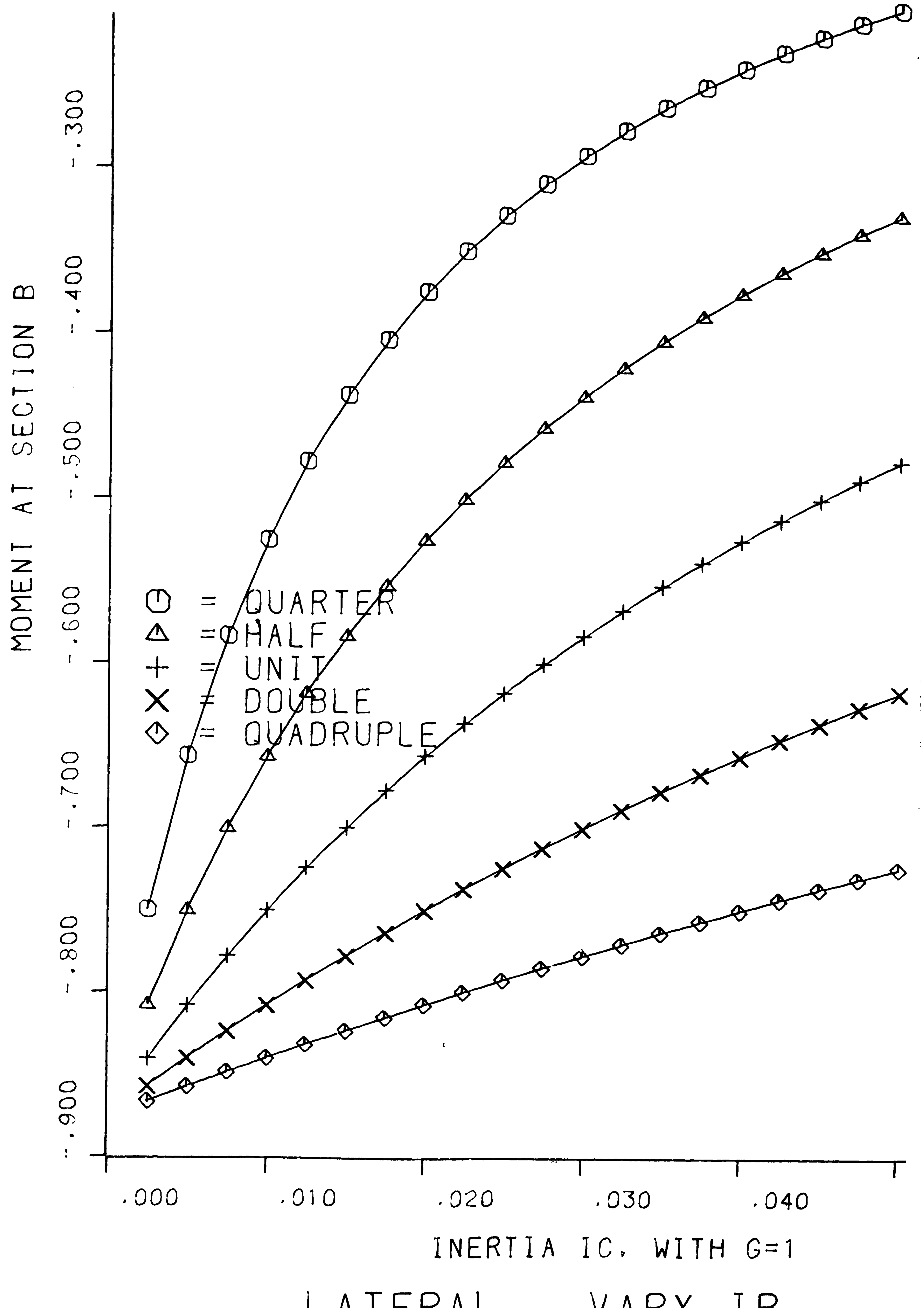


Figure 69: Corner Mom. vs Col. Inertia & Beam Inertia, G=2, Lat. Load

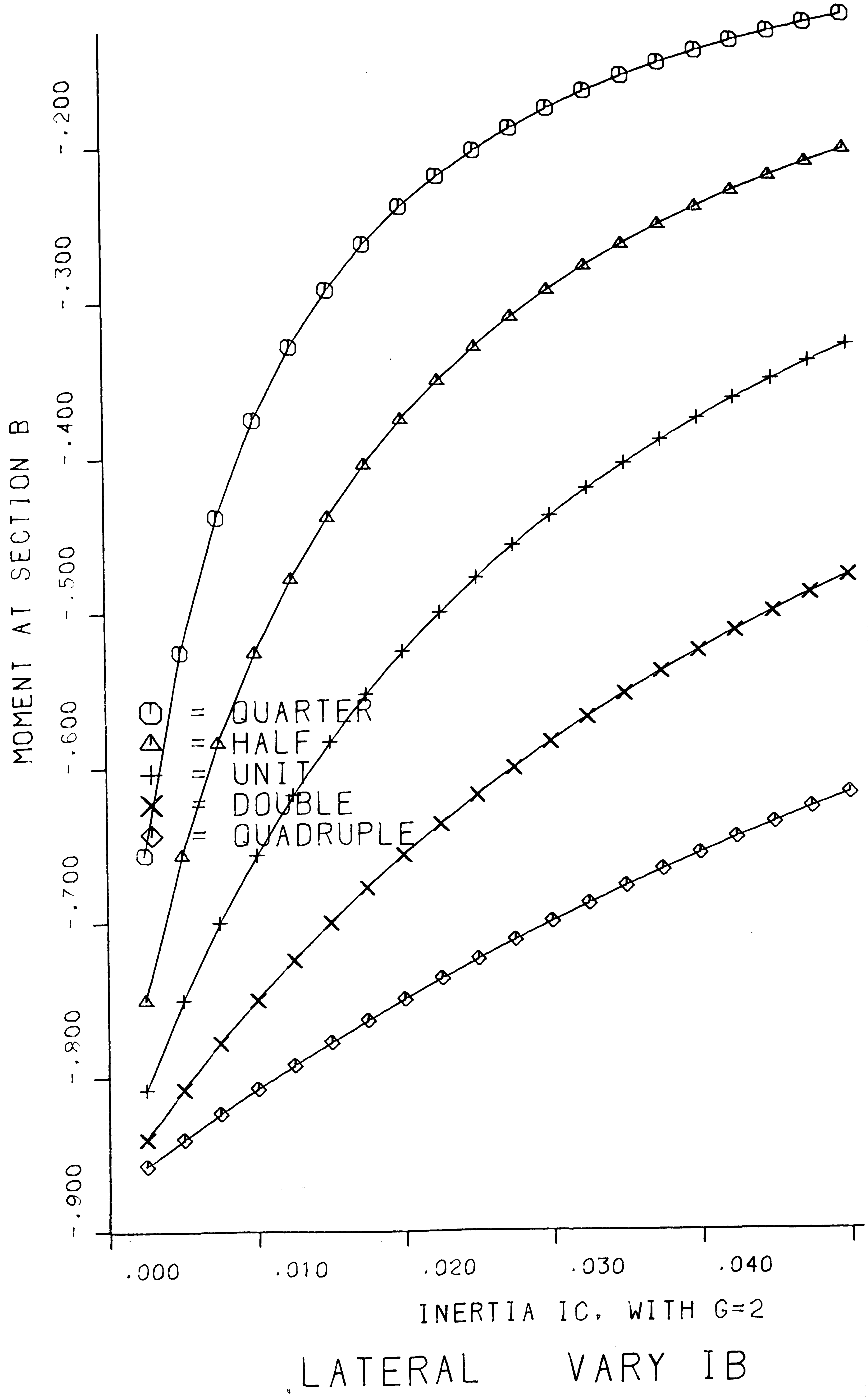


Figure 70: Corner Mom. vs Col. Inertia & Beam Inertia, G=3, Lat. Load

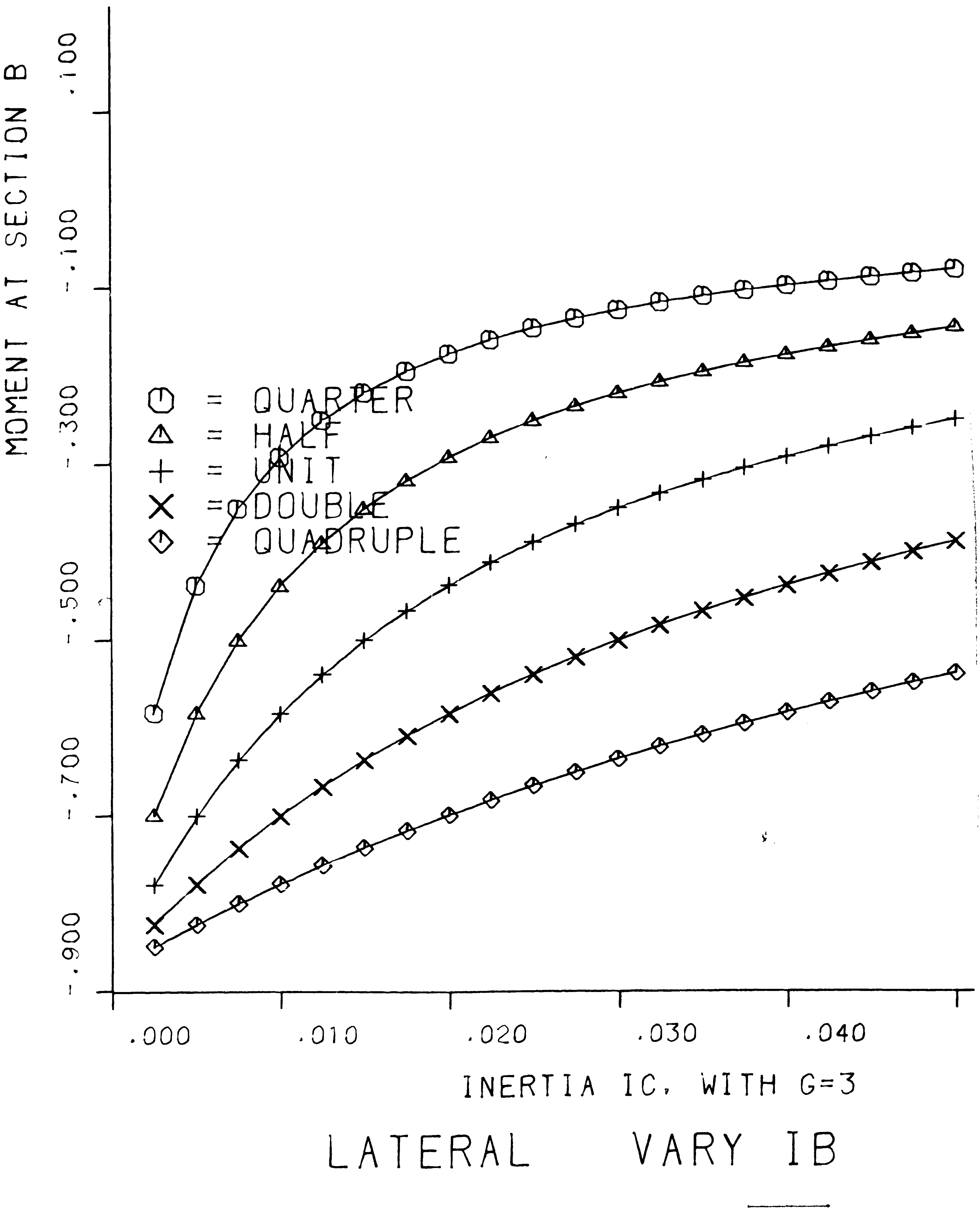


Figure 71: Corner Mom. vs Col. Inertia & Beam Inertia, G=4, Lat. Load

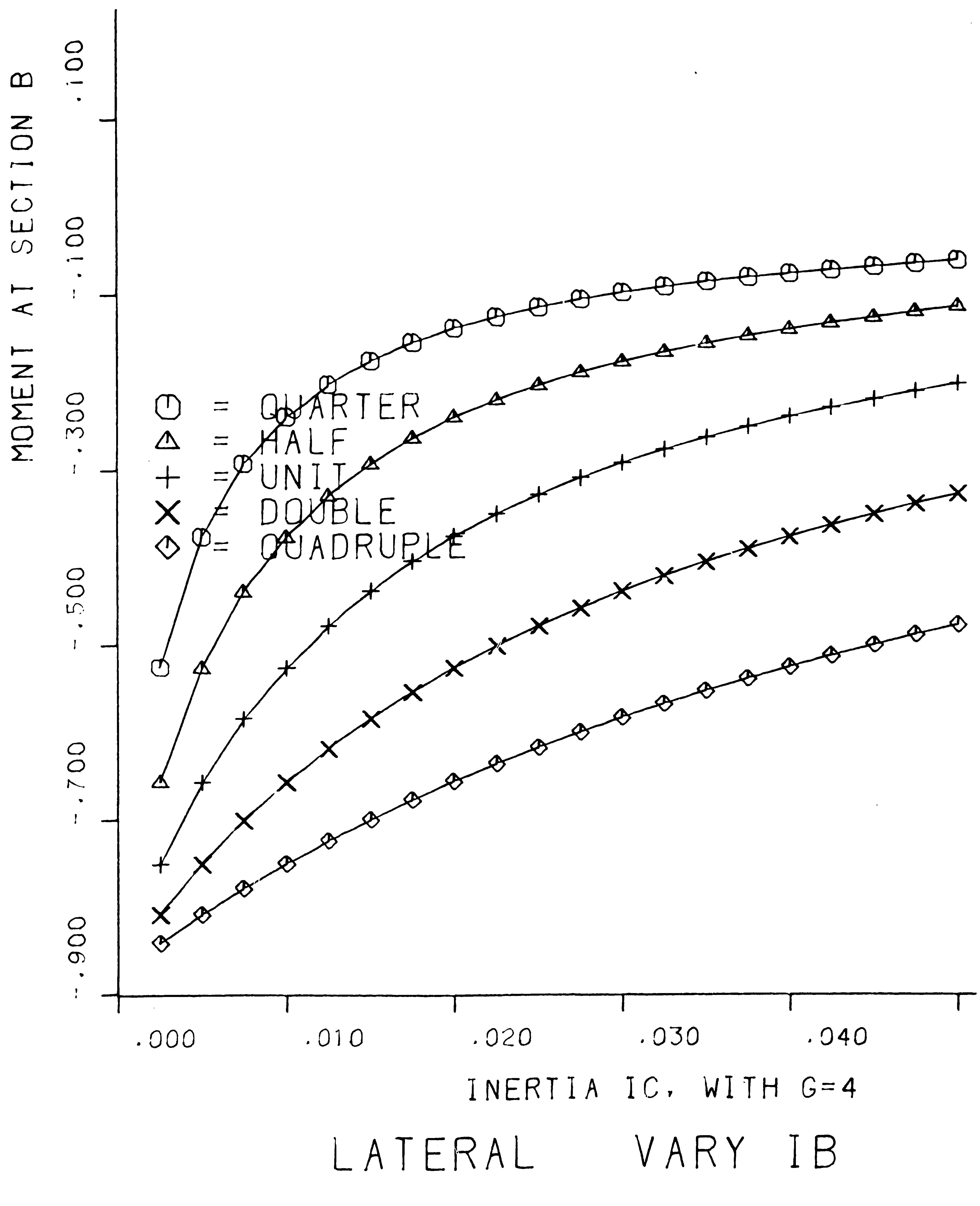


Figure 72: Corner Mom. vs Col. Inertia & Beam Inertia, G=5, Lat. Load

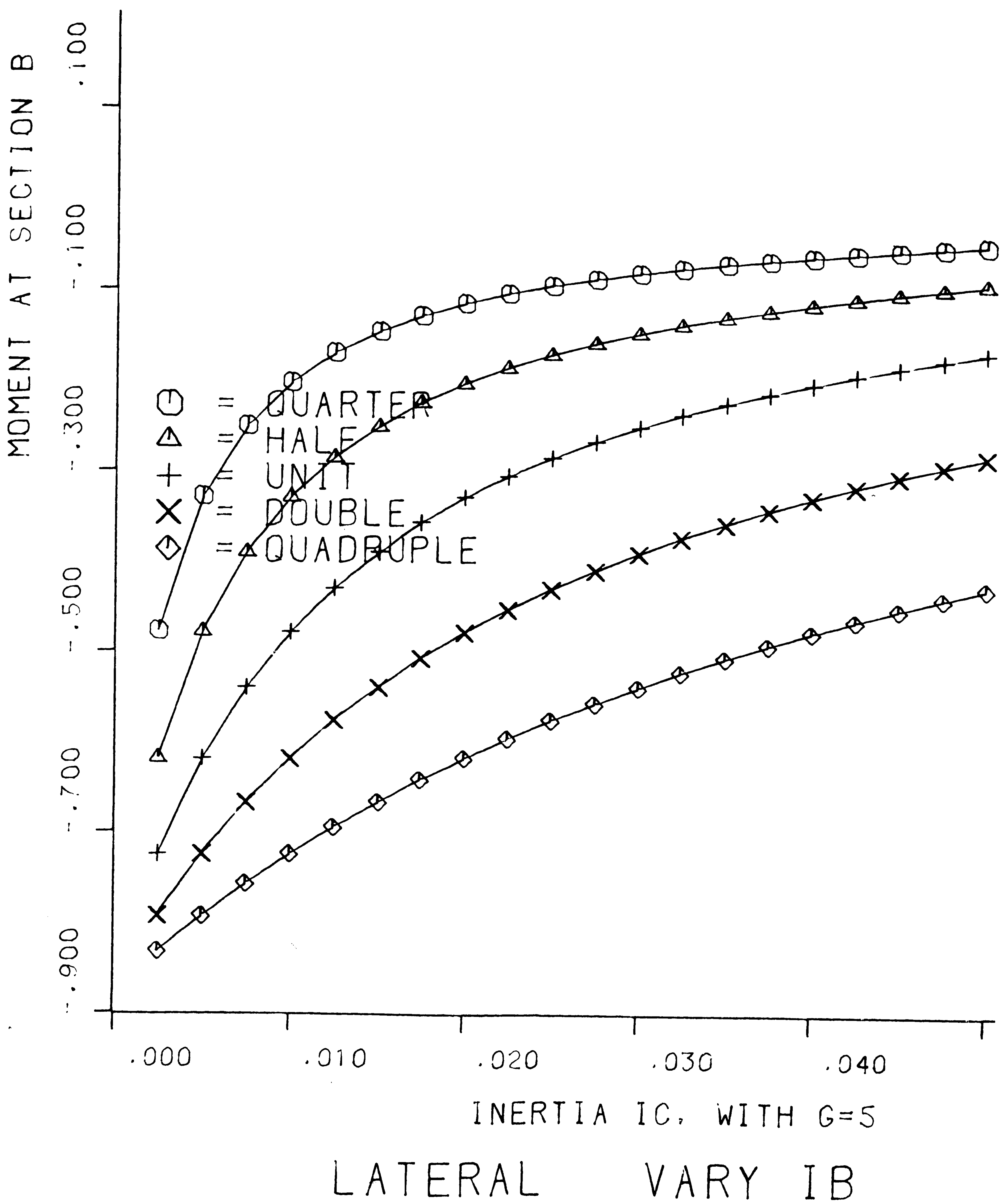


Figure 73: % Corner Mom. vs Col. Inertia & Beam Inertia, G=1, Lat. Load

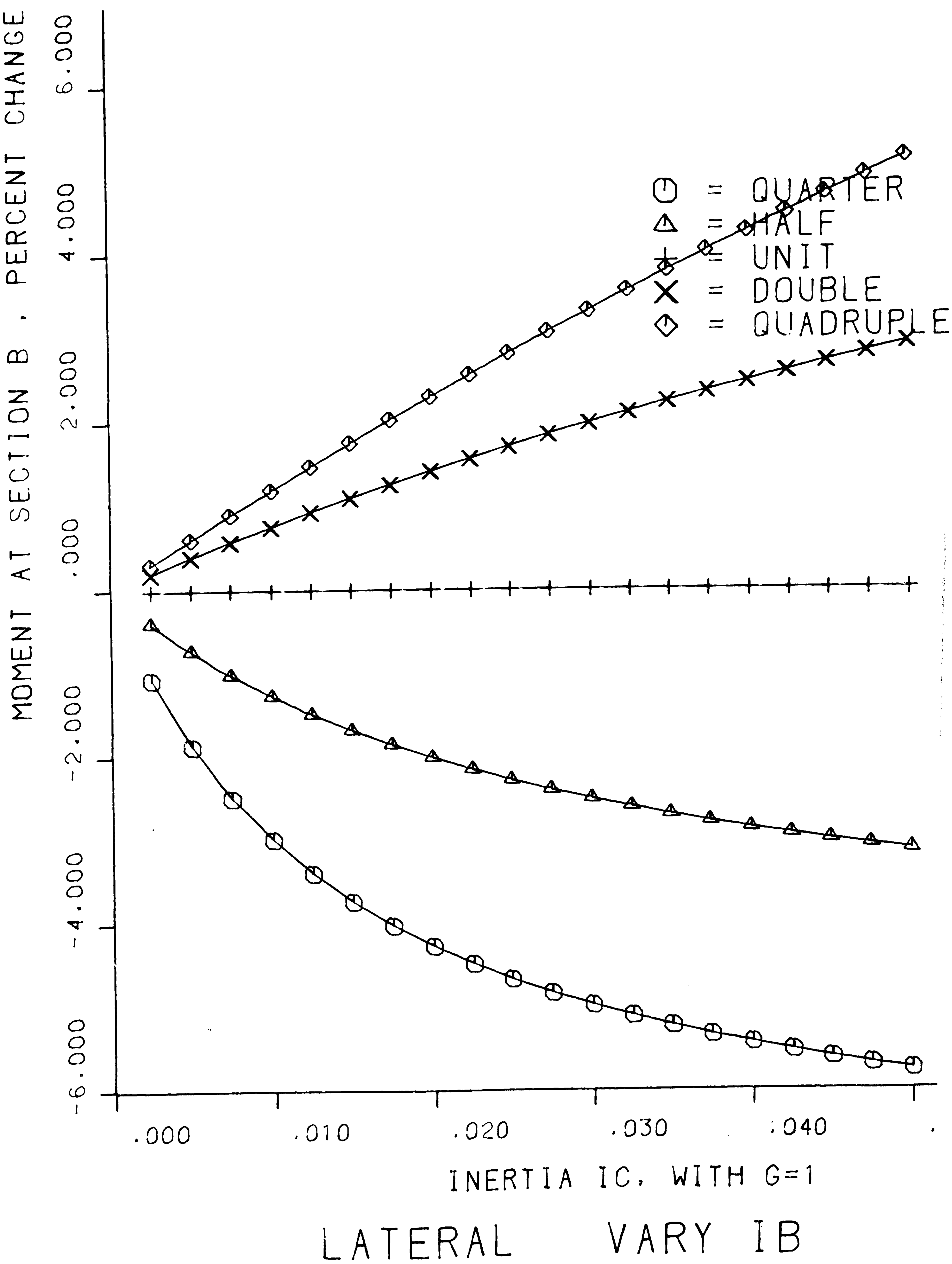


Figure 74: % Corner Mom. vs Col. Inertia & Beam Inertia, G=2, Lat. Load

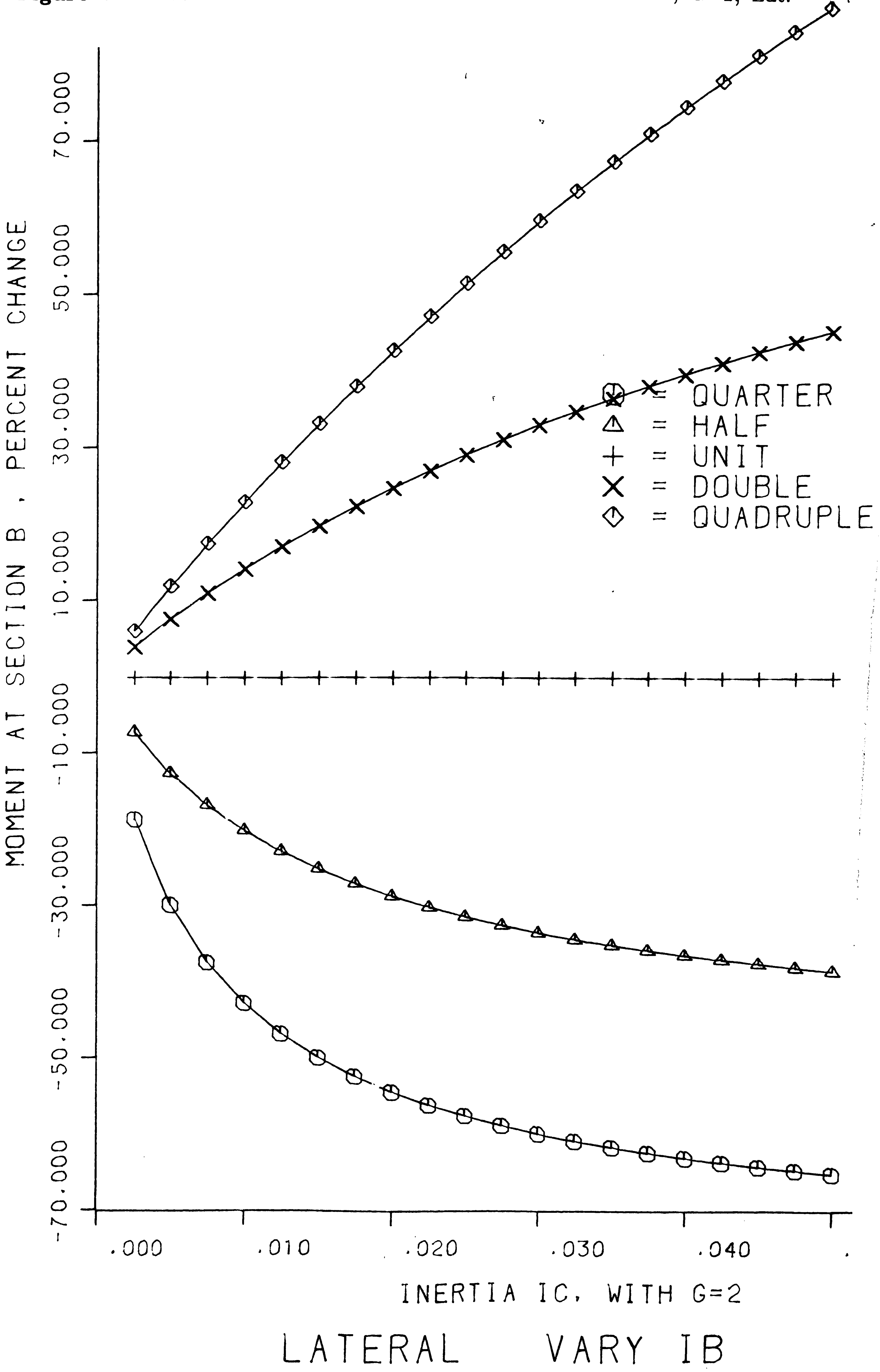


Figure 75: % Corner Mom. vs Col. Inertia & Beam Inertia, G=3, Lat. Load

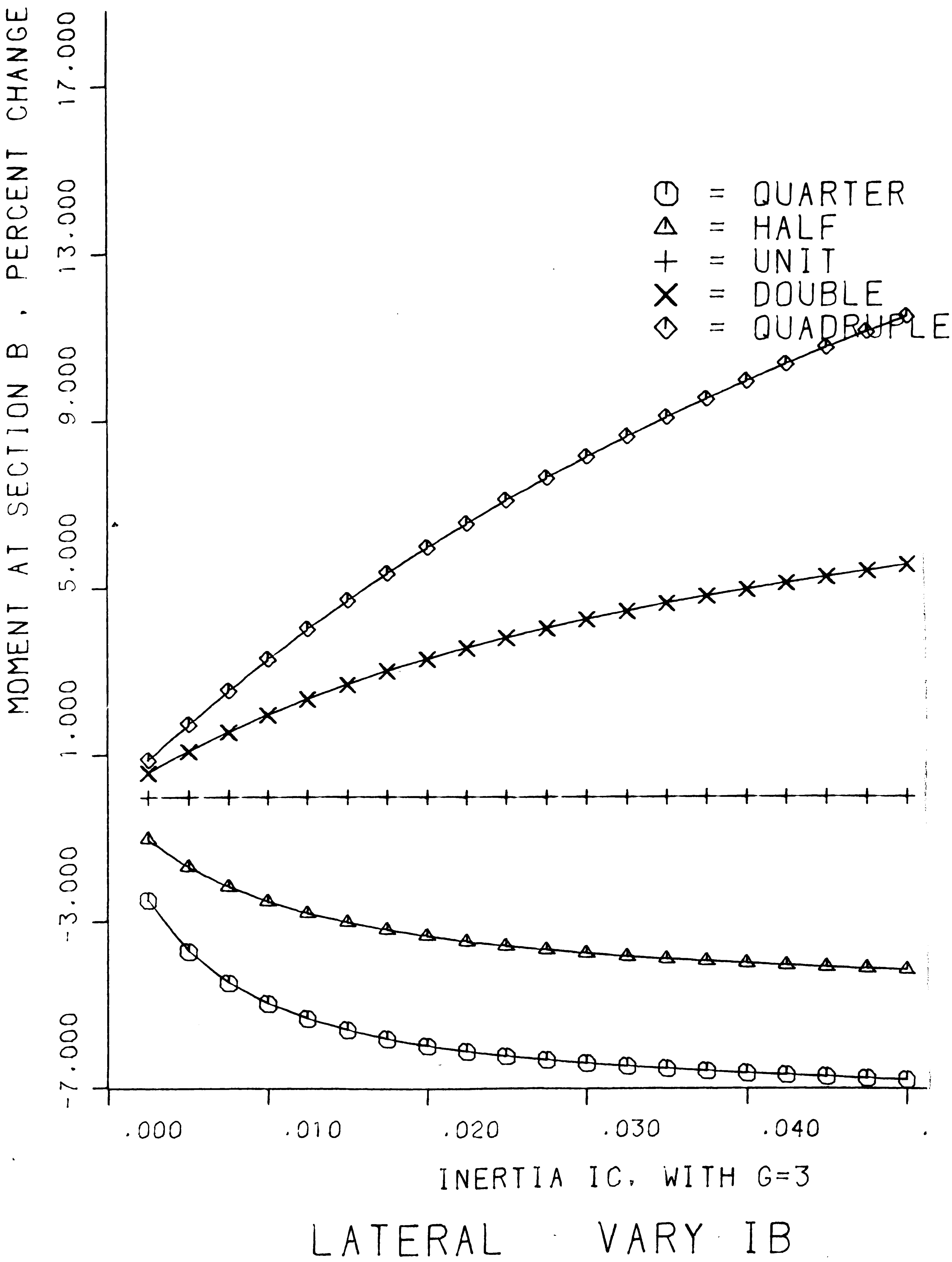


Figure 76: % Corner Mom. vs Col. Inertia & Beam Inertia, G=4, Lat. Load

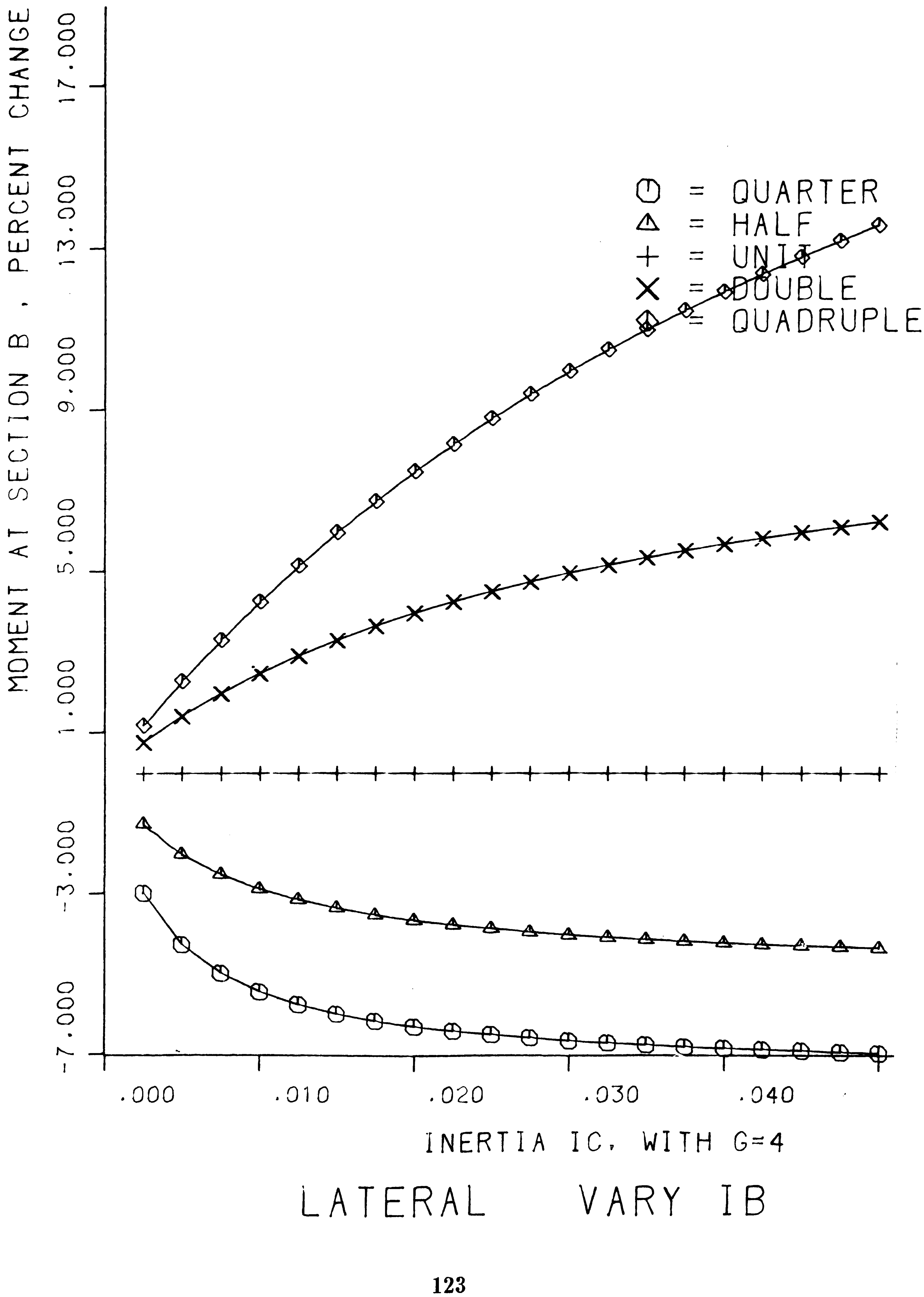


Figure 77: % Corner Mom. vs Col. Inertia & Beam Inertia, G=5, Lat. Load

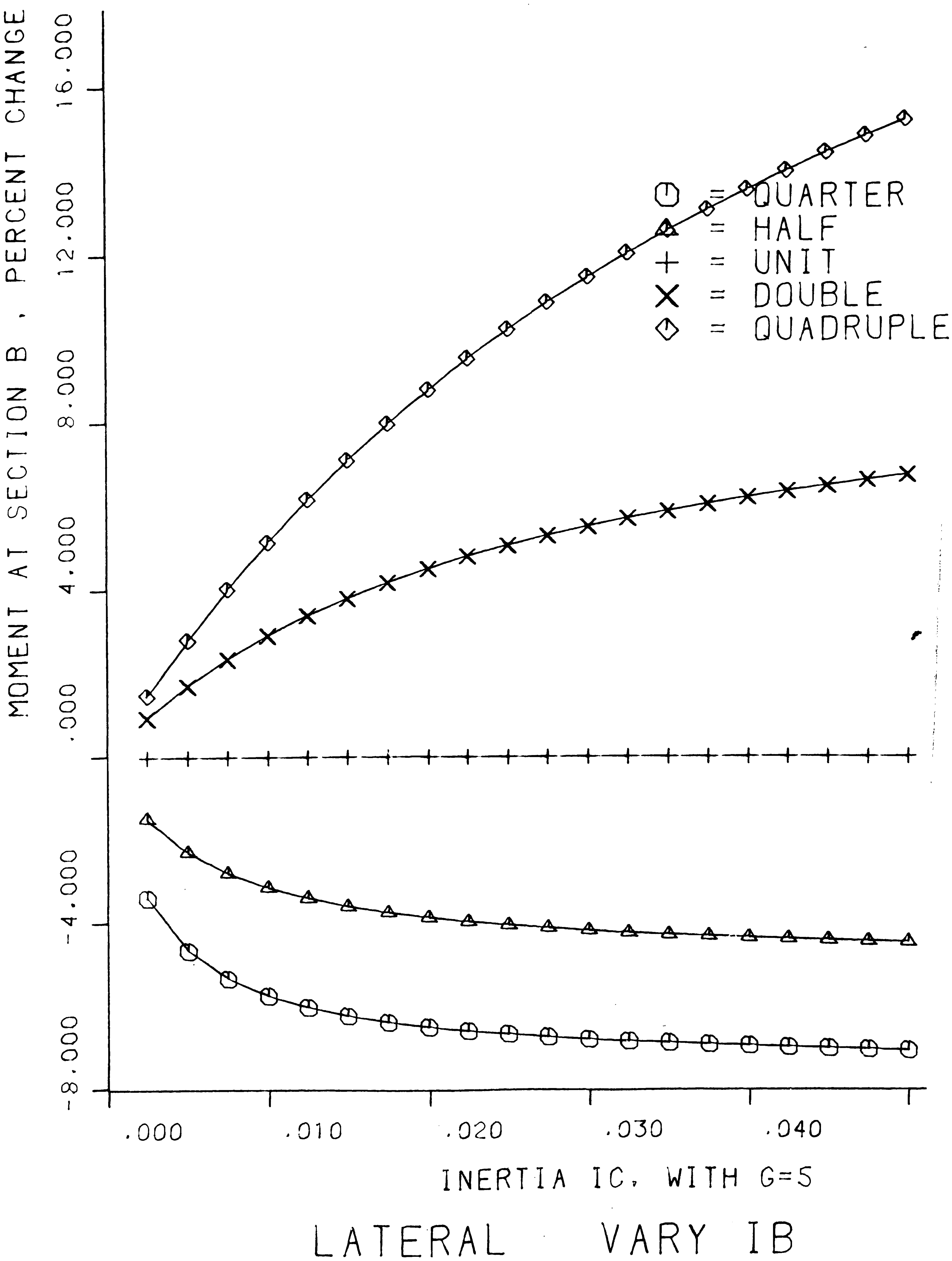


Figure 78: Sway vs Col. Inertia & Beam Inertia, G=1, Lat. Load

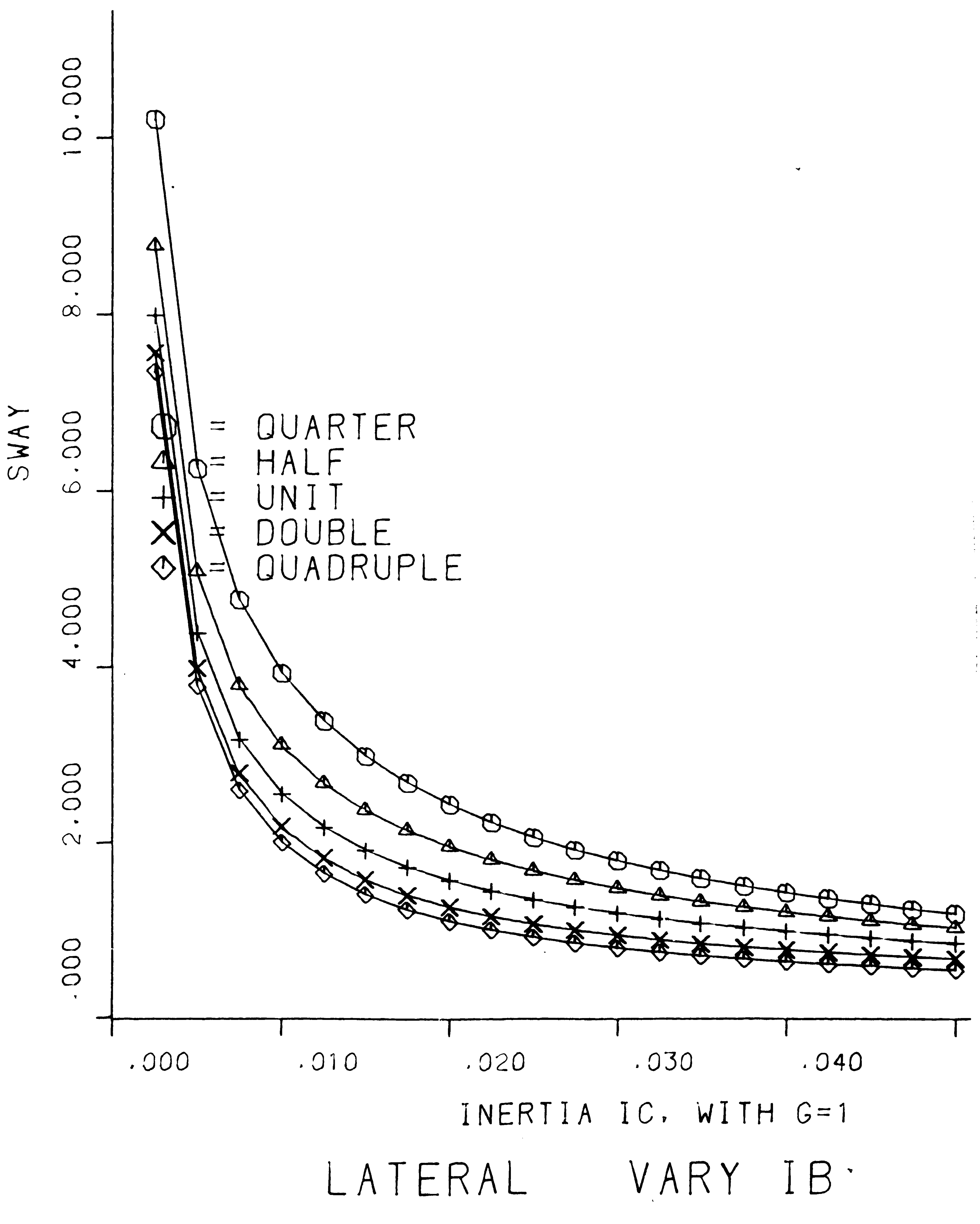


Figure 79: Sway vs Col. Inertia & Beam Inertia, G=2, Lat. Load

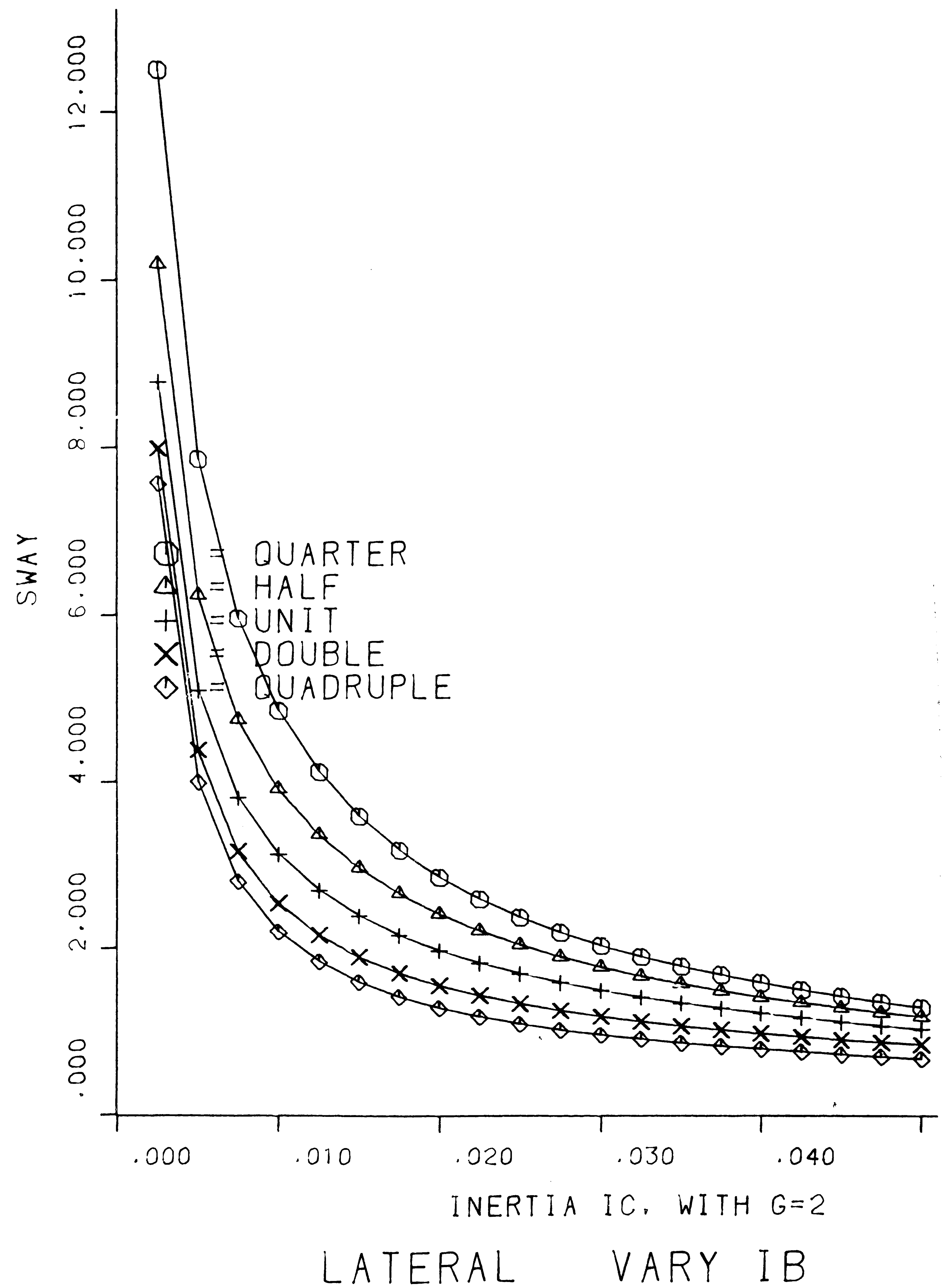


Figure 80: Sway vs Col. Inertia & Beam Inertia, G=3, Lat. Load

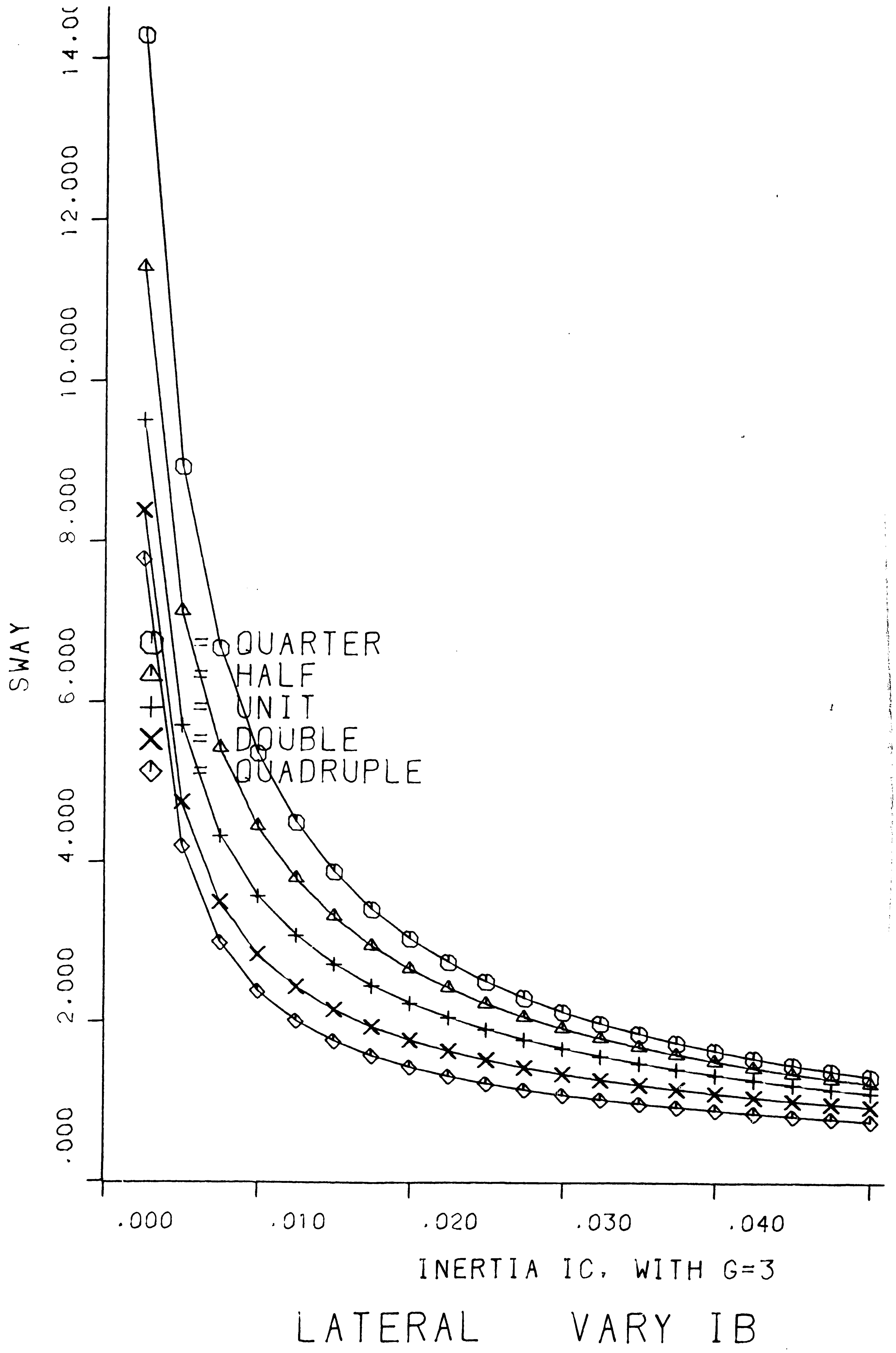


Figure 81: Sway vs Col. Inertia & Beam Inertia, G=4, Lat. Load

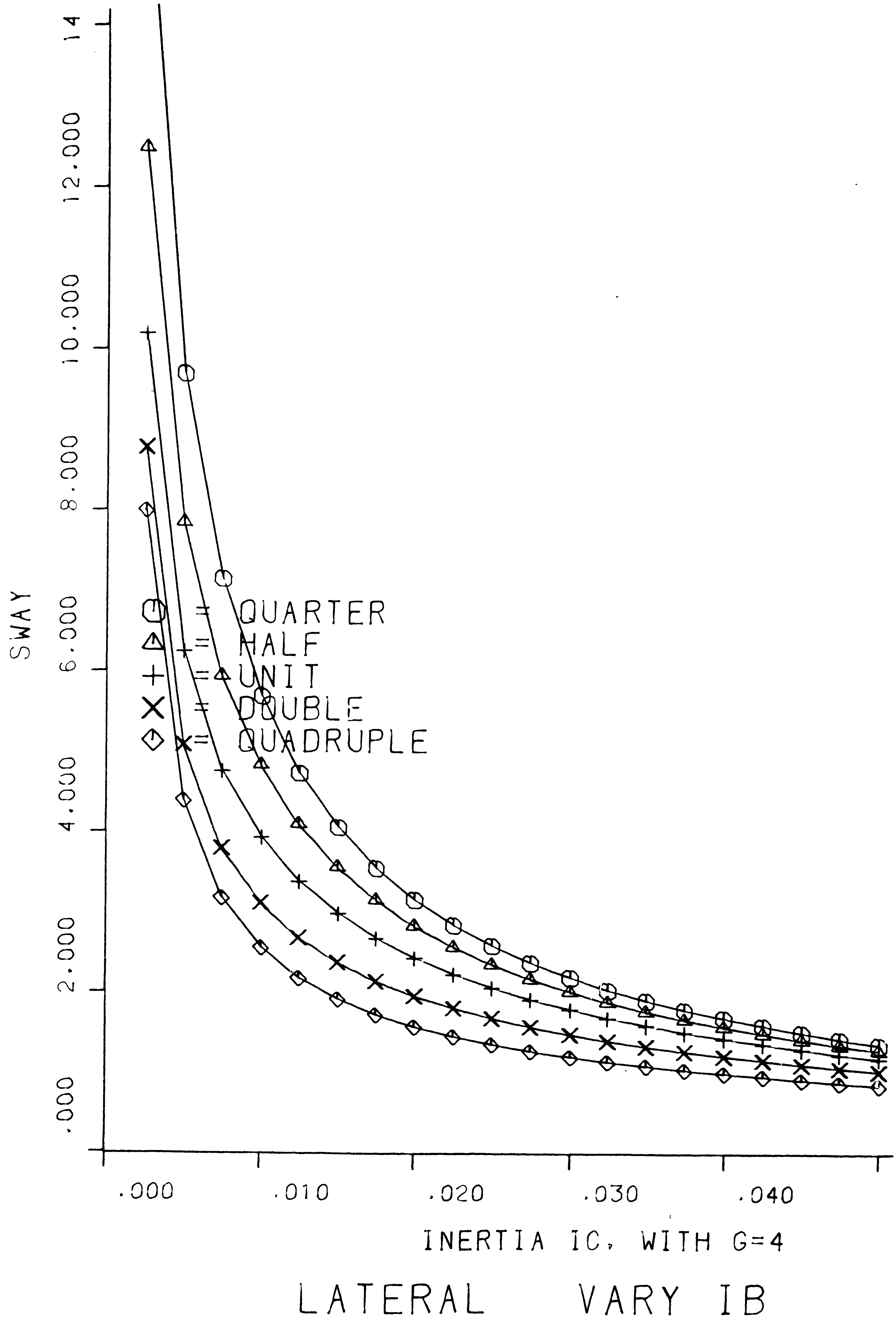


Figure 82: Sway vs Col. Inertia & Beam Inertia, G=5, Lat. Load

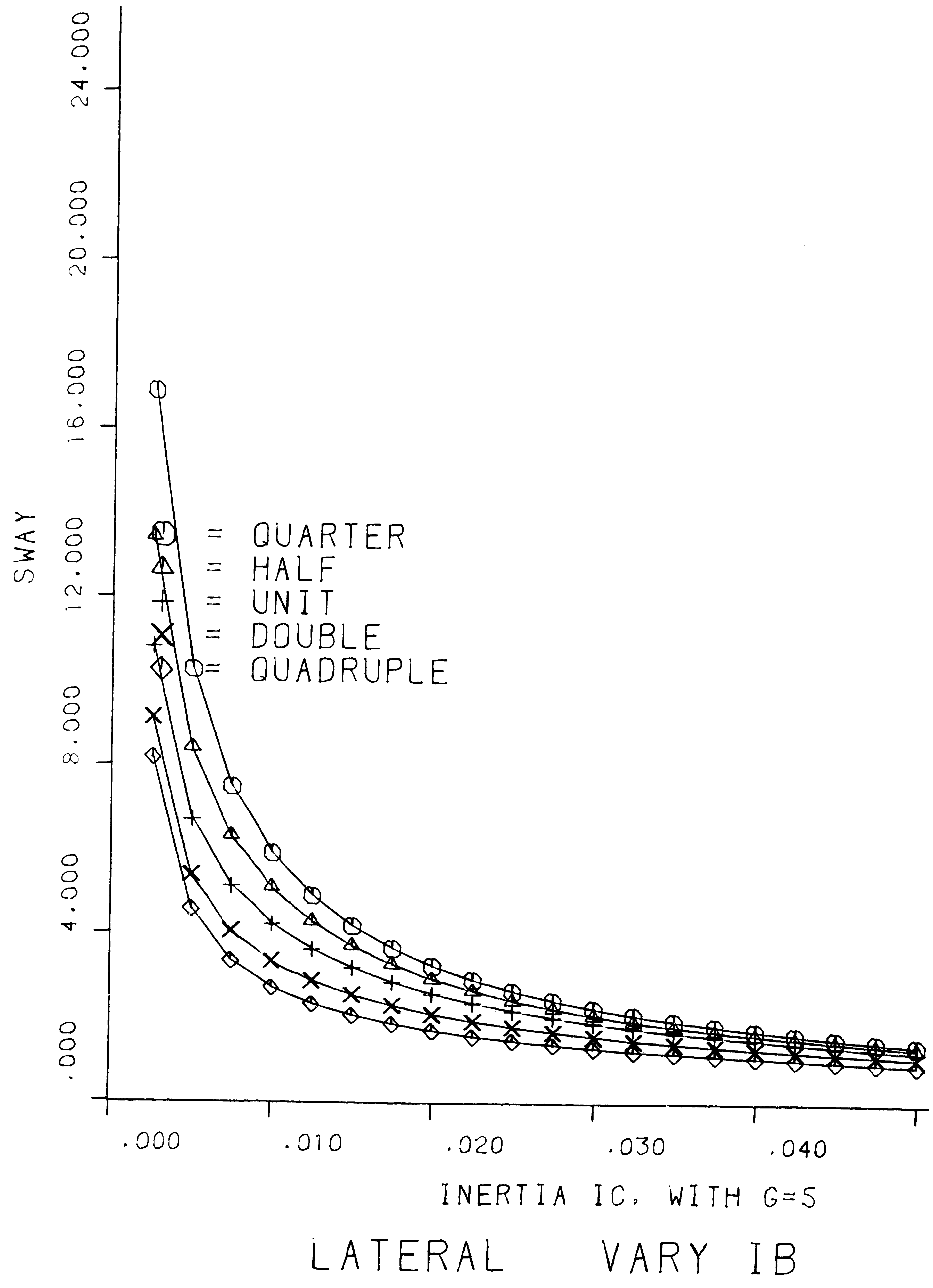


Figure 83: % Sway vs Col. Inertia & Beam Inertia, G=1, Lat. Load

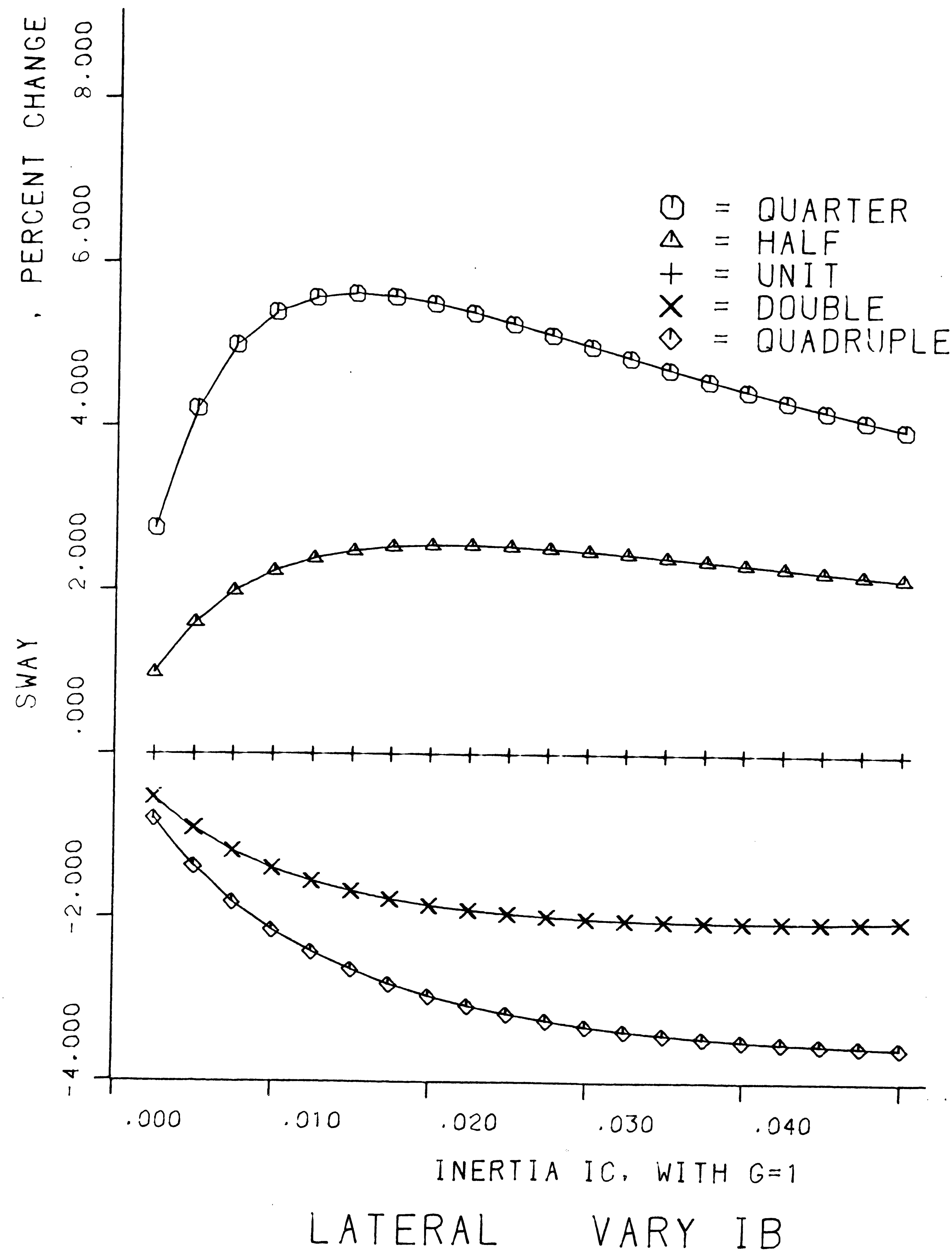


Figure 84: % Sway vs Col. Inertia & Beam Inertia, G=2, Lat. Load

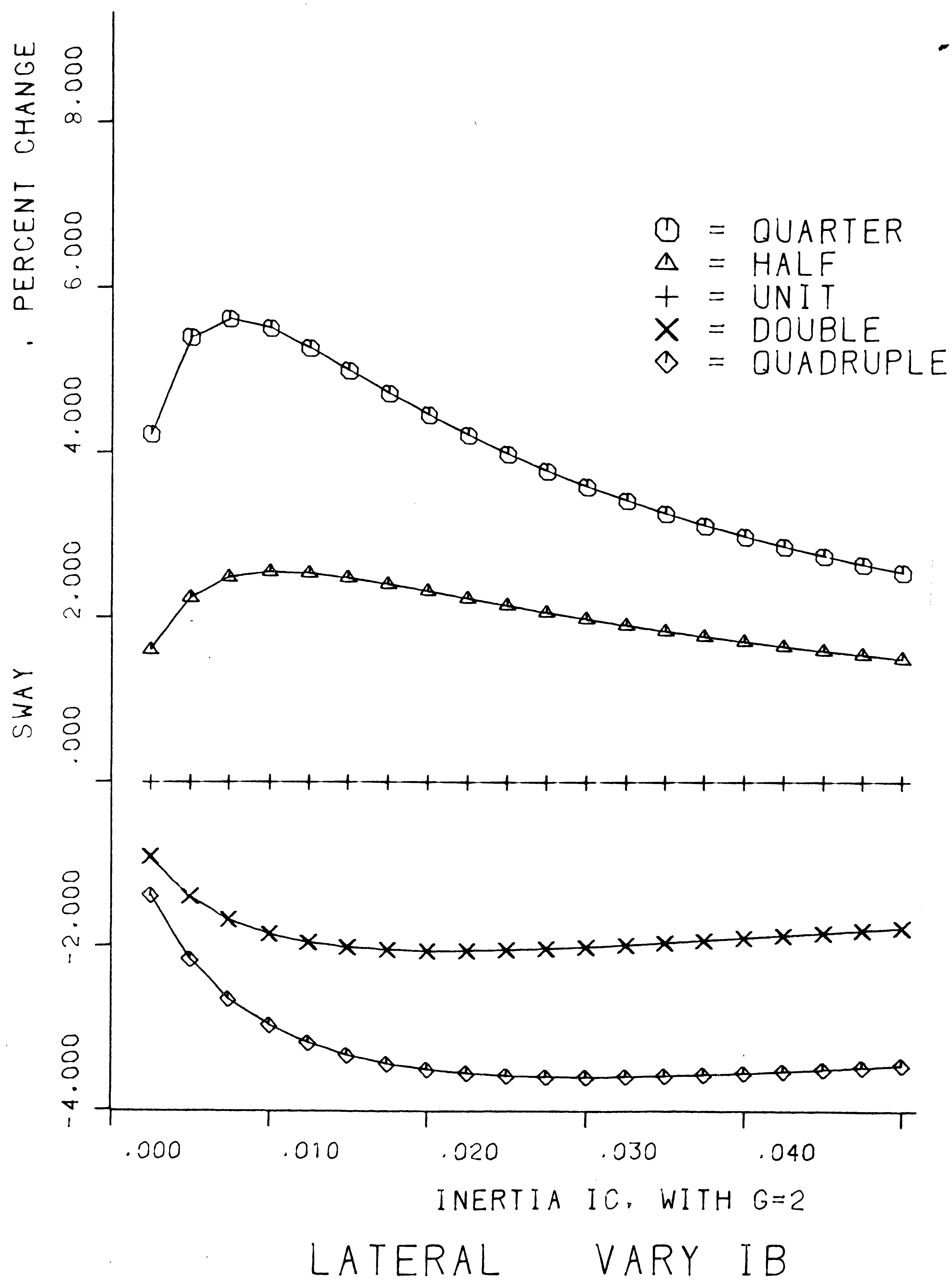


Figure 85: % Sway vs Col. Inertia & Beam Inertia, G=3, Lat. Load

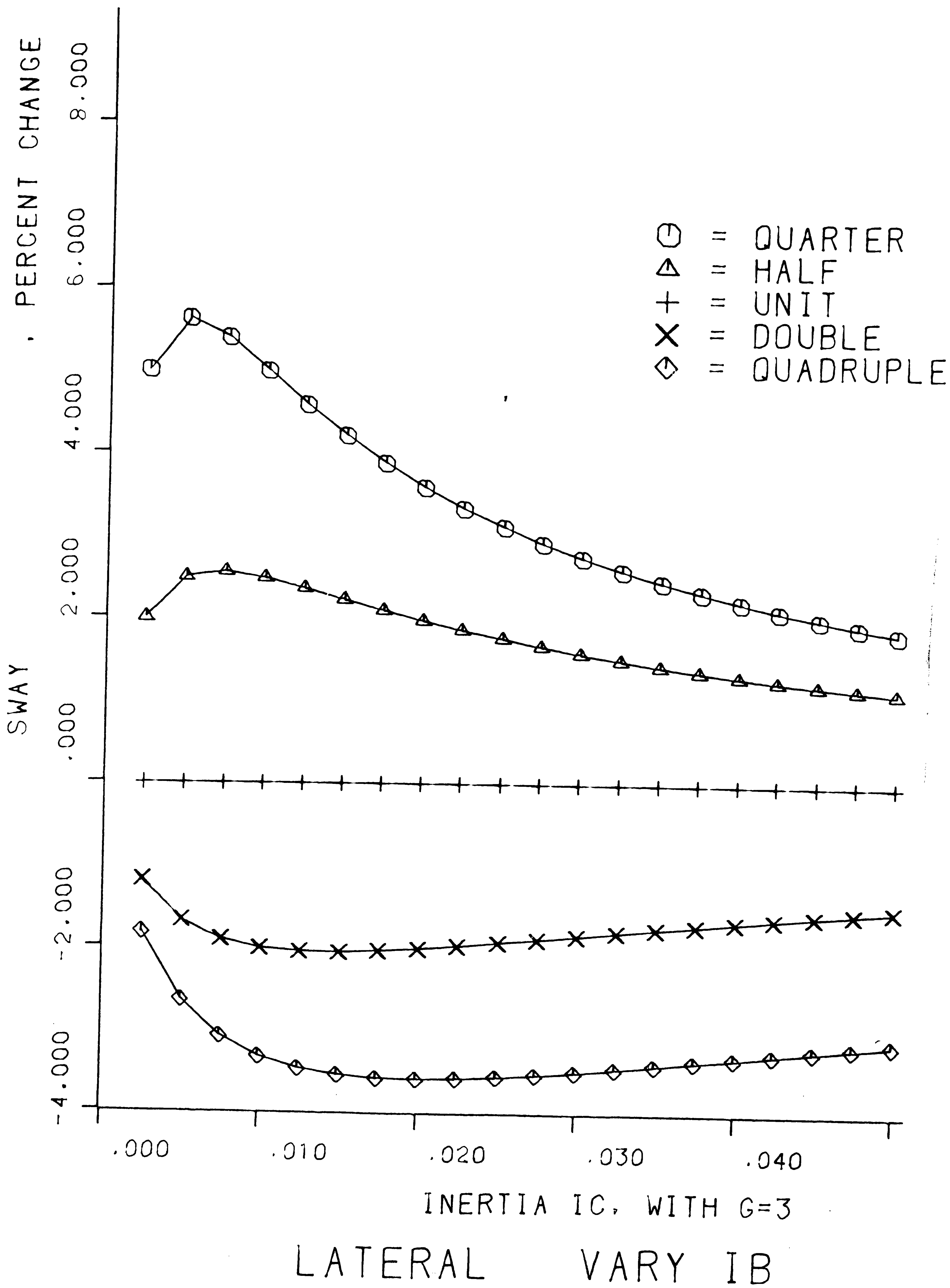


Figure 86: % Sway vs Col. Inertia & Beam Inertia, G=4, Lat. Load

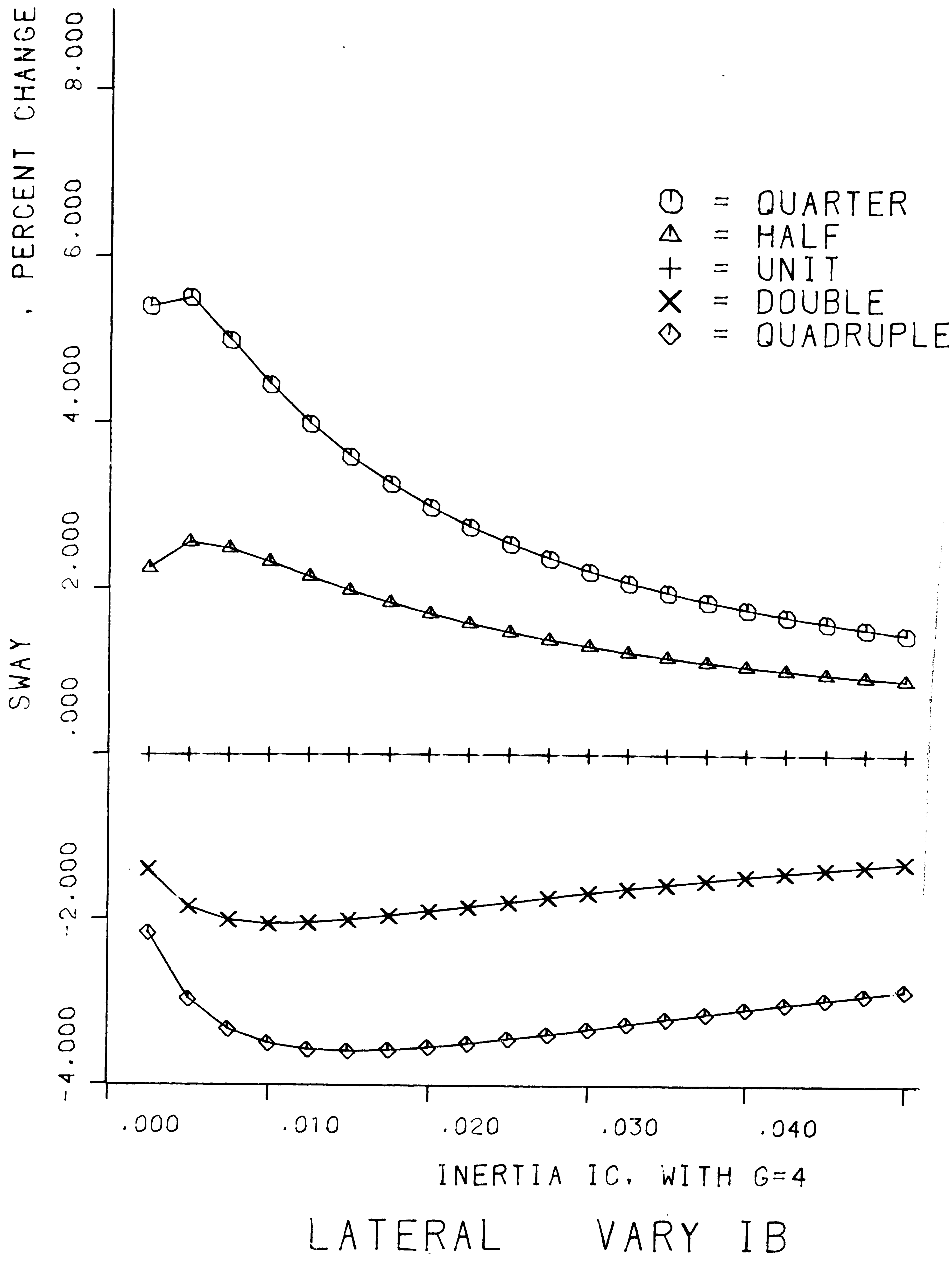


Figure 87: % Sway vs Col. Inertia & Beam Inertia, G=5, Lat. Load

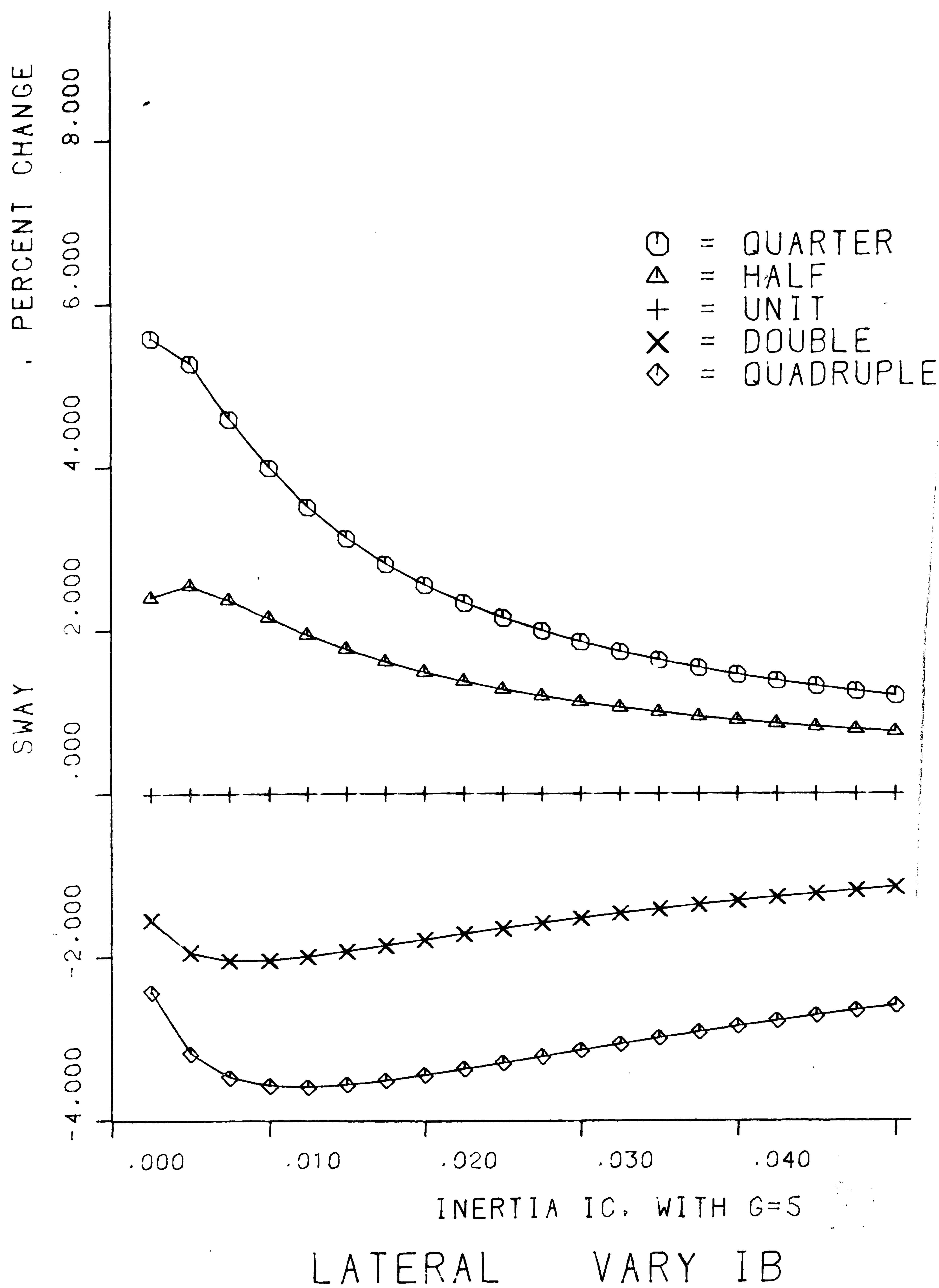


Figure 88: Base Mom. vs Col. Area & Beam Area under Grav. Load

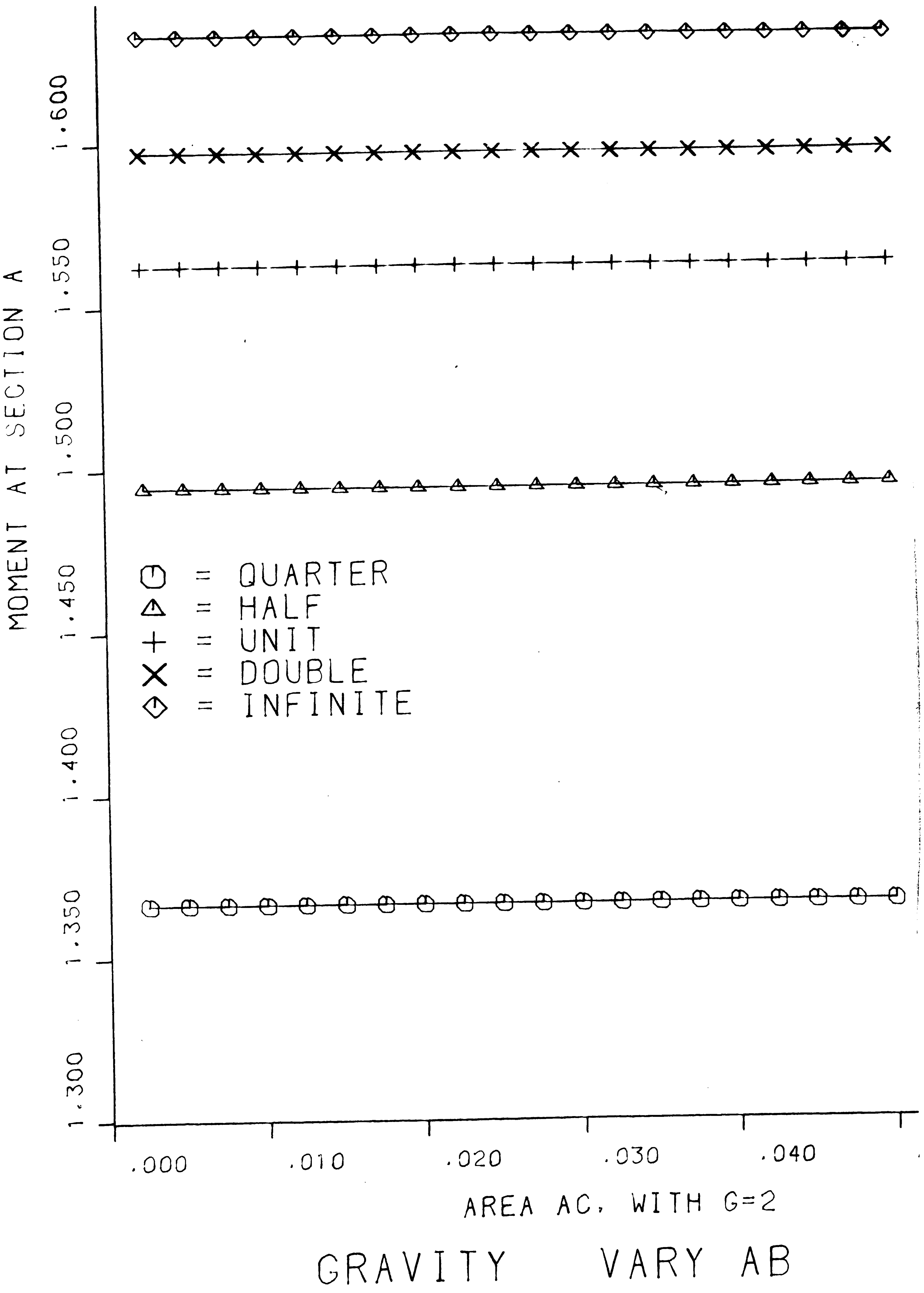


Figure 89: Midspan Mom. vs Col. Area & Beam Area under Grav. Load

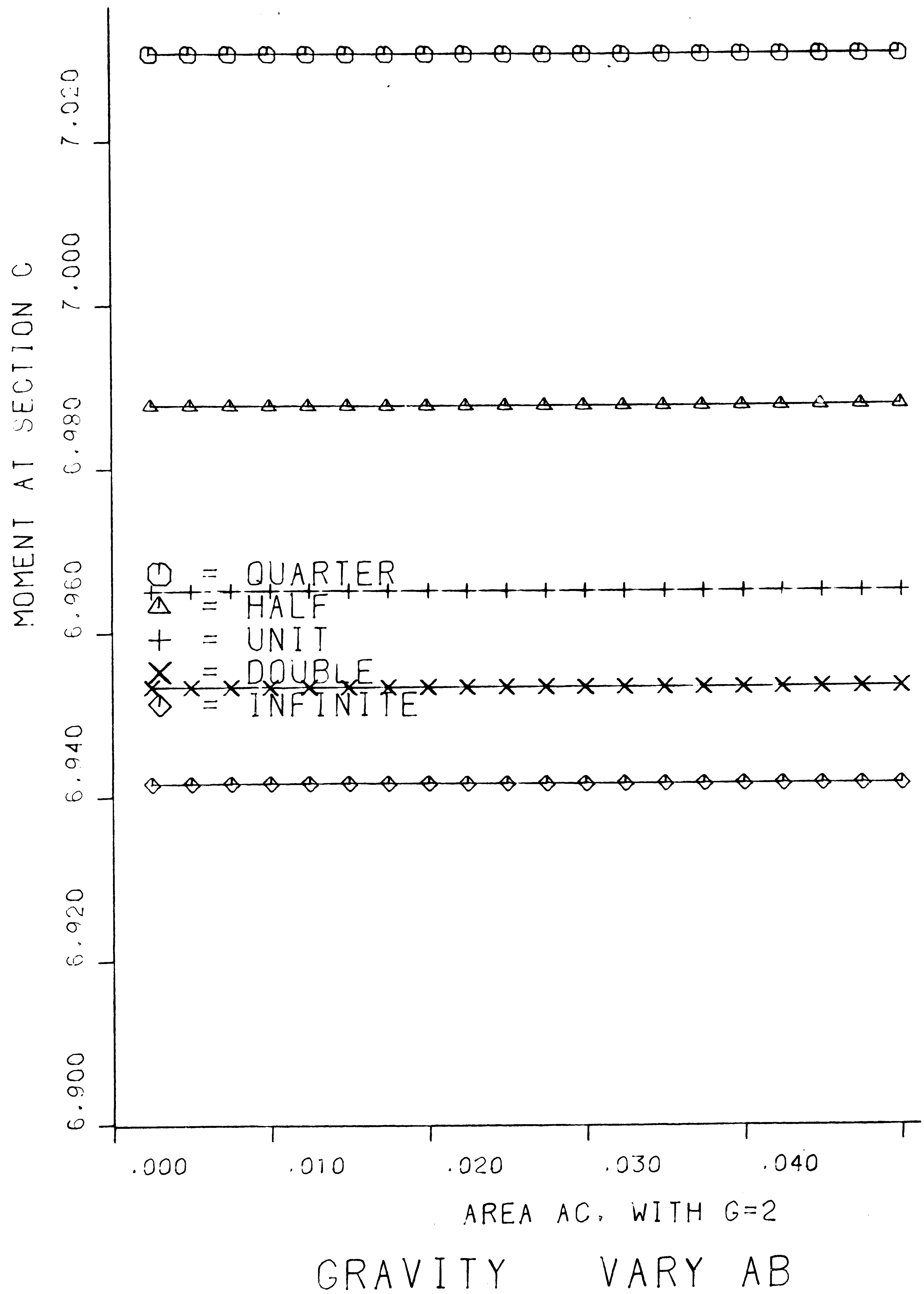


Figure 90: Sway vs Col. Area & Beam Area under Grav. Load

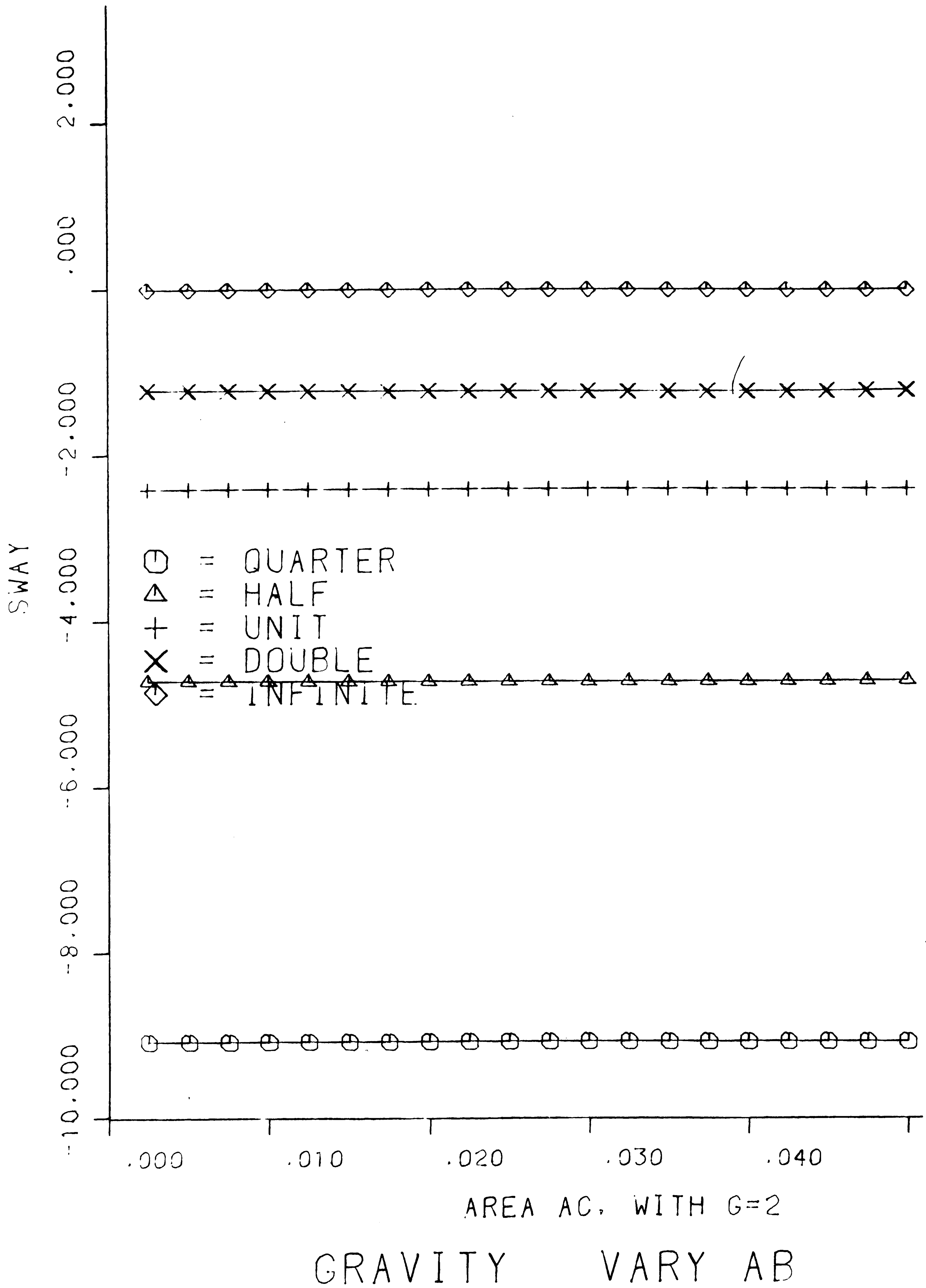


Figure 91: Base Mom. vs Col. Area & Beam Area under Lat. Load

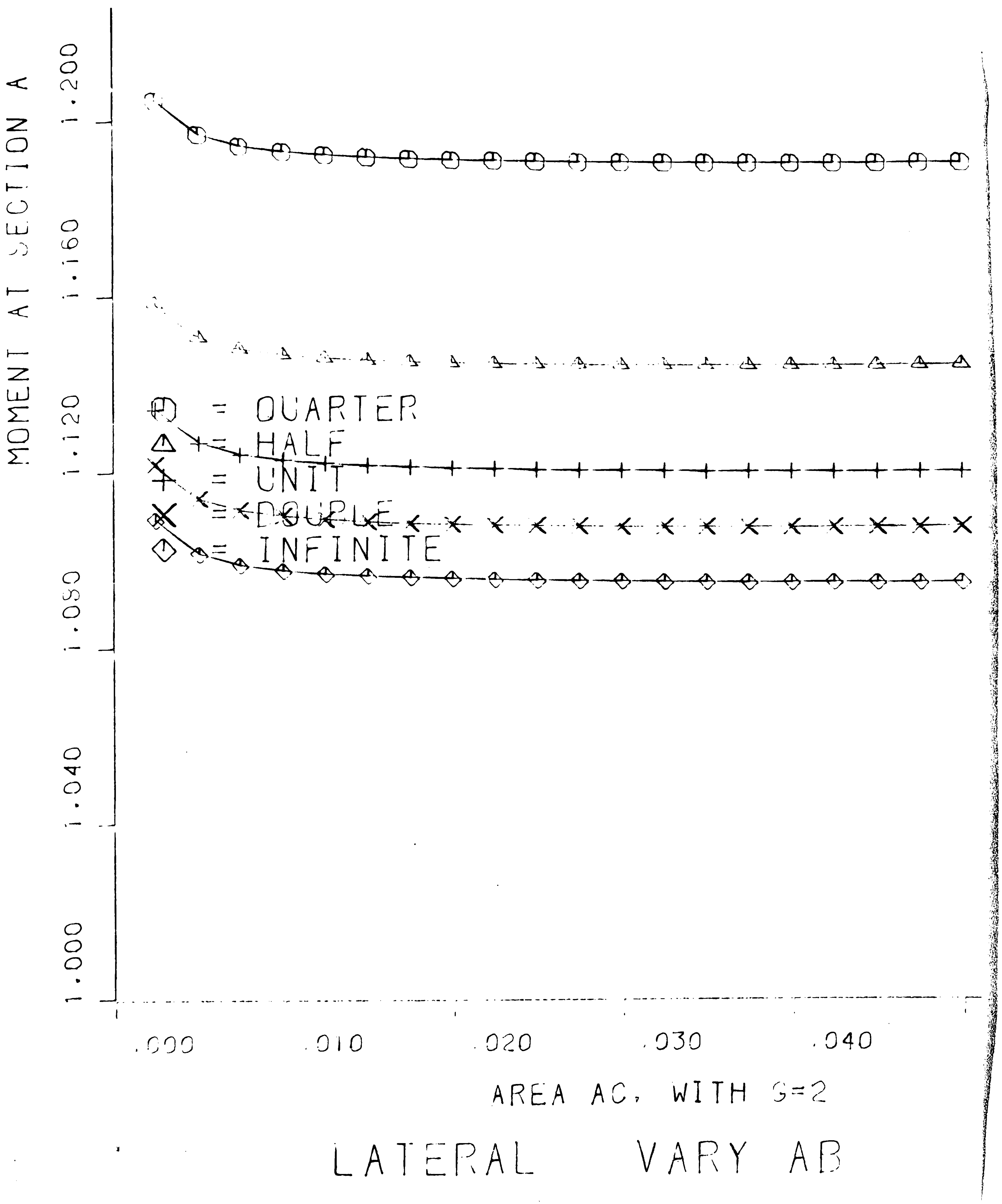


Figure 92: Corner Mom. vs Col. Area & Beam Area under Lat. Load

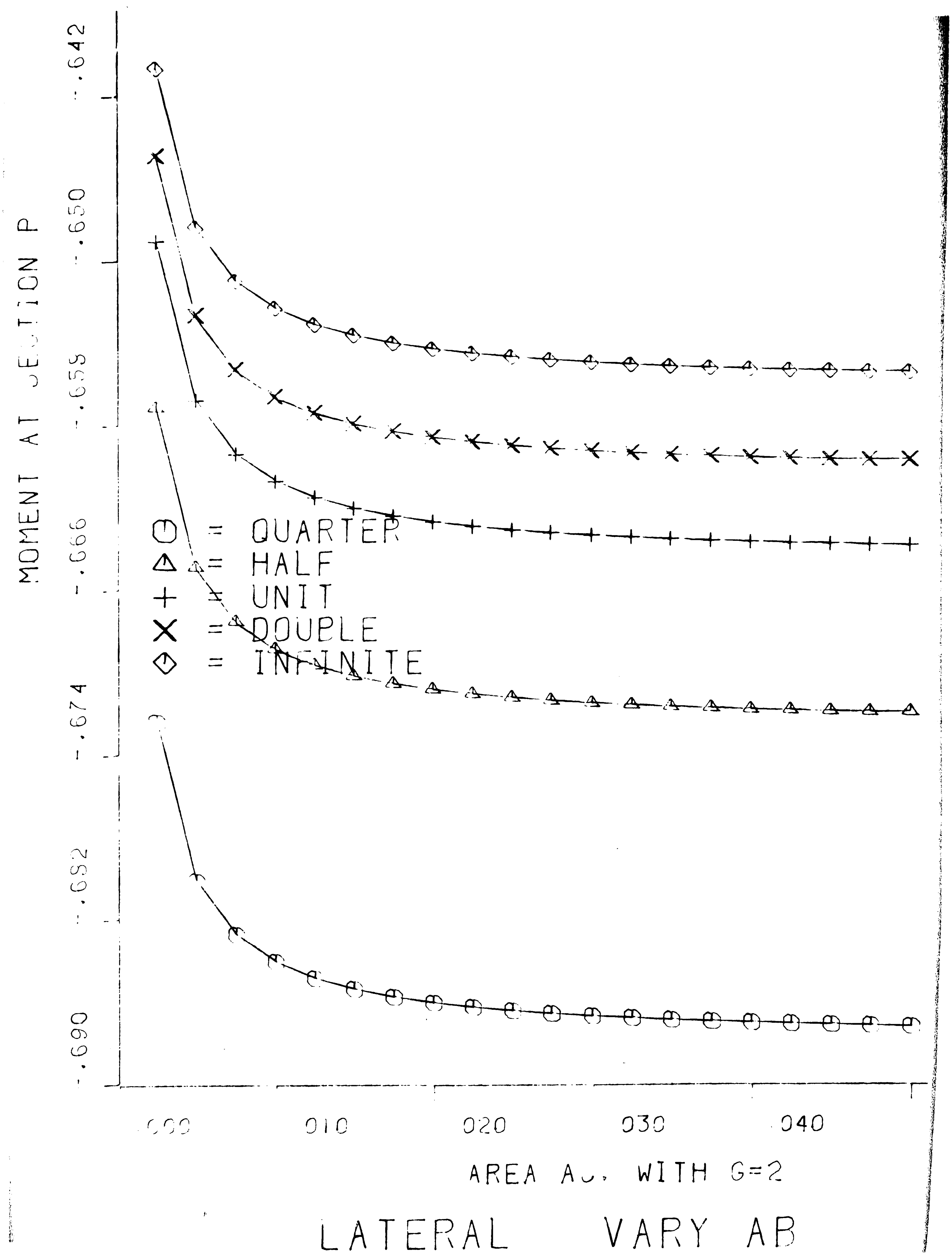
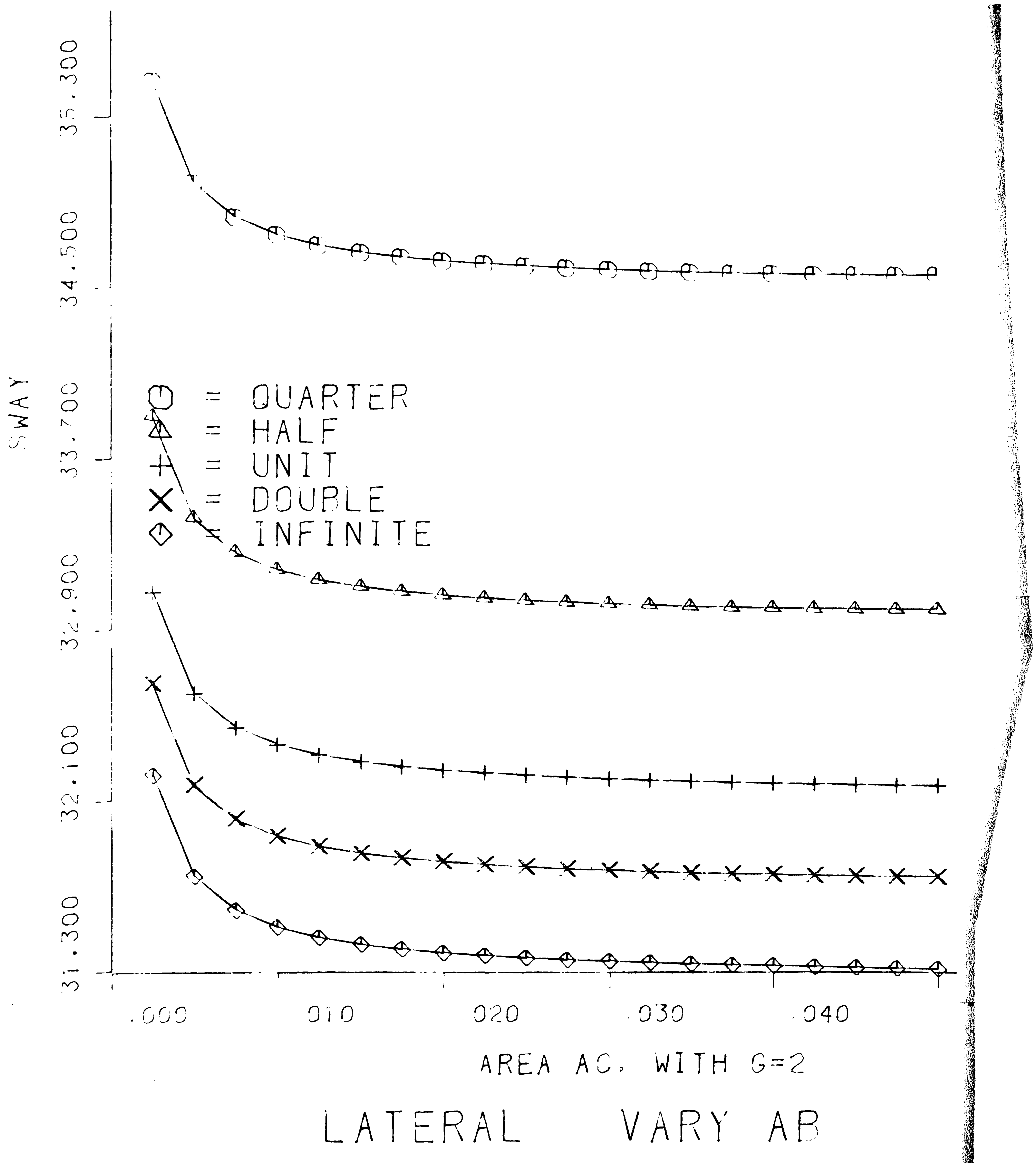


Figure 93: Sway vs Col. Area & Beam Area under Lat. Load



Appendix A
The Preliminary Topic Specification

A.1 Notation

The notation that is used utilizes superscripts and subsripts. Postfix superscripts denote the index to the 'floor level' of the parameter, and postfix subscripts denote the index to the 'bay number' of the parameter. Prefix superscripts appearing before the parameter refers to the 'load case' in question. A postfix subscript of 'b' or 'c' denotes 'beam' and 'column' respectively.

Briefly: ${}^{load}P^{floor}_{bay}$

If a parameter is generic (uniform) to a floor or bay, the relevant sub/superscript will be missing. If no sub/superscripts are used the parameter is generic to the total system, or is simply system independent.

- A^i_j
  AB(i,j),AC(i,j)
  AB,AR,AL - Axial Area of a beam/column
- n
  NBAY - number of bays horizontally (integer > 0)
- m
  MLEVEL - number of levels vertically (integer > 0)
- γ_j
  G(j),G - cell aspect ratio, beam span = $\gamma_j L$
- L - floor to floor height
- ${}^lW^i_j$
  W(i,j),W - uniformly distributed load on a floor beam
- ${}^lWD^i$
  WD,P - Wind load at a given floor
- I^i_j

IB(i,j),IC(i,j)
IB,IR,IL - Second Area Moment of a beam/column

- E,EE - Young's Modulus
- (u,v,θ) - Deflection vector at a node
- (P,V,M) - Primary Stress Resultants at a section
- [**K**],AKA - System Stiffness Matrix
- [**F**],FLEX - System Flexibility Matrix
- [**k**],$K_{1,1}..K_{3,3}$ - Primary Stiffness Matrix
- $[\mathbf{K}]^{ele}$
  KL,KH,KV - Elemental Stiffness Matrix
- $[\mathbf{S}]^{ele}$
  SL,SH,SV - Elemental Stress Resultant Transform Matrix
- [**T**] - Nodal Force/Displacement Transformation Matrix
- $[L_D]$ - Co-ordinate Transformation Matrix
- K_x,K_y,K_θ - Boundary Stiffness Coefficients
- P_a - Axial Stress Resultant
- V_a - Shear Stress Resultant
- M_a - Moment Stress Resultant

A.2 Overview of the System Model

The total structure is defined in the deflection domain, using assembled 'first order' member properties. This formulation produces a set of linear functions between the load and deflection terms. The use of linear functions eases the solution of these equations, so that the deflection terms may be expressed in terms of the applied loads, or visa versa, or some hybrid set. The functional relationship between geometric, structural properties, load and deflection terms is not linear, and so an arbitrary solution of one set of these terms with respect to another is non trivial.

The problem at hand is to express the deflection terms with respect to a set of formulae containing load, geometric and structural parameters. Thus in a linear model the assembled global stiffness matrix need only be inverted to achieve this goal.

Once the deflections are known in terms of the load and structural parameters, the member end stress resultants can also be formulated in terms of the load and structural parameters.

The four boundary conditions that parameterize the deflected shape, rotation, moment and shear diagrams for the member are then defined in terms of the loads and structural terms.

The maximal combined stress and deflection conditions for each member, taken over the range of all possible loading scenerios defines a set of design constraints particular to that member. Considering all members of the structure in this way, a set of structural system constraints is formed. Any design that satisfies all constraints is a feasible design. A design that minimizes the weight of the structure is an optimal design based on weight reduction as an objective. A design that ensures that as many constraints as possible for strict equalities under application of maximum loads, is an optimal design based on maximum straining of the system. The objective is the minimization of the total slack summed over all constraints. This objective might also include constraints that all slack variables of a similar class (ie. deflection, axial, moment, shear or combined stress) are to have equal value, which would form the rational of a 'balanced' design.

Figure A-1: The Representation of a 5 Level by 3 Bay Frame

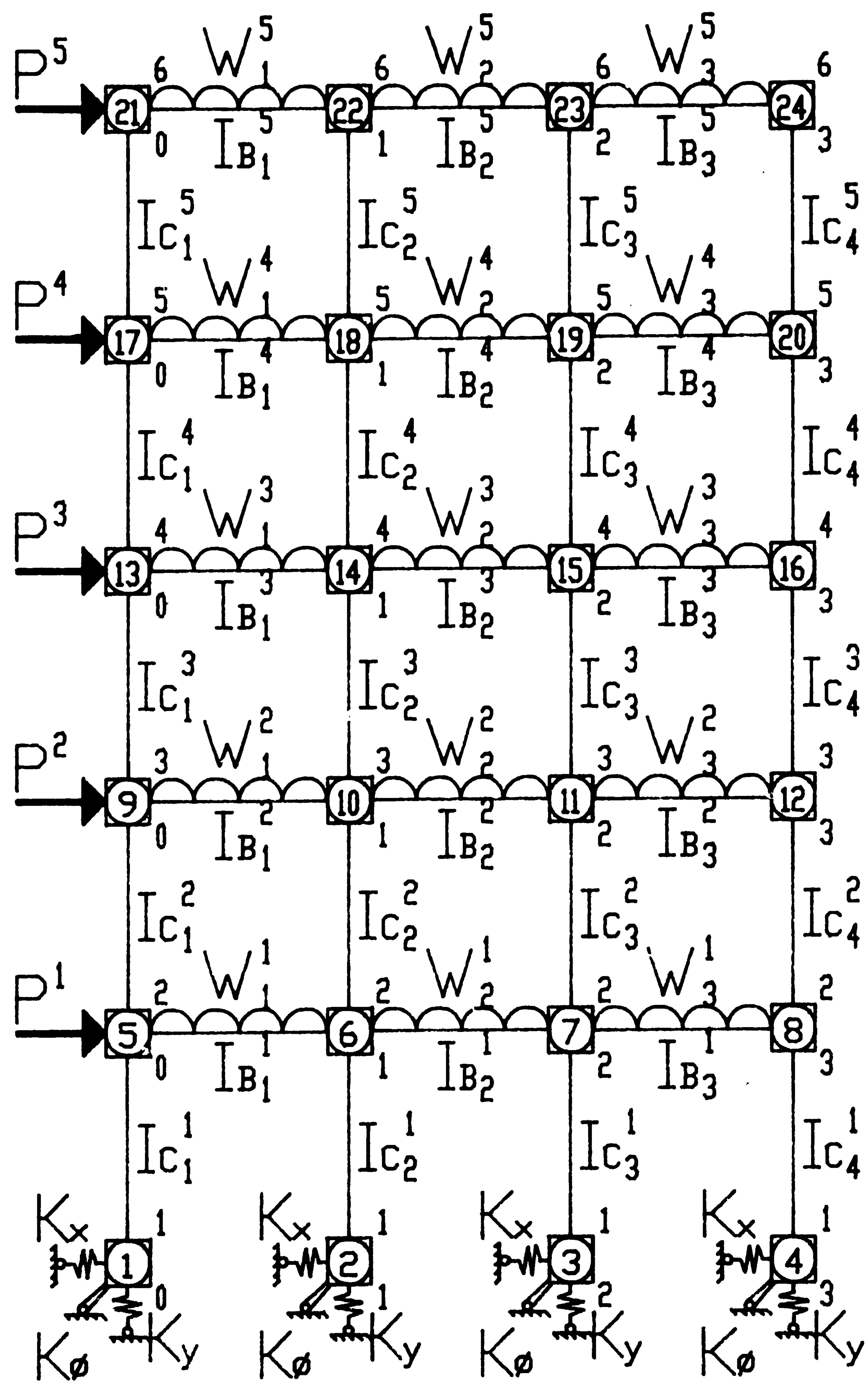
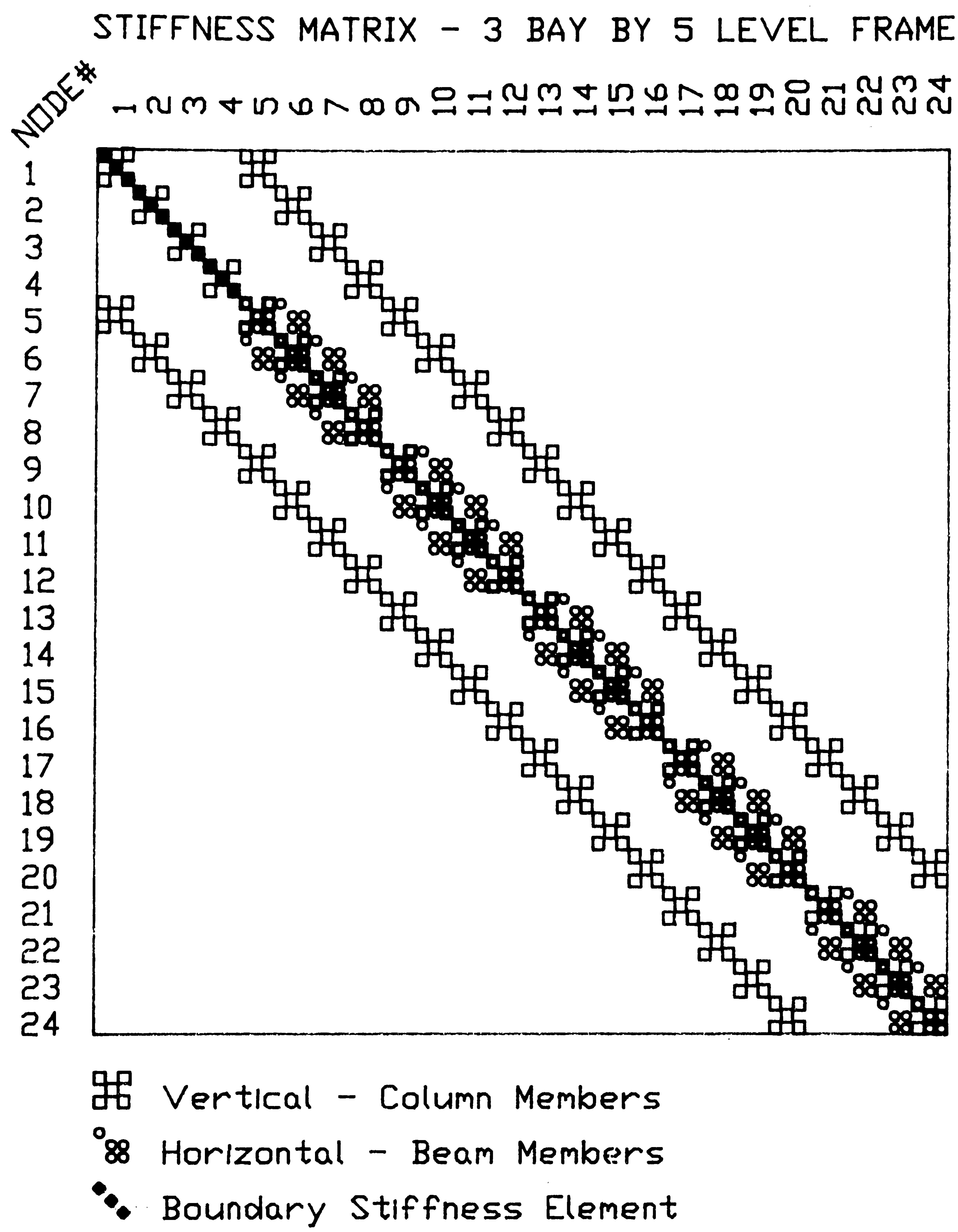


Figure A-2: System Matrix for a 5 Level by 3 Bay Frame



A.3 System Parameters

A.3.1 Geometric Parameters

The rectilinear structures in this investigation are parameterized by four variables $(\mathbf{n},\mathbf{m},\gamma_j,\mathbf{L})$, which have the following meanings:-

- **n** - number of bays horizontally (non-zero integer)
- **m** - number of levels vertically (non-zero integer)
- γ_j - cell aspect ratio, beam span $= \gamma_j\mathbf{L}$
- **L** - floor to floor height

A general investigation imposes the uniformity requirement that all floors are of similar height. Since floors are often of similar function this constraint is practical. Since the investigation deals only with rectilinear frames, the bay lengths remain constant vertically. The bays could be of different span length, thus γ_j could be different for each bay. However this investigation shall assume all bays to be of similar length, another uniformity constraint.

Realistic values of **n** and **m** are only weakly coupled to the design problem, and are more strongly controlled by height restrictions, the site's land area and architectual considerations about the number of columns in a frame.

A.3.2 Structural Parameters

These parameters are discussed in the thesis body, see Section 2.2.2.

- $\mathbf{A}^i_j$ - Section Area
- $\mathbf{I}^i_j$ - Second Area Moment.

A.3.3 Load Parameters

These parameters are discussed in the thesis body, see Section 2.2.3.

- $(\mathbf{F}_x,\mathbf{F}_y,\mathbf{M}_{zn})$ - Point Load applied at a node
- ${}^l\mathbf{w}^i_j$ - Uniform Load, Floor loading
- ${}^l\mathbf{W}^i$ - Wind load at each level

A.3.4 Deflection - Response Parameters

These parameters are discussed in the thesis body, see Section 2.2.4.

- ${}^l\delta^i_j={}^l(\mathbf{u},\mathbf{v},\theta^i_j$ - Global Deflection

A.3.5 Internal Stress Resultant Parameters

These parameters are discussed in the thesis body, see Section 2.2.5.

- ${}^l\Sigma^i_j={}^l(\mathbf{P},\mathbf{V},\mathbf{M})^i_j$ - Local End Stresses

A.4 The Structural Components

A.4.1 The Beam Element

The beam element is based on first order deflection-strain relationship. The lumped properties of each element are computed by integrating the strain energy of the member over its section and length and differentiating with respect to each deflection term.

A.4.2 Boundary Conditions

The support conditions for the problem are confined to the base of the structure. The boundary constraints may be treated in a general manner, so that variation in boundary conditions can be considered as part of the system's parameters. The boundary conditions are parameterized by direct stiffness coefficients (K_x, K_y, K_θ). A fixed support corresponds to infinite stiffness for each coefficient, and a hinge support corresponds to K_θ=0. The use of boundary stiffness provides a means to include first order foundation elastic settlement in the total structural model.

A.5 Loads

A.5.1 Gravity Loads

Gravity loads encompass all vertical loads acting on the beam members in the system. All gravity loads are considered to be uniformly distributed.

Gravity loads are to be placed to cause worst case:-

1. Positive moment and mid-beam deflection
2. Negative moment and sway deflections

Each is to be considered a load case and should be arranged to cause the worst case condition in all members, but not necessarily in simultaneity. These loads should be placed in conjunction with wind loads.

Gravity loads must be segregated into dead and live load components. These loads are collectively designated as ${}^{l}\mathbf{W}^{i}_{j}$, and in general are specific for a given load case l, level i, and bay j.

Rational design would apply the simple uniformity constraint that all such loading is equal to the minimum dead load alone, or the maximal dead and live loading.

A.5.2 Wind Loads

Wind loads encompass all horizontal loads acting on the column members in the system. Wind loads are frequentally defined in local building codes to form a certain force profile up the height of a structure. In this way, for a given force profile, the wind load on a structure can be parameterized by a single amplitude variable, which in this investigation is the wind pressure/force at the topmost level of the structure. The wind is modelled by applying horizontal point loads to the exterior of the structure, whose magnitude can be determined from the exposed area, and the pressure profile **WD**(WD_{max},i), at the i'th level.

Wind loads should be applied to achieve worst case effects, without live load, and in conjunction with live load.

A.6 Constraints

A.6.1 Maximum Longitudinal Stress (Strain)

The limiting constraints for moment and axial stress resultants are coupled, and shall be denoted as longitudinal stress (strain). For a stocky, prismatic Bernoulli-Navier beam the interaction relationship is:

$$\sigma_x(x)=\frac{P(x)}{A}\pm\frac{M(x)\cdot y'}{I}$$

There are other interaction formulae, such as I sections, which can also be used to formulate the longitudinal stress (strain) limitation criteria.

A.6.2 Maximum Shear Stress

The shear force diagram represents the integrated effect of shearing stress developed in a particular section. The distribution of the shear is strongly related to the section shape. Thus a particular section has a specific shear stress distribution, irrespective of the shear stress resultant magnitude.

A.6.3 Serviceability

For a given uniformly distributed load the member will experience only one point of maximum deflection. This point and the deflection is computable from the shape function of the member. No member must exceed some allowable value of deflection due to live loading and typical wind conditions. Typically this deflection is expressed as a ratio of deflection to member length in beams and slabs, or a measure of tilt from plumbness in the case of columns.

At the system level, there may be a global sway constraint that limits the magnitude of drift of the structure under wind loading.

A.6.4 Construction Uniformity

Construction uniformity is the outgrowth of the need for rationalized economic constraints that reduce the degree of dissimilarity of structural parameters in the overall system. These constraints require that members in a region of the structure should have equal structural properties. These constraints provide for construction economies in the prudent use of similar sized members, in order to streamline the time taken to erect the structure, and to reduce overall labor and tooling costs. An obvious uniformity constraint is the use of similar material properties for a given class of beams and columns. A design using many changes of material would be subject to scheduling delays, workmanship errors and extra handling costs that would make such a solution infeasible.

Geometric and uniformity of structural properties is another common construction constraint. Such constraints can be grouped by level, or vertically by common bays.

Mobile (live) loading on areas with similar function can also be uniformatized, which can have as it outgrowth the identification of member groups, that could be of uniform geometry and structural capacity.

The overall effect of uniformity constraints is to compact the potential design parameter set into a form that contains fewer parameters. The equations expressing the structural behavior of the system may then be compacted in terms of this reduced parameter set. Thus these constraints can reduce the complexity of the problem to a substancial degree.

A.6.5 Other Constraints

Other constraints would relate to design policy, and are not limit states of the structure or system. Uniformity is a constraint, which addresses architectural and construction limitations and rationalizes the structure to reduce such costs. The design office or building code for an area may require other constraints to be met. A ductility requirement is a constraint that would require that the value of all strain (or stress) slack variables not be reduced below certain values. To make allowance for system response, the ductility requirement may allow the sum of the slack variables to not be reduced below a certain value, which is a more relaxed requirement. A design office may introduce 'balanced' design constraints that would require a certain selection of the slack variables for stress(strain) to reach a limit state in simultaneity.

A structure will also have architectural constraints, that are related to the use of the structure, height, clearance and zoning laws. These constraints are site specific, and as they introduce arbitrary and extra complexity to the problem, they are not directly considered in the model.

A.6.6 Objective Function

The objective function is a function of the design parameters, in the structural synthesis problem. This scalar function assesses the worth or merit of a particular design. Different choices of design parameters will generate new objective values, which allows one to rank designs in order of their desirability.

Common objective functions are:

- Least Weight
- Least Volume
- Least Cost

These objective functions do not necessitate the explicit bounding of the structural design parameters, which are best kept unbounded, unless architectural requirements are violated.

The objective function cannot focus on maximizing load capacity or minimizing deflection, unless additional constraints are given to bound the problem in these parameters. Asking for minimized sway would produce infinitely rigid, infinitely thick columns, unless bounds were placed on the structural parameters.

Ready-made structural sections, such as rolled steel sections, limit the choice of feasible solutions, by placing a fixed relationship between the section properties A and I, and restrict the choice of I to discrete values. Furthermore 'slender' sections must also satisfy non-linear buckling deflection/strength constraints.

A.7 Specification in Alternative Parameters

The uniformity constraints may be of the form of direct equalities between various design parameters. This allows direct substitution into the solution set of equations, which would have the effect of reducing the number of unspecified design parameters in a given structural problem. Uniformity in floor loads and floor structural section sizes will greatly reduce the design parameter set.

Another means of reducing the number of unspecified design parameters is to use alternate parameters of a simpler form. One may also choose alternative parameters to re-express parameters in the original solution in another domain of similar scope. For instance using width and depth dimensions as an alternative to section area and moment of inertia. In general these parameters can be specified as functions along the member's length.

A.7.1 Simplification Using Rectangular Sections

Rectangular sections may be used to approximate monolithic and stocky column and beam-slab sections. With only two dimensional parameters specifying the section, then the substitution of section dimensions as alternative parameters in place of section moments (A,I), would involve no introduction of extra parameters into the equation set. In formulations where shear stiffness is considered there would be a simplification of terms from (A,I,A_y) to (B,D). Alternately one could introduce another section shape parameter (a third) to cover the extra terms involving shear effects.

The formulae expressing section area, moment of inertia and shear area in terms of beam width (B), and beam depth (D) are:-

$$A = B \cdot D$$

$$I = \frac{B \cdot D^3}{12} = \frac{A \cdot D^2}{12}$$

$$A_y = 2/3 \cdot A = \frac{2}{3} B \cdot D$$

A.7.2 Simplification Using Any Given Section Geometry

Once a section geometry is chosen, the all section properties of that family can be expressed in terms of one section property (the reference), and the remainder as dimensioned factors of the reference property. This can be expanded to utilize dimensionless factors, and a dimensionless reference property, see Section 3.

A.8 Floor Uniformity

In general floor uniformity follows into the following catagory of terms:

1. Beam span length pattern:
   a. Uniform from column to column
   b. Symmetric about the structure's centerline.
2. Beam depth
   a. Uniform from column to column
   b. Fixed functional of the span length
3. Beam width
   a. Free to vary to control service/stress constraints
   b. Uniform from column to column
   c. Fixed functional of the span length

4. Column size

a. Set as same width as overlying beam

b. Set as uniform for a floor

c. A fixed functional of the beam span

Appendix B
The Formulation of the Behavioral Model

The details of the formulation of the basic stiffness coefficients for the two dimensional beam element are listed in many reference texts on structural mechanics and finite element methods. The reader may consult any of the appropriate references in the bibliography.

B.1 Elastic Axial-Flexural Beam Element

B.2 Axial/Flexural Stiffness Coefficients

These coefficients are derivable using the minimum potential energy theorems. The following INPUT file, called **OLDSTIF**, contains the two dimensional flexure and axial stiffnesses for a planar beam element. No proportionality or dimensionless relationships are involved in this formulation:

```
%--- ELEMENTAL STIFFNESS COEFFICIENTS - ELASTIC FLEXURE/AXIAL ELEMENT
ELTITLE:="ELASTIC FLEXURAL/AXIAL ELEMENT";
OPERATOR EE,Q,Ax,Iz,Lx;

LET K11=EE*Ax/Lx;
LET K12=0;
LET K13=0;
LET K22=12*EE*Iz/Lx**3;
LET K23=6*EE*Iz/Lx**2;
LET K33=4*EE*Iz/Lx;

;END;
```

The following matrices were derived directly from the OUTPUT file <u>ELMAT</u>, and processed into a suitable numerical text format.

Table B-1: General Transformed Elemental Stiffness Matrix

Left-Hand Half

(u_i, v_i, θ_i)

$\frac{EA_zL_z^2\cos^2\alpha+12EI_z\sin^2\alpha}{L_z^3}$	$\frac{(EA_zL_z^2-12EI_z)\cos\alpha\sin\alpha}{L_z^3}$	$\frac{-6EI_z\sin\alpha}{L_z^2}$
$\frac{(EA_zL_z^2-12EI_z)\cos\alpha\sin\alpha}{L_z^3}$	$\frac{EA_zL_z^2\sin^2\alpha+12EI_z\cos^2\alpha}{L_z^3}$	$\frac{6EI_z\cos\alpha}{L_z^2}$
$\frac{-6EI_z\sin\alpha}{L_z^2}$	$\frac{6EI_z\cos\alpha}{L_z^2}$	$\frac{4EI_z}{L_z}$
$\frac{-EA_zL_z^2\cos^2\alpha-12EI_z\sin^2\alpha}{L_z^3}$	$\frac{(-EA_zL_z^2+12EI_z)\cos\alpha\sin\alpha}{L_z^3}$	$\frac{6EI_z\sin\alpha}{L_z^2}$
$\frac{(-EA_zL_z^2+12EI_z)\cos\alpha\sin\alpha}{L_z^3}$	$\frac{-EA_zL_z^2\sin^2\alpha-12EI_z\cos^2\alpha}{L_z^3}$	$\frac{-6EI_z\cos\alpha}{L_z^2}$
$\frac{-6EI_z\sin\alpha}{L_z^2}$	$\frac{6EI_z\cos\alpha}{L_z^2}$	$\frac{2EI_z}{L_z}$

Right-Hand Half

(u_j, v_j, θ_j)

$\frac{-EA_zL_z^2\cos^2\alpha-12EI_z\sin^2\alpha}{L_z^3}$	$\frac{(-EA_zL_z^2+12EI_z)\cos\alpha\sin\alpha}{L_z^3}$	$\frac{-6EI_z\sin\alpha}{L_z^2}$
$\frac{(-EA_zL_z^2+12EI_z)\cos\alpha\sin\alpha}{L_z^3}$	$\frac{-EA_zL_z^2\sin^2\alpha-12EI_z\cos^2\alpha}{L_z^3}$	$\frac{6EI_z\cos\alpha}{L_z^2}$
$\frac{6EI_z\sin\alpha}{L_z^2}$	$\frac{-6EI_z\cos\alpha}{L_z^2}$	$\frac{2EI_z}{L_z}$
$\frac{EA_zL_z^2\cos^2\alpha+12EI_z\sin^2\alpha}{L_z^3}$	$\frac{(EA_zL_z^2-12EI_z)\cos\alpha\sin\alpha}{L_z^3}$	$\frac{6EI_z\sin\alpha}{L_z^2}$
$\frac{(EA_zL_z^2-12EI_z)\cos\alpha\sin\alpha}{L_z^3}$	$\frac{EA_zL_z^2\sin^2\alpha+12EI_z\cos^2\alpha}{L_z^3}$	$\frac{-6EI_z\cos\alpha}{L_z^2}$
$\frac{6EI_z\sin\alpha}{L_z^2}$	$\frac{-6EI_z\cos\alpha}{L_z^2}$	$\frac{4EI_z}{L_z}$

Table B-2: General Normal Section Stress Resultant Transform Matrix

$\frac{EA_x \cos\alpha}{L_x}$	$\frac{EA_x \sin\alpha}{L_x}$	0	$\frac{-EA_x \cos\alpha}{L_x}$	$\frac{-EA_x \sin\alpha}{L_x}$	0
$\frac{-12EI_z \sin\alpha}{L_x^3}$	$\frac{12EI_z \cos\alpha}{L_x^3}$	$\frac{6EI_z}{L_x^2}$	$\frac{12EI_z \sin\alpha}{L_x^3}$	$\frac{-12EI_z \cos\alpha}{L_x^3}$	$\frac{6EI_z}{L_x^2}$
$\frac{-6EI_z \sin\alpha}{L_x^2}$	$\frac{6EI_z \cos\alpha}{L_x^2}$	$\frac{4EI_z}{L_x}$	$\frac{6EI_z \sin\alpha}{L_x^2}$	$\frac{-6EI_z \cos\alpha}{L_x^2}$	$\frac{2EI_z}{L_x}$
$\frac{-EA_x \cos\alpha}{L_x}$	$\frac{-EA_x \sin\alpha}{L_x}$	0	$\frac{EA_x \cos\alpha}{L_x}$	$\frac{EA_x \sin\alpha}{L_x}$	0
$\frac{12EI_z \sin\alpha}{L_x^3}$	$\frac{-12EI_z \cos\alpha}{L_x^3}$	$\frac{-6EI_z}{L_x^2}$	$\frac{-12EI_z \sin\alpha}{L_x^3}$	$\frac{12EI_z \cos\alpha}{L_x^3}$	$\frac{-6EI_z}{L_x^2}$
$\frac{-6EI_z \sin\alpha}{L_x^2}$	$\frac{6EI_z \cos\alpha}{L_x^2}$	$\frac{2EI_z}{L_x}$	$\frac{6EI_z \sin\alpha}{L_x^2}$	$\frac{-6EI_z \cos\alpha}{L_x^2}$	$\frac{4EI_z}{L_x}$

Table B-3: Horizontal Beam Element Stiffness Matrix

$\frac{EA_x}{L_x}$	0	0	$\frac{-EA_x}{L_x}$	0	0
0	$\frac{12EI_z}{L_x^3}$	$\frac{6EI_z}{L_x^2}$	0	$\frac{-12EI_z}{L_x^3}$	$\frac{6EI_z}{L_x^2}$
0	$\frac{6EI_z}{L_x^2}$	$\frac{4EI_z}{L_x}$	0	$\frac{-6EI_z}{L_x^2}$	$\frac{2EI_z}{L_x}$
$\frac{-EA_x}{L_x}$	0	0	$\frac{EA_x}{L_x}$	0	0
0	$\frac{-12EI_z}{L_x^3}$	$\frac{-6EI_z}{L_x^2}$	0	$\frac{12EI_z}{L_x^3}$	$\frac{-6EI_z}{L_x^2}$
0	$\frac{6EI_z}{L_x^2}$	$\frac{2EI_z}{L_x}$	0	$\frac{-6EI_z}{L_x^2}$	$\frac{4EI_z}{L_x}$

Table B-4: Vertical Column Element Stiffness Matrix

$\frac{12EI_z}{L_x^3}$	0	$\frac{-6EI_z}{L_x^2}$	$\frac{-12EI_z}{L_x^3}$	0	$\frac{-6EI_z}{L_x^2}$
0	$\frac{EA_x}{L_x}$	0	0	$\frac{-EA_x}{L_x}$	0
$\frac{-6EI_z}{L_x^2}$	0	$\frac{4EI_z}{L_x}$	$\frac{6EI_z}{L_x^2}$	0	$\frac{2EI_z}{L_x}$
$\frac{-12EI_z}{L_x^3}$	0	$\frac{6EI_z}{L_x^2}$	$\frac{12EI_z}{L_x^3}$	0	$\frac{6EI_z}{L_x^2}$
0	$\frac{-EA_x}{L_x}$	0	0	$\frac{EA_x}{L_x}$	0
$\frac{-6EI_z}{L_x^2}$	0	$\frac{2EI_z}{L_x}$	$\frac{6EI_z}{L_x^2}$	0	$\frac{4EI_z}{L_x}$

Table B-5: Horizontal Beam Element Stress Resultant Transform Matrix

$\frac{EA_x}{L_x}$	0	0	$\frac{-EA_x}{L_x}$	0	0
0	$\frac{12EI_z}{L_x^3}$	$\frac{6EI_z}{L_x^2}$	0	$\frac{-12EI_z}{L_x^3}$	$\frac{6EI_z}{L_x^2}$
0	$\frac{6EI_z}{L_x^2}$	$\frac{4EI_z}{L_x}$	0	$\frac{-6EI_z}{L_x^2}$	$\frac{2EI_z}{L_x}$

Table B-6: Vertical Column Element Stress Resultant Transform Matrix

0	$\frac{EA_x}{L_x}$	0	0	$\frac{-EA_x}{L_x}$	0
$\frac{-12EI_z}{L_x^3}$	0	$\frac{6EI_z}{L_x^2}$	$\frac{12EI_z}{L_x^3}$	0	$\frac{6EI_z}{L_x^2}$
$\frac{-6EI_z}{L_x^2}$	0	$\frac{4EI_z}{L_x}$	$\frac{6EI_z}{L_x^2}$	0	$\frac{2EI_z}{L_x}$

B.3 Dimensionless Stiffness

This basic element formulation, introduces dimensionless area and inertia, as $\frac{A}{L^2}$, $\frac{I}{L^4}$. The term Q has been introduced as the factor of proportionality between the dimensionless area and inertia. This formulation results in the section properties being converted to a dimensionless form, normalized with respect to the dimensionless inertia of the section. Young's modulus, which is a constant factor in the denominator of the deflection terms, and appears in the stress resultant terms has been factored out. In general the two unconstrained design parameters, still exist, however for a given choice of section geometry, (ie. I sections, box sections) the factor of proportionality between inertia and area becomes a fixed constant.

```
%--- ELEMENTAL STIFFNESS COEFFICIENTS - ELASTIC FLEXURE/AXIAL ELEMENT
%      USING DIMENSIONLESS FACTORS I/L**4 AND A/L**2

ELTITLE:="ELASTIC NON-DIMENSIONAL FLEXURAL/AXIAL ELEMENT";
OPERATOR EE,Q,Ax,Iz,Lx;

LET K11=Iz*Lx/Q;
LET K12=0;
LET K13=0;
LET K22=12*Iz/Lx;
LET K23=6*Iz;
LET K33=4*Iz/Lx;

;END;
```

The following matrices were derived directly from the OUTPUT file <u>ELMAT</u>, and processed into a suitable numerical text format.

Table B-7: General Transformed Elemental Stiffness Matrix

Left-Hand Half

(u_i, v_i, θ_i)

$\frac{I_zL_x^2\cos^2\alpha+12I_z\sin^2\alpha Q}{L_xQ}$	$\frac{(L_x^2-12Q)I_z\cos\alpha\sin\alpha}{L_xQ}$	$\underline{-6I_z\sin\alpha}$
$\frac{(L_x^2-12Q)I_z\cos\alpha\sin\alpha}{L_xQ}$	$\frac{I_zL_x^2\sin^2\alpha+12I_z\cos^2\alpha Q}{L_xQ}$	$\underline{6I_z\cos\alpha}$
$\underline{-6I_z\sin\alpha}$	$\underline{6I_z\cos\alpha}$	$\frac{4I_z}{L_x}$
$\frac{-I_zL_x^2\cos^2\alpha-12I_z\sin^2\alpha Q}{L_xQ}$	$\frac{(-L_x^2+12Q)I_z\cos\alpha\sin\alpha}{L_xQ}$	$\underline{6I_z\sin\alpha}$
$\frac{-L_x^2+12Q)I_z\cos\alpha\sin\alpha Q}{L_xQ}$	$\frac{-I_zL_x^2\sin^2\alpha-12I_z\cos^2\alpha Q}{L_xQ}$	$\underline{-6I_z\cos\alpha}$
$\underline{-6I_z\sin\alpha}$	$\underline{6I_z\cos\alpha}$	$\frac{6I_zL_x^2-4I_z}{L_x}$

Right-Hand Half

(u_j, v_j, θ_j)

$\frac{-I_zL_x^2\cos^2\alpha-12I_z\sin^2\alpha Q}{L_xQ}$	$\frac{(-L_x^2+12Q)I_z\cos\alpha\sin\alpha}{L_xQ}$	$\underline{-6I_z\sin\alpha}$
$\frac{(-L_x^2+12Q)I_z\cos\alpha\sin\alpha}{L_xQ}$	$\frac{-I_zL_x^2\sin^2\alpha-12I_z\cos^2\alpha Q}{L_xQ}$	$\underline{6I_z\cos\alpha}$
$\underline{6I_z\sin\alpha}$	$\underline{-6I_z\cos\alpha}$	$\frac{6I_zL_x^2-4I_z}{L_x}$
$\frac{I_zL_x^2\cos^2\alpha+12I_z\sin^2\alpha Q}{L_xQ}$	$\frac{(L_x^2-12Q)I_z\cos\alpha\sin\alpha}{L_xQ}$	$\underline{6I_z\sin\alpha}$
$\frac{(L_x^2-12Q)I_z\cos\alpha\sin\alpha}{L_xQ}$	$\frac{I_zL_x^2\sin^2\alpha+12I_z\cos^2\alpha Q}{L_xQ}$	$\underline{-6I_z\cos\alpha}$
$\underline{6I_z\sin\alpha}$	$\underline{-6I_z\cos\alpha}$	$\frac{4I_z}{L_x}$

Table B-8: General Normal Section Stress Resultant Transform Matrix

$$
\begin{bmatrix}
\frac{I_zL_x\cos\alpha}{Q} & \frac{I_zL_x\sin\alpha}{Q} & 0 & \frac{-I_zL_x\cos\alpha}{Q} & \frac{-I_zL_x\sin\alpha}{Q} & 0 \\
\frac{-12I_z\sin\alpha}{L_x} & \frac{12I_z\cos\alpha}{L_x} & 6I_z & \frac{12I_z\sin\alpha}{L_x} & \frac{-12I_z\cos\alpha}{L_x} & 6I_z \\
-6I_z\sin\alpha & 6I_z\cos\alpha & \frac{4I_z}{L_x} & 6I_z\sin\alpha & -6I_z\cos\alpha & \frac{6I_zL_x^2-4I_z}{L_x} \\
\frac{-I_zL_x\cos\alpha}{Q} & \frac{-I_zL_x\sin\alpha}{Q} & 0 & \frac{I_zL_x\cos\alpha}{Q} & \frac{I_zL_x\sin\alpha}{Q} & 0 \\
\frac{12I_z\sin\alpha}{L_x} & \frac{-12I_z\cos\alpha}{L_x} & -6I_z & \frac{-12I_z\sin\alpha}{L_x} & \frac{12I_z\cos\alpha}{L_x} & -6I_z \\
-6I_z\sin\alpha & 6I_z\cos\alpha & \frac{6I_zL_x^2-4I_z}{L_x} & 6I_z\sin\alpha & -6I_z\cos\alpha & \frac{4I_z}{L_x}
\end{bmatrix}
$$

Table B-9: Horizontal Beam Element Stiffness Matrix

$$
\begin{bmatrix}
\frac{I_zL_x}{Q} & 0 & 0 & \frac{-I_zL_x}{Q} & 0 & 0 \\
0 & \frac{12I_z}{L_x} & 6I_z & 0 & \frac{-12I_z}{L_x} & 6I_z \\
0 & 6I_z & \frac{4I_z}{L_x} & 0 & -6I_z & \frac{6I_zL_x^2-4I_z}{L_x} \\
\frac{-I_zL_x}{Q} & 0 & 0 & \frac{I_zL_x}{Q} & 0 & 0 \\
0 & \frac{-12I_z}{L_x} & -6I_z & 0 & \frac{12I_z}{L_x} & -6I_z \\
0 & 6I_z & \frac{6I_zL_x^2-4I_z}{L_x} & 0 & -6I_z & \frac{4I_z}{L_x}
\end{bmatrix}
$$

Table B-10: Vertical Column Element Stiffness Matrix

$\frac{12I_z}{L_x}$	0	$\frac{-6I_z}{\ }$	$\frac{-12I_z}{L_x}$	0	$\frac{-6I_z}{\ }$
0	$\frac{I_zL_x}{Q}$	0	0	$\frac{-I_zL_x}{Q}$	0
$\frac{-6I_z}{\ }$	0	$\frac{4I_z}{L_x}$	$\frac{6I_z}{\ }$	0	$\frac{6I_zL_x^2-4I_z}{L_x}$
$\frac{-12I_z}{L_x}$	0	$\frac{6I_z}{\ }$	$\frac{12I_z}{L_x}$	0	$\frac{6I_z}{\ }$
0	$\frac{-I_zL_x}{Q}$	0	0	$\frac{I_zL_x}{Q}$	0
$\frac{-6I_z}{\ }$	0	$\frac{6I_zL_x^2-4I_z}{L_x}$	$\frac{6I_z}{\ }$	0	$\frac{4I_z}{L_x}$

Table B-11: Horizontal Beam Element Stress Resultant Transform Matrix

$\frac{I_zL_x}{Q}$	0	0	$\frac{-I_zL_x}{Q}$	0	0
0	$\frac{12I_z}{L_x}$	$\frac{6I_z}{\ }$	0	$\frac{-12I_z}{L_x}$	$\frac{6I_z}{\ }$
0	$\frac{6I_z}{\ }$	$\frac{4I_z}{L_x}$	0	$\frac{-6I_z}{\ }$	$\frac{6I_zL_x^2-4I_z}{L_x}$

Table B-12: Vertical Column Element Stress Resultant Transform Matrix

0	$\frac{I_zL_x}{Q}$	0	0	$\frac{-I_zL_x}{Q}$	0
$\frac{12I_z}{L_x}$	0	$\frac{6I_z}{\ }$	$\frac{12I_z}{L_x}$	0	$\frac{6I_z}{\ }$
$\frac{-6I_z}{\ }$	0	$\frac{4I_z}{L_x}$	$\frac{6I_z}{\ }$	0	$\frac{6I_zL_x^2-4I_z}{L_x}$

Appendix C
REDUCE Problem Formulation

The REDUCE formulation consists of four major file types. This appendix contains brief copies of the INPUT files used in the problem solved in this study, and the system formulation, solution and analysis REDUCE files.

C.1 The Files Used

The following REDUCE data files are used:

- STIF Contains the basic stiffness terms
- STRUC Defines the size and special features of the structure

The following REDUCE data files are generated:

- ELSTIF Contains the 6x6 element stiffness matrices
- ELSTRS Contains the 3x6 stress resultant transform matrices
- DELTA Contains the deflection vector for the system

The following REDUCE output files are created:

- ELMAT Contains all elemental matrices
- OUTSR Contains Pa,Va,Ma,Mb stress resultants for all elements
- IFLSR Contains deflection and stress influence coefficients
- MAXSR Contains Mmax, the other maximal stress in beams
- AKAFLX Contains full printout of system solution
- SOL1SR Contains numerical solution for stress resultants
- PLTSR Contains FORTRAN compatible equations for influence coefficients

The following REDUCE command files generate the solutions:

- GENEL Generates ELSTIF and ELSTRS data files, using STIF
- SOLVE Solves the system, given by STRUC, generates DELTA
- STRESS Using DELTA, generates output file OUTSR
- INFLU Using DELTA, generates influence coefficients under unit loads
- MAXIM Using DELTA, generates output file MAXSR
- MAKELK Generates the output file ELMAT
- PLOTS Generates FORTRAN equations in output file PLTSR
- SOLFLX Generates output file AKAFLX, containing the full general solution fo the system, and substitutes in dimensionless parameters to aid in proving the dimensional consistency of the solution.
- SOL1 Generates an numerical test solution, in output file AKAFLX
- SOL1ST Generates an numerical test stress resultant solution into output file SOL1SR

C.2 System Definition - STRUC

The structure is defined by the parameters that are setup in the INPUT file **STRUC**, which is listed below. This listing shows the change in parameter naming convention that was used in all subsequent output. The REDUCE LET statement allows for easy parameter renaming, and equivalencing.

```
% ****  STRUC  **** SYSTEM PARAMETERS
SYSTITLE:="RIGID SUPPORTS, FULL 3 DOF PER NODE";

LET NBAY=1;
LET MLEVEL=1;

% -- ALTERNATIVE PARAMETER FORMS
LET G(1)=G;
LET IB(1,1)=IB, AB(1,1)=AB;
LET IC(1,1)=IL, AC(1,1)=AL;
LET IC(1,2)=IR, AC(1,2)=AR;
LET QC(1,1)=QL, QB(1,1)=QB, QC(1,2)=QR;
LET W(1,1)=W;
```

```
% -- OVERRIDE PROGRAM LINE LENGTH FOR LASER OUTPUTTING
LINELENGTH 230;
```

C.3 REDUCE Element Matrices Formulation

The system assembly only pertains to the use of vertical and horizontal elements. In general the element stiffness matrix can be left with angle α as unspecified, thereby permitting full system generality. GENEL produces the DATA files *ELSTIF* and *ELSTRS*, which are only pertinant for rectilinear systems.

```
% ****   GENEL   ****
%
%-- PROGRAM TO GENERATE THE ELEMENTAL STIFFNESS AND STRESS TRANSFORM MATRICES
%
% DATE: FEB-86
% AUTHOR: A. RANKINE
% MASTER'S THESIS WORK
%
%  INPUT FILES:  STIF   = LOCAL STIFFNESS COEFFICIENTS
%  OUTPUT FILES: ELSTIF = HORZ/VERT STIFFNESS MATRICES
%                ELSTRS = HORZ/VERT STRESS TRANSFORM MATRICES
%
LINELENGTH(115);
OFF ALLFAC;         % BRING ANY FACTORS OUT
ON EXP;             % USE EXPANDED POWERS     (ON)
OFF DIV;
OFF LIST;           % LIST NOT DOWN THE PAGE (OFF)
OFF FORT;           % NO FORTRAN TYPE OUTPUT (OFF)

%---------------------------------------------------------------
%--- SYSTEM CONSTANTS
 LET PDOF=3,
     DOF=6;

%---------------------------------------------------------------
%--- ELEMENTAL STIFFNESS, ROTATION AND FORCE TRANSFORM MATRICES

MATRIX KL(6,6),KH(6,6),KV(6,6);
MATRIX SL(6,6),SH(3,6),SV(3,6);

ORDER EE,AX,IZ,LX;    % ORDER OUTPUT AS REGULAR EA, EI FORMAT.
KORDER EE,IZ,AX,LX;   % SET AS MAJOR VARIABLES, SPEEDUP INVERSION TIMES

%--- ELEMENTAL STIFFNESS COEFFICIENTS
%
 IN STIF;      % READIN AND ECHO STIFFNESS COEFICIENTS
%---------------------------------------------------------------

%%%%%%%%%%%%%%%%%%%
PROCEDURE FORMELK();
```

```
%%%%%%%%%%%%%%%%%%%%
 BEGIN
  MATRIX LD(6,6), TT(6,3), K(3,3), TX(6,3);

WRITE "START - FORMATION OF BASIC ELEMENT STIFFNESS AND STRESS MATRICES";

LD := MAT((COS A,-SIN A, O  ,  O  ,  O  ,  O  ),
          (SIN A, COS A, O  ,  O  ,  O  ,  O  ),
          (  O  ,   O  , 1  ,  O  ,  O  ,  O  ),
          (  O  ,   O  , O  ,COS A,-SIN A, O  ),
          (  O  ,   O  , O  ,SIN A, COS A, O  ),
          (  O  ,   O  , O  ,  O  ,  O  ,  1  ))$

TT := MAT((  1  ,  O  ,  O  ),
          (  O  ,  1  ,  O  ),
          (  O  ,  O  ,  1  ),
          ( -1  ,  O  ,  O  ),
          (  O  , -1  ,  O  ),
          (  O  ,  Lx ,  -1  ))$

 K := MAT(( K11 , K12 , K13 ),
          ( K12 , K22 , K23 ),
          ( K13 , K23 , K33 ))$

  TX := LD*TT$

  KL := TX*K*TP(TX);  % DO ELEMENT TRANSFORMATION TO GET GENERAL FORM

  SL:= TT*K*TP(TX);

  FOR I:=1:DOF DO <<  % SUBSTITUTE IN ANGLES TO GET VERTICAL/HORIZONTAL FORMS
   FOR J:=1:DOF DO <<
    KH(I,J):=SUB(SIN A=O,COS A=1,KL(I,J));
    IF I<=PDOF THEN SH(I,J):=SUB(SIN A=O,COS A=1,SL(I,J));
   >>;
  >>;

  FOR I:=1:DOF DO <<
   FOR J:=1:DOF DO <<
    KV(I,J):=SUB(SIN A=1,COS A=O,KL(I,J));
    IF I<=PDOF THEN SV(I,J):=SUB(SIN A=1,COS A=O,SL(I,J));
   >>;
  >>;
 WRITE "END - FORMATION OF BASIC ELEMENT STIFFNESS AND STRESS MATRICES";
END;


%--- ****************************
%--- ******* MAIN PROGRAM *******
%--- ****************************

 FORMELK();     % CALL PROCEDURE TO DEFINE ELEMENT MATRICES

OFF ECHO$
OFF NAT$
OUT ELSTIF$          % CREATE THE 'ELSTIF' DATA FILE
LINELENGTH 72$
```

```
 WRITE "% **** ELSTIF **** KV,KH  DATA FILE";
 WRITE "% ",ELTITLE;
 WRITE "MATRIX KV(DOF,DOF),KH(DOF,DOF)";
 WRITE "%--  HORIZONTAL STIFFNESS MATRIX";
 KH:=KH;
 WRITE "%--  VERTICAL STIFFNESS MATRIX";
 KV:=KV;
 WRITE ";END;";
SHUT ELSTIF$

OUT ELSTRS$
 WRITE "% **** ELSTRS **** SV,SH  DATA FILE";
 WRITE "% ",ELTITLE;
 WRITE "MATRIX SV(PDOF,DOF),SH(PDOF,DOF)";
 WRITE "%--  HORIZONTAL STRESS RESULTANT TRANSFORM MATRIX";
 SH:=SH;
 WRITE "%--  VERTICAL STRESS RESULTANT TRANSFORM MATRIX";
 SV:=SV;
 WRITE ";END;";
SHUT ELSTRS$

ON NAT$

;END;
```

C.4 REDUCE Assembly and Solution

The system is assembled in accordance with the definitions given in the INPUT file **STRUC**. The REDUCE file SOLVE, uses this data and the DATA file *ELSTIF* to formulate the global system stiffness matrix, and the force vector. The system is solved and the DATA file *DELTA* is produced, which contains the solution in terms of the design, geometric and force parameters of the system.

```
%--PROGRAM TO ASSEMBLE AND SOLVE RECTILINEAR FRAME PROBLEM
%
% DATE: FEB-86
% AUTHOR: A. RANKINE
% MASTER'S THESIS WORK
%
%  INPUT FILES :  ELSTIF = ELEMENTAL STIFFNESS MATRICES
%                 STRUC  = STRUCTURAL PARAMETERS FOR THIS PROBLEM
%  OUTPUT FILE :  DELTA  = DEFLECTION VECTOR, GENERAL SOLUTION
%
LINELENGTH(80);
OFF ALLFAC;        % BRING ANY FACTORS OUT
ON EXP;           % USE EXPANDED POWERS     (ON)
OFF DIV;
OFF LIST;         % LIST NOT DOWN THE PAGE  (OFF)
OFF FORT;         % NO FORTRAN TYPE OUTPUT  (OFF)
```

```
%--------------------------------------------------------------
%--- SYSTEM CONSTANTS
LET PDOF=3,
    DOF=6;

%--------------------------------------------------------------
%--- GET DATA FROM INPUT FILES ---
 IN STIF$
%--- ELEMENTAL STIFFNESS
 IN ELSTIF$

%--- READ-IN GEOMETRIC PARAMETERS
 IN STRUC$

%--------------------------------------------------------------
%--- GLOBAL WORKING VARIABLE DECLARATION

LET NNODE=(NBAY+1)*(MLEVEL),
    GDOF=PDOF*NNODE,
    NPBAY=NBAY+1;

MATRIX AKA(GDOF,GDOF);     %- GLOBAL STIFFNESS MATRIX
MATRIX FLEX(GDOF,GDOF);    %- GLOBAL FLEXIBILITY MATRIX

MATRIX DV(GDOF,1);         %- DEFLECTION VECTOR
MATRIX FG(GDOF,1);         %- WIND AND GRAVITY LOAD VECTOR

OPERATOR W;       % SYMBOL FOR UDL
OPERATOR G;       % HEIGHT TO SPAN RATIO
OPERATOR AB,IB;   % SYMBOLS FOR AREA AND MOMENT OF INERTIA - BEAMS
OPERATOR AC,IC;   % SYMBOLS FOR AREA AND MOMENT OF INERTIA - COLUMNS

ORDER EE,AB,AC,IB,IC,L;    % ORDER OUTPUT AS REGULAR EA, EI FORMAT.
KORDER EE,IC,IB,AC,AB,L;   % SET AS MAJOR VARIABLES, SPEEDUP INVERSION TIMES

%--- ********************************************
%--- ***** ELEMENTAL TO GLOBAL ASSEMBLY PROC ******
%--- ********************************************

%%%%%%%%%%%%%%%%
PROCEDURE ASSEMKV(LEVEL,BAY,ATI,ATJ,FRMI,FRMJ);
%%%%%%%%%%%%%%%%
 BEGIN
   FOR I1:=0:2 DO
     FOR J1:=0:2 DO
      << WRITE "KV(",FRMI+I1,",",FRMJ+J1,")=",KV(FRMI+I1,FRMJ+J1),
               " --> AKA(",ATI+I1,",",ATJ+J1,")";
       AKA(ATI+I1,ATJ+J1):=AKA(ATI+I1,ATJ+J1)+SUB(Iz=IC(LEVEL,BAY),
           Q=QC(LEVEL,BAY),Ax=AC(LEVEL,BAY),Lx=L,KV(FRMI+I1,FRMJ+J1));
      >>;
 END$

%%%%%%%%%%%%%%%%
PROCEDURE ASSEMKH(LEVEL,BAY,ATI,ATJ,FRMI,FRMJ);
%%%%%%%%%%%%%%%%
 BEGIN
   FOR I1:=0:2 DO
     FOR J1:=0:2 DO
```

```
      << WRITE "KH(",FRMI+I1,",",FRMJ+J1,")=",KH(FRMI+I1,FRMJ+J1),
                 " --> AKA(",ATI+I1,",",ATJ+J1,")";
       AKA(ATI+I1,ATJ+J1):=AKA(ATI+I1,ATJ+J1)+SUB(Iz=IB(LEVEL,BAY),
           Q=QB(LEVEL,BAY),Ax=AB(LEVEL,BAY),Lx=G(BAY)*L,KH(FRMI+I1,FRMJ+J1));
      >>;
 END$

%%%%%%%%%%%%%
PROCEDURE WIND(LEV);    %-- lINEAR VARIATION OF WIND INTENSITY WITH HEIGHT
%%%%%%%%%%%%%
 BEGIN
  WD:=LEV/MLEVEL*WM/2;
  RETURN WD;
 END$

%%%%%%%%%%%%%%%%%
PROCEDURE ASSEMBLE();
%%%%%%%%%%%%%%%%%
 BEGIN
 WRITE "START ASSEMBLY PROCESS";

 FOR LEVEL:=1:MLEVEL DO
  FOR BAY:=1:NPBAY DO
   BEGIN

%--- ADD-IN COLUMN
    NI:=(LEVEL-2)*(NPBAY)+BAY-1;
    NJ:=NI+NPBAY;
    II:=PDOF*NI+1;
    JJ:=PDOF*NJ+1;
    WRITE "COL MEMBER ASSEMBLY AT EQU ",II," AND EQU ",JJ;
    IF LEVEL>1 THEN ASSEMKV(LEVEL,BAY,II,II,1,1);
    IF LEVEL>1 THEN ASSEMKV(LEVEL,BAY,II,JJ,1,PDOF+1);
    IF LEVEL>1 THEN ASSEMKV(LEVEL,BAY,JJ,II,PDOF+1,1);
    ASSEMKV(LEVEL,BAY,JJ,JJ,PDOF+1,PDOF+1);

    IF (BAY = NPBAY) THEN
     BEGIN
      IF LEVEL>1 THEN FG(II,1):=FG(II,1)+WIND(LEVEL-1);
      FG(JJ,1):=FG(JJ,1)+WIND(LEVEL);
     END
    ELSE
     BEGIN

%--- ADD-IN BEAM
      NI:=NJ;
      NJ:=NJ+1;
      II:=PDOF*NI+1;
      JJ:=PDOF*NJ+1;
      WRITE "BEAM MEMBER ASSEMBLY AT EQU ",II," AND EQU ",JJ;
      ASSEMKH(LEVEL,BAY,II,II,1,1);
      ASSEMKH(LEVEL,BAY,II,JJ,1,PDOF+1);
      ASSEMKH(LEVEL,BAY,JJ,II,PDOF+1,1);
      ASSEMKH(LEVEL,BAY,JJ,JJ,PDOF+1,PDOF+1);

%--- ADD-IN BEAM GRAVITY LOADS
      FG(II+1,1):=FG(II+1,1)-W(LEVEL,BAY)*L*G(BAY)/2;
      FG(II+2,1):=FG(II+2,1)-W(LEVEL,BAY)*L**2*G(BAY)**2/12;
```

```
      FG(JJ+1,1):=FG(JJ+1,1)-W(LEVEL,BAY)*L*G(BAY)/2;
      FG(JJ+2,1):=FG(JJ+2,1)+W(LEVEL,BAY)*L**2*G(BAY)**2/12;
     END
   END
 END;


%%%%%%%%%%%%%%%
PROCEDURE SETUP();
%%%%%%%%%%%%%%%
 BEGIN
  WRITE "% STRUCTURAL SPECIFICATIONS:";
  WRITE "% ELEMENT TYPE IS ",ELTITLE;
  WRITE "% SYSTEM TYPE IS ",SYSTITLE;
  WRITE "% NUMBER OF BAYS   = ",NBAY;
  WRITE "% NUMBER OF LEVELS= ",MLEVEL;
  WRITE "% NUMBER OF NODES = ",NNODE;
  WRITE "% GLOBAL D.O.F.    = ",GDOF;

 END;

%%%%%%%%%%%%%%%
PROCEDURE SOLVE();
%%%%%%%%%%%%%%%
 BEGIN
  WRITE "ASSEMBLY COMPLETED";

  WRITE "INVERTING AND SOLVING GLOBAL STIFFNESS FOR FORCE SET";
  FLEX:=1/AKA;

 END;

%--- ****************************
%--- ******* MAIN PROGRAM *******
%--- ****************************

 SETUP();           % SETUP ELEMENT K, AND GLOBAL K.
 ASSEMBLE();        % FORM GLOBAL STIFFNESS MATRIX
 SOLVE();           % RUN AND GET GENERAL SOLUTION

OFF ECHO$
OFF NATS$
OUT DELTA$
 DV:=FLEX*FG;
SHUT DELTA$

;END;
```

C.5 REDUCE Post-Solution Analysis - Stress Resultants

The system is fully defined by 'n' equations in many unspecified parameters:

1. Member Properties - E, $I^{2.\ i}_{3.\ j}$, $A^{2.\ i}_{3.\ j}$
2. Geometric Properties - L_i
3. Load Conditions - $W^{4.\ i}_{5.\ j}$, WD_j
4. Coefficients - reflecting the inter-connectivity of the system elements

These equations are the body of the DATA file *DELTA*, with zero coefficient terms not shown, see Appendix D, Section D.1. The equations are generalized multivariate expansions, with no simple common factors. All terms in a given equation are dimensionally consistent. This means that the product of all length dimensions in a term must multiply out to a common minimum form, which is consistent for all terms in the equation. In this case the simplest form is the dimensional form of the deflection term, on the right-hand side.

Another REDUCE procedure was required to apply simple matrix operations to obtain the member end stress resultants, this was the sole function of STRESS. The resulting 'n' equations, express the end stress resultants in terms of the same parameters as in *DELTA*, except that all terms are of different dimensional form, and now include a force factor.

```
% ****  STRESS   ****
%--PROGRAM TO FORMULATE THE STRESSES IN THE RECTILINEAR FRAME MEMBERS
%
% DATE: FEB-86
% AUTHOR: A. RANKINE
% MASTER'S THESIS WORK
%
%  INPUT FILES : ELSTRS = ELEMENT STIFFNESS AND STRESS TRANSFORM MATRICES
%                STRUC  = SYSTEM PARAMETERS
%                DELTA  = SYSTEM DEFLECTION VECTOR (GENERAL)
%  OUTPUT FILES: OUTSR  = STRESS RESULTANT OUTPUT
%
LINELENGTH(200);
```

```
OFF ALLFAC;          % BRING ANY FACTORS OUT
ON EXP;              % USE EXPANDED POWERS     (ON)
OFF DIV;
OFF LIST;            % LIST NOT DOWN THE PAGE (OFF)
OFF FORT;            % NO FORTRAN TYPE OUTPUT (OFF)

%----------------------------------------------------------------
%--- SYSTEM CONSTANTS
LET PDOF=3,
    DOF=6;

%----------------------------------------------------------------
%--- GEOMETRIC PARAMETERS
 IN STRUC$

%----------------------------------------------------------------
%--- GLOBAL WORKING VARABLES DECLARATION

LET NNODE=(NBAY+1)*(MLEVEL),
    GDOF=PDOF*NNODE,
    NPBAY=NBAY+1;

MATRIX DV(GDOF+PDOF,1);         %- DEFLECTION VECTOR PLUS ZERO VECTOR (RIGID)

OPERATOR W;        % SYMBOL FOR UDL
OPERATOR G;        % HEIGHT TO SPAN RATIO
OPERATOR AB,IB;    % SYMBOLS FOR AREA AND MOMENT OF INERTIA - BEAMS
OPERATOR AC,IC;    % SYMBOLS FOR AREA AND MOMENT OF INERTIA - COLUMNS

ORDER EE,AB,AC,IB,IC,L;    % ORDER OUTPUT AS REGULAR EA, EI FORMAT.
KORDER EE,IC,IB,AC,AB,L;   % SET AS MAJOR VARIABLES, SPEEDUP INVERSION TIMES

%----------------------------------------------------------------
%--- GET DATA FROM ALL INPUT FILES ----
 IN STIF$
%--- ELEMENTAL STRESS TRANSFORM MATRICES
 IN ELSTRS$
 IN DELTA$
%----------------------------------------------------------------

%%%%%%%%%%%%%%
PROCEDURE SETUP();
%%%%%%%%%%%%%%
 BEGIN
  WRITE "% STRUCTURAL SPECIFICATIONS:";
  WRITE "% ELEMENT TYPE IS ",ELTITLE;
  WRITE "% SYSTEM TYPE IS ",SYSTITLE;
  WRITE "% NUMBER OF BAYS   = ",NBAY;
  WRITE "% NUMBER OF LEVELS= ",MLEVEL;
  WRITE "% NUMBER OF NODES = ",NNODE;
  WRITE "% GLOBAL D.O.F.    = ",GDOF;

 END;

%%%%%%%%%%%%%%%%%
PROCEDURE STRESSES();
%%%%%%%%%%%%%%%%%
 BEGIN
```

```
  WRITE "BASIC ELEMENT STRESSES";

 FOR LEVEL:=1:MLEVEL DO
  FOR BAY:=1:NPBAY DO
   BEGIN
    CLEAR PA,VA,MA,MMAX,XMAX,VMAX,BENDSTR$

%--- COLUMN
    NI:=(LEVEL-2)*(NPBAY)+BAY-1;
    NJ:=NI+NPBAY;
    II:=PDOF*NI+1;
    JJ:=PDOF*NJ+1;
    IF II<1 THEN II:=GDOF+1;  % RIGID SUPPORT NODE

    WRITE "COLUMN MEMBER FROM NODE ",NI+1," TO ",NJ+1;
    PA:=SV(1,1)*DV(II,1)+SV(1,2)*DV(II+1,1)+SV(1,3)*DV(II+2,1)$
    PA:=PA+SV(1,4)*DV(JJ,1)+SV(1,5)*DV(JJ+1,1)+SV(1,6)*DV(JJ+2,1)$
    PA:=SUB(Q=QC(LEVEL,BAY),LX=L,AX=AC(LEVEL,BAY),IZ=IC(LEVEL,BAY),PA);
    WRITE "PA := ",PA;
CLEAR PA$
    VA:=SV(2,1)*DV(II,1)+SV(2,2)*DV(II+1,1)+SV(2,3)*DV(II+2,1)$
    VA:=VA+SV(2,4)*DV(JJ,1)+SV(2,5)*DV(JJ+1,1)+SV(2,6)*DV(JJ+2,1)$
    VA:=SUB(Q=QC(LEVEL,BAY),LX=L,AX=AC(LEVEL,BAY),IZ=IC(LEVEL,BAY),VA);

    MA:=SV(3,1)*DV(II,1)+SV(3,2)*DV(II+1,1)+SV(3,3)*DV(II+2,1)$
    MA:=MA+SV(3,4)*DV(JJ,1)+SV(3,5)*DV(JJ+1,1)+SV(3,6)*DV(JJ+2,1)$
    MA:=SUB(Q=QC(LEVEL,BAY),LX=L,AX=AC(LEVEL,BAY),IZ=IC(LEVEL,BAY),MA);

    WRITE "Va := ",VA;
    WRITE "Ma := ",MA;
    WRITE "Mb := ",VA*L-MA;
%     WRITE " MAX MOMENT Mmax = ",W(LEVEL,BAY)*L**2/8+VA*L/2+1/2*VA**2/W(LEVEL,BAY)
%     WRITE "   MAX AT    Xmax = ",L/2+VA/W(LEVEL,BAY);
%     WRITE " MAX SHEAR   Vmax = ",ABS(VA)+ABS(W(LEVEL,BAY)*L/2);
%     BENDSTR:=ABS(W(LEVEL,BAY)*L**2/8+VA*L/2+1/2*VA**2/W(LEVEL,BAY)-MA*D/2/IC(LEVE
%     BENDSTR:=SUB(IC(LEVEL,BAY)=B*D**3/12,AC(LEVEL,BAY)=B*D,BENDSTR);
%     WRITE " MAX BEND STRESS RECT SECTION= ",BENDSTR;
   END;

 FOR LEVEL:=1:MLEVEL DO
  FOR BAY:=1:NBAY DO
   BEGIN

    CLEAR PA,VA,MA,MMAX,XMAX,VMAX,BENDSTR$

%--- BEAM
    NI:=(LEVEL-1)*NPBAY+BAY-1;
    NJ:=NI+1;
    II:=PDOF*NI+1;
    JJ:=PDOF*NJ+1;

    WRITE "BEAM MEMBER FROM NODE ",NI+1," TO ",NJ+1;
    PA:=SH(1,1)*DV(II,1)+SH(1,2)*DV(II+1,1)+SH(1,3)*DV(II+2,1)$
    PA:=PA+SH(1,4)*DV(JJ,1)+SH(1,5)*DV(JJ+1,1)+SH(1,6)*DV(JJ+2,1)$
    PA:=SUB(Q=QB(LEVEL,BAY),LX=G(BAY)*L,AX=AB(LEVEL,BAY),IZ=IB(LEVEL,BAY),PA);
    WRITE "Pa := ",PA;
CLEAR PA$
    VA:=SH(2,1)*DV(II,1)+SH(2,2)*DV(II+1,1)+SH(2,3)*DV(II+2,1)$
```

```
    VA:=VA+SH(2,4)*DV(JJ,1)+SH(2,5)*DV(JJ+1,1)+SH(2,6)*DV(JJ+2,1)$
    VA:=SUB(Q=QB(LEVEL,BAY),LX=G(BAY)*L,AX=AB(LEVEL,BAY),IZ=IB(LEVEL,BAY),VA);

    MA:=SH(3,1)*DV(II,1)+SH(3,2)*DV(II+1,1)+SH(3,3)*DV(II+2,1)$
    MA:=MA+SH(3,4)*DV(JJ,1)+SH(3,5)*DV(JJ+1,1)+SH(3,6)*DV(JJ+2,1)$
    MA:=SUB(Q=QB(LEVEL,BAY),LX=G(BAY)*L,AX=AB(LEVEL,BAY),IZ=IB(LEVEL,BAY),MA);

    WRITE "Va := ",VA;
    WRITE "Ma := ",MA;
    WRITE "Mb := ",VA*(G(BAY)*L)-MA;
%    WRITE " MAX MOMENT Mmax = ",W(LEVEL,BAY)*(G(BAY)*L)**2/8+VA*(G(BAY)*L)/2+1/2*
%    WRITE "    MAX AT    Xmax = ",(G(BAY)*L)/2+VA/W(LEVEL,BAY);
%    WRITE " MAX SHEAR  Vmax = ",ABS(VA)+ABS(W(LEVEL,BAY)*(G(BAY)*L)/2);
%    BENDSTR:=ABS(W(LEVEL,BAY)*(G(BAY)*L)**2/8+VA*(G(BAY)*L)/2+1/2*VA**2/W(LEVEL,B
%    BENDSTR:=SUB(IB(LEVEL,BAY)=B*D**3/12,AB(LEVEL,BAY)=B*D,BENDSTR);
%    WRITE " MAX BEND STRESS RECT SECTION= ",BENDSTR;

   END;

 END;

%--- ****************************
%--- ******* MAIN PROGRAM *******
%--- ****************************


OFF NAT$
OUT OUTSR;
% ON FLOAT$
% LET AL=100, AB=100, AR=100;     % TESTING ONLY
% LET IL=500, IB=500, IR=500;
% LET G=1, L=12, EE=30000;
% LET W(1,1)=1, W=1, WD=5;

 SETUP();          % SETUP ELEMENT K, AND GLOBAL K.
 STRESSES();       % CALCULATE THE WORST CASE DEFLECTIONS AND STRESSES
SHUT OUTSR;

;END;
```

C.6 REDUCE Post-Solution Analysis - Influence Coefficients

The analysis is not complete until the stress resultants have been re-expressed in terms of the load conditions alone. This produces an 'n' by 'm' matrix of influence factors, relating unit distributed loads and lateral loads to the stresses that the unit load generates for the 'n' stress sections of the system. Due to REDUCE limitations, another REDUCE file called INFLU was required to carry out this process.

```
% ****  INFLU  ****
```

```
%--PROGRAM TO FORMULATE THE STRESSES IN THE RECTILINEAR FRAME MEMBERS
%      AND OUTPUT THE INFLUENCE TABLE, STRESSES VS. UNIT LOAD PLACEMENT
%
% DATE: FEB-86
% AUTHOR: A. RANKINE
% MASTER'S THESIS WORK
%
%  INPUT FILES  : ELSTRS = ELEMENT STIFFNESS AND STRESS TRANSFORM MATRICES
%                 STRUC  = SYSTEM PARAMETERS
%                 DELTA  = SYSTEM DEFLECTION VECTOR (GENERAL)
%  OUTPUT FILES: IFLSR  = INFLUENCE COEFFICIENTS FOR STRESS RESULTANTS
%
LINELENGTH(240);
OFF ALLFAC;        % BRING ANY FACTORS OUT
ON EXP;            % USE EXPANDED POWERS     (ON)
OFF DIV;
OFF LIST;          % LIST NOT DOWN THE PAGE (OFF)
OFF FORT;          % NO FORTRAN TYPE OUTPUT (OFF)

%-----------------------------------------------------------------
%--- SYSTEM CONSTANTS
LET PDOF=3,
    DOF=6;

%-----------------------------------------------------------------
%--- GEOMETRIC PARAMETERS
 IN STRUC$

%-----------------------------------------------------------------
%--- GLOBAL WORKING VARABLES DECLARATION

LET NNODE=(NBAY+1)*(MLEVEL),
    GDOF=PDOF*NNODE,
    NPBAY=NBAY+1;

MATRIX DV(GDOF+PDOF,1);        %- DEFLECTION VECTOR PLUS ZERO VECTOR (RIGID)

OPERATOR W;        % SYMBOL FOR UDL
OPERATOR G;        % HEIGHT TO SPAN RATIO
OPERATOR AB,IB;    % SYMBOLS FOR AREA AND MOMENT OF INERTIA - BEAMS
OPERATOR AC,IC;    % SYMBOLS FOR AREA AND MOMENT OF INERTIA - COLUMNS

ORDER EE,AB,AC,IB,IC,L;    % ORDER OUTPUT AS REGULAR EA, EI FORMAT.
KORDER EE,IC,IB,AC,AB,L;   % SET AS MAJOR VARIABLES, SPEEDUP INVERSION TIMES

%-----------------------------------------------------------------
%--- GET DATA FROM ALL INPUT FILES ----
 IN STIF$
%--- ELEMENTAL STRESS TRANSFORM MATRICES
 IN ELSTRS$

%--- SOLVED DEFLECTION VECTOR
 IN DELTA$   % <=== NO ECHO IN NATURAL FORMAT, HARD TO READ

%-----------------------------------------------------------------

%%%%%%%%%%%%%%%
PROCEDURE SETUP();
```

```
%%%%%%%%%%%%%%%
 BEGIN
  WRITE "% STRUCTURAL SPECIFICATIONS:";
  WRITE "% ELEMENT TYPE IS ",ELTITLE;
  WRITE "% SYSTEM TYPE IS ",SYSTITLE;
  WRITE "% NUMBER OF BAYS   = ",NBAY;
  WRITE "% NUMBER OF LEVELS= ",MLEVEL;
  WRITE "% NUMBER OF NODES = ",NNODE;
  WRITE "% GLOBAL D.O.F.    = ",GDOF;

 END;

%%%%%%%%%%%%%%%%%%
PROCEDURE INFLU();
%%%%%%%%%%%%%%%%%%
 BEGIN
%  FOR ALL I,J LET W(I,J)=0;  % ZERO OUT ALL FORCE TERMS
  LET WD=0;
  FOR I:=1:MLEVEL DO
   FOR J:=1:NBAY DO
    BEGIN
     WRITE "%***** FLOOR UDL CASE AT LEVEL=",I," BAY=",J;
%     LET W(I,J)=1;            % SET UP UNIT LOADS, ONE AT AT TIME
     LET W=1;
     STRESSES();
     LET W=0;
%     LET W(I,J)=0;
    END;
  LET WD=1;
  WRITE "%***** WIND LOAD CASE *****";
  STRESSES();
 END;

%%%%%%%%%%%%%%%%%%
PROCEDURE INFDEL();
%%%%%%%%%%%%%%%%%%
 BEGIN
%  FOR ALL I,J LET W(I,J)=0;  % ZERO OUT ALL FORCE TERMS
  LET WD=0;
  FOR I:=1:MLEVEL DO
   FOR J:=1:NBAY DO
    BEGIN
     WRITE "%***** FLOOR UDL CASE AT LEVEL=",I," BAY=",J;
%     LET W(I,J)=1;            % SET UP UNIT LOADS, ONE AT AT TIME
     LET W=1;
     WRITE DV;
     LET W=0;
%     LET W(I,J)=0;
    END;
  LET WD=1;
  WRITE "%***** WIND LOAD CASE *****";
  WRITE DV;
 END;

%%%%%%%%%%%%%%%%%%
PROCEDURE STRESSES();
%%%%%%%%%%%%%%%%%%
 BEGIN
```

```
WRITE "% BASIC ELEMENT STRESSES";

FOR LEVEL:=1:MLEVEL DO
 FOR BAY:=1:NPBAY DO
  BEGIN

%--- COLUMN
    NI:=(LEVEL-2)*(NPBAY)+BAY-1;
    NJ:=NI+NPBAY;
    II:=PDOF*NI+1;
    JJ:=PDOF*NJ+1;
    IF II<1 THEN II:=GDOF+1;  % RIGID SUPPORT NODE
    CLEAR PA,VA,MA,MMAX,XMAX,VMAX,BENDSTR$

    WRITE "COLUMN MEMBER FROM NODE ",NI+1," TO ",NJ+1;
    PA:=SV(1,1)*DV(II,1)+SV(1,2)*DV(II+1,1)+SV(1,3)*DV(II+2,1)$
    PA:=PA+SV(1,4)*DV(JJ,1)+SV(1,5)*DV(JJ+1,1)+SV(1,6)*DV(JJ+2,1)$
    PA:=SUB(Q=QC(LEVEL,BAY),LX=L,AX=AC(LEVEL,BAY),IZ=IC(LEVEL,BAY),PA);
    WRITE "Pa := ",PA;
CLEAR PA$
    VA:=SV(2,1)*DV(II,1)+SV(2,2)*DV(II+1,1)+SV(2,3)*DV(II+2,1)$
    VA:=VA+SV(2,4)*DV(JJ,1)+SV(2,5)*DV(JJ+1,1)+SV(2,6)*DV(JJ+2,1)$
    VA:=SUB(Q=QC(LEVEL,BAY),LX=L,AX=AC(LEVEL,BAY),IZ=IC(LEVEL,BAY),VA);

    MA:=SV(3,1)*DV(II,1)+SV(3,2)*DV(II+1,1)+SV(3,3)*DV(II+2,1)$
    MA:=MA+SV(3,4)*DV(JJ,1)+SV(3,5)*DV(JJ+1,1)+SV(3,6)*DV(JJ+2,1)$
    MA:=SUB(Q=QC(LEVEL,BAY),LX=L,AX=AC(LEVEL,BAY),IZ=IC(LEVEL,BAY),MA);

    WRITE "Va := ",VA;
    WRITE "Ma := ",MA;
    WRITE "Mb := ",VA*L-MA;
   END;

 FOR LEVEL:=1:MLEVEL DO
  FOR BAY:=1:NBAY DO
   BEGIN

%--- BEAM
    NI:=(LEVEL-1)*NPBAY+BAY-1;
    NJ:=NI+1;
    II:=PDOF*NI+1;
    JJ:=PDOF*NJ+1;
    CLEAR PA,VA,MA,MMAX,XMAX,VMAX,BENDSTR$

    WRITE "BEAM MEMBER FROM NODE ",NI+1," TO ",NJ+1;
    PA:=SH(1,1)*DV(II,1)+SH(1,2)*DV(II+1,1)+SH(1,3)*DV(II+2,1)$
    PA:=PA+SH(1,4)*DV(JJ,1)+SH(1,5)*DV(JJ+1,1)+SH(1,6)*DV(JJ+2,1)$
    PA:=SUB(Q=QB(LEVEL,BAY),LX=G(BAY)*L,AX=AB(LEVEL,BAY),IZ=IB(LEVEL,BAY),PA);
    WRITE "Pa := ",PA;
 CLEAR PA$
    VA:=SH(2,1)*DV(II,1)+SH(2,2)*DV(II+1,1)+SH(2,3)*DV(II+2,1)$
    VA:=VA+SH(2,4)*DV(JJ,1)+SH(2,5)*DV(JJ+1,1)+SH(2,6)*DV(JJ+2,1)$
    VA:=SUB(Q=QB(LEVEL,BAY),LX=G(BAY)*L,AX=AB(LEVEL,BAY),IZ=IB(LEVEL,BAY),VA);

    MA:=SH(3,1)*DV(II,1)+SH(3,2)*DV(II+1,1)+SH(3,3)*DV(II+2,1)$
    MA:=MA+SH(3,4)*DV(JJ,1)+SH(3,5)*DV(JJ+1,1)+SH(3,6)*DV(JJ+2,1)$
    MA:=SUB(Q=QB(LEVEL,BAY),LX=G(BAY)*L,AX=AB(LEVEL,BAY),IZ=IB(LEVEL,BAY),MA);
```

```
    WRITE "Va := ",VA;
    WRITE "Ma := ",MA;
    WRITE "Mb := ",VA*(G(BAY)*L)-MA;
%    WRITE " MAX MOMENT Mmax = ",W(LEVEL,BAY)*(G(BAY)*L)**2/8+VA*(G(BAY)*L)/2+1/2*

   END;
 END;

%--- ****************************
%--- ******* MAIN PROGRAM *******
%--- ****************************

% OFF NAT$
OUT IFLSR$
% ON FLOAT$
% LET AL=100, AB=100, AR=100;  % TESTING ONLY
% LET IL=500, IB=500, IR=500;
% LET G=1, L=12, EE=30000;

 SETUP()$
 WRITE "%** INFLUENCE COEFFICIENTS FOR DEFLECTIONS **"$
 INFDEL()$
 WRITE "%** INFLUENCE COEFFICIENTS FOR STRESS RESULTANTS **"$
 INFLU()$
SHUT IFLSR$

;END;
```

Appendix D
Important REDUCE Listings of Results

D.1 General Stiffness, Flexibility, Load and Deflection Matrices

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AVAILABLE

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/ **** AKA AND FLEX MATRICES ****

/ STRUCTURAL SPECIFICATIONS:

/ ELEMENT TYPE IS ELASTIC FLEXURAL/AXIAL ELEMENT

/ SYSTEM TYPE IS RIGID SUPPORTS, FULL 3 DOF PER NODE

/ NUMBER OF BAYS  = 1

/ NUMBER OF LEVELS= 1

/ NUMBER OF NODES = 2

/ GLOBAL D.O.F.   = 6

0

LET GDOF=6

MATRIX AKA(GDOF,GDOF);     /- GLOBAL STIFFNESS MATRIX

MATRIX FLEX(GDOF,GDOF);    /- GLOBAL FLEXIBILITY MATRIX

MATRIX DU(GDOF,1);         /- DEFLECTION VECTOR

MATRIX FG(GDOF,1);         /- WIND AND GRAVITY LOAD VECTOR

                    2                3
AKA(1,1) := (E .Ab.L  + 12.E .I!.G)/(L .G)

AKA(1,2) := 0

                      2
AKA(1,3) := (6.E .I!)/L

AKA(1,4) := ( - E .Ab)/(L.G)

AKA(1,5) := 0

AKA(1,6) := 0

AKA(2,1) := 0

                          2     3    3  3
AKA(2,2) := (12.E .Ib + E .L .A!.G )/(L .G )

                      2  2
AKA(2,3) := (6.E .Ib)/(L .G )

AKA(2,4) := 0

                          3  3
AKA(2,5) := ( - 12.E .Ib)/(L .G )

                      2  2
AKA(2,6) := (6.E .Ib)/(L .G )

                      2
AKA(3,1) := (6.E .I!)/L

                      2  2
```

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$AKA(3,2) := (6 \cdot E \cdot Ib)/(L \cdot G)$

$AKA(3,3) := (4 \cdot E \cdot Ib + 4 \cdot E \cdot Il \cdot G)/(L \cdot G)$

$AKA(3,4) := 0$

$AKA(3,5) := (-6 \cdot E \cdot Ib)/(L^{2} \cdot G^{2})$

$AKA(3,6) := (2 \cdot E \cdot Ib)/(L \cdot G)$

$AKA(4,1) := (-E \cdot Ab)/(L \cdot G)$

$AKA(4,2) := 0$

$AKA(4,3) := 0$

$AKA(4,4) := (E \cdot Ab \cdot L^{2} + 12 \cdot E \cdot Ir \cdot G)/(L^{3} \cdot G)$

$AKA(4,5) := 0$

$AKA(4,6) := (6 \cdot E \cdot Ir)/L^{2}$

$AKA(5,1) := 0$

$AKA(5,2) := (-12 \cdot E \cdot Ib)/(L^{3} \cdot G^{3})$

$AKA(5,3) := (-6 \cdot E \cdot Ib)/(L^{2} \cdot G^{2})$

$AKA(5,4) := 0$

$AKA(5,5) := (12 \cdot E \cdot Ib + E \cdot L^{2} \cdot Ar \cdot G^{3})/(L^{3} \cdot G^{3})$

$AKA(5,6) := (-6 \cdot E \cdot Ib)/(L^{2} \cdot G^{2})$

$AKA(6,1) := 0$

$AKA(6,2) := (6 \cdot E \cdot Ib)/(L^{2} \cdot G^{2})$

$AKA(6,3) := (2 \cdot E \cdot Ib)/(L \cdot G)$

$AKA(6,4) := (6 \cdot E \cdot Ir)/L^{2}$

$AKA(6,5) := (-6 \cdot E \cdot Ib)/(L^{2} \cdot G^{2})$

$AKA(6,6) := (4 \cdot E \cdot Ib + 4 \cdot E \cdot Ir \cdot G)/(L \cdot G)$

$FLEX(1,1) := (3 \cdot Ab \cdot Ib^{2} \cdot L^{7} \cdot Al \cdot Ar \cdot G^{2} + 12 \cdot Ab \cdot Ib^{2} \cdot L^{5} \cdot Il \cdot Al + 12 \cdot Ab \cdot Ib^{2} \cdot L^{5} \cdot Il \cdot Ar + 12 \cdot Ab \cdot Ib^{2} \cdot L^{5} \cdot Al \cdot Ir + 12 \cdot Ab \cdot Ib^{2} \cdot L^{5} \cdot Ir \cdot Ar + 4 \cdot Ab \cdot Ib \cdot L^{7} \cdot Il \cdot Al \cdot Ar \cdot G^{3} + 4 \cdot Ab \cdot Ib \cdot L^{7} \cdot Al \cdot Ir \cdot Ar \cdot G^{3} + 48 \cdot Ab \cdot Ib \cdot L^{5} \cdot Il \cdot Al \cdot Ir \cdot G + 48 \cdot Ab \cdot Ib \cdot L^{5} \cdot Il \cdot Ir \cdot Ar \cdot G + 4 \cdot Ab$

$\cdot L^{7} \cdot Il \cdot Al \cdot Ir \cdot Ar \cdot G^{4} + 36 \cdot Ib^{2} \cdot L^{5} \cdot Al \cdot Ir \cdot Ar \cdot G^{3} + 144 \cdot Ib^{2} \cdot L^{3} \cdot Il \cdot Al \cdot Ir \cdot G + 144 \cdot Ib^{2} \cdot L^{3} \cdot Il \cdot Ir \cdot Ar \cdot G + 36 \cdot Ib^{2} \cdot L^{3} \cdot Al \cdot Ir^{2} \cdot G + 36 \cdot Ib^{2} \cdot L^{3} \cdot Ir^{2} \cdot Ar \cdot G + 48 \cdot Ib \cdot L^{5} \cdot Il \cdot Al \cdot Ir \cdot Ar \cdot G^{4} + 12 \cdot Ib \cdot L^{5} \cdot Al \cdot Ir^{2} \cdot Ar \cdot G^{4} + 144 \cdot Ib \cdot L^{3} \cdot Il \cdot Al \cdot Ir^{2} \cdot G^{2} +$

$$144.Ib.L^{3}.Il.Ir^{2}.Ar.G^{2} + 12.L^{5}.Il.Al.Ir^{2}.Ar.G^{5})/(36.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 36.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 36.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 36.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 72.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 72.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar$$

$$+ 36.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 36.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 12.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 132.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 12.E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 144.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 144.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 144$$

$$.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 144.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 12.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 12.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 432.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 432.E.Ib^{2}.Il^{2}.Al.Ir.G + 432.E.Ib^{2}.Il^{2}.Ir.Ar.G + 432.E.Ib^{2}.$$

$$Il.Al.Ir^{2}.G + 432.E.Ib^{2}.Il.Ir^{2}.Ar.G + 144.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 144.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 432.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 432.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 36.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$FLEX(1,2) := (3.Ab.Ib^{2}.L^{5}.Il.Ar.G + 3.Ab.Ib^{2}.L^{5}.Ir.Ar.G + 12.Ab.Ib.L^{5}.Il.Ir.Ar.G^{2} + 36.Ib^{2}.L^{3}.Il.Ir.Ar.G^{2} + 18.Ib.L^{3}.Il.Ir^{2}.Ar.G^{3})/(6.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 6.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 6.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 6.E.Ab.Ib^{2}$$

$$.L^{2}.Il^{2}.Ar + 12.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 12.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 6.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 6.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 2.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 22.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 2.E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 24.$$

$$E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 24.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 24.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 24.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 2.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 2.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 72.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 72.E.$$

$$Ib^{2}.Il^{2}.Al.Ir.G + 72.E.Ib^{2}.Il^{2}.Ir.Ar.G + 72.E.Ib^{2}.Il.Al.Ir^{2}.G + 72.E.Ib^{2}.Il.Ir^{2}.Ar.G + 24.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 24.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 72.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 72.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 6.E.L^{2}.$$

$$Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$FLEX(1,3) := (- 6.Ab.Ib^{2}.L^{4}.Il.Al - 6.Ab.Ib^{2}.L^{4}.Il.Ar - 6.Ab.Ib^{2}.L^{4}.Al.Ir - 6.Ab.Ib^{2}.L^{4}.Ir.Ar - 2.Ab.Ib.L^{6}.Il.Al.Ar.G^{3} + Ab.Ib.L^{6}.Al.Ir.Ar.G^{3} - 24.Ab.Ib.L^{4}.Il.Al.Ir.G - 24.Ab.Ib.L^{4}.Il.Ir.Ar.G - 2.Ab.L^{6}.Il.Al.Ir.Ar.G^{4} - 72.$$

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$$Ib^{2}.L^{2}.Il.Al.Ir.G - 72.Ib^{2}.L^{2}.Il.Ir.Ar.G - 24.Ib.L^{4}.Il.Al.Ir.Ar.G^{4} - 72.Ib.L^{2}.Il.Al.Ir^{2}.G^{2} - 72.Ib.L^{2}.Il.Ir^{2}.Ar.G^{2} - 6.L^{4}.Il.Al.Ir^{2}.Ar.G^{5})/(12.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 12.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 12.E.Ab.$$

$$Ib^{2}.L^{2}.Il^{2}.Al + 12.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 24.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 24.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 12.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 12.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 4.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 44.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 4.$$

$$E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 48.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 48.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 48.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 48.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 4.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 4.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 144.E.$$

$$Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 144.E.Ib^{2}.Il^{2}.Al.Ir.G + 144.E.Ib^{2}.Il^{2}.Ir.Ar.G + 144.E.Ib^{2}.Il.Al.Ir^{2}.G + 144.E.Ib^{2}.Il.Ir^{2}.Ar.G + 48.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 48.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 144.E.Ib.Il^{2}.Al.Ir^{2}.G^{2}$$

$$+ 144.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 12.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$FLEX(1,4) := (3.Ab.Ib^{2}.L^{7}.Al.Ar.G^{2} + 12.Ab.Ib^{2}.L^{5}.Il.Al + 12.Ab.Ib^{2}.L^{5}.Il.Ar + 12.Ab.Ib^{2}.L^{5}.Al.Ir + 12.Ab.Ib^{2}.L^{5}.Ir.Ar + 4.Ab.Ib.L^{7}.Il.Al.Ar.G^{3} + 4.Ab.Ib.L^{7}.Al.Ir.Ar.G^{3} + 48.Ab.Ib.L^{5}.Il.Al.Ir.G + 48.Ab.Ib.L^{5}.Il.Ir.Ar.G + 4.Ab$$

$$.L^{7}.Il.Al.Ir.Ar.G^{4} + 108.Ib^{2}.L^{3}.Il.Al.Ir.G + 108.Ib^{2}.L^{3}.Il.Ir.Ar.G - 18.Ib.L^{5}.Il.Al.Ir.Ar.G^{4})/(36.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 36.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 36.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 36.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 72.E.$$

$$Ab.Ib^{2}.L^{2}.Il.Al.Ir + 72.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 36.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 36.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 12.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 132.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 12.E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 144.E.Ab.Ib.L^{2}.$$

$$Il^{2}.Al.Ir.G + 144.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 144.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 144.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 12.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 12.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 432.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 432.E.Ib^{2}.Il^{2}$$

$$.Al.Ir.G + 432.E.Ib^{2}.Il^{2}.Ir.Ar.G + 432.E.Ib^{2}.Il.Al.Ir^{2}.G + 432.E.Ib^{2}.Il.Ir^{2}.Ar.G + 144.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 144.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 432.E.Ib.Il^{2}.A\ Ir^{2}.G^{2} + 432.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 36.E.L^{2}.$$

$$Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

```
FLEX(1,5) := ( - 3.Ab.Ib^2.L^5.Il.Al.G - 3.Ab.Ib^2.L^5.Al.Ir.G - 12.Ab.Ib.L^5.Il.Al.Ir.G^2 - 36.Ib^2.L^3.Il.Al.Ir.G^2 - 18.Ib.L^3.Il.Al.Ir^2.G^3 )/(6.E.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 6.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 6.E.Ab.Ib^2.L^2.Il^2.Al + 6.E.Ab.
   Ib^2.L^2.Il^2.Ar + 12.E.Ab.Ib^2.L^2.Il.Al.Ir + 12.E.Ab.Ib^2.L^2.Il.Ir.Ar + 6.E.Ab.Ib^2.L^2.Al.Ir^2 + 6.E.Ab.Ib^2.L^2.Ir^2.Ar + 2.E.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 22.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 2.E.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 +
   24.E.Ab.Ib.L^2.Il^2.Al.Ir.G + 24.E.Ab.Ib.L^2.Il^2.Ir.Ar.G + 24.E.Ab.Ib.L^2.Il.Al.Ir^2.G + 24.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + 2.E.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 2.E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 72.E.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 72.E
   .Ib^2.Il^2.Al.Ir.G + 72.E.Ib^2.Il^2.Ir.Ar.G + 72.E.Ib^2.Il.Al.Ir^2.G + 72.E.Ib^2.Il.Ir^2.Ar.G + 24.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 24.E.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 72.E.Ib.Il^2.Al.Ir^2.G^2 + 72.E.Ib.Il^2.Ir^2.Ar.G^2 + 6.E.L^2.
   Il^2.Al.Ir^2.Ar.G^5 )

FLEX(1,6) := ( - 6.Ab.Ib^2.L^4.Il.Al - 6.Ab.Ib^2.L^4.Il.Ar - 6.Ab.Ib^2.L^4.Al.Ir - 6.Ab.Ib^2.L^4.Ir.Ar + Ab.Ib.L^6.Il.Al.Ar.G^3 - 2.Ab.Ib.L^6.Al.Ir.Ar.G^3 - 24.Ab.Ib.L^4.Il.Al.Ir.G - 24.Ab.Ib.L^4.Il.Ir.Ar.G - 2.Ab.L^6.Il.Al.Ir.Ar.G^4 - 72.
   Ib^2.L^2.Il.Al.Ir.G - 72.Ib^2.L^2.Il.Ir.Ar.G + 12.Ib.L^4.Il.Al.Ir.Ar.G^4 )/(12.E.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 12.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 12.E.Ab.Ib^2.L^2.Il^2.Al + 12.E.Ab.Ib^2.L^2.Il^2.Ar + 24.E.Ab.Ib^2.L^2.Il.Al.Ir + 24.E
   .Ab.Ib^2.L^2.Il.Ir.Ar + 12.E.Ab.Ib^2.L^2.Al.Ir^2 + 12.E.Ab.Ib^2.L^2.Ir^2.Ar + 4.E.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 44.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 4.E.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 + 48.E.Ab.Ib.L^2.Il^2.Al.Ir.G + 48.E.Ab.Ib.L^2.Il^2
   .Ir.Ar.G + 48.E.Ab.Ib.L^2.Il.Al.Ir^2.G + 48.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + 4.E.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 4.E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 144.E.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 144.E.Ib^2.Il^2.Al.Ir.G + 144.E.Ib^2.Il^2.Ir.Ar.G +
   144.E.Ib^2.Il.Al.Ir^2.G + 144.E.Ib^2.Il.Ir^2.Ar.G + 48.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 48.E.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 144.E.Ib.Il^2.Al.Ir^2.G^2 + 144.E.Ib.Il^2.Ir^2.Ar.G^2 + 12.E.L^2.Il^2.Al.Ir^2.Ar.G^5 )

FLEX(2,1) := (3.Ab.Ib^2.L^5.Il.Ar.G + 3.Ab.Ib^2.L^5.Ir.Ar.G + 12.Ab.Ib.L^5.Il.Ir.Ar.G^2 + 36.Ib^2.L^3.Il.Ir.Ar.G^2 + 18.Ib.L^3.Il.Ir^2.Ar.G^3 )/(6.E.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 6.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 6.E.Ab.Ib^2.L^2.Il^2.Al + 6.E.Ab.Ib^2
   .L^2.Il^2.Ar + 12.E.Ab.Ib^2.L^2.Il.Al.Ir + 12.E.Ab.Ib^2.L^2.Il.Ir.Ar + 6.E.Ab.Ib^2.L^2.Al.Ir^2 + 6.E.Ab.Ib^2.L^2.Ir^2.Ar + 2.E.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 22.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 2.E.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 + 24.
   E.Ab.Ib.L^2.Il^2.Al.Ir.G + 24.E.Ab.Ib.L^2.Il^2.Ir.Ar.G + 24.E.Ab.Ib.L^2.Il.Al.Ir^2.G + 24.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + 2.E.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 2.E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 72.E.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 72.E.
   Ib^2.Il^2.Al.Ir.G + 72.E.Ib^2.Il^2.Ir.Ar.G + 72.E.Ib^2.Il.Al.Ir^2.G + 72.E.Ib^2.Il.Ir^2.Ar.G + 24.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 24.E.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 72.E.Ib.Il^2.Al.Ir^2.G^2 + 72.E.Ib.Il^2.Ir^2.Ar.G^2 + 6.E.L^2.
   Il^2.Al.Ir^2.Ar.G^5 )

FLEX(2,2) := (3.Ab.Ib^2.L^5.Il.Ar.G^2 + 3.Ab.Ib^2.L^5.Ir.Ar.G^2 + 3.Ab.Ib^2.L^3.Il^2 + 6.Ab.Ib^2.L^3.Il.Ir + 3.Ab.Ib^2.L^3.Ir^2 + Ab.Ib.L^5.Il^2.Ar.G^3 + 11.Ab.Ib.L^5.Il.Ir.Ar.G^3 + Ab.Ib.L^5.Ir^2.Ar.G^3 + 12.Ab.Ib.L^3.Il^2.Ir.G + 12.Ab.Ib.L^3.Il.Ir^2
   .G + Ab.L^5.Il^2.Ir.Ar.G^4 + Ab.L^5.Il.Ir^2.Ar.G^4 + 36.Ib^2.L^3.Il.Ir.Ar.G^3 + 36.Ib^2.L.Il^2.Ir.G + 36.Ib^2.L.Il.Ir^2.G + 12.Ib.L^3.Il^2.Ir.Ar.G^4 + 12.Ib.L^3.Il.Ir^2.Ar.G^4 + 36.Ib.L.Il^2.Ir^2.G^2 + 3.L^3.Il^2.Ir^2.Ar.G^5 )/(3.E.Ab.
   Ib^2.L^4.Il.Al.Ar.G^2 + 3.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 3.E.Ab.Ib^2.L^2.Il^2.Al + 3.E.Ab.Ib^2.L^2.Il^2.Ar + 6.E.Ab.Ib^2.L^2.Il.Al.Ir + 6.E.Ab.Ib^2.L^2.Il.Ir.Ar + 3.E.Ab.Ib^2.L^2.Al.Ir^2 + 3.E.Ab.Ib^2.L^2.Ir^2.Ar + E.Ab.Ib.L^4
   .Il^2.Al.Ar.G^3 + 11.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + E.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 + 12.E.Ab.Ib.L^2.Il^2.Al.Ir.G + 12.E.Ab.Ib.L^2.Il^2.Ir.Ar.G + 12.E.Ab.Ib.L^2.Il.Al.Ir^2.G + 12.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + E.Ab.L^4.Il^2.Al.Ir
   .Ar.G^4 + E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 36.E.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 36.E.Ib^2.Il^2.Al.Ir.G + 36.E.Ib^2.Il^2.Ir.Ar.G + 36.E.Ib^2.Il.Al.Ir^2.G + 36.E.Ib^2.Il.Ir^2.Ar.G + 12.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 12.E.Ib.L^2.Il.Al
   .Ir^2.Ar.G^4 + 36.E.Ib.Il^2.Al.Ir^2.G^2 + 36.E.Ib.Il^2.Ir^2.Ar.G^2 + 3.E.L^2.Il^2.Al.Ir^2.Ar.G^5 )
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                          2 4                   2 4                           4         2          4  2   2       2 2          2          2    2    3            2 4            2           2 4          2
FLEX(2.3) := ( - 6.Ab.Ib .L .Il.Ar.G - 6.Ab.Ib .L .Ir.Ar.G - 21.Ab.Ib.L .Il.Ir.Ar.G  - 3.Ab.Ib.L .Ir .Ar.G  - 72.Ib .L .Il.Ir.Ar.G  - 36.Ib.L .Il.Ir .Ar.G )/(6.E .Ab.Ib .L .Il.Al.Ar.G  + 6.E .Ab.Ib .L .Al.Ir.Ar.G  + 6.E .Ab.

          2  2  2             2  2  2             2 2                     2 2            2 2    2             2  2  2            4  2       3               4              3
          Ib .L .Il .Al + 6.E .Ab.Ib .L .Il .Ar + 12.E .Ab.Ib .L .Il.Al.Ir + 12.E .Ab.Ib .L .Il.Ir.Ar + 6.E .Ab.Ib .L .Al.Ir  + 6.E .Ab.Ib .L .Ir .Ar + 2.E .Ab.Ib.L .Il .Al.Ar.G  + 22.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.E .

              4    2   3            2  2                 2 2                   2       2                2      2           4  2          4          4     2    4        2
          Ab.Ib.L .Al.Ir .Ar.G  + 24.E .Ab.Ib.L .Il .Al.Ir.G + 24.E .Ab.Ib.L .Il .Ir.Ar.G + 24.E .Ab.Ib.L .Il.Al.Ir .G + 24.E .Ab.Ib.L .Il.Ir .Ar.G + 2.E .Ab.L .Il .Al.Ir.Ar.G  + 2.E .Ab.L .Il.Al.Ir .Ar.G  + 72.E .Ib .

          2            3          2  2             2  2             2       2            2     2             2  2        4           2      2   4           2    2 2
          L .Il.Al.Ir.Ar.G  + 72.E .Ib .Il .Al.Ir.G + 72.E .Ib .Il .Ir.Ar.G + 72.E .Ib .Il.Al.Ir .G + 72.E .Ib .Il.Ir .Ar.G + 24.E .Ib.L .Il .Al.Ir.Ar.G  + 24.E .Ib.L .Il.Al.Ir .Ar.G  + 72.E .Ib.Il .Al.Ir .G  + 72.E .Ib

           2  2    2        2  2    2   5
          .Il .Ir .Ar.G  + 6.E .L .Il .Al.Ir .Ar.G )

                        2 5                 2 5                   5        2      2 3         2        3  2      3              2 4           2             2 4          2           2  2  2            2
FLEX(2.4) := (3.Ab.Ib .L .Il.Ar.G + 3.Ab.Ib .L .Ir.Ar.G + 12.Ab.Ib.L .Il.Ir.Ar.G  + 36.Ib .L .Il.Ir.Ar.G  + 18.Ib.L .Il .Ir.Ar.G )/(6.E .Ab.Ib .L .Il.Al.Ar.G  + 6.E .Ab.Ib .L .Al.Ir.Ar.G  + 6.E .Ab.Ib .L .Il .Al + 6.E .Ab.Ib

           2  2            2 2               2 2                    2 2    2            2  2  2            4  2       3               4              3              4    2   3
          .L .Il .Ar + 12.E .Ab.Ib .L .Il.Al.Ir + 12.E .Ab.Ib .L .Il.Ir.Ar + 6.E .Ab.Ib .L .Al.Ir  + 6.E .Ab.Ib .L .Ir .Ar + 2.E .Ab.Ib.L .Il .Al.Ar.G  + 22.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.E .Ab.Ib.L .Al.Ir .Ar.G  + 24.

                  2  2                2  2                  2       2              2      2            4  2         4          4       2    4          2 2             3
          E .Ab.Ib.L .Il .Al.Ir.G + 24.E .Ab.Ib.L .Il .Ir.Ar.G + 24.E .Ab.Ib.L .Il.Al.Ir .G + 24.E .Ab.Ib.L .Il.Ir .Ar.G + 2.E .Ab.L .Il .Al.Ir.Ar.G  + 2.E .Ab.L .Il.Al.Ir .Ar.G  + 72.E .Ib .L .Il.Al.Ir.Ar.G  + 72.E .

           2  2           2  2              2       2            2     2            2  2         4           2      2   4           2    2 2           2  2    2         2
          Ib .Il .Al.Ir.G + 72.E .Ib .Il .Ir.Ar.G + 72.E .Ib .Il.Al.Ir .G + 72.E .Ib .Il.Ir .Ar.G + 24.E .Ib.L .Il .Al.Ir.Ar.G  + 24.E .Ib.L .Il.Al.Ir .Ar.G  + 72.E .Ib.Il .Al.Ir .G  + 72.E .Ib.Il .Ir .Ar.G  + 6.E .L .

           2     2    5
          Il .Al.Ir .Ar.G )

                        2 3 2            2 3              2 3 2             3  2               3   2            2    2           2       2              2  2  2          2 4           2          2 4
FLEX(2.5) := (3.Ab.Ib .L .Il  + 6.Ab.Ib .L .Il.Ir + 3.Ab.Ib .L .Ir  + 12.Ab.Ib.L .Il .Ir.G + 12.Ab.Ib.L .Il.Ir .G + 36.Ib .L.Il .Ir.G + 36.Ib .L.Il.Ir .G + 36.Ib.L.Il .Ir .G )/(3.E .Ab.Ib .L .Il.Al.Ar.G  + 3.E .Ab.Ib .L .Al.

           2            2  2  2            2  2  2           2 2                  2 2              2 2   2             2  2  2          4  2       3               4
          Ir.Ar.G  + 3.E .Ab.Ib .L .Il .Al + 3.E .Ab.Ib .L .Il .Ar + 6.E .Ab.Ib .L .Il.Al.Ir + 6.E .Ab.Ib .L .Il.Ir.Ar + 3.E .Ab.Ib .L .Al.Ir  + 3.E .Ab.Ib .L .Ir .Ar + E .Ab.Ib.L .Il .Al.Ar.G  + 11.E .Ab.Ib.L .Il.Al.Ir

           3          4   2   3              2  2              2  2               2      2              2     2          4  2          4         4       2    4
          .Ar.G  + E .Ab.Ib.L .Al.Ir .Ar.G  + 12.E .Ab.Ib.L .Il .Al.Ir.G + 12.E .Ab.Ib.L .Il .Ir.Ar.G + 12.E .Ab.Ib.L .Il.Al.Ir .G + 12.E .Ab.Ib.L .Il.Ir .Ar.G + E .Ab.L .Il .Al.Ir.Ar.G  + E .Ab.L .Il.Al.Ir .Ar.G  + 36.

             2 2          3           2  2            2  2             2       2             2     2            2  2         4           2      2   4           2    2 2
          E .Ib .L .Il.Al.Ir.Ar.G  + 36.E .Ib .Il .Al.Ir.G + 36.E .Ib .Il .Ir.Ar.G + 36.E .Ib .Il.Al.Ir .G + 36.E .Ib .Il.Ir .Ar.G + 12.E  Ib.L .Il .Al.Ir.Ar.G  + 12.E .Ib.L .Il.Al.Ir .Ar.G  + 36.E .Ib.Il .Al.Ir .G  +

                 2  2    2        2  2    2   5
          36.E .Ib.Il .Ir .Ar.G  + 3.E .L .Il .Al.Ir .Ar.G )

                         2 4                   2 4                   4  2    2             4          2       2 2          2          2  2    3            2 4            2           2 4          2
FLEX(2.6) := ( - 6.Ab.Ib .L .Il.Ar.G - 6.Ab.Ib .L .Ir.Ar.G - 3.Ab.Ib.L .Il .Ar.G  - 21.Ab.Ib.L .Il.Ir.Ar.G  - 72.Ib .L .Il.Ir.Ar.G  - 36.Ib.L .Il .Ir.Ar.G )/(6.E .Ab.Ib .L .Il.Al.Ar.G  + 6.E .Ab.Ib .L .Al.Ir.Ar.G  + 6.E .Ab.

          2  2  2             2  2  2             2 2                     2 2            2 2    2             2  2  2            4  2       3               4              3
          Ib .L .Il .Al + 6.E .Ab.Ib .L .Il .Ar + 12.E .Ab.Ib .L .Il.Al.Ir + 12.E .Ab.Ib .L .Il.Ir.Ar + 6.E .Ab.Ib .L .Al.Ir  + 6.E .Ab.Ib .L .Ir .Ar + 2.E .Ab.Ib.L .Il .Al.Ar.G  + 22.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.E .

              4    2   3            2  2                 2 2                   2       2                2      2           4  2          4          4     2    4        2
          Ab.Ib.L .Al.Ir .Ar.G  + 24.E .Ab.Ib.L .Il .Al.Ir.G + 24.E .Ab.Ib.L .Il .Ir.Ar.G + 24.E .Ab.Ib.L .Il.Al.Ir .G + 24.E .Ab.Ib.L .Il.Ir .Ar.G + 2.E .Ab.L .Il .Al.Ir.Ar.G  + 2.E .Ab.L .Il.Al.Ir .Ar.G  + 72.E .Ib .

          2            3          2  2             2  2             2       2            2     2             2  2        4           2      2   4           2    2 2
          L .Il.Al.Ir.Ar.G  + 72.E .Ib .Il .Al.Ir.G + 72.E .Ib .Il .Ir.Ar.G + 72.E .Ib .Il.Al.Ir .G + 72.E .Ib .Il.Ir .Ar.G + 24.E .Ib.L .Il .Al.Ir.Ar.G  + 24.E .Ib.L .Il.Al.Ir .Ar.G  + 72.E .Ib.Il .Al.Ir .G  + 72.E .Ib

           2  2    2        2  2    2   5
          .Il .Ir .Ar.G  + 6.E .L .Il .Al.Ir .Ar.G )

                         2 4                 2 4                 2 4                2 4              6             3            6            3                4                     4                    6              4
FLEX(3.1) := ( - 6.Ab.Ib .L .Il.Al - 6.Ab.Ib .L .Il.Ar - 6.Ab.Ib .L .Al.Ir - 6.Ab.Ib .L .Ir.Ar - 2.Ab.Ib.L .Il.Al.Ar.G  + Ab.Ib.L .Al.Ir.Ar.G  - 24.Ab.Ib.L .Il.Al.Ir.G - 24.Ab.Ib.L .Il.Ir.Ar.G - 2.Ab.L .Il.Al.Ir.Ar.G  - 72.
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$$Ib^{2}.L^{2}.Il.Al.Ir.G - 72.Ib^{2}.L^{2}.Il.Ir.Ar.G - 24.Ib.L^{4}.Il.Al.Ir.Ar.G^{4} - 72.Ib.L^{2}.Il.Al.Ir^{2}.G^{2} - 72.Ib.L^{2}.Il.Ir^{2}.Ar.G^{2} - 6.L^{4}.Il.Al.Ir^{2}.Ar.G^{5})/(12.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 12.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 12.E.Ab.$$

$$Ib^{2}.L^{2}.Il^{2}.Al + 12.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 24.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 24.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 12.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 12.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 4.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 44.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 4.$$

$$E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 48.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 48.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 48.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 48.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 4.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 4.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 144.E.$$

$$Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 144.E.Ib^{2}.Il^{2}.Al.Ir.G + 144.E.Ib^{2}.Il^{2}.Ir.Ar.G + 144.E.Ib^{2}.Il.Al.Ir^{2}.G + 144.E.Ib^{2}.Il.Ir^{2}.Ar.G + 48.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 48.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 144.E.Ib.Il^{2}.Al.Ir^{2}.G^{2}$$

$$+ 144.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 12.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$FLEX(3,2) := (- 6.Ab.Ib^{2}.L^{4}.Il.Ar.G - 6.Ab.Ib^{2}.L^{4}.Ir.Ar.G - 21.Ab.Ib.L^{4}.Il.Ir.Ar.G^{2} - 3.Ab.Ib.L^{4}.Ir^{2}.Ar.G^{2} - 72.Ib^{2}.L^{2}.Il.Ir.Ar.G^{2} - 36.Ib.L^{2}.Il.Ir^{2}.Ar.G^{3})/(6.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 6.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 6.E.Ab.$$

$$Ib^{2}.L^{2}.Il^{2}.Al + 6.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 12.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 12.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 6.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 6.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 2.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 22.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 2.E.$$

$$Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 24.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 24.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 24.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 24.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 2.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 2.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 72.E.Ib^{2}.$$

$$L^{2}.Il.Al.Ir.Ar.G^{3} + 72.E.Ib^{2}.Il^{2}.Al.Ir.G + 72.E.Ib^{2}.Il^{2}.Ir.Ar.G + 72.E.Ib^{2}.Il.Al.Ir^{2}.G + 72.E.Ib^{2}.Il.Ir^{2}.Ar.G + 24.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 24.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 72.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 72.E.Ib$$

$$.Il^{2}.Ir^{2}.Ar.G^{2} + 6.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$FLEX(3,3) := (12.Ab.Ib^{2}.L^{3}.Il.Al + 12.Ab.Ib^{2}.L^{3}.Il.Ar + 12.Ab.Ib^{2}.L^{3}.Al.Ir + 12.Ab.Ib^{2}.L^{3}.Ir.Ar + 4.Ab.Ib.L^{5}.Il.Al.Ar.G^{3} + 4.Ab.Ib.L^{5}.Al.Ir.Ar.G^{3} + 48.Ab.Ib.L^{3}.Il.Al.Ir.G + 48.Ab.Ib.L^{3}.Il.Ir.Ar.G + 12.Ab.Ib.L^{3}.Al.Ir^{2}.G + 12.$$

$$Ab.Ib.L^{3}.Ir^{2}.Ar.G + 4.Ab.L^{5}.Il.Al.Ir.Ar.G^{4} + Ab.L^{5}.Al.Ir^{2}.Ar.G^{4} + 144.Ib^{2}.L.Il.Al.Ir.G + 144.Ib^{2}.L.Il.Ir.Ar.G + 48.Ib.L^{3}.Il.Al.Ir.Ar.G^{4} + 144.Ib.L.Il.Al.Ir^{2}.G^{2} + 144.Ib.L.Il.Ir^{2}.Ar.G^{2} + 12.L^{3}.Il.Al.Ir^{2}.Ar.G^{5})/($$

$$12.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 12.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 12.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 12.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 24.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 24.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 12.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 12.E.Ab.Ib^{2}.L^{2}.Ir^{2}.$$

$$Ar + 4.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 44.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 4.E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 48.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 48.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 48.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 48.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G$$

$$+ 4.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 4.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 144.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 144.E.Ib^{2}.Il^{2}.Al.Ir.G + 144.E.Ib^{2}.Il^{2}.Ir.Ar.G + 144.E.Ib^{2}.Il.Al.Ir^{2}.G + 144.E.Ib^{2}.Il.Ir^{2}.Ar.G + 48.E.Ib.L^{2}.Il^{2}.$$

$$Al.Ir.Ar.G^{4} + 48.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 144.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 144.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 12.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$FLEX(3,4) := (- 6.Ab.Ib^{2}.L^{4}.Il.Al - 6.Ab.Ib^{2}.L^{4}.Il.Ar - 6.Ab.Ib^{2}.L^{4}.Al.Ir - 6.Ab.Ib^{2}.L^{4}.Ir.Ar - 2.Ab.Ib.L^{6}.Il.Al.Ar.G^{3} + Ab.Ib.L^{6}.Al.Ir.Ar.G^{3} - 24.Ab.Ib.L^{4}.Il.Al.Ir.G - 24.Ab.Ib.L^{4}.Il.Ir.Ar.G - 2.Ab.L^{6}.Il.Al.Ir.Ar.G^{4} - 72.$$

$$Ib^{2}.L^{2}.Il.Al.Ir.G - 72.Ib^{2}.L^{2}.Il.Ir.Ar.G + 12.Ib.L^{4}.Il.Al.Ir.Ar.G^{4})/(12.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 12.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 12.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 12.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 24.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 24.E$$

$$.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 12.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 12.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 4.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 44.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 4.E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 48.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 48.E.Ab.Ib.L^{2}.Il^{2}$$

$$.Ir.Ar.G + 48.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 48.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 4.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 4.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 144.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 144.E.Ib^{2}.Il^{2}.Al.Ir.G + 144.E.Ib^{2}.Il^{2}.Ir.Ar.G +$$

$$144.E.Ib^{2}.Il.Al.Ir^{2}.G + 144.E.Ib^{2}.Il.Ir^{2}.Ar.G + 48.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 48.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 144.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 144.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 12.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

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FLEX(3,5) := (6.Ab.Ib^2.L^4.Il.Al.G + 6.Ab.Ib^2.L^4.Al.Ir.G + 21.Ab.Ib.L^4.Il.Al.Ir.G^2 + 3.Ab.Ib.L^4.Al.Ir^2.G^2 + 72.Ib^2.L^2.Il.Al.Ir.G^2 + 36.Ib.L^2.Il.Al.Ir^2.G^3)/(6.E.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 6.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 6.E.Ab.Ib^2.
   L^2.Il^2.Al + 6.E.Ab.Ib^2.L^2.Il^2.Ar + 12.E.Ab.Ib^2.L^2.Il.Al.Ir + 12.E.Ab.Ib^2.L^2.Il.Ir.Ar + 6.E.Ab.Ib^2.L^2.Al.Ir^2 + 6.E.Ab.Ib^2.L^2.Ir^2.Ar + 2.E.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 22.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 2.E.Ab.Ib
   .L^4.Al.Ir^2.Ar.G^3 + 24.E.Ab.Ib.L^2.Il^2.Al.Ir.G + 24.E.Ab.Ib.L^2.Il^2.Ir.Ar.G + 24.E.Ab.Ib.L^2.Il.Al.Ir^2.G + 24.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + 2.E.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 2.E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 72.E.Ib^2.L^2.Il.
   Al.Ir.Ar.G^3 + 72.E.Ib^2.Il^2.Al.Ir.G + 72.E.Ib^2.Il^2.Ir.Ar.G + 72.E.Ib^2.Il.Al.Ir^2.G + 72.E.Ib^2.Il.Ir^2.Ar.G + 24.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 24.E.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 72.E.Ib.Il^2.Al.Ir^2.G^2 + 72.E.Ib.Il^2.
   Ir^2.Ar.G^2 + 6.E.L^2.Il^2.Al.Ir^2.Ar.G^5)

FLEX(3,6) := (12.Ab.Ib^2.L^3.Il.Al + 12.Ab.Ib^2.L^3.Il.Ar + 12.Ab.Ib^2.L^3.Al.Ir + 12.Ab.Ib^2.L^3.Ir.Ar - 2.Ab.Ib.L^5.Il.Al.Ar.G^3 - 2.Ab.Ib.L^5.Al.Ir.Ar.G^3 + 36.Ab.Ib.L^3.Il.Al.Ir.G + 36.Ab.Ib.L^3.Il.Ir.Ar.G + 3.Ab.L^5.Il.Al.Ir.Ar.G^4 +
   144.Ib^2.L.Il.Al.Ir.G + 144.Ib^2.L.Il.Ir.Ar.G - 24.Ib.L^3.Il.Al.Ir.Ar.G^4)/(12.E.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 12.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 12.E.Ab.Ib^2.L^2.Il^2.Al + 12.E.Ab.Ib^2.L^2.Il^2.Ar + 24.E.Ab.Ib^2.L^2.Il.Al.Ir + 24
   .E.Ab.Ib^2.L^2.Il.Ir.Ar + 12.E.Ab.Ib^2.L^2.Al.Ir^2 + 12.E.Ab.Ib^2.L^2.Ir^2.Ar + 4.E.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 44.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 4.E.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 + 48.E.Ab.Ib.L^2.Il^2.Al Ir.G + 48.E.Ab.Ib.L^2.
   Il^2.Ir.Ar.G + 48.E.Ab.Ib.L^2.Il.Al.Ir^2.G + 48.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + 4.E.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 4.E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 144.E.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 144.E.Ib^2.Il^2.Al.Ir.G + 144.E.Ib^2.Il^2.Ir.Ar.G
   + 144.E.Ib^2.Il.Al.Ir^2.G + 144.E.Ib^2.Il.Ir^2.Ar.G + 48.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 48.E.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 144.E.Ib.Il^2.Al.Ir^2.G^2 + 144.E.Ib.Il^2.Ir^2.Ar.G^2 + 12.E.L^2.Il^2.Al.Ir^2.Ar.G^5)

FLEX(4,1) := (3.Ab.Ib^2.L^7.Al.Ar.G^2 + 12.Ab.Ib^2.L^5.Il.Al + 12.Ab.Ib^2.L^5.Il.Ar + 12.Ab.Ib^2.L^5.Al.Ir + 12.Ab.Ib^2.L^5.Ir.Ar + 4.Ab.Ib.L^7.Il.Al.Ar.G^3 + 4.Ab.Ib.L^7.Al.Ir.Ar.G^3 + 48.Ab.Ib.L^5.Il.Al.Ir.G + 48.Ab.Ib.L^5.Il.Ir.Ar.G + 4.Ab
   .L^7.Il.Al.Ir.Ar.G^4 + 108.Ib^2.L^3.Il.Al.Ir.G + 108.Ib^2.L^3.Il.Ir.Ar.G - 18.Ib.L^5.Il.Al.Ir.Ar.G^4)/(36.E.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 36.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 36.E.Ab.Ib^2.L^2.Il^2.Al + 36.E.Ab.Ib^2.L^2.Il^2.Ar + 72.E.
   Ab.Ib^2.L^2.Il.Al.Ir + 72.E.Ab.Ib^2.L^2.Il.Ir.Ar + 36.E.Ab.Ib^2.L^2.Al.Ir^2 + 36.E.Ab.Ib^2.L^2.Ir^2.Ar + 12.E.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 132.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 12.E.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 + 144.E.Ab.Ib.L^2.
   Il^2.Al.Ir.G + 144.E.Ab.Ib.L^2.Il^2.Ir.Ar.G + 144.E.Ab.Ib.L^2.Il.Al.Ir^2.G + 144.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + 12.E.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 12.E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 432.E.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 432.E.Ib^2.Il^2
   .Al.Ir.G + 432.E.Ib^2.Il^2.Ir.Ar.G + 432.E.Ib^2.Il.Al.Ir^2.G + 432.E.Ib^2.Il.Ir^2.Ar.G + 144.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 144.E.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 432.E.Ib.Il^2.Al.Ir^2.G^2 + 432.E.Ib.Il^2.Ir^2.Ar.G^2 + 36.E.L^2.
   Il^2.Al.Ir^2.Ar.G^5)

FLEX(4,2) := (3.Ab.Ib^2.L^5.Il.Ar.G + 3.Ab.Ib^2.L^5.Ir.Ar.G + 12.Ab.Ib.L^5.Il.Ir.Ar.G^2 + 36.Ib^2.L^3.Il.Ir.Ar.G^2 + 18.Ib.L^3.Il^2.Ir.Ar.G^3)/(6.E.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 6.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 6.E.Ab.Ib^2.L^2.Il^2.Al + 6.E.Ab.Ib^2
   .L^2.Il^2.Ar + 12.E.Ab.Ib^2.L^2.Il.Al.Ir + 12.E.Ab.Ib^2.L^2.Il.Ir.Ar + 6.E.Ab.Ib^2.L^2.Al.Ir^2 + 6.E.Ab.Ib^2.L^2.Ir^2.Ar + 2.E.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 22.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 2.E.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 + 24.
   E.Ab.Ib.L^2.Il^2.Al.Ir.G + 24.E.Ab.Ib.L^2.Il^2.Ir.Ar.G + 24.E.Ab.Ib.L^2.Il.Al.Ir^2.G + 24.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + 2.E.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 2.E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 72.E.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 72.E.
   Ib^2.Il^2.Al.Ir.G + 72.E.Ib^2.Il^2.Ir.Ar.G + 72.E.Ib^2.Il.Al.Ir^2.G + 72.E.Ib^2.Il.Ir^2.Ar.G + 24.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 24.E.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 72.E.Ib.Il^2.Al.Ir^2.G^2 + 72.E.Ib.Il^2.Ir^2.Ar.G^2 + 6.E.L^2.
   Il^2.Al.Ir^2.Ar.G^5)
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FLEX(4,3) := ( - 6.Ab.Ib^2.L^4.Il.Al - 6.Ab.Ib^2.L^4.Il.Ar - 6.Ab.Ib^2.L^4.Al.Ir - 6.Ab.Ib^2.L^4.Ir.Ar - 2.Ab.Ib.L^6.Il.Al.Ar.G^3 + Ab.Ib.L^6.Al.Ir.Ar.G^3 - 24.Ab.Ib.L^4.Il.Al.Ir.G - 24.Ab.Ib.L^4.Il.Ir.Ar.G - 2.Ab.L^6.Il.Al.Ir.Ar.G^4 - 72.
    Ib^2.L^2.Il.Al.Ir.G - 72.Ib^2.L^2.Il.Ir.Ar.G + 12.Ib.L^4.Il.Al.Ir.Ar.G^4 )/(12.E.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 12.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 12.E.Ab.Ib^2.L^2.Il^2.Al + 12.E.Ab.Ib^2.L^2.Il^2.Ar + 24.E.Ab.Ib^2.L^2.Il.Al.Ir + 24.E
    .Ab.Ib^2.L^2.Il.Ir.Ar + 12.E.Ab.Ib^2.L^2.Al.Ir^2 + 12.E.Ab.Ib^2.L^2.Ir^2.Ar + 4.E.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 44.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 4.E.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 + 48.E.Ab.Ib.L^2.Il^2.Al.Ir.G + 48.E.Ab.Ib.L^2.Il^2
    .Ir.Ar.G + 48.E.Ab.Ib.L^2.Il.Al.Ir^2.G + 48.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + 4.E.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 4.E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 144.E.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 144.E.Ib^2.Il^2.Al.Ir.G + 144.E.Ib^2.Il^2.Ir.Ar.G +
    144.E.Ib^2.Il.Al.Ir^2.G + 144.E.Ib^2.Il.Ir^2.Ar.G + 48.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 48.E.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 144.E.Ib.Il^2.Al.Ir^2.G^2 + 144.E.Ib.Il^2.Ir^2.Ar.G^2 + 12.E.L^2.Il^2.Al.Ir^2.Ar.G^5 )

FLEX(4,4) := (3.Ab.Ib^2.L^7.Al.Ar.G^2 + 12.Ab.Ib^2.L^5.Il.Al + 12.Ab.Ib^2.L^5.Il.Ar + 12.Ab.Ib^2.L^5.Al.Ir + 12.Ab.Ib^2.L^5.Ir.Ar + 4.Ab.Ib.L^7.Il.Al.Ar.G^3 + 4.Ab.Ib.L^7.Al.Ir.Ar.G^3 + 48.Ab.Ib.L^5.Il.Al.Ir.G + 48.Ab.Ib.L^5.Il.Ir.Ar.G + 4.Ab
    .L^7.Il.Al.Ir.Ar.G^4 + 36.Ib^2.L^5.Il.Al.Ar.G^3 + 36.Ib^2.L^3.Il^2.Al.G + 36.Ib^2.L^3.Il^2.Ar.G + 144.Ib^2.L^3.Il.Al.Ir.G + 144.Ib^2.L^3.Il.Ir.Ar.G + 12.Ib.L^5.Il^2.Al.Ar.G^4 + 48.Ib.L^5.Il.Al.Ir.Ar.G^4 + 144.Ib.L^3.Il^2.Al.Ir.G^2 +
    144.Ib.L^3.Il^2.Ir.Ar.G^2 + 12.L^5.Il^2.Al.Ir.Ar.G^5 )/(36.E.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 36.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 36.E.Ab.Ib^2.L^2.Il^2.Al + 36.E.Ab.Ib^2.L^2.Il^2.Ar + 72.E.Ab.Ib^2.L^2.Il.Al.Ir + 72.E.Ab.Ib^2.L^2.Il.Ir.Ar
    + 36.E.Ab. b^2.L^2.Al.Ir^2 + 36.E.Ab.Ib^2.L^2.Ir^2.Ar + 12.E.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 132.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 12.E.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 + 144.E.Ab.Ib.L^2.Il^2.Al.Ir.G + 144.E.Ab.Ib.L^2.Il^2.Ir.Ar.G + 144
    .E.Ab.Ib.L^2.Il.Al.Ir^2.G + 144.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + 12.E.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 12.E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 432.E.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 432.E.Ib^2.Il^2.Al.Ir.G + 432.E.Ib^2.Il^2.Ir.Ar.G + 432.E.Ib^2.
    Il.Al.Ir^2.G + 432.E.Ib^2.Il.Ir^2.Ar.G + 144.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 144.E.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 432.E.Ib.Il^2.Al.Ir^2.G^2 + 432.E.Ib.Il^2.Ir^2.Ar.G^2 + 36.E.L^2.Il^2.Al.Ir^2.Ar.G^5 )

FLEX(4,5) := ( - 3.Ab.Ib^2.L^5.Il.Al.G - 3.Ab.Ib^2.L^5.Al.Ir.G - 12.Ab.Ib.L^5.Il.Al.Ir.G^2 - 36.Ib^2.L^3.Il.Al.Ir.G^2 - 18.Ib.L^3.Il^2.Al.Ir.G^3 )/(6.E.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 6.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 6.E.Ab.Ib^2.L^2.Il^2.Al + 6.E.Ab.
    Ib^2.L^2.Il^2.Ar + 12.E.Ab.Ib^2.L^2.Il.Al.Ir + 12.E.Ab.Ib^2.L^2.Il.Ir.Ar + 6.E.Ab.Ib^2.L^2.Al.Ir^2 + 6.E.Ab.Ib^2.L^2.Ir^2.Ar + 2.E.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 22.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 2.E.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 +
    24.E.Ab.Ib.L^2.Il^2.Al.Ir.G + 24.E.Ab.Ib.L^2.Il^2.Ir.Ar.G + 24.E.Ab.Ib.L^2.Il.Al.Ir^2.G + 24.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + 2.E.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 2.E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 72.E.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 72.E
    .Ib^2.Il^2.Al.Ir.G + 72.E.Ib^2.Il^2.Ir.Ar.G + 72.E.Ib^2.Il.Al.Ir^2.G + 72.E.Ib^2.Il.Ir^2.Ar.G + 24.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 24.E.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 72.E.Ib.Il^2.Al.Ir^2.G^2 + 72.E.Ib.Il^2.Ir^2.Ar.G^2 + 6.E.L^2.
    Il^2.Al.Ir^2.Ar.G^5 )

FLEX(4,6) := ( - 6.Ab.Ib^2.L^4.Il.Al - 6.Ab.Ib^2.L^4.Il.Ar - 6.Ab.Ib^2.L^4.Al.Ir - 6.Ab.Ib^2.L^4.Ir.Ar + Ab.Ib.L^6.Il.Al.Ar.G^3 - 2.Ab.Ib.L^6.Al.Ir.Ar.G^3 - 24.Ab.Ib.L^4.Il.Al.Ir.G - 24.Ab.Ib.L^4.Il.Ir.Ar.G - 2.Ab.L^6.Il.Al.Ir.Ar.G^4 - 72.
    Ib^2.L^2.Il.Al.Ir.G - 72.Ib^2.L^2.Il.Ir.Ar.G - 24.Ib.L^4.Il.Al.Ir.Ar.G^4 - 72.Ib.L^2.Il^2.Al.Ir.G^2 - 72.Ib.L^2.Il^2.Ir.Ar.G^2 - 6.L^4.Il^2.Al.Ir.Ar.G^5 )/(12.E.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 12.E.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 12.E.Ab.
    Ib^2.L^2.Il^2.Al + 12.E.Ab.Ib^2.L^2.Il^2.Ar + 24.E.Ab.Ib^2.L^2.Il.Al.Ir + 24.E.Ab.Ib^2.L^2.Il.Ir.Ar + 12.E.Ab.Ib^2.L^2.Al.Ir^2 + 12.E.Ab.Ib^2.L^2.Ir^2.Ar + 4.E.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 44.E.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 4.
    E.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 + 48.E.Ab.Ib.L^2.Il^2.Al.Ir.G + 48.E.Ab.Ib.L^2.Il^2.Ir.Ar.G + 48.E.Ab.Ib.L^2.Il.Al.Ir^2.G + 48.E.Ab.Ib.L^2.Il.Ir^2.Ar.G + 4.E.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 4.E.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 144.E.
    Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 144.E.Ib^2.Il^2.Al.Ir.G + 144.E.Ib^2.Il^2.Ir.Ar.G + 144.E.Ib^2.Il.Al.Ir^2.G + 144.E.Ib^2.Il.Ir^2.Ar.G + 48.E.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 48.E.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 144.E.Ib.Il^2.Al.Ir^2.G^2
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$$+ 144.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 12.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$FLEX(5,1) := (- 3.Ab.Ib^{2}.L^{5}.Il.Al.G - 3.Ab.Ib^{2}.L^{5}.Al.Ir.G - 12.Ab.Ib.L^{5}.Il.Al.Ir.G^{2} - 36.Ib^{2}.L^{3}.Il.Al.Ir.G^{2} - 18.Ib.L^{3}.Il.Al.Ir^{2}.G^{3})/(6.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 6.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 6.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 6.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 12.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 12.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 6.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 6.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 2.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 22.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 2.E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 24.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 24.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 24.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 24.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 2.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 2.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 72.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 72.E.Ib^{2}.Il^{2}.Al.Ir.G + 72.E.Ib^{2}.Il^{2}.Ir.Ar.G + 72.E.Ib^{2}.Il.Al.Ir^{2}.G + 72.E.Ib^{2}.Il.Ir^{2}.Ar.G + 24.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 24.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 72.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 72.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 6.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$FLEX(5,2) := (3.Ab.Ib^{2}.L^{3}.Il^{2} + 6.Ab.Ib^{2}.L^{3}.Il.Ir + 3.Ab.Ib^{2}.L^{3}.Ir^{2} + 12.Ab.Ib.L^{3}.Il^{2}.Ir.G + 12.Ab.Ib.L^{3}.Il.Ir^{2}.G + 36.Ib^{2}.L.Il^{2}.Ir.G + 36.Ib^{2}.L.Il.Ir^{2}.G + 36.Ib.L.Il^{2}.Ir^{2}.G^{2})/(3.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 3.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 3.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 3.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 6.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 6.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 3.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 3.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 11.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 12.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 12.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 12.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 12.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar\ G + E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 36.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 36.E.Ib^{2}.Il^{2}.Al.Ir.G + 36.E.Ib^{2}.Il^{2}.Ir.Ar.G + 36.E.Ib^{2}.Il.Al.Ir^{2}.G + 36.E.Ib^{2}.Il.Ir^{2}.Ar.G + 12.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 12.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 36.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 36.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 3.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$FLEX(5,3) := (6.Ab.Ib^{2}.L^{4}.Il.Al.G + 6.Ab.Ib^{2}.L^{4}.Al.Ir.G + 21.Ab.Ib.L^{4}.Il.Al.Ir.G^{2} + 3.Ab.Ib.L^{4}.Al.Ir^{2}.G^{2} + 72.Ib^{2}.L^{2}.Il.Al.Ir.G^{2} + 36.Ib.L^{2}.Il.Al.Ir^{2}.G^{3})/(6.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 6.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 6.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 6.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 12.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 12.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 6.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 6.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 2.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 22.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 2.E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 24.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 24.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 24.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 24.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 2.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 2.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 72.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 72.E.Ib^{2}.Il^{2}.Al.Ir.G + 72.E.Ib^{2}.Il^{2}.Ir.Ar.G + 72.E.Ib^{2}.Il.Al.Ir^{2}.G + 72.E.Ib^{2}.Il.Ir^{2}.Ar.G + 24.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 24.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 72.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 72.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 6.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$FLEX(5,4) := (- 3.Ab.Ib^{2}.L^{5}.Il.Al.G - 3.Ab.Ib^{2}.L^{5}.Al.Ir.G - 12.Ab.Ib.L^{5}.Il.Al.Ir.G^{2} - 36.Ib^{2}.L^{3}.Il.Al.Ir.G^{2} - 18.Ib.L^{3}.Il^{2}.Al.Ir.G^{3})/(6.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 6.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 6.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 6.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 12.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 12.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 6.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 6.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 2.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 22.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 2.E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 24.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 24.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 24.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 24.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 2.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 2.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 72.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 72.E.Ib^{2}.Il^{2}.Al.Ir.G + 72.E.Ib^{2}.Il^{2}.Ir.Ar.G + 72.E.Ib^{2}.Il.Al.Ir^{2}.G + 72.E.Ib^{2}.Il.Ir^{2}.Ar.G + 24.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 24.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 72.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 72.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 6.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

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            2  5        2           2  5     2          2  3   2       2  3          2  3  2       5  2   3           5          3          5  2  3          3  2             3  2
FLEX(5,5) := (3.Ab.Ib .L .Il.Al.G  + 3.Ab.Ib .L .Al.Ir.G  + 3.Ab.Ib .L .Il  + 6.Ab.Ib .L .Il.Ir + 3.Ab.Ib .L .Ir  + Ab.Ib.L .Il .Al.G  + 11.Ab.Ib.L .Il.Al.Ir.G  + Ab.Ib.L .Al.Ir .G  + 12.Ab.Ib.L .Il .Ir.G + 12.Ab.Ib.L .Il.Ir

           5  2       4        5      2  4       2  3          3       2    2         2    2       3  2     4        3      2  4         2  2  2       3  2     2  5
      .G + Ab.L .Il .Al.Ir.G  + Ab.L .Il.Al.Ir .G  + 36.Ib .L .Il.Al.Ir.G  + 36.Ib .L.Il .Ir.G + 36.Ib .L.Il.Ir .G + 12.Ib.L .Il .Al.Ir.G  + 12.Ib.L .Il.Al.Ir .G  + 36.Ib.L.Il .Ir .G  + 3.L .Il .Al.Ir .G )/(3.E .Ab.

        2  4         2          2  4        2          2  2  2          2  2  2          2  2          2  2           2  2    2          2  2  2           4
      Ib .L .Il.Al.Ar.G  + 3.E .Ab.Ib .L .Al.Ir.Ar.G  + 3.E .Ab.Ib .L .Il .Al + 3.E .Ab.Ib .L .Il .Ar + 6.E .Ab.Ib .L .Il.Al.Ir + 6.E .Ab.Ib .L .Il.Ir.Ar + 3.E .Ab.Ib .L .Al.Ir  + 3.E .Ab.Ib .L .Ir .Ar + E .Ab.Ib.L

         2    3             4             3          4   2   3            2  2             2  2               2      2            2     2          4  2
      .Il .Al.Ar.G  + 11.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + E .Ab.Ib.L .Al.Ir .Ar.G  + 12.E .Ab.Ib.L .Il .Al.Ir.G + 12.E .Ab.Ib.L .Il .Ir.Ar.G + 12.E .Ab.Ib.L .Il.Al.Ir .G + 12.E .Ab.Ib.L .Il.Ir .Ar.G + E .Ab.L .Il .Al.Ir

       4       4        2    4         2  2            3         2  2           2  2             2     2          2     2          2  2        4              2
      .Ar.G  + E .Ab.L .Il.Al.Ir .Ar.G  + 36.E .Ib .L .Il.Al.Ir.Ar.G  + 36.E .Ib .Il .Al.Ir.G + 36.E .Ib .Il .Ir.Ar.G + 36.E .Ib .Il.Al.Ir .G + 36.E .Ib .Il.Ir .Ar.G + 12.E .Ib.L .Il .Al.Ir.Ar.G  + 12.E .Ib.L .Il.Al

        2    4          2     2  2          2  2     2       2  2    2    5
      .Ir .Ar.G  + 36.E .Ib.Il .Al.Ir .G  + 36.E .Ib.Il .Ir .Ar.G  + 3.E .L .Il .Al.Ir .Ar.G )

             2  4             2  4            4  2    2            4          2       2  2          2         2  2     3             2  4          2            2  4          2           2
FLEX(5,6) := (6.Ab.Ib .L .Il.Al.G + 6.Ab.Ib .L .Al.Ir.G + 3.Ab.Ib.L .Il .Al.G  + 21.Ab.Ib.L .Il.Al.Ir.G  + 72.Ib .L .Il.Al.Ir.G  + 36.Ib.L .Il .Al.Ir.G )/(6.E .Ab.Ib .L .Il.Al.Ar.G  + 6.E .Ab.Ib .L .Al.Ir.Ar.G  + 6.E .Ab.Ib .

       2   2            2  2  2             2  2             2  2             2  2    2          2  2  2             4  2     3              4              3
      L .Il .Al + 6.E .Ab.Ib .L .Il .Ar + 12.E .Ab.Ib .L .Il.Al.Ir + 12.E .Ab.Ib .L .Il.Ir.Ar + 6.E .Ab.Ib .L .Al.Ir  + 6.E .Ab.Ib .L .Ir .Ar + 2.E .Ab.Ib.L .Il .Al.Ar.G  + 22.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.E .Ab.Ib

        4    2    3              2  2                 2  2                 2       2               2      2            4  2                    4        2    4       2  2
      .L .Al.Ir .Ar.G  + 24.E .Ab.Ib.L .Il .Al.Ir.G + 24.E .Ab.Ib.L .Il .Ir.Ar.G + 24.E .Ab.Ib.L .Il.Al.Ir .G + 24.E .Ab.Ib.L .Il.Ir .Ar.G + 2.E .Ab.L .Il .Al.Ir.Ar.G  + 2.E .Ab.L .Il.Al.Ir .Ar.G  + 72.E .Ib .L .Il.

           3           2    2          2    2           2        2          2       2           2  2            4             2     2    4          2     2  2           2
      Al.Ir.Ar.G  + 72.E .Ib .Il .Al.Ir.G + 72.E .Ib .Il .Ir.Ar.G + 72.E .Ib .Il.Al.Ir .G + 72.E .Ib .Il.Ir .Ar.G + 24.E .Ib.L .Il .Al.Ir.Ar.G  + 24.E .Ib.L .Il.Al.Ir .Ar.G  + 72.E .Ib.Il .Al.Ir .G  + 72.E .Ib.Il .

       2    2          2   2    2    5
      Ir .Ar.G  + 6.E .L .Il .Al.Ir .Ar.G )

              2  4            2  4            2  4            2  4              6       3             6         3             4                     4                   6           4
FLEX(6,1) := ( - 6.Ab.Ib .L .Il.Al - 6.Ab.Ib .L .Il.Ar - 6.Ab.Ib .L .Al.Ir - 6.Ab.Ib .L .Ir.Ar + Ab.Ib.L .Il.Al.Ar.G  - 2.Ab.Ib.L .Al.Ir.Ar.G  - 24.Ab.Ib.L .Il.Al.Ir.G - 24.Ab.Ib.L .Il.Ir.Ar.G - 2.Ab.L .Il.Al.Ir.Ar.G  - 72.

         2  2             2  2                  4                   4            2  4            2            2  4          2           2  2  2            2  2  2            2  2
      Ib .L .Il.Al.Ir.G - 72.Ib .L .Il.Ir.Ar.G + 12.Ib.L .Il.Al.Ir.Ar.G )/(12.E .Ab.Ib .L .Il.Al.Ar.G  + 12.E .Ab.Ib .L .Al.Ir.Ar.G  + 12.E .Ab.Ib .L .Il .Al + 12.E .Ab.Ib .L .Il .Ar + 24.E .Ab.Ib .L .Il.Al.Ir + 24.E

           2  2               2  2    2            2  2  2          4  2    3             4           3            4   2   3            2  2              2   2
      .Ab.Ib .L .Il.Ir.Ar + 12.E .Ab.Ib .L .Al.Ir  + 12.E .Ab.Ib .L .Ir .Ar + 4.E .Ab.Ib.L .Il .Al.Ar.G  + 44.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 4.E .Ab.Ib.L .Al.Ir .Ar.G  + 48.E .Ab.Ib.L .Il .Al.Ir.G + 48.E .Ab.Ib.L .Il

                       2         2            2       2           4  2          4           4       2    4           2  2             3            2  2             2  2
      .Ir.Ar.G + 48.E .Ab.Ib.L .Il.Al.Ir .G + 48.E .Ab.Ib.L .Il.Ir .Ar.G + 4.E .Ab.L .Il .Al.Ir.Ar.G  + 4.E .Ab.L .Il.Al.Ir .Ar.G  + 144.E .Ib .L .Il.Al.Ir.Ar.G  + 144.E .Ib .Il .Al.Ir.G + 144.E .Ib .Il .Ir.Ar.G -

             2         2             2     2             2  2           4            2       2    4            2    2  2            2  2    2          2  2    2    5
      144.E .Ib .Il.Al.Ir .G + 144.E .Ib .Il.Ir .Ar.G + 48.E .Ib.L .Il .Al.Ir.Ar.G  + 48.E .Ib.L .Il.Al.Ir .Ar.G  + 144.E .Ib.Il .Al.Ir .G  + 144.E .Ib.Il .Ir .Ar.G  + 12.E .L .Il .Al.Ir .Ar.G )

             2  4             2  4            4  2    2             4          2       2  2          2         2  2     3             2  4          2            2  4          2
FLEX(6,2) := ( - 6.Ab.Ib .L .Il.Ar.G - 6.Ab.Ib .L .Ir.Ar.G - 3.Ab.Ib.L .Il .Ar.G  - 21.Ab.Ib.L .Il.Ir.Ar.G  - 72.Ib .L .Il.Ir.Ar.G  - 36.Ib.L .Il .Ir.Ar.G )/(6.E .Ab.Ib .L .Il.Al.Ar.G  + 6.E .Ab.Ib .L .Al.Ir.Ar.G  + 6.E .Ab.

         2  2  2           2  2  2             2  2             2  2             2  2    2          2  2  2             4  2     3              4              3
      Ib .L .Il .Al + 6.E .Ab.Ib .L .Il .Ar + 12.E .Ab.Ib .L .Il.Al.Ir + 12.E .Ab.Ib .L .Il.Ir.Ar + 6.E .Ab.Ib .L .Al.Ir  + 6.E .Ab.Ib .L .Ir .Ar + 2.E .Ab.Ib.L .Il .Al.Ar.G  + 22.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.E .

               4    2    3              2  2                 2  2                 2       2               2      2            4  2                    4        2    4          2
      Ab.Ib.L .Al.Ir .Ar.G  + 24.E .Ab.Ib.L .Il .Al.Ir.G + 24.E .Ab.Ib.L .Il .Ir.Ar.G + 24.E .Ab.Ib.L .Il.Al.Ir .G + 24.E .Ab.Ib.L .Il.Ir .Ar.G + 2.E .Ab.L .Il .Al.Ir.Ar.G  + 2.E .Ab.L .Il.Al.Ir .Ar.G  + 72.E .Ib .

       2          3           2    2          2    2           2        2          2       2           2  2            4             2     2    4          2     2  2
      L .Il.Al.Ir.Ar.G  + 72.E .Ib .Il .Al.Ir.G + 72.E .Ib .Il .Ir.Ar.G + 72.E .Ib .Il.Al.Ir .G + 72.E .Ib .Il.Ir .Ar.G + 24.E .Ib.L .Il .Al.Ir.Ar.G  + 24.E .Ib.L .Il.Al.Ir .Ar.G  + 72.E .Ib.Il .Al.Ir .G  + 72.E .Ib

         2    2    2           2   2    2    5
      .Il .Ir .Ar.G  + 6.E .L .Il .Al.Ir .Ar.G )
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             2 3                   2 3                  2 3                  2 3                 5          3           5          3          3                  3                5              4
FLEX(6,3) := (12.Ab.Ib .L .Il.Al + 12.Ab.Ib .L .Il.Ar + 12.Ab.Ib .L .Al.Ir + 12.Ab.Ib .L .Ir.Ar - 2.Ab.Ib.L .Il.Al.Ar.G - 2.Ab.Ib.L .Al.Ir.Ar.G + 36.Ab.Ib.L .Il.Al.Ir.G + 36.Ab.Ib.L .Il.Ir.Ar.G + 3.Ab.L .Il.Al.Ir.Ar.G +

        2                  2                  3              4              2 4              2            2 4             2             2 2  2             2 2  2           2 2
   144.Ib .L.Il.Al.Ir.G + 144.Ib .L.Il.Ir.Ar.G - 24.Ib.L .Il.Al.Ir.Ar.G )/(12.E .Ab.Ib .L .Il.Al.Ar.G + 12.E .Ab.Ib .L .Al.Ir.Ar.G + 12.E .Ab.Ib .L .Il .Al + 12.E .Ab.Ib .L .Il .Ar + 24.E .Ab.Ib .L .Il.Al.Ir + 24

      2 2                 2 2   2             2 2  2            4  2          3            4            3            4   2    3              2  2                 2
   .E .Ab.Ib .L .Il.Ir.Ar + 12.E .Ab.Ib .L .Al.Ir + 12.E .Ab.Ib .L .Ir .Ar + 4.E .Ab.Ib.L .Il .Al.Ar.G + 44.E .Ab.Ib.L .Il.Al.Ir.Ar.G + 4.E .Ab.Ib.L .Al.Ir .Ar.G + 48.E .Ab.Ib.L .Il .Al.Ir.G + 48.E .Ab.Ib.L .

    2              2            2              2   2             4 2           4             4   2   4               2 2            3                 2  2              2  2
   Il .Ir.Ar.G + 48.E .Ab.Ib.L .Il.Al.Ir .G + 48.E .Ab.Ib.L .Il.Ir .Ar.G + 4.E .Ab.L .Il .Al.Ir.Ar.G + 4.E .Ab.L .Il.Al.Ir .Ar.G + 144.E .Ib .L .Il.Al.Ir.Ar.G + 144.E .Ib .Il .Al.Ir.G + 144.E .Ib .Il .Ir.Ar.G

        2          2              2     2           2  2           4           2          2   4           2      2 2              2   2    2            2   2    2     5
   + 144.E .Ib .Il.Al.Ir .G + 144.E .Ib .Il.Ir .Ar.G + 48.E .Ib.L .Il .Al.Ir.Ar.G + 48.E .Ib.L .Il.Al.Ir .Ar.G + 144.E .Ib.Il .Al.Ir .G + 144.E .Ib.Il .Ir .Ar.G + 12.E .L .Il .Al.Ir .Ar.G )

              2 4                  2 4                 2 4                2 4               6          3            6           3              4                   4                   6             4
FLEX(6,4) := ( - 6.Ab.Ib .L .Il.Al - 6.Ab.Ib .L .Il.Ar - 6.Ab.Ib .L .Al.Ir - 6.Ab.Ib .L .Ir.Ar + Ab.Ib.L .Il.Al.Ar.G - 2.Ab.Ib.L .Al.Ir.Ar.G - 24.Ab.Ib.L .Il.Al.Ir.G - 24.Ab.Ib.L .Il.Ir.Ar.G - 2.Ab.L .Il.Al.Ir.Ar.G - 72.

  2 2                2 2                4              4            2 2        2            2  2       2    4  2              5                    2 4              2           2 4              2
   Ib .L .Il.Al.Ir.G - 72.Ib .L .Il.Ir.Ar.G - 24.Ib.L .Il.Al.Ir.Ar.G - 72.Ib.L .Il .Al.Ir.G - 72.Ib.L .Il .Ir.Ar.G - 6.L .Il .Al.Ir.Ar.G )/(12.E .Ab.Ib .L .Il.Al.Ar.G + 12.E .Ab.Ib .L .Al.Ir.Ar.G + 12.E .Ab.

   2 2  2            2 2  2              2 2                     2 2                  2 2    2             2 2  2            4  2         3               4             3
   Ib .L .Il .Al + 12.E .Ab.Ib .L .Il .Ar + 24.E .Ab.Ib .L .Il.Al.Ir + 24.E .Ab.Ib .L .Il.Ir.Ar + 12.E .Ab.Ib .L .Al.Ir + 12.E .Ab.Ib .L .Ir .Ar + 4.E .Ab.Ib.L .Il .Al.Ar.G + 44.E .Ab.Ib.L .Il.Al.Ir.Ar.G + 4.

     4   2    3                2  2                2  2               2       2              2    2            4  2          4             4      2    4
   E .Ab.Ib.L .Al.Ir .Ar.G + 48.E .Ab.Ib.L .Il .Al.Ir.G + 48.E .Ab.Ib.L .Il .Ir.Ar.G + 48.E .Ab.Ib.L .Il.Al.Ir .G + 48.E .Ab.Ib.L .Il.Ir .Ar.G + 4.E .Ab.L .Il .Al.Ir.Ar.G + 4.E .Ab.L .Il.Al.Ir .Ar.G + 144.E .

  2 2              3           2  2              2  2               2            2              2  2          4           2       2   4          2   2 2
   Ib .L .Il.Al.Ir.Ar.G + 144.E .Ib .Il .Al.Ir.G + 144.E .Ib .Il .Ir.Ar.G + 144.E .Ib .Il.Al.Ir .G + 144.E .Ib .Il.Ir .Ar.G + 48.E .Ib.L .Il .Al.Ir.Ar.G + 48.E .Ib.L .Il.Al.Ir .Ar.G + 144.E .Ib.Il .Al.Ir .G

          2   2    2       2  2    2   5
   + 144.E .Ib.Il .Ir .Ar.G + 12.E .L .Il .Al.Ir .Ar.G )

            2 4                2 4              4  2    2             4          2        2 2          2          2  2      3                 2 4            2             2 4           2          2
FLEX(6,5) := (6.Ab.Ib .L .Il.Al.G + 6.Ab.Ib .L .Al.Ir.G + 3.Ab.Ib.L .Il .Al.G + 21.Ab.Ib.L .Il.Al.Ir.G + 72.Ib .L .Il.Al.Ir.G + 36.Ib.L .Il .Al.Ir.G )/(6.E .Ab.Ib .L .Il.Al.Ar.G + 6.E .Ab.Ib .L .Al.Ir.Ar.G + 6.E .Ab.Ib .

  2 2            2 2  2             2 2              2 2               2 2  2            2 2  2          4  2         3               4            3
   L .Il .Al + 6.E .Ab.Ib .L .Il .Ar + 12.E .Ab.Ib .L .Il.Al.Ir + 12.E .Ab.Ib .L .Il.Ir.Ar + 6.E .Ab.Ib .L .Al.Ir + 6.E .Ab.Ib .L .Ir .Ar + 2.E .Ab.Ib.L .Il .Al.Ar.G + 22.E .Ab.Ib.L .Il.Al.Ir.Ar.G + 2.E .Ab.Ib

  4    2   3                2  2               2  2             2      2              2    2          4  2          4            4    2    4          2 2
   .L .Al.Ir .Ar.G + 24.E .Ab.Ib.L .Il .Al.Ir.G + 24.E .Ab.Ib.L .Il .Ir.Ar.G + 24.E .Ab.Ib.L .Il.Al.Ir .G + 24.E .Ab.Ib.L .Il.Ir .Ar.G + 2.E .Ab.L .Il .Al.Ir.Ar.G + 2.E .Ab.L .Il.Al.Ir .Ar.G + 72.E .Ib .L .Il.

      3            2  2              2  2             2         2            2       2             2  2          4           2      2  4          2   2 2          2
   Al.Ir.Ar.G + 72.E .Ib .Il .Al.Ir.G + 72.E .Ib .Il .Ir.Ar.G + 72.E .Ib .Il.Al.Ir .G + 72.E .Ib .Il.Ir .Ar.G + 24.E .Ib.L .Il .Al.Ir.Ar.G + 24.E .Ib.L .Il.Al.Ir .Ar.G + 72.E .Ib.Il .Al.Ir .G + 72.E .Ib.Il .

  2    2       2  2    2   5
   Ir .Ar.G + 6.E .L .Il .Al.Ir .Ar.G )

             2 3                2 3                 2 3                 2 3                5          3           5          3         3 2                3 2              3
FLEX(6,6) := (12.Ab.Ib .L .Il.Al + 12.Ab.Ib .L .Il.Ar + 12.Ab.Ib .L .Al.Ir + 12.Ab.Ib .L .Ir.Ar + 4.Ab.Ib.L .Il.Al.Ar.G + 4.Ab.Ib.L .Al.Ir.Ar.G + 12.Ab.Ib.L .Il .Al.G + 12.Ab.Ib.L .Il .Ar.G + 48.Ab.Ib.L .Il.Al.Ir.G + 48.Ab.

    3              5 2        4        5           4              2                   2                 3                 4              2           2              2          2         3  2          5
   Ib.L .Il.Ir.Ar.G + Ab.L .Il .Al.Ar.G + 4.Ab.L .Il.Al.Ir.Ar.G + 144.Ib .L.Il.Al.Ir.G + 144.Ib .L.Il.Ir.Ar.G + 48.Ib.L .Il.Al.Ir.Ar.G + 144.Ib.L.Il .Al.Ir.G + 144.Ib.L.Il .Ir.Ar.G + 12.L .Il .Al.Ir.Ar.G )/(12

     2 4              2            2 4             2            2 2  2             2 2  2             2 2                 2 2                2 2    2             2 2  2
   .E .Ab.Ib .L .Il.Al.Ar.G + 12.E .Ab.Ib .L .Al.Ir.Ar.G + 12.E .Ab.Ib .L .Il .Al + 12.E .Ab.Ib .L .Il .Ar + 24.E .Ab.Ib .L .Il.Al.Ir + 24.E .Ab.Ib .L .Il.Ir.Ar + 12.E .Ab.Ib .L .Al.Ir + 12.E .Ab.Ib .L .Ir .Ar

      4  2          3              4            3              4   2    3              2  2              2  2               2      2              2    2
   + 4.E .Ab.Ib.L .Il .Al.Ar.G + 44.E .Ab.Ib.L .Il.Al.Ir.Ar.G + 4.E .Ab.Ib.L .Al.Ir .Ar.G + 48.E .Ab.Ib.L .Il .Al.Ir.G + 48.E .Ab.Ib.L .Il .Ir.Ar.G + 48.E .Ab.Ib.L .Il.Al.Ir .G + 48.E .Ab.Ib.L .Il.Ir .Ar.G +

     4  2          4             4    2    4           2 2             3          2  2               2  2             2     2               2      2            2 2
   4.E .Ab.L .Il .Al.Ir.Ar.G + 4.E .Ab.L .Il.Al.Ir .Ar.G + 144.E .Ib .L .Il.Al.Ir.Ar.G + 144.E .Ib .Il .Al.Ir.G + 144.E .Ib .Il .Ir.Ar.G + 144.E .Ib .Il.Al.Ir .G + 144.E .Ib .Il.Ir .Ar.G + 48.E .Ib.L .Il .Al.
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$$Ir.Ar.G^{4} + 48.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 144.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 144.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 12.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

FG(1,1) := 0

FG(2,1) := (- L.G.W)/2

$$FG(3,1) := (- L^{2}.G^{2}.W)/12$$

FG(4,1) := P

FG(5,1) := (- L.G.W)/2

$$FG(6,1) := (L^{2}.G^{2}.W)/12$$

$$DU(1,1) := (12.Ab.Ib^{2}.L^{7}.Al.Ar.P.G^{2} + 36.Ab.Ib^{2}.L^{6}.Il.Al.G^{2}.W - 36.Ab.Ib^{2}.L^{6}.Il.Ar.G^{2}.W + 36.Ab.Ib^{2}.L^{6}.Al.Ir.G^{2}.W - 36.Ab.Ib^{2}.L^{6}.Ir.Ar.G^{2}.W + 48.Ab.Ib^{2}.L^{5}.Il.Al.P + 48.Ab.Ib^{2}.L^{5}.Il.Ar.P + 48.Ab.Ib^{2}.L^{5}.Al.Ir.P + 48.Ab.Ib^{2}.$$

$$L^{5}.Ir.Ar.P + 3.Ab.Ib.L^{8}.Il.Al.Ar.G^{5}.W - 3.Ab.Ib.L^{8}.Al.Ir.Ar.G^{5}.W + 16.Ab.Ib.L^{7}.Il.Al.Ar.P.G^{3} + 16.Ab.Ib.L^{7}.Al.Ir.Ar.P.G^{3} + 144.Ab.Ib.L^{6}.Il.Al.Ir.G^{3}.W - 144.Ab.Ib.L^{6}.Il.Ir.Ar.G^{3}.W + 192.Ab.Ib.L^{5}.Il.Al.Ir.P.G +$$

$$192.Ab.Ib.L^{5}.Il.Ir.Ar.P.G + 16.Ab.L^{7}.Il.Al.Ir.Ar.P.G^{4} + 432.Ib^{2}.L^{4}.Il.Al.Ir.G^{3}.W - 432.Ib^{2}.L^{4}.Il.Ir.Ar.G^{3}.W + 432.Ib^{2}.L^{3}.Il.Al.Ir.P.G + 432.Ib^{2}.L^{3}.Il.Ir.Ar.P.G + 36.Ib.L^{6}.Il.Al.Ir.Ar.G^{6}.W - 72.Ib.L^{5}.Il.Al.Ir.$$

$$Ar.P.G^{4} + 288.Ib.L^{4}.Il.Al.Ir^{2}.G^{4}.W - 144.Ib.L^{4}.Il.Ir^{2}.Ar.G^{4}.W + 6.L^{6}.Il.Al.Ir^{2}.Ar.G^{7}.W)/(144.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 144.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 144.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 144.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 288.E.Ab$$

$$.Ib^{2}.L^{2}.Il.Al.Ir + 288.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 144.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 144.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 48.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 528.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 48.E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 576.E.Ab.Ib.L^{2}.Il^{2}$$

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$$.Al.Ir.G + 576.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 576.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 576.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 48.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 48.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 1728.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 1728.E.Ib^{2}.Il^{2}.Al$$

$$.Ir.G + 1728.E.Ib^{2}.Il^{2}.Ir.Ar.G + 1728.E.Ib^{2}.Il.Al.Ir^{2}.G + 1728.E.Ib^{2}.Il.Ir^{2}.Ar.G + 576.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 576.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 1728.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 1728.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 144.E.L^{2}$$

$$.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$DU(2,1) := (- 12.Ab.Ib^{2}.L^{6}.Il.Ar.G^{3}.W - 12.Ab.Ib^{2}.L^{6}.Ir.Ar.G^{3}.W + 12.Ab.Ib^{2}.L^{5}.Il.Ar.P.G + 12.Ab.Ib^{2}.L^{5}.Ir.Ar.P.G - 24.Ab.Ib^{2}.L^{4}.Il^{2}.G.W - 48.Ab.Ib^{2}.L^{4}.Il.Ir.G.W - 24.Ab.Ib^{2}.L^{4}.Ir^{2}.G.W - 5.Ab.Ib.L^{6}.Il^{2}.Ar.G^{4}.W - 44.Ab.Ib.$$

$$L^{6}.Il.Ir.Ar.G^{4}.W - 3.Ab.Ib.L^{6}.Ir^{2}.Ar.G^{4}.W + 48.Ab.Ib.L^{5}.Il.Ir.Ar.P.G^{2} - 96.Ab.Ib.L^{4}.Il^{2}.Ir.G^{2}.W - 96.Ab.Ib.L^{4}.Il.Ir^{2}.G^{2}.W - 4.Ab.L^{6}.Il^{2}.Ir.Ar.G^{5}.W - 4.Ab.L^{6}.Il.Ir^{2}.Ar.G^{5}.W - 144.Ib^{2}.L^{4}.Il.Ir.Ar.G^{4}.W + 144.Ib^{2}.L^{3}.$$

$$Il.Ir.Ar.P.G^{2} - 288.Ib^{2}.L^{2}.Il^{2}.Ir.G^{2}.W - 288.Ib^{2}.L^{2}.Il.Ir^{2}.G^{2}.W - 60.Ib.L^{4}.Il^{2}.Ir.Ar.G^{5}.W - 36.Ib.L^{4}.Il.Ir^{2}.Ar.G^{5}.W + 72.Ib.L^{3}.Il^{2}.Ir.Ar.P.G^{3} - 288.Ib.L^{2}.Il^{2}.Ir^{2}.G^{3}.W - 12.L^{4}.Il^{2}.Ir^{2}.Ar.G^{6}.W)/(24.E.Ab.Ib^{2}.L^{4}.Il$$

$$.Al.Ar.G^{2} + 24.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 24.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 24.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 48.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 48.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 24.E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 24.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 8.E.Ab.Ib.L^{4}.$$

$$Il^{2}.Al.Ar.G^{3} + 88.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 8.E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 96.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 96.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 96.E.Ab.Ib.L^{2}.Il.Al.Ir^{2}.G + 96.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 8.E.Ab.L^{4}.Il^{2}.Al.$$

$$Ir.Ar.G^{4} + 8.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 288.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 288.E.Ib^{2}.Il^{2}.Al.Ir.G + 288.E.Ib^{2}.Il^{2}.Ir.Ar.G + 288.E.Ib^{2}.Il.Al.Ir^{2}.G + 288.E.Ib^{2}.Il.Ir^{2}.Ar.G + 96.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 96.E.Ib.$$

$$L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 288.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 288.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 24.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

```
               2  5         2              2  5          2              2  5         2                2  5         2               2  4                    2  4                    2  4                     2  4                   7
DU(3,1) := ( - 72.Ab.Ib .L .I1.A1.G .W + 72.Ab.Ib .L .I1.Ar.G .W - 72.Ab.Ib .L .A1.Ir.G .W + 72.Ab.Ib .L .Ir.Ar.G .W - 72.Ab.Ib .L .I1.A1.P  - 72.Ab.Ib .L .I1.Ar.P  - 72.Ab.Ib .L .A1.Ir.P  - 72.Ab.Ib .L .Ir.Ar.P  - 6.Ab.Ib.L

              5              7             5                 6              3             6              3              5              3               5              3               5       2 3               5    2     3
  ..I.A1.Ar.G .W - 6.Ab.Ib.L .A1.Ir.Ar.G .W - 24.Ab.Ib.L .I1.A1.Ar.P .G + 12.Ab.Ib.L .A1.Ir.Ar.P .G - 264.Ab.Ib.L .I1.A1.Ir.G .W + 240.Ab.Ib.L .I1.Ir.Ar.G .W - 48.Ab.Ib.L .A1.Ir .G .W + 24.Ab.Ib.L .Ir .Ar.G .W -

              4                  4                  7            6        7       2       6             6                  4           2  3           3          2  3             3          2  2
  288.Ab.Ib.L .I1.A1.Ir.P .G - 288.Ab.Ib.L .I1.Ir.Ar.P .G - Ab.L .I1.A1.Ir.Ar.G .W - Ab.L .A1.Ir .Ar.G .W - 24.Ab.L .I1.A1.Ir.Ar.P .G - 864.Ib .L .I1.A1.Ir.G .W + 864.Ib .L .I1.Ir.Ar.G .W - 864.Ib .L .I1.A1.Ir.P .G

            2  2                 5               6              4                 4            3       2  4          3       2     4         5           2    7                  2  4               2                 2
  - 864.Ib .L .I1.Ir.Ar.P .G - 72.Ib.L .I1.A1.Ir.Ar.G .W + 144.Ib.L .I1.A1.Ir.Ar.P .G - 576.Ib.L .I1.A1.Ir .G .W + 288.Ib.L .I1.Ir .Ar.G .W - 12.L .I1.A1.Ir .Ar.G .W)/(144.E .Ab.Ib .L .I1.A1.Ar.G + 144.E .Ab.Ib .

     4          2                 2  2  2                 2  2  2                 2  2                   2  2                   2  2    2                2  2  2               4    2        3
   L .A1.Ir.Ar.G + 144.E .Ab.Ib .L .I1 .A1 + 144.E .Ab.Ib .L .I1 .Ar + 288.E .Ab.Ib .L .I1.A1.Ir + 288.E .Ab.Ib .L .I1.Ir.Ar + 144.E .Ab.Ib .L .A1.Ir + 144.E .Ab.Ib .L .Ir .Ar + 48.E .Ab.Ib.L .I1 .A1.Ar.G + 528.

          4            3               4     2     3              2   2                    2   2                     2       2                 2      2               4   2          4
   E .Ab.Ib.L .I1.A1.Ir.Ar.G + 48.E .Ab.Ib.L .A1.Ir .Ar.G + 576.E .Ab.Ib.L .I1 .A1.Ir.G + 576.E .Ab.Ib.L .I1 .Ir.Ar.G + 576.E .Ab.Ib.L .I1.A1.Ir .G + 576.E .Ab.Ib.L .I1.Ir .Ar.G + 48.E .Ab.L .I1 .A1.Ir.Ar.G + 48

        4       2    4              2  2             3                2  2                 2  2                  2       2                2      2              2  2          4           2
   .E .Ab.L .I1.A1.Ir .Ar.G + 1728.E .Ib .L .I1.A1.Ir.Ar.G + 1728.E .Ib .I1 .A1.Ir.G + 1728.E .Ib .I1 .Ir.Ar.G + 1728.E .Ib .I1.A1.Ir .G + 1728.E .Ib .I1.Ir .Ar.G + 576.E .Ib.L .I1 .A1.Ir.Ar.G + 576.E .Ib.L .I1.

       2     4               2     2  2               2   2    2             2   2     2    5
   A1.Ir .Ar.G + 1728.E .Ib.I1 .A1.Ir .G + 1728.E .Ib.I1 .Ir .Ar.G + 144.E .L .I1 .A1.Ir .Ar.G )

             2  7            2              2  6          2              2  6          2              2  6         2              2  6         2               2  5                    2  5                   2  5                    2
DU(4,1) := (12.Ab.Ib .L .A1.Ar.P .G + 36.Ab.Ib .L .I1.A1.G .W - 36.Ab.Ib .L .I1.Ar.G .W + 36.Ab.Ib .L .A1.Ir.G .W - 36.Ab.Ib .L .Ir.Ar.G .W + 48.Ab.Ib .L .I1.A1.P  + 48.Ab.Ib .L .I1.Ar.P  + 48.Ab.Ib .L .A1.Ir.P  + 48.Ab.Ib .

    5               8             5               8              5               7              3             7              3                 6            3                6              3               5
   L .Ir.Ar.P + 3.Ab.Ib.L .I1.A1.Ar.G .W - 3.Ab.Ib.L .A1.Ir.Ar.G .W + 16.Ab.Ib.L .I1.A1.Ar.P .G + 16.Ab.Ib.L .A1.Ir.Ar.P .G + 144.Ab.Ib.L .I1.A1.Ir.G .W - 144.Ab.Ib.L .I1.Ir.Ar.G .W + 192.Ab.Ib.L .I1.A1.Ir.P .G +

              5             7              4            2  5             3          2  4          3           2  4         3          2  3  2           2  3  2           2  3
   192.Ab.Ib.L .I1.Ir.Ar.P .G + 16.Ab.L .I1.A1.Ir.Ar.P .G + 144.Ib .L .I1.A1.Ar.P .G + 432.Ib .L .I1.A1.Ir.G .W - 432.Ib .L .I1.Ir.Ar.G .W + 144.Ib .L .I1 .A1.P .G + 144.Ib .L .I1 .Ar.P .G + 576.Ib .L .I1.A1.Ir.P .

          2  3              6             6            5  2            4            5               4         4  2          4          4  2          4            3  2           2        3  2
   G + 576.Ib .L .I1.Ir.Ar.P .G - 36.Ib.L .I1.A1.Ir.Ar.G .W + 48.Ib.L .I1 .A1.Ar.P .G + 192.Ib.L .I1.A1.Ir.Ar.P .G + 144.Ib.L .I1 .A1.Ir.G .W - 288.Ib.L .I1 .Ir.Ar.G .W + 576.Ib.L .I1 .A1.Ir.P .G + 576.Ib.L .I1 .

          2     6  2        7      5  2       5         2  4             2          2  4            2           2  2  2             2  2  2             2  2
   Ir.Ar.P .G - 6.L .I1 .A1.Ir.Ar.G .W + 48.L .I1 .A1.Ir.Ar.P .G )/(144.E .Ab.Ib .L .I1.A1.Ar.G + 144.E .Ab.Ib .L .A1.Ir.Ar.G + 144.E .Ab.Ib .L .I1 .A1 + 144.E .Ab.Ib .L .I1 .Ar + 288.E .Ab.Ib .L .I1.A1.Ir + 288.

            2  2               2  2     2            2  2  2             4   2       3             4              3              4    2      3             2  2
   .E .Ab.Ib .L .I1.Ir.Ar + 144.E .Ab.Ib .L .A1.Ir + 144.E .Ab.Ib .L .Ir .Ar + 48.E .Ab.Ib.L .I1 .A1.Ar.G + 528.E .Ab.Ib.L .I1.A1.Ir.Ar.G + 48.E .Ab.Ib.L .A1.Ir .Ar.G + 576.E .Ab.Ib.L .I1 .A1.Ir.G + 576.E .Ab.Ib

     2  2                  2       2                 2       2              4   2            4              4        2   4           2  2             3            2   2                2  2
   .L .I1 .Ir.Ar.G + 576.E .Ab.Ib.L .I1.A1.Ir .G + 576.E .Ab.Ib.L .I1.Ir .Ar.G + 48.E .Ab.L .I1 .A1.Ir.Ar.G + 48.E .Ab.L .I1.A1.Ir .Ar.G + 1728.E .Ib .L .I1.A1.Ir.Ar.G + 1728.E .Ib .I1 .A1.Ir.G + 1728.E .Ib .I1

            2        2               2   2             2  2            4             2        2    4              2   2 2               2   2    2             2  2     2    5
   .Ir.Ar.G + 1728.E .Ib .I1.A1.Ir .G + 1728.E .Ib .I1.Ir .Ar.G + 576.E .Ib.L .I1 .A1.Ir.Ar.G + 576.E .Ib.L .I1.A1.Ir .Ar.G + 1728.E .Ib.I1 .A1.Ir .G + 1728.E .Ib.I1 .Ir .Ar.G + 144.E .L .I1 .A1.Ir .Ar.G )

               2  6         3              2  6          3              2  5                     2  5                   2  4  2             2  4                    2  4  2             6  2    4
DU(5,1) := ( - 12.Ab.Ib .L .I1.A1.G .W - 12.Ab.Ib .L .A1.Ir.G .W - 12.Ab.Ib .L .I1.A1.P .G - 12.Ab.Ib .L .A1.Ir.P .G - 24.Ab.Ib .L .I1 .G.W - 48.Ab.Ib .L .I1.Ir.G.W - 24.Ab.Ib .L .Ir .G.W - 3.Ab.Ib.L .I1 .A1.G .W - 44.Ab.Ib.

   6          4            6    2  4             5             2            4  2    2          4    2  2          6  2       5         6          2  5             2  4             4            2  3
   L .I1.A1.Ir.G .W - 5.Ab.Ib.L .A1.Ir .G .W - 48.Ab.Ib L .I1.A1.Ir.P .G - 96.Ab.Ib.L .I1 .Ir.G .W - 96.Ab.Ib.L .I1.Ir .G .W - 4.Ab.L .I1 .A1.Ir.G .W - 4.Ab.L .I1.A1.Ir .G .W - 144.Ib .L .I1.A1.Ir.G .W - 144.Ib .L .

        2          2  2  2     2            2  2     2  2          4  2         5            4           2  5          3  2           3            2  2  2  3         4  2      2  6                  2  4
   I1.A1.Ir.P .G - 288.Ib .L .I1 .Ir.G .W - 288.Ib .L .I1.Ir .G .W - 36.Ib.L .I1 .A1.Ir.G .W - 60.Ib.L .I1.A1.Ir .G .W - 72.Ib.L .I1 .A1.Ir.P .G - 288.Ib.L .I1 .Ir .G .W - 12.L .I1 .A1.Ir .G .W)/(24.E .Ab.Ib .L .I1

         2          2  4            2              2  2  2            2  2  2             2  2                  2  2                  2  2     2             2  2  2             4
   .A1.Ar.G + 24.E .Ab.Ib .L .A1.Ir.Ar.G + 24.E .Ab.Ib .L .I1 .A1 + 24.E .Ab.Ib .L .I1 .Ar + 48.E .Ab.Ib .L .I1.A1.Ir + 48.E .Ab.Ib .L .I1.Ir.Ar + 24.E .Ab.Ib .L .A1.Ir + 24.E .Ab.Ib .L .Ir .Ar + 8.E .Ab.Ib.L .

     2         3              4              3             4     2     3              2  2                  2  2                     2       2                2       2            4  2
   I1 .A1.Ar.G + 88.E .Ab.Ib.L .I1.A1.Ir.Ar.G + 8.E .Ab.Ib.L .A1.Ir .Ar.G + 96.E .Ab.Ib.L .I1 .A1.Ir.G + 96.E .Ab.Ib.L .I1 .Ir.Ar.G + 96.E .Ab.Ib.L .I1.A1.Ir .G + 96.E .Ab.Ib.L .I1.Ir .Ar.G + 8.E .Ab.L .I1 .A1.
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$$Ir.Ar.G^{4} + 8.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 288.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 288.E.Ib^{2}.Il^{2}.Al.Ir.G + 288.E.Ib^{2}.Il^{2}.Ir.Ar.G + 288.E.Ib^{2}.Il.Al.Ir^{2}.G + 288.E.Ib^{2}.Il.Ir^{2}.Ar.G + 96.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 96.E.Ib.$$

$$L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 288.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 288.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 24.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

$$DU(6,1) := (- 72.Ab.Ib^{2}.L^{5}.Il.Al.G^{2}.W + 72.Ab.Ib^{2}.L^{5}.Il.Ar.G^{2}.W - 72.Ab.Ib^{2}.L^{5}.Al.Ir.G^{2}.W + 72.Ab.Ib^{2}.L^{5}.Ir.Ar.G^{2}.W - 72.Ab.Ib^{2}.L^{4}.Il.Al.P - 72.Ab.Ib^{2}.L^{4}.Il.Ar.P - 72.Ab.Ib^{2}.L^{4}.Al.Ir.P - 72.Ab.Ib^{2}.L^{4}.Ir.Ar.P + 6.Ab.Ib.L^{7}$$

$$.Il.Al.Ar.G^{5}.W + 6.Ab.Ib.L^{7}.Al.Ir.Ar.G^{5}.W + 12.Ab.Ib.L^{6}.Il.Al.Ar.P^{3}.G - 24.Ab.Ib.L^{6}.Al.Ir.Ar.P^{3}.G - 24.Ab.Ib.L^{5}.Il^{2}.Al.G^{3}.W + 48.Ab.Ib.L^{5}.Il^{2}.Ar.G^{3}.W - 240.Ab.Ib.L^{5}.Il.Al.Ir.G^{3}.W + 264.Ab.Ib.L^{5}.Il.Ir.Ar.G^{3}.W -$$

$$288.Ab.Ib.L^{4}.Il.Al.Ir.P.G - 288.Ab.Ib.L^{4}.Il.Ir.Ar.P.G + Ab.L^{7}.Il^{2}.Al.Ar.G^{6}.W + Ab.L^{7}.Il.Al.Ir.Ar.G^{6}.W - 24.Ab.L^{6}.Il.Al.Ir.Ar.P.G^{4} - 864.Ib^{2}.L^{3}.Il.Al.Ir.G^{3}.W + 864.Ib^{2}.L^{3}.Il.Ir.Ar.G^{3}.W - 864.Ib^{2}.L^{2}.Il.Al.Ir.P.G$$

$$- 864.Ib^{2}.L^{2}.Il.Ir.Ar.P.G + 72.Ib.L^{5}.Il.Al.Ir.Ar.G^{6}.W - 288.Ib.L^{4}.Il.Al.Ir.Ar.P.G^{4} - 288.Ib.L^{3}.Il^{2}.Al.Ir.G^{4}.W + 576.Ib.L^{3}.Il^{2}.Ir.Ar.G^{4}.W - 864.Ib.L^{2}.Il^{2}.Al.Ir.P.G^{2} - 864.Ib.L^{2}.Il^{2}.Ir.Ar.P.G^{2} + 12.L^{5}.Il^{2}.Al.Ir$$

$$.Ar.G^{7}.W - 72.L^{4}.Il^{2}.Al.Ir.Ar.P.G^{5})/(144.E.Ab.Ib^{2}.L^{4}.Il.Al.Ar.G^{2} + 144.E.Ab.Ib^{2}.L^{4}.Al.Ir.Ar.G^{2} + 144.E.Ab.Ib^{2}.L^{2}.Il^{2}.Al + 144.E.Ab.Ib^{2}.L^{2}.Il^{2}.Ar + 288.E.Ab.Ib^{2}.L^{2}.Il.Al.Ir + 288.E.Ab.Ib^{2}.L^{2}.Il.Ir.Ar + 144.$$

$$E.Ab.Ib^{2}.L^{2}.Al.Ir^{2} + 144.E.Ab.Ib^{2}.L^{2}.Ir^{2}.Ar + 48.E.Ab.Ib.L^{4}.Il^{2}.Al.Ar.G^{3} + 528.E.Ab.Ib.L^{4}.Il.Al.Ir.Ar.G^{3} + 48.E.Ab.Ib.L^{4}.Al.Ir^{2}.Ar.G^{3} + 576.E.Ab.Ib.L^{2}.Il^{2}.Al.Ir.G + 576.E.Ab.Ib.L^{2}.Il^{2}.Ir.Ar.G + 576.E.Ab.$$

$$Ib.L^{2}.Il.Al.Ir^{2}.G + 576.E.Ab.Ib.L^{2}.Il.Ir^{2}.Ar.G + 48.E.Ab.L^{4}.Il^{2}.Al.Ir.Ar.G^{4} + 48.E.Ab.L^{4}.Il.Al.Ir^{2}.Ar.G^{4} + 1728.E.Ib^{2}.L^{2}.Il.Al.Ir.Ar.G^{3} + 1728.E.Ib^{2}.Il^{2}.Al.Ir.G + 1728.E.Ib^{2}.Il^{2}.Ir.Ar.G + 1728.E.Ib^{2}.Il.Al$$

$$.Ir^{2}.G + 1728.E.Ib^{2}.Il.Ir^{2}.Ar.G + 576.E.Ib.L^{2}.Il^{2}.Al.Ir.Ar.G^{4} + 576.E.Ib.L^{2}.Il.Al.Ir^{2}.Ar.G^{4} + 1728.E.Ib.Il^{2}.Al.Ir^{2}.G^{2} + 1728.E.Ib.Il^{2}.Ir^{2}.Ar.G^{2} + 144.E.L^{2}.Il^{2}.Al.Ir^{2}.Ar.G^{5})$$

D.2 Simplified Stiffness, Flexibility, and Deflection Matrices

```
LET Il=Ic, Ir=Ic, Al=Ac, Ar=Ac;

WRITE "AKA FOR UNIFORM COLUMNS";

AKA FOR UNIFORM COLUMNS

AKA:=AKA;

                        2                   3
AKA(1,1) := (Ab.E .L  + 12.Ic.E .G)/(G.L )

AKA(1,2) := 0

                          2
AKA(1,3) := (6.Ic.E )/L

AKA(1,4) := ( - Ab.E )/(G.L)

AKA(1,5) := 0

AKA(1,6) := 0

AKA(2,1) := 0

                                   3  2    3  3
AKA(2,2) := (12.Ib.E  + Ac.E .G .L )/(G .L )

                          2  2
AKA(2,3) := (6.Ib.E )/(G .L )

AKA(2,4) := 0

                               3  3
AKA(2,5) := ( - 12.Ib.E )/(G .L )

                          2  2
AKA(2,6) := (6.Ib.E )/(G .L )

                         2
AKA(3,1) := (6.Ic.E )/L

                          2  2
AKA(3,2) := (6.Ib.E )/(G .L )

AKA(3,3) := (4.Ib.E  + 4.Ic.E .G)/(G.L)

AKA(3,4) := 0

                              2  2
AKA(3,5) := ( - 6.Ib.E )/(G .L )

AKA(3,6) := (2.Ib.E )/(G.L)

AKA(4,1) := ( - Ab.E )/(G.L)

AKA(4,2) := 0

AKA(4,3) := 0

                        2                   3
AKA(4,4) := (Ab.E .L  + 12.Ic.E .G)/(G.L )
```

$AKA(4,5) := 0$

$AKA(4,6) := (6 \cdot Ic \cdot E)/L^{2}$

$AKA(5,1) := 0$

$AKA(5,2) := (-12 \cdot Ib \cdot E)/(G^{3} \cdot L^{3})$

$AKA(5,3) := (-6 \cdot Ib \cdot E)/(G^{2} \cdot L^{2})$

$AKA(5,4) := 0$

$AKA(5,5) := (12 \cdot Ib \cdot E + Ac \cdot E \cdot G^{3} \cdot L^{2})/(G^{3} \cdot L^{3})$

$AKA(5,6) := (-6 \cdot Ib \cdot E)/(G^{2} \cdot L^{2})$

$AKA(6,1) := 0$

$AKA(6,2) := (6 \cdot Ib \cdot E)/(G^{2} \cdot L^{2})$

$AKA(6,3) := (2 \cdot Ib \cdot E)/(G \cdot L)$

$AKA(6,4) := (6 \cdot Ic \cdot E)/L^{2}$

$AKA(6,5) := (-6 \cdot Ib \cdot E)/(G^{2} \cdot L^{2})$

$AKA(6,6) := (4 \cdot Ib \cdot E + 4 \cdot Ic \cdot E \cdot G)/(G \cdot L)$

WRITE "FLEX FOR UNIFORM COLUMNS";

FLEX FOR UNIFORM COLUMNS

FLEX:=FLEX;

$$FLEX(1,1) := (48 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot L^{5} + 3 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot G^{2} \cdot L^{7} + 360 \cdot Ib^{2} \cdot Ic^{2} \cdot G \cdot L^{3} + 36 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot G^{3} \cdot L^{5} + 96 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot G \cdot L^{5} + 8 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot G^{3} \cdot L^{7} + 288 \cdot Ib \cdot Ic^{3} \cdot G^{2} \cdot L^{3} + 60 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot G^{4} \cdot L^{5} + 4 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot G^{4} \cdot L^{7} + 12 \cdot Ic^{3} \cdot Ac \cdot G^{5} \cdot L^{5})/(288 \cdot Ib^{2} \cdot Ab \cdot Ic^{2} \cdot E \cdot L^{2} + 72 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 1728 \cdot Ib^{2} \cdot Ic^{3} \cdot E \cdot G + 432 \cdot Ib^{2} \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 576 \cdot Ib \cdot Ab \cdot Ic^{3} \cdot E \cdot G \cdot L^{2} + 156 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 864 \cdot Ib \cdot Ic^{4} \cdot E \cdot G^{2} + 288 \cdot Ib \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 24 \cdot Ab \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 36 \cdot Ic^{4} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(1,2) := (3 \cdot Ib^{2} \cdot Ab \cdot G \cdot L^{5} + 18 \cdot Ib^{2} \cdot Ic \cdot G^{2} \cdot L^{3} + 6 \cdot Ib \cdot Ab \cdot Ic \cdot G^{2} \cdot L^{5} + 9 \cdot Ib \cdot Ic^{2} \cdot G^{3} \cdot L^{3})/(24 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot E \cdot L^{2} + 6 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 144 \cdot Ib^{2} \cdot Ic^{2} \cdot E \cdot G + 36 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 48 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot E \cdot G \cdot L^{2} + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 72 \cdot Ib \cdot Ic^{3} \cdot E \cdot G^{2} + 24 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{[illegible]} + 2 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 3 \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(1,3) := (-24 \cdot Ib^{2} \cdot Ab \cdot L^{4} - 144 \cdot Ib^{2} \cdot Ic \cdot G \cdot L^{2} - 48 \cdot Ib \cdot Ab \cdot Ic \cdot G \cdot L^{4} - Ib \cdot Ab \cdot Ac \cdot G^{3} \cdot L^{6} - 144 \cdot Ib \cdot Ic^{2} \cdot G^{2} \cdot L^{2} - 24 \cdot Ib \cdot Ic \cdot Ac \cdot G^{4} \cdot L^{4} - 2 \cdot Ab \cdot Ic \cdot Ac \cdot G^{4} \cdot L^{6} - 6 \cdot Ic^{2} \cdot Ac \cdot G^{5} \cdot L^{4})/(96 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot E \cdot L^{2} + 24 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 576 \cdot Ib^{2} \cdot Ic^{2} \cdot E \cdot G + 144 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 192 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot E \cdot G \cdot L^{2} + 52 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 288 \cdot Ib \cdot Ic^{3} \cdot E \cdot G^{2} + 96 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 8 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 12 \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(1,4) := (48 \cdot Ib^2 \cdot Ab \cdot Ic \cdot L^5 + 3 \cdot Ib^2 \cdot Ab \cdot Ac \cdot G^2 \cdot L^7 + 216 \cdot Ib^2 \cdot Ic^2 \cdot G \cdot L^3 + 96 \cdot Ib \cdot Ab \cdot Ic^2 \cdot G \cdot L^5 + 8 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot G^3 \cdot L^7 - 18 \cdot Ib \cdot Ic^2 \cdot Ac \cdot G^4 \cdot L^5 + 4 \cdot Ab \cdot Ic^2 \cdot Ac \cdot G^4 \cdot L^7)/(288 \cdot Ib^2 \cdot Ab \cdot Ic^2 \cdot E \cdot L^2 + 72 \cdot Ib^2 \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^2 \cdot L^4 + 1728 \cdot Ib^2 \cdot Ic^3 \cdot E \cdot G + 432 \cdot Ib^2 \cdot Ic^2 \cdot Ac \cdot E \cdot G^3 \cdot L^2 + 576 \cdot Ib \cdot Ab \cdot Ic^3 \cdot E \cdot G \cdot L^2 + 156 \cdot Ib \cdot Ab \cdot Ic^2 \cdot Ac \cdot E \cdot G^3 \cdot L^4 + 864 \cdot Ib \cdot Ic^4 \cdot E \cdot G^2 + 288 \cdot Ib \cdot Ic^3 \cdot Ac \cdot E \cdot G^4 \cdot L^2 + 24 \cdot Ab \cdot Ic^3 \cdot Ac \cdot E \cdot G^4 \cdot L^4 + 36 \cdot Ic^4 \cdot Ac \cdot E \cdot G^5 \cdot L^2)$$

$$FLEX(1,5) := (- 3 \cdot Ib^2 \cdot Ab \cdot G \cdot L^5 - 18 \cdot Ib^2 \cdot Ic \cdot G^2 \cdot L^3 - 6 \cdot Ib \cdot Ab \cdot Ic \cdot G^2 \cdot L^5 - 9 \cdot Ib \cdot Ic^2 \cdot G^3 \cdot L^3)/(24 \cdot Ib^2 \cdot Ab \cdot Ic \cdot E \cdot L^2 + 6 \cdot Ib^2 \cdot Ab \cdot Ac \cdot E \cdot G^2 \cdot L^4 + 144 \cdot Ib^2 \cdot Ic^2 \cdot E \cdot G + 36 \cdot Ib^2 \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^2 + 48 \cdot Ib \cdot Ab \cdot Ic^2 \cdot E \cdot G \cdot L^2 + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^4 + 72 \cdot Ib \cdot Ic^3 \cdot E \cdot G^2 + 24 \cdot Ib \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^2 + 2 \cdot Ab \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^4 + 3 \cdot Ic^3 \cdot Ac \cdot E \cdot G^5 \cdot L^2)$$

$$FLEX(1,6) := (- 24 \cdot Ib^2 \cdot Ab \cdot L^4 - 144 \cdot Ib^2 \cdot Ic \cdot G \cdot L^2 - 48 \cdot Ib \cdot Ab \cdot Ic \cdot G \cdot L^4 - Ib \cdot Ab \cdot Ac \cdot G^3 \cdot L^6 + 12 \cdot Ib \cdot Ic \cdot Ac \cdot G^4 \cdot L^4 - 2 \cdot Ab \cdot Ic \cdot Ac \cdot G^4 \cdot L^6)/(96 \cdot Ib^2 \cdot Ab \cdot Ic \cdot E \cdot L^2 + 24 \cdot Ib^2 \cdot Ab \cdot Ac \cdot E \cdot G^2 \cdot L^4 + 576 \cdot Ib^2 \cdot Ic^2 \cdot E \cdot G + 144 \cdot Ib^2 \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^2 + 192 \cdot Ib \cdot Ab \cdot Ic^2 \cdot E \cdot G \cdot L^2 + 52 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^4 + 288 \cdot Ib \cdot Ic^3 \cdot E \cdot G^2 + 96 \cdot Ib \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^2 + 8 \cdot Ab \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^4 + 12 \cdot Ic^3 \cdot Ac \cdot E \cdot G^5 \cdot L^2)$$

$$FLEX(2,1) := (3 \cdot Ib^2 \cdot Ab \cdot G \cdot L^5 + 18 \cdot Ib^2 \cdot Ic \cdot G^2 \cdot L^3 + 6 \cdot Ib \cdot Ab \cdot Ic \cdot G^2 \cdot L^5 + 9 \cdot Ib \cdot Ic^2 \cdot G^3 \cdot L^3)/(24 \cdot Ib^2 \cdot Ab \cdot Ic \cdot E \cdot L^2 + 6 \cdot Ib^2 \cdot Ab \cdot Ac \cdot E \cdot G^2 \cdot L^4 + 144 \cdot Ib^2 \cdot Ic^2 \cdot E \cdot G + 36 \cdot Ib^2 \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^2 + 48 \cdot Ib \cdot Ab \cdot Ic^2 \cdot E \cdot G \cdot L^2 + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^4 + 72 \cdot Ib \cdot Ic^3 \cdot E \cdot G^2 + 24 \cdot Ib \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^2 + 2 \cdot Ab \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^4 + 3 \cdot Ic^3 \cdot Ac \cdot E \cdot G^5 \cdot L^2)$$

$$FLEX(2,2) := (12 \cdot Ib^2 \cdot Ab \cdot Ic \cdot L^3 + 6 \cdot Ib^2 \cdot Ab \cdot Ac \cdot G^2 \cdot L^5 + 72 \cdot Ib^2 \cdot Ic^2 \cdot G \cdot L + 36 \cdot Ib^2 \cdot Ic \cdot Ac \cdot G^3 \cdot L^3 + 24 \cdot Ib \cdot Ab \cdot Ic^2 \cdot G \cdot L^3 + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot G^3 \cdot L^5 + 36 \cdot Ib \cdot Ic^3 \cdot G^2 \cdot L + 24 \cdot Ib \cdot Ic^2 \cdot Ac \cdot G^4 \cdot L^3 + 2 \cdot Ab \cdot Ic^2 \cdot Ac \cdot G^4 \cdot L^5 + 3 \cdot Ic^3 \cdot Ac \cdot G^5 \cdot L^3)/(24 \cdot Ib^2 \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot L^2 + 6 \cdot Ib^2 \cdot Ab \cdot Ac^2 \cdot E \cdot G^2 \cdot L^4 + 144 \cdot Ib^2 \cdot Ic^2 \cdot Ac \cdot E \cdot G + 36 \cdot Ib^2 \cdot Ic \cdot Ac^2 \cdot E \cdot G^3 \cdot L^2 + 48 \cdot Ib \cdot Ab \cdot Ic^2 \cdot Ac \cdot E \cdot G \cdot L^2 + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac^2 \cdot E \cdot G^3 \cdot L^4 + 72 \cdot Ib \cdot Ic^3 \cdot Ac \cdot E \cdot G^2 + 24 \cdot Ib \cdot Ic^2 \cdot Ac^2 \cdot E \cdot G^4 \cdot L^2 + 2 \cdot Ab \cdot Ic^2 \cdot Ac^2 \cdot E \cdot G^4 \cdot L^4 + 3 \cdot Ic^3 \cdot Ac^2 \cdot E \cdot G^5 \cdot L^2)$$

$$FLEX(2,3) := (- 6 \cdot Ib^2 \cdot Ab \cdot G \cdot L^4 - 36 \cdot Ib^2 \cdot Ic \cdot G^2 \cdot L^2 - 12 \cdot Ib \cdot Ab \cdot Ic \cdot G^2 \cdot L^4 - 18 \cdot Ib \cdot Ic^2 \cdot G^3 \cdot L^2)/(24 \cdot Ib^2 \cdot Ab \cdot Ic \cdot E \cdot L^2 + 6 \cdot Ib^2 \cdot Ab \cdot Ac \cdot E \cdot G^2 \cdot L^4 + 144 \cdot Ib^2 \cdot Ic^2 \cdot E \cdot G + 36 \cdot Ib^2 \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^2 + 48 \cdot Ib \cdot Ab \cdot Ic^2 \cdot E \cdot G \cdot L^2 + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^4 + 72 \cdot Ib \cdot Ic^3 \cdot E \cdot G^2 + 24 \cdot Ib \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^2 + 2 \cdot Ab \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^4 + 3 \cdot Ic^3 \cdot Ac \cdot E \cdot G^5 \cdot L^2)$$

$$FLEX(2,4) := (3 \cdot Ib^2 \cdot Ab \cdot G \cdot L^5 + 18 \cdot Ib^2 \cdot Ic \cdot G^2 \cdot L^3 + 6 \cdot Ib \cdot Ab \cdot Ic \cdot G^2 \cdot L^5 + 9 \cdot Ib \cdot Ic^2 \cdot G^3 \cdot L^3)/(24 \cdot Ib^2 \cdot Ab \cdot Ic \cdot E \cdot L^2 + 6 \cdot Ib^2 \cdot Ab \cdot Ac \cdot E \cdot G^2 \cdot L^4 + 144 \cdot Ib^2 \cdot Ic^2 \cdot E \cdot G + 36 \cdot Ib^2 \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^2 + 48 \cdot Ib \cdot Ab \cdot Ic^2 \cdot E \cdot G \cdot L^2 + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^4 + 72 \cdot Ib \cdot Ic^3 \cdot E \cdot G^2 + 24 \cdot Ib \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^2 + 2 \cdot Ab \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^4 + 3 \cdot Ic^3 \cdot Ac \cdot E \cdot G^5 \cdot L^2)$$

$$FLEX(2,5) := (12 \cdot Ib^2 \cdot Ab \cdot Ic \cdot L^3 + 72 \cdot Ib^2 \cdot Ic^2 \cdot G \cdot L + 24 \cdot Ib \cdot Ab \cdot Ic^2 \cdot G \cdot L^3 + 36 \cdot Ib \cdot Ic^3 \cdot G^2 \cdot L)/(24 \cdot Ib^2 \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot L^2 + 6 \cdot Ib^2 \cdot Ab \cdot Ac^2 \cdot E \cdot G^2 \cdot L^4 + 144 \cdot Ib^2 \cdot Ic^2 \cdot Ac \cdot E \cdot G + 36 \cdot Ib^2 \cdot Ic \cdot Ac^2 \cdot E \cdot G^3 \cdot L^2 + 48 \cdot Ib \cdot Ab \cdot Ic^2 \cdot Ac \cdot E \cdot G \cdot L^2 + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac^2 \cdot E \cdot G^3 \cdot L^4 + 72 \cdot Ib \cdot Ic^3 \cdot Ac \cdot E \cdot G^2 + 24 \cdot Ib \cdot Ic^2 \cdot Ac^2 \cdot E \cdot G^4 \cdot L^2 + 2 \cdot Ab \cdot Ic^2 \cdot Ac^2 \cdot E \cdot G^4 \cdot L^4 + 3 \cdot Ic^3 \cdot Ac^2 \cdot E \cdot G^5 \cdot L^2)$$

$$FLEX(2,6) := (- 6 \cdot Ib^2 \cdot Ab \cdot G \cdot L^4 - 36 \cdot Ib^2 \cdot Ic \cdot G^2 \cdot L^2 - 12 \cdot Ib \cdot Ab \cdot Ic \cdot G^2 \cdot L^4 - 18 \cdot Ib \cdot Ic^2 \cdot G^3 \cdot L^2)/(24 \cdot Ib^2 \cdot Ab \cdot Ic \cdot E \cdot L^2 + 6 \cdot Ib^2 \cdot Ab \cdot Ac \cdot E \cdot G^2 \cdot L^4 + 144 \cdot Ib^2 \cdot Ic^2 \cdot E \cdot G + 36 \cdot Ib^2 \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^2 + 48 \cdot Ib \cdot Ab \cdot Ic^2 \cdot E \cdot G \cdot L^2 + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^4 + 72 \cdot Ib \cdot Ic^3 \cdot E \cdot G^2 + 24 \cdot Ib \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^2 + 2 \cdot Ab \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^4 + 3 \cdot Ic^3 \cdot Ac \cdot E \cdot G^5 \cdot L^2)$$

$$FLEX(3,1) := (- 24 \cdot Ib^2 \cdot Ab \cdot L^4 - 144 \cdot Ib^2 \cdot Ic \cdot G \cdot L^2 - 48 \cdot Ib \cdot Ab \cdot Ic \cdot G \cdot L^4 - Ib \cdot Ab \cdot Ac \cdot G^3 \cdot L^6 - 144 \cdot Ib \cdot Ic^2 \cdot G^2 \cdot L^2 - 24 \cdot Ib \cdot Ic \cdot Ac \cdot G^4 \cdot L^4 - 2 \cdot Ab \cdot Ic \cdot Ac \cdot G^4 \cdot L^6 - 6 \cdot Ic^2 \cdot Ac \cdot G^5 \cdot L^4)/(96 \cdot Ib^2 \cdot Ab \cdot Ic \cdot E \cdot L^2 + 24 \cdot Ib^2 \cdot Ab \cdot Ac \cdot E \cdot G^2 \cdot L^4 + 576 \cdot Ib^2 \cdot Ic^2 \cdot E \cdot G + 144 \cdot Ib^2 \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^2 + 192 \cdot Ib \cdot Ab \cdot Ic^2 \cdot E \cdot G \cdot L^2 + 52 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^3 \cdot L^4 + 288 \cdot Ib \cdot Ic^3 \cdot E \cdot G^2 + 96 \cdot Ib \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^2 + 8 \cdot Ab \cdot Ic^2 \cdot Ac \cdot E \cdot G^4 \cdot L^4 + 12 \cdot Ic^3 \cdot Ac \cdot E \cdot G^5 \cdot L^2)$$

$$FLEX(3,2) := (-6 \cdot Ib^{2} \cdot Ab \cdot G \cdot L^{4} - 36 \cdot Ib^{2} \cdot Ic \cdot G^{2} \cdot L^{2} - 12 \cdot Ib \cdot Ab \cdot Ic \cdot G^{2} \cdot L^{4} - 18 \cdot Ib \cdot Ic^{2} \cdot G^{3} \cdot L^{2})/(24 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot E \cdot L^{2} + 6 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 144 \cdot Ib^{2} \cdot Ic^{2} \cdot E \cdot G + 36 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 48 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot E \cdot G \cdot L^{2} + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 72 \cdot Ib \cdot Ic^{3} \cdot E \cdot G^{2} + 24 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 2 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 3 \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(3,3) := (48 \cdot Ib^{2} \cdot Ab \cdot L^{3} + 288 \cdot Ib^{2} \cdot Ic \cdot G \cdot L + 120 \cdot Ib \cdot Ab \cdot Ic \cdot G \cdot L^{3} + 8 \cdot Ib \cdot Ab \cdot Ac \cdot G^{3} \cdot L^{5} + 288 \cdot Ib \cdot Ic^{2} \cdot G^{2} \cdot L + 48 \cdot Ib \cdot Ic \cdot Ac \cdot G^{4} \cdot L^{3} + 5 \cdot Ab \cdot Ic \cdot Ac \cdot G^{4} \cdot L^{5} + 12 \cdot Ic^{2} \cdot Ac \cdot G^{5} \cdot L^{3})/(96 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot E \cdot L^{2} + 24 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 576 \cdot Ib^{2} \cdot Ic^{2} \cdot E \cdot G + 144 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 192 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot E \cdot G \cdot L^{2} + 52 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 288 \cdot Ib \cdot Ic^{3} \cdot E \cdot G^{2} + 96 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 8 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 12 \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(3,4) := (-24 \cdot Ib^{2} \cdot Ab \cdot L^{4} - 144 \cdot Ib^{2} \cdot Ic \cdot G \cdot L^{2} - 48 \cdot Ib \cdot Ab \cdot Ic \cdot G \cdot L^{4} - Ib \cdot Ab \cdot Ac \cdot G^{3} \cdot L^{6} + 12 \cdot Ib \cdot Ic \cdot Ac \cdot G^{4} \cdot L^{4} - 2 \cdot Ab \cdot Ic \cdot Ac \cdot G^{4} \cdot L^{6})/(96 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot E \cdot L^{2} + 24 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 576 \cdot Ib^{2} \cdot Ic^{2} \cdot E \cdot G + 144 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 192 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot E \cdot G \cdot L^{2} + 52 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 288 \cdot Ib \cdot Ic^{3} \cdot E \cdot G^{2} + 96 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 8 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 12 \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(3,5) := (6 \cdot Ib^{2} \cdot Ab \cdot G \cdot L^{4} + 36 \cdot Ib^{2} \cdot Ic \cdot G^{2} \cdot L^{2} + 12 \cdot Ib \cdot Ab \cdot Ic \cdot G^{2} \cdot L^{4} + 18 \cdot Ib \cdot Ic^{2} \cdot G^{3} \cdot L^{2})/(24 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot E \cdot L^{2} + 6 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 144 \cdot Ib^{2} \cdot Ic^{2} \cdot E \cdot G + 36 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 48 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot E \cdot G \cdot L^{2} + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 72 \cdot Ib \cdot Ic^{3} \cdot E \cdot G^{2} + 24 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 2 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 3 \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(3,6) := (48 \cdot Ib^{2} \cdot Ab \cdot L^{3} + 288 \cdot Ib^{2} \cdot Ic \cdot G \cdot L + 72 \cdot Ib \cdot Ab \cdot Ic \cdot G \cdot L^{3} - 4 \cdot Ib \cdot Ab \cdot Ac \cdot G^{3} \cdot L^{5} - 24 \cdot Ib \cdot Ic \cdot Ac \cdot G^{4} \cdot L^{3} + 3 \cdot Ab \cdot Ic \cdot Ac \cdot G^{4} \cdot L^{5})/(96 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot E \cdot L^{2} + 24 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 576 \cdot Ib^{2} \cdot Ic^{2} \cdot E \cdot G + 144 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 192 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot E \cdot G \cdot L^{2} + 52 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 288 \cdot Ib \cdot Ic^{3} \cdot E \cdot G^{2} + 96 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 8 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 12 \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(4,1) := (48 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot L^{5} + 3 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot G^{2} \cdot L^{7} + 216 \cdot Ib^{2} \cdot Ic^{2} \cdot G \cdot L^{3} + 96 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot G \cdot L^{5} + 8 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot G^{3} \cdot L^{7} - 18 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot G^{4} \cdot L^{5} + 4 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot G^{4} \cdot L^{7})/(288 \cdot Ib^{2} \cdot Ab \cdot Ic^{2} \cdot E \cdot L^{2} + 72 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 1728 \cdot Ib^{2} \cdot Ic^{3} \cdot E \cdot G + 432 \cdot Ib^{2} \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 576 \cdot Ib \cdot Ab \cdot Ic^{3} \cdot E \cdot G \cdot L^{2} + 156 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 864 \cdot Ib \cdot Ic^{4} \cdot E \cdot G^{2} + 288 \cdot Ib \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 24 \cdot Ab \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 36 \cdot Ic^{4} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(4,2) := (3 \cdot Ib^{2} \cdot Ab \cdot G \cdot L^{5} + 18 \cdot Ib^{2} \cdot Ic \cdot G^{2} \cdot L^{3} + 6 \cdot Ib \cdot Ab \cdot Ic \cdot G^{2} \cdot L^{5} + 9 \cdot Ib \cdot Ic^{2} \cdot G^{3} \cdot L^{3})/(24 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot E \cdot L^{2} + 6 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 144 \cdot Ib^{2} \cdot Ic^{2} \cdot E \cdot G + 36 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 48 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot E \cdot G \cdot L^{2} + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 72 \cdot Ib \cdot Ic^{3} \cdot E \cdot G^{2} + 24 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 2 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 3 \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(4,3) := (-24 \cdot Ib^{2} \cdot Ab \cdot L^{4} - 144 \cdot Ib^{2} \cdot Ic \cdot G \cdot L^{2} - 48 \cdot Ib \cdot Ab \cdot Ic \cdot G \cdot L^{4} - Ib \cdot Ab \cdot Ac \cdot G^{3} \cdot L^{6} + 12 \cdot Ib \cdot Ic \cdot Ac \cdot G^{4} \cdot L^{4} - 2 \cdot Ab \cdot Ic \cdot Ac \cdot G^{4} \cdot L^{6})/(96 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot E \cdot L^{2} + 24 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 576 \cdot Ib^{2} \cdot Ic^{2} \cdot E \cdot G + 144 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 192 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot E \cdot G \cdot L^{2} + 52 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 288 \cdot Ib \cdot Ic^{3} \cdot E \cdot G^{2} + 96 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 8 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 12 \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(4,4) := (48 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot L^{5} + 3 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot G^{2} \cdot L^{7} + 360 \cdot Ib^{2} \cdot Ic^{2} \cdot G \cdot L^{3} + 36 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot G^{3} \cdot L^{5} + 96 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot G \cdot L^{5} + 8 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot G^{3} \cdot L^{7} + 288 \cdot Ib \cdot Ic^{3} \cdot G^{2} \cdot L^{3} + 60 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot G^{4} \cdot L^{5} + 4 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot G^{4} \cdot L^{7} + 12 \cdot Ic^{3} \cdot Ac \cdot G^{5} \cdot L^{5})/(288 \cdot Ib^{2} \cdot Ab \cdot Ic^{2} \cdot E \cdot L^{2} + 72 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 1728 \cdot Ib^{2} \cdot Ic^{3} \cdot E \cdot G + 432 \cdot Ib^{2} \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 576 \cdot Ib \cdot Ab \cdot Ic^{3} \cdot E \cdot G \cdot L^{2} + 156 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 864 \cdot Ib \cdot Ic^{4} \cdot E \cdot G^{2} + 288 \cdot Ib \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 24 \cdot Ab \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 36 \cdot Ic^{4} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

$$FLEX(4,5) := (-3 \cdot Ib^{2} \cdot Ab \cdot G \cdot L^{5} - 18 \cdot Ib^{2} \cdot Ic \cdot G^{2} \cdot L^{3} - 6 \cdot Ib \cdot Ab \cdot Ic \cdot G^{2} \cdot L^{5} - 9 \cdot Ib \cdot Ic^{2} \cdot G^{3} \cdot L^{3})/(24 \cdot Ib^{2} \cdot Ab \cdot Ic \cdot E \cdot L^{2} + 6 \cdot Ib^{2} \cdot Ab \cdot Ac \cdot E \cdot G^{2} \cdot L^{4} + 144 \cdot Ib^{2} \cdot Ic^{2} \cdot E \cdot G + 36 \cdot Ib^{2} \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{2} + 48 \cdot Ib \cdot Ab \cdot Ic^{2} \cdot E \cdot G \cdot L^{2} + 13 \cdot Ib \cdot Ab \cdot Ic \cdot Ac \cdot E \cdot G^{3} \cdot L^{4} + 72 \cdot Ib \cdot Ic^{3} \cdot E \cdot G^{2} + 24 \cdot Ib \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{2} + 2 \cdot Ab \cdot Ic^{2} \cdot Ac \cdot E \cdot G^{4} \cdot L^{4} + 3 \cdot Ic^{3} \cdot Ac \cdot E \cdot G^{5} \cdot L^{2})$$

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                   2     4         2      2                4            3  6         2  2  2            4  4              4  6      2      5  4         2            2       2          2  4        2  2
FLEX(4,6) := ( - 24.Ib .Ab.L  - 144.Ib .Ic.G.L  - 48.Ib.Ab.Ic.G.L  - Ib.Ab.Ac.G .L  - 144.Ib.Ic .G .L  - 24.Ib.Ic.Ac.G .L  - 2.Ab.Ic.Ac.G .L  - 6.Ic .Ac.G .L )/(96.Ib .Ab.Ic.E .L  + 24.Ib .Ab.Ac.E .G .L  + 576.Ib .Ic .E .G +

                  2          3  2              2      2                 3  4           3    2          2          4  2          2        4  4        3         5  2
              144.Ib .Ic.Ac.E .G .L  + 192.Ib.Ab.Ic .E .G.L  + 52.Ib.Ab.Ic.Ac.E .G .L  + 288.Ib.Ic .E .G  + 96.Ib.Ic .Ac.E .G .L  + 8.Ab.Ic .Ac.E .G .L  + 12.Ic .Ac.E .G .L )

                 2      5        2    2  3             2  5           2  3  3      2           2        2           2  4        2   2             2           3  2             2      2                     3  4
FLEX(5,1) := ( - 3.Ib .Ab.G.L  - 18.Ib .Ic.G .L  - 6.Ib.Ab.Ic.G .L  - 9.Ib.Ic .G .L )/(24.Ib .Ab.Ic.E .L  + 6.Ib .Ab.Ac.E .G .L  + 144.Ib .Ic .E .G + 36.Ib .Ic.Ac.E .G .L  + 48.Ib.Ab.Ic .E .G.L  + 13.Ib.Ab.Ic.Ac.E .G .L  + 72

                3     2          2      4  2          2       4  4       3        5  2
              .Ib.Ic .E .G  + 24.Ib.Ic .Ac.E .G .L  + 2.Ab.Ic .Ac.E .G .L  + 3.Ic .Ac.E .G .L )

                 2          3       2    2               2    3          3  7          2            2       2     2    2  4          2   2             2      2    3  2          2           2                    2
FLEX(5,2) := (12.Ib .Ab.Ic.L  + 72.Ib .Ic .G.L + 24.Ib.Ab.Ic .G.L  + 36.Ib.Ic .G .L)/(24.Ib .Ab.Ic.Ac.E .L  + 6.Ib .Ab.Ac .E .G .L  + 144.Ib .Ic .Ac.E .G + 36.Ib .Ic.Ac .E .G .L  + 48.Ib.Ab.Ic .Ac.E .G.L  + 13.Ib.Ab.Ic.Ac .E

                3  4          3       2          2   2    4  2        2   2      4  4     3    2     5  2
              .G .L  + 72.Ib.Ic .Ac.E .G  + 24.Ib.Ic .Ac .E .G .L  + 2.Ab.Ic .Ac .E .G .L  + 3.Ic .Ac .E .G .L )

                2       4         2      2  2              2  4         2  3  2        2            2       2           2  4        2   2             2           3  2             2      2                     3  4
FLEX(5,3) := (6.Ib .Ab.G.L  + 36.Ib .Ic.G .L  + 12.Ib.Ab.Ic.G .L  + 18.Ib.Ic .G .L )/(24.Ib .Ab.Ic.E .L  + 6.Ib .Ab.Ac.E .G .L  + 144.Ib .Ic .E .G + 36.Ib .Ic.Ac.E .G .L  + 48.Ib.Ab.Ic .E .G.L  + 13.Ib.Ab.Ic.Ac.E .G .L  + 72.

                 3     2          2      4  2          2       4  4       3        5  2
              Ib.Ic .E .G  + 24.Ib.Ic .Ac.E .G .L  + 2.Ab.Ic .Ac.E .G .L  + 3.Ic .Ac.E .G .L )

                 2      5        2    2  3             2  5           2  3  3      2           2        2           2  4        2   2             2           3  2             2      2                     3  4
FLEX(5,4) := ( - 3.Ib .Ab.G.L  - 18.Ib .Ic.G .L  - 6.Ib.Ab.Ic.G .L  - 9.Ib.Ic .G .L )/(24.Ib .Ab.Ic.E .L  + 6.Ib .Ab.Ac.E .G .L  + 144.Ib .Ic .E .G + 36.Ib .Ic.Ac.E .G .L  + 48.Ib.Ab.Ic .E .G.L  + 13.Ib.Ab.Ic.Ac.E .G .L  + 72

                3     2          2      4  2          2       4  4       3        5  2
              .Ib.Ic .E .G  + 24.Ib.Ic .Ac.E .G .L  + 2.Ab.Ic .Ac.E .G .L  + 3.Ic .Ac.E .G .L )

                 2          3        2        2  5      2   2           2         3  3          2    3            3  5          3  2          2     4  3          2     4  5       3      5  3          2
FLEX(5,5) := (12.Ib .Ab.Ic.L  + 6.Ib .Ab.Ac.G .L  + 72.Ib .Ic .G.L + 36.Ib .Ic.Ac.G .L  + 24.Ib.Ab.Ic .G.L  + 13.Ib.Ab.Ic.Ac.G .L  + 36.Ib.Ic .G .L + 24.Ib.Ic .Ac.G .L  + 2.Ab.Ic .Ac.G .L  + 3.Ic .Ac.G .L )/(24.Ib .Ab.Ic.Ac.

                 2       2      2    2  4       2   2               2      2    3  2            2           2                   2    3  4         3       2          2   2    4  2          2   2      4  4       3
              E .L  + 6.Ib .Ab.Ac .E .G .L  + 144.Ib .Ic .Ac.E .G + 36.Ib .Ic.Ac .E .G .L  + 48.Ib.Ab.Ic .Ac.E .G.L  + 13.Ib.Ab.Ic.Ac .E .G .L  + 72.Ib.Ic .Ac.E .G  + 24.Ib.Ic .Ac .E .G .L  + 2.Ab.Ic .Ac .E .G .L  + 3.Ic .

                2    5  2
              Ac .E .G .L )

                2       4         2      2  2              2  4         2  3  2        2            2       2           2  4        2   2             2           3  2             2      2                     3  4
FLEX(5,6) := (6.Ib .Ab.G.L  + 36.Ib .Ic.G .L  + 12.Ib.Ab.Ic.G .L  + 18.Ib.Ic .G .L )/(24.Ib .Ab.Ic.E .L  + 6.Ib .Ab.Ac.E .G .L  + 144.Ib .Ic .E .G + 36.Ib .Ic.Ac.E .G .L  + 48.Ib.Ab.Ic .E .G.L  + 13.Ib.Ab.Ic.Ac.E .G .L  + 72.

                 3     2          2      4  2          2       4  4       3        5  2
              Ib.Ic .E .G  + 24.Ib.Ic .Ac.E .G .L  + 2.Ab.Ic .Ac.E .G .L  + 3.Ic .Ac.E .G .L )

                   2     4         2      2                4             3  6          4  4              4  6      2           2      2          2  4        2  2             2            3  2
FLEX(6,1) := ( - 24.Ib .Ab.L  - 144.Ib .Ic.G.L  - 48.Ib.Ab.Ic.G.L  - Ib.Ab.Ac.G .L  + 12.Ib.Ic.Ac.G .L  - 2.Ab.Ic.Ac.G .L )/(96.Ib .Ab.Ic.E .L  + 24.Ib .Ab Ac.E .G .L  + 576.Ib .Ic .E .G + 144.Ib .Ic.Ac.E .G .L  + 192.Ib.Ab.

                2      2                3  4           3    2          2          4  2          2        4  4        3         5  2
              Ic .E .G.L  + 52.Ib.Ab.Ic.Ac.E .G .L  + 288.Ib.Ic .E .G  + 96.Ib.Ic .Ac.E .G .L  + 8.Ab.Ic .Ac.E .G .L  + 12.Ic .Ac.E .G .L )

                 2      4        2    2  2             2  4          2  3  2        2            2       2           2  4        2   2             2           3  2             2      2                     3  4
FLEX(6,2) := ( - 6.Ib .Ab.G.L  - 36.Ib .Ic.G .L  - 12.Ib.Ab.Ic.G .L  - 18.Ib.Ic .G .L )/(24.Ib .Ab.Ic.E .L  + 6.Ib .Ab.Ac.E .G .L  + 144.Ib .Ic .E .G + 36.Ib .Ic.Ac.E .G .L  + 48.Ib.Ab.Ic .E .G.L  + 13.Ib.Ab.Ic.Ac.E .G .L  +

                 3     2          2      4  2          2       4  4       3        5  2
              72.Ib.Ic .E .G  + 24.Ib.Ic .Ac.E .G .L  + 2.Ab.Ic .Ac.E .G .L  + 3.Ic .Ac.E .G .L )

                 2     3          2                        3               3  5             4  3             4  5      2           2      2          2  4        2  2             2            3  2                  2
FLEX(6,3) := (48.Ib .Ab.L  + 288.Ib .Ic.G.L + 72.Ib.Ab.Ic.G.L  - 4.Ib.Ab.Ac.G .L  - 24.Ib.Ic.Ac.G .L  + 3.Ab.Ic.Ac.G .L )/(96.Ib .Ab.Ic.E .L  + 24.Ib .Ab.Ac.E .G .L  + 576.Ib .Ic .E .G + 144.Ib .Ic.Ac.E .G .L  + 192.Ib.Ab.Ic

                   2                3  4           3    2          2          4  2          2        4  4        3         5  2
              .E .G.L  + 52.Ib.Ab.Ic.Ac.E .G .L  + 288.Ib.Ic .E .G  + 96.Ib.Ic .Ac.E .G .L  + 8.Ab.Ic .Ac.E .G .L  + 12.Ic .Ac.E .G .L )
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FLEX(6,4) := ( - 24.Ib^2.Ab.L^4 - 144.Ib^2.Ic.G.L^2 - 48.Ib.Ab.Ic.G.L^4 - Ib.Ab.Ac.G^3.L^6 - 144.Ib.Ic^2.G^2.L^2 - 24.Ib.Ic.Ac.G^4.L^4 - 2.Ab.Ic.Ac.G^4.L^6 - 6.Ic^2.Ac.G^5.L^4 )/(96.Ib^2.Ab.Ic.E .L^2 + 24.Ib^2.Ab.Ac.E .G^2.L^4 + 576.Ib^2.Ic^2.E .G +

        144.Ib^2.Ic.Ac.E .G^3.L^2 + 192.Ib.Ab.Ic^2.E .G.L^2 + 52.Ib.Ab.Ic.Ac.E .G^3.L^4 + 288.Ib.Ic^3.E .G^2 + 96.Ib.Ic^2.Ac.E .G^4.L^2 + 8.Ab.Ic^2.Ac.E .G^4.L^4 + 12.Ic^3.Ac.E .G^5.L^2 )

FLEX(6,5) := (6.Ib^2.Ab.G.L^4 + 36.Ib^2.Ic.G^2.L^2 + 12.Ib.Ab.Ic.G^2.L^4 + 18.Ib.Ic^2.G^3.L^2 )/(24.Ib^2.Ab.Ic.E .L^2 + 6.Ib^2.Ab.Ac.E .G^2.L^4 + 144.Ib^2.Ic^2.E .G + 36.Ib^2.Ic.Ac.E .G^3.L^2 + 48.Ib.Ab.Ic^2.E .G.L^2 + 13.Ib.Ab.Ic.Ac.E .G^3.L^4 + 72.

        Ib.Ic^3.E .G^2 + 24.Ib.Ic^2.Ac.E .G^4.L^2 + 2.Ab.Ic^2.Ac.E .G^4.L^4 + 3.Ic^3.Ac.E .G^5.L^2 )

FLEX(6,6) := (48.Ib^2.Ab.L^3 + 288.Ib^2.Ic.G.L + 120.Ib.Ab.Ic.G.L^3 + 8.Ib.Ab.Ac.G^3.L^5 + 288.Ib.Ic^2.G^2.L + 48.Ib.Ic.Ac.G^4.L^3 + 5.Ab.Ic.Ac.G^4.L^5 + 12.Ic^2.Ac.G^5.L^3 )/(96.Ib^2.Ab.Ic.E .L^2 + 24.Ib^2.Ab.Ac.E .G^2.L^4 + 576.Ib^2.Ic^2.E .G +

        144.Ib^2.Ic.Ac.E .G^3.L^2 + 192.Ib.Ab.Ic^2.E .G.L^2 + 52.Ib.Ab.Ic.Ac.E .G^3.L^4 + 288.Ib.Ic^3.E .G^2 + 96.Ib.Ic^2.Ac.E .G^4.L^2 + 8.Ab.Ic^2.Ac.E .G^4.L^4 + 12.Ic^3.Ac.E .G^5.L^2 )

WRITE "DU FOR UNIFORM COLUMNS";

DU FOR UNIFORM COLUMNS

DU:=DU;

DU(1,1) := (96.Ib^2.Ab.Ic.L^5.P + 6.Ib^2.Ab.Ac.G^2.L^7.P + 432.Ib^2.Ic^2.G.L^3.P + 192.Ib.Ab.Ic^2.G.L^5.P + 16.Ib.Ab.Ic.Ac.G^3.L^7.P + 72.Ib.Ic^3.G^4.L^4.W + 18.Ib.Ic^2.Ac.G^6.L^6.W - 36.Ib.Ic^2.Ac.G^4.L^5.P + 8.Ab.Ic^2.Ac.G^4.L^7.P + 3.Ic^3.

        Ac.G^7.L^6.W)/(576.Ib^2.Ab.Ic^2.E .L^2 + 144.Ib^2.Ab.Ic.Ac.E .G^2.L^4 + 3456.Ib^2.Ic^3.E .G + 864.Ib^2.Ic^2.Ac.E .G^3.L^2 + 1152.Ib.Ab.Ic^3.E .G.L^2 + 312.Ib.Ab.Ic^2.Ac.E .G^3.L^4 + 1728.Ib.Ic^4.E .G^2 + 576.Ib.Ic^3.Ac.E .G^4.L^2 + 48.Ab

        .Ic^3.Ac.E .G^4.L^4 + 72.Ic^4.Ac.E .G^5.L^2 )

DU(2,1) := ( - 24.Ib^2.Ab.Ic.G.L^4.W - 6.Ib^2.Ab.Ac.G^3.L^6.W + 6.Ib^2.Ab.Ac.G.L^5.P - 144.Ib^2.Ic^2.G^2.L^2.W - 36.Ib^2.Ic.Ac.G^4.L^4.W + 36.Ib^2.Ic.Ac.G^2.L^3.P - 48.Ib.Ab.Ic^2.G^2.L^4.W - 13.Ib.Ab.Ic.Ac.G^4.L^6.W + 12.Ib.Ab.Ic.Ac.G^2.L^5.P -

        72.Ib.Ic^3.G^3.L^2.W - 24.Ib.Ic^2.Ac.G^5.L^4.W + 18.Ib.Ic^2.Ac.G^3.L^3.P - 2.Ab.Ic^2.Ac.G^5.L^6.W - 3.Ic^3.Ac.G^6.L^4.W)/(48.Ib^2.Ab.Ic.Ac.E .L^2 + 12.Ib^2.Ab.Ac^2.E .G^2.L^4 + 288.Ib^2.Ic^2.Ac.E .G + 72.Ib^2.Ic.Ac^2.E .G^3.L^2 + 96.Ib.Ab.

        Ic^2.Ac.E .G.L^2 + 26.Ib.Ab.Ic.Ac^2.E .G^3.L^4 + 144.Ib.Ic^3.Ac.E .G^2 + 48.Ib.Ic^2.Ac^2.E .G^4.L^2 + 4.Ab.Ic^2.Ac^2.E .G^4.L^4 + 6.Ic^3.Ac^2.E .G^5.L^2 )

DU(3,1) := ( - 144.Ib^2.Ab.L^4.P - 864.Ib^2.Ic.G.L^2.P - 24.Ib.Ab.Ic.G^3.L^5.W - 288.Ib.Ab.Ic.G.L^4.P - 6.Ib.Ab.Ac.G^5.L^7.W - 6.Ib.Ab.Ac.G^3.L^6.P - 144.Ib.Ic^2.G^4.L^3.W - 36.Ib.Ic.Ac.G^6.L^5.W + 72.Ib.Ic.Ac.G^4.L^4.P - Ab.Ic.Ac.G^6.L^7.W

        - 12.Ab.Ic.Ac.G^4.L^6.P - 6.Ic^2.Ac.G^7.L^5.W)/(576.Ib^2.Ab.Ic.E .L^2 + 144.Ib^2.Ab.Ac.E .G^2.L^4 + 3456.Ib^2.Ic^2.E .G + 864.Ib^2.Ic.Ac.E .G^3.L^2 + 1152.Ib.Ab.Ic^2.E .G.L^2 + 312.Ib.Ab.Ic.Ac.E .G^3.L^4 + 1728.Ib.Ic^3.E .G^2 + 576.

        Ib.Ic^2.Ac.E .G^4.L^2 + 48.Ab.Ic^2.Ac.E .G^4.L^4 + 72.Ic^3.Ac.E .G^5.L^2 )

DU(4,1) := (96.Ib^2.Ab.Ic.L^5.P + 6.Ib^2.Ab.Ac.G^2.L^7.P + 720.Ib^2.Ic^2.G.L^3.P + 72.Ib^2.Ic.Ac.G^3.L^5.P + 192.Ib.Ab.Ic^2.G.L^5.P + 16.Ib.Ab.Ic.Ac.G^3.L^7.P - 72.Ib.Ic^3.G^4.L^4.W + 576.Ib.Ic^3.G^2.L^3.P - 18.Ib.Ic^2.Ac.G^6.L^6.W + 120.Ib.

        Ic^2.Ac.G^4.L^5.P + 8.Ab.Ic^2.Ac.G^4.L^7.P - 3.Ic^3.Ac.G^7.L^6.W + 24.Ic^3.Ac.G^5.L^5.P )/(576.Ib^2.Ab.Ic^2.E .L^2 + 144.Ib^2.Ab.Ic.Ac.E .G^2.L^4 + 3456.Ib^2.Ic^3.E .G + 864.Ib^2.Ic^2.Ac.E .G^3.L^2 + 1152.Ib.Ab.Ic^3.E .G.L^2 + 312.Ib.Ab.

        Ic^2.Ac.E .G^3.L^4 + 1728.Ib.Ic^4.E .G^2 + 576.Ib.Ic^3.Ac.E .G^4.L^2 + 48.Ab.Ic^3.Ac.E .G^4.L^4 + 72.Ic^4.Ac.E .G^5.L^2 )

      2       4       2      3 6      2       5        2 2 2 2       2      4 4       2      2 3            2 2 4              4 6            2 5
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 DU(5,1) := ( - 24.Ib .Ab.Ic.G.L .W - 6.Ib .Ab.Ac.G .L .W - 6.Ib .Ab.Ac.G L .P  - 144.Ib .Ic .G .L .W - 36.Ib .Ic.Ac.G .L .W - 36.Ib .Ic.Ac.G .L .P  - 48.Ib.Ab.Ic .G .L .W - 13.Ib.Ab.Ic.Ac.G .L .W - 12.Ib.Ab.Ic.Ac.G .L .P  -

                     3  3  2            2     5  4            2     3  3            2     5  6         3    6   4         2             2        2      2     2  4        2   2                2     2     3  2
             72.Ib.Ic .G .L .W - 24.Ib.Ic .Ac.G .L .W - 18.Ib.Ic .Ac.G .L .P  - 2.Ab.Ic .Ac.G .L .W - 3.Ic .Ac.G .L .W)/(48.Ib .Ab.Ic.Ac.E .L  + 12.Ib .Ab.Ac .E .G .L  + 288.Ib .Ic .Ac.E .G + 72.Ib .Ic.Ac .E .G .L  + 96.Ib.Ab.

                 2         2                 2    3  4            3       2           2   2     4  2         2   2     4  4       3  2     5  2
               Ic .Ac.E .G.L  + 26.Ib.Ab.Ic.Ac .E .G .L  + 144.Ib.Ic .Ac.E .G  + 48.Ib.Ic .Ac .E .G .L  + 4.Ab.Ic .Ac .E .G .L  + 6.Ic .Ac .E .G .L )

                      2    4            2       2                  3  5                    4                 5  7                 3 6               2  4  3             2  2  2                  6  5                  4
 DU(6,1) := ( - 144.Ib .Ab.L .P  - 864.Ib .Ic.G.L .P  + 24.Ib.Ab.Ic.G .L .W - 288.Ib.Ab.Ic.G.L .P  + 6.Ib.Ab.Ac.G .L .W - 6.Ib.Ab.Ac.G .L .P  + 144.Ib.Ic .G .L .W - 864.Ib.Ic .G .L .P  + 36.Ib.Ic.Ac.G .L .W - 144.Ib.Ic.Ac.G .

              4               5  7                 4  6          2    7  5          2     5  4           2           2        2           2  4          2  2              2          3  2                2       2
            L .P  + Ab.Ic.Ac.G .L .W - 12.Ab.Ic.Ac.G .L .P  + 6.Ic .Ac.G .L .W - 36.Ic .Ac.G .L .P )/(576.Ib .Ab.Ic.E .L  + 144.Ib .Ab.Ac.E .G .L  + 3456.Ib .Ic .E .G + 864.Ib .Ic.Ac.E .G .L  + 1152.Ib.Ab.Ic .E .G.L  + 312.Ib

                            3  4             3    2            2        4 2           2        4  4       3        5  2
              .Ab.Ic.Ac.E .G .L  + 1728.Ib.Ic .E .G  + 576.Ib.Ic .Ac.E .G .L  + 48.Ab.Ic .Ac.E .G .L  + 72.Ic .Ac.E .G .L )


LET Ab=A, Ac=A;

LET Ib=Iz, Ic=Iz;

WRITE "AKA FOR AIL ELEMENTS UNIFORM";

AKA FOR AIL ELEMENTS UNIFORM

AKA:=AKA;

                                2       3
AKA(1,1) := (12.E .Iz.G + E .A.L  )/(G.L )

AKA(1,2) := 0

                       2
AKA(1,3) := (6.E .Iz)/L

AKA(1,4) := ( - E .A)/(G.L)

AKA(1,5) := 0

AKA(1,6) := 0

AKA(2,1) := 0

                              3  2      3  3
AKA(2,2) := (12.E .Iz + E .A.G .L  )/(G .L  )

                        2  2
AKA(2,3) := (6.E .Iz)/(G .L )

AKA(2,4) := 0

                            3  3
AKA(2,5) := ( - 12.E .Iz)/(G .L )

                        2  2
AKA(2,6) := (6.E .Iz)/(G .L )

                       2
AKA(3,1) := (6.E .Iz)/L

                        2  2
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AKA(3,2) := (6.E .Iz)/(G .L )

AKA(3,3) := (4.E .Iz.G + 4.E .Iz)/(G.L)

AKA(3,4) := 0

                           2  2
AKA(3,5) := ( - 6.E .Iz)/(G .L )

AKA(3,6) := (2.E .Iz)/(G.L)

AKA(4,1) := ( - E .A)/(G.L)

AKA(4,2) := 0

AKA(4,3) := 0

                                  2      3
AKA(4,4) := (12.E .Iz.G + E .A.L )/(G.L )

AKA(4,5) := 0

                      2
AKA(4,6) := (6.E .Iz)/L

AKA(5,1) := 0

                            3  3
AKA(5,2) := ( - 12.E .Iz)/(G .L )

                           2  2
AKA(5,3) := ( - 6.E .Iz)/(G .L )

AKA(5,4) := 0

                              3  2    3  3
AKA(5,5) := (12.E .Iz + E .A.G .L )/(G .L )

                           2  2
AKA(5,6) := ( - 6.E .Iz)/(G .L )

AKA(6,1) := 0

                       2  2
AKA(6,2) := (6.E .Iz)/(G .L )

AKA(6,3) := (2.E .Iz)/(G.L)

                      2
AKA(6,4) := (6.E .Iz)/L

                           2  2
AKA(6,5) := ( - 6.E .Iz)/(G .L )

AKA(6,6) := (4.E .Iz.G + 4.E .Iz)/(G.L)

WRITE "FLEX FOR AlL ELEMENTS UNIFORM";

FLEX FOR AlL ELEMENTS UNIFORM

FLEX:=FLEX;
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$$\mathrm{FLEX}(1,1) := (288 \cdot Iz^{2} \cdot G^{2} \cdot L^{3} + 360 \cdot Iz^{2} \cdot G \cdot L^{3} + 12 \cdot Iz \cdot A \cdot G^{5} \cdot L^{5} + 60 \cdot Iz \cdot A \cdot G^{4} \cdot L^{5} + 36 \cdot Iz \cdot A \cdot G^{3} \cdot L^{5} + 96 \cdot Iz \cdot A \cdot G \cdot L^{5} + 48 \cdot Iz \cdot A \cdot L^{5} + 4 \cdot A^{2} \cdot G^{4} \cdot L^{7} + 8 \cdot A^{2} \cdot G^{3} \cdot L^{7} + 3 \cdot A^{2} \cdot G^{2} \cdot L^{7}) / (864 \cdot E \cdot Iz^{3} \cdot G^{2} + 1728 \cdot E \cdot Iz^{3} \cdot G + 36 \cdot E \cdot Iz^{2} \cdot A \cdot G^{5} \cdot L^{2} + 288 \cdot E \cdot Iz^{2} \cdot A \cdot G^{4} \cdot L^{2} + 432 \cdot E \cdot Iz^{2} \cdot A \cdot G^{3} \cdot L^{2} + 576 \cdot E \cdot Iz^{2} \cdot A \cdot G \cdot L^{2} + 288 \cdot E \cdot Iz^{2} \cdot A \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A^{2} \cdot G^{4} \cdot L^{4} + 156 \cdot E \cdot Iz \cdot A^{2} \cdot G^{3} \cdot L^{4} + 72 \cdot E \cdot Iz \cdot A^{2} \cdot G^{2} \cdot L^{4})$$

$$\mathrm{FLEX}(1,2) := (9 \cdot Iz \cdot G^{3} \cdot L^{3} + 18 \cdot Iz \cdot G^{2} \cdot L^{3} + 6 \cdot A \cdot G^{2} \cdot L^{5} + 3 \cdot A \cdot G \cdot L^{5}) / (72 \cdot E \cdot Iz^{2} \cdot G^{2} + 144 \cdot E \cdot Iz^{2} \cdot G + 3 \cdot E \cdot Iz \cdot A \cdot G^{5} \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A \cdot G^{4} \cdot L^{2} + 36 \cdot E \cdot Iz \cdot A \cdot G^{3} \cdot L^{2} + 48 \cdot E \cdot Iz \cdot A \cdot G \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A \cdot L^{2} + 2 \cdot E \cdot A^{2} \cdot G^{4} \cdot L^{4} + 13 \cdot E \cdot A^{2} \cdot G^{3} \cdot L^{4} + 6 \cdot E \cdot A^{2} \cdot G^{2} \cdot L^{4})$$

$$\mathrm{FLEX}(1,3) := (-144 \cdot Iz^{2} \cdot G^{2} \cdot L^{2} - 144 \cdot Iz^{2} \cdot G \cdot L^{2} - 6 \cdot Iz \cdot A \cdot G^{5} \cdot L^{4} - 24 \cdot Iz \cdot A \cdot G^{4} \cdot L^{4} - 48 \cdot Iz \cdot A \cdot G \cdot L^{4} - 24 \cdot Iz \cdot A \cdot L^{4} - 2 \cdot A^{2} \cdot G^{4} \cdot L^{6} - A^{2} \cdot G^{3} \cdot L^{6}) / (288 \cdot E \cdot Iz^{3} \cdot G^{2} + 576 \cdot E \cdot Iz^{3} \cdot G + 12 \cdot E \cdot Iz^{2} \cdot A \cdot G^{5} \cdot L^{2} + 96 \cdot E \cdot Iz^{2} \cdot A \cdot G^{4} \cdot L^{2} + 144 \cdot E \cdot Iz^{2} \cdot A \cdot G^{3} \cdot L^{2} + 192 \cdot E \cdot Iz^{2} \cdot A \cdot G \cdot L^{2} + 96 \cdot E \cdot Iz^{2} \cdot A \cdot L^{2} + 8 \cdot E \cdot Iz \cdot A^{2} \cdot G^{4} \cdot L^{4} + 52 \cdot E \cdot Iz \cdot A^{2} \cdot G^{3} \cdot L^{4} + 24 \cdot E \cdot Iz \cdot A^{2} \cdot G^{2} \cdot L^{4})$$

$$\mathrm{FLEX}(1,4) := (216 \cdot Iz^{2} \cdot G \cdot L^{3} - 18 \cdot Iz \cdot A \cdot G^{4} \cdot L^{5} + 96 \cdot Iz \cdot A \cdot G \cdot L^{5} + 48 \cdot Iz \cdot A \cdot L^{5} + 4 \cdot A^{2} \cdot G^{4} \cdot L^{7} + 8 \cdot A^{2} \cdot G^{3} \cdot L^{7} + 3 \cdot A^{2} \cdot G^{2} \cdot L^{7}) / (864 \cdot E \cdot Iz^{3} \cdot G^{2} + 1728 \cdot E \cdot Iz^{3} \cdot G + 36 \cdot E \cdot Iz^{2} \cdot A \cdot G^{5} \cdot L^{2} + 288 \cdot E \cdot Iz^{2} \cdot A \cdot G^{4} \cdot L^{2} + 432 \cdot E \cdot Iz^{2} \cdot A \cdot G^{3} \cdot L^{2} + 576 \cdot E \cdot Iz^{2} \cdot A \cdot G \cdot L^{2} + 288 \cdot E \cdot Iz^{2} \cdot A \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A^{2} \cdot G^{4} \cdot L^{4} + 156 \cdot E \cdot Iz \cdot A^{2} \cdot G^{3} \cdot L^{4} + 72 \cdot E \cdot Iz \cdot A^{2} \cdot G^{2} \cdot L^{4})$$

$$\mathrm{FLEX}(1,5) := (-9 \cdot Iz \cdot G^{3} \cdot L^{3} - 18 \cdot Iz \cdot G^{2} \cdot L^{3} - 6 \cdot A \cdot G^{2} \cdot L^{5} - 3 \cdot A \cdot G \cdot L^{5}) / (72 \cdot E \cdot Iz^{2} \cdot G^{2} + 144 \cdot E \cdot Iz^{2} \cdot G + 3 \cdot E \cdot Iz \cdot A \cdot G^{5} \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A \cdot G^{4} \cdot L^{2} + 36 \cdot E \cdot Iz \cdot A \cdot G^{3} \cdot L^{2} + 48 \cdot E \cdot Iz \cdot A \cdot G \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A \cdot L^{2} + 2 \cdot E \cdot A^{2} \cdot G^{4} \cdot L^{4} + 13 \cdot E \cdot A^{2} \cdot G^{3} \cdot L^{4} + 6 \cdot E \cdot A^{2} \cdot G^{2} \cdot L^{4})$$

$$\mathrm{FLEX}(1,6) := (-144 \cdot Iz^{2} \cdot G \cdot L^{2} + 12 \cdot Iz \cdot A \cdot G^{4} \cdot L^{4} - 48 \cdot Iz \cdot A \cdot G \cdot L^{4} - 24 \cdot Iz \cdot A \cdot L^{4} - 2 \cdot A^{2} \cdot G^{4} \cdot L^{6} - A^{2} \cdot G^{3} \cdot L^{6}) / (288 \cdot E \cdot Iz^{3} \cdot G^{2} + 576 \cdot E \cdot Iz^{3} \cdot G + 12 \cdot E \cdot Iz^{2} \cdot A \cdot G^{5} \cdot L^{2} + 96 \cdot E \cdot Iz^{2} \cdot A \cdot G^{4} \cdot L^{2} + 144 \cdot E \cdot Iz^{2} \cdot A \cdot G^{3} \cdot L^{2} + 192 \cdot E \cdot Iz^{2} \cdot A \cdot G \cdot L^{2} + 96 \cdot E \cdot Iz^{2} \cdot A \cdot L^{2} + 8 \cdot E \cdot Iz \cdot A^{2} \cdot G^{4} \cdot L^{4} + 52 \cdot E \cdot Iz \cdot A^{2} \cdot G^{3} \cdot L^{4} + 24 \cdot E \cdot Iz \cdot A^{2} \cdot G^{2} \cdot L^{4})$$

$$\mathrm{FLEX}(2,1) := (9 \cdot Iz \cdot G^{3} \cdot L^{3} + 18 \cdot Iz \cdot G^{2} \cdot L^{3} + 6 \cdot A \cdot G^{2} \cdot L^{5} + 3 \cdot A \cdot G \cdot L^{5}) / (72 \cdot E \cdot Iz^{2} \cdot G^{2} + 144 \cdot E \cdot Iz^{2} \cdot G + 3 \cdot E \cdot Iz \cdot A \cdot G^{5} \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A \cdot G^{4} \cdot L^{2} + 36 \cdot E \cdot Iz \cdot A \cdot G^{3} \cdot L^{2} + 48 \cdot E \cdot Iz \cdot A \cdot G \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A \cdot L^{2} + 2 \cdot E \cdot A^{2} \cdot G^{4} \cdot L^{4} + 13 \cdot E \cdot A^{2} \cdot G^{3} \cdot L^{4} + 6 \cdot E \cdot A^{2} \cdot G^{2} \cdot L^{4})$$

$$\mathrm{FLEX}(2,2) := (36 \cdot Iz^{2} \cdot G^{2} \cdot L + 72 \cdot Iz^{2} \cdot G \cdot L + 3 \cdot Iz \cdot A \cdot G^{5} \cdot L^{3} + 24 \cdot Iz \cdot A \cdot G^{4} \cdot L^{3} + 36 \cdot Iz \cdot A \cdot G^{3} \cdot L^{3} + 24 \cdot Iz \cdot A \cdot G \cdot L^{3} + 12 \cdot Iz \cdot A \cdot L^{3} + 2 \cdot A^{2} \cdot G^{4} \cdot L^{5} + 13 \cdot A^{2} \cdot G^{3} \cdot L^{5} + 6 \cdot A^{2} \cdot G^{2} \cdot L^{5}) / (72 \cdot E \cdot Iz^{2} \cdot A \cdot G^{2} + 144 \cdot E \cdot Iz^{2} \cdot A \cdot G + 3 \cdot E \cdot Iz \cdot A^{2} \cdot G^{5} \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A^{2} \cdot G^{4} \cdot L^{2} + 36 \cdot E \cdot Iz \cdot A^{2} \cdot G^{3} \cdot L^{2} + 48 \cdot E \cdot Iz \cdot A^{2} \cdot G \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A^{2} \cdot L^{2} + 2 \cdot E \cdot A^{3} \cdot G^{4} \cdot L^{4} + 13 \cdot E \cdot A^{3} \cdot G^{3} \cdot L^{4} + 6 \cdot E \cdot A^{3} \cdot G^{2} \cdot L^{4})$$

$$\mathrm{FLEX}(2,3) := (-18 \cdot Iz \cdot G^{3} \cdot L^{2} - 36 \cdot Iz \cdot G^{2} \cdot L^{2} - 12 \cdot A \cdot G^{2} \cdot L^{4} - 6 \cdot A \cdot G \cdot L^{4}) / (72 \cdot E \cdot Iz^{2} \cdot G^{2} + 144 \cdot E \cdot Iz^{2} \cdot G + 3 \cdot E \cdot Iz \cdot A \cdot G^{5} \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A \cdot G^{4} \cdot L^{2} + 36 \cdot E \cdot Iz \cdot A \cdot G^{3} \cdot L^{2} + 48 \cdot E \cdot Iz \cdot A \cdot G \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A \cdot L^{2} + 2 \cdot E \cdot A^{2} \cdot G^{4} \cdot L^{4} + 13 \cdot E \cdot A^{2} \cdot G^{3} \cdot L^{4} + 6 \cdot E \cdot A^{2} \cdot G^{2} \cdot L^{4})$$

$$\mathrm{FLEX}(2,4) := (9 \cdot Iz \cdot G^{3} \cdot L^{3} + 18 \cdot Iz \cdot G^{2} \cdot L^{3} + 6 \cdot A \cdot G^{2} \cdot L^{5} + 3 \cdot A \cdot G \cdot L^{5}) / (72 \cdot E \cdot Iz^{2} \cdot G^{2} + 144 \cdot E \cdot Iz^{2} \cdot G + 3 \cdot E \cdot Iz \cdot A \cdot G^{5} \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A \cdot G^{4} \cdot L^{2} + 36 \cdot E \cdot Iz \cdot A \cdot G^{3} \cdot L^{2} + 48 \cdot E \cdot Iz \cdot A \cdot G \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A \cdot L^{2} + 2 \cdot E \cdot A^{2} \cdot G^{4} \cdot L^{4} + 13 \cdot E \cdot A^{2} \cdot G^{3} \cdot L^{4} + 6 \cdot E \cdot A^{2} \cdot G^{2} \cdot L^{4})$$

$$\mathrm{FLEX}(2,5) := (36 \cdot Iz^{2} \cdot G^{2} \cdot L + 72 \cdot Iz^{2} \cdot G \cdot L + 24 \cdot Iz \cdot A \cdot G \cdot L^{3} + 12 \cdot Iz \cdot A \cdot L^{3}) / (72 \cdot E \cdot Iz^{2} \cdot A \cdot G^{2} + 144 \cdot E \cdot Iz^{2} \cdot A \cdot G + 3 \cdot E \cdot Iz \cdot A^{2} \cdot G^{5} \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A^{2} \cdot G^{4} \cdot L^{2} + 36 \cdot E \cdot Iz \cdot A^{2} \cdot G^{3} \cdot L^{2} + 48 \cdot E \cdot Iz \cdot A^{2} \cdot G \cdot L^{2} + 24 \cdot E \cdot Iz \cdot A^{2} \cdot L^{2} + 2 \cdot E \cdot A^{3} \cdot G^{4} \cdot L^{4} + 13 \cdot E \cdot A^{3} \cdot G^{3}$$

```
      4         3  2  4
    .L  + 6.E .A .G .L  )

                        3  2          2  2         2  4          4           2  2           2                5  2               4  2               3  2                 2               2         2  4  4          2  3  4
FLEX(2,6) := ( - 18.Iz.G .L  - 36.Iz.G .L  - 12.A.G .L  - 6.A.G.L )/(72.E .Iz .G + 144.E .Iz .G + 3.E .Iz.A.G .L  + 24.E .Iz.A.G .L  + 36.E .Iz.A.G .L  + 48.E .Iz.A.G.L  + 24.E .Iz.A.L  + 2.E .A .G .L  + 13.E .A .G .L  + 6.
        2  2  4
    E .A .G .L )

                       2  2  2         2    2           5  4            4  4              4            4      2  4  6    2  3  6            3  2            3             2    5  2           2    4  2            2    3  2
FLEX(3,1) := ( - 144.Iz .G .L  - 144.Iz .G.L  - 6.Iz.A.G .L  - 24.Iz.A.G .L  - 48.Iz.A.G.L  - 24.Iz.A.L  - 2.A .G .L  - A .G .L )/(288.E .Iz .G  + 576.E .Iz .G + 12.E .Iz .A.G .L  + 96.E .Iz .A.G .L  + 144.E .Iz .A.G .L  +
             2      2           2    2            2  4  4             2  3  4             2  2  4
    192.E .Iz .A.G.L  + 96.E .Iz .A.L  + 8.E .Iz.A .G .L  + 52.E .Iz.A .G .L  + 24.E .Iz.A .G .L )

                        3  2          2  2         2  4          4           2  2           2                5  2               4  2               3  2                 2               2         2  4  4          2  3  4
FLEX(3,2) := ( - 18.Iz.G .L  - 36.Iz.G .L  - 12.A.G .L  - 6.A.G.L )/(72.E .Iz .G + 144.E .Iz .G + 3.E .Iz.A.G .L  + 24.E .Iz.A.G .L  + 36.E .Iz.A.G .L  + 48.E .Iz.A.G.L  + 24.E .Iz.A.L  + 2.E .A .G .L  + 13.E .A .G .L  + 6.
        2  2  4
    E .A .G .L )

                    2  2           2                5  3            4  3               3            3      2  4  5      2  3  5            3  2            3             2    5  2           2    4  2            2    3  2
FLEX(3,3) := (288.Iz .G .L + 288.Iz .G.L + 12.Iz.A.G .L  + 48.Iz.A.G .L  + 120.Iz.A.G.L  + 48.Iz.A.L  + 5.A .G .L  + 8.A .G .L )/(288.E .Iz .G  + 576.E .Iz .G + 12.E .Iz .A.G .L  + 96.E .Iz .A.G .L  + 144.E .Iz .A.G .L  + 192
          2      2           2    2            2  4  4             2  3  4             2  2  4
    .E .Iz .A.G.L  + 96.E .Iz .A.L  + 8.E .Iz.A .G .L  + 52.E .Iz.A .G .L  + 24.E .Iz.A .G .L )

                       2    2            4  4              4            4      2  4  6    2  3  6            3  2            3             2    5  2           2    4  2            2    3  2            2      2           2
FLEX(3,4) := ( - 144.Iz .G.L  + 12.Iz.A.G .L  - 48.Iz.A.G.L  - 24.Iz.A.L  - 2.A .G .L  - A .G .L )/(288.E .Iz .G  + 576.E .Iz .G + 12.E .Iz .A.G .L  + 96.E .Iz .A.G .L  + 144.E .Iz .A.G .L  + 192.E .Iz .A.G.L  + 96.E .Iz .A.
     2            2  4  4             2  3  4             2  2  4
    L  + 8.E .Iz.A .G .L  + 52.E .Iz.A .G .L  + 24.E .Iz.A .G .L )

                     3  2          2  2         2  4          4           2  2           2                5  2               4  2               3  2                 2               2         2  4  4          2  3  4
FLEX(3,5) := (18.Iz.G .L  + 36.Iz.G .L  + 12.A.G .L  + 6.A.G.L )/(72.E .Iz .G + 144.E .Iz .G + 3.E .Iz.A.G .L  + 24.E .Iz.A.G .L  + 36.E .Iz.A.G .L  + 48.E .Iz.A.G.L  + 24.E .Iz.A.L  + 2.E .A .G .L  + 13.E .A .G .L  + 6.E .
     2  2  4
    A .G .L )

                    2                4  3              3            3      2  4  5      2  3  5            3  2            3             2    5  2           2    4  2            2    3  2            2      2           2    2
FLEX(3,6) := (288.Iz .G.L - 24.Iz.A.G .L  + 72.Iz.A.G.L  + 48.Iz.A.L  + 3.A .G .L  - 4.A .G .L )/(288.E .Iz .G  + 576.E .Iz .G + 12.E .Iz .A.G .L  + 96.E .Iz .A.G .L  + 144.E .Iz .A.G .L  + 192.E .Iz .A.G.L  + 96.E .Iz .A.L
               2  4  4             2  3  4             2  2  4
    + 8.E .Iz.A .G .L  + 52.E .Iz.A .G .L  + 24.E .Iz.A .G .L )

                    2    3            4  5              5            5      2  4  7      2  3  7      2  2  7            3  2             3             2    5  2            2    4  2            2    3  2            2      2
FLEX(4,1) := (216.Iz .G.L  - 18.Iz.A.G .L  + 96.Iz.A.G.L  + 48.Iz.A.L  + 4.A .G .L  + 8.A .G .L  + 3.A .G .L )/(864.E .Iz .G  + 1728.E .Iz .G + 36.E .Iz .A.G .L  + 288.E .Iz .A.G .L  + 432.E .Iz .A.G .L  + 576.E .Iz .A.G.L
               2    2             2  4  4              2  3  4             2  2  4
    + 288.E .Iz .A.L  + 24.E .Iz.A .G .L  + 156.E .Iz.A .G .L  + 72.E .Iz.A .G .L )

                    3  3          2  3        2  5          5           2  2           2                5  2               4  2               3  2                 2               2         2  4  4          2  3  4         2
FLEX(4,2) := (9.Iz.G .L  + 18.Iz.G .L  + 6.A.G .L  + 3.A.G.L )/(72.E .Iz .G + 144.E .Iz .G + 3.E .Iz.A.G .L  + 24.E .Iz.A.G .L  + 36.E .Iz.A.G .L  + 48.E .Iz.A.G.L  + 24.E .Iz.A.L  + 2.E .A .G .L  + 13.E .A .G .L  + 6.E .A .
     2  4
    G .L )

                       2    2            4  4              4            4      2  4  6    2  3  6            3  2            3             2    5  2           2    4  2            2    3  2            2      2           2
FLEX(4,3) := ( - 144.Iz .G.L  + 12.Iz.A.G .L  - 48.Iz.A.G.L  - 24.Iz.A.L  - 2.A .G .L  - A .G .L )/(288.E .Iz .G  + 576.E .Iz .G + 12.E .Iz .A.G .L  + 96.E .Iz .A.G .L  + 144.E .Iz .A.G .L  + 192.E .Iz .A.G.L  + 96.E .Iz .A.
     2            2  4  4             2  3  4             2  2  4
    L  + 8.E .Iz.A .G .L  + 52.E .Iz.A .G .L  + 24.E .Iz.A .G .L )
```

$$FLEX(4,4) := (288 Iz^2 G^2 L^3 + 360 Iz^2 G L^3 + 12 Iz A G^5 L^5 + 60 Iz A G^4 L^5 + 36 Iz A G^3 L^5 + 96 Iz A G L^5 + 48 Iz A L^5 + 4 A^2 G^4 L^7 + 8 A^2 G^3 L^7 + 3 A^2 G^2 L^7)/(864 E Iz^3 G^2 + 1728 E Iz^3 G + 36 E Iz^2 A G^5 L^2 + 288 E Iz^2 A G^4 L^2 + 432 E Iz^2 A G^3 L^2 + 576 E Iz^2 A G L^2 + 288 E Iz^2 A L^2 + 24 E Iz A^2 G^4 L^4 + 156 E Iz A^2 G^3 L^4 + 72 E Iz A^2 G^2 L^4)$$

$$FLEX(4,5) := (- 9 Iz G^3 L^3 - 18 Iz G^2 L^3 - 6 A G^2 L^5 - 3 A G L^5)/(72 E Iz^2 G^2 + 144 E Iz^2 G + 3 E Iz A G^5 L^2 + 24 E Iz A G^4 L^2 + 36 E Iz A G^3 L^2 + 48 E Iz A G L^2 + 24 E Iz A L^2 + 2 E A^2 G^4 L^4 + 13 E A^2 G^3 L^4 + 6 E A^2 G^2 L^4)$$

$$FLEX(4,6) := (- 144 Iz^2 G^2 L^2 - 144 Iz^2 G L^2 - 6 Iz A G^5 L^4 - 24 Iz A G^4 L^4 - 48 Iz A G L^4 - 24 Iz A L^4 - 2 A^2 G^4 L^6 - A^2 G^3 L^6)/(288 E Iz^3 G^2 + 576 E Iz^3 G + 12 E Iz^2 A G^5 L^2 + 96 E Iz^2 A G^4 L^2 + 144 E Iz^2 A G^3 L^2 + 192 E Iz^2 A G L^2 + 96 E Iz^2 A L^2 + 8 E Iz A^2 G^4 L^4 + 52 E Iz A^2 G^3 L^4 + 24 E Iz A^2 G^2 L^4)$$

$$FLEX(5,1) := (- 9 Iz G^3 L^3 - 18 Iz G^2 L^3 - 6 A G^2 L^5 - 3 A G L^5)/(72 E Iz^2 G^2 + 144 E Iz^2 G + 3 E Iz A G^5 L^2 + 24 E Iz A G^4 L^2 + 36 E Iz A G^3 L^2 + 48 E Iz A G L^2 + 24 E Iz A L^2 + 2 E A^2 G^4 L^4 + 13 E A^2 G^3 L^4 + 6 E A^2 G^2 L^4)$$

$$FLEX(5,2) := (36 Iz^2 G^2 L + 72 Iz^2 G L + 24 Iz A G L^3 + 12 Iz A L^3)/(72 E Iz^2 A G^2 + 144 E Iz^2 A G + 3 E Iz A^2 G^5 L^2 + 24 E Iz A^2 G^4 L^2 + 36 E Iz A^2 G^3 L^2 + 48 E Iz A^2 G L^2 + 24 E Iz A^2 L^2 + 2 E A^3 G^4 L^4 + 13 E A^3 G^3 L^4 + 6 E A^3 G^2 L^4)$$

$$FLEX(5,3) := (18 Iz G^3 L^2 + 36 Iz G^2 L^2 + 12 A G^2 L^4 + 6 A G L^4)/(72 E Iz^2 G^2 + 144 E Iz^2 G + 3 E Iz A G^5 L^2 + 24 E Iz A G^4 L^2 + 36 E Iz A G^3 L^2 + 48 E Iz A G L^2 + 24 E Iz A L^2 + 2 E A^2 G^4 L^4 + 13 E A^2 G^3 L^4 + 6 E A^2 G^2 L^4)$$

$$FLEX(5,4) := (- 9 Iz G^3 L^3 - 18 Iz G^2 L^3 - 6 A G^2 L^5 - 3 A G L^5)/(72 E Iz^2 G^2 + 144 E Iz^2 G + 3 E Iz A G^5 L^2 + 24 E Iz A G^4 L^2 + 36 E Iz A G^3 L^2 + 48 E Iz A G L^2 + 24 E Iz A L^2 + 2 E A^2 G^4 L^4 + 13 E A^2 G^3 L^4 + 6 E A^2 G^2 L^4)$$

$$FLEX(5,5) := (36 Iz^2 G^2 L + 72 Iz^2 G L + 3 Iz A G^5 L^3 + 24 Iz A G^4 L^3 + 36 Iz A G^3 L^3 + 24 Iz A G L^3 + 12 Iz A L^3 + 2 A^2 G^4 L^5 + 13 A^2 G^3 L^5 + 6 A^2 G^2 L^5)/(72 E Iz^2 A G^2 + 144 E Iz^2 A G + 3 E Iz A^2 G^5 L^2 + 24 E Iz A^2 G^4 L^2 + 36 E Iz A^2 G^3 L^2 + 48 E Iz A^2 G L^2 + 24 E Iz A^2 L^2 + 2 E A^3 G^4 L^4 + 13 E A^3 G^3 L^4 + 6 E A^3 G^2 L^4)$$

$$FLEX(5,6) := (18 Iz G^3 L^2 + 36 Iz G^2 L^2 + 12 A G^2 L^4 + 6 A G L^4)/(72 E Iz^2 G^2 + 144 E Iz^2 G + 3 E Iz A G^5 L^2 + 24 E Iz A G^4 L^2 + 36 E Iz A G^3 L^2 + 48 E Iz A G L^2 + 24 E Iz A L^2 + 2 E A^2 G^4 L^4 + 13 E A^2 G^3 L^4 + 6 E A^2 G^2 L^4)$$

$$FLEX(6,1) := (- 144 Iz^2 G L^2 + 12 Iz A G^4 L^4 - 48 Iz A G L^4 - 24 Iz A L^4 - 2 A^2 G^4 L^6 - A^2 G^3 L^6)/(288 E Iz^3 G^2 + 576 E Iz^3 G + 12 E Iz^2 A G^5 L^2 + 96 E Iz^2 A G^4 L^2 + 144 E Iz^2 A G^3 L^2 + 192 E Iz^2 A G L^2 + 96 E Iz^2 A L^2 + 8 E Iz A^2 G^4 L^4 + 52 E Iz A^2 G^3 L^4 + 24 E Iz A^2 G^2 L^4)$$

$$FLEX(6,2) := (- 18 Iz G^3 L^2 - 36 Iz G^2 L^2 - 12 A G^2 L^4 - 6 A G L^4)/(72 E Iz^2 G^2 + 144 E Iz^2 G + 3 E Iz A G^5 L^2 + 24 E Iz A G^4 L^2 + 36 E Iz A G^3 L^2 + 48 E Iz A G L^2 + 24 E Iz A L^2 + 2 E A^2 G^4 L^4 + 13 E A^2 G^3 L^4 + 6.$$

```
          2  2  4
        E .A .G .L )

                2                 4  3               3            3     2  4  5     2  3  5           3   2          3              2   5  2            2   4  2              2   3  2             2      2           2  2
FLEX(6,3) := (288.Iz .G.L - 24.Iz.A.G .L  + 72.Iz.A.G.L  + 48.Iz.A.L  + 3.A .G .L  - 4.A .G .L )/(288.E .Iz .G  + 576.E .Iz .G + 12.E .Iz .A.G .L  + 96.E .Iz .A.G .L  + 144.E .Iz .A.G .L  + 192.E .Iz .A.G.L  + 96.E .Iz .A.L

                   2  4  4           2  3  4           2  2  4
        + 8.E .Iz.A .G .L  + 52.E .Iz.A .G .L  + 24.E .Iz.A .G .L )

                   2  2  2         2   2          5  4          4  4          4            4        2  4  6    2  3  6           3  2           3             2   5  2            2   4  2              2   3  2
FLEX(6,4) := ( - 144.Iz .G .L  - 144.Iz .G.L  - 6.Iz.A.G .L  - 24.Iz.A.G .L  - 48.Iz.A.G.L  - 24.Iz.A.L  - 2.A .G .L  - A .G .L )/(288.E .Iz .G  + 576.E .Iz .G + 12.E .Iz .A.G .L  + 96.E .Iz .A.G .L  + 144.E .Iz .A.G .L  +

              2     2            2    2          2  4  4           2  3  4           2  2  4
        192.E .Iz .A.G.L  + 96.E .Iz .A.L  + 8.E .Iz.A .G .L  + 52.E .Iz.A .G .L  + 24.E .Iz.A .G .L )

                 3 2             2 2           2  4          4             2 2             2               5  2             4  2             3  2               2              2         2  4  4         2  3  4
FLEX(6,5) := (18.Iz.G .L  + 36.Iz.G .L  + 12.A.G .L  + 6.A.G.L )/(72.E .Iz .G  + 144.E .Iz .G + 3.E .Iz.A.G .L  + 24.E .Iz.A.G .L  + 36.E .Iz.A.G .L  + 48.E .Iz.A.G.L  + 24.E .Iz.A.L  + 2.E .A .G .L  + 13.E .A .G .L  + 6.E .

          2  2  4
        A .G .L )

                  2  2           2              5  3           4  3             3             3       2  4  5     2  3  5             3  2           3             2   5  2            2   4  2              2   3  2
FLEX(6,6) := (288.Iz .G .L + 288.Iz .G.L + 12.Iz.A.G .L  + 48.Iz.A.G .L  + 120.Iz.A.G.L  + 48.Iz.A.L  + 5.A .G .L  + 8.A .G .L )/(288.E .Iz .G  + 576.E .Iz .G + 12.E .Iz .A.G .L  + 96.E .Iz .A.G .L  + 144.E .Iz .A.G .L  + 192

          2     2            2    2          2  4  4           2  3  4           2  2  4
        .E .Iz .A.G.L  + 96.E .Iz .A.L  + 8.E .Iz.A .G .L  + 52.E .Iz.A .G .L  + 24.E .Iz.A .G .L )

WRITE "DU FOR AIL ELEMENTS UNIFORM";

DU FOR AIL ELEMENTS UNIFORM

DU:=DU;

                 2  4  4          2  3           7  6            6  6            4  5               5            5       2  4  7     2  3  7      2  2  7            3  2            3
DU(1,1) := (72.Iz .G .L .W + 432.Iz .G.L .P  + 3.Iz.A.G .L .W + 18.Iz.A.G .L .W - 36.Iz.A.G .L .P  + 192.Iz.A.G.L .P  + 96.Iz.A.L .P  + 8.A .G .L .P  + 16.A .G .L .P  + 6.A .G .L .P )/(1728.E .Iz .G  + 3456.E .Iz .G + 72.E .

           2   5  2           2   4  2            2   3  2             2      2          2    2           2  4  4            2  3  4            2  2  4
        Iz .A.G .L  + 576.E .Iz .A.G .L  + 864.E .Iz .A.G .L  + 1152.E .Iz .A.G.L  + 576.E .Iz .A.L  + 48.E .Iz.A .G .L  + 312.E .Iz.A .G .L  + 144.E .Iz.A .G .L )

                   2  3  2         2  2  2          6  4            5  4            4  4            3  3            2  4            2  3                 4       2  5  6      2  4  6      2  3  6
DU(2,1) := ( - 72.Iz .G .L .W - 144.Iz .G .L .W - 3.Iz.A.G .L .W - 24.Iz.A.G .L .W - 36.Iz.A.G .L .W + 18.Iz.A.G .L .P  - 48.Iz.A.G .L .W + 36.Iz.A.G .L .P  - 24.Iz.A.G.L .W - 2.A .G .L .W - 13.A .G .L .W - 5.A .G .L .W + 12.

          2  2  5       2   5            2   2          2            2  5  2          2  4  2            2  3  2              2     2           2 2        3  4  4        3  3  4        3  2  4
        A .G .L .P  + 6.A .G.L .P )/(144.E .Iz .A.G  + 288.E .Iz .A.G + 6.E .Iz.A .G .L  + 48.E .Iz.A .G .L  + 72.E .Iz.A .G .L  + 96.E .Iz.A .G.L  + 48.E .Iz.A .L  + 4.E .A .G .L  + 26.E .A .G .L  + 12.E .A .G .L )

                    2  4  3          2   2           7  5            6  5            4  4             3  5             4               4       2  6  7      2  5  7      2  4  6      2  3  6
DU(3,1) := ( - 144.Iz .G .L .W - 864.Iz .G.L .P  - 6.Iz.A.G .L .W - 36.Iz.A.G .L .W + 72.Iz.A.G .L .P  - 24.Iz.A.G .L .W - 288.Iz.A.G.L .P  - 144.Iz.A.L .P  - A .G .L .W - 6.A .G .L .W - 12.A .G .L .P  - 6.A .G .L .P )/(1728.

          3   2           3             2   5  2           2   4  2           2   3  2             2      2          2    2           2  4  4            2  3  4            2  2  4
        E .Iz .G  + 3456.E .Iz .G + 72.E .Iz .A.G .L  + 576.E .Iz .A.G .L  + 864.E .Iz .A.G .L  + 1152.E .Iz .A.G.L  + 576.E .Iz .A.L  + 48.E .Iz.A .G .L  + 312.E .Iz.A .G .L  + 144.E .Iz.A .G .L )

                   2  4  4         2  2  3          2   3           7  6            6  6            5  5             4  5            3  5              5           5       2  4  7      2  3  7
DU(4,1) := ( - 72.Iz .G .L .W + 576.Iz .G .L .P  + 720.Iz .G.L .P  - 3.Iz.A.G .L .W - 18.Iz.A.G .L .W + 24.Iz.A.G .L .P  + 120.Iz.A.G .L .P  + 72.Iz.A.G .L .P  + 192.Iz.A.G.L .P  + 96.Iz.A.L .P  + 8.A .G .L .P  + 16.A .G .L .

          2  2  7              3   2           3             2   5  2           2   4  2           2   3  2             2      2          2    2           2  4  4            2  3  4            2  2
        P  + 6.A .G .L .P )/(1728.E .Iz .G  + 3456.E .Iz .G + 72.E .Iz .A.G .L  + 576.E .Iz .A.G .L  + 864.E .Iz .A.G .L  + 1152.E .Iz .A.G.L  + 576.E .Iz .A.L  + 48.E .Iz.A .G .L  + 312.E .Iz.A .G .L  + 144.E .Iz.A .G .

         4
        L )

                   2  3  2         2  2  2          6  4            5  4            4  4            3  3            2  4            2  3                 4       2  5  6      2  4  6      2  3  6
DU(5,1) := ( - 72.Iz .G .L .W - 144.Iz .G .L .W - 3.Iz.A.G .L .W - 24.Iz.A.G .L .W - 36.Iz.A.G .L .W - 18.Iz.A.G .L .P  - 48.Iz.A.G .L .W - 36.Iz.A.G .L .P  - 24.Iz.A.G.L .W - 2.A .G .L .W - 13.A .G .L .W - 6.A .G .L .W - 12.
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```
     2  2  5       2    5                2    2              2                2  5  2             2  4  2            2  3  2           2    2            2 2         3  4  4        3  3  4         3  2  4
   A .G .L .P  - 6.A .G.L .P )/(144.E .Iz .A.G  + 288.E .Iz .A.G + .E .Iz.A .G .L  + 48.E .Iz.A .G .L  + 72.E .Iz.A .G .L  + 96.E .Iz.A .G.L  + 48.E .Iz.A .L  + 4.E .A .G .L  + 26.E .A .G .L  + 12.E .A .G .L )

             2  4  3          2  2  2            2    2            7  5             6  5             5  4               4  4              3  5                 4                 4       2  6  7        2  5  7
DU(6,1) := (144.Iz .G .L .W - 864.Iz .G .L .P  - 864.Iz .G.L .P  + 6.Iz.A.G .L .W + 36.Iz.A.G .L .W - 36.Iz.A.G .L .P  - 144.Iz.A.G .L .P  + 24.Iz.A.G .L .W - 288.Iz.A.G.L .P  - 144.Iz.A.L .P  + A .G .L .W + 6.A .G .L .W - 12

       2  4  6        2  3  6                3  2             3            2   5  2            2   4  2            2   3  2             2     2             2    2            2  4  4             2  3  4
    .A .G .L .P  - 6.A .G .L .P )/(1728.E .Iz .G  + 3456.E .Iz .G + 72.E .Iz .A.G .L  + 576.E .Iz .A.G .L  + 864.E .Iz .A.G .L  + 1152.E .Iz .A.G.L  + 576.E .Iz .A.L  + 48.E .Iz.A .G .L  + 312.E .Iz.A .G .L  + 144.E .

         2  2  4
      Iz.A .G .L )


LET G=1;

WRITE "AKA FOR UNIFORM ELEMENTS AND G=1";

AKA FOR UNIFORM ELEMENTS AND G=1

AKA:=AKA;

                                2   3
AKA(1,1) := (12.E .Iz + E .A.L )/L

AKA(1,2) := 0

                       2
AKA(1,3) := (6.E .Iz)/L

AKA(1,4) := ( - E .A)/L

AKA(1,5) := 0

AKA(1,6) := 0

AKA(2,1) := 0

                                2   3
AKA(2,2) := (12.E .Iz + E .A.L )/L

                       2
AKA(2,3) := (6.E .Iz)/L

AKA(2,4) := 0

                           3
AKA(2,5) := ( - 12.E .Iz)/L

                       2
AKA(2,6) := (6.E .Iz)/L

                       2
AKA(3,1) := (6.E .Iz)/L

                       2
AKA(3,2) := (6.E .Iz)/L

AKA(3,3) := (8.E .Iz)/L

AKA(3,4) := 0
```

```
AKA(3,5) := ( - 6.E.Iz)/L^2

AKA(3,6) := (2.E.Iz)/L

AKA(4,1) := ( - E.A)/L

AKA(4,2) := 0

AKA(4,3) := 0

AKA(4,4) := (12.E.Iz + E.A.L^2)/L^3

AKA(4,5) := 0

AKA(4,6) := (6.E.Iz)/L^2

AKA(5,1) := 0

AKA(5,2) := ( - 12.E.Iz)/L^3

AKA(5,3) := ( - 6.E.Iz)/L^2

AKA(5,4) := 0

AKA(5,5) := (12.E.Iz + E.A.L^2)/L^3

AKA(5,6) := ( - 6.E.Iz)/L^2

AKA(6,1) := 0

AKA(6,2) := (6.E.Iz)/L^2

AKA(6,3) := (2.E.Iz)/L

AKA(6,4) := (6.E.Iz)/L^2

AKA(6,5) := ( - 6.E.Iz)/L^2

AKA(6,6) := (8.E.Iz)/L

WRITE "FLEX FOR UNIFORM ELEMENTS AND G=1";

FLEX FOR UNIFORM ELEMENTS AND G=1

FLEX:=FLEX;

FLEX(1,1) := (216.Iz^2.L^3 + 84.Iz.A.L^5 + 5.A^2.L^7)/(864.E.Iz^3 + 540.E.Iz^2.A.L^2 + 84.E.Iz.A^2.L^4)

FLEX(1,2) := (9.Iz.L^3 + 3.A.L^5)/(72.E.Iz^2 + 45.E.Iz.A.L^2 + 7.E.A^2.L^4)
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$FLEX(1,3) := (-96 \cdot Iz^2 \cdot L^2 - 34 \cdot Iz \cdot A \cdot L^4 - A^2 \cdot L^6)/(288 \cdot E \cdot Iz^3 + 180 \cdot E \cdot Iz^2 \cdot A \cdot L^2 + 28 \cdot E \cdot Iz \cdot A^2 \cdot L^4)$

$FLEX(1,4) := (72 \cdot Iz^2 \cdot L^3 + 42 \cdot Iz \cdot A \cdot L^5 + 5 \cdot A^2 \cdot L^7)/(864 \cdot E \cdot Iz^3 + 540 \cdot E \cdot Iz^2 \cdot A \cdot L^2 + 84 \cdot E \cdot Iz \cdot A^2 \cdot L^4)$

$FLEX(1,5) := (-9 \cdot Iz \cdot L^3 - 3 \cdot A \cdot L^5)/(72 \cdot E \cdot Iz^2 + 45 \cdot E \cdot Iz \cdot A \cdot L^2 + 7 \cdot E \cdot A^2 \cdot L^4)$

$FLEX(1,6) := (-48 \cdot Iz^2 \cdot L^2 - 20 \cdot Iz \cdot A \cdot L^4 - A^2 \cdot L^6)/(288 \cdot E \cdot Iz^3 + 180 \cdot E \cdot Iz^2 \cdot A \cdot L^2 + 28 \cdot E \cdot Iz \cdot A^2 \cdot L^4)$

$FLEX(2,1) := (9 \cdot Iz \cdot L^3 + 3 \cdot A \cdot L^5)/(72 \cdot E \cdot Iz^2 + 45 \cdot E \cdot Iz \cdot A \cdot L^2 + 7 \cdot E \cdot A^2 \cdot L^4)$

$FLEX(2,2) := (36 \cdot Iz^2 \cdot L + 33 \cdot Iz \cdot A \cdot L^3 + 7 \cdot A^2 \cdot L^5)/(72 \cdot E \cdot Iz^2 \cdot A + 45 \cdot E \cdot Iz \cdot A^2 \cdot L^2 + 7 \cdot E \cdot A^3 \cdot L^4)$

$FLEX(2,3) := (-18 \cdot Iz \cdot L^2 - 6 \cdot A \cdot L^4)/(72 \cdot E \cdot Iz^2 + 45 \cdot E \cdot Iz \cdot A \cdot L^2 + 7 \cdot E \cdot A^2 \cdot L^4)$

$FLEX(2,4) := (9 \cdot Iz \cdot L^3 + 3 \cdot A \cdot L^5)/(72 \cdot E \cdot Iz^2 + 45 \cdot E \cdot Iz \cdot A \cdot L^2 + 7 \cdot E \cdot A^2 \cdot L^4)$

$FLEX(2,5) := (36 \cdot Iz^2 \cdot L + 12 \cdot Iz \cdot A \cdot L^3)/(72 \cdot E \cdot Iz^2 \cdot A + 45 \cdot E \cdot Iz \cdot A^2 \cdot L^2 + 7 \cdot E \cdot A^3 \cdot L^4)$

$FLEX(2,6) := (-18 \cdot Iz \cdot L^2 - 6 \cdot A \cdot L^4)/(72 \cdot E \cdot Iz^2 + 45 \cdot E \cdot Iz \cdot A \cdot L^2 + 7 \cdot E \cdot A^2 \cdot L^4)$

$FLEX(3,1) := (-96 \cdot Iz^2 \cdot L^2 - 34 \cdot Iz \cdot A \cdot L^4 - A^2 \cdot L^6)/(288 \cdot E \cdot Iz^3 + 180 \cdot E \cdot Iz^2 \cdot A \cdot L^2 + 28 \cdot E \cdot Iz \cdot A^2 \cdot L^4)$

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$FLEX(3,2) := (-18 \cdot Iz \cdot L^2 - 6 \cdot A \cdot L^4)/(72 \cdot E \cdot Iz^2 + 45 \cdot E \cdot Iz \cdot A \cdot L^2 + 7 \cdot E \cdot A^2 \cdot L^4)$

$FLEX(3,3) := (576 \cdot Iz^2 \cdot L + 228 \cdot Iz \cdot A \cdot L^3 + 13 \cdot A^2 \cdot L^5)/(864 \cdot E \cdot Iz^3 + 540 \cdot E \cdot Iz^2 \cdot A \cdot L^2 + 84 \cdot E \cdot Iz \cdot A^2 \cdot L^4)$

$FLEX(3,4) := (-48 \cdot Iz^2 \cdot L^2 - 20 \cdot Iz \cdot A \cdot L^4 - A^2 \cdot L^6)/(288 \cdot E \cdot Iz^3 + 180 \cdot E \cdot Iz^2 \cdot A \cdot L^2 + 28 \cdot E \cdot Iz \cdot A^2 \cdot L^4)$

$FLEX(3,5) := (18 \cdot Iz \cdot L^2 + 6 \cdot A \cdot L^4)/(72 \cdot E \cdot Iz^2 + 45 \cdot E \cdot Iz \cdot A \cdot L^2 + 7 \cdot E \cdot A^2 \cdot L^4)$

$FLEX(3,6) := (288 \cdot Iz^2 \cdot L + 96 \cdot Iz \cdot A \cdot L^3 - A^2 \cdot L^5)/(864 \cdot E \cdot Iz^3 + 540 \cdot E \cdot Iz^2 \cdot A \cdot L^2 + 84 \cdot E \cdot Iz \cdot A^2 \cdot L^4)$

$FLEX(4,1) := (72 \cdot Iz^2 \cdot L^3 + 42 \cdot Iz \cdot A \cdot L^5 + 5 \cdot A^2 \cdot L^7)/(864 \cdot E \cdot Iz^3 + 540 \cdot E \cdot Iz^2 \cdot A \cdot L^2 + 84 \cdot E \cdot Iz \cdot A^2 \cdot L^4)$

$FLEX(4,2) := (9 \cdot Iz \cdot L^3 + 3 \cdot A \cdot L^5)/(72 \cdot E \cdot Iz^2 + 45 \cdot E \cdot Iz \cdot A \cdot L^2 + 7 \cdot E \cdot A^2 \cdot L^4)$

$FLEX(4,3) := (-48 \cdot Iz^2 \cdot L^2 - 20 \cdot Iz \cdot A \cdot L^4 - A^2 \cdot L^6)/(288 \cdot E \cdot Iz^3 + 180 \cdot E \cdot Iz^2 \cdot A \cdot L^2 + 28 \cdot E \cdot Iz \cdot A^2 \cdot L^4)$

$FLEX(4,4) := (216 \cdot Iz^2 \cdot L^3 + 84 \cdot Iz \cdot A \cdot L^5 + 5 \cdot A^2 \cdot L^7)/(864 \cdot E \cdot Iz^3 + 540 \cdot E \cdot Iz^2 \cdot A \cdot L^2 + 84 \cdot E \cdot Iz \cdot A^2 \cdot L^4)$

$FLEX(4,5) := (-9 \cdot Iz \cdot L^3 - 3 \cdot A \cdot L^5)/(72 \cdot E \cdot Iz^2 + 45 \cdot E \cdot Iz \cdot A \cdot L^2 + 7 \cdot E \cdot A^2 \cdot L^4)$

$$FLEX(4,6) := (-96\, Iz^{2} L^{2} - 34\, Iz\, A\, L^{4} - A^{2} L^{6})/(288\, E\, Iz^{3} + 180\, E\, Iz^{2} A\, L^{2} + 28\, E\, Iz\, A^{2} L^{4})$$

$$FLEX(5,1) := (-9\, Iz\, L^{3} - 3\, A\, L^{5})/(72\, E\, Iz^{2} + 45\, E\, Iz\, A\, L^{2} + 7\, E\, A^{2} L^{4})$$

$$FLEX(5,2) := (36\, Iz^{2} L + 12\, Iz\, A\, L^{3})/(72\, E\, Iz^{2} A + 45\, E\, Iz\, A^{2} L^{2} + 7\, E\, A^{3} L^{4})$$

$$FLEX(5,3) := (18\, Iz\, L^{2} + 6\, A\, L^{4})/(72\, E\, Iz^{2} + 45\, E\, Iz\, A\, L^{2} + 7\, E\, A^{2} L^{4})$$

$$FLEX(5,4) := (-9\, Iz\, L^{3} - 3\, A\, L^{5})/(72\, E\, Iz^{2} + 45\, E\, Iz\, A\, L^{2} + 7\, E\, A^{2} L^{4})$$

$$FLEX(5,5) := (36\, Iz^{2} L + 33\, Iz\, A\, L^{3} + 7\, A^{2} L^{5})/(72\, E\, Iz^{2} A + 45\, E\, Iz\, A^{2} L^{2} + 7\, E\, A^{3} L^{4})$$

$$FLEX(5,6) := (18\, Iz\, L^{2} + 6\, A\, L^{4})/(72\, E\, Iz^{2} + 45\, E\, Iz\, A\, L^{2} + 7\, E\, A^{2} L^{4})$$

$$FLEX(6,1) := (-48\, Iz^{2} L^{2} - 20\, Iz\, A\, L^{4} - A^{2} L^{6})/(288\, E\, Iz^{3} + 180\, E\, Iz^{2} A\, L^{2} + 28\, E\, Iz\, A^{2} L^{4})$$

$$FLEX(6,2) := (-18\, Iz\, L^{2} - 6\, A\, L^{4})/(72\, E\, Iz^{2} + 45\, E\, Iz\, A\, L^{2} + 7\, E\, A^{2} L^{4})$$

$$FLEX(6,3) := (288\, Iz^{2} L + 96\, Iz\, A\, L^{3} - A^{2} L^{5})/(864\, E\, Iz^{3} + 540\, E\, Iz^{2} A\, L^{2} + 84\, E\, Iz\, A^{2} L^{4})$$

$$FLEX(6,4) := (-96\, Iz^{2} L^{2} - 34\, Iz\, A\, L^{4} - A^{2} L^{6})/(288\, E\, Iz^{3} + 180\, E\, Iz^{2} A\, L^{2} + 28\, E\, Iz\, A^{2} L^{4})$$

$$FLEX(6,5) := (18\, Iz\, L^{2} + 6\, A\, L^{4})/(72\, E\, Iz^{2} + 45\, E\, Iz\, A\, L^{2} + 7\, E\, A^{2} L^{4})$$

$$FLEX(6,6) := (576\, Iz^{2} L + 228\, Iz\, A\, L^{3} + 13\, A^{2} L^{5})/(864\, E\, Iz^{3} + 540\, E\, Iz^{2} A\, L^{2} + 84\, E\, Iz\, A^{2} L^{4})$$

WRITE "DU FOR UNIFORM ELEMENTS AND G=1";

DU FOR UNIFORM ELEMENTS AND G=1

DU:=DU;

$$DU(1,1) := (24\, Iz^{2} L^{4} W + 144\, Iz^{2} L^{3} P + 7\, Iz\, A\, L^{6} W + 84\, Iz\, A\, L^{5} P + 10\, A^{2} L^{7} P)/(1728\, E\, Iz^{3} + 1080\, E\, Iz^{2} A\, L^{2} + 168\, E\, Iz\, A^{2} L^{4})$$

$$DU(2,1) := (-72\, Iz^{2} L^{2} W - 45\, Iz\, A\, L^{4} W + 18\, Iz\, A\, L^{3} P - 7\, A^{2} L^{6} W + 6\, A^{2} L^{5} P)/(144\, E\, Iz^{2} A + 90\, E\, Iz\, A^{2} L^{2} + 14\, E\, A^{3} L^{4})$$

$$DU(3,1) := (-144\, Iz^{2} L^{3} W - 864\, Iz^{2} L^{2} P - 66\, Iz\, A\, L^{5} W - 360\, Iz\, A\, L^{4} P - 7\, A^{2} L^{7} W - 18\, A^{2} L^{6} P)/(5184\, E\, Iz^{3} + 3240\, E\, Iz^{2} A\, L^{2} + 504\, E\, Iz\, A^{2} L^{4})$$

$$DU(4,1) := (-24\, Iz^{2} L^{4} W + 432\, Iz^{2} L^{3} P - 7\, Iz\, A\, L^{6} W + 168\, Iz\, A\, L^{5} P + 10\, A^{2} L^{7} P)/(1728\, E\, Iz^{3} + 1080\, E\, Iz^{2} A\, L^{2} + 168\, E\, Iz\, A^{2} L^{4})$$

$$DU(5,1) := (-72\, Iz^{2} L^{2} W - 45\, Iz\, A\, L^{4} W - 18\, Iz\, A\, L^{3} P - 7\, A^{2} L^{6} W - 6\, A^{2} L^{5} P)/(144\, E\, Iz^{2} A + 90\, E\, Iz\, A^{2} L^{2} + 14\, E\, A^{3} L^{4})$$

$$DU(6,1) := (144\, Iz^{2} L^{3} W - 1728\, Iz^{2} L^{2} P + 66\, Iz\, A\, L^{5} W - 612\, Iz\, A\, L^{4} P + 7\, A^{2} L^{7} W - 18\, A^{2} L^{6} P)/(5184\, E\, Iz^{3} + 3240\, E\, Iz^{2} A\, L^{2} + 504\, E\, Iz\, A^{2} L^{4})$$

```
LET A=1/AA;

AKA:=AKA8

FLEX:=FLEX8

DU:=DU8

LET AA=0;
   /-- REMOVE AXIAL TERMS

/ WRITE "AKA FOR UNIFORM FLEXURAL ELEMENTS AND G=1";  /<--- RESULTS IN ZERO DENOMINATOR
/ AKA:=AKA;
WRITE "FLEX FOR UNIFORM FLEXURAL ELEMENTS AND G=1";

FLEX FOR UNIFORM FLEXURAL ELEMENTS AND G=1

FLEX:=FLEX;

                   3
FLEX(1,1) := (5.L  )/(84.E .Iz)

FLEX(1,2) := 0

                    2
FLEX(1,3) := ( - L  )/(28.E .Iz)

                   3
FLEX(1,4) := (5.L  )/(84.E .Iz)

FLEX(1,5) := 0

                    2
FLEX(1,6) := ( - L  )/(28.E .Iz)

FLEX(2,1) := 0

FLEX(2,2) := 0

FLEX(2,3) := 0

FLEX(2,4) := 0

FLEX(2,5) := 0

FLEX(2,6) := 0

                    2
FLEX(3,1) := ( - L  )/(28.E .Iz)

FLEX(3,2) := 0

FLEX(3,3) := (13.L)/(84.E .Iz)

                    2
FLEX(3,4) := ( - L  )/(28.E .Iz)

FLEX(3,5) := 0

FLEX(3,6) := ( - L)/(84.E .Iz)
```

```
                    3
FLEX(4,1) := (5.L )/(84.E .Iz)

FLEX(4,2) := 0

                     2
FLEX(4,3) := ( - L )/(28.E .Iz)

                    3
FLEX(4,4) := (5.L )/(84.E .Iz)

FLEX(4,5) := 0

                     2
FLEX(4,6) := ( - L )/(28.E .Iz)

FLEX(5,1) := 0

FLEX(5,2) := 0

FLEX(5,3) := 0

FLEX(5,4) := 0

FLEX(5,5) := 0

FLEX(5,6) := 0

                     2
FLEX(6,1) := ( - L )/(28.E  Iz)

FLEX(6,2) := 0

FLEX(6,3) := ( - L)/(84.E .Iz)

                     2
FLEX(6,4) := ( - L )/(28.E .Iz)

FLEX(6,5) := 0

FLEX(6,6) := (13.L)/(84.E .Iz)

WRITE "DU FOR UNIFORM FLEXURAL ELEMENTS AND G=1";

DU FOR UNIFORM FLEXURAL ELEMENTS AND G=1

DU:=DU;

                 3
DU(1,1) := (5.L .P )/(84.E .Iz)

DU(2,1) := 0

                     3          2
DU(3,1) := ( - 2.L .W - 18.L .P )/(504.E .Iz)

                 3
DU(4,1) := (5.L .P )/(84.E .Iz)

DU(5,1) := 0
```

```
                     3         2
DU(6,1) := (7.L .W - 18.L .P )/(504.E .Iz)


SHUT SIMAKA;
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213(a)

D.3 Dimensionally Reduced System Matrices

NOTE: Lateral coefficients are for 1/2 Loads

```
/ **** AKA AND FLEX MATRICES *****$

/ STRUCTURAL SPECIFICATIONS:$

/ ELEMENT TYPE IS ELASTIC FLEXURAL/AXIAL ELEMENT$

/ SYSTEM TYPE IS RIGID SUPPORTS, FULL 3 DOF PER NODE$

/ NUMBER OF BAYS  = 1$

/ NUMBER OF LEVELS= 1$

/ NUMBER OF NODES = 2$

/ GLOBAL D.O.F.   = 6$

0$

/ **** DIMENSIONAL CHECK *****$

/ **** F=) FORCE, L=) LENGTH ****$

AKA(1,1) := (13.F)/L$
AKA(1,2) := 0$
AKA(1,3) := 6.F$
AKA(1,4) := ( - F)/L$
AKA(1,5) := 0$
AKA(1,6) := 0$
AKA(2,1) := 0$
AKA(2,2) := (13.F)/L$
AKA(2,3) := 6.F$
AKA(2,4) := 0$
AKA(2,5) := ( - 12.F)/L$
AKA(2,6) := 6.F$
AKA(3,1) := 6.F$
AKA(3,2) := 6.F$
AKA(3,3) := 8.L.F$
AKA(3,4) := 0$
AKA(3,5) :=  - 6.F$
AKA(3,6) := 2.L.F$
AKA(4,1) := ( - F)/L$
AKA(4,2) := 0$
AKA(4,3) := 0$
AKA(4,4) := (13.F)/L$
AKA(4,5) := 0$
AKA(4,6) := 6.F$
AKA(5,1) := 0$
AKA(5,2) := ( - 12.F)/L$
AKA(5,3) :=  - 6.F$
AKA(5,4) := 0$
AKA(5,5) := (13.F)/L$
AKA(5,6) :=  - 6.F$
AKA(6,1) := 0$
AKA(6,2) := 6.F$
AKA(6,3) := 2.L.F$
AKA(6,4) := 6.F$
AKA(6,5) :=  - 6.F$
AKA(6,6) := 8.L.F$

FLEX(1,1) := (305.L)/(1488.F)$
FLEX(1,2) := (3.L)/(31.F)$
FLEX(1,3) := ( - 131)/(496.F)$
FLEX(1,4) := (119.L)/(1488.F)$
```

```
FLEX(1,5) := ( - 3.L)/(31.F)$
FLEX(1,6) := ( - 69)/(496.F)$
FLEX(2,1) := (3.L)/(31.F)$
FLEX(2,2) := (19.L)/(31.F)$
FLEX(2,3) := ( - 6)/(31.F)$
FLEX(2,4) := (3.L)/(31.F)$
FLEX(2,5) := (12.L)/(31.F)$
FLEX(2,6) := ( - 6)/(31.F)$
FLEX(3,1) := ( - 131)/(496.F)$
FLEX(3,2) := ( - 6)/(31.F)$
FLEX(3,3) := 817/(1488.L.F)$
FLEX(3,4) := ( - 69)/(496.F)$
FLEX(3,5) := 6/(31.F)$
FLEX(3,6) := 383/(1488.L.F)$
FLEX(4,1) := (119.L)/(1488.F)$
FLEX(4,2) := (3.L)/(31.F)$
FLEX(4,3) := ( - 69)/(496.F)$
FLEX(4,4) := (305.L)/(1488.F)$
FLEX(4,5) := ( - 3.L)/(31.F)$
FLEX(4,6) := ( - 131)/(496.F)$
FLEX(5,1) := ( - 3.L)/(31.F)$
FLEX(5,2) := (12.L)/(31.F)$
FLEX(5,3) := 6/(31.F)$
FLEX(5,4) := ( - 3.L)/(31.F)$
FLEX(5,5) := (19.L)/(31.F)$
FLEX(5,6) := 6/(31.F)$
FLEX(6,1) := ( - 69)/(496.F)$
FLEX(6,2) := ( - 6)/(31.F)$
FLEX(6,3) := 383/(1488.L.F)$
FLEX(6,4) := ( - 131)/(496.F)$
FLEX(6,5) := 6/(31.F)$
FLEX(6,6) := 817/(1488.L.F)$

FG(1,1) := 0$
FG(2,1) :=  - 5.F$
FG(3,1) := ( - 5.L.F)/6$
FG(4,1) := F$
FG(5,1) :=  - 5.F$
FG(6,1) := (5.L.F)/6$

DU(1,1) := 0$
DU(2,1) := 0$
DU(3,1) := 0$
DU(4,1) := 0$
DU(5,1) := 0$
DU(6,1) := 0$
```

D.4 General Extremal Stress Resultant Solutions

NOTE: Lateral coefficients are for 1/2 Loads

```
SETUP( );

/ STRUCTURAL SPECIFICATIONS:

/ ELEMENT TYPE IS ELASTIC FLEXURAL/AXIAL ELEMENT

/ SYSTEM TYPE IS RIGID SUPPORTS, FULL 3 DOF PER NODE

/ NUMBER OF BAYS   = 1

/ NUMBER OF LEVELS= 1

/ NUMBER OF NODES = 2

/ GLOBAL D.O.F.    = 6

0
        / SETUP ELEMENT K, AND GLOBAL K.
 STRESSES( );

BASIC ELEMENT STRESSES

COLUMN MEMBER FROM NODE (-1) TO 1

PA := (12.Ab.Ib^2.L^5.Il.Al.Ar.G^3.W + 12.Ab.Ib^2.L^5.Al.Ir.Ar.G^3.W - 6.Ab.Ib^2.L^4.Il.Al.Ar.P.G - 6.Ab.Ib^2.L^4.Al.Ir.Ar.P.G + 24.Ab.Ib^2.L^3.Il^2.Al.G.W + 48.Ab.Ib^2.L^3.Il.Al.Ir.G.W + 24.Ab.Ib^2.L^3.Al.Ir^2.G.W + 5.Ab.Ib.L^5.Il^2.Al.Ar.G^4.
    W + 44.Ab.Ib.L^5.Il.Al.Ir.Ar.G^4.W + 3.Ab.Ib.L^5.Al.Ir^2.Ar.G^4.W - 24.Ab.Ib.L^4.Il.Al.Ir.Ar.P^2.G + 96.Ab.Ib.L^3.Il^2.Al.Ir.G^2.W + 96.Ab.Ib.L^3.Il.Al.Ir^2.G^2.W + 4.Ab.L^5.Il^2.Al.Ir.Ar.G^5.W + 4.Ab.L^5.Il.Al.Ir^2.Ar.G^5.W + 144.Ib^2.
    L^3.Il.Al.Ir.Ar.G^4.W - 72.Ib^2.L^2.Il.Al.Ir.Ar.P^2.G + 288.Ib^2.L.Il^2.Al.Ir.G^2.W + 288.Ib^2.L.Il.Al.Ir^2.G^2.W + 60.Ib.L^3.Il^2.Al.Ir.Ar.G^5.W + 36.Ib.L^3.Il.Al.Ir^2.Ar.G^5.W - 36.Ib.L^2.Il^2.Al.Ir.Ar.P^3.G + 288.Ib.L.Il^2.Al.Ir^2.G^3.W
     + 12.L^3.Il^2.Al.Ir^2.Ar.G^6.W)/(24.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 24.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 24.Ab.Ib^2.L^2.Il^2.Al^2 + 24.Ab.Ib^2.L^2.Il^2.Ar + 48.Ab.Ib^2.L^2.Il.Al.Ir + 48.Ab.Ib^2.L^2.Il.Ir.Ar + 24.Ab.Ib^2.L^2.Al.Ir^2 + 24.Ab.Ib^2.L^2.Ir^2.Ar
      + 8.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 88.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 8.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 + 96.Ab.Ib.L^2.Il^2.Al.Ir.G^2 + 96.Ab.Ib.L^2.Il^2.Ir.Ar.G + 96.Ab.Ib.L^2.Il.Al.Ir^2.G + 96.Ab.Ib.L^2.Il.Ir^2.Ar.G + 8.Ab.L^4.Il^2.Al.Ir.Ar.G^4 + 8.
     Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 288.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 288.Ib^2.Il^2.Al.Ir.G + 288.Ib^2.Il^2.Ir.Ar.G + 288.Ib^2.Il.Al.Ir^2.G + 288.Ib^2.Il.Ir^2.Ar.G + 96.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 96.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 288.Ib.Il^2.Al.Ir^2.G^2
      + 288.Ib.Il^2.Ir^2.Ar.G^2 + 24.L^2.Il^2.Al.Ir^2.Ar.G^5)

UA := (12.Ab.Ib^2.L^4.Il.Al.Ar.P^2.G + 12.Ab.Ib^2.L^2.Il^2.Al.P + 12.Ab.Ib^2.L^2.Il^2.Ar.P + 12.Ab.Ib^2.L^2.Il.Al.Ir.P + 12.Ab.Ib^2.L^2.Il.Ir.Ar.P - 12.Ab.Ib.L^5.Il.Al.Ir.Ar.G^5.W + 4.Ab.Ib.L^4.Il^2.Al.Ar.P^3.G + 22.Ab.Ib.L^4.Il.Al.Ir.Ar.
    P^3.G + 24.Ab.Ib.L^3.Il^2.Al.Ir.G^3.W - 48.Ab.Ib.L^3.Il^2.Ir.Ar.G^3.W - 48.Ab.Ib.L^3.Il.Al.Ir^2.G^3.W + 24.Ab.Ib.L^3.Il.Ir^2.Ar.G^3.W + 48.Ab.Ib.L^2.Il^2.Al.Ir.P.G + 48.Ab.Ib.L^2.Il^2.Ir.Ar.P.G - Ab.L^5.Il^2.Al.Ir.Ar.G^6.W - Ab.L^5.Il.
    Al.Ir^2.Ar.G^6.W + 4.Ab.L^4.Il^2.Al.Ir.Ar.P^4.G)/(24.Ab.Ib^2.L^4.Il.Al.Ar.G^2 + 24.Ab.Ib^2.L^4.Al.Ir.Ar.G^2 + 24.Ab.Ib^2.L^2.Il^2.Al + 24.Ab.Ib^2.L^2.Il^2.Ar + 48.Ab.Ib^2.L^2.Il.Al.Ir + 48.Ab.Ib^2.L^2.Il.Ir.Ar + 24.Ab.Ib^2.L^2.Al.Ir^2 + 24.
    Ab.Ib^2.L^2.Ir^2.Ar + 8.Ab.Ib.L^4.Il^2.Al.Ar.G^3 + 88.Ab.Ib.L^4.Il.Al.Ir.Ar.G^3 + 8.Ab.Ib.L^4.Al.Ir^2.Ar.G^3 + 96.Ab.Ib.L^2.Il^2.Al.Ir.G + 96.Ab.Ib.L^2.Il^2.Ir.Ar.G + 96.Ab.Ib.L^2.Il.Al.Ir^2.G + 96.Ab.Ib.L^2.Il.Ir^2.Ar.G + 8.Ab.L^4.Il^2.
    Al.Ir.Ar.G^4 + 8.Ab.L^4.Il.Al.Ir^2.Ar.G^4 + 288.Ib^2.L^2.Il.Al.Ir.Ar.G^3 + 288.Ib^2.Il^2.Al.Ir.G + 288.Ib^2.Il^2.Ir.Ar.G + 288.Ib^2.Il.Al.Ir^2.G + 288.Ib^2.Il.Ir^2.Ar.G + 96.Ib.L^2.Il^2.Al.Ir.Ar.G^4 + 96.Ib.L^2.Il.Al.Ir^2.Ar.G^4 + 288.Ib
    .Il^2.Al.Ir^2.G^2 + 288.Ib.Il^2.Ir^2.Ar.G^2 + 24.L^2.Il^2.Al.Ir^2.Ar.G^5)
```

```
          2 5           2        2 4  2   2          2 4  2   2          2 4        2          2 4        2          2 3  2          2 3  2          2 3
MA := (18.Ab.Ib .L .Il.Al.Ar.P .G  + 36.Ab.Ib .L .Il .Al.G .W - 36.Ab.Ib .L .Il .Ar.G .W + 36.Ab.Ib .L .Il.Al.Ir.G .W - 36.Ab.Ib .L .Il.Ir.Ar.G .W + 36.Ab.Ib .L .Il .Al.P  + 36.Ab.Ib .L .Il .Ar.P  + 36.Ab.Ib .L .Il.Al.Ir.P

          2 3                 6  2      5          6           5          5  2      3          5            3           4  2      3           4  2      3
   + 36.Ab.Ib .L .Il.Ir.Ar.P  + 3.Ab.Ib.L .Il .Al.Ar.G .W - 15.Ab.Ib.L .Il.Al.Ir.Ar.G .W + 12.Ab.Ib.L .Il .Al.Ar.P .G  + 30.Ab.Ib.L .Il.Al.Ir.Ar.P .G  + 168.Ab.Ib.L .Il .Al.Ir.G .W - 192.Ab.Ib.L .Il .Ir.Ar.G .W - 48.Ab.

    4       2 3          4     2    3            3  2             3  2             6  2           6          6     2   6           5  2          4      2 2 2      3
   Ib.L .Il.Al.Ir .G .W + 24.Ab.Ib.L .Il.Ir .Ar.G .W + 144.Ab.Ib.L .Il .Al.Ir.P .G + 144.Ab.Ib.L .Il .Ir.Ar.P .G - Ab.L .Il .Al.Ir.Ar.G .W - Ab.L .Il.Al.Ir .Ar.G .W + 12.Ab.L .Il .Al.Ir.Ar.P .G  + 432.Ib .L .Il .Al.Ir.G .

        2  2  2       3         2    2              2    2             4  2          6         3  2            4         2   2    2 4           2  2  2     4      4  2     2
   W - 432.Ib .L .Il .Ir.Ar.G .W + 216.Ib .L.Il .Al.Ir.P .G + 216.Ib .L.Il .Ir.Ar.P .G + 36.Ib.L .Il .Al.Ir.Ar.G .W - 36.Ib.L .Il .Al.Ir.Ar.P .G  + 288.Ib.L .Il .Al.Ir .G .W - 144.Ib.L .Il .Ir .Ar.G .W + 6.L .Il .Al.Ir .

    7        2 4          2         2 4       2          2  2  2        2  2  2         2  2               2  2              2  2     2          2  2  2         4  2
   Ar.G .W)/(72.Ab.Ib .L .Il.Al.Ar.G  + 72.Ab.Ib .L .Al.Ir.Ar.G  + 72.Ab.Ib .L .Il .Al + 72.Ab.Ib .L .Il .Ar + 144.Ab.Ib .L .Il.Al.Ir + 144.Ab.Ib .L .Il.Ir.Ar + 72.Ab.Ib .L .Al.Ir  + 72.Ab.Ib .L .Ir .Ar + 24.Ab.Ib.L .Il .

          3           4          3           4   2     3             2  2               2  2              2       2              2     2            4  2            4
         Al.Ar.G  + 264.Ab.Ib.L .Il.Al.Ir.Ar.G  + 24.Ab.Ib.L .Al.Ir .Ar.G  + 288.Ab.Ib.L .Il .Al.Ir.G + 288.Ab.Ib.L .Il .Ir.Ar.G + 288.Ab.Ib.L .Il.Al.Ir .G + 288.Ab.Ib.L .Il.Ir .Ar.G + 24.Ab.L .Il .Al.Ir.Ar.G  + 24.Ab

          4       2    4       2 2         3       2  2          2  2          2        2         2      2         2  2        4        2       2    4          2
         .L .Il.Al.Ir .Ar.G  + 864.Ib .L .Il.Al.Ir.Ar.G  + 864.Ib .Il .Al.Ir.G + 864.Ib .Il .Ir.Ar.G + 864.Ib .Il.Al.Ir .G + 864.Ib .Il.Ir .Ar.G + 288.Ib.L .Il .Al.Ir.Ar.G  + 288.Ib.L .Il.Al.Ir .Ar.G  + 864.Ib.Il .Al.

          2  2          2  2    2        2  2    2    5
         Ir .G  + 864.Ib.Il .Ir .Ar.G  + 72.L .Il .Al.Ir .Ar.G )

          2 5           2        2 4  2   2          2 4  2   2          2 4        2          2 4        2         6  2      5          6           5          5
MB := (18.Ab.Ib .L .Il.Al.Ar.P .G  - 36.Ab.Ib .L .Il .Al.G .W + 36.Ab.Ib .L .Il .Ar.G .W - 36.Ab.Ib .L .Il.Al.Ir.G .W + 36.Ab.Ib .L .Il.Ir.Ar.G .W - 3.Ab.Ib.L .Il .Al.Ar.G .W - 21.Ab.Ib.L .Il.Al.Ir.Ar.G .W + 36.Ab.Ib.L .Il.Al

      3              4  2      3           4  2      3            4       2 3         4     2    3            6  2          6          6     2   6           2  2  2      3
   .Ir.Ar.P .G  - 96.Ab.Ib.L .Il .Al.Ir.G .W + 48.Ab.Ib.L .Il .Ir.Ar.G .W - 96.Ab.Ib.L .Il.Al.Ir .G .W + 48.Ab.Ib.L .Il.Ir .Ar.G .W - 2.Ab.L .Il .Al.Ir.Ar.G .W - 2.Ab.L .Il.Al.Ir .Ar.G .W - 432.Ib .L .Il .Al.Ir.G .W + 432

     2  2  2      3         2    2              2    2             4  2          6         3  2             4         2   2    2 4           2  2  2     4      4  2     2   7
   .Ib .L .Il .Ir.Ar.G .W - 216.Ib .L.Il .Al.Ir.P .G - 216.Ib .L.Il .Ir.Ar.P .G - 36.Ib.L .Il .Al.Ir.Ar.G .W + 36.Ib.L .Il .Al.Ir.Ar.P .G  - 288.Ib.L .Il .Al.Ir .G .W + 144.Ib.L .Il .Ir .Ar.G .W - 6.L .Il .Al.Ir .Ar.G .W)

        2 4          2         2 4       2          2  2  2        2  2  2         2  2               2  2              2  2     2          2  2  2         4  2     3
   /(72.Ab.Ib .L .Il.Al.Ar.G  + 72.Ab.Ib .L .Al.Ir.Ar.G  + 72.Ab.Ib .L .Il .Al + 72.Ab.Ib .L .Il .Ar + 144.Ab.Ib .L .Il.Al.Ir + 144.Ab.Ib .L .Il.Ir.Ar + 72.Ab.Ib .L .Al.Ir  + 72.Ab.Ib .L .Ir .Ar + 24.Ab.Ib.L .Il .Al.Ar.G

          4           3           4   2     3             2  2               2  2              2       2              2     2            4  2            4        4      2
   + 264.Ab.Ib.L .Il.Al.Ir.Ar.G  + 24.Ab.Ib.L .Al.Ir .Ar.G  + 288.Ab.Ib.L .Il .Al.Ir.G + 288.Ab.Ib.L .Il .Ir.Ar.G + 288.Ab.Ib.L .Il.Al.Ir .G + 288.Ab.Ib.L .Il.Ir .Ar.G + 24.Ab.L .Il .Al.Ir.Ar.G  + 24.Ab.L .Il.Al.Ir .Ar.

   4       2 2         3       2  2          2  2          2        2         2      2         2  2        4        2       2    4          2     2 2           2  2
   G  + 864.Ib .L .Il.Al.Ir.Ar.G  + 864.Ib .Il .Al.Ir.G + 864.Ib .Il .Ir.Ar.G + 864.Ib .Il.Al.Ir .G + 864.Ib .Il.Ir .Ar.G + 288.Ib.L .Il .Al.Ir.Ar.G  + 288.Ib.L .Il.Al.Ir .Ar.G  + 864.Ib.Il .Al.Ir .G  + 864.Ib.Il .Ir .Ar

    2       2  2    2    5
   .G  + 72.L .Il .Al.Ir .Ar.G )

COLUMN MEMBER FROM NODE 0 TO 2

          2 5          3            2 5          3           2 4                2 4                 2 3  2          2 3               2 3  2          5  2      4
PA := (12.Ab.Ib .L .Il Al.Ar.G .W + 12.Ab.Ib .L .Al.Ir.Ar.G .W + 6.Ab.Ib .L .Il.Al.Ar.P .G + 6.Ab.Ib .L .Al.Ir.Ar.P .G + 24.Ab.Ib .L .Il .Ar.G.W + 48.Ab.Ib .L .Il.Ir.Ar.G.W + 24.Ab.Ib .L .Ir .Ar.G.W + 3.Ab.Ib.L .Il .Al.Ar.G .

           5           4          5    2    4          4             2        3  2       2            3    2    2          5  2          5          5      2    5        2
   W + 44.Ab.Ib.L .Il.Al.Ir.Ar.G .W + 5.Ab.Ib.L .Al.Ir .Ar.G .W + 24.Ab.Ib.L .Il.Al.Ir.Ar.P .G  + 96.Ab.Ib.L .Il .Ir.Ar.G .W + 96.Ab.Ib.L .Il.Ir .Ar.G .W + 4.Ab.L .Il .Al.Ir.Ar.G .W + 4.Ab.L .Il.Al.Ir .Ar.G .W + 144.Ib .

   3          4        2 2               2        2     2          2      2       2          3  2          5          3     2    5          2  2            3         2  2     3
   L .Il.Al.Ir.Ar.G .W + 72.Ib .L .Il.Al.Ir.Ar.P .G  + 288.Ib .L.Il .Ir.Ar.G .W + 288.Ib .L.Il.Ir .Ar.G .W + 36.Ib.L .Il .Al.Ir.Ar.G .W + 60.Ib.L .Il.Al.Ir .Ar.G .W + 36.Ib.L .Il .Al.Ir.Ar.P .G  + 288.Ib.L.Il .Ir .Ar.G .W

        3  2     2    6         2 4          2          2 4       2            2  2  2        2  2  2         2  2               2  2              2  2     2           2  2  2
   + 12.L .Il .Al.Ir .Ar.G .W)/(24.Ab.Ib .L .Il.Al.Ar.G  + 24.Ab.Ib .L .Al.Ir.Ar.G  + 24.Ab.Ib .L .Il .Al + 24.Ab.Ib .L .Il .Ar + 48.Ab.Ib .L .Il.Al.Ir + 48.Ab.Ib .L .Il.Ir.Ar + 24.Ab.Ib .L .Al.Ir  + 24.Ab.Ib .L .Ir .Ar

           4  2       3           4           3           4   2     3             2  2             2  2             2       2            2     2           4  2           4
   + 8.Ab.Ib.L .Il .Al.Ar.G  + 88.Ab.Ib.L .Il.Al.Ir.Ar.G  + 8.Ab.Ib.L .Al.Ir .Ar.G  + 96.Ab.Ib.L .Il .Al.Ir.G + 96.Ab.Ib.L .Il .Ir.Ar.G + 96.Ab.Ib.L .Il.Al.Ir .G + 96.Ab.Ib.L .Il.Ir .Ar.G + 8.Ab.L .Il .Al.Ir.Ar.G  + 8.

     4        2    4       2 2         3       2  2          2  2          2        2         2      2         2  2        4        2       2    4          2     2 2
```

```
Ab.L .Il.Al.Ir .Ar.G  + 288.Ib .L .Il.Al.Ir.Ar.G  + 288.Ib .Il .Al.Ir.G + 288.Ib .Il .Ir.Ar.G + 288.Ib .Il.Al.Ir .G + 288.Ib .Il.Ir .Ar.G + 96.Ib.L .Il .Al.Ir.Ar.G  + 96.Ib.L .Il.Al.Ir .Ar.G  + 288.Ib.Il .Al.Ir .G

          2   2     2        2   2       2    5
 + 288.Ib.Il .Ir .Ar.G  + 24.L .Il .Al.Ir .Ar.G )

        2 4            2          2 2                    2 2                  2 2    2             2 2  2               5               5              4             3           4   2
UA := (12.Ab.Ib .L .Al.Ir.Ar.P .G  + 12.Ab.Ib .L .Il.Al.Ir.P  + 12.Ab.Ib .L .Il.Ir.Ar.P  + 12.Ab.Ib .L .Al.Ir .P  + 12.Ab.Ib .L .Ir .Ar.P  + 12.Ab.Ib.L .Il.Al.Ir.Ar.G .W + 22.Ab.Ib.L .Il.Al.Ir.Ar.P .G  + 4.Ab.Ib.L .Al.Ir .Ar.

  3          3  2     3           3  2     3              3         2 3           3    2    3              2         2              2    2          5  2        6        5
P .G  - 24.Ab.Ib.L .Il .Al.Ir.G .W + 48.Ab.Ib.L .Il .Ir.Ar.G .W + 48.Ab.Ib.L .Il.Al.Ir .G .W - 24.Ab.Ib.L .Il.Ir .Ar.G .W + 48.Ab.Ib.L .Il.Al.Ir .P .G + 48.Ab.Ib.L .Il.Ir .Ar.P .G + Ab.L .Il .Al.Ir.Ar.G .W + Ab.L .Il.

  2    6         4        2        4       2 2              3          2 2            2 2             2       2            2     2             2  2              4         2
Al.Ir .Ar.G .W + 4.Ab.L .Il.Al.Ir .Ar.P .G  + 144.Ib .L .Il.Al.Ir.Ar.P .G  + 144.Ib .Il .Al.Ir.P .G + 144.Ib .Il .Ir.Ar.P .G + 144.Ib .Il.Al.Ir .P .G + 144.Ib .Il.Ir .Ar.P .G + 48.Ib.L .Il .Al.Ir.Ar.P .G  + 48.Ib.L .Il

   2       4        2     2     2        2  2     2      2    2    2    5        2 4           2        2 4         2           2 2  2          2 2  2           2 2
.Al.Ir .Ar.P .G  + 144.Ib.Il .Al.Ir .P .G  + 144.Ib.Il .Ir .Ar.P .G  + 12.L .Il .Al.Ir .Ar.P .G )/(24.Ab.Ib .L .Il.Al.Ar.G  + 24.Ab.Ib .L .Al.Ir.Ar.G  + 24.Ab.Ib .L .Il .Al + 24.Ab.Ib .L .Il .Ar + 48.Ab.Ib .L .Il.Al.Ir

        2 2             2 2  2         2 2  2         4  2      3            4            3        4   2    3            2 2                   2  2                  2
 + 48.Ab.Ib .L .Il.Ir.Ar + 24.Ab.Ib .L .Al.Ir  + 24.Ab.Ib .L .Ir .Ar + 8.Ab.Ib.L .Il .Al.Ar.G  + 88.Ab.Ib.L .Il.Al.Ir.Ar.G  + 8.Ab.Ib.L .Al.Ir .Ar.G  + 96.Ab.Ib.L .Il .Al.Ir.G + 96.Ab.Ib.L .Il .Ir.Ar.G + 96.Ab.Ib.L .

      2             2      2         4  2        4         4      2   4       2 2             3       2 2           2 2                 2         2          2     2            2
Il.Al.Ir .G + 96.Ab.Ib.L .Il.Ir .Ar.G + 8.Ab.L .Il .Al.Ir.Ar.G + 8.Ab.L .Il.Al.Ir .Ar.G  + 288.Ib .L .Il.Al.Ir.Ar.G  + 288.Ib .Il .Al.Ir.G + 288.Ib .Il .Ir.Ar.G + 288.Ib .Il.Al.Ir .G + 288.Ib .Il.Ir .Ar.G + 96.Ib.L

    2          4          2        2    4          2        2 2            2 2    2        2  2     2    5
.Il .Al.Ir.Ar.G  + 96.Ib.L .Il.Al.Ir .Ar.G  + 288.Ib.Il .Al.Ir .G  + 288.Ib.Il .Ir .Ar.G  + 24.L .Il .Al.Ir .Ar.G )

        2 5           2          2 4          2           2 4           2           2 4     2 2          2 4   2    2           2 3                        2 3             2 3    2
MA := (18.Ab.Ib .L .Al.Ir.Ar.P .G  + 36.Ab.Ib .L .Il.Al.Ir.G .W - 36.Ab.Ib .L .Il.Ir.Ar.G .W + 36.Ab.Ib .L .Al.Ir .G .W - 36.Ab.Ib .L .Ir .Ar.G .W + 36.Ab.Ib .L .Il.Al.Ir     + 36.Ab.Ib .L .Il.Ir.Ar.P  + 36.Ab.Ib .L .Al.Ir .P

       2 3  2              6               5            6    2    5                5              3             5    2      3             4  2       3              4  2       3
+ 36.Ab.Ib .L .Ir .Ar.P  + 15.Ab.Ib.L .Il.Al.Ir.Ar.G .W - 3.Ab.Ib.L .Al.Ir .Ar.G .W + 30.Ab.Ib.L .Il.Al.Ir.Ar.P .G  + 12.Ab.Ib.L .Al.Ir .Ar.P .G  - 24.Ab.Ib.L .Il .Al.Ir.G .W + 48.Ab.Ib.L .Il .Ir.Ar.G .W + 192.Ab.Ib.

4       2 3             4  2    3              3        2                3      2              6  2           6          6           2   6           5          2         4        2 3
L .Il.Al.Ir .G .W - 168.Ab.Ib.L .Il.Ir .Ar.G .W + 144.Ab.Ib.L .Il.Al.Ir .P .G + 144.Ab.Ib.L .Il.Ir .Ar.P .G + Ab.L .Il .Al.Ir.Ar.G .W + Ab.L .Il.Al.Ir .Ar.G .W + 12.Ab.L .Il.Al.Ir .Ar.P .G  + 216.Ib .L .Il.Al.Ir.Ar.P .

3         2 2        2 3           2 2       2        3         2                  2  2                     2             2              2              4        2    6          3  2
G  + 432.Ib .L .Il.Al.Ir .G .W - 432.Ib .L .Il.Ir .Ar.G .W + 216.Ib .L.Il .Al.Ir.P .G + 216.Ib .L.Il .Ir.Ar.P .G + 432.Ib .L.Il.Al.Ir .P .G + 432.Ib .L.Il .Ir .Ar.P .G - 36.Ib.L .Il.Al.Ir .Ar.G .W + 72.Ib.L .Il .Al.Ir.

     4          3         2          2   2     2 4          2 2  2    4              2      2           2  2       2     4    2     2       3   2      2         5          2
Ar.P .G  + 144.Ib.L .Il.Al.Ir .Ar.P .G  + 144.Ib.L .Il .Al.Ir .G .W - 288.Ib.L .Il .Ir .Ar.G .W + 432.Ib.L.Il .Al.Ir .P .G  + 432.Ib.L.Il .Ir .Ar.P .G  - 6.L .Il .Al.Ir .Ar.G .W + 36.L .Il .Al.Ir .Ar.P .G )/(72.Ab.Ib .

4      2        2 4          2          2 2  2             2 2  2             2 2                2 2              2 2    2            2 2  2                4  2       3
L .Il.Al.Ar.G  + 72.Ab.Ib .L .Al.Ir.Ar.G  + 72.Ab.Ib .L .Il .Al + 72.Ab.Ib .L .Il .Ar + 144.Ab.Ib .L .Il.Al.Ir + 144.Ab.Ib .L .Il.Ir.Ar + 72.Ab.Ib .L .Al.Ir  + 72.Ab.Ib .L .Ir .Ar + 24.Ab.Ib.L .Il .Al.Ar.G  + 264.Ab.

    4          3             2   3            2 2                2 2                   2             2         2             4  2              4          4        2    4
Ib.L .Il.Al.Ir.Ar.G  + 24.Ab.Ib.L .Al.Ir .Ar.G  + 288.Ab.Ib.L .Il .Al.Ir.G + 288.Ab.Ib.L .Il .Ir.Ar.G + 288.Ab.Ib.L .Il.Al.Ir .G + 288.Ab.Ib.L .Il.Ir .Ar.G + 24.Ab.L .Il .Al.Ir.Ar.G  + 24.Ab.L .Il.Al.Ir .Ar.G  + 864.

 2 2          3          2 2            2 2             2               2             2      2           2 2            4               2          2    4           2      2 2             2   2    2
Ib .L .Il.Al.Ir.Ar.G  + 864.Ib .Il .Al.Ir.G + 864.Ib .Il .Ir.Ar.G + 864.Ib .Il.Al.Ir .G + 864.Ib .Il.Ir .Ar.G + 288.Ib.L .Il .Al.Ir.Ar.G  + 288.Ib.L .Il.Al.Ir .Ar.G  + 864.Ib.Il .Al.Ir .G  + 864.Ib.Il .Ir .Ar.G  + 72

 2  2     2    5
.L .Il .Al.Ir .Ar.G )

        2 5           2          2 4          2          2 4          2           2 4     2 2         2 4   2    2              6              5             6    2    5                5
MB := (18.Ab.Ib .L .Al.Ir.Ar.P .G  - 36.Ab.Ib .L .Il.Al.Ir.G .W + 36.Ab.Ib .L .Il.Ir.Ar.G .W - 36.Ab.Ib .L .Al.Ir .G .W + 36.Ab.Ib .L .Ir .Ar.G .W + 21.Ab.Ib.L .Il.Al.Ir.Ar.G .W + 3.Ab.Ib.L .Al.Ir .Ar.G .W + 36.Ab.Ib.L .Il.Al

      3          4  2      3          4  2     3              4        2 3            4   2    3           6  2          6          2    6          2 3                3
.Ir.Ar.P .G  - 48.Ab.Ib.L .Il .Al.Ir.G .W + 96.Ab.Ib.L .Il .Ir.Ar.G .W - 48.Ab.Ib.L .Il.Al.Ir .G .W + 96.Ab.Ib.L .Il.Ir .Ar.G .W + 2.Ab.L .Il .Al.Ir.Ar.G .W + 2.Ab.L .Il.Al.Ir .Ar.G .W + 216.Ib .L .Il.Al.Ir.Ar.P .G  -

    2 2        2 3           2 2      2     3          2     2                   4          2    6           3  2              4           2  2      2 4          2 2  2
432.Ib .L .Il.Al.Ir .G .W + 432.Ib .L .Il.Ir .Ar.G .W + 216.Ib .L.Il .Al.Ir.P .G + 216.Ib .L.Il .Ir.Ar.P .G + 36.Ib.L .Il.Al.Ir .Ar.G .W + 72.Ib.L .Il .Al.Ir.Ar.P .G  - 144.Ib.L .Il .Al.Ir .G .W + 288.Ib.L .Il .Ir .Ar.

4      4  2     2    7          2 4          2         2 4          2          2 2  2            2 2  2           2 2             2 2                2 2    2          2 2
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G .W + 6.L .Il .Al.Ir .Ar.G .W)/(72.Ab.Ib .L .Il.Al.Ar.G  + 72.Ab.Ib .L .Al.Ir.Ar.G  + 72.Ab.Ib .L .Il .Al + 72.Ab.Ib .L .Il .Ar + 144.Ab.Ib .L .Il.Al.Ir + 144.Ab.Ib .L .Il.Ir.Ar + 72.Ab.Ib .L .Al.Ir  + 72.Ab.Ib .L .

  2              4  2         3              4                3               4   2    3                  2  2                   2  2                     2    2                   2      2              4  2
Ir .Ar + 24.Ab.Ib.L .Il .Al.Ar.G  + 264.Ab.Ib.L .Il.Al.Ir.Ar.G  + 24.Ab.Ib.L .Al.Ir .Ar.G  + 288.Ab.Ib.L .Il .Al.Ir.G + 288.Ab.Ib.L .Il .Ir.Ar.G + 288.Ab.Ib.L .Il.Al.Ir .G + 288.Ab.Ib.L .Il.Ir .Ar.G + 24.Ab.L .Il .Al

      4          4         2    4          2  2                3         2  2                  2  2                  2         2              2       2                 2  2          4             2          2   4
.Ir.Ar.G  + 24.Ab.L .Il.Al.Ir .Ar.G  + 864.Ib .L .Il.Al.Ir.Ar.G  + 864.Ib .Il .Al.Ir.G + 864.Ib .Il .Ir.Ar.G + 864.Ib .Il.Al.Ir .G + 864.Ib .Il.Ir .Ar.G + 288.Ib.L .Il .Al.Ir.Ar.G  + 288.Ib.L .Il.Al.Ir .Ar.G  + 864.

    2     2 2          2  2    2     2  2    2   5
Ib.Il .Al.Ir .G  + 864.Ib.Il .Ir .Ar.G  + 72.L .Il .Al.Ir .Ar.G )

BEAM MEMBER FROM NODE 1 TO 2

               2 4            2           2  2  2            2  2  2            2  2                 2  2                   5                   5              4  2             3              4
PA := ( - 12.Ab.Ib .L .Il.Al.Ar.P .G  - 12.Ab.Ib .L .Il .Al.P  - 12.Ab.Ib .L .Il .Ar.P  - 12.Ab.Ib .L .Il.Al.Ir.P  - 12.Ab.Ib .L .Il.Ir.Ar.P  + 12.Ab.Ib.L .Il.Al.Ir.Ar.G .W - 4.Ab.Ib.L .Il .Al.Ar.P .G  - 22.Ab.Ib.L .Il.Al.Ir.

  3             3  2        3            3  2        3             3       2 3              3      2      3                2  2                    2  2                    5  2           6        5
Ar.P .G  - 24.Ab.Ib.L .Il .Al.Ir.G .W + 48.Ab.Ib.L .Il .Ir.Ar.G .W + 48.Ab.Ib.L .Il.Al.Ir .G .W - 24.Ab.Ib.L .Il.Ir .Ar.G .W - 48.Ab.Ib.L .Il .Al.Ir.P .G - 48.Ab.Ib.L .Il .Ir.Ar.P .G + Ab.L .Il .Al.Ir.Ar.G .W + Ab.L .

   2     6         4  2            4             2 4            2          2 4          2             2  2  2             2  2  2            2  2                2  2               2  2      2
Il.Al.Ir .Ar.G .W - 4.Ab.L .Il .Al.Ir.Ar.P .G )/(24.Ab.Ib .L .Il.Al.Ar.G  + 24.Ab.Ib .L .Al.Ir.Ar.G  + 24.Ab.Ib .L .Il .Al + 24.Ab.Ib .L .Il .Ar + 48.Ab.Ib .L .Il.Al.Ir + 48.Ab.Ib .L .Il.Ir.Ar + 24.Ab.Ib .L .Al.Ir  +

     2  2  2            4  2         3              4                3            4   2    3               2  2                2  2                 2          2               2      2               4
24.Ab.Ib .L .Ir .Ar + 8.Ab.Ib.L .Il .Al.Ar.G  + 88.Ab.Ib.L .Il.Al.Ir.Ar.G  + 8.Ab.Ib.L .Al.Ir .Ar.G  + 96.Ab.Ib.L .Il .Al.Ir.G + 96.Ab.Ib.L .Il .Ir.Ar.G + 96.Ab.Ib.L .Il.Al.Ir .G + 96.Ab.Ib.L .Il.Ir .Ar.G + 8.Ab.L .

 2          4          4        2    4          2  2                3           2  2                 2  2                 2         2               2       2               2  2          4            2        2   4
Il .Al.Ir.Ar.G  + 8.Ab.L .Il.Al.Ir .Ar.G  + 288.Ib .L .Il.Al.Ir.Ar.G  + 288.Ib .Il .Al.Ir.G + 288.Ib .Il .Ir.Ar.G + 288.Ib .Il.Al.Ir .G + 288.Ib .Il.Ir .Ar.G + 96.Ib.L .Il .Al.Ir.Ar.G  + 96.Ib.L .Il.Al.Ir .Ar.G  +

     2     2 2          2  2    2     2  2    2   5
288.Ib.Il .Al.Ir .G  + 288.Ib.Il .Ir .Ar.G  + 24.L .Il .Al.Ir .Ar.G )

              2 4                      2 4                      2  3  2                 2  3  2               2  3                    2  3                   2  3     2                 2  3  2
UA := ( - 6.Ab.Ib .L .Il.Al.Ar.P .G - 6.Ab.Ib .L .Al.Ir.Ar.P .G + 12.Ab.Ib .L .Il .Al.G.W - 12.Ab.Ib .L .Il .Ar.G.W + 24.Ab.Ib .L .Il.Al.Ir.G.W - 24.Ab.Ib .L .Il.Ir.Ar.G.W + 12.Ab.Ib .L .Al.Ir .G.W - 12.Ab.Ib .L .Ir .Ar.G.W

         5  2         4          5  2    4              4                  2            3  2       2            3  2       2            3      2  2            3     2     2         2  2
 + Ab.Ib.L .Il .Al.Ar.G .W - Ab.Ib.L .Al.Ir .Ar.G .W - 24.Ab.Ib.L .Il.Al.Ir.Ar.P .G  + 48.Ab.Ib.L .Il .Al.Ir.G .W - 48.Ab.Ib.L .Il .Ir.Ar.G .W + 48.Ab.Ib.L .Il.Al.Ir .G .W - 48.Ab.Ib.L .Il.Ir .Ar.G .W - 72.Ib .L .Il.Al

      2        2     2         2          2     2         2          2         2  2         2          2     2          3  2         5          3      2   5            2  2            3
.Ir.Ar.P .G  + 144.Ib .L.Il .Al.Ir.G .W - 144.Ib .L.Il .Ir.Ar.G .W + 144.Ib .L.Il.Al.Ir .G .W - 144.Ib .L.Il.Ir .Ar.G .W + 12.Ib.L .Il .Al.Ir.Ar.G .W - 12.Ib.L .Il.Al.Ir .Ar.G .W - 36.Ib.L .Il .Al.Ir.Ar.P .G  + 144.Ib.

  2     2 3             2  2    3           2 4            2          2 4          2            2  2  2           2  2  2            2  2                2  2               2  2      2
L.Il .Al.Ir .G .W - 144.Ib.L.Il .Ir .Ar.G .W)/(24.Ab.Ib .L .Il.Al.Ar.G  + 24.Ab.Ib .L .Al.Ir.Ar.G  + 24.Ab.Ib .L .Il .Al + 24.Ab.Ib .L .Il .Ar + 48.Ab.Ib .L .Il.Al.Ir + 48.Ab.Ib .L .Il.Ir.Ar + 24.Ab.Ib .L .Al.Ir  + 24.

   2  2  2           4  2        3              4                3            4   2    3              2  2                2  2                  2         2               2       2               4  2
Ab.Ib .L .Ir .Ar + 8.Ab.Ib.L .Il .Al.Ar.G  + 88.Ab.Ib.L .Il.Al.Ir.Ar.G  + 8.Ab.Ib.L .Al.Ir .Ar.G  + 96.Ab.Ib.L .Il .Al.Ir.G + 96.Ab.Ib.L .Il .Ir.Ar.G + 96.Ab.Ib.L .Il.Al.Ir .G + 96.Ab.Ib.L .Il.Ir .Ar.G + 8.Ab.L .Il .

        4          4        2    4          2  2                3          2  2                 2  2                 2          2              2       2               2  2          4            2        2   4
Al.Ir.Ar.G  + 8.Ab.L .Il.Al.Ir .Ar.G  + 288.Ib .L .Il.Al.Ir.Ar.G  + 288.Ib .Il .Al.Ir.G + 288.Ib .Il .Ir.Ar.G + 288.Ib .Il.Al.Ir .G + 288.Ib .Il.Ir .Ar.G + 96.Ib.L .Il .Al.Ir.Ar.G  + 96.Ib.L .Il.Al.Ir .Ar.G  + 288.Ib

  2     2 2          2  2    2      2  2    2   5
.Il .Al.Ir .G  + 288.Ib.Il .Ir .Ar.G  + 24.L .Il .Al.Ir .Ar.G )

              2 6          4              2 6         4              2 5              2               2  4  2   2               2  4  2   2             2 4               2              2 4             2           2 4
MA := ( - 6.Ab.Ib .L .Il.Al.Ar.G .W - 6.Ab.Ib .L .Al.Ir.Ar.G .W - 18.Ab.Ib .L .Il.Al.Ar.P .G  + 30.Ab.Ib .L .Il .Al.G .W - 42.Ab.Ib .L .Il .Ar.G .W + 24.Ab.Ib .L .Il.Al.Ir.G .W - 48.Ab.Ib .L .Il.Ir.Ar.G .W - 6.Ab.Ib .L .Al.

 2 2             2  4  2    2          6  2           5            6            5              6     2    5              5                  3               4  2       3               4  2        3
Ir .G .W - 6.Ab.Ib .L .Ir .Ar.G .W + Ab.Ib.L .Il .Al.Ar.G .W - Ab.Ib.L .Il.Al.Ir.Ar.G .W - 2.Ab.Ib.L .Al.Ir .Ar.G .W - 36.Ab.Ib.L .Il.Al.Ir.Ar.P .G  + 72.Ab.Ib.L .Il .Al.Ir.G .W - 72.Ab.Ib.L .Il .Ir.Ar.G .W + 72.Ab.Ib.

 4         2 3                4       2     3          2 4                5               2  2  2       3              2  2  2       3             2  2        2  3             2  2      2     3           2      2
L .Il.Al.Ir .G .W - 72.Ab.Ib.L .Il.Ir .Ar.G .W - 72.Ib .L .Il.Al.Ir.Ar.G .W + 360.Ib .L .Il .Al.Ir.G .W - 504.Ib .L .Il .Ir.Ar.G .W - 72.Ib .L .Il.Al.Ir .G .W - 72.Ib .L .Il.Ir .Ar.G .W + 216.Ib .L.Il .Al.Ir.P .G + 216

  2   2                  4  2           6             4        2    6             3  2                4            2    2       2 4               2   2   2       4             2 4         2             2 4
.Ib .L.Il .Ir.Ar.P .G + 12.Ib.L .Il .Al.Ir.Ar.G .W - 24.Ib.L .Il.Al.Ir .Ar.G .W - 36.Ib.L .Il .Al.Ir.Ar.P .G  + 216.Ib.L .Il .Al.Ir .G .W - 216.Ib.L .Il .Ir .Ar.G .W)/(72.Ab.Ib .L .Il.Al.Ar.G  + 72.Ab.Ib .L .Al.Ir.Ar.
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 2          2  2  2            2  2  2            2  2                 2  2             2  2  2           4  2          3                4              3              4
G  + 72.Ab.Ib .L .Il .Al + 72.Ab.Ib .L .Il .Ar + 144.Ab.Ib .L .Il.Al.Ir + 144.Ab.Ib .L .Il.Ir.Ar + 72.Ab.Ib .L .Al.Ir  + 72.Ab.Ib .L .Ir .Ar + 24.Ab.Ib.L .Il .Al.Ar.G  + 264.Ab.Ib.L .Il.Al.Ir.Ar.G  + 24.Ab.Ib.L .Al.

 2   3                   2  2                  2  2                 2       2                  2     2             4  2             4         4        2   4           2  2            3          2  2
Ir .Ar.G  + 288.Ab.Ib.L .Il .Al.Ir.G + 288.Ab.Ib.L .Il .Ir.Ar.G + 288.Ab.Ib.L .Il.Al.Ir .G + 288.Ab.Ib.L .Il.Ir .Ar.G + 24.Ab.L .Il .Al.Ir.Ar.G  + 24.Ab.L .Il.Al.Ir .Ar.G  + 864.Ib .L .Il.Al.Ir.Ar.G  + 864.Ib .Il .Al

            2  2                2       2            2     2            2  2           4                2         2   4            2      2  2             2  2      2          2  2     2   5
.Ir.G + 864.Ib .Il .Ir.Ar.G + 864.Ib .Il.Al.Ir .G + 864.Ib .Il.Ir .Ar.G + 288.Ib.L .Il .Al.Ir.Ar.G  + 288.Ib.L .Il.Al.Ir .Ar.G  + 864.Ib.Il .Al.Ir .G  + 864.Ib.Il .Ir .Ar.G  + 72.L .Il .Al.Ir .Ar.G )

             2 6           4             2 6            4             2 5           2            2 4   2   2              2 4  2  2              2 4              2             2 4           2             2 4      2 2
MB := (6.Ab.Ib .L .Il.Al.Ar.G .W + 6.Ab.Ib .L .Al.Ir.Ar.G .W - 18.Ab.Ib .L .Al.Ir.Ar.P .G  + 6.Ab.Ib .L .Il .Al.G .W + 6.Ab.Ib .L .Il .Ar.G .W + 48.Ab.Ib .L .Il.Al.Ir.G .W - 24.Ab.Ib .L .Il.Ir.Ar.G .W + 42.Ab.Ib .L .Al.Ir .G

             2 4  2    2              6  2      5            6              5          6     2    5              5              3             4  2       3                4  2      3              4
 .W - 30.Ab.Ib .L .Ir .Ar.G .W + 2.Ab.Ib.L .Il .Al.Ar.G .W + Ab.Ib.L .Il.Al.Ir.Ar.G .W - Ab.Ib.L .Al.Ir .Ar.G .W - 36.Ab.Ib.L .Il.Al.Ir.Ar.P .G  + 72.Ab.Ib.L .Il .Al.Ir.G .W - 72.Ab.Ib.L .Il .Ir.Ar.G .W + 72.Ab.Ib.L .Il

      2 3             4     2   3           2 4            5         2 3            3       2  2  2        3          2  2  2       3           2 2          2 3          2  2    2   3
 .Al.Ir .G .W - 72.Ab.Ib.L .Il.Ir .Ar.G .W - 72.Ib .L .Il.Al.Ir.Ar.G .W - 216.Ib .L .Il.Al.Ir.Ar.P .G  + 72.Ib .L .Il .Al.Ir.G .W + 72.Ib .L .Il .Ir.Ar.G .W + 504.Ib .L .Il.Al.Ir .G .W - 360.Ib .L .Il.Ir .Ar.G .W - 216.

  2    2                 2    2                    4  2          6           4      2   6            3  2           4          2  2      2 4          2  2  2    4           2 4          2
Ib .L.Il .Al.Ir.P .G - 216.Ib .L.Il .Ir.Ar.P .S + 24.Ib.L .Il .Al.Ir.Ar.G .W - 12.Ib.L .Il.Al.Ir .Ar.G .W - 72.Ib.L .Il .Al.Ir.Ar.P .G  + 216.Ib.L .Il .Al.Ir .G .W - 216.Ib.L .Il .Ir .Ar.G .W)/(72.Ab.Ib .L .Il.Al.Ar.G

            2 4        2               2  2  2             2  2  2              2 2                  2 2              2 2       2            2  2  2           4  2         3              4
 + 72.Ab.Ib .L .Al.Ir.Ar.G  + 72.Ab.Ib .L .Il .Al + 72.Ab.Ib .L .Il .Ar + 144.Ab.Ib .L .Il.Al.Ir + 144.Ab.Ib .L .Il.Ir.Ar + 72.Ab.Ib .L .Al.Ir  + 72.Ab.Ib .L .Ir .Ar + 24.Ab.Ib.L .Il .Al.Ar.G  + 264.Ab.Ib.L .Il.Al.Ir

   3           4     2   3              2  2                2  2                 2       2                2     2             4  2            4          4       2   4           2 2
 .Ar.G  + 24.Ab.Ib.L .Al.Ir .Ar.G  + 288.Ab.Ib.L .Il .Al.Ir.G + 288.Ab.Ib.L .Il .Ir.Ar.G + 288.Ab.Ib.L .Il.Al.Ir .G + 288.Ab.Ib.L .Il.Ir .Ar.G + 24.Ab.L .Il .Al.Ir.Ar.G  + 24.Ab.L .Il.Al.Ir .Ar.G  + 864.Ib .L .Il.Al.

     3          2  2               2  2               2       2            2     2             2  2          4              2        2   4          2      2  2             2  2     2         2  2
Ir.Ar.G  + 864.Ib .Il .Al.Ir.G + 864.Ib .Il .Ir.Ar.G + 864.Ib .Il.Al.Ir .G + 864.Ib .Il.Ir .Ar.G + 288.Ib.L .Il .Al.Ir.Ar.G  + 288.Ib.L .Il.Al.Ir .Ar.G  + 864.Ib.Il .Al.Ir .G  + 864.Ib.Il .Ir .Ar.G  + 72.L .Il .Al.

 2   5
Ir .Ar.G )

0
    / CALCULATE THE WORST CASE DEFLECTIONS AND STRESSES
SHUT OUTSR;
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 WRITE "/ **** MAXIMUM STRESS IN BEAM ELEMENTS - UDL ****";

/ **** MAXIMUM STRESS IN BEAM ELEMENTS - UDL ****

 SETUP( );

/ STRUCTURAL SPECIFICATIONS:

/ ELEMENT TYPE IS ELASTIC FLEXURAL/AXIAL ELEMENT

/ SYSTEM TYPE IS RIGID SUPPORTS, FULL 3 DOF PER NODE

/ NUMBER OF BAYS  = 1

/ NUMBER OF LEVELS= 1

/ NUMBER OF NODES = 2

/ GLOBAL D.O.F.   = 6

0
       / SETUP ELEMENT K, AND GLOBAL K.
 STRESSES( );

BASIC ELEMENT STRESSES

BEAM MEMBER FROM NODE 1 TO 2

                     2   4  10   2   2   2  6  2          2   4  10      2      2  6  2          2   4  10   2   2   2  6  2          2   4   9   2   2   2     4           2   4   9   2   2   2     4          2   4   8   3
 MAX MOMENT MMAX = (720.Ab .Ib .L  .Il .Al .Ar .G .W  + 1440.Ab .Ib .L  .Il.Al ..Ir.Ar .G .W  + 720.Ab .Ib .L  .Al .Ir .Ar .G .W  + 432.Ab .Ib .L .Il .Al .Ar .P .G .W - 432.Ab .Ib .L .Al .Ir .Ar .P .G .W + 576.Ab .Ib .L .Il .

     2    4  2          2   4   8   3      2   4  2          2   4   8   2   2       4  2          2   4   8   2   2   2   2  2          2   4   8   2          2   4  2          2   4   8     2   2      4  2
 Al .Ar.G .W  + 2304.Ab .Ib .L .Il .Al.Ar .G .W  + 3456:Ab .Ib .L .Il .Al .Ir.Ar.G .W  + 108.Ab .Ib .L .Il .Al .Ar .P  .G  + 5184.Ab .Ib .L .Il .Al.Ir.Ar .G .W  + 5184.Ab .Ib .L .Il.Al .Ir .Ar.G .W  + 216

   2   4   8      2      2   2  2        2   4   8        2   2  4  2          2   4   8   2   3     4  2          2   4   8   2   2   2   2  2         2   4   8      3   2   4   2         2   4   7   3      2
 .Ab .Ib .L .Il.Al .Ir.Ar .P  .G  + 3456.Ab .Ib .L .Il.Al.Ir .Ar .G .W  + 2304.Ab .Ib .L .Al .Ir .Ar.G .W  + 108.Ab .Ib .L .Al .Ir .Ar .P  .G  + 576.Ab .Ib .L .Al.Ir .Ar .G .W  + 864.Ab .Ib .L .Il .Al.Ar

     2           2   4   7   2   2          2          2   4   7   2          2     2          2   4   7      2   2      2          2   4   7       2   2     2          2   4   7   2   3      2
 .P .G .W - 864.Ab .Ib .L .Il .Al .Ir.Ar.P .G .W + 1728.Ab .Ib .L .Il .Al.Ir.Ar .P .G .W - 1728.Ab .Ib .L .Il.Al .Ir .Ar.P .G .W + 864.Ab .Ib .L .Il.Al.Ir .Ar .P .G .W - 864.Ab .Ib .L .Al .Ir .Ar.P .G .W

         2   4   6   4   2   2   2         2   4   6   4        2  2         2   4   6   4   2   2   2          2   4   6   3   2      2  2          2   4   6   3          2  2          2   4   6   3      2   2   2
 + 288.Ab .Ib .L .Il .Al .G .W  + 576.Ab .Ib .L .Il .Al.Ar.G .W  + 2016.Ab .Ib .L .Il .Ar .G .W  + 2880.Ab .Ib .L .Il .Al .Ir.G .W  + 2304.Ab .Ib .L .Il .Al.Ir.Ar.G .W  + 6336.Ab .Ib .L .Il .Ir.Ar .G .W

          2   4   6   2   2   2   2   2          2   4   6   2      2     2  2          2   4   6   2   2   2   2   2          2   4   6      2   3   2   2          2   4   6          3      2  2          2   4   6
 + 6912.Ab .Ib .L .Il .Al .Ir .G .W  + 3456.Ab .Ib .L .Il .Al.Ir .Ar.G .W  + 6912.Ab .Ib .L .Il .Ir .Ar .G .W  + 6336.Ab .Ib .L .Il.Al .Ir .G .W  + 2304.Ab .Ib .L .Il.Al.Ir .Ar.G .W  + 2880.Ab .Ib .L .Il

     3   2   2   2          2   4   6   2   4   2   2         2   4   6      4      2  2          2   4   6   4   2   2   2          2   3  10   3   2   2  7  2          2   3  10   2   2      2  7  2          2   3
 .Ir .Ar .G .W  + 2016.Ab .Ib .L .Al .Ir .G .W  + 576.Ab .Ib .L .Al.Ir .Ar.G .W  + 288.Ab .Ib .L .Ir .Ar .G .W  + 408.Ab .Ib .L  .Il .Al .Ar .G .W  + 4680.Ab .Ib .L  .Il .Al .Ir.Ar .G .W  + 4680.Ab .Ib .

  10     2   2   2  7  2          7   3  10   2   3   2  7  2          2   3   9   3   2   2     5          2   3   9   2   2      2     5          2   3   9       2   2   2     5          2   3   9   2
 L  .Il.Al .Ir .Ar .G .W  + 408.Ab .Ib .L  .Al .Ir .Ar .G .W  + 108.Ab .Ib .L .Il .Al .Ar .P .G .W + 1404.Ab .Ib .L .Il .Al .Ir.Ar .P .G .W - 1404.Ab .Ib .L .Il.Al .Ir .Ar .P .G .W - 108.Ab .Ib .L .Al .

   3   2     5          2   3   8   4   2     5  2          2   3   8   4      2  5  2          2   3   8   3   2      5  2          2   3   8   3          2  5  2          2   3   8   2   2   2     5  2
 Ir .Ar .P .G .W + 192.Ab .Ib .L .Il .Al .Ar.G .W  + 624.Ab .Ib .L .Il .Al.Ar .G .W  + 6672.Ab .Ib .L .Il .Al .Ir.Ar.G .W  + 12720.Ab .Ib .L .Il .Al.Ir.Ar .G .W  + 18576.Ab .Ib .L .Il .Al .Ir .Ar.G .W  +

      2   3   8   2   2      2   2   3          2   3   8   2      2   2  5  2          2   3   8      2   3      5  2          2   3   8      2   2   2   2   3          2   3   8         3   2  5  2          2
 864.Ab .Ib .L .Il .Al .Ir.Ar .P  .G  + 18576.Ab .Ib .L .Il .Al.Ir .Ar .G .W  + 12720.Ab .Ib .L .Il.Al .Ir .Ar.G .W  + 864.Ab .Ib .L .Il.Al .Ir .Ar .P  .G  + 6672.Ab .Ib .L .Il.Al.Ir .Ar .G .W  + 624.Ab .

   3   8   2   4     5  2          2   3   8      4   2  5  2          2   3   7   3   2          3          2   3   7   3          2     3          2   3   7   2   2   2        3          2   3   7   2
 Ib .L .Al .Ir .Ar.G .W  + 192.Ab .Ib .L .Al.Ir .Ar .G .W  - 1728.Ab .Ib .L .Il .Al .Ir.Ar.P .G .W + 5184.Ab .Ib .L .Il .Al.Ir.Ar .P .G .W - 6912.Ab .Ib .L .Il .Al .Ir .Ar.P .G .W + 6912.Ab .Ib .L .Il .Al

    2   2     3          2   3   7      2   3      3          2   3   7       3   2     3          2   3   6   4   2      3  2          2   3   6   4          3  2          2   3   6   4      2   3
 .Ir .Ar .P .G .W - 5184.Ab .Ib .L .Il.Al .Ir .Ar.P .G .W + 1728.Ab .Ib .L .Il.Al.Ir .Ar .P .G .W + 4608.Ab .Ib .L .Il .Al .Ir.G .W  + 2304.Ab .Ib .L .Il .Al.Ir.Ar.G .W  + 11520.Ab .Ib .L .Il .Ir.Ar .G .
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2          2  3 6  3  2  2 3 2        2  3 6  3    2     3 2           2  3 6  3  2  2 3 2          2  3 6  2  2  3 3 2         2  3 6  2     3  3  2        2
W + 20736.Ab .Ib .L .I! .Al .Ir .G .W + 6912.Ab .Ib .L .I! .Al.Ir .Ar.G .W + 27648.Ab .Ib .L .I! .Ir .Ar .G .W + 27648.Ab .Ib .L .I! .Al .Ir .G .W + 6912.Ab .Ib .L .I! .Al.Ir .Ar.G .W + 20736.Ab .

 3 6  2  3  2 3 2          2  3 6    2  4 3 2        2  3 6     4   3 2         2  3 6   4  2 3 2        2  2 10  4  2  2 8 2        2  2 10  3  2    2
Ib .L .I! .Ir .Ar .G .W + 11520.Ab .Ib .L .I!.Al .Ir .G .W + 2304.Ab .Ib .L .I!.Al.Ir .Ar.G .W + 4608.Ab .Ib .L .I!.Ir .Ar .G .W + 59.Ab .Ib .L .I! .Al .Ar .G .W + 1544.Ab .Ib .L .I! .Al .Ir.Ar .

8 2         2  2 10  2  2  2  2 8 2        2  2 10      2  3  2 8 2       2  2 10  2  4  2 8 2          2  2 8  4  2       6 2        2  2 8  4      2 6 2
G .W + 6858.Ab .Ib .L .I! .Al .Ir .Ar .G .W + 1544.Ab .Ib .L .I!.Al .Ir .Ar .G .W + 59.Ab .Ib .L .Al .Ir .Ar .G .W + 1632.Ab .Ib .L .I! .Al .Ir.Ar.G .W + 1632.Ab .Ib .L .I! .Al.Ir.Ar .G .W +

       2  2 8  3  2  2     6 2          2  2 8  3      2  2 6 2           2  2 8  2  2  3     6 2          2  2 8  2  2  2  2  2 4          2  2 8  2     3  2 6 2
15264.Ab .Ib .L .I! .Al .Ir .Ar.G .W + 15264.Ab .Ib .L .I! .Al.Ir .Ar .G .W + 15264.Ab .Ib .L .I! .Al .Ir .Ar.G .W + 1728.Ab .Ib .L .I! .Al .Ir .Ar .P .G + 15264.Ab .Ib .L .I! .Al.Ir .Ar .G .W +

      2  2 8     2  4    6 2          2  2 8      4  2 6 2          2  2 7  3  2  2       4          2  2 7  3     2  2    4          2  2 7  2  2  3       4
1632.Ab .Ib .L .I!.Al .Ir .Ar.G .W + 1632.Ab .Ib .L .I!.Al.Ir .Ar .G .W - 6912.Ab .Ib .L .I! .Al .Ir .Ar.P .G .W + 6912.Ab .Ib .L .I! .Al.Ir .Ar .P .G .W - 6912.Ab .Ib .L .I! .Al .Ir .Ar.P .G .W + 6912

  2  2 7  2     3  2    4          2  2 6  4  2  2 4 2          2  2 6  4  2  2 4 2          2  2 6  3  2  3 4 2          2  2 6  3  3  2 4 2          2  2 6
.Ab .Ib .L .I! .Al.Ir .Ar .P .G .W + 13824.Ab .Ib .L .I! .Al .Ir .G .W + 13824.Ab .Ib .L .I! .Ir .Ar .G .W + 27648.Ab .Ib .L .I! .Al .Ir .G .W + 27648.Ab .Ib .L .I! .Ir .Ar .G .W + 13824.Ab .Ib .L .

 2  2  4 4 2          2  2 6  2  4  2 4 2          2    10  4  2      2 9 2          2    10  3  2  2  2 9 2          2    10  2  2  3  2 9 2          2    10     2  4
I! .Al .Ir .G .W + 13824.Ab .Ib .L .I! .Ir .Ar .G .W + 104.Ab .Ib.L .I! .Al .Ir.Ar .G .W + 1176.Ab .Ib.L .I! .Al .Ir .Ar .G .W + 1176.Ab .Ib.L .I! .Al .Ir .Ar .G .W + 104.Ab .Ib.L .I!.Al .Ir .

 2 9 2          2    8  4  2  2     7 2          2    8  4     2  2  2 2          2    8  3  2  3     7 2          2    8  3     3  2 7 2          2    8  2  2  4    7 2
Ar .G .W + 1152.Ab .Ib.L .I! .Al .Ir .Ar.G .W + 1152.Ab .Ib.L .I! .Al.Ir .Ar .G .W + 2304.Ab .Ib.L .I! .Al .Ir .Ar.G .W + 2304.Ab .Ib.L .I! .Al.Ir .Ar .G .W + 1152.Ab .Ib.L .I! .Al .Ir .Ar.G .W +

      2    8  2     4  2 7 2          2 10  4  2  2  2 10 2          2 10  3  2  3  2 10 2          2 10  2  2  4  2 10 2          4  8  2  2     2 7 2          4  8
1152.Ab .Ib.L .I! .Al.Ir .Ar .G .W + 48.Ab .L .I! .Al .Ir .Ar .G .W + 96.Ab .L .I! .Al .Ir .Ar .G .W + 48.Ab .L .I! .Al .Ir .Ar .G .W + 17280.Ab.Ib .L .I! .Al .Ir.Ar .G .W + 17280.Ab.Ib .L .I!

 2  2  2 7 2          4 7      2  2  2     5          4  6  3  2       5 2          4  6  3       2 5 2          4  6  2  2  2     5 2          4  6  2  2
.Al .Ir .Ar .G .W - 10368.Ab.Ib .L .I!.Al .Ir .Ar .P .G .W + 13824.Ab.Ib .L .I! .Al .Ir.Ar.G .W + 55296.Ab.Ib .L .I! .Al.Ir.Ar .G .W + 69120.Ab.Ib .L .I! .Al .Ir .Ar.G .W + 2592.Ab.Ib .L .I! .Al .Ir.

 2  2 3          4  6  2     2  2 5 2          4  6     2  3     5 2          4  6     2  2  2  2 3          4  6     3  2 5 2          4  5  3  2        3
Ar .P .G + 69120.Ab.Ib .L .I! .Al.Ir .Ar .G .W + 55296.Ab.Ib .L .I!.Al .Ir .Ar.G .W + 2592.Ab.Ib .L .I!.Al .Ir .Ar .P .G + 13824.Ab.Ib .L .I!.Al.Ir .Ar .G .W - 20736.Ab.Ib .L .I! .Al .Ir.Ar.P .G .

         4  5  2  2  2     3          4  5     2  3     3          4  4  4  2     3 2          4  4  4        3 2          4  4  4     2 3 2        4
W - 41472.Ab.Ib .L .I! .Al .Ir .Ar.P .G .W - 20736.Ab.Ib .L .I!.Al .Ir .Ar.P .G .W + 6912.Ab.Ib .L .I! .Al .Ir.G .W + 13824.Ab.Ib .L .I! .Al.Ir.Ar.G .W + 48384.Ab.Ib .L .I! .Ir.Ar .G .W + 62208.Ab.Ib

 4  3  2  2 3 2          4  4  3     2  3 2          4  4  3  2  2 3 2          4  4  2  2  3 3 2          4  4  2     3  3 2          4  4  2 3 2
.L .I! .Al .Ir .G .W + 41472.Ab.Ib .L .I! .Al.Ir .Ar.G .W + 103680.Ab.Ib .L .I! .Ir .Ar .G .W + 103680.Ab.Ib .L .I! .Al .Ir .G .W + 41472.Ab.Ib .L .I! .Al.Ir .Ar.G .W + 62208.Ab.Ib .L .I! .Ir .Ar .

3 2          4  4     2  4 3 2          4  4     4  3 2          4  4     4  2 3 2          4  3  4  2          4  3  4          4 3
G .W + 48384.Ab.Ib .L .I!.Al .Ir .G .W + 13824.Ab.Ib .L .I!.Al.Ir .Ar.G .W + 6912.Ab.Ib .L .I!.Ir .Ar .G .W - 10368.Ab.Ib .L .I! .Al .Ir.P .G.W - 20736.Ab.Ib .L .I! .Al.Ir.Ar.P .G.W - 10368.Ab.Ib .L

 4     2          4  3  3  2  2          4  3  3     2          4  3  3  2  2          4  3  2  2  3          4  3  2     3
.I! .Ir.Ar .P .G.W - 20736.Ab.Ib .L .I! .Al .Ir .P .G.W - 41472.Ab.Ib .L .I! .Al.Ir .Ar.P .G.W - 20736.Ab.Ib .L .I! .Ir .Ar .P .G.W - 10368.Ab.Ib .L .I! .Al .Ir .P .G.W - 20736.Ab.Ib .L .I! .Al.Ir .Ar.P

          4  3  2  3  2          3  8  3  2     2 8 2          3  8  2  2  2  2 8 2          3  8     2  3  2 8 2          3  7  3  2     2  6
.G.W - 10368.Ab.Ib .L .I! .Ir .Ar .P .G.W + 9792.Ab.Ib .L .I! .Al .Ir.Ar .G .W + 61056.Ab.Ib .L .I! .Al .Ir .Ar .G .W + 9792.Ab.Ib .L .I!.Al .Ir .Ar .G .W - 1728.Ab.Ib .L .I! .Al .Ir.Ar .P .G .W -

        3  7  2  2  2  2     6          3  7     2  3  2     6          3  6  4  2       6 2          3  6  4       2 6 2          3  6  3  2  2     6 2
19872.Ab.Ib .L .I! .Al .Ir .Ar .P .G .W - 2592.Ab.Ib .L .I!.Al .Ir .Ar .P .G .W + 4608.Ab.Ib .L .I! .Al .Ir.Ar.G .W + 14976.Ab.Ib .L .I! .Al.Ir.Ar .G .W + 103680.Ab.Ib .L .I! .Al .Ir .Ar.G .W + 1296.

   3  6  3  2     2  2 4          3  6  3     2  2 6 2          3  6  2  2  3     6 2          3  6  2  2  2  2  2 4          3  6  2     3  2 6 2
Ab.Ib .L .I! .Al .Ir.Ar .P .G + 176256.Ab.Ib .L .I! .Al.Ir .Ar .G .W + 176256.Ab.Ib .L .I! .Al .Ir .Ar.G .W + 11664.Ab.Ib .L .I! .Al .Ir .Ar .P .G + 103680.Ab.Ib .L .I! .Al.Ir .Ar .G .W + 14976.Ab

 3  6     2  4     6 2          3  6     4  2 6 2          3  5  4  2       4          3  5  4     2  4          3  5  3  2  2     4          3  5
.Ib .L .I!.Al .Ir .Ar.G .W + 4608.Ab.Ib .L .I!.Al.Ir .Ar .G .W - 6912.Ab.Ib .L .I! .Al .Ir.Ar.P .G .W - 1728.Ab.Ib .L .I! .Al.Ir.Ar .P .G .W - 107136.Ab.Ib .L .I! .Al .Ir .Ar.P .G .W - 3456.Ab.Ib .L .

3  2  2  4          3  5  2  2  3     4          3  5  2     3  2  4          3  4  4  2  2 4 2          3  4  4     2  4 2          3  4  4
I! .Al.Ir .Ar .P .G .W - 79488.Ab.Ib .L .I! .Al .Ir .Ar.P .G .W + 19008.Ab.Ib .L .I! .Al.Ir .Ar .P .G .W + 69120.Ab.Ib .L .I! .Al .Ir .G .W + 34560.Ab.Ib .L .I! .Al.Ir .Ar.G .W + 172800.Ab.Ib .L .I! .

 2  2 4 2          3  4  3  2  3 4 2          3  4  3     3  4 2          3  4  3  3  2 4 2          3  4  2  2  4 4 2          3  4  2     4  4 2
Ir .Ar .G .W + 241920.Ab.Ib .L .I! .Al .Ir .G .W + 69120.Ab.Ib .L .I! .Al.Ir .Ar.G .W + 241920.Ab.Ib .L .I! .Ir .Ar .G .W + 172800.Ab.Ib .L .I! .Al .Ir .G .W + 34560.Ab.Ib .L .I! .Al.Ir .Ar.G .W +
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          3   4   2   4   2  4   2             3   3   4   2   2    2            3  3   4      2       2            3  3   4   2   2    2            3  3   3   2   3    2           3
69120.Ab.Ib .L .Il .Ir .Ar .G .W  - 41472.Ab.Ib .L .Il .Al .Ir .P .G .W - 82944.Ab.Ib .L .Il .Al.Ir .Ar.P .G .W - 41472.Ab.Ib .L .Il .Ir .Ar .P .G .W - 41472.Ab.Ib .L .Il .Al .Ir .P .G .W - 82944.Ab.Ib .
 3   3      3        2               3   3   3   3   2     2            2  8   4   2     2  9  2            2  8   3   2   2  2  9  2            2  8   2   2   3   2  9  2            2  8
L .Il .Al.Ir .Ar.P ..G .W - 41472.Ab.Ib .L .Il .Ir .Ar .P .G .W + 1416.Ab.Ib .L .Il .Al .Ir.Ar .G .W  + 20952.Ab.Ib .L .Il .Al .Ir .Ar .G .W  + 20952.Ab.Ib .L .Il .Al .Ir .Ar .G .W  + 1416.Ab.Ib .L .Il.
 2   4   2  9  2           2  7   4   2     2    7            2  7   3   2   2   2     7            2  7   2   2   3   2     7            2  6   4   2   2     7  2            2  6   4
Al .Ir .Ar .G .W  - 504.Ab.Ib .L .Il .Al .Ir.Ar .P .G .W - 6192.Ab.Ib .L .Il .Al .Ir .Ar .P .G .W - 504.Ab.Ib .L .Il .Al .Ir .Ar .P .G .W + 24480.Ab.Ib .L .Il .Al .Ir .Ar.G .W  + 24480.Ab.Ib .L .Il .Al.
 2   2  7  2            2  6   3   2   3     7  2            2  6   3   2   2   2   2  5            2  6   3      3   2  7  2            2  6   2   2   4     7  2            2  6   2      4  2
Ir .Ar .G .W  + 80064.Ab.Ib .L .Il .Al .Ir .Ar.G .W  + 5184.Ab.Ib .L .Il .Al .Ir .Ar .P  .G  + 80064.Ab.Ib .L .Il .Al.Ir .Ar .G .W  + 24480.Ab.Ib .L .Il .Al .Ir .Ar.G .W  + 24480.Ab.Ib .L .Il .Al.Ir .Ar
 7  2             2  5   4   2   2       5            2  5   4     2   2     5            2  5   3   2   3       5            2  5   3     3   2     5            2  4   4   2   3  5
.G .W  - 17280.Ab.Ib .L .Il .Al .Ir .Ar.P .G .W + 3456.Ab.Ib .L .Il .Al.Ir .Ar .P .G .W - 38016.Ab.Ib .L .Il .Al .Ir .Ar.P .G .W + 24192.Ab.Ib .L .Il .Al.Ir .Ar .P .G .W + 82944.Ab.Ib .L .Il .Al .Ir .G .
 2              2  4   4   3   2  5  2            2  4   3   2   4  5  2            2  4   3   4   2  5  2            8   4   2   2   2  10  2            8   3   2   3   2  10  2
W  + 82944.Ab.Ib .L .Il .Ir .Ar .G .W  + 82944.Ab.Ib .L .Il .Al .Ir .G .W  + 82944.Ab.Ib .L .Il .Ir .Ar .G .W  + 1560.Ab.Ib.L .Il .Al .Ir .Ar .G  .W  + 5712.Ab.Ib.L .Il .Al .Ir .Ar .G  .W  + 1560.Ab.Ib.
 8   2   2   4   2  10  2            7   4   2   2   2     8            7   3   2   3   2     8            6   4   2   3     8  2            6   4     3   2  8  2            6   3   2
L .Il .Al .Ir .Ar .G  .W  - 288.Ab.Ib.L .Il .Al .Ir .Ar .P .G .W - 288.Ab.Ib.L .Il .Al .Ir .Ar .P .G .W + 6912.Ab.Ib.L .Il .Al .Ir .Ar.G .W  + 6912.Ab.Ib.L .Il .Al.Ir .Ar .G .W  + 6912.Ab.Ib.L .Il .Al .
 4     8  2            6   3     4   2  8  2            8   4   2   3   2  11  2            8   3   2   4   2  11  2            4  6   2   2   2   2  8  2            4  5   2   2   2   2     6
Ir .Ar.G .W  + 6912.Ab.Ib.L .Il .Al.Ir .Ar .G .W  + 288.Ab.L .Il .Al .Ir .Ar .G  .W  + 288.Ab.L .Il .Al .Ir .Ar .G  .W  + 103680.Ib .L .Il .Al .Ir .Ar .G .W  - 62208.Ib .L .Il .Al .Ir .Ar .P .G .W +
        4   4   3   2   2     6  2            4   4   3      2   2  6  2            4   4   2   2   3     6  2            4   4   2   2   2   2   2  4            4   4   2     3   2  6  2            4   3   3
82944.Ib .L .Il .Al .Ir .Ar.G .W. + 331776.Ib .L .Il .Al.Ir .Ar .G .W  + 331776.Ib .L .Il .Al .Ir .Ar.G .W  + 15552.Ib .L .Il .Al .Ir .Ar .P  .G  + 82944.Ib .L .Il .Al.Ir .Ar .G .W  - 248832.Ib .L .Il .
 2   2       4              4   3   3     2   2     4            4   3   2   2   3       4            4   2   4   2   2  4  2            4   2   4     2     4  2            4   2   4   2   2  4  2
Al .Ir .Ar.P .G .W - 124416 Ib .L .Il .Al.Ir .Ar .P .G .W - 124416.Ib .L .Il .Al .Ir .Ar.P .G .W + 41472.Ib .L .Il .Al .Ir .G .W  + 82944.Ib .L .Il .Al.Ir .Ar.G .W  + 290304.Ib .L .Il .Ir .Ar .G .W +
         4   2   3   2   3  4  2            4   2   3     3     4  2            4   2   3   3   2  4  2            4   2   2   2   4  4  2            4   2   2     4     4  2            4   2   2   4   2  4  2
331776.Ib .L .Il .Al .Ir .G .W  + 165888.Ib .L .Il .Al.Ir .Ar.G .W  + 331776.Ib .L .Il .Ir .Ar .G .W  + 290304.Ib .L .Il .Al .Ir .G .W  + 82944.Ib .L .Il .Al.Ir .Ar.G .W  + 41472.Ib .L .Il .Ir .Ar .G .W
           4   4   2   2     2            4   4     2     2            4   4   2   2     2            4   3   2   3     2            4     3     3     2            4   3   3
- 124416.Ib .L.Il .Al .Ir .P .G .W - 248832.Ib .L.Il .Al.Ir .Ar.P .G .W - 124416.Ib .L.Il .Ir .Ar .P .G .W - 124416.Ib .L.Il .Al .Ir .P .G .W - 248832.Ib .L.Il .Al.Ir .Ar.P .G .W - 124416.Ib .L.Il .Ir .
 2     2              3   6   3   2   2   2  9  2            3   6   2   2   3   2  9  2            3   5   3   2   2   2     7            3   5   2   2   3   2     7            3   4   4   2   2     7  2
Ar .P .G .W + 58752.Ib .L .Il .Al .Ir .Ar .G .W  + 58752.Ib .L .Il .Al .Ir .Ar .G .W  - 36288.Ib .L .Il .Al .Ir .Ar .P .G .W - 15552.Ib .L .Il .Al .Ir .Ar .P .G .W + 27648.Ib .L .Il .Al .Ir .Ar.G .W +
        3   4   4     2   2  7  2            3   4   3   2   3     7  2            3   4   3   2   2   2   2  5            3   4   3     3   2  7  2            3   4   2   2   4     7  2            3   4   2
89856.Ib .L .Il .Al.Ir .Ar .G .W  + 283392.Ib .L .Il .Al .Ir .Ar.G .W  + 15552.Ib .L .Il .Al .Ir .Ar .P  .G  + 283392.Ib .L .Il .Al.Ir .Ar .G .W  + 89856.Ib .L .Il .Al .Ir .Ar.G .W  + 27648.Ib .L .Il .Al
 4   2  7  2            3   3   4   2   2       5            3   3   4     2   2     5            3   3   3   2   3       5            3   3   3     3   2     5            3   2   4   2   3  5  2
.Ir .Ar .G .W  - 82944.Ib .L .Il .Al .Ir .Ar.P .G .W - 20736.Ib .L .Il .Al.Ir .Ar .P .G .W - 207360.Ib .L .Il .Al .Ir .Ar.P .G .W - 20736.Ib .L .Il .Al.Ir .Ar .P .G .W + 165888.Ib .L .Il .Al .Ir .G .W
         3   2   4     3     5  2            3   2   4   3   2  5  2            3   2   3   2   4  5  2            3   2   3     4     5  2            3   2   3   4   2  5  2            3     4   2   3
+ 82944.Ib .L .Il .Al.Ir .Ar.G .W  + 414720.Ib .L .Il .Ir .Ar .G .W  + 414720.Ib .L .Il .Al .Ir .G .W  + 82944.Ib .L .Il .Al.Ir .Ar.G .W  + 165888.Ib .L .Il .Ir .Ar .G .W  - 124416.Ib .L.Il .Al .Ir .P .
 3              3   4   3     3            3   4   3   2     3            2  6   4   2   2   2  10  2            2  6   3   2   3   2  10  2            2  6   2   2   4   2  10  2            2  5
G .W - 248832.Ib .L.Il .Al.Ir .Ar.P .G .W - 124416.Ib .L.Il .Ir .Ar .P .G .W + 8496.Ib .L .Il .Al .Ir .Ar .G  .W  + 29088.Ib .L .Il .Al .Ir .Ar .G  .W  + 8496.Ib .L .Il .Al .Ir .Ar .G  .W  - 6048.Ib .L .
 4   2   2   2     8            2  5   3   2   3   2     8            2  4   4   2   3     8  2            2  4   4   2   2   2   2  6            2  4   4     3   2  8  2            2  4   3   2   4     8
Il .Al .Ir .Ar .P .G .W - 6048.Ib .L .Il .Al .Ir .Ar .P .G .W + 58752.Ib .L .Il .Al .Ir .Ar.G .W  + 3888.Ib .L .Il .Al .Ir .Ar .P  .G  + 58752.Ib .L .Il .Al.Ir .Ar .G .W  + 58752.Ib .L .Il .Al .Ir .Ar.G
 2              2  4   3     4   2  8  2            2  3   4   2   3       6            2  3   4     3   2     6            2  2   4   2   4  6  2            2  2   4   4   2  6  2            6   4
.W  + 58752.Ib .L .Il .Al.Ir .Ar .G .W  - 51840.Ib .L .Il .Al .Ir .Ar.P .G .W + 10368.Ib .L .Il .Al.Ir .Ar .P .G .W + 124416.Ib .L .Il .Al .Ir .G .W  + 124416.Ib .L .Il .Ir .Ar .G .W  + 3744.Ib.L .Il .
 2   3   2  11  2            6   3   2   4   2  11  2            5   4   2   3   2     9            4   4   2   4     9  2            4   4     4   2  9  2            6   4   2   4   2  12  2         2
Al .Ir .Ar .G  .W  + 3744.Ib.L .Il .Al .Ir .Ar .G  .W  - 864.Ib.L .Il .Al .Ir .Ar .P .G .W + 10368.Ib.L .Il .Al .Ir .Ar.G .W  + 10368.Ib.L .Il .Al.Ir .Ar .G .W  + 432.L .Il .Al .Ir .Ar .G  .W )/(3456.Ab
  4   8   2   2   2  4            2   4   8     2     2  4            2   4   8   2   2   2  4            2   4   6   3   2     2            2   4   6   3     2  2            2   4   6   2   2      2
.Ib .L .Il .Al .Ar .G .W + 6912.Ab .Ib .L .Il.Al .Ir.Ar .G .W + 3456.Ab .Ib .L .Al .Ir .Ar .G .W + 6912.Ab .Ib .L .Il .Al .Ar.G .W + 6912.Ab .Ib .L .Il .Al.Ar .G .W + 20736.Ab .Ib .L .Il .Al .Ir.Ar.G .W
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       2  4  6  2        2 2           2  4  6     2  2    2          2  4  6       2  2  2        2  4  6  2  3    2         2  4  6    3  2  2          2  4  4
+ 20736.Ab .Ib .L .Il .Al.Ir.Ar .G .W + 20736.Ab .Ib .L .Il.Al .Ir .Ar.G .W + 20736.Ab .Ib .L .Il.Al.Ir .Ar .G .W + 6912.Ab .Ib .L .Al .Ir .Ar.G .W + 6912.Ab .Ib .L .Al.Ir .Ar .G .W + 3456.Ab .Ib .L .

4  2        2  4  4  4           2  4  4  4  2          2  4  4  3  2          2  4  4  3            2  4  4  3     2         2  4  4  2  2  2
Il .Al .W + 6912.Ab .Ib .L .Il .Al.Ar.W + 3456.Ab .Ib .L .Il .Ar .W + 13824.Ab .Ib .L .Il .Al .Ir.W + 27648.Ab .Ib .L .Il .Al.Ir.Ar.W + 13824.Ab .Ib .L .Il .Ir.Ar .W + 20736.Ab .Ib .L .Il .Al .Ir .W +

      2  4  4  2   2       2  4  4  2  2  2       2  4  4    2  3       2  4  4      3          2  4  4    3  2       2  4  4  2  4      2
41472.Ab .Ib .L .Il .Al.Ir .Ar.W + 20736.Ab .Ib .L .Il .Ir .Ar .W + 13824.Ab .Ib .L .Il.Al .Ir .W + 27648.Ab .Ib .L .Il.Al.Ir .Ar.W + 13824.Ab .Ib .L .Il.Ir .Ar .W + 3456.Ab .Ib .L .Al .Ir .W + 6912.Ab .

4  4    4        2  4  4  4  2        2  3  8  3  2  2  5       2  3  8  2  2    2  5       2  3  8    2  2  2  5       2  3  8  2  3  2  5
Ib .L .Al.Ir .Ar.W + 3456.Ab .Ib .L .Ir .Ar .W + 2304.Ab .Ib .L .Il .Al .Ar .G .W + 27648.Ab .Ib .L .Il .Al .Ir.Ar .G .W + 27648.Ab .Ib .L .Il.Al .Ir .Ar .G .W + 2304.Ab .Ib .L .Al .Ir .Ar .G .W + 2304.

2  3  6  4  2   3        2  3  6  4    2  3       2  3  6  3  2     3        2  3  6  3     2  3       2  3  6  2  2  2   3        2  3  6  2
Ab .Ib .L .Il .Al .Ar.G .W + 2304.Ab .Ib .L .Il .Al.Ar .G .W + 57600.Ab .Ib .L .Il .Al .Ir.Ar.G .W + 57600.Ab .Ib .L .Il .Al.Ir.Ar .G .W + 110592.Ab .Ib .L .Il .Al .Ir .Ar.G .W + 110592.Ab .Ib .L .Il .Al

 2  2  3       2  3  6    2  3   3      2  3  6     3  2  3        2  3  6  2  4    3       2  3  6    4  2  3       2  3  4  4  2      2
.Ir .Ar .G .W + 57600.Ab .Ib .L .Il.Al .Ir .Ar.G .W + 57600.Ab .Ib .L .Il.Al.Ir .Ar .G .W + 2304.Ab .Ib .L .Al .Ir .Ar.G .W + 2304.Ab .Ib .L .Al.Ir .Ar .G .W + 27648.Ab .Ib .L .Il .Al .Ir.G.W + 55296.Ab

 3  4  4          2  3  4  4    2        2  3  4  3  2  2       2  3  4  3    2        2  3  4  3  2  2       2  3  4  2  2  3
.Ib .L .Il .Al.Ir.Ar.G.W + 27648.Ab .Ib .L .Il .Ir.Ar .G.W + 82944.Ab .Ib .L .Il .Al .Ir .G.W + 165888.Ab .Ib .L .Il .Al.Ir .Ar.G.W + 82944.Ab .Ib .L .Il .Ir .Ar .G.W + 82944.Ab .Ib .L .Il .Al .Ir .G.W

         2  3  4  2    3        2  3  4  2  3  2       2  3  4    2  4       2  3  4      4        2  3  4    4  2       2  2  8  4  2
+ 165888.Ab .Ib .L .Il .Al.Ir .Ar.G.W + 82944.Ab .Ib .L .Il .Ir .Ar .G.W + 27648.Ab .Ib .L .Il.Al .Ir .G.W + 55296.Ab .Ib .L .Il.Al.Ir .Ar.G.W + 27648.Ab .Ib .L .Il.Ir .Ar .G.W + 384.Ab .Ib .L .Il .Al .

2  6       2  2  8  3  2    2  6       2  2  8  2  2  2  2  6       2  2  8    2  3  2  6       2  2  8  2  4  2  6       2  2  6  4  2      4
Ar .G .W + 10752.Ab .Ib .L .Il .Al .Ir.Ar .G .W + 51840.Ab .Ib .L .Il .Al .Ir .Ar .G .W + 10752.Ab .Ib .L .Il.Al .Ir .Ar .G .W + 384.Ab .Ib .L .Al .Ir .Ar .G .W + 11520.Ab .Ib .L .Il .Al .Ir.Ar.G .W +

      2  2  6  4      2  4        2  2  6  3  2  2    4        2  2  6  3    2  2  4        2  2  6  2  2  3    4        2  2  6  2    3  2  4
11520.Ab .Ib .L .Il .Al.Ir.Ar .G .W + 117504.Ab .Ib .L .Il .Al .Ir .Ar.G .W + 117504.Ab .Ib .L .Il .Al.Ir .Ar .G .W + 117504.Ab .Ib .L .Il .Al .Ir .Ar.G .W + 117504.Ab .Ib .L .Il .Al.Ir .Ar .G .W + 11520

 2  2  6    2  4    4       2  2  6    4  2  4       2  2  4  4  2  2  2       2  2  4  4    2    2       2  2  4  4  2  2  2       2  2  4  3
.Ab .Ib .L .Il.Al .Ir .Ar.G .W + 11520.Ab .Ib .L .Il.Al.Ir .Ar .G .W + 55296.Ab .Ib .L .Il .Al .Ir .G .W + 110592.Ab .Ib .L .Il .Al.Ir .Ar.G .W + 55296.Ab .Ib .L .Il .Ir .Ar .G .W + 110592.Ab .Ib .L .Il

 2  3  2        2  2  4  3    3  2        2  2  4  3  3  2  2       2  2  4  2  2  4  2       2  2  4  2    4  2       2  2  4  2  4  2  2
.Al .Ir .G .W + 221184.Ab .Ib .L .Il .Al.Ir .Ar.G .W + 110592.Ab .Ib .L .Il .Ir .Ar .G .W + 55296.Ab .Ib .L .Il .Al .Ir .G .W + 110592.Ab .Ib .L .Il .Al.Ir .Ar.G .W + 55296.Ab .Ib .L .Il .Ir .Ar .G .W +

    2  8  4  2   2  7        2  8  3  2  2  2  7       2  8  2  2  3  2  7       2  8    2  4  2  7       2  6  4  2  2    5       2  6  4
768.Ab .Ib.L .Il .Al .Ir.Ar .G .W + 9216.Ab .Ib.L .Il .Al .Ir .Ar .G .W + 9216.Ab .Ib.L .Il .Al .Ir .Ar .G .W + 768.Ab .Ib.L .Il.Al .Ir .Ar .G .W + 9216.Ab .Ib.L .Il .Al .Ir .Ar.G .W + 9216.Ab .Ib.L .Il

 2  2  5        2  6  3  2  3    5       2  6  3    3  2  5       2  6  2  2  4    5       2  6  2    4  2  5       2  8  4  2  2  2  8
.Al.Ir .Ar .G .W + 18432.Ab .Ib.L .Il .Al .Ir .Ar.G .W + 18432.Ab .Ib.L .Il .Al.Ir .Ar .G .W + 9216.Ab .Ib.L .Il .Al .Ir .Ar.G .W + 9216.Ab .Ib.L .Il .Al.Ir .Ar .G .W + 384.Ab .L .Il .Al .Ir .Ar .G .W +

    2  8  3  2  3  2  8       2  8  2  2  4  2  8       4  6  2  2    2  5       4  6    2  2  2  5       4  4  3  2      3       4  4  3
768.Ab .L .Il .Al .Ir .Ar .G .W + 384.Ab .L .Il .Al .Ir .Ar .G .W + 82944.Ab.Ib .L .Il .Al .Ir.Ar .G .W + 82944.Ab.Ib .L .Il.Al .Ir .Ar .G .W + 165888.Ab.Ib .L .Il .Al .Ir.Ar.G .W + 165888.Ab.Ib .L .Il .

  2  3        4  4  2  2  2    3       4  4  2    2  2  3       4  4    2  3    3       4  4    3  2  3       4  2  4  2
Al.Ir.Ar .G .W + 331776.Ab.Ib .L .Il .Al .Ir .Ar.G .W + 331776.Ab.Ib .L .Il .Al.Ir .Ar .G .W + 165888.Ab.Ib .L .Il.Al .Ir .Ar.G .W + 165888.Ab.Ib .L .Il.Al.Ir .Ar .G .W + 82944.Ab.Ib .L .Il .Al .Ir.G.W

          4  2  4          4  2  4    2       4  2  3  2  2       4  2  3    2       4  2  3  2  2       4  2  2  2
+ 165888.Ab.Ib .L .Il .Al.Ir.Ar.G.W + 82944.Ab.Ib .L .Il .Ir.Ar .G.W + 248832.Ab.Ib .L .Il .Al .Ir .G.W + 497664.Ab.Ib .L .Il .Al.Ir .Ar.G.W + 248832.Ab.Ib .L .Il .Ir .Ar .G.W + 248832.Ab.Ib .L .Il .Al

 3        4  2  2    3        4  2  2  3  2       4  2    2  4       4  2      4       4  2    4  2       3  6
.Ir .G.W + 497664.Ab.Ib .L .Il .Al.Ir .Ar.G.W + 248832.Ab.Ib .L .Il .Ir .Ar .G.W + 82944.Ab.Ib .L .Il.Al .Ir .G.W + 165888.Ab.Ib .L .Il.Al.Ir .Ar.G.W + 82944.Ab.Ib .L .Il.Ir .Ar .G.W + 55296.Ab.Ib .L .

3  2    2  6       3  6  2  2  2  2  6       3  6    2  3  2  6       3  4  4  2    4       3  4  4    2  4       3  4  3  2  2
Il .Al .Ir.Ar .G .W + 359424.Ab.Ib .L .Il .Al .Ir .Ar .G .W + 55296.Ab.Ib .L .Il.Al .Ir .Ar .G .W + 55296.Ab.Ib .L .Il .Al .Ir.Ar.G .W + 55296.Ab.Ib .L .Il .Al.Ir.Ar .G .W + 829440.Ab.Ib .L .Il .Al .Ir .

  4        3  4  3  2  2  4       3  4  2  2  3    4       3  4  2    3  2  4       3  4    2  4    4       3  4      4  2  4
Ar.G .W + 829440.Ab.Ib .L .Il .Al.Ir .Ar .G .W + 829440.Ab.Ib .L .Il .Al .Ir .Ar.G .W + 829440.Ab.Ib .L .Il .Al.Ir .Ar .G .W + 55296.Ab.Ib .L .Il.Al .Ir .Ar.G .W + 55296.Ab.Ib .L .Il.Al.Ir .Ar .G .W +

       3  2  4  2  2  2        3  2  4    2    2       3  2  4  2  2  2       3  2  3  2  3  2       3  2  3    3  2       3  2
414720.Ab.Ib .L .Il .Al .Ir .G .W + 829440.Ab.Ib .L .Il .Al.Ir .Ar.G .W + 414720.Ab.Ib .L .Il .Ir .Ar .G .W + 829440.Ab.Ib .L .Il .Al .Ir .G .W + 1658880.Ab.Ib .L .Il .Al.Ir .Ar.G .W + 829443.Ab.Ib .L .
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 3   3   2 2                    3  2  2  2  4  2                 3  2   2      4     2                   3  2   2   4   2 2                  2 6   4   2     2 7                 2 6   3   2   2  2 7
Il .Ir .Ar .G .W + 414720.Ab.Ib .L .Il .Al .Ir .G .W + 829440.Ab.Ib .L .Il .Al.Ir .Ar.G .W + 414720.Ab.Ib .L .Il .Ir .Ar .G .W + 9216.Ab.Ib .L .Il .Al .Ir.Ar .G .W + 145152.Ab.Ib .L .Il .Al .Ir .Ar .G .W

               2 6   2   2   3   2 7               2 6     2  4   2 7                  2 4   4   2   2     5                  2 4   4      2   2 5                 2 4   3   2   3     5
 + 145152.Ab.Ib .L .Il .Al .Ir .Ar .G .W + 9216.Ab.Ib .L .Il.Al .Ir .Ar .G .W + 172800.Ab.Ib .L .Il .Al .Ir .Ar.G .W + 172800.Ab.Ib .L .Il .Al.Ir .Ar .G .W + 594432.Ab.Ib .L .Il .Al .Ir .Ar.G .W + 594432

    2  4   3     3  2  5                2  4   2   2   4     5                2  4   2      4  2 5                2 2   4   2  3 3                2 2   4      3     3                   2 2
.Ab.Ib .L .Il .Al.Ir .Ar .G .W + 172800.Ab.Ib .L .Il .Al .Ir .Ar.G .W + 172800.Ab.Ib .L .Il .Al.Ir .Ar .G .W + 331776.Ab.Ib .L .Il .Al .Ir .G .W + 663552.Ab.Ib .L .Il .Al.Ir .Ar.G .W + 331776.Ab.Ib .L .

 4   3   2  3                    2  2   3   2   4  3                 2  2   3      4     3                   2  2   3   4   2  3              6   4   2   2   2 8                6   3   2   3   2 8
Il .Ir .Ar .G .W + 331776.Ab.Ib .L .Il .Al .Ir .G .W + 663552.Ab.Ib .L .Il .Al.Ir .Ar.G .W + 331776.Ab.Ib .L .Il .Ir .Ar .G .W + 11520.Ab.Ib.L .Il .Al .Ir .Ar .G .W + 43776.Ab.Ib.L .Il .Al .Ir .Ar .G .W

              6   2   2   4   2 8                4   4   2   3     6                4   4      3   2 6                4   3   2   4     6                4   3      4   2 6                6  4
 + 11520.Ab.Ib.L .Il .Al .Ir .Ar .G .W + 55296.Ab.Ib.L .Il .Al .Ir .Ar.G .W + 55296.Ab.Ib.L .Il .Al.Ir .Ar .G .W + 55296.Ab.Ib.L .Il .Al .Ir .Ar.G .W + 55296.Ab.Ib.L .Il .Al.Ir .Ar .G .W + 2304.Ab.L .Il

   2   3   2 9              6   3   2  4   2 9                4   4   2   2   2   2 6                4  2   3   2   2     4                4  2   3      2  2  4                4  2   2   2   3     4
.Al .Ir .Ar .G .W + 2304.Ab.L .Il .Al .Ir .Ar .G .W + 497664.Ib .L .Il .Al .Ir .Ar .G .W + 995328.Ib .L .Il .Al .Ir .Ar.G .W + 995328.Ib .L .Il .Al.Ir .Ar .G .W + 995328.Ib .L .Il .Al .Ir .Ar.G .W +

          4  2   2     3  2 4                 4   4   2   2 2                 4   4      2     2                  4   4   2   2 2                 4   3   2   3 2                 4   3      3     2             4
995328.Ib .L .Il .Al.Ir .Ar .G .W + 497664.Ib .Il .Al .Ir .G .W + 995328.Ib .Il .Al.Ir .Ar.G .W + 497664.Ib .Il .Ir .Ar .G .W + 995328.Ib .Il .Al .Ir .G .W + 1990656.Ib .Il .Al.Ir .Ar.G .W + 995328.Ib .

 3   3   2 2                 4   2   2   4  2                4   2      4     2                  4  2   4   2 2                  3 4   3   2   2   2 7                3  4   2   2   3   2 7                3  2  4
Il .Ir .Ar .G .W + 497664.Ib .Il .Al .Ir .G .W + 995328.Ib .Il .Al.Ir .Ar.G .W + 497664.Ib .Il .Ir .Ar .G .W + 331776.Ib .L .Il .Al .Ir .Ar .G .W + 331776.Ib .L .Il .Al .Ir .Ar .G .W + 331776.Ib .L .Il .

 2   2      5                3  2   4     2  2 5                  3  2   3   2   3     5                  3  2   3     3  2  5                3  2   2   2   4     5                3  2   2      4  2  5
Al .Ir .Ar.G .W + 331776.Ib .L .Il .Al.Ir .Ar .G .W + 1658880.Ib .L .Il .Al .Ir .Ar.G .W + 1658880.Ib .L .Il .Al.Ir .Ar .G .W + 331776.Ib .L .Il .Al .Ir .Ar.G .W + 331776.Ib .L .Il .Al.Ir .Ar .G .W +

          3   4   2   3 3                  3   4      3     3                3   4   3   2  3                3   3   2   4  3                 3   3      4     3                3   3   4   2  3              2  4   4
995328.Ib .Il .Al .Ir .G .W + 1990656.Ib .Il .Al.Ir .Ar.G .W + 995328.Ib .Il .Ir .Ar .G .W + 995328.Ib .Il .Al .Ir .G .W + 1990656.Ib .Il .Al.Ir .Ar.G .W + 995328.Ib .Il .Ir .Ar .G .W + 55296.Ib .L .Il .

 2   2   2 8                2  4   3   2   3   2 8               2  4   2   2   4   2 8                2 2   4   2   3     6                2 2   4      3   2 6                2 2   3   2   4     6
Al .Ir .Ar .G .W + 193536.Ib .L .Il .Al .Ir .Ar .G .W + 55296.Ib .L .Il .Al .Ir .Ar .G .W + 414720.Ib .L .Il .Al .Ir .Ar.G .W + 414720.Ib .L .Il .Al.Ir .Ar .G .W + 414720.Ib .L .Il .Al .Ir .Ar.G .W +

          2 2   3      4  2  6                 2   4   2   4 4                 2   4      4     4                 2   4   4   2 4                 4   4   2   3   2 9               4   3   2   4   2 9
414720.Ib .L .Il .Al.Ir .Ar .G .W + 497664.Ib .Il .Al .Ir .G .W + 995328.Ib .Il .Al.Ir .Ar.G .W + 497664.Ib .Il .Ir .Ar .G .W + 27648.Ib.L .Il .Al .I- .Ar .G .W + 27648.Ib.L .Il .Al .Ir .Ar .G .W + 82944

     2   4   2   4     7                2   4      4   2 7                4   4   2   4   2 10
.Ib.L .Il .Al .Ir .Ar.G .W + 82944.Ib.L .Il .Al.Ir .Ar .G .W + 3456.L .Il .Al .Ir .Ar .G  .W)

0
    / CAICULATE THE WORST CASE MID-SPAN MOMENT
SHUT MAXSR:
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D.5 General Influence Coefficient Solution

NOTE: Lateral coefficients are for 1/2 Loads

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/ ON FLOAT$
/ LET AL=100, AB=100, AR=100;  / TESTING ONLY
/ LET IL=500, IB=500, IR=500;
/ LET G=1, L=12, EE=30000;

 SETUP( )$

/ STRUCTURAL SPECIFICATIONS:

/ ELEMENT TYPE IS ELASTIC FLEXURAL/AXIAL ELEMENT

/ SYSTEM TYPE IS RIGID SUPPORTS, FULL 3 DOF PER NODE

/ NUMBER OF BAYS  = 1

/ NUMBER OF LEVELS= 1

/ NUMBER OF NODES = 2

/ GLOBAL D.O.F.   = 6

 WRITE "/.. INFLUENCE COEFFICIENTS FOR DEFLECTIONS .."$

/.. INFLUENCE COEFFICIENTS FOR DEFLECTIONS ..

 INFDEL( )$

/..... FLOOR UDL CASE AT LEVEL=1 BAY=1

                2 6        2           2 6        2           2 6        2           2 6        2           8           5           8           5           6           3           6           3           2 4
MAT(1,1) := (12.Ab.Ib .L .Il.Al.G  - 12.Ab.Ib .L .Il.Ar.G  + 12.Ab.Ib .L .Al.Ir.G  - 12.Ab.Ib .L .Ir.Ar.G  + Ab.Ib.L .Il.Al.Ar.G  - Ab.Ib.L .Al.Ir.Ar.G  + 48.Ab.Ib.L .Il.Al.Ir.G  - 48.Ab.Ib.L .Il.Ir.Ar.G  + 144.Ib .L .Il.Al.

         3      2 4          3         6           6         4       2 4          4       2      4       6           2     7                      2 4           2                  2 4           2                  2 2  2
        Ir.G  - 144.Ib .L .Il.Ir.Ar.G  + 12.Ib.L .Il.Al.Ir.Ar.G  + 96.Ib.L .Il.Al.Ir .G  - 48.Ib.L .Il.Ir .Ar.G  + 2.L .Il.Al.Ir .Ar.G )/(48.E .Ab.Ib .L .Il.Al.Ar.G  + 48.E .Ab.Ib .L .Al.Ir.Ar.G  + 48.E .Ab.Ib .L .Il .Al

            2  2  2             2  2                2  2                2  2     2               2  2  2            4  2        3              4             3                 4
         + 48.E .Ab.Ib .L .Il .Ar + 96.E .Ab.Ib .L .Il.Al.Ir + 96.E .Ab.Ib .L .Il.Ir.Ar + 48.E .Ab.Ib .L .Al.Ir  + 48.E .Ab.Ib .L .Ir .Ar + 16.E .Ab.Ib.L .Il .Al.Ar.G  + 176.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 16.E .Ab.Ib.L .

         2     3             2  2              2  2               2        2                 2   2              4  2           4              4       2      4            2 2
        Al.Ir .Ar.G  + 192.E .Ab.Ib.L .Il .Al.Ir.G + 192.E .Ab.Ib.L .Il .Ir.Ar.G + 192.E .Ab.Ib.L .Il.Al.Ir .G + 192.E .Ab.Ib.L .Il.Ir .Ar.G + 16.E .Ab.L .Il .Al.Ir.Ar.G  + 16.E .Ab.L .Il.Al.Ir .Ar.G  + 576.E .Ib .L .

            3         2  2             2  2              2        2             2     2           2  2         4           2      2   4           2     2  2
        Il.Al.Ir.Ar.G  + 576.E .Ib .Il .Al.Ir.G + 576.E .Ib .Il .Ir.Ar.G + 576.E .Ib .Il.Al.Ir .G + 576.E .Ib .Il.Ir .Ar.G + 192.E .Ib.L .Il .Al.Ir.Ar.G  + 192.E .Ib.L .Il.Al.Ir .Ar.G  + 576.E .Ib.Il .Al.Ir .G  + 576.

           2  2     2         2  2     2    5
        E .Ib.Il .Ir .Ar.G  + 48.E .L .Il .Al.Ir .Ar.G )

                2 6       3           2 6       3          2  4   2          2 4               2  4   2            6  2     4            6          4           6  2     4              4  2     2
MAT(2,1) := ( - 12.Ab.Ib .L .Il.Ar.G  - 12.Ab.Ib .L .Ir.Ar.G  - 24.Ab.Ib .L .Il .G - 48.Ab.Ib .L .Il.Ir.G - 24.Ab.Ib .L .Ir .G - 5.Ab.Ib.L .Il .Ar.G  - 44.Ab.Ib.L .Il.Ir.Ar.G  - 3.Ab.Ib.L .Ir .Ar.G  - 96.Ab.Ib.L .Il .Ir.G  -

           4     2  2       6   2       5        6     2    5         2 4         4          2  2  2     2          2  2     2  2        4  2       5        4    2    5          2  2  2 3
        96.Ab.Ib.L .Il.Ir .G  - 4.Ab.L .Il .Ir.Ar.G  - 4.Ab.L .Il.Ir .Ar.G  - 144.Ib .L .Il.Ir.Ar.G - 288.Ib .L .Il .Ir.G  - 288.Ib .L .Il.Ir .G  - 60.Ib.L .Il .Ir.Ar.G  - 36.Ib.L .Il.Ir .Ar.G  - 288.Ib.L .Il .Ir .G  -

          4   2   2    6             2 4           2            2 4          2           2  2  2            2  2  2          2  2              2  2             2  2     2
        12.L .Il .Ir .Ar.G )/(24.E .Ab.Ib .L .Il.Al.Ar.G  + 24.E .Ab.Ib .L .Al.Ir.Ar.G  + 24.E .Ab.Ib .L .Il .Al + 24.E .Ab.Ib .L .Il .Ar + 48.E .Ab.Ib .L .Il.Al.Ir + 48.E .Ab.Ib .L .Il.Ir.Ar + 24.E .Ab.Ib .L .Al.Ir  +

           2  2  2              4  2      3           4          3          4   2   3              2  2               2  2              2        2
         24.E .Ab.Ib .L .Ir .Ar + 8.E .Ab.Ib.L .Il .Al.Ar.G  + 88.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 8.E .Ab.Ib.L .Al.Ir .Ar.G  + 96.E .Ab.Ib.L .Il .Al.Ir.G + 96.E .Ab.Ib.L .Il .Ir.Ar.G + 96.E .Ab.Ib.L .Il.Al.Ir .G + 96.E .Ab

          2    2           4  2        4        4      2    4        2  2          3          2  2              2  2              2        2               2     2
        .Ib.L .Il.Ir .Ar.G + 8.E .Ab.L .Il .Al.Ir.Ar.G  + 8.E .Ab.L .Il.Al.Ir .Ar.G  + 288.E .Ib .L .Il.Al.Ir.Ar.G  + 288.E .Ib .Il .Al.Ir.G + 288.E .Ib .Il .Ir.Ar.G + 288.E .Ib .Il.Al.Ir .G + 288.E .Ib .Il.Ir .Ar.G +

            2  2        4       2       2   4        2    2 2        2  2    2        2  2    2    5
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96.E .Ib.L .Il .Al.Ir.Ar.G  + 96.E .Ib.L .Il.Al.Ir .Ar.G  + 288.E .Ib.Il .Al.Ir .G  + 288.E .Ib.Il .Ir .Ar.G  + 24.E .L .Il .Al.Ir .Ar.G )

               2 5         2            2 5         2            2 5         2            2 5         2              7            5              7              5                5              3              5              3
MAT(3,1) := ( - 72.Ab .Ib .L .Il.Al.G  + 72.Ab.Ib .L .Il.Ar.G  - 72.Ab.Ib .L .Al.Ir.G  + 72.Ab.Ib .L .Ir.Ar.G  - 6.Ab.Ib.L .Il.Al.Ar.G  - 6.Ab.Ib.L .Al.Ir.Ar.G  - 264.Ab.Ib.L .Il.Al.Ir.G  + 240.Ab.Ib.L .Il.Ir.Ar.G  - 48.Ab.Ib.

5       2 3              5  2     3        7                 6          7       2      6               2 3                3              2 3              3               5                 6                 3            2  4             3     2     4
L .Al.Ir .G  + 24.Ab.Ib.L .Ir .Ar.G  - Ab.L .Il.Al.Ir.Ar.G  - Ab.L .Al.Ir .Ar.G  - 864.Ib .L .Il.Al.Ir.G  + 864.Ib .L .Il.Ir.Ar.G  - 72.Ib.L .Il.Al.Ir.Ar.G  - 576.Ib.L .Il.Al.Ir .G  + 288.Ib.L .Il.Ir .Ar.G  - 12.

5          2    7                 2 4              2               2 4               2                 2 2   2                2 2   2                 2 2                        2 2                        2 2
L .Il.Al.Ir .Ar.G )/(144.E .Ab.Ib .L .Il.Al.Ar.G  + 144.E .Ab.Ib .L .Al.Ir.Ar.G  + 144.E .Ab.Ib .L .Il .Al + 144.E .Ab.Ib .L .Il .Ar + 288.E .Ab.Ib .L .Il.Al.Ir + 288.E .Ab.Ib .L .Il.Ir.Ar + 144.E .Ab.Ib .L .Al.

2              2 2   2                   4  2          3                   4                   3                 4     2     3                  2  2                        2  2                        2     2
Ir  + 144.E .Ab.Ib .L .Ir .Ar + 48.E .Ab.Ib.L .Il .Al.Ar.G  + 528.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 48.E .Ab.Ib.L .Al.Ir .Ar.G  + 576.E .Ab.Ib.L .Il .Al.Ir.G + 576.E .Ab.Ib.L .Il .Ir.Ar.G + 576.E .Ab.Ib.L .Il.Al.Ir

        2     2                    4   2              4                 4          2     4                2  2                  3                  2  2                  2  2                         2           2
.G + 576.E .Ab.Ib.L .Il.Ir .Ar.G + 48.E .Ab.L .Il .Al.Ir.Ar.G  + 48.E .Ab.L .Il.Al.Ir .Ar.G  + 1728.E .Ib .L .Il.Al.Ir.Ar.G  + 1728.E .Ib .Il .Al.Ir.G + 1728.E .Ib .Il .Ir.Ar.G + 1728.E .Ib .Il.Al.Ir .G + 1728.

   2      2                   2   2            4                2            2      4                 2      2 2                  2    2     2               2     2     2     5
E .Ib .Il.Ir .Ar.G + 576.E .Ib.L .Il .Al.Ir.Ar.G  + 576.E .Ib.L .Il.Al.Ir .Ar.G  + 1728.E .Ib.Il .Al.Ir .G  + 1728.E .Ib.Il .Ir .Ar.G  + 144.E .L .Il .Al.Ir .Ar.G )

               2 6        2            2 6        2            2 6        2            2 6        2            8              5                 6                3               6               3            2  4
MAT(4,1) := (12.Ab.Ib .L .Il.Al.G  - 12.Ab.Ib .L .Il.Ar.G  + 12.Ab.Ib .L .Al.Ir.G  - 12.Ab.Ib .L .Ir.Ar.G  + Ab.Ib.L .Il.Al.Ar.G  - Ab.Ib.L .Al.Ir.Ar.G  + 48.Ab.Ib.L .Il.Al.Ir.G  - 48.Ab.Ib.L .Il.Ir.Ar.G  + 144.Ib .L .Il.Al.

3           2  4           3            6               6                4   2            4              4   2          4        6   2            7                 2  4               2                 2  4              2                 2  2  2
Ir.G  - 144.Ib .L .Il.Ir.Ar.G  - 12.Ib.L .Il.Al.Ir.Ar.G  + 48.Ib.L .Il .Al.Ir.G  - 96.Ib.L .Il .Ir.Ar.G  - 2.L .Il .Al.Ir.Ar.G )/(48.E .Ab.Ib .L .Il.Al.Ar.G  + 48.E .Ab.Ib .L .Al.Ir.Ar.G  + 48.E .Ab.Ib .L .Il .Al

          2  2  2                2  2                     2  2                    2  2    2                2  2   2                 4   2            3                 4                 3                 4
 + 48.E .Ab.Ib .L .Il .Ar + 96.E .Ab.Ib .L .Il.Al.Ir + 96.E .Ab.Ib .L .Il.Ir.Ar + 48.E .Ab.Ib .L .Al.Ir  + 48.E .Ab.Ib .L .Ir .Ar + 16.E .Ab.Ib.L .Il .Al.Ar.G  + 176.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 16.E .Ab.Ib.L .

    2      3                    2   2                   2   2                    2        2                    2     2                4   2           4             4          2     4            2 2
Al.Ir .Ar.G  + 192.E .Ab.Ib.L .Il .Al.Ir.G + 192.E .Ab.Ib.L .Il .Ir.Ar.G + 192.E .Ab.Ib.L .Il.Al.Ir .G + 192.E .Ab.Ib.L .Il.Ir .Ar.G + 16.E .Ab.L .Il .Al.Ir.Ar.G  + 16.E .Ab.L .Il.Al.Ir .Ar.G  + 576.E .Ib .L .

            3           2   2                   2   2                  2        2                  2     2                2   2           4                2          2     4                2     2 2
Il.Al.Ir.Ar.G  + 576.E .Ib .Il .Al.Ir.G + 576.E .Ib .Il .Ir.Ar.G + 576.E .Ib .Il.Al.Ir .G + 576.E .Ib .Il.Ir .Ar.G + 192.E .Ib.L .Il .Al.Ir.Ar.G  + 192.E .Ib.L  Il.Al.Ir .Ar.G  + 576.E .Ib.Il .Al.Ir .G  + 576.

     2     2             2    2    2    5
E .Ib.Il .Ir .Ar.G  + 48.E .L .Il .Al.Ir .Ar.G )

               2 6        3             2 6       3              2 4   2             2 4              2 4   2              6   2     4            6              4               6       2  4              4   2    2
MAT(5,1) := ( - 12.Ab.Ib .L .Il.Al.G  - 12.Ab.Ib .L .Al.Ir.G  - 24.Ab.Ib .L .Il .G - 48.Ab.Ib .L .Il.Ir.G - 24.Ab.Ib .L .Ir .G - 3.Ab.Ib.L .Il .Al.G  - 44.Ab.Ib.L .Il.Al.Ir.G  - 5.Ab.Ib.L .Al.Ir .G  - 96.Ab.Ib.L .Il .Ir.G -

        4      2 2           6  2        5          6             2 5             2 4              4             2 2  2    2              2 2    2 2              4  2           5            4            2 5              2  2  2 3
96.Ab.Ib.L .Il.Ir .G  - 4.Ab.L .Il .Al.Ir.G  - 4.Ab.L .Il.Al.Ir .G  - 144.Ib .L .Il.Al.Ir.G  - 288.Ib .L .Il .Ir.G  - 288.Ib .L .Il.Ir .G  - 36.Ib.L .Il .Al.Ir.G  - 60.Ib.L .Il.Al.Ir .G  - 288.Ib.L .Il .Ir .G -

   4   2      2 6                   2 4               2                2 4              2                 2 2  2                 2 2   2                 2 2                      2 2                       2  2     2
12.L .Il .Al.Ir .G )/(24.E .Ab.Ib .L .Il.Al.Ar.G  + 24.E .Ab.Ib .L .Al.Ir.Ar.G  + 24.E .Ab.Ib .L .Il .Al + 24.E .Ab.Ib .L .Il .Ar + 48.E .Ab.Ib .L .Il.Al.Ir + 48.E .Ab.Ib .L .Il.Ir.Ar + 24.E .Ab.Ib .L .Al.Ir  +

             2  2  2                4   2           3                  4                  3                 4     2     3                 2  2                     2  2                        2          2
 24.E .Ab.Ib .L .Ir .Ar + 8.E .Ab.Ib.L .Il .Al.Ar.G  + 88.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 8.E .Ab.Ib.L .Al.Ir .Ar.G  + 96.E .Ab.Ib.L .Il .Al.Ir.G + 96.E .Ab.Ib.L .Il .Ir.Ar.G + 96.E .Ab.Ib.L .Il.Al.Ir .G + 96.E .Ab.

  2      2                 4   2              4               4          2     4               2 2                 3                 2 2                 2 2                     2          2                 2     2
.Ib.L .Il.Ir .Ar.G + 8.E .Ab.L .Il .Al.Ir.Ar.G  + 8.E .Ab.L .Il.Al.Ir .Ar.G  + 288.E .Ib .L .Il.Al.Ir.Ar.G  + 288.E .Ib .Il .Al.Ir.G + 288.E .Ib .Il .Ir.Ar.G + 288.E .Ib .Il.Al.Ir .G + 288.E .Ib .Il.Ir .Ar.G +

       2   2              4                2            2     4                2      2 2                 2    2    2              2    2     2     5
96.E .Ib.L .Il .Al.Ir.Ar.G  + 96.E .Ib.L .Il.Al.Ir .Ar.G  + 288.E .Ib.Il .Al.Ir .G  + 288.E .Ib.Il .Ir .Ar.G  + 24.E .L .Il .Al.Ir .Ar.G )

               2 5         2            2 5         2            2 5         2            2 5         2              7            5              7              5                5    2     3               5    2     3               5
MAT(6,1) := ( - 72.Ab.Ib .L .Il.Al.G  + 72.Ab.Ib .L .Il.Ar.G  - 72.Ab.Ib .L .Al.Ir.G  + 72.Ab.Ib .L .Ir.Ar.G  + 6.Ab.Ib.L .Il.Al.Ar.G  + 6.Ab.Ib.L .Al.Ir.Ar.G  - 24.Ab.Ib.L .Il .Al.G  + 48.Ab.Ib.L .Il .Ar.G  - 240.Ab.Ib.L .Il

      3               5             3       7  2         6           7                 6             2 3              3              2 3              3               5                 6               3  2          4               3  2         4
.Al.Ir.G  + 264.Ab.Ib.L .Il.Ir.Ar.G  + Ab.L .Il .Al.Ar.G  + Ab.L .Il.Al.Ir.Ar.G  - 864.Ib .L .Il.Al.Ir.G  + 864.Ib .L .Il.Ir.Ar.G  + 72.Ib.L .Il.Al.Ir.Ar.G  - 288.Ib.L .Il .Al.Ir.G  + 576.Ib.L .Il .Ir.Ar.G  + 12.

5   2           7                2 4              2               2 4              2                 2 2   2                2 2   2                2 2                         2 2                         2 2
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L .I! .Al.Ir.Ar.G )/(144.E .Ab.Ib .L .I!.Al.Ar.G  + 144.E .Ab.Ib .L .Al.Ir.Ar.G  + 144.E .Ab.Ib .L .I! .Al + 144.E .Ab.Ib .L .I! .Ar + 288.E .Ab.Ib .L .I!.Al.Ir + 288.E .Ab.Ib .L .I!.Ir.Ar + 144.E .Ab.Ib .L .Al.

  2                    2  2   2                 4   2          3                  4                 3                 4     2     3                      2   2                      2   2                        2      2
 Ir  + 144.E .Ab.Ib .L .Ir .Ar + 48.E .Ab.Ib.L .I! .Al.Ar.G  + 528.E .Ab.Ib.L .I!.Al.Ir.Ar.G  + 48.E .Ab.Ib.L .Al.Ir .Ar.G  + 576.E .Ab.Ib.L .I! .Al.Ir.G + 576.E .Ab.Ib.L .I! .Ir.Ar.G + 576.E .Ab.Ib.L .I!.Al.Ir

              2      2                 4   2            4              4            2    4               2  2                3                 2   2                                                       2
.G + 576.E .Ab.Ib.L .I!.Ir .Ar.G + 48.E .Ab.L .I! .Al.Ir.Ar.G  + 48.E .Ab.L .I!.Al.Ir .Ar.G  + 1728.E .Ib .L .I!.Al.Ir.Ar.G  + 1728.E .Ib .I! .Al.Ir.G + 1728.E .Ib .I! .Ir.Ar.G + 1728.E .Ib .I!.Al.Ir .G + 1728.

    2      2               4  2   2             4                2        2    4                2       2  2                    2    2      2                2    2       2    5
 E .Ib .I!.Ir .Ar.G + 576.E .Ib.L .I! .Al.Ir.Ar.G  + 576.E .Ib.L .I!.Al.Ir .Ar.G  + 1728.E .Ib.I! .Al.Ir .G  + 1728.E .Ib.I! .Ir .Ar.G  + 144.E .L .I! .Al.Ir .Ar.G )

MAT(7,1) := 0

MAT(8,1) := 0

MAT(9,1) := 0

/..... WIND LOAD CASE .....

              2  7       2            2  5              2  5              2  5              2  5                  7           3                 7          3                  5                   5
MAT(1,1) := (3.Ab.Ib .L .Al.Ar.G  + 12.Ab.Ib .L .I!.Al + 12.Ab.Ib .L .I!.Ar + 12.Ab.Ib .L .Al.Ir + 12.Ab.Ib .L .Ir.Ar + 4.Ab.Ib.L .I!.Al.Ar.G  + 4.Ab.Ib.L .Al.Ir.Ar.G  + 48.Ab.Ib.L .I!.Al.Ir.G + 48.Ab.Ib.L .I!.Ir.Ar.G + 4.Ab.

            7              4           2  3               2  3                5              4                2  4               2               2  4               2                2  2  2                2  2  2
           L .I!.Al.Ir.Ar.G  + 108.Ib .L .I!.Al.Ir.G + 108.Ib .L .I!.Ir.Ar.G - 18.Ib.L .I!.Al.Ir.Ar.G )/(36.E .Ab.Ib .L .I!.Al.Ar.G  + 36.E .Ab.Ib .L .Al.Ir.Ar.G  + 36.E .Ab.Ib .L .I! .Al + 36.E .Ab.Ib .L .I! .Ar + 72.E .Ab

           2  2                    2  2                     2  2      2                 2  2  2                4   2         3                  4                3               4     2     3                    2  2
           .Ib .L .I!.Al.Ir + 72.E .Ab.Ib .L .I!.Ir.Ar + 36.E .Ab.Ib .L .Al.Ir  + 36.E .Ab.Ib .L .Ir .Ar + 12.E .Ab.Ib.L .I! .Al.Ar.G  + 132.E .Ab.Ib.L .I!.Al.Ir.Ar.G  + 12.E .Ab.Ib.L .Al.Ir .Ar.G  + 144.E .Ab.Ib.L .I! .

                         2  2                   2      2                  2      2              4   2          4             4        2     4               2  2               3                2  2
           Al.Ir.G + 144.E .Ab.Ib.L .I! .Ir.Ar.G + 144.E .Ab.Ib.L .I!.Al.Ir .G + 144.E .Ab.Ib.L .I!.Ir .Ar.G + 12.E .Ab.L .I! .Al.Ir.Ar.G  + 12.E .Ab.L .I!.Al.Ir .Ar.G  + 432.E .Ib .L .I!.Al.Ir.Ar.G  + 432.E .Ib .I! .Al.

                    2  2                2       2              2     2                   2  2              4                2        2    4                2    2  2                 2    2     2              2  2
           Ir.G + 432.E .Ib .I! .Ir.Ar.G + 432.E .Ib .I!.Al.Ir .G + 432.E .Ib .I!.Ir .Ar.G + 144.E .Ib.L .I! .Al.Ir.Ar.G  + 144.E .Ib.L .I!.Al.Ir .Ar.G  + 432.E .Ib.I! .Al.Ir .G  + 432.E .Ib.I! .Ir .Ar.G  + 36.E .L .I! .

              2    5
           Al.Ir .Ar.G )

              2  5              2  5              5            2       2  3            2          3   2         3                2  4              2              2  4               2                2  2  2               2
MAT(2,1) := (3.Ab.Ib .L .I!.Ar.G + 3.Ab.Ib .L .Ir.Ar.G + 12.Ab.Ib.L .I!.Ir.Ar.G  + 36.Ib .L .I!.Ir.Ar.G  + 18.Ib.L .I! .Ir.Ar.G )/(6.E .Ab.Ib .L .I!.Al.Ar.G  + 6.E .Ab.Ib .L .Al.Ir.Ar.G  + 6.E .Ab.Ib .L .I! .Al + 6.E .Ab.Ib .

            2  2                   2  2                2  2                  2  2      2                 2  2  2              4   2         3                  4                 3                4     2    3
           L .I! .Ar + 12.E .Ab.Ib .L .I!.Al.Ir + 12.E .Ab.Ib .L .I!.Ir.Ar + 6.E .Ab.Ib .L .Al.Ir  + 6.E .Ab.Ib .L .Ir .Ar + 2.E .Ab.Ib.L .I! .Al.Ar.G  + 22.E .Ab.Ib.L .I!.Al.Ir.Ar.G  + 2.E .Ab.Ib.L .Al.Ir .Ar.G  + 24.E .

                 2  2                  2  2                   2      2                2      2             4   2          4             4        2     4               2  2             3                2
           Ab.Ib.L .I! .Al.Ir.G + 24.E .Ab.Ib.L .I! .Ir.Ar.G + 24.E .Ab.Ib.L .I!.Al.Ir .G + 24.E .Ab.Ib.L .I!.Ir .Ar.G + 2.E .Ab.L .I! .Al.Ir.Ar.G  + 2.E .Ab.L .I!.Al.Ir .Ar.G  + 72.E .Ib .L .I!.Al.Ir.Ar.G  + 72.E .Ib .

            2                2  2                 2      2               2      2                  2  2              4                2        2    4                2    2  2                 2    2     2             2  2
           I! .Al.Ir.G + 72.E .Ib .I! .Ir.Ar.G + 72.E .Ib .I!.Al.Ir .G + 72.E .Ib .I!.Ir .Ar.G + 24.E .Ib.L .I! .Al.Ir.Ar.G  + 24.E .Ib.L .I!.Al.Ir .Ar.G  + 72.E .Ib.I! .Al.Ir .G  + 72.E .Ib.I! .Ir .Ar.G  + 6.E .L .I! .Al

              2    5
           .Ir .Ar.G )

                  2  4             2  4             2  4             2  4            6          3          6          3            4                                       4                     6            4        2
MAT(3,1) := ( - 6.Ab.Ib .L .I!.Al - 6.Ab.Ib .L .I!.Ar - 6.Ab.Ib .L .Al.Ir - 6.Ab.Ib .L .Ir.Ar - 2.Ab.Ib.L .I!.Al.Ar.G  + Ab.Ib.L .Al.Ir.Ar.G  - 24.Ab.Ib.L .I!.Al.Ir.G - 24.Ab.Ib.L .I!.Ir.Ar.G - 2.Ab.L .I!.Al.Ir.Ar.G  - 72.Ib

            2              2  2             4            4               2  4              2              2  4              2               2  2  2               2  2  2
           .L .I!.Al.Ir.G - 72.Ib .L .I!.Ir.Ar.G + 12.Ib.L .I!.Al.Ir.Ar.G )/(12.E .Ab.Ib .L .I!.Al.Ar.G  + 12.E .Ab.Ib .L .Al.Ir.Ar.G  + 12.E .Ab.Ib .L .I! .Al + 12.E .Ab.Ib .L .I! .Ar + 24.E .Ab.Ib .L .I!.Al.Ir + 24.E .Ab.

            2  2                     2  2      2                2  2  2               4   2         3                  4                 3               4     2     3                  2  2                       2  2
           Ib .L .I!.Ir.Ar + 12.E .Ab.Ib .L .Al.Ir  + 12.E .Ab.Ib .L .Ir .Ar + 4.E .Ab.Ib.L .I! .Al.Ar.G  + 44.E .Ab.Ib.L .I!.Al.Ir.Ar.G  + 4.E .Ab.Ib.L .Al.Ir .Ar.G  + 48.E .Ab.Ib.L .I! .Al.Ir.G + 48.E .Ab.Ib.L .I! .Ir.

                  2       2             2      2              4  2          4           4        2    4           2  2              3              2  2                  2  2
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Ar.G + 48.E .Ab.Ib.L .Il.Al.Ir .G + 48.E .Ab.Ib.L .Il.Ir .Ar.G + 4.E .Ab.L .Il .Al.Ir.Ar.G  + 4.E .Ab.L .Il.Al.Ir .Ar.G  + 144.E +Ib .L .Il.Al.Ir.Ar.G  + 144.E .Ib .Il .Al.Ir.G + 144.E .Ib .Il .Ir.Ar.G + 144.E

 2      2            2     2            2   2          4            2        2    4             2      2 2                2   2     2          2    2     2    5
.Ib .Il.Al.Ir .G + 144.E .Ib .Il.Ir .Ar.G + 48.E .Ib.L .Il .Al.Ir.Ar.G  + 48.E .Ib.L .Il.Al.Ir .Ar.G  + 144.E .Ib.Il .Al.Ir .G  + 144.E .Ib.Il .Ir .Ar.G  + 12.E .L .Il .Al.Ir .Ar.G )

                  2 7       2          2 5           2 5          2 5          2 5              7         3           7          3          5                    5
MAT(4,1) := (3.Ab.Ib .L .Al.Ar.G  + 12.Ab.Ib .L .Il.Al + 12.Ab.Ib .L .Il.Ar + 12.Ab.Ib .L .Al.Ir + 12.Ab.Ib .L .Ir.Ar + 4.Ab.Ib.L .Il.Al.Ar.G  + 4.Ab.Ib.L .Al.Ir.Ar.G  + 48.Ab.Ib.L .Il.Al.Ir.G + 48.Ab.Ib.L .Il.Ir.Ar.G + 4.Ab.

 7             4       2 5         3        2 3  2         2 3  2          2 3             2 3           5  2       4        5             4        3   2     2
L .Il.Al.Ir.Ar.G  + 36.Ib .L .Il.Al.Ar.G  + 36.Ib .L .Il .Al.G + 36.Ib .L .Il .Ar.G + 144.Ib .L .Il.Al.Ir.G + 144.Ib .L .Il.Ir.Ar.G + 12.Ib.L .Il .Al.Ar.G  + 48.Ib.L .Il.Al.Ir.Ar.G  + 144.Ib.L .Il .Al.Ir.G  + 144

 3  2     2     5  2      5          2 4         2            2 4         2           2  2  2              2  2  2           2 2                2 2
.Ib.L .Il .Ir.Ar.G  + 12.L .Il .Al.Ir.Ar.G )/(36.E .Ab.Ib .L .Il.Al.Ar.G  + 36.E .Ab.Ib .L .Al.Ir.Ar.G  + 36.E .Ab.Ib .L .Il .Al + 36.E .Ab.Ib .L .Il .Ar + 72.E .Ab.Ib .L .Il.Al.Ir + 72.E .Ab.Ib .L .Il.Ir.Ar + 36

  2 2   2          2 2  2          4  2        3               4            3             4   2    3              2 2
.E .Ab.Ib .L .Al.Ir  + 36.E .Ab.Ib .L .Ir .Ar + 12.E .Ab.Ib.L .Il .Al.Ar.G  + 132.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 12.E .Ab.Ib.L .Al.Ir .Ar.G  + 144.E .Ab.Ib.L .Il .Al.Ir.G + 144.E .Ab.Ib.L .Il .Ir.Ar.G + 144.E .Ab

  2        2          2     2          4  2         4          4        2    4          2 2             3         2 2              2 2               2
.Ib.L .Il.Al.Ir .G + 144.E .Ab.Ib.L .Il.Ir .Ar.G + 12.E .Ab.L .Il .Al.Ir.Ar.G  + 12.E .Ab.L .Il.Al.Ir .Ar.G  + 432.E .Ib .L .Il.Al.Ir.Ar.G  + 432.E .Ib .Il .Al.Ir.G + 432.E .Ib .Il .Ir.Ar.G + 432.E .Ib .Il.Al.

 2          2    2            2  2          4          2        2    4            2      2 2             2  2    2          2  2   2    5
Ir .G + 432.E .Ib .Il.Ir .Ar.G + 144.E .Ib.L .Il .Al.Ir.Ar.G  + 144.E .Ib.L .Il.Al.Ir .Ar.G  + 432.E .Ib.Il .Al.Ir .G  + 432.E .Ib.Il .Ir .Ar.G  + 36.E .L .Il .Al.Ir .Ar.G )

                    2 5            2 5               5          2       2 3         2          3   2      3           2 4         2          2 4        2          2  2  2
MAT(5,1) := ( - 3.Ab.Ib .L .Il.Al.G - 3.Ab.Ib .L .Al.Ir.G - 12.Ab.Ib.L .Il.Al.Ir.G - 36.Ib .L .Il.Al.Ir.G - 18.Ib.L .Il .Al.Ir.G )/(6.E .Ab.Ib .L .Il.Al.Ar.G  + 6.E .Ab.Ib .L .Al.Ir.Ar.G  + 6.E .Ab.Ib .L .Il .Al + 6.E .Ab.

 2  2  2            2  2          2 2           2 2    2        2  2  2         4  2     3           4            3          4   2   3
Ib .L .Il .Ar + 12.E .Ab.Ib .L .Il.Al.Ir + 12.E .Ab.Ib .L .Il.Ir.Ar + 6.E .Ab.Ib .L .Al.Ir  + 6.E .Ab.Ib .L .Ir .Ar + 2.E .Ab.Ib.L .Il .Al.Ar.G  + 22.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.E .Ab.Ib.L .Al.Ir .Ar.G  + 24

    2  2                2  2              2       2              2     2           4  2          4            4        2    4           2  2            3
.E .Ab.Ib.L .Il .Al.Ir.G + 24.E .Ab.Ib.L .Il .Ir.Ar.G + 24.E .Ab.Ib.L .Il.Al.Ir .G + 24.E .Ab.Ib.L .Il.Ir .Ar.G + 2.E .Ab.L .Il .Al.Ir.Ar.G  + 2.E .Ab.L .Il.Al.Ir .Ar.G  + 72.E .Ib .L .Il.Al.Ir.Ar.G  + 72.E .

 2 2            2 2           2        2           2      2           2 2          4           2        2    4           2      2 2             2  2    2         2
Ib .Il .Al.Ir.G + 72.E .Ib .Il .Ir.Ar.G + 72.E .Ib .Il.Al.Ir .G + 72.E .Ib .Il.Ir .Ar.G + 24.E .Ib.L .Il .Al.Ir.Ar.G  + 24.E .Ib.L .Il.Al.Ir .Ar.G  + 72.E .Ib.Il .Al.Ir .G  + 72.E .Ib.Il .Ir .Ar.G  + 6.E .L .

 2      2    5
Il .Al.Ir .Ar.G )

                  2 4          2 4          2 4          2 4              6          3            6          3            4                4                  6            4       2
MAT(6,1) := ( - 6.Ab.Ib .L .Il.Al - 6.Ab.Ib .L .Il.Ar - 6.Ab.Ib .L .Al.Ir - 6.Ab.Ib .L .Ir.Ar + Ab.Ib.L .Il.Al.Ar.G  - 2.Ab.Ib.L .Al.Ir.Ar.G  - 24.Ab.Ib.L .Il.Al.Ir.G - 24.Ab.Ib.L .Il.Ir.Ar.G - 2.Ab.L .Il.Al.Ir.Ar.G  - 72.Ib

 2             2 2          4          4       2   2       2          2  2       2   4  2         5           2 4        2            2 4         2          2 2
.L .Il.Al.Ir.G - 72.Ib .L .Il.Ir.Ar.G - 24.Ib.L .Il.Al.Ir.Ar.G  - 72.Ib.L .Il .Al.Ir.G  - 72.Ib.L .Il .Ir.Ar.G  - 6.L .Il .Al.Ir.Ar.G )/(12.E .Ab.Ib .L .Il.Al.Ar.G  + 12.E .Ab.Ib .L .Al.Ir.Ar.G  + 12.E .Ab.Ib .L

 2          2  2  2          2 2             2 2           2 2    2        2  2  2         4  2      3            4            3
.Il .Al + 12.E .Ab.Ib .L .Il .Ar + 24.E .Ab.Ib .L .Il.Al.Ir + 24.E .Ab.Ib .L .Il.Ir.Ar + 12.E .Ab.Ib .L .Al.Ir  + 12.E .Ab.Ib .L .Ir .Ar + 4.E .Ab.Ib.L .Il .Al.Ar.G  + 44.E .Ab.Ib.L .Il.Al.Ir.Ar.G  + 4.E .Ab.Ib

 4   2   3          2 2              2 2             2       2             2     2          4  2         4           4        2    4            2 2
.L .Al.Ir .Ar.G  + 48.E .Ab.Ib.L .Il .Al.Ir.G + 48.E .Ab.Ib.L .Il .Ir.Ar.G + 48.E .Ab.Ib.L .Il.Al.Ir .G + 48.E .Ab.Ib.L .Il.Ir .Ar.G + 4.E .Ab.L .Il .Al.Ir.Ar.G  + 4.E .Ab.L .Il.Al.Ir .Ar.G  + 144.E .Ib .L .Il.

      3          2 2           2 2           2        2            2     2           2 2            4           2        2   4            2     2 2
Al.Ir.Ar.G  + 144.E .Ib .Il .Al.Ir.G + 144.E .Ib .Il .Ir.Ar.G + 144.E .Ib .Il.Al.Ir .G + 144.E .Ib .Il.Ir .Ar.G + 48.E .Ib.L .Il .Al.Ir.Ar.G  + 48.E .Ib.L .Il.Al.Ir .Ar.G  + 144.E .Ib.Il .Al.Ir .G  + 144.E .Ib.

 2   2    2        2  2    2    5
Il .Ir .Ar.G  + 12.E .L .Il .Al.Ir .Ar.G )

MAT(7,1) := 0

MAT(8,1) := 0

MAT(9,1) := 0
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WRITE "/.. INFLUENCE COEFFICIENTS FOR STRESS RESULTANTS .."$

/.. INFLUENCE COEFFICIENTS FOR STRESS RESULTANTS ..

 INFLU( )$

/..... FLOOR UDL CASE AT LEVEL=1 BAY=1

/ BASIC ELEMENT STRESSES

COLUMN MEMBER FROM NODE (-1) TO 1

                2 5          3           2 5          3           2 3  2               2 3                         2 3    2              5  2      4           5           4          5    2   4
PA := (12.Ab.Ib .L .I1.A1.Ar.G  + 12.Ab.Ib .L .A1.Ir.Ar.G  + 24.Ab.Ib .L .I1 .A1.G + 48.Ab.Ib .L .I1.A1.Ir.G + 24.Ab.Ib .L .A1.Ir .G + 5.Ab.Ib.L .I1 .A1.Ar.G  + 44.Ab.Ib.L .I1.A1.Ir.Ar.G  + 3.Ab.Ib.L .A1.Ir .Ar.G  + 96.Ab.Ib.

        3  2      2              3          2 2         5  2           5           5        2    5         2 3              4           2     2          2            2             2 2          3  2            5          3
       L .I1 .A1.Ir.G  + 96.Ab.Ib.L .I1.A1.Ir .G  + 4.Ab.L .I1 .A1.Ir.Ar.G  + 4.Ab.L .I1.A1.Ir .Ar.G  + 144.Ib .L .I1.A1.Ir.Ar.G  + 288.Ib .L.I1 .A1.Ir.G  + 288.Ib .L.I1.A1.Ir .G  + 60.Ib.L .I1 .A1.Ir.Ar.G  + 36.Ib.L .I1.A1.

        2    5               2    2 3    3  2    2    6          2 4          2            2 4           2             2 2  2            2 2  2            2 2                2 2                 2
       Ir .Ar.G  + 288.Ib.L.I1 .A1.Ir .G  + 12.L .I1 .A1.Ir .Ar.G )/(24.Ab.Ib .L .I1.A1.Ar.G  + 24.Ab.Ib .L .A1.Ir.Ar.G  + 24.Ab.Ib .L .I1 .A1 + 24.Ab.Ib .L .I1 .Ar + 48.Ab.Ib .L .I1.A1.Ir + 48.Ab.Ib .L .I1.Ir.Ar + 24.Ab.Ib .

        2     2         2 2  2          4  2      3          4             3           4   2    3        2 2               2 2              2        2            2    2
       L .A1.Ir  + 24.Ab.Ib .L .Ir .Ar + 8.Ab.Ib.L .I1 .A1.Ar.G  + 88.Ab.Ib.L .I1.A1.Ir.Ar.G  + 8.Ab.Ib.L .A1.Ir .Ar.G  + 96.Ab.Ib.L .I1 .A1.Ir.G + 96.Ab.Ib.L .I1 .Ir.Ar.G + 96.Ab.Ib.L .I1.A1.Ir .G + 96.Ab.Ib.L .I1.Ir .Ar.G

           4  2         4        4          2    4       2 2           3       2  2              2  2            2       2          2     2            2 2          4         2        2
        + 8.Ab.L .I1 .A1.Ir.Ar.G  + 8.Ab.L .I1.A1.Ir .Ar.G  + 288.Ib .L .I1.A1.Ir.Ar.G  + 288.Ib .I1 .A1.Ir.G + 288.Ib .I1 .Ir.Ar.G + 288.Ib .I1.A1.Ir .G + 288.Ib .I1.Ir .Ar.G + 96.Ib.L .I1 .A1.Ir.Ar.G  + 96.Ib.L .I1.A1.Ir

          4            2      2 2        2  2    2     2  2    2   5
       .Ar.G  + 288.Ib.I1 .A1.Ir .G  + 288.Ib.I1 .Ir .Ar.G  + 24.L .I1 .A1.Ir .Ar.G )

                  5              5          3  2       3          3  2      3            3         2 3              3    2    3     5  2         6      5      2    6          2 4         2
UA := ( - 12.Ab.Ib.L .I1.A1.Ir.Ar.G  + 24.Ab.Ib.L .I1 .A1.Ir.G  - 48.Ab.Ib.L .I1 .Ir.Ar.G  - 48.Ab.Ib.L .I1.A1.Ir .G  + 24.Ab.Ib.L .I1.Ir .Ar.G  - Ab.L .I1 .A1.Ir.Ar.G  - Ab.L .I1.A1.Ir .Ar.G )/(24.Ab.Ib .L .I1.A1.Ar.G  + 24.

        2 4         2        2 2  2          2 2  2          2 2            2 2             2 2     2          2 2  2          4  2      3          4              3
       Ab.Ib .L .A1.Ir.Ar.G  + 24.Ab.Ib .L .I1 .A1 + 24.Ab.Ib .L .I1 .Ar + 48.Ab.Ib .L .I1.A1.Ir + 48.Ab.Ib .L .I1.Ir.Ar + 24.Ab.Ib .L .A1.Ir  + 24.Ab.Ib .L .Ir .Ar + 8.Ab.Ib.L .I1 .A1.Ar.G  + 88.Ab.Ib.L .I1.A1.Ir.Ar.G  + 8

        4   2    3          2 2             2 2              2       2          2      2          4  2        4       4    2    4      2 2            3       2
       .Ab.Ib.L .A1.Ir .Ar.G  + 96.Ab.Ib.L .I1 .A1.Ir.G + 96.Ab.Ib.L .I1 .Ir.Ar.G + 96.Ab.Ib.L .I1.A1.Ir .G + 96.Ab.Ib.L .I1.Ir .Ar.G + 8.Ab.L .I1 .A1.Ir.Ar.G  + 8.Ab.L .I1.A1.Ir .Ar.G  + 288.Ib .L .I1.A1.Ir.Ar.G  + 288.Ib

        2           2 2        2     2         2    2          2 2          4          2      2   4      2   2 2        2 2  2      2 2    2  5
       .I1 .A1.Ir.G + 288.Ib .I1 .Ir.Ar.G + 288.Ib .I1.A1.Ir .G + 288.Ib .I1.Ir .Ar.G + 96.Ib.L .I1 .A1.Ir.Ar.G  + 96.Ib.L .I1.A1.Ir .Ar.G  + 288.Ib.I1 .A1.Ir .G  + 288.Ib.I1 .Ir .Ar.G  + 24.L .I1 .A1.Ir .Ar.G )

            2 4  2    2          2 4  2.  2          2 4         2           2 4          2           6  2       5          6          5           4  2       3            4  2      3
MA := (36.Ab.Ib .L .I1 .A1.G  - 36.Ab.Ib .L .I1 .Ar.G  + 36.Ab.Ib .L .I1.A1.Ir.G  - 36.Ab.Ib .L .I1.Ir.Ar.G  + 3.Ab.Ib.L .I1 .A1.Ar.G  - 15.Ab.Ib.L .I1.A1.Ir.Ar.G  + 168.Ab.Ib.L .I1 .A1.Ir.G  - 192.Ab.Ib.L .I1 .Ir.Ar.G  - 48.

          4     2 3          4     2    3      6  2         6      6        2    6       2 2  2       3         2 2  2       3        4  2        6          2   2       2 4       2 2
       Ab.Ib.L .I1.A1.Ir .G  + 24.Ab.Ib.L .I1.Ir .Ar.G  - Ab.L .I1 .A1.Ir.Ar.G  - Ab.L .I1.A1.Ir .Ar.G  + 432.Ib .L .I1 .A1.Ir.G  - 432.Ib .L .I1 .Ir.Ar.G  + 36.Ib.L .I1 .A1.Ir.Ar.G  + 288.Ib.L .I1 .A1.Ir .G  - 144.Ib.L .I1 .

        2   4   4  2   2   7          2 4         2          2 4          2          2 2  2          2 2  2           2 2            2 2              2 2      2          2 2
       Ir .Ar.G  + 6.L .I1 .A1.Ir .Ar.G )/(72.Ab.Ib .L .I1.A1.Ar.G  + 72.Ab.Ib .L .A1.Ir.Ar.G  + 72.Ab.Ib .L .I1 .A1 + 72.Ab.Ib .L .I1 .Ar + 144.Ab.Ib .L .I1.A1.Ir + 144.Ab.Ib .L .I1.Ir.Ar + 72.Ab.Ib .L .A1.Ir  + 72.Ab.Ib .L

        2           4  2       3             4              3           4   2   3           2 2             2 2                2       2               2      2          4  2
       .Ir .Ar + 24.Ab.Ib.L .I1 .A1.Ar.G  + 264.Ab.Ib.L .I1.A1.Ir.Ar.G  + 24.Ab.Ib.L .A1.Ir .Ar.G  + 288.Ab.Ib.L .I1 .A1.Ir.G + 288.Ab.Ib.L .I1 .Ir.Ar.G + 288.Ab.Ib.L .I1.A1.Ir .G + 288.Ab.Ib.L .I1.Ir .Ar.G + 24.Ab.L .I1 .

          4        4        2   4        2 2          3        2 2             2 2            2      2          2     2          2 2          4           2      2   4
       A1.Ir.Ar.G  + 24.Ab.L .I1.A1.Ir .Ar.G  + 864.Ib .L .I1.A1.Ir.Ar.G  + 864.Ib .I1 .A1.Ir.G + 864.Ib .I1 .Ir.Ar.G + 864.Ib .I1.A1.Ir .G + 864.Ib .I1.Ir .Ar.G + 288.Ib.L .I1 .A1.Ir.Ar.G  + 288.Ib.L .I1.A1.Ir .Ar.G  + 864

          2     2 2           2   2   2      2   2    2   5
       .Ib.I1 .A1.Ir .G  + 864.Ib.I1 .Ir .Ar.G  + 72.L .I1 .A1.Ir .Ar.G )

              2 4  2    2            2 4  2    2           2 4          2           2 4          2          6  2        5           6           5            4  2       3           4  2      3
MB := ( - 36.Ab.Ib .L .I1 .A1.G  + 36.Ab.Ib .L .I1 .Ar.G  - 36.Ab.Ib .L .I1.A1.Ir.G  + 36.Ab.Ib .L .I1.Ir.Ar.G  - 3.Ab.Ib.L .I1 .A1.Ar.G  - 21.Ab.Ib.L .I1.A1.Ir.Ar.G  - 96.Ab.Ib.L .I1 .A1.Ir.G  + 48.Ab.Ib.L .I1 .Ir.Ar.G  - 96
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      4         2 3          4     2    3       6  2        6        6       2   6        2 2  2        3        2 2  2        3        4  2           6           2  2       2 4         2
 .Ab.Ib.L .Il.Al.Ir .G  + 48.Ab.Ib.L .Il.Ir .Ar.G  - 2.Ab.L .Il .Al.Ir.Ar.G  - 2.Ab.L .Il.Al.Ir .Ar.G  - 432.Ib .L .Il .Al.Ir.G  + 432.Ib .L .Il .Ir.Ar.G  - 36.Ib.L .Il .Al.Ir.Ar.G  - 288.Ib.L .Il .Al.Ir .G  + 144.Ib.L

  2  2     4     4   2     2    7          2 4          2          2 4         2           2 2  2            2 2  2              2 2               2 2                2 2     2
 .Il .Ir .Ar.G  - 6.L .Il .Al.Ir .Ar.G )/(72.Ab.Ib .L .Il.Al.Ar.G  + 72.Ab.Ib .L .Al.Ir.Ar.G  + 72.Ab.Ib .L .Il .Al + 72.Ab.Ib .L .Il .Ar + 144.Ab.Ib .L .Il.Al.Ir + 144.Ab.Ib .L .Il.Ir.Ar + 72.Ab.Ib .L .Al.Ir  + 72.Ab.

   2 2  2           4   2         3           4          3          4    2    3           2  2             2  2            2         2           2     2            4
  Ib .L .Ir .Ar + 24.Ab.Ib.L .Il .Al.Ar.G  + 264.Ab.Ib.L .Il.Al.Ir.Ar.G  + 24.Ab.Ib.L .Al.Ir .Ar.G  + 288.Ab.Ib.L .Il .Al.Ir.G + 288.Ab.Ib.L .Il .Ir.Ar.G + 288.Ab.Ib.L .Il.Al.Ir .G + 288.Ab.Ib.L .Il.Ir .Ar.G + 24.Ab.L

   2           4          4          2    4      2 2          3          2  2            2  2            2         2          2     2          2  2          4          2       2  4
  .Il .Al.Ir.Ar.G  + 24.Ab.L .Il.Al.Ir .Ar.G  + 864.Ib .L .Il.Al.Ir.Ar.G  + 864.Ib .Il .Al.Ir.G + 864.Ib .Il .Ir.Ar.G + 864.Ib .Il.Al.Ir .G + 864.Ib .Il.Ir .Ar.G + 288.Ib.L .Il .Al.Ir.Ar.G  + 288.Ib.L .Il.Al.Ir .Ar.G

          2     2 2          2  2    2      2  2    2    5
  + 864.Ib.Il .Al.Ir .G  + 864.Ib.Il .Ir .Ar.G  + 72.L .Il .Al.Ir .Ar.G )

COLUMN MEMBER FROM NODE 0 TO 2

        2 5        3           2 5          3           2  3  2               2 3                2 3  2          5  2        4           5            4          5   2  4
PA := (12.Ab.Ib .L .Il.Al.Ar.G  + 12.Ab.Ib .L .Al.Ir.Ar.G  + 24.Ab.Ib .L .Il .Ar.G + 48.Ab.Ib .L .Il.Ir.Ar.G + 24.Ab.Ib .L .Ir .Ar.G + 3.Ab.Ib.L .Il .Al.Ar.G  + 44.Ab.Ib.L .Il.Al.Ir.Ar.G  + 5.Ab.Ib.L .Al.Ir .Ar.G  + 96.Ab.Ib.

   3  2       2            3    2    2        5  2         5       5         2    5          2 3               4        2    2          2     2         2    2          3  2          5       3
  L .Il .Ir.Ar.G  + 96.Ab.Ib.L .Il.Ir .Ar.G  + 4.Ab.L .Il .Al.Ir.Ar.G  + 4.Ab.L .Il.Al.Ir .Ar.G  + 144.Ib .L .Il.Al.Ir.Ar.G  + 288.Ib .L.Il .Ir.Ar.G  + 288.Ib .L.Il.Ir .Ar.G  + 36.Ib.L .Il .Al.Ir.Ar.G  + 60.Ib.L .Il.Al.

  2     5            2  2    3       3    2     2    6           2 4          2          2 4           2            2 2  2            2 2  2              2 2               2 2         2
 Ir .Ar.G  + 288.Ib.L.Il .Ir .Ar.G  + 12.L .Il .Al.Ir .Ar.G )/(24.Ab.Ib .L .Il.Al.Ar.G  + 24.Ab.Ib .L .Al.Ir.Ar.G  + 24.Ab.Ib .L .Il .Al + 24.Ab.Ib .L .Il .Ar + 48.Ab.Ib .L .Il.Al.Ir + 48.Ab.Ib .L .Il.Ir.Ar + 24.Ab.Ib

  2     2          2 2  2           4   2      3            4            3           4   2    3           2  2              2  2             2         2             2     2
  L .Al.Ir  + 24.Ab.Ib .L .Ir .Ar + 8.Ab.Ib.L .Il .Al.Ar.G  + 88.Ab.Ib.L .Il.Al.Ir.Ar.G  + 8.Ab.Ib.L .Al.Ir .Ar.G  + 96.Ab.Ib.L .Il .Al.Ir.G + 96.Ab.Ib.L .Il .Ir.Ar.G + 96.Ab.Ib.L .Il.Al.Ir .G + 96.Ab.Ib.L .Il.Ir .Ar.G

      4   2          4        4          2     4          2 2           3           2   2           2  2           2         2          2     2           2  2          4            2        2
  + 8.Ab.L .Il .Al.Ir.Ar.G  + 8.Ab.L .Il.Al.Ir .Ar.G  + 288.Ib .L .Il.Al.Ir.Ar.G  + 288.Ib .Il .Al.Ir.G + 288.Ib .Il .Ir.Ar.G + 288.Ib .Il.Al.Ir .G + 288.Ib .Il.Ir .Ar.G + 96.Ib.L .Il .Al.Ir.Ar.G  + 96.Ib.L .Il.Al.Ir

     4          2     2 2          2   2    2       2  2     2     5
  .Ar.G  + 288.Ib.Il .Al.Ir .G  + 288.Ib.Il .Ir .Ar.G  + 24.L .Il .Al.Ir .Ar.G )

            5             5            3  2        3          3  2       3           3          2 3            3     2     3      5  2           6       5         2    6         2 4          2
UA := (12.Ab.Ib.L .Il.Al.Ir.Ar.G  - 24.Ab.Ib.L .Il .Al.Ir.G  + 48.Ab.Ib.L .Il .Ir.Ar.G  + 48.Ab.Ib.L .Il.Al.Ir .G  - 24.Ab.Ib.L .Il.Ir .Ar.G  + Ab.L .Il .Al.Ir.Ar.G  + Ab.L .Il.Al.Ir .Ar.G )/(24.Ab.Ib .L .Il.Al.Ar.G  + 24.Ab.

  2 4          2           2 2  2            2 2  2            2 2               2 2               2 2     2           2 2  2          4   2        3           4            3
 Ib .L .Al.Ir.Ar.G  + 24.Ab.Ib .L .Il .Al + 24.Ab.Ib .L .Il .Ar + 48.Ab.Ib .L .Il.Al.Ir + 48.Ab.Ib .L .Il.Ir.Ar + 24.Ab.Ib .L .Al.Ir  + 24.Ab.Ib .L .Ir .Ar + 8.Ab.Ib.L .Il .Al.Ar.G  + 88.Ab.Ib.L .Il.Al.Ir.Ar.G  + 8.Ab

   4    2    3            2  2              2  2             2         2           2     2           4  2           4        4           2    4         2 2              3          
 .Ib.L .Al.Ir .Ar.G  + 96.Ab.Ib.L .Il .Al.Ir.G + 96.Ab.Ib.L .Il .Ir.Ar.G + 96.Ab.Ib.L .Il.Al.Ir .G + 96.Ab.Ib.L .Il.Ir .Ar.G + 8.Ab.L .Il .Al.Ir.Ar.G  + 8.Ab.L .Il.Al.Ir .Ar.G  + 288.Ib .L .Il.Al.Ir.Ar.G  + 288.Ib .

  2          2  2           2         2          2     2           2  2          4            2        2   4          2     2 2          2 2   2         2 2   2    5
 Il .Al.Ir.G + 288.Ib .Il .Ir.Ar.G + 288.Ib .Il.Al.Ir .G + 288.Ib .Il.Ir .Ar.G + 96.Ib.L .Il .Al.Ir.Ar.G  + 96.Ib.L .Il.Al.Ir .Ar.G  + 288.Ib.Il .Al.Ir .G  + 288.Ib.Il .Ir .Ar.G  + 24.L .Il .Al.Ir .Ar.G )

       2 4          2            2 4         2          2 4    2 2         2 4   2    2          6            5           6    2     5          4  2        3            4  2     3
MA := (36.Ab.Ib .L .Il.Al.Ir.G  - 36.Ab.Ib .L .Il.Ir.Ar.G  + 36.Ab.Ib .L .Al.Ir .G  - 36.Ab.Ib .L .Ir .Ar.G  + 15.Ab.Ib.L .Il.Al.Ir.Ar.G  - 3.Ab.Ib.L .Al.Ir .Ar.G  - 24.Ab.Ib.L .Il .Al.Ir.G  + 48.Ab.Ib.L .Il .Ir.Ar.G  + 192.

     4        2 3           4      2     3      6  2        6       6        2    6         2 2          2 3          2 2     2    3           4        2    6           2  2       2 4        2  2
 Ab.Ib.L .Il.Al.Ir .G  - 168.Ab.Ib.L .Il.Ir .Ar.G  + Ab.L .Il .Al.Ir.Ar.G  + Ab.L .Il.Al.Ir .Ar.G  + 432.Ib .L .Il.Al.Ir .G  - 432.Ib .L .Il.Ir .Ar.G  - 36.Ib.L .Il.Al.Ir .Ar.G  + 144.Ib.L .Il .Al.Ir .G  - 288.Ib.L .Il

  2     4     4    2    2   7          2 4          2          2 4         2           2 2  2            2 2  2              2 2               2 2                2 2     2         2
 .Ir .Ar.G  - 6.L .Il .Al.Ir .Ar.G )/(72.Ab.Ib .L .Il.Al.Ar.G  + 72.Ab.Ib .L .Al.Ir.Ar.G  + 72.Ab.Ib .L .Il .Al + 72.Ab.Ib .L .Il .Ar + 144.Ab.Ib .L .Il.Al.Ir + 144.Ab.Ib .L .Il.Ir.Ar + 72.Ab.Ib .L .Al.Ir  + 72.Ab.Ib .

  2  2           4   2        3           4          3          4    2    3           2  2             2  2            2         2           2     2           4  2
  L .Ir .Ar + 24.Ab.Ib.L .Il .Al.Ar.G  + 264.Ab.Ib.L .Il.Al.Ir.Ar.G  + 24.Ab.Ib.L .Al.Ir .Ar.G  + 288.Ab.Ib.L .Il .Al.Ir.G + 288.Ab.Ib.L .Il .Ir.Ar.G + 288.Ab.Ib.L .Il.Al.Ir .G + 288.Ab.Ib.L .Il.Ir .Ar.G + 24.Ab.L .Il

      4          4        2    4       2 2          3          2  2            2  2            2         2          2     2          2  2          4          2       2  4
  .Al.Ir.Ar.G  + 24.Ab.L .Il.Al.Ir .Ar.G  + 864.Ib .L .Il.Al.Ir.Ar.G  + 864.Ib .Il .Al.Ir.G + 864.Ib .Il .Ir.Ar.G + 864.Ib .Il.Al.Ir .G + 864.Ib .Il.Ir .Ar.G + 288.Ib.L .Il .Al.Ir.Ar.G  + 288.Ib.L .Il.Al.Ir .Ar.G  +

            2     2 2          2  2    2      2  2    2    5
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864.Ib.Il .Al.Ir .G  + 864.Ib.Il .Ir .Ar.G  + 72.L .Il .Al.Ir .Ar.G )

            2 4         2         2 4         2         2 4    2 2         2 4  2   2          6           5          6    2   5          4  2       3          4  2     3
MB := ( - 36.Ab.Ib .L .Il.Al.Ir.G  + 36.Ab.Ib .L .Il.Ir.Ar.G  - 36.Ab.Ib .L .Al.Ir .G  + 36.Ab.Ib .L .Ir .Ar.G  + 21.Ab.Ib.L .Il.Al.Ir.Ar.G  + 3.Ab.Ib.L .Al.Ir .Ar.G  - 48.Ab.Ib.L .Il .Al.Ir.G  + 96.Ab.Ib.L .Il .Ir.Ar.G  - 48

      4        2 3          4      2    3        6  2          6          6        2    6        2 2          2 3        2 2      2     3          4        2    6          2   2      2 4      2
.Ab.Ib.L .Il.Al.Ir .G  + 96.Ab.Ib.L .Il.Ir .Ar.G  + 2.Ab.L .Il .Al.Ir.Ar.G  + 2.Ab.L .Il.Al.Ir .Ar.G  - 432.Ib .L .Il.Al.Ir .G  + 432.Ib .L .Il.Ir .Ar.G  + 36.Ib.L .Il.Al.Ir .Ar.G  - 144.Ib.L .Il .Al.Ir .G  + 288.Ib.L

 2  2     4     4   2     2     7          2 4          2          2 4          2          2 2  2           2 2  2           2 2                2 2                2 2     2
.Il .Ir .Ar.G  + 6.L .Il .Al.Ir .Ar.G )/(72.Ab.Ib .L .Il.Al.Ar.G  + 72.Ab.Ib .L .Al.Ir.Ar.G  + 72.Ab.Ib .L .Il .Al + 72.Ab.Ib .L .Il .Ar + 144.Ab.Ib .L .Il.Al.Ir + 144.Ab.Ib .L .Il.Ir.Ar + 72.Ab.Ib .L .Al.Ir  + 72.Ab.

 2  2  2          4  2       3           4               3           4    2   3             2  2              2  2                2      2                2      2               4
Ib .L .Ir .Ar + 24.Ab.Ib.L .Il .Al.Ar.G  + 264.Ab.Ib.L .Il.Al.Ir.Ar.G  + 24.Ab.Ib.L .Al.Ir .Ar.G  + 288.Ab.Ib.L .Il .Al.Ir.G + 288.Ab.Ib.L .Il .Ir.Ar.G + 288.Ab.Ib.L .Il.Al.Ir .G + 288.Ab.Ib.L .Il.Ir .Ar.G + 24.Ab.L

 2          4          4         2    4          2 2          3          2 2            2 2          2       2         2     2          2 2          4         2      2   4
.Il .Al.Ir.Ar.G  + 24.Ab.L .Il.Al.Ir .Ar.G  + 864.Ib .L .Il.Al.Ir.Ar.G  + 864.Ib .Il .Al.Ir.G + 864.Ib .Il .Ir.Ar.G + 864.Ib .Il.Al.Ir .G + 864.Ib .Il.Ir .Ar.G + 288.Ib.L .Il .Al.Ir.Ar.G  + 288.Ib.L .Il.Al.Ir .Ar.G

        2     2 2          2  2    2      2   2     2     5
 + 864.Ib.Il .Al.Ir .G  + 864.Ib.Il .Ir .Ar.G  + 72.L .Il .Al.Ir .Ar.G )

BEAM MEMBER FROM NODE 1 TO 2

             5              5             3 2        3              3 2       3              3         2 3             3     2    3       5  2          6     5         2    6         2 4          2
PA := (12.Ab.Ib.L .Il.Al.Ir.Ar.G  - 24.Ab.Ib.L .Il .Al.Ir.G  + 48.Ab.Ib.L .Il .Ir.Ar.G  + 48.Ab.Ib.L .Il.Al.Ir .G  - 24.Ab.Ib.L .Il.Ir .Ar.G  + Ab.L .Il .Al.Ir.Ar.G  + Ab.L .Il.Al.Ir .Ar.G )/(24.Ab.Ib .L .Il.Al.Ar.G  + 24.Ab.

 2 4          2          2 2  2          2 2  2          2 2               2 2              2 2    2          2 2  2          4  2         3           4          3
Ib .L .Al.Ir.Ar.G  + 24.Ab.Ib .L .Il .Al + 24.Ab.Ib .L .Il .Ar + 48.Ab.Ib .L .Il.Al.Ir + 48.Ab.Ib .L .Il.Ir.Ar + 24.Ab.Ib .L .Al.Ir  + 24.Ab.Ib .L .Ir .Ar + 8.Ab.Ib.L .Il .Al.Ar.G  + 88.Ab.Ib.L .Il.Al.Ir.Ar.G  + 8.Ab

   4    2   3            2  2                2  2                 2      2              2      2         4  2           4        4        2    4         2  2            3      2
.Ib.L .Al.Ir .Ar.G  + 96.Ab.Ib.L .Il .Al.Ir.G + 96.Ab.Ib.L .Il .Ir.Ar.G + 96.Ab.Ib.L .Il.Al.Ir .G + 96.Ab.Ib.L .Il.Ir .Ar.G + 8.Ab.L .Il .Al.Ir.Ar.G  + 8.Ab.L .Il.Al.Ir .Ar.G  + 288.Ib .L .Il.Al.Ir.Ar.G  + 288.Ib .

 2           2  2          2      2        2      2          2  2          4         2        2    4         2    2 2          2   2   2         2   2    2    5
Il .Al.Ir.G + 288.Ib .Il .Ir.Ar.G + 288.Ib .Il.Al.Ir .G + 288.Ib .Il.Ir .Ar.G + 96.Ib.L .Il .Al.Ir.Ar.G  + 96.Ib.L .Il.Al.Ir .Ar.G  + 288.Ib.Il .Al.Ir .G  + 288.Ib.Il .Ir .Ar.G  + 24.L .Il .Al.Ir .Ar.G )

    2 3  2          2 3  2          2 3               2 3             2 3     2          2 3  2          5  2         4         5     2   4          3  2
UA := (12.Ab.Ib .L .Il .Al.G - 12.Ab.Ib .L .Il .Ar.G + 24.Ab.Ib .L .Il.Al.Ir.G - 24.Ab.Ib .L .Il.Ir.Ar.G + 12.Ab.Ib .L .Al.Ir .G - 12.Ab.Ib .L .Ir .Ar.G + Ab.Ib.L .Il .Al.Ar.G  - Ab.Ib.L .Al.Ir .Ar.G  + 48.Ab.Ib.L .Il .Al.Ir.

2          3  2       2           3       2 2          3    2   2        2    2       2      2    2      2       2       2 2        2      2    2        3  2          5
G  - 48.Ab.Ib.L .Il .Ir.Ar.G  + 48.Ab.Ib.L .Il.Al.Ir .G  - 48.Ab.Ib.L .Il.Ir .Ar.G  + 144.Ib .L.Il .Al.Ir.G  - 144.Ib .L.Il .Ir.Ar.G  + 144.Ib .L.Il.Al.Ir .G  - 144.Ib .L.Il.Ir .Ar.G  + 12.Ib.L .Il .Al.Ir.Ar.G  - 12.Ib

3      2    5          2     2 3         2  2    3         2 4        2          2 4          2          2 2  2         2 2  2          2 2               2 2
.L .Il.Al.Ir .Ar.G  + 144.Ib.L.Il .Al.Ir .G  - 144.Ib.L.Il .Ir .Ar.G )/(24.Ab.Ib .L .Il.Al.Ar.G  + 24.Ab.Ib .L .Al.Ir.Ar.G  + 24.Ab.Ib .L .Il .Al + 24.Ab.Ib .L .Il .Ar + 48.Ab.Ib .L .Il.Al.Ir + 48.Ab.Ib .L .Il.Ir.Ar +

      2 2     2        2 2  2          4  2       3           4          3          4    2   3            2  2              2  2                2      2               2
24.Ab.Ib .L .Al.Ir  + 24.Ab.Ib .L .Ir .Ar + 8.Ab.Ib.L .Il .Al.Ar.G  + 88.Ab.Ib.L .Il.Al.Ir.Ar.G  + 8.Ab.Ib.L .Al.Ir .Ar.G  + 96.Ab.Ib.L .Il .Al.Ir.G + 96.Ab.Ib.L .Il .Ir.Ar.G + 96.Ab.Ib.L .Il.Al.Ir .G + 96.Ab.Ib.L .

  2          4 2          4        4        2    4         2 2          3          2 2             2 2           2      2          2     2           2  2          4
Il.Ir .Ar.G + 8.Ab.L .Il .Al.Ir.Ar.G  + 8.Ab.L .Il.Al.Ir .Ar.G  + 288.Ib .L .Il.Al.Ir.Ar.G  + 288.Ib .Il .Al.Ir.G + 288.Ib .Il .Ir.Ar.G + 288.Ib .Il.Al.Ir .G + 288.Ib .Il.Ir .Ar.G + 96.Ib.L .Il .Al.Ir.Ar.G  + 96.Ib.

 2      2   4         2     2 2          2   2   2        2 2      2    5
L .Il.Al.Ir .Ar.G  + 288.Ib.Il .Al.Ir .G  + 288.Ib.Il .Ir .Ar.G  + 24.L .Il .Al.Ir .Ar.G )

         2 6         4          2 6          4          2 4  2   2          2 4  2   2          2 4          2          2 4          2          2 4    2 2          2 4  2   2       6
MA := ( - 6.Ab.Ib .L .Il.Al.Ar.G  - 6.Ab.Ib .L .Al.Ir.Ar.G  + 30.Ab.Ib .L .Il .Al.G  - 42.Ab.Ib .L .Il .Ar.G  + 24.Ab.Ib .L .Il.Al.Ir.G  - 48.Ab.Ib .L .Il.Ir.Ar.G  - 6.Ab.Ib .L .Al.Ir .G  - 6.Ab.Ib .L .Ir .Ar.G  + Ab.Ib.L .

 2        5          6           5          6    2   5         4  2       3          4  2     3          4        2 3          4    2    3        2 4            5         2 2  2
Il .Al.Ar.G  - Ab.Ib.L .Il.Al.Ir.Ar.G  - 2.Ab.Ib.L .Al.Ir .Ar.G  + 72.Ab.Ib.L .Il .Al.Ir.G  - 72.Ab.Ib.L .Il .Ir.Ar.G  + 72.Ab.Ib.L .Il.Al.Ir .G  - 72.Ab.Ib.L .Il.Ir .Ar.G  - 72.Ib .L .Il.Al.Ir.Ar.G  + 360.Ib .L .Il .

 3         2 2  2       3          2 2         2 3        2 2       2   3         4   2        6         4        2   6          2  2       2 4          2 2  2       4          2 4           2
Al.Ir.G  - 504.Ib .L .Il .Ir.Ar.G  - 72.Ib .L .Il.Al.Ir .G  - 72.Ib .L .Il.Ir .Ar.G  + 12.Ib.L .Il .Al.Ir.Ar.G  - 24.Ib.L .Il.Al.Ir .Ar.G  + 216.Ib.L .Il .Al.Ir .G  - 216.Ib.L .Il .Ir .Ar.G )/(72.Ab.Ib .L .Il.Al.Ar.G

     2 4          2          2 2  2          2 2  2          2 2               2 2              2 2    2           2 2  2          4  2       3           4
 + 72.Ab.Ib .L .Al.Ir.Ar.G  + 72.Ab.Ib .L .Il .Al + 72.Ab.Ib .L .Il .Ar + 144.Ab.Ib .L .Il.Al.Ir + 144.Ab.Ib .L .Il.Ir.Ar + 72.Ab.Ib .L .Al.Ir  + 72.Ab.Ib .L .Ir .Ar + 24.Ab.Ib.L .Il .Al.Ar.G  + 264.Ab.Ib.L .Il.Al.Ir
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          3                4    2    3             2    2                2    2                 2        2                 2      2           4  2          4        4       2    4       2  2
.Ar.G  + 24.Ab.Ib.L .Al.Ir .Ar.G  + 288.Ab.Ib.L .Il .Al.Ir.G + 288.Ab.Ib.L .Il .Ir.Ar.G + 288.Ab.Ib.L .Il.Al.Ir .G + 288.Ab.Ib.L .Il.Ir .Ar.G + 24.Ab.L .Il .Al.Ir.Ar.G  + 24.Ab.L .Il.Al.Ir .Ar.G  + 864.Ib .L .Il.Al.

      3           2  2              2  2              2        2         2     2              2  2          4            2       2    4            2     2  2          2     2     2      2  2
Ir.Ar.G  + 864.Ib .Il .Al.Ir.G + 864.Ib .Il .Ir.Ar.G + 864.Ib .Il.Al.Ir .G + 864.Ib .Il.Ir .Ar.G + 288.Ib.L .Il .Al.Ir.Ar.G + 288.Ib.L .Il.Al.Ir .Ar.G  + 864.Ib.Il .Al.Ir .G  + 864.Ib.Il .Ir .Ar.G  + 72.L .Il .Al.

  2    5
Ir .Ar.G )
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           2  6           4          2  6          4          2  4    2     2           2  4   2     2           2  4           2          2  4          2          2  4     2  2          2  4   2    2               6  2
MB := (6.Ab.Ib .L .Il.Al.Ar.G  + 6.Ab.Ib .L .Al.Ir.Ar.G  + 6.Ab.Ib .L .Il .Al.G  + 6.Ab.Ib .L .Il .Ar.G  + 48.Ab.Ib .L .Il.Al.Ir.G  - 24.Ab.Ib .L .Il.Ir.Ar.G  + 42.Ab.Ib .L .Al.Ir .G  - 30.Ab.Ib .L .Ir .Ar.G  + 2.Ab.Ib.L .Il

        5          6          5        6     2    5            4   2      3            4   2      3             4          2  3            4      2     3       2  4             5      2  2  2
.Al.Ar.G  + Ab.Ib.L .Il.Al.Ir.Ar.G  - Ab.Ib.L .Al.Ir .Ar.G  + 72.Ab.Ib.L .Il .Al.Ir.G  - 72.Ab.Ib.L .Il .Ir.Ar.G  + 72.Ab.Ib.L .Il.Al.Ir .G  - 72.Ab.Ib.L .Il.Ir .Ar.G  + 72.Ib .L .Il.Al.Ir.Ar.G  + 72.Ib .L .Il .Al.Ir.

3          2  2  2        3          2  2         2  3          2  2     2     3         4   2          6          4       2     6           2    2     2  4           2   2   2     4          2  4         2
G  + 72.Ib .L .Il .Ir.Ar.G  + 504.Ib .L .Il.Al.Ir .G  - 360.Ib .L .Il.Ir .Ar.G  + 24.Ib.L .Il .Al.Ir.Ar.G  - 12.Ib.L .Il.Al.Ir .Ar.G  + 216.Ib.L .Il .Al.Ir .G  - 216.Ib.L .Il .Ir .Ar.G )/(72.Ab.Ib .L .Il.Al.Ar.G  + 72.

   2  4          2          2  2  2          2  2  2           2  2                2  2               2  2      2          2  2  2           4   2        3                4         3
Ab.Ib .L .Al.Ir.Ar.G  + 72.Ab.Ib .L .Il .Al + 72.Ab.Ib .L .Il .Ar + 144.Ab.Ib .L .Il.Al.Ir + 144.Ab.Ib .L .Il.Ir.Ar + 72.Ab.Ib .L .Al.Ir  + 72.Ab.Ib .L .Ir .Ar + 24.Ab.Ib.L .Il .Al.Ar.G  + 264.Ab.Ib.L .Il.Al.Ir.Ar.G

           4     2    3            2  2             2  2               2       2              2    2           4  2          4        4       2    4       2  2
 + 24.Ab.Ib.L .Al.Ir .Ar.G  + 288.Ab.Ib.L .Il .Al.Ir.G + 288.Ab.Ib.L .Il .Ir.Ar.G + 288.Ab.Ib.L .Il.Al.Ir .G + 288.Ab.Ib.L .Il.Ir .Ar.G + 24.Ab.L .Il .Al.Ir.Ar.G  + 24.Ab.L .Il.Al.Ir .Ar.G  + 864.Ib .L .Il.Al.Ir.Ar.

  3          2  2             2  2              2        2          2     2              2  2          4            2        2    4            2     2  2         2     2     2        2  2     2
G  + 864.Ib .Il .Al.Ir.G + 864.Ib .Il .Ir.Ar.G + 864.Ib .Il.Al.Ir .G + 864.Ib .Il.Ir .Ar.G + 288.Ib.L .Il .Al.Ir.Ar.G  + 288.Ib.L .Il.Al.Ir .Ar.G  + 864.Ib.Il .Al.Ir .G  + 864.Ib.Il .Ir .Ar.G  + 72.L .Il .Al.Ir .Ar.

 5
G )
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..... WIND LOAD CASE .....

/ BASIC ELEMENT STRESSES

COLUMN MEMBER FROM NODE (-1) TO 1

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              2  4                  2  4                    4                2       2  2            2         2  2          3          2  4          2          2  4          2          2  2  2           2
PA := ( - 3.Ab.Ib .L .Il.Al.Ar.G - 3.Ab.Ib .L .Al.Ir.Ar.G - 12.Ab.Ib.L .Il.Al.Ir.Ar.G  - 36.Ib .L .Il.Al.Ir.Ar.G  - 18.Ib.L .Il .Al.Ir.Ar.G )/(6.Ab.Ib .L .Il.Al.Ar.G  + 6.Ab.Ib .L .Al.Ir.Ar.G  + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib

   2  2              2  2             2  2              2  2     2          2  2  2            4   2        3           4            3          4   2       3            2  2
L .Il .Ar + 12.Ab.Ib .L .Il.Al.Ir + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir  + 6.Ab.Ib .L .Ir .Ar + 2.Ab.Ib.L .Il .Al.Ar.G  + 22.Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.Ab.Ib L .Al.Ir .Ar.G  + 24.Ab.Ib.L .Il .Al.Ir.G + 24.Ab.

     2  2              2         2             2    2            4   2          4           4       2    4          2  2            3       2  2               2   2            2        2
Ib.L .Il .Ir.Ar.G + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L .Il .Al.Ir.Ar.G  + 2.Ab.L .Il.Al.Ir .Ar.G  + 72.Ib .L .Il.Al.Ir.Ar.G  + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib .Il.Al.Ir .G + 72

      2     2            2  2           4          2        2    4          2       2  2          2  2     2       2  2     2    5
.Ib .Il.Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G  + 24.Ib.L .Il.Al.Ir .Ar.G  + 72.Ib.Il .Al.Ir .G  + 72.Ib.Il .Ir .Ar.G  + 6.L .Il .Al.Ir .Ar.G )
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          2  4            2           2  2  2           2  2  2          2  2              2  2               4   2        3            4            3             2  2               2  2
UA := (6.Ab.Ib .L .Il.Al.Ar.G  + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib .L .Il .Ar + 6.Ab.Ib .L .Il.Al.Ir + 6.Ab.Ib .L .Il.Ir.Ar + 2.Ab.Ib.L .Il .Al.Ar.G  + 11.Ab.Ib.L .Il.Al.Ir.Ar.G  + 24.Ab.Ib.L .Il .Al.Ir.G + 24.Ab.Ib.L .Il .Ir.Ar.G

        4  2          4          2  4           2          2  4            2          2  2  2          2  2  2            2  2                2  2               2  2     2            2  2  2
 + 2.Ab.L .Il .Al.Ir.Ar.G )/(6.Ab.Ib .L .Il.Al.Ar.G  + 6.Ab.Ib .L .Al.Ir.Ar.G  + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib .L .Il .Ar + 12.Ab.Ib .L .Il.Al.Ir + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir  + 6.Ab.Ib .L .Ir .Ar + 2.Ab.

    4  2      3            4            3           4   2      3           2  2                 2  2                2        2               2        2         4  2          4      4
Ib.L .Il .Al.Ar.G  + 22.Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.Ab.Ib.L .Al.Ir .Ar.G  + 24.Ab.Ib.L .Il .Al.Ir.G + 24.Ab.Ib.L .Il .Ir.Ar.G + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L .Il .Al.Ir.Ar.G  + 2.Ab.L .Il

    2    4         2  2           3         2  2           2  2               2         2         2   2          2  2        4            2        2    4          2       2  2        2  2
.Al.Ir .Ar.G  + 72.Ib .L .Il.Al.Ir.Ar.G  + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib .Il.Al.Ir .G + 72.Ib .Il.Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G  + 24.Ib.L .Il.Al.Ir .Ar.G  + 72.Ib.Il .Al.Ir .G  + 72.Ib.Il .Ir

  2      2  2   2    5
.Ar.G  + 6.L .Il .Al.Ir .Ar.G )
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          2 5          2        2  3  2          2  3  2          2 3            2 3             5  2        3            5              3             3  2               3  2
MA := (3.Ab.Ib .L .Il.Al.Ar.G  + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib .L .Il .Ar + 6.Ab.Ib .L .Il.Al.Ir + 6.Ab.Ib .L .Il.Ir.Ar + 2.Ab.Ib.L .Il .Al.Ar.G  + 5.Ab.Ib.L .Il.Al.Ir.Ar.G  + 24.Ab.Ib.L .Il .Al.Ir.G + 24.Ab.Ib.L .Il .Ir.Ar.G

           5  2            4        2      2           2     2            3  2           4        2 4          2           2 4           2          2  2  2          2  2  2          2  2
     + 2.Ab.L .Il .Al.Ir.Ar.G  + 36.Ib .L.Il .Al.Ir.G + 36.Ib .L.Il .Ir.Ar.G - 6.Ib.L .Il .Al.Ir.Ar.G )/(6.Ab.Ib .L .Il.Al.Ar.G  + 6.Ab.Ib .L .Al.Ir.Ar.G  + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib .L .Il .Ar + 12.Ab.Ib .L .Il.Al.Ir

             2  2             2  2      2          2  2  2            4  2        3              4                 3         4   2    3                 2  2                   2  2                   2
     + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir  + 6.Ab.Ib .L .Ir .Ar + 2.Ab.Ib.L .Il .Al.Ar.G  + 22.Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.Ab.Ib.L .Al.Ir .Ar.G  + 24.Ab.Ib.L .Il .Al.Ir.G + 24.Ab.Ib.L .Il .Ir.Ar.G + 24.Ab.Ib.L .Il

        2             2       2          4  2             4           4           2     4           2  2              3         2  2            2  2              2          2          2        2              2  2
     .Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L .Il .Al.Ir.Ar.G  + 2.Ab.L .Il.Al.Ir .Ar.G  + 72.Ib .L .Il.Al.Ir.Ar.G  + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib .Il.Al.Ir .G + 72.Ib .Il.Ir .Ar.G + 24.Ib.L .Il .Al

        4         2         2    4          2     2  2           2   2      2        2   2       2      5
     .Ir.Ar.G  + 24.Ib.L .Il.Al.Ir .Ar.G  + 72.Ib.Il .Al.Ir .G  + 72.Ib.Il .Ir .Ar.G  + 6.L .Il .Al.Ir .Ar.G )

          2 5          2              5                 3       2     2                2     2         3  2           4        2 4           2            2 4            2          2  2  2          2  2  2
MB := (3.Ab.Ib .L .Il.Al.Ar.G  + 6.Ab.Ib.L .Il.Al.Ir.Ar.G  - 36.Ib .L.Il .Al.Ir.G - 36.Ib .L.Il .Ir.Ar.G + 6.Ib.L .Il .Al.Ir.Ar.G )/(6.Ab.Ib .L .Il.Al.Ar.G  + 6.Ab.Ib .L .Al.Ir.Ar.G  + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib .L .Il .Ar

             2  2              2  2             2  2      2          2  2  2            4  2        3              4                 3         4   2    3                 2  2                  2  2
     + 12.Ab.Ib .L .Il.Al.Ir + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir  + 6.Ab.Ib .L .Ir .Ar + 2.Ab.Ib.L .Il .Al.Ar.G  + 22.Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.Ab.Ib.L .Al.Ir .Ar.G  + 24.Ab.Ib.L .Il .Al.Ir.G + 24.Ab.Ib.L .Il .

                       2             2             2     2         4  2            4           4           2    4          2  2             3         2  2            2  2              2          2          2
     Ir.Ar.G + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L .Il .Al.Ir.Ar.G  + 2.Ab.L .Il.Al.Ir .Ar.G  + 72.Ib .L .Il.Al.Ir.Ar.G  + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib .Il.Al.Ir .G + 72.Ib .Il.

       2           2  2          4          2           2    4          2     2  2          2   2      2      2   2      2      5
     Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G  + 24.Ib.L .Il.Al.Ir .Ar.G  + 72.Ib.Il .Al.Ir .G  + 72.Ib.Il .Ir .Ar.G  + 6.L .Il .Al.Ir .Ar.G )

COLUMN MEMBER FROM NODE 0 TO 2

          2 4            2 4                  4               2       2  2              2        2  2        3         2 4           2           2 4           2          2  2  2          2  2
PA := (3.Ab.Ib .L .Il.Al.Ar.G + 3.Ab.Ib .L .Al.Ir.Ar.G + 12.Ab.Ib.L .Il.Al.Ir.Ar.G  + 36.Ib .L .Il.Al.Ir.Ar.G  + 18.Ib.L .Il .Al.Ir.Ar.G )/(6.Ab.Ib .L .Il.Al.Ar.G  + 6.Ab.Ib .L .Al.Ir.Ar.G  + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib .L .

       2             2  2              2  2             2  2      2          2  2  2            4  2        3              4                 3         4   2    3                 2  2
     Il .Ar + 12.Ab.Ib .L .Il.Al.Ir + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir  + 6.Ab.Ib .L .Ir .Ar + 2.Ab.Ib.L .Il .Al.Ar.G  + 22.Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.Ab.Ib.L .Al.Ir .Ar.G  + 24.Ab.Ib.L  Il .Al.Ir.G + 24.Ab.Ib.

      2  2                    2             2          2     2          4  2            4           4          2    4          2  2            3         2  2             2  2            2           2
     L .Il .Ir.Ar.G + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L .Il .Al.Ir.Ar.G  + 2.Ab.L .Il.Al.Ir .Ar.G  + 72.Ib .L .Il.Al.Ir.Ar.G  + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib .Il.Al.Ir .G + 72.

       2     2              2  2          4          2           2    4          2     2  2          2   2      2      2   2      2      5
     Ib .Il.Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G  + 24.Ib.L .Il.Al.Ir .Ar.G  + 72.Ib.Il .Al.Ir .G  + 72.Ib.Il .Ir .Ar.G  + 6.L .Il .Al.Ir .Ar.G )

          2 4            2              2  2              2  2             2  2      2          2  2  2             4              3           4  2    3               2         2                  2     2
UA := (6.Ab.Ib .L .Al.Ir.Ar.G  + 6.Ab.Ib .L .Il.Al.Ir + 6.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir  + 6.Ab.Ib .L .Ir .Ar + 11.Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.Ab.Ib.L .Al.Ir .Ar.G  + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G

         4      2    4          2  2              3         2  2            2  2              2          2             2  2          4          2           2    4          2      2  2
     + 2.Ab.L .Il.Al.Ir .Ar.G  + 72.Ib .L .Il.Al.Ir.Ar.G  + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib .Il.Al.Ir .G + 72.Ib .Il.Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G  + 24.Ib.L .Il.Al.Ir .Ar.G  + 72.Ib.Il .Al Ir .G  + 72

        2   2      2       2   2      2     5           2 4           2           2 4           2          2  2  2          2  2              2  2             2  2      2          2  2
     .Ib.Il .Ir .Ar.G  + 6.L .Il .Al.Ir .Ar.G )/(5.Ab.Ib .L .Il.Al.Ar.G  + 6.Ab.Ib .L .Al.Ir.Ar.G  + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib .L .Il .Ar + 12.Ab.Ib .L .Il.Al.Ir + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir  + 6.Ab.Ib .L

       2         4  2        3              4                 3         4   2    3                 2  2                    2  2                    2           2             2     2          4  2
     .Ir .Ar + 2.Ab.Ib.L .Il .Al.Ar.G  + 22.Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.Ab.Ib.L .Al.Ir .Ar.G  + 24.Ab.Ib.L .Il .Al.Ir.G + 24.Ab.Ib.L .Il .Ir.Ar.G + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L .Il .Al.Ir.Ar.

     4       4     2    4          2  2              3         2  2            2  2              2          2             2  2          4          2           2    4          2      2  2
     G  + 2.Ab.L .Il.Al.Ir .Ar.G  + 72.Ib .L .Il.Al.Ir.Ar.G  + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib .Il.Al.Ir .G + 72.Ib .Il.Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G  + 24.Ib.L .Il.Al.Ir .Ar.G  + 72.Ib.Il .Al.Ir .G

               2   2      2       2   2      2      5
     + 72.Ib.Il .Ir .Ar.G  + 6.L .Il .Al.Ir .Ar.G )

          2 5          2           2 3                   2 3               2 3      2          2  3  2              5                 3            5  2    3                 3          2                 3     2
MA := (3.Ab.Ib .L .Al.Ir.Ar.G  + 6.Ab.Ib .L .Il.Al.Ir + 6.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir  + 6.Ab.Ib .L .Ir .Ar + 5.Ab.Ib.L .Il.Al.Ir.Ar.G  + 2.Ab.Ib.L .Al.Ir .Ar.G  + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G
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          5        2    4       2 3          3      2    2              2    2            2           2         2       2            3 2         4         3        2    4           2
 + 2.Ab.L .Il.Al.Ir .Ar.G + 36.Ib .L .Il.Al.Ir.Ar.G + 36.Ib .L.Il .Al.Ir.G + 36.Ib .L.Il .Ir.Ar.G + 72.Ib .L.Il.Al.Ir .G + 72.Ib .L.Il.Ir .Ar.G + 12.Ib.L .Il .Al.Ir.Ar.G + 24.Ib.L .Il.Al.Ir .Ar.G + 72.Ib.L.Il .Al.

 2 2           2  2   2     3  2     2    5        2 4           2        2 4          2         2  2  2          2  2  2             2 2              2 2            2  2    2
Ir .G + 72.Ib.L.Il .Ir .Ar.G + 6.L .Il .Al.Ir .Ar.G )/(6.Ab.Ib .L .Il.Al.Ar.G + 6.Ab.Ib .L .Al.Ir.Ar.G + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib .L .Il .Ar + 12.Ab.Ib .L .Il.Al.Ir + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir

         2  2  2          4  2        3           4              3          4   2    3        2  2                 2  2             2          2             2        2                 4
 + 6.Ab.Ib .L .Ir .Ar + 2.Ab.Ib.L .Il .Al.Ar.G + 22.Ab.Ib.L .Il.Al.Ir.Ar.G + 2.Ab.Ib.L .Al.Ir .Ar.G + 24.Ab.Ib.L .Il .Al.Ir.G + 24.Ab.Ib.L .Il .Ir.Ar.G + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L

 2          4          4          2    4        2 2           3      2  2         2  2              2           2            2     2              2  2           4         2         2    4
.Il .Al.Ir.Ar.G + 2.Ab.L .Il.Al.Ir .Ar.G + 72.Ib .L .Il.Al.Ir.Ar.G + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib .Il.Al.Ir .G + 72.Ib .Il.Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G + 24.Ib.L .Il.Al.Ir .Ar.G + 72.Ib.

 2     2 2        2   2     2      2  2     2    5
Il .Al.Ir .G + 72.Ib.Il .Ir .Ar.G + 6.L .Il .Al.Ir .Ar.G )

            2 5           2           5              3       2 3            3      2     2             2   2                3 2          4      2 4          2            2 4          2      2 2
MB := (3.Ab.Ib .L .Al.Ir.Ar.G + 6.Ab.Ib.L .Il.Al.Ir.Ar.G + 36.Ib .L .Il.Al.Ir.Ar.G + 36.Ib .L.Il .Al.Ir.G + 36.Ib .L.Il .Ir.Ar.G + 12.Ib.L .Il .Al.Ir.Ar.G )/(6.Ab.Ib .L .Il.Al.Ar.G + 6.Ab.Ib .L .Al.Ir.Ar.G + 6.Ab.Ib .L .

 2            2  2  2          2 2               2  2               2  2     2          2  2  2         4  2       3           4            3           4   2   3           2  2
Il .Al + 6.Ab.Ib .L .Il .Ar + 12.Ab.Ib .L .Il.Al.Ir + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir + 6.Ab.Ib .L .Ir .Ar + 2.Ab.Ib.L .Il .Al.Ar.G + 22.Ab.Ib.L .Il.Al.Ir.Ar.G + 2.Ab.Ib.L .Al.Ir .Ar.G + 24.Ab.Ib.L .Il

                 2  2                2           2              2     2          4  2          4        4          2    4        2  2           3     2  2         2  2            2
.Al.Ir.G + 24.Ab.Ib.L .Il .Ir.Ar.G + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L .Il .Al.Ir.Ar.G + 2.Ab.L .Il.Al.Ir .Ar.G + 72.Ib .L .Il.Al.Ir.Ar.G + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib

       2         2    2            2  2         4         2        2   4        2     2 2          2  2   2     2  2     2    5
.Il.Al.Ir .G + 72.Ib .Il.Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G + 24.Ib.L .Il.Al.Ir .Ar.G + 72.Ib.Il .Al.Ir .G + 72.Ib.Il .Ir .Ar.G + 6.L .Il .Al.Ir .Ar.G )

BEAM MEMBER FROM NODE 1 TO 2

               2 4          2           2  2  2         2  2  2          2 2              2 2               4  2        3             4         3           2  2             2  2
PA := ( - 6.Ab.Ib .L .Il.Al.Ar.G - 6.Ab.Ib .L .Il .Al - 6.Ab.Ib .L .Il .Ar - 6.Ab.Ib .L .Il.Al.Ir - 6.Ab.Ib .L .Il.Ir.Ar - 2.Ab.Ib.L .Il .Al.Ar.G - 11.Ab.Ib.L .Il.Al.Ir.Ar.G - 24.Ab.Ib.L .Il .Al.Ir.G - 24.Ab.Ib.L .Il .Ir.

             4  2         4        2 4           2         2 4         2         2  2  2          2  2  2           2 2              2 2             2 2     2          2  2  2
Ar.G - 2.Ab.L .Il .Al.Ir.Ar.G )/(6.Ab.Ib .L .Il.Al.Ar.G + 6.Ab.Ib .L .Al.Ir.Ar.G + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib .L .Il .Ar + 12.Ab.Ib .L .Il.Al.Ir + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir + 6.Ab.Ib .L .Ir .Ar + 2

       4  2        3            4            3          4   2    3           2  2                2  2                 2         2              2        2         4  2           4
.Ab.Ib.L .Il .Al.Ar.G + 22.Ab.Ib.L .Il.Al.Ir.Ar.G + 2.Ab.Ib.L .Al.Ir Ar.G + 24.Ab.Ib.L .Il .Al.Ir.G + 24.Ab.Ib.L .Il .Ir.Ar.G + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L .Il .Al.Ir.Ar.G + 2.Ab.

 4        2    4       2  2           3        2  2          2  2           2        2        2       2            2  2           4        2        2   4        2     2 2
L .Il.Al.Ir .Ar.G + 72.Ib .L .Il.Al.Ir.Ar.G + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib .Il.Al.Ir .G + 72.Ib .Il.Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G + 24.Ib.L .Il.Al.Ir .Ar.G + 72.Ib.Il .Al.Ir .G + 72.Ib.

 2   2    2     2  2     2    5
Il .Ir .Ar.G + 6.L .Il .Al.Ir .Ar.G )

              2 4            2 4                  4               2      2 2              2     2  2          3        2 4         2           2 4          2          2  2  2             2
VA := ( - 3.Ab.Ib .L .Il.Al.Ar.G - 3.Ab.Ib .L .Al.Ir.Ar.G - 12.Ab.Ib.L .Il.Al.Ir.Ar.G - 36.Ib .L .Il.Al.Ir.Ar.G - 18.Ib.L .Il .Al.Ir.Ar.G )/(6.Ab.Ib .L .Il.Al.Ar.G + 6.Ab.Ib .L .Al.Ir.Ar.G + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib .

 2  2           2 2              2 2              2 2    2          2  2  2         4  2       3            4            3          4   2    3          2  2
L .Il .Ar + 12.Ab.Ib .L .Il.Al.Ir + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir + 6.Ab.Ib .L .Ir .Ar + 2.Ab.Ib.L .Il .Al.Ar.G + 22.Ab.Ib.L .Il.Al.Ir.Ar.G + 2.Ab.Ib.L .Al.Ir .Ar.G + 24.Ab.Ib.L .Il .Al.Ir.G + 24.Ab.

    2  2                2          2               2     2         4  2          4        4          2    4        2  2          3      2  2         2  2           2         2
Ib.L .Il .Ir.Ar.G + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L .Il .Al.Ir.Ar.G + 2.Ab.L .Il.Al.Ir .Ar.G + 72.Ib .L .Il.Al.Ir.Ar.G + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib .Il.Al.Ir .G + 72

 2     2            2  2        4         2        2   4        2     2 2          2  2   2     2  2     2    5
.Ib .Il.Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G + 24.Ib.L .Il.Al.Ir .Ar.G + 72.Ib.Il .Al.Ir .G + 72.Ib.Il .Ir .Ar.G + 6.L .Il .Al.Ir .Ar.G )

             2 5         2            5              3      2     2             2    2             3 2          4         2 4          2          2 4          2          2  2  2          2  2  2
MA := ( - 3.Ab.Ib .L .Il.Al.Ar.G - 6.Ab.Ib.L .Il.Al.Ir.Ar.G + 36.Ib .L.Il .Al.Ir.G + 36.Ib .L.Il .Ir.Ar.G - 6.Ib.L .Il .Al.Ir.Ar.G )/(6.Ab.Ib .L .Il.Al.Ar.G + 6.Ab.Ib .L .Al.Ir.Ar.G + 6.Ab.Ib .L .Il .Al + 6.Ab.Ib .L .Il .

          2 2                2 2              2 2     2          2  2  2         4  2       3            4            3          4   2    3           2  2                 2
Ar + 12.Ab.Ib .L .Il.Al.Ir + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir + 6.Ab.Ib .L .Ir .Ar + 2.Ab.Ib.L .Il .Al.Ar.G + 22.Ab.Ib.L .Il.Al.Ir.Ar.G + 2.Ab.Ib.L .Al.Ir .Ar.G + 24.Ab.Ib.L .Il .Al.Ir.G + 24.Ab.Ib.L .

 2                 2        2             2      2          4  2          4        4          2    4        2  2            3      2  2            2  2            2          2           2
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Il .Ir.Ar.G + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L .Il .Al.Ir.Ar.G + 2.Ab.L .Il.Al.Ir .Ar.G + 72.Ib .L .Il.Al.Ir.Ar.G + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.Ib .Il.Al.Ir .G + 72.Ib .

      2              2   2         4          2         2    4            2      2 2          2  2   2      2  2     2     5
Il.Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G + 24.Ib.L .Il.Al.Ir .Ar.G + 72.Ib.Il .Al.Ir .G + 72.Ib.Il .Ir .Ar.G + 6.L .Il .Al.Ir .Ar.G )

                2 5            2                5              3       2 3             3       2     2               2     2              3  2          4           2 4            2          2 4           2          2
M6 := ( - 3.Ab.Ib .L .Al.Ir.Ar.G - 6.Ab.Ib.L .Il.Al.Ir.Ar.G - 36.Ib .L .Il.Al.Ir.Ar.G - 36.Ib .L.Il .Al.Ir.G - 36.Ib .L.Il .Ir.Ar.G - 12.Ib.L .Il .Al.Ir.Ar.G )/(6.Ab.Ib .L .Il.Al.Ar.G + 6.Ab.Ib .L .Al.Ir.Ar.G + 6.Ab.Ib .

 2   2             2  2   2             2 2                2 2                 2 2     2          2 2  2           4  2      3               4         3               4    2    3         2
L .Il .Al + 6.Ab.Ib .L .Il .Ar + 12.Ab.Ib .L .Il.Al.Ir + 12.Ab.Ib .L .Il.Ir.Ar + 6.Ab.Ib .L .Al.Ir + 6.Ab.Ib .L .Ir .Ar + 2.Ab.Ib.L .Il .Al.Ar.G + 22.Ab.Ib.L .Il.Al.Ir.Ar.G + 2.Ab.Ib.L .Al.Ir .Ar.G + 24.Ab.Ib.L .

 2                   2  2                 2         2                 2       2              4  2            4         4          2      4        2  2              3       2  2               2  2
Il .Al.Ir.G + 24.Ab.Ib.L .Il .Ir.Ar.G + 24.Ab.Ib.L .Il.Al.Ir .G + 24.Ab.Ib.L .Il.Ir .Ar.G + 2.Ab.L .Il .Al.Ir.Ar.G + 2.Ab.L .Il.Al.Ir .Ar.G + 72.Ib .L .Il.Al.Ir.Ar.G + 72.Ib .Il .Al.Ir.G + 72.Ib .Il .Ir.Ar.G + 72.

 2         2          2      2               2   2          4           2         2    4          2     2 2          2  2     2      2  2     2     5
Ib .Il.Al.Ir .G + 72.Ib .Il.Ir .Ar.G + 24.Ib.L .Il .Al.Ir.Ar.G + 24.Ib.L .Il.Al.Ir .Ar.G + 72.Ib.Il .Al.Ir .G + 72.Ib.Il .Ir .Ar.G + 6.L .Il .Al.Ir .Ar.G )

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Appendix E
Vita

The author was born in Sydney, Australia, on the last day in the month of June, in the year of our lord 1957, the first son to John and Patricia Rankine.

He attended Cranbrook School, at the kindergarten level, before studying at a school in Berkeley, California, USA, for a year. The remainder of his education from 1963 to 1975 was at Cranbrook, in Sydney, Australia. In his final year he was awarded Dux of School, as well as other Foundation prizes in Physics and Chemistry. The author's education continued at the University of Sydney, and in 1979 he was awarded the degree of Bachelor of Science, for majoring in Computer Science and Advanced Physics. Following two more years of study, concentrating on structures, he was awarded the degree of Bachelor of Engineering with First Class Honors, and was awarded the Roderick Thesis and Reinforced Concrete prizes.

His first job was with Ove Arup and Partners, where he designed a post-tensioned cantilevered carpark, and the pile system for a multistorey building in Kuala Lumpur, Malaysia. He left Ove Arup to start up his own business dealing in microcomputer equipment, with two other associates. This lead to his settling in Denver, Colorado, where in further pursuit of this business, he setup an operation in the USA. The course of this work exposed him to the rigors of business practise, marketing, banking, sales support, purchasing, electronic and software design, manufacture and quality control. The most taxing product, that he formulated was a five-functioned peripheral for the IBM PC family of computers.

In 1984, the author sold his interests in both businesses, after the market in the USA started to soften. At the start of 1985, Anthony resumed his engineering education, by attending Lehigh University, and in June 1986 was awarded a Master of Science degree, for work in civil engineering and expert systems.