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Computerized two-dimensional shape sieving of carbonate sands using the multivariate rotation method /

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COMPUTERIZED TWO-DIMENSIONAL SHAPE SIEVING OF CARBONATE SANDS USING THE MULTIVARIATE ROTATION METHOD

by

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ABSTRACT

Thirty-six equiangular radial lengths, as interpolated by the multivariate rotation method of Parks (1982), are sufficient to describe the shapes of the two-dimensional peripheries (outlines) of grains from thirty-seven carbonate sand samples collected along four traverses extending across the backreef sub-environment, adjacent to the Florida Keys. Parks' multivariate rotation method as used by Collins (1983) is modified in order to enhance the precision of its principal objective: to orient shapes, as represented by unprocessed equiangular radial lengths, to standard and comparable positions. This modification is accomplished by using a triangle with a three-to-two height-to-base ratio as a standard reference shape for outline rotation and by a test which compares the northern half of the outline of each grain to the southern half of the outline as a criterion for shape flipping.

Q-mode cluster analysis of the processed radial lengths or factor scores groups (clusters) similar shapes together so that average shapes may be computed from the clustered shapes for each sample. Q-mode cluster analysis of an array containing the average shapes from all samples results in a dendrogram that describes which average shapes may be compared between samples. Eight final average shapes are calculated from this dendrogram.

Percentages of sample grains, belonging to each of the eight final average shapes, are tabulated for each sample. This

computerized two-dimensional shape sieving procedure is used to determine if general shape-type trends occur along the traverses extending accross the backreef. No consistent trends were observed, due to the many environment types within the backreef sub-environment, the many possible shape-types resulting from skeletal breakdown, and the effects on hydrodynamic sorting caused by bulk density variations between sand grains.

INTRODUCTION

PURPOSE

Sands from various sub-environments of carbonate environments may be distinguished from each other on the basis of their constituent compositions. Since the constituents are of biogenic origin, the shapes of the particles depend, to some extent, on the constituent types of which they are composed. There are relatively few major sand-sized particle types involved in carbonate beach and subtidal environments. They include Halimeda, coral, coralline algae, mollusks, and foraminifera.

Comparison of subtidal carbonate sands with beach sands should show fundamental differences in the two (Folk and Robles, 1964), such as a greater percentage of finer material along with poorer sorting of the subtidal sediments. In this study the fundamental difference of interest between these two sediment types is shape. Numerous quantitative shape studies have been performed previously, but this is the first to be applied to biogenically-derived sands.

The most widely used limestone classification schemes are those by Folk (1962) and Dunham (1961). Both schemes include grain shape as an important factor. However, there is little quantitative information as to whether an ancient calcarenite was formed on a beach or within a subtidal environment (Folk and Robles, 1964). A quantitative study is needed to determine if these environments are distinguishable on the basis of shape. Size distributions, constituent compositions, and attrition are other important factors that are considered in this study.

Sedimentological analysis of sediments in carbonate environments has mainly been accomplished by the time-consuming method of point counting under the microscope. However, recent developments in computer hardware and software has made image processing both practical and affordable. This study is an initial attempt to quantify shape properties of carbonate sands and obtain information concerning shape variation within the backreef carbonate sub-environment.

A reasonable shape analysis study of sand-sized particles would result in a descriptive set of shape percentages that could be compared between samples. The descriptive set of shapes would be obtained by computerized shape sieving, a procedure to tabulate shape percentages within samples. The ideal consequence of that study for carbonate sands would be an association between each particular shape and a specific biogenic constituent. This ideal consequence may not occur because of the large percentage of abraded carbonate sand particles within the reef tract environment. This study applies a variation of Parks' (1981; 1982; 1983a; 1983b) method to describe the shapes of carbonate sands.

The working hypotheses of this investigation were: (1) The shape of a particular carbonate particle depends to some extent on the constituent of which it is composed. (2) Beach sediments are derived from the bottom sediments of the backreef and modified by shape-sorting and abrasion/breakage during transport from offshore onto the beach.

With the above hypotheses in mind, the objectives of this study were: (1) To modify Parks' (1982) multivariate rotation method as used by Collins (1983) in order to develop a procedure to sieve shapes by computer; (2) To determine the extent to which the shapes of carbonate particles depend on the constituent types of which they are composed; (3) To determine if there is a shape factor by which modern carbonate beach deposits can be distinquished from adjacent non-beach carbonate sands or if there are general trends in the percentages of various shapes along traverses extending across the backreef.

STUDY AREA

The problem was studied within the backreef sub-environment (Ginsburg, 1956) of the Holocene south Florida carbonate shelf (Fig. 1). This is a zone approximately 6.5 kilometers wide, extending from the landward edge of the outer living reef to the Florida Keys. It is the only area in the continental United States where dominantly carbonate deposition is taking place. The geographic name for the area between the reefs and Keys is termed "Hawk Channel". It typically slopes very gradually from the Keys to a depth of about five meters (15-20 ft), where it deepens into depressions eight to fifteen meters (25-50 ft) deep.

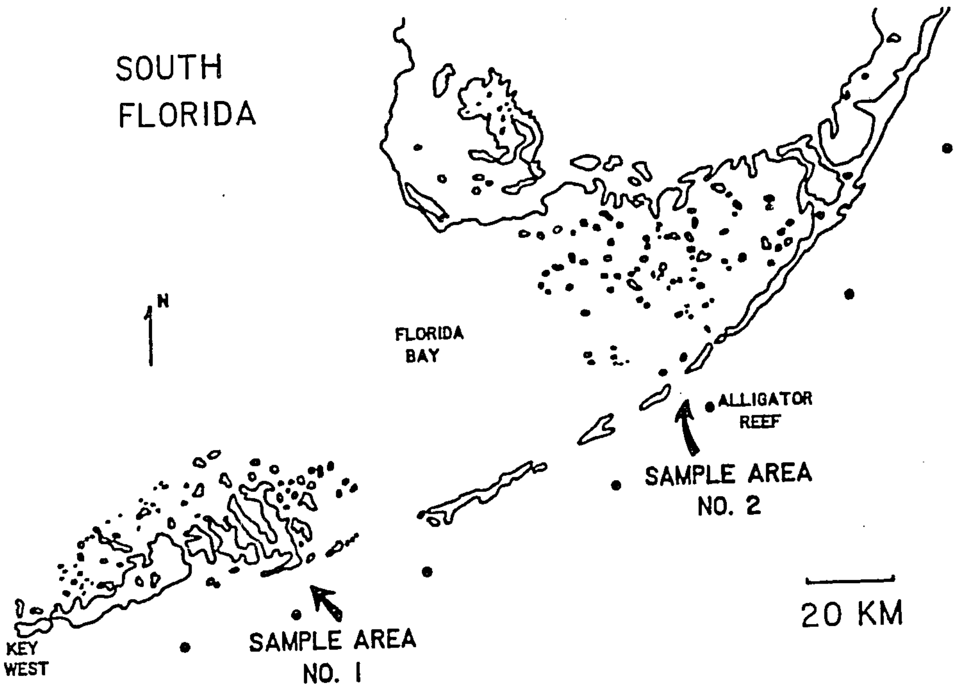

Figure 1. The Holocene south Florida carbonate shelf with the southern sample area (no. 1) and the northern sample area (no. 2) (After Enos and Perkins, 1977).

Maximum wave energy occurs at the shelf break and decreases rapidly towards the inner shelf margin due to the dissipation of energy at the outer living reefs. Vaughn (1935) found that currents within the backreef area generally range from 5-15 cm/sec, but are occasionally or locally as strong as 34 cm/sec. Semidiurnal tidal exchange causes these currents to reverse, however southeast trade winds in the summer and northeasterly storm winds in the winter result in a net southwestward current flow (Enos and Perkins, 1977).

Ripple marks, common in the subtidal areas, are evidence of a moving substrate. Thorp (1935) suggested that waves and currents are of paramount importance in sorting the sediments exposed to their action and that the Gulf Stream, although indirectly important to sedimentation, is less effective on the Florida coast than the southwestward-flowing countercurrent. Tides and winds tend to carry material from the reefs to the areas behind the Keys, while countercurrent moves material southward and westward (Thorp, 1935).

Organisms of the south Florida reef tract are responsible for the production of almost all of the sediment present. There is a lack of sand-sized non-skeletal components such as lithoclasts, pelletoids, ooids, aggregates, and cryptocrystalline lumps in the area. Enos and Perkins (1977) state that substrate is a primary control on the organism habitat communities; organisms, in turn, produce virtually all the substrate sediment.

The major constituents typically reported from the Florida reef tract are calacareous green algae (Halimeda and dasyclads), mollusks (bivalves and gastropods), corals, coralline algae, and foraminifera. Since carbonate sand is composed of relatively few constituent types, the shape of a particular sand grain is dependent to some extent on the constituent type of which it is composed.

Four species of Halimeda are found in the backreef area of the Keys. They are H. incrassata, H. optuntia, H. monile, and H. tuna. Halimeda spp. belong to the family, Codiaceae, and appear to be sufficiently similar in shape to be grouped together under the genus, Halimeda, for purposes of simplicity in this study. The family, Dasycladacea (dasyclads), is similar to Halimeda in appearance, but more elongate.

There are two main groups of branching (coralline) red algae within the study area. They include Goniolithon and some Lithothamnium. Again, these branching constituents have shapes similar enough to be grouped under one constituent type, coralline algae.

The remains of mollusks, chiefly bivalves and gastropods, are grouped under either bivalves or gastropods because of the easily recognizable differences in appearance of these two groups. Codakia obicularis, Codakia orbiculata, Chione cancellata, and Laevicardium spp. are very common pelecypod species in the area (Enos and Perkins, 1977). Common species of gastropods include Modulus modulus, Astrea americana, A. longispina and various cerithids.

All species of foraminifera were analyzed as a group in this study due to the obvious shape similarities between them. Two families, Miliolidae and Peneroplidae, make up the majority of foraminifera in the backreef sub-environment. All species belonging to these families are mobile benthonic forms which live on the grass and in or on the bottom sediment (Enos and Perkins, 1977).

The most important group of coelenterates in the reef environment is, of course, the corals. Millepora, the stinging coral, is the only hydrozoan that secretes a calcareous skeleton. Since there are many species of corals, all species were classified under one group for purposes of simpliciy.

Enos and Perkins (1977) listed four primary controls on sediment distribution patterns, in apparent order of importance. They included skeletal productivity, mechanical redistribution, pre-existing rock topography, and contemporary sediment topography. They stated that the relative importance of these factors may be qualitatively evaluated for the various types of accumulations.

Beaches and subtidal areas may vary considerably. Both may range from muddy to sandy to rocky, with a variety of current intensities, depths, and benthic life form densities. The main relief features are patch reefs, small sediment banks, and tidal deltas (Enos and Perkins, 1977). The thickest sediment accumulations, up to seven meters (23 ft), are associated with these features. Ebanks and Bubb (1975) studied a very large (5 by 8 km) complex tidal delta between Upper and Lower Matecumbe Keys.

Marine grass beds do not supply much sediment to adjacent bare sand environments, but do produce considerable sediment in place

(Clack, 1976). Thallassia testudinum and Syringodium filiforme protect the bottom from currents, indirectly influencing sedimentation by allowing finer grain sizes to settle and providing shelter and grazing areas for various organisms, especially mollusks.

PREVIOUS WORK

CARBONATE ENVIRONMENTS

Much of the previous work involving the Florida reef tract has been discussed in the previous section in order to describe the study area. Enos and Perkins (1977) listed five habitual communities and their sedimentologically important inhabitants on the south Florida shelf:

(1) Rock or dead reef.
- mainly encrusting and boring organisms.

(2) Mud.
(a) bare - a few green algae, foraminifera.
(b) grass (Thallasia) covered - green algae, miliolid foraminifera, browsing gastropods, and burrowing pelecypods.

(3) Sand.
(a) bare - burrowing echinoids.
(b) grass covered - Halimeda, peneroplid foraminifera, browsing gastropods and burrowing pelecypods.

(4) Patch reef.
- head corals.

(5) Outer reef.
- corals (Acopora, Monastrea, Diploria, Porites), Millepora, Halimeda opuntia.

They estimated the relative skeletal productivity by habitat community as being; 5 > 4 >> 2b >> 3b > 1 > 2a > 3a.

The most well-known study of the Florida reef tract is that of Ginsburg (1956). He subdivided the reef tract into three sub-environments: backreef, outer reef arc, and the forereef. The backreef is the area between the landward side of the outer reef arc and the Keys. It is the study area of this investigation. Ginsburg's sub-environments correspond with changes in the relative abundances of constituents and to a lesser degree with variations in the percentages of fines (less than 1/8 mm). However, in Ginsburg's study these trends are not observed within the sub-environments, notably, the backreef sub-environment. Weight percentages of particles less than 1/8 mm in diameter average 17 percent in the reef tract, but range from zero to 68 percent. Two of Ginsburg's conclusions are relevant to this study: (1) Major differences in environment, as between the reef tract and Florida Bay, produce sediments with quite different grain size and constituent composition, but where the changes in the environment are small and gradual, as within the reef tract, the sediments are less distinctive, and their differentiation requires recognition of gradual changes in relative abundances of the major constituents. (2) Local variations within a major environment may be so large that their effect on grain size and constituent composition of the sediments obscure small and subtle variations produced by gradual changes across a large area. Enos and Perkins (1977) noted that the inner shelf margin (backreef) sediments are finer, more poorly sorted, and more variable than the sediments of the outer shelf margin (forereef).

SHAPE

The lack of carbonate grain shape studies may be due to the inadequacy of current quartz particle shape analysis techniques to describe carbonate sand shapes. Maiklem (1968), in what was perhaps the most in-depth analysis of carbonate grain shapes, demonstrated experimentally that shape and density of bioclastic carbonate grains are important factors affecting the grain-size distribution of a clastic carbonate sediment deposited in water.

Shape studies prior to the 1970's employed charts for visual comparisons of roundness (Powers, 1953). Calculations were performed for sphericity. The concepts of roundness and sphericity were originally described by Wentworth (1919) and Wadell (1932; 1935) and were usually employed as the only two qualities that were necessary and sufficient to describe shape.

Zing (1935) made an attempt to describe semi-quantitatively the forms of particles. His diagram (the "Zing diagram"), based on ratios of the three principal diameters of a particle, defined four main shape classes: oblate, equant, bladed, and prolate. Equant particles have the highest sphericity, but oblate, bladed, and prolate particles may all have the same sphericity values. The problems with these pre-1970 methods of shape analysis are that they are very time consuming, don't uniquely define shapes, and are highly subjective to operator error.

Methods that are currently being applied (Ehrlich and Weinberg, 1970; Parks, 1982) utilize two-dimensional digitized projections of grain boundaries as a basis for shape representation. Evidence indicates that the two-dimensional outline of a quartz grain is representative of its three-dimensional shape (Schwarcz and Shane, 1969; Tilmann, 1973). Microscopic observation of carbonate grains suggests that their two-dimensional projection outlines may not be as representative of the total shape as the two-dimensional outlines of quartz grains.

The Fourier method (Ehrlich and Weinberg, 1970) of shape analysis is currently the most widely utilized method of quantitative shape studies. Numerous workers (Ehrlich et al., 1974; Yarus et al., 1976; Van Nieuwenhuise et al., 1978; Porter et al., 1979; Brown et al., 1980; Riester et al., 1982) have demonstrated that quantitative Fourier quartz grain shape analysis is useful as a natural tracer of sediment transport and for discriminating between sediments from different environments. Mrakovitch (et al., 1976) and Mazzulo and Ehrlich (1980) indicated the usefulness of Fourier analysis for stratigraphic correlation.

The Fourier method starts with a center of gravity calculated from digitized coordinates of the two-dimensional grain outlines. A harmonic Fourier series of the expansion of the radius as a function of the angle about the center of gravity is then calculated. Each harmonic is described by:

$$R_n \cos(n\theta - \Phi_n)$$

where, R_n is the amplitude of the nth harmonic and $\cos(n\theta-\Phi_n)$ is the angle at which each shape component is oriented (Brown et al., 1980). Each harmonic amplitude represents the contribution of a specific shape component to the overall two-dimensional shape. The second harmonic represents elongation and the third harmonic represents triangularity. The harmonic amplitude spectrum is used as a set of variables for further analysis by graphical and statistical techniques (Ehrlich and Weinberg, 1970).

Harmonic amplitudes are rotation-invariant (Clark, 1981), but each associated phase angle is rotation dependent. For this reason, phase angle information is not utilized, resulting in a loss of useful information. Parks (1981) has shown that it is possible to have two or more different shapes with the same amplitude values because of different phase angles. Because of that discovery, the Fourier method was not used in this investigation.

Mitiche and Aggarwal (1983) state that shape matching is a much easier task when shapes are properly registered. Registration is the act of adjusting the positions of two or more outline shapes so that they correspond exactly or conform as closely as possible. Their registration technique computes shape-specific points in the spatial or picture domain for the purpose of registration. These points are easy to compute and are not used directly for recognition, but rather for registration. After shapes are registered, the task of matching should become much easier and several matching algorithms, such as template matching, may be used that could not have been used on unregistered images (Mitiche and Aggarwal, 1983). This procedure is tested in this study by the computerized matching of unknown shapes with one of several prestored reference shapes.

Collins (1983) has shown that Parks' (1982) multivariate rotation method produces results that are at least comparable to those obtained by semi-quantitative analysis of Fourier-derived shape data for discrimination of sediments from the diverse sources and environments sampled. Parks' method focuses on gross shape variations. The two-dimensional digitized coordinates of the grain outline are used to calculate thirty-six equianqular radial lengths extending from the center of gravity to the grain outline. These radial lengths, representative of the gross shape, are oriented to a standard position so that they may be compared with oriented radials of another grain shape or group of grain shapes. These radials serve as descriptors of two-dimensional shape which may be used as input into multivariate analysis programs.

METHODS

SAMPLING

Thirty-seven random grab samples were taken along four traverses, extending from the beaches to the near-reef areas, across the backreef sub-environment. Sample stations were located by taking compass readings on landmarks, buoys, a microwave tower, and a lighthouse, all of which are marked on NOAA National Ocean Survey Maps 11445 and 11449. The northern sample area includes traverses A and B (Fig. 2). Traverse A includes samples #1 through #17 and extends from Upper Matecumbe Key to Alligator Reef. Traverse B includes samples #18 through #26 and extends from Lower Matecumbe Key to Alligator Reef. The southern sample area is approximately 113 kilometers southwest of the northern sample area and includes traverses C and D (Fig. 3). Traverse C includes samples #27 through #33 and extends from the channel between Sunshine Key (formerly Ohio Key) and Bahia Honda Key, onto Bahia Honda State Park Beach, and to Big Pine Shoal. Traverse D includes samples #34 through #37 and extends from Long Beach to Big Pine Shoal. In two areas (sample stations #13,14,15 and #22,23,24), three samples were collected at the corners of a ten-foot equilateral triangular grid to check for within-sample-area variability. Grab samples, one to three centimeters-deep, were scooped into plastic ziplock bags. SCUBA gear was used in order to collect deep-water samples.

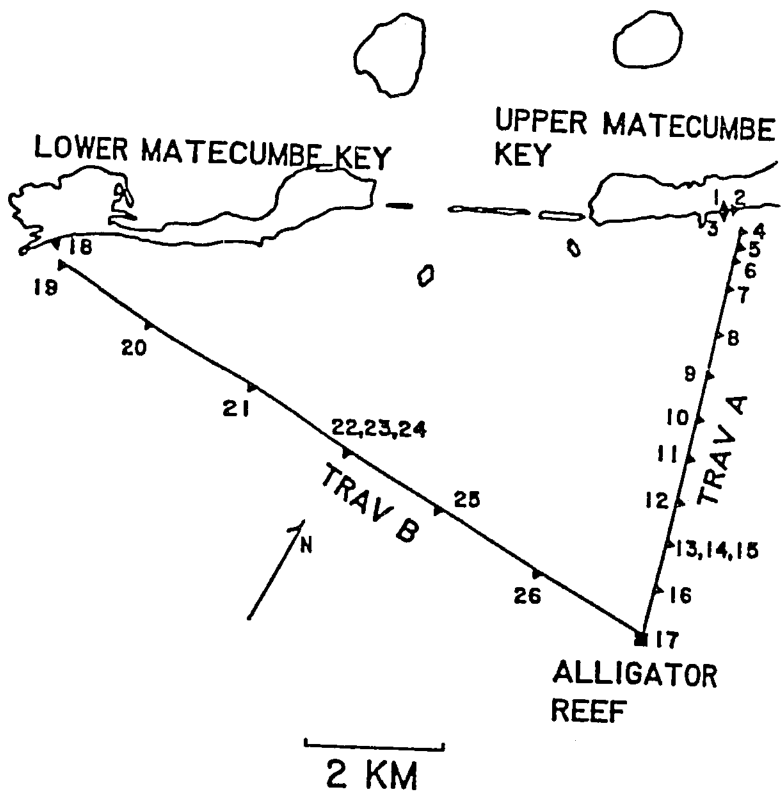

Figure 2. The northern sample area: Traverse A includes sample stations #1 through 17. Traverse B includes sample stations #18 through 26 (After NOAA National Ocean Survey Map #11449).

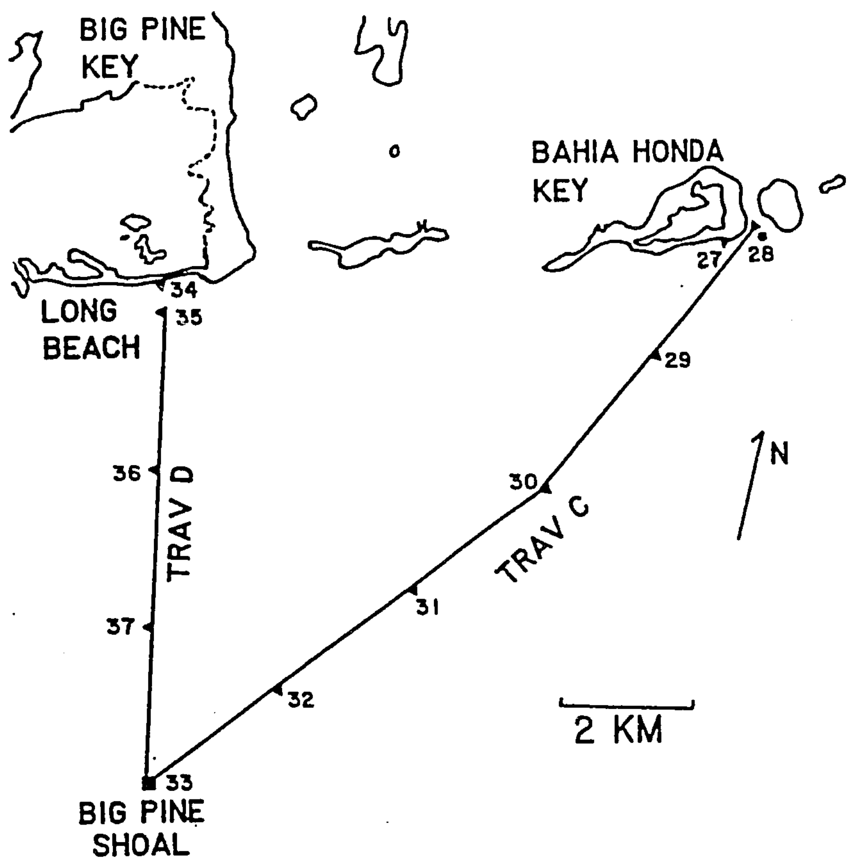

Figure 3. The southern sample area: Traverse C includes sample stations #27 through 33 and traverse D includes sample stations #34 through 37 (After NOAA National Ocean Survey Map #11445).

In the laboratory, samples were dried and split using the cone and quarter method. Subsamples were soaked in clorox for twenty-four hours so that aggregates due to drying were destroyed. Size separation was performed on an automatic vibrating shaker by wet or dry sieving, depending on the concentration of fines in the sample, for ten minutes. Size fractions were weighed to the nearest 0.01 g. Size results were used to estimate the relative current intensities of the sample areas by assuming that relatively low mean phi-size values are indicative of areas with relatively high current intensities. Swinchatt (1965) assumed that the presence of a significant amount of silt and clay-sized material in carbonates indicated low current intensity. The 0 to -1 phi size fraction (very coarse sand) was used for the shape analysis. Sand grains within the size interval were very carefully split down to 200 grain subsamples and mounted on glass slides in their stable orientation by using transparent double-stick tape. This orientation appeared to show the maximum two-dimensional area of the shape when projected.

SIZE ANALYSIS

The percentage of material finer than 4 phi (silt and clay-sized) was determined by subtracting the weight of the material coarser than 4 phi from the initial sample weight. Two size statistics of Folk and Ward (1957) were computed from the size distributions. The first is the mean (phi) diameter as given by:

$$M_z = .33(\Phi_{16} + \Phi_{50} + \Phi_{84}).$$

The second is the inclusive graphic standard deviation (sorting) given by:

$$\sigma_I = 0.25(\Phi_{84} - \Phi_{16}) + 0.1515(\Phi_{95} - \Phi_5).$$

CONSTITUENT ANALYSIS

Mounted 200-grain subsamples representing the sample stations from traverses A and D were analyzed under the binocular microscope with reflected light in order to obtain the relative percentages of constituents in these samples. Identification of Halimeda plates is generally easy due to their small pores (generally $\leq$ 35 microns) and internal tubular cell structures. Dasyclad stems are similar in appearance to Halimeda plates, but have a more cylindrical shape and layer of lime crust on their exterior. Coralline algae are easily identified due to their distinctive stem and partial-stem forms. Millepora were also grouped under corals. Pore sizes of corals are typically over twice the size as those of Halimeda. Star-shaped pores are often seen on Millepora grains while relatively large individual corallites with septa are typically visible on corals. Coral grains also tend to be semi-translucent and of lighter shade than the other grains. Recognition of bivalves and gastropods are relatively easy due to their interior nacreous lining and rough exterior. Broken and unbroken gastropods typically show some degree of spiralling. Foraminifera are the easiest constituents to indentify due to their planispiral tests.

DATA COLLECTION

The equipment used for collecting raw shape data (X-Y coordinate pairs) is illustrated in Figure 4. The hardware included a Fisher microprojector, a Houston Hipad II electronic digitizing tablet with stylus, and an IMS 5000 microcomputer with a floppy disc drive and Winchester hard disc drive. Communication between the digitizing tablet and the microprojector was implemented by a FORTRAN digitizing program. Each of the slides containing a sample of mounted constituent particles were placed on the mechanical stage above the fifteen millimeter objective of the microprojector (an inverted microscope arrangement), resulting in the projection of a two-dimensional shaded image, approximately seven to eight centimeters in diameter, onto the digitizing tablet. Coordinates for 150 to 200 points along the periphery of the projected image were obtained by using the stylus to manually trace the edge of the image within a ten to fifteen second period of time while in the digitizer's steam mode. In this mode, the position of the stylus is sampled in 25 millisecond intervals. The shape-descriptive coordinates were temporarily stored on the hard

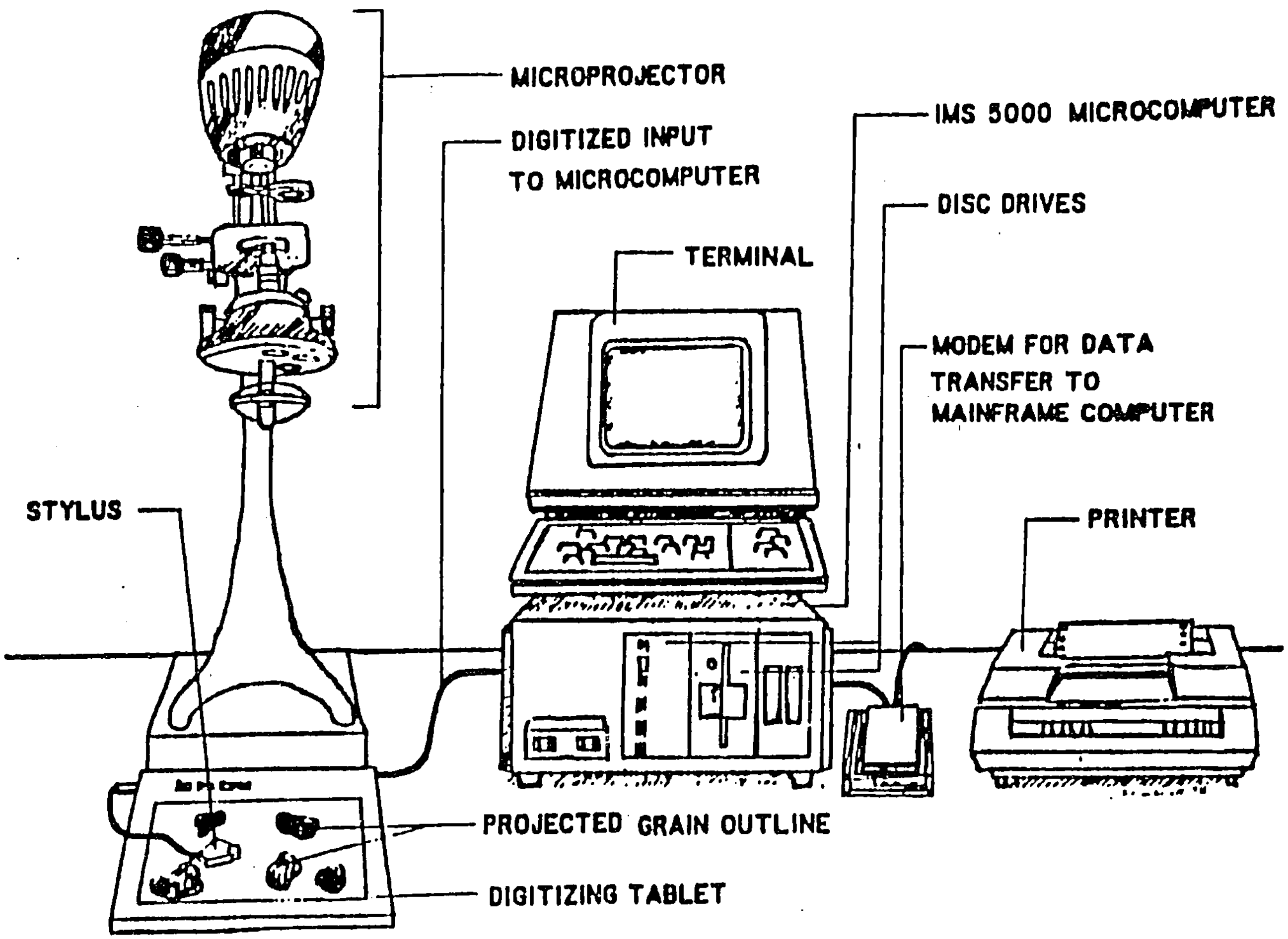

Figure 4. Equipment used for data collection and processing. (Illustration by Jessica Smith; from Collins, 1983)

disc. These coordinates are shape data which must be reduced in quantity.

DATA PROCESSING

Registration technique. An initial shape separation was performed on a -0.75 to -1.0 phi-sized composite sample. A settling tube was used in order to sort hydrodynamically the constituents according to shape and possibly, to a lesser degree, bulk density. However, regardless of density differences, this procedure should have closely approximated the sorting patterns that occur in the natural environment. Size was not a major sorting factor because of the narrow range of sand size used.

Carbonate sand from the composite sample, representative of the backreef sub-environment, was dropped into the settling tube and collected in plastic tubing as it reached the bottom. The tubing, and collected sand within it, were then clamped and frozen in an upright position. Sections of the tubing were cut open and the sand within them digitized. Shapes that were expected from the bottom-most sections of the tubing were those which approach spheres, and primarily consist of abraded massive corals, representing those shapes that require relatively greater current intensities in order to be transported. Shapes that were expected from the middle sections of the tubing were rod-like shapes which would primarily involve coralline algae and gastropods. Shapes from the top sections of the tubing were expected to be primarily platy and probably involve foraminifera, bivalves, and Halimeda. These shapes would represent the constituents which require relatively lesser current intensities in order to be transported.

After digitization of the two-dimensional projection shapes of the constituent particles of each tubing section, the X-Y coordinate pairs were registered according to the method of Mitiche and Aggarwal (1983). They claim that registration can be easily achieved by using shape-specific points computed from the given shapes and note that by using any shape-specific point, and radii from it to points on the shape, one can obtain a new shape specific point. Two examples of shape specific points were described by Mitiche and Aggarwal:

> "Given a shape S described by the discrete set of points $\{(x_i, y_i)\ \ i = 1,\ldots,n\}$, consider the unweighted mean point (centroid) A, the coordinates of which are given by:
>
> $$x_A = 1/n\ \Sigma\ x_i$$
>
> $$y_b = 1/n\ \Sigma\ y_i$$
>
> It is clear that this point is a shape-specific point of S.
> Another example of a shape-specific point is the radius weighted mean point B, the coordinates of which are:
>
> $$x_B = 1/R\ \Sigma\ r_i x_i$$
>
> $$y_B = 1/R\ \Sigma\ r_i y_i$$
>
> where r_i is the distance from the centroid A, as defined in the previous example, to the point (x_i, y_i) on the shape and $R = \Sigma\ r_i$."

These and other shape-specific points were calculated and tested in this study.

After shape-specific points "A" and "B" have been calculated for each grain, the grain coordinates were rotated about the center of gravity (Hall, 1976) so that the line which connected shape-specific points "A" and "B" was vertical and "A" was above "B" (Fig. 5). As a result, all the "A's" were in close proximity to each other and all the "B's" were in close proximity to each other along the same vertical line for all grains. The location of the center of gravity was used as a test for determining whether the shape needed to be flipped. If the center of gravity was on the right side of the grain, the shape was flipped. Thirty-six equianqular radial lengths were then interpolated for each particle from the center of mass to the boundary outline by a FORTRAN program developed by Parks (in preparation). The sizes of these radial lengths were normalized to reduce the effect due to size variations between grains within the size interval by an algorithm which adjusts the area of each shape, as represented by radials, so that it equals the area of a unit circle. At this stage, the grain outlines have been registered and data substantially reduced (from 150-200 coordinate pairs to 36 processed radials).

Average shapes were calculated from the settled grain sections from the settling tube procedure described above. These shapes were to be used as templates for further shape analyses of the carbonate

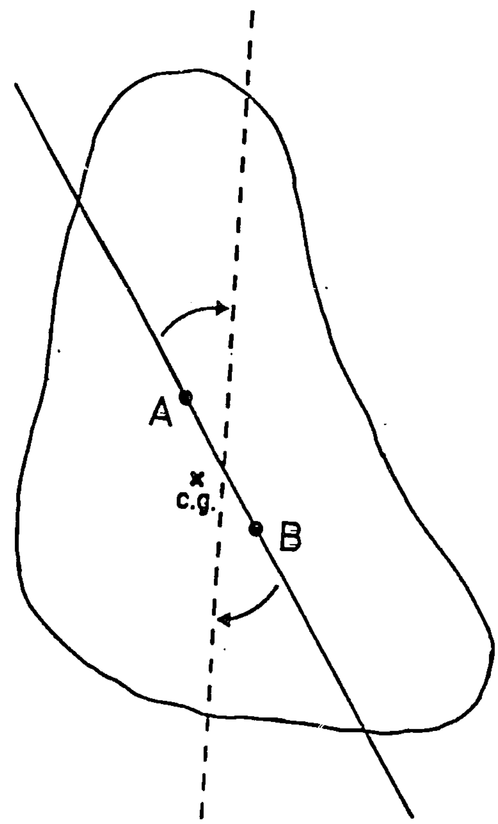

Figure 5. Shape-specific points A and B, the line connecting them, the center of gravity, and the rotation to the vertical position are shown.

sand samples and were calculated from the thirty-six processed (registered) radials by:

$$\bar{R} = \sum_{i=1}^{n} r_i/n$$

where $\bar{R}$ is the average radial length of the first radial length, r, of all the grains in the section and n is the number of grains in the sample. Calculations of $\bar{R}$ for all radials in the sample were performed in a parallel manner and formed the average shapes when combined.

Subsequent shape analysis of samples was performed by template matching to determine the percentages of particles in the sample which best fit each template. In the study by Mitiche and Aggarwal (1983), similarity between contours is expressed in terms of the fraction s=C/N, where N is the total number of contour points in both contours and C is the count of points from either contour that have at least one point from the other contour in its 3 X 3 neighborhood. A variation of this template matching technique was used in this investigation by calculating the correlation coefficient between each sample grain shape and each template in order to determine and tabulate which template was "best fit" by each sample grain shape. Other templates that were chosen arbitrarily or obtained by computing average shapes of the various constituents were also tested.

Multivariate rotation technique. The multivariate rotation method (Parks, 1982; Collins, 1983) was used to process all of the raw data, which consisted of the digitized coordinates of the two-dimensional boundaries for 200 carbonate grains in each of thirty-seven samples. Collins (1983) and Blanchard (1985) used three FORTRAN programs by Parks (in preparation) in order to process their raw data. These programs were altered and combined to form one relatively large program, resulting in a substantial savings of processing time.

The first main algorithm in the program is from Hall (1976) and calculates the centers of gravity of the outlines represented by the X-Y coordinates for each grain. The next main algorithm calculates radial lengths extending from the center of gravity to every third pair of X-Y coordinates on the shape outline, resulting in the calculation of approximately sixty radial lengths. This number of radial lengths was subsequently reduced to thirty-six radial lengths equally spaced at ten degree intervals about the center of gravity (Fig. 6) by a cubic interpolation procedure (Parks, in preparation). This procedure results in an eighty percent data reduction. Remaining size effects within the 0 to -1 phi fraction were eliminated according to the normalizing procedure described in the previous technique.

Grain outlines, now represented by thirty-six equianqular radial lengths, are then rotated at ten degree intervals about their centers of gravity by the third main algorithm until a best fit position is located, relative to a reference shape, by a least squares procedure.

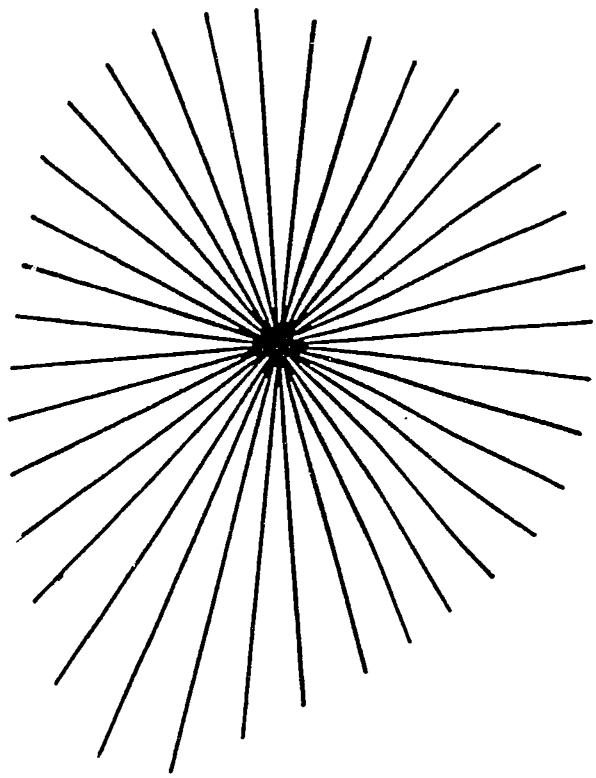

Figure 6. Interpolated equiangular radials about the center of gravity.

The initial reference shape used was that of Collins and Parks (1984; Fig. 7). Several other empirical reference shapes were also tested.

The next algorithm reorders the radials such that the grain shape is, in effect, flipped over. The flipped shape is then rotated in the same manner as the unflipped shape was. If the correlation coefficient of the best fit position relative to the reference shape was greater than that of the unflipped shape, the shape stayed flipped. Otherwise, the radials were reordered back to their original locations.

The final algorithm is used to adjust the rotation in order to correct for the ambiguity caused by the rotation at ten degree intervals. A one degree best fit position is determined. The radials are then renumbered clockwise so that radial #1 is the most elongate. At this stage, all grains have a common orientation, allowing more meaningful comparisons between either individual grains or groups of grains (Parks, 1981; 1983b; Collins, 1983).

MULTIVARIATE ANALYSIS

After processing (rotating and, perhaps, flipping) each grain shape by the multivariate rotation method, the thirty-six radial lengths served as input variables for multivariate analysis. The main multivariate statistical technique used was cluster analysis. Factor analysis was performed on a composite data set in order to examine its usefulness for future studies.

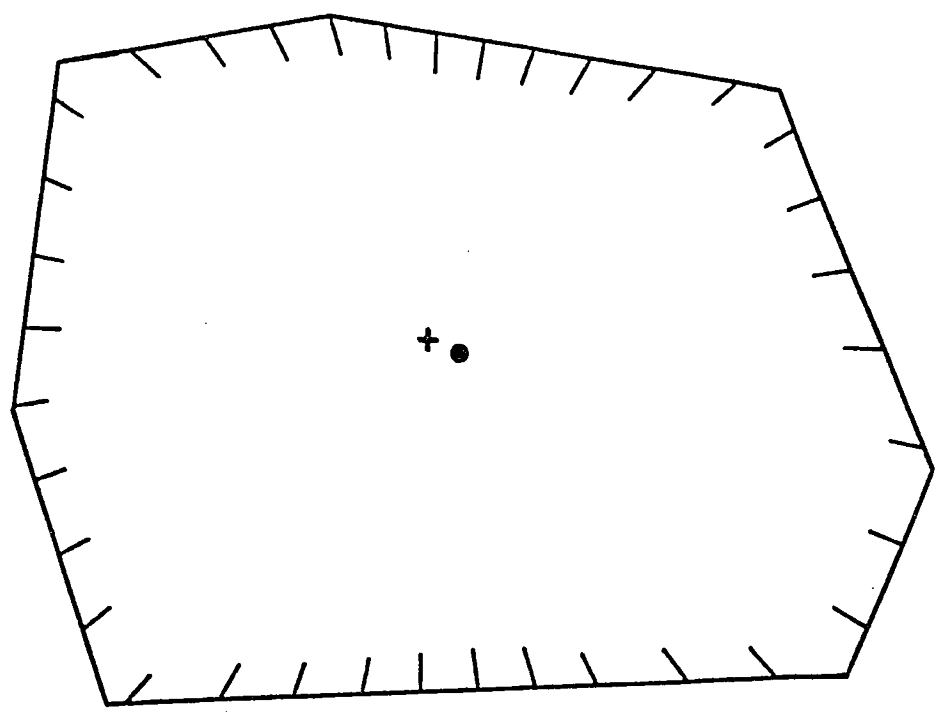

Figure 7. Initial reference shape as used by Collins and Parks (1984) (From Collins, 1983).

Cluster Analysis. Q-mode cluster analysis, a quantitative method for numerical multivariate classification, was performed on each 200-grain sample shape-data set. It was used to group together or cluster similar shapes by using the processed radials as input. Cluster analysis involves two principal steps.

The first step is to compute similarity coefficients, based on shape, between all possible pairs of grains. The coefficient D_{AB} describes the similarity between grains A and B. Two types of similarity coefficients are commonly used: the correlation coefficient and the simple distance function (Sokal, 1961). The distance function typically produces more effective cluster diagrams or dendrograms since its value in not constrained within the range of + and -1.0 and since it yields higher cophenetic correlations (Davis, 1973).

A Q-type cluster analysis program by Parks (1966) was used in this study. In this program, the formula for the Euclidian distance (D) between two objects (shapes) in a multidimensional hyperspace of M dimensions (M = 36 = number of variables = number of processed radials) is:

$$D_{1,2} = \sqrt{\sum_{i=1}^{M} (x_{i1} - x_{i2})^2 / M}$$

where x equals the normalized (transformed to range from 0.0 to 1.0) values of the variables. A low distance indicates relative similarity, whereas a large distance is indicative of dissimilarity.

The second principal step of cluster analysis is to arrange the objects into a hierarchy so shapes with the highest mutual similaity are clustered (grouped) together. Parks (1966) uses a weighted pair-group method, a method that Davis (1973) suggests to be superior to either single-linkage or unweighted average methods. The program terminates after printing out a two-dimensional heirarchical dendrogram (Fig. 8) on a line printer, which is faster and less expensive than a plotter. A shape can appear in only a single cluster because non-overlapping clusters are used in the program.

The cut-off value of the distance function needed to define clusters was determined visually from the dendrograms. Average shapes were then calculated, according to the formula in the registration technique section of this paper, from each cluster of similar shapes, resulting in a set of percentages of average shape types for each sample. A cluster analysis was then performed on a data array containing all of the average shapes previously computed from all samples, resulting in a dendrogram that describes which shapes could be statistically compared between samples.

Factor analysis. R-mode factor analysis resolves the interrelationships between multivariables and, in effect, reduces data. Factors are uncorrelated composite variables formed by using linear combinations of the original variables (radials in this study). A few factors (5 or 6) typically account for 80 to 90 percent of the

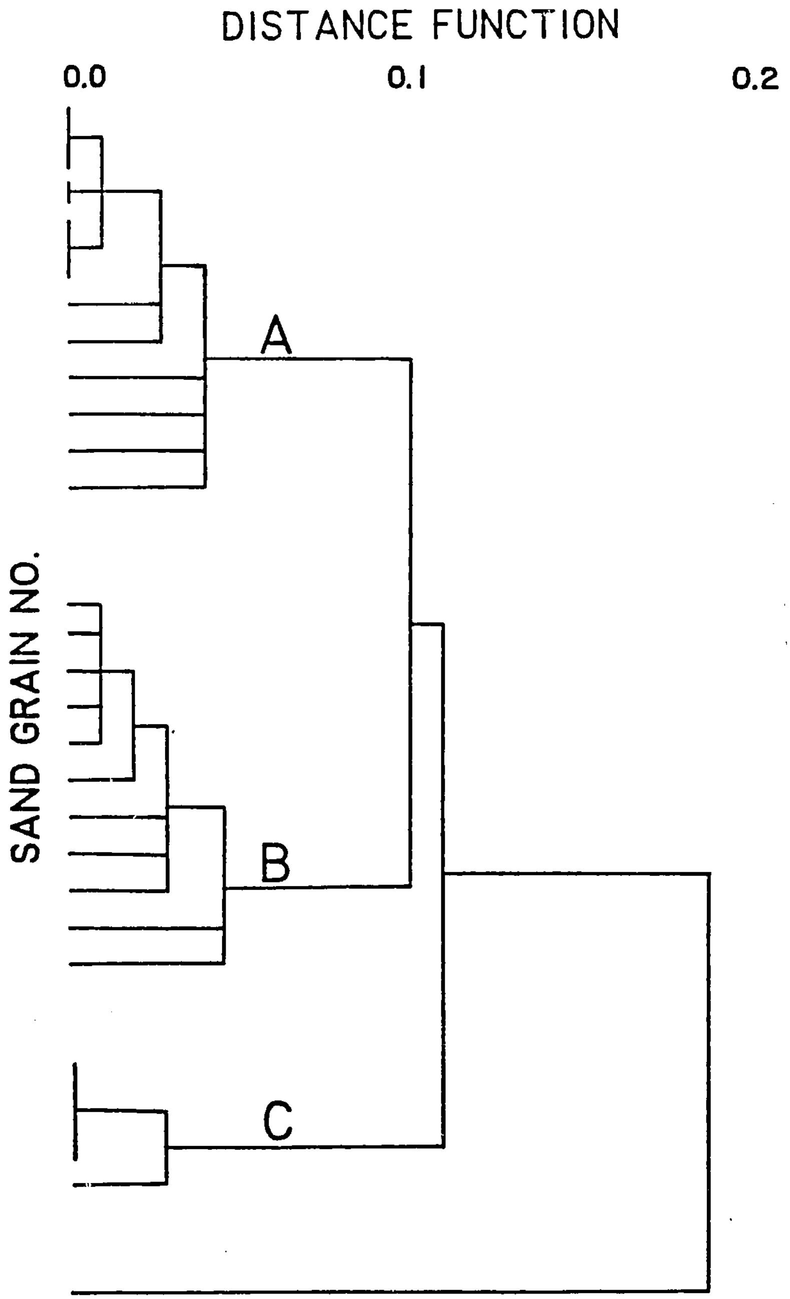

Figure 8. Two-dimensional heirarchical dendrogram with non-overlapping clusters. Similar shapes are grouped within each cluster, A, B, and C.

variance in the sample. Klovan (1975) explains the R-mode factor model in detail.

The basic factor equation (Joreskog et al., 1976) is:

$$Z = FA'$$

where Z is an n x v standardized data matrix (where n is the number of objects or grains in the sample and v is the number of original variables), F is the n x p factor score matrix (p = no. of factors), and A' is the v x p factor loadings matrix. F can be derived from:

$$F = Z(A')^{-1}$$

The factor loadings are the weightings (influence) of each variable onto each factor and are used in order to calculate the factor scores. The n x p factor score matrix contains p columns of elements in standardized form (mean = 0, variance = 1), f_{nj}, that describe the amount of factor j in object (grain) n. This matrix can be considered as a new and reduced variable matrix which describes the original data. Each factor score measures the amount of a particular factor in each object in units of standard deviation.

Factor analysis, both with and without rotation (Varimax), was performed on a composite data set in this study to test its usefulness for future studies. The method of factor analysis used in this study is maximum-liklihood factor analysis. It is an option in the BMDP statistical software package (BMDP4M of Dixon, 1981). Joreskog (et al., 1976) declares that it is a more efficient method for computing the estimate of the factor loadings matrix and asserts that it is a very complicated computational procedure.

RESULTS

SIZE ANALYSIS

The results of the size analysis are shown in Appendix 1. The mean phi diameter (M_z) and the standard deviation were calculated for each sample from the size frequency distributions. Standard deviations are presented in terms of Folk's (1964) scale of sorting values:

< 0.35	very well sorted	(VWS)
0.35 - 0.50	well sorted	(WS)
0.50 - 0.71	moderately well sorted	(MWS)
0.71 - 1.0	moderately sorted	(MS)
1.00 - 2.00	poorly sorted	(PS)
2.00 - 4.00	very poorly sorted	(VPS)
> 4.00	extremely poorly sorted	(EPS)

No general trends could be observed in mean phi sizes or sorting values along the traverses or between them. Figure 9 is the graph of the phi mean along traverse A from the beach to the reef. Sorting values are also indicated in this figure. This graph is typical of all four traverses.

DATA PROCESSING

Registration technique. This investigation discovered two main problems with the registration method. The first was finding suitable

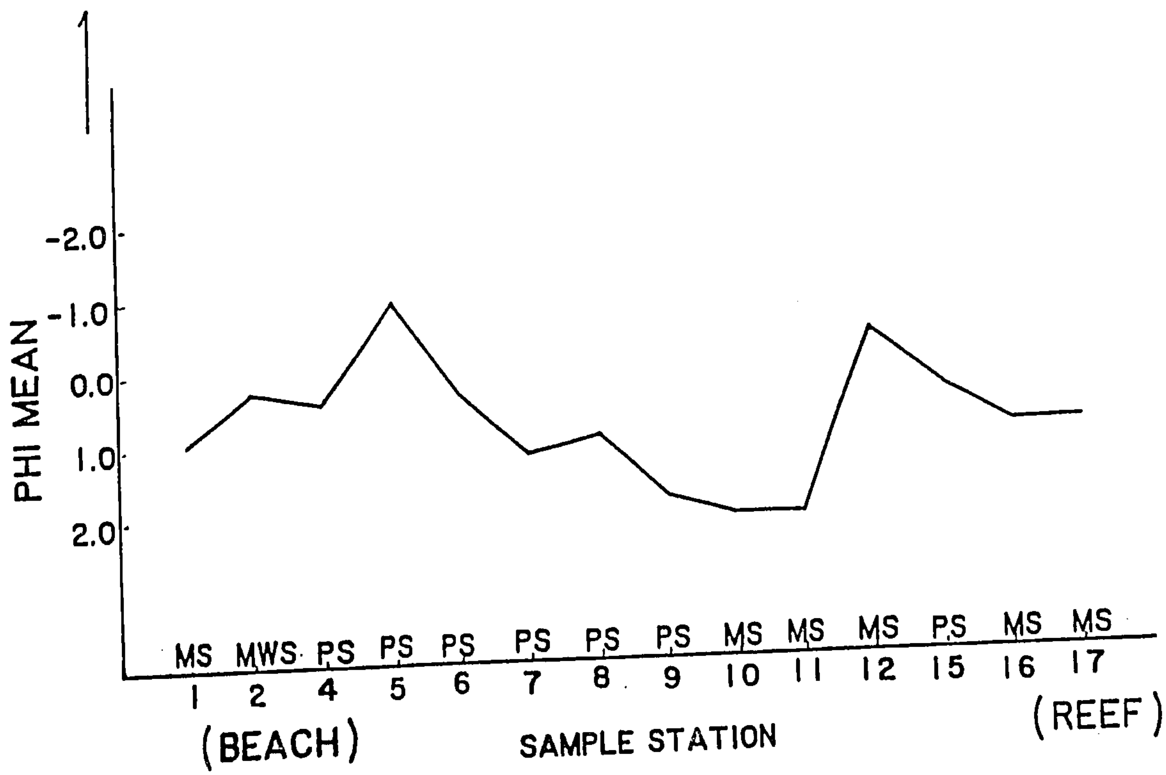

Figure 9. Graph of the phi mean along traverse A from the beach to Alligator Reef. Sorting values are indicated as MS (moderately sorted), MWS (moderately well sorted) or PS (poorly sorted).

template shapes to categorize the shape date in each sample. The other was the inability of the registration method to orient all of the sample shapes to similar positions.

The settling tube failed to sort hydrodynamically the composite sample by shape. The calculated average shapes from the different tubing sections were all very similar and egg-like. Since size was not a significant sorting factor, bulk density differences between and within various constituent types must have strongly interfered with genuine shape sorting. Obviously, these average shapes were not suitable for templates.

Other attempts to choose templates ranged from microscopically picking out various shapes that were visually selected to be representative of the constituents, to using arbitrary shapes as templates. However, by using these methods, the choice of template shapes that represented the shape types of carbonate sand was conjectural. The uncertainty of results demonstrated a need for an unbiased statistical method of extracting shape-type percentages from samples.

The other main problem, namely, the inability of Mitiche and Aggarwal's registration method to orient all shapes to similar positions, could not be solved. Many shape-specific points including those described by Mitiche and Aggarwal were tried, but there were always shape types within the samples that would not orient themselves in the same elongate direction as the other computer-plotted shapes. For this reason the registration method was abandoned.

Multivariate rotation technique. The processing of the 150 to 200 X-Y coordinate pairs in order to interpolate thirty-six equiangular radial lengths for each shape results in a data reduction of approximately eighty percent which still accurately describes the gross shapes of the sand grains. However, there are also two main problems with the multivariate rotation technique as used by Collins (1983). First, the final rotation positions of all grains are not in the same comparable elongate position and secondly, the final flipping orientations of all shapes are not visually comparable or logical.

The original asymmetrical reference shape (Fig. 7) of Parks (1982) and Collins (1983) is a unique shape, but not one that points to a specific elongation and widening direction. Sample grains are not rotated about the center of gravity of this reference shape, but about a point slightly offset from it. Many reference shape-types were tested. It was found that a symmetrical reference shape would be more appropriate for a standard elongate orientation position. It was also determined that rotation about a point that is offset from the center of gravity is unnecessary and, in effect, changes the shape of the reference shape as originally conceived. The ideal reference shape appears to be a triangle with a height to base ratio of three to two (Fig. 10).

The second main problem was due to the method of flipping. With the method as used by Collins and Parks, shape outlines are also flipped in order to "best-fit" the reference shape. Of all the many reference shapes tested in this study, none were fully satisfactory as

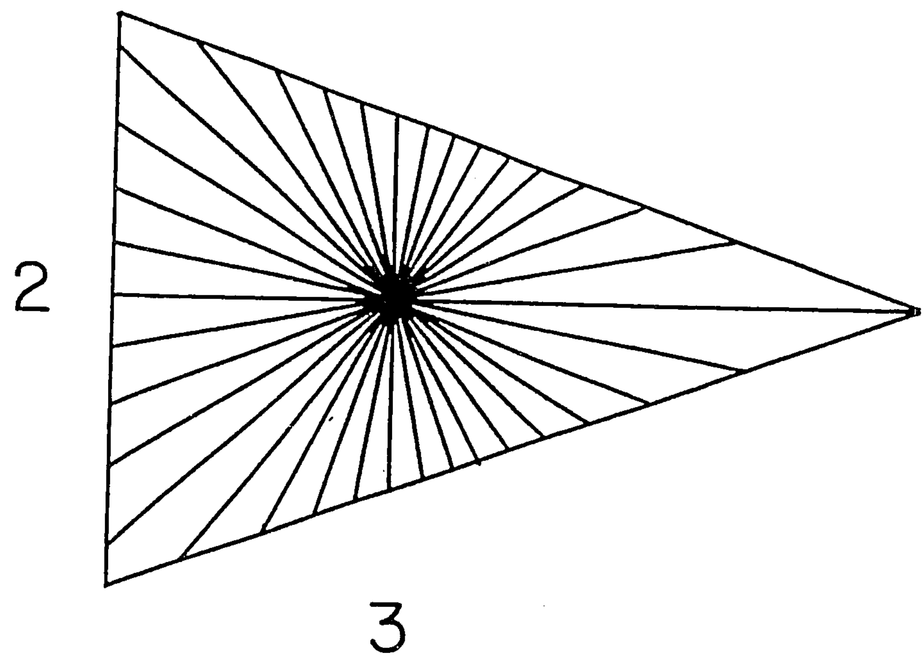

Figure 10. The current reference shape is a symmetrical triangle with a height to base ratio of three to two.

a flipping-test shape. The new reference shape described above orients shapes so that they are in a standard elongate orientation along an east (radial #1) - west (radial #19) axis. This general elongate direction is satifactory, but a criterion was needed to determine whether a particular grain should be flipped about the east-west axis in a north-south direction so that it was also oriented in a standard position relative to its triangularity. It was thought that a second triangular reference shape, oriented ninety degrees from the first reference shape, would serve as a flipping-test shape, however, none of the numerous triangular shapes experimented with was fully satisfactory.

The only alternative was to somehow quantify the radials on the northern side of the grain versus the radials on the southern (half) side of the grain. The most simple test is: if the sum of the northern radials is greater than the sum of the southern radials, then flip the grain (reorder the radials such that the grain outline is, in effect, flipped). This test works very well.

This revised multivariate rotation procedure involves six steps: (1) each two-dimensional sand grain shape is digitized, (2) a center of gravity is calculated for each shape, (3) X-Y coordinate data is reduced to thirty-six radial lengths, (4) each outline is size normalized, (5) the grain outlines are rotated to best fit (least squares) a 3:2 (height:base) triangle (standard orientation), (6) and if the criteria as explained above are met, the outline is flipped. Entire processing of the digitized coordinates (steps 2 through 6) takes less than one minute per grain on the microcomputer. A partial computer plot of a sample before rotations and flippings is shown in Fig. 11 and a plot of the same sample shapes after processing is shown in Fig. 12. From these plots it is easy to see the need for a standard orientation for purposes of shape comparison techniques. After processing, the outlines can easily be statistically compared by using the radials as variables for input into multivariate analysis programs. An example of radial sets for five processed grain outlines is shown in Appendix 2.

CLUSTER ANALYSIS

Dendrograms typically exhibited eight to ten well-defined clusters per sample. On occasion, there were samples with one or two peculiar shapes (outliers) which had the effect of combining clusters on the dendrogram. This problem was readily solved by removing these peculiar shapes and re-clustering the sample. The 0.5 value of the distance function was the typical cut-off point used to define, divide, and space the clusters on the dendrogram. It should be noted that no statistical tests of significance are available for cluster analysis.

The plots containing the groups of clustered sample shapes along with the calculated average shape for each clustered group are impressive (Fig. 13). It was visually obvious that the cluster analysis program did the task expected of it; clustering (grouping)

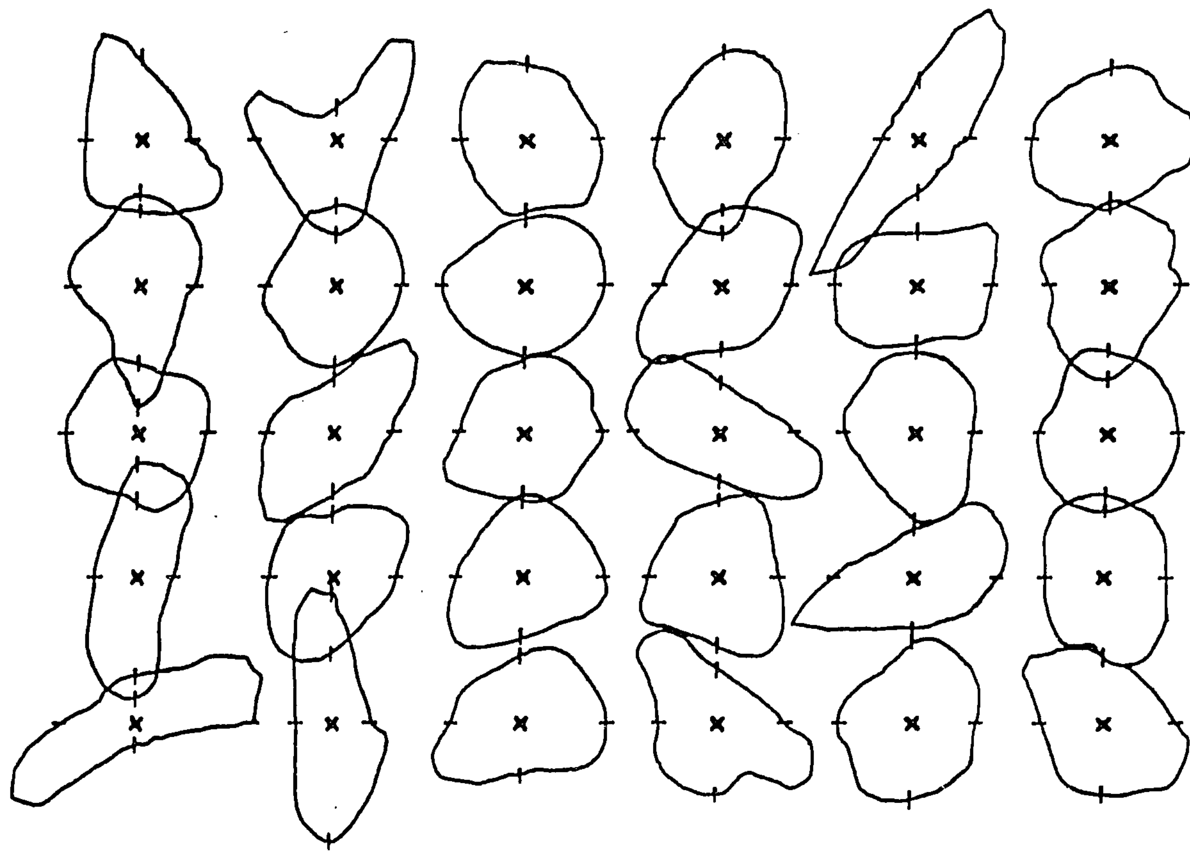

Figure 11. Partial computer plot of a sample before rotations and flippings.

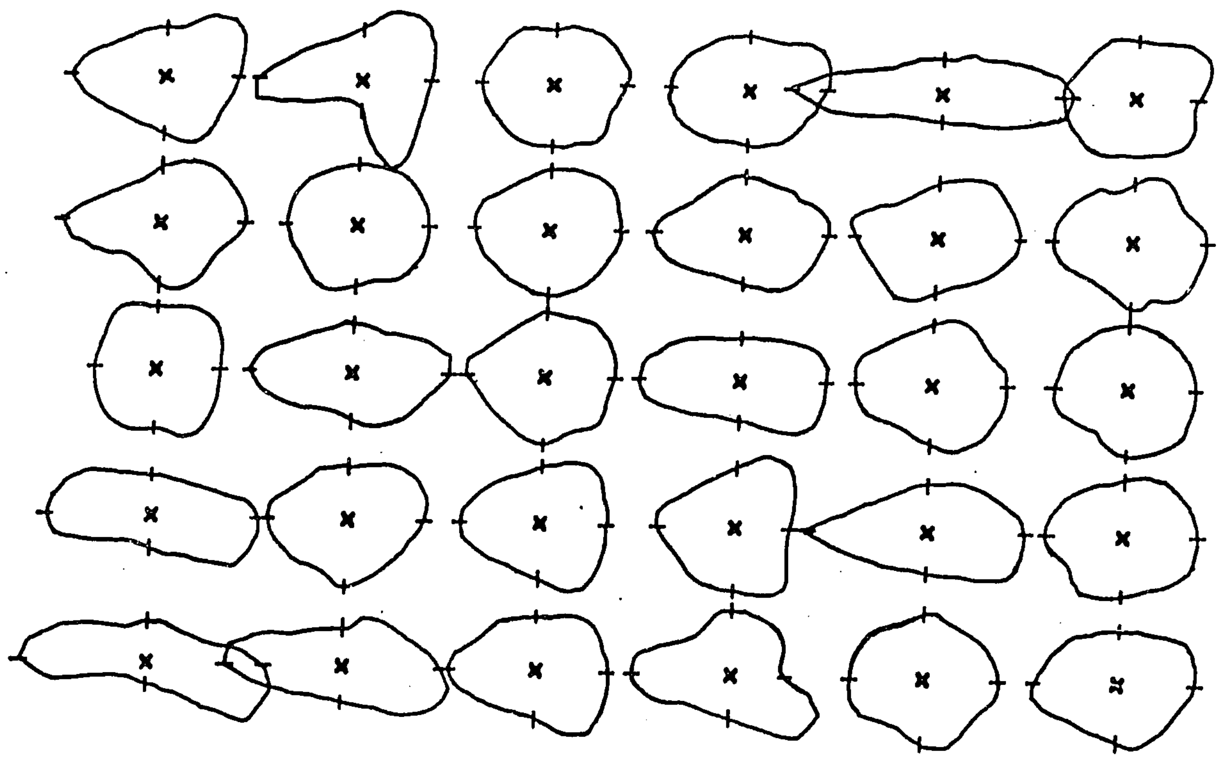

Figure 12. A partial computer plot of the same sample shown in Figure 11 after processing with the multivariate rotation method.

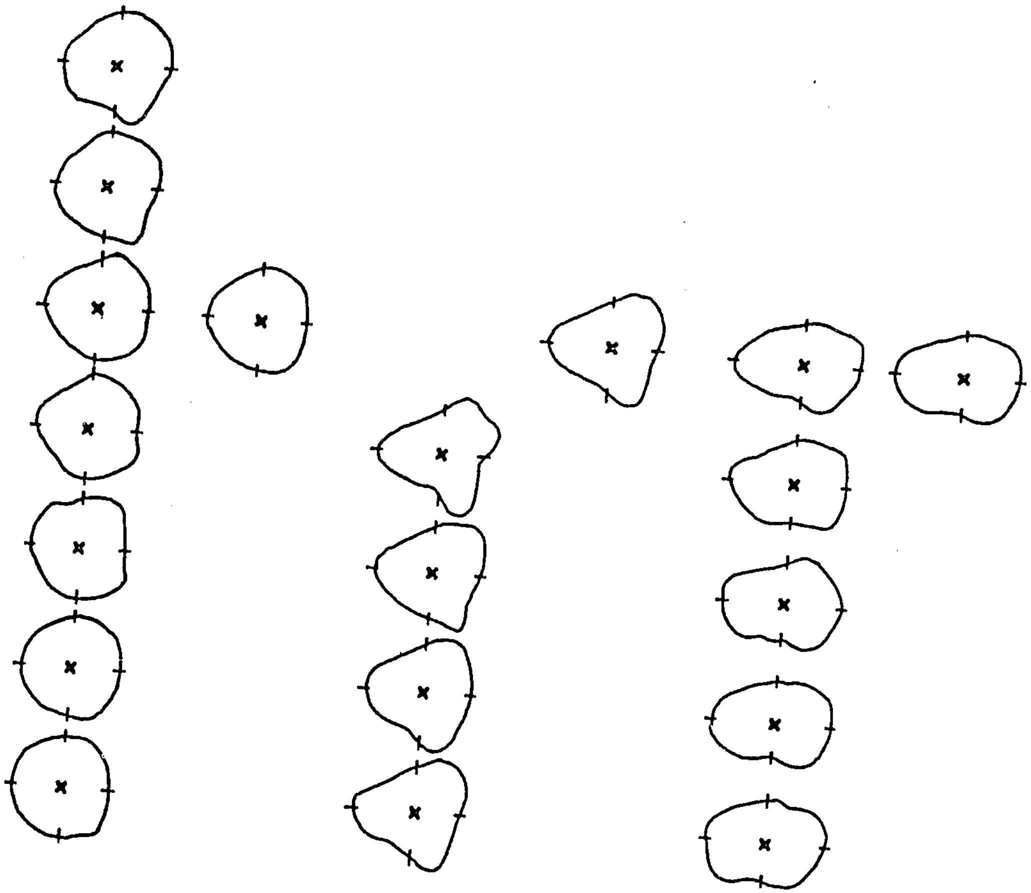

Figure 13. Computer plot for a small test sample showing the groups of clustered grain shapes with their associated average shapes.

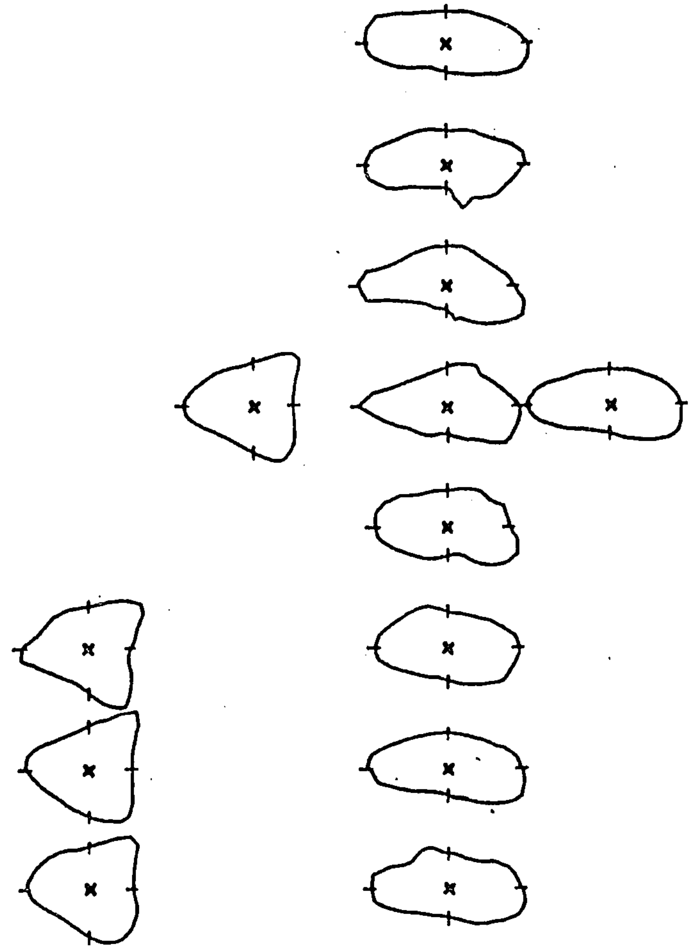

Figure 13 (continued).

similar shapes together. This task would have been impossible without using the multivariate rotation method. There were distict groupings of shape types within the samples. The average shapes calculated from these groupings were undoubtedly representitive of all sample shapes within the cluster groups.

Sample comparisons. It is difficult to compare between samples by merely looking at the average shapes for each sample. Consequently, all the average shapes previously computed from all of the samples were combined into a single data array. A cluster analysis was then performed on this array, resulting in a dendrogram that described which shapes were statistically comparable between samples, and to a lesser degree, within samples. Computing the average shapes from the clustered groups of average shapes resulted in a set of eight final average shapes, A through H, shown in Fig. 14. These shapes could have been used in the template matching program discussed earlier in order to tabulate shape percentages within samples, but were not because there was no a priori way to know that these were the shapes necessary.

Figure 15 is the graph of the frequency of occurrence of final shape "A" (circular shape) along traverse A from the beach to the reef. This type of graph is typical for all of the shape frequencies along all four traverses as tabulated in Table 1. Shape frequency graphs were reminiscent of the mean phi size graphs. In both, relatively high between-sample-area variabilities and no smooth general trends were observed from the beaches to the reefs. This was

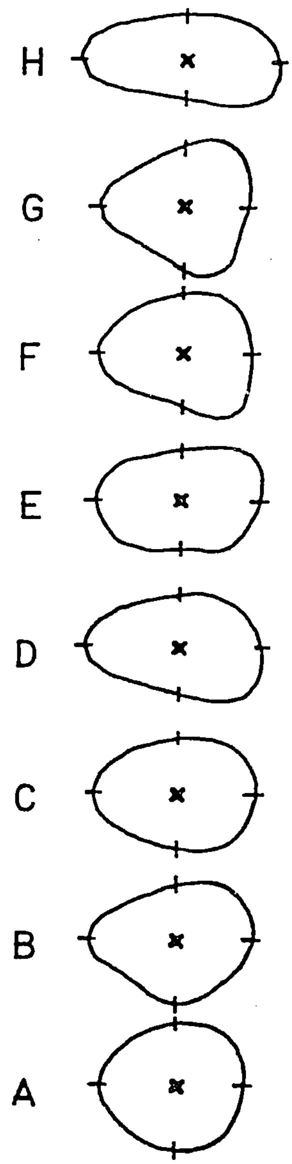

Figure 14. The set of the eight final average shapes ("A" through "H").

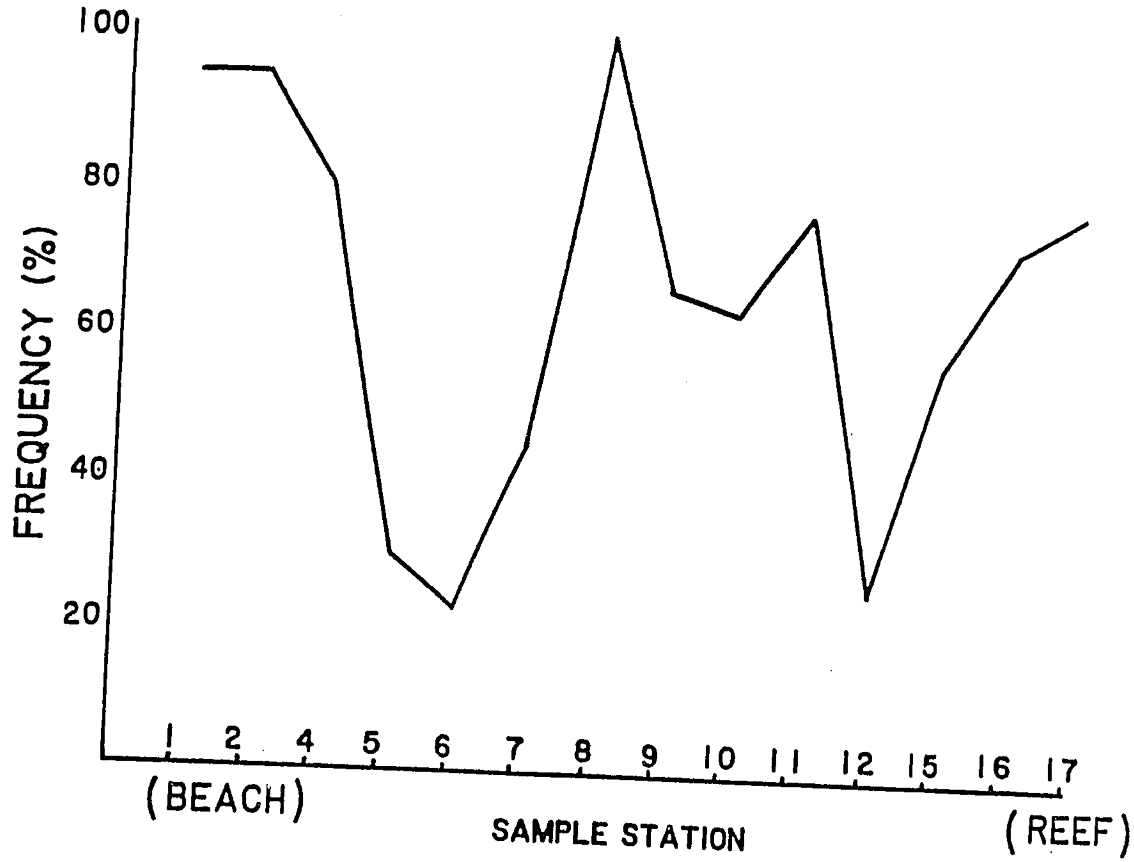

Figure 15. The graph of the frequency of occurrence of final shape "A" (circular shape) along traverse A from the beach to Alligator reef.

SAMPLE STATION	A	B	C	D	E	F	G	H	MIS
				SHAPE TYPE					
1	94	0	49	0	0	14	0	33	10
2	94	27	30	2	5	19	2	12	9
3	51	0	70	17	0	12	2	34	14
4	80	0	51	18	0	20	0	24	7
5	31	0	69	16	0	57	5	6	16
6	22	2	119	30	0	17	0	0	10
7	45	6	51	44	0	25	7	9	13
8	100	0	27	0	4	46	0	11	12
9	66	0	38	0	7	50	6	25	8
10	63	0	63	4	0	26	5	31	8
11	77	0	71	0	2	28	0	14	8
12	26	2	98	5	0	41	0	10	18
13	74	6	35	22	0	25	19	10	11
14	70	4	29	30	0	21	32	3	18
15	64	8	37	28	0	19	16	10	14
16	73	0	75	3	0	22	2	11	14
17	82	4	77	0	0	14	6	2	15
18	52	0	72	0	0	41	0	22	13
19	16	0	98	41	2	30	3	0	10
20	73	0	75	0	2	13	11	18	8
21	82	0	66	0	0	5	16	25	6
22	101	2	65	2	0	21	9	0	0
23	90	0	44	8	0	28	17	3	10
24	111	0	50	9	0	16	7	0	7
25	102	0	42	23	0	9	18	0	6
26	48	3	64	0	6	35	11	17	16
27	100	0	63	5	0	4	3	20	5
28	36	0	93	2	0	30	1	20	18
29	61	0	53	23	0	38	2	10	13
30	71	0	28	5	0	53	10	17	16
31	31	0	77	25	17	11	12	13	14
32	36	0	89	10	0	37	0	7	21
33	77	0	43	6	3	20	13	19	19
34	85	0	21	19	10	47	0	12	6
35	128	0	30	6	2	19	5	0	10
36	64	2	75	0	0	28	0	18	13
37	53	3	71	0	0	38	0	26	9

Table 1. Percentages of the eight final average shapes accounted for within each 200-grain sample (MIS = miscellaneous shapes).

statistically confirmed by using the eight final average shapes as variables for input into Q-mode cluster analysis. However, the samples that were collected at the corners of the ten-foot equilateral trianqular grids clustered together according to the grids with which they were associated, indicating low within-sample-location-area variability.

Shape and constituent-type relationships. Results of the constituent analysis as performed on the samples from the stations of traverses A and D are tabulated in table 2. The most abundant constituent type, accounting for 34.9 percent of all sample grains, is coral with a range of one to 90.5 percent in each sample. Bivalves are the second most abundant constituent type; accounting for 23.2 percent of all sample grains.

A cluster analysis using the nine constituent types as variables was performed on the samples and resulted in three clusters. Beach samples (#1,2,3,15) were grouped within a cluster due to the fact that they are all relatively high in coralline algae content. The other two clusters were not indicative of specific environmental areas of the backreef sub-environment. Five of the sample stations were not incorporated into clusters by the program.

The relationships between the constituents and the final average shapes as described above were tabulated in Appendix 3 and are presented graphically by the histograms in Figure 16. As can be seen in Figure 16, coral is the most abundant constituent of the backreef

SAMPLE STATION	COSTITUENT TYPE HAL	C-ALG	COR	BIV	GAS	DAS	FOR	MISC
1	20	41	50	28	6	11	38	6
2	26	35	69	28	3	3	32	3
4	18	51	55	22	5	5	42	2
5	166	2	2	0	10	19	0	1
6	22	4	43	31	33	9	49	8
7	39	1	87	41	10	10	4	7
8	22	2	77	65	11	7	12	3
9	19	0	72	78	11	3	6	10
10	28	0	52	84	9	7	10	10
11	32	1	57	40	5	13	26	26
12	0	0	181	8	5	3	1	2
15	18	1	159	13	4	3	1	1
16	37	0	96	45	2	1	13	5
17	61	0	36	86	1	1	14	1
34	30	23	59	36	1	0	35	11
35	17	1	3	12	4	1	161	1
36	38	0	29	97	8	4	9	15
37	14	0	22	156	2	0	4	2
33	7	2	173	9	0	1	1	6
SUMS	614	165	1322	879	130	101	452	120
% TOTAL	16.2	4.4	34.9	23.2	3.4	2.7	11.9	3.2
RANGE (%)	0-83	0-25.5	1-90.5	0-78	0-16.5	0-9.5	0-80.5	0.5-13

Table 2. Percentages of constituents accounted for within the samples of traverses A (sample stations #1 through 17) and D (sample stations #33 through 37).

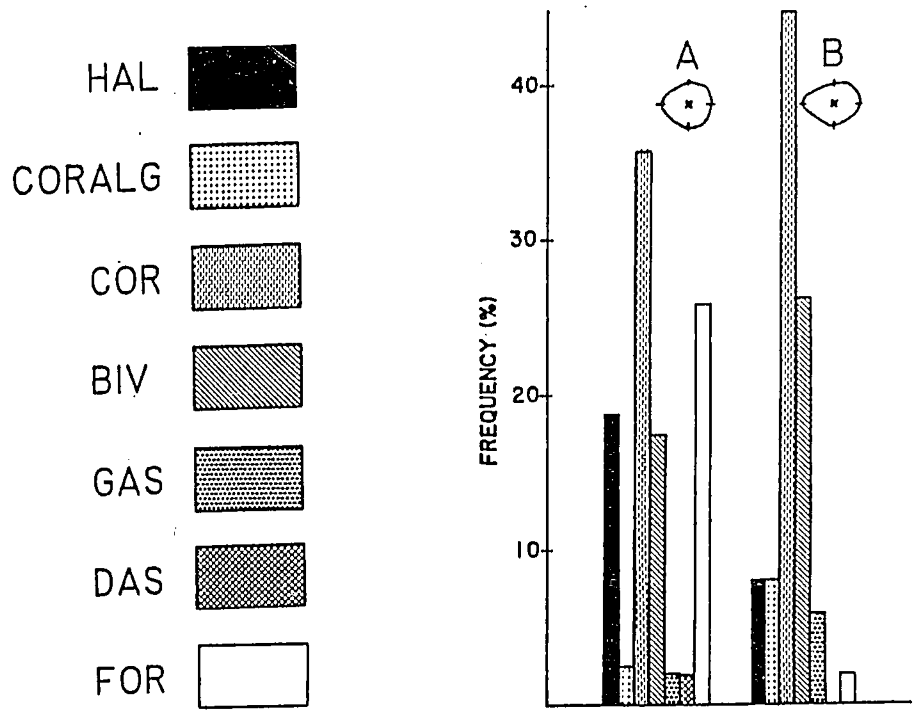

Figure 16. Histograms describing the relationships between the constituents and the final average shapes.

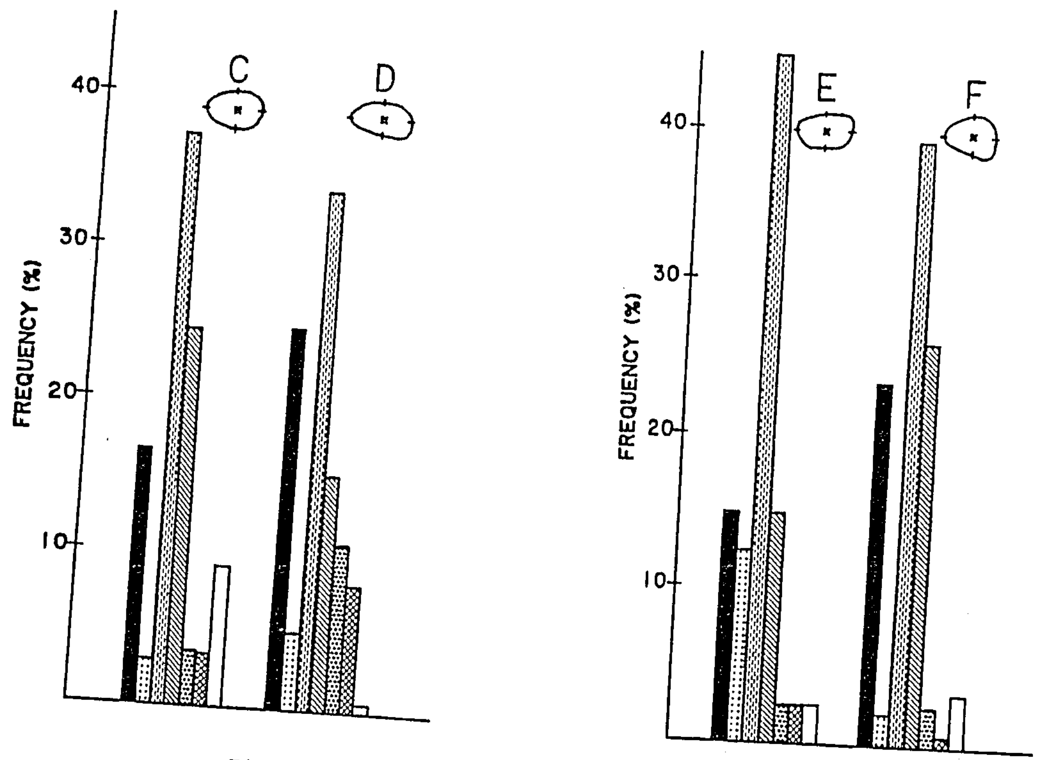

Figure 16 (continued).

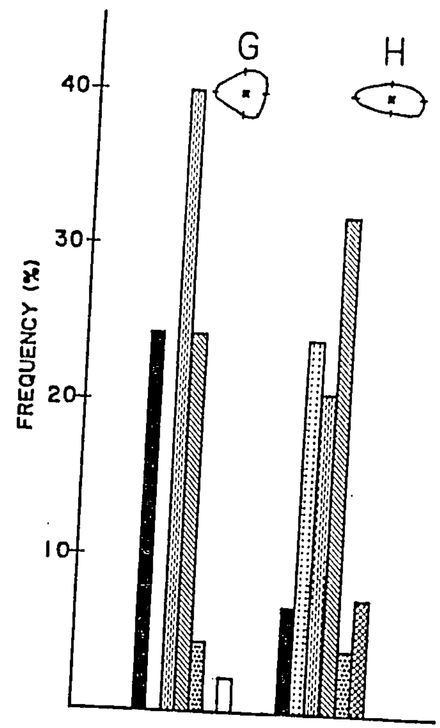

Figure 16 (continued).

sub-environment and breaks down into many shape types. Surprisingly, bivalves also tend to break into many shape types.

By looking at the same information in a different manner, the percentages of shapes accounted for within each constituent type may be inspected (Fig. 17; Table 3). The frequency of Halimeda shapes changes with an approximate slope of one, which implies that this constituent has a defined shape preference. Shape "H" is very dominant for coralline algae. Gastropods, dasyclads, and especially foraminifera have relatively well-defined shape preferences. Shape "A" accounts for 56.7 percent of the foraminifera.

FACTOR ANALYSIS

The applicability of R-mode factor analysis for the purpose of data reduction, as applied to a random composite sample of processed shape data (radials) for 486 grains, has been considered by analyzing the output of the maximum-liklihood method program described above. The mean and range of each ordered radial (variable) is given in Appendix 4. Six factors were requested of the program, but only five were defined, due to the low variance explained by more factors. Canonical correlations are the multiple correlations of each factor with the original variables. They are very high (Table 4). This fact suggests that all five factors are well-defined (Frane and Hill, 1974). Therefore, less than five factors would not be suitable. The communality of each variable (Table 5) is the proportion of that

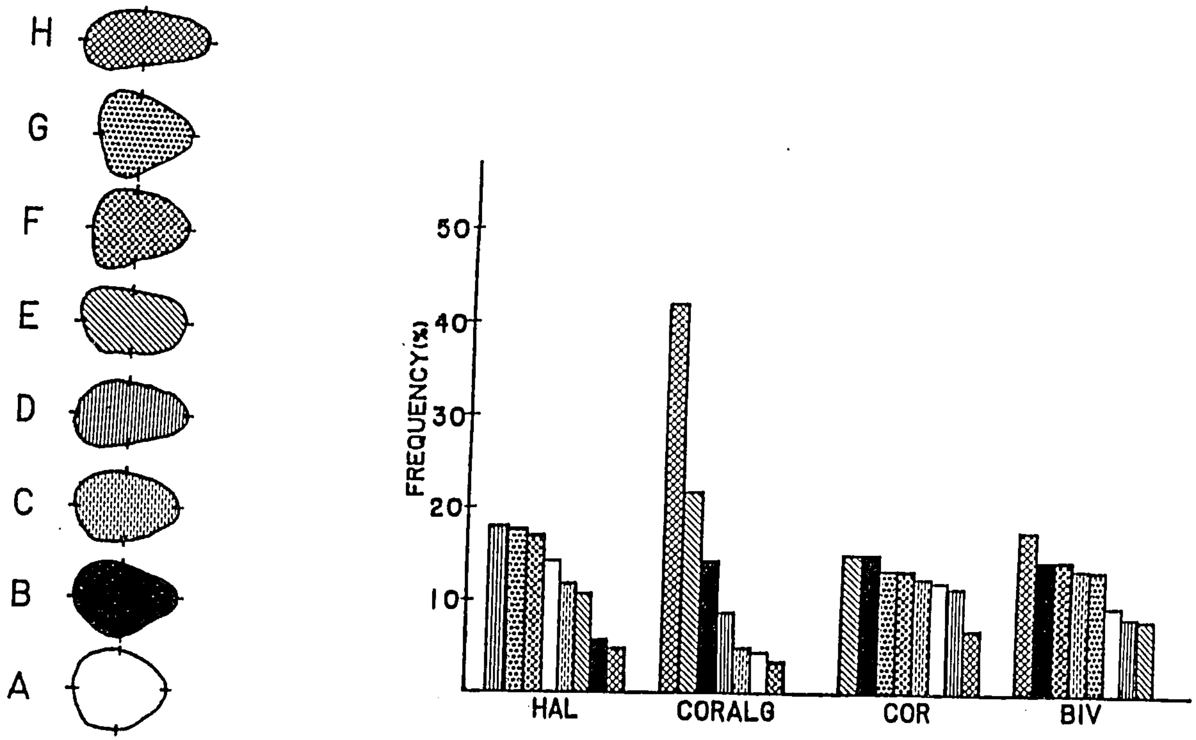

Figure 17. Histograms showing the percentages of shapes accounted for within each constituent type.

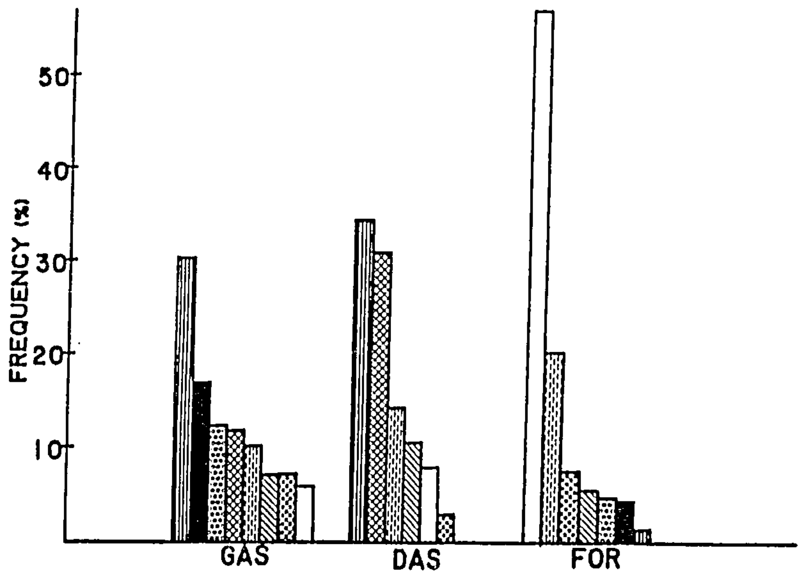

Figure 17 (continued).

CONSTITUENT TYPE	SHAPE TYPE A	B	C	D	E	F	G	H
HAL	14.3	5.9	11.9	17.9	10.7	17.0	17.5	4.9
CORALG	4.5	14.3	5.0	8.9	21.7	3.5	0.0	42.1
COR	12.1	15.1	12.6	11.4	15.1	13.3	13.4	7.0
BIV	9.7	14.6	13.6	8.5	8.2	14.5	13.4	17.6
GAS	5.8	16.8	9.9	30.0	6.9	6.9	12.1	11.6
DAS	7.8	0.0	14.0	34.2	10.3	2.9	0.0	30.8
FOR	56.7	4.3	20.0	1.3	5.4	7.4	4.8	0.0

Table 3. The percentages of the eight final shapes accounted for within each of the constituent types.

FACTOR	CANONICAL CORRELATION
FACTOR 1	.9989
FACTOR 2	.9954
FACTOR 3	.9905
FACTOR 4	.9821
FACTOR 5	.9784

Table 4. Canonical correlations of each factor as determined by maximum-liklihood factor analysis.

VARIABLE (RADIAL #)	COMMUNALITLY
1	.9767
2	.9622
3	.9003
4	.8319
5	.8596
6	.9148
7	.9474
8	.9684
9	.9665
10	.9678
11	.9801
12	.9849
13	.9718
14	.9375
15	.8461
16	.7573
17	.8620
18	.9382
19	.9090
20	.9280
21	.5288
22	.5838
23	.7582
24	.8793
25	.9577
26	.9565
27	.9027
28	.8532
29	.8471
30	.8648
31	.8870
32	.7909
33	.7549
34	.9191
35	.9850
36	.9822

Table 5. The communality of each variable (radial) as determined by maximum-liklihood factor analysis.

variable that can be explained by the five factors. Radial numbers 21 and 22 have the least to do with the other radials or with the five factors.

The variance and cumulative proportion of total variance explained by each factor are given in Table 6. The analysis resulted in the reduction of thirty-six radials or variables to five variables or factors, accounting for 88.5 percent of the total variance.

Interpretations of factors are typically made by inspection of the factor loadings matrix, A' (Joreskog et al., 1976). Joreskog (et al., 1976) states that this matrix should, more properly, be called the factor pattern. The unrotated factor loadings for this study are given in Appendix 5. R. Gibson and J. M. Parks (personal communication, 1985) have plotted the unrotated factor loadings of each factor for processed quartz grain shape data on polar coordinate grids. That same procedure was done in this study. Resulting patterns are shown in Figure 18. Surprisingly, the patterns formed for each factor were practically indistinquishable between quartz and carbonate sand grains. Factor one represents an elongate (figure-eight shape) component of shape and is similar to the second harmonic of Fourier analysis. Factor two probably represents a slightly distorted and rotated Factor one. Factor three represents a triangular shape component and is very similar to the third harmonic. Factors four and five represent rectangular shape components, respectively oriented in different positions, and are very similar to the fourth Fourier harmonic.

FACTOR	VARIANCE EXPLAINED	CUMULATIVE PROPORTION OF TOTAL VARIANCE
1	17.995980	.499888
2	5.962969	.665526
3	3.697094	.768223
4	2.217826	.829830
5	1.987702	.885044

Table 6. The variance and cumulative proportion of total variance explained by each factor.

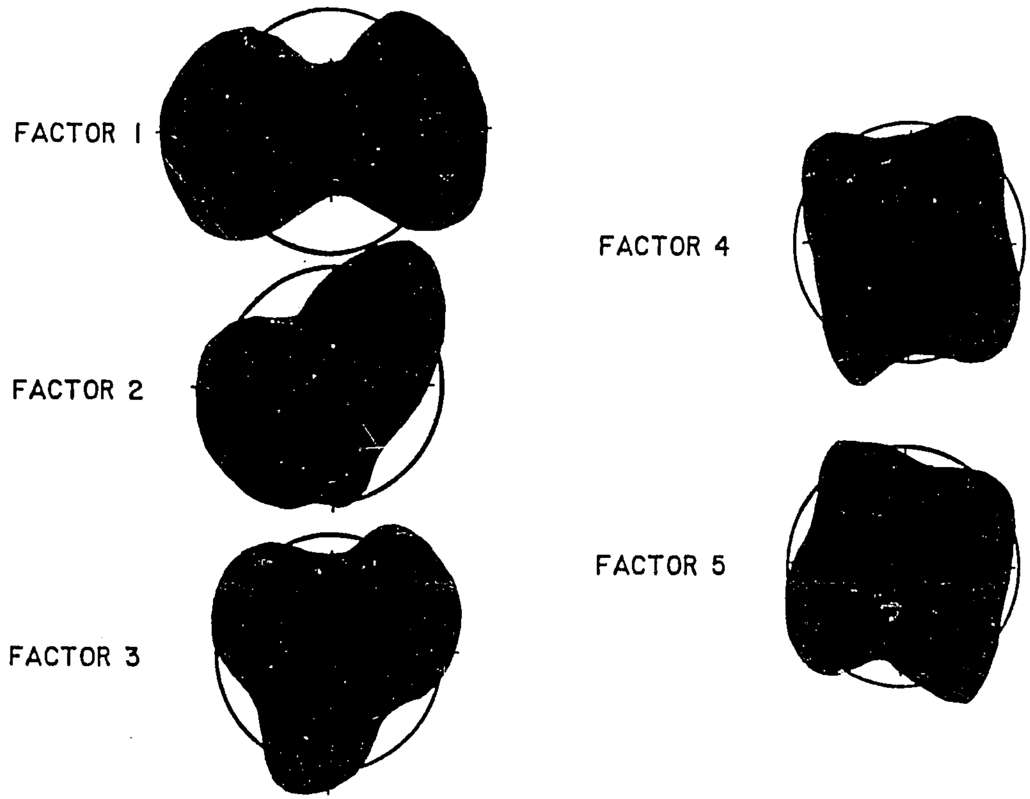

Figure 18. Polar coordinate plots of the unrotated factor loadings for factors one through five.

Methods of rotating the factor loadings did not produce recognizable factor patterns, therefore the factor score matrix (condensed data matrix) was computed by using the unrotated factor loadings. Factor score plots as output by the program were indicative of a normal distribution of factor scores due to their "shot gun blast" appearance (Frane and Hill, 1974). A Q-mode cluster analysis of the factor scores for this 486 grain data set resulted in cluster groups that generally contained 85 to 90 percent of the same grains as the cluster groups resulting from a Q-mode cluster analysis of the original variables (radials) for the 486 shape outlines.

DISCUSSION

SIZE ANALYSIS

The frequencies of the mean (phi) diameters along the traverses as well as the sorting values at each sample station were not indicative of a consistent trend in relative current intensities. This situation may indicate that the relative current intensities at the sample stations along the traverses are quite variable or that size frequency distributions for carbonate sands within a particular sub-environment are of little value for current intensity estimates.

Maiklem (1968) found that calculated settling velocities of bioclastic carbonate grains are up to four times greater than actual velocities. He noted that differences in settling velocities can be accounted for by shape, bulk density, and settling motion and that settling motion, in turn, is related to shape. Swinchatt (1965) stated that particles produced in place do not necessarily reflect the action of the current.

DATA PROCESSING

Two of the initial steps of the multivariate rotation method by Parks (1982), namely, the center of gravity determination and the radial interpolation procedure (including size normalization) were also used with the registration technique of Mitiche and Aggarwal

(1983). The purpose of these steps is data reduction. Plots resulting from the trigonometric conversion of equiangular radial lengths to connecting X-Y coordinate pairs (points) show that the radials still very accurately describe the gross shapes of the sample grains. This fact could be observed by visually comparing the grain shapes on the plots with the actual grain shape outlines as projected by the inverted microscope arrangement.

Registration method. The failure of all sample shapes to orient in the same elongate direction precluded the use of the registration method for processing (rotating and possibly flipping) radial lengths. Shape-specific points as defined by Mitiche and Aggarwal (1983) may not be as specific as they had proposed. For this reason, the multivariate rotation method was used and modified in this study.

The failure of the attempt to hydrodynamically sort the composite sample of carbonate sand according to shape for the purpose of producing template shapes was dissapointing, but also useful, with regard to understanding the effects of bulk density on shape and size sorting. Bulk density effects are considerable for bioclastic materials. Jell (et al., 1965) quantified the bulk densities (specific gravities) for various carbonate bioclastc particles (Fig. 19). Bulk densities range from approximately 1.6 to 2.9 for the various constituents that he analyzed. Variation within a particular genus type was considerable. The bulk density of the foram, *Marginopora*, ranges from 2.0 to 2.4. The bulk density of the foram, *Calcarina*, ranges from 1.62 to 2.02. Surprisingly abraded *Calcarina*

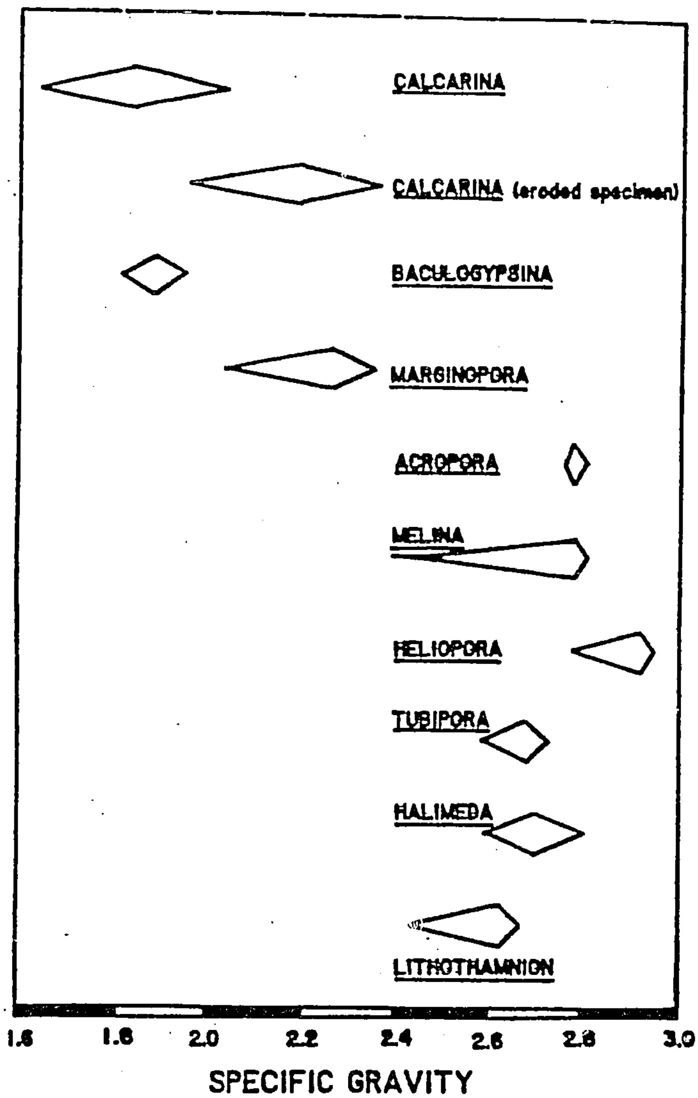

Figure 19. Specific gravities (bulk densities) for various carbonate bioclastic particles (after Jell and others, 1965; from Orme, 1977).

have bulk densities ranging from 1.98 to 2.39. Bulk density changes, within carbonate particles, are very sensitive to porosity. Therefore, porosity differences within the various broken and unbroken constituents are very complicating factors and have very strong effects on hydrodynamic size and shape sorting.

Multivariate rotation technique. Joreskog (et al., 1976) asserts that a data matrix must contain sufficient information to portray adequately the problem under study. The multivariate rotation method reduces the original data matrix of digitized X-Y coordinate pairs, which is much too large for most computers to handle as input into multivariate analysis programs, to a much smaller data matrix of equiangular radial lengths. As explained above, processed radials appear to sufficiently portray the problem under study: the gross shape types of the samples.

The standardized orientation of all shape outlines is the primary objective of the multiple rotation method. If two similar triangular shapes are to be compared, it would be meaningless to compare the widened part of the first shape with the pointy part of the other. The multiple rotation method rotates shapes in order to best fit a three by two (height to base ratio) triangle and, if certain criteria are met, flips them. It also renumbers the radials in a clockwise manner with radial #19 located on the pointed side of the grain where a best fit position with the pointed tip of the reference shape occurs. Q-mode multivariate analysis techniques can compare these similarly oriented shapes efficiently by comparing radial #19 of grain A with radial #19 of grain B with radial # 19 of grain C, et cetera.

CLUSTER ANALYSIS

The dendrogram resulting from the Q-mode cluster analysis program demonstrates the effectiveness of Parks' multivariate rotation method. Similar shapes were grouped together within distinct clusters. This computerized two-dimensional shape sieving could not have been achieved without standardizing the orientations of the sample shape outlines.

Sample comparisons. Vinopal (1976) noticed that, compared to quartzose sands, carbonate particles are extremely diverse in shape due to their biotic origin. The absence of general shape trends between or along traverses in this study was due to the many environmentally diverse areas within the backreef sub-environment and the surprisingly multiple breakage patterns of most constituent types.

Within the backreef sub-environment, complex interactions between physical and biological processes greatly complicate the understanding of sedimentary distributions, even over short distances (Hubbard, et al., 1981). Beach and subtidal areas may vary considerably. Both areas may range from very muddy, to very coarse sandy, to rocky. They may have a variety of current intensities, depths, benthic life form densities, and sediment features such as tidal deltas. Sea grass acts as a current inhibitor and sediment binder (Ginsburg and Lowenstam,

1958) and also supports an extensive micro-fauna which, in turn, exerts control on the presence of macrofauna. There are also patch reefs, an additional constituent and shape analysis complication, between the Keys and the reefs. Briefly, various areas within the backreef sub-environment vary markedly.

Chave (1964) listed many biological destructive processes. All of these occur within the backreef and include the dropping of shells on rocks by sea birds; the gnawing activities of fish, crabs, and other animals; boring activities of sponges, algae, pelecypods, and snails, burrowing activities of sediment-feeding worms, mollusks, and echinoderms; and bacterial destruction of organic integuments within calcareous skeletons. Cousteau (1963) recorded observations of fish knocking off and injesting ostrich egg-sized coral fragments, but only evacuating much finer sized material. Many of the activities of organisms result in the sorting and reworking of the sediment, such as the burrowing shrimp, Callianassa, whose mounds are very common within Hawk Channel.

An analogy to the variation of environment within the backreef sub-environment is the restricted sub-environment of Florida Bay. The backreef is, in a sense, semi-restricted in that it is bounded on one side by the Keys and on the other side by the reefs. Ginsburg (1956) stated that the rapid changes in physical and biologic environment within Florida Bay from the near-surface grass-covered mud banks to the deeper water of the adjacent rock-floored depressions are so large that they produce variations in grain size and constituent composition of the sediments as large as, or larger than, those produced by progressive changes across the bay. Results of this study are indicative of the same condition for the backreef sub-environment.

Shape and constituent-type relationships. The Sorby principal (1879) has long been accepted as an explanation of the breakage patterns of skeletal particles. It stresses that skeletal particles breakdown according to their microarchitectural structure. However, Mitchell-Tapping (1981) examined microparticles of the major constituents of carbonate sand shoals with the scanning electron microscope and found no particular breakage patterns in any of the main constituents.

As previously mentioned, coral and bivalves are especially problematic in that they easily abrade into several shape types. Speciation may very well have a considerable effect on shape type. The effects of speciation may also have been increased in this study by grouping species into major constituent-type categories. A photograph of assorted gastropods and bivalves are shown in Plate 1. From this plate it is easy to see how abrasion strongly affects the shape types of bivalves.

LIMITATIONS OF METHOD

There are two limitations to the method as used in this study. The first is the sampling pattern. A dense sampling grid covering the entire backreef sub-environment of the south Florida carbonate shelf

Plate 1. Gastropods and gastropod fragments are shown on the top half of this photograph. A bivalve and bivalve fragments are shown on the bottom. (Photograph was taken by Mary Volpi.)

would have been optimal, but probably impractical for an initial shape analysis study of carbonate sands. This study was only concerned with the general shape trends across the backreef from the beach to the reef areas.

The two-dimensional aspect of quantitative shape analysis is also a limitation. Computer hardware for automatic three-dimensional particle shape data collection is at present unavailable. Each particle shape must be reoriented by hand in order to quantitativily collect data concerning the third-dimension. However, it is quite possible that the act of taking the third dimension into account would be of immense help. Folk (1962) and Swinchatt (1965) remark that the significance of shape is little understood in the case of siliceous sediments; the problems involved in interpretations of carbonates are immeasurably more complex (Swinchatt, 1965).

FUTURE STUDIES

Factor Analysis. In this study, R-mode factor analysis was performed on a composite processed radial data set for the purpose of testing its usefulness for future studies. Joreskog (et al., 1976) states that a factor score matrix, in the R-mode, is a condensed data matrix, the columns of which are linear combinations of the original variables (radials) and the rows are the original objects (grains) in the analysis. The similarity of the dendrogram, resulting from the Q-mode cluster analysis of the factor scores, to the dendrogram resulting from the Q-mode cluster analysis of the original variables, supports his statement and shows that a few factor scores (five in this study) may be used to replace the original variables (thirty-six in this study).

The similarity between the factor loading patterns for siliceous and carbonate sands is, most likely, a result of the multivariate rotation method as modified in this study. The patterns show that only the first four or five harmonics, which are descriptive of gross shape, are important for studies of sand shapes. However, many of the earlier studies, in which Fourier analysis was used, concentrated on the higher harmonics. Factor loading patterns along with other information, such as the variance accounted for by each factor, may be important in discriminating between sediments.

CONCLUSIONS

(1) As visually determined from grain outline plots, as plotted using interpolated equiangular radial lengths, thirty-six equiangular radial lengths are sufficient to describe the shape of the maximum two-dimensional periphery (projection) of a sand particle.

(2) The multivariate rotation technique of Parks (1982) was improved upon in this study by changing the reference shape and shape outline-flipping procedure. The best rotation results were achieved by using a triangle with a height to base ratio of three to two as a reference shape. The sum of the radials on the southern side of the rotated outline (#2 through #18) versus the sum of the radials on the northern side of the outline (#20 through #36) is a adequate test for flipping.

(3) Q-mode cluster analysis, of processed radial lengths or factor scores, groups similar shapes together so that average shapes may be computed for each sample. A Q-mode cluster analysis performed on an array containing all of the average shapes from all samples, results in a dendrogram that describes which average shapes are statistically comparable

(4) The systematic utilization of Parks' multivariate rotation technique and his Q-mode cluster analysis program resulted in the computerized shape sieving of two-dimensional outlines for the purpose of analyzing shapes of carbonate sands.

(5) The geologic interpretation of shape analysis results for carbonate sands of the Florida backreef sub-environment is very difficult, due to the many possible shapes resulting from skeletal particle breakdown and the rapid and inconsistent physical and biological changes throughout the backreef.

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SAMPLE STATION	COARSER THAN (CUMULATIVE %): -1 PHI	0 PHI	1 PHI	2 PHI	3 PHI	4 PHI	MEAN	SORTING
1	1.23	11.83	51.13	88.82	99.39	99.95	.9832	.9307
2	2.46	25.57	95.17	99.87	99.98	100.00	.2588	.5994
3	62.23	91.96	98.26	99.85	99.97	100.00	-1.0690	.7329
4	3.97	33.66	74.31	90.94	94.74	98.66	.4633	1.1553
5	65.45	81.93	90.81	95.38	97.89	99.06	-.9195	1.0790
6	23.00	32.03	64.29	89.34	96.40	98.95	.3465	1.4637
7	13.06	18.27	29.05	64.84	92.83	99.31	1.2780	1.5304
8	20.63	27.34	38.55	64.46	90.74	98.26	.9870	1.7987
9	4.75	7.37	18.77	51.44	89.05	98.95	1.8597	1.2097
10	2.26	3.89	9.49	36.29	89.06	99.26	2.1356	.9281
11	1.70	3.75	13.12	31.65	90.20	99.47	2.1210	.9473
12	23.49	68.29	94.25	99.26	99.88	99.97	-.3740	.9261
13	12.19	31.51	55.89	76.77	99.96	99.99	.7559	1.4417
14	10.63	37.18	74.74	94.35	99.96	100.00	.3386	1.1199
15	8.68	33.80	71.75	93.25	99.89	99.95	.4293	1.1283
16	2.80	15.34	46.78	93.40	99.98	100.00	.9628	.9093
17	1.56	9.55	54.12	99.07	99.97	99.99	.9057	.7556
18	26.69	45.66	71.37	78.85	90.43	99.61	.4044	1.7660
19	3.17	4.63	14.21	56.40	90.10	98.31	1.9032	.9833
20	14.23	17.94	27.02	46.70	70.63	82.87	1.8935	2.1103
21	7.58	10.03	18.10	43.49	87.46	99.11	1.9362	1.3011
22	0.67	3.80	39.02	90.11	99.18	100.00	1.1472	.7630
23	0.68	3.06	29.34	80.32	98.48	99.94	1.3668	.8419
24	2.44	5.53	34.15	83.45	97.54	99.77	1.2422	.8712
25	0.54	3.75	23.44	77.07	99.60	99.96	1.4751	.8353
26	2.64	10.11	27.88	87.52	99.93	99.98	1.2144	.9004
27	15.43	35.16	57.66	77.23	96.67	99.99	.6788	1.5253
28	60.17	72.65	83.74	92.47	96.50	98.61	-.6243	1.3796
29	19.01	28.12	50.82	83.35	94.57	98.30	.6211	1.5391
30	9.67	19.49	54.32	78.35	92.33	98.21	.9749	1.4378
31	1.08	2.95	4.84	6.82	12.44	22.62	4.1657	.9446
32	16.42	43.43	94.74	99.71	99.99	100.00	-.0357	.8704
33	13.74	46.57	90.37	98.68	99.21	99.96	.0007	.9303
34	5.76	14.79	82.50	97.69	99.95	99.99	.5456	.7179
35	0.35	3.94	25.09	36.66	88.81	99.92	1.9113	1.1157
36	9.64	19.08	33.21	70.48	97.05	99.75	1.2111	1.3760
37	1.33	8.27	27.17	64.79	88.55	94.06	1.6080	1.3013

Appendix I. Results of the size analysis. The mean refers to the mean (phi) diameter and sorting refers to the inclusive graphic standard deviation, both of Folk and Wark (1957)

GRAIN #	PROCESSED RADIAL LENGTHS							
1	34.727	35.578	36.208	36.640	36.701	35.910	34.515	32.827
	31.122	29.952	29.379	29.317	29.882	30.877	32.505	34.633
	36.999	39.222	40.171	39.068	36.544	34.137	32.347	31.217
	30.634	30.335	30.404	30.885	31.847	32.943	33.970	34.816
	35.092	34.713	34.323	34.328				
2	40.680	37.350	35.062	32.813	31.540	31.216	30.575	29.533
	29.056	28.734	28.974	29.202	30.355	31.393	33.365	36.734
	40.934	43.863	44.345	40.615	36.754	33.166	29.404	28.156
	28.589	28.899	27.847	26.103	24.876	25.818	28.423	32.586
	37.492	41.365	43.499	42.890				
3	36.032	37.084	37.876	38.084	36.976	34.648	32.416	30.248
	28.766	28.162	28.947	32.473	33.821	33.625	33.219	33.378
	33.872	37.318	40.624	41.137	37.724	32.417	29.478	28.997
	29.630	31.008	32.927	33.696	34.132	34.253	33.527	32.769
	32.555	32.487	33.554	34.824				
4	39.741	40.791	40.640	39.159	36.793	33.648	30.676	28.323
	26.821	26.112	25.903	26.383	27.572	29.346	31.979	35.519
	38.752	42.445	44.756	43.731	39.996	35.225	31.420	29.103
	27.646	26.944	27.011	27.038	28.044	29.738	31.861	33.447
	34.566	35.497	36.786	38.099				
5	30.324	30.944	32.413	35.315	38.222	39.134	38.083	35.523
	33.017	30.323	29.014	28.710	29.610	30.982	32.592	34.533
	36.761	39.779	41.599	40.632	37.177	33.248	30.290	28.249
	26.537	26.522	27.974	30.725	34.096	37.475	39.241	38.909
	36.939	34.413	31.821	30.556				

Appendix 2. Example of processed radial sets for five sand grain outlines.

CONSTITUENT TYPE	SHAPE TYPE A	B	C	D	E	F	G	H
HAL	19.0	8.2	16.6	25.0	15.0	23.7	24.4	6.8
CORALG	8.2	8.2	2.9	5.1	12.5	2.0	0.0	24.2
COR	35.9	44.9	37.4	34.0	45.0	39.6	40.0	20.8
BIV	17.6	26.5	24.7	15.4	15.0	26.3	24.4	32.1
GAS	2.1	6.1	3.6	10.9	2.5	2.5	4.4	4.2
DAS	1.9	0.0	3.4	8.3	2.5	0.7	0.0	7.5
FOR	26.1	2.0	9.2	0.6	2.5	3.4	2.2	0.0

Appendix 3. The percentages of constituent types accounted for within each final average shape.

VARIABLE (PROCESSED RADIAL)	MEAN	RANGE
1	37.606	27.019 - 59.3730
2	37.738	28.883 - 52.381
3	37.515	29.855 - 47.599
4	36.871	28.910 - 49.132
5	35.760	25.095 - 46.607
6	34.186	21.884 - 44.485
7	32.264	19.903 - 42.099
8	30.371	18.502 - 37.993
9	28.812	17.701 - 35.873
10	27.790	17.669 - 35.544
11	27.317	17.605 - 34.304
12	27.411	17.982 - 34.739
13	28.119	18.733 - 34-527
14	29.500	20.957 - 35.121
15	31.617	23.920 - 38.052
16	34.637	26.458 - 42.278
17	38.611	32.067 - 51.459
18	42.945	34.556 - 63.484
19	45.373	35.084 - 71.114
20	43.353	33.378 - 59.028
21	38.214	21.525 - 49.482
22	33.237	14.907 - 41.845
23	29.719	13.141 - 41.707
24	27.452	11.140 - 36.015
25	26.135	9.617 - 36.047
26	25.672	9.535 - 35.514
27	25.902	9.637 - 37.518
28	26.817	9.998 - 37.535
29	28.276	10.194 - 39.893
30	30.249	11.667 - 41.127
31	32.685	13.152 - 41.965
32	35.170	17.266 - 45.397
33	36.859	25.796 - 51.738
34	37.456	27.500 - 60.941
35	37.444	25.747 - 62.422
36	37.397	24.451 - 60.960

Appendix 4. The mean and range of each ordered variable from the test data set used as input into R-mode factor analysis.

VARIABLE (RADIAL)	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5
1	-.914	-.086	.292	-.172	-.137
2	-.834	.191	.407	-.039	-.250
3	-.572	.534	.422	.211	-.255
4	-.074	.779	.215	.403	-.106
5	.414	.747	-.111	.332	.019
6	.695	.538	-.320	.086	.182
7	.818	.289	-.390	-.140	.153
8	.879	.072	-.344	-.262	.063
9	.905	-.130	-.212	-.289	-.038
10	.899	-.295	-.054	-.239	-.109
11	.870	-.429	.072	-.120	-.137
12	.819	-.521	.168	.031	-.118
13	.726	-.588	.239	.200	-.033
14	.577	-.615	.289	.346	.150
15	.274	-.602	.257	.409	.419
16	-.270	-.466	.107	.305	.617
17	-.821	-.149	-.133	-.017	.385
18	-.903	-.026	-.294	-.189	.007
19	-.847	-.042	-.355	-.148	-.205
20	-.821	-.019	-.297	.098	-.395
21	-.299	.168	.189	.550	-.270
22	.281	.302	.483	.405	.130
23	.490	.358	.527	.160	.295
24	.600	.367	.525	-.045	.326
25	.663	.374	.509	-.200	.280
26	.709	.379	.443	-.276	.194
27	.746	.381	.337	-.290	-.065
28	.777	.379	.210	-.236	-.079
29	.819	.335	.061	-.135	-.205
30	.849	.241	-.117	.008	-.270
31	.820	.081	-.343	.198	-.227
32	.572	-.223	-.521	.378	-.004
33	.042	-.544	-.516	.342	.272
34	-.543	-.646	-.299	.071	.335
35	-.835	-.490	-.045	-.132	.168
36	-.914	-.294	.143	-.198	.009

Appendix 5. The unrotated factor loadings as derived form maximum-liklihood factor analysis and plotted in Figure 18.

VITA

Martin E. Mengel was born to Mr. and Mrs. Martin E. Mengel Jr. in Easton, Pennsylvania on November 11, 1959. He resided in Nazareth, Pennsylvania and graduated from Nazareth Area Senior High School in 1977. He recieved a Bachelor of Arts degree in Geology from Kutztown University of Pennsylvania in 1981.

From July 1981 through May 1982 he was employed by Exploration Logging Inc., U.S.A. as a geologist. On June 12, 1982 he married Sheila Evans of Reading, Pennsylvania. Also, in June 1982 he entered Lehigh University as a research assistant and received a Master of Science degree in Geological Sciences in June 1985.