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# On the selection of control lines for x charts on an economic basis

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ON THE SELECTION OF CONTROL

LINES FOR  $\bar{X}$  CHARTS ON AN

ECONOMIC BASIS

by

Vincent James Graziano

A Thesis

Presented to the Graduate Faculty

of Lehigh University

In Candidacy For The Degree of

Master of Science

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1965

This thesis is accepted and approved in partial fulfillment of  
the requirements for the degree of Master of Science.

May 17, 1965  
Date

Wallace Richardson  
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Head of the Department

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ABSTRACT

"The real basis for the use of 3-sigma limits on control charts for variables in industrial quality control is experience that when closer limits, such as 2-sigma, are used, the control chart often gives indication of assignable causes of variation that simply cannot be found, whereas when 3-sigma limits are used and points fall out of control, a diligent search will usually disclose the assignable causes of variation." - E. L. Grant

"If the model for the change in process conditions is more complex, it may be essential to do a detailed cost analysis ...." - E. S. Page

These two sentences state concisely the problem this paper explored. All the recent work in this area gives rather detailed explanations for methods of general comparison of inspection schemes and not for detailed studies of costs involved for particular cases. It is the writer's contention that optimum process control using a control chart for variables can be attained through an economic study and design.

This paper proposes a procedure for determining, through simulation, the multiple of sigma that should be used to maintain the current control of a process. This is done by minimizing the cost of the operation of the control chart program and any income loss which may accrue from the interaction of the causal variables. The process is subjected to four typical causes of mean variation and the criterion to be used as a basis for establishing these values of sigma will be the maximization of long run net income for the process.

### Historical Background

During the last quarter century the control chart technique has been employed with considerable success in statistical control of industrial processes. The principal type of control chart for process inspection is basically that suggested by Shewhart (1) with various modifications that have been made since the original scheme was proposed. Originally the charts were furnished with what came to be called "control lines", so that when any point fell outside these lines, action of some sort was required by the user. Simply then, the samples themselves were classified as being good or bad. The good points, those falling within the control lines, suggested that the process should continue without interruption or any special action. The bad points demanded some kind of corrective action. For the successful operation of such a process inspection scheme the critical decision was that of the position at which the control lines should be placed once the sample size was determined. The positions most frequently chosen were the "three-sigma limits." For example, if it were decided to control the mean dimension of a process at some target value  $\mu$ , the lines were then drawn at  $\mu \pm 3.0 \frac{\sigma}{\sqrt{n}}$  for samples of size  $n$ . Placing the lines at these positions meant that if the process mean remained unchanged, the probability that the sample mean of  $n$  observations would fall outside the control lines and cause action to be taken would be .0027, so that action would be taken unnecessarily about once in every 370 times.

While the contribution of Shewhart's charts has been vital to the development and satisfactory control of production methods, his original work caused a fresh approach to be taken in considering the sort of

measures that one should use for assessing the statistical properties of process inspection schemes. There was consideration of not only the number of times action was unnecessarily taken but the effect of the size of samples to be taken and the frequency of taking them. These considerations were compared against the disadvantages and costs of failing to detect departures from product specifications and, on the other hand, the alternate disadvantage of searching for a "false alarm" cause, where the process has by chance produced a bad sample.

In 1950, Aroian and Levene (2) undertook a new consideration of the statistical properties of process inspection schemes and the measures one should use in their assessment of them. In essence they felt that if there is an abrupt change in the quality of the product or in the departure from specification we need to know the distribution of the amount produced by the process before the deterioration is noticed by the inspection rule being used. From this distribution a detailed study of the costs involved could be made in any particular case.

However, for a general comparison they decided something rather simpler was preferable as a measurement criteria and called this function the "average stoppage spacing number" or A.S.S.N. (2). This function supplies the average amount produced before action is taken, as prescribed by the inspection scheme, so that, for example, if the rate at which the process is sampled is constant, this will be proportional to the average number of items sampled before the action is taken.

The name suggested by Aroian and Levene has undergone a slight modification and, as suggested by Page (3), is now called the Average Run Length (A.R.L.). Barnard (4) and Ewan (5) and Kemp (6) have taken

the A.R.L. to be the average number of samples taken before action is demanded. Of course if the units are sampled singly the two are identical and if each sample contains more than one unit they differ.

The inspection scheme, to be effective, should have the ability to call attention quickly to any deterioration of product quality considered as serious by the user whether it be a rapid and large deterioration of quality or a slight change that is constantly increasing, however slowly. A slight deterioration of quality on a Shewhart Chart is portrayed by a sequence of points departing consistently from the target value but often insufficiently extreme to fall outside the action lines. This brought about suggested modifications to the charts in which warning lines would be placed at limits closer to the target value, with the additional rule that if some "K out of the last N points fall between the warning and action lines, then an investigation would be demanded." (2) This scheme presented smaller values of the A.R.L. function than was previously attainable by the Shewhart scheme. (7)

Other modifications with an eye toward the best choice of rule and positions of the control lines were presented by Moore (8), Weiler (9), (10), (11) and Roberts (12).

Moore adopted the rule as suggested by Weiler of stopping production when a specified number of means in succession fell over the control limits set up for the scheme. He then varied the position of a single control line with the number of successive means decided upon. The method adopted was to make the average number of samples drawn before a stoppage occurred the same whatever the number of successive means being used.



Roberts concerned himself with the statistical properties of tests composed of the standard control chart test supplemented by one or more tests for runs of points falling into various zones into which the control chart has been partitioned. The basic properties of the resultant tests, called zone tests, were then illustrated graphically and a procedure for determining the properties of many zone tests of practical interest there described.

Then following Wald's introduction of sequential methods in hypothesis testing (13), Page introduced a scheme whereby the actual positions of the points on the chart and not just the classifications into which division of the chart the point fell were taken into account. These were called Cumulative Sum Charts (14).

The operation of cumulative sum charts is in practice very similar to the operation of the usual charts. The differences lie in the type of visual record made and the criteria for deciding to take action. For example, if one wishes to detect a positive shift in a process parameter, such as the mean, then the mean path of the sum

$$S_r = \sum_i^r (x_i - k)$$

will take a turn upwards if the process mean increases above  $k$ . Any negative increments,  $x_i - k < 0$ , and parts of the path pointing downwards will give no indication of an increase. When the path tends downwards, the value of the process mean is satisfactory.

Most of the work developing this technique has been done in Great Britain by Page, G. A. Barnard, K. W. Kemp, and P. L. Goldsmith

and H. Whitfield (15). In the United States, H. M. Truax (16) has also found some useful applications.

### Introduction to Problem

"The real basis for the use of 3-sigma limits on control charts for variables in industrial quality control is experience that where closer limits, such as 2-sigma, are used, the control chart often gives indication of assignable causes of variation that simply cannot be found, whereas when 3-sigma limits are used and points fall out of control, a diligent search will usually disclose the assignable causes of variation."(17)

This sentence, by E. L. Grant, is typical of much that has been written concerning the placing of control limits on control charts for variables. Different points of view abound. Some writers like H. Weiler, W. D. Ewan and K. W. Kemp have discussed various schemes for the optimum position of control limits, paying more attention to particular manufacturing process restrictions on sample sizes and sampling intervals rather than strict adherence to a 3-sigma dogma.

Others like E. S. Page and G. A. Barnard and S. W. Roberts have proposed completely new schemes like the Cumulative Sum Charts and Geometric Moving Average Charts respectively.

It is generally agreed, however, that a control chart is a device for describing a state of statistical control, attaining that state of statistical control and finally judging whether that statistical control has indeed been attained.

The control chart describes statistical control in the following way. If samples of a given size are taken from a process at approximately regular intervals and some statistic of the sample is computed, because it is a sample it will be subject to sampling fluctuations. If there are no assignable causes present, then the fluctuations will distribute themselves in a definite statistical pattern. If enough samples are



taken then it becomes possible to estimate the governing parameters of these distributions.

If it is the goal of those managing the process to detect particular patterns of parameter fluctuations, then sample values of this parameter can be plotted for a significant range of output and time and if these values all fall within the limits set by the managers then it can be said that the process is in a state of statistical control at that designated level.

For example, if the population being sampled was considered a normal population and the statistic computed was the mean of the sample and the sample means conformed to a pattern of random variation within the control limits, then the process would be judged as being in control at a level equal to the mean line on the chart. If the data do not conform to this pattern then departures from the pattern are investigated and assignable causes tracked down. If the cause is favorable, an effort is made to extenuate the cause. If the cause is unfavorable then an effort is made to eliminate it. In this way statistical control is both attained and maintained.

Note that if no points fall outside the assigned control limits and there is no further evidence of nonrandom fluctuations within the limits, it does not mean that assignable causes are not present. It only means that statistically speaking you have made a favorable hypothesis in assuming random chance causes are at work in your process and it will be unprofitable to look for specific assignable causes of fluctuation.

In general, however, two different points of view have developed with respect to the real basis for setting limits on control charts for variables.

The first point of view, as mentioned above, is one of experience, for through extensive operation of variable control charts one strikes an economic balance between at least two kinds of possible errors that the chart introduces. In one instance, the chart may signal for action to be initiated to detect the cause of condition that has driven the process out of control. This cause may or may not be detectable. If ignored, however, the average net income of the process may deteriorate if indeed the cause exists. Experience such as this has led in general to the use of 3-sigma limits.

The second point of view is a statistical one. It prefers to discuss the expected number of samples before one can expect to take action if the universe being sampled does not change. The proponents of this point of view have frequently used .998 as the desired probability that any point will fall within the control limits so long as the sample population remains unchanged. Since 99.8% of the area under the normal curve falls within the limits  $\bar{X} \pm 3.09 \sigma$  this has been the cause of the limits on  $\bar{X}$  charts being set precisely at  $\bar{X} \pm 3.09 \sigma_{\bar{X}}$  as opposed to  $\bar{X} \pm 3.09 \sigma_{\bar{X}}$ .

Quite simply then the proponents of the 3-sigma approach feel their choice is justified on the grounds that in the case of variables, the sampling distribution of a variable is frequently not known well enough to compute probability limits and that 3-sigma limits have been found to give good results. They feel that the choice of limits on a proba-

bility basis is a poor one since the limit is chosen precisely by a probability that really is uncertain, for in many industrial operations the sampling is from populations that are unknown to those sampling. For instance it would be almost impossible to detect the difference between the effects of probabilities of .0010 and .00135 that a point would fall above the control limit by chance. The real basis seems to be experience over a long period of time where the operator feels that the control limits provide a satisfactory basis for action.

Both points of view do, however, concur on the contention that if the model for the change in process conditions is at all complex it may be essential to do a detailed cost analysis. All the recent work such as the ARL (defined earlier) are done for general comparisons of inspection schemes and not for detailed studies of costs involved for particular cases. It is the writer's contention that optimum process control using a control chart for variables can be attained through an economic study and design.

In 1956, A. J. Duncan (18) established "a criterion that measures approximately the average net income of a process under surveillance of an  $\bar{X}$  chart when the process is subject to random shifts in the process mean." He assumed a quality control rule that an assignable cause of process error was looked for whenever a point fell outside the control limits. The process was not shut down while the cause was searched for, nor was the cost of adjustment or repair and the cost of bringing the process back into a state of control after the assignable cause was discovered charged to the control chart program.

The paper sought a theoretical basis for determining the sample size, the interval between samples, and the control limits that would yield approximately maximum average net income. It also presented numerical examples of optimum designs to see how the variation in the various risk and cost factors affected the optimum.

### Statement of the Problem

As explained in the previous section, control charts used in statistical quality control are essentially of two kinds; those that bring a process under control, and those that assist in maintaining control of a process. This paper will seek a procedure for determining, through simulation, the multiple of sigma that should be used to maintain current control of a process.

The process will be subjected to four typical causal variables.

These are

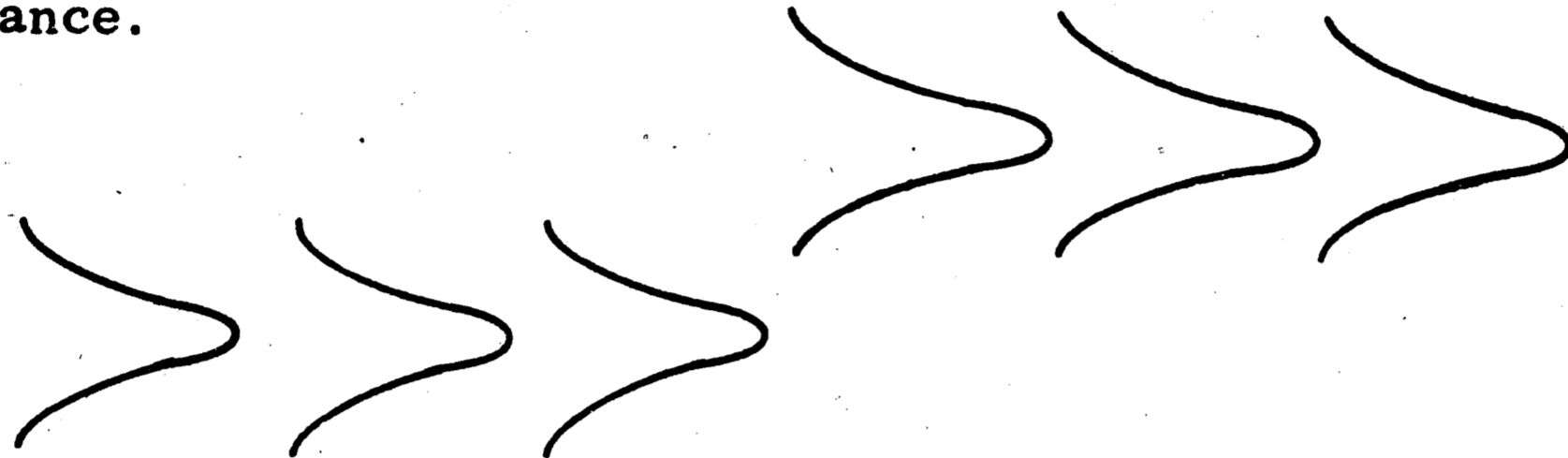
- (1) A sustained linear shift occurring in the population mean,
- (2) A probable step shift occurring in the population mean,
- (3) A change in the probability of a step shift occurring in the population mean,
- (4) A change occurring in the variance of the population.

It seems reasonable to assume that the objective of most business enterprises is the maximization of long run net income. Based on this premise the criterion to be used for establishing these values of sigma will be the maximization of long run net income for the process by minimizing the cost of the operation of the control chart program and any income loss which may accrue from the interaction of the causal variables.

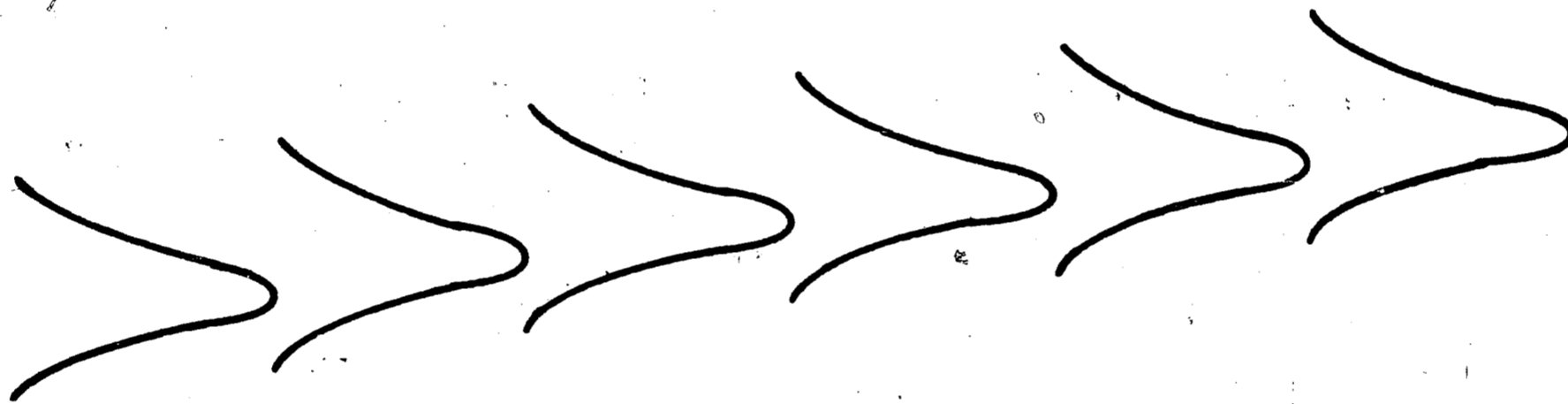
On  $\bar{X}$  charts used to maintain current control of a process, the target line is set at  $\bar{X}'$  and the control limits are taken as  $\bar{X}' \pm Z \left( \frac{\sigma'}{\sqrt{n}} \right)$  where  $\bar{X}'$  and  $\sigma'$  are the mean and standard deviation of the process population based on past experience. If a sample  $\bar{X}$  falls outside the control limits it is assumed that some change in the process average

$\bar{X}$  has occurred and a search is initiated for the assignable cause of this fluctuation in the mean.

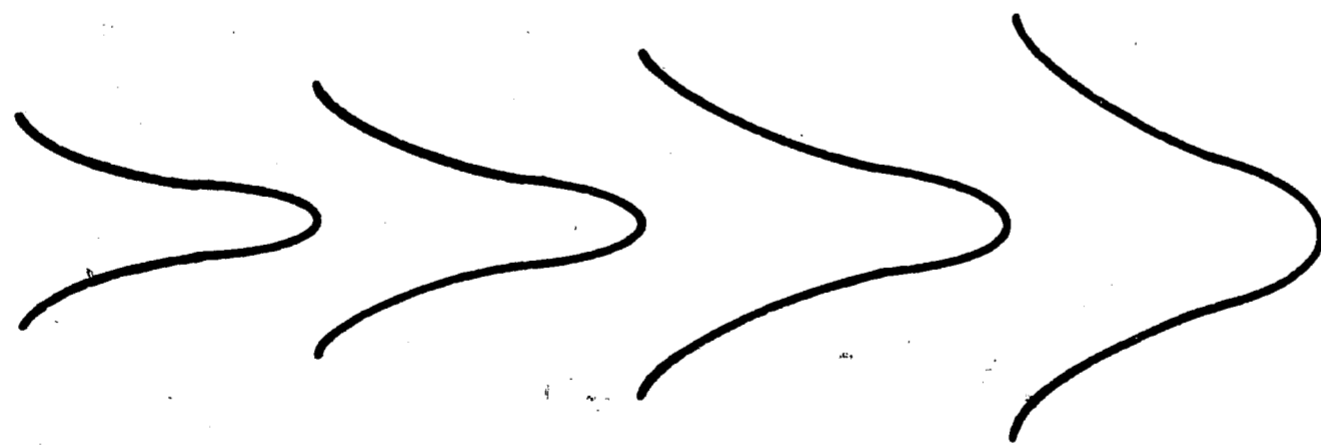
A chance cause system may change in different ways. For example, there may occur a sustained shift in the population average with constant variance.



or a steady trend in the population mean with a constant variance,



or a change in the population with no change in the average,



It will be assumed at the start that the process is in a state of control and the control chart is used to detect a single assignable cause that occurs as prescribed by the causal variables. Samples of  $N$  will be taken from the process every  $H$  hours and the  $\bar{X}$  of the sample recorded on an  $\bar{X}$  chart. If a sample  $\bar{X}$  falls outside the assigned control limit it will be assumed that some change in the process average has occurred and a search will be undertaken for the assignable cause. It



will be supposed that the rate of production is sufficiently high so that the possibility of a step change occurring in the mean of the process during sampling can be neglected. The process will be allowed to continue in operation during the search for the assignable cause and the cost of adjustment or repair and the cost of bringing the process back to a state of control after discovery of the cause will not be charged against the control chart operation.

Other more complex rules exist that do not require action unless two or more points in succession fall outside the control limits, or action is taken when ever unusual patterns occur outside the limits. The question of optimality as regards these rules will not be discussed here.

Finally the resulting multiple of sigma that determines the "least cost" process operation will be expressed as a function of the above causal variables.

### Experimental Procedure

This experiment required the examination of the effects of varying four factors. In a complete exploration of such a situation it is not sufficient to vary one factor at a time. All combinations of the different factor levels must be examined in order to analyze the effect of each factor and the possible ways in which each factor may be modified by the variation of the others. In the analysis of the experimental results the effect of each factor can be determined with the same accuracy as if only one factor had been varied at a time, and the interaction effects between the factors can also be evaluated. If each factor were tested at two levels these requirements would be met by a factorial experiment, in this case a  $2^4$  factorial experiment.

A program to simulate the operation of an  $\bar{X}$  chart used to maintain current control of a process was initiated. The  $\bar{X}$  chart would be subjected to the aforementioned input mean fluctuations and the program would accumulate the amount of income loss and cost associated with these fluctuations. After a sample was taken and a mean calculated and plotted, based on the assumptions made in the previous section, the control chart operation could find itself in any one of four distinct situations. These situations would be determined by a decision on whether the process was or was not in a "state of control." It would have to be determined at the earliest possible point in the operation that an assignable cause of error be detected. When the mean of the sampled population reached this point the assignable cause could be detected if an investigation were undertaken; however, only a sample



point outside the control limit could initiate this investigation. It was decided to use one standard deviation as this "out of control" point.

Therefore if the last sample point fell below the control limit and the process was in a state of control, the cost associated with this sample would be stored in register 1.

If the last sample point fell outside a control limit and the process was in a state of control the cost associated with this sample would be stored in register 2.

When the last sample point fell inside the control limit but the process was considered out of control the cost would be stored in register 4.

Finally, when the last sample point fell outside the control limit and the process was considered out of control the subsequent costs were placed in register 3.

To attach specific charges to cover all the assumptions made in the previous section the following model parameters were defined:

#### Model Parameters

N - sample size

E - the rate at which the time between the taking of a sample and the plotting of a point on the  $\bar{X}$  chart increases with the sample size.

$$\text{Total Delay} = EN$$

D - the average time taken to find the assignable cause after a point plotted on the control chart falls outside the control chart.

- T** - the cost per occasion of looking for an assignable cause when none exists.
- W** - the average cost per occasion of finding the assignable cause when it exists.
- H** - sampling interval.
- B** - the cost per sample of sampling and plotting that is independent of the sample size.
- C** - the cost per unit of sampling, testing and computation that is related to the sample size. The relationship is assumed to be linear.

These parameters would then be subjected to the four mean fluctuations listed below

- XNCR** - A sustained linear shift is occurring in the mean of the process population. The amount of this shift is  $(XNCR) \cdot \sigma'$ .
- VALT** - There is a probability of a step shift occurring in the mean of the process population. The amount of the step shift, if it occurs, is  $(VALT) \cdot \sigma'$ .
- PROB** - The value of the probability of a step shift occurring in the mean of the process population.
- SDEV** - The value of the standard deviation ( $\sigma'$ ) of the process population being sampled.

To establish the control limit for each operational run of the control chart the model variable  $Z$  was established

- Z** - The control chart limits for the  $\bar{X}$  chart were placed at

$$\bar{X}' \pm Z \left( \frac{\sigma'}{\sqrt{n}} \right)$$

There remained the problem of deciding on the calculation of the rate of loss in income that was attributed to the assignable cause of error. These values were derived from the assumption that the rate of production is constant and the specification limits fall at  $\bar{X}' \pm 3 \sigma'$ . When the mean of the process shifts by  $2 \sigma'$  the loss-rate was arbitrarily given the value \$100 per hour. It was also assumed that the quality characteristic is normally distributed and that the loss-rate is proportional to the increase in the percentage of defective items.

This means that register 1 would accumulate the cost of sampling plus the loss-rate specific to its condition.

Register 2 would also accumulate the sampling and loss-rate costs but would also store all "false alarm" costs (T) as defined above in the model variables.

Register 4 would store loss-rates and sampling costs for undetected out of control conditions.

Finally, register 3 would accumulate loss-rates and sampling costs as the other registers do but would include the cost of looking for the assignable cause (W) and the additional loss rate levied during the search.

Likely numerical values were chosen for the model parameters as follows

E - .05 hours per unit per sample

D - 2 hours

T - \$50

W - \$25

B - \$.50 per sample

C - \$.10 per unit

The levels chosen for the mean fluctuations were:

XNCR (1) - .01  $\sigma'$

XNCR (2) - .03  $\sigma'$

PROB (1) - .01

PROB (2) - .05

VALT (1) - .3  $\sigma'$

VALT (2) - .6  $\sigma'$

SDEV (1) - .50

SDEV (2) - 1.00

It was decided that Z would run from .75 to 3.50 in increments of .25 and that there would be 5 runs made for every value of Z. A replication would be performed for a reliable estimate of error variance.

When all 32 runs were completed a regression analysis will be performed on each of the resulting data plots to find the value of Z that allowed the minimum cost for the run.

An analysis of variance will also be performed on these  $Z_{\min}$  values to uncover all main effects and interactions found significant at the 5% level (4.49 = value of F for  $\theta_1 = 1$ ,  $\theta_2 = 16$ ). Finally, a linear response surface will be fitted to these values of  $Z_{\min}$  through a regression analysis to discover the best possible method of fit that allowed for satisfactory extrapolation for intermediate values of the four mean fluctuations in the future.

Sample Model Calculations

The following is the derivation of the expression used to calculate the loss in income experienced through a shift in the process mean. As explained earlier the process being sampled was considered a normal population. The specifications of the process were set at  $\mu \pm 3\sigma$  where

$\mu$  - process mean

$\sigma$  - standard deviation of the process

This means that if the process was considered in a state of control one could expect the percentage of defective product to be

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\mu-3\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx + \frac{1}{\sqrt{2\pi}} \int_{\mu+3\sigma}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Shifting to the standard normal distribution this becomes

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-3} e^{-\frac{t^2}{2}} dt + \frac{1}{\sqrt{2\pi}} \int_{3}^{\infty} e^{-\frac{t^2}{2}} dt = .0027 = .27\%$$

If the mean of the process shifts and the amount of the shift is expressed in terms of standard deviations of the process the percent defective then becomes

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\mu-3\sigma} e^{-\frac{1}{2}\left(\frac{x-(\mu+\Delta\sigma)}{\sigma}\right)^2} dx + \frac{1}{\sqrt{2\pi}} \int_{\mu+3\sigma}^{\infty} e^{-\frac{1}{2}\left(\frac{x-(\mu+\Delta\sigma)}{\sigma}\right)^2} dx$$

where  $\Delta$  = number of standard deviations shift in the process mean

To discover how this has affected the limits when evaluating this integral let

$$x - \frac{(\mu + \Delta\sigma)}{\sigma} = t$$

then

$$x = \sigma t + \mu + \Delta \sigma$$

and

$$dx = \sigma dt$$

then the limits become

when

$$x = -\infty$$

$$t = -\infty$$

when

$$x = \mu - 3\sigma$$

$$t = \mu - 3\sigma - \mu - \Delta\sigma$$

$$= -3\sigma - \Delta\sigma$$

and when

$$x = \infty$$

$$t = \infty$$

Therefore in terms of the standard normal distribution the percentage defective becomes

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-3-\Delta\sigma} e^{-\frac{t^2}{2}} dt + \frac{1}{\sqrt{2\pi}} \int_{3-\Delta\sigma}^{\infty} e^{-\frac{t^2}{2}} dt$$

In this problem the loss in income was based on a  $2\sigma$  change in the process mean and the amount of income loss established at \$100 per hour while this shift was in effect. The quality characteristic was assumed normal and any subsequent loss in income was proportional to the increase in defective material over and above the normal expected amount.



This meant that when the mean shifted to  $2\sigma$  the percent defective became

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-5} e^{-\frac{t^2}{2}} dt + \frac{1}{\sqrt{2\pi}} \int_1^{\infty} e^{-\frac{t^2}{2}} dt = .15866 = 15.866\%$$

This indicates a percentage increase in defective material of

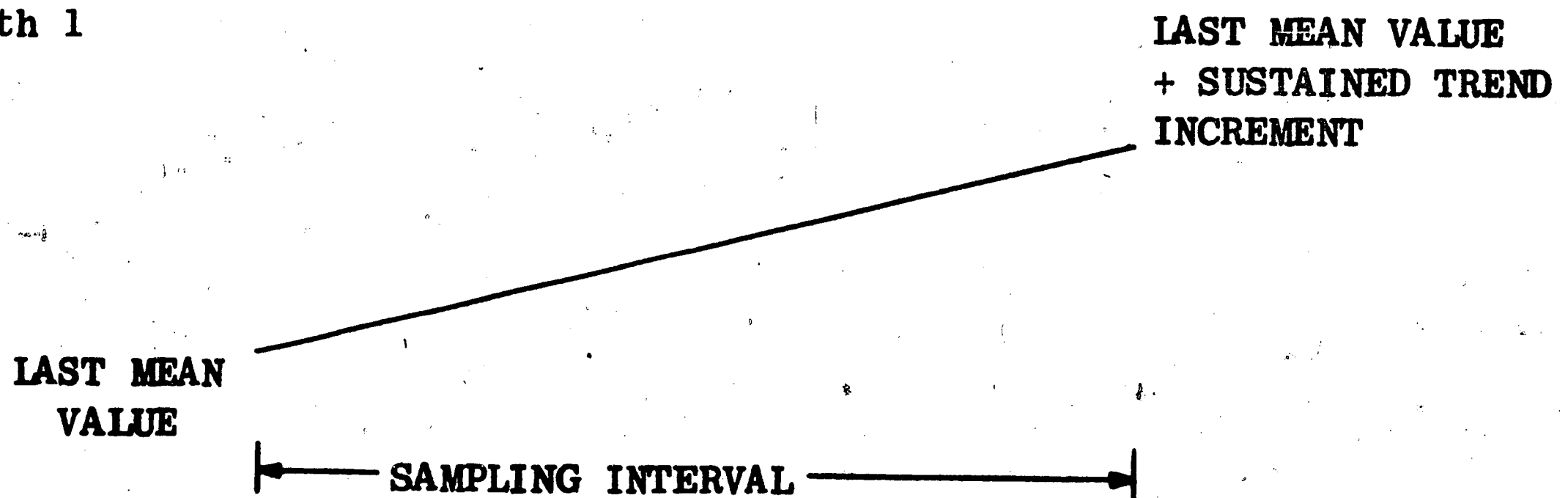
$$15.866\% - .27\% = 15.596\%$$

since the percentage change in defective material was considered proportional to the \$100 cost per hour levied on the process. The proportionality constant of future calculations became

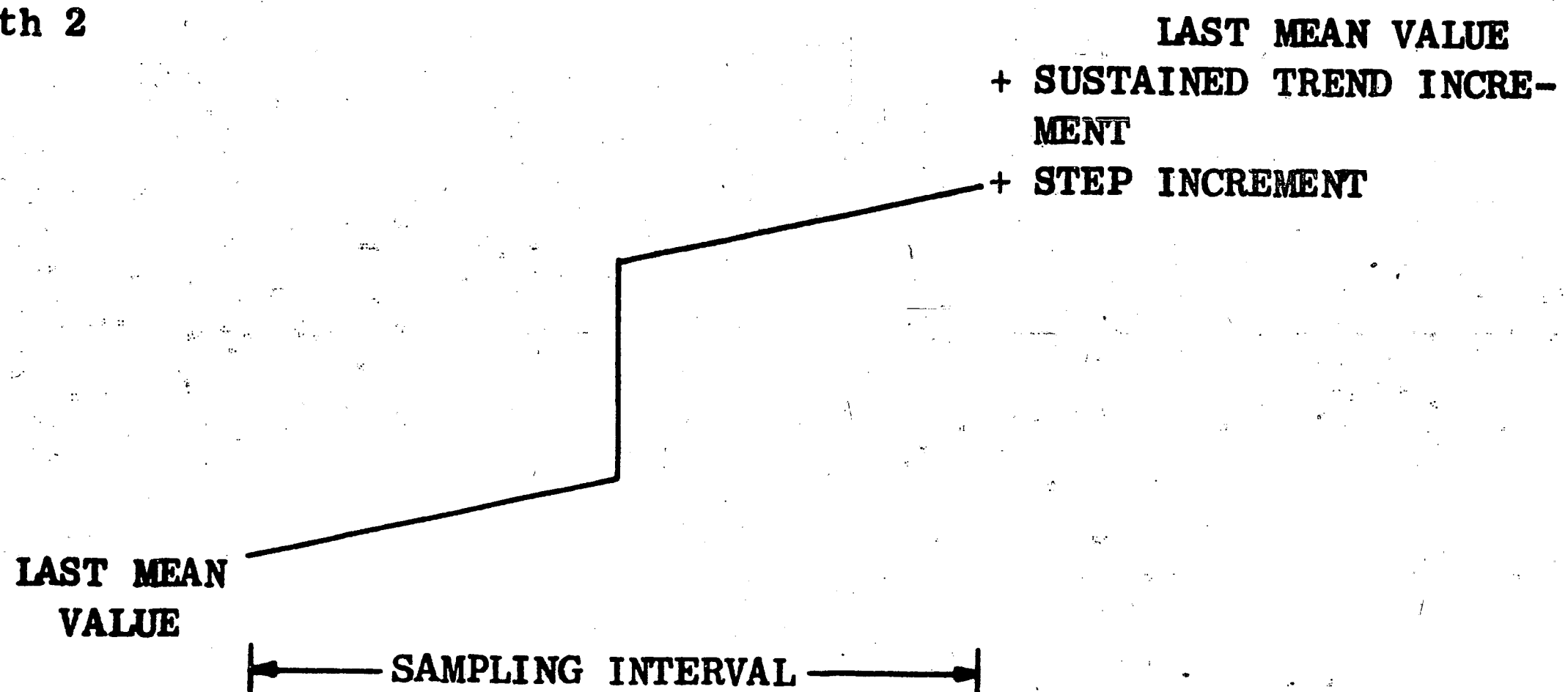
$$k = \$100/15.596$$

The problem remained how to calculate the loss when the mean could change in anyone of two possible paths.

Path 1

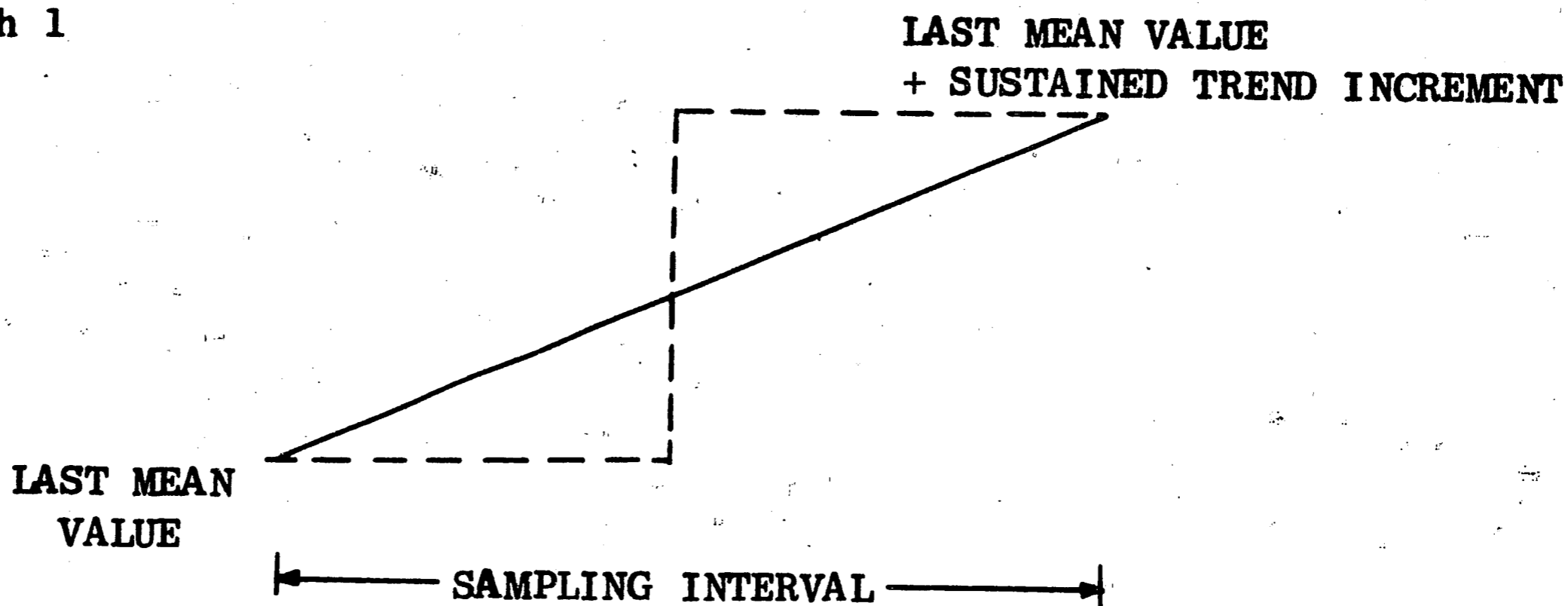


Path 2

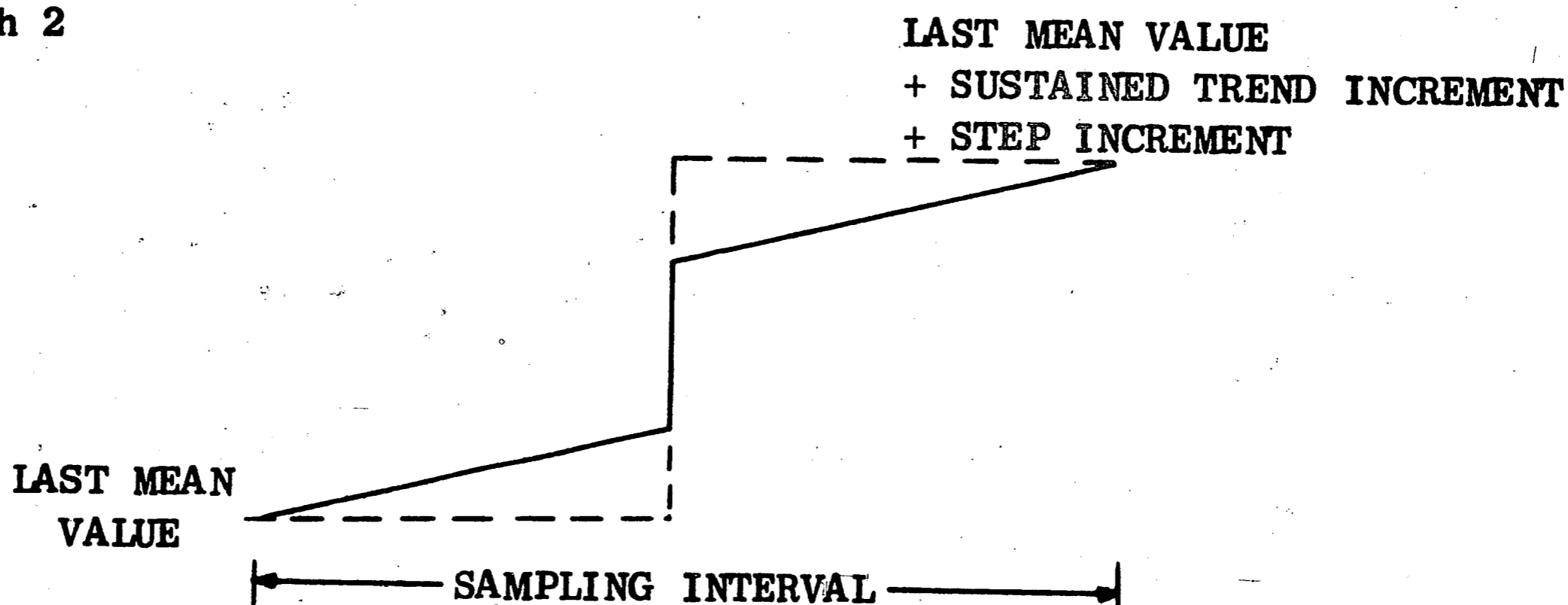


It was found that an accurate estimate could be obtained if the 1 hour sampling interval was divided in two 30 minute intervals and the cost calculations made as shown below

Path 1



Path 2



The final general expression for calculating the income loss for any path became

$$((((1. - P)100) - .27)100) / 15.596$$

where P = value from standard normal tables to establish percent change in defective material dependent upon latest value of mean.



### Experimental Results

The program simulations were run on the IBM 1620 and IBM 1410 computers.

For purposes of illustration a typical simulation run for the set of causal variables

$$\text{XNCR} = .03 \sigma'$$

$$\text{PROB} = .01$$

$$\text{VALT} = .6 \sigma'$$

$$\text{SDEV} = 1.00$$

is displayed in Figures 1 and 2. These figures list the cost associated with each value of Z as it progresses from .75 to 3.5. They also show the value of the control limit expressed in terms of standard deviations of the distribution of the sample means, the actual displacement path of the mean (solid line) and the sample averages taken from the parent population.

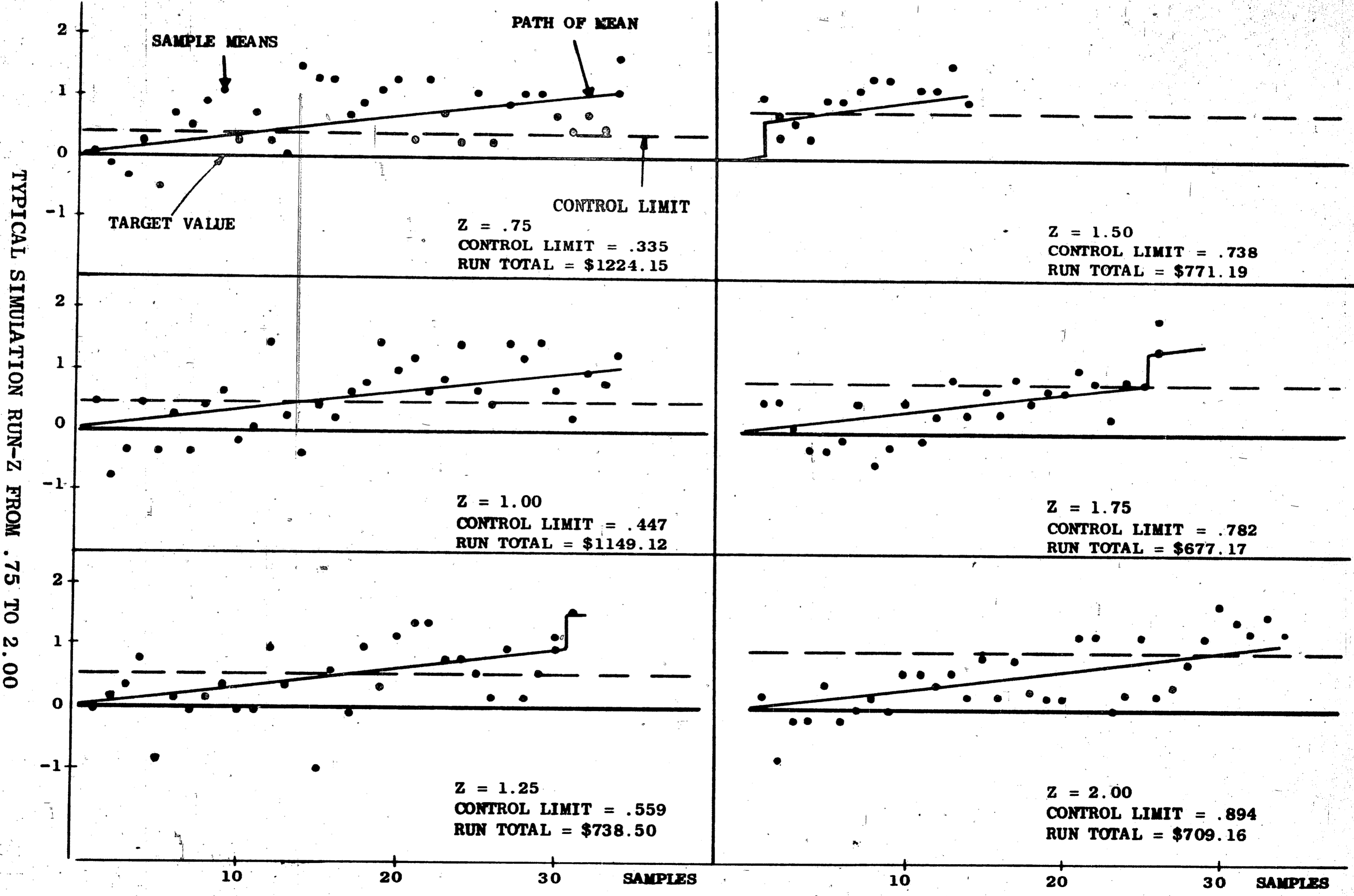
Figure 3 illustrates a plot of these cost values and the associated least squares curves together with pertinent analysis information. Also indicated is the value of Z that determines the position of the control limit for a "least cost" control chart operation for the tested causal variables.

The simulation values for the 32 runs (1 run for each of  $2^4$  combinations plus 1 replication) are listed in Appendix I.

The results from the initial regression analysis are located in Appendix II.

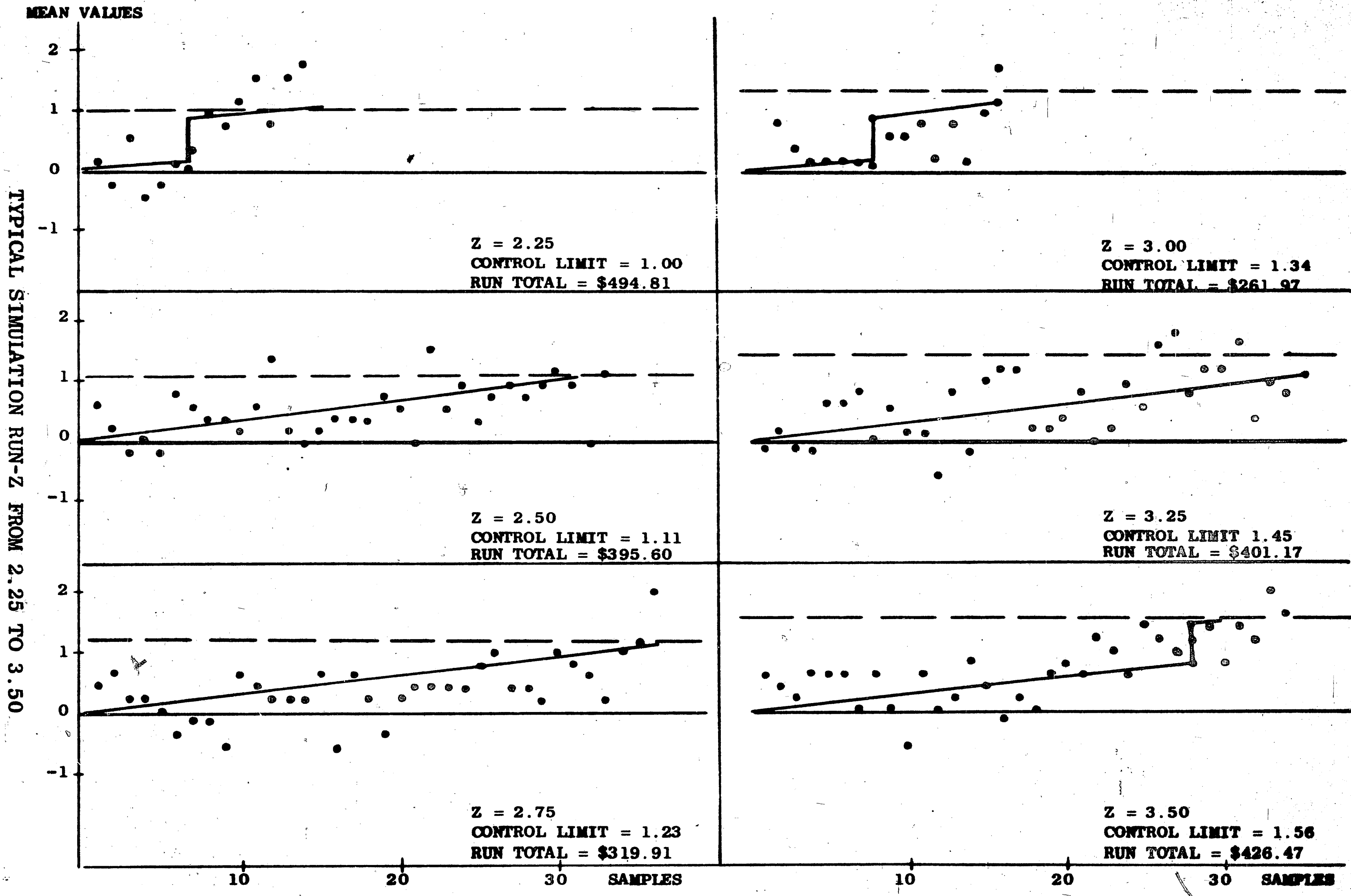
The values of Z that determine the least cost position for the control limit are presented in experimental factorial form in Figure 4.

MEAN VALUES



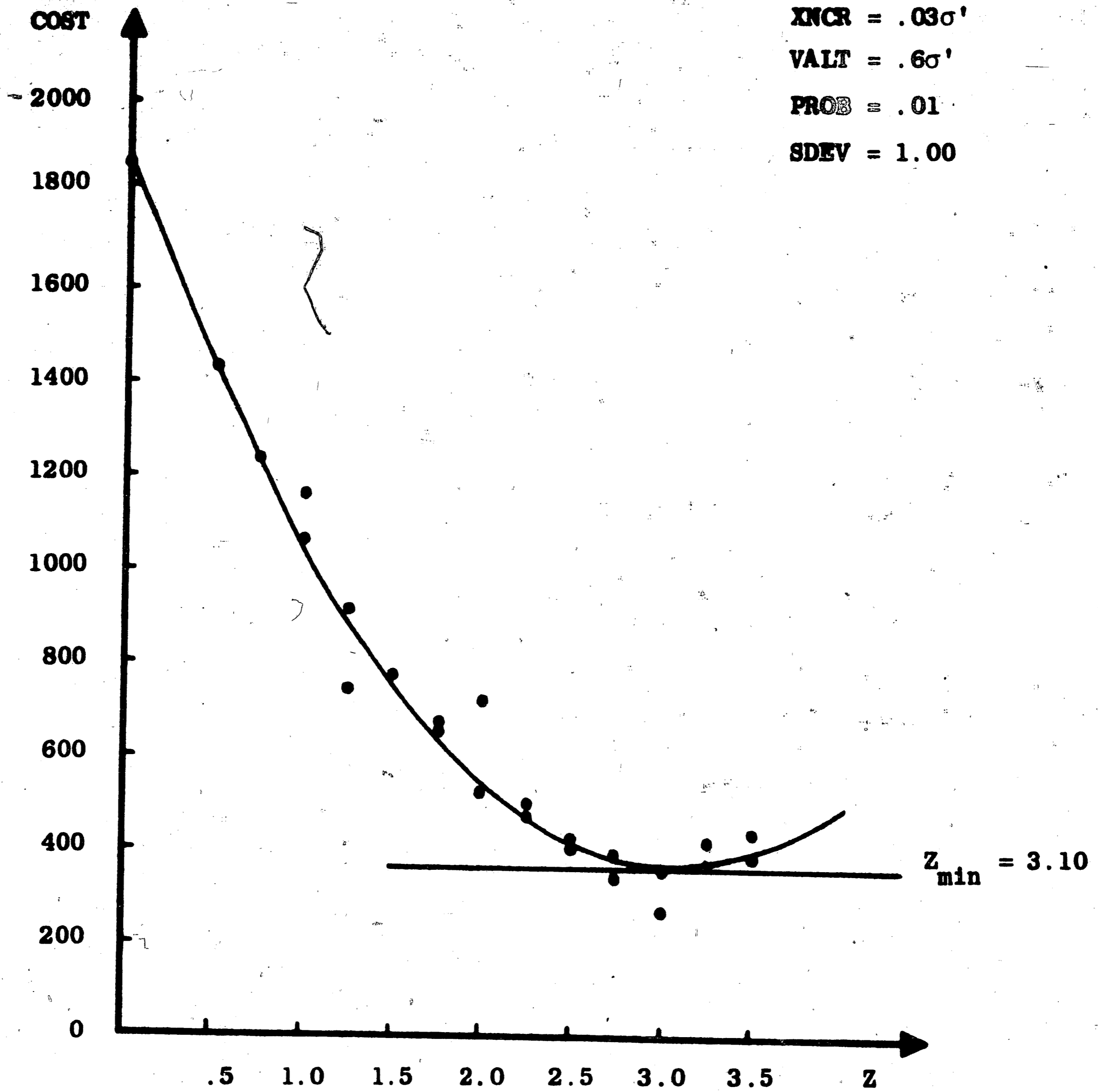
TYPICAL SIMULATION RUN-Z FROM .75 TO 2.00

Figure 1



TYPICAL SIMULATION RUN-Z FROM 2.25 TO 3.50

Figure 2



**COEFFICIENT OF MULT. CORRELATION, R SQUARE**  
**.92646**

**REGRESSION EQUATION**  
**COST = 1873.08 - 967.87 x + 154.82 x<sup>2</sup>**

**REGRESSION CURVE FOR TYPICAL SIMULATION RUN**

**Figure 3**

STEP SHIFT IN MEAN	STANDARD DEVIATION OF PROCESS	XNCR - TREND SHIFT IN MEAN			
		XNCR(1) = $.01\sigma'$		XNCR(2) = $.03\sigma'$	
		PROBABILITY		PROBABILITY	
		.01	.05	.01	.05
<b>.3<math>\sigma'</math></b>	<b>.50</b>	<b>2.880</b>	<b>3.096</b>	<b>2.772</b>	<b>3.351</b>
		<b>3.167</b>	<b>2.987</b>	<b>4.010</b>	<b>2.709</b>
		<b>AVG - 3.024</b>	<b>AVG - 3.0415</b>	<b>AVG - 3.390</b>	<b>AVG - 3.030</b>
	<b>1.00</b>	<b>3.818</b>	<b>3.220</b>	<b>3.555</b>	<b>3.393</b>
		<b>3.506</b>	<b>3.472</b>	<b>3.823</b>	<b>3.289</b>
		<b>AVG - 3.662</b>	<b>AVG - 3.346</b>	<b>AVG - 3.689</b>	<b>AVG - 3.341</b>
<b>.6<math>\sigma'</math></b>	<b>.5</b>	<b>2.772</b>	<b>3.138</b>	<b>2.958</b>	<b>5.534</b>
		<b>3.942</b>	<b>3.298</b>	<b>3.580</b>	<b>5.863</b>
		<b>AVG - 3.357</b>	<b>AVG - 3.218</b>	<b>AVG - 3.269</b>	<b>AVG - 5.697</b>
	<b>1.00</b>	<b>3.578</b>	<b>3.369</b>	<b>3.833</b>	<b>3.310</b>
		<b>3.271</b>	<b>3.584</b>	<b>3.617</b>	<b>2.971</b>
		<b>AVG - 3.425</b>	<b>AVG - 3.477</b>	<b>AVG - 3.725</b>	<b>AVG - 3.141</b>

TRUE MEAN OF ALL TRIALS

$$\mu = 3.4895$$

Figure 4

### Analysis of Results

A more thorough appreciation of cost action in the operation of a control chart can be gained from Figures 5 and 6 which depict the cost path for a typical simulation.

Figure 5 shows the accumulation of costs in Register 1 (defined earlier) while Figure 6 shows the accumulated costs in Register 2 (also defined earlier).

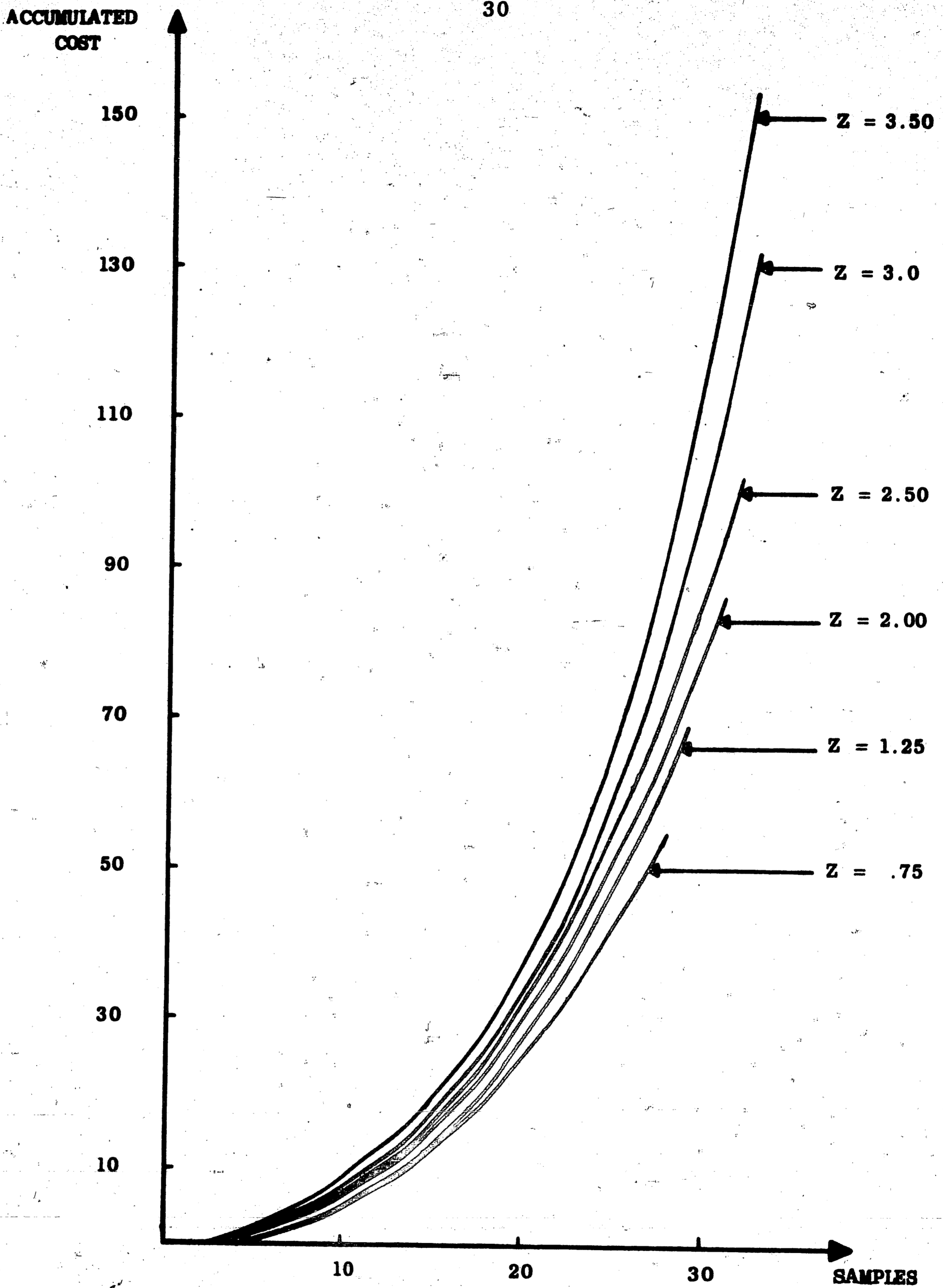
For the smaller values of Z the most prominent cost is that of looking for an assignable cause of process error where none exists (T). This cost appears early and often for the smaller values of X and since it is one of the more expensive items it keeps the overall cost curve high in this area.

As Z moves toward larger values the overall cost plots show the gradual diminution of the "false alarm" costs and the increasing values, though they are still relatively small compared to the earlier costs, associated with an increase in the production of defective material.

The paths of accumulated costs for Registers 1 and 2 show this process action. Both registers retain their quadratic shape but as Z moves toward larger values Register 1 rises higher and higher for the same number of samples while Register 2 gradually loses its influence.

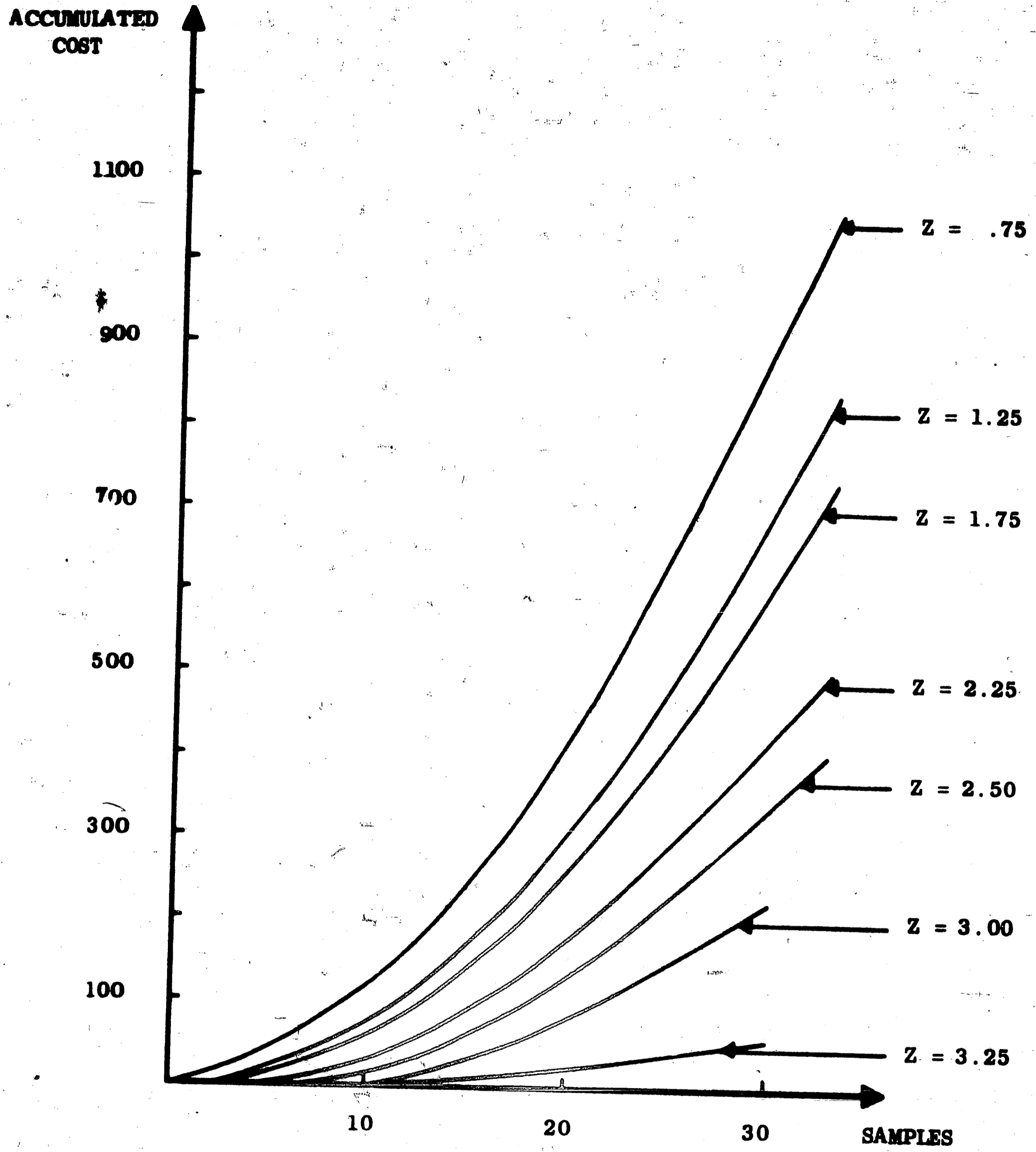
Then as Z reaches its largest values Register 4 begins to influence the cost path since the amount of defective material reaches appreciable heights and the path begins to rise even higher. This action is also displayed in Figures 1 and 2.





COST ACCUMULATION FOR REGISTER 1 FOR TYPICAL RUN

Figure 5



COST ACCUMULATION FOR REGISTER 2 FOR TYPICAL RUN

Figure 6



Figure 7 shows the results of an analysis of variance performed on the  $Z_{\min}$  values of Figure 4. Note that in the figure

A = XNCR

B = PROB

C = VALT

D = SDEV

For 1 and 16 degrees of freedom the 5% value of F is 4.49. A mean square based on 1 degree of freedom is thus significant at the 5% level if it is as great as

$$4.49 \times 139902. = 628049.$$

The analysis of variance shows that Factors A and C are significant at the 5% level. Interactions AD, BC, ABC, ABD and BCD are also significant at the 5% level while interactions BD, CD and ABCD are significant at the 1% level as well ( $8.53 \times 139902. = 1193364.$ ).

Note, however, that if the analysis of variance indicates, as it does here, that while interaction AD is significant and main effect D is not, we cannot conclude that D has no effect. The existence of AD means that both A and D affect the response but not independently. The non-existence of D simply means that D affects the response in different ways at the various levels of A and that when its effect is averaged over the values of A used in the experiment the average effect is small. In quoting the effect of D it is thus necessary to state also the level of A, and vice versa.

To find the main effect of a factor, say A, we average the response corresponding to all treatments containing the higher level of A and do the same for all treatments containing the lower level of A.

SOURCE OF VARIATION		SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE
Main Effect	A	933661.	1	933661.
	B	70125.	1	70125.
	C	968136.	1	968135.
	D	6161.	1	6161.
Two Factor Interactions	AB	288421.	1	288421.
	AC	488567.	1	488567.
	AD	952200.	1	952200.
	BC	954272.	1	954272.
	BD	1234020.	1	1234020.
	CD	1382784.	1	1382784.
Three Factor Interactions	ABC	685033.	1	685033.
	ABD	1019592.	1	1019592.
	ACD	547058.	1	547058.
	BCD	781250.	1	781250.
Four Factor Interactions	ABCD	1575313.	1	1575313.
Sum		11886593.	16	-
Residual		2238435.	15	139902.
Total		14125028.		

ANALYSIS OF VARIANCE OF FIGURE 4

Figure 7

For example, referring to Figure 4 we have:

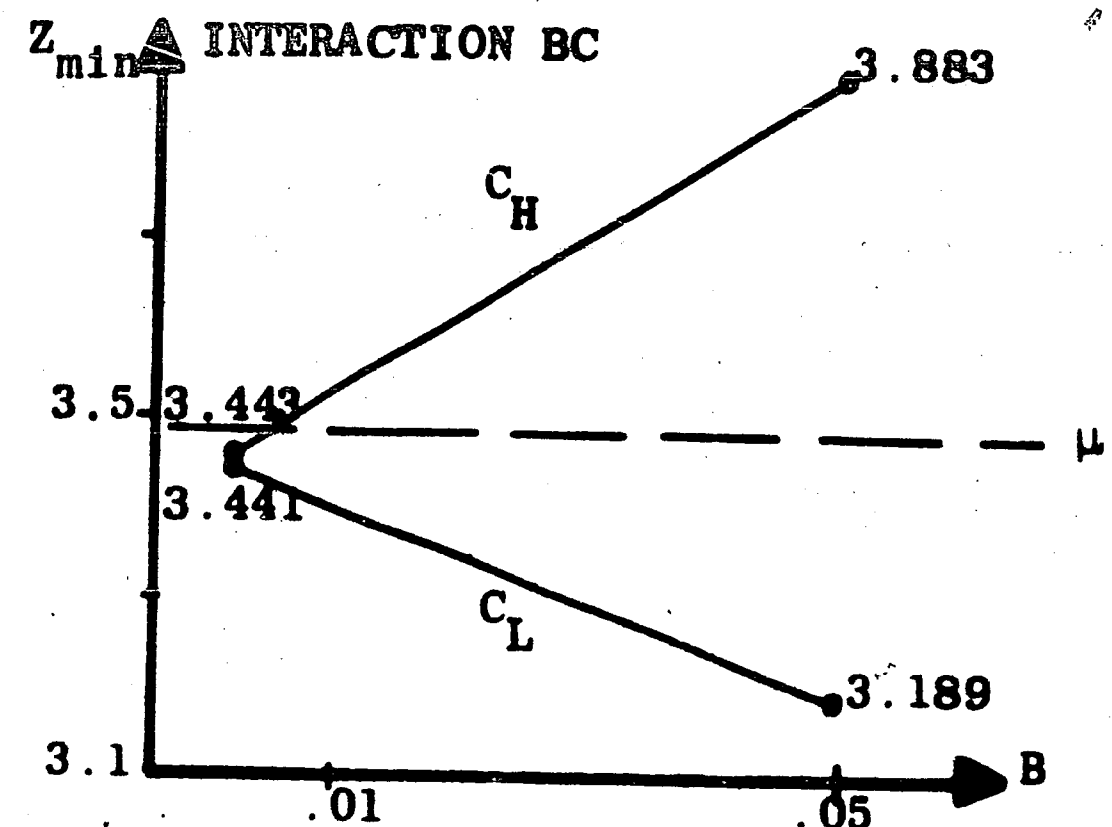
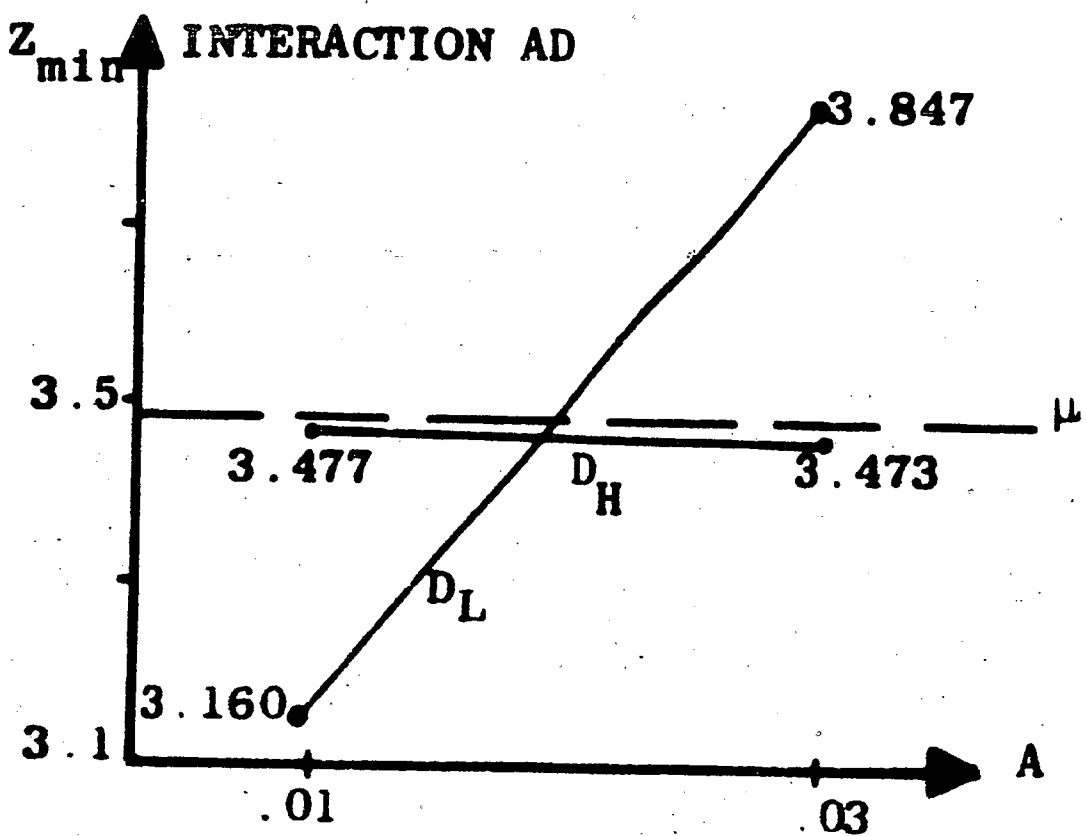
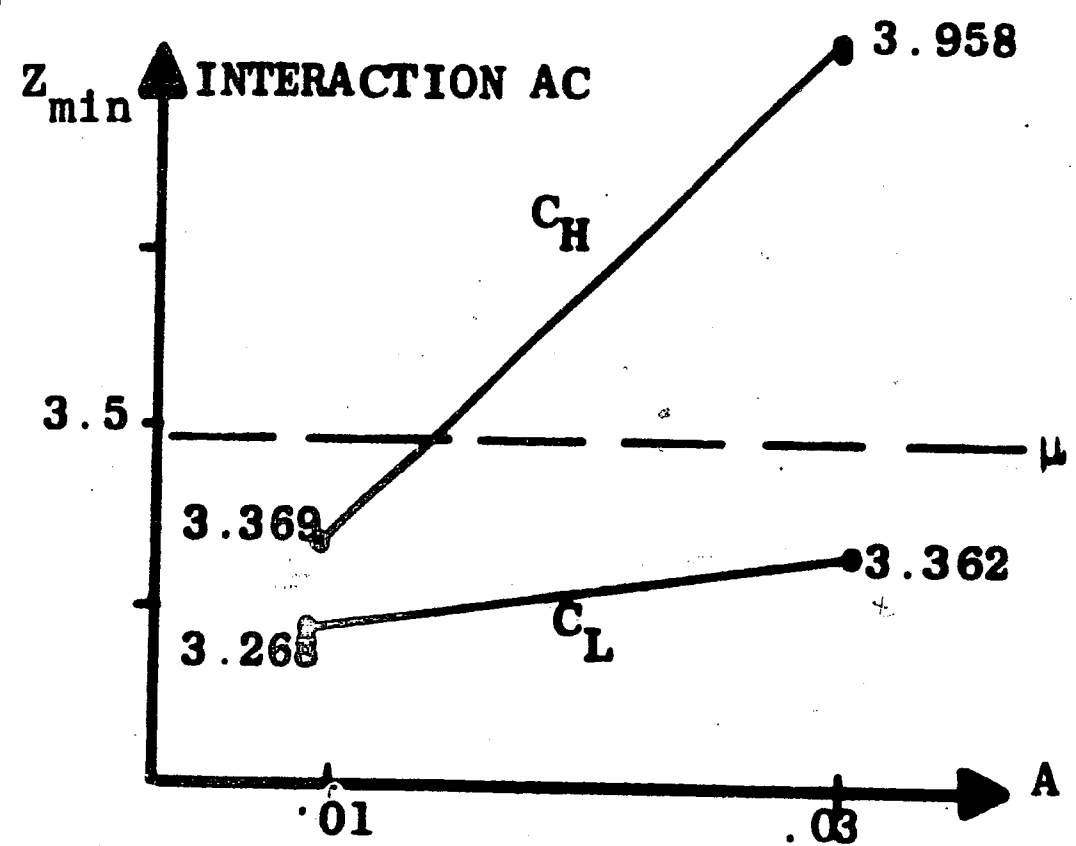
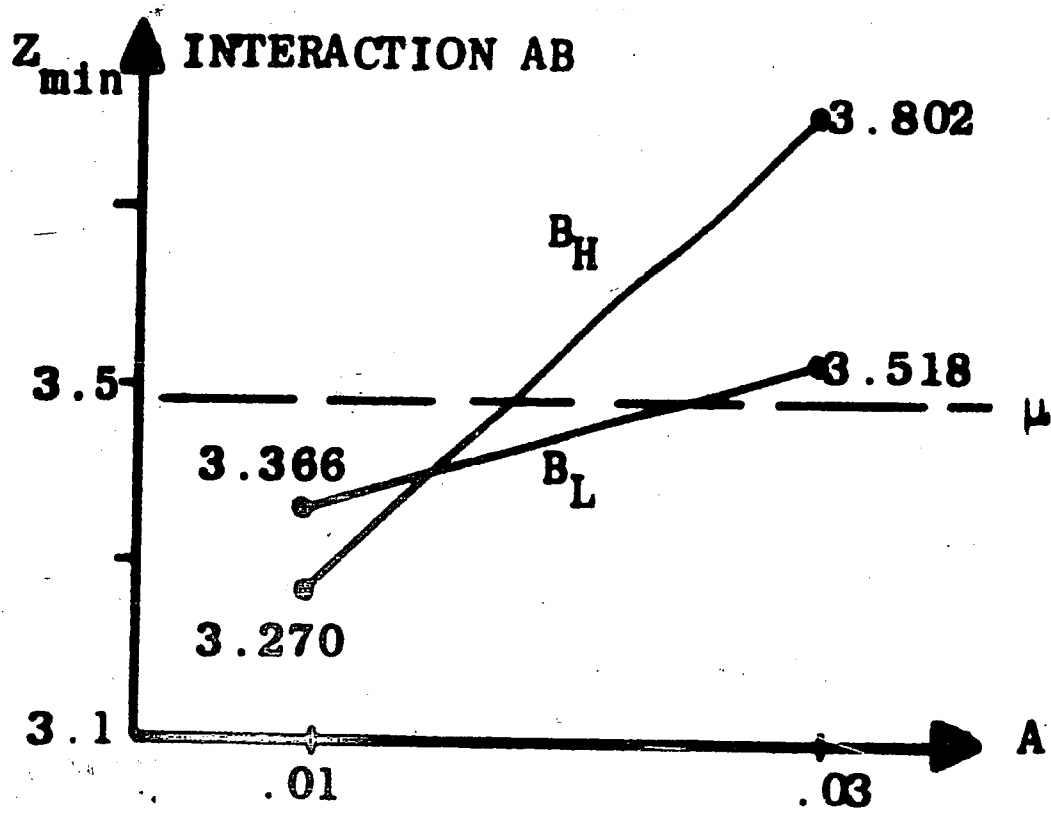
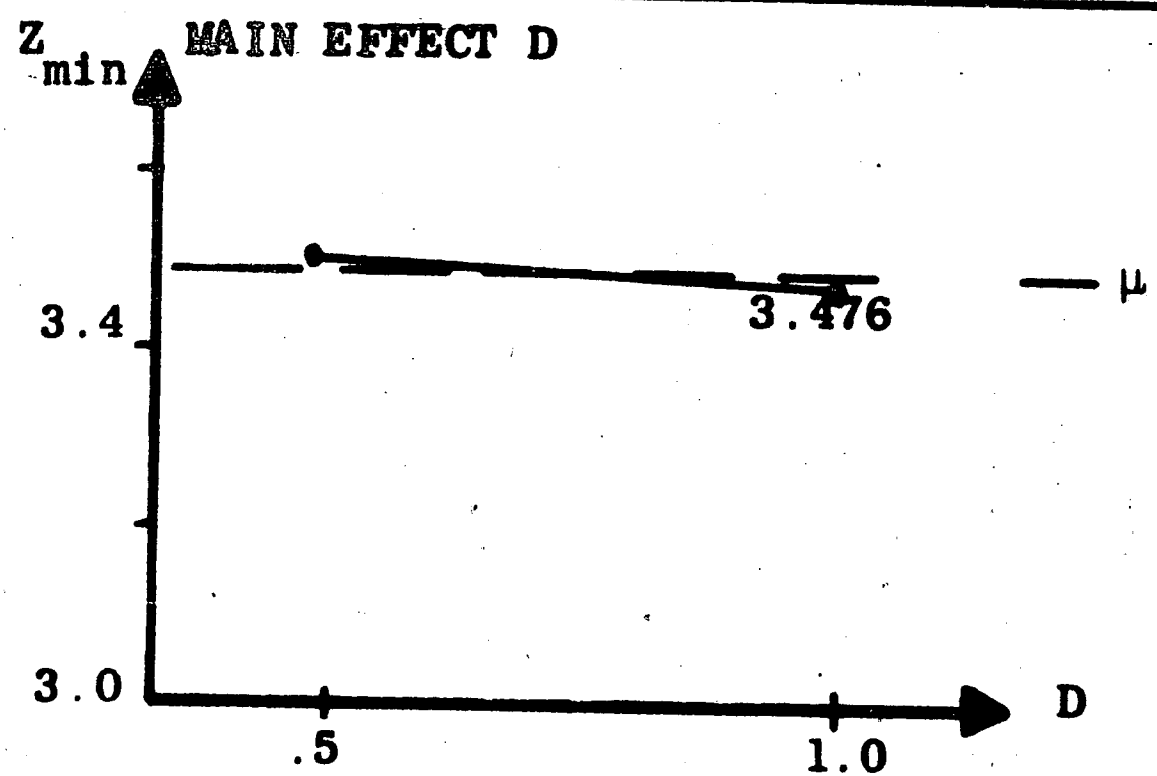
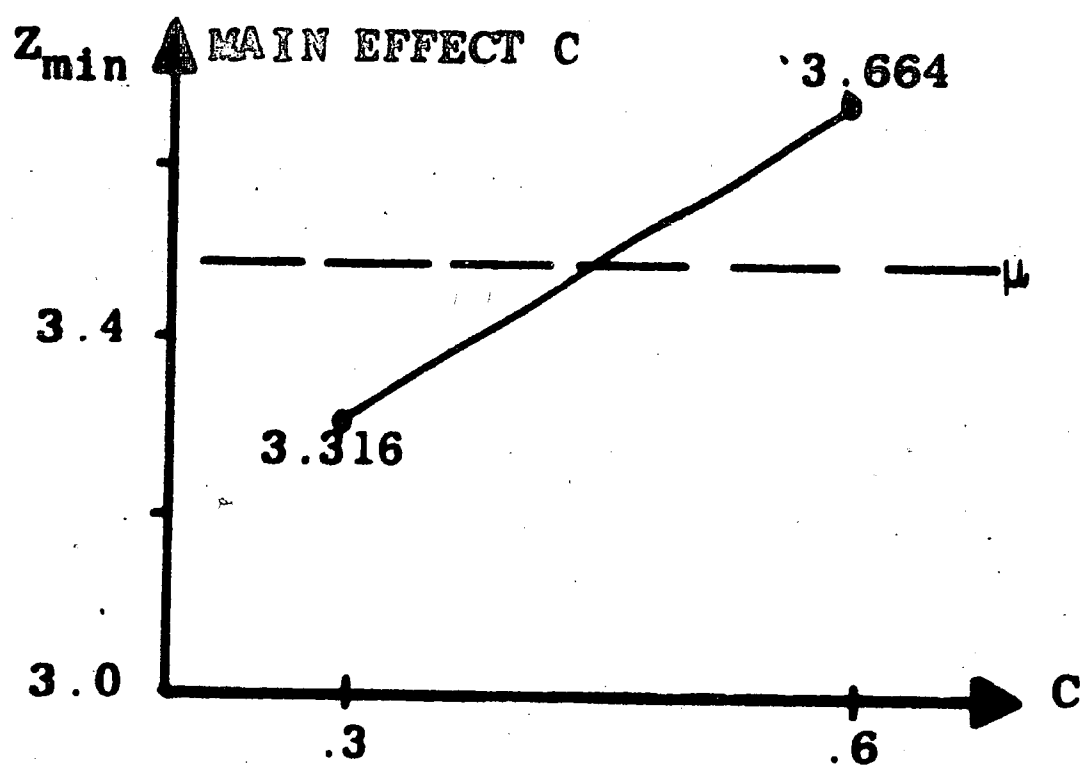
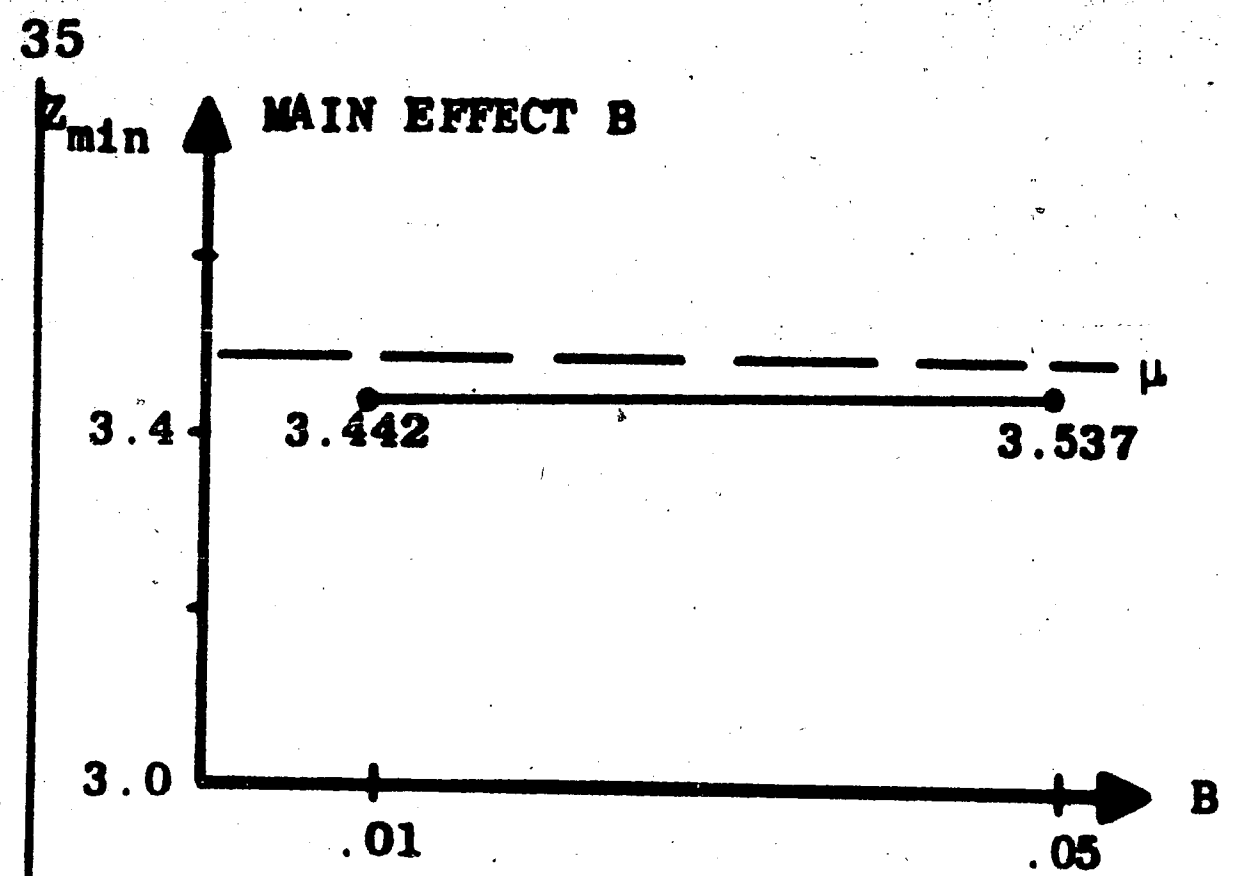
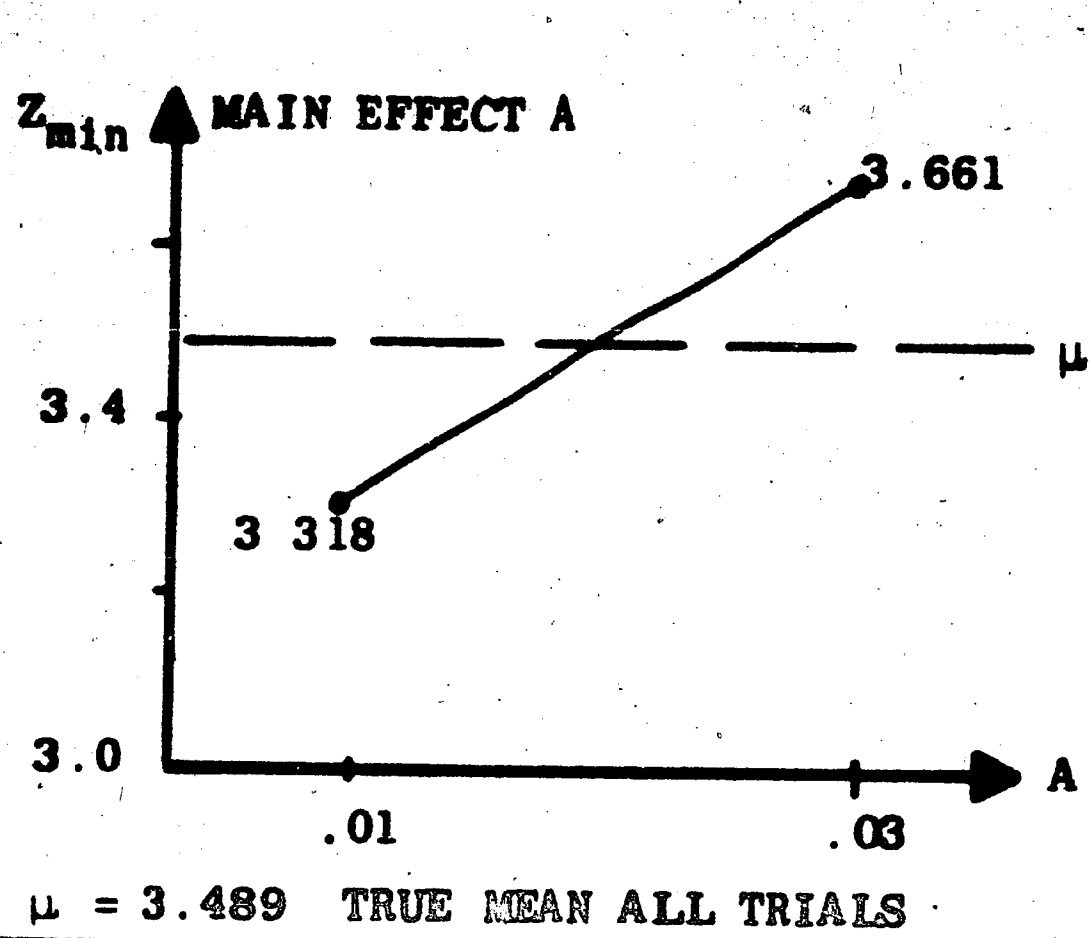
$$\begin{aligned} & \text{Average response at higher level of A} \\ & = 1/16 (2.880 + 3.167 + 3.818 + 3.506 + 2.772 + 3.942 \\ & \quad + 3.578 + 3.271 + 3.096 + 2.987 + 3.220 + 3.472 \\ & \quad + 3.138 + 3.298 + 3.369 + 3.584) = 3.318 \end{aligned}$$

Similarly the average response at the lower level of A = 3.661

These points are then subtracted to determine the main effect of A. The sign of the difference denoting the path of the main effect.

Interaction effects are defined in a similar manner. For example the inter-action between A and B denoted AB is defined as one-half of the difference between the effect of A when B is at the higher level, and the effect of A when B is at the lower level. All the main effects and interactions are shown plotted in Figures 8 and 9. These plots illustrate the strong interaction between the probability and standard deviation chosen (BD) as well as the interaction between the step increment and the standard deviation (CD). On the other hand, while the trend increment interacts with the standard deviation (AD) rather significantly as does the probability of a step increment and the amount of the step increment (BC), the reactions are in no way as significant as the others.

In the final analysis, however, having inferred from a factorial experiment that some combinations of input process variables provide greater resulting effects than others, it remains to estimate as precisely as possible what response can be expected for the other combinations of inputs as yet untried. Toward this end a regression analysis (SCRAP) (19) was performed on the factors and interactions considered signifi-



MAIN EFFECTS AND INTERACTIONS

Figure 8

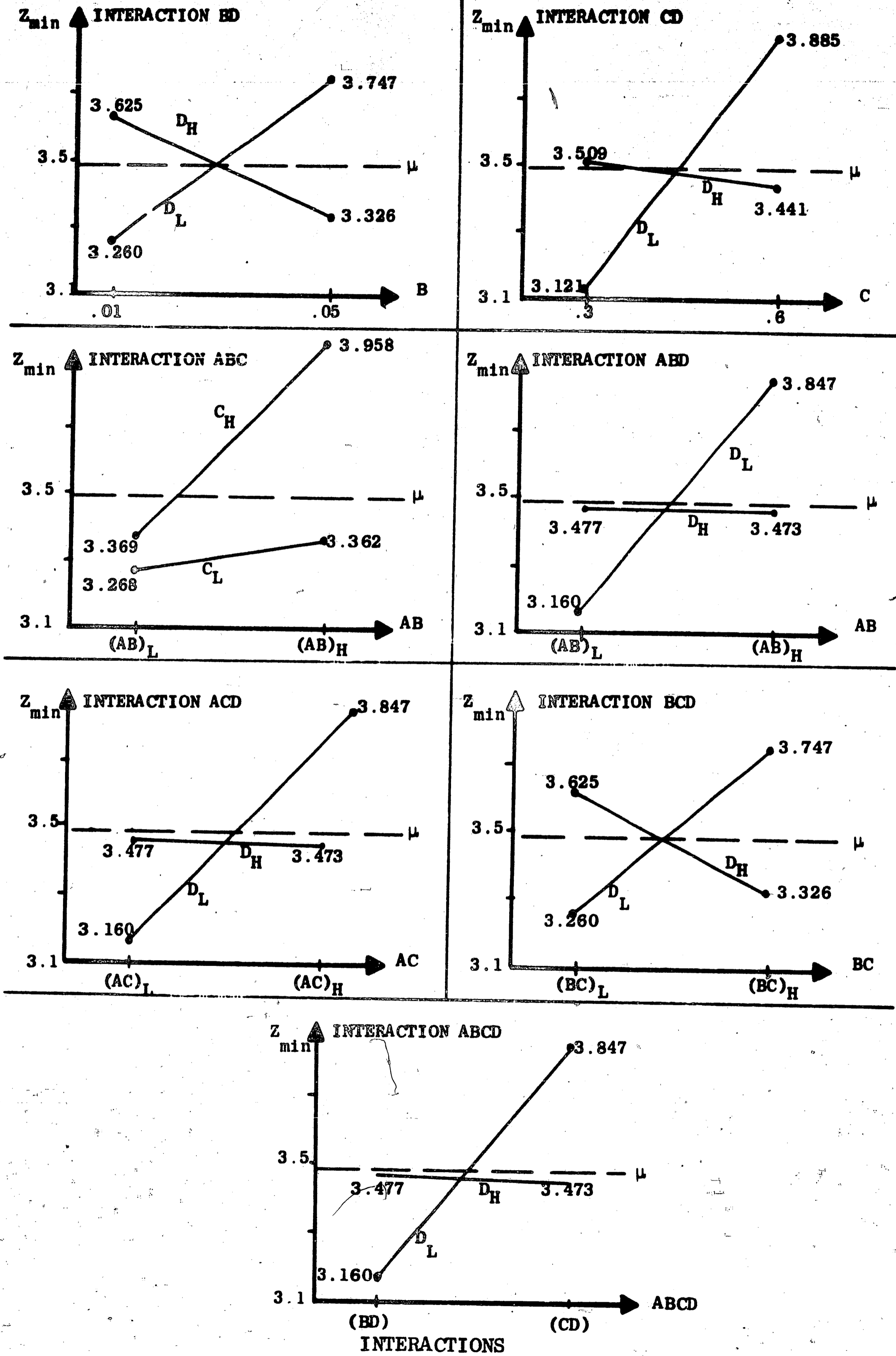


Figure 9

cant by the analysis of variance. Another regression analysis (STRAP) (20) was performed on the entire set of factors and interactions included in the analysis of variance be they considered significant or not.

The regression analysis for the significant factors only produced the equation:

$$Z_{\min} = 2.96 - 26.35A + .17C + 50.07AD - 17.49BC + 46.43BD \\ + .31CD + 5124.29ABC - 2813.63ABD - 79.72BCD$$

$$\text{Coefficient of Multiple Correlation, } R^2 = .6516$$

The interaction ABCD was not included in this analysis since the program was not equipped to handle variables of this size.

The regression analysis for the entire set of factors and interactions produced

$$Z_{\min} = 3.04 + 31.56A + 31.43B + .71C - 5488.71AB - 152.88AC \\ - 153.17BC - 27.47BD + 19791.32ABC + 5026.28ABD + 118.70ACD \\ + 150.94BCD - 19887.94ABCD$$

$$\text{Coefficient of Multiple Correlation, } R^2 = .97386$$

Both surfaces were evaluated at the experimental levels and the results are noted below. The actual input levels can be found from Figure 4.



<u>Z<sub>min</sub></u> <u>SIMULATION</u> <u>RESULTS</u>	<u>Z<sub>min</sub></u> <u>SCRAP</u> <u>ANALYSIS</u> <u>RESULTS</u>	<u>Z<sub>min</sub></u> <u>SCRAP</u> <u>ANALYSIS</u> <u>RESULTS</u>
3.024	3.11	3.22
3.662	3.38	3.44
3.357	3.19	3.22
3.425	3.39	3.55
3.042	3.41	2.99
3.346	3.56	3.38
3.218	3.41	3.24
3.477	3.02	3.44
3.390	3.11	3.29
3.689	3.60	3.77
3.269	3.50	3.31
3.725	3.91	3.66
3.030	3.51	3.04
3.341	2.76	3.31
5.697	5.05	5.68
3.141	3.75	3.14

These data show a more favorable response from the STRAP generated response surface than from the SCRAP response surface.

For further comparisons Appendix III shows some extrapolations of these surfaces for various values of causal variables.

### Conclusions

1. The cost of looking for a cause of mean variation when none exists (T) is the primary factor in determining the path of the cost curve for smaller values of Z. The cost of locating a cause of mean variation when it can be detected (W) with the accompanying cost of increased defective material determines the path of the cost curve for the larger values of Z.
  - a. It is reasonable to assume that T will usually be higher than W, but if both tend to be small so will the value of  $Z_{\min}$ . The converse is also true. If T and W rise proportionately, so does  $Z_{\min}$ .
  - b. This conclusion confirmed a preliminary intuition that the general cost path would be parabolic in nature with "false alarm" cost dominating the accumulated costs for small values of Z. Then as Z took on larger values the costs of increased amounts of defective material would be a prime factor in raising the cost path once some minimum cost was achieved.
  
2. If T is large, while W is perhaps 50% of T, and the cost of defective material is not excessive, it may be economically feasible to run an  $\bar{X}$  chart with a  $Z_{\min}$  value greater than the specification imposed on the process.
  - a. This becomes possible because the cost of looking for the cause of some mean variation can become so excessive as to permit small departures in quality from the assigned specification conditions.

- b. This higher value of  $Z_{\min}$  is justified in this experiment because of the distribution of the quality characteristic of the process output. Small shifts in the mean resulted in only a small buildup of defective material costs. This normal buildup of costs becomes serious only in a process that accepts very little defective material. However, if this were so, it seems improbable that an  $\bar{X}$  chart would be used to control the process.
- c. For example, in cases where a specification is placed at  $\bar{X} \pm 2\sigma'$  and small shifts in the process mean seriously affect the percentage defective the control chart would most likely be supplemented by a 100% inspection for attributes or some other attribute sampling plan.
3. As the value of the sustained linear shift and the step shift in the process mean increase, so may the value of  $Z_{\min}$ . If the false alarm cost (T) dominates the selection of the  $Z_{\min}$  value, as it did in this experiment, then the control chart seeks a higher control limit value to escape its consequences.
4. The analysis of variance showed that the standard deviation of the process population interacted strongly with all the levels of the remaining input factors. The paths of the interactions, however, also varied with the size of the standard deviation.
- a. For the 2 factor interactions, when the standard deviation was at its smaller value, an increase in each of the other

variables drove up the value of  $Z_{\min}$ , while the higher level of the standard deviation scarcely affected the  $Z_{\min}$  value.

- b. In the 3 factor interactions, however, the higher level of standard deviation began to drive down the value of  $Z_{\min}$ .
  - c. This points to a rather critical situation when one is dealing with distributions in which the shift in the standard deviation is toward smaller values. Care must be taken to insure as precise an estimate of process mean variation as possible. Small errors in the estimation of these variables can lead to large errors in the selection of  $Z_{\min}$ .
5. In this experiment the levels of the factors were widely spaced. Further experimental work at intermediate levels is necessary to establish patterns of interaction.
- a. When an interaction is large, the corresponding main effects do not convey as much meaning as they could have. (21) In the present example then, it is of no great advantage to know that on the average (i.e., averaged over all levels of the other variables), the sustained linear trend changes at its lower level. The existence of these large interactions means that the effect of one factor is markedly dependent on the level of the other.
  - b. Some interaction, however, is expected by the very nature of the experiment and one should not be dismayed by this. The input factors have an additive effect on system response and this additive effect is heightened even further as the variance of the sampled distribution decreases.

RECOMMENDATIONS FOR FUTURE STUDY

1. As mentioned in the conclusion, further simulation should be accomplished on intermediate values of all the causal variables used in the present model to ascertain the path of  $Z$  between  $\min$  the values chosen for the experiment.
2. A study of an existing process under the maintenance of an  $\bar{X}$  chart should be undertaken to define all costs of operation and losses in income. A simulation model could be designed and past  $\bar{X}$  plots used as inputs to the model. A cost comparison between the present limit values and the values found through simulation might then prove profitable.
3. A comparison of the economic feasibility of Cumulative Sum Charts as opposed to Control Charts with a single control limit and Control Charts with warning lines should be investigated using the same criterion as those employed in this paper.
4. If there is a cyclic nature to the  $\bar{X}$  plots of a process then a study should be attempted to completely design through simulation, the sample size, the sampling interval and control limit of a Control Chart operation.

**APPENDIX I**

**Results from Cost Model Simulations**



APPENDIX IRUN TOTALS-SAMPLES

RUN 1  
 XNCR =  $.01\sigma'$   
 PROB = .01  
 VALT =  $.30\sigma'$   
 SDEV = 1.00  
 Z = .75  
 2101.46      72  
 3616.41      100  
 3366.41      100  
 2215.14      70  
 2070.72      70  
 RUN AVE.    2674.03  
 Z = 1.00  
 1377.40      46  
 1937.17      70  
 2439.06      72  
 2511.33      70  
 3016.41      100  
 RUN AVE.    2256.28  
 Z = 1.25  
 2707.94      70  
 1703.59      70  
 2816.41      100  
 2666.41      100  
 2037.89      71  
 RUN AVE.    2386.45  
 Z = 1.50  
 1427.60      40  
 2180.60      101  
 2172.44      70  
 1128.80      70  
 1835.77      71  
 RUN AVE.    1749.11  
 Z = 1.75  
 1966.41      100  
   425.25      40  
 1201.68      71  
   944.50      70  
 1636.22      70  
 RUN AVE.    1234.82  
 Z = 2.00  
 1856.87      72  
 1566.41      100  
   804.68      60  
 1845.81      102  
 1375.16      98  
 RUN AVE.    1489.79

RUN TOTALS-SAMPLES

Z = 2.25  
 1216.41      100  
 1616.41      100  
   990.69      71  
 1566.41      100  
 1576.68      104  
 RUN AVE.    1393.33  
Z = 2.50  
   851.03      70  
   585.24      66  
   976.22      70  
   400.59      47  
   344.21      71  
 RUN AVE.    631.46  
Z = 2.75  
 1043.07      108  
   578.80      70  
 1161.06      103  
   835.89      74  
   916.41      100  
 RUN AVE.    907.05  
Z = 3.00  
   366.71      73  
   175.75      45  
   880.92      101  
   761.99      71  
   856.88      100  
 RUN AVE.    608.42  
Z = 3.25  
   678.17      77  
 1059.07      106  
   330.91      47  
   534.43      76  
   699.16      72  
 RUN AVE.    660.35  
Z = 3.50  
   291.26      51  
 1155.39      125  
   458.64      88  
   510.60      73  
   725.87      107  
RUN 2  
 XNCR =  $.01\sigma'$   
 PROB = .01  
 VALT =  $.3\sigma'$   
 SDEV = .50

RUN TOTALS-SAMPLES

Z = .75  
 1535.29      50  
   597.91      22  
 1735.29      50  
 1835.29      53  
 1985.29      58  
 RUN AVE.    1537.81  
Z = 1.00  
 1535.29      50  
 1685.29      55  
   737.18      20  
 1635.29      48  
   592.85      20  
 RUN AVE.    1237.18  
Z = 1.25  
   487.29      20  
   368.95      26  
 1635.29      50  
   388.93      23  
   478.54      26  
 RUN AVE.    671.81  
Z = 1.50  
 1167.03      52  
   337.29      20  
 1085.29      50  
 1100.77      51  
 1385.29      50  
 RUN AVE.    1015.14  
Z = 1.75  
   378.54      20  
 1235.29      50  
 1085.29      47  
   581.18      23  
   885.29      26  
 RUN AVE.    833.12  
Z = 2.00  
   885.29      50  
   885.29      50  
   885.29      50  
   252.24      31  
   412.34      42  
 RUN AVE.    664.09  
Z = 2.25  
   620.23      20  
   400.77      51  
   635.29      50  
   226.62      25

RUN TOTALS-SAMPLES

970.73	55
RUN AVE.	570.73
Z = 2.50	
450.77	51
205.63	24
240.31	21
535.29	50
485.29	50
RUN AVE.	383.46
Z = 2.75	
600.77	51
376.32	39
365.20	23
518.52	49
485.29	50
RUN AVE.	469.22
Z = 3.00	
244.01	26
265.40	38
137.90	29
300.77	51
385.29	50
RUN AVE.	266.68
Z = 3.25	
775.66	67
534.06	48
266.08	26
560.94	57
554.99	59
RUN AVE.	538.35
Z = 3.50	
808.90	68
256.69	23
663.19	65
490.37	56
528.56	60
RUN AVE.	549.54

RUN 3

$XNCR = .01\sigma'$   
 $PROB = .01$   
 $VALT = .6\sigma'$   
 $SDEV = 1.00$   
 $Z = .75$

2141.03	72
3616.41	100
3366.41	100
1596.72	58
1867.78	69
RUN AVE.	2517.68

RUN TOTALS-SAMPLES

Z = 1.00	
932.99	40
1298.75	40
1052.54	49
1467.29	45
994.14	47
RUN AVE.	1149.15
Z = 1.25	
2616.41	100
1641.03	72
2916.41	105
1534.28	40
2816.41	103
RUN AVE.	2304.92
Z = 1.50	
2316.41	100
1273.12	40
238.21	18
2116.41	100
732.41	44
RUN AVE.	1335.32
Z = 1.75	
1830.92	101
1000.29	40
1795.81	102
2551.88	99
1284.44	40
RUN AVE.	1692.67
Z = 2.00	
398.44	40
570.16	62
332.41	44
624.68	40
1595.81	102
RUN AVE.	704.30
Z = 2.25	
1080.92	101
442.83	16
1693.07	108
1114.89	92
1216.41	100
RUN AVE.	1109.63
Z = 2.50	
1316.41	100
307.13	57
1411.06	103
1166.41	100
375.72	54
RUN AVE.	915.35
Z = 2.75	

RUN TOTALS-SAMPLES

367.78	69
432.99	40
709.02	44
236.59	48
744.55	44
RUN AVE.	498.19
Z = 3.00	
118.24	40
1035.40	113
325.05	60
1016.41	100
569.01	42
RUN AVE.	612.83
Z = 3.25	
861.06	103
877.66	109
125.12	42
926.91	63
596.40	85
RUN AVE.	677.43
Z = 3.50	
946.18	116
544.20	30
711.06	103
524.86	90
448.95	41
RUN AVE.	635.05

RUN 4

$XNCR = .01\sigma'$   
 $PROB = .01$   
 $VALT = .6\sigma'$   
 $SDEV = .50$   
 $Z = .75$

1535.29	50
706.00	22
1735.29	50
1835.29	53
1985.29	58
RUN AVE.	1559.43
Z = 1.00	
1535.29	50
1685.29	55
167.30	8
1635.29	53
432.79	19
RUN AVE.	1091.20
Z = 1.25	
511.38	28
694.26	31

RUN TOTALS-SAMPLES

1635.29	50
490.23	20
376.23	16
RUN AVE.	741.48
Z = 1.50	
1317.03	52
232.79	19
1217.03	52
1050.77	51
1235.29	50
RUN AVE.	1010.59
Z = 1.75	
282.79	19
1335.29	50
1135.29	50
143.74	13
935.29	45
RUN AVE.	766.48
Z = 2.00	
885.29	50
935.29	55
884.09	53
602.61	39
586.63	42
RUN AVE.	778.79
Z = 2.25	
87.18	1
600.77	51
700.77	51
886.63	42
835.29	50
RUN AVE.	622.13
Z = 2.50	
550.77	51
341.46	26
193.66	15
650.77	51
435.29	50
RUN AVE.	434.36
Z = 2.75	
600.77	51
769.34	48
91.10	2
735.29	50
190.23	20
RUN AVE.	477.35
Z = 3.00	
335.29	50
240.23	20
588.58	44

RUN TOTALS-SAMPLES

271.92	29
300.77	51
RUN AVE.	347.36
Z = 3.25	
335.29	50
775.66	67
698.14	47
230.09	15
410.94	57
RUN AVE.	490.03
Z = 3.50	
317.03	52
410.94	57
331.14	34
351.98	54
401.98	54
RUN AVE.	362.62
<u>RUN 5</u>	
XNCR = .01 $\sigma$ '	
PROB = .05	
VALT = .3 $\sigma$ '	
SDEV = 1.00	
Z = .75	
2035.01	5
413.69	15
113.96	38
1993.41	65
2508.06	79
RUN AVE.	1612.83
Z = 1.00	
1674.30	60
2554.25	67
1142.65	36
1755.69	40
554.08	10
RUN AVE.	1536.20
Z = 1.25	
1427.35	61
2068.42	68
607.20	19
1409.02	65
983.16	58
RUN AVE.	1299.03
Z = 1.50	
1303.92	34
330.08	16
1292.21	41
1328.13	65
1392.87	52

RUN TOTALS-SAMPLES

RUN AVE.	1129.45
Z = 1.75	
1121.95	40
458.65	41
1200.24	40
689.91	41
267.49	29
RUN AVE.	747.65
Z = 2.00	
298.79	24
766.41	40
292.48	43
1473.56	70
1090.10	40
RUN AVE.	784.27
Z = 2.25	
454.10	40
641.48	70
330.52	28
415.21	43
845.85	67
RUN AVE.	537.43
Z = 2.50	
544.50	70
719.76	40
384.33	44
526.82	57
565.48	30
RUN AVE.	548.18
Z = 2.75	
244.40	19
178.30	24
453.32	73
571.67	63
220.27	19
RUN AVE.	333.60
Z = 3.00	
474.48	61
576.50	45
217.45	26
400.75	35
363.24	34
RUN AVE.	406.49
Z = 3.25	
327.00	72
250.98	20
262.22	45
749.23	67
683.86	44
RUN AVE.	454.66

RUN TOTALS-SAMPLES

Z = 3.50  
 847.22 72  
 306.89 27  
 302.00 41  
 269.53 48  
 349.28 77  
 RUN AVE. 414.99

RUN 6

XNCR = .01σ'  
 PROB = .05  
 VALT = .3σ'  
 SDEV = .50  
 Z = .75  
 603.51 20  
 1155.00 37  
 355.67 10  
 1086.42 19  
 502.75 23  
 RUN AVE. 740.68  
 Z = 1.00  
 770.23 20  
 1105.00 37  
 781.18 20  
 1635.29 50  
 575.18 21  
 RUN AVE. 973.38  
 Z = 1.25  
 1239.32 44  
 453.51 20  
 696.67 21  
 976.32 39  
 511.55 18  
 RUN AVE. 775.48  
 Z = 1.50  
 159.70 20  
 292.41 8  
 402.24 31  
 144.93 2  
 1135.29 50  
 RUN AVE. 426.92  
 Z = 1.75  
 714.12 18  
 278.54 20  
 1051.23 48  
 246.52 8  
 418.11 20  
 RUN AVE. 541.71  
 Z = 2.00  
 539.32 44

RUN TOTALS-SAMPLES

426.49 21  
 512.34 42  
 346.88 14  
 757.89 20  
 RUN AVE. 516.59  
 Z = 2.25  
 169.90 12  
 243.25 11  
 138.93 20  
 527.78 21  
 768.52 49  
 RUN AVE. 369.68  
 Z = 2.50  
 368.39 22  
 532.97 46  
 502.83 20  
 635.29 50  
 306.46 11  
 RUN AVE. 469.19  
 Z = 2.75  
 280.05 13  
 551.23 48  
 218.11 20  
 115.05 21  
 192.41 8  
 RUN AVE. 271.37  
 Z = 3.00  
 423.34 21  
 311.09 23  
 195.45 7  
 355.16 43  
 251.41 15  
 RUN AVE. 307.29  
 Z = 3.25  
 462.97 26  
 367.85 44  
 238.95 25  
 216.24 36  
 226.49 21  
 RUN AVE. 302.50  
 Z = 3.50  
 285.29 50  
 839.87 29  
 193.30 15  
 643.56 37  
 178.13 8  
 RUN AVE. 428.03

RUN 7

XNCR = .01σ'

RUN TOTALS-SAMPLES

PROB = .05  
 VALT = .60σ'  
 SDEV = 1.00  
 Z = .75  
 1484.28 40  
 827.79 22  
 331.45 10  
 1583.64 37  
 1902.63 41  
 RUN AVE. 1225.96  
 Z = 1.00  
 816.18 25  
 2202.86 79  
 994.14 47  
 930.25 29  
 1061.41 51  
 RUN AVE. 1200.97  
 Z = 1.25  
 405.70 18  
 332.14 23  
 1562.63 36  
 141.86 2  
 1035.92 56  
 RUN AVE. 695.66  
 Z = 1.50  
 923.38 28  
 716.05 52  
 228.44 8  
 1244.78 40  
 704.27 36  
 RUN AVE. 763.39  
 Z = 1.75  
 411.41 51  
 305.05 14  
 1098.51 27  
 281.45 10  
 932.77 26  
 RUN AVE. 605.84  
 Z = 2.00  
 1195.12 41  
 734.94 44  
 202.04 8  
 302.59 46  
 449.20 19  
 RUN AVE. 576.79  
 Z = 2.25  
 316.05 52  
 264.10 13  
 809.34 40  
 436.84 29



RUN TOTALS-SAMPLES  
 389.57 32  
 RUN AVE. 443.18  
 Z = 2.50  
 131.45 10  
 203.73 19  
 259.29 24  
 176.73 46  
 194.79 15  
 RUN AVE. 193.20  
 A = 2.75  
 791.16 41  
 382.99 40  
 910.71 45  
 277.76 18  
 455.81 70  
 RUN AVE. 563.69  
 Z = 3.00  
 412.60 28  
 128.02 9  
 220.11 38  
 235.92 56  
 719.14 41  
 RUN AVE. 343.17  
 Z = 3.25  
 129.12 40  
 152.04 8  
 356.12 68  
 840.16 45  
 230.50 20  
 RUN AVE. 281.59  
 Z = 3.50  
 130.55 11  
 219.96 24  
 275.62 14  
 151.99 22  
 166.32 5  
 RUN AVE. 188.89

RUN 8  
 XNCR = .01 $\sigma$ '  
 PROB = .05  
 VALT = .6 $\sigma$ '  
 SDEV = .50  
 Z = .75  
 243.74 13  
 1738.58 44  
 103.56 5  
 103.56 5  
 203.56 8  
 RUN AVE. 478.60

RUN TOTALS-SAMPLES  
 Z = 1.00  
 399.64 14  
 514.36 23  
 87.18 1  
 1585.29 50  
 157.99 6  
 RUN AVE. 548.89  
 A = 1.25  
 172.22 9  
 1535.29 50  
 771.92 29  
 1598.14 47  
 193.74 13  
 RUN AVE. 854.27  
 Z = 1.50  
 376.23 16  
 1235.29 50  
 87.18 1  
 299.64 14  
 99.28 4  
 RUN AVE. 419.53  
 A = 1.75  
 175.64 18  
 134.14 6  
 99.28 4  
 544.26 31  
 87.18 1  
 RUN AVE. 208.11  
 Z = 2.00  
 112.93 2  
 735.29 50  
 134.14 6  
 182.58 11  
 176.23 16  
 RUN AVE. 268.24  
 A = 2.25  
 1162.68 52  
 146.06 4  
 99.28 4  
 132.58 11  
 517.03 52  
 RUN AVE. 411.53  
 Z = 2.50  
 87.18 1  
 138.05 12  
 788.14 51  
 145.81 8  
 139.88 7  
 RUN AVE. 259.82  
 Z = 2.75

RUN TOTALS-SAMPLES  
 95.13 3  
 264.36 23  
 103.56 5  
 199.27 8  
 117.99 3  
 RUN AVE. 156.07  
 Z = 3.00  
 240.23 20  
 152.51 5  
 470.73 55  
 164.88 11  
 155.76 15  
 RUN AVE. 236.83  
 Z = 3.25  
 139.88 7  
 557.72 45  
 146.06 4  
 247.41 17  
 788.14 51  
 RUN AVE. 375.85  
 Z = 3.50  
 151.95 9  
 183.09 6  
 139.88 7  
 619.34 48  
 358.61 16  
 RUN AVE. 290.58

RUN 9  
 XNCR = .03 $\sigma$ '  
 PROB = .01  
 VALT = .3 $\sigma$ '  
 SDEV = 1.00  
 Z = .75  
 969.12 34  
 1319.12 34  
 952.77 24  
 1169.12 34  
 1169.12 34  
 RUN AVE. 1115.85  
 Z = 1.00  
 1169.12 34  
 1119.12 34  
 1069.12 34  
 1119.12 34  
 1108.69 34  
 RUN AVE. 1117.04  
 Z = 1.25  
 919.12 34  
 819.12 34

RUN TOTALS-SAMPLES

723.43	25
920.06	25
690.87	24
RUN AVE.	814.52
Z = 1.50	
619.12	34
605.03	25
581.82	24
986.41	35
383.04	25
RUN AVE.	635.09
Z = 1.75	
669.12	34
719.12	34
786.41	35
569.12	34
928.16	35
RUN AVE.	734.39
Z = 2.00	
919.12	34
586.41	35
824.88	37
547.76	24
604.98	36
RUN AVE.	696.63
Z = 2.25	
554.98	36
704.98	36
519.12	34
536.41	35
536.41	35
RUN AVE.	570.38
Z = 2.50	
331.82	24
307.58	31
492.00	26
254.98	36
574.88	37
RUN AVE.	392.25
Z = 2.75	
624.88	37
343.39	24
469.12	34
469.03	39
269.12	34
RUN AVE.	435.11
Z = 3.00	
458.30	27
319.12	34
558.64	44

RUN TOTALS-SAMPLES

269.12	34
369.12	34
RUN AVE.	394.86
Z = 3.25	
304.98	36
328.10	22
419.12	34
276.80	32
336.41	35
RUN AVE.	333.09
Z = 3.50	
427.45	36
642.36	43
179.31	27
274.88	37
393.44	40
RUN AVE.	383.49
RUN 10	
XNCR = .03σ'	
PROB = .01	
VALT = .3σ'	
SDEV = .50	
Z = .75	
487.50	17
587.50	17
737.50	17
687.50	17
298.48	7
RUN AVE.	559.70
Z = 1.00	
558.02	18
487.50	17
437.50	17
637.50	17
587.50	17
RUN AVE.	541.61
Z = 1.25	
487.50	17
587.50	16
537.50	19
537.50	18
637.50	17
RUN AVE.	557.50
Z = 1.50	
408.02	18
387.50	17
337.50	17
387.50	17
439.73	17

RUN TOTALS-SAMPLES

RUN AVE.	392.05
Z = 1.75	
287.50	17
487.50	17
287.50	17
337.50	17
214.57	7
RUN AVE.	322.92
Z = 2.00	
308.02	18
269.00	8
281.54	19
80.96	8
387.50	17
RUN AVE.	265.41
Z = 2.25	
237.50	17
237.50	17
237.50	17
210.17	7
116.05	11
RUN AVE.	207.74
Z = 2.50	
258.02	18
408.02	18
237.50	17
187.50	17
308.60	9
RUN AVE.	279.93
Z = 2.75	
288.96	21
208.39	20
258.02	18
308.39	20
238.96	21
RUN AVE.	260.55
Z = 3.00	
187.50	17
137.50	17
323.60	22
104.48	9
231.54	19
RUN AVE.	196.92
Z = 3.25	
208.39	20
258.39	20
238.96	21
258.02	18
217.26	15
RUN AVE.	236.21



RUN TOTALS-SAMPLES

Z = 3.25  
 208.39 20  
 258.39 20  
 238.96 21  
 258.02 18  
 217.26 15  
 RUN AVE. 236.21

Z = 3.50  
 187.50 17  
 406.83 24  
 273.60 22  
 456.30 25  
 288.96 21  
 RUN AVE. 322.64

RUN 11

XNCR = .03σ'  
 PROB = .01  
 VALT = .6σ'  
 SDEV = 1.00

Z = .75  
 969.12 34  
 1319.12 34  
 711.92 14  
 1019.12 34  
 1269.12 34  
 RUN AVE. 1057.68

Z = 1.00  
 1369.12 34  
 1069.12 34  
 1169.12 34  
 1169.12 34  
 1119.12 34  
 RUN AVE. 1179.12

Z = 1.25  
 371.89 14  
 969.12 34  
 767.30 31  
 615.04 28  
 917.30 31  
 RUN AVE. 728.14

Z = 1.50  
 1019.12 34  
 1019.12 34  
 671.17 14  
 479.21 15  
 986.41 35  
 RUN AVE. 835.01

Z = 1.75  
 424.79 25

RUN TOTALS-SAMPLES

669.12 34  
 719.12 34  
 786.41 35  
 569.12 34  
 RUN AVE. 633.72

Z = 2.00  
 646.25 35  
 919.12 34  
 586.41 35  
 824.88 37  
 344.40 14  
 RUN AVE. 664.21

Z = 2.25  
 536.41 35  
 519.12 34  
 519.12 34  
 604.98 36  
 736.41 35  
 RUN AVE. 583.21

Z = 2.50  
 536.41 35  
 237.31 26  
 369.12 34  
 198.73 9  
 419.12 34  
 RUN AVE. 352.14

Z = 2.75  
 274.88 37  
 436.41 35  
 386.41 35  
 184.06 15  
 369.12 34  
 RUN AVE. 330.18

Z = 3.00  
 354.98 36  
 274.88 37  
 137.49 14  
 173.35 16  
 319.12 34  
 RUN AVE. 251.97

Z = 3.25  
 558.64 44  
 269.12 34  
 269.12 34  
 304.98 36  
 246.69 13  
 RUN AVE. 329.71

Z = 3.50  
 254.98 36  
 433.80 35

RUN TOTALS-SAMPLES

304.98 36  
 352.66 33  
 304.98 36  
 RUN AVE. 330.28

RUN 12

XNCR = .03σ'  
 PROB = .01  
 VALT = .6σ'  
 SDEV = .50

Z = .75  
 487.50 17  
 587.50 17  
 737.50 17  
 687.50 17  
 178.76 4  
 RUN AVE. 535.76

Z = 1.00  
 587.50 17  
 437.50 17  
 558.02 18  
 537.50 17  
 637.50 17  
 RUN AVE. 551.61

Z = 1.25  
 537.50 17  
 637.50 17  
 537.50 17  
 487.50 17  
 537.50 17  
 RUN AVE. 547.50

Z = 1.50  
 408.02 18  
 437.50 17  
 408.02 18  
 481.54 19  
 643.69 15  
 RUN AVE. 475.76

Z = 1.75  
 287.50 17  
 487.50 17  
 287.50 17  
 337.50 17  
 93.12 1  
 RUN AVE. 298.63

Z = 2.00  
 281.54 19  
 317.41 9  
 358.02 18  
 474.64 13

RUN TOTALS-SAMPLES

387.50	17
RUN AVE.	363.82
Z = 2.25	
237.50	17
237.50	17
237.50	17
142.99	3
493.69	15
RUN AVE.	269.84
Z = 2.50	
258.02	18
408.02	18
237.50	17
187.50	17
93.12	1
RUN AVE.	236.83
Z = 2.75	
506.30	25
356.83	24
258.02	18
308.39	20
238.96	21
RUN AVE.	333.71
Z = 3.00	
187.50	17
137.50	17
323.60	22
272.11	9
231.54	19
RUN AVE.	230.45
Z = 3.25	
208.39	20
258.39	20
238.96	21
258.02	18
342.08	14
RUN AVE.	261.17
Z = 3.50	
208.02	18
406.83	24
273.60	22
456.30	25
288.96	21
RUN AVE.	326.75

RUN 13  
 XNCR = .03σ'  
 PROB = .05  
 VALT = .3σ'  
 SDEV = 1.00

RUN TOTALS-SAMPLES

Z = .75	
781.82	24
1290.47	33
382.31	14
620.06	15
620.45	24
RUN AVE.	739.03
Z = 1.00	
1069.12	34
787.74	24
997.76	24
1069.12	34
916.23	28
RUN AVE.	968.00
Z = 1.25	
1036.41	35
844.14	25
670.25	24
1019.12	34
664.74	23
RUN AVE.	846.94
Z = 1.50	
557.11	22
482.31	14
454.32	27
422.11	14
619.12	34
RUN AVE.	507.00
Z = 1.75	
695.21	22
470.25	24
869.12	34
209.49	14
316.69	15
RUN AVE.	512.16
Z = 2.00	
519.12	34
373.01	25
576.72	28
329.03	29
269.48	15
RUN AVE.	413.48
Z = 2.25	
454.07	24
306.17	12
718.58	25
165.28	14
586.41	35
RUN AVE.	446.11
Z = 2.50	

RUN TOTALS-SAMPLES

305.03	25
266.14	20
546.21	38
184.36	14
420.96	26
RUN AVE.	344.55
Z = 2.75	
484.98	36
323.95	16
195.68	24
579.57	37
447.76	24
RUN AVE.	400.39
Z = 3.00	
112.28	16
218.17	16
158.39	17
262.99	23
219.12	34
RUN AVE.	194.19
Z = 3.25	
407.87	24
338.63	26
336.41	35
344.97	25
319.12	34
RUN AVE.	349.40
Z = 3.50	
313.29	26
558.64	44
227.23	28
252.29	24
239.59	19
RUN AVE.	318.21
<u>RUN 14</u>	
XNCR = .03σ'	
PROB = .05	
VALT = .3σ'	
SDEV = .50	
Z = .75	
348.35	13
587.50	17
587.50	17
502.77	10
240.98	7
RUN AVE.	453.42
Z = 1.00	
254.84	7
414.57	7

RUN TOTALS-SAMPLES

280.96	8
487.50	17
236.90	7
RUN AVE.	334.96
Z = 1.25	
437.50	17
587.50	17
587.50	17
240.98	7
122.10	7
RUN AVE.	395.12
Z = 1.50	
537.50	17
387.50	17
487.50	17
487.50	17
266.05	11
RUN AVE.	433.22
Z = 1.75	
387.50	17
508.02	18
181.17	12
198.35	13
514.43	16
RUN AVE.	357.90
Z = 2.00	
258.02	18
337.50	17
314.43	16
317.78	14
169.00	8
RUN AVE.	279.35
Z = 2.25	
167.78	14
140.98	7
154.84	7
281.54	19
80.96	8
RUN AVE.	165.23
Z = 2.50	
147.39	3
358.02	18
258.02	18
557.82	19
220.33	13
RUN AVE.	308.32
Z = 2.75	
189.73	15
258.02	18
308.02	18

RUN TOTALS-SAMPLES

764.43	16
110.17	7
RUN AVE.	226.08
Z = 3.00	
135.09	8
98.48	7
323.60	22
356.83	24
169.37	10
RUN AVE.	216.68
Z = 3.25	
401.04	15
237.50	17
273.60	22
406.14	11
126.27	9
RUN AVE.	288.91
Z = 3.50	
158.60	9
189.73	15
90.98	7
316.50	9
396.38	20
<u>RUN 15</u>	
XNCR = .03 $\sigma$ '	
PROB = .05	
VALT = .6 $\sigma$ '	
SDEV = 1.00	
Z = .75	
293.58	14
1169.12	34
784.66	14
384.19	10
432.69	14
RUN AVE.	612.85
Z = 1.00	
502.23	23
592.42	15
1069.12	34
537.49	14
666.77	14
RUN AVE.	673.61
Z = 1.25	
969.12	34
1173.78	34
1036.41	35
253.72	14
263.15	14
RUN AVE.	739.24

RUN TOTALS-SAMPLES

Z = 1.50	
310.87	15
886.41	35
182.69	14
232.69	14
461.92	14
RUN AVE.	414.92
Z = 1.75	
371.03	13
592.45	16
273.82	20
98.09	2
1019.12	34
RUN AVE.	470.91
Z = 2.00	
471.76	23
163.16	14
143.58	14
636.41	35
164.32	17
RUN AVE.	315.85
Z = 2.25	
206.71	7
163.15	14
604.98	36
82.69	14
153.72	14
RUN AVE.	242.25
Z = 2.50	
504.98	36
160.87	15
424.09	13
384.06	15
314.18	16
RUN AVE.	357.64
Z = 2.75	
235.41	11
173.82	20
216.77	14
496.21	38
130.86	19
RUN AVE.	250.62
Z = 3.00	
126.80	17
367.68	17
459.51	34
330.88	17
242.42	15
RUN AVE.	305.46
Z = 3.25	

RUN TOTALS-SAMPLES

304.98 36  
 318.99 12  
 334.27 13  
 254.98 36  
 139.58 16  
 RUN AVE. 270.57

Z = 3.50

230.26 16  
 418.28 24  
 184.19 10  
 290.54 19  
 152.23 23  
 RUN AVE. 255.10

RUN 16

XNCR = .03σ'

PROB = .05

VALT = .6σ'

SDEV = .5

Z = .75

524.64 13  
 587.50 17  
 587.50 17  
 640.75 10  
 RUN AVE. 516.83

Z = 1.00

143.21 5  
 143.21 5  
 243.21 5  
 692.08 14  
 487.50 17  
 RUN AVE. 341.85

Z = 1.25

159.17 6  
 93.12 1  
 437.50 17  
 587.50 17  
 587.50 17  
 RUN AVE. 372.96

Z = 1.50

193.21 5  
 267.41 9  
 537.50 17  
 387.50 17  
 487.50 17  
 RUN AVE. 374.63

Z = 1.75

437.50 17  
 366.31 11  
 387.50 17

RUN TOTALS-SAMPLES

508.02 18  
 344.21 12  
 RUN AVE. 408.71

Z = 2.00

374.64 13  
 682.55 16  
 258.02 18  
 337.50 17  
 532.55 16  
 RUN AVE. 437.05

Z = 2.25

442.08 14  
 128.76 4  
 570.44 18  
 198.56 6  
 128.76 4  
 RUN AVE. 293.72

Z = 2.50

281.54 19  
 294.21 12  
 93.12 1  
 127.77 2  
 358.02 8  
 RUN AVE. 230.94

Z = 2.75

208.02 18  
 769.63 19  
 344.21 12  
 482.55 16  
 258.02 18  
 RUN AVE. 412.48

Z = 3.00

323.60 22  
 294.21 12  
 187.08 4  
 178.23 5  
 240.75 10  
 RUN AVE. 244.78

Z = 3.25

456.30 25  
 356.83 24  
 128.76 4  
 294.21 12  
 362.75 23  
 RUN AVE. 319.78

Z = 3.50

406.30 25  
 209.24 5  
 198.56 6  
 289.45 8

RUN TOTALS-SAMPLES

187.08 4  
 RUN AVE. 258.13

REPLICATION RUNSRUN 1

XNCR = .01σ'

PROB = .01

VALT = .3σ'

SDEV = 1.00

Z = .75

4066.41 100  
 2108.91 73  
 2786.79 70  
 3716.42 100  
 3556.42 100  
 RUN AVE. 3248.99

Z = 1.00

1462.18 51  
 3416.42 100  
 1807.87 70  
 1674.53 74  
 2205.92 67  
 RUN AVE. 2113.38

Z = 1.25

1109.64 40  
 2182.26 70  
 1873.39 70  
 1408.69 70  
 2616.42 100  
 RUN AVE. 1838.08

Z = 1.50

2077.48 70  
 1388.82 72  
 2616.42 100  
 2193.88 72  
 1220.72 70  
 RUN AVE. 1899.46

Z = 1.75

1431.68 70  
 887.77 70  
 2216.42 100  
 1358.07 79  
 2216.42 100  
 RUN AVE. 1621.95

Z = 2.00

870.72 70  
 1073.39 69  
 1716.42 100  
 1451.03 70



RUN TOTALS-SAMPLES

1566.42	100
RUN AVE.	1335.60
Z = 2.25	
951.22	71
1580.92	101
1430.93	101
824.99	60
1366.42	100
RUN AVE.	1230.90
Z = 2.50	
995.81	102
497.62	70
962.78	71
748.01	88
1195.81	102
RUN AVE.	880.01
Z = 2.75	
710.15	43
152.43	40
1053.75	71
632.01	60
961.06	103
RUN AVE.	701.88
Z = 3.00	
887.74	74
570.45	70
570.95	90
911.06	103
688.78	76
RUN AVE.	725.80
Z = 3.25	
377.90	76
743.97	80
563.86	74
730.93	101
358.20	72
RUN AVE.	554.97
Z = 3.50	
680.93	101
624.85	77
380.76	58
695.81	102
666.04	76
RUN AVE.	609.68

RUN 2

$\overline{XNCR} = .01\sigma'$   
 PROB = .01  
 VALT =  $.3\sigma'$   
 SDEV = .50

RUN TOTALS-SAMPLES

Z = .75	
1535.29	50
1585.29	50
1149.77	41
2085.29	50
1835.29	50
RUN AVE.	1638.19
Z = 1.00	
1885.29	50
1685.29	50
1685.29	50
931.89	21
787.19	20
RUN AVE.	1394.99
Z = 1.25	
1385.29	50
1185.29	50
1285.29	50
1385.29	50
1405.43	51
RUN AVE.	1329.32
Z = 1.50	
402.24	31
1267.04	52
1285.29	50
435.66	35
1235.29	50
RUN AVE.	925.11
Z = 1.75	
1100.78	51
503.19	20
770.23	20
546.20	34
374.83	20
RUN AVE.	659.05
Z = 2.00	
228.55	20
875.52	43
800.78	51
542.85	20
685.29	50
RUN AVE.	626.60
Z = 2.25	
639.40	45
750.78	51
755.44	51
585.29	50
685.29	51
RUN AVE.	683.24
Z = 2.50	

RUN TOTALS-SAMPLES

174.95	27
685.29	50
684.09	53
120.65	24
517.04	52
RUN AVE.	436.41
570.73	55
302.68	21
137.91	29
485.29	50
550.78	51
RUN AVE.	409.48
Z = 3.00	
423.63	23
567.04	52
435.29	50
534.09	53
316.91	23
RUN AVE.	455.39
Z = 3.25	
420.73	55
610.94	57
355.86	39
400.78	51
231.89	21
RUN AVE.	404.04
Z = 3.50	
248.15	22
350.78	51
451.98	54
254.20	27
350.78	51
RUN AVE.	331.18
<u>RUN 3</u>	
$\overline{XNCR} = .01\sigma'$	
PROB = .05	
VALT = $.3\sigma'$	
SDEV = 1.00	
Z = .75	
785.67	41
2115.14	70
990.15	29
1252.92	37
666.55	27
RUN AVE.	1162.08
Z = 1.00	
3666.41	100
384.33	10

RUN TOTALS-SAMPLES

2010.35	61
1179.39	41
1252.81	40
RUN AVE.	1698.66
Z = 1.25	
876.83	37
1076.42	40
750.48	40
1524.85	69
909.76	41
RUN AVE.	1027.67
Z = 1.50	
1058.04	37
1412.10	40
846.38	42
391.55	32
1881.68	70
RUN AVE.	1117.95
Z = 1.75	
612.42	53
1294.72	51
1189.29	36
1360.79	40
268.32	13
RUN AVE.	945.11
Z = 2.00	
1163.78	71
845.93	58
315.01	23
753.37	52
574.49	40
RUN AVE.	730.52
Z = 2.25	
107.62	18
514.57	48
559.64	40
251.18	44
513.85	58
RUN AVE.	389.37
Z = 2.50	
367.45	44
652.99	40
793.26	59
476.77	60
297.91	18
RUN AVE.	517.68
Z = 2.75	
518.66	57
298.38	41
318.66	18

RUN TOTALS-SAMPLES

352.87	46
369.90	44
RUN AVE.	371.69
Z = 3.00	
211.29	42
471.03	57
326.33	47
458.37	41
706.72	48
RUN AVE.	434.75
Z = 3.25	
640.81	65
130.81	12
573.16	65
385.68	46
707.18	42
RUN AVE.	487.51
Z = 3.50	
387.31	34
582.05	64
553.41	65
316.21	37
327.58	39
RUN AVE.	433.31
<u>RUN 4</u>	
XNCR = .01 $\sigma$ '	
PROB = .05	
VALT = .3 $\sigma$ '	
SDEV = .50	
Z = .75	
536.07	18
998.19	20
1935.29	50
1165.41	38
99.25	3
RUN AVE.	946.84
Z = 1.00	
687.91	29
1005.01	37
724.95	27
626.49	21
383.09	13
RUN AVE.	685.50
Z = 1.25	
1618.88	46
476.77	11
518.95	26
857.90	20
252.76	23

RUN TOTALS-SAMPLES

RUN AVE.	745.05
Z = 1.50	
807.90	20
812.34	42
209.70	20
93.30	21
453.51	20
RUN AVE.	475.35
Z = 1.75	
351.70	16
199.56	20
350.35	8
868.88	46
324.95	27
RUN AVE.	419.09
Z = 2.00	
237.91	29
149.25	3
411.47	30
750.78	51
296.27	26
RUN AVE.	369.13
Z = 2.25	
346.90	20
816.42	47
318.89	10
211.05	28
702.84	20
RUN AVE.	479.22
Z = 2.50	
143.30	21
185.02	10
625.14	52
275.19	21
275.50	35
RUN AVE.	300.83
Z = 2.75	
109.56	22
414.13	18
320.22	12
475.52	43
219.01	29
RUN AVE.	307.69
Z = 3.00	
306.58	22
108.27	5
752.04	32
225.22	27
361.10	23
RUN AVE.	350.64



RUN TOTALS-SAMPLES

Z = 3.25  
 491.82 32  
 298.69 21  
 289.32 44  
 195.61 20  
 125.19 21  
 RUN AVE. 280.13

Z = 3.50  
 315.32 29  
 533.01 21  
 201.97 31  
 216.24 36  
 216.24 36  
 RUN AVE. 296.56

RUN 5XNCR = .01  $\sigma'$ 

PROB = .01

VALT = .6  $\sigma'$ 

SDEV = 1.00

Z = .75  
 871.63 41  
 3466.42 100  
 1844.78 40  
 3716.42 100  
 1206.92 50  
 RUN AVE. 2221.23

Z = 1.00  
 3416.42 100  
 1091.25 57  
 1302.34 59  
 1200.01 41  
 3116.42 100  
 RUN AVE. 2025.29

Z = 1.25  
 1400.01 41  
 2316.42 100  
 965.43 66  
 2716.42 100  
 3016.42 100  
 RUN AVE. 2082.94

Z = 1.50  
 711.42 51  
 1005.61 41  
 2530.93 101  
 2366.42 100  
 448.29 48  
 RUN AVE. 1412.53

Z = 1.75  
 671.93 41

RUN TOTALS-SAMPLES

1995.81 102  
 656.69 40  
 1637.68 91  
 1031.81 41  
 RUN AVE. 1198.78

Z = 2.00  
 661.08 38  
 649.00 40  
 439.26 40  
 311.42 51  
 1861.06 103  
 RUN AVE. 784.36

Z = 2.25  
 1098.96 41  
 496.29 66  
 1316.42 100  
 541.09 40  
 1466.42 100  
 RUN AVE. 983.84

Z = 2.50  
 928.93 43  
 816.82 87  
 121.63 41  
 1130.93 101  
 964.30 41  
 RUN AVE. 792.52

Z = 2.75  
 1230.93 101  
 144.14 47  
 1061.06 103  
 237.91 57  
 285.92 56  
 RUN AVE. 591.99

Z = 3.00  
 450.00 41  
 942.69 105  
 611.94 43  
 960.69 109  
 170.58 50  
 RUN AVE. 627.18

Z = 3.25  
 716.42 100  
 642.69 105  
 191.73 49  
 428.70 41  
 978.72 110  
 RUN AVE. 591.65

Z = 3.50  
 793.07 108  
 232.99 40

RUN TOTALS-SAMPLES

409.02 44  
 938.58 112  
 676.68 104  
 RUN AVE. 610.07

RUN 6XNCR = .01  $\sigma'$ 

PROB = .01

VALT = .6  $\sigma'$ 

SDEV = .50

Z = .75  
 91.10 2  
 312.14 16  
 1735.29 50  
 91.10 2  
 332.59 11  
 RUN AVE. 512.44

Z = 1.00  
 791.47 26  
 2048.14 47  
 1535.29 50  
 167.31 8  
 1735.29 50  
 RUN AVE. 1255.50

Z = 1.25  
 1485.29 50  
 153.56 5  
 1585.29 50  
 1335.29 50  
 103.56 5  
 RUN AVE. 932.60

Z = 1.50  
 1085.29 50  
 1285.29 50  
 451.24 27  
 1185.29 50  
 1285.29 50  
 RUN AVE. 1058.48

Z = 1.75  
 391.47 26  
 885.29 50  
 835.29 50  
 162.14 16  
 836.64 42  
 RUN AVE. 622.17

Z = 2.00  
 785.29 50  
 800.78 51  
 177.31 10  
 1235.29 50

RUN TOTALS-SAMPLES

835.29 50  
 RUN AVE. 766.79  
 Z = 2.25  
 885.29 50  
 421.93 29  
 800.77 51  
 817.04 52  
 1214.41 54  
 RUN AVE. 827.89  
 Z = 2.50  
 751.99 54  
 103.56 5  
 255.22 22  
 635.29 50  
 600178 51  
 RUN AVE. 469.37  
 Z=2.75  
 552.61 39  
 517.04 52  
 567.04 52  
 617.04 52  
 347.60 30  
 RUN AVE. 520.27  
 Z=3.00  
 440.37 56  
 634.09 53  
 400.78 51  
 500.78 51  
 448.09 40  
 RUN AVE. 484.82  
 Z = 3.25  
 251.24 27  
 482.46 58  
 520.73 55  
 528.56 60  
 534.09 53  
 RUN AVE. 463.42  
 Z = 3.50  
 372.42 37  
 294.27 31  
 555.00 59  
 384.09 53  
 232.09 25  
 RUN AVE. 367.57

RUN 7

XNCR = .01σ'  
 PROB = .05  
 VALT = .6σ'  
 SDEV = 1.00

RUN TOTALS-SAMPLES

Z = .75  
 1346.57 30  
 1925.00 76  
 483.79 22  
 1240.12 46  
 447.28 11  
 RUN AVE. 1088.55  
 Z = 1.00  
 707.45 30  
 1596.76 39  
 278.02 9  
 870.81 53  
 1605.12 40  
 RUN AVE. 1011.63  
 Z = 1.25  
 1438.64 37  
 750.34 19  
 235.81 9  
 540.12 46  
 926.56 39  
 RUN AVE. 787.29  
 Z = 1.50  
 184.93 17  
 1114.99 32  
 657.20 30  
 999.00 40  
 480.80 33  
 RUN AVE. 687.39  
 Z = 1.75  
 432.41 44  
 1048.77 38  
 1146.76 39  
 1488.69 42  
 439.39 43  
 RUN AVE. 910.21  
 Z = 2.00  
 507.42 40  
 385.01 21  
 312.74 12  
 1198.96 41  
 978.79 40  
 RUN AVE. 676.59  
 Z = 2.25  
 351.69 14  
 181.70 16  
 806.92 32  
 557.20 30  
 449.00 40  
 RUN AVE. 469.30  
 Z = 2.50

RUN TOTALS-SAMPLES

415.44 35  
 125.12 42  
 748.77 38  
 469.74 30  
 619.63 41  
 RUN AVE. 475.74  
 Z = 2.75  
 247.03 10  
 200.02 44  
 270.82 53  
 290.94 8  
 634.29 40  
 RUN AVE. 328.62  
 Z = 3.00  
 299.00 40  
 333.17 25  
 233.79 22  
 341.09 40  
 317.26 14  
 RUN AVE. 304.86  
 Z = 3.25  
 552.08 33  
 335.66 44  
 235.14 59  
 220.86 23  
 263.27 29  
 RUN AVE. 321.40  
 Z = 3.50  
 297.93 26  
 213.26 20  
 598.81 45  
 783.42 65  
 533.72 55  
 RUN AVE. 485.43

RUN 8  
 XNCR = .01σ'  
 PROB = .05  
 VALT = .60σ'  
 SDEV = .50  
 Z = .75  
 232.59 11  
 901.24 27  
 388.06 12  
 362.14 16  
 157.99 6  
 RUN AVE. 408.40  
 Z = 1.00  
 1877.58 46  
 95.14 3

RUN TOTALS-SAMPLES

117.31	8
691.46	26
99.28	4
RUN AVE.	576.16
Z = 1.25	
1052.61	39
149.28	4
1450.78	51
112.57	7
325.64	18
RUN AVE.	618.18
Z = 1.50	
264.36	23
193.74	13
325.64	18
103.56	5
232.80	19
RUN AVE.	224.02
Z = 1.75	
141.10	2
112.57	7
1077.58	46
451.24	27
371.93	29
RUN AVE.	430.88
Z = 2.00	
91.10	2
87.19	1
521.93	29
767.04	52
373.05	24
RUN AVE.	368.06
Z = 2.25	
162.14	16
107.99	6
977.58	46
123.21	4
112.57	7
RUN AVE.	296.70
Z = 2.50	
241.47	26
149.28	4
268.13	18
247.96	21
128.59	5
RUN AVE.	207.09
Z = 2.75	
128.59	5
812.69	52
226.90	19

RUN TOTALS-SAMPLES

226.90	19
394.39	35
214.36	23
RUN AVE.	355.39
Z = 3.00	
145.81	8
171.70	12
227.78	6
237.83	7
498.96	41
RUN AVE.	256.42
Z = 3.25	
395.14	29
164.89	11
155.77	15
617.49	4
425.85	31
RUN AVE.	351.83
Z = 3.50	
269.39	19
195.89	11
233.05	24
486.64	42
134.15	6
RUN AVE.	261.82
<u>RUN 9</u>	
XNCR = .03σ'	
PROB = .01	
VALT = .3σ'	
SDEV = 1.00	
Z = .75	
1286.41	35
1219.12	34
1219.12	34
865.76	24
1269.12	34
RUN AVE.	1171.91
Z = 1.00	
1019.12	34
936.41	35
899.63	24
573.55	24
969.12	34
RUN AVE.	879.67
Z = 1.25	
1069.12	34
919.12	34
1060.69	25
1269.12	34
1119.12	34

RUN TOTALS-SAMPLES

RUN AVE.	1087.44
Z = 1.50	
769.12	34
886.41	35
643.29	26
919.12	34
1119.12	34
RUN AVE.	867.41
Z = 1.75	
736.41	35
786.41	35
373.02	25
719.12	34
555.73	24
RUN AVE.	634.14
Z = 2.00	
270.26	24
636.41	35
586.41	35
619.12	34
479.04	29
RUN AVE.	518.25
Z = 2.25	
624.88	37
504.98	36
536.41	35
404.98	36
524.88	37
RUN AVE.	519.23
Z = 2.50	
370.07	25
336.41	35
569.12	34
619.12	34
369.12	34
RUN AVE.	452.77
Z = 2.75	
286.41	35
604.98	36
266.74	26
123.55	24
424.88	37
RUN AVE.	341.31
Z = 3.00	
354.98	36
424.88	37
469.04	39
254.98	36
569.51	41
RUN AVE.	414.68

RUN TOTALS-SAMPLES

Z = 3.25  
 361.65 38  
 218.87 31  
 396.21 38  
 369.12 34  
 369.03 39  
 RUN AVE. 342.98

Z = 3.50  
 240.07 30  
 346.22 38  
 236.41 35  
 347.05 33  
 205.41 25  
 RUN AVE. 275.03

RUN 10

XNCR = .03σ'  
 PROB = .01  
 VALT = .3σ'  
 SDEV = .50

Z = .75  
 687.50 17  
 737.50 17  
 219.00 8  
 272.10 7  
 487.50 17  
 RUN AVE. 480.72

Z = 1.00  
 341.13 9  
 737.50 17  
 637.50 17  
 698.35 13  
 587.50 17  
 RUN AVE. 600.40

Z = 1.25  
 537.50 17  
 437.50 17  
 122.10 7  
 637.50 17  
 558.02 18  
 RUN AVE. 458.53

Z = 1.50  
 337.50 17  
 437.50 17  
 387.50 17  
 387.50 17  
 260.17 7  
 RUN AVE. 362.04

Z = 1.75  
 408.02 18

RUN TOTALS-SAMPLES

387.50 17  
 487.50 17  
 408.02 18  
 531.54 19  
 RUN AVE. 444.52

Z = 2.00  
 116.06 11  
 458.02 18  
 331.54 19  
 237.50 17  
 408.02 18  
 RUN AVE. 310.23

Z = 2.25  
 387.50 17  
 231.54 19  
 187.50 17  
 160.17 7  
 281.54 19  
 RUN AVE. 249.65

Z = 2.50  
 258.39 20  
 331.54 19  
 287.50 17  
 308.02 18  
 237.50 17  
 RUN AVE. 284.59

Z = 2.75  
 150.70 12  
 331.54 19  
 187.50 17  
 187.50 17  
 187.50 17  
 RUN AVE. 208.95

Z = 3.00  
 137.50 17  
 208.40 20  
 104.49 9  
 231.54 19  
 308.40 20  
 RUN AVE. 198.07

Z = 3.25  
 204.21 9  
 181.54 19  
 181.54 19  
 406.84 24  
 187.50 17  
 RUN AVE. 232.32

Z = 3.50  
 208.02 18  
 122.10 7  
 181.54 19

RUN TOTALS-SAMPLES

208.02 18  
 308.40 20  
 RUN AVE. 205.62

RUN 11

XNCR = .03σ'  
 PROB = .05  
 VALT = .3σ'  
 SDEV = 1.00

Z = .75  
 1001.30 24  
 947.76 24  
 993.29 26  
 1002.77 24  
 773.55 24  
 RUN AVE. 943.74

Z = 1.00  
 778.27 24  
 678.27 24  
 721.45 20  
 760.93 24  
 710.93 24  
 RUN AVE. 729.97

Z = 1.25  
 904.07 24  
 415.76 24  
 1019.12 34  
 782.28 23  
 617.89 14  
 RUN AVE. 747.83

Z = 1.50  
 755.19 24  
 857.58 31  
 554.33 27  
 685.10 24  
 433.80 14  
 RUN AVE. 657.20

Z = 1.75  
 430.88 24  
 477.23 22  
 647.76 24  
 560.69 25  
 404.33 27  
 RUN AVE. 504.18

Z = 2.00  
 320.07  
 961.65 38  
 268.73 24  
 420.26 24



RUN TOTALS-SAMPLES

572.65 23  
 RUN AVE. 508.67  
 Z = 2.25  
 440.87 24  
 341.80 14  
 585.41 35  
 619.12 34  
 389.06 29  
 RUN AVE. 475.45  
 Z = 2.50  
 322.12 14  
 383.06 25  
 357.35 32  
 123.55 24  
 285.10 24  
 RUN AVE. 294.24  
 Z = 2.75  
 183.79 14  
 436.41 35  
 347.54 19  
 233.04 25  
 470.07 25  
 RUN AVE. 334.17  
 Z = 3.00  
 215.27 22  
 269.12 34  
 250.65 28  
 378.19 29  
 261.99 22  
 RUN AVE. 275.04  
 Z = 3.25  
 199.78 23  
 184.61 20  
 505.67 33  
 150.14 24  
 280.79 29  
 RUN AVE. 264.20  
 Z = 3.50  
 189.93 25  
 206.14 24  
 629.88 39  
 419.51 41  
 439.85 25

RUN 12

XNCR = .03σ'  
 PROB = .05  
 VALT = .3σ'  
 SDEV = .5  
 Z = .75

RUN TOTALS-SAMPLES

787.50 17  
 587.50 17  
 366.06 11  
 254.84 7  
 348.48 7  
 RUN AVE. 468.89  
 Z = 1.00  
 587.50 17  
 232.24 7  
 404.84 7  
 348.35 13  
 787.50 17  
 RUN AVE. 472.09  
 Z = 1.25  
 290.99 7  
 364.84 7  
 587.50 17  
 487.50 17  
 387.50 17  
 RUN AVE. 423.67  
 Z = 1.50  
 310.17 7  
 448.35 13  
 487.50 17  
 337.50 17  
 260.17 7  
 RUN AVE. 368.74  
 Z = 1.75  
 673.19 18  
 92.62 8  
 266.06 11  
 110.17 7  
 160.17 7  
 RUN AVE. 260.44  
 Z = 2.00  
 358.02 18  
 337.50 17  
 132.24 7  
 187.50 17  
 198.55 11  
 RUN AVE. 242.77  
 Z = 2.25  
 102.77 10  
 314.44 16  
 144.86 4  
 358.02 18  
 189.85 13  
 RUN AVE. 221.99  
 Z = 2.50  
 392.17 17

RUN TOTALS-SAMPLES

116.06 11  
 308.40 20  
 176.28 9  
 258.40 20  
 RUN AVE. 250.26  
 Z = 2.75  
 137.50 17  
 264.84 7  
 281.54 19  
 258.40 20  
 140.99 7  
 RUN AVE. 216.65  
 Z = 3.00  
 131.18 12  
 187.50 17  
 142.99 10  
 217.79 14  
 208.02 18  
 RUN AVE. 177.50  
 Z = 3.25  
 148.48 7  
 183.70 11  
 137.50 17  
 189.85 13  
 456.84 24  
 RUN AVE. 223.27  
 Z = 3.50  
 170.33 13  
 137.50 17  
 349.17 19  
 181.54 19  
 696.60 11  
 RUN AVE. 307.03

RUN 13

XNCR = .03σ'  
 PROB = .01  
 VALT = .6σ'  
 SDEV = 1.00  
 Z = .75  
 413.15 14  
 1219.12 34  
 1269.12 34  
 1269.12 34  
 600.76 14  
 RUN AVE. 954.26  
 Z = 1.00  
 1269.12 34  
 1019.12 34  
 1069.12 34

RUN TOTALS-SAMPLES  
 1219.12 34  
 1019.12 34  
 RUN AVE. 1119.12  
 Z = 1.25  
 675.13 14  
 919.12 34  
 969.12 34  
 919.12 34  
 1069.12 34  
 RUN AVE. 910.32  
 Z = 1.50  
 237.49 14  
 819.12 34  
 465.64 19  
 410.46 22  
 819.12 34  
 RUN AVE. 550.37  
 Z = 1.75  
 736.41 35  
 819.12 34  
 619.12 34  
 516.77 14  
 769.12 34  
 RUN AVE. 692.11  
 Z = 2.00  
 696.22 38  
 586.41 35  
 669.12 34  
 559.51 34  
 519.12 34  
 RUN AVE. 606.08  
 Z = 2.25  
 369.12 34  
 586.41 35  
 619.12 34  
 469.12 34  
 519.12 34  
 RUN AVE. 512.58  
 Z = 2.50  
 386.41 35  
 113.15 14  
 163.12 24  
 419.12 34  
 574.88 37  
 RUN AVE. 331.34  
 Z = 2.75  
 369.12 34  
 574.88 37  
 396.21 38  
 386.41 35

RUN TOTALS-SAMPLES  
 319.12 34  
 RUN AVE. 409.15  
 Z = 3.00  
 182.61 19  
 225.35 27  
 354.98 36  
 319.12 34  
 419.51 41  
 RUN AVE. 300.32  
 Z = 3.25  
 174.79 25  
 346.22 38  
 254.98 36  
 443.48 34  
 233.09 21  
 Z = 3.50  
 346.22 38  
 180.27 16  
 421.56 21  
 444.79 21  
 409.51 34  
 RUN AVE. 360.47  
 RUN 14  
 XNCR = .03σ'  
 PROB = .01  
 VALT = .6σ'  
 SDEV = .50  
 Z = .75  
 637.50 17  
 687.50 17  
 737.50 17  
 209.18 6  
 787.50 17  
 RUN AVE. 611.84  
 Z = 1.00  
 587.50 17  
 587.50 17  
 487.50 17  
 637.50 17  
 687.50 17  
 RUN AVE. 597.50  
 Z = 1.25  
 165.69 3  
 408.02 18  
 587.50 17  
 487.50 17  
 337.50 17  
 RUN AVE. 397.24

RUN TOTALS-SAMPLES  
 Z = 1.50  
 487.50 17  
 487.50 17  
 103.84 2  
 587.50 17  
 287.50 17  
 RUN AVE. 390.77  
 Z = 1.75  
 508.02 18  
 558.02 18  
 487.50 17  
 381.54 19  
 437.50 17  
 RUN AVE. 474.52  
 Z = 2.00  
 178.77 4  
 258.02 18  
 508.02 18  
 458.40 20  
 358.02 18  
 RUN AVE. 352.25  
 Z = 2.25  
 308.02 18  
 287.50 17  
 624.87 17  
 237.50 17  
 287.50 17  
 RUN AVE. 349.08  
 Z = 2.50  
 181.54 19  
 258.02 18  
 237.50 17  
 258.40 20  
 196.14 8  
 RUN AVE. 226.32  
 187.50 17  
 412.76 23  
 103.84 2  
 137.50 17  
 331.54 19  
 RUN AVE. 234.63  
 Z = 3.00  
 238.96 21  
 181.54 19  
 308.40 20  
 443.69 15  
 181.54 19  
 RUN AVE. 270.83  
 Z = 3.25



RUN TOTALS-SAMPLES

208.02	18
308.40	20
208.40	20
158.02	18
258.40	20
RUN AVE.	228.25
Z = 3.50	
273.60	22
373.60	22
137.50	17
115.69	3
237.50	17
RUN AVE.	227.58

RUN 15

XNCR = .03σ'  
 PROB = .05  
 VALT = .60σ'  
 SDEV = 1.00  
 Z = .75

532.61	21
500.97	17
293.58	14
550.97	17
663.12	24
RUN AVE.	508.25
Z = 1.00	
479.99	14
1152.66	33
1119.12	34
874.79	25
288.18	7
RUN AVE.	782.95
Z = 1.25	
789.81	15
869.12	34
458.04	18
600.71	27
253.72	14
RUN AVE.	594.28
Z = 1.50	
199.99	15
492.45	16
294.42	16
390.05	20
398.77	10
RUN AVE.	355.14
Z = 1.75	
325.35	27
746.22	38

RUN TOTALS-SAMPLES

207.84	8
804.98	36
321.76	23
RUN AVE.	481.23
Z = 2.00	
122.30	18
456.59	14
354.79	15
350.76	14
171.90	14
RUN AVE.	291.27
Z = 2.25	
375.13	14
334.94	10
356.59	14
619.12	34
469.12	34
RUN AVE.	430.98
Z = 2.50	
210.60	26
98.09	2
530.37	29
269.12	34
316.77	14
RUN AVE.	284.99
Z = 2.75	
94.43	16
209.48	17
149.01	16
376.13	14
409.51	34
RUN AVE.	247.51
Z = 3.00	
246.86	11
224.79	25
311.92	14
230.37	29
336.25	20
RUN AVE.	270.04
Z = 3.25	
386.41	35
244.32	19
218.06	15
260.93	22
671.08	23
RUN AVE.	356.16
Z = 3.50	
609.15	24
277.82	10
304.98	36

RUN TOTALS-SAMPLES

165.86	16
358.48	26
RUN AVE.	343.26
<u>RUN 16</u>	
XNCR = .03σ'	
PROB = .05	
VALT = .60σ'	
SDEV = .50	
Z = .75	
737.50	17
159.18	6
153.84	2
93.12	1
537.50	17
RUN AVE.	336.23
Z = 1.00	
544.22	12
587.50	17
487.50	17
537.50	17
687.50	17
Z = 1.25	
246.14	8
632.55	16
159.18	6
737.50	17
537.50	17
RUN AVE.	462.58
Z = 1.50	
444.21	12
159.77	4
226.77	7
437.50	17
267.42	9
RUN AVE.	307.13
Z = 1.75	
178.77	4
674.87	17
487.50	17
193.22	5
128.77	4
RUN AVE.	332.63
Z = 2.00	
143.22	5
337.50	17
387.50	17
719.63	19
437.50	17
RUN AVE.	405.07

RUN TOTALS-SAMPLES

Z = 2.25

127.77	2
337.50	17
670.44	18
337.50	17
159.18	6
RUN AVE.	326.48

Z = 2.50

245.34	8
366.31	11
187.08	4
328.84	8
208.02	18
RUN AVE.	267.12

Z = 2.75

158.02	18
226.77	7
158.02	18
217.41	9
294.22	12
RUN AVE.	210.89

Z = 3.00

432.55	16
731.60	4
406.31	25
233.58	6
474.87	17
RUN AVE.	455.78

Z = 3.25

266.31	11
208.40	20
159.18	6
461.65	26
482.55	16
RUN AVE.	315.62

Z = 3.50

143.21	5
281.54	19
208.40	20
143.22	5
342.09	14
RUN AVE.	223.69

**APPENDIX II**

**Results from Regression Analysis Performed on**

**32 Simulations of Cost Model**

$$\text{XNCR} = .01\sigma', \text{PROB} = .01, \text{VALT} = .3\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 3896.57 - 1749.30Z + 229.19Z^2$$

$$Z_{\min} = 3818$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .9274$$

$$\text{XNCR} = .01\sigma', \text{PROB} = .01, \text{VALT} = .3\sigma', \text{SDEV} = .50$$

$$\text{COSTS} = 2296.18 - 1295.81Z + 225.36Z^2$$

$$Z_{\min} = 2.880$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8534$$

$$\text{XNCR} = .01\sigma', \text{PROB} = .01, \text{VALT} = .6\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 3233.63 - 1478.05Z + 206.56Z^2$$

$$Z_{\min} = 3.578$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .6910$$

$$\text{XNCR} = .01\sigma', \text{PROB} = .01, \text{VALT} = .6\sigma', \text{SDEV} = .50$$

$$\text{COSTS} = 960.07 - 522.69Z + 94.27Z^2$$

$$Z_{\min} = 2.772$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8819$$

$$\text{XNCR} = .01\sigma', \text{PROB} = .05, \text{VALT} = .3\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 2645.18 - 1397Z + 216.97Z^2$$

$$Z_{\min} = 3.220$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .9709$$

$$\text{XNCR} = .01\sigma', \text{PROB} = .05, \text{VALT} = .3\sigma', \text{SDEV} = .50$$

$$\text{COSTS} = 1300.74 - 619.82Z + 100.11Z^2$$

$$Z_{\min} = 3.09$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .7562$$

$$\text{XNCR} = .01\sigma', \text{PROB} = .05, \text{VALT} = .6\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 1836.60 - 928.64Z + 137.81Z^2$$

$$Z_{\min} = 3.369$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8739$$

$$\text{XNCR} = .01\sigma', \text{PROB} = .05, \text{VALT} = .6\sigma, \text{SDEV} = .50$$

$$\text{COSTS} = 934.02 - 454.32Z + 77.39Z^2$$

$$Z_{\min} = 3.138$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .4288$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .01, \text{VALT} = .3\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 1608.04 - 702.28Z + 98.76Z^2$$

$$Z_{\min} = 3.555$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .9269$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .01, \text{VALT} = .3\sigma', \text{SDEV} = .50$$

$$\text{COSTS} = 960.04 - 522.67Z + 94.27Z^2$$

$$Z_{\min} = 2.772$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8819$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .01, \text{VALT} = .6\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 1619.16 - 703.97Z + 91.83Z^2$$

$$Z_{\min} = 3.833$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .9029$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .01, \text{VALT} = .6\sigma', \text{SDEV} = .50$$

$$\text{COSTS} = 867.89 - 403.45Z + 68.20Z^2$$

$$Z_{\min} = 2.958$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8184$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .05, \text{VALT} = .3\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 1299.96 - 590.24Z + 86.98Z^2$$

$$Z_{\min} = 3.393$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8032$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .05, \text{VALT} = .3\sigma', \text{SDEV} = .50$$

$$\text{COSTS} = 584.91 - 207.05Z + 30.90Z^2$$

$$Z_{\min} = 3.351$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .6230$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .05, \text{VALT} = .6\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 1023.65 - 462.89Z + 69.97Z^2$$

$$Z_{\min} = 3.310$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .7899$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .05, \text{VALT} = .6\sigma', \text{SDEV} = .50$$

$$\text{COSTS} = 516.87 - 100.55Z + 9.087Z^2$$

$$Z_{\min} = 5.534$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .4249$$

#### REPLICATION RUN

$$\text{XNCR} = .01\sigma', \text{PROB} = .01, \text{VALT} = .3\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 4283.60 - 2099.39Z + 299.27Z^2$$

$$Z_{\min} = 3.506$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .9406$$



$$XNCR = .01\sigma', \text{ PROB} = .01, \text{ VALT} = .3\sigma', \text{ SDEV} = .50$$

$$\text{COSTS} = 2568.73 - 1386.22Z + 218.88Z^2$$

$$Z_{\min} = 3.167$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .9633$$

$$XNCR = .01\sigma', \text{ PROB} = .01, \text{ VALT} = .6\sigma', \text{ SDEV} = 1.00$$

$$\text{COSTS} = 3582.35 - 1833.68Z + 280.28Z^2$$

$$Z_{\min} = 3.271$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .9463$$

$$XNCR = .01\sigma', \text{ PROB} = .01, \text{ VALT} = .6\sigma', \text{ SDEV} = .50$$

$$\text{COSTS} = 1811.99 - 723.35Z + 91.75Z^2$$

$$Z_{\min} = 3.942$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8469$$

$$XNCR = .01\sigma', \text{ PROB} = .05, \text{ VALT} = .3\sigma', \text{ SDEV} = 1.00$$

$$\text{COSTS} = 2177.43 - 1024.57Z + 147.54Z^2$$

$$Z_{\min} = 3.472$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .7996$$

$$XNCR = .01\sigma', \text{ PROB} = .05, \text{ VALT} = .3\sigma', \text{ SDEV} = .50$$

$$\text{COSTS} = 1354.74 - 709.92Z + 118.84Z^2$$

$$Z_{\min} = 2.987$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .9014$$

$$XNCR = .01\sigma', \text{ PROB} = .05, \text{ VALT} = .6\sigma', \text{ SDEV} = 1.00$$

$$\text{COSTS} = 1562.13 - 670.79Z + 93.57Z^2$$

$$Z_{\min} = 3.584$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8614$$

$$\text{XNCR} = .01\sigma', \text{PROB} = .05, \text{VALT} = .6\sigma', \text{SDEV} = .50$$

$$\text{COSTS} = 689.07 - 245.66Z + 37.242Z^2$$

$$Z_{\min} = 3.298$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .4036$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .01, \text{VALT} = .3\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 1647.01 - 710.92Z + 92.97Z^2$$

$$Z_{\min} = 3.823$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .9219$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .01, \text{VALT} = .3\sigma', \text{SDEV} = .50$$

$$\text{COSTS} = 749.94 - 281.47Z + 35.07Z^2$$

$$Z_{\min} = 4.01$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8496$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .01, \text{VALT} = .6\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 1540.84 - 682.77Z + 94.37Z^2$$

$$Z_{\min} = 3.617$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8794$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .01, \text{VALT} = .6\sigma', \text{SDEV} = .50$$

$$\text{COSTS} = 839.99 - 342.75Z + 47.86Z^2$$

$$Z_{\min} = 3.580$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8760$$

$$\text{XNCR} = .03\sigma', \text{PROB} = .05, \text{VALT} = .3\sigma', \text{SDEV} = 1.00$$

$$\text{COSTS} = 1347.91 - 632.73Z + 96.18Z^2$$

$$Z_{\min} = 3.280$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .9426$$

$$XNCR = .03\sigma', \text{ PROB} = .05, \text{ VALT} = .3\sigma', \text{ SDEV} = .50$$

$$\text{COSTS} = 793.40 - 425.08Z + 78.45Z^2$$

$$Z_{\min} = 2.709$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .8981$$

$$XNCR = .03\sigma', \text{ PROB} = .05, \text{ VALT} = .6\sigma', \text{ SDEV} = 1.00$$

$$\text{COSTS} = 954.78 - 437.67Z + 73.65Z^2$$

$$Z_{\min} = 2.971$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .6095$$

$$XNCR = .03\sigma', \text{ PROB} = .05, \text{ VALT} = .6\sigma', \text{ SDEV} = .50$$

$$\text{COSTS} = 509.89 - 94.78Z + 8.087Z^2$$

$$Z_{\min} = 5.863$$

$$\text{Coeff. of Mult. Corr.}, R^2 = .2725$$

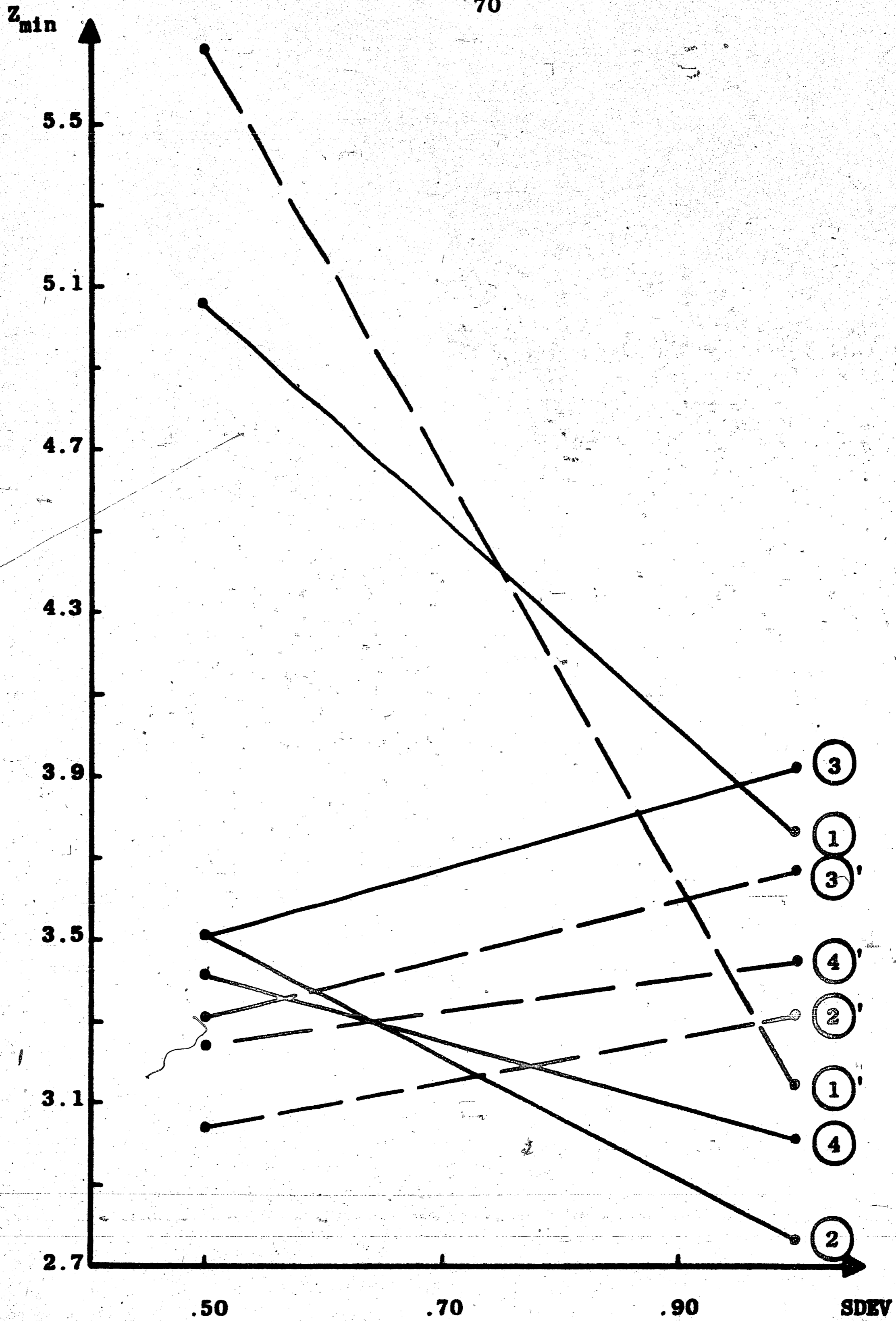
**APPENDIX III**

**Extrapolations of SCRAP and STRAP Response Surfaces**

APPENDIX III

	<u>XNCR</u>	<u>PROB</u>	<u>VALT</u>	
1	.03σ'	.05	.6σ'	scrap surface
1'	.03σ'	.05	.6σ'	strap surface
2	.03σ'	.05	.3σ'	scrap surface
2'	.03σ'	.05	.3σ'	strap surface
3	.03σ'	.01	.6σ'	scrap surface
3'	.03σ'	.01	.6σ'	strap surface
4	.01σ'	.05	.6σ'	scrap surface
4'	.01σ'	.05	.6σ'	strap surface



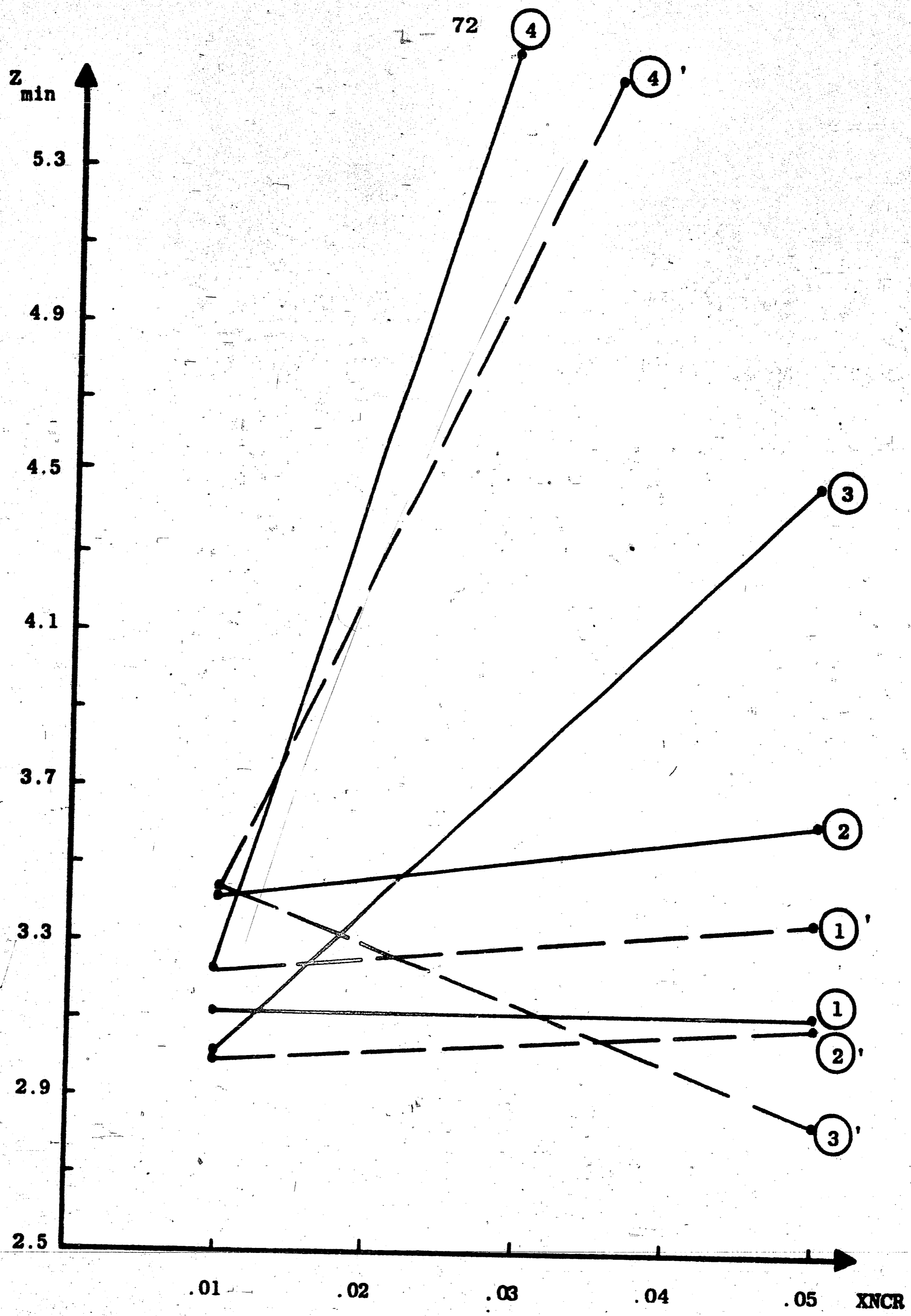


EXTRAPOLATION OF RESPONSE SURFACES

Figure 10



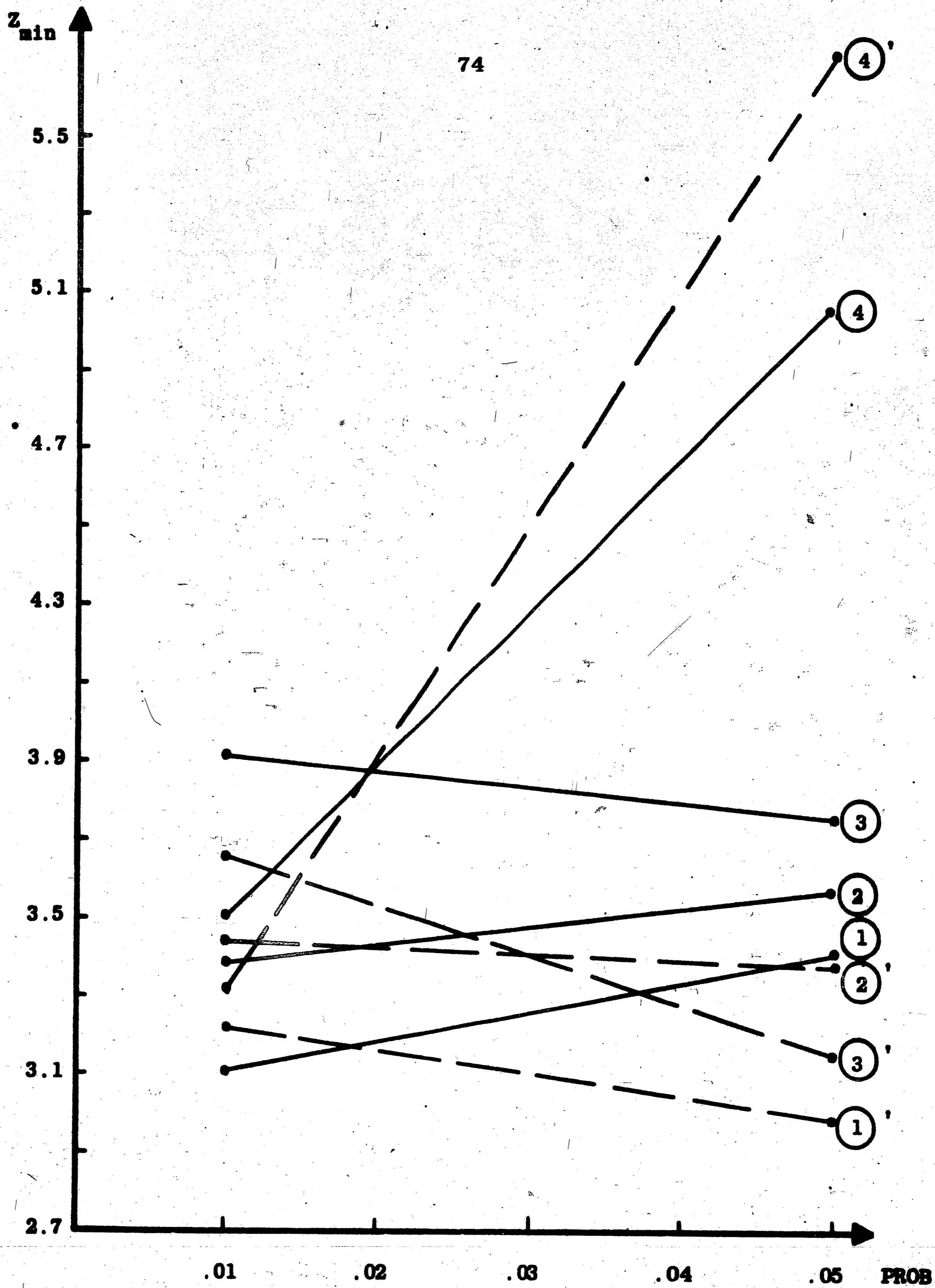
	<u>PROB</u>	<u>VALT</u>	<u>SDEV</u>	
1	.01	.3 $\sigma$ '	.50	scrap surface
1'	.01	.3 $\sigma$ '	.50	strap surface
2	.05	.3 $\sigma$ '	.50	scrap surface
2'	.05	.3 $\sigma$ '	.50	strap surface
3	.05	.6 $\sigma$ '	1.00	scrap surface
3'	.05	.6 $\sigma$ '	1.00	strap surface
4	.05	.6 $\sigma$ '	.50	scrap surface
4'	.05	.6 $\sigma$ '	.50	strap surface



EXTRAPOLATION OF RESPONSE SURFACES

Figure 11

	<u>XNCR</u>	<u>VALT</u>	<u>SDEV</u>	
1	.01σ'	.3σ'	.50	scrap surface
1'	.01σ'	.3σ'	.50	strap surface
2	.01σ'	.3σ'	1.00	scrap surface
2'	.01σ'	.3σ'	1.00	strap surface
3	.03σ'	.6σ'	1.00	scrap surface
3'	.03σ'	.6σ'	1.00	strap surface
4	.03σ'	.6σ'	.50	scrap surface
4'	.03σ'	.6σ'	.50	strap surface

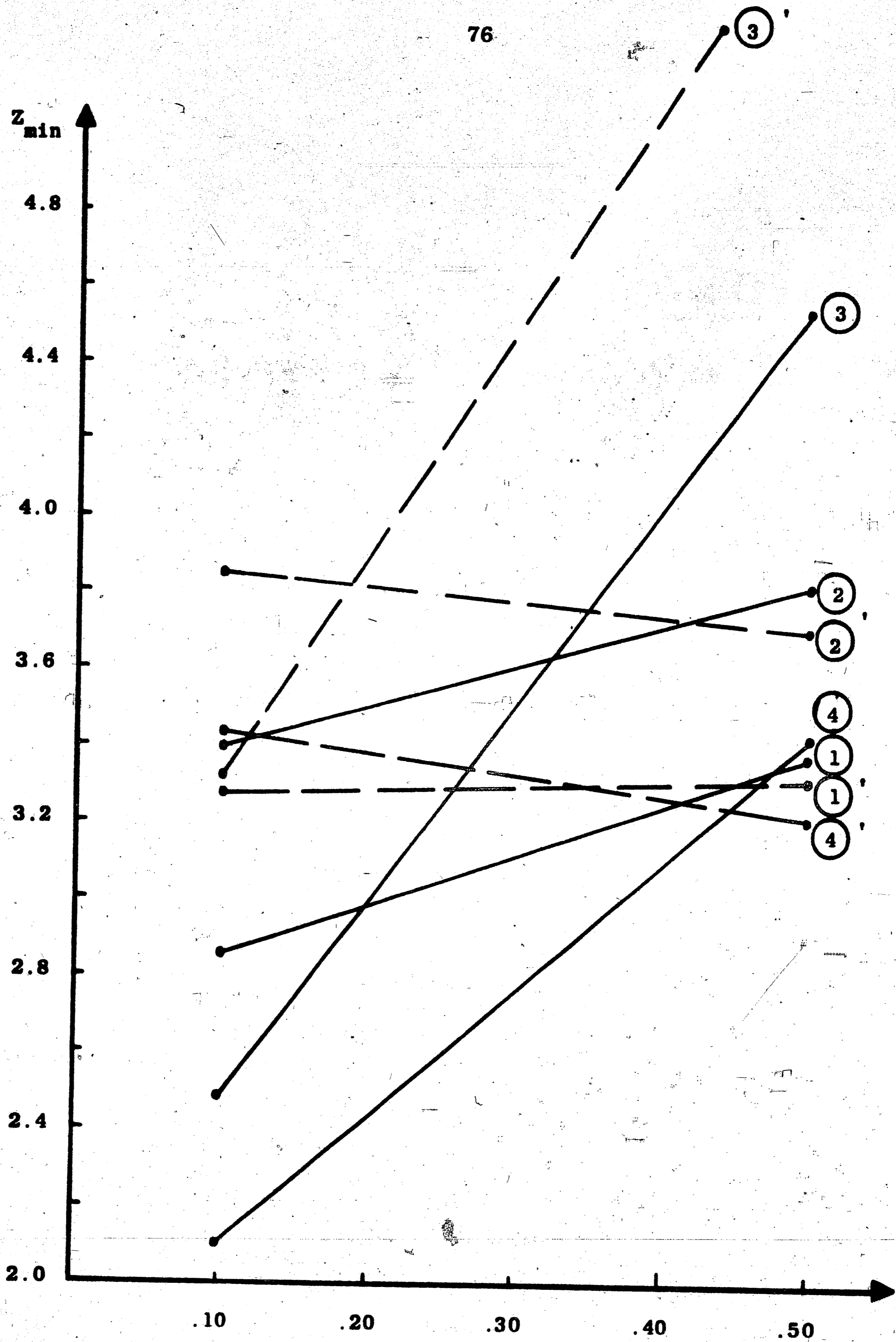


EXTRAPOLATION OF RESPONSE SURFACES

Figure 12

	<u>XNCR</u>	<u>PROB</u>	<u>SDEV</u>	
1	.03σ'	.01	.50	scrap surface
1'	.03σ'	.01	.50	strap surface
2	.03σ'	.01	1.00	scrap surface
2'	.03σ'	.01	1.00	strap surface
3	.03σ'	.05	.50	scrap surface
3'	.03σ'	.05	.50	strap surface
4	.03σ'	.05	1.00	scrap surface
4'	.03σ	.05	1.00	strap surface





EXTRAPOLATION OF RESPONSE SURFACES

Figure 13



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