1962

A test of a statistical learning model for multiple choice behavior

Susan Arnold Beil
Lehigh University

A TEST OF A STATISTICAL LEARNING MODEL FOR MULTIPLE CHOICE BEHAVIOR

by

Susan Arnold Beil

A Thesis

Presented to the Graduate Faculty

of Lehigh University

In Candidacy for the Degree of

Master of Science

Lehigh University

1962

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

8/31/62
(date)

Arthur L. Brody
Professor in Charge

Sept. 19, 1962

Head of the Department

ACKNOWLEDGEMENTS

The writer wishes to express her appreciation to Dr. Arthur L. Brody for his helpful advice and criticism during the formulation and completion of this study and for making laboratory subjects, apparatus and facilities available for the experiment.

This experiment was financed by funds from a National Institute of Mental Health grant, contract number M3342.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure | Page No.

INTRODUCTION

The purpose of the present study was twofold; first, to study the multiple choice behavior of animal subjects (Ss) under partial reinforcement, and second, to test certain predictions of Estes' statistical learning theory in this situation.

Food deprived albino rats were run in a hexagonally shaped maze with either two, three, or six choices available to them. A forced choice correction procedure was used. That is, the aminal had a free choice from the complete set of choices (2,3, or 6) as its initial response on any trial. If a correct response was given, a food reinforcement was obtained and the trial ended. If an incorrect response was given, no reinforcement was obtained, the animal was replaced in the start box, and forced to make the correct choice by reducing the complete set of choices to one available choice. Following the forced correction, reinforcement was obtained and the trial ended. Thus, each trial was terminated with a reinforcement. The correct response on each trial was determined by the experimenter according to a prearranged random schedule and was in no way contingent upon the animal's behavior. Partial reinforcement schedules were used with the restriction that on each trial one choice was designated the correct choice. That is, there were no trials on which more than one choice could result in reinforcement nor trials on which none of the choices resulted in reinforcement. One of the choices from the complete set was designated A_1 and the probability of that choice being correct on each trial was designated π_1. All Ss, regardless of the number of choices in the complete set, had the same π_1 value. The remaining or non-A_1 choice for the two choice Ss was correct with a probability of $1-\pi_1$. The remaining choices for the three and six choice Ss were correct with a probability that summed to $1-\pi_1$. Thus the effect of the number of choices on the learning of the A_1 response could be investigated. In addition, one group of three choice Ss and the group of six choice Ss had an equal probability of being correct on all non-A_1 choices. That is, the probability of the non-A_1 choices being correct, $1-\pi_1$, was evenly divided over the non-A_1 choices. Another group of three choice Ss had the probability of the non-A_1 choices being correct unevenly divided over the non-A_1 choices. The effect of this difference in partial schedules with respect to non-A_1 responses on both the learning of the A_1 response and the learning of the non-A_1 responses was studied. If, as is shown below, the learning of the A_1 response depends solely on the probability of reinforcing an A_1 response, the different partial schedules on non-A_1 responses should not have resulted in any differences in final level of A_1 responding among the different groups. By a similar line of reasoning, the different partial schedules on non-A_1 responses should have resulted in different final levels of non-A_1 responding.

Turning to the predictions derived from Estes' statistical learning model, a brief outline of the model employed will be presented first (See Estes, 1959 for further details). The stimulating situation is represented by a population of stimulus elements. On any trial, in a series of discrete trials, an independent ramdon sample of elements is drawn from the population in which each element has an equal probability of being sampled. It is assumed that the experimental situation remains constant during the series of trials so that the same population of elements is sampled on each trial and so that the size of the sample remains constant. All of the elements sampled on a trial become connected to the response reinforced on that trial. The response is a member of one response class and the experimental situation determines the set of mutually exclusive and exhaustive response classes. The probability of occurrence of a response class is the basic theoretical dependent variable and is defined as the proportion of elements in the population connected to that response class. Thus when a response is reinforced, all of the elements sampled on that trial become connected to that response; and there is a resultant increase in the probability of that response (provided only that the probability is not already equal to unity or that all the elements are not already connected to that response). After enough random samples have been drawn, each element will be connected to one of the response classes and since, then, the sum of the probabilities of all response classes must be unity, an increment in the probability of one response class will result in a corresponding decrement in the probability of all other classes.

In this multiple choice situation, there was a series of discrete trials, each trial terminating with the reinforcement of one response class from a set of mutually exclusive and exhaustive response classes; the stimulating conditions throughout the series of trials remained relatively constant. Thus the conditions of the model were met.

For simplicity, we will follow the changes in probability of one of the response classes, A_1. With respect to the A_1 response, there were only two kinds of trials. Either A_1 was the correct response and was reinforced or it was not correct and one of the other response classes was correct and therefore reinforced. Notice that postulating only two kinds of trials is a simplifying assumption. Thus, one kind of trial is when the A_1 response is reinforced regardless of whether the A_1 occurs as the initial response or as the forced correction response. The second kind of trial is when the A_1 response is not reinforced regardless of whether the A_1 does or does not occur as the initial response. The essential feature is whether the A_1 response is or is not reinforced following its occurence. The change in the probability of an A_1 following the first kind of trial can be represented by the following difference equation:

$$P(n+1) = P(n) + \theta \left[1-P(n) \right] \qquad (1)$$

where P(n) is the probability of an A_1 prior to the nth trial, P(n+ 1) is the probability of an A_1 after the nth trial, and θ is the parameter representing the proportion of elements sampled from the population on each trial. Since the probability of an A_1 is defined as the proportion of elements in the population connected to A_1, 1-P(n) represents the proportion of elements not connected to A_1. Of the new sample of elements, $\theta \left[1-P(n) \right]$ represents the proportion of elements previously not connected to A_1 which are now sampled and become connected to A_1. This proportion is added to the proportion of elements already connected to A_1 prior to the trial resulting in an increment in the probability of an A_1. Similarly, the change in probability of an A_1 response following the second kind of trial can be represented by the following difference equation:

$$P(n+1) = P(n) - \theta P(n) \qquad (2)$$

which shows that the proportion of elements connected to A_1 prior to the trial is reduced by an amount corresponding to $\theta P(n)$, the proportion of elements which were connected to A_1 which are now sampled and become connected to some other response class. This results in a decrement in the probability of A_1.

The mean probability of an A_1 after trial n, can be obtained by weighting each of the difference equations by the proportion of trials on which each should apply and summing. That is, since the first kind of trial (A_1 correct and reinforced) occurs with a probability π_1 and since equation (1) represents the change in probability of A_1 which occurs on this kind of trial, equation (1) is weighted by π_1. Similarly, equation (2) is weighted by $1-\pi_1$. We then have

$$P(n+1) = \pi_1 \left\{ P(n) + \theta \left[1-P(n)\right] \right\} + (1-\pi_1)\left[P(n)-\theta P(n)\right]$$

$$= (1-\theta)\, P(n) + \theta\, \pi_1$$

It can be shown by mathematical induction, that at the end of the nth trial the probability of an A_1 response is

$$P(n) = \pi_1 - \left[\pi_1 - P(o)\right]\left[1-\theta\right]^n \qquad (3)$$

Since $1 > \theta > 0$, the equation describes a negatively accelerating curve. With n sufficiently large, the asymptotic level of A_1 responses is seen to be π_1. This prediction has been supported by empirical findings for two, three, four and eight choice situations employing human Ss (Detambel, 1955; Estes and Straughan, 1954; Neimark, 1956).

The predictions derived from the model for this experimental situation were: (1) The terminal level of A_1 responses will be the same for all Ss regardless of the number of available choices (2,3, or 6); (2) The terminal level of the non-A_1 responses (three choice and six choice groups) will be equal to the proportion of trials on which each of the non-A_1 responses is correct whether there is an equal or unequal division of the probability of the non-A_1 choices being correct $(1-\pi_1)$; (3) The terminal level of the non-A_1 responses will be equal to each other and to $(1-\pi_1)$/number of non-A_1 response alternatives where there is an even division of the probability of the non-A_1 responses being correct; (4) The theoretical learning curve, equation (3), should provide a good fit to the empirical data; (5) The θ value will be an increasing function of the number of available choices if the stimulation of the choices is a large proportion of the total stimulation of the experimental situation.

METHOD

Subjects. The Ss were 48 naive, female, albino rats, between the ages of 90 and 120 days old.

Apparatus. The apparatus was a six alley hexagonal maze constructed of wood and covered with plexiglass. The start box in the center of the maze was a plexiglass hexagon with 5-1/8" sides, 6-1/2" depth, and had a hinged top. Each alley of the maze was 14" long, 6-1/2" deep, 4-1/2" wide, and had a hinged top. At the end of each alley was a food dish extending 1-1/2" into the alley, centered, and raised 2-1/8" above the floor. Six sheet metal guillotine doors were located 1-3/8" from the beginning of each alley. One side of each door had a different painted design of horizontal or vertical black and white stripes plus some arbitrary figure (e.g. triangles, circles, etc.) superimposed on the stripes. The other side was unpainted. The start box and doors were operated manually using a system of pulleys. The doors could be raised or lowered simultaneously in combination or individually. The start box and doors could be raised to a height of 3" from the floor. With the appropriate doors raised, raising the start box permitted access to the appropriate alleys. The maze was centered beneath a rectangular flourescent ceiling light so that alleys two and five were directly beneath the light. The Experimenter stood between alleys one and six during each trial.

Procedure. Ss were randomly assigned to one of four groups with 12 Ss per group. Reinforcement probabilities for the four groups are outlined in Table 1. Reinforcement schedules for 150 acquisition trials were determined randomly for each S so that each choice was correct with the probability outlined in Table 1 in each block of 50

Table 1

The proportion of trials on which each choice was correct for each of the four groups.

Group	Choice 1	Choice 2	Choice 3	Choice 4	Choice 5	Choice 6
I	.60	.40				
II	.60	.20	.20			
III	.60	.30	.10			
IV	.60	.08	.08	.08	.08	.08

trials. The alleys used in each group were equidistant (opposite alleys for the two choice group, alleys at 120° for the three choice groups, and all alleys for the six choice group). All combinations of probabilities for the set of alleys in each group were randomized across all Ss in the group as follows: With the alleys numbered consecutively from one to six in a clockwise direction, six randomly selected Ss in Group I had alley number one correct with probability .60 and alley four correct with probability .40. The remaining six Ss in Group I had alley four correct with probability .60 and alley one correct with probability .40. Three randomly selected sets of four Ss each in Groups II and III had respectively alley one, three, and five correct with probability .60. For Ss in Group II, the other two alleys were correct with probability .20. For Ss in Group III, within a set of four Ss all having the same alley correct with probability .60, two randomly selected Ss had one of the remaining alleys correct with probability .30 and the other alley correct with probability .10; the remaining two Ss had the reverse condition. For Group IV, sets of two randomly selected Ss were assigned a different one of the six alleys to be correct with probability .60, the remaining five alleys to be correct with probability .08 each.

Each S was handled approximately five minutes daily for three days during which time she could explore the maze with all doors removed and no reinforcement present. The Ss were on approximately a 15 hour deprivation schedule that continued for the entire experiment. During these three days after handling was completed for all animals and just before the regular food (Purina Chow) was provided, each animal was given two 97 mg dextrose tablets. This was done to familiarize the Ss with the tablets which were used as reinforcements.

Six pre-training trials were then given using a procedure analogous to the correction procedure (described below). Reinforcement was presented with equal probability in each alley being used in the group to which S had been assigned.

Following the training trials, each S was given 150 acquisition trials, four trials per day. On each trial S was placed in the start box with the appropriate doors open and with the painted side of these doors facing the start box. The remaining doors were closed with the unpainted side of these doors facing the start box. After five seconds, the start box was raised allowing S to run into any one of the appropriate alleys. When S had entered one of the alleys, the open doors were all lowered. If reinforcement was present (a single 97 mg dextrose tablet placed in the food dish), S was allowed 20 seconds to consume the tablet and was then removed from the alley and returned to the home cage. If reinforcement was not present, S was confined in the alley for 10 seconds, then removed for correction. The correction procedure used involved replacing S in the start box with all doors lowered except the one to the alley containing the reinforcement. The arrangement of the doors with respect to painted and unpainted sides facing the start box remained the same as that at the start of the trial. After five seconds, the start box was raised allowing S to gain reinforcement. After 20 seconds, S was removed from the alley and returned to the home cage. This procedure is analogous to the forced-choice correction procedure used in a previous study (LoGiudice, 1962). One-half hour following the daily trial series of four trials, the Ss were given food for one and one-half hours. All Ss had continuous access to water in their home cages. Since the total time of the four trials was approximately seven hours, different Ss had different deprivation schedules. In order to balance differential amounts of deprivation for the four groups of Ss, the order of running the Ss was randomized across the four groups and this became the fixed order of running Ss.

RESULTS AND DISCUSSION

During the course of the experiment, one of the 48 Ss died. The data presented are based on 12 Ss in Group I, 11 Ss in Group II, 12 Ss in Group III and 12 Ss in Group IV.

In all cases, the datum to be discussed was the initial response on a trial, not the corrected response which may have followed. Defining an A_1 response as entering the alley which was correct with probability .60(π_1), the first results to be presented are the terminal levels of A_1 responding for the four groups. To see whether the A_1 responses reached a stable terminal level, separate t-tests for each group were run on the difference between the mean proportion of A_1 responses in the next to last block of 10 trials (trials 131 - 140) and the last block of 10 trials (trials 141 - 150). The results of these t-tests were, Group I (t = .034, df = 11), Group II (t = .114, df =10), Group III (t = .158, df = 11), and Group IV (t = .016, df =11). None of these t-tests showed a significant difference (.05 level of confidence) in level of responding in the two terminal 10 trial blocks. It was therefore concluded that all groups reached asymptote within 150 trials, and because of the finding of no differences, the data for the two terminal 10 trial blocks were combined for further tests.

Table 2 shows the terminal proportion of A_1 responses in the last block of 20 trials for each group. An analysis of variance was done on this terminal level for the four groups. The F of .570 with 3 and 43 df was not significant at the .05 level. Therefore, the terminal level of A_1 responses was the same for all groups regardless of the number of available choices (2,3, or 6), and regardless of the different partial schedules of reinforcement of the non-A_1 responses.

Table 2

Terminal mean proportion of A_1 responses for each group

	$\bar{p}$ trials (131-150)	π_1	t
Group I	.60	.60	0
Group II	.62	.60	.415
Group III	.56	.60	.884
Group IV	.52	.60	1.053

$F = .570$

Separate $\underline{t}$-tests for each group were run between the terminal level of A_1 responses and the probability of reinforcement (π_1). Group I ($t = 0$), Group II ($t = .415$, $df = 10$), Group III ($t = .884$, $df = 11$), and Group IV ($t = 1.053$, $df = 11$) all showed no significant difference (.05 level of confidence). Thus, the asymptotic level of A_1 responding did not differ significantly from π_1, (the probability of reinforcement of an A_1 response) for any of the four groups. Again, the terminal level of A_1 responses is seen to be independent of the number of available choices and the partial schedules of reinforcement of the non-A_1 responses. This is taken as a confirmation of the first prediction derived from the Estes' model.

Let us turn now to the terminal level of non-A_1 responses. In Group II, there was an equal division of probability of reinforcement among the non-A_1 response alternatives. Recall that the two non-A_1 alternatives were selected from three alleys. In order to test the difference between terminal response proportions on the non-A_1 alternatives, one of the two alleys used for each S was randomly assigned to what was called the A_2 alternative and the other alley to what was called the A_3 alternative.

Table 3 shows the terminal mean proportion of responses to each of the non-A_1 response alternatives in Group II. A $\underline{t}$-test was run between the terminal proportion of A_2 responses and the terminal proportion of A_3 responses. The $\underline{t}$ of .488 with 10 df showed that there was no significant difference in terminal level of A_2 and A_3 responding. A separate $\underline{t}$-test was run between the mean proportion of A_2 responses and the proportion of trials on which that response was correct ($1-\pi_1/2 = .20$). The $\underline{t}$ value of .134 with 10 df was not significant. Similarly,

Table 3

Terminal mean proportion for each of the non-A_1 responses in Group II.

	A_2	A_3	
Mean Proportion	.205	.182	$\underline{t}$ = .488
Expected Proportion	.200	.200	
	$\underline{t}$ = .134	$\underline{t}$ = .625	

a $\underline{t}$-test was run between the mean proportion of A_3 responses and the proportion of trials on which that response was correct ($1-\pi_1/2 = .20$). The $\underline{t}$ value of .625 with 10 df was also non-significant. It is concluded, therefore, that for Group II, the terminal levels of the two non-A_1 responses were equal to each other and that they were both equal to $1-\pi_1/2$.

Table 4 shows the terminal mean proportion of responses to each of the non-A_1 response alternatives in Group IV where there was also an equal division of $(1-\pi_1)$ among the remaining alternatives. Here, the five non-A_1 alternatives (which were selected from the six alleys) were randomly assigned for each S to the alternatives A_2, A_3, A_4, A_5, and A_6. The mean proportion of responses to each of the five non-A_1 response alternatives in the terminal block of 20 trials showed no significant differences ($\underline{F}$ = .642 with 4 and 55 df). None of the five terminal mean response proportions differed from the proportion of trials on which each alternative was correct ($1-\pi_1/5 = .08$). The separate $\underline{t}$ values of .121, 1.467, .135, .050, and .909 each with 11 df were all nonsignificant. For Group IV, the analogous conclusion to that of Group II is therefore reached, namely that the terminal levels of the five non-A_1 responses were equal to each other and that they were all equal to $1-\pi_1/5$.

The mean proportion of responses to each of the two non-A_1 response alternatives in the terminal block of 20 trials for Group III is shown in Table 5. Here there was an unequal division of $1-\pi_1$ and the alley with probability of reinforcement .30 was called the A_2 alternative and the alley with probability of reinforcement .10 was called the A_3 alternative. A $\underline{t}$-test was run to test the difference

Table 4

Terminal mean proportion for each of the non-A_1 responses in Group IV.

	A_2	A_3	A_4	A_5	A_6	
Mean Proportion	.083	.129	.083	.079	.104	$\underline{F}$ = .642
Expected Proportion	.080	.080	.080	.080	.080	
t =	.121	1.467	.135	.050	.909	

Table 5

Terminal mean proportion for each of the non-A_1 responses in Group III.

	A_2	A_3	
Mean Proportion	.296	.146	$t = 3.198$*
Expected Proportion	.300	.100	
	$t = .116$	$t = 1.450$	

* Significant at the .05 level.

between the terminal response proportions to A_2 and A_3. The t value of 3.198 with 11 df was significant. The mean proportion of A_2 responses did not differ significantly from .30 (proportion of trials on which the response was correct). The t value of .116 with 11 df was non-significant. Similarly, the mean proportion of A_3 responses did not differ significantly from .10 (proportion of trials on which that response was correct). The t value of 1.45 with 11 df was not significant. It is therefore concluded that for Group III, the terminal levels of the two non-A_1 responses were different from each other and that each was equal to the proportion of trials on which that response was correct.

In summary, comparing the results on the non-A_1 responses with the predictions derived from Estes' statistical learning model, it can be seen that the terminal level of responding of the non-A_1 responses did not differ from the proportion of trials on which each of the non-A_1 responses was correct whether there was an equal or unequal division of $1-\pi_1$. Further, the terminal levels of responding of the non-A_1 responses did not differ from each other where there was an even division of the probability of the non-A_1 being correct and did differ from each other where there was an uneven division. Thus, the second and the third predictions of the model are confirmed.

The results, discussed above, which were all based on group means support the predictions of the model. According to the model, the predictions should also hold for individual Ss. Therefore, each S's terminal level of responding to each of the response alternatives was tested against the predicted terminal level of responding. The observed frequencies of responses to each of the response alternatives in the terminal block of 20 trials were tested against the theoretical frequency distribution given by the model by the Chi-square technique. Tables 6 through 9 present for Groups I through IV respectively the tests of the adequacy of the model for each of the 47 Ss. Since the Chi-squares for individual Ss are independent of each other, they can be summed to provide a Chi-square to test the adequacy of the model for the group. One S in Group I, one S in Group II, two Ss in Group III and nine Ss in Group IV had observed terminal frequency distributions which differed significantly from the theoretical frequencies.

The group Chi-squares showed no significant departures for Groups I and II, but both Group III ($\chi^2 = 51.67$, df = 24) and Group IV ($\chi^2 = 147.04$, df = 24) showed significant departures from the model. While the non-significant Chi-squares of Groups I and II are in line with the previous t-tests on group means, the significant departures of Groups III and IV are not. Inspection of Table 8, Group III, shows that the significant departure for this group was a result of large deviations of only two Ss. The model, therefore, seems to be fairly satisfactory. Inspection of Table 9, Group IV, however, shows that there were many large individual deviations. Ss responded to the A_1 alternative with frequencies ranging from one and five to 18 and 17 in the terminal block of 20 trials. Similar deviations are seen on the remaining non-A_1 alternatives. However, when these frequencies are averaged (as for the t-tests on means) the group frequencies do not differ from the predicted frequencies of responding. It therefore seems reasonable to conclude that the model did not adequately predict the behavior of individual Ss in Group IV, (the six choice situation with the particular reinforcement probabilities used); although it was

Table 6

The frequency distribution and Chi-square values for the responses of individual subjects in the terminal block of 20 trials - Group I

		A_1	A_2	Chi-Square
Theoretical Frequency		12	8	
Observed Frequency	S #1	9	11	1.88
	S #2	11	9	.21
	S #3	14	6	.83
	S #4	15	5	1.88
	S #5	10	10	.83
	S #6	15	5	1.88
	S #7	12	8	0
	S #8	8	12	3.33
	S #9	17	3	5.21
	S 10	17	3	5.21
	S 11	11	9	.21
	S 12	6	14	7.50 *
				Σ = 28.97

* Significant at the .05 level.

Table 7

The frequency distribution and Chi-square values for the responses of individual subjects in the terminal block of 20 trials - Group II

		A_1	A_2	A_3	Chi-square
Theoretical Frequency		12	4	4	
Observed Frequency	S #1	17	2	1	5.33
	S #2	11	6	3	1.33
	S #3	16	2	2	3.33
	S #4	14	5	1	2.83
	S #5	12	5	3	.50
	S #6	12	4	4	0
	S #7	16	4	0	5.33
	S #8	8	9	3	7.83 *
	S #9	8	7	5	3.83
	S #10	12	4	4	0
	S #11	9	7	4	3.00
					Σ = 33.31

*Significant at the .05 level.

Table 8

The frequency distribution and Chi-square values for the responses of individual subjects in the terminal block of 20 trials - Group III

		A_1	A_2	A_3	Chi-square
Theoretical Frequency		12	6	2	
Observed Frequency	S #1	9	4	7	13.92 **
	S #2	12	4	4	2.67
	S #3	12	8	0	2.67
	S #4	11	4	5	5.25
	S #5	5	10	5	11.25 **
	S #6	16	4	0	4.00
	S #7	10	9	1	2.33
	S #8	12	6	2	0
	S #9	10	6	4	2.33
	S #10	10	7	3	1.00
	S #11	10	7	3	1.00
	S #12	17	2	1	5.25
					Σ = 51.67 **

** Significant at the .01 level.

Table 9

The frequency distribution and Chi-square values for the responses of individual subjects in the terminal block of 20 trials - Group IV.

		A_1	A_2	A_3	A_4	A_5	A_6	Chi-square
Theoretical Frequency		12	1.6	1.6	1.6	1.6	1.6	
Observed Frequency	S #1	14	1	0	2	2	1	2.58
	S #2	18	0	0	1	1	0	8.25 *
	S #3	5	3	1	5	3	3	15.21**
	S #4	10	4	1	4	0	1	9.58**
	S #5	17	0	0	2	0	1	7.21 *
	S #6	13	0	1	5	0	1	10.96**
	S #7	8	3	1	6	1	1	15.33**
	S #8	5	2	1	7	4	1	26.46**
	S #9	14	3	0	0	1	2	5.08
	S #10	13	3	2	1	1	0	3.46
	S #11	7	3	5	0	3	2	13.46**
	S #12	1	6	4	3	3	3	29.46**
							$\Sigma =$	147.04**

* Significant at the .05 level.

** Significant at the .01 level.

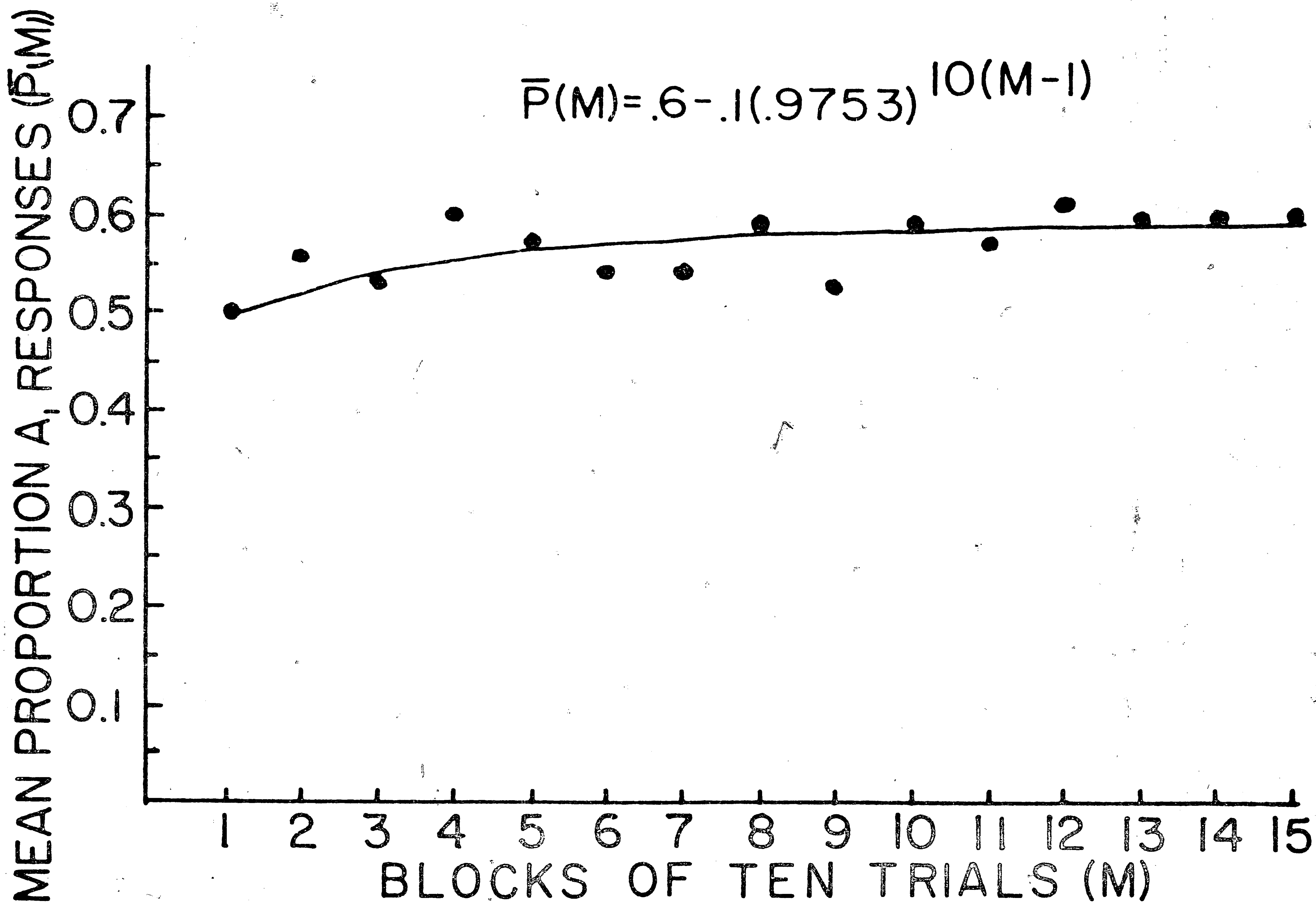
adequate for predicting the mean behavior of Ss in this group. Perhaps, the results from Group IV should be taken as evidence that tests of models for individual behavior which are based on averages over a group of Ss are at best a first approximation to a test, and that the crucial feature of how many individual Ss show the same trend as the group mean cannot be overlooked. In attempting to account for the discrepancies observed in Group IV, it is important to note that the choice of reinforcement probabilities was severely limited by the conditions of the experiment. In order to have the same π_1 value for all groups and, at the same time, not to have this value too close to .50 (so that a reasonable two choice situation would result), it was necessary to have quite small reinforcement probabilities for the five non-A_1 alternatives of Group IV. Perhaps there are upper and lower bounds on reinforcement probabilities which restrict, somewhat, the usefulness of this model. That is, it might turn out that if π_1 is set equal to a value greater than .90 or less than .10 the model breaks down. Unfortunately, there is no way of knowing, on the basis of the present study, whether such is the case or whether the model breaks down when the number of available choices is greater than three.

Turning from the terminal level of response to the A_1 responses throughout the entire 150 trial acquisition series, Figures 1 through 4 show the mean proportion of A_1 responses per block of 10 trials each for Groups I through IV respectively. The Figures also show theoretical curves fit to the data. The equations for these curves is

$$\bar{P}(m) = \pi_1 - [\pi_1 - \bar{P}(1)][1-\theta]^{10(m-1)} \qquad (4)$$

in which $\bar{P}(m)$ represents the theoretical mean proportion of A_1 responses

Fig. 1. The mean proportion of A_1 responses by blocks of 10 trials and the theoretical curve fit to the data of subjects in Group I.



$\bar{P}(M) = .6 - .1(.9753)^{10(M-1)}$
MEAN PROPORTION A_1 RESPONSES ($\bar{P}(M)$)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
BLOCKS OF TEN TRIALS (M)

Fig. 1

Fig 2. The mean proportion of A_1 responses by blocks of 10 trials and the theoretical curve fit to the data of subjects in Group II.

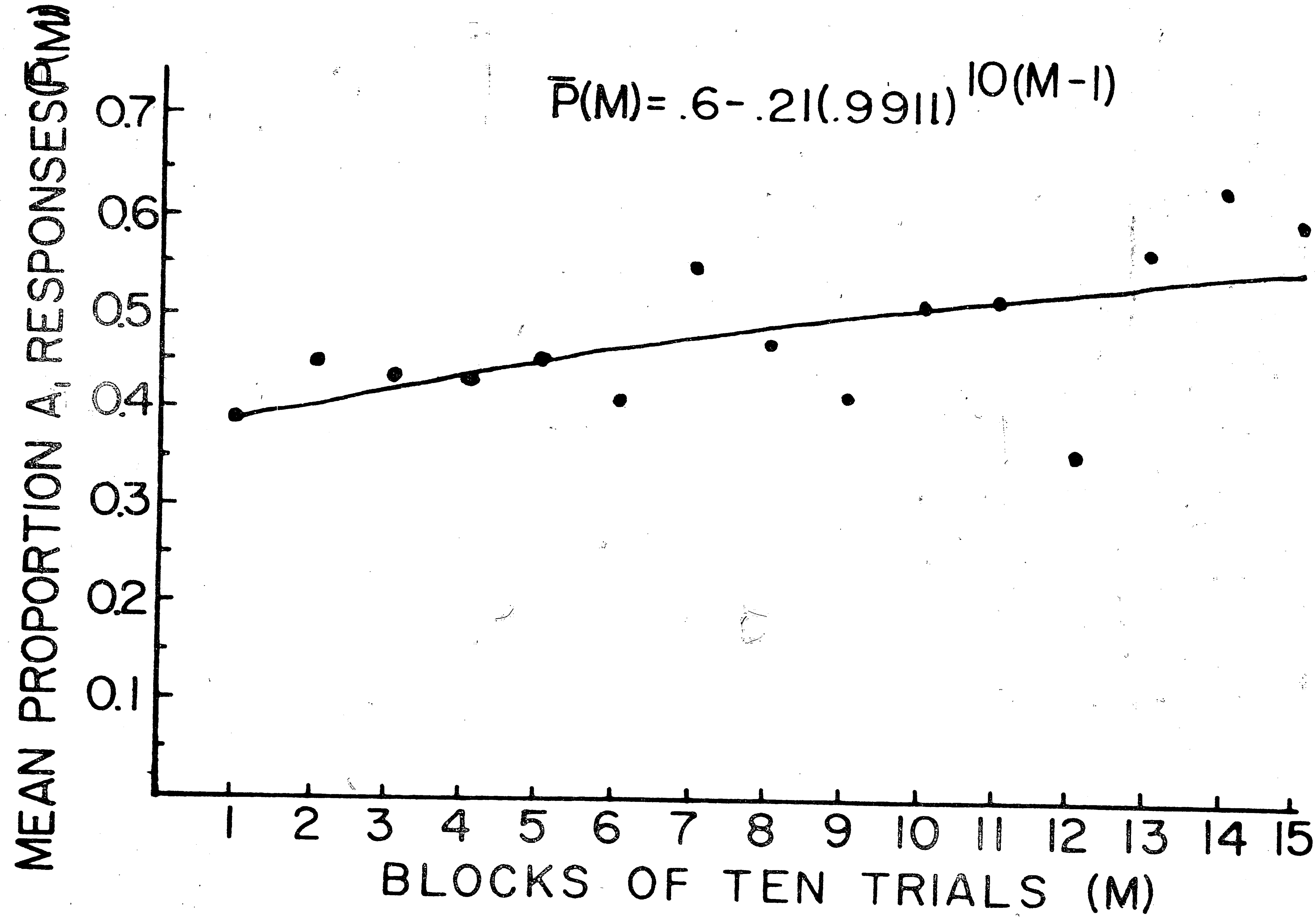
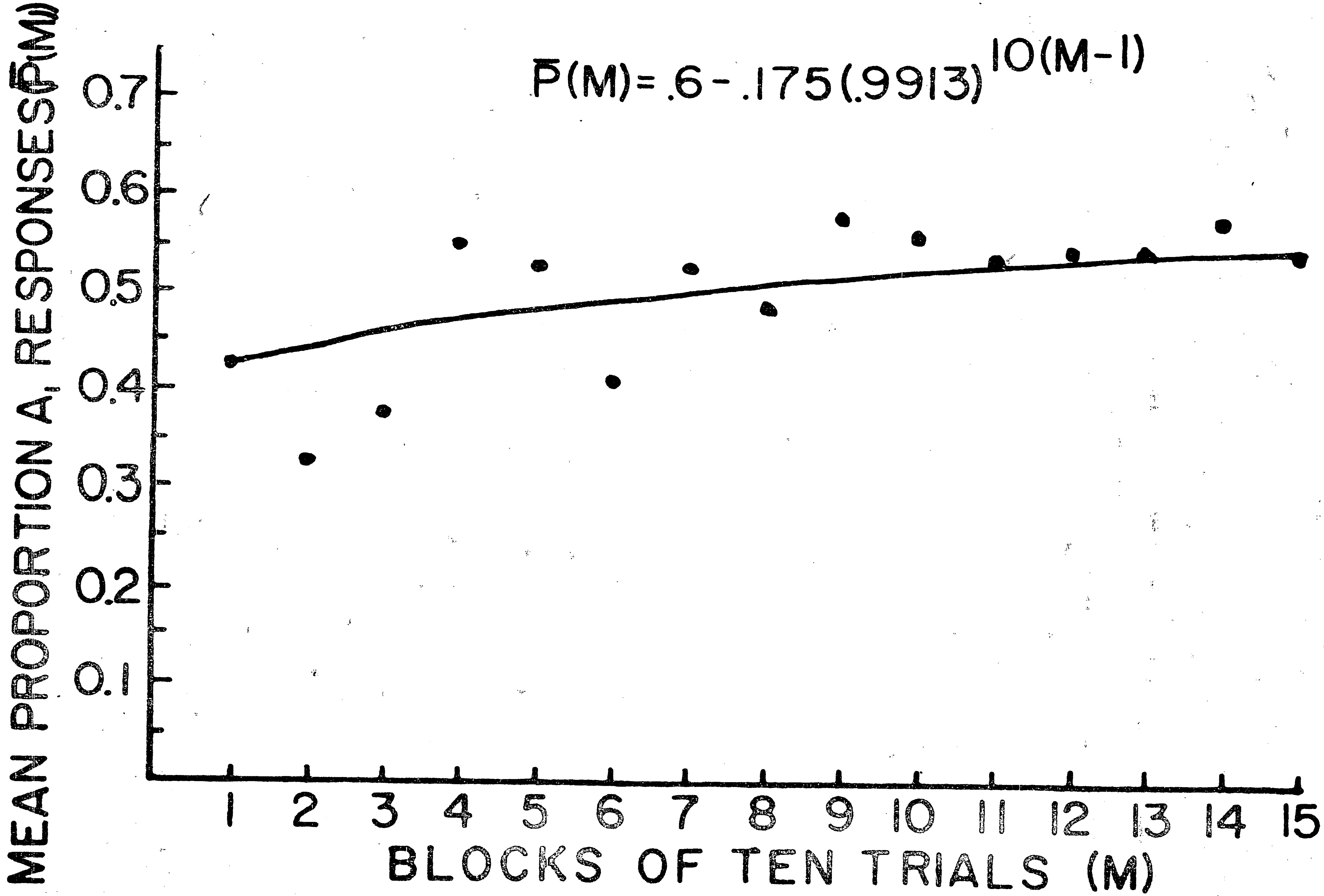

$\bar{P}(M) = .6 - .21(.9911)^{10(M-1)}$
MEAN PROPORTION A_1 RESPONSES ($\bar{P}$(M))
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
BLOCKS OF TEN TRIALS (M)

Fig. 2

Fig. 3. The mean proportion of A_1 responses by blocks of 10 trials and the theoretical curve fit to the data of subjects in Group III.

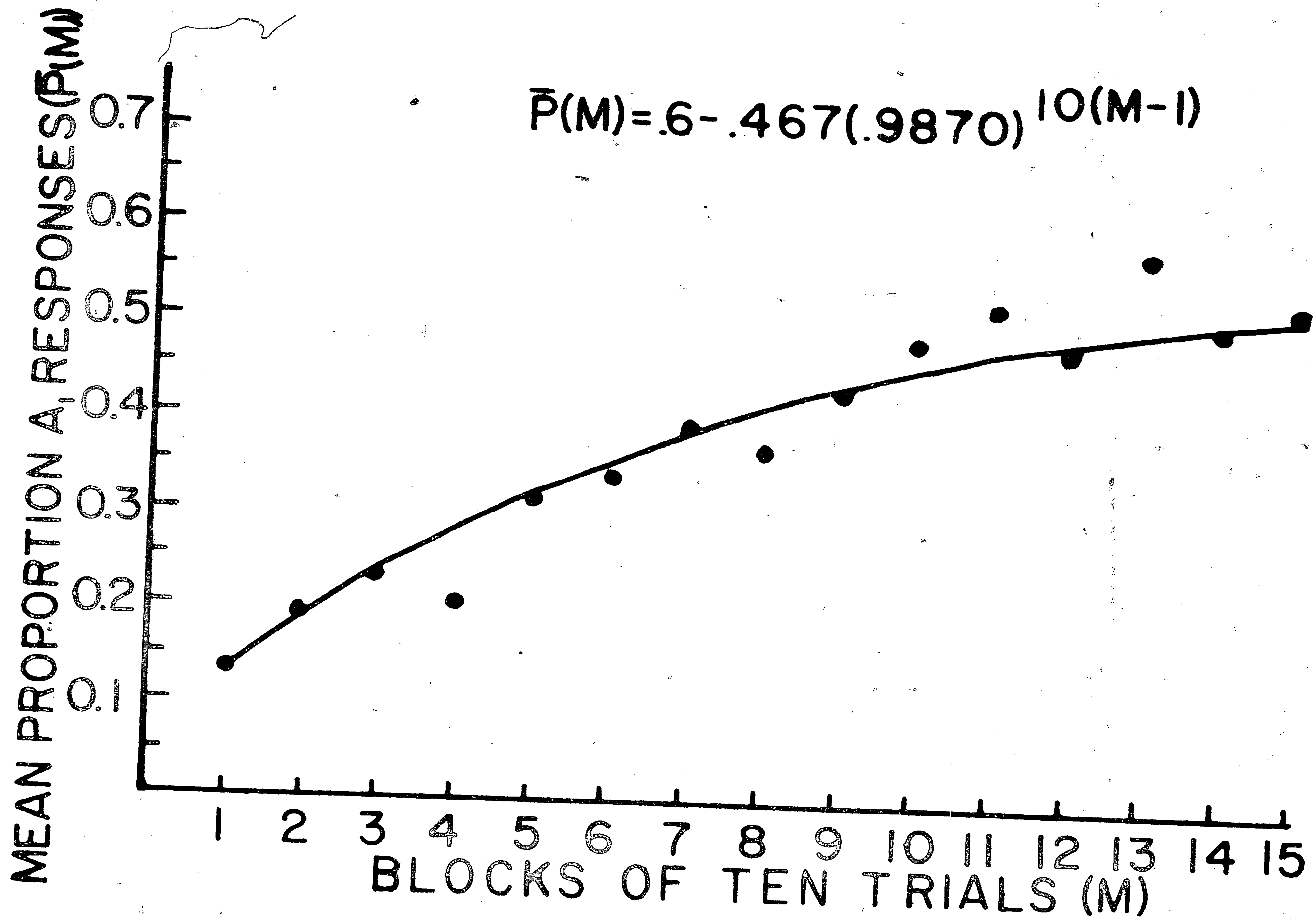


P̄(M)=.6-.175(.9913)^10(M-1)
MEAN PROPORTION A1 RESPONSES P̄(M)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
BLOCKS OF TEN TRIALS (M)

Fig. 3

Fig. 4. The mean proportion of A_1 responses by blocks of 10 trials and the theoretical curve fit to the data of subjects in Group IV.

Fig. 4



P̄(M)=.6-.467(.9870)^10(M-1)
MEAN PROPORTION A1 RESPONSES (P̄(M))
0.1
0.2
0.3
0.4
0.5
0.6
0.7
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
BLOCKS OF TEN TRIALS (M)

on the mth block of trials; $\bar{P}(1)$ represents the observed mean proportion of A_1 responses on the first block of trials; π_1 is the probability of an A_1 being correct and is equal to .60; and θ represents the proportion of stimulus elements sampled on each trial. This equation is derived from equation (3) (see Estes and Straughan, 1954 for details of the derivation). There are two parameters, $\bar{P}(1)$ and θ, which must be estimated from the data of each group separately. The estimate for $\bar{P}(1)$ is simply the observed mean proportion of A_1 responses on the first block of 10 trials. The estimate of θ can be obtained by summing equation (4) over the 15 blocks of trials and setting this sum equal to the observed mean total A_1 responses over all trials divided by the number of trials per block (10). Inserting the estimate for $\bar{P}(1)$, the equation can be solved for the one unknown, θ. Table 10 shows the $\bar{P}(1)$ and θ values thus estimated from the data.

F-goodness of fit tests for repeated measures were run for each group to determine whether or not the theoretical equation provided a good fit to the empirical data. The F values for Group I (F = .305, df = 13 and 154), Group II (F = 1.366, df = 13 and 140), Group III (F = 1.510, df = 13 and 154), and Group IV (F = .5636, df = 13 and 154) were all non-significant at the .05 level of confidence. Thus there is no adequate basis for rejecting hypothesis (4) that the theoretical learning curve satisfactorily fits the data.

The last hypothesis was concerned with the possibility of an increase in θ as a function of the number of available choices. An inspection of Table 10 shows no such relationship. Therefore, either the relationship does hold but the stimulation of the choices is not

Table 10

Parameters $\overline{P}(1)$ and θ estimated from the data for each group and used in obtaining the theoretical equation.

	$\overline{P}(1)$	θ
Group I	.5000	.0247
Group II	.3900	.0089
Group III	.4250	.0087
Group IV	.1330	.0130

a large enough proportion of the total stimulus of the experimental situation to make a difference in the θ value or the stimulation of the choices is the same regardless of the number of available choices.

To sum up, for all groups (2, 3, and 6 choice), the group mean data on terminal responding and A_1 responding throughout the acquisition series supported the predictions of the model without exception. Investigation of individual Ss showed that the model provided an adequate fit to terminal frequencies of responding for Ss in Groups I, II and III, but that individual Ss in Group IV showed large deviations from expected frequencies given by the model. The conclusions based on the results of this experiment are that the behavior of animal Ss was adequately accounted for by the Estes' statistical learning model employed, as far as the two and three choice situations; but, the adequacy of the model in accounting for the behavior of animals in the six choice situation leaves something to be desired.

SUMMARY

This experiment was designed to study the multiple choice behavior of animal _S_s under partial reinforcement and to test certain predictions of Estes' statistical learning theory in this situation.

Forty-eight food-deprived albino rats were run in a hexagonally shaped maze with either two, three, or six choices available to them. _S_s were randomly assigned to one of four groups (two groups of three choice _S_s) representing different reinforcement probabilities. For all groups one response, designated A_1, had a probability of reinforcement equal to .60 (π_1). The remaining choices were correct with probability that summed to $1-\pi_1$. For one group of three choice _S_s and the six choice _S_s, the probability of reinforcement on the remaining non-A_1 choices was evenly divided; and for another group of three choice _S_s the probability of reinforcement was unevenly divided among the remaining choices.

Statistical analyses for the four groups indicated that the terminal level of A_1 responding did not differ significantly from π_1, and that there were no differences in terminal levels of A_1 responding among the groups. This confirmed the predictions of the model that the terminal level of A_1 responding depends only upon the value of π_1, and is independent of the number of available choices and the partial schedules of reinforcement of the non-A_1 choices. There were also no differences between terminal level of responding to the non-A_1 responses and the probability with which each non-A_1 response was reinforced. This confirmed the predictions of the model on the terminal levels of non-A_1 responses.

The data for individual _S_s showed that the model was adequate in

accounting for the behavior of Ss in Groups I, II, and III. However, in Group IV, nine Ss deviated significantly from the predictions of the model, with respect to terminal frequencies of response. The model could not be said to give an adequate account of the behavior of these Ss.

A theoretical learning curve based on the model, and involving the estimation of two parameters, was fitted to the data for each group. F-goodness of fit tests showed that the theoretical curve provided a good fit to the empirical data for all groups.

It is concluded that Estes' statistical learning model provides a good fit to the behavior of animal Ss in the two and three choice situations, but not in the six choice situation.

REFERENCES

1. Detambel, M.H. A test of a model for multiple-choice behavior, J. exp. Psychol., 1955, 49, 97-104.

2. Estes, W.K. The statistical approach to learning theory. In S. Koch (Ed.), Psychology: a study of a science, Vol. 2., New York: McGraw-Hill, 1959. Pp 380-491.

3. Estes, W.K. & Straughan, J.H. Analysis of a verbal conditioning situation in terms of statistical learning theory, J. exp. Psychol., 1954, 47, 225-234.

4. LoGiudice, J.K. Unpublished study, Lehigh University, 1962.

5. Neimark, Edith D. Effects of type of non-reinforcement and number of alternative responses in two verbal conditioning situations, J. exp. Psychol., 1956, 52, 209-220.

VITA

Susan Arnold Beil, daughter of Susan and Thomas Beil, was born in Reading, Pennsylvania, on May 6, 1938.

A 1956 graduate of Exeter High School, Reiffton, Pennsylvania, she attended the Mary Washington College of the University of Virginia and graduated in June, 1960, with a major in Mathematics and a major in Psychology.

In September, 1960, she began graduate work at Lehigh University in the department of psychology. She was a graduate assistant in the University Counseling and Testing Services under Dr. Andrew J. Edmiston from July, 1961 until July, 1962. In October, 1962, she obtained the degree of Master of Science from Lehigh University.