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Variance of the estimates in performance of active flat plate solar collectors using the F-chart /

Greg F. Schmidt
Lehigh University

VARIANCE OF THE ESTIMATES IN PERFORMANCE OF ACTIVE FLAT PLATE SOLAR COLLECTORS USING THE F-CHART

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Greg F. Schmidt

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Professor In Charge

F. Erdogan

Chairman Of Department

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Abstract

A method of estimating the standard deviation of output from an active flat plate solar collector which uses liquid as the heat transfer medium is presented. The technique was developed as an extension of the 'f' factor method and is capable of estimating the range of outputs from common solar collection systems by having a knowledge of the system design parameters and the statistical information about the weather. 'f' is the proportion of the heating load supplied by the collector and the deviation of 'f' is the percentage change in output due to variations in weather. Since the deviation of 'f' was developed using the 'f' factor method, the ease of calculation associated with the 'f' factor method is also a feature of the deviation of 'f'.

The standard deviation of 'f' is dependent upon y and the standard deviation of y. Y is the ratio of energy collected to the total heating requirement. The standard deviation of y is a function of the variability of insolation and temperature and the dependence of temperature on insolation. 'f' is also dependent upon x, the ratio of collector losses to heating load. However, the deviation of x has an insignificant effect on the deviation of 'f'.

Of significance is the correlation coefficient which is an indicator of the dependence of temperature on insolation. The degree of dependence is shown by the correlation coefficient and this dependence can increase or decrease the variability of 'f' depending upon whether the correlation coefficient is positive or negative. A positive correlation exists from April to August, while the correlation is negative from September until February. The resultant decrease in the variability of 'f', caused by the correlation coefficient, is significant and is especially important because the correlation reduces the variability of 'f' during the heating season.

Graphs of the deviation of 'f' are developed which demonstrate the effects of the deviation of temperature, insolation, and the correlation coefficient on the collector output. In addition, equations are provided through which the deviation of 'f' can be calculated.

1.0 INTRODUCTION

Estimation of the power output from an active solar collection system using flat plate collectors can be accomplished by developing analytical models for each of the system's components and simulating performance using actual temperature and insolation data. However, the complexity of these models coupled with the need for an extensive weather data base, mandates the use of a computer. Due to the extensive amount of computer time necessary to run a simulation, optimizing the performance of the system by varying design parameters and rerunning the simulation can only be justified for the largest systems or for the development of a solar collector product line.

The 'f' factor design technique developed by Klein (Ref. 1) provides a method of estimating the solar gain from certain common solar heating systems without resorting to complicated computer models. It enables the designer to calculate the fraction of heating load ('f') the solar collection system will carry using system design specification and average monthly inputs of sun and temperature. It is useful for optimizing the design of the collection system because of the ease with which the output can be calculated.

Using averaged data creates uncertainty in the calculated value of 'f'. This uncertainty has two causes. First, the design parameters used in the calculation of 'f' may not vary proportionally with the insolation and temperature over the entire load range and second, insolation and temperature have an inherent variability. This paper deals mainly with deviations in energy output caused by the second type of uncertainty. It attempts to provide the designer with an estimate of the range through which the actual collector output can vary from average values on a monthly basis.

Because of the variability of insolation and temperature, there is a corresponding statistical variation in the value of 'f' about its mean value. In addition, the correlation between insolation and temperature, where the variability of temperature depends (at least partially) on previous levels of insolation, has an effect on the variability of 'f'.

1.1 Purpose Of The Study

The purpose of this study is to determine the extent to which the normally occuring random variations of temperature and insolation will affect the estimates of solar collector performance provided by the 'f' chart design technique. A brief description of the 'f' chart design method is given in the next section to provide background on how 'f' is determined. Subsequent sections will develop a method for estimating the deviations of 'f'.

2.0 THE 'f' FACTOR DESIGN TECHNIQUE

The majority of active systems which are being designed today are smaller systems. For these systems, the use of a computer simulation can not be cost justified and these are the types of systems where the 'f' factor has widespread popularity.

There are two different classifications of active solar systems for which the 'f' factor has been developed. These classifications result from the type of heat transfer medium used by the system - either air or liquid. Separate 'f' factor tables must be employed when using either medium. Only the liquid system will be discussed since the model for the air system is the same in principle.

Figure (2.0.1) shows a detail of a standard flat plate collector and Figure (2.0.2) shows the whole solar energy collector system including the collector, primary heat exchanger and storage system. Energy for heating is obtained from insolation which is absorbed by the plate (1), and is transferred to a working fluid (usually an antifreeze solution) in the tubes (2) in contact with the plate. The heated fluid passes through the primary loop (3) to a heat exchanger (4) which transfers heat to storage (5). As it is needed,

Figure 2.0.1

TYPICAL FLAT PLATE COLLECTOR

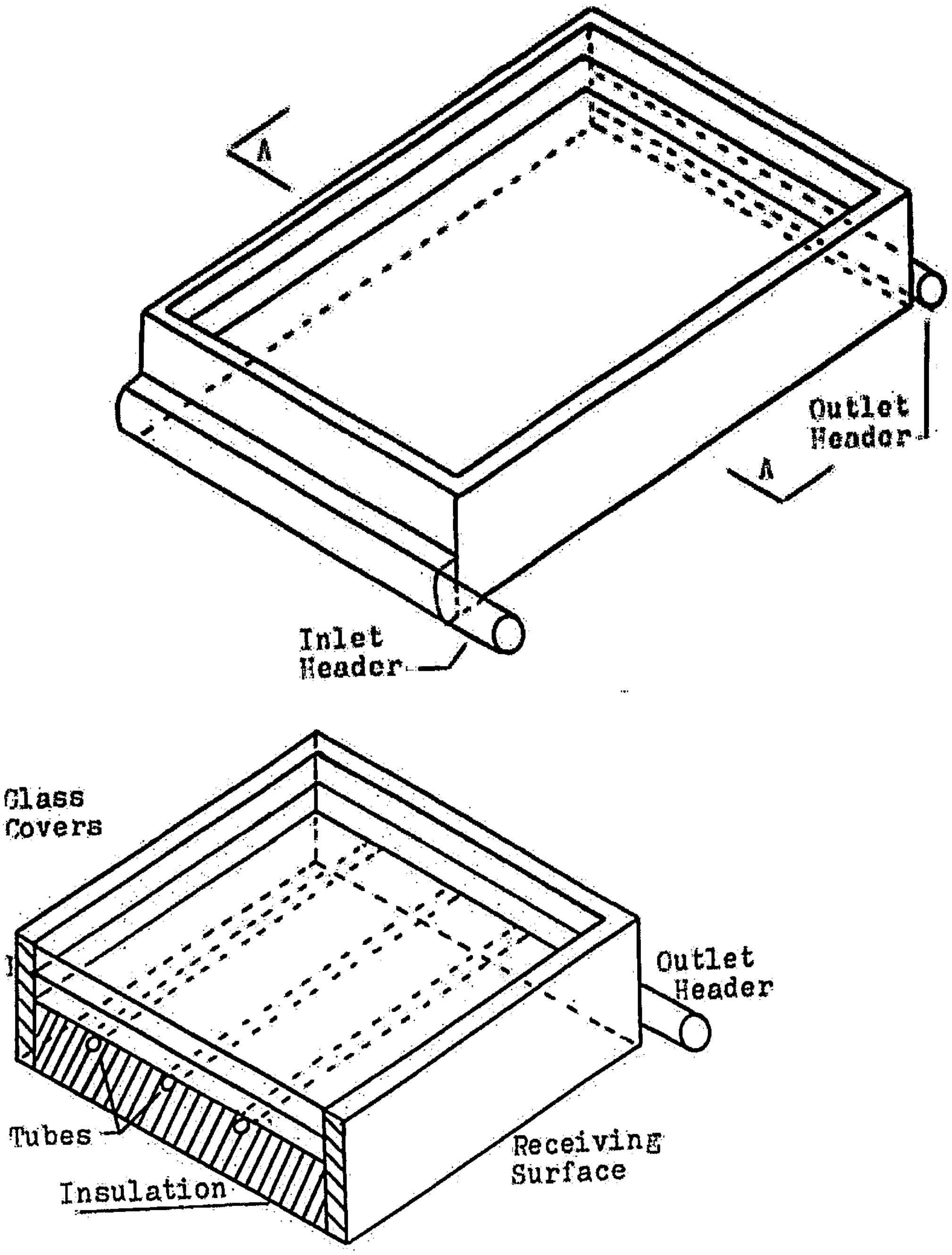
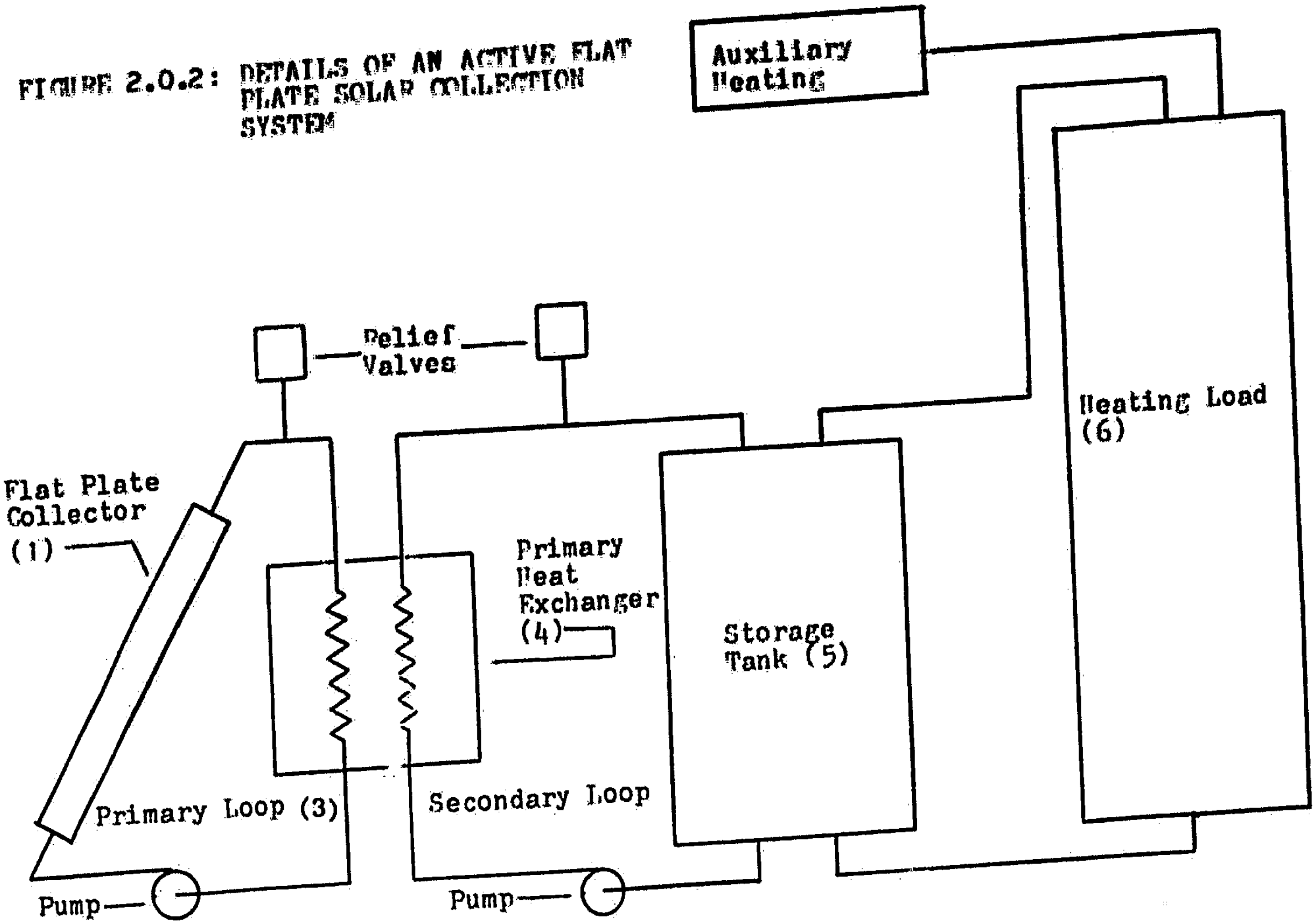


FIGURE 2.0.2: DETAILS OF AN ACTIVE FLAT PLATE SOLAR COLLECTION SYSTEM



the stored energy is delivered to the distribution system and the heating load (L). The system shown is one of many possible configurations but the essential elements remain the same, a collector, heat exchanger(s), a storage and a distribution system.

2.1 The Development Of The 'f' Factor

The 'f' factor was developed from computer simulations of typical collector configurations. Simulations were run using the modelling program TRNSYS, developed by Klein (Ref. 1), and the results were correlated against non-dimensional collector loss and collector gain parameters.

Klein (Ref. 1) developed the non-dimensional parameters from the basic equation for the output from an active solar collection system which is given in Ref. 1 as:

$$Qu = A_c \, Fr' \quad (S - U_L \, (T_{in} - T_a)) \qquad (2.1.1)$$

where:

Qu = represents the actual rate of heat output from the solar collector excluding any

losses from the distribution system.
(BTU/hour or Watts)

S = the amount of solar energy absorbed by the collector plate, after cover absorption and incident angle correction.
($BTU/Hr\text{-}Ft^2$ or $Watts/M^2$)

T_{in} = the inlet working fluid temperature
(^oF or ^oC)

T_a = ambient temperature (^oF or ^oC)

U_L = heat transfer loss coefficient from the cover plate(s) to the atmosphere due to convection, conduction and radiation. It is a function of hardware, dimensions and material. It also depends upon the operating temperature of the collector
($BTU/Hr\text{-}Ft^2\ ^oF$ or $Watts/M^2\ ^oC$)

Ac = total collector area (Ft^2 or M^2)

Fr' = heat removal factor. Represents the heat transfer efficiency between the collector plate and the working fluid. Fr' also corrects for the variation in working fluid temperature between the inlet and outlet of the collector. It is the ratio of heat supplied by the collector to heat which would be supplied if all working fluid were maintained at the inlet (lower) temperature throughout the collector. It also accounts for the efficiency of heat transfer in the primary heat exchanger and is always less than one.

If the total heating load (L) for the month is divided into equation (2.1.2) and the two terms in the brackets are separated, the result is:

$$f = y - x \tag{2.1.2}$$

where:

$$f = \frac{(Qu)_T}{L} \tag{2.1.2a}$$

f = the monthly fraction of the heating load carried by the collector

$$y = \frac{A_c \, Fr' \, (\tau\alpha) \, H_T \, N}{L} \qquad (2.1.2b)$$

y = the monthly total solar energy absorbed by the collector as a fraction of the total heating load

H_T = monthly average daily radiation (Btu/day or Watts)

N = number of days in the month

$\tau\alpha$ = transmittance-absorptance product corrects for cover plate optical effects

$$x = \frac{A_c \, Fr' \, U_L \, (T_{in} - T_a) \, \Delta t}{L} \qquad (2.1.2c)$$

x = the monthly losses of the collector plate and primary heat exchanger as a fraction of the total heating load.

A further simplification is necessary before x, given by Equation (2.1.2c) can be calculated. T_{in} is a variable that is a function of collector output, storage tank size, and heating requirements. There are only two methods by which T_{in} can be obtained, either through a computer simulation or actual field measurements. Neither of these is acceptable if 'f' is to be an inexpensive estimating technique.

Klein (Ref. 1) removed T_{in} from Equation (2.1.2c) by multiplying x by:

$$z = \frac{T_{in} - T_a}{T_{ref} - T_a} \qquad (2.1.3)$$

where:

T_{ref} = an arbitrarialy chosen constant equal to 100 °C (212 °F)

which yields:

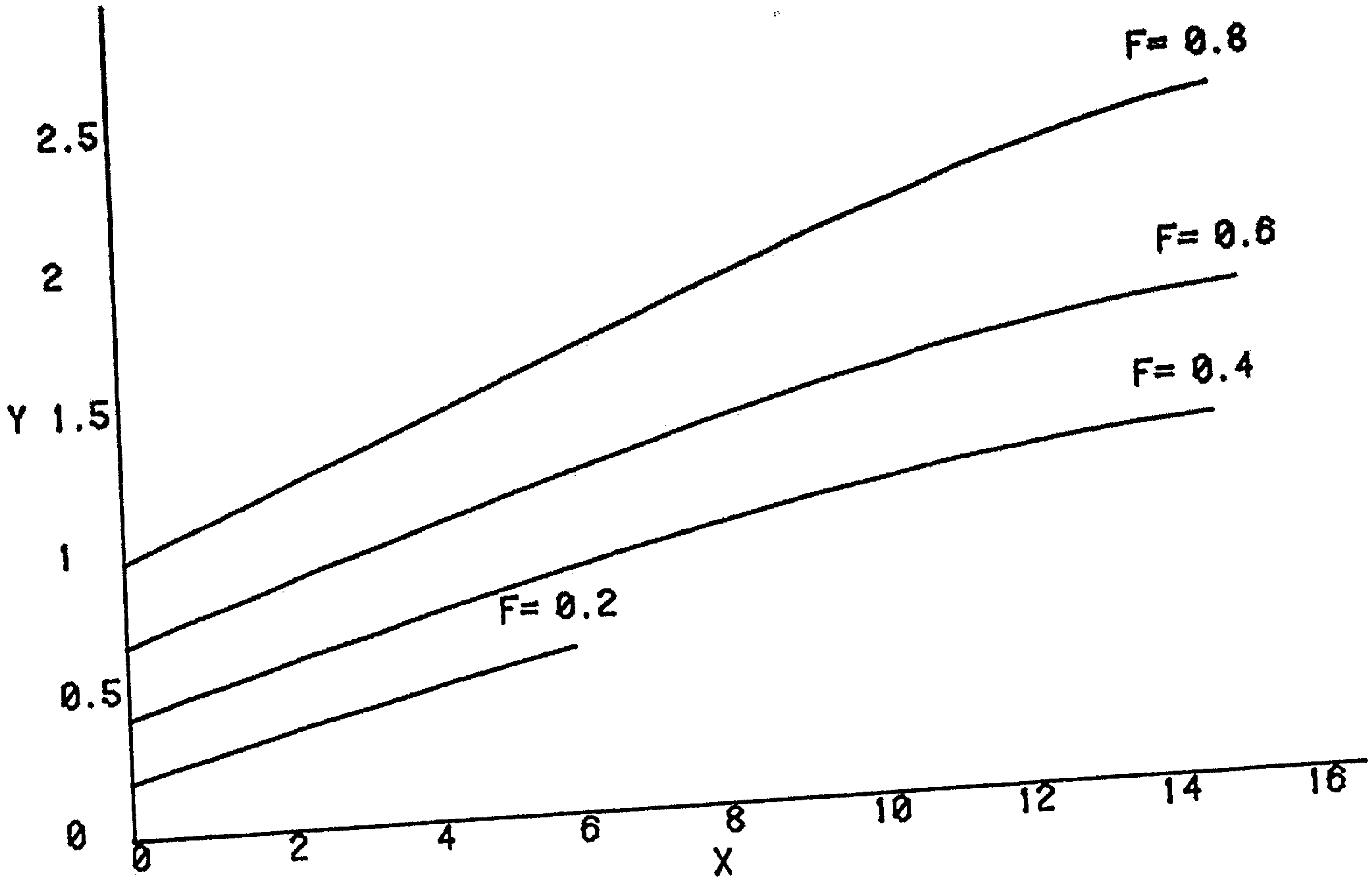
$$x = \frac{A_c \, Fr' \, U_L \, (T_{ref} - T_a) \, \Delta t}{L} \qquad (2.1.4)$$

In essence, a constant is being substituted for a variable. This causes an error to be introduced which will be discussed in a subsequent section.

Over three hundred simulations were run using TRNSYS and the results were plotted against x and y for constant values of 'f'. These are shown in Figure (2.1.1) for the liquid system. In addition, Klein (Ref. 1) through curve fitting techniques developed the following equation for 'f':

FIGURE 2.1.1

F FACTOR FOR LIQUID SYSTEMS



$$f = 1.029 - .065x - .245y + .0018x^2 + .0215y^3 \qquad (2.1.5)$$

for liquid systems.

To find the fraction of heating load supplied by the solar collector, monthly average values of $(\tau\alpha)$, U_L and Fr are calculated. Monthly average values of insolation and ambient temperature are obtained from organizations, such as the National Oceanic and Atmospheric Administration (NOAA) in the United States. House heating load can be estimated from previous years' heating requirements or by calculations approved by the American Society of Heating, Refrigeration, and Air Conditioning Engineers (ASHRAE) (Ref. 3). These values are now used to calculate x and y and 'f' is found from either Figure (2.1.1) or Equation (2.1.5). This process is repeated for each month of the year where heating is required.

The advantages of the 'f' factor approach lies in its ability to calculate the fraction of heating load, supplied by a solar collection system, using readily available data and design information. Only a few straight forward calculations are required to estimate the yearly performance of the collector. Compare this approach with the alternative of calculating collector performance on an hourly basis using computer simulations and actual insolation and temperature data and the advantages of the 'f' factor are obvious.

2.2 The Effect Of Assumptions Used In Developing 'f'

The assumptions used in the development of 'f' affect the relationship between 'f' and x and y. This is evident from Equation (2.1.2) which indicates that plots of x versus y should be a straight line for constant values of 'f'.

$$f = y - x \qquad (2.1.2)$$

(repeated)

Figure (2.1.1) shows that the lines of constant 'f' deviate as the values of x and y increase. The deviation is primarialy caused by the removal of T_{in} from the equation for x. Of secondary importance is the treatment of hardware parameters such as U_L and Fr' as constants. Both U_L and Fr' are indirect functions of insolation and ambient temperature.

2.2.1 The Effects Of Substituting T_{ref} for T_{in}

The most important simplification made in the development of the 'f' factor method was the removal of the T_{in} term. Mathematically, T_{in} was eliminated from x by using Equation (2.1.3). In actuality, this manipulation resulted in substituting T_{ref} for T_{in}. Without this substitution, the 'f' factor method would only be valid for a limited number of systems where the collector inlet temperature remains constant.

T_{in} is a variable that is a function of house load, insolation, and storage capacity. It can vary continuously when the collector is operating. T_{ref} was chosen by Klein (Ref. 1) to be a constant of 100^{o} C.

The substitution is valid at lower values of 'f' and for systems with large storage capacity. When the collector carries a lower fraction of the monthly heating load, the inlet temperature will approach the temperature of the house and remain at or near that temperature for the entire month. The daily fluctuations will be minimal and T_{in} will approach a constant value.

Even though this constant value will be different than the reference temperature of 100^{o} C, it will have only a slight effect on 'f' because of the way the 'f' factor method was developed. 'f' was developed by correlating x and y. While the value of x is different than the value that would have been calculated using the actual inlet temperature, they are proportional by the factor z from Equation (2.1.3). The correlation will compensate for this proportionality.

However, when the collector is carrying a large percentage of the monthly heating load T_{in} can vary significantly throughout the day and from day to day. Under these conditions, T_{in} is at least partially related to T_a and to the rate of insolation which occurred over the last few days. The substitution of T_{ref} for T_a artificially removes a portion of the variability of x, which reduces the variability of 'f'.

The use of a constant in place of T_{in} will cause some error in our calculation of the deviation of 'f'. The error will increase as the value of 'f' increases and that error will cause an under prediction of the deviation of 'f'. The error is inherent in the method and cannot be removed. However, the error is judged to be slight.

2.2.2 Variations In Hardware Parameters

The heat removal factor (Fr'), the transmittance-absorptance product ($\tau\alpha$) and the loss coefficient (U_L) are the materials and design variables which affect collector performance. These are weak functions of the operating temperature and incident solar energy. In actual simulations, they do have an effect on x and y and therefore on 'f'. As a result, the straight line relationship between x and y is slightly altered.

2.2.3 Variations Due To Heating Load, L

Several methods are available for calculating the heating load. These include estimating heating requirements from previous years' fuel consumption and the degree day method contained in ASHRAE (Ref. 3). Regardless of the method used, the heating load is a function of the ambient temperature and deviations in ambient temperature result in deviations in the required heating load. This affects x, y and 'f'.

3.0 THE PARAMETERS AFFECTED BY VARIABILITY

The 'f' factor method predicts the collector systems' output for the "average" month. The method uses values of insolation and temperature, averaged over several years, to calculate the collector loss coefficient (U_L), the heat removal factor (Fr') and the residental heating load (L). The method also uses averaged values of insolation and temperature to determine x and y.

Since it is rare that any month behaves like the average month, it is also important to determine the range through which 'f' can vary because of variations in insolation and temperature. A brief discussion of the physical causes of this variability will be presented in this section. Subsequent sections will present the statistical aspects.

3.1 The Variability Of The Sun

Solar gain results from solar energy reaching the collector surface and being absorbed by the collector plate. The amount of energy absorbed is a function of collector design and materials parameters and can be controlled by the designer. The amount of energy reaching the surface is highly variable and is beyond the control of the designer.

In order to obtain a good estimate of collector performance, the insolation must be accurately predicted. However, insolation levels are location dependent and are influenced by time of year, cloud cover, airborne particles, and the elevation and topography of the location.

These changes in insolation can be grouped into two classifications - those which can be trended and those which occur randomly. Both are important; however, the random component causes the greatest concern. The designer can adjust his estimate of insolation to account for the trends but no adjustment can be made to fully account for the random component.

The following sections give a brief description of the causes of the variation in insolation.

3.2 Variations In Extraterrestial Radiation Caused By Trends

The sun emits radiation in a spherical pattern and as this radiation "spreads out", its intensity is reduced in accordance with the inverse square law. Only a small portion of this radiation is intercepted by the earth because of the relative size of the earth in comparison to the distance from the sun. The average extraterrestial intensity of radiation is on the order of 1353 W/m^2 (428 Btu/ft^2-hr). This is referred to as the "solar constant".

The solar constant must be adjusted to account for the eccentric, but predictable, orbit of the earth about the sun. The inverse square law predicts that as this distance increases, the extraterrestial radiation decreases in proportion to the distance squared. The variation in extraterrestial radiation caused by the eccentric orbit is predictable and is given by

$$E = E_o \left(1 + .033 \cos \frac{360n}{365}\right) \qquad (3.2.1)$$

where:

E = actual extraterrestial radiation ($Watts/m^2$ or $Btu/hr\text{-}ft^2$)

E_o = solar constant

(Watts/m^2 or Btu/hr-ft^2)

n = calendar number of the day

(i.e. January 1st = 1, December 31st = 365)

Since this change in insolation is cyclic with a period of one year, it is a trend.

Another yearly trend is responsible for the four seasons. In addition to being eccentric, the earth's orbit is not planer. Therefore, the earth moves up and down with respect to the sun. Since the earth is spherical, when the earth is above the sun, a portion of the insolation is prevented from reaching the northern hemisphere by the southern hemisphere. This blocking is responsible for the shorter days during the winter. In the summer, the earth is below the sun and the opposite occurs.

Effectively, the sun appears to move between 23.45 S latitude and 23.45 N latitude during the course of the year and this movement is described by:

$$\delta = 23.45 \sin \frac{360}{365} (284 - n) \qquad (3.2.2)$$

where:

δ = angle of declination (Degrees)

+ is north and - is south

n = calendar day of the year (days)

3.3 Attenuation Of Radiation By The Atmosphere

Not all of the extraterrestial radiation incident on the earth's atmosphere reaches the surface of the earth. As the insolation, in the form of electromagnetic radiation, travels through the atmosphere, it can be reflected, absorbed or diffracted by the constituents of the atmosphere. This can significantly reduce the amount of energy available for solar gain.

Reflection, scattering and absorption affect both the sun's radiation and the longwave radiation being emitted from the earth. Radiative energy flows from a warmer body to a colder body. Since the earth is, in general, warmer than the atmosphere and a night sky, energy will be emitted from the earth to the atmosphere and space. This energy can be interfered with in the same manner as the sun's energy.

Absorption, reflection and scattering can either have properties of a trend or be random. For instance, reflection of the clouds varies with the type of cloud and the amount of cloud cover, neither of which can be predicted; whereas, the reflection of a given location on the earth's surface remains relatively constant for a given time of the year because the characteristics of the earth's surface do not change drastically. The former results in a random change in energy, while the latter results in a predictable reduction in energy which depends upon the time of the year. Reflection, scattering and absorption will be discussed in a general nature before discussing their interaction and effect on the temperature of the earth.

3.3.1 Absorption

The wavelength distribution of radiant energy emitted by a blackbody is a function of its temperature. As the temperature of the body increases, the percentage of energy emitted as short wavelength radiation increases. The predominant wavelength is given by Planck's Law, which is:

$$E_{\lambda b} = \frac{3.74 \times 10^{-16}}{\lambda^5 \left(e^{.01439/\lambda T} - 1 \right)} \qquad (3.3.1)$$

where:

$E_{\lambda b}$ = energy per unit time, area and wavelength ($Watt/M^2\ m$)

λ = wavelength (m)

The sun, with an effective blackbody temperature of approximately 5760 ^oK, emits virtually all its energy in the 0 to 2900 angstrom (A) wavelength. Colder bodies, such as the earth, emit energy at predominately longer wavelengths. This is important since a particle, a molecule or a surface's ability to absorb radiation is a function of the wavelength of the radiation incident upon it.

As energy from the sun enters the atmosphere, a portion of the energy is absorbed by the constituents of the atmosphere. However, because the ability of a molecule to absorb radiation depends on the wavelength of the radiation, only certain wavelengths of energy are attenuated.

Attenuation of the sun's energy by absorption occurs mainly by ozone and water vapor. Ozone is a strong absorber of radiation in the 0 to 300 A range and accounts for the almost total attenuation of radiation in this band. Water vapor has strong absorption bands at 1000 A, 1400 A, 1800 A and 2300 A wavelengths. Barry, etal (Ref. 4) found that the combined effect of water vapor and ozone result in an eighteen percent annual decrease in energy reaching the surface of the earth.

While absorption has a significant effect on the attenuation of the direct energy of the sun, it also effects the long wave radiation reradiated from the surface of the earth. Due to the temperature of the earth radiation emitted by the earth is predominately longwave or infrared radiation. While energy from the sun was, for the most part, not absorbed by the atmosphere, the longer wave radiation being emitted by the earth will be more susceptible to absorption in the atmoshphere, and this absorption will increase the temperature of the atmosphere.

Atmospheric moisture and CO_2 are responsible for absorbing the majority of longwave radiation in the atmosphere. Water vapor is a strong absorber of radiation in the 4,000 to 8,000 A range and again for radiation longer than about 16,000 A. This coupled with the effects of CO_2, which is a strong absorber of radiation between about 13,000 to 17,000 A, prevents the majority of radiation in the 4,000 to 8,000 A and the 13,000 A and longer wavelengths from escaping directly from the earth. The amount captured will be a function of the earth's temperature (i.e. predominate wavelength of transmittance) as well as the composition of the atmosphere. Water vapor can vary significantly and can affect the amount of radiation captured by the atmoshpere.

Absorption in the atmosphere causes the atmosphere to become significantly warmer. Byers (Ref. 5) estimated that if absorption by the atmosphere did not occur, the temperature of the earth's surface would fall by at least 40 ^{o}C (70^{o}F).

There is a range between 8,000 and 14,000 A where, except for a slight absorption by ozone, no other appreciable absorption takes place. This is known as the "atmospheric window". This window occurs near the predominate wavelength of the emittance from the earth and does allow the earth to reradiate a large percentage of its energy to space.

3.3.2 Reflection

Byers (Ref. 5) found that about thirty percent of the extraterrestial radiation is reflected back to space. Cloud cover is responsible for the majority of this decrease and reflection from the surface of the earth is the second largest source. Reflection occurs when energy from the sun strikes a dust particle, gas molecules or other surface and is deflected away from the receiving surface, which in this case is the earth.

The amount of energy reflected by the earth is dependant upon the earth's surface. For instance, a freshly fallen snow will reflect more radiation than earth covered with vegetation.

3.3.3 Scattering

Fine dusts and gas molecules can scatter or diffuse the energy from the sun. When light hits the scattering particle, the light is diffused in all directions and the intensity is lessened due to the "spreading out" of the light. Depending upon the literature, there are two general theories. The first being that the light is forward scattered; that is the principle direction of the scattered light is still the direction in which the beam was travelling. The second theory is that the light is scattered in a spherical pattern with equal intensity in all directions.

If the forward scattering theory is applied, scattering does not cause any loss of energy from the sun. However, this energy is no longer concentrated in the sun beam. Rather, it is "spread out" over a wide area; and since only a portion of this energy can be collected (i.e. finite collector area), scattering reduces the energy available for solar gain; however, it has little effect on the energy reaching the surface of the earth.

3.4 Causes Of The Random Component Of Insolation

Clouds, moisture and the atmosphere are the major causes of the random variation in insolation with water vapor and clouds causing most of the variability. Both vary randomly from day to day. Clouds attenuate insolation through absorption, scattering and reflection. The type of cloud, the thickness of the cloud and the amount of sky covered by the clouds all influence the variability of insolation. For instance, a sky totally covered by thin clouds may transmit more radiation than a sky partially covered with thicker clouds. Moisture vapor relies mainly on absorption to attenuate the sun's energy; however, moisture vapor can also reflect and scatter the electromagnetic radiation.

Finally, there is the atmosphere which attenuates radiation through absorption, scattering and reflection depending upon the costituents of the atmosphere.

3.5 The Effects Of Reflection, Scattering And Absorption On Temperature

In recent history, the annual variations in the average temperature of the earth have been measured to be on the order of tenths of degrees. These minor variations suggest that, on an annual basis, an energy balance exists between energy received and emitted by the earth. This energy balance is taken over the entire earth for a period of one year. However, at any given location at any particular time, this energy is likely not to be in balance and these locations are either heating or cooling.

The energy at any location is in a constant state of flux, with locations in the northern hemisphere heating from February to August and cooling during the remainder of the year. The energy to heat the earth comes totally (ignoring small contributions from the combustion of fossil fuels, etc.) from the sun and this energy is attenuated by absorption scattering and reflection as discussed previously. The effects of absorption, scattering and reflection of the sun beam are obvious because they effect energy in the visible spectrum (i.e. we can see the effect of clouds by the darker sky). However, their effect on the longwave radiation is less evident but of equal importance. The cooling of the earth is accomplished solely by the emittance of longwave radiation by the earth, the atmosphere and the clouds, and this longwave radiation is interferred with by the various constituents of the atmosphere through the same processes of absorption, scattering and reflection that attenuate the sun's energy.

As with incident solar radiation, clouds have the greatest impact on the longwave radiation. They can, depending upon size, act as black bodies absorbing all incident radiation from the earth and atmosphere, including radiation in the "atmospheric window" range. Since the clouds become warmer than the atmosphere, they effectively reradiate a portion of this energy back to the atmosphere. During the day clouds prevent insolation from reaching the earth. This results in lower levels of insolation on earth than would occur under the same conditions on a cloudless day. However, at night, the clouds block radiation from the earth and atmosphere from being lost to space, resulting in more heat being retained by the earth than on a cloudless night under the same conditions.

Whether cloud cover is more likely to cause a net increase or decrease in energy on earth, when compared to a clear day depends, in part, on the length of the day and energy levels at the location. During the winter with longer nights, cloud cover will help the earth retain more heat; whereas in summer, cloud cover will cause a net reduction in the earth's energy when compared to a clear day.

3.6 Discussion Of Relationships Between Temperature And Insolation

The purpose of the previous sections was to describe the major causes of the variation of insolation, to determine whether the effect was random or a trend, and to suggest the possibility of a relationship between insolation and temperature.

For the short term, any major effects on extraterrestial radiation can be considered to be trends. Their existence can be mathematically determined and their effects modelled. In general, they have a period of one year. Other effects such as the absorption of radiation by ozone and CO_2 can also be considered a trend since these absorb all radiation at particular wavelengths and the quantity of these gases does not change appreciabely in the short term.

Cloud cover and water vapor have the greatest effect on radiation emitted from both the sun and the earth. Cloud cover and water vapor are random. Their influence, on a daily basis, cannot be predicted.

Insolation is responsible for providing all the energy necessary to maintain the temperature of the earth. It is also at least partially responsible for temperature increases at certain locations during certain periods of the year. In addition, the effects of clouds, moisture, and the atmosphere cause variability in both insolation and temperature. This suggests that insolation and temperature may be correlated, which means that the variability of one variable (temperature) could be, at least partially, influenced by the other (insolation).

The correlation will change the deviation of 'f'. If the variables have a positive correlation (i.e. the variability of temperature increases with the variability of insolation), then the deviation of 'f' will increase. However, if the variables have a negative correlation, the variability of 'f' will be reduced.

Lund (Ref. 8) examined daily insolation and temperature data in the Massachusetts area and found that the relationship between daily insolation and temperature varied from month to month. He found negative correlations of insolation and temperature from September through March and positive correlations during the rest of the year.

4.0 DATA ACQUISITION

The staff at Lehigh University collected solar insolation data for the Bethlehem, Pennsylvania, area from 1975 until 1979. Bethlehem, a city in northeastern Pennsylvania, is at a latitude of $40^{o}36'$ and a longitude of $75^{o}23'$. It is 118 meters (389 feet) above sea level. The climate and rainfall can be considered moderate; however, the climate is somewhat modified by the 500 to 1000 foot high South Mountain.

Total radiation on horizontal and vertical planes was integrated over fifteen minute intervals from September 1975 until September 1978. This data along with wind speed and direction, taken over the same time interval and for the same time period, is available through Lehigh University.

The solar insolation was gather by Eppley pyranometers located atop a 9.1 meter tower situated on the roof of Packard Laboratory. The pyranometer is a differential thermocouple made of 48 plated, copper on constantin junctions located radially around the pyranometer. The hot junctions are coated with a stable black compound while the cold junctions are whitened with non-hygroscopic barium sulphate.

The data from the pyranometer was integrated by an Esterline-Angus model D2020 sample data acquisition system and integrator. The resulting data was recorded on magnetic tape using a Kennedy incremental tape recorder. This data was converted from EBCDIC to ASCII characters. Both the original and converted data are stored on magnetic tape at the Lehigh University computer facility. Table 4.0.1 contains a listing of available data.

4.1 Processing The Solar Data

The solar insolation data was summed for each day this information was available and complete. A day's data was considered complete only if all data points for that day were available between the sunrise and sunset hour angles. The sunrise or sunset hour angle is the angular displacement of the sun, east or west of the local meridian, which results in the sun beam being parallel to a horizontal collecting surface. When the beam is parallel to the collecting surface, no energy is incident upon the collecting surface and no energy is collected. The sunrise and sunset angles are given by:

TABLE 4.0.1

Table Of Available Data

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
						Month						
1975	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	3-30	1-31	1 4-30	1-25 29-31
1976	1-31	1-23 26-29	1-11 16-28	1-30	1-17	N/A	N/A	17-31	1-30	1-4 9-14 23-27 30-31	1-6 23-30	1-31
1977	1-31	1-3 8-9 14-28	1-31	1-28	1-31	1-30	1-31	1-16	17-20	1-31	1-27 30	1-31
1978	1-17 28-31	1-3 7-28	1-31	1-13 19-30	1-31	2-30	1-12	24-31	1-30	N/A	N/A	N/A

N/A - Not Available

$$\omega_S = \cos^{-1}(-\tan\phi \tan\delta) \qquad (4.1.1)$$

where: ω_S = sunrise or sunset angle (Degrees)

ϕ = latitude (Degrees)

δ = declination angle (Degrees) given by Equation (3.2.2)

This daily data was then processed to find the monthly average daily value, which represents the mean daily insolation for the month under consideration. The mean is defined by Bowker (Ref. 9) as:

$$H_T = \lim_{N\to\infty} \sum_{i=1}^{N} \frac{H_{Ti}}{N} \qquad (4.1.2)$$

Data grouping can significantly influence the statistical properties of the data. For the remainder of this paper, when data is referred to as average daily data, all the data will be grouped by month regardless of year with each day constituting one data point. Statistical properties will be determined for the average day of the month. Monthly average daily data results from averaging all daily data from a given month of a given year and obtaining a single average value for that month. All like months, for instance all Januaries, are then evaluated to determine the statistical properties of the average day in January. The mean values of these two groupings will be equal. However, the standard deviation of data in the first grouping will be significantly larger.

The average daily insolation for each month it was available is shown in Table 4.1.1. Table 4.1.1 also contains the monthly average daily radiation for the four years of data.

4.2 Temperature Data

No temperature data was taken at the Lehigh University facility. Therefore, temperature data recorded at the Allentown Bethlehem Easton Airport and made available by the National Oceanic and Atmospheric Administration was used. The airport is located in close proximity to the Lehigh University facility with a latitude of N 40° 39' and a longitude of W 75° 26'.

The mean daily temperature was determined using the generalized form of Equation (4.1.2). Table 4.2.1 shows the

TABLE 4.1.1

Total Insolation On A Horizontal Surface

For Bethlehem, Pennsylvania

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
						Month						
1975									11227	8707	6550	4314
1976	5994	9399	12226	16676				18788	13021			5392
1977	6993	9161	13245	16858	20287	18618	19696	13873		8423	4132	4836
1978	5948	11137	12556	16121	14190	19742	19856	11875	13975			
Average	6312	9899	12669	16552	17233	19174	19776	10841	12741	8571	5347	4847
Average (Ref.1)*	5987	8665	12238	15998	18576	20166	20030	17546	14051	10509	6449	4885

* - Taken from Duffie for the Allentown Area

TABLE 4.2.1

Mean Temperature

(^{o}C)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
						Month						
1975									16.4	13.9	9.4	.5
1976	-4.3	2.6	5.6	11.3	14.8	22.7	22.1	21.8	17.6	10.2	3.2	-2.7
1977	-7.2	-.6	7.6	13.7	17.8	20.6	24.3	22.9	19.4	12.0	6.9	-.9
1978	-3.7	-5.3	2.6	10.4	16.1	21.7	22.6	23.1	18.0	N/A	N/A	N/A
Monthly Ave	-5.2	-1.1	5.2	11.1	17.0	21.1	23.3	22.2	17.8	12.9	8.2	-1.1
Monthly Ave *	-2	-1	3	10	16	21	23	22	18	12	6	-1

* - As Published In Duffie, etal (Reference 2)

average daily temperature for each month and the monthly average daily temperature. Also shown is data made available in Duffie (Ref. 2) for comparison.

4.3 The Standard Deviation Of Insolation And Temperature

In the general form, the variance of a random variable "a" is defined in Croxten, etal, (Ref. 10) as:

$$\sigma^2 = \lim_{N\to\infty} \frac{\sum_{i=1}^{N} (a_i - a^*)^2}{N} \qquad (4.3.1)$$

The square root of the variance, σ_a, is called the standard deviation, which is a measure of the dispersion of the variable "a". Using this formula, the standard deviation of insolation and temperature can be calculated. However, in order to obtain the appropriate standard deviation, it is necessary to establish the frequency of data collection

The standard deviation of insolation and temperature will vary depending on the interval chosen for data collection. Insolation data was integrated over fifteen minute intervals. If any other interval had been chosen and both sources of data were manipulated to find an average daily insolation, the means of the two samples would be the same. However, the standard deviation could differ significantly.

Both insolation and temperature are continuous functions. By summing them over a period, they are treated as discrete functions. Unless data is gathered continuously, some of the variability will be lost. This is because a single value is used to represent, in our case, fifteen minutes worth of data. During those fifteen minutes, insolation was varying; however, this information is represented by an average value and the user has no knowledge of the variability during the period. The fifteen minutes worth of data is taken as a single data point and is, either knowingly or unknowingly, assumed to have no variability. Therefore, finding the standard deviation of daily insolation using the fifteen minute data periods will show less variability than the deviaiton determined using continuous data.

The period over which the standard deviations of insolation and temperature are calculated must be the same as the period for which 'f' is calculated. Since 'f' calculates the monthly average daily collector output, the deviation of 'f' must be the deviation in the monthly average daily output. Therefore, the appropriate period to use in the calculation of deviations of insolation and temperature is one month.

4.4 The Correlation Coefficient

The existence of a relationship between insolation and temperature was discussed earlier. If a relationship exists, its existence would be shown by the correlation coefficient, which is given by Croxton, etal (Ref. 10) as:

$$r = \frac{\sum_{i=1}^{N} (a_i - a^*)\ (b_i - b^*)}{\sum_{i=1}^{N} (a_i - a^*)^2 \ \sum_{i=1}^{N} (b_i - b^*)^2} \tag{4.4.1}$$

where a and b are two random variables.

The correlation coefficient is the ratio of the explained deviation to the total deviation. It shows the degree of agreement between groups of data. The correlation coefficient is also a function of the slope of a linear curve fit.

The correlation coefficient can be either positive or negative depending upon whether temperature varies directly or inversely with insolation. A coefficient of zero would indicate that the variables are totally independent, while a value of 1 indicates total dependency.

Ideally, for this study the correlation coefficient should be obtained on a monthly average daily basis for each month of the year as was done for the standard deviation. If data were available for several Januaries, a correlation coefficient could be obtained for January. However, since data was available for only three years and in some cases data gaps reduced the available data to only two like months, a correlation coefficient for monthly average daily data would be misleading. This is obvious when one considers that the correlation coefficient is derived from the slope of a linear curve fit and if only two data points were available, the correlation coefficient would show a perfect fit, even if no relationship exists.

With the amount of data available for the Bethlehem area it is possible to obtain a correlation between insolation and temperature on a daily basis. This coefficient varies widely for like months; however, some of this variability is reduced when all the data from like months is combined to form a correlation coefficient for that month (i.e. all the days from the three Januaries are considered data points and are used to form a single correlation coefficient for the month of January). To distinguish this coefficient, it will be referred to as the "correlation coefficient for the month". The reduction in variability is caused by an increase in sample size. The correlation coefficients for daily data for individual months that data was available are shown in Table 4.4.1.

Table 4.4.1 also shows correlation coefficients obtained by Lund (Ref. 8) for Blue Hill, Massachusettes using twenty years of data. These coefficients trend in the same manner as those obtained for the Bethlehem area except for the summer months where much of the data for the Bethlehem area is missing.

4.5 Effect Of Sample Size On The Correlation Coefficient

The correlation coefficient is an important statistical property when evaluating the deviation of 'f'. As occurs with the standard deviation, the grouping of data will significantly

TABLE 4.4.1

Correlation Coefficients

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
						Month						
1975									.28	-.07	.22	-.35
1976	-.50	.13	.19	.29	-.45			-.44	.12		-.16	-.40
1977	-.50	-.07	.03	.18	.34	.27	-.03	.47	-.66	-.05	-.27	-.47
1978	-.68	-.34	.19	.28	.43	.09	.38	.35	-.09	-	-	-
Coefficient for the month	-.55	-.07	.16	.29	.37	.19	.24	.44	.11	-.05	-.10	-.44
Blue Hill	-.51	-.35	-.10	.22	.38	.35	.32	.13	.00	-.08	-.30	-.40

influence the correlation coefficient. The ideal situation would have been to find a correlation between the monthly average daily temperature and insolation. However, this would have required several years of data and this amount of data is not available. Therefore, the only way the correlation coefficient could be employed in this study was to use the correlation coefficient for the month.

The use of the "daily coefficient for the month" will cause some error to be introduced since the daily values will have more scatter than monthly average daily values. This will cause the coefficients to be numerically lower, or show less of a correlation than would have occurred had the monthly average daily correlations been available.

Using the wrong data base to establish the correlation allows us to accomplish two objectives, it establishes the existence of a correlation, and it allows the use of the correlation coefficient in the calculation of the deviation of 'f' for illustrative purposes only. However, it must be recognized that the "monthly average daily correlation coefficient" will be of a greater absolute value than the "correlation coefficient for the month". Using the "correlation coefficient for the month" will underpredict the effects of the correlation.

The correlation coefficient trends the changes in energy at a particular location. Negative coefficients occur during periods when the location is cooling (September to February) and positive coefficients occur during the heating of the earth (March to August). Of particular interest is the negative coefficient which occurs during a large percentage of the residential heating season. The negative coefficient indicates that temperature rises as insolation decreases. This implies that when considering auxiliary heating it may be more beneficial for the owner of a small solar collector system to have a cloudy day. On a cloudy day, the solar collector system's performance will deminish; however, the ambient temperature will be higher and less auxiliary power will be needed.

5.0 THE STANDARD DEVIATION OF X AND Y

To fully characterize data, information about the average and the scatter of the data is necessary. Knowing the scatter can be as important as knowing the average since the scatter provides information on the range through which the variable has varied. Using average data will predict the average collector output for an average month. However, since rarely will an actual month behave like the average month, a band exists around the average 'f' which predicts with certain confidence intervals the expected minimum and maximum values of 'f' for a specific month. This band is caused by the deviations of insolation and temperature and is referred to as the standard deviation of 'f'.

The standard deviation of 'f' is difficult to evaluate because 'f' is a function of x and y, which in turn are complex functions of insolation and ambient temperature. In addition, there is the possibility of temperature being a function of insolation which further complicates the calculation. Any correlation between temperature and insolation will affect the deviation of 'f'.

For this reason, the correlation terms must be included in the development of the standard deviation of 'f'. The correlation will directly affect the standard deviation of y since y is a function of insolation and temperature. X does not include an insolation term and no correlation term exists in the deviation of x. This does not imply that x is independent of insolation, since if temperature and insolation are correlated, insolation is implicity included.

5.1 Calculation Of The Standard Deviation Of X And Y

Both x and y are composed of an average quantity and the random component which can be expressed as follows:

$$x = x^* + \Delta x \qquad (5.1.1)$$

$$y = y^* + \Delta y \qquad (5.1.2)$$

where: x^* = mean value of x

y^* = mean value of y

Δx = perturbations of x about the mean

Δy = perturbations of y about the mean

Because the terms x and y are complex nonlinear functions of insolation and temperature, the calculation of their standard deviations is more difficult and it is necessary to approximate the deviations of x and y around insolation and temperature using a Taylor series.

A Taylor polynomial approximates a function by expanding the function about the point of interest. The order of the polynomial indicates the number of derivatives taken to approximate the function. A higher order polynomial will be more accurate; however, it will also require increased computational effort. Therefore, it is advantageous to keep the order as low as possible while maintaining sufficient accuracy.

The first degree Taylor polynomial was found to be sufficiently accurate for the Bethlehem data since there was, a maximum of a ten percent difference in the deviations of x and y when compared to a second order polynomial.

The large error occurs during May and September when the residental heating load approaches zero. Since x and y are both functions of the reciprocal of heating load (L), they approach infinity as the heating load (L) approaches zero. With the low heating requirements, small changes in T_a cause large changes in x and y and resulted in the large difference between the first and second order approximations. Increasing the order of the Taylor polynomial will improve the accuracy of the deviation of 'f'. However, to obtain a sufficiently accurate estimate the order must approach infinity to accomodate (L) approaching zero. Therefore, the errors encountered when calculating the deviation of 'f' can be quite large for months when heating load (L) approaches zero. However, there is no way to account for these errors. This does not invalidate the approach being developed since the error occurs only when the need for solar energy is approaching zero. When demand on the solar collection system is high there will be less than one percent difference between the first and second order approximations.

From Equations (2.1.2b) and (2.1.4), x is given by:

$$x = \frac{Ac\ Fr'\ U_L\ (T_{ref} - T_a)\ \Delta t}{L} \qquad (5.1.3)$$

and y is given by:

$$y = \frac{Ac\ Fr'\ (\tau\alpha)\ H_T\ N}{L} \qquad (5.1.4)$$

Before expanding x and y around insolation and temperature, all terms in x and y which could be functions of insolation and temperature must be identified. These are the system heating load (L), the collector loss coefficient (U_L), and the collector efficiency factor (Fr').

Fr' and U_L will be considered constants. There is some error introduced by considering Fr' and U_L to be constants; however, Klein (Ref. 1) judged this error to be slight, and our analysis showed that this assumption does not significantly affect the results provided that the ranges over which the functions can vary are limited.

The heating load term is contained in both x and y and it is calculated by using the degree day method described in ASHRAE (Ref. 3). This method calculates an overall conduction coefficient for the house and using the area of the house, a loss coefficient-area product is determined. This loss coefficient-area product is a function of the house design, materials and size only. To find the total heat loss, the loss coefficient-area product is multiplied by the degree days, where degree days are defined, by the United States Weather Service, as the difference between 68^{o} F and the mean daily ambient temperature, with only positive values included in the compilation. This allows house heating loads to be treated as a function of the ambient temperature.

Using the ASHRAE method to determine load, Equation (2.1.4) becomes:

$$x = \frac{Ac\ Fr'\ U_L\ (T_{ref} - T_a)}{U_h\ A_h\ (T_h - T_a)} \tag{5.1.5}$$

And Equation (2.1.2b becomes:

$$y = \frac{Ac\ Fr'\ (\tau\alpha)\ H_T\ N}{U_h\ A_h\ (T_h - T_a)} \tag{5.1.6}$$

Removing all constant terms (and those assumed constant):

$$x = K\,\frac{(T_{ref} - T_a)}{(T_h - T_a)} \tag{5.1.7}$$

where:

$$K = \frac{Ac\ Fr'\ U_L}{U_h\ A_h} \tag{5.1.8}$$

And, y becomes:

$$y = \frac{C\ H_T}{(T_h - T_a)} \tag{5.1.9}$$

where:

$$C = \frac{Ac\ Fr'\ (\tau\alpha)\ N}{U_h\ A_h} \tag{5.1.10}$$

Expanding x and y around insolation and temperature using the first order Taylor polynomial:

$$x = K\frac{(T_{ref} - T_a^*)}{(T_h - T_a^*)} + (T_a - T_a^*)\left.\frac{dx}{dT_a}\right|_{T_a = T_a^*} \tag{5.1.11}$$

$$y = \frac{C\ H_T^*}{(T_h - T_a^*)} + (H_T - H_T^*)\left.\frac{dy}{dH_T}\right|_{H_T = H_T^*} +$$

$$(T_a - T_a^*)\left.\frac{dy}{dT_a}\right|_{T_a = T_a^*} \tag{5.1.12}$$

Taking the appropriate derivatives and evaluating both equations around the point of interest yields:

$$x = K\frac{(T_{ref} - T_a^*)}{(T_h - T_a^*)} + \sum_{i=1}^{N}\left[\frac{K\ (T_{ai} - T_a^*)\ (T_{ref} - T_a^*)}{(T_h - T_a^*)^2} - \frac{K\ (T_a - T_a^*)}{(T_h - T_a^*)}\right] \tag{5.1.13}$$

$$y = \frac{C\,H_T^*}{(T_h - T_a^*)} + \sum_{i=1}^{N} \frac{C\,(H_{Ti} - H_T^*)}{(T_h - T_a^*)} +$$

$$\sum_{i=1}^{N} \frac{C\,H_T^*\,(T_{ai} - T_a^*)}{(T_h - T_a^*)^2} \qquad (5.1.14)$$

Equations (5.1.13) and (5.1.14) are of the same format as equations (5.1.1) and (5.1.2), respectively. Further investigation of (5.1.13) and (5.1.14) indicate that the first terms in each, when evaluated at the mean values of temperature and insolation, give the mean value of x and y or to be more specific, x^* and y^*. The second terms in equations (5.1.13) and (5.1.14) are the residues or perturbations of x and y, respectively.

The variance, given by Bowker (Ref. 9) is defined by;

$$\sigma^2 = \sum_{i=1}^{N} \frac{(a_i - a^*)^2}{(N-1)} \qquad (5.1.15)$$

By rearranging Equation (5.1.1):

$$\Delta x = x - x^* \qquad (5.1.16)$$

the perturbation is determined and the numerator in Equation (5.1.15) is solved for. Rearranging Equation (5.1.13) to conform with Equation (5.1.16) and substituting Δx into Equation

$$\sigma_x^2 = \sum_{i=1}^{N} \frac{(\Delta x)^2}{(N - 1)} \qquad (5.1.17)$$

$$= \sum_{i=1}^{N} \frac{\dfrac{K^2 (T_{ref} - T_h^*)^2 (T_{ai} - T_a^*)^2}{(T_h - T_a^*)^4}}{(N - 1)} \qquad (5.1.18)$$

which is the square of the standard deviation of x.

Using the same argument for y:

$$\sigma_y^2 = \sum_{i=1}^{N} \frac{\left[\dfrac{C (H_{Ti} - H_T^*)}{(T_h - T_a^*)} + \dfrac{C H_T (T_{ai} - T_a^*)}{(T_h - T_a^*)^2}\right]^2}{(N - 1)} \qquad (5.1.19)$$

After performing the mathematics and some simplifications, the resultant equations are:

$$\sigma_x^2 = \frac{K^2 (T_{ref} - T_h^*)^2}{(T_h - T_a^*)^4} \sum_{i=1}^{N} \frac{(T_{ai} - T_a^*)^2}{(N - 1)} \qquad (5.1.20)$$

$$\sigma_y^2 = \frac{c^2}{(T_h - T_a^*)^2} \sum_{i=1}^{N} \left[\frac{\frac{H_T^{*2} (T_{ai} - T_a^*)^2}{(T_h - T_a^*)^2} + (H_{Ti} - H_T^*)^2}{(N - 1)} \right.$$

$$\left. + \frac{\frac{2 H_T^* (T_{ai} - T_a^*) (H_{Ti} - H_T^*)}{(T_h - T_a^*)}}{(N - 1)} \right] \qquad (5.1.21)$$

Since the variability of x and y are caused by the variability of insolation and temperature, the standard deviation of x and y should be a function of the standard deviations of insolation and temperature. The variance, which is the standard deviation of insolation squared is given by:

$$\sigma_{HT}^2 = \frac{\sum_{i=1}^{N} (H_{Ti} - H_T^*)^2}{(N - 1)} \qquad (5.1.22)$$

and the variance of temperature is given by:

$$\sigma_{Ta}^2 = \frac{\sum_{i=1}^{N} (T_{ai} - T_a^*)}{(N - 1)} \qquad (5.1.23)$$

Substituting Equations (5.1.22) and (5.1.23) into Equations (5.1.17) and (5.1.18) and taking the square root gives the standard deviation of x and y:

$$\sigma_X = K \frac{(T_{ref} - T_h^*)}{(T_h - T_a^*)^2} \sigma_{Ta} \qquad (5.1.24)$$

$$\sigma_Y = \frac{C}{(T_h - T_a^*)} \left[\frac{H_T^*}{(T_h - T_a^*)} \sigma_{Ta} + \sigma_{HT} + \sum_{i=1}^{N} \left[\frac{2 H_T^* (T_{ai} - T_a^*) (H_{Ti} - H_T^*)}{(T_h - T_a^*) (N - 1)} \right] \right]^{1/2} \qquad (5.1.25)$$

Replacing the generalized random variables in Equation (4.4.1) with insolation and temperature and recognizing that the denominator is a function of the standard deviations of insolation and temperature, Equation (4.4.1) becomes:

$$r = \frac{\sum_{i=1}^{N} (T_{ai} - T_a^*) (H_{Ti} - H_T^*)}{(N - 1) (\sigma_{Ta}) (\sigma_{HT})^{1/2}} \qquad (5.1.26)$$

Rearranging Equation (5.1.26), we obtain:

$$\sum_{i=1}^{N} (T_{ai} - T_a^*) (H_{Ti} - H_T^*) = (N - 1) r \sigma_{Ta} \sigma_{HT} \qquad (5.1.27)$$

Substituting into Equation (5.1.25) gives:

$$\sigma_y = \frac{C}{(T_h - T_a^*)} \left[\frac{H_T}{(T_h - T_a^*)} \sigma_{Ta} + \sigma_{HT} + \left[\frac{2 H_T^* r \sigma_{Ta} \sigma_{HT}}{(T_h - T_a^*)} \right] \right]^{1/2} \quad (5.1.29)$$

Equations (5.1.27) and (5.1.28) can be simplified by using Equations (5.1.7) and (5.1.9) to:

$$\sigma_x = x \frac{T_{ref} - T_h}{T_h - T_a^*} \left[\frac{\sigma_{Ta}}{T_h - T_a^*} \right] \quad (5.1.29)$$

$$\sigma_y = y \left| \frac{\sigma_{Ta}}{T_h - T_a^*} + \frac{\sigma_{HT}}{H_T^*} + \left[\frac{2 r \sigma_{HT}}{H_T^*} \frac{\sigma_{Ta}}{T_h - T_a^*} \right] \right]^{1/2} \quad (5.1.30)$$

The standard deviation of x is a function of the standard deviation of temperature only. Whereas, the standard deviation of y is a function of the standard deviations of both temperature and insolation. In addition, there is another term in the deviation of y which reflects the correlation of temperature and insolation. If temperature and insolation are independent of each other, this term drops out. However, if a dependency exists, the term gets proportionately stronger as the dependency increases. This term can either increase or decrease the standard deviation of y depending upon whether the correlation has a positive or negative slope.

5.2 The Standard Deviation Of 'F'

Calculating the standard deviation of 'f' is accomplished in the same manner as calculating the standard deviation of x and y. 'f' is composed of a mean value and perturbations about 'f'. In equation form:

$$'f' = f^{*} + \Delta f \qquad (5.2.1)$$

Klein (Ref. 1) used a least squares curve fit to obtain an equation for 'f' in terms of x and y, which is:

$$'f' = 1.029y - .065x - .245y^2 + .0018x^2 + .0215y^3 \qquad (5.2.2)$$

This equation is an approximation of the actual simulation data. Expanding the equation for 'f' around the point x equals x^* and y equals y^*, using the first degree Taylor polynomial:

$$f = f^* + (x - x^*) \left.\frac{df}{dx}\right|_{x = x^*} + (y - y^*) \left.\frac{df}{dy}\right|_{y = y^*} \quad (5.2.3)$$

Taking derivatives of Equation (5.2.2) with respect to x and y and using Equation (5.2.1) to solve for Δf, the following is obtained:

$$\Delta f = \sum_{i=1}^{N} \left[(x_i - x^*) \; (-.065 + .0036x^*) \; + \; (y_i - y^*) \; (1.029 - .490y^* + .0645y^{*2}) \right] \quad (5.2.4)$$

The standard deviation squared of 'f' is described by:

$$\sigma_f^2 = \sum_{i=1}^{N} \frac{(\Delta f)^2}{(N - 1)} \quad (5.2.5)$$

Substituting Δf into Equation (5.2.5) and squaring yields:

$$\Delta f^2 = \sum_{i=1}^{N} (x_i - x^*)^2 (.00422 - .000234x^* + 1.3x10^{-5}x^{*\,2}) + \sum_{i=1}^{N} (y_i - y^*)^2 (1.0588 - 1.008y^* + .3728y^{*\,2} - .0632y^{*\,3} + .00416y^{*\,4}) + \sum_{i=1}^{N} (x_i - x^*)(y_i - y^*) (-.0668 + .0318y^* - .00419y^{*\,2} + .0037x^* - .0017x^*y^* + .000232x^*y^{*\,2}) \quad (5.2.6)$$

Portions of the first and second terms of equation (5.2.6) can be put into the same form as Equation (5.1.18) and can be replaced by the standard deviation of x and y respectively. The third term, however, requires some consideration. This is the correlation term between x and y. If x and y are uncorrelated, this term drops out. However, any relationship between x and y will cause this term to influence the deviation of 'f'.

For the uncorrelated case the last term of Equation (5.2.6) goes to zero and the following is obtained:

$$\sigma_f = \left[(.00422 - .000234x^* + 1.3\text{x}10^{-5} x^{*2})\, \sigma_x^2 + (1.0588 - 1.008y^* + .3728y^{*2} - .0632y^{*3} + .00416y^{*4})\, \sigma_y^2 \right]^{1/2} \qquad (5.2.7)$$

For the correlated case using the definition of correlation given in Equation (5.2.2):

$$\sigma_f = \left[(.00422 - .000234x^* + 1.3\text{x}10^{-5} x^{*2})\, \sigma_x^2 + (1.0588 - 1.008y^* + .3728y^{*2} - .0632y^{*3} + .00416y^{*4})\, \sigma_y^2 + 2\,(-.0668 + .0318y^* - .00419y^{*\,2} + .0037x^* - .0017x^*y^* + .000232x^*y^{*\,2})\, r\, \sigma_x \sigma_y \right]^{1/2} \qquad (5.2.8)$$

6.0 THE RELATIVE IMPORTANCE OF TERMS USED IN THE CALCULATION OF X, Y AND F

Ultimately the standard deviation of 'f' can be traced to the standard deviations of insolation and temperature, since the standard deviations of x and y are functions of the variations and correlations of insolation and temperature. By evaluating the effect of the variability and correlations on x, y and 'f', it will be possible to determine which has the greatest effect on 'f' and whether any effects are insignificant.

6.1 The Causes Of The Variability Of X

The standard deviation of x is a function of the design parameters and the mean and standard deviation of ambient temperature. Therefore, the standard deviation of temperature causes all the variation in x for a given system design during a given month.

6.2 The Causes Of The Variability Of Y

The deviation of y shown in Equation (5.1.30) is dependent upon the design parameters, the means and standard deviations of temperature and insolation and the correlation between insolation and temperature.

Equation (5.1.30) contains three sources of variability. These are the normalized temperature deviation:

$$\frac{\sigma_{Ta}}{T_h - T_a} \tag{6.2.1}$$

which is caused by the variability of temperature.

The normalized insolation deviation:

$$\frac{\sigma_{HT}}{H_T} \tag{6.2.2}$$

which is caused by the variability of insolation; and:

$$\left[2\, r \frac{\sigma_{Ta}\, \sigma_{HT}}{(T_h - T_a)\, H_T} \right]^{1/2} \qquad (6.2.3)$$

which results from the correlation between insolation and temperature.

An order of magnitude analysis of these terms using the Bethlehem area weather data indicates that, in the calculation of the standard deviation of y, the order of magnitude of the correlation term is the same as the contributions from the terms resulting from the variability of temperature and insolation for the colder months. These months include November, December and January, a major portion of the heating season.

As the monthly mean insolation and temperature increases, the terms resulting from correlation and the variability of temperature increase at about the same rate while the term resulting from the deviation of insolation remains relatively constant throughout the year. Therefore, as the months become warmer, the variability of insolation becomes less significant and the variability of temperature and the correlation predominate.

The standard deviation of insolation is relatively constant throughout the year. However, the deviation of y is a function of the normalized deviation of insolation (obtained by dividing the standard deviation of insolation by the average insolation), which decreases with increasing insolation. This causes the deviation of insolation to have less impact on y as the insolation increases.

The standard deviation of temperature is also relatively constant throughout the year. The deviation of y is a function of the normalized deviation of temperature which was obtained by dividing the standard deviation of temperature by the difference between the house temperature and ambient temperature. Conversely, the normalized temperature distribution given by Equation (6.2.1) increases with increasing ambient temperature. This causes the relative importance of the deviation of temperature on the deviation of y to increase as the ambient temperature increases.

The correlation coefficient is a product of the deviations of both insolation and temperature. It is of the same magnitude as the deviations of temperature and insolation. Therefore, all the terms used in the calculation of σ_y are important and cannot be neglected.

6.3 The Deviation Of 'F'

The same approach used in determining the relative effects of terms in the calculation of the standard deviation of y can be used in determining the terms' effects on the standard deviation of 'f'. It can be seen from equation (5.2.8) that the standard deviation squared of 'f' is a function of the following three terms:

$$(.00422 - .000234x^{*} + 1.3x10^{-5}x^{*2})\ \sigma_x^{2} \qquad (6.3.1)$$

$$(1.0588 - 1.008y^{*} + .3728y^{*2} - .0632y^{*3} +$$

$$.00416y^{*4})\ \sigma_y^{2} \qquad (6.3.2)$$

$$(-.0668 + .318y^* - .00419y^{*2} + .0037x^* - .0017x^*y^* +$$

$$.000234x^*y^{*2}) \quad r\,\sigma_x\,\sigma_y \qquad (6.3.3)$$

Determining the contributions of each of these terms to the deviation of 'f' will show the relative importance of that term.

Before begining an order of magnitude analysis, some discussions of the term due to the correlation of x and y shown in Equation (6.3.3) is necessary. This term is different than the correlation of insolation and temperature and would exist even if insolation and temperature were uncorrelated, since x and y are both functions of the residential heating load. Changes in residential heating load will cause both x and y to vary and, because of this, a correlation will exist.

Expanding:

$$\sum_{i=1}^{N} (x_i - x^*) \quad (y_i - y^*) \qquad (6.3.4)$$

from Equation (5.2.8) and using Equations (5.1.13), (5.1.14), (5.1.15) and (5.1.16), the complete term due to the correlation is:

$$C\,K \frac{(T_{ref} - T_h)}{(T_h - T_a^*)^3} \left[\frac{H_T^* \sigma_{Ta}}{(T_h - T_a^*)} + \left[r\, \sigma_{Ta}\, \sigma_{HT} \right]^{1/2} \right] \qquad (6.3.5)$$

which can be reduced to:

$$x\,y \frac{(T_{ref} - T_h)}{(T_{ref} - T_a)} \left[\frac{\sigma_{Ta}}{(T_h - T_a)} + \left[r \frac{\sigma_{Ta}\ \sigma_{HT}}{(T_h - T_a)\ H_T} \right]^{1/2} \right] \qquad (6.3.6)$$

The first term in Equation (6.3.5) is due to the dependency of both x and y on temperature, while the second term results from the correlation of weather data.

Of particular interest is the effect of this correlation on the deviation of 'f'. The first part of Equation (6.3.3):

$$(-.0668 + .318y^* - .00419y^{*2} + .0037x^* - .0017x^*y^* + .00023x^*y^{*2}) \qquad (6.3.7)$$

will be negative for all values of x and y which are possible when calculating 'f'. This means that there will be a reduction in the deviation of 'f' caused by the first term in Equation (6.3.1) as well as for all positive values of the correlation of insolation and temperature. This is opposite the effect the correlation had on the value of 'y'.

An order of magnitude assessment of the three terms shows that the deviation of y has the greatest influence on the deviation of 'f'. This term is shown in Equation (6.3.2), and as mentioned previously, is a function of the insolation, temperature, and the correlation of insolation and temperature. The effect of the correlation term shown in Equation (6.3.3) is one order of magnitude less than the deviation of 'f' caused by the deviation of y; however, it is one order of magnitude greater than that caused by the deviation of x. Thus, when estimating the deviation of 'f', the most significant term is the deviation of y, followed by the correlation of x and y. Finally, of little importance is the deviation of x.

The small effect caused by the deviation of x does have a physical explaination. X is the ratio of collector losses to house heating load. Both numerator and denominator are functions of the difference between the respective operating temperatures and ambient temperature. Changes in ambient temperature cause both the collector losses (numerator) and the heating load (denominator) to change. However, the changes tend to cancel. Therefore, slight to moderate changes in ambient temperature cause the standard deviation of x to be insignificant. That any deviation at all occurs is attributed to the different operating temperatures (i.e. T_{ref} = 100 ^{o}C; T_H = 20 ^{o}C).

6.4 Applicability

The deviation of 'f' can be calculated by summing Equations (6.3.1), (6.3.2) and (6.3.3) and taking the square root. Due to the number of variables, a simple chart, as was developed for the 'f' factor, cannot be developed; however, the equations can be rearranged to facilitate calculation.

Equation (6.3.1) can be rewritten as:

$$x \text{ term} = (.00422 - .000234x^{*} + 1.3\text{x}10^{-5}x^{*\,2})\ x^{*\,2}$$

$$\frac{(T_{ref} - T_h)^2}{(T_{ref} - T_a)^2}\ \frac{\sigma_{Ta}^{\ 2}}{(T_h - T_a)^2} \qquad (6.4.1)$$

Equation (6.3.2) can be rewritten as:

$$\text{y term} = (1.0588 - 1.008y^* + .3728y^{*\,2} - .0632y^{*\,3} + .00416y^{*\,4})\ y^{*\,2}\left[\frac{\sigma_{Ta}^{\ 2}}{(T_h - T_a^*)^2} + \frac{\sigma_{HT}^{\ 2}}{H_T^{\ 2}} + \frac{r\ \sigma_{HT}\ \sigma_{Ta}}{H_T\ (T_h - T_a^*)}\right] \tag{6.4.2}$$

Finally, Equation (6.3.3) can be rewritten as:

$$\text{xy term} = (-.0668 + .318y^* - .00419y^{*\,2} + .0037x^* - .0017x^*y^* + .000234x^*y^{*\,2})\ x^*\ y^*\left[\frac{\sigma_{Ta}^{\ 2}}{(T_h - T_a^*)^2} + \frac{r\ \sigma_{Ta}\ \sigma_{HT}}{(T_h - T_a)\ H_T}\right] \tag{6.4.3}$$

$$\sigma_f = (\text{x term} + \text{y term} + \text{xy term})^{1/2} \tag{6.4.4}$$

In this format, the equations are readilly calculable and are shown to be functions of x and y. Only three additional terms are needed to calculate σ_f. These are the normalized deviations of temperature and insolation and the correlation term. Finally, a good estimate of the deviation of 'f' can be obtained by taking the square root of Equation (6.4.2). This is at least one order of magnitude greater than the other terms.

6.5 Results

The method presented is capable of predicting within certain confidence intervals, the variations in the monthly output of active solar collection heating systems. A series of graphs have been developed which illustrate the effects of variations in insolation, temperature and the correlation coefficient. The graphs were developed using correlation coefficients of -.5, 0, and .5 for values of the normalized temperature and insolation standard deviations of .1, .2, and .3. These were done for values of 'f' of .4, .6, and .8 and are shown in Figures (6.5.1) through (6.5.27). The x's depict the range through which 'f' can vary for one standard deviation of 'f'. According to Croxten, etal (Ref. 10), a standard deviation of one has a confidence interval of 68%.

The graphs are useful for demonstrating the effects of changes in the deviations of insolation and temperature as well as changes in the correlation coefficient. As the variations in insolation and temperature increase, so does the variability of 'f'. This can be seen by comparing Figures (6.5.1), (6.5.4), and (6.5.7). These figures have the same correlation coefficient but

FIGURE 6.5.1

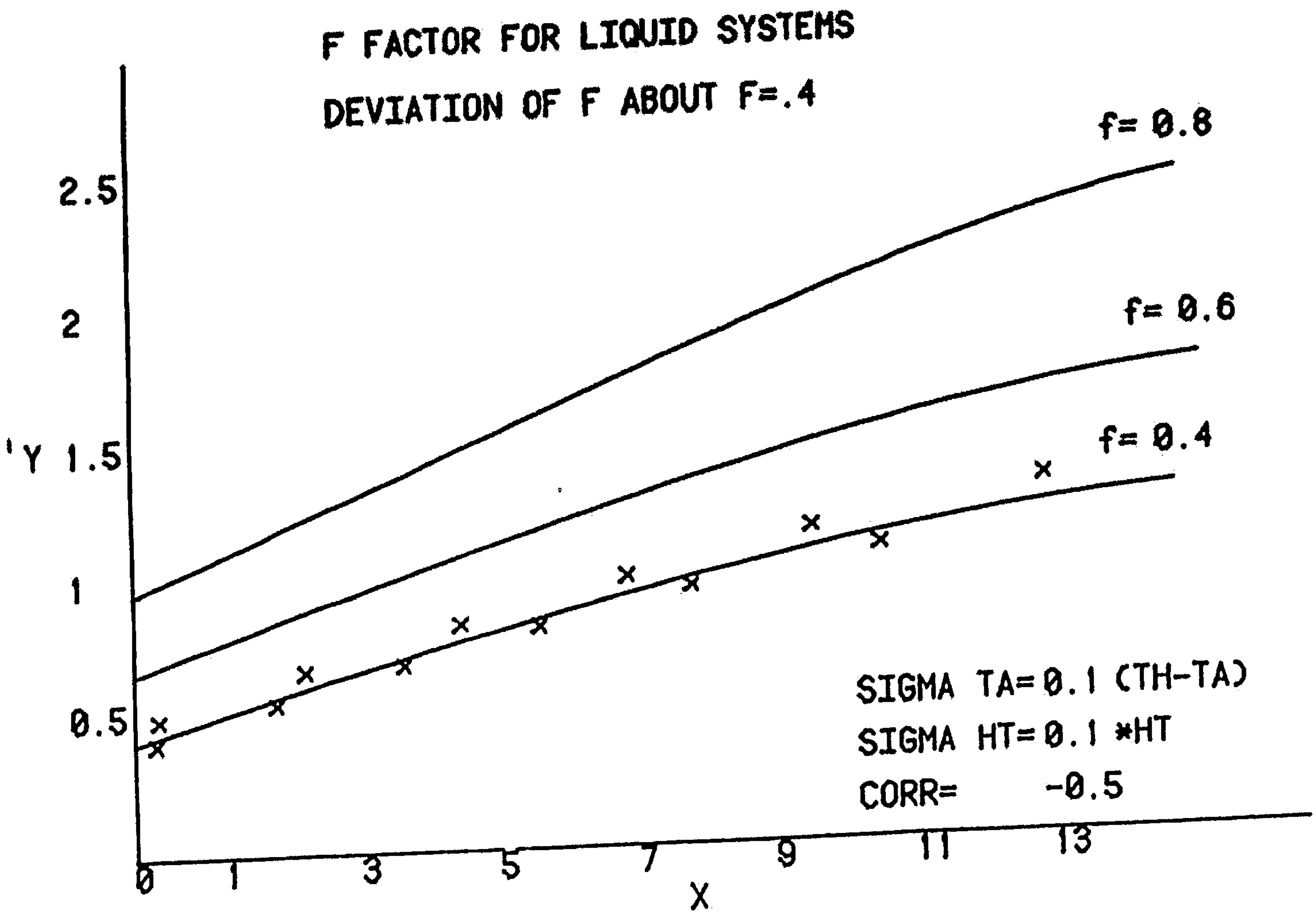

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT F=.4
f= 0.8
f= 0.6
f= 0.4
2.5
2
Y 1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA=0.1 (TH-TA)
SIGMA HT=0.1 *HT
CORR= -0.5

FIGURE 6.5.2

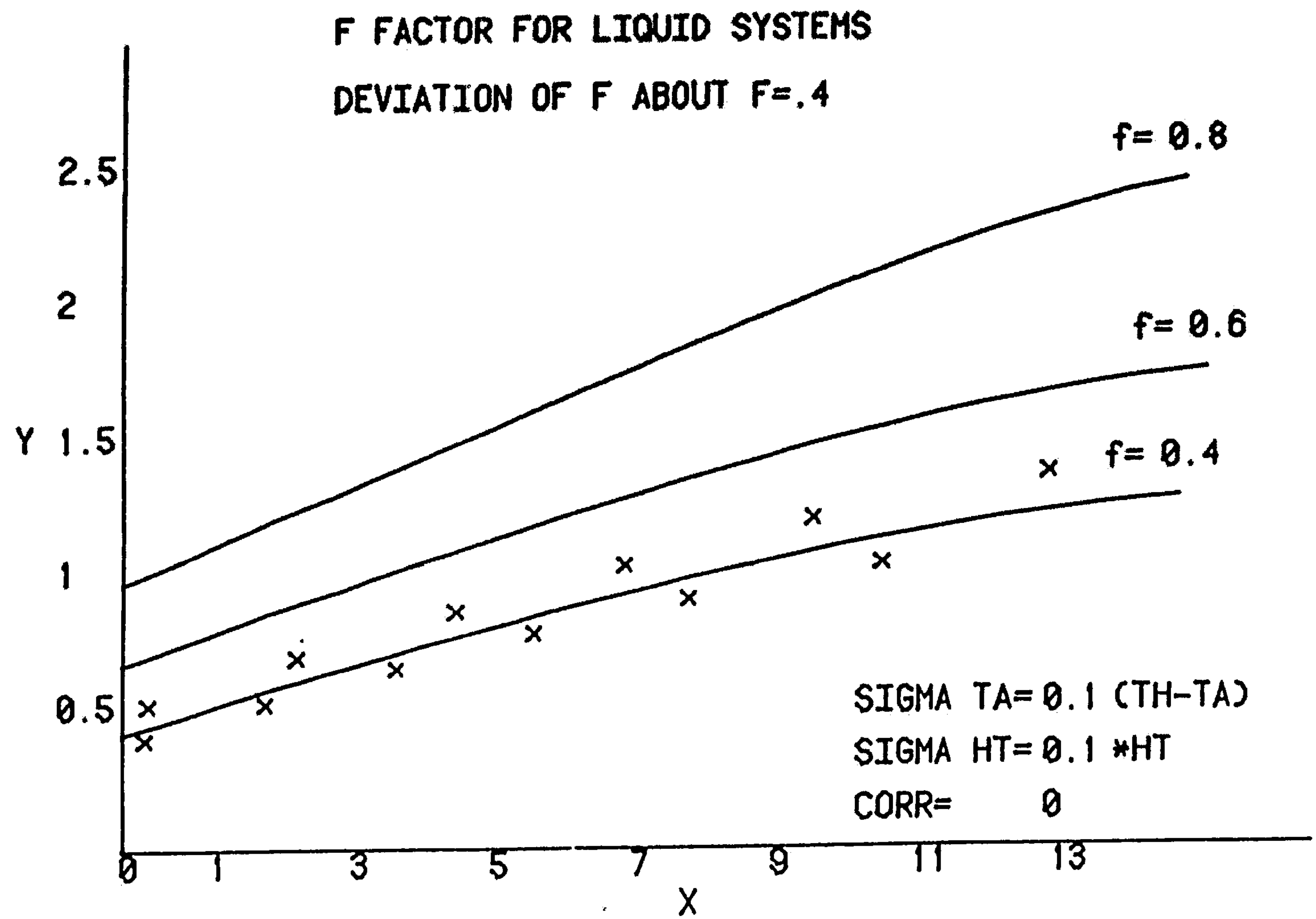

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT F=.4
f= 0.8
f= 0.6
f= 0.4
2.5
2
Y 1.5
1
0.5
SIGMA TA= 0.1 (TH-TA)
SIGMA HT= 0.1 *HT
CORR= 0
0
1
3
5
7
9
11
13
X

FIGURE 6.5.3

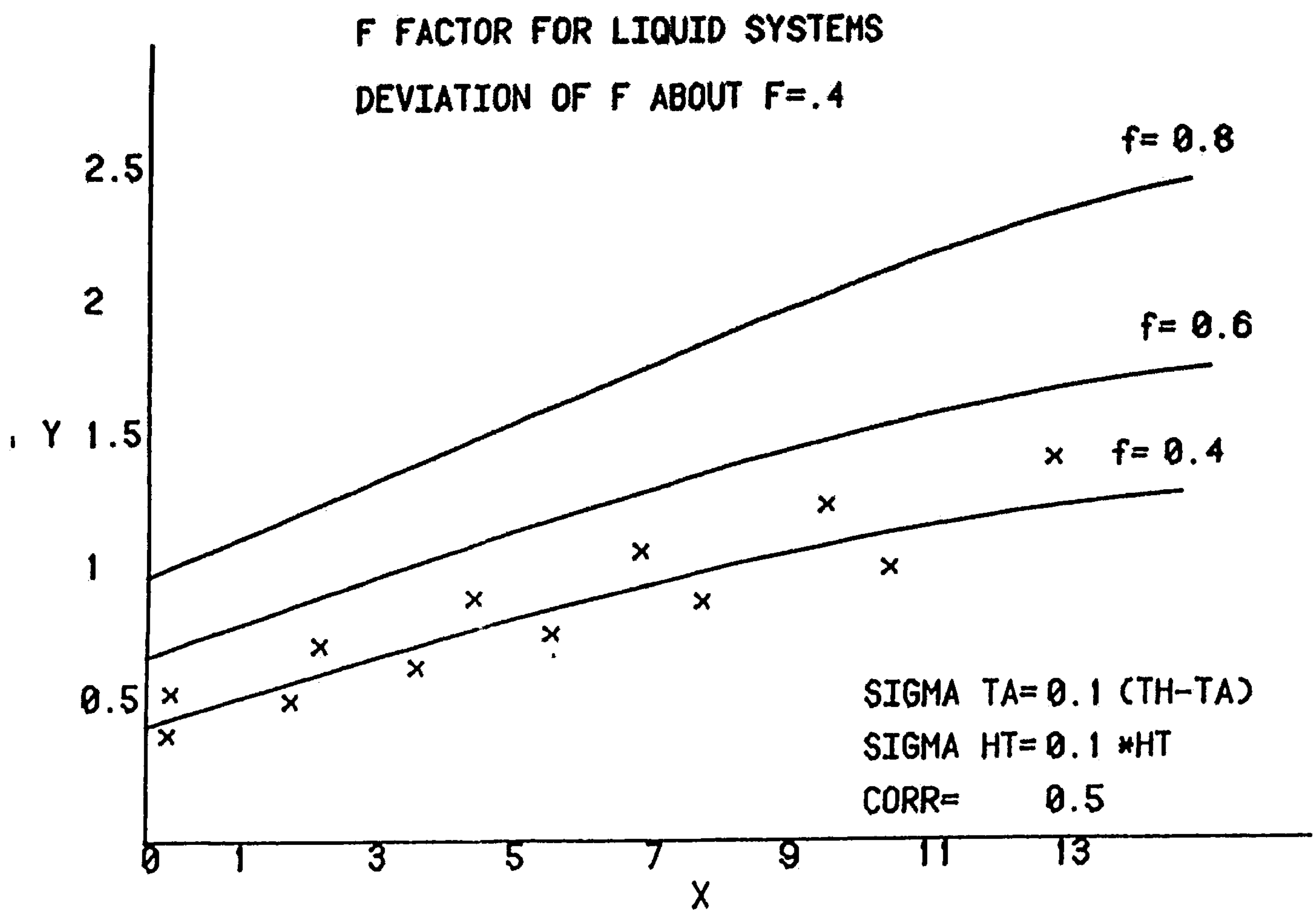
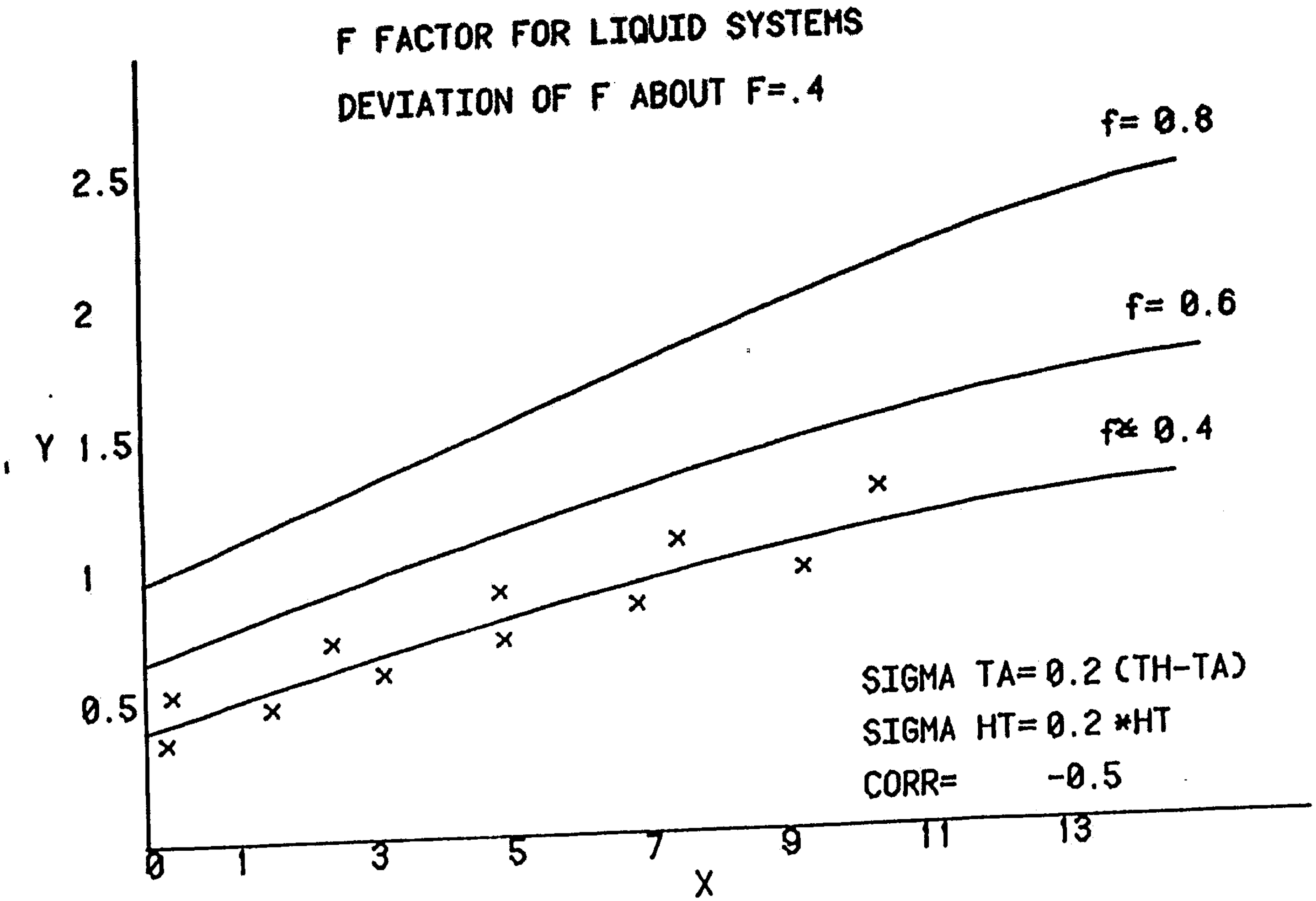

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT F=.4
f= 0.8
f= 0.6
f= 0.4
SIGMA TA= 0.1 (TH-TA)
SIGMA HT= 0.1 *HT
CORR= 0.5
Y
2.5
2
1.5
1
0.5
X
0
1
3
5
7
9
11
13

FIGURE 6.5.4


F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT F=.4
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
SIGMA TA= 0.2 (TH-TA)
SIGMA HT= 0.2 *HT
CORR= -0.5
0
1
3
5
7
9
11
13
X

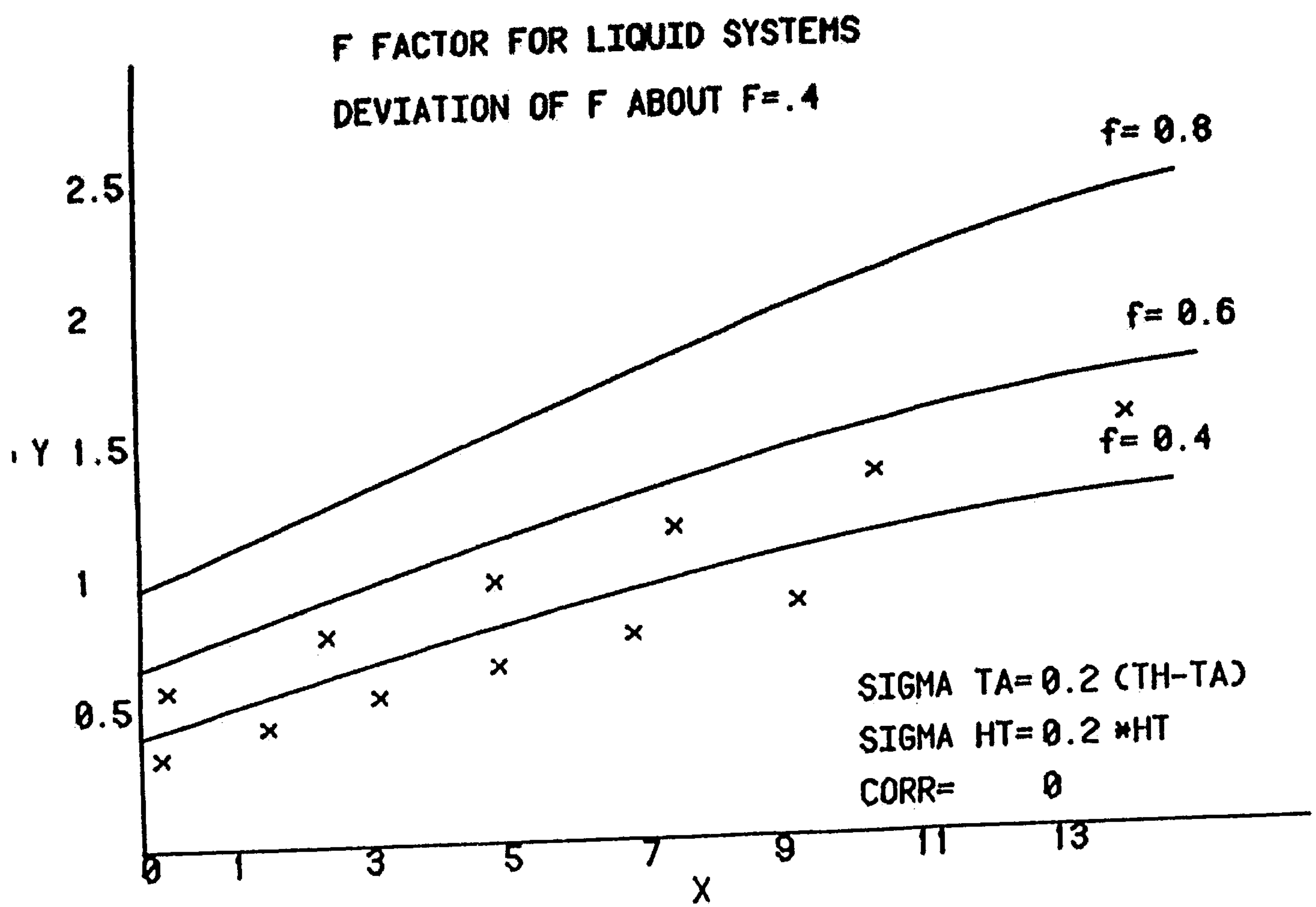
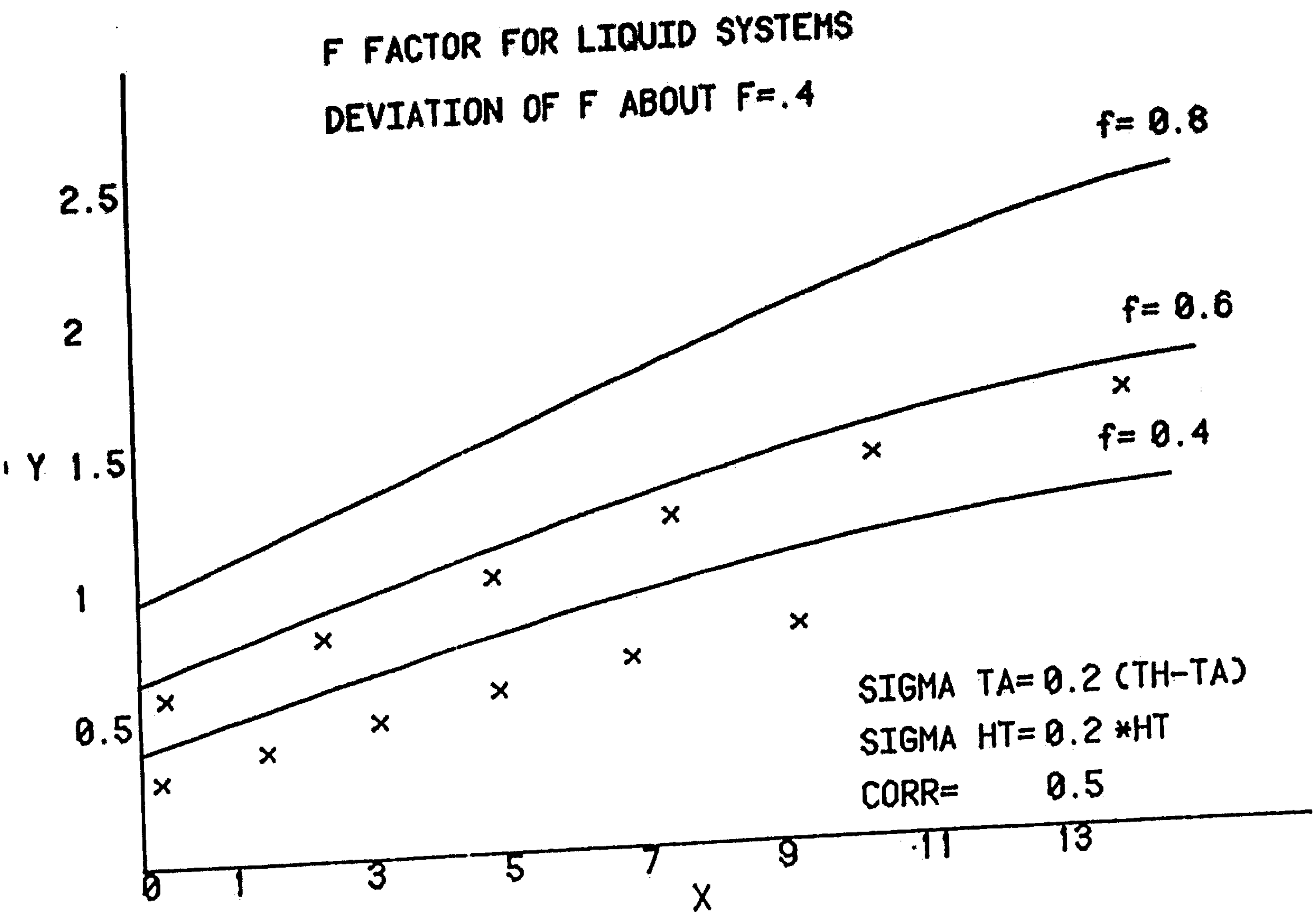

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT F=.4
f= 0.8
f= 0.6
f= 0.4
2.5
2
Y 1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.2 (TH-TA)
SIGMA HT= 0.2 *HT
CORR= 0

FIGURE 6.5.5

FIGURE 6.5.6



F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT F=.4
f= 0.8
f= 0.6
f= 0.4
2.5
2
Y 1.5
1
0.5
SIGMA TA= 0.2 (TH-TA)
SIGMA HT= 0.2 *HT
CORR= 0.5
0
1
3
5
7
9
11
13
X

FIGURE 6.5.7

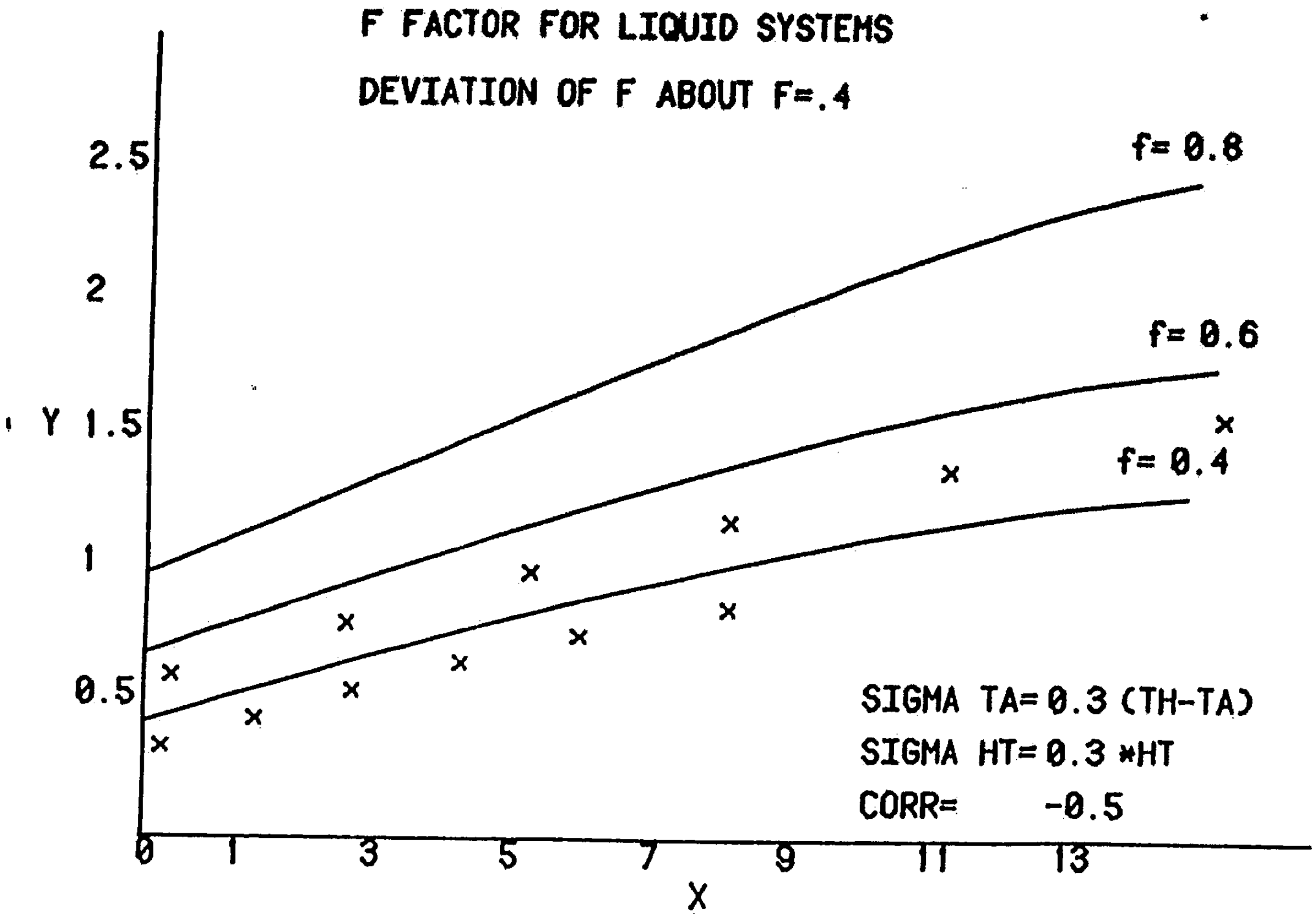

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT F=.4
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.3 (TH-TA)
SIGMA HT= 0.3 *HT
CORR= -0.5

FIGURE 6.5.8

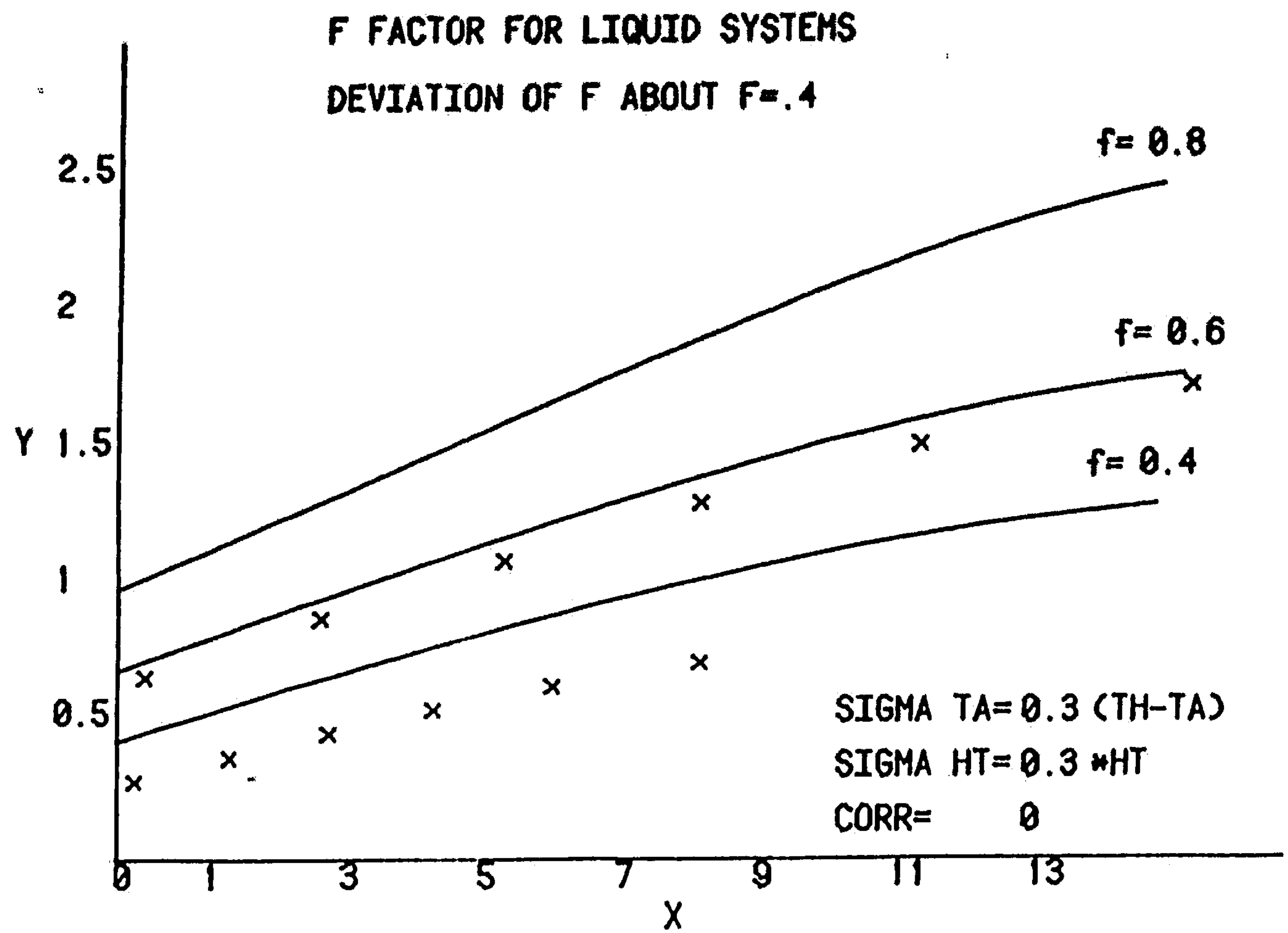

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT F=.4
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.3 (TH-TA)
SIGMA HT= 0.3 *HT
CORR= 0

FIGURE 6.5.9

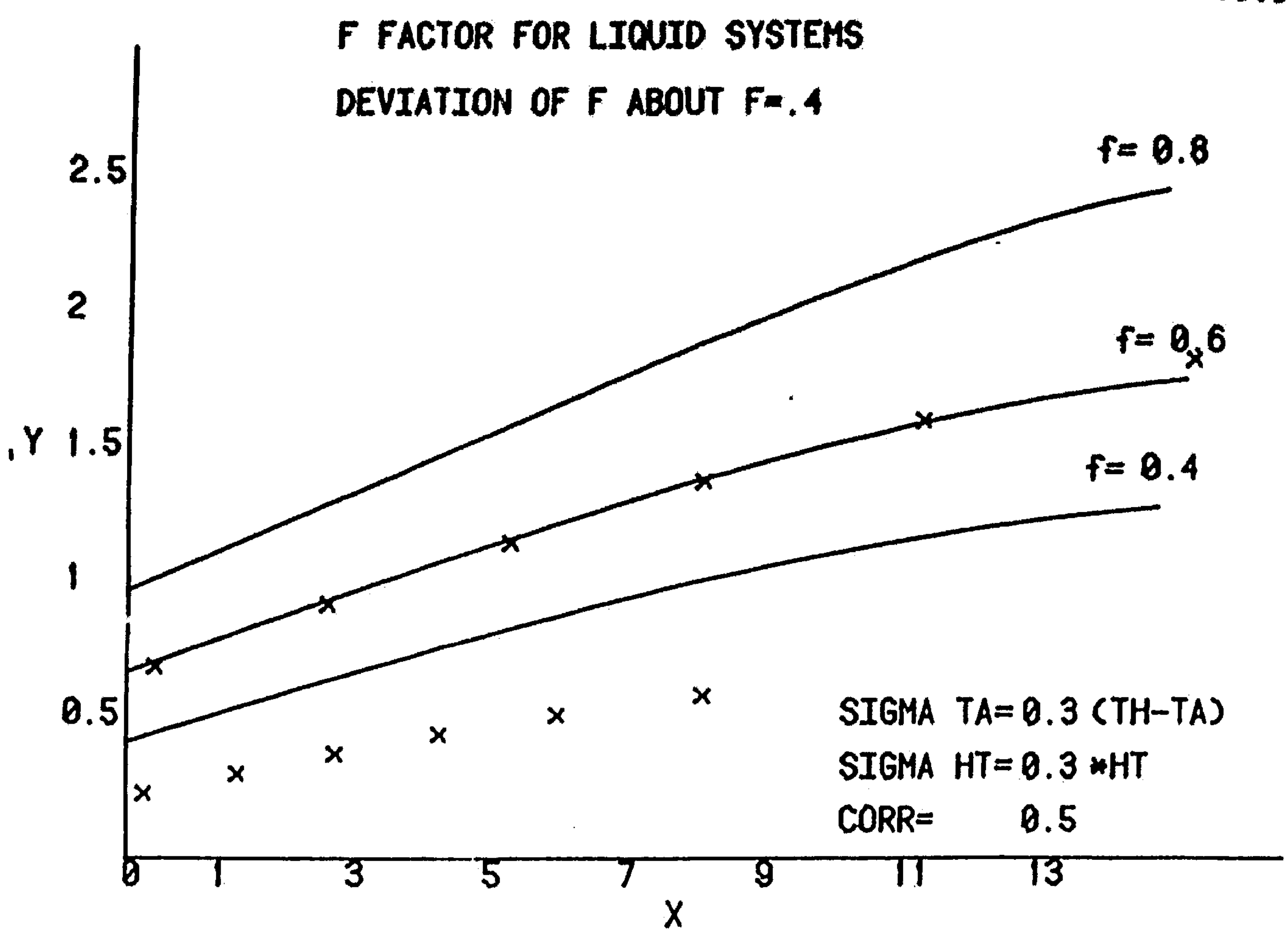

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT F=.4
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA=0.3 (TH-TA)
SIGMA HT=0.3 *HT
CORR= 0.5

FIGURE 8.5.12

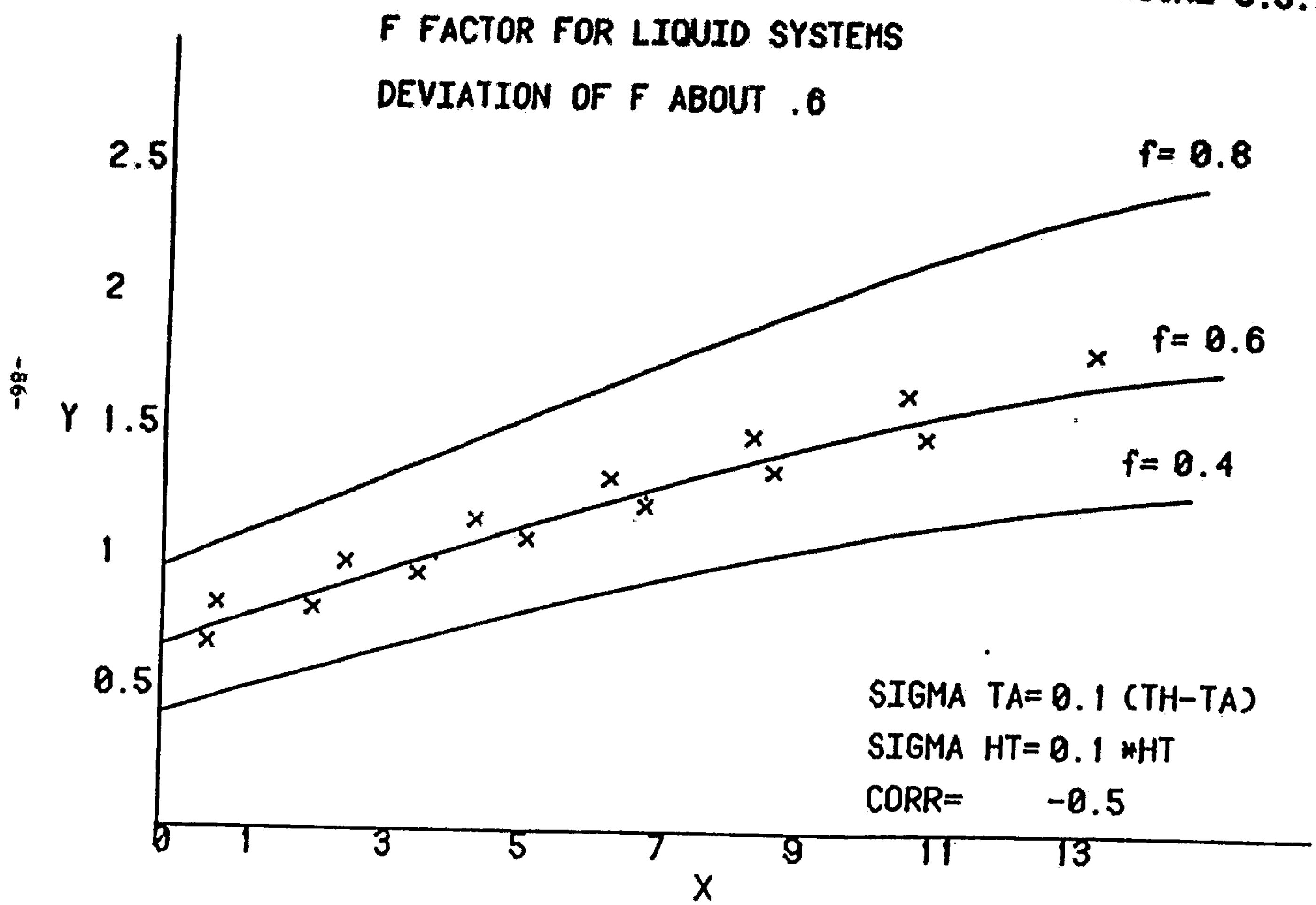

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .6
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.1 (TH-TA)
SIGMA HT= 0.1 *HT
CORR= -0.5

FIGURE 8.5.11

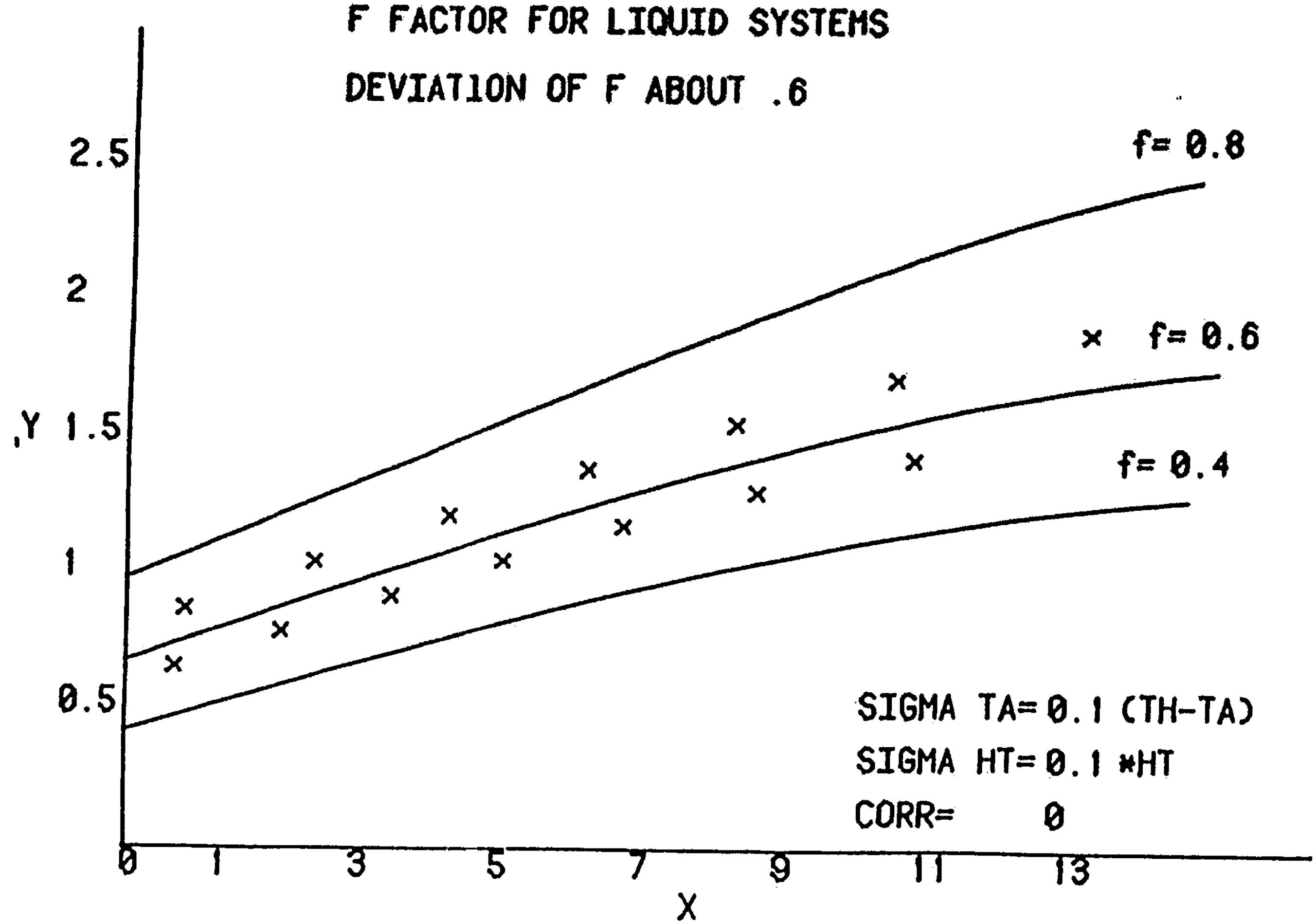

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .6
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.1 (TH-TA)
SIGMA HT= 0.1 *HT
CORR= 0

FIGURE 6.5.12

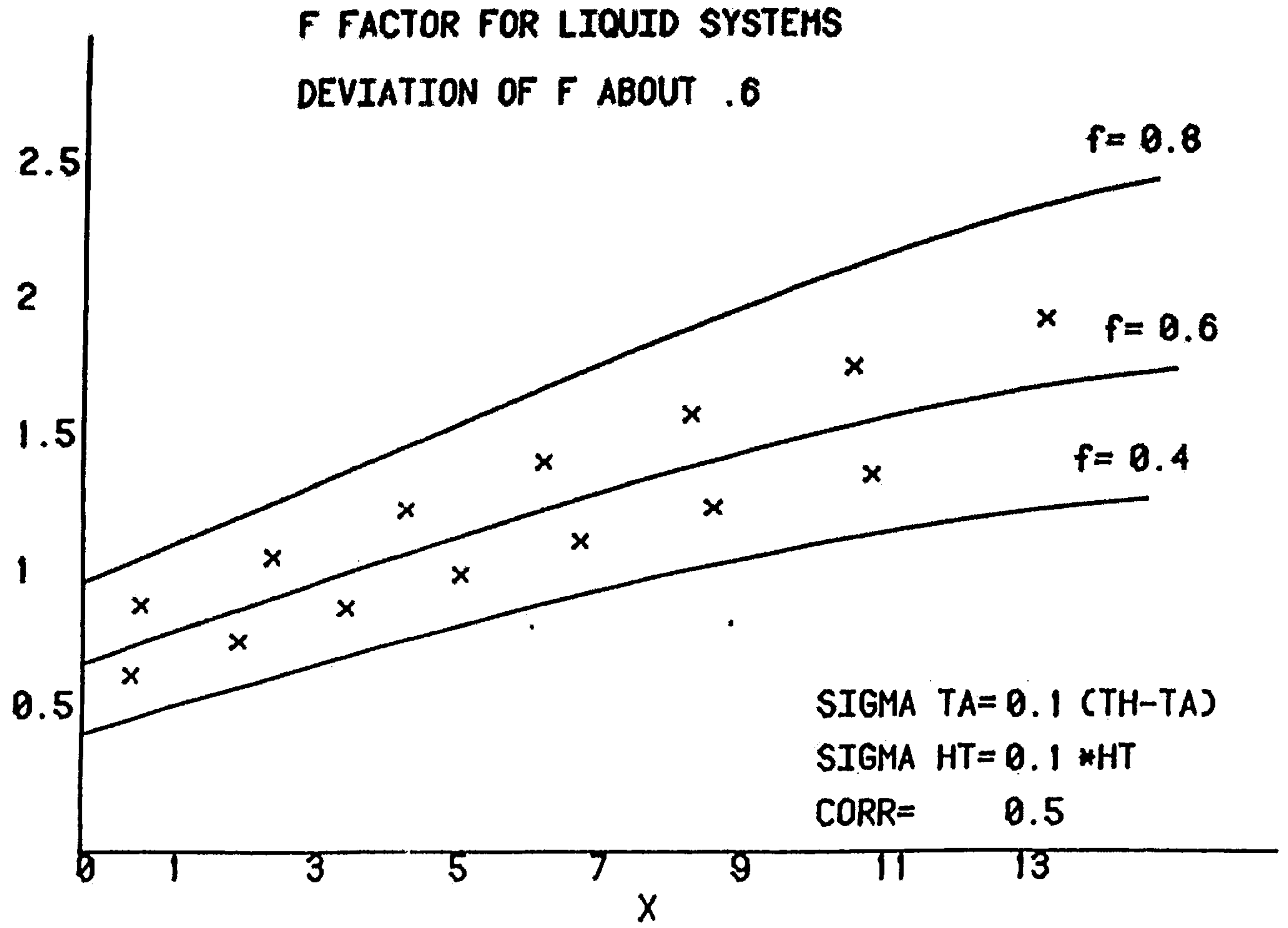
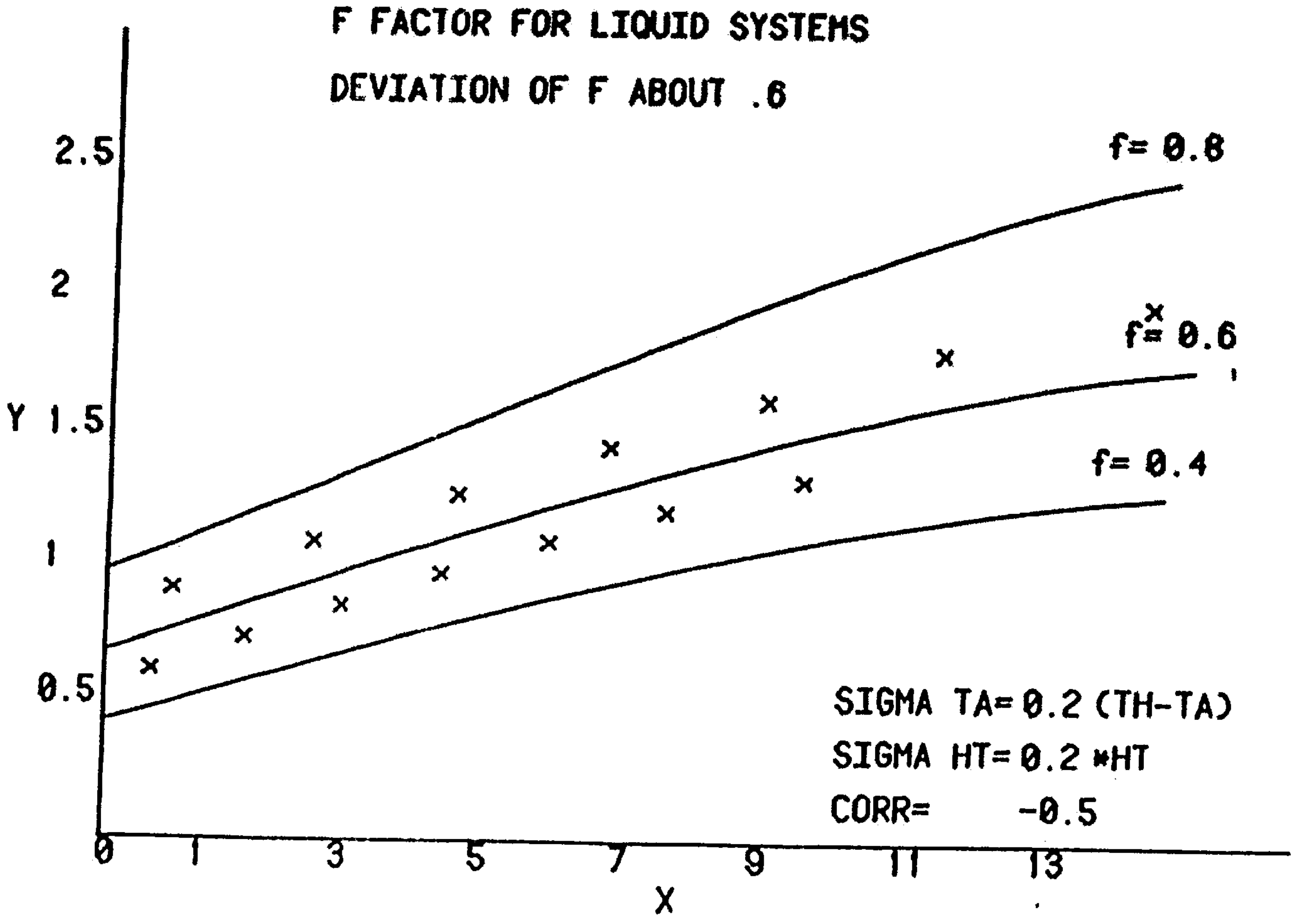

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .6
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
X
0
1
3
5
7
9
11
13
SIGMA TA= 0.1 (TH-TA)
SIGMA HT= 0.1 *HT
CORR= 0.5

FIGURE 6.5.13



F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .6
f= 0.8
f= 0.6
f= 0.4
Y
0.5
1
1.5
2
2.5
X
0
1
3
5
7
9
11
13
SIGMA TA= 0.2 (TH-TA)
SIGMA HT= 0.2 *HT
CORR= -0.5

FIGURE 6.5.14

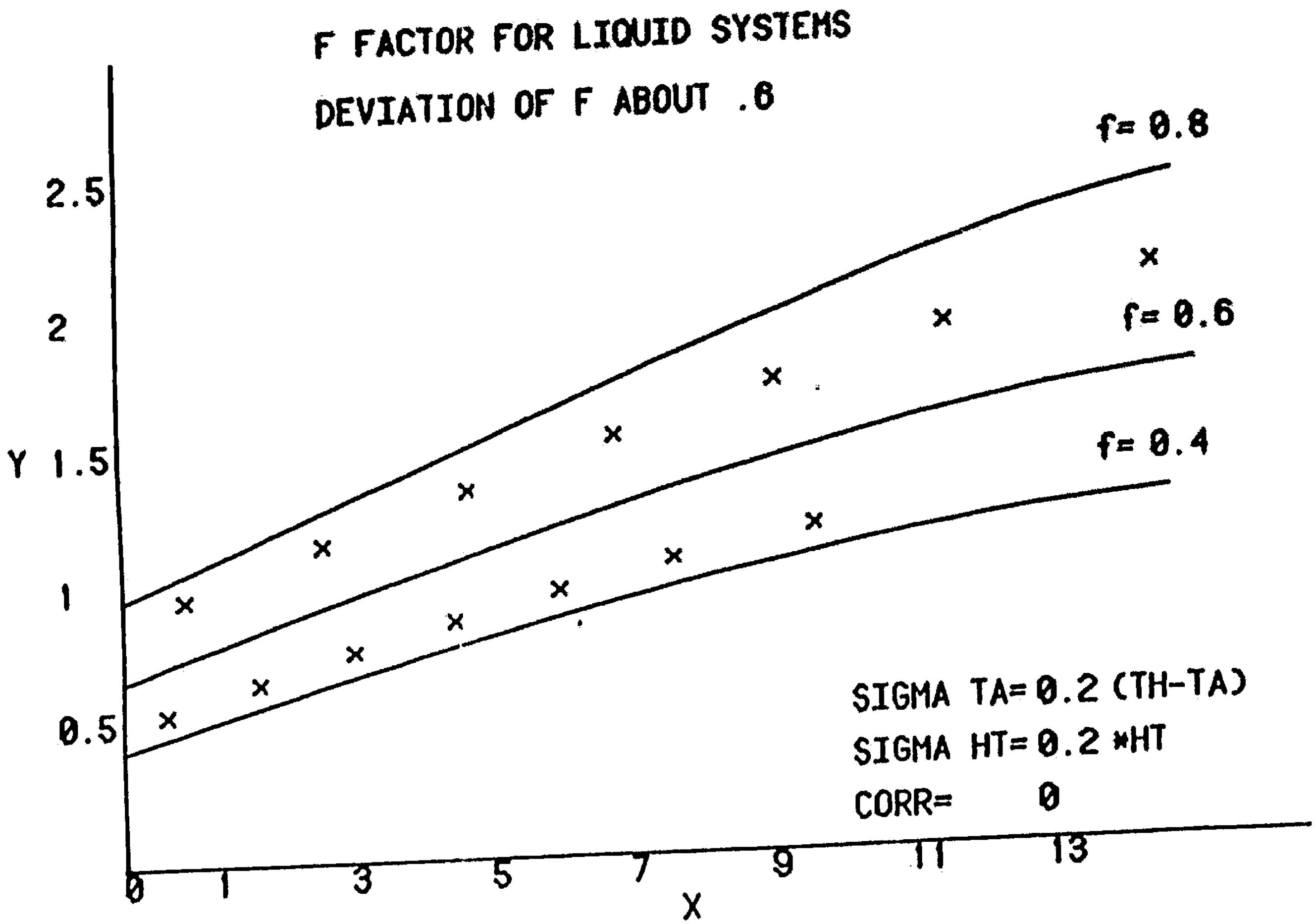

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .6
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
X
0
1
3
5
7
9
11
13
SIGMA TA= 0.2 (TH-TA)
SIGMA HT= 0.2 *HT
CORR= 0

FIGURE 6.5.15

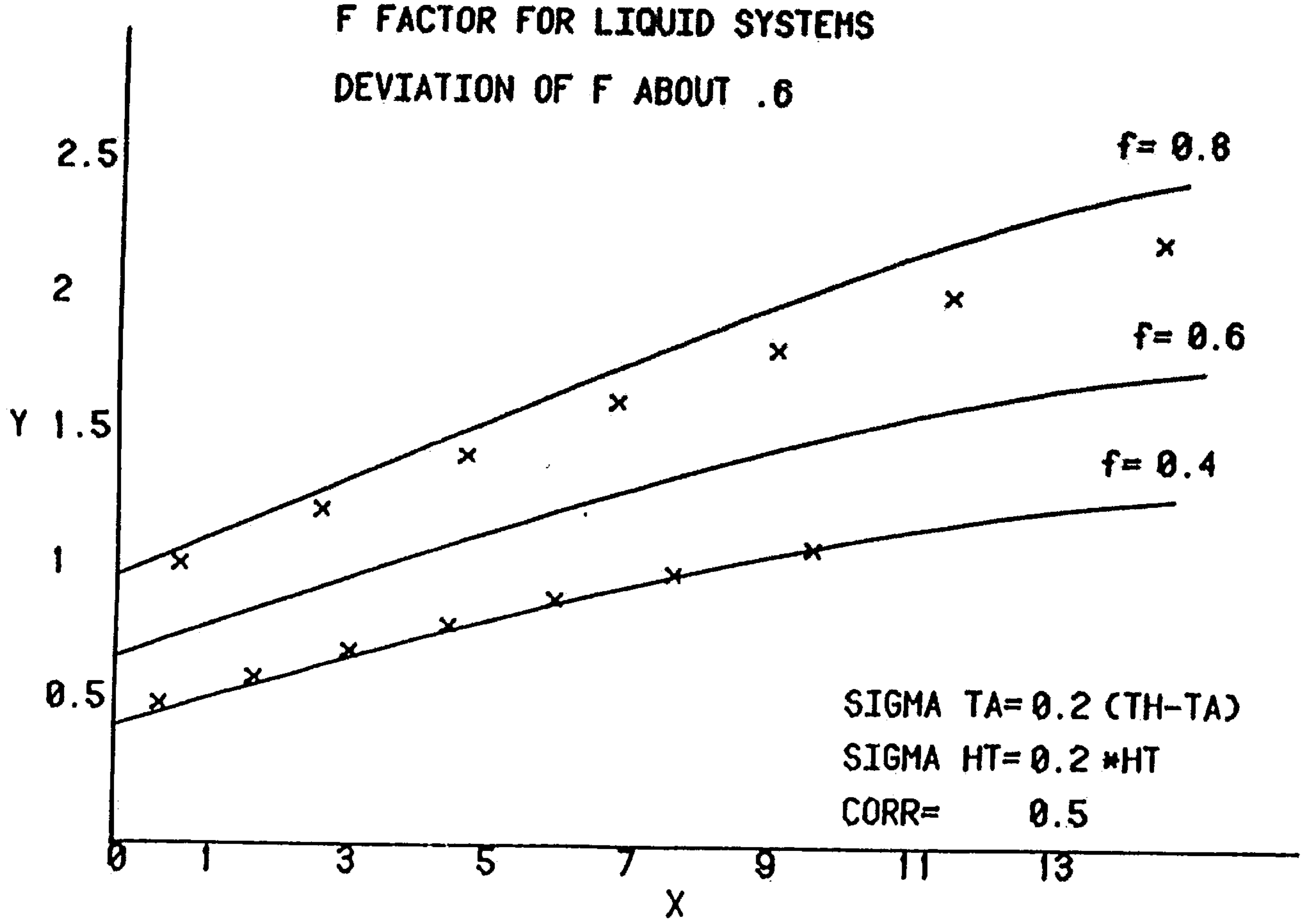

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .6
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
X
0
1
3
5
7
9
11
13
SIGMA TA= 0.2 (TH-TA)
SIGMA HT= 0.2 *HT
CORR= 0.5

FIGURE 6.5.16

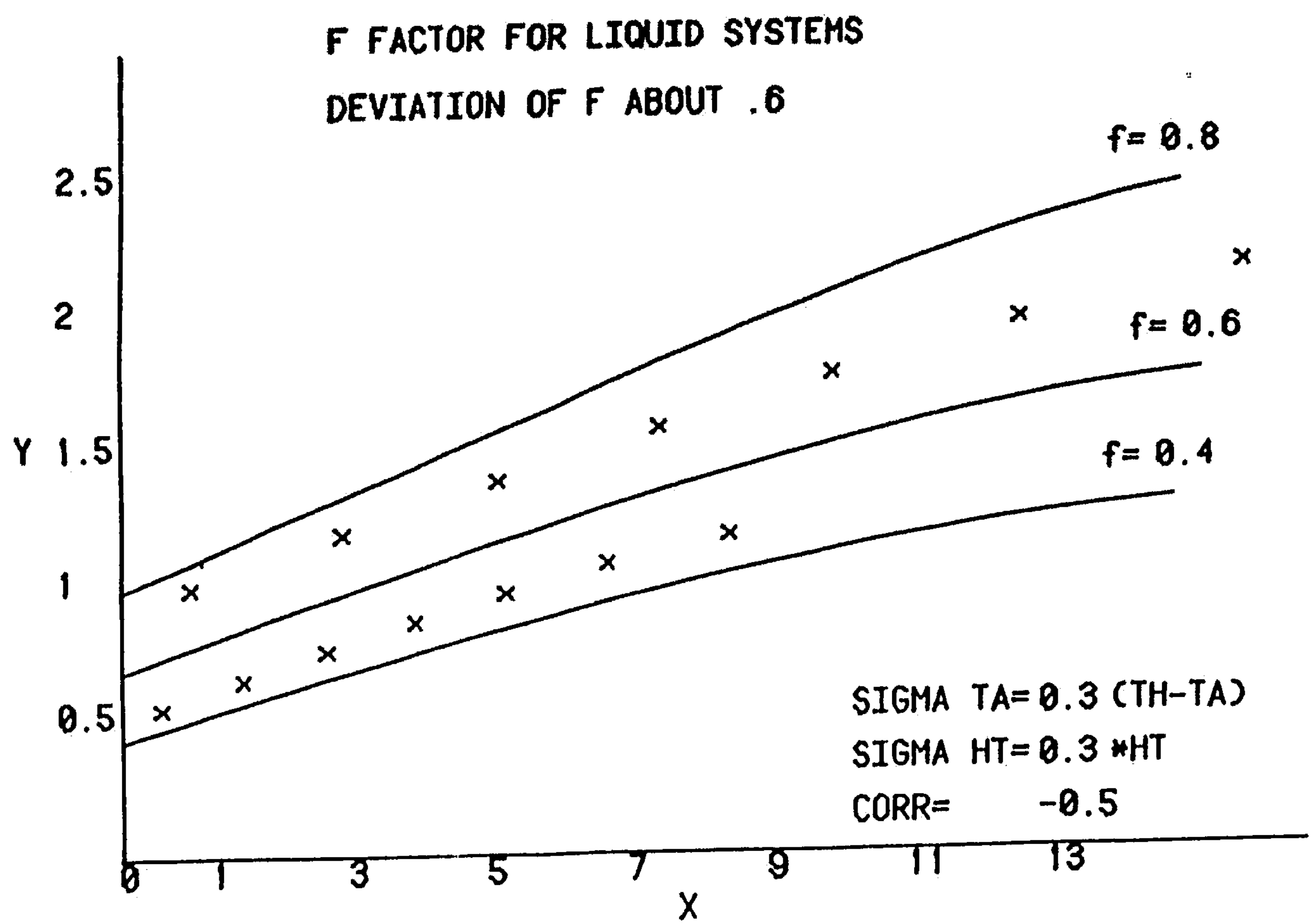

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .6
f= 0.8
f= 0.6
f= 0.4
2.5
2
Y 1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA=0.3 (TH-TA)
SIGMA HT=0.3 *HT
CORR= -0.5

FIGURE 6.5.17

F FACTOR FOR LIQUID SYSTEMS

DEVIATION OF F ABOUT .6

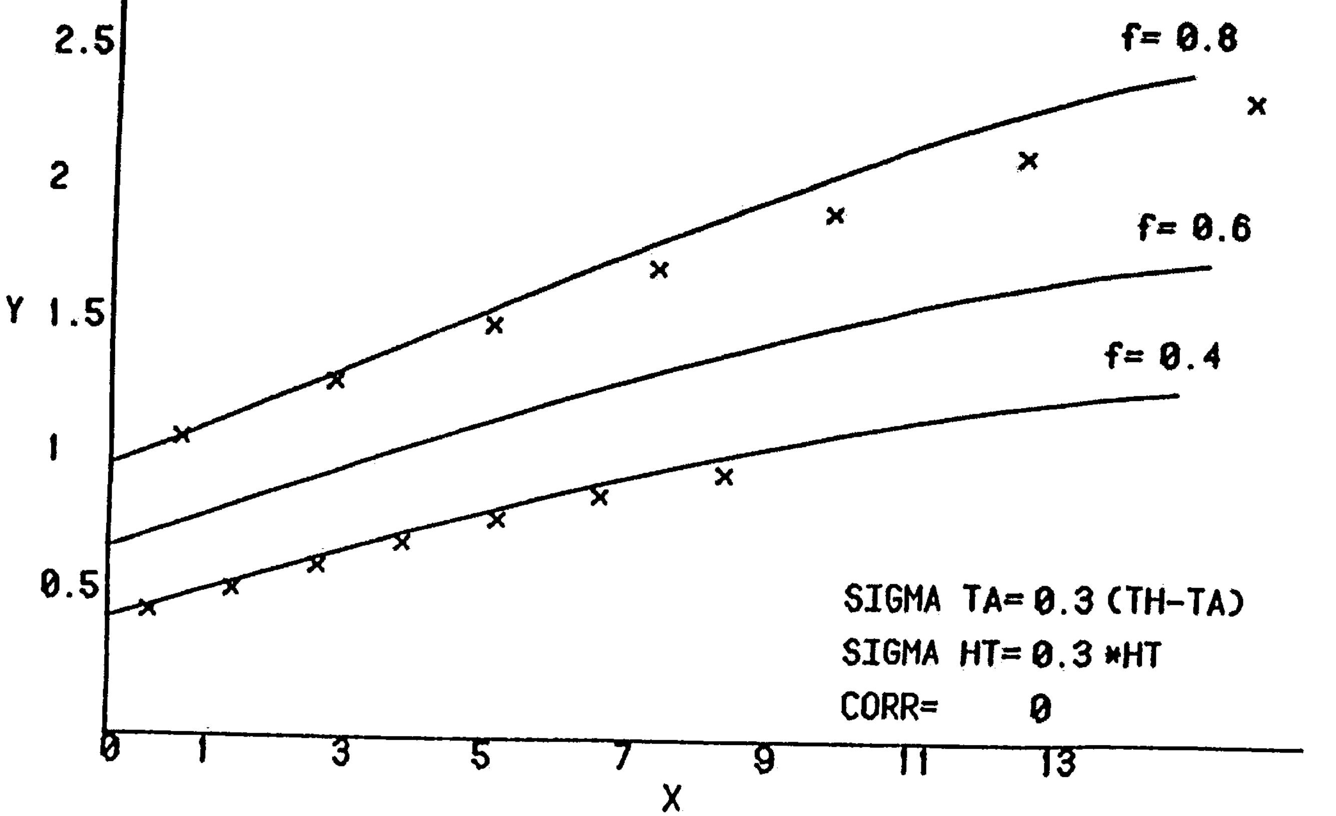
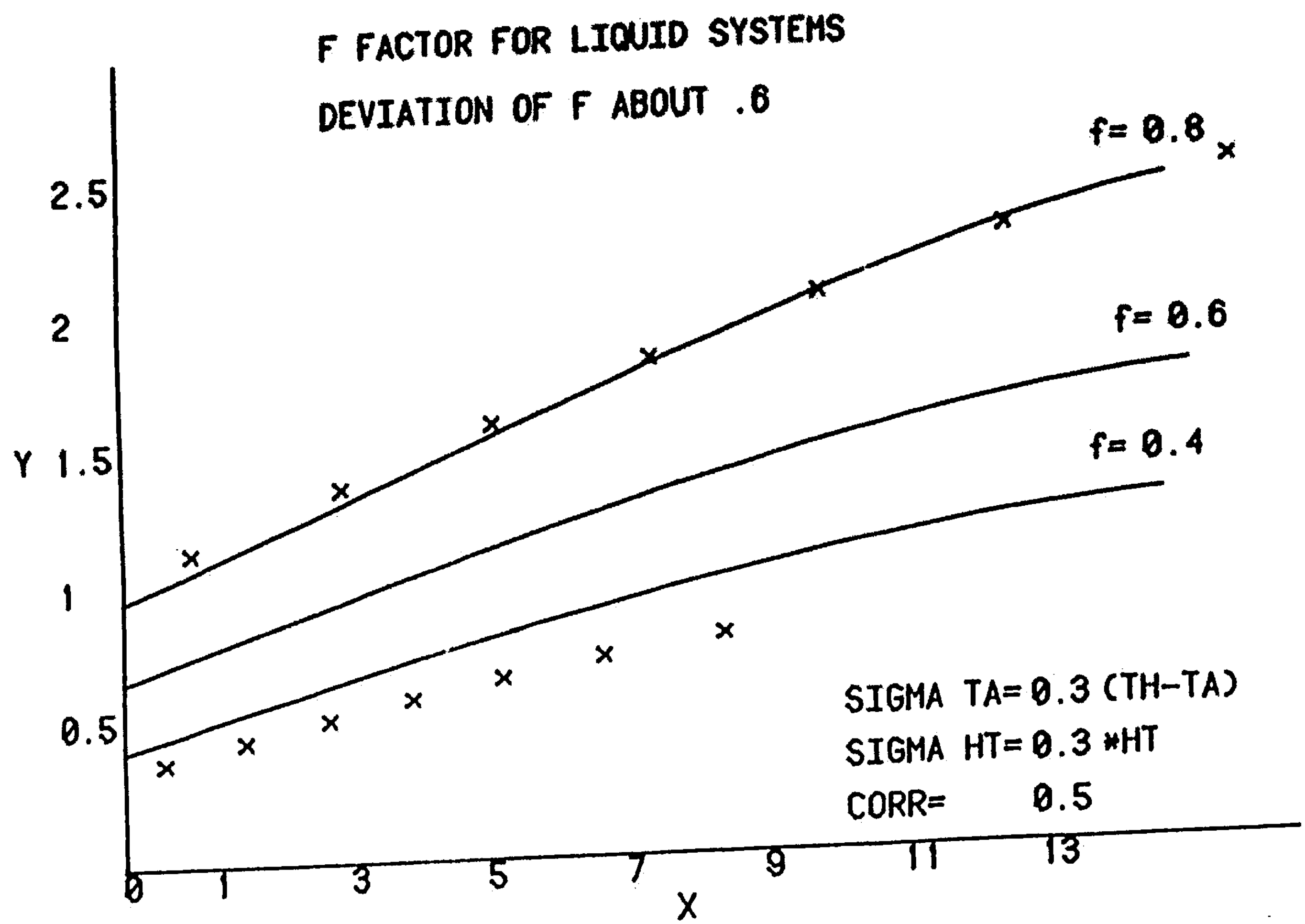
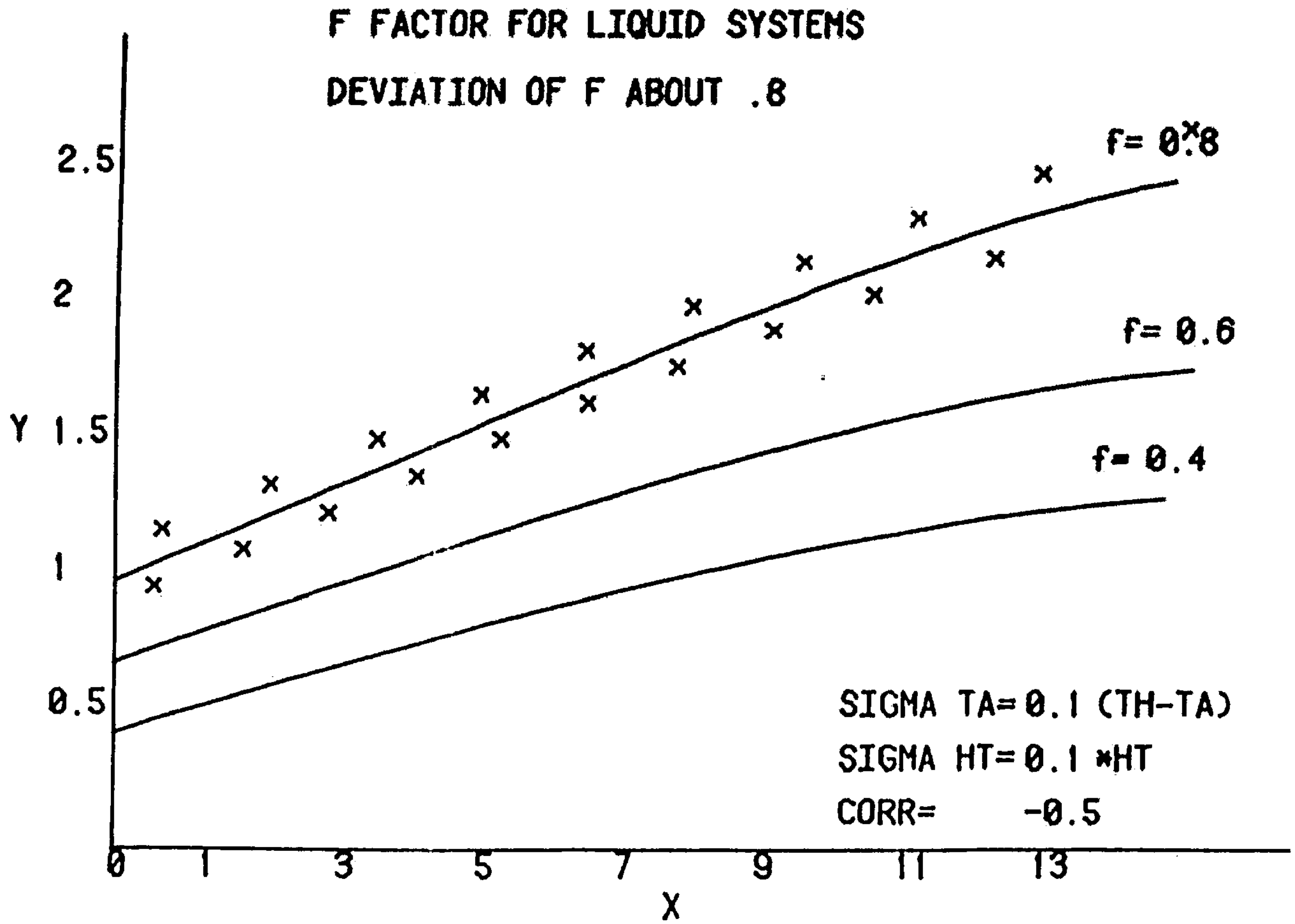


FIGURE 6.5.18



F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .6
f= 0.8
f= 0.6
f= 0.4
2.5
2
Y 1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.3 (TH-TA)
SIGMA HT= 0.3 *HT
CORR= 0.5

FIGURE 6.5.10



F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .8
f= 0.8
f= 0.6
f= 0.4
2.5
2
Y 1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.1 (TH-TA)
SIGMA HT= 0.1 *HT
CORR= -0.5

FIGURE 6.5.20

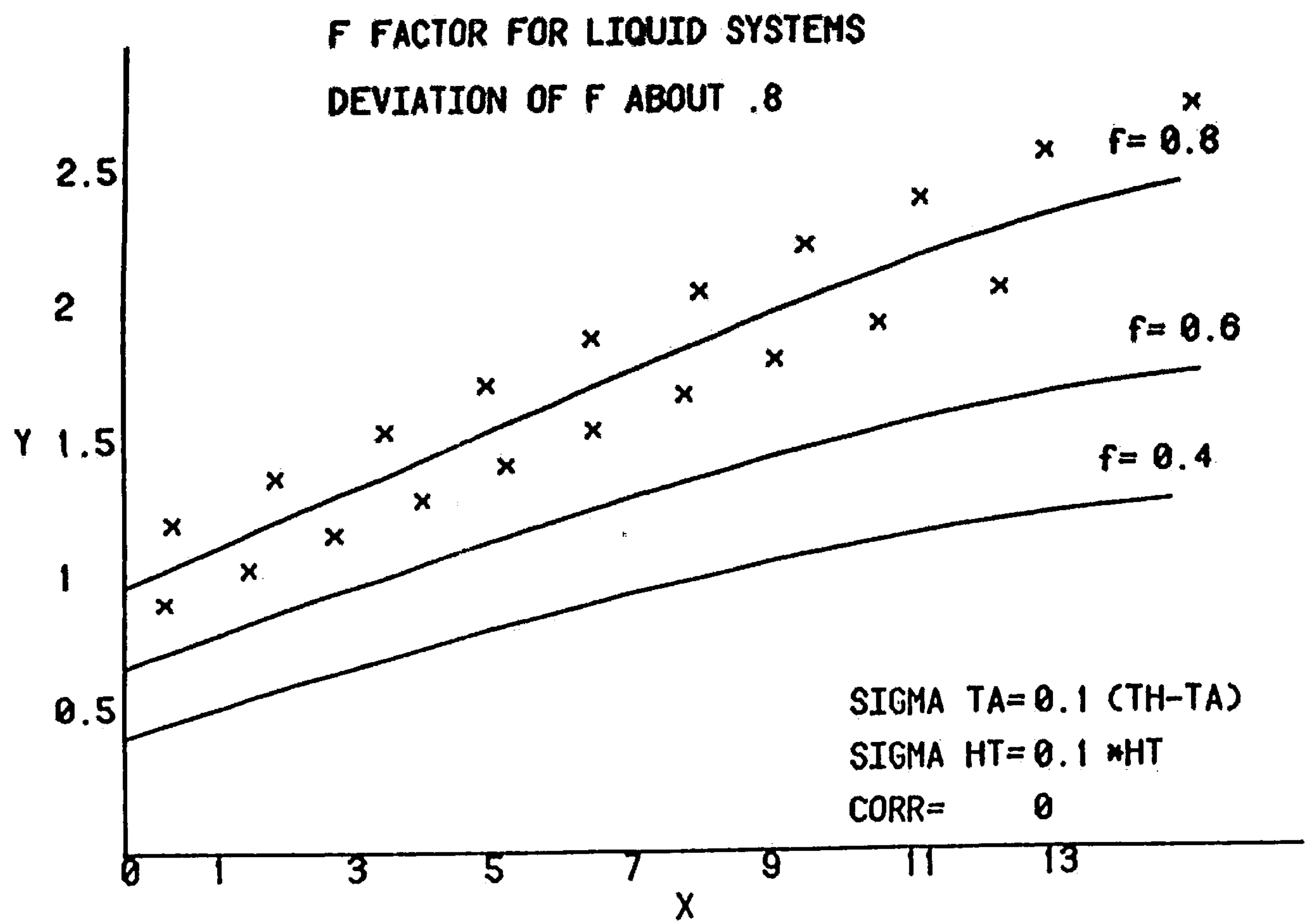

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .8
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA=0.1 (TH-TA)
SIGMA HT=0.1 *HT
CORR= 0

FIGURE 8.5.21

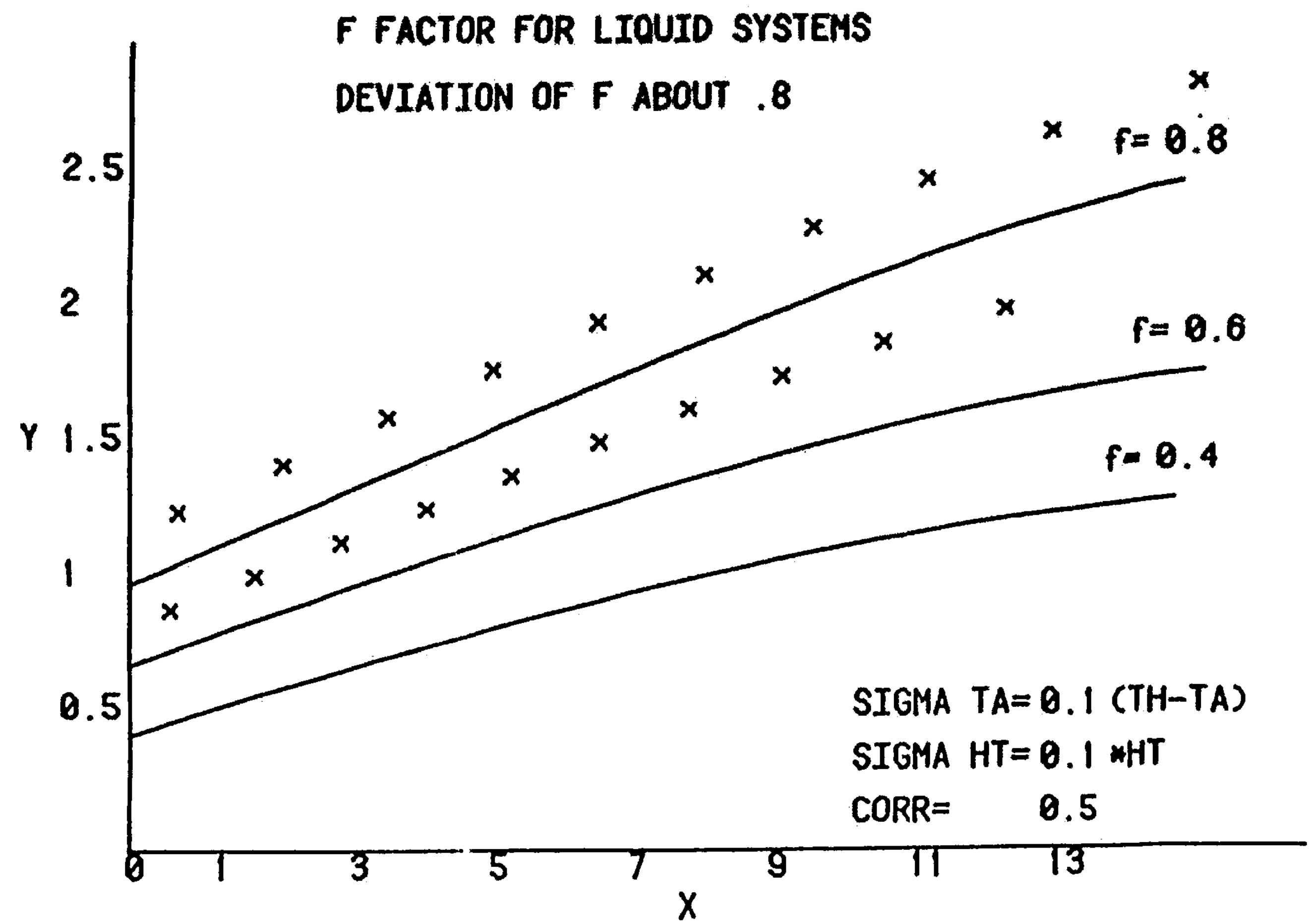

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .8
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.1 (TH-TA)
SIGMA HT= 0.1 *HT
CORR= 0.5

FIGURE 8.5.22

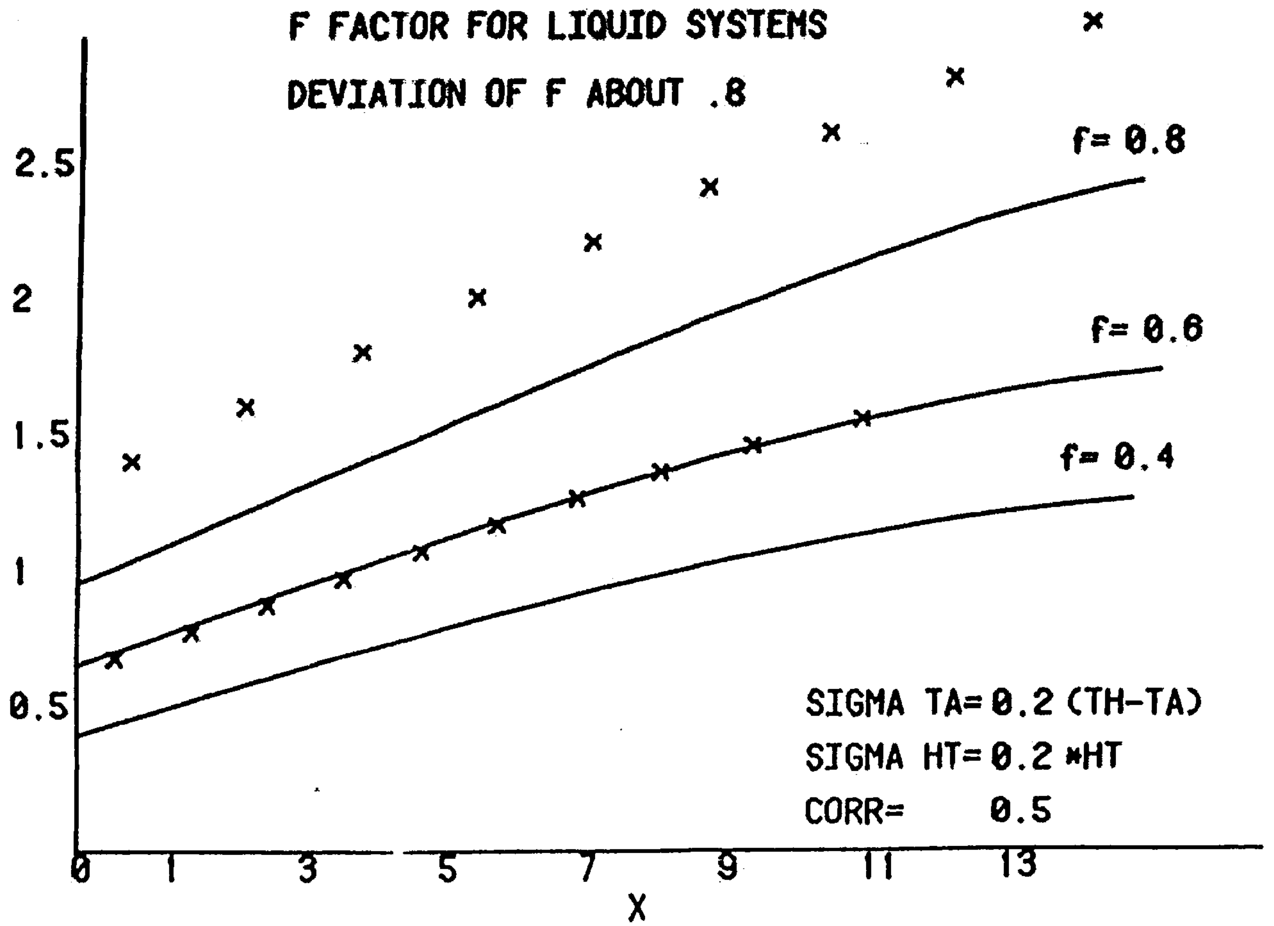
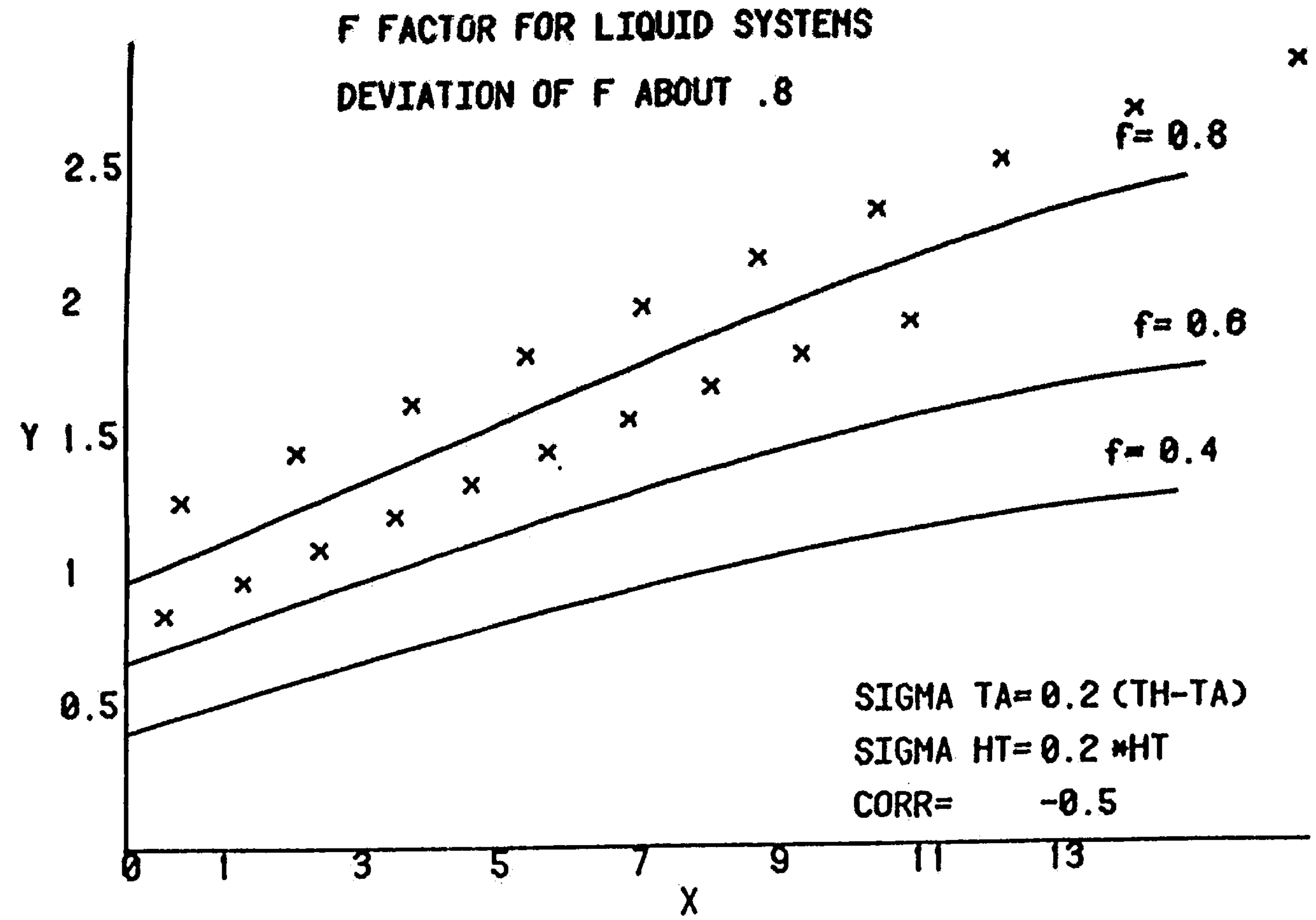

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .8
f= 0.8
f= 0.6
f= 0.4
2.5
2
Y 1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.2 (TH-TA)
SIGMA HT= 0.2 *HT
CORR= 0.5

FIGURE 6.5.23



F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .8
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
X
0
1
3
5
7
9
11
13
SIGMA TA= 0.2 (TH-TA)
SIGMA HT= 0.2 *HT
CORR= -0.5

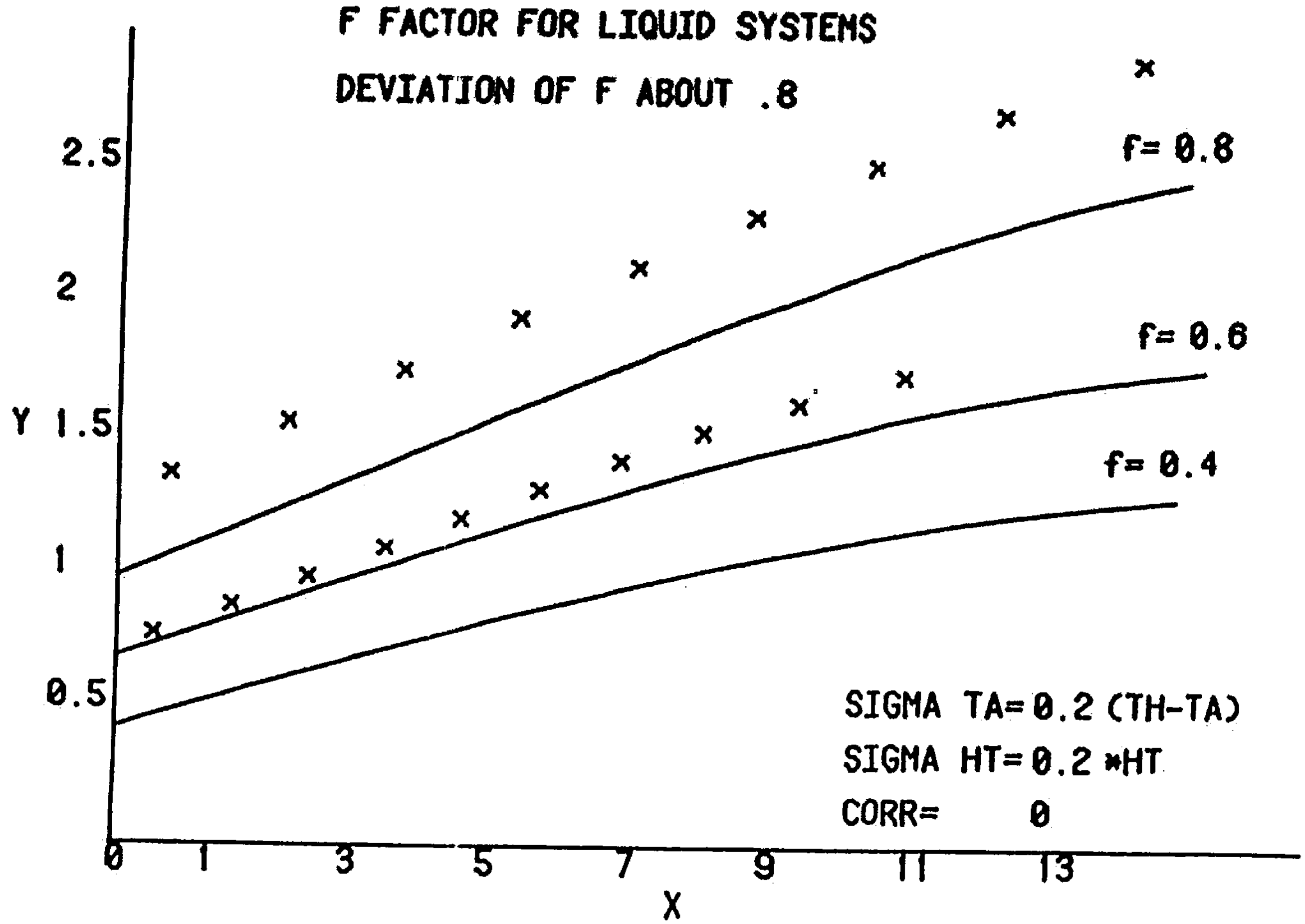

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .8
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.2 (TH-TA)
SIGMA HT= 0.2 *HT
CORR= 0

FIGURE 6.5.24

FIGURE 0.5.25

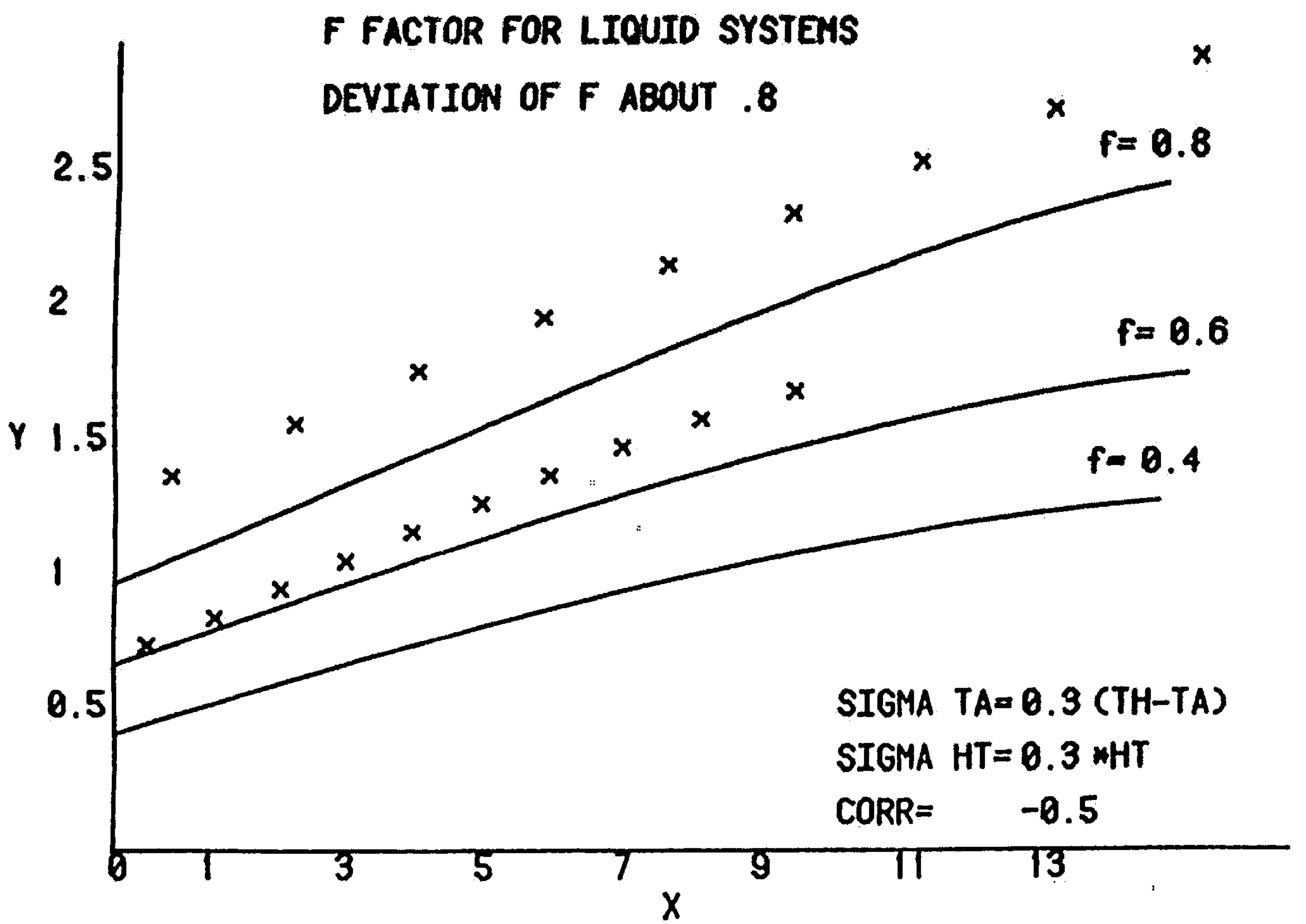
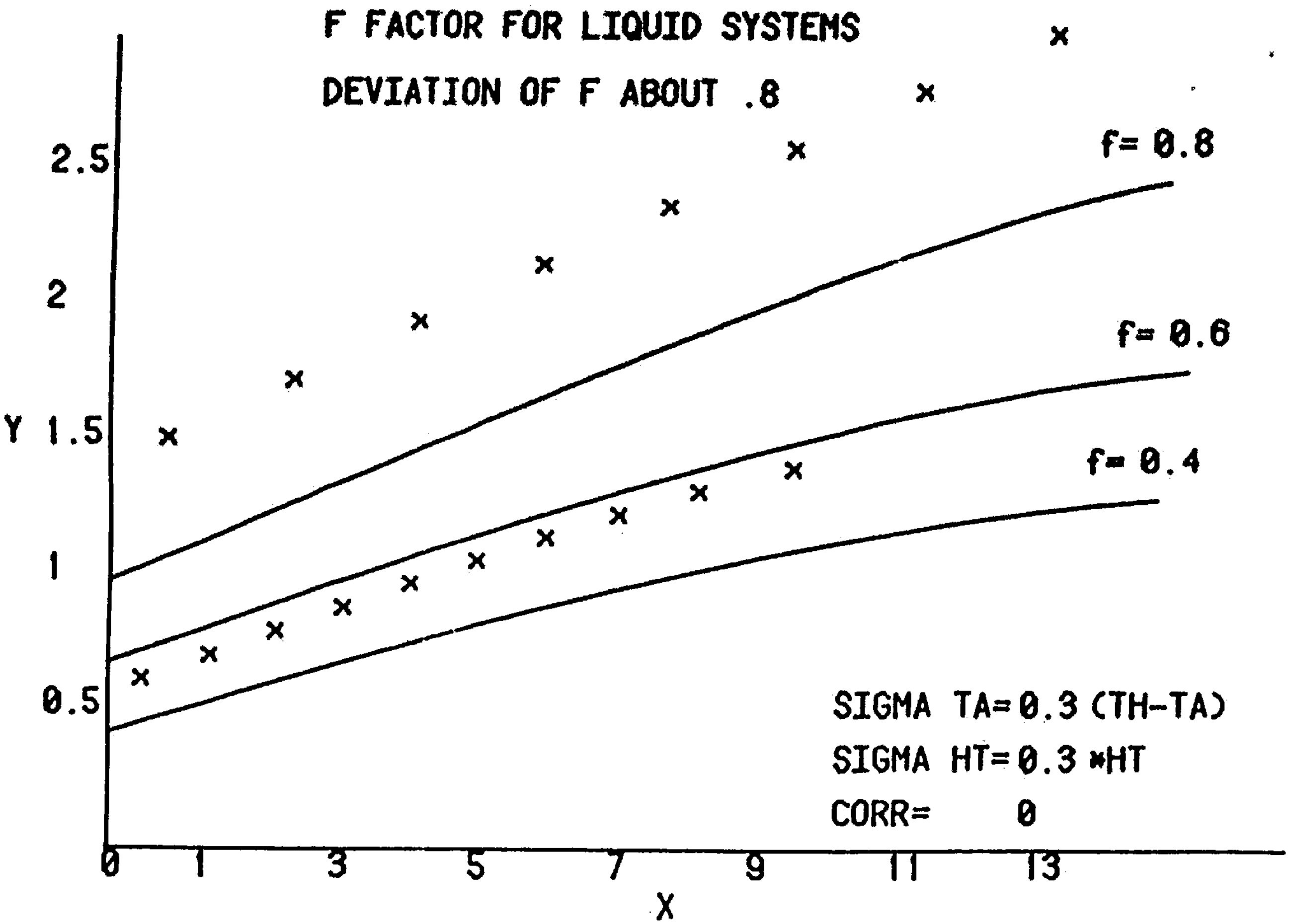
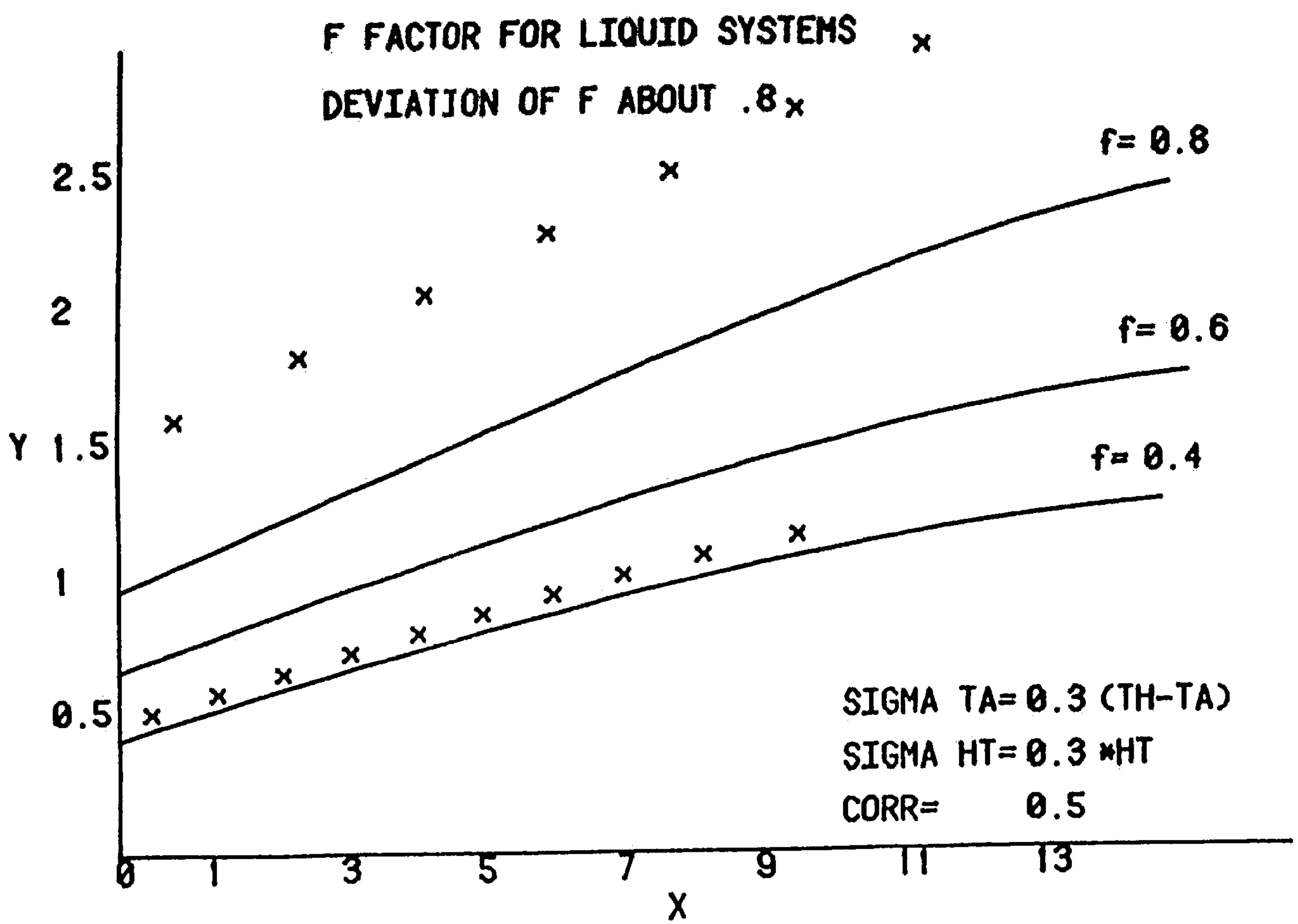

F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .8
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.3 (TH-TA)
SIGMA HT= 0.3 *HT
CORR= -0.5

FIGURE 6.5.26


F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .8
f= 0.8
f= 0.6
f= 0.4
Y
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.3 (TH-TA)
SIGMA HT= 0.3 *HT
CORR= 0


F FACTOR FOR LIQUID SYSTEMS
DEVIATION OF F ABOUT .8
f= 0.8
f= 0.6
f= 0.4
2.5
2
Y 1.5
1
0.5
0
1
3
5
7
9
11
13
X
SIGMA TA= 0.3 (TH-TA)
SIGMA HT= 0.3 *HT
CORR= 0.5

the values of the normalized deviations of insolation and temperature are .1, .2, and.3, respectively.

The effect of the correlation coefficient is significant. This can be seen by reviewing Figures (6.5.1), (6.5.2), and (6.5.3). The negative correlation, which occurs as the particular location on earth is cooling, causes the deviation of 'f' to be less than the positive correlations. This is important since it lessens the deviations of 'f' during the time when residental heating demands are at their highest. The effect of the negative correlation coefficient during the heating season demonstrates the compensating effect of the earth. When heating demands are high, the earth has a buffering effect. On days when levels of insolation are higher, the ambient temperature is lower. While the higher insolation results in more heat available from the solar collector, the lower ambient temperature increases the need for residental heat. When lower levels of insolation occur, the ambient temperature tends to be higher. Less energy is available for heating but the requirements for it are reduced.

Figures (6.5.1) through (6.5.9) show the deviations of 'f' about 'f'=.4, (6.5.10) through (6.5.18) show the deviations of 'f' about 'f'=.6 and (6.5.19) through (6.5.27) show the deviations about 'f'=.8, for various values of the standard deviations of insolation and temperature and the correlation. The deviation of 'f' about 'f'=.4 when compared to the deviation of 'f' about 'f'=.6 and 'f'=.8 show that for the same deviations of normalized temperature and insolation and the same correlation coefficient the deviation of 'f' is about the same.

The graphs are useful for comparative purposes and to graphically illustrate the effect of changes in the standard deviations of insolation and temperature and the correlation coefficients on 'f'. However, the graphs cannot be used to estimate the deviation of 'f' due to the number of variables which exist. The deviation of 'f' must be calculated as described in Section 6.4

7.0 Conclusion

A method has been developed to predict the deviation of 'f' using common statistical techniques. The method relies on the variability of insolation and temperature to determine the variability of 'f' for an active solar collection system using liquid as the heat transfer medium

The variability of 'f' as a function of x and y was examined and it was determined that the standard deviation of 'f' is a stronger function of the standard deviation of y than of x. Although, x influences the standard deviation of 'f', its influence is two orders of magnitude less than the effect of y and can be neglected.

The standard deviation of y is a function of the standard deviation of insolation and temperature and a correlation between insolation and temperature. The effect of these terms varies from month to month with insolation having a greater impact on 'f' during the colder months and the deviation of temperature and the correlation having greater importance during the warmer heating months. All terms are important and none can be neglected.

Cloud cover is responsible for the majority of the variability of insolation and at least partially responsible for the variability of temperature. Clouds can absorb most of the short wave radiation from the sun as well as the long wave radiation emitted by the earth. Due to the capacitence effects of the earth, the effects of changes in cloud cover are less noticable on the ambient temperature than on the insolation and the capacitence effects of the earth tend to reduce the variability of temperature.

The correlation coefficient is an important term in the variability of 'f'. It can be positive or negative. A negative term will reduce the variability of 'f', while a positive term increases the variability. Correlations are negative from October to March and positive for the rest of the year. The sign of the correlation coefficient indicates whether that particular location on earth is heating (positive) or cooling (negative) and depends mainly upon the length of the day and cloud cover.

The correlation coefficient measures the dependency of temperature upon insolation. In the winter, temperature tends to vary inversely with insolation, which results in the negative coefficient. Days with low insolation levels tend to have higher ambient temperatures. As the insolation decreases, the need for heat also decreases. One effect compensates for the other and the variability of 'f' is reduced. During months with positive corelation coefficients, the opposite occurs and the variability is increased.

8.0 Recommendations

- Data collection should be continued so that a representative data base is acquired. The data base used in the course of this work consisted of three years. This is not sufficient to accurately determine the standard deviations of insolation and temperature or the correlation coefficient between insolation and temperature.

- The correlation coefficient should be accurately determined for each location insolation and temperature is collected in order to obtain an accurate standard deviation of 'f'. The results presented in this paper used a correlation coefficient that was obtained from a different data base and used a different sample time period. It was the best available data and the data was only used to show the existence of correlation coefficients.

  After an appropriate data base is established, the standard deviation of 'f' should be compared to outputs of active collector systems to verify the accuracy of the standard deviation.

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VITA

The author was born in Northampton, Pennsylvania, on August 11, 1954. He graduated from Villanova University, Villanova, Pennsylvania, in 1976 with the Degree of Bachelor of Science (Mechanical Engineering). He is a member of Pi Tau Sigma and has been employed by Pennsylvania Power and Light Company since 1976. The work described in this report was performed between January, 1982, and May, 1985, while attending Lehigh University.