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The Inmate Transportation Problem

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Abstract

The Inmate Transportation Problem (ITP) is a common complex problem in any correctional system. In this project we studied the present policies and practices used by the Pennsylvania Department of Corrections (PADoC) to transport inmates between 25 different state Correctional Institutions (CIs) across the state of Pennsylvania. As opposed to the current practice of manually deciding about transportation we propose a mathematical optimization approach.

We develop a weighted multi-objective mixed integer linear optimization (MILO) model. The MILO model optimizes the transportation of the inmates within a correctional system. Particularly, the MILO model assigns inmates, who needs to be transported from a particular CI to another, to routes and vehicles while considering all legal restrictions and best business practices. By using real data instances, we tested the performance of the MILO model and show that the transportation need in a correctional system can be organized efficiently using classic vehicle routing and assignment optimization models. As a proof of concept, this master's thesis proves that operations research is an effective tool to solve a complicated business problem in a correctional system, and save significant time and money along with ensuring safety of people involved in transportation.

Keywords: inmate transportation problem; mixed integer linear optimization; multi-objective optimization; vehicle routing problem

Chapter 1

Introduction

According to the International Centre for Prison Studies, the U.S. incarcerates 698 people for every 100,000 of its population. Having approximately 4.5% of the world's population, the U.S. has 21.4 % of the world's incarcerated population [27].

Population Management of the inmates is one of the most critical operations within a correctional system involving the inmate assignment to CIs and transportation within CIs. The efficient management of the inmate population and transportation results in substantial savings. Appropriate assignment of the inmates to the CIs is a key element of population management. It can lead to significant savings and enhancing the security of the CIs. Assignment system was optimized with the help of Inmate Assignment and Decision Support System (IADSS) [25] which was developed studying the assignment and scheduling operation within the PADoC. Another significant monetary savings can be achieved by optimizing the transportation of inmates within CIs. The transportation category of expenditure throughout the PADoC encompasses the price of fuel, the fixed and variable costs for using a vehicle, and the cost of labor. Security of the staff and inmates is the most important aspect of the transportation. In particular, we want to curtail the total transportation cost without compromising security and considering all the regulations and business practices. Here, we study the inmate transportation process and develop a mathematical optimization model for the Inmate Transportation Problem (ITP). This complex problem can be studied in the framework of a novel formulation of the classic vehicle routing problem [4].

Conventionally, inmate transportation planning has been a manual and subjective process at the PADoC, where a staff member creates trips and assigns inmates to those trips considering all the transportation criteria and policies. While the general guidelines are known, the large number of possible routes, and the complexity of the transportation problem makes it extremely difficult, if not impossible, to manually determine optimal routes for a fleet of vehicles.

In this thesis, we formulate a multi-objective mixed integer linear optimization (MILO) model for the ITP. The model is validated by solving various datasets of PADoC. The goal of the ITP is to optimize the inmate transportation process to achieve the following objectives:

- reduce the number of inmates not transported in the given time period,
- reduce the total number of seats used for the inmate transportation.

1.1 Literature Review

In this chapter, we provide the relevant literature related to the ITP. We provide traditional model and definitions of Linear Optimization (LO), Mixed Integer Linear Optimization (MILO), Traveling Salesman Problem (TSP) and Vehicle Routing Problem (VRP).

Mathematical optimization has been around for more than 150 years, though much of the field has matured in the second half of the 20th century due to the advent of computers and the growth of computational speed. "Famous French mathematician Joseph Fourier in 1823 and Belgian mathematician de la Vallee Paussin in 1911 each wrote a paper on linear optimization" writes Dantzig in 2002 [9].

LO formulation of a problem which can be equivalent to the general LO problem was first formulated by Leonid Kantorovich in *Mathematical Methods in the Organization and Planning of Production* [14] in 1939. In recognition of his pioneering work in the 1940's with LO he was awarded the Nobel Prize in Economics along with T. Koopmans in 1975. Since then many people have worked on LO [1].

In a general form of a LO model, x represents the vector of decision variables, c and b are vectors of known data based on the problem at hand, A is the coefficient matrix. c^T is the transpose of the vector c. In LO problem the objective is either to minimize or maximize certain objectives which are expressed as a linear function. In problem (1.1), the objective is to minimize

$c^T x$, the two sets of inequalities $Ax \leq b$ and $x \geq 0$ are the constraints which specify a polyhedron over which the objective function needs to be optimized:

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b, \\ & x \geq 0. \end{aligned} \tag{1.1}$$

Since 1950's, a wide variety of complex technological and business problems have been optimized using LO. *A case of sustainable intensification* [2] uses stochastic optimization, which was first introduced in *Linear Programming under Uncertainty* [8], studies a diversification in agriculture strategy reducing the overall production risk and increase profitability of organic farms. *Large-Scale Portfolio Optimization* [22] proposes a practical algorithm for large-scale mean-variance portfolio optimization to create a portfolio of stocks and options for investment, in order to minimize the risk associated with the total investment. Thousands of papers have been published since then and Operations Research (OR) have grown into becoming one of the most important disciplines for decision making.

Frequently, real numbers as a solution doesn't make sense when it comes to making decisions. For an instance, a value of 16.33 staff member doesn't make sense when trying to find the optimal number of staff individuals needed to do a certain job. Discrete values, for such staff assignment problem, would be more appropriate while making business decisions. Thus, optimization with integer variables was formalized as Integer Optimization (IO). The general form of an IO problem is similar to the general form of an LO problem (1.1), only here the second inequality becomes $x \in \mathbb{N}$, as seen in equation (1.2), where $\mathbb{N}$ denotes the set of nonnegative integers. If not all the variables are required to be integer the problem becomes a Mixed Integer Linear Optimization (MILO) problem. Much of the developments in MILO started along with LO, early in the second half of the 20th century. The classic assignment problem using MILO and the algorithms employed to solve it were studied extensively during the 1950s [7, 20]. Kuhn [15] suggests the —by now well-known— Hungarian method for solving the assignment problem. Assignment models have been used in a large variety of applications of optimization including transportation models.

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b, \\ & x \in \mathbb{N}. \end{aligned} \tag{1.2}$$

Unlike LOs which can be solved efficiently [24], MILO problems in many cases are extremely difficult to solve. Much work has been done on solving MILO problems. The Branch and Bound (B&B) algorithm developed in early 1950's is widely used to solve MILO models. The B&B method was first proposed in "An Automatic Method of Solving Discrete Programming Problems" [17] for discrete optimization and since then has become the most commonly used tool for solving NP-hard integer optimization problems. Later, Gomory showed how to systematically generate the *cutting planes* [12]. Cuts yield another tool, when repeatedly added to an existing system of inequalities, guarantee that the optimal solution of the continuous problem will be integer.

Transportation optimization has been extensively studied in the past under the umbrella of TSP and VRP. The TSP was considered mathematically already in the 1930s, e.g., by Flood who was looking to solve a school bus routing problem [6, 18]. He later formalized the problem in 1956 in his paper "Traveling-Salesman Problem" [11]. The goal of the TSP model is to find the shortest possible route which visits each city once in a given set of cities while returning to the origin city. The general form of a traveling salesman problem is shown in problem (1.3). Here, the set of n cities to be visited is $V = \{1, .., n\}$ and for all $i, j \in V$, x_{ij} is the decision variable which is 1 if a path of the tour goes from city i to city j; 0, otherwise. Here, c_{ij} is the distance between city i and city j. For all $i \in V$, a dummy variable u_i is introduced [3] to eliminate sub tours and enforces that all the cities are visited only once as represented in the last inequality in the optimization model (1.3).

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{n} \sum_{j\neq i, j=1}^{n} c_{ij} x_{ij} & \\
\text{s.t.} \quad & \sum_{i=1, i\neq j}^{n} x_{ij} = 1 & j \in V, \\
& \sum_{j=1, j\neq i}^{n} x_{ij} = 1 & i \in V, \\
& u_i - u_j + n x_{ij} \leq n - 1 & 2 \leq i \neq j \leq n, \\
& x_{ij} \in \{0, 1\} & i, j \in V, i \neq j, \\
& u_i \in \mathbb{N} & i \in V.
\end{aligned} \tag{1.3}
$$

The TSP is an NP-hard problem [3]. The most direct solution would be to try all permutations i.e. all possible ordered combination of the cities and see which one has the minimum distance. This approach would yield enumerating $n!$ possible routes, where n is the number of cities. Thus for 20 cities the number of possible permutations is already more than 2×10^{18}, which is beyond the capacity of today's most powerful computers. Various branch and cut algorithms have been developed in the past [21] to solve large scale instances of TSP with significant advances [3]. Heuristics for the TSP have made significant advances in recent years achieving near optimal solutions of large scale TSP's [23].

VRP is a combinatorial optimization problem which finds the optimal set of routes for a fleet of vehicles to traverse in order to deliver to a given set of customers and return back to depot. Dantzig in his paper "The Truck Dispatching Problem" [5] formulated a generalization of TSP first as a VRP. There are different mathematical optimization formulations for a variety of VRPs depending on the nature of problem [16, 26]. VRP has many variants depending on what application need to be solved. The *capacitated VRP* has a capacity attached to a vehicle; *VRP with time windows* has time limitation attached to delivery at various locations, and in the *open VRP* vehicles do not have to return to their depots.

Like TSP, VRP is difficult to solve and various algorithms based on branch and cut, and also on heuristics have been employed to solve large scale instances of VRP [10]. Every logistic operation today has its own version of VRP to solve depending on its unique features of operation.

As far as we know, until recently there has been no application of OR methodologies, in the field of corrections. The first known application of OR in the field of corrections was an assignment model which assigns inmates to different CIs across the State of Pennsylvania [19]. Li et al. [19] created a decision tree model which gives a ranked ordered list of CIs for an inmate while considering all the business regulations.

This was further studied extensively in the later study of the inmate assignment and scheduling of the treatment programs which lead to developing the award winning IADSS [25]. The IADSS simultaneously assigns a batch of inmates to the most appropriate CIs and schedules their rehabilitation programs during the course of their sentence while considering all the legal requirements and best operational practices.

Due to the nature of the ITP, as discussed in detail in Chapter 2 and Chapter 3, we have formulated a weighted multi-objective MILO model to solve the ITP at the PADoC optimally.

This thesis is structured as follows. Chapter 2 contains the problem description. It explains the transportation guidelines and business constraints that the PADoC follows in order to assign inmates to routes and vehicles. It further explains the manual way of assignment and scheduling of these routes and vehicles. Chapter 3 contains the mathematical optimization model. Here, first we define the terms used to develop the mathematical model. Then we discuss the development of the mathematical model which is used to solve the real data instances. In Chapter 4 we discuss the results that we got by testing and validating the model, and we compare the results to that of the manual way of transportation. Further, in Chapter 5 we discuss the anticipated benefits and impacts of utilizing the proposed multi-objective MILO approach. Finally, in Chapter 6, we summarize the main findings of this thesis, and discuss potential improvements and opportunities for future research. Detailed computational results are presented in the Appendix A.

Chapter 2

Problem Description

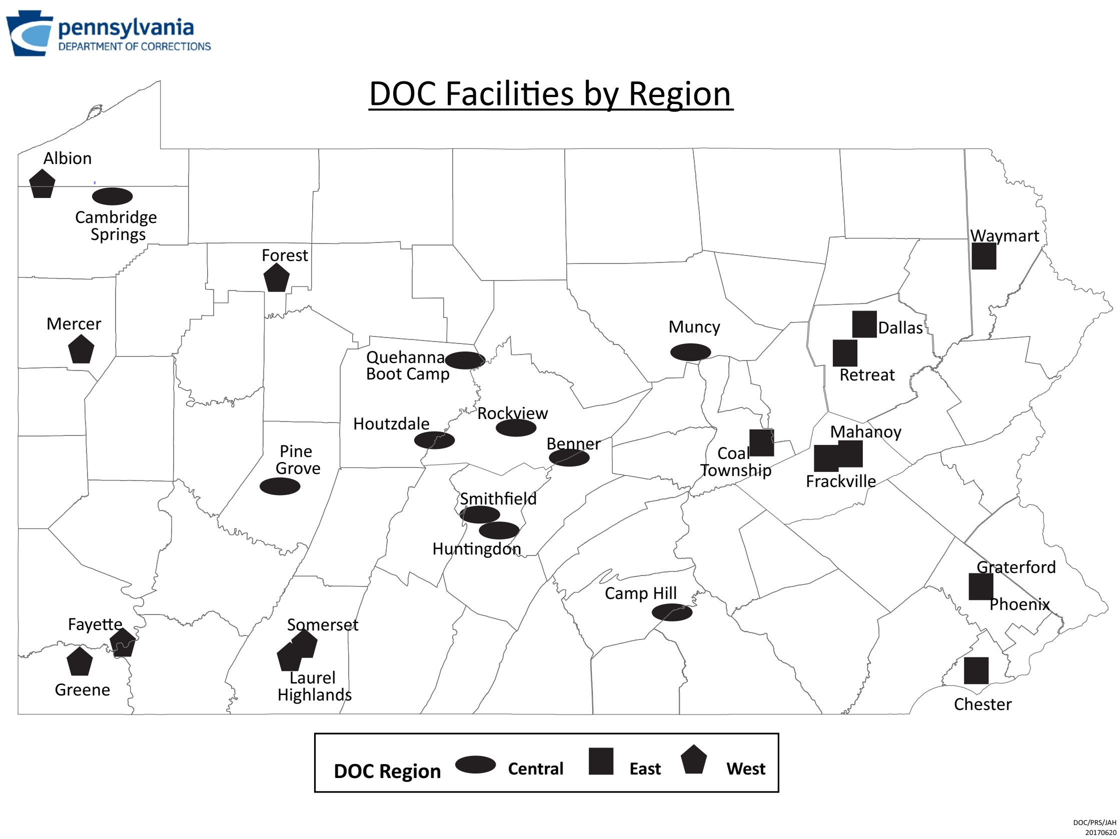
In this chapter, we discuss the transportation process, guidelines and constraints that the PADoC considers while transporting inmates.

The Office of Population Management (OPM) is responsible for the transportation of inmates at the PADoC. There are 25 CIs in PA, which are managed and operated by the PADoC as shown in Figure 2.1. On average 35,000 transportations are scheduled annually, yielding about 650 transportations in a week.

Conventionally, a staff member of OPM with his experience and judgment manually makes the decisions about the transportation of inmates. The decisions are made in two steps. First, the routes are specified for the vehicles, and then inmates are assigned to the vehicle based on their origin and destination CIs. One of the critical restrictions of the manual assignment is that there is a small set of predefined routes, which are fixed to a certain day of the week, and the trips are currently scheduled based only on those predefined routes. The limited number of predefined routes in the current policy significantly limits the flexibility of the transportation decisions. This manual way of planning for the transportation is clearly not efficient.

Next, we define ITP. Given a time horizon, the set of inmates who need to be transported within the PADoC are identified. For each inmate the origin and the destination is predefined. In other words, the decision about the assignment of an inmate to a CI is made prior to deciding on his/her transportation. In the ITP, we decide on the vehicles used at each transportation day, their routes, and the number of inmates that are going to be assigned to the vehicles at each day.

Figure 2.1: The Map of Pennsylvania shows 25 State CIs of the PADoC and their Placement in one of the States Three Main Regions



Vehicles visit a sequence of CIs and they need to return to their starting CI, because the vehicles are maintained by the respective CIs, and the drivers need to return home at the end of the day. Trips should be scheduled in the time window [7 a.m., 7 p.m.]. This means that every route should start and finish at the same CI, and transport inmates within the time frame of the 12 hours. Considering the travel time limit, there are a few pairs of CIs which can not be visited in a single trip. In order to be able to transport inmates between any two arbitrary CIs, the PADoC has one transfer hub. The transfer hub is located in the central region of the state to assist in the transportation process. By introducing a transfer hub, an inmate can be transported through two trips with two different vehicles. Such combined trips helps reduce the total cost of transportation.

The time horizon adds another level of complexity to the problem. Right now the time horizon considered for the trips is a week. The time horizon depends on the frequency of the transportation days and the number of inmates which need to be transported. The MILO model allows to consider longer time horizon.

Another important element is to consider the custody level of inmates. Not all of the transportation vehicles are equipped with the needed security to transport high custody level inmates. The ones that can transport these high custody level inmates have special seat types, and seat type capacities are given for each vehicle.

In addition, other limitations and constraints need to be considered before creating trips and assigning inmates to them. The limitations and constraints are listed as follows:

- Capacity of CIs,
- Capacity and type of vehicle along with it's location,
- Qualification of vehicles to transport high custody level inmates,
- Time horizon of the trips,
- Separations of inmates from other inmates or staff,
- Special conditions for inmates with medical condition and mental instability,
- Special cases, such as court hearing trips,
- CIs are gender specific, thus separate transports are required for different genders.

Chapter 3

Model Development

In this chapter, we introduce the MILO mathematical model. Specifically, the model constructs the optimal routes for a fleet of vehicles and minimizes the total number of seats used for transportation, while ensuring that maximum number of inmates are assigned to routes in the given time horizon. We also define the terms and assumptions that we have used to develop the MILO model.

Definition 3.0.1. *A **route** is a sequence of CIs which starts and ends at the same CI. The starting CI of a route is the **origin** of the route, and two consecutive CIs of the route form a **leg**.*

Example 3.0.2. *Let* $\{1, 2, 3, 4\}$ *be set a of CIs. A route can be 1-2-3-2-1 or 4-2-3-4 or 1-2-4-3-1 or any other loop. The route 1-2-4-3-1 has the origin '1' and legs are '1-2', '2-4', '4-3' and '3-1'.*

Definition 3.0.3. *A **trip** is specified with a vehicle along with its capacity and location at a given CI, a given transportation day, and a route. The given CI is the **origin** and the final destination of the trip.*

Example 3.0.4. *If a vehicle with capacity 40 is located at CI 4. Then a trip scheduled on day 1, e.g., can be Vehicle_40_Day1_4-2-3-4. The origin and final destination of this trip is '4'.*

Definition 3.0.5. *A **potential trip** is a trip where the vehicle with its capacity, the origin CI, and the transportation day is specified, but the route is not specified.*

In ITP, we define the set of potential trips. One of the main decisions to be made is to assign a route –if any– to potential trips and use those trips for inmate transportation.

3.1 Assumptions

In this section, we discuss the various assumptions that we have considered to develop the MILO model.

Due to various policy restrictions and business practices we limited the set of possible routes. We used Google Maps API to calculate the pessimistic travel time between the facilities to create the distance matrix, which is then further used to create routes. In order to comply with the business practices as mentioned in Chapter 2, we made the following assumptions in generating the set of possible routes:

- Every route should start and end at the same CI.
- The overall time of a route should not exceed 12 hours.
- We allocate a predefined time duration for getting on and off the vehicle at each CI, except the route origin.
- We only consider routes starting from a CI with a vehicle.
- The hub may only be visited at most once in a route.
- No consecutive pairs of CIs should be visited more than once.
- Only the legs that are currently used by the PADoC are considered in generating the set of the routes. In this case the vehicles will travel only on the paths that are approved by the PADoC.

  Example 3.1.1. *If the predefined route for a set of CIs* $\{1, 2, 3, 4\}$ *is 1-2-3-4-3-2-1, then we are only considering legs 1-2, 2-3, 3-4 and so forth. We are not considering 1-3 as a possible leg in our routes, since there is no approved direct path to visit from CI 1 to 3.*

For assigning inmates to trips we have considered following inmate specific assumptions:

- We do not consider special cases of inmate transfer types, such as medical transfer, since such requests form a small percentage of the total transportation requests, and are handled with different vehicles.

- We do not consider over night stay for an inmate, i.e., all the inmates assigned to a trip will reach their destination at the same day.

One hub is used for inmate transportation as a transfer point within the PADoC CIs. Adding a transfer point is necessary, because considering all the route assumptions it is clear that there is no acceptable route for inmates to move between some CI pairs which are at the furthest to each other. Furthermore, using the hub helps to reduce the cost of the transportation. The time horizon considered for the MILO mathematical model is a week.

We have two main objectives. We aim to minimize the number of the allocated trips and minimize the number of inmates not assigned to a trip.

3.2 Mathematical Model

In this section, we present the multi-objective MILO model for the ITP. As mentioned earlier, we have three main decisions to make. We need to allocate trips for transportation, assign routes to the allocated trips, and specify the number of inmates that are going to be transported on each trip.

A natural modeling option for routing problems with a hub and multiple depots is using binary variables. Here, our main decision variables are both binary and integer. Binary variables represent if a trip used for the transportation of inmates has a route or not. Integer variables are used to define the number of inmates assigned to a trip.

We first explain the constraints for allocating inmates to routes without using the hub and then the ones which use the hub, and finally the objectives of the problem.

3.2.1 Constraints without considering the hub

First, we define the general constraints. Let $\mathcal{P}$ be the set of all the potential trips. The constraints in (3.1) ensures that at most one route is allocated to a potential trip. Here, let x_{pr} for all $p \in \mathcal{P}$ and $r \in \mathcal{R}$ be a binary variable and is equal to 1 if route r is allocated to potential trip p; otherwise, $x_{pr} = 0$

$$\sum_{r \in \mathcal{R}} x_{pr} \leq 1 \quad \forall p \in \mathcal{P}. \tag{3.1}$$

Second, we define the constraints for direct transportation, i.e., without using the hub for transportation.

Let $\mathcal{C}$ be the set of all the CIs at the PADoC, let $\mathcal{R}$ be the set of all possible routes, and let $\mathcal{K}_{ri}$ be the set of the stops corresponding to CI i in route r. For all $i, j \in \mathcal{C}$ and $p \in \mathcal{P}$, let y_{ijp} be the number of inmates moving directly from the inmate's origin CI i to destination CI j in trip p. Also, for all $r \in \mathcal{R}$, let η_r be the number of stops or CIs in the route r. For all $p \in \mathcal{P}$, $r \in \mathcal{R}$ and $1 \leq n_1 < n_2 \leq \eta_r$, let $u_{prn_1n_2}$ be the number of inmates going from the n_1-th CI to the n_2-th CI in route r and trip p. The constraints in (3.2) makes sure that the number of inmates directly moving between any two CIs in a trip is equal to the sum of all the inmates moving between those two CIs in the route allocated to the trip

$$y_{ijp} = \sum_{r \in \mathcal{R}} \sum_{n_1 \in \mathcal{K}_{ri}} \sum_{n_2 \in \mathcal{K}_{rj}} u_{prn_1n_2} \quad \forall i, j \in \mathcal{C}, \forall p \in \mathcal{P}, i \neq j. \tag{3.2}$$

Now we define the constraints to balance the number of inmates at each CI by introducing a state variable. Let $\mathcal{R}'$ be the set of routes which goes through the hub or visits the hub. Let g_{prn} be an integer variable for all $p \in \mathcal{P}$, $r \in \mathcal{R}$ and $n \leq \eta_r$, which is the number of inmates at the n-th CI in route r and trip p. The constraints in (3.3) represent the balance equation corresponding to the first stop of a route in a trip

$$g_{pr0} = \sum_{n=1}^{\eta_r} u_{pr0n} \quad \forall p \in \mathcal{P}, r \in \mathcal{R} \setminus \mathcal{R}'. \tag{3.3}$$

The constraints in (3.4) enforce that the number of inmates at each CI should be equal to the number of inmates at the previous stop visited in the route plus number of inmates *getting on* the trip on that stop minus the number of inmates *getting off* the trip on that stop

$$g_{prn} = g_{pr,n-1} + \sum_{n_2 > n} u_{prnn_2} - \sum_{n_1 < n} u_{prn_1n} \quad \forall p \in \mathcal{P}, r \in \mathcal{R} \setminus \mathcal{R}', n < \eta_r. \tag{3.4}$$

Furthermore, we have capacity constraints for direct transportations. Let S_p for all $p \in \mathcal{P}$ be the seat capacity of the vehicle used in trip p. The constraints in (3.5) enforce a bound on the state variable g_{prn} for every CI, making sure that at any given point of time during the transportation the number of inmates on a trip does not exceed the capacity of the vehicle

$$g_{prn} \leq S_p x_{pr} \quad \forall p \in \mathcal{P}, r \in \mathcal{R}, n \leq \eta_r. \tag{3.5}$$

The constraints in (3.6) ensure that the number of inmates moving between any two arbitrary CIs does not exceed the capacity of the vehicle for all trips used for transportation

$$u_{prn_1n_2} \leq S_p x_{pr} \quad \forall p \in \mathcal{P}, r \in \mathcal{R}, 0 \leq n_1 < n_2 \leq \eta_r. \tag{3.6}$$

We also need to put a bound on the number of inmates moving between any two consecutive CIs in a trip. Let, w_{ijr} be a binary parameter for all $i, j \in \mathcal{C}$ and $r \in \mathcal{R}$, which is equal to 1 if CI i is before CI j in route r; 0, otherwise. The constraints in (3.7) ensure that the number of inmates moving between any two consecutive CIs is not more than $S^{\max}$, the maximum capacity of the vehicle

$$y_{ijp} \leq S^{\max} \sum_{r \in \mathcal{R}} w_{ijr} x_{pr} \quad \forall i, j \in \mathcal{C}, p \in \mathcal{P}, i \neq j. \tag{3.7}$$

3.2.2 Constraints considering the hub

In this section, we define the constraints for the transportation of the inmates who need to go through the hub. Here, inmates need to be assigned to two separate trips. The first trip transports inmates to the hub, and the second trip picks them up from the hub to transport them to their final destination.

The general constraint (3.1) also holds true for all the transportation through the hub.

For all $r \in \mathcal{R}'$, let η_r^h be the stop number of the hub. Constraints (3.8)-(3.10) are equivalent to constraint (3.4) for the transportation through the hub. Here, there are three constraints as opposed to one for the direct transportation. Constraints (3.8)-(3.10) enforce that the number of inmates *getting on* at each stop is equal to the number of inmates at the previous stop plus the ones that are *getting on* at the stop minus the ones that are *getting off* at that stop.

Here, the state variable is the same g_{prn} for all $p \in \mathcal{P}$, $r \in \mathcal{R}$ and $1 \leq n \leq \eta_r$. For all $p \in \mathcal{P}$, $r \in \mathcal{R}'$, $1 \leq n \leq \eta_r^h$ and $i \in \mathcal{C}$, let $\overline{v}_{prni}$ be the number of inmates in trip p and route r moving from the n-th CI to the hub with final destination i. Similarly, for all $p \in \mathcal{P}$, $r \in \mathcal{R}'$, $\eta_r^h \leq n \leq \eta_r$ and $i \in \mathcal{C}$, let $\overline{\overline{v}}_{prni}$ be the number of inmates in trip p and route r moving from the hub to the n-th CI with origin i.

Similar to transportation through the hub, constraints in (3.8) represent the balance equation corresponding to the first stop of a route in a trip

$$g_{pr0} = \sum_{n=1}^{\eta_r} u_{pr0n} + \sum_{i \in \mathcal{C}} \overline{v}_{pr0i} \quad \forall p \in \mathcal{P}, r \in \mathcal{R}'. \tag{3.8}$$

The constraints in (3.9) represent the case when the n-th stop is before the hub and $\sum_{i \in \mathcal{C}} \overline{v}_{prni}$ is the total number of inmates *getting on* the trip p and route r, and are *getting off* at the hub

$$g_{prn} = g_{pr,n-1} + \sum_{n_2 > n} u_{prnn_2} - \sum_{n_1 < n} u_{prn_1n} + \sum_{i \in \mathcal{C}} \overline{v}_{prni} \quad \forall p \in \mathcal{P}, r \in \mathcal{R}', n < \eta_r^h. \tag{3.9}$$

The constraints in (3.10) represent the case when the n-th stop is the hub, where the inmates *get off* and *get on*

$$\begin{aligned} g_{pr\eta_r^h} = g_{pr,\eta_r^h-1} + \sum_{n_2 > \eta_r^h} u_{pr\eta_r^h n_2} - \sum_{n_1 < \eta_r^h} u_{prn_1\eta_r^h} - \\ \sum_{i \in \mathcal{C}} \sum_{n_1 < \eta_r^h} \overline{v}_{prn_1i} + \sum_{i \in \mathcal{C}} \sum_{n_2 > \eta_r^h} \overline{\overline{v}}_{prn_2i} \quad \forall p \in \mathcal{P}, r \in \mathcal{R}'. \end{aligned} \tag{3.10}$$

The constraints in (3.11) represent the case when the n-th stop is after the hub and $\sum_{i \in \mathcal{C}} \overline{\overline{v}}_{prni}$ is the total number of inmates getting on trip p and route r at the hub

$$g_{prn} = g_{pr,n-1} + \sum_{n_2 > n} u_{prnn_2} - \sum_{n_1 < n} u_{prn_1n} - \sum_{i \in \mathcal{C}} \overline{\overline{v}}_{prni} \quad \forall p \in \mathcal{P}, r \in \mathcal{R}', n > \eta_r^h. \tag{3.11}$$

Let $\mathcal{T}$ be the set of the days of the transportation and let $\mathcal{P}_t$ be the set of all the potential trips corresponding to day $t \in \mathcal{T}$. The constraints in (3.12) enforce that at each transportation day the total number of inmates *getting off* a trip at the hub is equal to the total number of inmates *getting on* a trip

$$\sum_{p\in\mathcal{P}_t}\sum_{r\in\mathcal{R}'}\sum_{n_1\in\mathcal{K}_{ri}}\overline{v}_{prn_1j}=\sum_{p\in\mathcal{P}_t}\sum_{r\in\mathcal{R}'}\sum_{n_2=\mathcal{K}_{rj}}\overline{\overline{v}}_{prn_2i}\quad\forall i,j\in\mathcal{C},t\in\mathcal{T},i\neq j. \tag{3.12}$$

Furthermore, constraints (3.13) and (3.14) are the capacity constraints for the transportation of inmates through the hub

$$\overline{v}_{prn_1i}\leq S_px_{pr}\quad\forall p\in\mathcal{P},r\in\mathcal{R}',1\leq n_1\leq\eta_r^h,i\in\mathcal{C}, \tag{3.13}$$

$$\overline{\overline{v}}_{prn_2i}\leq S_px_{pr}\quad\forall p\in\mathcal{P},r\in\mathcal{R}',\eta_r^h\leq n_2\leq\eta_r,i\in\mathcal{C}. \tag{3.14}$$

3.2.3 Objective Function

The ITP is a multi-objective problem. The PADoC primarily uses two types of vehicles to transport inmates between CIs, buses and vans. Here, we consider two main objectives, to reduce the number of inmates not transported in a given week and to reduce the total number of seats utilized for the inmate transportation. For all $i,j\in\mathcal{C}$, let $\overline{N}_{ij}$ be the number of inmates not assigned to any trip which is defined in equation (3.15)

$$\overline{N}_{ij}=N_{ij}-\sum_{p\in\mathcal{P}}y_{ijp}+\sum_{p\in\mathcal{P}}\sum_{r\in\mathcal{R}'}\sum_{n_1\in\mathcal{K}_{ri}}\overline{v}_{prn_1j}\quad\forall i,j\in\mathcal{C},i\neq j. \tag{3.15}$$

Our aim is to minimize the weighted sum of the two objectives of the MILO model presented in (3.16). Here, α is the weight of the total seats used for transportation

$$\alpha\sum_{p\in\mathcal{P}}\sum_{r\in\mathcal{R}}S_px_{pr}+\sum_{i,j\in\mathcal{C}|i\neq j}\overline{N}_{ij}. \tag{3.16}$$

3.2.4 MILO Model

In this section, we present the mathematical optimization model for the ITP. The lists of sets, decision variables and parameters of the ITP are summarized in Table 3.1. We utilize the weighted sum method to combine the two objectives. The MILO model is as follows:

$$\min \alpha \sum_{p\in\mathcal{P}} \sum_{r\in\mathcal{R}} S_p x_{pr} + \sum_{i,j\in\mathcal{C}|i\neq j} \overline{N}_{ij}$$

subject to

$$\sum_{r\in\mathcal{R}} x_{pr} \leq 1 \qquad \forall p\in\mathcal{P},$$

$$y_{ijp} = \sum_{r\in\mathcal{R}} \sum_{n_1\in\mathcal{K}_{ri}} \sum_{n_2\in\mathcal{K}_{rj}} u_{prn_1n_2} \qquad \forall i,j\in\mathcal{C}, p\in\mathcal{P}, i\neq j,$$

$$g_{pr0} = \sum_{n=1}^{\eta_r} u_{pr0n} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}\setminus\mathcal{R}',$$

$$g_{prn} = g_{pr,n-1} + \sum_{n_2>n} u_{prnn_2} - \sum_{n_1<n} u_{prn_1n} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}\setminus\mathcal{R}', n<\eta_r,$$

$$g_{pr0} = \sum_{n=1}^{\eta_r} u_{pr0n} + \sum_{i\in\mathcal{C}} \overline{v}_{pr0i} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}',$$

$$g_{prn} = g_{pr,n-1} + \sum_{n_2>n} u_{prnn_2} - \sum_{n_1<n} u_{prn_1n} + \sum_{i\in\mathcal{C}} \overline{v}_{prni} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}', n<\eta_r^h,$$

$$g_{pr\eta_r^h} = g_{pr,\eta_r^h-1} + \sum_{n_2>\eta_r^h} u_{pr\eta_r^hn_2} - \sum_{n_1<\eta_r^h} u_{prn_1\eta_r^h} - \sum_{i\in\mathcal{C}} \sum_{n_1<\eta_r^h} \overline{v}_{prn_1i} + \sum_{i\in\mathcal{C}} \sum_{n_2>\eta_r^h} \overline{\overline{v}}_{prn_2i} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}',$$

$$g_{prn} = g_{pr,n-1} + \sum_{n_2>n} u_{prnn_2} - \sum_{n_1<n} u_{prn_1n} - \sum_{i\in\mathcal{C}} \overline{\overline{v}}_{prni} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}', n>\eta_r^h,$$

$$g_{prn} \leq S_p x_{pr} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}, 1\leq n\leq\eta_r,$$

$$u_{prn_1n_2} \leq S_p x_{pr} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}, 1\leq n_1<n_2\leq\eta_r,$$

$$\sum_{p\in\mathcal{P}_t} \sum_{r\in\mathcal{R}'} \sum_{n_1\in\mathcal{K}_{ri}} \overline{v}_{prn_1j} = \sum_{p\in\mathcal{P}_t} \sum_{r\in\mathcal{R}'} \sum_{n_2=\mathcal{K}_{rj}} \overline{\overline{v}}_{prn_2i} \qquad \forall i,j\in\mathcal{C}, t\in\mathcal{T}, i\neq j,$$

$$\overline{v}_{prn_1i} \leq S_p x_{pr} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}', 1\leq n_1\leq\eta_r^h, i\in\mathcal{C},$$

$$\overline{\overline{v}}_{prn_2i} \leq S_p x_{pr} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}', \eta_r^h\leq n_2\leq\eta_r, i\in\mathcal{C},$$

$$N_{ij} = \sum_{p\in\mathcal{P}} y_{ijp} + \sum_{p\in\mathcal{P}} \sum_{r\in\mathcal{R}'} \sum_{n_1\in\mathcal{K}_{ri}} \overline{v}_{prn_1j} + \overline{N}_{ij} \qquad \forall i,j\in\mathcal{C}, i\neq j,$$

$$y_{ijp} \leq S^{\max} \sum_{r\in\mathcal{R}} \omega_{ijr} x_{pr} \qquad \forall i,j\in\mathcal{C}, p\in\mathcal{P}, i\neq j,$$

$$x_{pr} = \{0,1\} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R},$$

$$y_{ijp} \in \mathbb{N} \qquad \forall i,j\in\mathcal{C}, p\in\mathcal{P}, i\neq j,$$

$$g_{prn} \in \mathbb{N} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}, 1\leq n\leq\eta_r,$$

$$\overline{v}_{prnj} \in \mathbb{N} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}', 1\leq n\leq\eta_r, j\in\mathcal{C},$$

$$\overline{\overline{v}}_{prni} \in \mathbb{N} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}', 1\leq n\leq\eta_r, i\in\mathcal{C},$$

$$u_{prn_1n_2} \in \mathbb{N} \qquad \forall p\in\mathcal{P}, r\in\mathcal{R}, 1\leq n_1<n_2\leq\eta_r,$$

$$\overline{N}_{ij} \in \mathbb{N} \qquad \forall i,j\in\mathcal{C}, i\neq j.$$

Table 3.1: The sets, decision variables, and parameters of the MILO model

Sets	
$\mathcal{C}$	Set of all CIs
$\mathcal{R}$	Set of all possible routes
$\mathcal{R}'$	Set of all possible routes visiting the hub
$\mathcal{T}$	Set of days of the transportation
$\mathcal{P}_t$	Set of the potential trips on day t
$\mathcal{P}$	Set of the all the potential trips ($\mathcal{P} = \bigcup_{t\in\mathcal{T}} \mathcal{P}_t$)
$\mathcal{K}_{ri}$	Set of the stops corresponding to CI i on route r
Variables	
x_{pr}	1, if route r is assigned to potential trip p; 0, otherwise
y_{ijp}	Number of inmates moving directly (without going to the hub) from CI i to CI j on trip p
$u_{prn_1n_2}$	Number of inmates directly going from the n_1-th CI to the n_2-th CI of route r on trip p
$\overline{v}_{prn_1j}$	Number of inmates on trip p going from the n_1-th CI of route r to the hub with final destination j
$\overline{\overline{v}}_{prn_2i}$	Number of inmates on trip p going from the hub to the n_2-th CI of route r with origin i
g_{prn}	Number of inmates on the vehicle at the n-th CI of route r on trip p
$\overline{N}_{ij}$	Number of inmates that need to move from CI i to CI j, but not assigned to any trip
Parameters	
N_{ij}	Number of inmates that need to move from CI i to CI j
S_p	Number of seats of the vehicle of trip p
$S^{\max}$	Maximum number of available seats among all the vehicles
η_r	Number of stops (CIs) on route r
η_r^h	Stop number of the hub on route r if the route visits the hub; ∞, otherwise
ω_{ijr}	1, if CI i is before CI j on route$r$; 0, otherwise

The ITP is a multi-objective optimization problem. We had to specify and fine-tune the weights of the objectives and ensure robustness of the model in assigning inmates to trips for various datasets.

As it is clear from the decisions that we have to make, the two objectives are competing. The more the number of trip the less the number of inmates not going to be transported and vice-versa. In addition, the decisions are dependent on the number of inmates who need to move and the CIs they need to move from and to i.e. N_{ij}.

Chapter 4

Computational Results

In this chapter, we discuss our computational experiments with the MILO model, and compare the performance of the MILO model with that of the manual transportation process. For testing of the model we used a dataset of 4682 inmates which were transported between 1st April 2018 to 26th May 2018. As mentioned earlier in Chapters 2 and 3, the transportation of inmates is scheduled on a weekly basis. The number of inmates which were transported in each week between 1st April 2018 to 26th May 2018 are presented in Table 4.1.

Table 4.1: The total number of inmates transported in each week between 1st April 2018 to 26th May 2018

Date	Number of Week	Inmates transported
1st April 2018 - 7th April 2018	Week 1	550
8th April 2018 - 14th April 2018	Week 2	530
15th April 2018 - 21st April 2018	Week 3	668
22nd April 2018 - 28th April 2018	Week 4	657
29th April 2018 - 5th May 2018	Week 5	499
6th May 2018 - 12th May 2018	Week 6	554
13th May 2018 - 19th May 2018	Week 7	581
20th May 2018 - 26th May 2018	Week 8	643
Total number of inmates transported		4682

For computational experiments a computer with Dual Intel Xeon® CPU E5-2630 @ 2.20 GHz (20 cores) and 64 GB of RAM is used. Gurobi [13] is used to solve the MILO model with its default parameters and is set to use 10 threads. The solution time limit of Gurobi is set to 43,200 seconds, i.e. 12 hours for all datasets.

There are two vehicle types, buses and vans, available at the CIs. Depending on their make and model, the capacities of these buses and vans are different. The capacities of buses are generally larger than those of the vans. Since we minimize the total number of seats used for transportation, the model tends to minimize the number of allocated trips with buses as opposed to vans.

For comparison between the output of the model and the manual way of organizing transportation, we looked at the following indicators:

- Total number of trips allocated.
- Total number of buses and vans used in allocated trips.
- Total number of seats in the vehicles used in allocated trips.

- Total number of inmates transported and not transported.
- Percentage of inmates using the hub for transportation compared to that of the total inmates transported.

  If an inmate uses the hub in order to be transported to the destination CI then that inmate is considered to take two trips. One trip drops the inmate at the hub, and another trip picks the inmate up from the hub to drop at the destination CI.
- Seat utilization ratio.

  The seat utilization ratio is the ratio of the total number of inmates transported to the total number of seats used in trips for the transportation. The seat utilization ratio can be greater than one, since multiple inmates can occupy the same seat in a trip, as they get on and get off at different stops. We consider two types of seat utilization ratio:
  - Without hub.

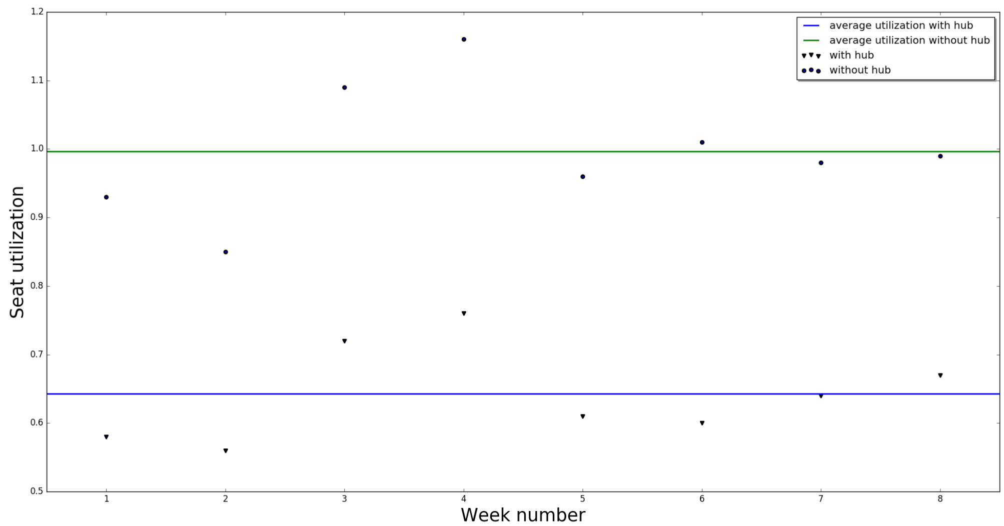
    This represents the utilization ratio when we consider the inmates moving from the hub as occupying one seat.
  - With hub.

    This represents the ratio when the inmate who is moving through the hub is considered to take two seats instead of a single seat.

The results of the manual allocation of the trips and the assignment of the inmates to the trips for 8 weeks are presented in Table A.1. The average number of the trips scheduled during the 8 weeks is 38.75. The seat utilization ratio with hub and without hub for all the weeks are presented in Figure 4.1.

The results of the MILO model with 1800 seconds time limit are presented in Tables A.2-A.9. The parameter α is the coefficient used in the objective function to penalize the allocation of vehicles for transportation. As α increases, the penalty associated with allocating a vehicle for transportation increases. Thus, the number of the allocated trips and more importantly the number of allocated buses for transportation decreases as α increases. There is a trade-off between the two objectives of the model: minimize the number of the inmates not transported and minimize the number of seats used in the allocated trips. The relative penalty of not assigning

Figure 4.1: Seat utilization ratio in manual transportation process



inmates to trips decreases as α increases. Thus, the number of inmates that are not assigned to a trip increases as α increases. Additionally, the number of inmates assigned to a trip increases, thus the utilization ratio increases. We tested the MILO model for $\alpha = 0.10$, $\alpha = 0.25$, $\alpha = 0.50$, $\alpha = 0.75$ and $\alpha = 1.00$.

The results of the MILO model with 43200 seconds (12 hours) time limit is presented in Tables A.10-A.17. As we can see in the results, none of the data and α instances are solved to global optimality. The gap has decreased for all the instances with different values of α as the time limit increases. Though the improvements differ from an instance to another. The biggest improvement is seen when $\alpha = 0.1$ while the smallest improvement is seen when $\alpha = 1$. As the decisions about inmate transportation are made on a weekly basis we can let the solver run for longer time duration (e.g., 12 hours) to obtain a better solution.

One of the most important decisions to make is to select the appropriate value for α. Since there is a trade off we need to make sure that we select an α which leads to a small number of inmates not transported with the smallest possible number of trips necessary to transport the inmates. Figure 4.2 is a plot illustrating the pay off between the percentage of inmates not transported and the relative co-efficient α for the 8 weeks of data. In the figure we can see that as α increases the percentage of inmates not transported increases. Since for the data that we consider, all the inmates are already transported in those respective weeks, we need to make sure that the number of inmates not transported is small. Thus, we can say that any $\alpha \leq 0.25$ would be appropriate, as for all weeks for $\alpha \leq 0.25$, less than 5% of inmates were not transported i.e. not assigned to any trip.

In addition, we need to make sure that value of α does not compromise on the number of trips allocated to transport the set of inmates who need to be transported in a given week. Figure 4.3 is a plot which illustrates seat utilization with hub, for all values of α, for the 8 weeks of data. In the figure we can see that there is a significant improvement in seat utilization for all values α, particularly $\alpha = 0.25$ and $\alpha = 0.10$.

Figure 4.2: Inmates not transported to total inmates who need to be transported

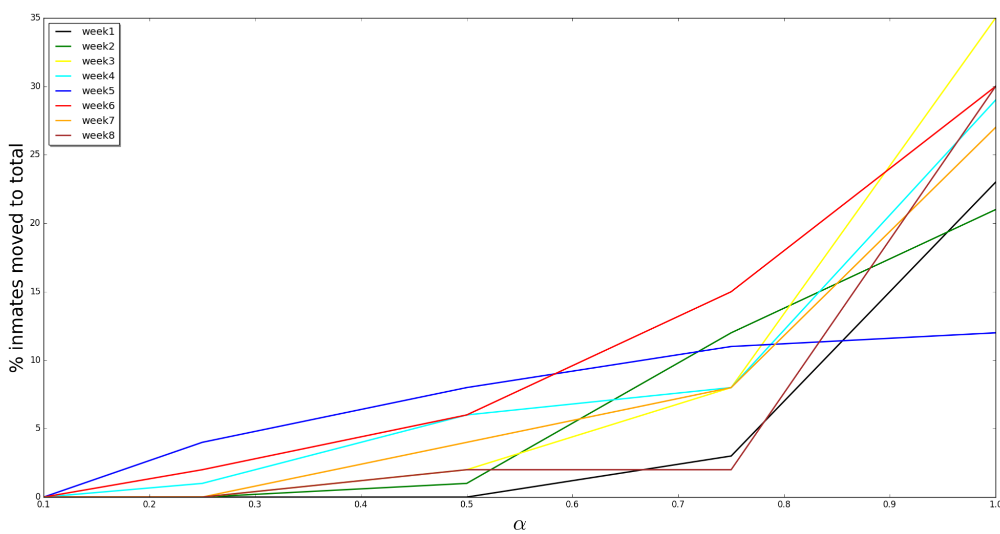
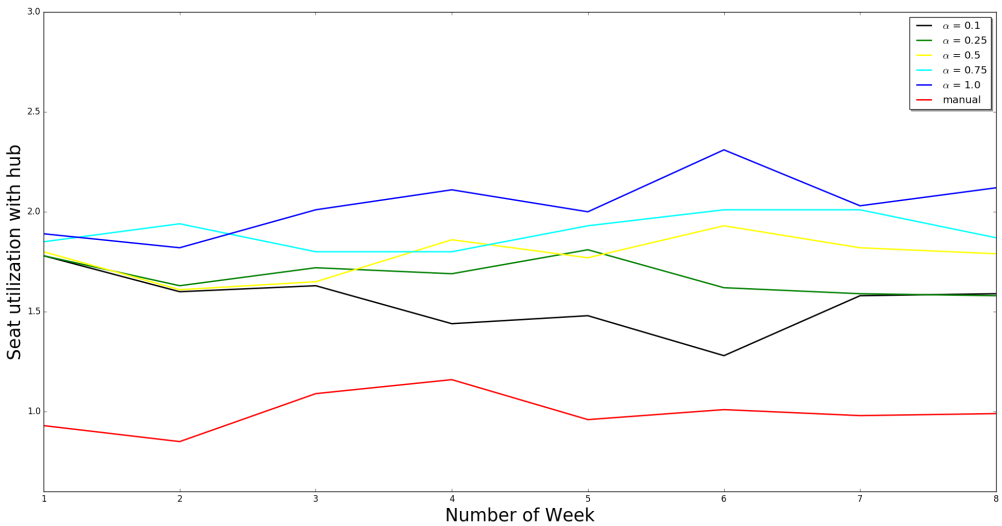


Figure 4.3: Seat utilization rate for all values of α and manual transportation



Figures 4.4, 4.5 and 4.6, represent the total number of trips scheduled each week for $\alpha = 0.1$, $\alpha = 0.25$ and $\alpha = 1.00$, respectively. The striped bars represents the number of the trips scheduled manually to transport the inmates. In Figure 4.6 we can see the number of buses allocated for the transportation of inmates are ≤ 5 for all weeks, but the inmates that did not move are significantly more, see Tables A.10-A.17 for more details. In the figures we can also see that there is a significant drop for the number of trips scheduled with $\alpha = 0.10$ to $\alpha = 0.25$. The biggest improvement is obtained in week 5, the total number of trips were 26 for $\alpha = 0.1$ and 22 for $\alpha = 0.25$, while the manual transportation had 38 trips scheduled.

Figure 4.4: Number of trips allocated for $\alpha = 0.10$

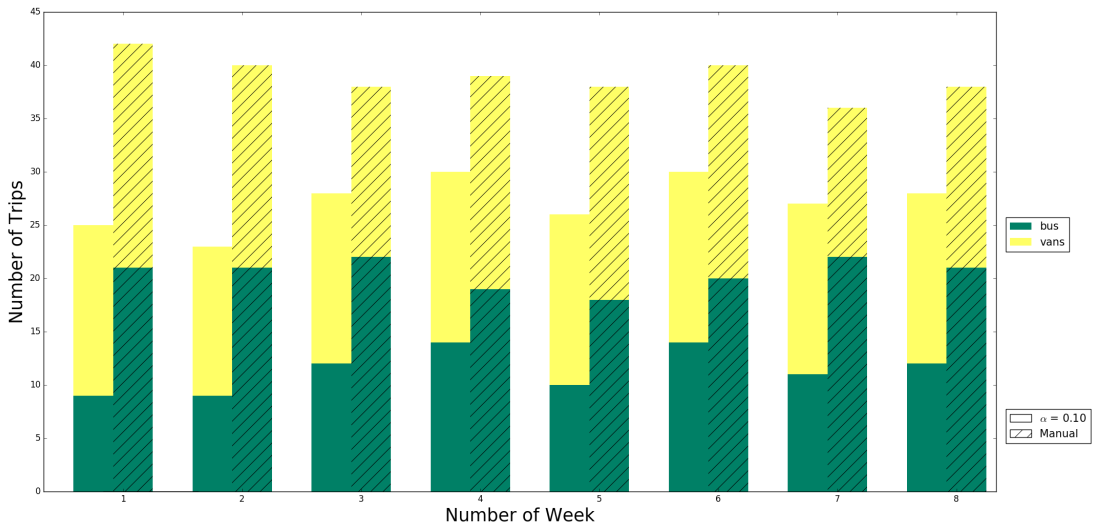


Figure 4.5: Number of trips allocated for $\alpha = 0.25$

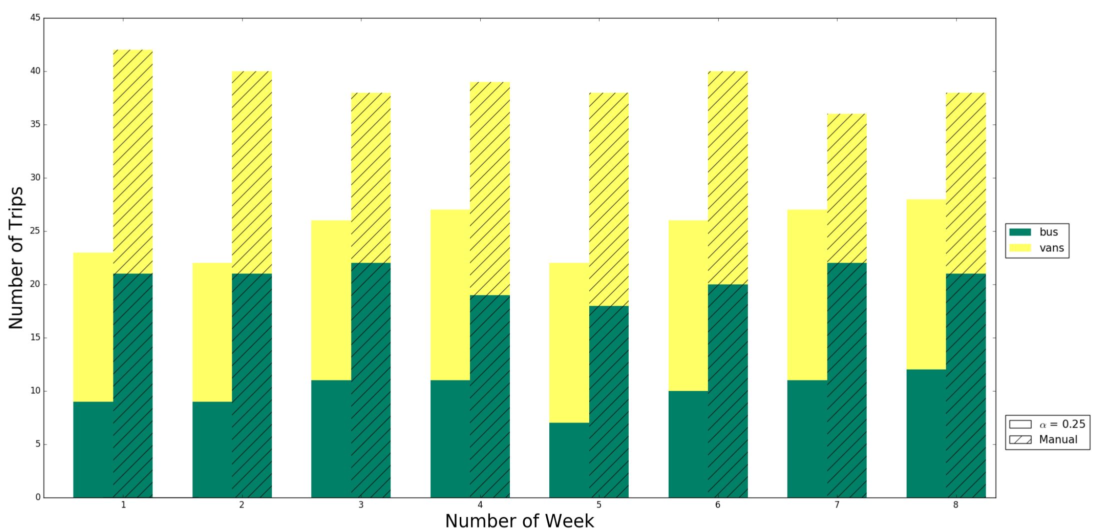
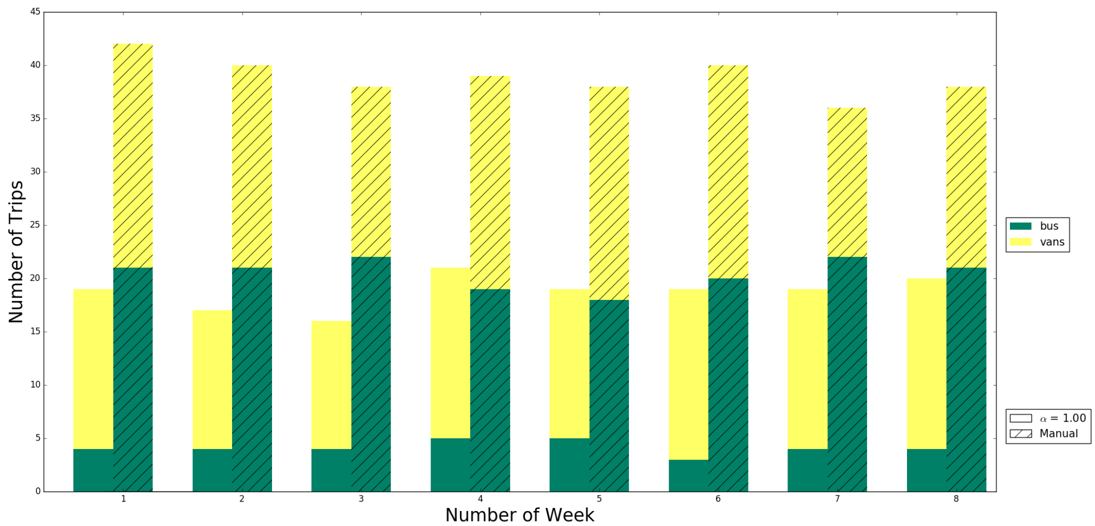


Figure 4.6: Number of trips allocated for $\alpha = 1.00$



After evaluating the trade-off between the two objectives for different values of α, the most appropriate value of α was determined to be 0.25. On average for, $\alpha = 0.25$, 25.13 trips were allocated each week for transportation and less than 1.4% of inmates were not transported. The inmates who were not assigned to any trip can further be transported in the following week. The average results of the MILO model for all weeks with $\alpha = 0.25$, and the average of the manual transportation schedule are presented in Tables 4.2 and 4.3, respectively.

Table 4.2: Average results for all weeks with $\alpha = 0.25$

Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Utilization ratio Without hub	Utilization ratio With hub
25	481	10	15	8	577	39.63	1.20	1.68

Table 4.3: Average results for all weeks for manual transportation

Trips	Seats used	Buses	Vans	Inmates moved	% moved with hub	Utilization ratio	
						Without hub	With hub
39	912	21	18	585	57.55	0.64	1.00

In the worst case the MILO model, for $\alpha = 0.25$, allocates 12 buses and 16 vans to transport inmates in a week. While the worst case for the manual transportation process schedules 22 buses and 21 vans for inmate transportation in a week. The worst case for both the manual transportation and the MILO model are presented in Table 4.4.

Table 4.4: Worst case scenarios for manual transportation and the MILO model

	Number of Buses	Number of Vans
Manual	22	21
MILO model	12	16

As seen in Table 4.2 and 4.3, in average weekly transportation of inmates can be done by using just about half of the buses and 3 fewer vans. The optimized transportation significantly improve the seat utilization ratio, while the only disadvantage is that in average less than 1.4 % of inmates are not assigned to trips on a week. In addition, for the worst case as seen in Table 4.4, the MILO model uses 10 less buses and 5 fewer vans to transport inmates in a week as compared to the manual transportation.

Chapter 5

Benefits and Impact

In this chapter, we quantify the expected savings that can be achieved by using the MILO model for the inmate transportation process. We have identified two main areas of savings that can be achieved by optimizing the transportation process. In order to compute the savings, we compare the average and the worst case scenarios of manual transportation and MILO model output. For average, we compare the results in Table 4.3 for manual transportation and Table 4.2 for the MILO model output. While for the worst case scenario we compare the results presented in Table 4.4.

- Gas & Maintenance:

  It was reported by the PADoC in 2013 that the gas and maintenance costs for 21 buses was $500,000.

  - Average:

    Using the MILO model, the number of the buses used for transportation reduced from 21 to 10. The model reduces the number of buses by 11. Thus, the savings from the gas and maintenance is projected to be $261,900 annually.

  - Worst Case:

    The number of buses used for transportation reduced by 10. In the worst case scenario the MILO model projects a saving of $238,000 annually.

- Salary:

Each bus and van, used for the transportation, needs three and two correctional officers, respectively. The average salary and benefits of a correctional officer is $135,000.

- Average:

  Since the number of buses and vans used for transportation reduced from 21 to 10 and 18 to 15, respectively. There is a reduction of 11 bus-trips and 3 van-trips. This would translate in a saving of 39 man-day which can translate to 7.5 full-time correctional officer positions. Thus, the saving from the salary would be $1,012,500, annually.

- Worst Case:

  For the worst case, the number of buses and vans used for transportation reduced by 10 and 5, respectively. This could translate in a saving of 40 man-day which is equivalent to 8 full-time correctional officer positions. Thus, the saving would be $1,080,000, annually.

The projected quantified savings in one year and over five years for a week in average are summarized in Table 5.1. The quantified savings for the comparison between the worst case scenario of the manual transportation process and the worst case scenario of the MILO model output is presented Table 5.2.

Table 5.1: Quantified savings for a week in average

Savings	One Year ($)	Five years ($)
Gas & Maintenance	261,900	1,309,500
Salary	1,012,500	5,062,500
Sum	1,274,400	6,372,000

Table 5.2: Quantified savings for the worst case scenarios

Savings	One Year ($)	Five years ($)
Gas & Maintenance	238,000	1,190,000
Salary	1,080,000	5,400,000
Sum	1,318,000	6,590,000

In addition to this, since the optimized schedule would use the hub less than what was done manually. The cost of using the hub less can also contribute to significant savings.

Another big saving can be achieved by reducing overtime salaries of correctional officers required in transports. Often trips are scheduled for irregular time which are leading to required extra hours for the correctional officers. Overtime salaries are usually very high as compared to normal work hour salaries. The discussions we have had with the PADoC shows that overtime payments have become a significant monetary burden on the PADoC. To quantify the savings for reduced use of the hub and reduced overtime payment requires the collection and analysis of additional data. The quantification of these savings remains for future analysis.

Conventionally the PADoC uses a set of about 40 routes to transfer inmates, out of these predefined routes some routes are also fixed to a certain day of the week. The model, since it considers only the number of inmates who wants to move and the resources (vehicles) available to move them assigns inmates to routes from a set of about 1200 routes. Changing the routes and letting the model decide which route to take can make the entire transportation safer, as the routes might change every week depending on the inputs and resources.

Chapter 6

Summary and Future Work

In this thesis, we studied the inmate transportation process as a proof of concept at the PADoC as it is done manually, and suggest an alternative to optimize the process system-wide. We developed a multi-objective MILO model to optimize the ITP. Numerical results demonstrate that significant savings can be achieved by using the model for the ITP. Throughout the model development and discussions with the Office of Population Management at the PADoC, we realized that transportation indeed is a crucial operation at the PADoC.

The work presented here can be advanced further to incorporate additional elements of transportation which we have not considered here. Some of those are listed below.

- Flexible and longer time horizons can be considered for assigning inmates to trips for better results.
- More flexible routes can be used in the future for optimal trip assignments.
- Incorporating other petition types such as medical trips and court hearings.
- PADoC has recently started GPS tracking of vehicles on the road, studying real time movement of vehicles can be further used to obtain better solutions.
- Using the model and further testing it might help us determine better locations of vehicles, thus changing the input to enhance system performance.

- A GUI based decision support system can be developed to assist PADoC personnel to do trip assignments.
- Integrating the transportation system with the IADSS (Inmate Assignment and Decision Support System[25]) can further help in achieving a system-wide optimal operation.

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Appendix A

MILO Model Output

Table A.1: Week 1 output received from the PADoC database

Week	Trips	Seats used	Buses	Vans	Inmates moved	% moved with hub	Utilization ratio	
							Without hub	With hub
Week 1	42	948	21	21	550	58	0.58	0.93
Week 2	40	943	21	19	530	53	0.56	0.85
Week 3	37	931	22	16	668	52	0.72	1.09
Week 4	39	862	19	20	657	55	0.76	1.16
Week 5	38	823	18	20	499	62	0.61	0.96
Week 6	40	925	20	20	554	70	0.60	1.01
Week 7	36	912	22	14	581	58	0.64	0.98
Week 8	38	955	21	17	643	52	0.67	0.99

Table A.2: Week 1 output when Gurobi time-limit is set to 1800 seconds

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio Without hub	Seat utilization ratio With hub	Opt. gap %
0.10	27	557	12	15	2	548	43	0.98	1.41	33.40
0.25	28	583	13	15	4	546	41	0.94	1.32	35.50
0.50	24	437	9	15	24	526	39	1.20	1.68	20.80
0.75	22	404	8	14	30	520	36	1.29	1.75	13.20
1.00	18	225	3	15	179	371	16	1.65	1.91	7.72

Table A.3: Week 2 output when Gurobi time-limit is set to 1800 seconds

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio Without hub	Seat utilization ratio With hub	Opt. gap %
0.10	29	578	13	16	1	529	33	0.92	1.12	35.10
0.25	29	562	13	16	7	523	35	0.93	1.26	36.20
0.50	28	550	12	16	15	515	34	0.94	1.25	36.90
0.75	23	378	7	16	38	492	32	1.30	1.72	15.70
1.00	18	258	4	14	122	408	26	1.58	1.83	10.40

Table A.4: Week 3 output when Gurobi time-limit is set to 1800 seconds

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio		Opt. gap %
								Without hub	With hub	
0.10	34	769	18	16	2	666	36	0.87	1.18	41.40
0.25	31	642	15	16	17	651	36	1.01	1.38	34.70
0.50	27	482	11	16	61	607	25	1.26	1.58	23.40
0.75	24	404	8	16	96	572	21	1.42	1.72	14.10
1.00	16	249	5	11	225	443	9	1.78	1.95	7.13

Table A.5: Week 4 output when Gurobi time-limit is set to 1800 seconds

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio		Opt. gap %
								Without hub	With hub	
0.10	33	736	17	16	9	648	37	0.88	1.21	49.10
0.25	27	510	11	16	35	622	40	1.22	1.71	35.40
0.50	25	458	9	16	60	597	43	1.30	1.86	27.50
0.75	24	418	8	16	104	553	32	1.32	1.75	25.30
1.00	21	284	5	16	211	446	31	1.57	2.06	18.20

Table A.6: Week 5 output when Gurobi time-limit is set to 1800 seconds

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio: Without hub	Seat utilization ratio: With hub	Opt. gap %
0.10	28	583	13	15	3	496	44	0.85	1.22	40.70
0.25	26	484	10	16	14	485	41	1.00	1.42	34.70
0.50	22	338	6	16	55	444	30	1.31	1.71	25.10
0.75	20	258	4	16	111	388	27	1.50	1.90	19.30
1.00	19	251	4	15	114	385	23	1.53	1.88	12.50

Table A.7: Week 6 output when Gurobi time-limit is set to 1800 seconds

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio: Without hub	Seat utilization ratio: With hub	Opt. gap %
0.10	29	604	13	16	14	540	53	0.89	1.37	46.50
0.25	25	458	9	16	36	518	49	1.13	1.68	33.80
0.50	24	404	8	16	44	510	44	1.26	1.88	19.30
0.75	22	338	6	16	105	449	46	1.33	1.92	18.40
1.00	16	145	1	15	288	266	15	1.83	2.12	13.00

Table A.8: Week 7 output when Gurobi time-limit is set to 1800 seconds

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio		Opt. gap %
								Without hub	With hub	
0.10	31	642	15	16	3	578	45	0.90	1.30	40.20
0.25	27	510	11	16	13	568	48	1.11	1.64	28.90
0.50	24	437	9	15	44	537	38	1.23	1.70	23.80
0.75	20	258	4	16	185	396	21	1.53	1.86	21.50
1.00	19	251	4	15	169	412	22	1.64	2.00	9.75

Table A.9: Week 8 output when Gurobi time-limit is set to 1800 seconds

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio		Opt. gap %
								Without hub	With hub	
0.10	34	762	18	16	5	638	39	0.84	1.17	46.10
0.25	29	649	15	14	9	634	39	0.98	1.36	36.70
0.50	32	696	16	16	7	636	43	0.91	1.31	39.10
0.75	25	444	9	16	59	584	36	1.32	1.78	17.00
1.00	20	272	4	16	198	445	28	1.64	2.09	10.60

Table A.10: Week 1 output when Gurobi time-limit is set to 43,200 seconds (12 hours)

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio		Opt. gap %
								Without hub	With hub	
0.10	25	444	9	16	1	549	44	1.24	1.78	13.60
0.25	23	430	9	14	4	546	40	1.27	1.78	12.10
0.50	23	430	9	14	1	549	41	1.28	1.80	9.11
0.75	22	404	8	14	14	536	39	1.33	1.85	7.49
1.00	19	265	4	15	129	421	19	1.59	1.89	3.92

Table A.11: Week 2 output when Gurobi time-limit is set to 43,200 seconds (12 hours)

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio		Opt. gap %
								Without hub	With hub	
0.10	23	430	9	14	0	530	30	1.23	1.60	10.00
0.25	22	437	9	13	1	529	35	1.21	1.63	13.90
0.50	24	418	8	16	10	520	30	1.24	1.61	15.00
0.75	21	312	5	16	65	465	30	1.49	1.94	8.29
1.00	17	251	4	13	116	414	11	1.65	1.82	6.11

Table A.12: Week 3 output when Gurobi time-limit is set to 43,200 seconds (12 hours)

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio: Without hub	Seat utilization ratio: With hub	Opt. gap %
0.10	28	550	12	16	0	668	34	1.12	1.63	14.10
0.25	26	517	11	15	5	663	34	1.28	1.72	12.10
0.50	27	510	11	16	19	649	29	1.27	1.65	13.80
0.75	23	437	9	14	60	608	30	1.39	1.80	9.91
1.00	16	230	4	12	240	428	8	1.86	2.01	5.21

Table A.13: Week 4 output when Gurobi time-limit is set to 43,200 seconds (12 hours)

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio: Without hub	Seat utilization ratio: With hub	Opt. gap %
0.10	30	644	14	16	1	656	41	1.02	1.44	33.30
0.25	27	531	11	16	13	644	39	1.21	1.69	25.10
0.50	25	458	9	16	41	616	39	1.34	1.86	19.30
0.75	25	444	9	16	54	603	33	1.36	1.80	15.90
1.00	21	284	5	16	197	460	30	1.62	2.11	12.50

Table A.14: Week 5 output when Gurobi time-limit is set to 43,200 seconds (12 hours)

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio		Opt. gap %
								Without hub	With hub	
0.10	26	484	10	16	1	498	43	1.03	1.48	24.70
0.25	22	371	7	15	24	475	41	1.28	1.81	22.30
0.50	22	338	6	16	41	458	30	1.36	1.77	18.20
0.75	21	298	5	16	56	443	30	1.49	1.93	10.10
1.00	19	284	5	14	61	438	29	1.54	2.00	6.00

Table A.15: Week 6 output when Gurobi time-limit is set to 43,200 seconds (12 hours)

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio		Opt. gap %
								Without hub	With hub	
0.10	30	644	14	16	0	554	49	0.86	1.28	36.60
0.25	26	505	10	16	13	541	52	1.07	1.62	26.70
0.50	24	404	8	16	38	516	51	1.28	1.93	15.10
0.75	22	338	6	16	87	467	46	1.38	2.01	11.70
1.00	19	232	3	16	171	383	40	1.65	2.31	4.86

Table A.16: Week 7 output when Gurobi time-limit is set to 43,200 seconds (12 hours)

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio		Opt. gap %
								Without hub	With hub	
0.10	27	510	11	16	2	579	39	1.14	1.58	22.60
0.25	27	510	11	16	2	579	40	1.14	1.59	20.70
0.50	25	444	9	16	27	554	45	1.25	1.82	17.50
0.75	22	371	7	15	48	533	40	1.44	2.01	6.73
1.00	19	251	4	15	159	422	21	1.68	2.03	6.22

Table A.17: Week 8 output when Gurobi time-limit is set to 43,200 seconds (12 hours)

α	Trips	Seats used	Buses	Vans	Inmates not moved	Inmates moved	% moved with hub	Seat utilization ratio		Opt. gap %
								Without hub	With hub	
0.10	28	571	12	16	1	642	42	1.12	1.59	23.50
0.25	28	550	12	16	5	638	36	1.16	1.58	22.10
0.50	25	491	10	15	18	625	40	1.27	1.79	16.10
0.75	26	470	10	16	15	628	40	1.34	1.87	10.00
1.00	20	272	4	16	193	450	28	1.65	2.12	8.19

Biography

Anshul Sharma was born in Indore, Madhya Pradesh, India in 1991. He obtained his Bachelors in Mechanical Engineering from Chameli Devi School of Engineering, CDGI affiliated with Rajeev Gandhi Technical University, Bhopal, Madhya Pradesh, India in 2014. He then joined Lehigh University's Industrial & Systems Engineering Department in 2016. At Lehigh, he worked under the guidance of Prof. Tamás Terlaky.

He also worked on the Inmate Assignment and Scheduling Project along with Prof Tamás Terlaky, Mohammad Shahabsafa, Chaitanya Gudapati, Prof Louis Plebani and Prof George Wilson.

He likes playing chess, independent films and The Beatles.