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An improved parametric programming methodology :

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AN IMPROVED PARAMETRIC PROGRAMMING METHODOLOGY

for the Purpose of

Understanding Demand Point Shadow Prices

to Establish a Foundation for Customer Pricing

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TABLE OF CONTENTS

CHAPTER I

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THE PROBLEM DEFINITION

Problem Definition

A problem at many large corporations involves the establishing of a laid-in cost for new customers. A laid-in cost is the incremental cost associated with producing and then distributing product to a customer location. This cost is used as the basis for establishing customer contract prices. Obviously, the competition will be developing contract prices as well. Thus, the calculated price must be fair (profitable) for the corporation but also competitive.

A Profit and Loss (P & L) analysis might suggest a selling price for new business based on laid-in costs from the closest available source for a product. For geographic areas with facilities at maximum capacity, a better laid-in cost can be determined when dislocation costs are considered. A dual variable or shadow price analysis is a means of considering the dislocation. The P & L calculation of laid-in costs is:

$$L\$ = D\$ + P\$$$

where L$ = total laid-in costs from closest facility

D$ = distribution costs (round trip miles * cost per mile)

P$ = production costs (volume * cost per unit of volume)

However, this algorithm does not consider the times when a corporation's production/distribution system becomes constrained. For example, it may not have the capacity at a specific facility to support another customer, or it can't continue to supply increasing amounts before capacity (and therefore costs) needs to be revised. In either case, the laid-in cost would change as facilities not at capacity are used to satisfy the new demand.

Linear programming models are often used to suggest sourcing patterns for a distribution system by minimizing total incremental costs (production and distribution). Constraints might include:

- Demand --------> The amount of product shipped to a location must be greater than or equal to the demand.
- Production ----> Total production at a facility must be less than or equal to capacity.
- Vehicles ------> The number of vehicles used to distribute product from a facility must be less than or equal to the vehicle count there.

Demand constraints are one of the three basic linear programming model limitations. The amount of product shipped to a specified location must be at least equal to a minimum value. The point (geographically) at which a demand occurs helps to define the costs of supplying product to the necessary constraint. That is, the greater the distance a demand point is from a product source, the greater the cost of supplying product there. Although there is a demand point referred to by each demand constraint, note that demand constraints are associated with the question of 'how much?' while demand points with 'where?'.

Because the L.P. model minimizes cost, each demand constraint is at its lower bound when optimality is reached and therefore, each constraint has a dual cost associated with it. By definition, this dual or demand point shadow price is the maximum amount one would be willing to pay for an additional amount of the input (demand). For this situation, the dual represents the amount that the objective function will decrease if the constraints were relaxed, or the amount that the objective function will increase if the constraint were increased by one unit. The shadow price for each demand location can be interpreted as the incremental cost to produce and deliver an additional unit of product to the specified location.

This approach can be used to help establish laid-in costs for new business. However, a stability in the value of the shadow price must be present before it is an accepted approach (stable in that it is at least predictable and understandable). When periodic updates to information are made to reflect possible reassignments in distribution fleets and/or fluctuations in production capacities due to planned or unexpected shutdowns, are the shadow price values predictable? And if the shadow prices are to be used for long term pricing strategies, is it relevant to even consider a short term constraint variation?

The basic theme to be addressed is the proper use of shadow prices to establish a foundation for customer pricing. This notion concerns itself not only with the possible usefulness of shadow prices, but also proper use. The extent to which one should rely upon shadow prices to understand laid-in costs is extremely important. Knowing the limitations of both linear programming and sensitivity analyses should help define the stability of a shadow price and hence its reliability.

Guidelines to suggest proper use of the results from sensitivity analyses must be developed and should help limit the overextension of the model/analyses results. That is, results are not to be used to help in decisions that are outside model boundaries. To assure proper use, the previously stated guidelines must be presented in an orderly and understandable package which includes information about data, model concepts, and results.

CHAPTER II

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ABOUT LINEAR PROGRAMMING AND SENSITIVITY ANALYSES

An Appreciation of Linear Programming

There is little question that linear programming has been poorly applied in some instances in the past, but misapplication does not lessen the ultimate utility of linear programming techniques. The corporation that refuses to allow linear programming to be attempted under favorable circumstances may be correct in assuming that it will not be successful in that particular area. The corporation that gambles all on linear programming, but is too busy to take the time to understand its use, may also prove linear programming can be unsuccessful. However, the corporation that appreciates what linear programming can do, and applies it well in situations for which it represents the appropriate viable technique, will make impressive inroads into improved design and operation of productive systems.

An optimization problem ultimately faced by a decision-maker is one of choosing from many alternatives the one that yields a maximum or minimum value of some numerically measurable criterion of performance. The necessity of coming to a decision and implementing it in the real world gives focus and meaning to the optimization problem, but the actual work of solving it often is performed by someone other than the decision-maker.

In general, there are a great many alternative ways to solve a problem. The conditions and restrictions that determine which courses of action can be adopted and which cannot are called constraints. The measure of effectiveness of each alternative is the crucial component of the optimization problem. It must be explicitly stated and must take on a single numerical value for each feasible policy. This implies that the effectiveness of each element of every policy must be measurable on the same scale. If it is not possible, then the definition of an optimization problem is not satisfied. By far the most frequently used measure of effectiveness is total dollar cost, which serves as the principal yardstick of an effective operation. Measurement in terms of dollars is quite natural and convenient. Most of the required cost data (wages, fuel cost, raw-material prices, etc.) are readily available, and those that are not (inventory carrying charge, customer ill will, etc. .) usually can be estimated in some rational manner.

Certain specific structural elements must be present for an optimization problem, including a numerical measure of effectiveness. This requirement is somewhat restrictive, but serves an important purpose in guaranteeing that any optimization problem can be expressed in terms of mathematical relationships and then solved by means of computational methods.

The process of translating a real-world situation into mathematical language is referred to as formulating the problem. An analyst must select equations and inequalities that define permissible or feasible sets of values for all the variables, ruling out those sets of values that are prohibited by the constraints of the problem. The art of modeling optimization problems plays a major role in developing a well structured representation of the problem.

Linear programming deals with the problem of allocating limited resources among competing activities in the best possible way. This problem of allocation can arise whenever one must select the level of certain activities that must compete for certain scarce resources necessary to perform these activities. The variety of situations to which this description applies is diverse, ranging from the allocation of production facilities to products to the solution of parlor games. However, the one common ingredient in each of these many situations is the necessity for allocating resources to activities.

Shadow Prices

If an inequality is of the form

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n < b_i$$

an equation can be made by adding what is known as a slack (<) or surplus (>) variable as follows

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n + x_s = b_i$$

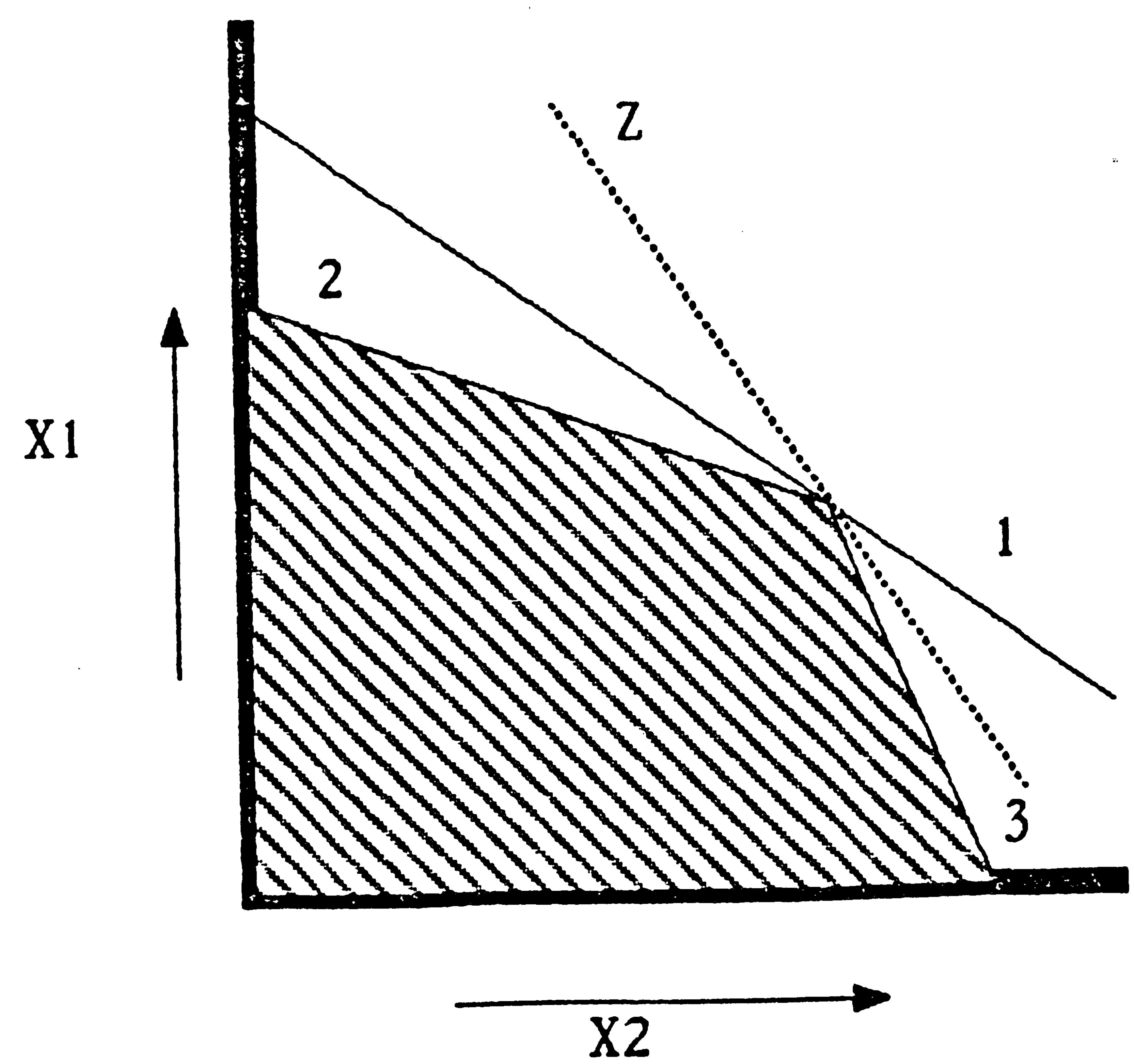
In linear programming, this necessity to convert to an equation has an important benefit. If the right-hand-side (b_i) is viewed as a resource, a slack variable in the optimal solution at a positive value indicates that the resource is not completely used.(19) Similarly a positive surplus variable in the optimal solution suggests that there is room for the left-hand-side to decrease. If the slack (surplus) variable is zero in the optimal solution the constraint holds as an equality and is binding. It binds down the objective function and prevents it from assuming a greater (lesser) value.(22) Shadow prices are a means of understanding the affect of a binding constraint.

Let's define the shadow price, p_i, of the ith resource b_i, to be the achievable rate of increase in the objective function per unit increase in resource i.(3) This definition may be formally stated as follows:

$$p = \delta z^* / \delta b_1$$

where z* denotes the optimal value of the objective function, provided only increases in b_1 are allowed. A realistic definition of the shadow price of the original surplus requirement is the rate of change in z per unit of decrease in this requirement.(1)

A geometric interpretation of shadow prices is illustrated through the following figure:



Note that increasing resource 1 does not alter the feasible region at all. Hence, the shadow price of resource 1 is 0. Also observe that although constraint 1 is redundant, its presence does affect achievable gain in the objective function resulting from increasing either resource 2 or resource 3. That is, the shadow price for constraint 2 and/or 3 is limited by the presence of constraint 1.

In many applications involving linear programming problems, the shadow prices are more important than the solution of the problem. They allow the model user to determine whether certain changes in the optimal model requirements might actually increase the objective function.

If primal degeneracy exists (primal degeneracy occurs when one or more of the basic variables equals 0), the left- and right-hand-side derivatives of z with respect to a resource may not be equal.(2) If the optimal primal solution is nondegenerate, however, these two values will be the same. There are two shadow prices then, for each resource or combinations of resources: positive and negative.(1) Each one can be determined either by solving a much simpler linear program over the set of optimal dual solutions or equivalently, and perhaps much more easily, by parametric programming. In addition, there is an interval of positive length for which a given shadow price is valid.

It is of interest to know what information regarding shadow prices is provided by commercial linear programming software packages. In order to ascertain the correct shadow prices using MPSX (IBM's Mathematical Programming System), the parametric analysis option (PARARHS) must be employed for each constraint for which RANGE has indicated the given shadow price may be valid. Hence, the problem must be run twice; first using RANGE and then using PARARHS.(2)

As previously stated, shadow prices are not necessarily equal to dual variables except in the case when the primal problem is nondegenerate. In all cases, the ith shadow price always equals the smallest value of the ith dual variable in the set of optimal dual extreme point solutions.(3) The widespread assumption that shadow prices and dual variables are identical may lead practitioners to the erroneous conclusion that increasing the value of a particular resource would be profitable (in some cases). This situation is made worse by the fact that there are commercial software packages that assume dual variables and shadow prices are synonymous.

Sensitivity Analyses

One of the advantages of linear programming is the amount of other sensitivity information that is available besides finding the optimal solution. Sensitivity analysis of the solution (often referred to as post-optimal analysis) permits the evaluation of the effect in changing a quantity.(19)

The study of how sensitive a given optimal solution is to various changes in the input parameters is usually called sensitivity analysis or parametric analysis. Sensitivity analysis, along with the investigation of how specific changes in the input parameter affect the optimal solution is called post-optimal analysis.

The general linear programming problem is to find non-negative values of n variables which maximize a given linear function of the variables, subject to m given linear constraints. It is presumed that the constants of the problem are known with absolute precision and do not change with time.(20) However, in many cases only estimates of these values are available, and the values may have to be changed when better estimates are available. The values may also change with time. Further, the optimal solution obtained may have to be changed to satisfy secondary objectives such as customer good will etc...

Again, when a practical problem has been formulated and solved as a linear programming problem, it is frequently the case that not all of the input parameters are known exactly. Typically, some of these parameters have been estimated or calculated only approximately. Thus it is important to know how sensitive the optimal solution is to changes in such parameters. For example, if it is known that a particular c_j is accurate to within ±5 per cent, then we must also determine whether the computed optimal solution remains optimal for all values of j in this range; without this additional information, it is not all certain that the computed optimal solution is indeed the true optimal solution to the actual problem.(8)

There are also many practical situations which arise in which a linear programming model is used periodically to find, for example, the optimal production quantities for the next period. In such cases, a few changes in the cost coefficients and/or the right-hand-sides are not uncommon.

Another type of modification in the linear programming model which sometimes occurs is the addition of a new constraint or variable to the original formulation; either because the original formulation was erroneous or because the model situation has changed.

An important aspect, then, to many practical problems is a sensitivity analysis in order to evaluate the consequences of a change in a constant or of deviating from the optimal solution. The objective of post-optimality analysis is to study the effect of discrete changes in coefficients of the linear programming problem on the optimal solution.(23) Parametric linear programming investigates behavior of the optimal solution as a result of predetermined linear variations in the parameter of the problem. The purpose of sensitivity analysis is to obtain new and informative results through a minimal amount of additional computational effort.(8)

Sensitivity analysis is performed to answer two basic questions:

1. What is the optimal solution when one of the constants of the problem (either some c_j, B_i, or A_{ij}) is changed?

2. When the value of one of the variables is changed by a given amount, what changes are necessary in the values of the other variables for the reduction in value of the objective function to be minimal?(20)

The study of the affect of changing the constants can result in a better understanding of the problem by providing a keener insight into the limitations involved. It can help in planning to meet changing conditions, and hence, result in a more profitable operation. When studying the cost coefficients of variables in the optimal basis, one understands the worth of a resource without changing optimality.(21)

What is the effect on the optimal solution when a set of given data of the problem is changed? Remember, problems of post-optimality are concerned with a discrete modification of the given data. That is, one of the following occurs:

1. b changes
2. c changes
3. a Column of A is varied
4. a row of A is varied

5. a new constraint is added

6. a new variable is added

where: A = coefficient matrix

b = requirements vector

c = cost vector

Parametric problems are concerned with these same six changes, however the data changes vary in a continuous manner. The purpose of parametric programming is to study the variation of the optimal program as a function of the values of certain data.

For example, let's make b vary continuously as a linear function of a parameter θ:

$$b = b_0 + \theta \delta$$

where δ and b_0 are fixed vectors.

When θ varies, the optimality criterion remains satisfied as long as the present basis is maintained. There also exists a critical value $\theta = \theta_1$ beyond which the problem ceases to be an optimal program. The question then surfaces on what must be done when θ passes the critical value θ_1 in order to re-optimize the problem.

Let's start from an optimal basic program corresponding to the basis B_0 and the value $\theta_0 = 0$ of the parameter, and make θ increase (decrease) through positive (negative) values. The series of distinct values θ_1, θ_2, ... θ_p of θ and a series of bases B_1, B_2, ... B_p are determined which have the following properties.(22)

1. Each basis B_i is deduced from B_{i-1} by substituting a single vector A_{ji} for A_{ji-1} if θ_i is determined by a unique minimum and not more than one vector.

2. The basic solution x_B associated with the basis B_i and the vector $b_0 + \theta_i$ is an optimal program for every value of θ taken on segment $\theta_i < \theta < \theta_{i+1}$

3. Each iteration of the preceding procedure is characterized by the fact that θ is given a finite increase and the dual algorithm is applied for a fixed value of θ.

Discussions of parametric analysis can be kept simple due to the fact a single parameter is introduced at a time. When discussions of simultaneous variation of many parameters occurs, critical points (i.e. θ) are replaced by critical hyperplanes.

A summary then, of the four basic areas where sensitivity analyses are performed are as follows.

1. The first focuses attention on the non-basic variables other than slack or surplus variables. The question is, in the case of

minimization, how much would the original cost coefficient of a variable have to be lowered in order for it to enter the optimal solution?(19)

2. Concerning slack variables, the relative cost factor often referred to as the shadow price, is a measure of the value of one additional unit of the right-hand-side. In terms of a resource, the shadow price is the value of one additional unit of that resource.

3. Another type of sensitivity analysis centers on the cost coefficient of variables in the optimal basis. What is the range of values that a particular cost coefficient can take on without affecting the optimal solution?

4. A fourth sensitivity that can be performed addresses the constants on the right-hand-side. What is the range permissible in a constant without changing the variables in the optimal solution?(19)

It is not necessary to solve a modified problem from the beginning to obtain the desired sensitivity information; instead the information can be found performing relatively few computations using data in the optimal tableau to the originally solved problem.

Suppose we have solved the linear programming problem; maximize $z = cx$, subject to $Ax = b$, $x > 0$, and have obtained an optimal basic feasible solution, x_B. If we denote by c_B the cost vector corresponding to x_B, then the current values of $z_j - c_j$ are

$$z_j - c_j = \quad c_B y_j - c_j \qquad j = 1, 2, \ldots m$$

where the y_j also correspond to the current optimal solution.

Now, suppose we wish to change one of the c_j, say c_k, to a new value, c_k, where

$$c_k = c_k - \delta_k$$

Since a change in the objective function in no way alters the set of feasible solutions, the solution x_B will remain optimal provided that the new values of $z_j - c_j$ are still non-negative.

If c_k corresponds to a variable which is currently nonbasic, then it is obvious from previous equation that only $z_k - c_k$ will be changed, all other $z_j - c_j$ will remain unchanged (since c_B has not been changed) and hence non-negative. Moreover

$$\begin{aligned} z_k - c_k &= c_B y_k - c_k \\ &= c_B y_k - (c_k + \delta_k) \\ &= (c_B y_k - c_k) - \delta_k \\ &= (z^*_k - c_k) - \delta_k \end{aligned}$$

Thus, $z_k - c_k > 0 \quad \text{if} \quad \delta_k < (z^*_k - c_k)$

Accordingly, if the cost c_k of any nonbasic variable x_k is increased by an amount up to $(z^*_k - c_k)$, the current optimal solution will remain optimal.

If such a cost c_k is increased by more than the quantity $(z^*_k - c_k)$ then the resulting $z_k - c_k$ will become negative, and a few more simplex iterations may be needed to determine the new optimal solution. Note that the cost of any nonbasic variable can be decreased without bound, without affecting the optimality of x^*_B.

Consider now the case in which we wish to change the cost c_k corresponding to a basic variable x_k. Suppose that x_k is the pth basic variable x^*_{Bp}. Let

$$c_k = c_k + \delta_k = c_{Bp} = c_{Bp} + \delta_k$$

Then, for $j = k$,

$$\begin{aligned} z_j - c_j &= c_B y_j - c_j \\ &= c_{Bi} y_{ij} - c_j \\ &= c_{Bi} y_{ij} + \delta_k y_{pj} - c_j \\ &= z^*_j + \delta_k y_{pj} - c_j \end{aligned}$$

hence,

$$\begin{aligned} z_j - c_j &= (z^*_j - c_j) + \delta_k y_{pj} && j = k \\ &= 0 && j = k \end{aligned}$$

$(z_k - c_k = 0$ because x is basic). In order that all $z_k - c_k > 0$ it is necessary that

$$-\delta_k y_{pj} < (z^*_k - c_j) \qquad j = 1, 2, \ldots, n \qquad j = k$$

Note that δ_k must satisfy all of the inequalities simultaneously. For each $y_{pj} > 0$ we have that

$$\delta_k > -(z^*_j - c_j) / y_{pj}$$

And hence $\delta_k > \max \quad (z^*_j - c_j) / -y_{pj} \qquad y_{pj} > 0$

Similarly, we obtain

$$\delta_k < \min \quad (z^*_j - c_j) / y_{pj} \qquad y_{pj} < 0$$

Thus, if δ_k lies in the range determined by the previous two equations, then x^*_B remains optimal. If δ_k falls outside this range, at least one $z_j - c_j$ will be negative.

$$\text{Maximize} \qquad z = cx$$
$$Ax = b$$
$$x > 0$$

Suppose we wish to modify the requirements vector b. If we wish to change the ith component of b by an amount f_i (positive or negative), then $b_i = b_i + f_i$, $i = 1, 2, \ldots, m$; or, in vector notation, we have $b = b + f$. Now, we must recompute the values of the basic variables corresponding to the vectors in the current basic feasible solution. If we denote the current basis matrix by B, then

$$
\begin{aligned}
x_B &= B^{-1}b \\
&= B^{-1}(b + f) \\
&= B^{-1}b + B^{-1}f \\
&= x^*_B + B^{-1}f
\end{aligned}
$$

Hence, depending on $B^{-1}f$, the new basic solution may or may not be feasible. However, the $z^*_j - c_j$ are unaffected by a change in the requirements vector and therefore if z_B is feasible, it is also optimal.(8)

When it is necessary to change one or more of the elements of the coefficient matrix A, the situation becomes much more complicated than making changes in c or b. This is particularly true if we wish to change an element from a column of A which is basic in the optimal solution, since in this case the optimal basis matrix must be recomputed.

Linear Programming Problems and Data Uncertainty

The general linear programming problem is deterministic. The model is formulated and solved with a given set of parameters. This general assumption makes linear programming an extremely easy technique to use. However, this assumption is also the primary problem in the practical application of linear programming. The true values of model parameters are usually not known until after the decision based upon the linear programming solution is actually implemented. Often, all or some of the parameters may be random variables, which are influenced by random events in the decision environment.

Sensitivity analyses and parametric programming can be used for examining the effects of changes in model parameters. The major disadvantage of these methods is their inability to take into account the randomness of the parameters as governed by specific probability distributions.

There have been many different approaches suggested for formulating the linear programming problems under uncertainty. Basically, two approaches to formulating these linear programming problems under uncertainty have shown some merits.(15) The first approach, which is generally referred to as stochastic programming, attempts to solve the problem through making one or more decisions by selecting model parameters at different points in time. Although this approach sounds very logical, its practical application is enormously complex, especially when the model is larger and involves a number of time periods. Characteristically, stochastic programming problems evolve into nonlinear problems as several of the parameters become random. This attribute directly effects the difficulty of solving the problem.(24) The second approach, called chance-constrained programming, involves the formulation of a deterministically equivalent model to the problem under uncertainty.(15)

Stochastic Programming solves problems under uncertainty that involves making two or more decisions at different points in time with the condition that at least one of the later decisions depends not only on an earlier decision but also on the value of random parameters observed in the time intervening between the two decisions.(24) After the random event occurs over time, the parameters can be revised according to decision rules to account for the resolved uncertainties. The following general Stochastic Programming formulation is as follows.(15)

$$\text{Minimize } Z = \sum_{j} E(C_j)X_j + \sum_{q} P_q(C_{qj}X_{qj})$$

Subject to

$$\sum_{j=1}^{k} a_{ij}X_j = b_i \qquad i = 1, \ldots, g$$

$$\sum_{j=1}^{K} a_{qij}X_j + \sum_{j=K+1}^{m} a_{qij}X_{qj} = b_{qi}$$

$X_j \geq 0$

$X_{qj} \geq 0$

$i = g+1, \ldots, m$

$q = 1, \ldots, Q$

$j = 1, \ldots, K, \ldots n$

Where X_j ($j = 1, \ldots, K \leq n$) = the fixed levels of X

X_{qj} ($j = K+1, \ldots, M$) = the levels of X after all random values are known

Q = a possible set of values for C_j, A_{ij}, and b_i

P_q = probability of occurrence

The primary difficulty in using this approach is related to the extremely large model that results, where additional variables and constraints are necessary for proper formulation. These additions make the problem more complex, resulting in a large incremental amount of computation. This drawback reduces the pragmatic application of the stochastic programming model to real-world situations.(23)

A nonsequential approach to stochastic programming is a one-stage technique that does not allow for intermediate revisions in the model formulation as previously described. One method used in this approach is the expected value, which transforms the original nonlinear, probabilistic problem into a deterministic linear programming model. Essentially the expected value is used in place of a random parameter and might be a more realistic value than the mean value approach.(15)

Chance-constrained programming, a special approach to stochastic programming, was pioneered and later extended by Charnes and Cooper.(7) This approach is concerned with selecting certain random variables as functions of random variables with known distributions. Constraints on these variables must be maintained at prescribed levels of probability.

The primary purpose of the chance-constrained method is to reduce the problem of planning in light of an uncertain future.(7) In developing the chance-constrained stochastic model, a deterministic equivalence to the original stochastic problem is derived.

$$\text{Minimize } Z = \sum_{j=1}^{K} E\,(C_j)X_j$$

Subject to

$$\sum_{j=1}^{K} a_{ij}\,X_j = b_i \qquad i = 1, \ldots, g$$

$$P\,[\sum_{j=1}^{K} a_{ij}\,X_j \leq b_i] \geq 1-\alpha_i \qquad i = g+1, \ldots, m$$

Where

- α_i = risk level for constraint i

- C_j and b_i are random variables

- All parameters are normally distributed with known means and variances

The equivalent model, while nonlinear, can be approximated using separable programming techniques. The only information necessary for each random variable b_i is the $(1-\alpha_i)$ fractile for the unconditional distribution of the b_i value. The ability of chance-constrained programming methods to handle larger problems is an advantage over other existing stochastic programming techniques.

The principal weakness of the chance-constrained model is that it only indirectly evaluates the economic consequences of violating a constraint. In most situations, specifying the acceptable values for $(1-\alpha_i)$ should be part of the optimization problem. Therefore, when faced with a choice between the stochastic programming techniques and chance-constrained programming, you will have to compare the serious limitation on the problem size against the restricted meaning of optimality.(15)

Consider the chance-constrained problem:

Maximize $Z = 5X_1 + 6X_2 + 3X_3$

Subject to

$$P\{a_{11}X_1 + a_{12}X_2 + a_{13}X_3 \leq 8\} \geq 0.95$$

$$P\{5X_1 + X_2 + 6X_3 \leq b_2\} \geq 0.10$$

with all $X_j \geq 0$. Suppose that the a_{ij}'s are independent, normally distributed random variables with the following means and variances:

$E\{a_{11}\} = 1$ $\quad$ $E\{a_{12}\} = 3$ $\quad$ $E\{a_{13}\} = 9$

$\text{var}\{a_{11}\} = 25$ $\quad$ $\text{var}\{a_{12}\} = 16$ $\quad$ $\text{var}\{a_{13}\} = 4$

The parameter b_2 is normally distributed with mean 7 and variance 9. From standard normal tables,

$$K_{\alpha_1} = K_{.05} = 1.645 \quad K_{\alpha_2} = K_{.10} = 1.285$$

For the first constraint the equivalent deterministic constraint is given by

$$X_1 + 3X_2 + 9X_3 + 1.645\,(25X_1^2 + 16X_2^2 + 4X_3^2)^{0.5} \leq 8$$

and for the second constraint

$$5X_1 + X_2 + 6X_3 \leq 7 + 1.285 (3) = 10.855$$

If we let

$$y = 25X_1^2 + 16X_2^2 + 4X_3^2$$

the complete problem then becomes:

$$\text{Maximize } Z = 5X_1 + 6X_1 + 3X_3$$

Subject to

$$X_1 + 3X_2 + 9X_3 + 1.645y \leq 8$$

$$25X_1^2 + 16X_2^2 + 4X_3^2 - y^2 = 0$$

$$5X_1 + X_2 + 6X_3 \leq 10.855$$

$$X_1, X_2, X_3, y \geq 0$$

which can be solved by separable programming. However, the additional variables and constraints make the problem much more complex (problem size and solving technique), resulting in a large incremental amount of computation. The realistic application of stochastic programming techniques, including chance-constrained, breaks down when the size of the original formulation is large or when the final formulation is nonlinear.

CHAPTER III

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THE MODEL INPUTS AND THEIR VARIABILITIES

Customer Demand Variability

Demands are generally projected through the use of some forecasting tool. From a theoretical viewpoint, forecasters use accepted methodology to decompose time series into components, classify trends, and produce a forecast. Often aggregate demands can be reasonably accurate; however forecasts at the customer level may be inadequate. Manual adjustments by knowledgable personnel are sometimes not even enough. Because of the knowledge of events in the field, their modifications can result in improved forecast accuracy, although it is often impossible for these individuals to scrutinize each of the many many forecasts.

Exception reporting which identifies customers whose forecasts differ significantly from historic information can be used to help narrow that information that is necessary to review. While exception reporting does recognize large changes in predicted shipments, it does not relate directly to the accuracy of a customer's forecast. However, exception reporting can be important when identifying demand pattern shifts that might cause large forecast errors.

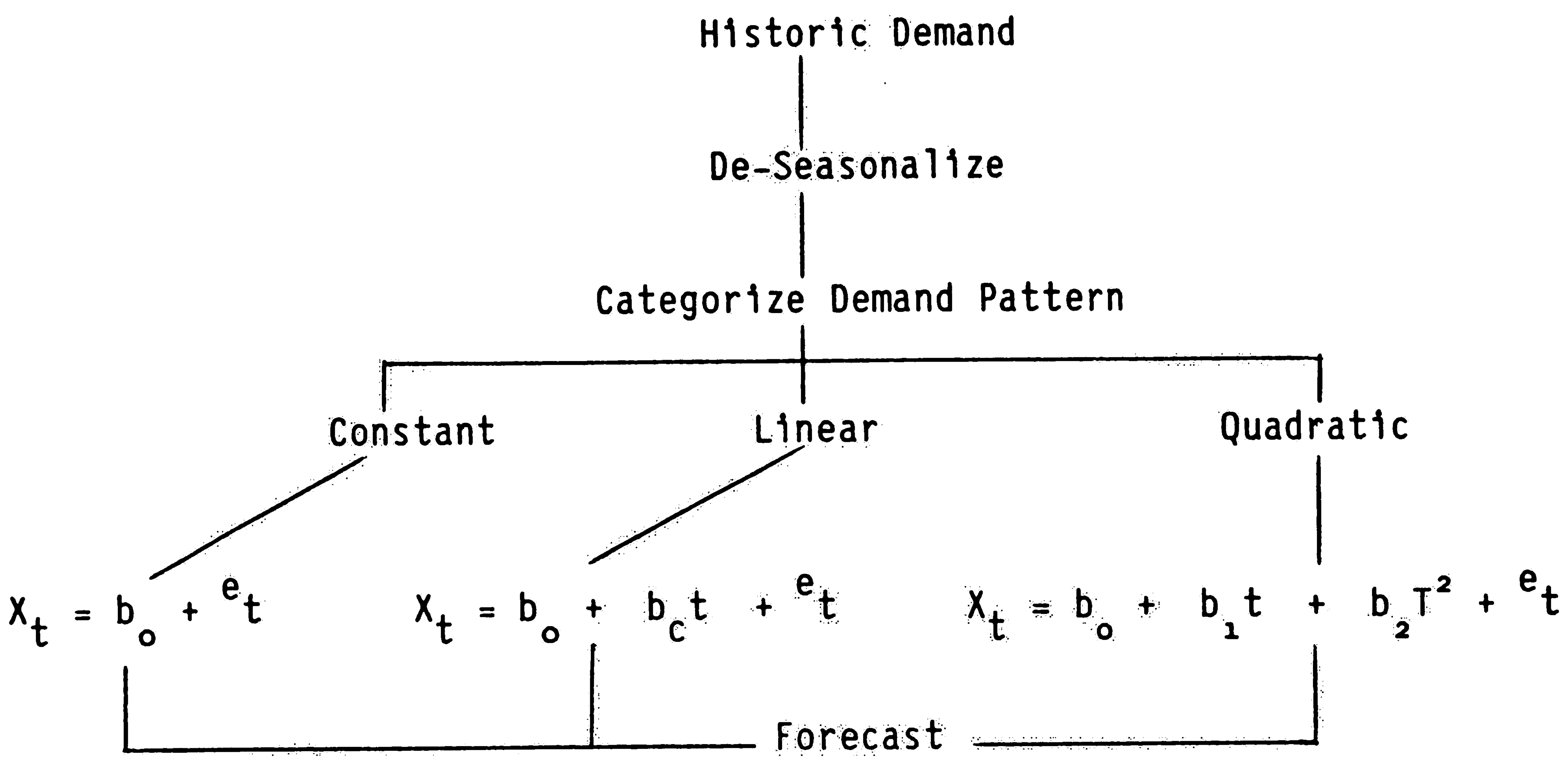
There are several potential forecasting procedures. To begin with suppose the underlying process is relatively stable and there exists little dependence in the estimate from period to period. Then, the "last value" is an example of an estimator. However, it has the disadvantage of being imprecise and is worth considering only if the conditional distribution has a very small variance or the process is changing so rapidly that anything before time t is almost irrelevant or may even be misleading.

An average is another estimator. This is an excellent estimate if the process is entirely stable. However, one does not want to use data that are too old because at minimum, occasional demand shifts are to be expected. A moving-average estimate uses only the last n periods. This estimator is easily updated from period to period; the first observation is dropped and the last one added. The moving-average estimator combines the advantages that it uses only recent history and represents multiple observations. A disadvantage of this procedure is that it places as much weight upon the first data item as on the last. Intuitively one would expect more weight on the most recent observation.

Exponential smoothing is yet another estimating technique. It represents a recursive relationship and can be expressed as $E(X_t) = \alpha X_t + \alpha(1-\alpha)(X_{t-1}) + \alpha(1-\alpha)^2(X_{t-2})$. In this form it becomes evident that exponential smoothing gives more weight to X_t and decreasing weights to earlier observations. An important drawback of exponential smoothing is that it lags behind a continuing trend. If the mean is increasing steadily, then the forecast will be several periods behind. Another disadvantage is the problem of choosing an appropriate smoothing constant α. If α is chosen to be too small, response to change is slow. If α is chosen to be too large, response to change is fast.

Many of the forementioned standard forecasting techniques are employed in many different situations to help in the prediction of demands. In addition, most tools incorporate the concept of seasonality. Seasonality can be traced through the decomposition of historical data and/or smoothing techniques.

The following documents the flow of information when seasonality is tracked through decomposition.



Historic data is put into a forecasting system and deseasonalized. The deseasonalized data is analyzed and a trend factor is isolated and categorized. After the trend factor is calculated the adjusted data is exponentially smoothed.

An alternative to decomposing the data into its seasonal and trend components is to perform the estimates of these factors simultaneously. An added advantage is that the models' sensitivity to changes in demand can be altered by simply changing smoothing factors.

Historic Demand

- Calculate Smooth Average
- Calculate Seasonal Factor
- Calculate Trend

$f_t = (U_t + b_t)\ F_{t-11}$ (linear trend)

U_t = Smoothed Average

b_t = Trend

F_{t-11} = Seasonal Factor

A decomposition model often yields a more accurate fit of the historical time series than elementary smoothing methods. However, the computer resources needed to perform the necessary calculations increases both the time and cost of producing a forecast.

Smoothing methods are used extensively because of their relative simplicity and low cost. The greater number of smoothing factors and the ability to seasonalize add to the complexity of smoothing methods. The Winters forecasting algorithm, for example, has the ability to recognize seasonality and contains three smoothing factors. So, not all smoothing methods are as simple as first explained.

Once a system of producing forecasts has been developed, there should be a monitoring technique to indicate when a change in demand pattern is causing forecast errors. When a monitoring method indicates that the forecast is out of control, the cause for change should be investigated. If it is found that the new demand pattern is likely to continue, then the forecasts can be modified to reflect this change.

Trigg's method of monitoring, proposed in 1964, was an improvement on a method proposed by Brown in 1962. Because the method is based on the calculation of the exponentionally weighted average of the error it is sometimes referred to as the "Smoothed Error Method," but is more generally known in this country by its author's name.(16)

The Trigg monitoring system utilizes a tracking signal whose value indicates, with degrees of statistical confidence, the failure of a forecasting system due to a change in demand pattern. The sign of the tracking signal indicates whether the forecast is higher or lower than the actual demands. The tracking signal is also an ideal smoothing factor, because it becomes larger as the data becomes more volatile and decreases when the data is more stable. A large value gives more weight to recent data, a smaller value will cause more weight to be associated with older data.

Estimating customer demand levels is paramount in producing good output from a distribution linear programming model. Often demand levels are the biggest set of data going into the model and also the most difficult to predict. Poor estimation of demand levels can at minimum suggest misleading results. It is necessary to understand demands well enough to put a range around the estimate and also suggest a confidence in the estimate. This confidence interval will establish a set of inputs that can be used to help calculate final results.

Production Capacity Variability

Capacity constraints are normally of known quantities. That is, the distribution system generally has a set of facilities with a given output at each location, and even maintenance work and other scheduled shutdowns at specific facilities can be incorporated into a L.P. system for planning purposes. Unexpected shutdowns, however, can force significant changes to a distribution plan due to the rather large percent of the total system capacity a single facility represents. If a system is small enough, shutdown of a single facility can lead to catastrophic variations to production levels at neighboring facilities.

In addition to considering loss of capacity, increases in product availability will also shift distribution patterns. Increased production might come in the way of efficiency improvements or new plant construction. Again capacity increases are planned and therefore can be incorporated in the distribution system. Distribution patterns will change significantly as new facilities are added, but because efficiency improvements are less dramatic increases, distribution shifts caused by these improvements are often explained. The greater the capacity increase the greater the amount of distribution pattern shifts.

Another less obvious situation that can effect L.P. results is the plant operation. Each facility in a distribution system has its own operation mode(s). Product interaction, minimum plant turndown, available raw material, etc.. are all factors which can put some restrictions on the obvious constraint of total plant capacity. These peripheral factors must be evaluated for applicability to the situation in question. The fewer the number of extra factors the less complicated the L.P. structure is. Conversely, the greater number of extra factors that can be incorporated (to a limited extent) the more accurate the final results.

Being able to understand the time frame of capacity changes (increases and decreases) is necessary for the achievement of good results. This is a much different situation than estimating other parameters. For example, demand variability needs to be considered at an individual input level. The time frame is important for the estimate of an overall demand, but even more important is recognizing the inherent variability of the forecast at a specific period of time.

Capacity constraint variability (from month to month) need not be heavily considered as a problem in defining distribution shifts. If total capacity constraints are reached as demand increases, distribution shifts can be numerous but explainable. Although capacity constraint variability need not be considered, capacity additions and shutdowns can have monumental effects on a system's distribution pattern. Being able to foresee a shutdown, or in the case of preventive maintenance plan for it, is a most important criterion in understanding the effects of capacity constraints upon final distribution patterns.

Distribution Requirement Variability

Many distribution L.P. systems have the capacity to consider both vehicle and driver constraints. For short-term planning (a monthly operational plan; plant shutdown due to equipment failure) using the distribution requirements as constraints is important. For longer term planning (strategic operations) the distribution requirements can be used to suggest the number of vehicles and drivers that will be necessary to properly operate the system over time.

The number of vehicles and drivers available for use is a known quantity. This quantity changes upon vehicle purchases, equipment problems, hiring of new drivers, contract disputes, etc... Generally these variations are known for short-term considerations. (Equipment failures would be a situation where an operation plan could be effected in the short term). Although these constraint availabilities are known, they can have a large effect upon L.P. results. For example, if a facility has a limited number of vehicles for distribution, its capacity will never be a concern, and for this reason it is important for a system to have an adequate supply of vehicles. Again, by analyzing results of long-term studies, the vehicle availability/production capacity balance should be kept controlled.

Understanding the effect of the variability of the distribution constraints on final L.P. results is important; but it is critical to recognize this variability is often full increments of vehicles or drivers. A loss of a vehicle at a facility already constrained by the number of vehicles might cause a rather large change to the operating system. Neighboring facilities will have to distribute the product not able to be distributed due to the loss of the vehicle. If all neighboring facilities are already constrained a ripple effect can take place. The neighboring facilities will pick up accounts from the facility which lost a vehicle. They in turn will have to drop certain deliveries to keep within constraints. This effect will continue until unconstrained facilities are reached.

Vehicle/driver constraints can have major effects on L.P. results. However, if the results are to be interpreted for long term situations, distribution constraints need not be employed but rather recorded for support of vehicle purchases, hiring of drivers, and vehicle/driver location transfers. It is necessary to recognize the major effects distribution constraints can have on an L.P. system, but it is equally important to recognize that these constraints may not need to be employed for given situations (specifically long-term planning).

Distribution/Product Costs

Input costs are the parameters that drive the distribution L.P. model, and since linear programming models minimize costs, the input costs obviously have a direct effect on model results. For this reason alone it is necessary to detail the input costs on an individual basis, as well as on how each cost compares with another. The relative cost of distributing the product as compared with producing the product must also be analyzed and correctly represented. If it is not, the system results will favor either the minimization of distribution costs or the minimization of production costs. The system should reflect the minimization of the total (distribution and production) system costs.

Potential for variability of and lack of confidence in the input cost parameters need to be addressed due to their direct effects upon results. Distribution cost parameters have particular potential for variability because of the many individual input costs that represent the total distribution charge. Vehicles cost components, for example, might be fuel, maintenance, depreciation, etc.. Labor charges for drivers must also be considered with the vehicle components to represent a total distribution charge. Uncertainty in each component can compound to a rather large variability in the total distribution cost parameter.

The production cost parameters on the surface would seem to have less potential for uncertainty. However, the components that represent production costs are utility costs, labor costs, inventory charges, etc... and having this many possible components represents potential for variability. Utility and labor costs are based upon contracts, therefore these components are relatively straight forward with respect to their effect upon total production costs. However, a complicated contract or inventory charge are components which may inject variability into the production cost parameter. The number of components which put variability into the final production cost parameter is less than the number which effect distribution costs. However, some of the production cost components can be very complicated, more complicated than any individual distribution cost component.

There exists a reasonable possibility for either production costs or distribution costs to show variability. All costs and their components must be scrutinized. A good representation of each cost component is paramount in the attainment of good results. Good representation might dictate parameters with little variability, or it might dictate a parameter with a large but understandable variability. Confidence in the value of the parameter itself and an understanding in the possible variability in the value must be grasped. A thorough knowledge of each is the means to assure the model is driven accurately.

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CHAPTER IV

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AN IMPROVED PARAMETRIC PROGRAMMING METHODOLOGY

Parametric Programming

Production capacities, vehicle constraints, and demand can all be monitored through parametric analyses of the right-hand-sides (in MPSX, this is the PARARHS option). Demand rows might also be monitored by understanding the range of demand values where the shadow prices will hold (RANGE option).

PARARHS is used post-optimally to perform parametric programming on the right-hand-sides. From any linear programming problem a series of related problems can be defined by replacing the right-hand-side by the original right-hand-side plus a multiple of a change column. This multiple is the parameter. Thus each value of the parameter defines a different related L.P. problem.

The function of PARARHS is to scan a whole series of solutions to such problems varying the parameter from zero up to a defined maximum. The parameter is gradually increased and the solution is kept optimal and feasible for values of the parameter by changing the basis when necessary.

RANGE is used post-optimally to generate and put out an analysis of the current solution. The analysis includes:

1. The effects of cost changes on optimum activity levels.

2. The cost of changing a column (row) activity from an optimum level and the activity range for which this cost is valid.

This type of information is important because it indicates:

1. How much the activity will vary when a discount is made on the variable.

2. How a problem may be adjusted to increase profits or reduce costs.

As just discussed, RANGE and PARARHS are important options of MPSX to obtain sensitivity information. The RANGE option gives information concerning the range of values over which a shadow price holds true. However, this range is true only if no other parameter is changed. Therefore, to discuss variability of input parameters PARARHS (or PARACOL or PARARIM) is the more important MPSX option. A methodology of using parametric programming to understand the variability of constraints and how the variability effects final results is the concept to be detailed.

Parametric Analyses to Explain Forecast Variability

$$b_i = \bar{b}_i + \alpha_i \theta$$

where

b_i = modified forecast of customer i

Σb_i = modified total forecast

$\bar{b}_i$ = original forecast of customer i

$\Sigma \bar{b}_i$ = original total forecast

θ = constant necessary to obtain the stated difference of $(\Sigma b_i - \Sigma \bar{b}_i)$

α_i = coefficient for customer i to assign a necessary part of the constant θ.

$\alpha_i \theta$ = the increase (decrease) in the forecast of customer i.

If one assumes that the per cent increase (decrease) in each customer forecast is equivalent to its initial ratio of forecast to total forecast, then,

$$b_i = \bar{b}_i + \bar{b}_i / \Sigma \bar{b}_i \; (\theta_*)$$

where $\theta_* =$ change from the original total forecast

Therefore,

$$\alpha_i = \bar{b}_i / \Sigma \bar{b}_i$$

because the ratio $\bar{b}_i / \Sigma \bar{b}_i$ represents the per cent of the total forecast that is represented by customer i.

However, this α_i does not allow for the possibility of forecast uncertainty at the customer level.

Assume that a normal distribution holds at the customer level, then

$$b_{i*} = \bar{b}_i + (RN_i)\,(S_i)$$

and,

$$\Sigma\, b_{i*} = \Sigma\, \bar{b}_i + \Sigma((RN_i)\,(S_i))$$

where

RN_i = standard normal random variable with $\bar{X} = 0$, $S = 1$

S_i = standard deviation for the ith customer distribution

Finally,

$$\alpha_{i*} = \frac{b_{i*}}{\Sigma\, b_{i*}} = \frac{\bar{b}_i + (RN_i)\ (S_i)}{\Sigma\, \bar{b}_i + \Sigma((RN_i)\ (S_i))}$$

The previous descriptions reflect the methodology used to help explain variability in both the total forecast (θ) and the individual forecast (α_i). An example follows to further explain the methodology.

Example:

Forecasts are calculated for inputs to a distribution L.P. model. We believe that a normal curve represents the uncertainty of the total forecast within $\pm 5\%$, and of the individual forecasts within $\pm 10\%$. In both instances we assume 95% confidence limits (i.e. $\pm 2\sigma$).

1. First let's calculate α_{i*} for each customer.

$$\alpha_{i*} = \frac{\bar{b}_i + (RN_i)\ (S_i)}{\Sigma\, b_i + \Sigma((RN_i)\ (S_i))}$$

where $\bar{b}_i = 10$

$\Sigma\, \bar{b}_i = 100$

$RN_1 = 1.56$ (1.56 Standard deviations above average)

$$S_1 = (\sigma)\ (\bar{b}_1)$$

$$= (.05)\ (10)$$

$$= .5$$

One can assume that

$$\Sigma\ ((RN_1)\ (S_1)) = 0$$

because of the important properties of the normal distribution. Its physical appearance is that of a symmetrical bell-shaped curve extending infinitely far in both positive and negative directions. And the sampling distribution of the means of random samples will be approximately normal if the sample size is sufficiently large.

Then,

$$\alpha_1{*} = \frac{10 + (1.56)\ (.5)}{100 + 0}$$

$$= 0.1078$$

2. Now, let's calculate N values of θ to be used for the L.P. runs;

$$N = \left\{ \frac{(t_{1-1/2\alpha})\ (S_N)}{d} \right\}^2$$

Where t = percentile of the student t distribution

d = the confidence interval will be of length 2d; the precision of the estimate is $\pm$d.

Assume: S_N, has been previously calculated from test runs.

Note: The sample size N may need to be reassessed when:

1. The variation in the total forecast changes

2. The variation of an individual forecast changes significantly.

$$\theta_{j*} = \theta_j + (RN_j)(S_j)$$

Where RN_j = standard normal random variable

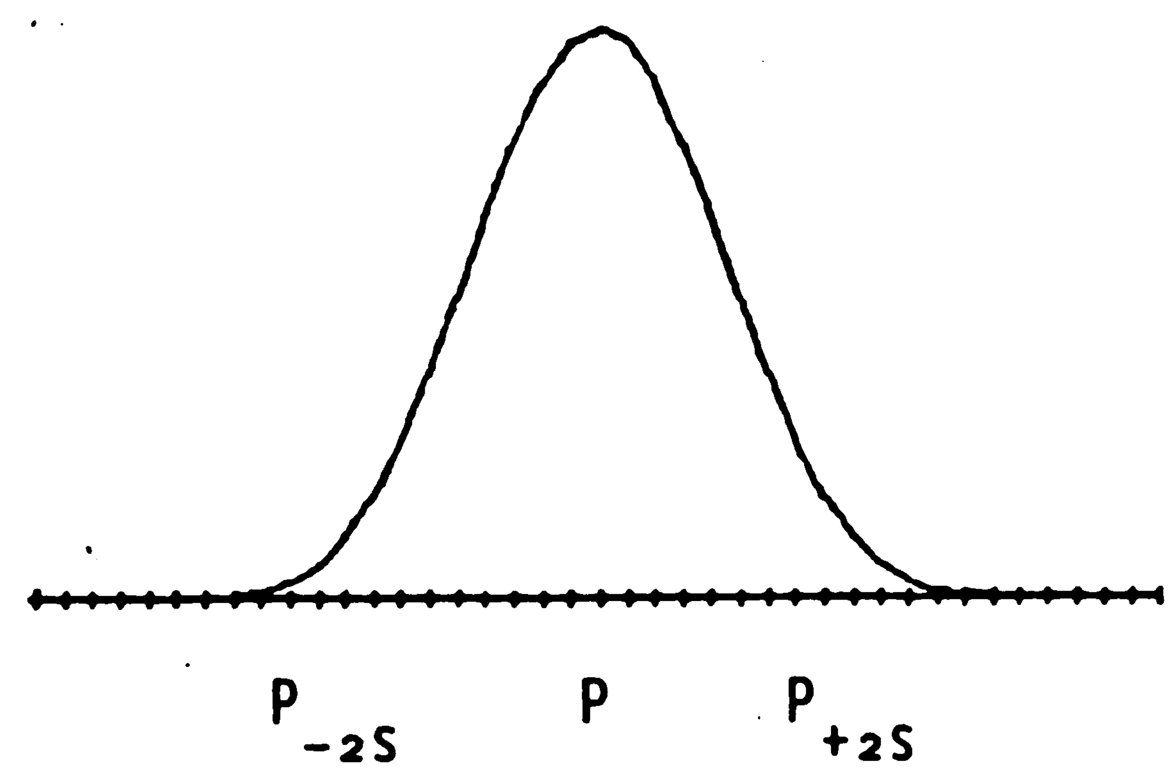
S_j = standard deviation for the total forecast distribution

j = 1, 2, ..., N

3. Run the necessary L.P. cases. Have the L.P. calculate N sets of customer laid-in costs using the α_{i*} values from Step 1 and the θ_{j*} values from Step 2.

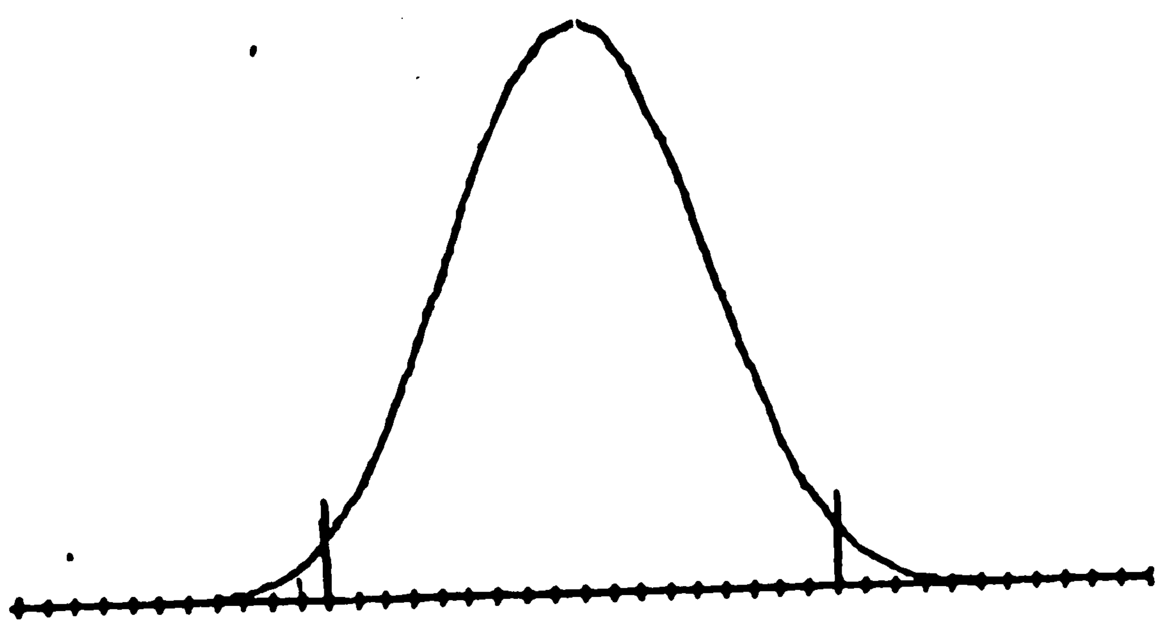
4. Use the results from Step 3 to estimate the value for the laid-in costs (P_i), as well as to explain their variability. We can now make a statement about the laid-in cost for customer i based upon forecast uncertainty: "With 95% confidence, the values of the laid-in costs for customer i range from P_{-2S} to P_{+2S}. The most likely outcome of the cost for customer i is P_o."

After obtaining the parameters from the previous statement we can draw a normal curve and present final results graphically:



The Methodology

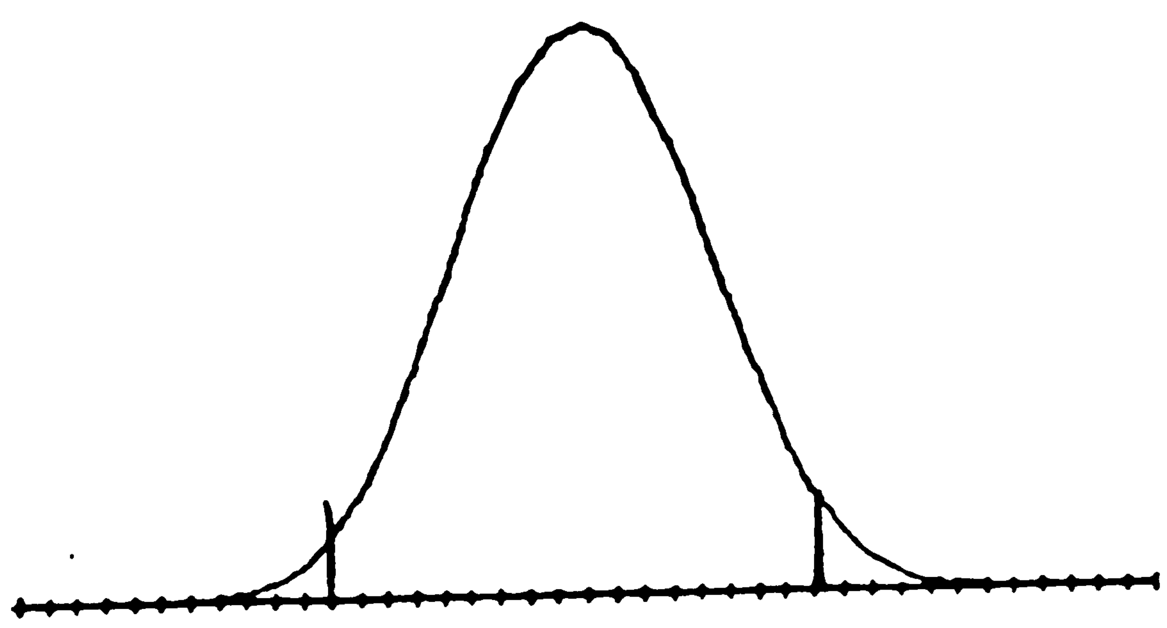
1.



α_{i*}

- Calculate α_{i*}
- Represents forecast variability at the customer level.

2.



θ_{j*}

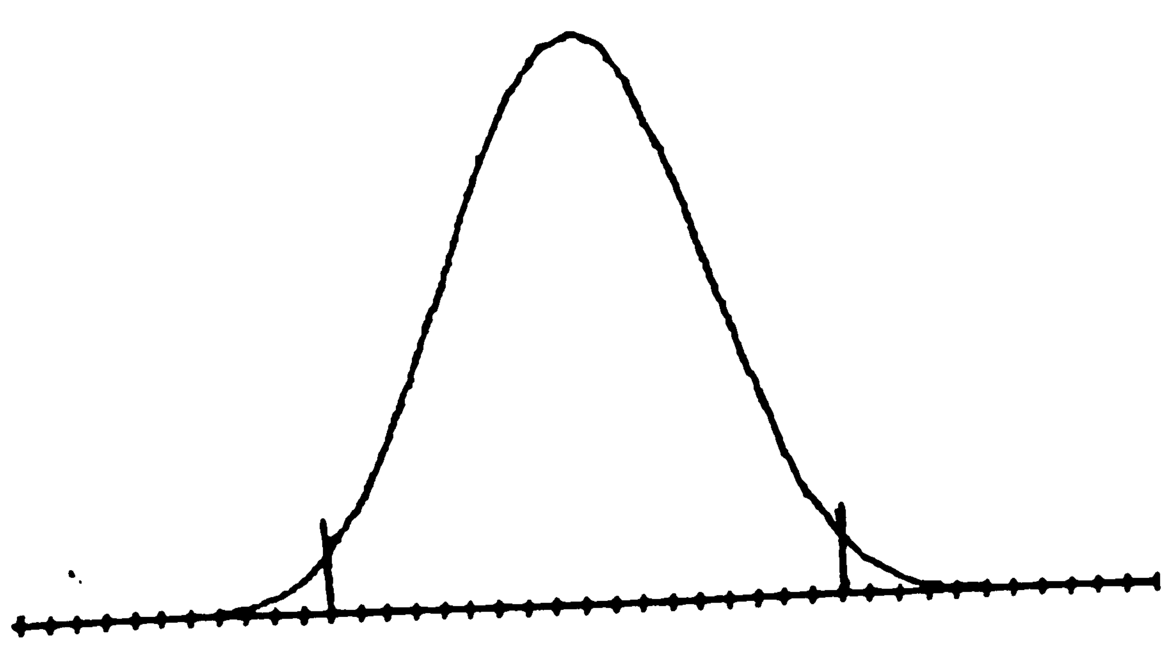
- Calculate θ_{j*}
- Represents forecast variability of the total forecast.

3.

M P S X

- Calculate P_{ij*}
- N sets of P_{ij*} using α_{i*} and θ_{j*}

4.



P_*

- Establish confidence limits for $P_{i\Sigma j*}$.

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Parametric Analyses to Explain Production Capacity Variability and Distribution Constraint Variability

$$b_i = \bar{b}_i + \alpha_i \theta$$

where

b_i = modified capacity of facility i

Σb_i = modified system capacity

$\bar{b}_i$ = original capacity of facility i

$\Sigma \bar{b}_i$ = original system capacity

θ = constant necessary to obtain the stated difference of $(\Sigma b_i - \Sigma \bar{b}_i)$

α_i = coefficient for facility i to assign a necessary part of the constant θ.

$\alpha_i \theta$ = the increase (decrease) in the forecast of facility i.

If one is to step back and ask the question "is it necessary to use parametric analyses to represent capacity changes when determining demand point shadow prices?", the answer will probably be "no". A simple rerun of the L.P. model with the capacity change (plant outage, new facility) considered will accomplish the same goal. Again, when establishing a base for customer prices one needs to consider the average product availability over time. From historical data, calculate average uptime (capacity) for each facility. If there is a large variability, it may become necessary to use parametric analyses.

α_i will represent individual facility variability.

θ will represent global variability.

The same considerations hold for distribution constraints. Review of historic data should define variability in both driver and vehicle availability. If the variability seems high enough to address:

α_i will represent terminal i distribution constraint variability.

θ will represent global distribution constraint variability.

If variability is addressed for either production capacity or distribution constraints, steps similar to those for addressing variability in forecast accuracy will be followed.

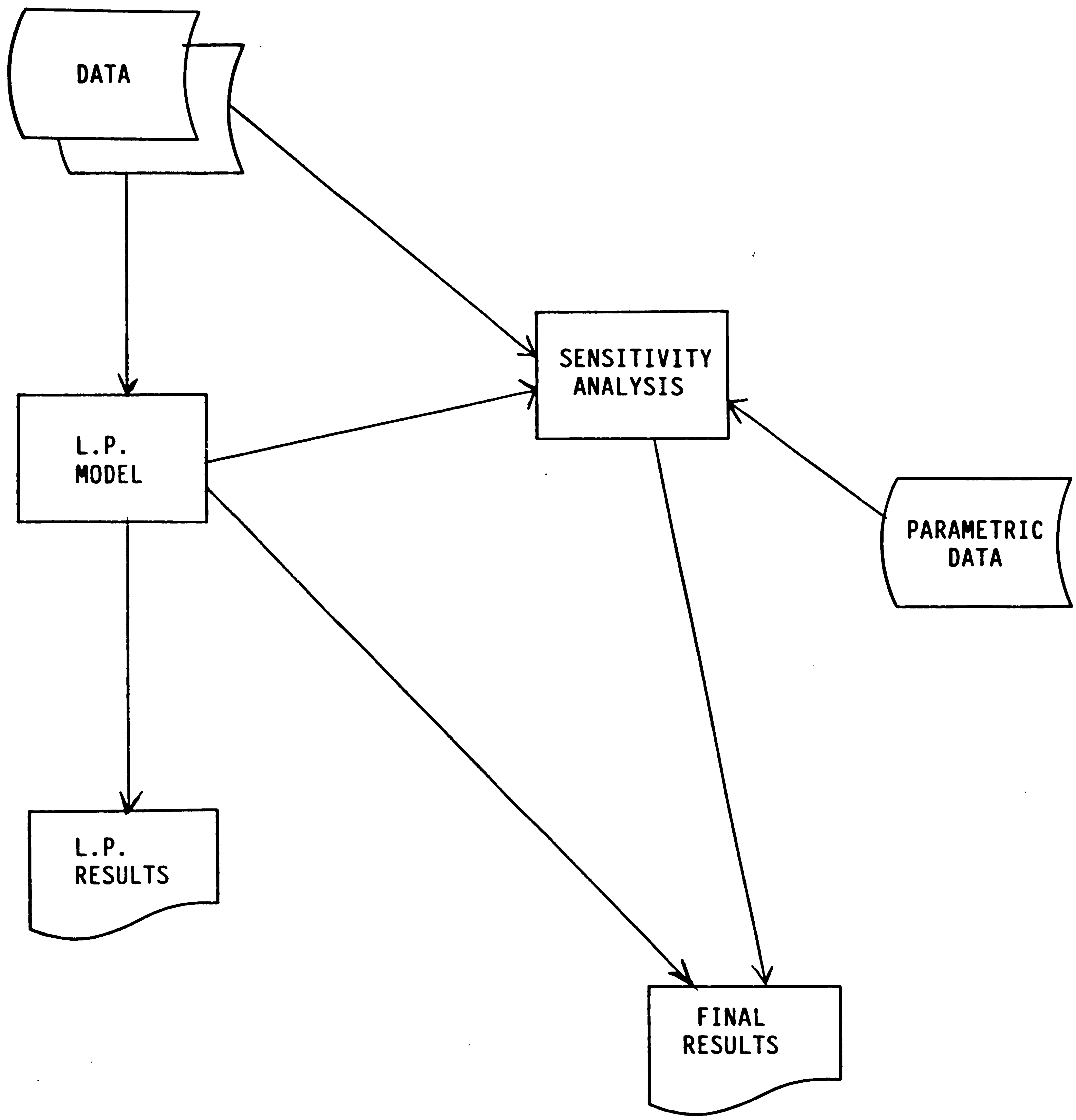
Parametric Analysis and Cost Coefficients

Cost coefficients are often input into a L.P. system on a per product unit basis. However, the costs that make up the coefficients are often not initially in the same per unit basis, and the variations that might be recognized in the costs are also not in this per unit basis. For example, wage rates, a part of the cost per unit of delivering a product, would probably vary on a per hour basis not a per unit. Fuel cost variability happens on a per mile base, not a per product unit. The point is that it becomes very difficult to identify values of α and θ for distribution cost coefficients.

Product cost coefficients and their variability can be more easily defined and represented by the parametric programming methodology. Most corporations are able to identify a cost per unit for producing a product. Although, utilities, labor, and raw materials (components of the final product cost coefficients) are not initially examined on a per unit basis. Again, it becomes difficult to use parametric analyses to represent the component variabilities.

Cost parameters are generally more stable than some of the constraint parameters. If contracts are renegotiated, fuel costs change, etc... then L.P. runs must be redone to reflect the change. At times the change to results will be obvious (only a change in total operating cost). When the cost changes are global, there will be no effect to the distribution patterns.

SYSTEM DESIGN



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```
DATA A1(DROP=GNUM); INFILE A MISSOVER;
     INPUT @1   GROUP    4.
           @16 VOLU     10.1;
     RANDOM = NORMAL(1961273);
     PERCNT = .10;
     STDDEV = PERCNT/2;
     APROD1 = RANDOM *(STDDEV * VOLU);
     SKEY = 1;
     GNUM + 1;
     FILE B; PUT @1 GROUP 4. GNUM 3.;

PROC SORT; BY GROUP;

DATA A2; INFILE B MISSOVER:
     INPUT @1 GROUP 4. GNUM $CHAR3.;
     IF SUBSTR(GNUM,1,1)=' ' THEN SUBSTR(GNUM,1,1)='0';
     IF SUBSTR(GNUM,2,1)=' ' THEN SUBSTR(GNUM,2,1)='0';
     IF SUBSTR(GNUM,3,1)=' ' THEN SUBSTR(GNUM,3,1)='0';

PROC SORT; BY GROUP;

DATA A;
     MERGE A1 A2;
     BY GROUP;

PROC MEANS NOPRINT;
     ID SKEY;
     VAR VOLU APROD1;
     OUTPUT OUT=B SUM=SVOL APROD2;

DATA C;
     MERGE A(IN=A) B(IN=B); BY SKEY;
     IF A;

DATA C; SET C;
     ALPHA = (VOLU + APROD1) / (SVOL + APROD2) ;

PROC PRINT; SUM VOLU ;

DATA _NULL_; SET C; FILE C;
     PROD=1; IF GROUP>=3000 THEN PROD=2;
     PUT @5 'ALPHA'
         @15 'DEM' PROD 1. GNUM $3.
         @25 ALPHA 10.8
         @79 '46' ;
```

```
PROC MEANS DATA=C NOPRINT;
     VAR RANDOM;
     OUTPUT OUT=D SUM=RSUM MEAN=RMEAN;

PROC PRINT;

PROC MEANS DATA=C NOPRINT;
     VAR ALPHA;
     OUTPUT OUT=D SUM=ASUM MEAN=AMEAN;

PROC PRINT;
```

CHAPTER V

****

THE RESULTS

Improved Parametric Programming and Stochastic Programming

Stochastic programming techniques stipulate that with a probability of $(1-\alpha_i)$ we want to be able to sell all the output produced (or, we must deliver at least b_i to customer i). We are predicting, with a degree of certainty, that a specified occurrence must happen. This approach is important, especially when we wish to suggest that high profit levels are essential, but secondary to the necessity of selling all our product. However, the inherent variability in the various inputs is not specifically addressed. Inherent variability refers to the individual as well as the overall input variabilities. For example, uncertainty associated with product demand must be addressed at the customer level to help describe our lack of knowledge concerning individual needs and at the product level to describe our inability to perfectly forecast total system-wide demand. Even those variabilities that are being considered by Stochastic Programming are often not linearly represented, forcing the use of separable or linear fractional programming techniques.

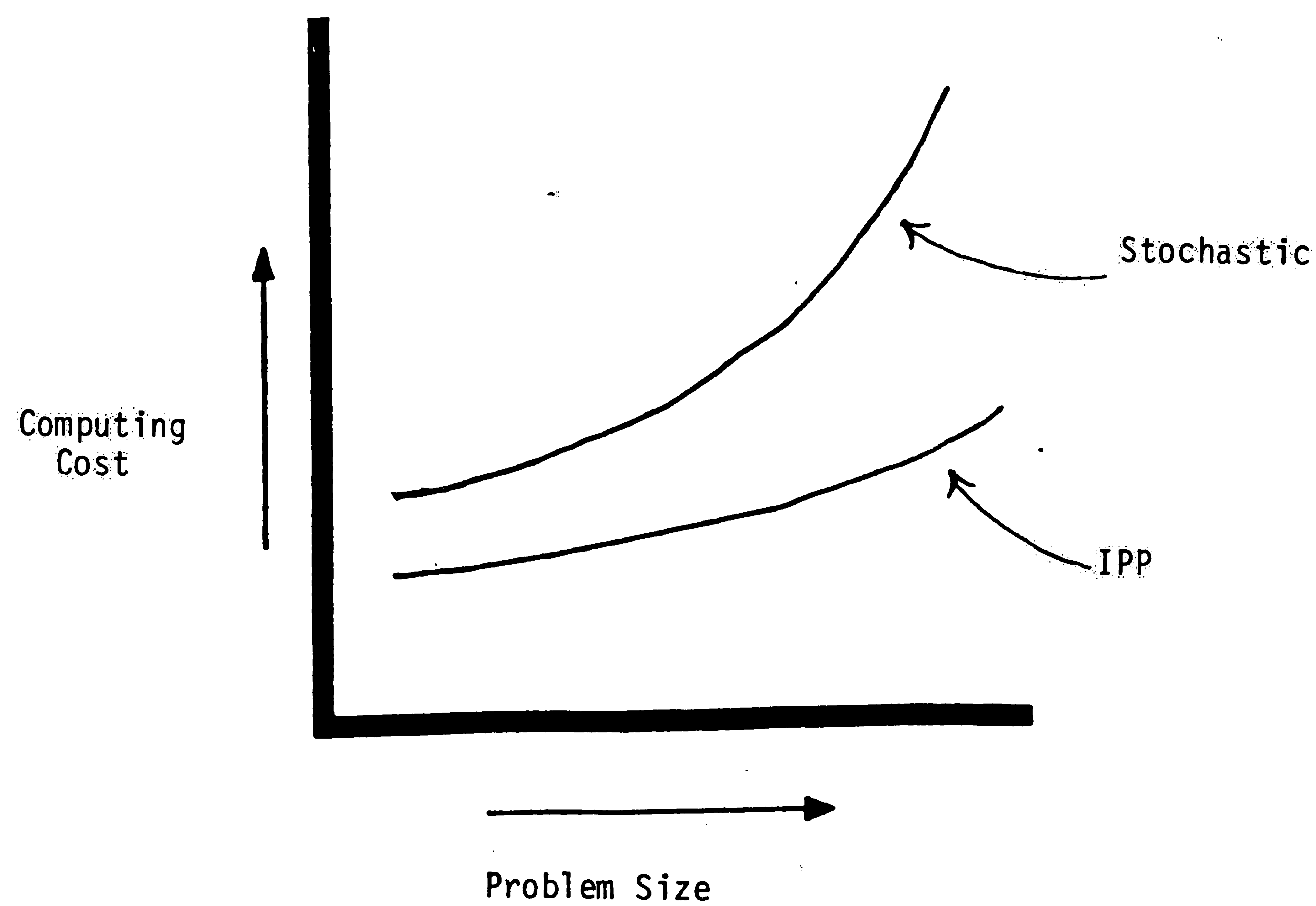
The Improved Parametric Programming (IPP) methodology addresses 3 basic concerns that are inherent problems with stochastic programming techniques. The improved parametric methodology:

1. attempts to address the inherent variability of inputs on both an individual and overall basis;

2. uses parametric programming approaches available in most linear programming software packages; and

3. derives final results in a reasonable amount of additional computing time and expense.

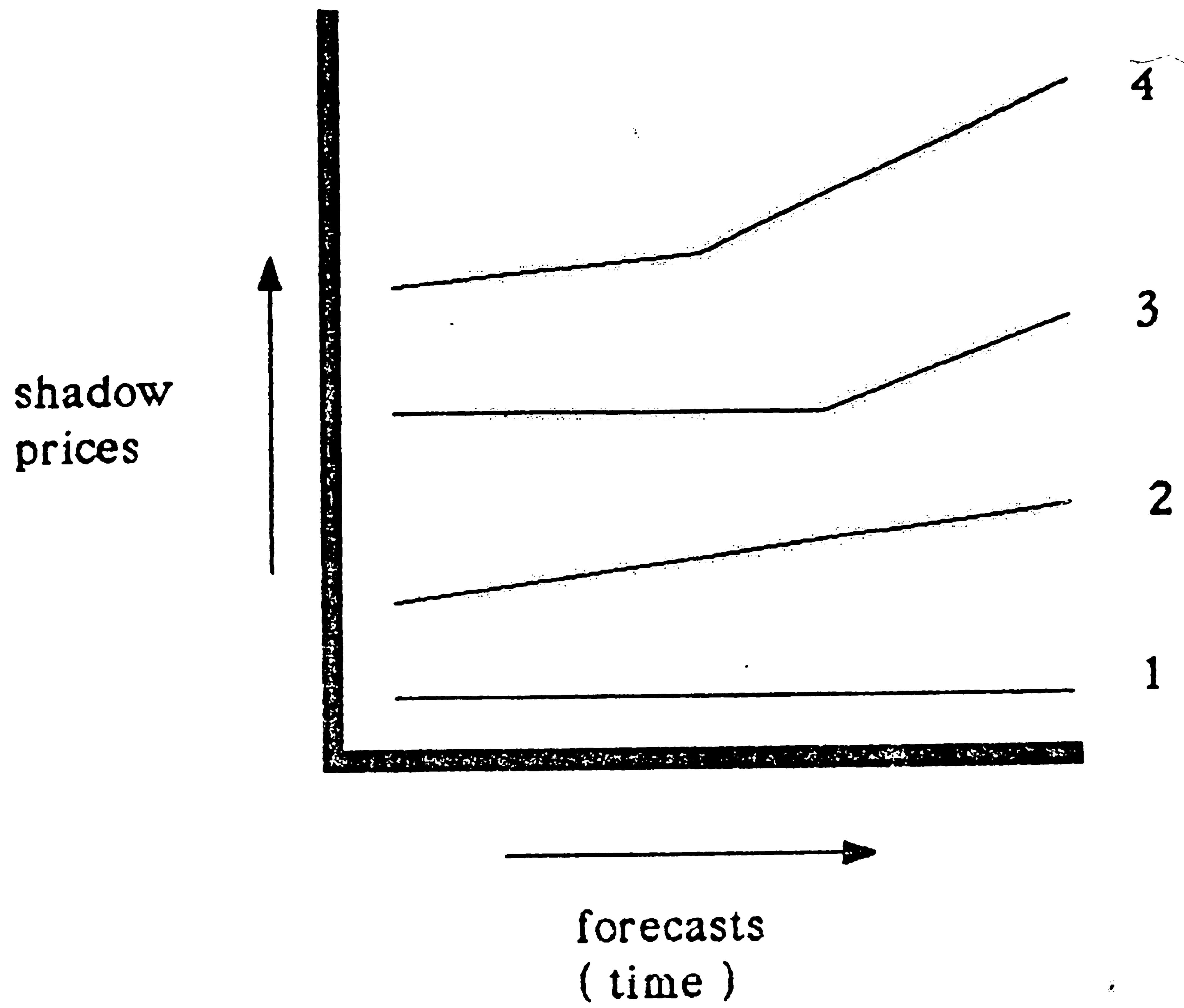
IPP results compare favorably with results achieved using Stochastic Programming. Refer to the appendix for a table of test results. Tests were performed on a distribution linear programming system where Stochastic Programming techniques were used to describe inherent input variability. When RHS variability is addressed, results are equivalent and IPP actually becomes more effective as additional cases are run. What is important is that for equivalent computing charges more cases can be run under the IPP environment and hence better results for an equal expense. As the initial problem grows in size (rows and columns) the cost savings using IPP over stochastic programming widens.



When cost coefficients' and allocation coefficients' variabilities were tested, results were mixed. When quadratic tendencies were observed, stochastic programming combined with linear fractional programming techniques displayed obvious advantages. However, the computing time to address the quadratic problems were extremely large; even to the point that the problems were not permitted to finish due to the excessive charges. Although stochastic programming and linear fractional techniques might effectively describe some variability, excessive costs from computing resources make it an unacceptable approach. IPP may not fully describe the variability in either C_j or a_{ij} (due to nonlinearity), but can relate important tendencies not shown by a single L.P. execution.

	PROS	CONS
Improved Parametric Methodology	• Deterministic • Linear Representation • Uses Parametric Programming procedures available in most linear programming software packages • Will derive results similar to proven techniques in most circumstances • Improved information due to the ability to address both overall and individual data uncertainties simultaneously	• Forces linear representation when considering cost coefficients or allocation coefficients • A large number of runs are necessary to correctly represent the input variabilities.
Stochastic Programming	• Deterministic • Describes variability of cost coefficients, allocation coefficients, and right hand sides	• Often a nonlinear (quadratic) representation and must be solved using separable or linear fractional programming techniques • Can add a large number of rows and columns to the original problem which suggests a large increase in computing time and costs. • A large number of runs are necessary to correctly represent the input variabilities.

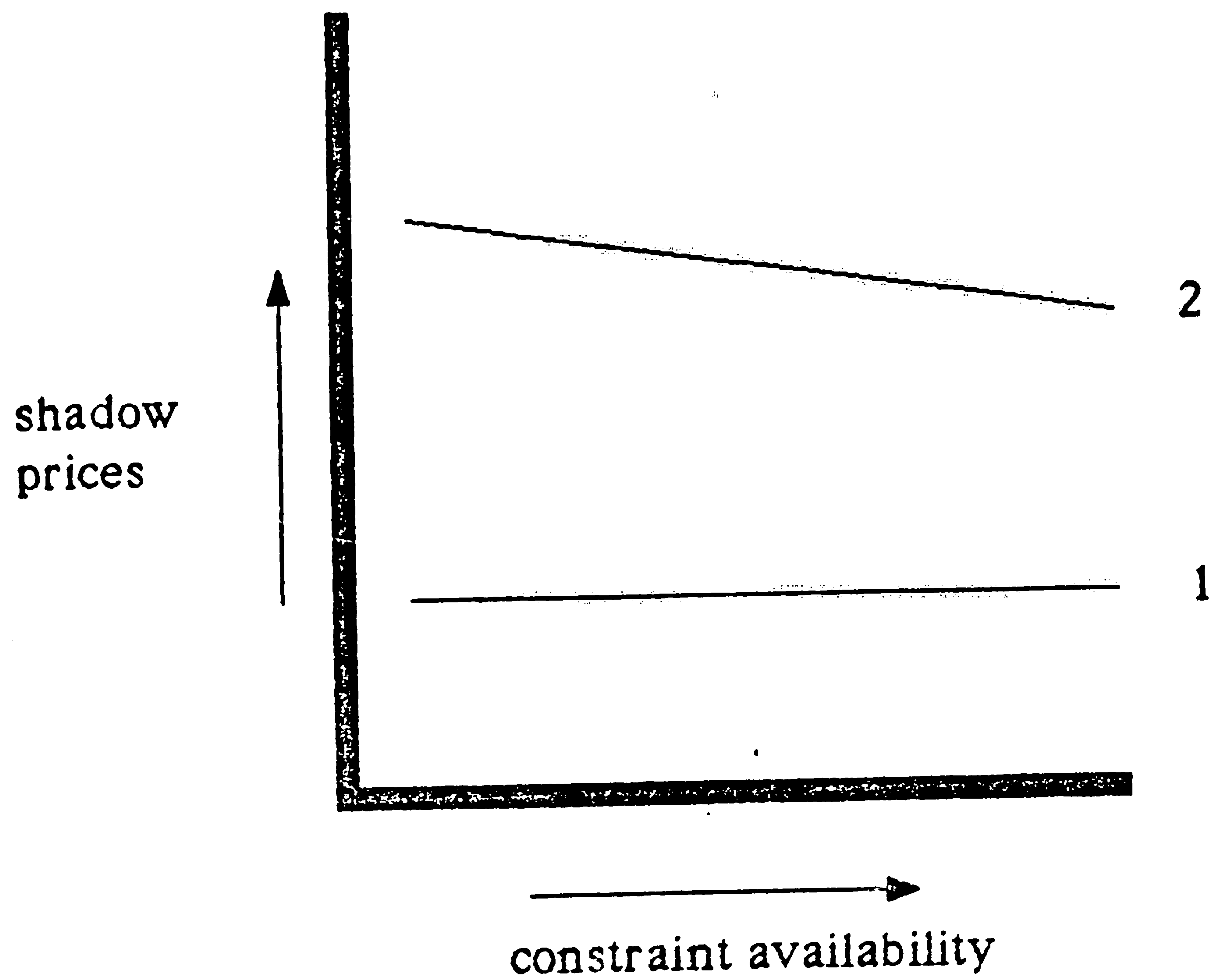
Demand Variations



Results from test data show that when studying demand variations, shadow prices plotted versus the forecast range can follow any of four lines, or some combination of them. That part of the function which is increasing represents a constrained system. As forecasts increase, the system must incorporate the extra pull for product and sacrifice total system cost and individual shadow prices.

Curve 1 represents an unconstrained system. For the expected uncertainty there is no system constraint which effects final results. In this situation, over the range of forecast values for customer 1 neither distribution nor production constraints have any effect upon the distribution pattern. Curve 2 represents a constrained system. The constraint that forces results at the lower forecast also effects results at the upper forecast. Curve 4 also represents a constrained system. However, the constraint that causes the initial effect is overridden by a second constraint at the point where the break in the slope occurs. For example, vehicle availability might cause the initial increasing function until plant capacity is reached. The capacity constraint takes effect at the slope break. Now the capacity constraint becomes the limiting constraint. Curve 3 represents a pattern similar to that of Curve 4. The difference is that for some initial period there is no constraint limitation. As a constraint is reached the slope break occurs and the function changes course.

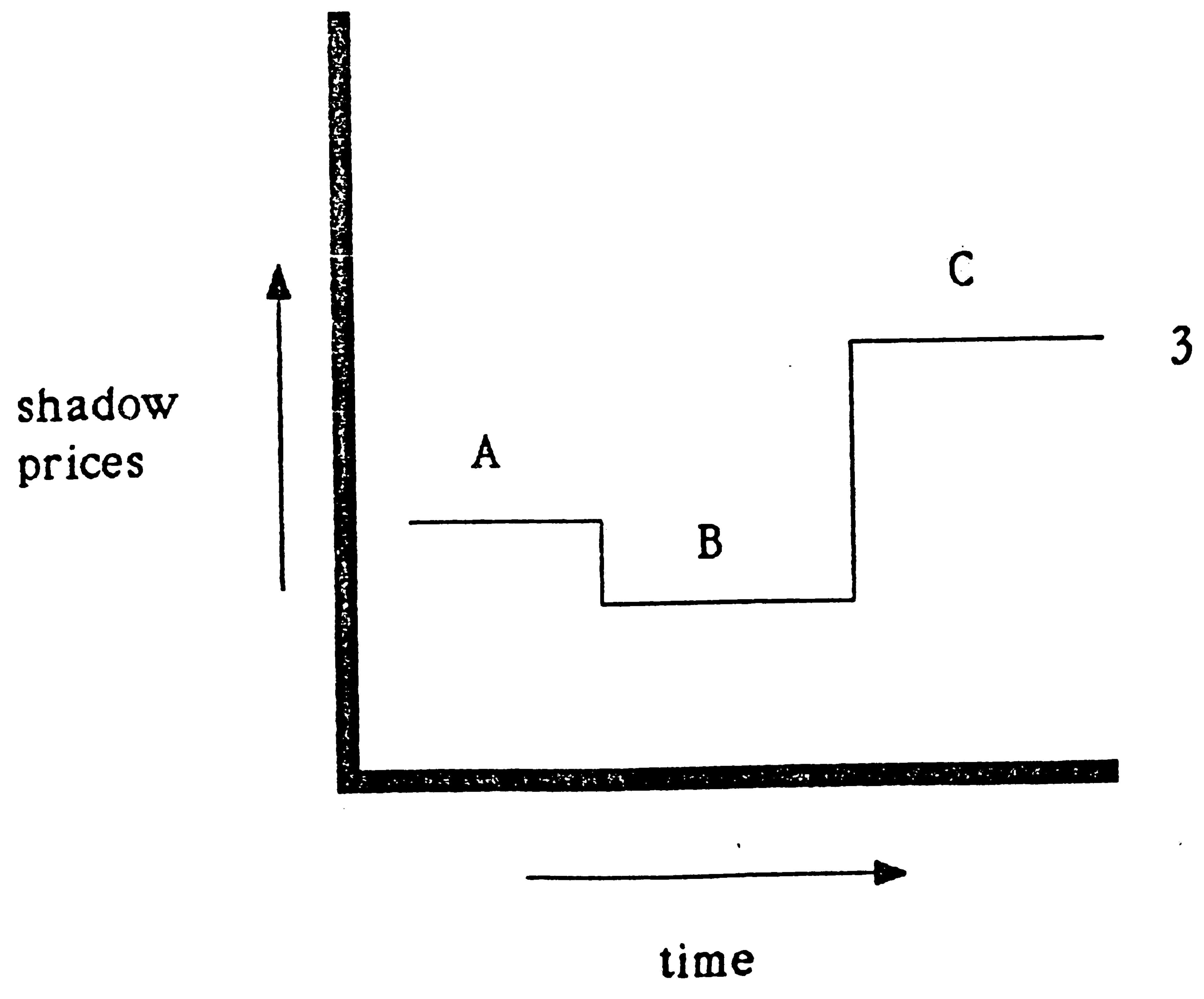
Constraint Changes



Constraint (product capacity, vehicles, and drivers) variability tends to be discrete changes rather than a distribution. Production increases, plant shutdowns, vehicle purchases, etc. all represent these discrete changes. Results often show dramatic changes when the constraint variation occurred. Test cases were run assuming a variability distribution to better understand effects upon final results.

Curve 1 represents a L.P. system where changes to constraints have no effect. Because a terminal was not yet constrained by vehicle numbers, the loss of one vehicle due to mechanical failure has no effect. A vehicle presently not utilized is put into operation. If a facility has not yet reached its capacity, efficiency improvements are irrelevant. Making more product available will not change any distribution patterns.

Curve 2 demonstrates the effect constraints can have on results. The curve has a slope that represents an increase in the shadow price as a constraint becomes more limiting. This might be the per cent loss of capacity for a month. The longer a facility is to be shutdown for the planning period, the greater the loss of capacity, and the more limiting the constraint.



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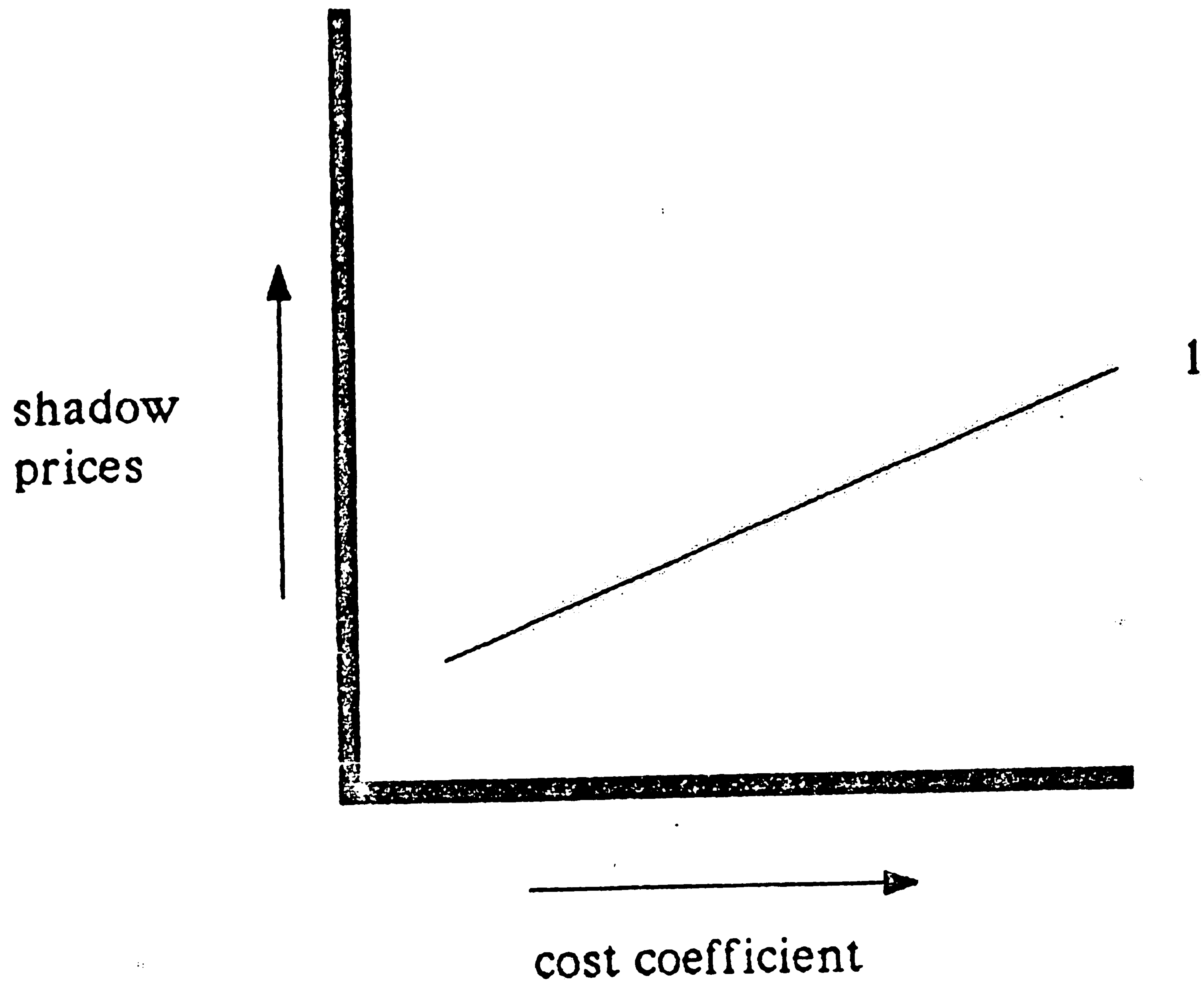
Whether one was examining forecast variability or demand increase over time, the effects upon shadow prices were consistent. However, because production and distribution constraint changes tend to be more abrupt when considered over time, the plotted function tended to show some stair-step effects.

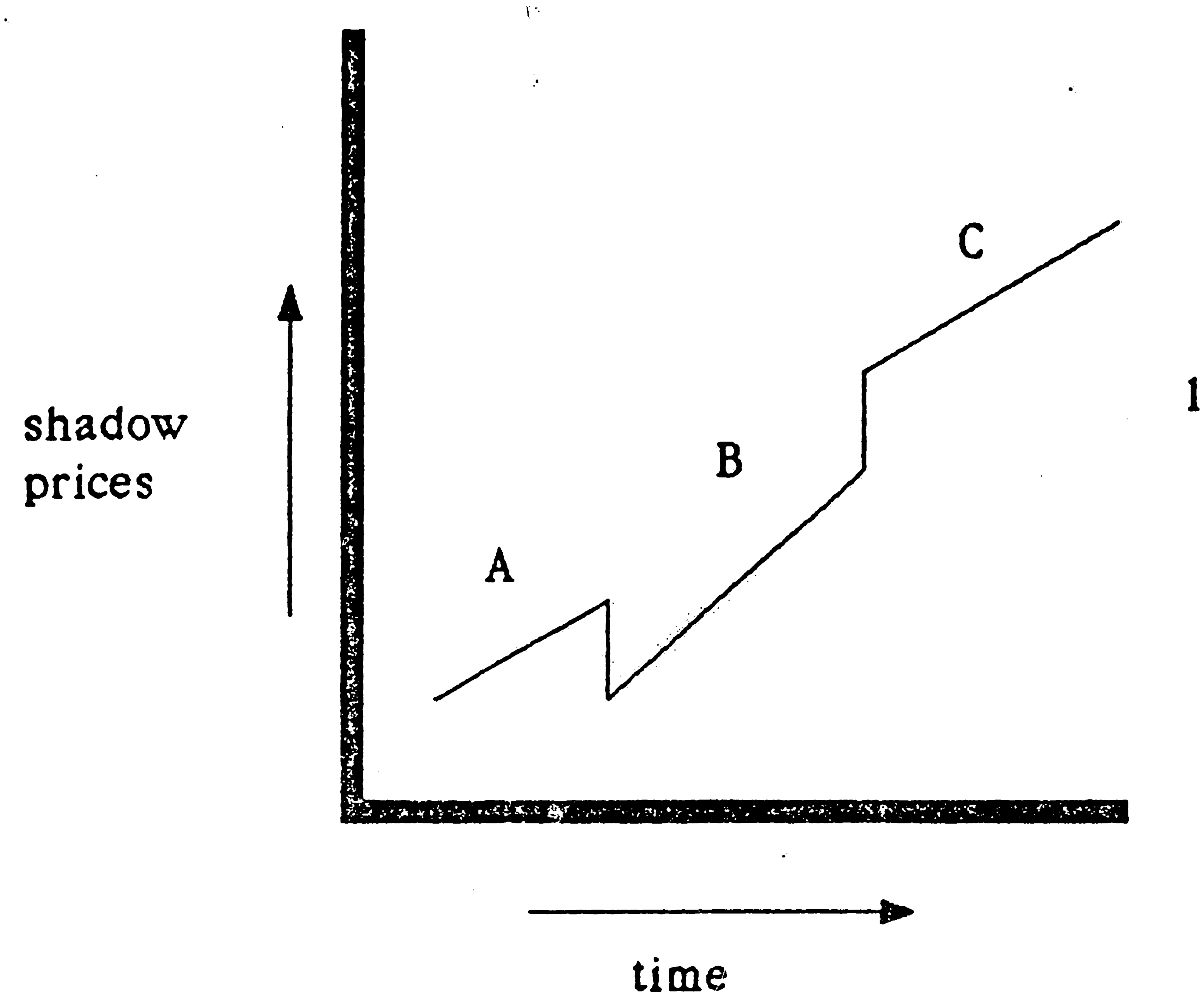
The following explanation refers to Curve 3. At the point where Part A of the curve meets Part B, and equivalently where Part B meets Part C, the stair-step effect is prevalent. The downward step might represent an addition of a new facility or the purchase of a vehicle. The immediate relaxation of a constraint. The upward step represents the limiting of some constraint, possibly the shutdown of an old facility. What is necessary to recognize is that it is extremely difficult to be able to equate time and the parameter change as was done when discussing forecast variability. If one can assume product demand will increase over time, then one can state that trends in shadow prices will be similar whether comparing them against demands or against time. However, comparing shadow prices against constraint variation is very different from comparing them against time.

Cost Coefficient Variation



Cost coefficient variation has a direct effect on final results. As cost coefficients increase, shadow prices increase; as costs decrease, prices decrease. If one assumes that costs increase over time, then one can also state that shadow prices increase as cost coefficients increase over time. What is important here is that basically one category of functions (increasing) represents the results of how cost coefficients effect shadow prices.

Shadow Prices and a Planning Horizon

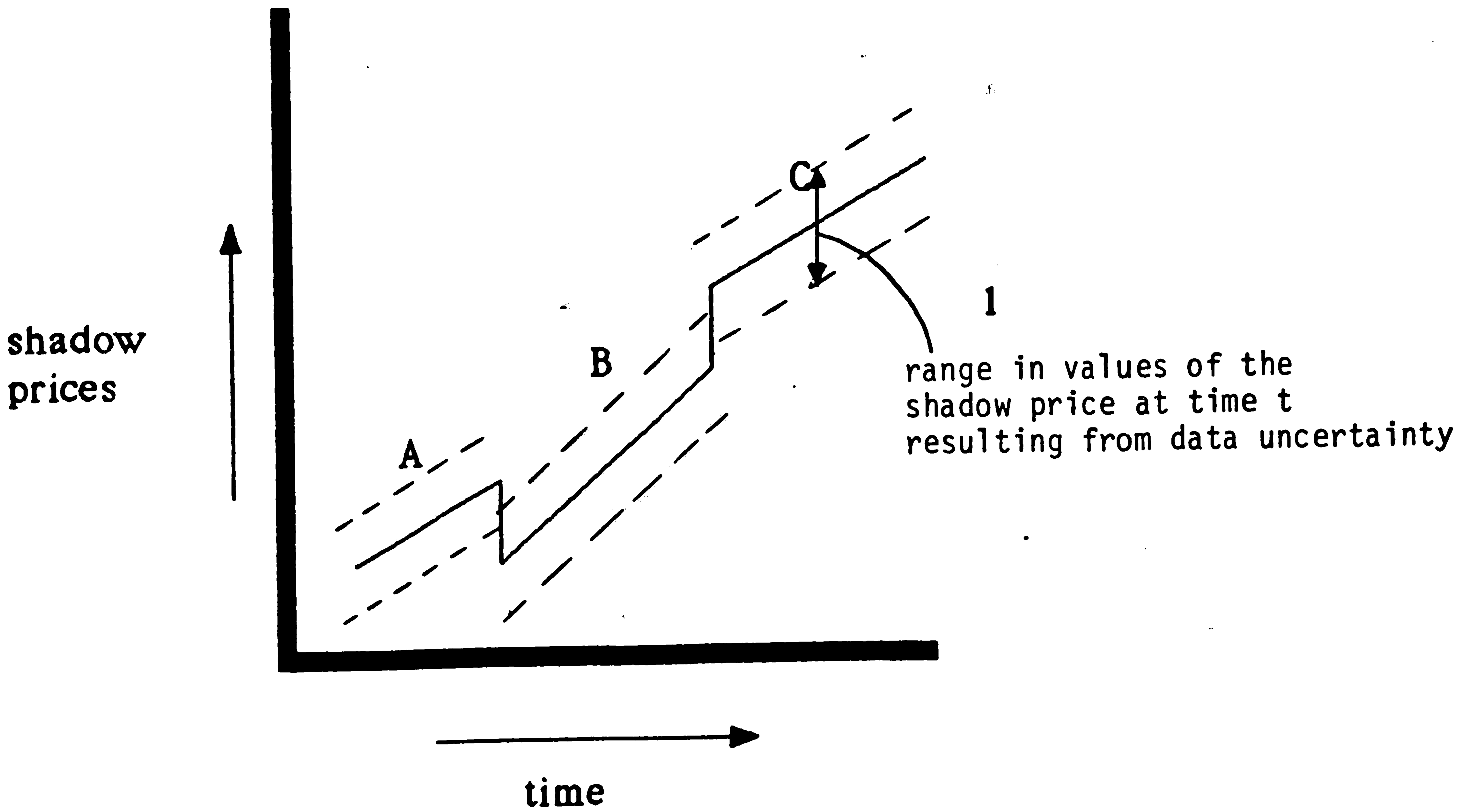


When the test case was set up to represent a long range planning situation shadow prices followed a pattern similar to the previous curve. For long range planning, distribution constraints were relaxed and results concerned with these constraints will be used only for suggestions for the number of necessary vehicles and drivers. Average available production capacities, based on historic data, were used for production constraints. Forecasts were projected over the planning horizon and costs were escalated to represent inflation and increased operating costs.

The first portion of the curve (Part A) shows the increasing value in the shadow price over an initial period of time. At the point where Part A meets Part B of the curve, an addition of a new facility causes a decrease in the shadow price. The decrease in price from increased availability of product dominates the increase in price from cost inflation and causes the step down in price. Part B of the curve represents the increase in shadow prices until a period in time when a facility is shut down (where Part B meets Part C). The remaining part of the curve (C) again displays the increases in the shadow price due to general inflation and more limiting production constraints.

Applying Improved Parametric Programming

The plots over the previous few pages display the trends of shadow prices from effects in data changes. Using the Improved Parametric Programming techniques to describe input variability, we will display some other very important trends on the same plot as detailed below.



Along with the general trend of the shadow price over time, the 95% (any acceptable level) confidence interval is plotted as well. At any point in time, one can locate the expected value of the shadow price as well as the range of values that might occur due to the uncertainty in the input data.

These trend plots, along with diagrams of the distribution of the shadow prices, are important in describing the risk involved with pricing a customer at a specific level.

It is also necessary to understand the possible consequences involved with suggesting prices at different levels. When a customer price is suggested based upon the expected value of a shadow price rather than some value less than this expected value, there is less risk involved with obtaining future profits. Although this is an obvious statement, it is important. Just as important, is making use of information from the Improved Parametric Programming techniques to define the risk involved with moving away from the expected value (i.e. define the risk involved with obtaining a specified level of profit). Large scale problems with data uncertainties are solved more effectively and more efficiently using IPP, providing the potential for increased corporate profits and better risk assessment. IPP can be a highly effective technique for describing data uncertainty in large-scale problems because it is similar to existing (i.e. accepted) technology. It is not necessary to "sell" anyone on new mathematical techniques; simply provide for an explanation of the benefits of IPP and show that it is only an extension of already accepted linear programming practices.

CHAPTER VI

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THE CONCLUSIONS AND FINAL REMARKS

Conclusions and Final Remarks

Results from the various tests suggest the following:

1. The parameters that have the largest effect upon shadow prices are capacity changes. This should be recognized because capacity changes present the largest relative change of any of the tested constraints and coefficients. Although variability in product output is minimal, new facilities and plant shutdowns represent the capacity change that show such a large effect upon shadow prices. It is important when considering strategic location of new facilities that they are placed properly. Even proposed new facilities that are to be onstream five years into the future should be highly scrutinized. A "good guess" as to where the facility might be located may not be good enough.

2. Cost coefficients' variabilities place a direct effect on shadow prices. If one is off just a little bit on the estimate of the value of a cost coefficient, then at least one shadow price will be off a little bit. However, because the cost coefficients are the parameters that have the most available data, their estimates are reasonably accurate. So, even though cost coefficients do have a direct effect upon final results, the fact that their estimates are very good suggest this is not an area of major concern. The various constraints have a much larger effect upon final results.

3. Distribution constraints can have a major effect upon results. If however, the system is to be a strategic planning tool, then these constraints should be relaxed and final results used to help plan for vehicle and driver needs. When these constraints were considered in the tests, they did represent some major effects. If vehicle availability was exhausted at a terminal, other terminals were forced to service new areas and the effects upon shadow prices were apparent. Obviously, the tighter the constraint the more drastic its effect. The effect was not linear; an exponential effect was prevalent.

4. Demand constraints represented the potential for most uncertainty. Forecast variability exists at both the customer and system levels. It is important to employ the modified parametric programming methodology so that individual shadow prices are better understood. Variability in the forecasts presented a rather wide range of effects upon final results. In areas where capacity was already a constraint, relaxing the forecast caused a large decrease in the shadow price. Increasing the forecast had an equivalent negative effect upon the shadow price. When capacity is not a limiting factor, forecast variability had little or no effect upon final results. It is necessary to recognize what can happen to a system if changes occur simply because of data uncertainty. To obtain a new customer account with assurance that a corporation will be more profitable based upon a single L.P. run is naive. To understand the probability of failure at different demand/price levels is critical.

In summary, to assure good results from a distribution L.P. model the following points must be addressed:

1. variability of demand forecasts
2 location of capacity increases or decreases
3. relevance of distribution constraints.

Using some of the sensitivity analyses presented can be a means to a good service, that is a good information system. However, the technical aspects are really just the beginning of a successful application. There are peripheral requirements for the success of any technology, not the least of these is to find an area that really provides some value to a client. The technology of using shadow prices as a base for customer pricing has proved to provide value to a corporation and has been generally accepted as an important methodology.

Because data is an essential element of the system, a second necessary requirement is how to manage the data. System inputs and results must be actively managed in an environment separate from production systems. The task of management might include providing for new data, purging old data, controlling access to confidential data, and communicating with clients on the relevance of the data.

Training and support represent another aspect of a successful system. The reason a system will be used is if someone (analyst) can demonstrate to the client how it helps get his/her job done. Technical people are motivated to learn technical material in order to do their jobs, this is not true for most clients. Their motivations might be to become a better financial analyst or product manager.

Odds are small that the system will fully satisfy the client's needs, and so it is necessary that the system remain flexible as not to limit its potential. The ultimate features of the system have been conceptualized but implementation should take place with only a few capabilities at a time. This gives the client time to become familiar with the system without becoming immediately overwhelmed. Choosing the right technology and using it well are important, but even more critical is having the methodology accepted by the client. If a client can make frequent use of the system with few or no complications, then the system will be a vehicle for providing a significant value to the corporation.

CHAPTER VII

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CHAPTER VIII

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THE APPENDIX

IMPROVED PARAMETRIC PROGRAMMING (IPP)
VS.
STOCHASTIC PROGRAMMING (SP)

Case	Parameter Addressed	# of Row	# of Columns	IPP Computing Time	SP Computing Time	IPP Objective ($000,000)	SP Objective ($000,000)
1	b_i	1,200	9,000	38	70	7.5	7.5
2	b_i	1,500	12,000	42	94	8.6	8.6
3	b_i	1,700	13,000	61	?	9.4	?
4	b_i	1,900	18,000	90	?	10.5	?
5	b_i	2,000	18,000	107	?	11.1	?
1	C_j	1,200	9,000	32	63	7.4	7.6
2	C_j	1,500	12,000	43	101	8.6	8.7
3	C_j	1,700	13,000	65	119	9.4	9.6
4	C_j	1,900	18,000	91	?	10.5	?
5	C_j	2,000	18,000	104	?	11.1	?
1	a_{ij}	1,200	9,000	36	63	7.5	7.6
2	a_{ij}	1,500	12,000	43	91	8.6	8.5
3	a_{ij}	1,700	13,000	69	118	9.4	9.4
4	a_{ij}	1,900	18,000	98	?	10.5	?
5	a_{ij}	2,000	18,000	109	?	11.1	?

Note: 1) All cases started execution from an optimal basis where input parameters were at their expected values.

2) Computing time is a measure of I/O and CPU time.

3) Each objective function cost is an average of the N runs for each case.

VITA

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