# Cable assembly shop throughput as a function of the point of order identity: 

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CABLE ASSEMBLY SHOP THROUGHPUT AS A FUNCTION OF THE POINT OF ORDER IDENTITY: AN LP APPROACH
by
Jerry H. Johnson


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## ABSTRACT

Within a flow assembly shop a multiple component item can be identified for specific customer orders at the start or end of the assembly production process. If customer orders are identified at the beginning, then raw material is dedicated at this point for the manufacturing of these orders. An alternative method would be to move the point of customer order identity to the end of the assembly process. The assembly shop would then become a generator of unallocated inventories against which customer orders could be matched. This point of identity, when used to specify specific customer orders, affects both machine throughput and in-process inventory levels. To date, there have been no studies made to determine which approach, if either, affords the most optimal shop operation.

To study the effect of moving the point of customer order identity a mathematical model was developed, and actual shop data from a Western Electric Cable Plant was used to determine the model's feasibility. An extension of a mathematical model presented by Kornbluth and Lepage [23] was used in the formulation of the cable shop under consideration. The solution of the model was via a restricted entry linear programming computer program.

The measure of effect on shop performance was machine throughput.

The results of this initial investigation for a limited number of cable core types indicated that machine throughput could, in fact, be increased over planning horizons of 24 and 36 hours when the point of customer order identity was moved to a later operation. This increase in throughput was due, in part, to a reduction of set-up changes with a slight increase in unallocated in-process inventories. With a planning horizon of 48 hours greater machine throughput was realized, with an increase in machine idle time at the second assembly operation, by leaving the point of customer order identity at the beginning of the assembly operation.

## CHAPTER I

## INTRODUCTION

Consider a cable manufacturing plant which is divided into two areas (see Figure 1), the wire shop and the assembly shop. Although the areas are physically under the same roof there is a distinct difference in the nature of their operation. The wire shop produces twisted wire for inventory which is the buffer stock between the two shops, and the assembly shop generates customer orders that are due to be delivered within a one week horizon time. Theoretically, this means the assembly shop does not create unallocated in-process or finished goods inventories.

The first operation in the wire shop is the reduction of copper rod to 12 or 13 gauge wire. This intermediate size wire is further reduced to a final size of 19, 22,24 , or 26 gauge, and a plastic insulation is applied at a tandem insulating operation. Finally, the single conductors are twisted together to form pairs. In the assembly shop the twisted pairs are again twisted together to form units or small cable cores with a range in size of 6 to 100 pairs at the stranding operation. If larger cable cores are desired the units are again twisted together at the cabling operation to form large
cable cores with up to 900 pairs. The cable cores, large and small, are then sent to the sheathing operation where a protective sheath is applied to form a finished product.


Figure 1
PRODUCT FLOW IN CABLE MANUFACTURING

- The twisted wire inventory must be maintained in such a manner as to ensure the stranding operation has the right twisted pairs available. Inventory control is accomplished by means of a simple "smoke-stack" procedure. At the start of each shift a physical inventory report is posted on a board for each twish length. If the inventory for a particular twist length is less than a certain level, then set-up changes are made from those twisting machines whose inventory is not being drawn from to increase the reduced inventory. This is a simple means by which the wire shop can plan its production so the assembly shop has the right type of wire for its operation.

As stated previously, the assembly shop production is based on customer orders that are identified at the stranding operation. As shown in Figure 1, an alternative method would be to move the point of identity to the sheathing operation. The stranding and cabling operations would now become generators of inventory against which specific orders are matched at the sheathing operation. In other words, the boundaries of the wire shop would be extended into the assembly shop. To date, there have been no studies made to determine which approach, if either, affords the most optimal sequencing schedule. Since orders are now sequenced through the three assembly
operations, this ensures, at the expense of shop efficiency, that service is met with a minimum of in-process and finished goods inventories.

It is worth pointing out that the wire shop maintains high throughput and capacity utilization because their operation does not depend directly on specific customer orders. Their main concern is generating buffer inventory which is later applied to orders.

The objective of this investigation is to measure the effect of moving the point of customer identity from the stranding to the sheathing operation. A measure of the effect is the extent the assembly shop increases throughput and capacity utilization. Multi-unit cables will be split into sub-units which, in turn, would be handled as individual jobs. As a tool for analysis, a mathematical model of the stranding and cabling operation is formulated. Since inventory and set-up times have a direct effect in determining the feasibility of moving the point of job identity, the article by Kornbluth and Lepage [23] leads itself, with modification, to such an investigation. In their multiple stage, multiple parallel machine continuous flow production model the question the model attempted to answer was, can a minimum number of items be produced in a given schedule horizon? And if. so, how many more could be produced before the system
capacity were reached? Set-up times and in-process inventory were included in their model for determining the optimal sequence. The solution to the model was obtained using a restricted entry technique similar to the one used in separable programming. This technique overcomes the size limitations set by zero-one integer programming formulations in the solution of large realistic problems.

Because actual shop data from a cable manufacturing plant will be used to evaluate the model, the results will be scaled to preserve their proprietary nature. While the absolute meaning of the results is destroyed, the evaluation of the feasibility of the model for the particular application remains intact.

## CHAPTER II

## REVIEW OF THE STATE OF the art

Sequencing has been a topic of considerable research in recent years. Though the basic problem of optimally sequencing production is generally the same, the individual research differs significantly by the assumptions made with respect to the production system and the nature of the work to be performed within the system. Usually the system to be studied can be described by making the appropriate choice from each of the following five classifications:

1. Type of production environment.
a. Flow shop
b. Job shop
2. Jobs available for processing.
a. N-finite deterministic jobs
b. An undetermined number of jobs arriving continuously, but randomly. This is often referred to as a stochastic system.
3. Number of component parts comprising a job.
a. Single-component jobs
b. Multiple-component jobs
4. Number of stages a component part passes through until a desired state is reached. a. Single-stage operation
b. Multiple-stage operations
5. Number of machines within a given stage.
a. Single-machine stage
b. Multiple-parallel machine stages

Thus a (1a-2a-3a-4a-5a) classification of a system would describe one in which $N$ deterministic single-component jobs are to be sequenced through a one machine, one stage flow shop. In the literature most of the work that has been done with regard to the sequencing problem appears to be of the (1a-2a-3a-4a-5a), (1a-2a-3a-4a-5b), (1a-2a-3a-4b-5a), and (1a-2a-3b-4a-5b) variety. For representive examples of these systems see Bowmann [5], Dantzig [6], Dudek and Ghare [8], Elmaghraby [11], Gilmore and Gomory [14], G1assey [15], Manne [25], Smith [31], and Young [35].

A stochastic system differs from a deterministic system in that probabilistic elements enter intof the formulation in one of the three forms: (1) the set of $N$ jobs is dynamically varying in a stochastic fashion, (2) the requirements of each job (concerning route, processing times, engineering content, etc.) vary stochastically, (3) the characteristics of the processors
(availability, suitability, number of processors, and so forth) change stochastically [10]. The order in which the machine numbers appear in the operation of individual jobs determines whether a shop is a flow shop or a job shop. A flow shop is one in which all the jobs follow essentially the same path from one machine to another. This is contrasted by the job shop where each job has its own individual route over the machines in the shop.

Four of the basic approaches to the solution of the deterministic sequencing problems are: (1) Combinatorial analysis, (2) Graphical analysis, (3) Heuristic algorithms, (4) Mathematical programming. Combinatorial approaches are based on the changing of one permutation to another by "switching around" of jobs that satisfy a given criterion. The fundamental concept in this approach can best be expressed by a theorem which was developed by Smith [31]. Their efficiency depends on how effectively enumeration is curtailed. To date, the effort with these approaches has proved the most successful in the search of exact solutions. Literature references for combinatorial approaches include papers by Bellman [4], Gapp, Mannkekar, and Mitten [12], Gilmore and Gomory [14], and Smith [31].

Graphical approaches are based upon a geometric interpretation of feasible schedules that are represented by paths in an $N$-dimensional rectangle. The algorithm is not limited by the number of machines, but it becomes unwieldy as the problem size increases. Hardgrave and Nemhauser [18] developed the approach for a 2 -job M-machine problem but were quick to point out that hand computations appeared to be practical for at most three jobs.

According to Elmaghraby [10] heuristic approaches are based on two principle concepts:

1. The use of controlled enumeration techniques for considering all potential solutions.
2. The elimination from explicit consideration of particular potential solutions which are known to be unacceptable.

Heuristic algorithms have presented the best approaches, with regard to computational effort, to very large problems yielding near optimal solutions but with no guarantee of optimality. If carried to completion, they do guarantee the discovery of an acceptable solution if one exists, or the knowledge that none exist. Their basic advantage has been the relatively small computation effort required for any size problem. Tonge [32] suggests that this approach is at best an art since there is no
underlying analytic framework.
General mathematical programming approaches include linear, dynamic, convex, quadratic programming, integer programming, networks of flow, Lagrangian methods, and the like. The following are representative of the great number of articles that have been published on this subject: Bowman [5], Dantzig [6], Dudek [8], Elmaghraby [11], Glassey [15], Gorenstein [16], Greenbery [17], Harris [19], Kornbluth and Lepage [23], Manne [25], Palmer [27], Rothlsofp [28], Senju and Toyoda [29], Von Lanzenover [33], and Wagner [34]. Mathematical programming approaches seem to have great potential for the solutions of the general problem. Linear programming and zero-one integer programming seem to be the most commonly used mathematical programming approaches used in attacking scheduling problems.

A principal characteristic of all algorithms, aimed at the solution of sequencing problems, is the magnitude of the computation effort involved. Although this effort is very small compared to total enumeration, it increases very rapidly as the size of the problem increases. According to Gere [13] and Sisson [30], to overcome this dimensionality problem most prior sequencing formulations which are carried to solution impose in part or in whole the following simplifying assumptions:

1. There are no random or uncertain elements.
2. The time to process each job on each machine is known.
3. The technological ordering for each job is given. Once the job routing is given no alternative routings are permitted.
4. Each job is an entity, even though it might be composed of individual parts. This eliminates "Job Splitting" between machines. It also eliminates assembly operating.
5. A machine may not process more than one job at a time.
6. Once a machine has begun to process a job, it must complete that job before starting on another.
7. Manpower of uniform ability is always available and machines never break down.
8. Due dates are known and fixed.
9. There is only one of each type of machine in a process.

Research in the one machine deterministic sequencing problems where the above assumptions have been imposed has been extensive. According to Elmaghraby [7], the optimal sequence has been found that minimizes the (1) maximum tardiness, (2) weighted sum of completion times,
(3) weighted sum of tardiness, (4) total cost of tardiness, (5) total penalty if the jobs are "related" to each other, (6) total setup time or setup cost (when either is sequence-dependent), (7) total cost of processing when the processor is characterized by a single state-variable, (8) number of changeovers when the products are subject to a demand schedule, (9) total cost of production to produce, but independent of sequence. Undoubtedly the study of the single machine case has shed light on the more complicated multiple machine problem.

Jobs can be identified at either the start or end of a process. This point of job identity, when used to specify specific customer orders for multi-component items, will determine machine utilization and in-process and finished goods inventory costs. To date no one has specifically examined the problems associated with and trade offs of job identity within a process. For an extensive review and bibliography of sequencing see [7] and [10].

## MATHEMATICAL MODEL FORMULATION

## Problem Formulation

The material flow between the stranding (Stage I) and cabling (Stage II) operations can be illustrated as shown in Figure 2 . Cable units which are produced


Figure 2
MATERIAL FLOW
at Stage $I$ go into intermediate storage between the two stages. These units remain in inventory until there is sufficient stock to construct a particular cable core. Output, a standard linear footage based on gauge and pair size, of the two stages is taken up on unit and core trucks.

Sequencing through the system is constrained by the machine capacity at each stage, the amount of in-process inventory which is allowed to accumulate between the stages and minimum and maximum number of cable cores which are required in a given planning horizon.

In the subsequent formulation of the model, the notation and definition of terms will be, where applicable, consistent with the continuous flow model presented by Kornbluth and Lepage [23]. To handle the case of a discrete-multiple unit process some of the continuous flow constraint equations in the original model were modified and a constraint was added for the removal of twisted wire.

The production capacity of a machine for each discrete time period is divided into two parts. (1) a penalized capacity which is equivalent to the maximum throughput for a period less the throughput lost during
a set-up change, and (2) a changeover capacity which is equivalent to the capacity lost during a set-up. Thus a machine's throughput is given by the equation:
penalized capacity + changeover capacity $=$ throughput.

A set-up in a given period will not be required if the following two conditions hold: (1) only one material has been scheduled for production in the period under consideration, and (2) the machine was working on the same material at the end of the previous period. By utilizing a restricted entry into the basis for the linear programming solution, the model is able to consider set-up times (changeover capcaity) in its attempt at optimization. The changeover capacity will be able to enter the basis only if the above two conditions hold. The machines throughput, if a set-up is not required, is given by equation (1). If a set-up change is required in the period on the machine specified, then the changeover capacity is not allowed to enter the basis, and the machine capacity is given by the equation:

> penalized capacity < throughput

In the development and solution of the problem it was assumed that all units and cable cores could be
produced on all machines in the respective stages. In the system this model was developed for, the above assumption is realistic. For other applications this restriction could be relaxed with minor adjustments in the constraints of the model.

## Notation

(1) Let $n$ be the number of equal discrete time periods in a given make-span of total length N, $\quad n=1,2, \ldots, N$.
(2) A set of $L$ different cable cores are manufactured from a combination of $J$ units. Cable cores are differentiated by number of units and gauge.
(3) $C_{j \ell}$ is the number of units $j$ required to assemble a particular cable code $\ell$.
(4) $S_{j}$ is the number of units $j$ which can be processed immediately at Stage II at the beginning of the sequencing run.
(5) $G_{j}$ is the upper limit on the number of units j which can be in storage at the end of each time period $n$. The maximum total intermediate storage of all units j at any time is limited to $G$.
(6) $E_{\ell}{ }^{M I N}$ and $E_{\ell}{ }^{M A X}$ are the minimum and maximum limits placed on the cable cores to be produced in a given planning horizon. The minimum and maximum units are denoted by $M_{j}{ }^{\text {MIN }}$ and $M_{j}{ }^{\text {MAX }}$. The relationship of these limits is given by:

$$
\begin{aligned}
& M_{j}^{M_{j}} \geq M_{j}^{M I N} \\
& M_{j}^{\text {MAX }} \leq \min \begin{array}{l}
\left\{\begin{array}{l}
\text { number of unit trucks } \\
\text { available, machine } \\
\text { capacity }\}
\end{array}\right.
\end{array}
\end{aligned}
$$

$$
\mathbf{E}_{\ell}^{\text {MAX }} \geq \mathrm{E}_{\ell}^{\text {MIN }}
$$

$$
E_{\ell}{ }^{M A X} \leq \min \text { \{demand during make span, }
$$ number of core trucks available, machine capacity\}

$$
\begin{aligned}
& C_{j \ell} *\left(E_{\ell}^{M A X}\right) \leq M_{j}^{M A X}+S_{j} \\
& C_{j \ell} *\left(E_{\ell}^{M I N}\right) \leq M_{j}^{M I N}
\end{aligned}
$$

(7) Let $M_{i j}$ represent a penalized throughput capacity for machine $i$ and unit $j$ in Stage $I$. $M_{i j}$ is equivalent to the maximum output in one time period less the lost production due to a set-up change in that period. $Q_{i j}$ represents a changeover capacity for machine $i$ and unit $j$ in Stage $I, Q_{i j}$ is equivalent to the lost output during a set-up change.
(8) Similarly $E_{k \ell}$ and $Z_{k \ell}$ refer to the penalized and changeover capacities when processing
cable core $\ell$ on machine $k$ in Stage II. $E_{k \ell}$ is equivalent to the maximum output in one time period less the lost production due to a set-up change in that period. $Z_{k \ell}$ is equivalent to the lost output during a set-up change.
(9) Let $D_{g n}$ be the number of twisted reels in inventory ahead of the stranding operation of gauge $g$ in period $n . A_{g j}$ is the number of twisted reels of gauge $g$ required to make one unit $j$.

## Variables

The variables of the model in Stage $I$ are:
$T_{i j n}$ represents the fraction of machine i's penalized capacity which is allocated to processing unit $j$ in period $n$ for Stage $I:$

$$
\begin{equation*}
0 \leq T_{i j n} \leq 1, \text { for all i,j,n. } \tag{3}
\end{equation*}
$$

$R_{i j n}$ represents the fraction of machine i's changeover capacity (added throughput) which is allocated to unit $j$ in period $n$ for Stage $I$ if a set-up is not required:

$$
\begin{equation*}
0 \leq R_{i j n} \leq 1, \text { for all } i, j, n \tag{4}
\end{equation*}
$$

$T_{i j n}$ and $R_{i j n}$ are tied by the restricted entry condition:

$$
R_{\text {ijn }}\left[\begin{array}{l}
>0 \\
=0
\end{array}\right] \text { implies } T_{i j n}\left[\begin{array}{l}
=1 \\
<1
\end{array}\right] \text {, for all } i, j, n .
$$

Thus no changeover capacity can be claimed, a set-up will be required, unless the program chooses to schedule unit $j$ on machine $i$ for the entire period $n$. The variables of the model in Stage II are: $P_{k \ell n}$ represents the fraction of machine $k^{\prime} s$ penalized capacity which is allocated to processing cable core $\ell$ in period $\mathfrak{n}$ for Stage II:

$$
0 \leq P_{k \ell n} \leq 1, \text { for all } k, \ell, n
$$

$W_{k \ell n}$ represents the fraction of machines $k$ 's changeover capacity which is allocated to cable core $\ell$ in period $n$ for Stage II if a set-up is not required:

$$
0 \leq W_{k \ell n} \leq 1, \text { for all } k, \ell, n
$$

$P_{k \ell n}$ and $W_{k \ell n}$ are tied by the restricted entry

$$
W_{k \ell n}\left[\begin{array}{l}
>0 \\
=0
\end{array}\right] \text { implies } P_{k \ell n}\left[\begin{array}{l}
=1 \\
<1
\end{array}\right] \text {, for all } k, \ell, n .
$$

Thus a changeover capacity (added throughput) cannot be claimed, a set-up will be required, unless the program chooses to schedule cable core $\ell$ on machine $k$ for the entire period n.

In all equations summation is over all possible values of the indices unless otherwise noted (e.g., $\left.\sum_{j}=\sum_{j=1}^{J}\right)$.

## Constraints

The constraint equations of the model are as follows

1. Machine Availability. The fractions of a machine's capacity in any period that is allocated to each product must total up to one or less.

Thus for Stage $I$,
and similarly for Stage $I I$,

$$
\begin{equation*}
\sum_{\ell} P_{k \ell n} \leq 1, \quad \text { for all } k, n \tag{8}
\end{equation*}
$$

These equations allow, due to fractional quantities, more than one product to be scheduled on a machine in a given time period. Ideally of course one would like as many $\mathrm{T}_{\mathrm{ijn}}{ }^{\prime} \mathrm{s}$ and $\mathrm{P}_{\mathrm{k} \mathrm{\ell n}}{ }^{\prime} \mathrm{s}$ as possible to be at their upper limit of 1 , so that set-up changes are not required.
2. Changeover Allowance. Added capacity can be obtained in the time period $n$ if a set-up on machine $i$ is not required. The changeover equations for Stage $I$ are:

$$
\begin{equation*}
-T_{i j(n-1)}+R_{i j n} \leq 0, \quad \text { for all } i, j, n \tag{9}
\end{equation*}
$$

$R_{i j n}$ can only enter the basis if $T_{i j n}=1$. The following two conditions must be considered for equation (9):
(a) If $0 \leq T_{i j n}<1$, then $R_{i j n}=0$. A changeover capacity cannot be claimed in period $n$ on machine $i$ because a set-up will be required to process another material $j$ on machine $i$ in period $n$.
(b) If $T_{i j n}=1$, then $R_{i j n} \geq 0$ because a set-up was not required in period $n$ on machine $i$. $R_{i j n}$ can only be greater than zero if the material $j$ was scheduled on machine $i$ in the previous period. This is covered by the term:

$$
T_{i j}(n-1)
$$

which can take values between 0 and 1 . If:

$$
T_{i j(n-1)}=0,
$$

then

$$
R_{i j n}=0,
$$

because a set-up will have to be made in period $n$ before machine $i$ can start to process material $j$.

On the other hand, if:

$$
T_{i j(n-1)}=1
$$

then

$$
R_{i j n}=1,
$$

and an added throughput can be claimed because the model chose to continue processing the same material for two consecutive time periods. If:

$$
0<T_{i j(n-1)}<1
$$

then

$$
R_{i j n} \leq T_{i j(n-1)}
$$

because the machine had been working on material $j$ for part of the previous period, and has scheduled to process material $j$ for the entire current period. Since $R_{i j n}$ can claim an amount equal to the proportion of the time spent by machine $i$ on material
$j$ in the previous period, this is at best a conservative approximation to the real system.

The state of the system at the beginning of the planning horizon can be handled by setting the indices in equation (9) to:

$$
R_{i j 1} \leq T_{i j 0}
$$

The optimizing LP mechanism has a choice to continue to process material $j$ on machine $i$ in period 1 with a changeover capacity equal to one.

The changeover equations for Stage II are:

$$
\begin{equation*}
-P_{k \ell(n-1)}+W_{k \ell n} \leq 0, \text { for all } k, \ell, n, \tag{10}
\end{equation*}
$$

which act in the same manner as discussed for Stage $I$.
3. Units Processed. The total number of units which can be processed in Stage $I$ is constrained by:

$$
\begin{equation*}
\sum_{i} \sum_{n} M_{i j} T_{i j n}+Q_{i j} R_{i j n} \leq M_{j}^{M A X}, \text { for all } j \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
\sum_{i} \sum_{n} M_{i j} T_{i j n}+Q_{i j} R_{i j n} \geq M_{j}^{M I N} \text {, for all } j \tag{12}
\end{equation*}
$$

4. Cable Cores Processed. The total number of cable cores which can be processed in Stage II is constrained by:

$$
\begin{equation*}
\sum_{k} \sum_{n} E_{k \ell} P_{k \ell n}+Z_{k \ell} W_{k \ell n} \leq E_{\ell}^{M A X}, \text { for all } \ell \text {, } \tag{13}
\end{equation*}
$$

and

$$
\begin{equation*}
\sum_{k} \sum_{n} E_{k \ell} P_{k \ell n}+Z_{k \ell} W_{k \ell n} \geq E_{\ell}{ }^{M I N} \text {, for all } \ell . \tag{14}
\end{equation*}
$$

5. Cable Core Processing. At the end of time period $n$ the total amount of cable cores that can be processed through Stage II must be less than or equal to the total number of units that were processed through Stage $I$ in the previous time periods plus the starting stock. Therefore,

$$
\begin{align*}
& \quad \sum_{k} \sum_{\ell} \sum_{n}^{n} C_{j \ell}\left[E_{k \ell} \stackrel{P}{P}_{k \ell n}+z_{k \ell} W_{k \ell n}\right]- \\
& {\left[\sum_{i} \sum_{n}^{n^{\prime}} M_{i j} T_{i j(n-1)}+Q_{i j} R_{i j(n-1)}\right] \leq s_{j}, \text { for }} \\
& \quad \text { all } j, \text { and } n^{\prime}=1,2, \ldots N . \tag{15}
\end{align*}
$$

6. In-Process Inventory Leve1. The number of units of each code which are allowed to remain in inventory at the end of a time period is given by:

$$
\begin{align*}
& \sum_{i} \sum_{n}^{n^{\prime}} M_{i j} T_{i j(n-1)}+Q_{i j} R_{i j(n-1)^{-}} \\
& {\left[\sum_{k} \sum_{\ell} \sum_{n}^{n^{\prime}} C_{j \ell}\left[E_{k \ell} P_{k \ell n}+Z_{k \ell} W_{k \ell n}\right]\right] \leq G_{j},} \\
& \quad \text { for all } j, \text { and } n^{\prime}=1,2, \ldots, N . \tag{16}
\end{align*}
$$

The cumulative amount of in-process inventory can not be greater than the total available storage space, Thus:

$$
\begin{align*}
& \sum_{i} \sum_{j} \sum_{n}^{n^{\prime}} M_{i j} T_{i j n}+Q_{i j} R_{i j n}- \\
& {\left[\sum_{k} \sum_{\ell} \sum_{n}^{n^{\prime}} C_{j \ell}\left[E_{k \ell} P_{k \ell n}+Z_{k \ell} W_{k \ell n}\right]\right] \leq G,} \\
& \quad \text { for } n^{\prime}=1,2, \ldots, N . \tag{17}
\end{align*}
$$

7. Twisted Wire Inventory Removal. The number of units $f$ which can be made in time period $n$ cannot be greater than the amount of twisted wire, which is in inventory at that time. Thus:

$$
\begin{align*}
& \sum_{i} \sum_{j} \sum_{n}^{n^{\prime}} A_{g j}\left[M_{i j} T_{i j n}+Q_{i j} R_{i j n}\right] \leq \sum_{n}^{n^{\prime}} D_{g(n-1)^{\prime}} \\
& \quad \text { for all g and } n^{\prime}=1,2, \ldots, N . \tag{18}
\end{align*}
$$

8. The Common Non-Negativity LP Conditions Must Hold.

Thus:

$$
\begin{equation*}
T_{i j n}, R_{i j n} \geq 0, \text { for all } i, j, n, \tag{19}
\end{equation*}
$$

and

$$
\begin{equation*}
P_{k \ell n}, W_{k \ell n} \geq 0, \text { for all } k, \ell, n \tag{20}
\end{equation*}
$$

Objective Function
The objective function of the model is to maximize throughput and is given by
$\sum_{i} \sum_{j} \sum_{n} M_{i j} T_{i j n}+Q_{i j} R_{i j n}+\sum_{k} \sum_{\ell} \sum_{n} E_{k \ell} P_{k \ell n}+Z_{k \ell} W_{k \ell n}$. It stands to reason, that this objective cannot be accomplished if set-up changes are excessive. Therefore, with the given constraints, the model will try to set machines up with long production runs of the same type of product. The results obtained with this model will be discussed in the next chapter.

For the case under investigation the maximization of throughput is consistent with present operating conditions. Management is concerned on a daily basis with output at the various cable operations. The objective function could be changed to minimize make span by assigning to each of the processing variables $\left(T_{i j n}, P_{k \ell n}\right)$ and changeover variables $\left(R_{i j n}, W_{k \ell n}\right)$ a cost term which is proportional to the throughput generated by each variable and which increases with time. The computer program would have to be changed to minimize total cost which would result in the termination of production runs at the earliest possible time with minimum output requirements being met.

Equations (15), (16), and (17) were modified, from the original model presented by Kornbluth and Lepage [23], to handle the case of a discrete-multiple unit process. This was accomplished by changing the time
subscript from $n$ to $n-1$ for the Stage $I$ variables in equations (15) and (16), and the component variable $C_{j \ell}$ was added to the Stage II variables in all three equations. Also the relationship between the maximum and minimum production limits $E_{\ell}{ }^{M A X}, E_{\ell}{ }^{M I N}, M_{j}{ }^{M A X}$, $M_{j}{ }^{\text {MIN }}$ was established so their interaction was consistent with the technological configurations of a cable shop. Equation (18) was added to handle the case of twisted wire inventory removal. All other equations are the same as presented by Kornbluth and Lepage [23].

## CHAPTER IV

## COMPUTATIONAL PROCEDURES

To evaluate the effect of the two methods of identifying customer órders (hereafter referred to as STPID for identity at stranding and SHPID for identity at sheathing) thirteen measures of effectiveness were calculated and summarized in Table 2. These measurements are as follows:
(I) Stage I results.

1. Set-ups
2. Units output
3. Units output/Processing time
(II) Stage II results
4. Set-ups
5. Cable cores output
6. Unit of equivalent cable core output
7. Million conductor feet (MCF) output
8. Unit output/Processing time
9. MCF output/Processing time
10. Unit in-process inventory after period 1
11. Unit in-process inventory after period 2
(III) System results
12. Total units output
13. Total units output/Total processing time

The common measure of output in Western Electric Cable Shops, million conductor feet (MCF), is given by the expression:

$$
M C F=(2 * \text { No. of Pairs * Linear Footage }) / 10^{6}
$$

Processing time is the length of the planning horizon times the number of machines in the stage.

Results for five schedules (nos. 1 thru 5), each with varying model input parameters, were evaluated. Scaled processing rates, job configuration, length of processing intervals, initial starting stocks, number of machines in each stage, and the amount of material (min. and max. limits) required to be produced in each stage for each schedule are given in Appendix A. Within each schedule two passes were made, one each for SHPID and STPID jobs. Schedules 1, 2, 3 and 4 were executed using an objective function designed to maximize output from both stages. The results of these schedules led to Schedule 5 which used an objective function designed to maximize output through stage II on1y.

Although the model has the capability for initial machine set-ups (see equations 9 and 10) none were used in the five schedules to avoid any bias in the results. Also to avoid bias, initial starting stocks was used in Schedule 2 only. In all schedules the minimum limit of material to be processed was set equal to one. The maximum limits on materials were varied between schedules and were based, in part, on actual weekly loads.

Due to computer core limitations, the number of intervals were limited to three and the number of different cable codes sequenced was set at six. The six codes, from a possible set of 24 codes that have to be cabled, required over $50 \%$ of the cabling effort in the month under study. To determine which method (SHPID or STPID) would allow the most material to be sequenced in the given planning horizon, the length of the processing intervals was set relatively short in comparison to the amount of material that had to be processed. Even though material produced in the last time period is not allocated (in the current sequence) to Stage II, it can be used as initial starting stock if subsequent schedules are dovetailed. Therefore, Stage I results are based on all three time periods while Stage II results are based on time Periods 2 and 3. In this way, the effectiveness of sequencing at Stage $I$ in the first
two time periods can be determined. If the units are not made in Stage $I$, they cannot be scheduled at Stage II.

The initial basic solution to the LP Freblem is given with the penalized and changeover variables set at their lower bound of zero. As the model goes to optimality, changeover variables ( $\mathrm{R}_{\mathrm{ijn}}, W_{k \ell n}$ ) are forced in and out of solution according to the zalues assigned to the penalized variables ( $\mathrm{T}_{\mathrm{ijn}}, \mathrm{P}_{\mathrm{k} \ell \mathrm{n}}$ ).

Since the LP technique used to solve the various schedules gives fractional answers for amount of product to be produced, a rounding rule was applied to the solutions to give integer values. A fractional quantity (e.g., cable cores) at Stage II was rounded to the next highest value if sufficient material for that code was produced in Stage $I$ during the appropriate time periods; if hot, the quantity of material was rounded down. Fractional units that were at most .5 were rounded up in Stage $I$ if they could be processed within Periods 1 and 2. Fractional units of . 5 or greater were rounded up in the last time period and allowed to extend past the planning horizon. The processing times were adjusted to reflect the rounding. The results of the different sequences were based on the planning horizon only. Therefore, those portions of jobs which
exceeded the last planning period were not included in the results.

Solution of the schedules was executed on an IBM 370/145 Computer, and the results were analyzed on a DEC PDP-10 Computer. Program descriptions are given in Appendix $B$.

The results of applying the two methods (SHPID and STPID), by schedule, are given in Table 2. In general, for the cases considered, it can be concluded that with a planning horizon of 24 and 36 hours (Schedules 1,2 and 4) the SHPID Method allowed more units to be processed in the given time frame. With a 48 hour planning horizon (Schedule 3) greater system output is realized with the STPID Method. In Schedule 5 for a 48 hour planning horizon and an objective function to maximize throughput (units) for Stage II only, the system output was the same for both methods. In all schedules the number of units output during Periods 1 and 2 for Stage $I$ machines was greater with the SHPID Methods. An example of this for Schedule 4 can be seen in the Gantt Charts in Figure 3. This explains, in part, why the output (number of units) in the schedules for Stage $I$ is somewhat less for STPID jobs. An example of actual and rounded results is given in Table 1.

|  | PERIOD 1 | PROCESSING <br> TIME | LENGTH OF <br> INTERVAL | IDLE <br> TIME |
| :--- | :--- | :---: | :---: | :---: |
| Actual | M3 $(.31)$ | 6.39 | 16.00 | .50 |
| Schedule | M5 $(.69)$ | 9.14 |  |  |
| Rounded | M5 $(1.00)$ | 12.36 | 16.00 | 3.64 |
| Schedule |  |  |  |  |

TABLE 1. STPID, Schedule 4 Actual And Rounded Quantities On Strander 1 Of Stage 1.

|  | SHPID SCHEDULES |  |  |  |  | STPID SCHEDULES |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| STAGE I | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| Set-ups | 15 | 13 | 18 | 15 | 13 | 14 | 11 | 13 | 10 | 6 |
| Units Output | 170 | 152 | 261 | 146 | 137 | 168 | 129 | 265 | 10 120 | 144 |
| $\begin{aligned} & \text { Processing } \\ & \text { Time } \\ & \text { Units/Time } \end{aligned}$ | 180 .94 | 180 .84 | 240 1.1 | 160 .91 | 144 .95 | 180 .93 | 180 180 .72 | 265 240 | 120 160 75 | 144 |
| STAGE II |  |  |  |  |  |  |  |  |  |  |
| Set-ups | 8 | 10 | 6 | 5 | 7 | 8 | 10 | 9 | 6 | 7 |
| Cores Output | 13 | 18 | 16 | 7 | 12 | 13 | 20 | 23 | 8 | 11 |
| Units Output | 118 | 178 | 147 | 79 | 104 | 105 | 177 | 181 | 69 | 97 |
| MCF | 88 | 144 | 115 | 56 | 71 | 83 | 146 | 114 | 60 | 68 |
| Processing |  |  |  |  |  |  |  |  |  |  |
| Units/ifme | 2.5 | 3.7 | 2.3 | 2.5 | 1.6 | 2.2 | 3.7 | 64 2.8 | 32 2.2 | 64 1 |
| MCF/Time | 1.8 | 3.0 | 1.8 | 1.8 | 1.1 | 1.7 | 3.0 | 1.8 1.8 | 2.2 1.9 | 1.5 1.1 |
| Inventory |  |  |  |  |  | 1.7 | 3.0 | 1.8 | 1.9 |  |
| After Period 1 Inventory | 7 | 4 | 13 | 5 | 8 | - | 2 | - | - | - |
| After Period 2 | 5 | 6 | 5 | 3 | - | 5 |  |  |  |  |
| TOTALS |  |  |  |  |  |  |  |  |  |  |
| Units Output Processing | 288 | 330 | 408 | 225 | 241 | 273 | 306 | 446 | 189 | 241 |
| Time | 228 | 228 | 304 | 192 | 208 | 228 | 228 | 304 | 192 | 208 |
| Units/Time | 1.3 | 1.5 | 1.3 | 1.2 | 1.2 | 1.2 | 1.3 | 1.5 | 1.0 | 1.2 |

TABLE 2. Summary Of Results For SHPID And STPID Sequencing Methods

GANTT CHART OF MACHINE ACTIUITY
GANTT CHART OF MACHINE ACTIUITY


Because the processing time of some of the Stage $I$ STPID jobs was greater than the length of the planning interval when the units were rounded, the length of processing extended into the next interval. This was the case in all five schedules for the M6-STPID job. The only schedule that had to be revised to reflect this condition is shown in Figure 4. Cable core type E6 could not be started on Cabler 6 until M6 was completed on Strander 1 , sometime after 35 hours.

Starting times on Stage II jobs could be ${ }^{\text {npulled }}$ up" (i.e., to reflect a real feasible sequence) to start as soon as they were completed on Stage $I$ machines.
 This would overcome the delay constraint of the model, but it would create more idle time between jobs at Stage II because job queues would not be allowed to build up ahead of Stage II. The results point out the fact that the stranding operation was not able to produce enough units to keep two cablers busy. The queue ing of jobs for longer production runs more realistically represents cable shop operation. Therefore, no attempt was made at "pulling up".

The amount of twisted wire was held constant (2000 reels/gauge/time period) over all 5 schedules, as was the amount of in-process inventroy (100 units/ period/ code and 100 total units/planning horizon). The

## GANTT CHART OF MACHINE ACTIVITY



FIGURE
SCHEDULE 1, WEEK 1 OUTPUT FOR STPID METHOD
results of the schedules showed that the in-process inventory constraint values were set so loese that they were, in fact, not constraining.

Since initial machine set-ups were not used in the sequences, a set-up had to be made (and was counted as such) at the start of each schedule. An additional set-up was not counted when $M 1$ and M4 STPID jobs were sequenced. In the actual construction of these jobs a set-up has to be made when going from 25-pair to 50-pair units. For example, in Schedule 3 only 13 set-ups were counted as being made at the Stage $I$ operation. Because there were $6-M 1$ and $3-M 4$ jobs scheduled, there were, in fact, 22 set-ups made. It is common practice in a cable shop not to count set-ups as being made when processing M1 and M4 type of jobs. Thus, the results tend to penalize SHPID jobs since all set-ups were counted.

In Schedules 1 thru 4 the optimizing LP mechanism always sequenced more M1 and M4 STPID jobs because of their shorter processing time in comparision to M2, M3, M5 and M6 STPID jobs. This accounts for the fact that while the number of cable cores in these schedules is greater for STPID jobs the total MCF is less than or equal to the MCF for $S H P I D$ jobs (see Table 3). Therefore, to maximize total unit throughput, the model
chose jobs with shorter processing times and with less MCF at Stage $I$ to sequence through Stage II. On the other hand, the SHPID Method allows the flexibility of splitting cabie core unit components on machines in Stage $I$ so that while the total processing time for units is equivalent, the model has a greater choice of cable cores to sequence. The difference in processing times between $E 4$ and $E 6$ jobs is not significant at Stage II.

| SHPID METHOD | E1 | E2 | E3 | E4 | E5 | E6 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cores Output 8 1 2 1 <br> 1 3 16   <br> Unit Equivalent <br> Output 56 12 24 7 <br> MCF 27 10 20 5 | 8 | 45 | 115 |  |  |  |  |

STPID METHOD

| Cores Output | 15 | 1 | 1 | 4 | 1 | 1 | 23 |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Unit Equivalent |  |  |  |  |  |  |  |
| Output | 105 | 12 | 12 | 28 | 12 | 12 | 181 |
| MCF | 51 | 10 | 10 | 20 | 8 | 15 | 114 |

TABLE 3. An Example Of The Output At Stage II, (From Schedule 3 of Appendix A)

As Table 4 shows, the combination of El and E4 jobs was always greater than any other combination of jobs in
the STPID Model. The SHPID Model tried to sequence as many cores as possible from those cores which had the largest maximum value in the set of $E_{\ell}$ jobs ( $\ell=1,2, \ldots, 6$ ). Thus, the STPID Model gave biased cable core outputs with respect to E1 and E4 jobs while the SHPID Model tended to give a better distribution of putput with respect to weekly load requirements.

| Week | Core <br> Type | Weekly <br> Load | SHPID <br> Cores <br> Output | STPID <br> Cores Output |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  | E1 | 4 | 1 | 1 |
|  | E2 | 1 | - | 1 |
|  | E3 | 12 | 4 | 1 |
|  | E4 | 6 | 2 | 5 |
|  | E5 | 3 | 1 | 1 |
|  | E6 | 4 | 1 | 1 |
| 2 | E1 | 2 | - | 2 |
|  | E2 | 2 | 1 | 1 |
|  | E3 | 1 | 8 | 1 |
|  | E4 | 8 | 2 | 7 |
|  | E5 | 2 | 1 | 1 |
|  | E6 | 5 | 8 | 1 |
| 3 | E1 | 8 | 3 | 6 |
|  | E2 | 4 | 1 | 3 |
|  | E3 | 6 | 1 | 1 |
|  | E4 | 2 | 1 | 2 |
|  | E5 | 1 | 1 | 1 |
|  | E6 | 1 |  | 1 |

TABLE 4. An Example Of The Distribution Of Core Output At Stage II (From Schedule 1 of Appendix A)

Schedules 1 thru 4 tended to indicate that the output at Stage $I$ was greater for SHPID jobs while the output at Stage II was greater for STPID jobs. Also there were more units left in in-process inventory at the end of Period 2 for the SHPID Method. This indicated that the units were being made in Stage $I$ but were not being sequenced through Stage II. Therefore, the objective function for Schedule 5 was changed to maximize output through Stage II. Although in Kornbluth and Lepage's [23] paper greater system output was obtained when maximizing both stages ("push-pull" effort) the results of Schedule 5 indicate that greater ${ }^{\circ}$ Stage II throughput for the SHPID Method is realized for the "pull" effort only. With this objective function the $L P$ mechanism is not concerned with getting added throughput (changeover capacity) in Stage I. The effect in the planning horizon of increased set-ups (54\%) for SHPID over STPID jobs can be seen in the reduction of output in Stage $I$ for this schedule. However, cable cores, units equivalent, and MCF was greater in Stage II. Also there were no units left in inventory at the end of Period 2. An explanation why output (cores, MCF and units) is greater when the "pull" objective function is used is that the two stages are not tied together
by a continuous process in a cable shop as they were
in Kornbluth and Lepage's model [23].

## CHAPTER VI

## conclusion and extensions

This thesis dealt with the development of a procedure for loading machines in an assembly operation (cable shop) that allowed machine throughput to be measured when the point of customer order identity was moved from the first operation (stranding) to a "downstream" operation (sheathing). It was hypothesized that this "downstream" shift would increase machine throughput because the system would become a generator of unallocated inventories which would later be matched to specific orders at the beginning of the sheathing operation. As a tool for analysis, a mathematical model of the stranding and cabling operations was formulated.

The mathematical model that was developed for this analysis was an extension of the model presented by Kornbluth and Lepage [23] for the case of a continuous flow production line. Their model was modified to handle the case of a multiple-discrete item assembly operation, and a raw material input constraint was added. A restricted entry feature was incorporated into a standard linear programming package to allow for set-up times in the solution of the scheduling problem.

Actual shop data from a cable shop was used to, demonstrate the model's feasibility. The initial investigation showed that moving the point of customer order identity from the stranding operation to the sheathing operation (STPID and SHPID) did, in fact, increase throughput in most schedules, with a slight increase in inventory.

The percentage of increased system output (units) of the SHPID versus the STPID Methodologies is given below. As the percentage of increased output indicates, system throughput in most cases is better when the SHPID Method is used. This is not the case in Schedules 3 and 5 when the length of the planning horizon ( 48 hours) was longer than in the other three schedules.

| SCHEDULE | TOTAL UNITS <br> SHPID | OUTPUT <br> STPID | E INCREASE OF <br> SHPID UNIT OUTPUT |
| :---: | :---: | :---: | :---: |
| 1 | 288 | 273 | 5.2 |
| 2 | 330 | 306 | 7.2 |
| 3 | 408 | 446 | -9.3 |
| 4 | 225 | 189 | 16.0 |
| 5 | 241 | 241 | 0.0 |

* 48 hour planning horizon allows the stranders to build up more inventory in Period 1 , thereby keeping the cablers busier in Period 2. If there is a limited amount of starting stock at the beginning of the planning period,
the disadvantage to a 48 hour planning horizon as compared to a 24 or 36 hour horizon is the amount of cabler wait time increases ( $50 \%$ and $25 \%$ respectively) while the inventory builds up.

In general, for the cases considered, it can be concluded that when the planning horizon is short ( 24 to 36 hours) the SHPID Method increases system throughput. The greatest percent increase (16.0\%) occurred with a short planning horizon of 24 hours. As the Gantt Charts in Appendix A show for this schedule, the lost output due to rounding is not significant for the STPID Method. It is worth pointing out that the present sequencing mode in most, Western Electric Cable Plants is over a 3 shift, 24 hour period.

Output was increased, in part, as Table 1 shows, by having fewer set-up changes at the cabling operation with an increase in unallocated in-process inventories for the SHPID Method. Since this inventory can be controlled with constraint equation 18, an upper limit can be set on its level so a feasible operating state can be mantained. Other advantages of the SHPID Method are as follows:

1. A better distribution of cable cores output with respect to weekly load requirements. This is due to the fact that the cablers have the flexibility to choose a wider variety of cable cores to make.
2. In a production environment if units are found defective after the stranding operation (unit testing is in fact performed here), they could be "swapped" with unallocated units from inventory. In this way, the cablers do not have to wait until the defective units are repaired before they start their operation.

The cable shop under consideration in this report was restricted in terms of the number of different types of cable codes and time periods in the computational results. Also in determining the effect of moving the point of customer identity, the last operation (sheathing) was not considered in the analysis. Even though the SHPID Method showed favorable results for this initial investigation, additional experiments need to be performed, with the above considerations, before any real conclusion to the question of increasing machine throughput in a cable shop by moving the point of customer order identity can be drawn.

From a solution standpoint the number of different jobs and planning intervals is limited in this model, like most algorithms, to computer core size. For the system presented the number of constraint equations was 213 while the number of variables was 477. To overcome
the computer dimensionality problem so additional cable codes and time periods could be added in the analysis, Manne's concept of "dominant schedules" [26] might be of use. He suggests that only a basic subset of "dominant" or efficient possibilities should be included in the model since in all likelihood the optimizing routine will choose one of these. This procedure can reduce the problem size quite considerably without seriously reducing its optir mizing potential.

The impact at the sheathing operation of sequencing jobs based on the SHPID Method should be investigated to determine system (strand, cable and sheath) feasibility. For example, at one extreme, the cable core queues could build up to such an extent at the sheathing operation that the cabling operation would run out of empty core trucks for their operation and would be forced to shut down. This situation, if allowed to continue, would force all the operations to shut down. On the other hand, the sheathing machine idle time (waiting for work) could become excessive and customers orders would not be completed in a given time frame for a service criterion to be meet.

The sheathing operation could be included in the model by simply defining a set of variables that are
similar to the Stage $I I$ variables and adding the appropriate constraint equations. This would increase the present two stage system to a three stage system (a . 50 increase in problem dimensionality). An alternative would be to adapt a sequencing rule such as first-come-first-serve (FCFS) and measure the core truck turn around time and sheathing machine idle time.

Possible other two stage areas for application of the model in a cable shop are: (1) the cable test and sealing operations, (2) strander-cable (this is a one stage operation in some cable plants) and sheathing operations, (3) stranding and twisting operations, and (4) insulating and twisting operations. The application of the mathematical model in this report is not limited in use to only the presented system. It can be used with very little re-programming in any two stage multiplediscrete item and multiple-machine system as an aid in production planning. The model has the flexibility to allow the user to choose for his specific system the metholodogy which gives the best operating results (i.e., identifying jobs at the start of the assembly operation or at a later "downstream" operation). After this determination the model can be used as an alytical guide for sequencing jobs through the production process.

## APPENDIX A

## NUMERICAL DATA FOR THE CABLE SHOP <br> UNDER CONSIDERATION

## Material Definition

(1) Standard linear footage (strander load) for material at each stage. The difference in footage between the Stage $I$ and Stage $I I$ output is due to the helix when units are twisted together to form cable cores.

| STAGE | GAUGE |  | PAIR SIZE | LENGTH |
| :---: | :---: | :---: | :---: | :---: |
|  | 22 |  | 25,50 | 17000 |
| I | 24 | 25,50 | 26000 |  |
| II | 22 |  | 200,300 | 16800 |
|  |  | 64 |  | 600 |
|  | 24 |  | $200,300,600$ | 25200 |

(2) Stage I SHPID Codes (Units).

| Symbol | Material |  |
| :--- | :--- | :--- |
| M1 | 25 pair, 22 gauge |  |
| M2 | 50 | " |
| M3 | 25 | pair, 24 gauge |
| M4 | 50 | " |

(3) Stage II SHPID Codes (Cable cores).

Symbol
E1
E2
E3
E4
E5
E6

Material
200 300 600 $\begin{array}{cc}200 \\ 300 & \text { pair, } 24 \text { gauge } \\ 600 & \text { " }\end{array}$

| pair, | 22 gauge |
| :---: | :---: |
| " | " |
| " | " |
| pair, | 24 gauge |
| " | " |
| " | " |

Construction
(6) M1, (1) M2
(12) M1
(12) M2
(6) M3, (1) M4
(12) M3
(12) M4
(4) Stage I STPID Codes (Units).

Symbol

| M1 | 200 | pair, | 22 gauge |
| :---: | :---: | :---: | :---: |
| M2 | 300 | " | " |
| M3 | 600 | $"$ | " |
| M4 | 200 | pair, | 24 gauge |
| M5 | 300 | $"$ | " |
| M6 | 600 | $"$ | $"$ |

(5) Stage II STPID Codes (Cable Cores).

Symbo1 E

E3
E4 E5 E6

Material

| $200$ | $\text { pair, } 22$ |  |
| :---: | :---: | :---: |
| 600 | " | " |
| 200 | pair, 24 | 4 |
| 300 |  |  |
| 600 |  |  |

## Construction

(1) M1
(1) M2
(1) M3
(1) M4
(1) M5
(1) M6

## Processing Rates

(6) SHPID job processing rates (strander loads per hour) for Stage $I$ machines.

MATERIAL

|  | M1 | M2 | M3 | M4 |
| :--- | :--- | :--- | :--- | :--- |
| Machine i, <br> $(i=1, \ldots, 5)$ | 4.9 | 2.7 | 3.1 | 2.0 |

(7) STPID job processing rates (strander loads per hour) for Stage $I$ machines.

## MATERIAL

|  | $\underline{\text { M1 }}$ | $\underline{\text { M2 }}$ | $\underline{\text { M3 }}$ | $\underline{\text { M4 }}$ | $\underline{\text { M5 }}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Machine i, <br> $(i=1, \ldots, 5)$ | 0.6 | 0.2 | 0.4 | 0.3 | 0.1 |

(8) Processing rates (strander loads per hour) for Stage II machines. These rates are the same for both SHPID and STPID jobs.

MATERIAL

| $\underset{\substack{\text { Machine } \\ (k=1,2)}}{ } k$, | $\frac{\text { E1 }}{2.1}$ | $\frac{\text { E2 }}{0.9}$ | $\frac{\text { E3 }}{1.4}$ | E4 | E5 | E6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1.5 | 1.2 | 1.0 |  |  |  |  |

(9) Changeover time was assumed constant at . 5 hours per set-up change per stage.

## Constant values

(10) Initial starting stock. Starting stock was assumed to be zero in all schedules except Schedule 2. The amount of material available is equivalent for both methods.

MATERIAL
S1 S2 S3 S4 S5 S6
$\begin{array}{lrrrrrr}\text { SHPID Method } & 24 & 14 & 24 & 14 & - & - \\ \text { STPID Method } & 2 & 1 & 1 & 2 & 1\end{array}$
(11) The amount of twisted wire available in each sequence.

MATERIAL

| Period | D1(22 gauge) |  |
| :---: | :---: | :---: |
| 1 | 2000 |  |
| 2 | 4000 | 2000 |
| 3 | 6000 |  |
|  |  | 6000 |

(12) Intermediate stock limits (Units).

|  | $\underline{1}$ | $\underline{2}$ | $\underline{3}$ | $\underline{4}$ | $\underline{5}$ | $\underline{6}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Finished Stock (G $)$ | 100 | 100 | 100 | 100 | 100 | 100 |
| Maximum Stock ( $\mathrm{F}_{\mathrm{j}}$ ) | 100 | 100 | 100 | 100 | 100 | 100 |
| Maximum allowable stock (G) on all Materials | $=100$ |  |  |  |  |  |

Material Limits
(13) Product requirements (units) Stage $I$. A minimum limit of one unit was placed on all material.

## SHPID METHOD MAXIMUM LIMITS

| Schedule | Week | M1 | M2 | M3 | M4 | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 36 | 148 | 72 | 54 | 310 |
|  | 2 | 36 | 14 | 72 | 68 | 190 |
| $\bigcirc$ | 3 | 96 | 80 | 24 | 14 | 214 |
| 2 | 1 | 12 | 134 | 48 | 40 | 234 |
|  | 2 | 12 | - | 48 | 54 | 114 |
| - | 3 | 72 | 66 | - | - | 138 |
| 3,4 | 4 | 174 | 55 | 48 | 40 | 317 |
| 5 | 3 | 96 | 80 | 24 | 14 | 214 |

STPID METHOD MAXIMUM LIMITS
Schedule Week M1 M2 M3 M4 M5 M6 Total

| 1 | 1 | 4 | 1 | 12 | 6 | 3 | 4 | 30 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 2 | 2 | 2 | 1 | 8 | 2 | 5 | 20 |
|  | 3 | 8 | 4 | 6 | 2 | 1 | 1 | 22 |
| 2 | 1 | 2 | - | 11 | 4 | 2 | 3 | 22 |
|  | 2 | - | 1 | - | 6 | 1 | 4 | 12 |
|  | 3 | 6 | 3 | 5 | - | - | - | 14 |
|  |  | 4 | 19 | 5 | 3 | 4 | 2 | 3 |
| 3 |  |  |  |  |  |  |  |  |
| 5 | 3 | 8 | 4 | 6 | 2 | 1 | 1 | 25 |

```
Product requirements (cable cores) Stage II. A
minimum limit of one cable core was placed in all
material.
```

MAXIMUM LIMITS

| Schedule | Week | El | E2 | ES | EL | E5 | E6 |
| :---: | :---: | ---: | :---: | ---: | ---: | ---: | ---: |
| 1,2 | 1 | 4 | 1 | 12 | 6 | 3 | 4 |
|  | 2 | 2 | 2 | 1 | 8 | 2 | 5 |
|  | 3 | 8 | 4 | 6 | 2 | 1 | 1 |
| 3,4 | 4 | 19 | 5 | 3 | 4 | 2 | 3 |
| 5 | 3 | 8 | 4 | 6 | 2 | 1 | 1 |

Schedule Description
The results of Schedules 1 and 2 are an average of three separate sequences. Each sequence had a different maximum amount of material (weekly load) to be made. Because Schedule 2 was allowed starting stock, the maximum limits at the Stage $I$ operation were reduced accordingly.

The number of Stage II machines (cablers) was the same for all schedules, 2 machines. Schedules 1 thru 4 had 5 machines (strangers) available for processing units. The number was reduced to 3 machines for Schedule 5. The length (hours) of each processing intervals is as follows:

| SCHEDULE | TIME <br> INTERVAL |
| :---: | :---: |
| 1,2 | 12 |
| 3,5 | 16 |
| 4 | 8 |

## Material Produced

The Gantt Charts that follow are the rounded results as discussed in Chapter IV of each schedule for each method considered. Stage $I$ output is represented by the quanity under each column headed ST i ( $i=1, \ldots, 5$ ), and Stage II output is under the columns $C B k(k=1,2)$. The type of material scheduled is represented by a three digit number ( $A B C$ ) with the following nomenclature:
(1) A: The stage (1,2) the material is scheduled.
(2) B: Type of material $\left(M_{b}, E_{b}\right)$ scheduled.
(3) C: Time period (1,2,3) material is scheduled. The five digit number at the bottom of each material scheduled represents the quantity made.

The Gantt Charts were executed by a routine which is part of a heuristic sequencing system [2] that was developed at the Western Electric Engineering Research Center.
ganfy ghant of machine activity
GANPT CHART OF MACHINE ACPIVITY


Schedule 1, Week 1, SHPID and STPID Output
ganty chart of machine activipy
GANTY CHART OP MACHINE ACYIVITY


Schedule 1, Heek 2, SHPID and STPID Output


Schedule 1, Meek, 3, SHPID and STPID Output
gANTT CHART OF MACHINE ACTIVITY


Schpdule 2, Week 1, SHPID and STPID Output

GANTY CHART OF MACHINE ACPIVIPY
ganfy chant of machine activity


Schedule 2, Heek 2, SHPID and STPID Output
gantt chart of machine activity
GANTT CHART OF MAGHINE ACTIVITY


Schedule 2, week. 3. SHPID and STPID Output


Schedule 3, Heek 4, SHPID and STPID Output

GANPT CHART OF MAENINE ACPIVIPY
GANPT CHART OF MACHINE ACPIVITY


Schedule 4, Keek 4, SHPID and STPID Output
gantt chart or machine activity

gantt chart of machine activity


Schedule 5, Heek 3, SHPID and STPID Output

## APPENDIX B

## COMPUTER PROGRAMS

The computer programs used to generate the tableau and subsequently to solve the restricted entry LP problem consists of a main program and five subroutines. The programs are written in a general nature so that any 2-stage production system can be described with a minimum of re-programming.

The parameters with regard to number of machines and number of products in each stage and the number of time periods in the planning horizon are set in the main program (e.g. $I, J, K, L, N)$. The appropriate subroutines are then called from the main program. The subroutines are: (1) RATE21, (2) LPRM21, (3) TAB22, (4) STAG2 and (5) SIMPZ.

Penalized ( $M_{i j}, E_{k \ell}$ ) and Changeover ( $Q_{i j}, Z_{k \ell}$ parameters are specified in Subroutine RATE21, along with the length of the planning interval and the time required to make a set-up change at each stage. The multiple-component matrix $\left(\mathrm{C}_{\boldsymbol{j} \ell}\right)$ is defined in Subroutine LPRM21. With the appropriate values Subroutine TAB22 generates the initial non-basic matrix shown in Figure 5 . The size of each individual matrix with the corresponding constraint equation numbers from Chapter III is shown.

If a $s$ ub-matrix does not have a dashed line through it, then it is a null matrix. The cost coefficient vector associated with the objective function is also generated in this subroutine. This matrix is then passed to Subroutine STAG2l which adds the initial basic matrix to the left side of the initial non-basic matrix, and adds the right-hand side to the LP tableau. Solution to the tableau is performed in Subroutine SIMZ. Results of the $L P$ solution are punched into data cards which are later read into a PDP-10 Computer for analysis.


Figure 5
Initial Non-Basic Tableau

The CPU execution times of each of the five multiple pass schedules is given below.

| Schedule | Execution <br> Time $($ Mini, |  |
| :---: | :---: | :---: |
| 1 | 1 | 28.20 |
|  | 2 | 24.46 |
|  | 3 | 25.58 |
| 2 | 1 | 28.59 |
|  | 2 | 24.20 |
| 3 | 3 | 26.24 |
|  | 4 | 25.57 |
| 4 | 4 | 27.23 |
| 5 | 3 | 22.36 |

To relax the assumption that all products can be made on all machines in the appropriate stages, the corresponding columns which represent the variables in the tableau could be set to zero or removed. If those columns were removed, then the dimensionality of the tableau would be reduced. To describe other two stage systems the values associated with the constant variables $\left(M_{i j}, Q_{i j}, E_{k \ell}, Z_{k \ell}, C_{j \ell}\right)$ would have to be changed in the appropriate subroutines.

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