1972

Development and investigation of an inventory control and procurement policy for a multi-item, multi-source, stochastic environment

James L. Kingsley
Lehigh University

DEVELOPMENT AND INVESTIGATION OF AN INVENTORY CONTROL AND PROCUREMENT POLICY FOR A MULTI-ITEM, MULTI-SOURCE, WAREHOUSE CONSTRAINED, STOCHASTIC ENVIRONMENT

by

JAMES L. KINGSLEY

A THESIS

PRESENTED TO THE GRADUATE COMMITTEE

OF LEHIGH UNIVERSITY

IN CANDIDACY FOR THE DEGREE OF

MASTER OF SCIENCE

IN

INDUSTRIAL ENGINEERING

LEHIGH UNIVERSITY

1972

CERTIFICATION OF APPROVAL

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

April 25, 1972

Date

John W. Adams

Professor in Charge

Arthur F. Gould

Chairman of the Department of Industrial Engineering

ACKNOWLEDGEMENTS

The author extends his thanks to Professor J. W. Adams for serving as the primary advisor in the preparation of this work.

Thanks are also due to Messrs. P. T. Lele and E. A. Siecienski of Western Electric Company's Engineering Research Center for their assistance and suggestions throughout the course of this work.

Finally, my special thanks to my wife, Georgia, without whose patience, encouragement, and understanding the work could not have come to successful completion.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

Procedures are presented which provide management with a tool to make integrated decisions which are consistent with the interacting constraints of a multi-item, multi-source, warehouse constrained, stochastic environment. These procedures reflect the necessity of an integrated decision making process for a class of items which, due to their low unit cost and high storage requirements, intensify the ever-present conflicts between the purchasing, production control, and warehouse organizations within a firm.

A procedure which accounts for the fluctuations in demand by varying the frequency with which the orders are placed is developed and illustrated. The procedure outputs, for each product, the optimal vendor selection, and the ordering policy. The procedure requires that the demand be identically distributed and that the products considered are stored in unit loads, such as pallets.

Finally, a procedure which accounts for the fluctuations in demand by varying the order quantity is developed and discussed.

CHAPTER I

BACKGROUND, OBJECTIVES, AND PROCEDURES

INTRODUCTION

Inventory control and procurement is of interest to any firm which must purchase or manufacture items. In general, a stock of a certain item is maintained to meet an internal or external demand. When that stock is diminished by usage it is necessary to take action to replenish the stock so as to avoid an "out of stock" situation. The customary action consists of initiating an order for that particular item. The order may be sent to an external supplier or it may be sent to one of the production shops within the firm itself. The control variables in such a situation are:

(1) When to order,

(2) How much to order,

(3) From whom to order.

By selecting the correct decision rule an inventory control and procurement system may be operated at a specific service level for the minimum cost. Many factors enter into determining the correct decision rule. These are the various cost parameters associated with the system, the type and amount of demand for the item, the limits on supply and production sources, the limits on storage capacity, the objectives of the system, and many more factors which must be taken into consideration if the optimal decision rule is to be formulated.

The optimal decision rule must be a rule which is optimal for the firm and not a particular subset of the firm. If one were to examine the organizations within a firm he would find the following four organizations:

(1) Purchasing,

(2) Production Control ,

(3) Warehousing,

(4) Manufacturing.

Although the organizational divisions are not always clear, these four organizations are basic to any manufacturing firm. If the firm is small, these four organizations may be combined into a single department. If the firm is large and complex, these organizations may exist separately, but their combined responsibility is to provide a service, so that the firm may meet the demands of their customers.

In most firms the operating cost is used as a measure of performance, therefore, the manager of the Purchasing Organization makes decisions so as to reduce the total cost of purchasing items for the firm. The manager of the Production Control Organization makes decisions so as to keep the number of stockouts and the inventory investment at a minimum. The manager of the Warehousing Organization makes decisions to provide for fast delivery and efficient space utilization. The effort of the Purchasing Organization to reduce costs means large order quantities and the effort of the Production Control Organization to reduce inventory investment means frequent ordering of small lots; both of these strategies place conflicting demands on the Warehouse Organization, resulting in inefficient space utilization. It is clearly seen that in such an environment there is a need for an integrated decision process: a process which is optimal for the firm.

PROBLEM TO BE INVESTIGATED

In many manufacturing firms there is one particular class of items which clearly demonstrates the conflicts between the Purchasing, Production Control, and Warehousing Organizations. This class contains those items which have a low dollar value, but a high storage space requirement. Packing material is one such class of material. Due to the fact that this class of items intensifies the conflict between the three organizations, there is a need for a model which will provide an integrated method of decision making.

OBJECTIVES

The primary objective of this thesis is to develop and investigate an inventory control and procurement model which will provide: (1) when to order, (2) how much to order, (3) from whom to order, and (4) the warehouse space to allocate to the items, for a system which has multiple items, multiple sources of supply, limited storage capacity, and stochastic demand and replenishment time. Development of such a model will provide an analytical method by which optimal decision rules can be derived. Once formulated, these decision rules can be applied to the operation of an inventory control and procurement system.

PROCEDURE

All of the pertinent variables and constraints will be identified and a model will be developed for an inventory system with multiple items, multiple sources of supply, limited storage capacity, and stochastic demand and replenishment time. The model to be developed will be an extension of the model developed by P. T. Lele and E. A. Siecienski[22] for the case of known and constant demand and lead time. A sample problem of a system with three items and multiple sources of supply for each item will be illustrated and the results discussed.

CHAPTER II

STATE OF THE ART AND PROBLEM STATEMENT

Since the beginning of time man has held various resources in inventory. Early man no doubt recognized the advantage of carrying extra weapons, and the disadvantage of carrying too many extra weapons. Not only did he have to spend the time and effort to construct the extra weapons, but he had to carry them until they were needed. In our ancestors' day too few or too many weapons could mean the difference between life and death. Fortunately not all inventory decisions are as final as the one our ancestors faced. Inventory decisions, however, are an important segment of any society. As societies advanced, more inventory problems were discovered. At one period in time, the merchant with the greatest number of warehouses and the largest stock of goods was thought to be the most prosperous. In current times excess inventories are usually viewed with disfavor since they represent an investment which produces no revenue. With society's advancement, businesses in the society became larger and more complex. Businessmen who once relied on their personal knowledge to manage their firms found that due to an increase in size, complexity, and competition, their personal knowledge was no longer adequate to manage the business. Recognizing this inadequacy they began to search for alternate means of control. The need for better methods of control, together with the increasing technical knowledge, resulted in sound analytical methods to aid in controlling inventory and production scheduling problems.

STATE OF THE ART

The first analytical method was applied to a firm which produced several different types of items in lots to be stored at its warehouse for future use. Manufacture of each different type of item required that the production facilities be altered. This alteration (setup) was costly, but so was storing (carrying) a large quantity of items in the warehouse. The problem was to determine the quantity of each item to produce before altering the facilities for the production of a different item. The first solution to this problem was provided by F. W. Harris[17]. His derivation is often referred to as the "simple lot size" formula or the "Wilson formula" since R. H. Wilson also derived the formula and was responsible for its application in several manufacturing firms. The model derived by Harris minimizes the sum of the inventory carrying costs and the production setup cost.

The elementary model for determining economic order quantities makes the following major assumptions about reality:

(1) Single Item,

(2) Continuous and known rate of demand,

(3) Constant and known replenishment lead time,

(4) All demand be satisfied,

(5) Indefinite planning horizon,

(6) Unit cost be constant.

Since the elementary model was developed, the research in inventory theory has been aimed at relaxing the assumptions necessary for determining an optimal decision rule. F. E. Raymond[23] published a book in 1931 which presents various extensions and applications of the simple lot size formula. In 1951 a paper by Arrow, Harris, and Marschak[2] provided an introduction into the realm of stochastic inventory theory. This paper was followed by the papers of Dvoretzky, Kiefer and Wolfowitz[6,7,8]. These highly analytical papers were the beginning of a great many books and papers in the field of inventory theory. In 1953 T. M. Whitin[29] published a book which summarizes the developments in inventory theory prior to mid-1952. In his text, Whitin provides an extensive bibliography of publications in inventory theory. In 1958 Arrow, Karlin, and Scarf[1] published a collection of papers which had been written in the years from 1955 to 1958 at Stanford and the RAND corporation. This book provided further stimulus in the area of stochastic inventory theory. The bibliography in the book by Arrow, et.al. supplements the bibliographies in Whitin and in Gourary, Lewis, and Neeland[14]. By 1962 a number of books[4,5,13,15,16,18,24,27] had been published on the theory of inventory and production control for the single item, single echelon environment.

The first treatment of a multi-item, multi-echelon situation consisted of decomposing the problem into single item, single echelon systems which could be treated with known theory. This technique, however, required many assumptions which are unrealistic for a complex system. Interactions between items and echelons do exist and must be accounted for rather than ignored. Thus, the bulk of the effort in inventory control after 1962 was focused on multi-item, multi-echelon inventory and production problems. Summaries of the developments in inventory control theory up to 1969 are provided by Iglehart[19,20], Karlin[21], Scarf[25], and Veinott[28]. Papers published after 1969 provide further insight into multi-item, multi-echelon problems as well as some extensions to the single item problem.

Most of the theory of complex inventory systems can be divided into two groups. One group of literature deals with attempts to find optimal solutions to n-period stochastic inventory problems, while the second group analyzes the use of fixed ordering rules by studying the resulting stochastic process. Most techniques to find optimal ordering rules depend on dynamic programming and are quite complex. Often the advantage of a simple ordering rule, which is easily understood by all, counters the higher cost of operating a system suboptimally. The need for simple operating rules has lead to the stationary analysis approach of many researchers.

Even though the amount of research in the field of inventory and production control theory is considerable, there are still many unanswered questions. Many of the advanced mathematical models are difficult to put into practice.

All of the literature thus far mentioned has at least one point in common. That is, none of the analytical models available provide a satisfactory solution if the storage space available is limited and the items may be purchased from several suppliers. If the system consists of a single item which can be purchased from several sources, the decision can be made quite easily. The decision maker need only evaluate the total annual cost of purchasing from each supplier and choose the supplier which results in the minimum total annual cost.

W. J. Fabrycky[9] investigates a generalized approach to an inventory problem in which he assumed that any item in the system is available from more than one source. The particular sources Fabrycky had in mind were various alternative suppliers and the manufacture alternative. That is, the firm had the option of manufacturing the item or of purchasing the item from several suppliers outside the firm. In his dissertation Fabrycky shows how the existing inventory theory can be extended to include source parameters. Once these extensions have been explained Fabrycky uses computer simulation to investigate a system with random demand and lead time. An extension of Fabrycky's dissertation[10] provides a further look into the environment of the multi-source item. Again simulation was used to study the stochastic system. In both these works Fabrycky has investigated a single item with multiple sources. By assuming that there is no interaction between items, these methods may be used to find the optimal policy for a multi-item system by combining the optimal policy found for each individual item. In a system where the items are competing for storage space the assumption of independence between items is not valid. Fabrycky and Banks[11] published a treatise in 1965 which combined the results of Fabrycky's earlier investigations of a multi-source item with the results of works published in inventory control literature dealing with a multiple item, single source system subject to restrictions on the storage and source capacity. In this publication Fabrycky and Banks discuss the multi-item, multi-source system with and without restrictions. By various assumptions they are able to provide closed form solutions to several situations. One of the contributions of this publication is the formulation of a dynamic programming algorithm which provides an analytical technique for finding an optimal procurement policy in a multi-item, multi-source, warehouse constrained system. This formulation can be used for systems in which the demand and lead time are constant, or where the distribution of demand during lead time can be described by a uniform distribution. The formulation does not account for the cost of using more space than is available, a situation which could occur with alarming frequency in a stochastic environment.

A dissertation written by Banks[3] and published in 1965 classifies procurement and inventory systems in a hierarchial order. The simplest system being the single item, single source and the most complex the multi-item, multi-source. In his dissertation, Banks develops decision models to represent each system in the hierarchy. He also extends some of the work done in the lower levels of the hierarchy. The dynamic programming formulation is used to provide the optimal procurement policy for the multi-item, multi-source, warehouse constrained environment. Again, the demand over lead time is assumed to be uniformly distributed and the cost of warehouse space is assumed proportional to the investment in inventory.

Fabrycky and Banks[12] provided some additional insight into inventory and procurement systems in their textbooks. This book provides a progressive study of inventory and procurement systems starting with the simple and progressing to the more complex. The techniques used to determine the optimal inventory decision rules include calculus, linear programming, direct enumeration, lagrangian multipliers, and dynamic programming. In this text the deterministic multi-item, multi-source, warehouse constrained formulation is used to provide the optimal decision rule for a probabilistic system by using expected value of demand and lead time, instead of known values of the various parameters; however, the distributions of demand and lead time are not considered.

In all the investigations by Fabrycky and Banks the following limitations exist. First for the multi-item, multi-source stochastic system with warehouse restrictions only a very special case has been solved, namely the case where the demand during the lead time is uniformly distributed. Secondly, the inventory holding cost is assumed to be proportional to the investment in inventory at any point in time. The dimensions of the constant of proportionality are dollars per unit time per dollar of inventory investments. The inventory carrying cost of one unit is the product of the value of one unit and the constant of proportionality, giving the carrying cost the dimensions of dollars per unit per year. Based on this assumption, the annual carrying cost is the product of the average number of units on hand and the cost per unit per year of carrying one item. The assumption that the carrying cost is proportional to the investment implies that warehouse rental costs, handling costs, and investment costs are all proportional to the value of the inventory. P. T. Lele and E. A. Siecienski[22] point out that this is not necessarily true. The components of carrying cost are;

(1) The cost of the investigated capital and the overhead cost,

(2) The cost associated with the storage space,

(3) The handling cost. (See Figure 1.)

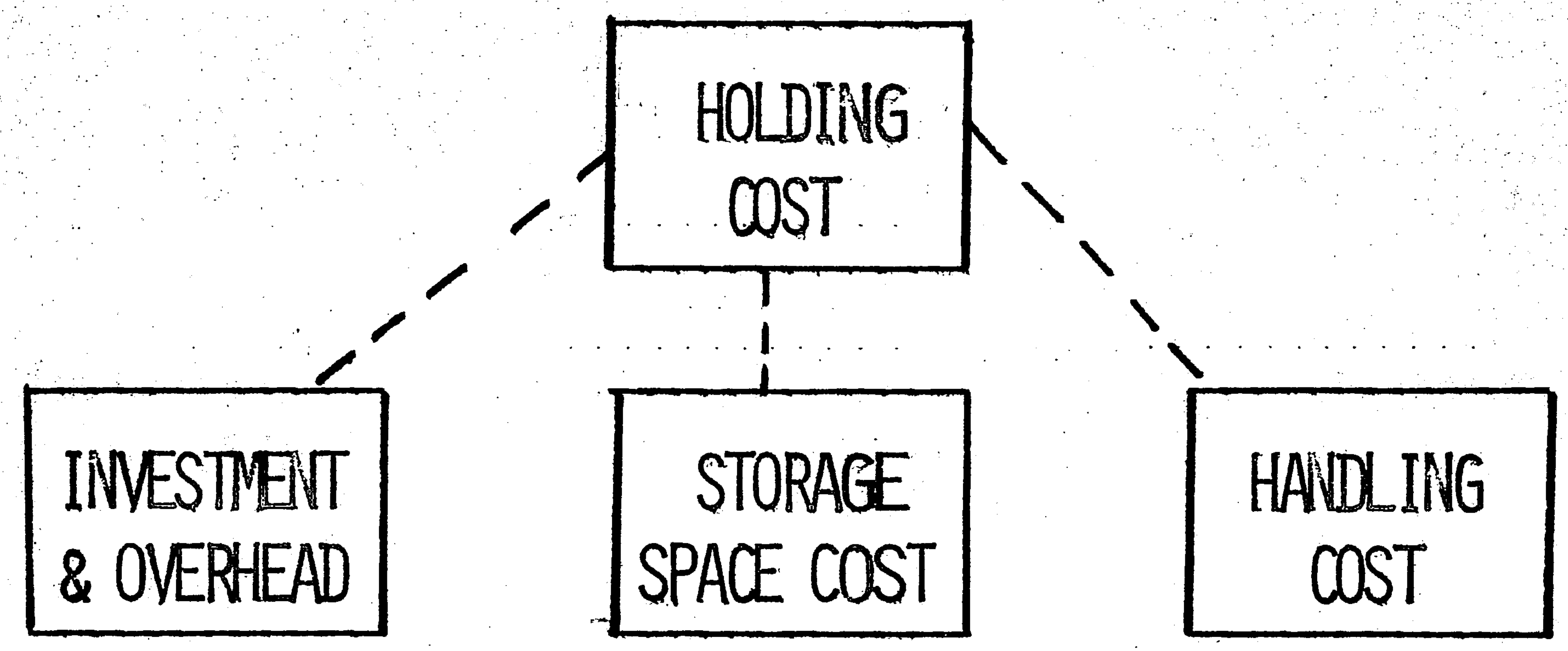

FIGURE 1.

COMPONENTS OF THE INVENTORY HOLDING COST

Of these three components, the handling cost and the costs of storage space are not proportional to the inventory investment. These costs are associated with the mode of storage and handling and not with the investment in inventory. Based on this decomposition of the handling cost, Lele and Siecienski have extended the work of Fabrycky and Banks to develop a model which deals separately with the various components of the carrying cost. The model not only provides an optimal decision rule for a given warehouse, but also determines the optimal warehouse size by varying the space allocations and total warehouse capacity until the minimum total cost configuration is found. This extension of the original work by Banks and Fabrycky provides the optimal inventory and procurement policy for a deterministic system. It is the objective of this thesis to extend the model developed by Lele and Siecienski to take into account stochastic demand and lead time.

THE PROBLEM

Consider a firm which can order certain items from a variety of suppliers and operates in the following manner. The standard procedure for the firm is to ask each of the suppliers to submit quotations on each of the items in question. The quotations contain the following supplier-related information:

(1) Unit Price,

(2) Minimum Order Quantity,

(3) Setup Cost for Each Lot,

(4) Lead Time Information.

Based on this information the Purchasing Organization must select a vendor for each item. In making this decision, Purchasing may consider the suppliers quality and quantity rating, (that is, does he ship the number ordered and are they of acceptable quality?) but the most important factor considered will be the unit cost. Purchasing will attempt to minimize the unit cost for each item. This usually means selecting the suppliers with the largest minimum order quantities since they offer the lowest unit cost. Once the vendor selection has been made the Production Control Organization has the responsibility of specifying the order lot size and the order frequency so as to minimize storeroom investment and still maintain a high level of service to the Using Organization - the production line. Many models are available to aid Production Control in making this decision. In fact, practically all inventory control models in existence have been designed from the view of the Production Control Organization. Once the lot size and frequency have been set the Warehousing Organization is asked to provide warehouse space. This in turn determines the size of the warehouse. If, however, there is a finite amount of warehouse space the decisions made by Production Control are affected and in turn Purchasing's decisions are affected. The classical decision process is:

(1) Vendor Selection,

(2) Lotsize and frequency of ordering determination,

(3) Space Allocation,

(4) Warehouse Capacity.

This is somewhat of a dilemma since the last decision affects earlier decisions. Since the models the Production Control Organization depends on assume that the cost of warehouse space and the cost of handling are proportional to the inventory investment, the models do not allow the components of the carrying cost to be accurately accounted for in the total system. The procedure developed by Lele and Siecienski[22] provides methods of examining the entire situation and of accounting for the various components of the holding cost. In this procedure a warehouse capacity is selected and a certain amount of space allocated to each item in the system. For each item the optimal vendor selection and ordering policy is determined. By varying the warehouse capacity, the optimal warehouse size, and the corresponding optimal order, policy and vendor are determined. The technique of dynamic programming is employed to perform the calculations. Figure 2 illustrates the procedure used by Lele and Siecienski.

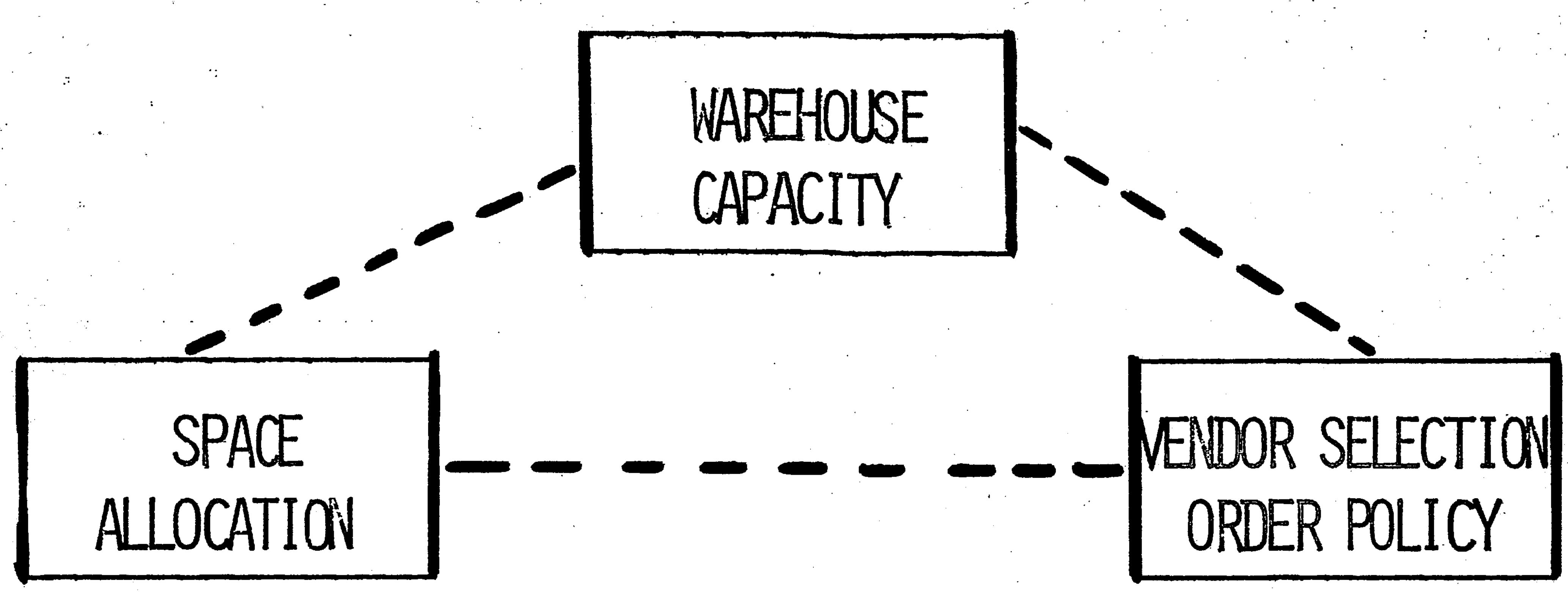

FIGURE 2.

LELE'S AND SIECIENSKI'S PROCEDURE

RELEVANT VARIABLES AND PARAMETERS

The system contains N items which are competing for the warehouse capacity W. Each item, i, has an annual demand with a mean of D_i. The distribution of demand is assumed to be known. A list of the expected demands are compiled and submitted to the various suppliers with a request for quotations. A supplier j(i), returns the quotas which contain the following data:

(1) Price, $c_{ij(i)}$, per unit of item i.

(2) Setup cost, $s_{ij(i)}$, to be charged for each order of item i.

(3) The minimum order quantity, $m_{ij(i)}$, which is the lower bound on the actual order quantity, $q_{ij(i)}$.

(4) The lead time for item i. The supplier will state, to the best of his knowledge the percent of time the item will be delievered in X days, X + 1 days, etc. (where X will depend on the situation).

All but one of the $q_{ij(i)}$ must be zero since order splitting will not be allowed.

Each time an order for an item is released to supplier j(i) for the order quantity $q_{ij(i)}$, a fixed charge, f_i, is incurred by the firm. This fixed charge is in addition to the setup charge, $s_{ij(i)}$, for item i, charged by supplier j(i). The fixed charge, f_i, can be thought of as the firm's cost of generating and placing an order. The fixed charge may be the same for all items.

The capital invested in inventory cannot be invested elsewhere, therefore, there is a cost of $\rho \cdot c_{ij(i)}$ for each unit of item i purchased from supplier j(i) and stored in the warehouse. The variable ρ denotes the current rate of interest which could be saved if the capital were not invested in inventory or the interest which could be drawn if the capital were invested elsewhere. The cost $\rho c_{ij(i)}$ is incurred for every unit of item i purchased from supplier j(i) and stored, including safety stock.

Since the system is stochastic in nature it is to be expected that some of the time there will be no stock on hand to meet the demand. One could assign a stockout cost which would represent the cost to the firm for each unit of item i not available when required. The stockout cost presents an interesting problem. Unlike the other cost parameters in the system, there is no accepted means of determining the cost of a stockout. It is quite difficult to place a price on such intangibles as loss of goodwill with a customer because his order was not met, or the cost of delaying an assembly line process because of a small but necessary item that was not available when needed. To avoid the specifications of a stockout cost, a probability of stockout, β_i, will be defined for item i. That is, management will agree that they are willing to be out of item i, $\beta_i \cdot 100$ percent of the time per cycle. It will be shown that by specifying the probability of a stockout, the stockout cost is implied. For example, if an item can never be short,the cost of a stockout must be very large (in fact infinite) and if an item never has to be on the shelf the cost of not having that item must be zero. Often the items in a system may be grouped by Pareto's law and β_A, β_B, β_C specified for the A, B, and C groups respectively. This allows the safety stocks to be controlled on a dollar value basis.

In addition to an out-of-stock condition there may exist the over-stock condition. This condition exists when the order quantity, $q_{ij(i)}$, arrives and there is not space available in the warehouse to store it. In this event it is necessary to move the extra items to a temporary storage area or perhaps store the items in the aisle. In either case there is the additional cost, e_i, for each unit of item i which is in excess of the storage area's capacity.

In modern storage practices items are stored in unit loads or pallets, therefore the assumption that the items are stored in pallets will be made. The variable p_i will denote the number of units of item i which can occupy a single pallet. The total warehouse space, W, will be expressed in pallet positions. Since a pallet which contains 1/2 p_i units of item i requires the same amount of storage space as a pallet which contains p_i units of item i, it is necessary to account for this fact. Let $[X]^+$ denote the smallest integer greater than X. The space requirement for $q_{ij(i)}$ units of item i is then $\left[\frac{q_{ij(i)}}{p_i}\right]^+$. The model developed by Lele and Siecienski assumes that the safety stock for each item is held in a separate storage area and there is a separate storage cost for carrying safety stock. In addition, they assume that the safety stock carried for item i is a fixed amount, based on the yearly demand for item i. Although these assumptions are realistic, the model developed in this thesis will investigate the case where the safety stock is competing for warehouse space and the amount of safety stock carried will be based on the predetermined probability of a stockout as given by the value of β_i for item i.

The sum of the maximum expected space requirements for the individual items is an upper limit on the total warehouse space. The actual warehouse capacity needs to equal this maximum only if (1) all orders for the N items arrive at the same time, or (2) the space is dedicated so that the space vacated by one item cannot be used by another. Both cases are unlikely in the real world. By the phasing of orders and a dynamic storage policy the actual warehouse capacity can be much less than the upper limit. To account for the order phasing and dynamic storage policy, a space utilization factor will be included in the model. The use of the space factor is presented by Lele and Siecienski[22] and by Siecienski[26]. The space utilization factor may be visualized by considering two items, item #1 and item #2, with order quantities q_1 and q_2 and safety stocks b_1 and b_2. (See Figure 3.)

The maximum total inventory is $(b_1 + b_2) + (q_1 + q_2)$. The average total inventory is $(b_1 + b_2) + (q_1 + q_2)/2$. The total inventory, since there is phasing of orders, must lie between $\left\{(b_1 + b_2) + (q_1 + q_2)\right\}$ and $\left\{(b_1 + b_2) + (q_1 + q_2)/2\right\}$. The total inventory in the system can then be written as $(b_1 + b_2) + (q_1 + q_2)/\alpha$ where $1 \leq \alpha \leq 2$. α is called the space utilization factor. Sicienski[26] investigates the value of the space utilization factor for various operating conditions through the use of a GPSS simulation program.

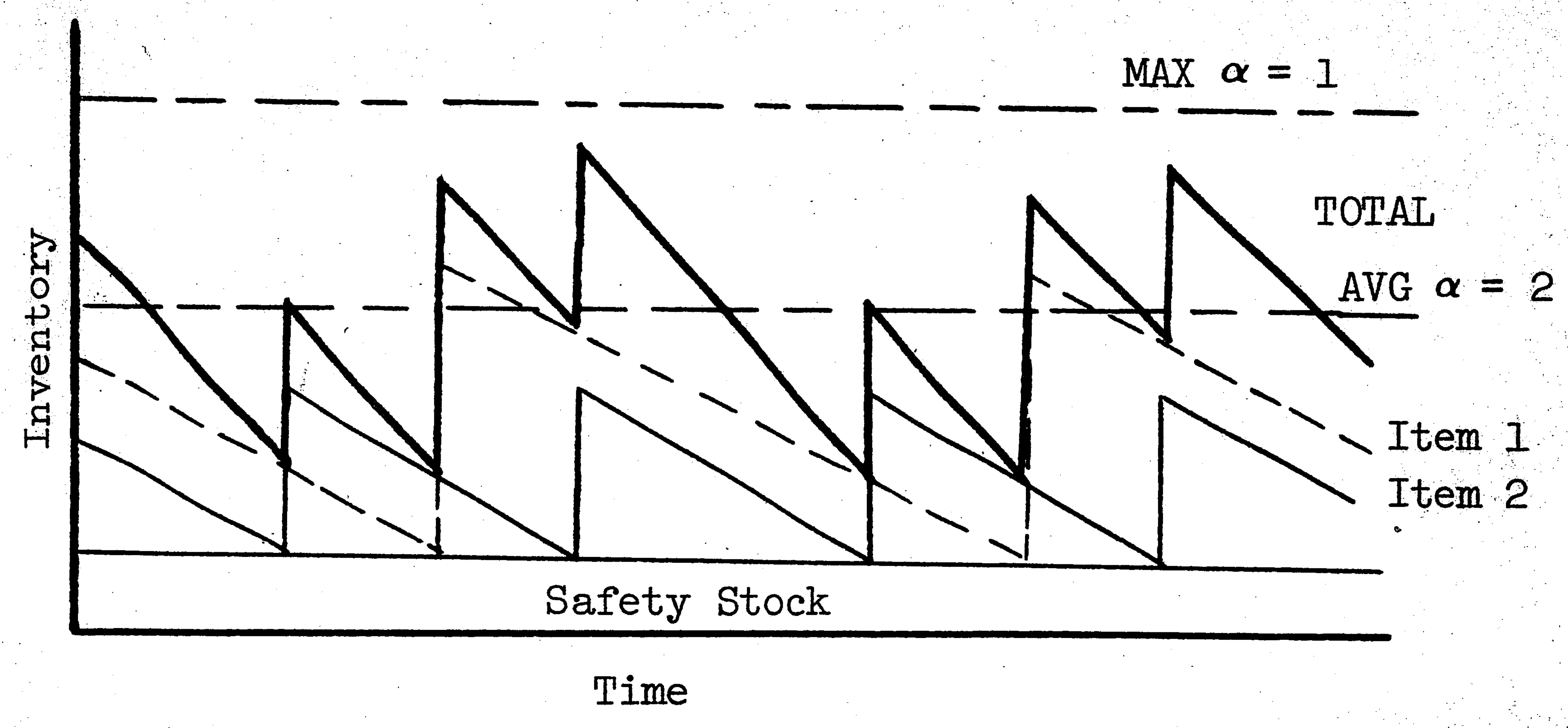

FIGURE 3.

INVENTORY SYSTEM WITH TWO ITEMS

Two additional cost parameters resulting from the decomposition of the holding cost are the cost of warehouse space, R(W), and the handling cost H(W). These two costs are functions of the total warehouse size as indicated by their notation.

Having defined the particular problem that is being investigated and the pertinent cost parameters, an analytical approach is given in the following chapter. The distribution of demand during lead time is first developed for the system independent of any particular supplier. Once this distribution is available the dynamic programming formulation which takes account of the various suppliers, the warehouse restraints, and the decomposition of the holding costs is developed.

CHAPTER III

(q, r) MODEL

In the inventory system described in Chapter II, there are three variables which the decision maker controls. In general it is desirable to choose the value of these control variables in such a way so as to minimize the total cost of operating the inventory and procurement system, and at the same time, maintain a specific level of service to the using organization. In the problem described, the decision maker may absorb the fluctuations in demand by varying:

(1) The amount ordered,

(2) The frequency with which orders are placed.

(q, r) MODEL

A solution to the inventory and procurement problem stated in Chapter II is a solution in which the policy of the decision maker is to vary the frequency with which orders are placed. In this policy, referred to as a (q, r) policy, the control variables are the order quantity, $q_{ij(i)}$, for item i and the frequency with which orders are placed for item i. For the time being the various suppliers and the warehouse restrictions will be ignored, however, these factors will be included in the final solution. The order frequency can be specified by stating a reorder print,

$r_{ij(i)}$, for item i when the order is placed with supplier $j(i)$. If the supplier has been determined to be $j(i)$ and the quantity on-hand is less than $r_{ij(i)}$, then an order for quantity $q_{ij(i)}$, is initiated. To describe the inventory policy for this system, it is necessary to

(1) Determine the supplier $j(i)$ for each item,

(2) Determine $q_{ij(i)}$,

(3) Determine $r_{ij(i)}$,

so that

(1) The operating cost is minimized,

(2) The warehouse restrictions are not violated,

(3) The minimum order quantity restrictions are not violated,

(4) The desired service level is maintained.

DETERMINATION OF DEMAND DURING LEAD TIME[30]

The value of $r_{ij(i)}$ represents the inventory of item i available to the user until the order quantity $q_{ij(i)}$ is placed in inventory. If the user requires more than $r_{ij(i)}$ units during the replenishment time (lead time), an out-of-stock condition will occur. Since the out-of-stock condition is to occur not more than $\beta_i \cdot 100$ percent of the time per cycle, $r_{ij(i)}$ must be large enough to insure this criteria. The usage during lead time, z_i, for item i consists of two components: (1) the usage per day, and (2) the number of days in the lead time period. If both of these components are known with certainty, the usage during lead time is the product of the usage per day and the number of lead time days. The reorder point, $r_{ij(i)}$, can then be set equal to this product and the result is zero inventory on-hand just prior to the new order's arrival. (See Figure 4.)

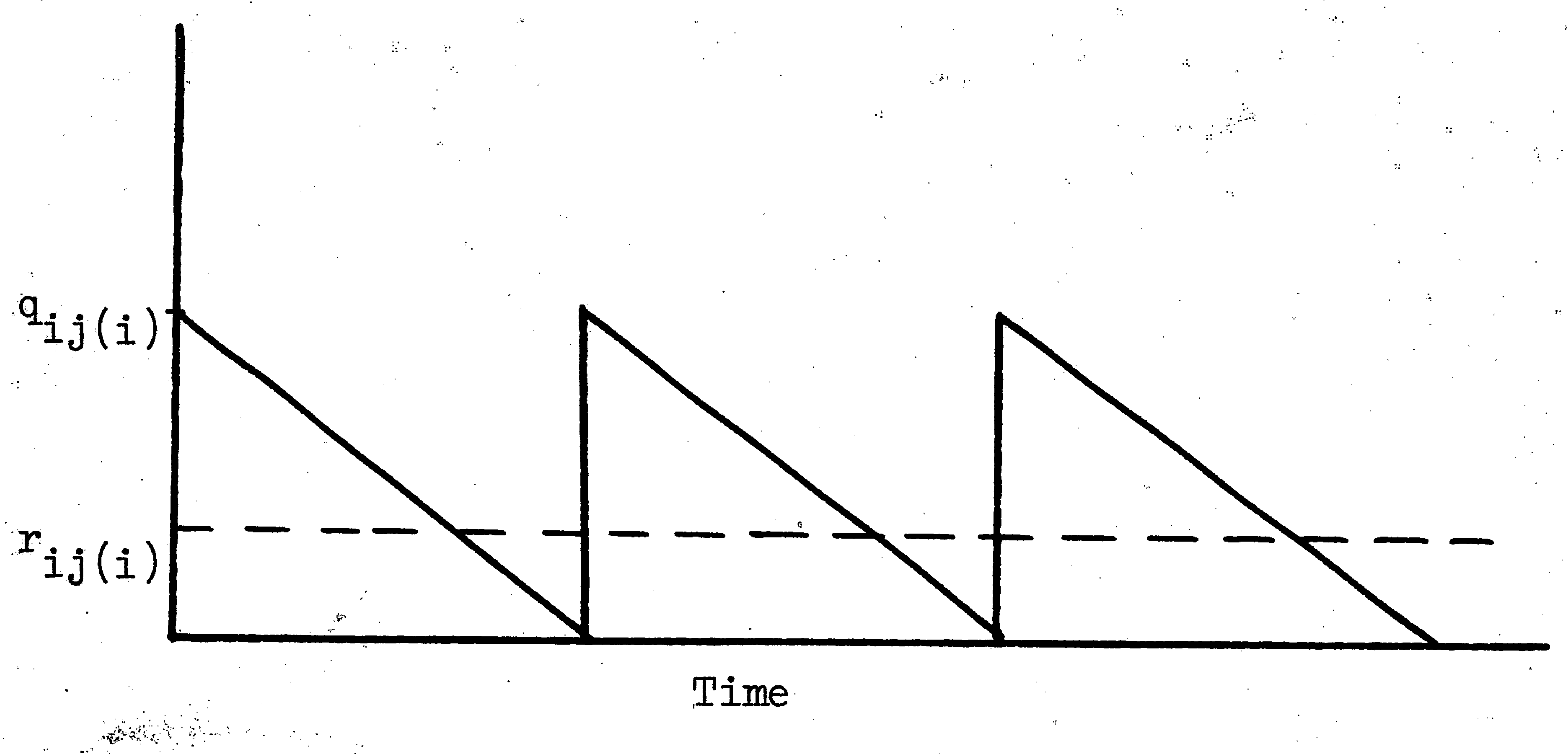

FIGURE 4.

SYSTEM WITH KNOWN DEMAND AND LEAD TIME

In our problem the magnitude of the demand is not known with certainty and the number of days for replenishing the inventory is a random variable. In order to provide a solution to the problem it is necessary that the probability distribution of the usage during lead time be found. Once this distribution has been established for item i and supplier j(i), it will be possible to determine the value of the reorder point, $r_{ij(i)}$, such that $Pr(z_i > r_{ij(i)}) \leq \beta_i$.

To avoid confusing notation the subscripts i and j(i) will be dropped in the following arguments, however, in the total system, all items and suppliers must be considered. The demand per day for any item may be described as a random variable, D, with $Pr(D = d) = f(d)$. When the inventory on-hand is depleted to a level r, an order is placed to replenish the inventory. The time required for the replenishment order to be placed in inventory may be described as a random variable, T, with $Pr(T = t) = g(t)$. During the T days, the total demand (usage during lead time) made upon the inventory is a random variable $z = D_1 + D_2 + \ldots + D_T$. In order to calculate the reorder point, r, such that $Pr(z > r) \leq \beta$, it is necessary to determine the distribution of z. In general this distribution can be found through the use of probability generating functions. The probability generating function (g.f.) of a probability density function (p.d.f.), f(d), is defined as

$$F_d(S) = f(0) + f(1)S + f(2)S^2 + \ldots$$

or

$$F_d(S) = \sum_{i=0}^{\infty} f(i)S^i,$$

and the g.f. of the p.d.f. g(t) is defined as

$$F_t(S) = g(0) + g(1)S + g(2)S^2 + \ldots$$

or

$$F_t(S) = \sum_{i=0}^{\infty} g(i)S^i.$$

The expected value and the variance of D are given by

$$E(D) = \left.\frac{dF_d(S)}{dS}\right|_{S=1}$$

and

$$V(D) = \left.\frac{dF_d(S)^2}{d^2S}\right|_{S=1} + E(D) - (E(D))^2$$

respectively.

Let w be the sum of independent random variables, x and y which have p.d.f.'s f(x) and g(y) respectively. The g.f. of w is given by

$$F_w(S) = F_x(S)\cdot F_y(S),$$

where $F_x(S)$ and $F_y(S)$ are the generating functions of f(x) and g(y) respectively. Since z is the sum of T independent identically distributed demands, D, each having f(d) as their density functions; for each value of T(T=t; t=0, t=1, ... t=∞)

$$F_z(S \mid t) = F_z(S \text{ given } t) = (F_d(S))^t.$$

Furthermore, the g.f. of z, $F_z(S)$ is given by:

$$F_z(S) = \sum_{z=0}^{\infty} S^z h(z) \quad \text{where } h(z) = Pr(Z=z) \text{ and}$$

$$F_z(S) = \sum_{z=0}^{\infty} S^z h(z) = \sum_{t=0}^{\infty} \sum_{z=0}^{\infty} S^z h(z \mid t) g(t)$$

or

$$F_z(S) = \sum_{t=0}^{\infty} g(t) \sum_{z=0}^{\infty} S^z h(z \mid t)$$

$$= \sum_{t=0}^{\infty} [F_d(S)]^t g(t)$$

$$= F_t[F_d(S)].$$

The expected value of Z and the variance of Z are found to be

$$E(Z) = E(D) \cdot E(T)$$

$$V(Z) = E(T)V(D) + (E(D))^2 V(T).$$

Therefore, for any given $f(d)$ and $g(t)$, $h(z)$ is given by the coefficient on the S^z term in the function

$F_t[F_d(S)]$ where

$$F_t[F_d(S)] = \sum_{t=0}^{\infty} g(t)(F_d(S))^t = \sum_{i=0}^{\infty} S^i h(i).$$

For example if $f(d)$ for $d = 1,2$ and 3 is defined as $f(1) = .3$, $f(2) = .3$ and $f(3) = .4$ and $g(t)$ for $t = 1$ and 2 is defined as $g(1) = .5$ and $g(2) = .5$ then

$$F_z(S) = .5(.3S + .3S^2 + .4S^3) + .5\,(.3S + .3S^2 + .4S^3)^2$$

The expected value of z, $E(z)$ is given by

$$E(z) = E(D) \cdot E(T)$$

where

$$E(D) = \left.\frac{dF_d(S)}{dS}\right|_{S=1} = \left.\frac{d(.3S + .3S^2 + .4S^3)}{dS}\right|_{S=1} = 2.1,$$

and

$$E(T) = \left.\frac{dF_t(S)}{dS}\right|_{S=1} = \left.\frac{d(.5S + .5S^2)}{dS}\right|_{S=1} = 1.5$$

therefore

$$E(z) = (2.1)\cdot(1.5) = 3.15.$$

The variance of z, V(z) is given by

$$V(z) = E(T)\ V(D) + (E(D))^2\ V(T)$$

where

$$E(T) = 1.5,$$

$$E(D) = 2.1,$$

$$V(D) = \left.\frac{d(.3S + .3S^2 + 4S^3)^2}{dS^2}\right|_{S=1} = 3.0$$

and

$$V(T) = \left.\frac{d(.5S + .5S^2)^2}{dS^2}\right|_{S=1} = 1.0$$

therefore

$$V(z) = (1.5)(3.0) + (2.1)^2(1.0)$$

$$= 8.91.$$

In summary, the distribution of z, ($z = D_1 + D_2 \ldots D_T$, where D and T are random variables), can be found through the use of probability generating functions. Once this distribution, h(z), has been determined, r can be determined so that $Pr(z \geq r) \leq \beta$.

SOLUTION PROCEDURE FOR THE TOTAL SYSTEM

As previously stated the technique of dynamic programming will be employed to provide a solution to the stated problem. The following symbols and definitions will be used throughout the remainder of this thesis. Although some of the variables have been referred to, and some will be defined in the text, all are included in this list for easy reference.

NOTATIONS

Symbol	Definition
β_i	Maximum stockout probability for item i.
$c_{ij(i)}$	Per unit cost of item i when purchased from supplier j(i).
D_i	Annual expected demand for item i.
e_i	Overstock cost per unit of item i.
f_i	Fixed order charge incurred by the firm for each order placed with a supplier for item i.
$f_n(w)$	The minimum expected cost of a system n items and total space w available when the space is allocated in an optimal manner.
$h(z)=Pr(Z=z)$	Probability of Z units being demanding during a lead time period.
$H(X)$	Annual handling cost for X pallets.
i	Item number.
j(i)	Supplier number for item i.
$m_{ij(i)}$	Minimum order quantity for item i when ordered from supplier j(i).
N	Number of items in the system
P_l	Pallet positions available to allocate to the l items.

Symbol	Definition
p_i	Quantity of item i required to fill a pallet.
$q_{ij(i)}$	Order quantity for item i from supplier j(i).
$(r_{ij(i)} - \bar{z}_i)$	Safety stock quantity.
$r_{ij(i)}$	Reorder level for item i when supplier j(i) is used.
$R_{ss}(w)$	Annual rental for safety stock space of size w.
$s_{ij(i)}$	Setup cost charged by supplier j(i) for each order of item i processed.
$TC(q_{ij(i)})$	Expected annual cost of procurement for item i given supplier j(i) and order quantity $q_{ij(i)}$.
u_i	Space allocated to item i at each state of the dynamic programming model.
V(W)	Total expected cost of the system given a warehouse of size W.
W	Total warehouse space.
W_i	Warehouse space occupied by item i.
$q_i(u_i)$	Minimum expected cost procurement policy given space u_i is available for item i.
z_i	Usage during lead time (A Random Variable).
$\bar{z}_i$	Expected value of z_i.

Using the preceding symbols and definitions the dynamic programming model can be described in the following general form. In a system with a finite amount of warehouse space, w, (expressed in pallet positions) where w_i represents the space occupied by item i, the total amount of space required by the N items is $\sum_{i=1}^{N} w_i$. The cost of the warehouse rental is then $R(\sum_{i=1}^{N} w_i)$. The cost of handling the annual expected demand for each item may be found by considering the total expected number of pallets handled for each item. The expected number of pallets per year for item i is

$$\left\{ \left[\frac{q_{ij(i)}}{p_i} \right]^{+} \cdot \frac{D_i}{q_{ij(i)}} \right\}$$

The annual expected cost for handling N items is then given by

$$H \left[\sum_{i=1}^{N} \left[\left[\frac{q_{ij(i)}}{p_i} \right]^{+} \cdot \left[\frac{D_i}{q_{ij(i)}} \right] \right]_{j(i)} \right] ,$$

where the extra subscript j(i) denotes the vendor selected. Let $TC(q_{ij(i)})$ be the minimum annual expected cost of ordering lots of size $q_{ij(i)}$ from supplier j(i) for item i. The total expected procurement costs for the N item is then given by

$$\sum_{i=1}^{N} \left[TC(q_{ij(i)}) \right]_{j(i)} .$$

The total expected cost of the system, V(W), is the sum of the rental cost, the expected handling costs, and the expected procurement cost. V(W) may be written as

$$V(W) = H\left[\sum_{i=1}^{N}\left[\left[\frac{q_{ij(i)}}{p_i}\right]^{+}\cdot\left[\frac{D_i}{q_{ij(i)}}\right]\right]_{j(i)}\right]$$

$$+\quad R\left[\sum_{i=1}^{N} W_i\right]$$

$$+\sum_{i=1}^{N}\left[TC(q_{ij(i)})\right]_{j(i)} .$$

It should be noted that TC is a function of the order quantity, $q_{ij(i)}$, and that $q_{ij(i)}$ must be greater than or equal to the minimum order quantity, $m_{ij(i)}$, required by supplier j(i).

Let u_i be the space to be allocated to item i and let the minimum expected cost procurement policy, which is a function of $q_{ij(i)}$ and hence u_i (since $q_{ij(i)} \leq u_i\ p_i$), be defined as

$$y_i(u_i) = \underset{j(i)q_{ij(i)}}{\text{Min.}} \left\{(TC(q_{ij(i)})\right\}$$

where $\qquad q_{ij(i)} \geq m_{ij(i)}$

and $\qquad \underline{q_{ij(i)}} \leq u_i\ p_i$ or $w_i \leq u_i$

$$\text{since } w_i = \frac{q_{ij(i)}}{p_i} .$$

Suppose P_1 pallet positions are available to allocate to Item 1 and that $u_1 \leq P_1$, are allocated to item 1. Associated with u_1 is an optimal procurement policy resulting in the minimum expected cost $y_1(u_1)$. The number of pallets remaining to be allocated to the remaining N-1 items is $P_2 = P_1 - u_1$. This procedure is represented in Figure 5 by a box that is characterized by five factors

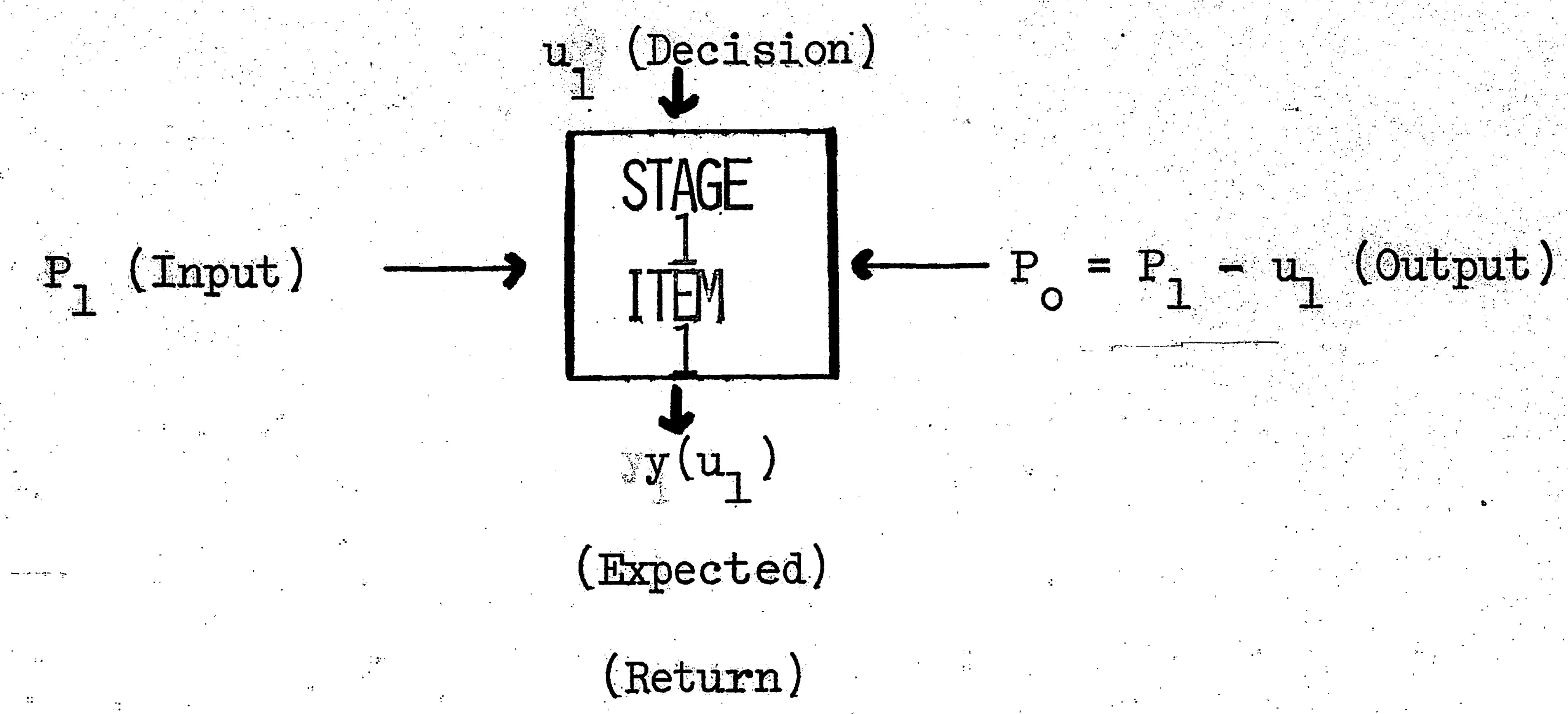

FIGURE 5.

SINGLE STAGE SYSTEM

This procedure can be continued until an optimal policy is known for the first k-1 items $(0 < k \leq N)$, as illustrated by Figure 6.

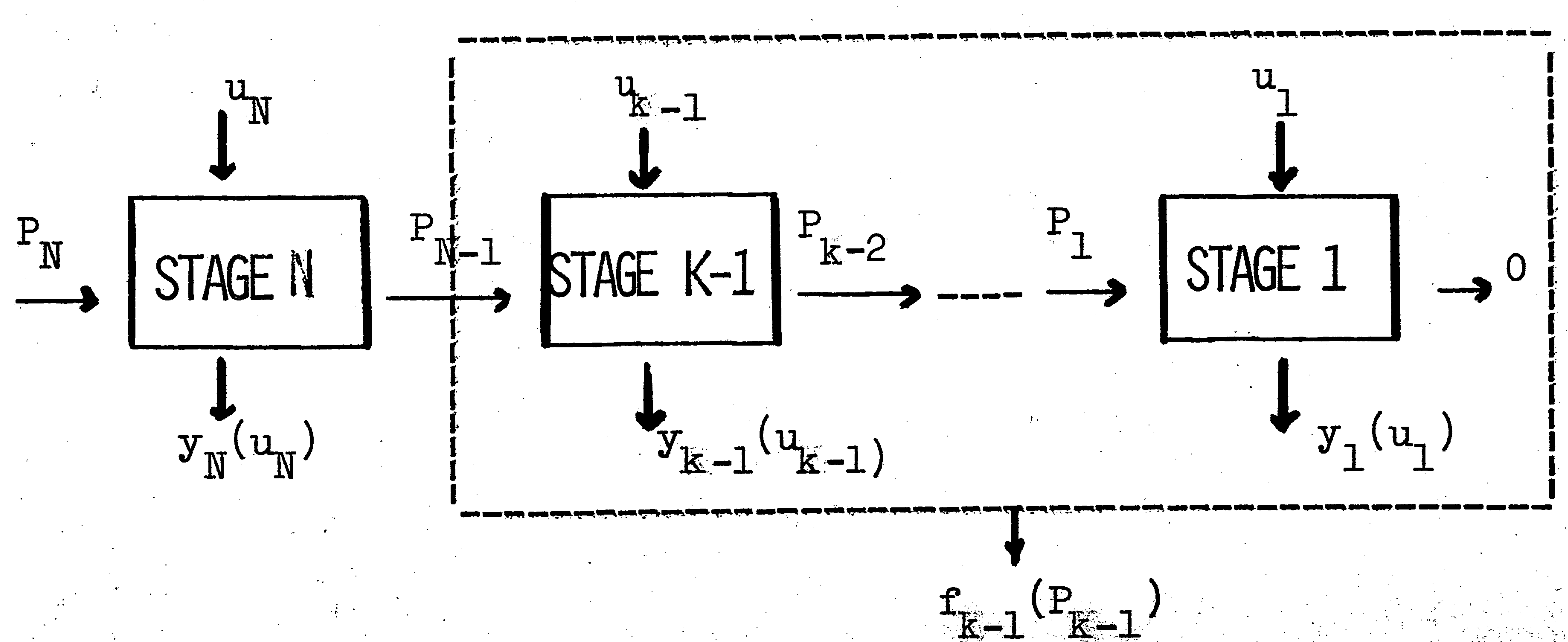

FIGURE 6.

MULTISTAGE SYSTEM

Suppose that the optimal policy for the first k-1 items has been calculated for every feasible warehouse size. Associated with these optimal policies are the minimum cost functions $f_{k-1}(P_{k-1})$. Thus the optimal amount of warehouse space to allocate to item k, given P_k is available for the k items, u_k, is the value of u_k which minimizes the expected cost function

$$\left\{y_k(u_k) + f_{k-1}(P_k - u_k)\right\} \text{ or } f_k(P_k) = \min_{0 \leq u_k \leq P_k} \left\{ y_k(u_k) + f_{k-1}(P_k - u_k)\right\}$$

where $f_o(P_o) = 0$ for all values of P_o. This is the functional equation of the dynamic programming model. Once $f_N(P_N)$ has been determined for all feasible warehouse sizes, P_N, the value of P_N, P_N^*, which minimizes the function

$$V(P_N) = H\left[\sum_{i=1}^{N}\left[\left[\frac{q_{ij(i)}}{p_i}\right]^{+}\left[\frac{D_i}{q_{ij(i)}}\right]\right]_{j(i)}\right]$$

$$+ R\left[\sum_{i=1}^{N} W_i\right]$$

$$+ \sum_{i=1}^{N}\left[TC(q_{ij(i)})_{j(i)}\right]$$

can be found.

This allows the optimal warehouse size P_N^* to be found.

Twice the phrase "all feasible warehouse sizes" has been used. It would seem well to define what will be considered a feasible warehouse size. The following assumptions will outline the range of possible warehouse sizes:

(1) Space is available to store the required safety stock.

(2) Space is available to store a lot equal to the minimum of the minimum quantities $m_{ij(i)}$.

(3) Not more than a year's expected demand of any item will be stored.

In view of these assumptions the lower bound on warehouse space for the system is

$$P_{Lower} = \sum_{i=1}^{N} \left[\frac{(r_{ij(i)} - \bar{z}_i)}{p_i} + \underset{j(i)}{\text{Min.}} \left(\frac{(m_{ij(i)})}{p_i} \right) \right]$$

and the upper

$$P_{Upper} = \sum_{i=1}^{N} \left[\frac{(r_{ij(i)} - \bar{z}_i)}{p_i} + D_i \right]$$

where $(r_{ij(i)} - \bar{z}_i)$ is the safety stock quantity.

In this general development of the dynamic programming algorithm several of the functions were assumed available for our use. If, however, the function $TC(q_{ij(i)})$, and the relationship between the amount of space occupied by item i, w_i, and the order quantities $q_{ij(i)}$ can be defined the model is complete. Therefore, the next logical step is to determine these relationships, but first a further and more detailed look at the warehouse itself is in order.

In a warehouse the storage space may be divided into two categories, dynamic storage area and static or dedicated storage area. If, for example, 10 pallet positions were reserved for item k and this space could not be used by any other item in the system, even though the inventory level of item k was zero, the space is said to be dedicated. If any available space can be used to store any item, then the space is said to be dynamic. In general the storage space is dynamic in nature, thus providing better space utilization. It is, however, not unreasonable to dedicate space in which to store the safety stock. In the following development of the function $TC(q_{ij(i)})$ two cases will be considered.

The two cases are based on the following assumptions:

Case 1: Space for safety stock is dedicated;

Case 2: Space for safety stock is dynamic.

Since the relationships are very similar the two arguments will be presented in parallel.

<u>SPACE OCCUPIED BY ITEM i.</u>

If the safety stock occupies dedicated space, the space (in pallet positions) occupied by item i is between zero and

$$\left[\frac{(r_{ij(i)} - \bar{z}_i)}{p_i} + \frac{q_{ij(i)}}{p_i}\right]$$

Since the safety stock is stored in dedicated space the dynamic space occupied by item i is between zero and $\frac{q_{ij(i)}}{p_i}$. Through the use of the previously discussed space utilization factor, the space occupied by item i, w_i, is given by

$$w_i = \frac{q_{ij(i)}}{\alpha p_i}$$

where α is the previously discussed space factor. If the safety stock is stored in dynamic space then

$$w_i = \left[\frac{q_{ij(i)}}{\alpha p_i} + \frac{(r_{ij(i)} - \bar{z}_i)}{p_i}\right]$$

The space required for the safety stock is not adjusted by the space factor α since the safety stock is expected to be part of the inventory for all items at all times. Based on these assumptions the rental cost for item i becomes

$$R(w_i) = R\left[\frac{q_{ij(i)}}{\alpha p_i}\right] + R_{ss}\left[\frac{(r_{ij(i)} - \bar{z}_i)}{p_i}\right]$$

if the safety stock is stored in dedicated space or

$$R(w_i) = R\left[\frac{q_{ij(i)}}{\alpha p_i} + \frac{(r_{ij(i)} - \bar{z}_i)}{p_i}\right]$$

if the safety stock is stored in the dynamic space. $R_{ss}(w)$ is the rent on the dedicated space of size w, since the cost of dedicated space may be different from the rent on dynamic space. The total rental cost is then given by

$$R(W) = \sum_{i=1}^{N} R(w_i)$$ where $R(w_i)$ is dependent on where the

safety stock is stored. Recalling that u_i space is available for item i, the constraint $w_i \leq u_i$ must not be violated. Using the values of w_i just developed the constraint becomes

$$\left[\frac{q_{ij(i)}}{\alpha p_i}\right]_{j(i)} \leq u_i \quad \text{or} \quad \left[\frac{q_{ij(i)}}{\alpha p_i} + \frac{(r_{ij(i)} - \bar{z}_i)}{p_i}\right]_{j(i)} \leq u_i$$

<u>$TC(q_{ij(i)})$</u>

The remaining function to be identified is $TC(q_{ij(i)})$. This function is also dependent on whether the space allocated for the safety stock is dynamic or dedicated. If the safety stock is stored in dedicated space then

$$TC(q_{ij(i)}) = c_{i(j)} D_i + \frac{D_i}{q_{ij(i)}} (f_i + s_{ij(i)})$$

$$+ \left[\frac{q_{ij(i)}}{2} + (r_{ij(i)} - \bar{z}_i)\right] \cdot \rho c_{ij(i)}$$

$$+ \frac{e_i D_i}{q_{ij(i)}} \sum_{x=0}^{(z_i + q_{ij(i)} - u_i p_i - 1)} (xh(x))$$

$$+ \frac{\Pi_i D_i}{q_{ij(i)}} \sum_{x=r_{ij(i)}+1}^{\infty} (x - r_{ij(i)})\, h(x)$$

As discussed briefly in Chapter II, the value of the variable Π_i is determined once a level of service has been set. The value of Π_i can be obtained from incremental analysis as follows.

The cost of holding one more unit of safety stock per cycle is

$$\begin{cases} \dfrac{\rho c_{ij(i)} q_{ij(i)}}{D_i} & \text{if } \left[\dfrac{r_{ij(i)}-\bar{z}_i}{p_i}\right] \neq \left[\dfrac{r_{ij(i)}-\bar{z}_i}{p_i}\right]^+ \\ \dfrac{\rho c_{ij(i)} + R_{ss}(1))q_{ij(i)}}{D_i} & \text{if } \left[\dfrac{r_{ij(i)}-\bar{z}_i}{p_i}\right] = \left[\dfrac{r_{ij(i)}-\bar{z}_i}{p_i}\right]^+ \end{cases}$$

Since the storage of one additional item requires a new pallet position, the cost of the rental is incurred. The gain from storing one additional unit of safety stock per cycle is

$$\Pi_i \Pr(z_i > r_{ij(i)}) = \Pi_i \ \beta_i$$

thus

$$\Pi_i = \begin{cases} \dfrac{\rho c_{ij(i)} q_{ij(i)}}{D_i \ \beta_i} & \text{if } \left[\dfrac{r_{ij(i)}-\bar{z}_i}{p_i}\right] \neq \left[\dfrac{r_{ij(i)}-\bar{z}_i}{p_i}\right]^+ \\ \dfrac{(\rho c_{ij(i)} + R_{ss}(1))q_{ij(i)}}{D_i \ \beta_i} & \text{if } \left[\dfrac{r_{ij(i)}-\bar{z}_i}{p_i}\right] = \left[\dfrac{r_{ij(i)}-\bar{z}_i}{p_i}\right]^+ \end{cases}$$

and $TC(q_{ij(i)})$ becomes

$$TC(q_{ij(i)}) = c_{ij(i)} D_i + \frac{D_i}{q_{ij(i)}} (f_i + s_i)$$
$$+ \rho c_{ij(i)} \left[\frac{(q_{ij(i)})}{2} + (r_{ij(i)} - \bar{z}_i) \right]$$
$$+ \frac{e_i D_i}{q_{ij(i)}} \sum_{x=0}^{\bar{z}_i + q_{ij(i)} - u_i p_i - 1)} xh(x)$$
$$+ \frac{c_{ij(i)}}{\beta_i} \sum_{x=r_{ij(i)}+1}^{\infty} \left[(x - r_{ij(i)}) \ h(x) \right]$$

if $\left[\frac{r_{ij(i)}-\bar{z}_i}{p_i}\right] \neq \left[\frac{r_{ij}-\bar{z}_i}{p_i}\right]^+$

and

$$TC(q_{ij(i)}) = c_{ij(i)}D_i + \frac{D_i}{q_{ij(i)}}(f_i + s_i)$$

$$+ \rho c_{ij(i)}\left[\frac{q_{ij(i)}}{2} + (r_{ij(i)}-\bar{z}_i)\right]$$

$$+ \frac{e_iD_i}{q_{ij(i)}} \sum_{x=0}^{\bar{z}_i + q_{ij(i)} - u_ip_i - 1} xh(x)$$

$$+ \frac{\rho c_{ij(i)} + R_{ss}(1)}{\beta_i} \sum_{x=r_{ij(i)}+1}^{\infty} (x-r_{ij(i)})\, h(x)$$

if $\left[\frac{r_{ij(i)}-\bar{z}_i}{p_i}\right] = \left[\frac{r_{ij(i)}-\bar{z}_i}{p_i}\right]^+$

Neither of these expressions presents any computational difficulties.

If the safety stock is stored in the dynamic space then $TC(q_{ij(i)})$ becomes

$$TC(q_{ij(i)}) = c_{ij(i)}D_i + \frac{D_i}{q_{ij(i)}}(f_i + s_i)$$

$$+\rho c_{ij(i)}\left[\frac{q_{ij(i)}}{z} + (r_{ij(i)}-\bar{z}_i)\right]$$

$$+ \frac{e_iD_i}{q_{ij(i)}} \sum_{x=0}^{r_{ij(i)} + q_{ij(i)} - u_ip_i - 1} xh(x)$$

$$+ \frac{\Pi_iD_i}{q_{ij(i)}} \sum_{x=r_{ij(i)}+1}^{\infty}\left[(x-r_{ij(i)})\, h(x)\right]$$

Again the value of Π_i can be determined from incremental analysis as follows. The cost per cycle of holding one additional unit of safety stock is

$$\frac{\rho c_{ij(i)} q_{ij(i)}}{D_i} + \begin{cases} 0 & \text{if } q_{ij(i)} + E(z_i < r_{ij(i)}) \le u_i p_i \\ e_i \Pr(z_i \;\; r_{ij(i)}) & \text{if } q_{ij(i)} + E(z_i < r_{ij(i)}) > u_i p_i . \end{cases}$$

The gain per cycle is

$$\Pi_i \Pr(z_i > r_{ij(i)}) = \Pi_i \quad \beta_i$$

thus

$$\Pi_i = \begin{cases} \dfrac{\rho c_{ij(i)} q_{ij(i)}}{D_i \quad i} & \text{if } q_{ij(i)} + E(z_i < r_{ij(i)}) \le u_i p_i \\[2ex] \dfrac{\rho c_{ij(i)} q_{ij(i)}}{D_i \quad i} + \dfrac{e_i \Pr(z_i > r_{ij(i)})}{i} & \text{if } q_{ij(i)} + E(z_i < r_{ij(i)}) > u_i p_i . \end{cases}$$

Therefore

$$TC(q_{ij(i)}) = c_{ij(i)} D_i + \frac{D_i}{q_{ij(i)}} (f_i + s_i)$$

$$+ \rho c_{ij(i)} \left[\frac{q_{ij(i)}}{2} + (r_{ij(i)} - \bar{z}_i) \right]$$

$$+ \frac{e_i D_i}{q_{ij(i)}} \sum_{x=0}^{(r_{ij(i)} + q_{ij(i)} - u_i p_i - 1)} x h(x)$$

$$+ A \sum_{x=r_{ij(i)}+1}^{\infty} (x - r_{ij(i)}) h(x)$$

where

$$A = \begin{cases} \dfrac{\rho\, c_{ij(i)}}{\beta_i} & \text{if } q_{ij(i)} + E(z_i \leq r_{ij(i)}) \leq u_i p_i \\[2ex] \dfrac{\rho\, c_{ij(i)}}{\beta_i} + \dfrac{e_i \Pr(z_i < r_{ij(i)})\, D_i}{q_{ij(i)}\ \beta_i} & \text{if } q_{ij(i)} + E(z_i \leq r_{ij(i)}) \geq u_i p_i \end{cases}$$

In summary the total expected cost of operating the system given a warehouse of size w is given by

$$V(W) = R(W) + H(Y) + \bar{T}C$$

where R(W) is the total rental cost, H(Y) is the total handling cost and $\bar{T}C$ is the minimum cost procurement policy. Each of these terms have been identified and the problem may now be solved.

CHAPTER IV

ILLUSTRATION OF (q, r) MODEL

In order to further examine the inventory control and procurement model developed in Chapter III, a small inventory control and procurement problem will be illustrated. In addition to the assumptions and constraints described in the development of the model, the following additional constraints exist in our sample system:

1. The order quantity must be a multiple of the quantity per pallet. That is, orders are placed in even pallet loads.
2. The safety stock carried must be a multiple of the quantity per pallet.
3. The maximum feasible order quantity is the minimum of One Year's Demand or the EOQ as determined by the Wilson Formula.
4. The minimum space requirement is space enough to hold the safety stock plus the expected demand during lead time.

These constraints were not considered in the development of the model because they are further restrictions to the more general problem described in Chapter II. These four additional restrictions are, however, quite realistic and can be easily incorporated into the model.

The value of the various supplier-oriented variables for the sample system are presented in Table I. As shown, item 1 is available from three suppliers and items 2 and 3 are available from four suppliers. Appendix I presents the demand distributions, the lead time distributions, and the resulting convoluted distribution of usage during lead time for each item in the system. The technique of probability generating functions described in the first section of Chapter III was used to determine the distribution of usage during lead time for each item, given a particular supplier. For purposes of illustration the distributions were kept quite simple. This is by no means necessary.

Solutions to the inventory control and procurement problem described by Table I and Appendix I will be illustrated for the following two cases:

Case I: The safety stock is stored in dedicated space.

Case II: The safety stock is stored in dynamic space.

These two cases correspond to the two cases for which the model was developed in Chapter III.

PART CODE	DEMAND/YR.	Q/PALLET	MOQ	P/M PCS	SETUP COST	VENDER
1	3000	100	1000	104.80	12.00	1(1)
			1500	117.55	19.00	2(1)
			500	129.00	21.00	3(1)
2	1600	80	1000	62.20	45.00	1(2)
			500	95.65	23.00	2(2)
			500	82.00	25.00	3(2)
			1000	80.00	32.00	4(2)
3	1000	40	1000	167.60	40.00	1(3)
			0	166.70	23.00	2(3)
			500	164.00	25.00	3(3)
			1000	178.25	32.00	4(3)

TABLE I

SUPPLIER ORIENTED VARIABLES

*Minimum Order Quantity
**Cost per 1000 Pieces

CASE 1: SAFETY STOCK STORED IN DEDICATED SPACE

Table II presents the values of the various parameters of the model.

FOR THE FOLLOWING VARIABLE CONDITIONS:

SAFETY STOCK LEVEL = .80
RATE OF RETURN= 0.25%
SPACE FACTOR= 1.00

COST OF WAREHOUSE SPACE/PALLET =z34.75
COST OF HANDLING/PALLET = 1.00
COST OF OVERSTOCK/PALLET = 50.00

SAFETY STOCK IS STORED IN DEDICATED SPACE

TABLE II

PARAMETER VALUES - CASE I

The output of the first stage of the dynamic programming algorithm is given in Table III.

The first row of Table I indicates that five pallet positions is the minimum feasible amount of space which must be allocated to item 1. Column one of Table III does not include the space required for safety stock, therefore the minimum space required for item 1 is six pallet positions. The fact that at least five pallet positions are required is due to the restrictions on minimum order quantities imposed by the suppliers of item 1. Table I indicates that at least 500 units of item 1 must be ordered regardless of which supplier is selected. As soon as the warehouse space allocated to item 1 is enough to allow an order quantity

ITEM NUMBER 1
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
5	5	500.	3(1)	1.	1094.45	1094.45
6	6	600.	3(1)	1.	1005.06	1005.06
7	7	700.	3(1)	1.	941.67	941.67
8	8	800.	3(1)	1.	894.54	894.54
9	9	900.	3(1)	1.	858.23	858.23
10	10	1000.	1(1)	1.	726.25	726.25
11	11	1100.	1(1)	1.	705.20	705.20
12	12	1200.	1(1)	1.	687.87	687.87
13	13	1300.	1(1)	1.	673.41	673.41
14	14	1400.	1(1)	1.	661.21	661.21
15	15	1500.	1(1)	1.	650.80	650.80
16	16	1000.	1(1)	1.	576.25	576.25
17	17	1100.	1(1)	1.	568.83	568.83
18	18	1200.	1(1)	1.	562.87	562.87
19	19	1300.	1(1)	1.	558.03	558.03
20	20	1400.	1(1)	1.	554.06	554.06
21	21	1500.	1(1)	1.	550.80	550.80
22	22	1600.	1(1)	1.	548.11	548.11
23	23	1700.	1(1)	1.	545.89	545.89
24	24	1800.	1(1)	1.	544.06	544.06
25	25	1900.	1(1)	1.	542.57	542.57
26	26	2000.	1(1)	1.	541.35	541.35
27	27	2100.	1(1)	1.	540.38	540.38
28	28	2200.	1(1)	1.	539.61	539.61
29	29	2300.	1(1)	1.	539.02	539.02

TABLE III

STAGE 1 OUTPUT - CASE I

ITEM NUMBER 2
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
12	7	560.	3(2)	2.	561.75	1656.20
13	7	560.	3(2)	2.	561.75	1566.81
14	7	560.	3(2)	2.	561.75	1503.42
15	7	560.	3(2)	2.	561.75	1456.29
16	7	560.	3(2)	2.	561.75	1419.98
17	7	560.	3(2)	2.	561.75	1288.00
18	8	640.	3(2)	2.	528.64	1254.89
19	9	720.	3(2)	2.	503.07	1229.32
20	9	720.	3(2)	2.	503.07	1208.27
21	10	800.	3(2)	2.	482.78	1187.98
22	10	800.	3(2)	2.	482.78	1170.65
23	7	560.	3(2)	2.	561.75	1138.00
24	8	640.	3(2)	2.	528.64	1104.89
25	9	720.	3(2)	2.	503.07	1079.32
26	10	800.	3(2)	2.	482.78	1059.03
27	11	880.	3(2)	2.	466.33	1042.58
28	12	960.	3(2)	2.	452.76	1029.01
29	13	1040.	1(2)	1.	435.85	1012.10
30	14	560.	3(2)	2.	418.89	995.14
31	15	640.	3(2)	2.	403.64	979.89
32	16	720.	3(2)	2.	391.96	968.21
33	17	800.	3(2)	2.	382.78	959.03
34	17	800.	3(2)	2.	382.78	951.62
35	18	880.	3(2)	2.	375.42	944.25
36	19	960.	3(2)	2.	369.42	938.26
37	19	960.	3(2)	2.	369.42	932.29
38	20	1040.	3(2)	2.	364.47	927.34
39	20	1040.	3(2)	2.	364.47	922.50
40	20	1040.	3(2)	2.	364.47	918.54
41	20	1040.	3(2)	2.	364.47	915.27
42	20	1040.	3(2)	2.	364.47	912.58
43	20	1040.	3(2)	2.	364.47	910.36
44	20	1040.	3(2)	2.	364.47	908.54
45	20	1040.	3(2)	2.	364.47	907.04
46	20	1040.	3(2)	2.	364.47	905.82
47	20	1040.	3(2)	2.	364.47	904.85
48	20	1040.	3(2)	2.	364.47	904.08
49	20	1040.	3(2)	2.	364.47	903.49

TABLE IV

STAGE 2 OUTPUT - CASE I

ITEM NUMBER 3
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
14	2	80.	2(3)	1.	1482.74	3138.94
15	3	120.	2(3)	1.	1096.08	2752.28
16	4	160.	2(3)	1.	903.16	2559.36
17	5	200.	2(3)	1.	787.74	2443.94
18	6	120.	2(3)	1.	679.41	2335.61
19	6	120.	2(3)	1.	679.41	2246.22
20	7	160.	2(3)	1.	590.66	2157.47
21	7	160.	2(3)	1.	590.66	2094.08
22	8	200.	2(3)	1.	537.74	2041.17
23	6	120.	2(3)	1.	679.41	1967.41
24	7	160.	2(3)	1.	590.66	1878.66
25	8	200.	2(3)	1.	537.74	1825.74
26	9	240.	2(3)	1.	502.74	1790.75
27	9	240.	2(3)	1.	502.74	1757.64
28	9	240.	2(3)	1.	502.74	1732.07
29	10	280.	2(3)	1.	477.98	1707.31
30	10	280.	2(3)	1.	477.98	1686.25
31	10	280.	2(3)	1.	477.98	1665.96
32	9	240.	2(3)	1.	502.74	1640.75
33	9	240.	2(3)	1.	502.74	1607.64
34	9	240.	2(3)	1.	502.74	1582.07
35	10	280.	2(3)	1.	477.98	1557.31
36	10	280.	2(3)	1.	477.98	1537.01
37	11	320.	2(3)	1.	459.62	1518.65
38	11	320.	2(3)	1.	459.62	1502.20
39	12	360.	2(3)	1.	445.52	1488.10
40	11	320.	2(3)	1.	459.62	1471.72
41	11	320.	2(3)	1.	459.62	1454.76
42	11	320.	2(3)	1.	459.62	1439.51
43	12	360.	2(3)	1.	445.52	1425.41
44	12	360.	2(3)	1.	445.52	1413.73
45	13	400.	2(3)	1.	434.41	1402.62
46	13	400.	2(3)	1.	434.41	1393.44
47	14	440.	2(3)	1.	425.47	1384.50
48	14	440.	2(3)	1.	425.47	1377.09
49	14	440.	2(3)	1.	425.47	1369.73
50	15	480.	2(3)	1.	418.16	1362.42
51	16	520.	2(3)	1.	412.10	1356.36
52	16	520.	2(3)	1.	412.10	1350.36
53	16	520.	2(3)	1.	412.10	1344.40
54	17	560.	2(3)	1.	407.03	1339.32
55	17	560.	2(3)	1.	407.03	1334.37

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
56	17	560.	2(3)	1.	407.03	1329.53
57	18	600.	2(3)	1.	402.74	1325.24
58	18	600.	2(3)	1.	402.74	1321.28
59	19	640.	2(3)	1.	399.10	1317.63
60	19	640.	2(3)	1.	399.10	1314.37
61	20	680.	3(3)	1.	395.96	1311.24
62	21	720.	3(3)	1.	393.11	1308.38
63	21	720.	3(3)	1.	393.11	1305.69
64	22	760.	3(3)	1.	390.64	1303.22
65	22	760.	3(3)	1.	390.64	1301.00
66	23	800.	3(3)	1.	388.49	1298.86
67	24	840.	3(3)	1.	386.64	1297.00
68	24	840.	3(3)	1.	386.64	1295.17
69	25	880.	3(3)	1.	385.02	1293.56
70	25	880.	3(3)	1.	385.02	1292.06
71	25	880.	3(3)	1.	385.02	1290.85
72	25	880.	3(3)	1.	385.02	1289.87
73	25	880.	3(3)	1.	385.02	1289.10
74	25	880.	3(3)	1.	385.02	1288.51

TABLE V.

STAGE 3 OUTPUT - CASE I

of 1000 units, supplier 1(1) was chosen over supplier 3(1). This change in supplier choice is due to the fact that supplier 1(1) offers a lower unit cost for orders of 1000 or more units. Supplier 2(1) was never selected due to the fact that supplier 3(1) offers a lower unit cost, a lower order setup cost, and the same minimum order quantity. This is obvious from the data in Table I, however, it is encouraging to note that the model follows the obvious.

Tables IV and V present the output of the second and third stages, respectively, of the dynamic programming algorithm. The total cost column is the minimum cost for the current item plus all preceding items, whereas the cost column represents the minimum cost procurement policy for the current item only.

Once the minimum cost procurement policy has been determined for each feasible value of warehouse space, two additional costs must be calculated. These are the rental costs and the handling costs.

Since the order quantity is a multiple of the quantity per pallet, p_i, for item i, the expected handling cost, given by $\sum_{i=1}^{3} \frac{D_i}{p_i}$, is constant. The rental cost is the combined rental cost of dynamic space and safety stock space. Table VI tabulates these costs and the total expected system costs for each feasible warehouse size. If the problem was to find a solution for a given warehouse size, one would find the appropriate row and trace back through Tables V, IV and III in order to determine the procurement policy. For example, if a total of forty pallet positions are available in which to store the dynamic stock, the optimal allocation for item 3 would be eleven pallet positions, as indicated by column two of Table V, leaving a remainder of 29 pallet positions to be allocated to items 1 and 2. The order quantity for item 3 is given in column three of Table V and is found to be 320 units or eight pallets. Table IV indicates that if 29 pallet positions are available to allocate to items 2 and 1, thirteen should be allocated for item 2 and the remaining sixteen will then be allocated to item 1. The order quantities for items 2 and 1 are 1040 units and 1000 units, respectively.

In addition, three pallet positions would be required for the safety stock, one pallet position for each item as indicated by the tables. The total expected cost of this system, given 49 pallet positions of dynamic space is $3026.72.

If the optimal warehouse size is the factor sought, one need only search the last column of Table VI for a minimum and determine the corresponding procurement policy. The optimal warehouse size for this example is a warehouse of 30 pallet positions, 26 of which are dynamic and four of which are for safety stock. Table VII presents a summary of the optimal policy for the minimum expected cost.

FINAL TABULATION

TOTAL SPACE	+SS	RENTAL AND HANDLING COST	PROCUREMENT COST	TOTAL COST
14	18	681.50	3138.94	3820.44
15	19	716.25	2752.28	3468.53
16	20	751.00	2559.36	3310.36
17	21	785.75	2443.94	3229.69
18	22	820.50	2335.61	3156.11
19	23	855.25	2246.22	3101.47
20	24	890.00	2157.47	3047.47
21	25	924.75	2094.08	3018.83
22	26	959.50	2041.17	3000.67
23	27	994.25	1967.41	2961.66
24	28	1029.00	1878.66	2907.66
25	29	1063.75	1825.74	2889.49
26	30	1098.50	1790.75	2889.25
27	31	1133.25	1757.64	2890.89
28	32	1168.00	1732.07	2900.07
29	33	1202.75	1707.31	2910.06
30	34	1237.50	1686.25	2923.75
31	35	1272.25	1665.96	2938.21
32	36	1307.00	1640.75	2947.75
33	37	1341.75	1607.64	2949.39
34	38	1376.50	1582.07	2958.57
35	39	1411.25	1557.31	2968.56
36	40	1446.00	1537.01	2983.01
37	41	1480.75	1518.65	2999.40
38	42	1515.50	1502.20	3017.70
39	43	1550.25	1488.10	3038.35
40	43	1555.00	1471.72	3026.72
41	45	1619.75	1454.76	3074.51
42	46	1654.50	1439.51	3094.01
43	47	1689.25	1425.41	3114.66
44	48	1724.00	1413.73	3137.73
45	49	1758.75	1402.62	3161.37
46	50	1793.50	1393.44	3186.94
47	51	1828.25	1384.50	3212.75
48	52	1863.00	1377.09	3240.09
49	53	1897.75	1369.73	3267.48
50	54	1932.50	1362.42	3294.92
51	55	1967.25	1356.36	3323.61
52	56	2002.00	1350.36	3352.36
53	57	2036.75	1344.40	3381.15
54	58	2071.50	1339.32	3410.82
55	59	2106.25	1334.37	3440.62
56	60	2141.00	1329.53	3470.53

TABLE VI.

FINAL TABULATION - CASE I

OPTIMAL POLICY

ITEM NUMBER	DEMAND PER YEAR	AVE SPACE	SAFETY STOCK	ORDER QTY	VENDER	REORDER POINT
3	1000	9	1	240.	2(3)	3
2	1600	7	2	560.	3(2)	3
1	3000	10	1	1000.	1(1)	3

TABLE VII

PROCUREMENT POLICY - CASE I

CASE II: SAFETY STOCK IS STORED IN DYNAMIC SPACE.

Although the tables presented for Case II closely resemble those previously presented for Case I where the safety stock was stored in dedicated space, they are presented to illustrate certain interesting differences between the two methods of handling safety stock. The supplier-oriented variables remain the same as for Case I, and can be found in Table I and Appendix I. Table VIII presents the values of the various control parameters of the model. These, too, are the same as in Case I. The variables were kept constant to allow a comparison of the two methods of handling safety stock.

Table IX presents the output of stage one of the dynamic programming model when the safety stock is stored in dynamic space.

FOR THE FOLLOWING VARIABLE CONDITIONS:

SAFETY STOCK LEVEL = .80
RATE OF RETURN= 0.25%
SPACE FACTOR= 1.00

COST OF SAFETY STOCKPALLET = 30.00
COST OF WAREHOUSE SPACE/PALLET = 34.75
COST OF HANDLING/PALLET = 1.00
COST OF OVERSTOCK/PALLET = 50.00

SAFETY STOCK IS STORED IN DYNAMIC SPACE

TABLE VIII

PARAMETER VALUES - CASE II

One point of major interest in this table is that when either ten or seventeen units of space are allocated to item 1 the optimal order quantity is 1000 units or ten pallets. According to the model this assures that the annual expected procurement costs are minimized; even though in the case of seventeen units of space available, seven are left unused at this time. The cost of space rental has not yet been considered. Bearing this fact in mind, let's examine the total cost curves for the two cases:

(1) when ten pallet positions are available for use and (2) when seventeen pallet positions are available. Figures 7 and 8 present graphs of the expected procurement costs for the various feasible order lot sizes of ten pallet positions available for item 1.

ITEM NUMBER 1
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
5	5	500.	3(1)	1.	944.45	944.45
6	6	600.	3(1)	1.	855.06	855.06
7	7	700.	3(1)	1.	791.67	791.67
8	8	800.	3(1)	1.	744.54	744.54
9	9	900.	3(1)	1.	708.23	708.23
10	10	1000.	1(1)	1.	576.25	576.25
11	11	1100.	1(1)	1.	555.20	555.20
12	12	1200.	1(1)	1.	537.87	537.87
13	13	1300.	1(1)	1.	523.41	523.41
14	14	1400.	1(1)	1.	511.21	511.21
15	15	1500.	1(1)	1.	500.80	500.80
16	16	1600.	1(1)	1.	491.86	491.86
17	17	1000.	1(1)	1.	426.25	426.25
18	18	1100.	1(1)	1.	418.83	418.83
19	19	1200.	1(1)	1.	412.87	412.87
20	20	1300.	1(1)	1.	408.03	408.03
21	21	1400.	1(1)	1.	404.06	404.06
22	22	1500.	1(1)	1.	400.80	400.80
23	23	1600.	1(1)	1.	398.11	398.11
24	24	1700.	1(1)	1.	395.89	395.89
25	25	1800.	1(1)	1.	394.06	394.06
26	26	1900.	1(1)	1.	392.57	392.57
27	27	2000.	1(1)	1.	391.35	391.35
28	28	2100.	1(1)	1.	390.38	390.38
29	29	2200.	1(1)	1.	389.61	389.61

TABLE IX

STAGE 1 OUTPUT - CASE II

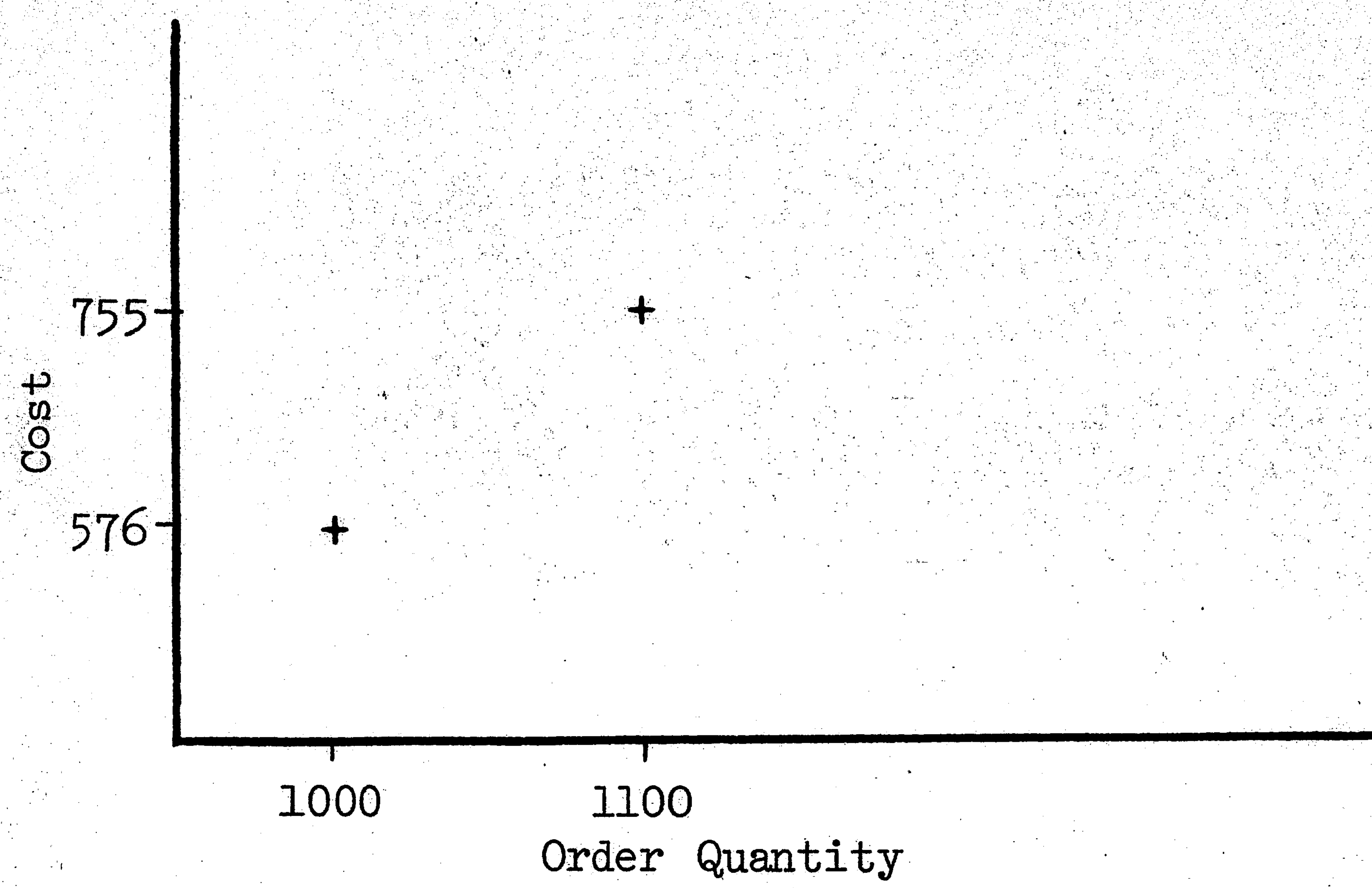

FIGURE 7.

PROCUREMENT COSTS VS. ORDER QUANTITY

SUPPLIER 1(1)

When eleven pallet positions (including one for safety stock) are allocated to item 1, the feasible order lot sizes range from 500 units to 1100 units in increments of 100 units. Figure 7 indicates that an order quantity of 1000 units results in the minimum expected procurement cost if supplier 1(1) is selected. The sharp increase in expected procurement cost when 1100 units are ordered is due to an increase in the imputed stockout cost, since the additional pallet of safety stock would create an overstock condition if it was carried. If only 1000 units are ordered, the imputed stockout cost is less due to the unused pallet position.

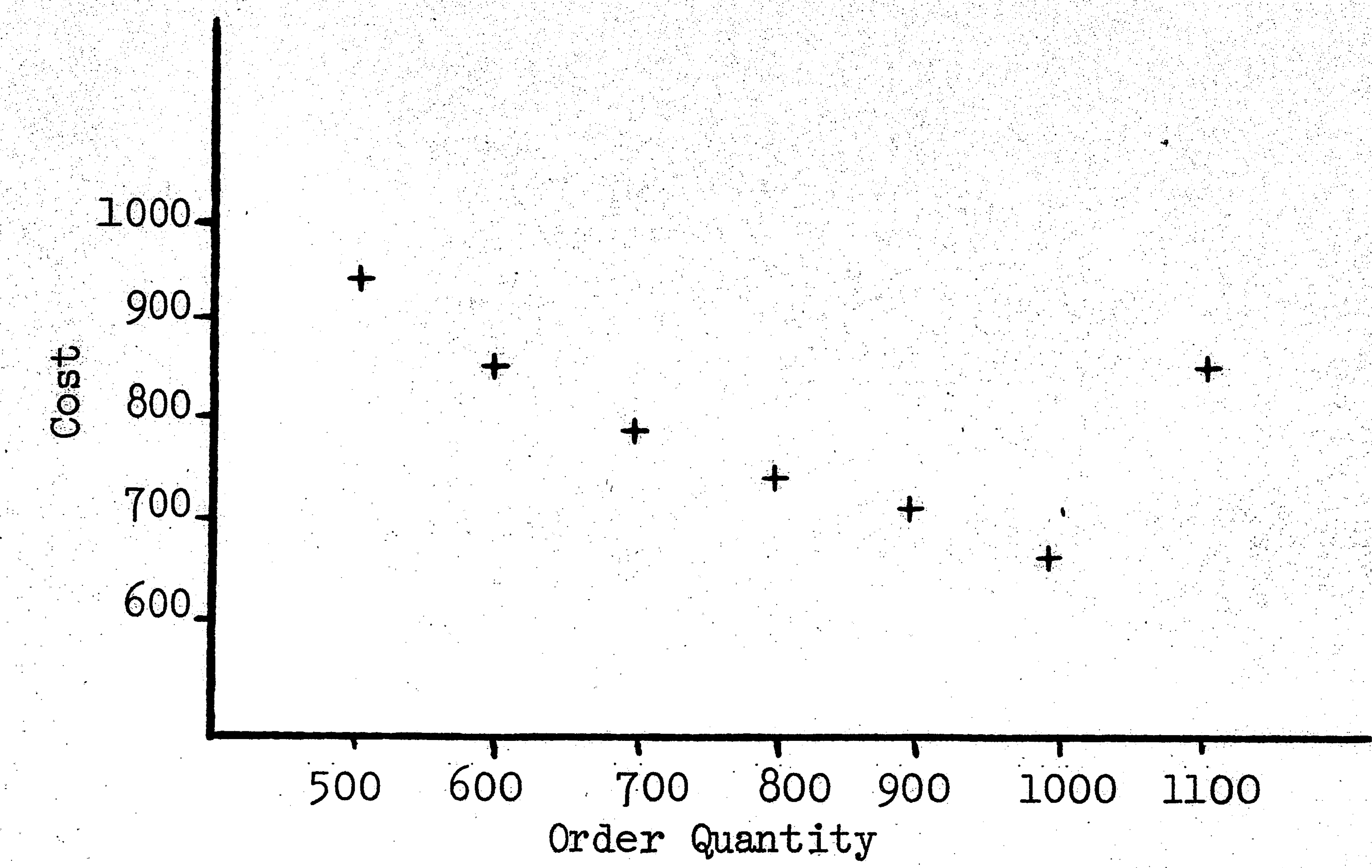

FIGURE 8.

PROCUREMENT COSTS VS. ORDER QUANTITY

SUPPLIER 3(1)

Figure 8 indicates that an order lot size of 1000 units results in the minimum expected procurement cost if supplier 3(1) is selected. Again the increase in expected procurement cost is due to an increase in the imputed stockout cost. The final decision between choosing supplier 1(1) and 3(1) is based on the minimum expected cost. Supplier 1(1) is chosen due to the fact that an order of 1000 units results in the lowest expected procurement cost for item 1.

Figures 9, 10, and 11 present graphs of the expected procurement cost for suppliers 1(1), 2(1), and 3(1) respectively when a total of 17 pallet positions (not including safety stock) are

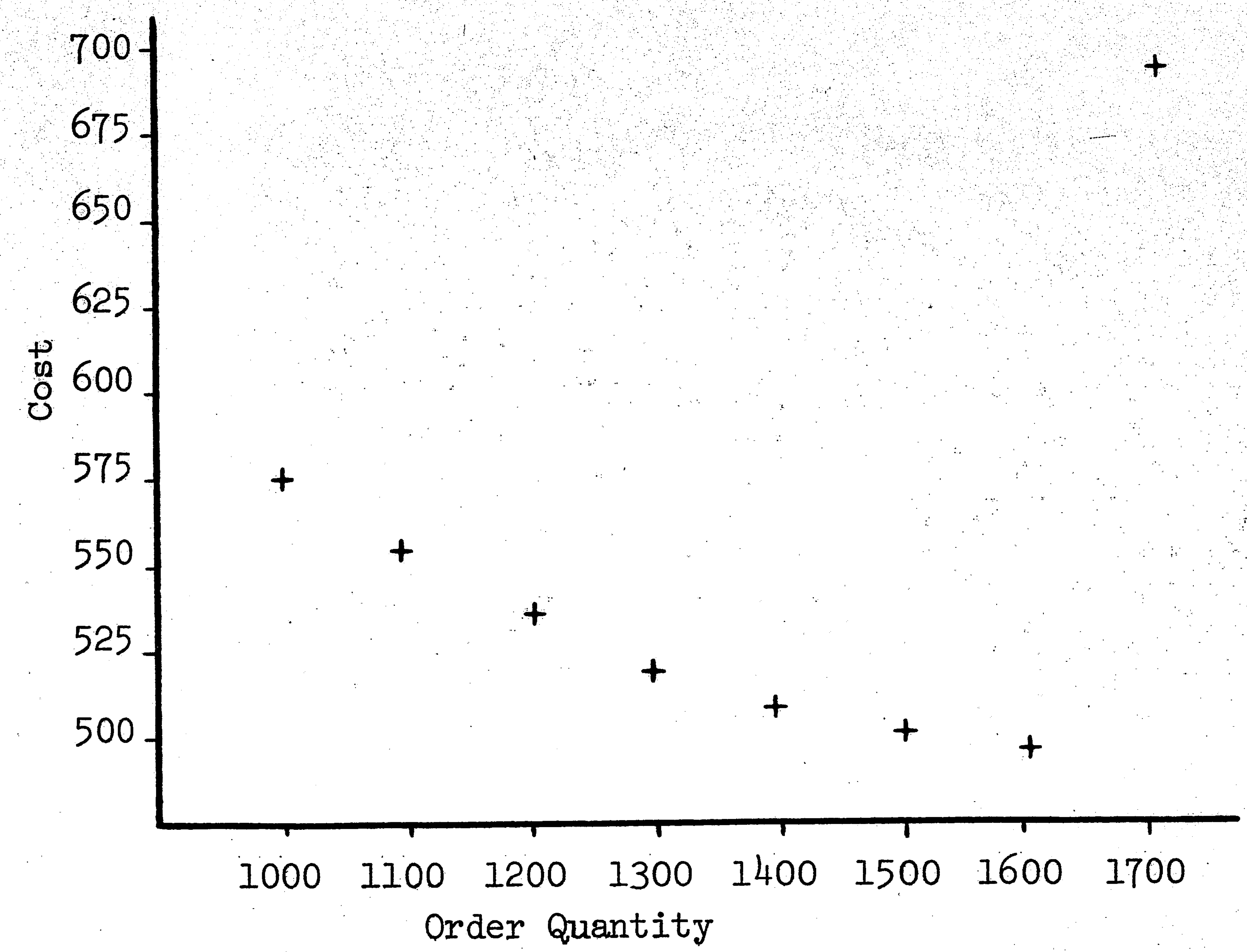

FIGURE 9.

PROCUREMENT COST VS. ORDER QUANTITY

SUPPLIER 1(1)

available in which to store item 1. The total space available for item 1 is dependent on the safety stock requirements of a particular supplier. Therefore the feasible ranges of order lot sizes are 1000 units to 1700 units for supplier 1(1), 1500 units to 1800 units for supplier 2(1), and 500 units to 1700 units for supplier 3(1). Supplier 2(1)'s lead time distribution is such that two pallet positions of safety stock are required to meet the stockout criteria, whereas supplier 1(1) and 3(1) only require one pallet of safety stock. The cost of spaae for safety stock is accounted for in the final tabulation.

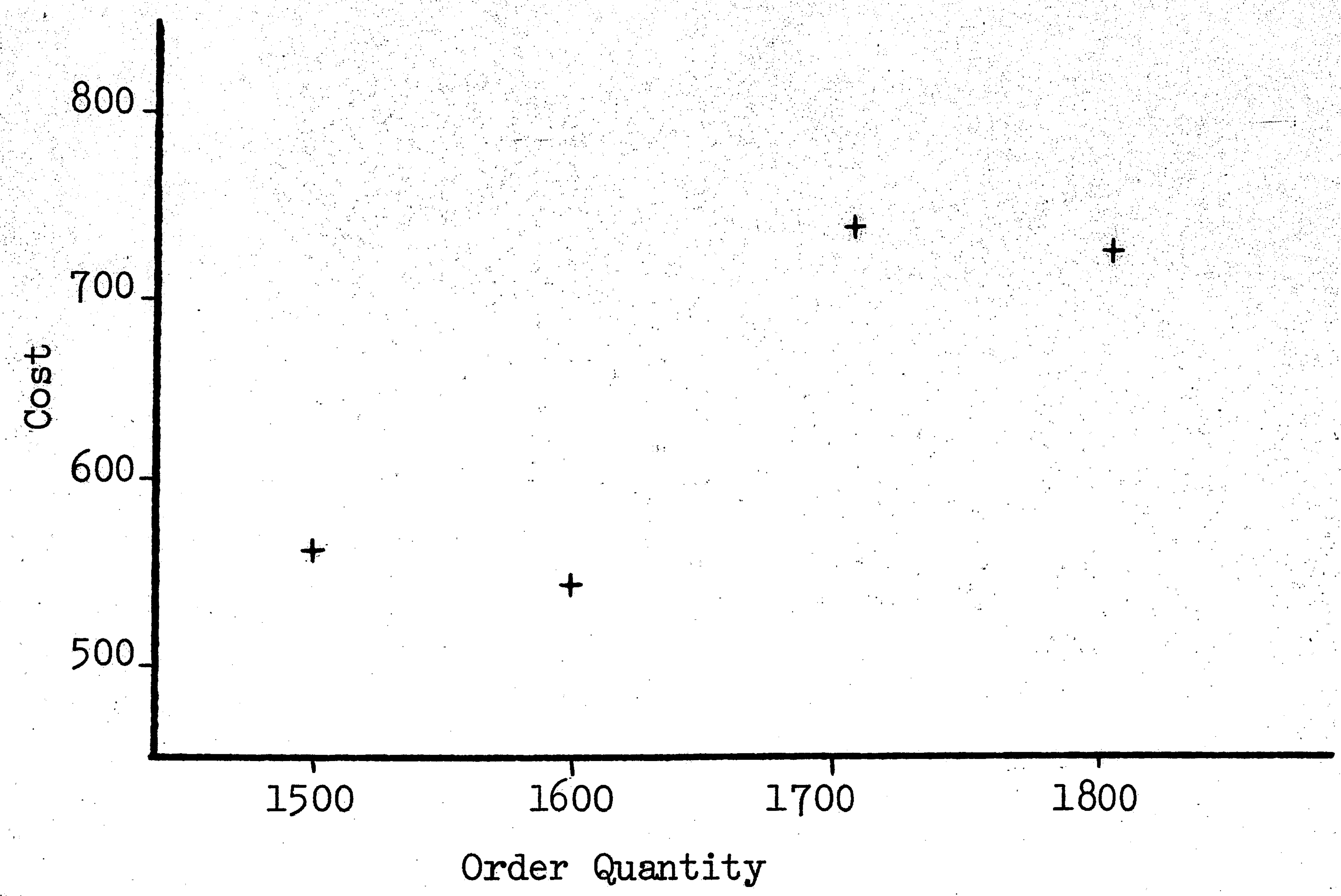

FIGURE 10.

PROCUREMENT COST VS. ORDER QUANTITY

SUPPLIER 2(1)

As indicated by Figure 9 the minimum cost order lot size for supplier 1(1) is 1600 units. The increase in the procurement cost incurred at a lot size of 1700 units is due to the increase in the imputed stockout cost. Figure 10 indicates the minimum cost order lot size for supplier 2(1) is 1600 units. Again the increase in cost incurred at 1700 units is due to an increase in the imputed stockout cost, however, as Figure 10 indicates once the additional cost has been incurred, a larger order quantity of 1800 results in fewer orders per year and thus a decline in the cost from quantity of 1700. Figure 11 can be summed up as follows; if the order quantity is less than or equal to 900 units, the expected overstock cost can be avoided.

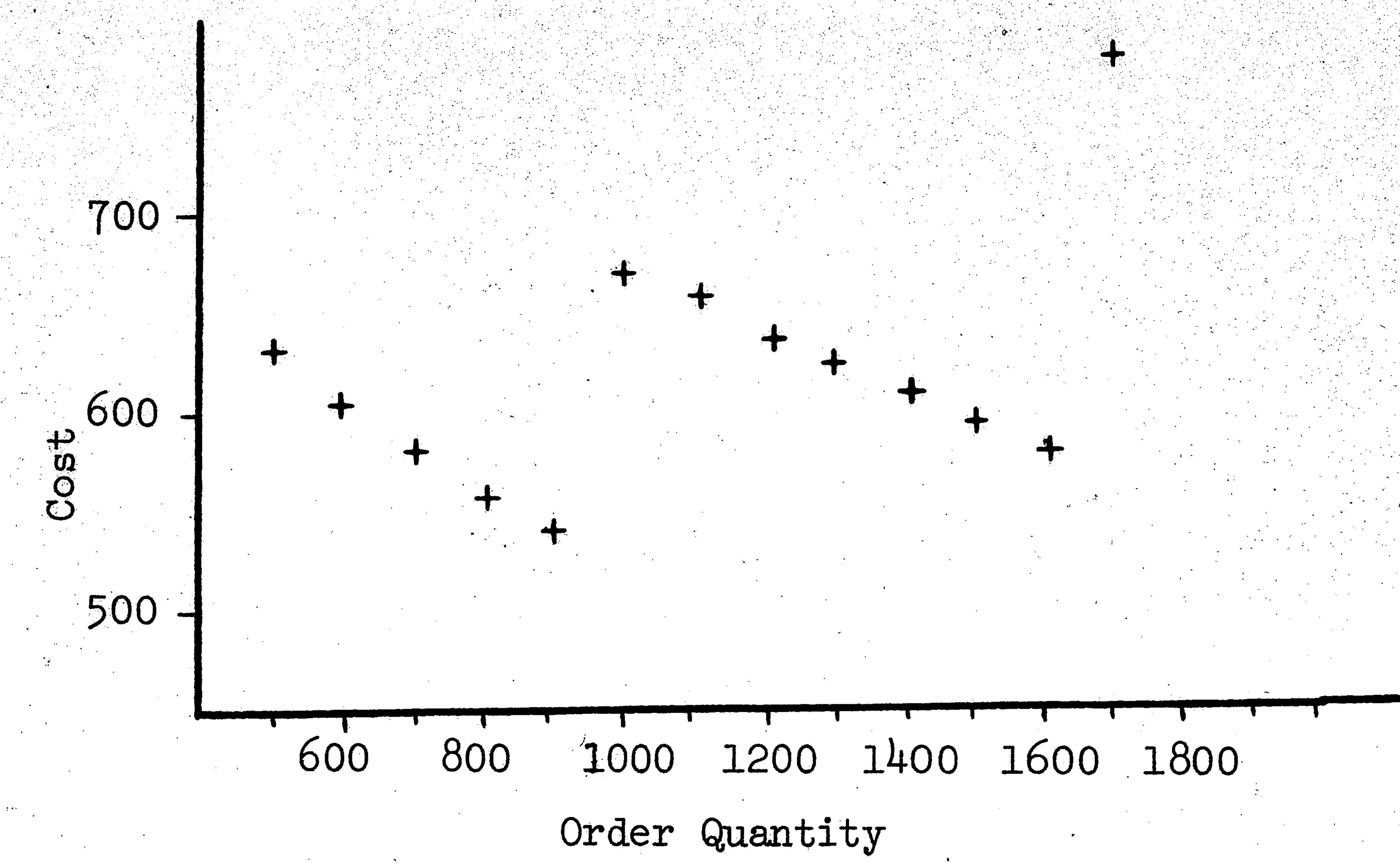

FIGURE 11.

PROCUREMENT COSTS VS. ORDER QUANTITY

SUPPLIER 3(1)

Once the order quantity exceeds 900 units an expected overstock cost is incurred as indicated by the marked increase in procurement cost at 1000 units. When the order quantity reaches 1700 units another increase in the cost is noted. This increase is due to the increase in imputed stockout cost.

Thus far, the cost of space rental has not been accounted for. This in done once the cost of the optimal procurement policy has been established for each feasible warehouse size. Tables X and XI present the output of stages two and three of the dynamic programming model.

ITEM NUMBER 2
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
12	7	560.	3(2)	2.	411.75	1356.20
13	7	560.	3(2)	2.	411.75	1266.81
14	7	560.	3(2)	2.	411.75	1203.42
15	7	560.	3(2)	2.	411.75	1156.29
16	7	560.	3(2)	2.	411.75	1119.98
17	7	560.	3(2)	2.	411.75	988.00
18	8	640.	3(2)	2.	378.64	954.89
19	9	720.	3(2)	2.	353.07	929.32
20	9	720.	3(2)	2.	353.07	908.27
21	10	800.	3(2)	2.	332.78	887.98
22	10	800.	3(2)	2.	332.78	870.65
23	11	880.	3(2)	2.	316.33	854.20
24	7	560.	3(2)	2.	411.75	838.00
25	8	640.	3(2)	2.	378.64	804.89
26	9	720.	3(2)	2.	353.07	779.32
27	10	800.	3(2)	2.	332.78	759.03
28	11	880.	3(2)	2.	316.33	742.58
29	12	960.	3(2)	2.	302.76	729.01
30	13	1040.	1(2)	1.	285.85	712.10
31	14	1120.	1(2)	1.	273.84	700.09
32	15	1200.	1(2)	1.	263.51	689.76
33	16	640.	3(2)	2.	253.64	679.89
34	17	720.	3(2)	2.	241.96	668.21
35	18	800.	3(2)	2.	232.78	659.03
36	18	800.	3(2)	2.	232.78	651.62
37	19	880.	3(2)	2.	225.42	644.25
38	20	960.	3(2)	2.	219.42	638.26
39	20	960.	3(2)	2.	219.42	632.29
40	20	960.	3(2)	2.	219.42	627.45
41	20	960.	3(2)	2.	219.42	623.48
42	20	960.	3(2)	2.	219.42	620.22
43	20	960.	3(2)	2.	219.42	617.53
44	20	960.	3(2)	2.	219.42	615.31
45	20	960.	3(2)	2.	219.42	613.49
46	20	960.	3(2)	2.	219.42	611.99
47	20	960.	3(2)	2.	219.42	610.77
48	20	960.	3(2)	2.	219.42	609.80
49	20	960.	3(2)	2.	219.42	609.03

TABLE X.

STAGE 2 OUTPUT - CASE II.

ITEM NUMBER 3
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
14	2	120.	2(3)	1.	1146.08	2502.28
15	3	120.	2(3)	1.	946.08	2302.28
16	4	160.	2(3)	1.	753.16	2109.36
17	5	200.	2(3)	1.	637.74	1993.94
18	5	200.	2(3)	1.	637.74	1904.56
19	6	240.	2(3)	1.	561.08	1827.89
20	6	240.	2(3)	1.	561.08	1764.50
21	8	160.	2(3)	1.	440.66	1707.47
22	5	200.	2(3)	1.	637.74	1625.74
23	6	240.	2(3)	1.	561.08	1549.08
24	7	280.	2(3)	1.	506.55	1494.55
25	8	160.	2(3)	1.	440.66	1428.66
26	9	200.	2(3)	1.	387.74	1375.74
27	10	240.	2(3)	1.	352.74	1340.75
28	10	240.	2(3)	1.	352.74	1307.64
29	10	240.	2(3)	1.	352.74	1282.07
30	11	280.	2(3)	1.	327.98	1257.31
31	11	280.	2(3)	1.	327.98	1236.25
32	11	280.	2(3)	1.	327.98	1215.96
33	12	320.	2(3)	1.	309.62	1197.60
34	12	320.	2(3)	1.	309.62	1180.27
35	10	240.	2(3)	1.	352.74	1157.64
36	10	240.	2(3)	1.	352.74	1132.07
37	11	280.	2(3)	1.	327.98	1107.31
38	11	280.	2(3)	1.	327.98	1087.01
39	12	320.	2(3)	1.	309.62	1068.65
40	12	320.	2(3)	1.	309.62	1052.20
41	13	360.	2(3)	1.	295.52	1038.10
42	12	320.	2(3)	1.	309.62	1021.72
43	13	360.	2(3)	1.	295.52	1007.62
44	13	360.	2(3)	1.	295.52	995.61
45	14	400.	2(3)	1.	284.41	984.50
46	14	400.	2(3)	1.	284.41	974.17
47	13	360.	2(3)	1.	295.52	963.73
48	14	400.	2(3)	1.	284.41	952.62
49	14	400.	2(3)	1.	284.41	943.44
50	15	440.	2(3)	1.	275.47	934.50
51	15	440.	2(3)	1.	275.47	927.09
52	15	440.	2(3)	1.	275.47	919.73
53	16	480.	2(3)	1.	268.16	912.42
54	17	520.	2(3)	1.	262.10	906.36
55	17	520.	2(3)	1.	262.10	900.36

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
56	17	520.	2(3)	1.	262.10	894.40
57	18	560.	2(3)	1.	257.03	889.32
58	18	560.	2(3)	1.	257.03	884.48
59	19	600.	2(3)	1.	252.74	880.19
60	19	600.	2(3)	1.	252.74	876.23
61	20	640.	2(3)	1.	249.10	872.58
62	20	640.	2(3)	1.	249.10	869.32
63	21	680.	3(3)	1.	245.96	866.18
64	22	720.	3(3)	1.	243.10	863.33
65	22	720.	3(3)	1.	243.10	860.64
66	23	760.	3(3)	1.	240.64	858.17
67	23	760.	3(3)	1.	240.64	855.95
68	24	800.	3(3)	1.	238.50	853.81
69	25	840.	3(3)	1.	236.64	851.95
70	25	840.	3(3)	1.	236.64	850.12
71	25	840.	3(3)	1.	236.64	848.63
72	25	840.	3(3)	1.	236.64	847.41
73	25	840.	3(3)	1.	236.64	846.43
74	25	840.	3(3)	1.	236.64	845.67

TABLE XI.

STAGE 3 OUTPUT - CASE II.

FINAL TABULATION

TOTAL SPACE	+SS	RENTAL AND HANDLING COST	PROCUREMENT COST	TOTAL COST
14	18	700.50	2502.28	3202.78
15	19	735.25	2302.28	3037.53
16	20	770.00	2109.36	2879.36
17	21	804.75	1993.94	2798.69
18	22	839.50	1904.56	2744.06
19	23	874.25	1827.89	2702.14
20	24	909.00	1764.50	2673.50
21	25	943.75	1707.47	2651.22
22	26	978.50	1625.74	2604.24
23	27	1013.25	1549.08	2562.33
24	28	1048.00	1494.55	2542.55
25	29	1082.75	1428.66	2511.41
26	30	1117.50	1375.74	2493.24
27	31	1152.25	1340.75	2493.00
28	32	1187.00	1307.64	2494.64
29	33	1221.75	1282.07	2503.82
30	34	1256.50	1257.31	2513.81
31	35	1291.25	1236.25	2527.50
32	36	1326.00	1215.96	2541.96
33	37	1360.75	1197.60	2558.35
34	38	1395.50	1180.27	2575.77
35	39	1430.25	1157.64	2587.89
36	40	1465.00	1132.07	2597.07
37	41	1499.75	1107.31	2607.06
38	42	1534.50	1087.01	2621.51
39	43	1569.25	1068.65	2637.90
40	44	1604.00	1052.20	2656.20
41	45	1638.75	1038.10	2676.85
42	45	1638.75	1021.72	2660.47
43	46	1673.50	1007.62	2681.12
44	47	1708.25	995.61	2703.86
45	48	1743.00	984.50	2727.50
46	49	1777.75	974.17	2751.92
47	51	1847.25	963.73	2810.98
48	52	1882.00	952.62	2834.62
49	53	1916.75	943.44	2860.19
50	54	1951.50	934.50	2886.00
51	55	1986.25	927.09	2913.34
52	56	2021.00	919.73	2940.73
53	57	2055.75	912.42	2968.17
54	58	2090.50	906.36	2996.86
55	59	2125.25	900.36	3025.61
56	60	2160.00	894.40	3054.40
57	61	2194.75	889.32	3084.07

TABLE XII.

FINAL TABULATION - CASE II.

OPTIMAL POLICY

ITEM NUMBER	DEMAND PER YEAR	AVE SPACE	SAFETY STOCK	ORDER QTY	VENDER	REORDER POINT
3	1000	10	1	240.	2(3)	3
2	1600	7	2	560.	3(2)	3
1	3000	10	1	1000.	1(1)	3

TABLE XIII.

PROCUREMENT POLICY - CASE II.

Table XII presents the final tabulation, including rental and handling costs for all feasible warehouse sizes. Table XIII presents the final procurement policy which results in the minimum cost. Such a policy required 30 pallet positions of warehouse space and the annual expected cost of such a system is $2,493.00.

Thus far, no comment has been made as to the effect of the space utilization factor which is an input parameter to the model. In the preceding illustrations a value of 1 has been assigned to the space utilization factor. This corresponds to the pessimistic viewpoint that all orders will arrive at the same time, thus requiring the maximum space. Siecienski[26] has made a thorough investigation of the effects of the space factor under varying conditions with the use of a GPSS simulator. In this investigation several different systems configurations, including stochastic systems, were analyzed.

Since the results of the investigation of the space utilization factor are available for reference, the reader is referred to these reports. However, to illustrate the effect of the space factor on the sample problem described in this chapter, outputs of the model are presented in Appendix II for the two cases of interest. The space factor was assigned a value of 2. This assignment corresponds to the viewpoint that only the average inventory will be on hand on any given day. The fundamental effect is that larger order quantities can be placed even though the space allocation is the same amount of space as for a space factor of 1. The tables in Appendix II illustrate this fact and also illustrate a change in the optimal procurement policy due to a change in the various procurement costs.

CHAPTER V

(R, r, T) MODEL

Implementation of the (q, r) model developed in Chapter III and illustrated in Chapter IV requires that a decision to place or not to place an order must be made each time a unit is withdrawn from inventory. In many situations this requirement presents no difficulty, however, under certain conditions the task of continuous review may present a serious management problem. In such a situation a system which allows periodic review may be desirable. For these reasons it is desirable to examine a second model which can provide a satisfactory solution to the initial problem described in Chapter II, and at the same time allow for periodic rather than continuous review of the items in the system. In a periodic review model the frequency with which the orders are placed is fixed, thus the fluctuations in demand must be absorbed by varying the amount ordered. This type of model is referred to as an (S, s) model in most of the literature. The model as presented in this thesis shall be denoted as the (R, r, T) model, where R is the order up to level, r the reorder level, and T the time between reviews expressed in fractions of a year.

(R, r, T) MODEL

Let I_i denote the inventory level in the warehouse for item i. Let M_i denote the amount of item i on order, but not yet received.

The operation of the (R, r, T) model is as follows. If the quantity $(I_i + M_i)$ is less than $r_{ij(i)}$, the reorder level at the time of the review, an order is placed for a quantity $q_{ij(i)}$ where $q_{ij(i)} = R_{ij(i)} - (I_i + M_i)$; thus the name "order up to" is applied to this model.

The following development of this model closely follows the development given in Hadley and Whitin[1] with the following exceptions: (1) the lead time, T, is assumed to be a random variable of known distribution, (2) the cost of a stockout is independent of the duration of the stockout, and (3) the notation is slightly different so as to conform with notation used in this thesis.

In addition to the symbols previously defined, the following symbols and definitions will be used in the development of the (R, r, T) model.

Notation	Definition
$q_{ij(i)}(T)$	Probability that the lead time for item i, if ordered from supplier j(i), is T.
J(i)	Cost of an order review for item i.

[1] Hadley and Whitin, Analysis of Inventory Systems, Prentice-Hall, Inc.; Englewood Cliffs, N.J., 1963, pp. 265-289.

Notation	Definition
$L(X: T)$	Expected cost of carrying inventory and stockout when an order arrives in a period of length T if the initial inventory for the period is X.
$p_{ij(i)}(X; T)$	Probability of X units of item i being demanded in a period of length T.
$p^{(n)}_{ij(i)}(X: T)$	Probability of X units of item i being demanded in n periods of length T. (n-fold convolution of $p_{ij(i)}(X: T)$).
$R_{ij(i)}$	Order up to level for item i if the supplier is j(i).
$T_{ij(i)}$	Mean lead time for item i when ordered from supplier j(i).
T_i	Time, in fractions of a year, between reviews of item i.

Again, as in the development of the (q, r) model, the subscripts i and j(i) will be eliminated except for the final results. Let t denote a review time. The inventory position after a decision to order or not has been made can be denoted as r + k where $k = 1, 2, \ldots R - r$. Let $L(r + k; T)$ be the expected cost of carrying inventory and stockouts from time $(t + T)$ to time $(t + T + T)$. Note that overstocks do not occur in this type of system since the quantity ordered is based, indirectly, on the available space. Since it may not be necessary to place an order at each review period, the time between orders will be of length nT where $n = 1, 2, 3 \ldots$ Let an order be placed at time t_0 and assume that the inventory position, as described by the on-hand quantity, is $(r + k)$ at time $(t_0 + nT)$. This implies that $R - (r + k)$ units have been demanded in time nT. The probability of $R - (r + k)$ units being demanded in n periods of length T is $p^{(n)}(R - (r + k); T)$. The expected cost of the system from time $(t_0 + nT + T)$ to time $(t_0 + (n + 1)T + T)$ is given by $L(r + k; T)$ for a given n. Therefore the expected cost in this one period is given by

$$\sum_{k=1}^{R-r} p^{(n)}(R - (r + k): T) \cdot L(r + k: T).$$

Since nT is also a random variable and the cost of the period from t_0 to $t_0 + T$ is $L(R: T)$ the expected cost per cycle is

$$L(R: T) + \sum_{n=1}^{\infty} \sum_{k=1}^{R-r} p^{(n)}(R - (r + k): T) \cdot L(r + k: T)$$

or

$$\sum_{n=0}^{\infty} \sum_{k=1}^{R-r} p^{(n)}(R - (r + k): T) \cdot L(r + k: T)$$

where

$$p^{(0)}(0: T) = 1, \quad p^{(0)}(x: T) = 0 \text{ for } x \neq 0.$$

The average number of cycles per year must be determined. The probability that a cycle contains n periods is

$$\sum_{k=1}^{R-r} p^{(n-1)}(R - (r + k))P(k: T)$$

where

$$P(X: T) = 1 - \sum_{x=0}^{X} p(x: T)$$

The probability that a cycle contains exactly one period is $P(R - r)$. Thus, the expected number of periods per cycle is

$$P(R - r: T) + \sum_{n=2}^{\infty} \sum_{k=1}^{R-r} np^{(n-1)}(R - (r + k): T) \cdot P(k: T)$$

or

$$\sum_{n=1}^{\infty} \sum_{k=1}^{R-r} np^{(n-1)}(R - (r + k): T) \cdot P(k: T)$$

where

$$p^{(0)}(X: T) = 1.$$

Letting $J(i)$ be the cost of a review for item i and including the subscripts i and $j(i)$ the total expected annual cost, $TC(R_{ij(i)}, r_{ij(i)}, T_i)$ is given by

$$\frac{J_i}{T_i} + \frac{(f_i + S_i)}{T_i \cdot A}$$

$$+ \frac{\sum_{n=0} \sum_{k=1}^{R_{ij(i)} - r_{ij(i)}} p_{ij(i)}^{(n)}(R_{ij(i)} - r_{ij(i)} - k: T_i) \cdot L(r_{ij(i)} + k: T_i)}{T_i \cdot A}$$

where

$$A = \sum_{n=1}^{\infty} \sum_{k=1}^{R_{ij(i)} - r_{ij(i)}} n \cdot p_{ij(i)}^{(n-1)}(R_{ij(i)} - r_{ij(i)} - k;\ T_i)\, p_{ij(i)}(k;\ T_i)$$

and

$$L(r_{ij(i)} + k;\ T_i) = \rho_i\, c_{ij(i)}(r_{ij(i)} + k - \bar{z}_{ij(i)} - \frac{D_i\ T_i}{2})$$

$$+ \Pi_i \left[\sum_{x=r_{ij(i)}+k}^{\infty} (x - r_{ij(i)} - k) \left[\sum_{T=0}^{\infty} p_{ij(i)}(x;\ T_i + T)\ g_{ij(i)}(T) \right. \right.$$

$$\left. \left. - \sum_{T=0}^{\infty} p_{ij(i)}(x;\ T)\ g_{ij(i)}(T) \right] \right]$$

Again the problem of specifying the cost of a stockout is present. This problem can be avoided by specifying an upper limit on the expected number of stockouts for the year and applying the technique of Lagrange multipliers to the total expected annual cost equation. This, however, presents an additional variable whose value must be found in order to minimize the total expected cost. An alternate method is to estimate Π_i by evaluating Π_i in the (q, r) model. This would allow an insight into what the cost of a stockout is and would probably be desirable over the Lagrange multiplier technique. It should be noted that the level of stockout protection will not be the same for the two models even if the same stockout cost is used.

Now that the total expected cost function has been identified, the dynamic programming formulation developed in Chapter III can be used to solve the problem, with two additional bits of information. The expected handling costs and the rental costs must be identified. The handling costs may be approximated by

$$H\left[\sum_{i=1}^{N} \frac{D_i}{p_i}\right]$$

and the rental costs are given by

$$R\left[\sum_{i=1}^{N} \frac{R_{ij(i)}}{\alpha}\right]$$

assuming all stock is stored in the same area. If some of the stock is stored in a dedicated area that amount must be defined.

Based on this formulation the problem is to find W* such that

$$V(W^*) = \underset{W}{\text{Min.}}\left\{H\left[\sum_{i=1}^{N} \frac{D_i}{p_i}\right] + R\left[\sum_{i=1}^{N} \frac{R_{ij(i)}}{\alpha}\right] + \sum_{i=1}^{N}\left[TC(R_{ij(i)}, r_{ij(i)}, T_i)\right]\right\}$$

where

$R_{ij(i)}$, $r_{ij(i)}$, and $j(i)$ have been determined from the d.p. formulation described in Chapter III when

$$y_i(u_i) = \underset{R_{ij(i)},\, r_{ij(i)},\, j(i)}{\text{Min.}} \left\{TC(R_{ij(i)}, r_{ij(i)}, T_i)\right\}$$

subject to

$$\frac{R_{ij(i)} - r_{ij(i)}}{\alpha} \geq m_{ij(i)}$$

$$\frac{R_{ij(i)}}{\alpha} \leq u_i p_i$$

As the functional equation indicates the search is now a three-dimensional search for each feasible warehouse size.

The (R, r, T) model as developed incorporates the following assumptions:

1. Lead times are independent random variables.
2. Orders are received in the same sequence as they were placed.
3. The range of lead time value is less than the review period.
4. The cost of a stockout must be known.
5. It is necessary to know the n-fold convolution of demand during review period. Hadley assumes the demand is Poisson which allows the n-fold convolution to be determined with relative ease. However, if this assumption cannot be made, the convolution must be done by the method outlined in Chapter III. This of course increases the amount of calculation necessary to determine the policy.
6. Overstock conditions are not expected to occur.
7. Review period must be determined. This would generally be dictated by outside factors.

The (R, r, T) model has been presented as an alternative to the fixed order quantity model. The assumptions necessary to develop the (R, r, T) model and the complex form of the annual expected cost equation do remove this model from the practical world in all but special cases. For example, if the demand can be assumed to be Poisson distributed the n-fold convolution $p^{(n)}(x: T)$ can be calculated with ease. If this is not the case the n-fold convolution does represent a serious barrier to the practical use of the (R, r, T). Also, the fact that the stockout cost must be identified is not desirable. The resolution of the functional equation requires a three-dimensional search. These two factors in addition to the other assumptions listed placed rather severe limitations on the practical use of such a model.

CHAPTER VI

SUMMARY AND CONCLUSIONS

This concluding chapter will be divided into three basic sections. The first section will summarize the material presented in this thesis. The second section will analyze the models developed herein. Proposals for further study will be discussed in the third section.

SUMMARY

Certain classes of items, due to their low unit cost and high space requirements (e.g. packing material) place conflicting demands on the purchasing, production control, and warehousing organizations within a firm. In order to provide an optimal policy for the firm, there was a need for an integrated decision making process, a process which circumvents organizational boundaries. P. T. Lele and E. A. Siecienski[22] have developed a mathematical model which provides an integrated method of decision making for a system which has:

(1) Multiple items,

(2) Multiple sources of supply,

(3) Known demand and lead time,

(4) Minimum order quantity restrictions,

(5) Limited warehouse capacities.

This model provides the following information:

(1) When to order,

(2) How much to order,

(3) From whom to order.

It has been the basic intent of this thesis to extend the concepts developed by P. T. Lele and E. A. Siecienski[23], to take account of stochastic demand and replenishment time.

The material in Chapter I and the first section of Chapter II serves to acquaint the reader briefly with major works in inventory control theory and extensively with investigations into the realm of the multi-item, multi-source, warehouse constrained, inventory control and procurement problem. The second section of Chapter II presented a detail problem description and definition. Pertinent variables were identified and discussed and a brief introduction to the analytical development presented in Chapter III was given.

An analytical method for determining the distribution of usage during lead time, given the distribution of demand per day and the distribution of lead time days, was presented in the first section of Chapter III. The models developed in the second section of Chapter III provide for the absorption of demand fluctuation by varying the frequency with which orders are placed. These models were developed under two basic but different methods of providing for the storage of the safety stock. The first model was developed under the assumption that no distinction is made between space for safety stock and space for regular stock. Since the models are closely related, their development was presented in a parallel manner. In addition to the standard parameters and variables found in most inventory models, the following special parameters were included in the models:

(1) The cost of an overstock,

(2) A space utilization factor,

(3) Space rental costs,

(4) Material handling costs.

In order to avoid the troublesome task of determining the cost of a stockout, both models allow for the specification of a specific service level. These models allow the desired inventory control and procurement policy to be determined.

Chapter IV examined specific applications of the analytical models developed in Chapter III. The effects on the procurement policy due to the value of the space utilization factor were illustrated. Although the problems considered were small in scale, they were quite complex in nature and do not lend themselves to any straightforward method of analysis.

A model which allows for the absorption of demand and lead time fluctuations through variable order quantities was presented in Chapter IV. This model requires certain limiting assumptions both in its development and in its use and is therefore of limited practicality.

CONCLUSIONS

The models developed in this thesis provide a method by which the desired inventory control and procurement policy may be determined for a system which has

(1) Multiple items,

(2) Multiple sources of supply,

(3) Stochastic demand and lead time,

(4) Minimum order quantity restrictions,

(5) Limited warehouse capacity.

The use of these models requires the following information:

(1) The distribution of demand per day for each item in the system.

(2) The distribution of lead time days for each supplier.

(3) The various supplier-oriented cost paramters.

(4) The various system cost parameters.

(5) The desired service level for the items or the stockout cost.

(6) The value of the space utilization factor.

In addition to the above information requirements, the models assume that the items are stored in unit loads such as pallets and that these unit loads are interchangeable. Although the models are quite general in concept, they do not consider the following major points:

(1) Procurement when a shortage condition exists,

(2) Stochastic demand that is not identically distributed,

(3) Stockout costs which are a function of the duration of the stockout,

(4) Stochastic unit costs.

Despite the fact that the models do not provide solutions for all situations, they are able to provide a basic tool in determining the inventory control and procurement policy for a multi-item, multi-source, stochastic, warehouse constrained system. The models not only provide policies for existing systems, but can be used in determining warehouse configurations for new systems. The models can easily be extended to account for multiple warehouses with limited capacities and different cost parameters, as would be the situation if special storage conditions were necessary for subclasses of the items. The incorporation of the space utilization factor and the decomposition of the inventory carrying cost into its various components adds considerable reality to the models.

Dynamic programming, unfortunately does not lend itself to solving systems which are extremely large. (See Appendix III for computational aspects of the model.) However, by identifying those items which contribute the most to the problem in terms of dollars and space utilization and by applying the model to those items, the model can be quite useful in reducing the operating costs of a system.

PROPOSALS FOR FUTURE STUDY

This thesis provides several basic tools for the determination of an inventory control and procurement policy for a multi-item, multi-source, warehouse constrained warehouse system with stochastic demand and lead time. The proposals for future study listed below are further refinements to the basic tools developed in this thesis.

(1) Extend the model to provide for demand distributions which are not identically distributed. This would allow for trends or seasonal effects in the demand distributions.

(2) Investigate the possibility of decomposing a large system with many items into several homogeneous groups each of which could be solved by the formula presented in this thesis.

(3) Make provision for procurement when the quantity on-hand becomes negative.

(4) Extend the model to allow multiple warehouses within the system. The additional warehouses would each have their set of cost parameters.

(5) Further investigation of the (R, r, T) model, particularly the effects of the space utilization factor on such a model.

(6) Investigation of search techniques in resolving the functional equation for specific conditions.

APPENDIX I

DISTRIBUTIONS OF USAGE DURING LEAD TIME

ITEM 1 SUPPLIER 1(1)

D	P(D)	T	P(T)
0	.880	10	.200
100	.120	11	.100
0	.000	12	.100
0	.000	13	.100
0	.000	14	.200
0	.000	15	.300
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000

MEAN ANNUAL DEMAND = 3000. MEAN LEAD TIME = 12.90

P(Z) FOR Z EQUAL 0 TO 1500 MEAN USAGE = 154.80

Z	P(Z)
0	0.19824977E+00
100	0.33561144E+00
200	0.26828207E+00
300	0.13469905E+00
400	0.47596979E-01
500	0.12549213E-01
600	0.25494240E-02
700	0.40576757E-03
800	0.50896388E-04
900	0.50178681E-05
1000	0.38466898E-06
1100	0.22467292E-07
1200	0.96613984E-09
1300	0.28842802E-10
1400	0.53411010E-12
1500	0.46221066E-14

ITEM 1 SUPPLIER 2(1)

D	P(D)	T	P(T)
0	.880	10	.100
100	.120	11	.100
0	.000	12	.100
0	.000	13	.100
0	.000	14	.100
0	.000	15	.500
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000

MEAN ANNUAL DEMAND = 3000.   MEAN LEAD TIME = 13.50

P(Z) FOR Z EQUAL 0 TO 1500 MEAN USAGE = 162.00

Z	P(Z)
0	0.18309287E+00
100	0.32587488E+00
200	0.27410889E+00
300	0.14472322E+00
400	0.53667783E-01
500	0.14803877E-01
600	0.31352831E-02
700	0.51842799E-03
800	0.67365554E-04
900	0.68668062E-05
1000	0.54368906E-06
1100	0.32788515E-07
1200	0.14562304E-08
1300	0.44924309E-10
1400	0.86022539E-12
1500	0.77035109E-14

ITEM 1 SUPPLIER 3(1)

D	P(D)	T	P(T)
0	.880	10	.500
100	.120	11	.100
0	.000	12	.100
0	.000	13	.100
0	.000	14	.100
0	.000	15	.100
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000

MEAN ANNUAL DEMAND = 3000. MEAN LEAD TIME = 11.50

P(Z) FOR Z EQUAL 0 TO 1500 MEAN USAGE = 138.00

Z	P(Z)
0	0.23570372E+00
100	0.35753317E+00
200	0.25254079E+00
300	0.11079269E+00
400	0.34009182E-01
500	0.78032370E-02
600	0.13938113E-02
700	0.19845400E-03
800	0.22733804E-04
900	0.20877297E-05
1000	0.15143536E-06
1100	0.84599173E-08
1200	0.35038507E-09
1300	0.10124981E-10
1400	0.18231643E-12
1500	0.15407022E-14

ITEM 2 SUPPLIER 1(2)

D	P(D)	T	P(T)
0	.920	10	.300
80	.080	12	.100
0	.000	14	.100
0	.000	16	.100
0	.000	18	.200
0	.000	20	.200
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	W00

MEAN ANNUAL DEMAND = 1600. MEAN LEAD TIME = 14.80

P(Z) FOR Z EQUAL 0 TO 1600 MEAN USAGE = 94.72

Z	P(Z)
0	0.30686776E+00
80	0.36163557E+00
160	0.21380431E+00
240	0.84957279E-01
320	0.25382122E-01
400	0.59981098E-02
480	0.11483419E-02
560	0.18034932E-03
640	0.23411046E-04
720	0.25240896E-05
800	0.22662211E-06
880	0.16946878E-07
960	0.10527479E-08
1040	0.54003342E-10
1120	0.22646217E-11
1200	0.76428688E-13
1280	0.20271395E-14
1360	0.40724445E-16
1440	0.58300359E-18
1520	0.53034388E-20
1600	0.23058430E-22

ITEM 2 SUPPLIER 2(2)

D	P(D)	T	P(T)
0	.920	10	.800
80	.080	13	.075
0	.000	16	.050
0	.000	19	.050
0	.000	25	.025
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000

MEAN ANNUAL DEMAND = 1600. MEAN LEAD TIME = 11.35

P(Z) FOR Z EQUAL 0 TO 2000 MEAN USAGE = 72.64

Z	P(Z)
0	0.39941360E+00
80	0.37288637E+00
160	0.16547052E+00
240	0.48274812E-01
320	0.11101377E-01
400	0.22978545E-02
480	0.45430575E-03
560	0.84479675E-04
640	0.14238578E-04
720	0.21238515E-05
800	0.27742367E-06
880	0.31581905E-07
960	0.31230918E-08
1040	0.26742240E-09
1120	0.19756844E-10
1200	0.12538783E-11
1280	0.67985004E-13
1360	0.31264789E-14
1440	0.12078287E-15
1520	0.38690417E-17
1600	0.10092964E-18
1680	0.20896406E-20
1760	0.33037795E-22
1840	0.37471978E-24
1920	0.27153608E-26
2000	0.94447332E-29

ITEM 2 SUPPLIER 3(2)

D	P(D)	T	P(T)
0	.920	10	.200
80	.080	11	.200
0	.000	12	.300
0	.000	13	.100
0	.000	14	.100
0	.000	15	.100
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000

MEAN ANNUAL DEMAND = 1600. MEAN LEAD TIME = 12.00

P(Z) FOR Z EQUAL 0 TO 1200 MEAN USAGE = 76.80

Z	P(Z)
0	0.37067942E+00
80	0.38055854E+00
160	0.18194067E+00
240	0.53855398E-01
320	0.11071083E-01
400	0.16803893E-02
480	0.19535813E-03
560	0.17790729E-04
640	0.12844388E-05
720	0.73711668E-07
800	0.33381165E-08
880	0.11703426E-09
960	0.30668503E-11
1040	0.56488509E-13
1120	0.65091088E-15
1200	0.35184372E-17

ITEM 2 SUPPLIER 4(2)

D	P(D)	T	P(T)
0	.920	10	.400
80	.080	15	.300
0	.000	20	.300
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000

MEAN ANNUAL DEMAND = 1600. MEAN LEAD TIME = 14.50

P(Z) FOR Z EQUAL 0 TO 1600 MEAN USAGE = 92.80

Z	P(Z)
0	0.31625261E+00
80	0.36156975E+00
160	0.20864182E+00
240	0.81836682E-01
320	0.24470649E-01
400	0.58635142E-02
480	0.11502034E-02
560	0.18653245E-03
640	0.25143408E-04
720	0.28254243E-05
800	0.26490210E-06
880	0.20688541E-07
960	0.13401223E-08
1040	0.71467942E-10
1120	0.31027498E-11
1200	0.10786896E-12
1280	0.29309348E-14
1360	0.59967974E-16
1440	0.86910107E-18
1520	0.79551583E-20
1600	0.34587646E-22

ITEM 3 SUPPLIER 1(3)

D	P(D)	T	P(T)
0	.900	10	.500
40	.100	11	.500
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000

MEAN ANNUAL DEMAND = 1000. MEAN LEAD TIME = 10.50

P(Z) FOR Z EQUAL 0 TO 440 MEAN USAGE = 42.00

Z	P(Z)
0	0.33124451E+00
40	0.38548338E+00
80	0.20339576E+00
120	0.64211360E-01
160	0.13472029E-01
200	0.19716461E-02
240	0.20529369E-03
280	0.15199650E-04
320	0.78367498E-06
360	0.26774999E-07
400	0.54499998E-09
440	0.49999998E-11

ITEM 3 SUPPLIER 2(3)

D	P(D)	T	P(T)
0	.900	10	.400
40	.100	11	.400
0	.000	12	.200
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000

MEAN ANNUAL DEMAND = 1000. MEAN LEAD TIME = 10.80

P(Z) FOR Z EQUAL 0 TO 480 MEAN USAGE = 43.20

Z	P(Z)
0	0.32148151E+00
40	0.38370124E+00
80	0.20874216E+00
120	0.68415589E-01
160	0.15039249E-01
200	0.23349391E-02
240	0.26244524E-03
280	0.21513081E-04
320	0.12764790E-05
360	0.53495998E-07
400	0.15052000E-08
440	0.25599999E-10
480	0.19999999E-12

ITEM 3 SUPPLIER 3(3)

D	P(D)	T	P(T)
0	.900	10	.400
40	.100	11	.600
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000

MEAN ANNUAL DEMAND = 1000. MEAN LEAD TIME = 10.60

P(Z) FOR Z EQUAL 0 TO 440 MEAN USAGE = 42.40

Z	P(Z)
0	0.32775773E+00
40	0.38509596E+00
80	0.20533286E+00
120	0.65574505E-01
160	0.13934383E-01
200	0.20683684E-02
240	0.21879622E-03
280	0.16489980E-04
320	0.86750998E-06
360	0.30329999E-07
400	0.63399998E-09
440	0.59999997E-11

ITEM 3 SUPPLIER 4(3)

D	P(D)	T	P(T)
0	.900	10	.600
40	.100	11	.400
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000
0	.000	0	.000

MEAN ANNUAL DEMAND = 1000. MEAN LEAD TIME = 10.40

P(Z) FOR Z EQUAL 0 TO 440 MEAN USAGE = 41.60

Z	P(Z)
0	0.33473130E+00
40	0.38587080E+00
80	0.20145865E+00
120	0.62848213E-01
160	0.13009676E-01
200	0.18749238E-02
240	0.19179115E-03
280	0.13909320E-04
320	0.69983998E-06
360	0.23219999E-07
400	0.45599999E-09
440	0.39999998E-11

APPENDIX II

SPACE FACTOR = 2.0

CASE I: SAFETY STOCK STORED IN DEDICATED SPACE

ITEM NUMBER 1
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
3	3	600.	3(1)	1.	1005.06	1005.06
4	4	800.	3(1)	1.	894.54	894.54
5	5	1000.	1(1)	1.	726.25	726.25
6	6	1200.	1(1)	1.	687.87	687.87
7	7	1400.	1(1)	1.	661.21	661.21
8	8	1600.	1(1)	1.	641.86	641.86
9	9	1000.	1(1)	1.	576.25	576.25
10	10	1200.	1(1)	1.	562.87	562.87
11	11	1400.	1(1)	1.	554.06	554.06
12	12	1600.	1(1)	1.	548.11	548.11
13	13	1800.	1(1)	1.	544.06	544.06
14	14	2000.	1(1)	1.	541.35	541.35
15	15	2200.	1(1)	1.	539.61	539.61

ITEM NUMBER 2
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
7	4	640.	3(2)	2.	528.64	1533.70
8	4	640.	3(2)	2.	528.64	1423.18
9	4	640.	3(2)	2.	528.64	1254.89
10	5	800.	3(2)	2.	482.78	1209.03
11	5	800.	3(2)	2.	482.78	1170.65
12	6	960.	3(2)	2.	452.76	1140.63
13	4	640.	3(2)	2.	528.64	1104.89
14	5	800.	3(2)	2.	482.78	1059.03
15	6	960.	3(2)	2.	452.76	1029.01
16	7	1120.	1(2)	1.	423.84	1000.09
17	8	640.	3(2)	2.	403.64	979.89
18	9	800.	3(2)	2.	382.78	959.03
19	9	800.	3(2)	2.	382.78	945.65
20	10	960.	3(2)	2.	369.42	932.29
21	10	960.	3(2)	2.	369.42	923.48
22	10	960.	3(2)	2.	369.42	917.53
23	10	960.	3(2)	2.	369.42	913.49
24	10	960.	3(2)	2.	369.42	910.77
25	10	960.	3(2)	2.	369.42	909.03

ITEM NUMBER 3
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
9	2	160.	2(3)	1.	903.16	2436.86
10	3	240.	2(3)	1.	711.08	2244.78
11	3	240.	2(3)	1.	711.08	2134.26
12	3	240.	2(3)	1.	711.08	1965.97
13	4	320.	2(3)	1.	615.87	1870.76
14	5	400.	2(3)	1.	559.41	1814.30
15	6	240.	2(3)	1.	502.74	1757.64
16	6	240.	2(3)	1.	502.74	1711.78
17	7	320.	2(3)	1.	459.62	1668.65
18	7	320.	2(3)	1.	459.62	1630.27
19	7	320.	2(3)	1.	459.62	1600.25
20	6	240.	2(3)	1.	502.74	1561.78
21	7	320.	2(3)	1.	459.62	1518.65
22	7	320.	2(3)	1.	459.62	1488.63
23	7	320.	2(3)	1.	459.62	1459.70
24	8	400.	2(3)	1.	434.41	1434.50
25	8	400.	2(3)	1.	434.41	1414.30
26	8	400.	2(3)	1.	434.41	1393.44
27	9	480.	2(3)	1.	418.16	1377.19
28	9	480.	2(3)	1.	418.16	1363.81
29	9	480.	2(3)	1.	418.16	1350.45
30	10	560.	2(3)	1.	407.03	1339.32
31	10	560.	2(3)	1.	407.03	1330.52
32	11	640.	2(3)	1.	399.10	1322.58
33	12	720.	3(3)	1.	393.11	1316.59
34	12	720.	3(3)	1.	393.11	1310.64
35	13	800.	3(3)	1.	388.49	1306.03
36	13	800.	3(3)	1.	388.49	1301.98
37	13	800.	3(3)	1.	388.49	1299.27
38	13	800.	3(3)	1.	388.49	1297.52

FINAL TABULATION

TOTAL SPACE	+SS	RENTAL AND HANDLING COST	PROCUREMENT COST	TOTAL COST
9	13	507.75	2436.86	2944.61
10	14	542.50	2244.78	2787.28
11	15	577.25	2134.26	2711.51
12	16	612.00	1965.97	2577.97
13	17	646.75	1870.76	2517.51
14	18	681.50	1814.30	2495.80
15	19	716.25	1757.64	2473.89
16	20	751.00	1711.78	2462.78
17	21	785.75	1668.65	2454.40
18	22	820.50	1630.27	2450.77
19	23	855.25	1600.25	2455.50
20	24	890.00	1561.78	2451.78
21	25	924.75	1518.65	2443.40
22	26	959.50	1488.63	2448.13
23	26	964.25	1459.70	2423.95
24	27	999.00	1434.50	2433.50
25	29	1063.75	1414.30	2478.05
26	30	1098.50	1393.44	2491.94
27	31	1133.25	1377.19	2510.44
28	32	1168.00	1363.81	2531.81
29	33	1202.75	1350.45	2553.20
30	34	1237.50	1339.32	2576.82
31	35	1272.25	1330.52	2602.77
32	36	1307.00	1322.58	2629.58
33	37	1341.75	1316.59	2658.34
34	38	1376.50	1310.64	2687.14
35	39	1411.25	1306.03	2717.28
36	40	1446.00	1301.98	2747.98
37	41	1480.75	1299.27	2780.02
38	42	1515.50	1297.52	2813.02

OPTIMAL POLICY

ITEM NUMBER	DEMAND PER YEAR	AVE SPACE	SAFETY STOCK	ORDER QTY	VENDER	REORDER POINT
3	1000	7	1	320.	2(3)	3
2	1600	7	1	1120.	1(2)	3
1	3000	9	1	1000.	1(1)	3

CASE II: SAFETY STOCK STORED IN DYNAMIC SPACE

ITEM NUMBER 1
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
3	3	600.	3(1)	1.	855.06	855.06
4	4	800.	3(1)	1.	744.54	744.54
5	5	1000.	1(1)	1.	576.25	576.25
6	6	1200.	1(1)	1.	537.87	537.87
7	7	1400.	1(1)	1.	511.21	511.21
8	8	1600.	1(1)	1.	491.86	491.86
9	9	1000.	1(1)	1.	426.25	426.25
10	10	1200.	1(1)	1.	412.87	412.87
11	11	1400.	1(1)	1.	404.06	404.06
12	12	1600.	1(1)	1.	398.11	398.11
13	13	1800.	1(1)	1.	394.06	394.06
14	14	2000.	1(1)	1.	391.35	391.35
15	15	2200.	1(1)	1.	389.61	389.61

ITEM NUMBER 2
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
7	4	640.	3(2)	2.	378.64	1233.70
8	4	640.	3(2)	2.	378.64	1123.18
9	4	640.	3(2)	2.	378.64	954.89
10	5	800.	3(2)	2.	332.78	909.03
11	5	800.	3(2)	2.	332.78	870.65
12	6	960.	3(2)	2.	302.76	840.63
13	4	640.	3(2)	2.	378.64	804.89
14	5	800.	3(2)	2.	332.78	759.03
15	6	960.	3(2)	2.	302.76	729.01
16	7	1120.	1(2)	1.	273.84	700.09
17	8	640.	3(2)	2.	253.64	679.89
18	9	800.	3(2)	2.	232.78	659.03
19	9	800.	3(2)	2.	232.78	645.65
20	10	960.	3(2)	2.	219.42	632.29
21	10	960.	3(2)	2.	219.42	623.48
22	10	960.	3(2)	2.	219.42	617.53
23	10	960.	3(2)	2.	219.42	613.49
24	10	960.	3(2)	2.	219.42	610.77
25	10	960.	3(2)	2.	219.42	609.03

ITEM NUMBER 3
TABULATION

TOTAL SPACE	SPACE ALLOCATED	ORDER QTY	VENDER	SAFETY STOCK	COST	TOTAL COST
9	2	160.	2(3)	1.	753.16	1986.86
10	3	240.	2(3)	1.	561.08	1794.78
11	3	240.	2(3)	1.	561.08	1684.26
12	3	240.	2(3)	1.	561.08	1515.97
13	4	320.	2(3)	1.	465.87	1420.76
14	5	400.	2(3)	1.	409.41	1364.30
15	6	240.	2(3)	1.	352.74	1307.64
16	6	240.	2(3)	1.	352.74	1261.78
17	7	320.	2(3)	1.	309.62	1218.65
18	7	320.	2(3)	1.	309.62	1180.27
19	7	320.	2(3)	1.	309.62	1150.25
20	6	240.	2(3)	1.	352.74	1111.78
21	7	320.	2(3)	1.	309.62	1068.65
22	7	320.	2(3)	1.	309.62	1038.63
23	7	320.	2(3)	1.	309.62	1009.70
24	8	400.	2(3)	1.	284.41	984.50
25	8	400.	2(3)	1.	284.41	964.30
26	8	400.	2(3)	1.	284.41	943.44
27	9	480.	2(3)	1.	268.16	927.19
28	9	480.	2(3)	1.	268.16	913.81
29	9	480.	2(3)	1.	268.16	900.45
30	10	560.	2(3)	1.	257.03	889.32
31	10	560.	2(3)	1.	257.03	880.52
32	11	640.	2(3)	1.	249.10	872.58
33	12	720.	3(3)	1.	243.10	866.59
34	12	720.	3(3)	1.	243.10	860.64
35	13	800.	3(3)	1.	238.50	856.03
36	13	800.	3(3)	1.	238.50	851.98
37	13	800.	3(3)	1.	238.50	849.27
38	13	800.	3(3)	1.	238.50	847.52

FINAL TABULATION

TOTAL SPACE	+SS	RENTAL AND HANDLING COST	PROCUREMENT COST	TOTAL COST
9	13	526.75	1986.86	2513.61
10	14	561.50	1794.78	2356.28
11	15	596.25	1684.26	2280.51
12	16	631.00	1515.97	2146.97
13	17	665.75	1420.76	2086.51
14	18	700.50	1364.30	2064.80
15	19	735.25	1307.64	2042.89
16	20	770.00	1261.78	2031.78
17	21	804.75	1218.65	2023.40
18	22	839.50	1180.27	2019.77
19	23	874.25	1150.25	2024.50
20	24	909.00	1111.78	2020.78
21	25	943.75	1068.65	2012.40
22	26	978.50	1038.63	2017.13
23	26	978.50	1009.70	1988.20
24	27	1013.25	984.50	1997.75
25	29	1082.75	964.30	2047.05
26	30	1117.50	943.44	2060.94
27	31	1152.25	927.19	2079.44
28	32	1187.00	913.81	2100.81
29	33	1221.75	900.45	2122.20
30	34	1256.50	889.32	2145.82
31	35	1291.25	880.52	2171.77
32	36	1326.00	872.58	2198.58
33	37	1360.75	866.59	2227.34
34	38	1395.50	860.64	2256.14
35	39	1430.25	856.03	2286.28
36	40	1465.00	851.98	2316.98
37	41	1499.75	849.27	2349.02
38	42	1534.50	847.52	2382.02

OPTIMAL POLICY

ITEM NUMBER	DEMAND PER YEAR	AVE SPACE	SAFETY STOCK	ORDER QTY	VENDER	REORDER POINT
3	1000	7	1	320.	2(3)	3
2	1600	7	1	1120.	1(2)	3
1	3000	9	1	1000.	1(1)	3

APPENDIX III

COMPUTATIONAL ASPECTS

The computational aspects of the algorithms are dependent on the particular situation to which the model is being applied. There are, however, certain aspects of the dynamic programming formulation which, if considered, can result in more efficient execution of the algorithm. It is the purpose of this appendix to briefly examine those aspects.

The order in which the items are processed is the major factor which determines the number of rows in the dynamic programming tableau at any given stage. The number of rows in the tableau, at any given stage, is equivalent to the difference between the minimum and the maximum number of pallets required for all items thus far considered (including the current item). By processing the items with the smallest range requirements first, the number of rows in the dynamic programming tableau is kept to a minimum at each stage. This accomplishes two things. First, the number of rows in the tableau at each stage is kept to a minimum thus reducing search time in the early stages; secondly, if, as in any large application, the tableaus must be stored on an external medium, the total external storage is kept at a minimum.

The amount of data which must be stored at each stage to allow the final traceback through the tableaus is also of interest. Although the procurement policy could be stored for every feasible warehouse size at each stage, it is not necessary. All that is required is the cost of the best policy associated with each given warehouse size. The allocation of space can then be determined for each item, through traceback, which gives the minimum expected cost for the system. Once the space allocated to each item is known, the procurement policy can be recalculated. This recalculation takes extra time, but reduces the amount of data which must be stored.

In examining the search procedure at each stage the following was observed. For the first row in each tableau the vendor must be determined given that u_1 units of space are available. For the second row the vendor must be selected given that u_1+1 units of space are available. However to determine the supplier, given that u_1+1 units of space are available, requires considering allocation of u_1 and u_1+1 units of space. Since the best policy is already known given that u_1 units of space are available, it is only necessary to search the vendors given that u_1+1 units of space are available, find the best policy and compare it to the preceding row.

Since the determination of the minimum expected cost policy depends on the distributions of the usage during lead time, these distributions must be available. It is only necessary to have the distributions for a given item in core at any one time. If the range of usage during lead time is extremely large, it may be necessary to scale the distributions. The scaling would be dependent on the amount of core available and the accuracy required.

Although the dynamic programming formulation is quite time consuming for a computation aspect, it should be noted that the use of the algorithm should be quite infrequent. The algorithm does not have to be executed every time an order is placed, but is executed once to determine the procurement policy. Once the policy has been determined the algorithm is not required until another decision on procurement policy has to be made.

BIBLIOGRAPHY

1. Arrow, K. J., S. Karlin, and H. Scarf, Studies in the Mathematical Theory of Inventory and Production, Stanford, Calf.: Stanford University Press, 1958.

2. Arrow, K. J., T. Harris and J. Marschak, "Optimal Inventory Policy," Econometrica, XIX (1959), pp. 250-272.

3. Banks, J., A Hierarchy of Procurement Inventory Systems, Ph.D. Dissertation, Oklahoma State University, 1966.

4. Buchan, J. and E. Koenigsberg, Scientific Inventory Management, Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963.

5. Churchman, C. W., R. L. Ackoff, and E. L. Arnoff, Introduction to Operations Research, New York: Wiley and Sons, 1957.

6. Dvoretzky, A., J. Kiefer, and J. Wolfowitz, "The Inventory Problem I: Case of Known Distributions of Demand," Econometrica, XX (1952), pp. 187-222.

7. Dvoretzky, A., J. Kiefer and J. Wolfowitz, "The Inventory Problem II: Case of Unknown Distributions of Demand." Econometrica, XX (1952), pp. 450-466.

8. Dvoretzky, A., J. Kiefer and J. Wolfowitz,"On the Optimal Character of the (s, S) Policy in Inventory Theory," Econometrica, XXI (1953), pp. 586-596.

9. Fabrycky, W. J., Optimal Inventory Policy for the Multi-Source Item, Ph.D. Dissertation, Oklahoma State University, 1962.

10. Fabrycky, W. J., "Procurement and Inventory Theory Vol.I - The Multi-Source Item Concept," Oklahoma State University Engineering Bulletin #140, 1964.

11. Fabrycky, W. J. and J. Banks, "Procurement and Inventory Theory Vol. III - The Multi-Item, Multi-Source Concept," Oklahoma State University Engineering Bulletin #146, 1965.

12. Fabrycky, W. J. and J. Banks, Procurement and Inventory Systems, New York: Reinhold Publishing Corp., 1967.

13. Fetter, R. B. and W. C. Dalleck, Decision Models for Inventory Management, Homewood, Ill.: Richard D. Irwin, Inc., 1961.

14. Gourary, M., R. Lewis, and F. Neeland, "An Inventory Control Bibliography," Naval Research Logistics Quarterly, III (December 1959), pp. 295-304.

15. Hadley, G. and T. M. Whitin, Analysis of Inventory Systems, Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963.

16. Hanssmann, F., Operations Research in Production and Inventory Control, New York: Wiley and Sons, 1962.

17. Harris, F., Operations and Cost, Factory Management Series, Chicago: A. W. Shaw Co., 1916.

18. Holt, C. C., et.al., Planning Production Inventories and Work Forces, Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1960.

19. Inglehart, D. L., "Recent Results in Inventory Theory," Journal of Industrial Engineering, XVIII (Jan. 1967), pp. 48-51.

20. Inglehart, D. L., "Recent Developments in Stochastic Inventory Control," Annual Meeting of ORSA, June 19, 1969.

21. Karlin, S., The Mathematical Theory of Inventory Processer in Modern Mathematics for the Engineer, New York: McGraw-Hill, 1960.

22. Lele, P. T., and E. A. Siecienski, "Multiple-Item, Multiple-Source Procurement Policy with several Warehouses," 40th Meeting of Operations Research Society, Los Angeles, Calif., October 1971.

23. Raymond, F. E., Quantity and Economy in Manufacturing, New York: McGraw-Hill, 1931.

24. Scarf, H., D. Gilford, and M. Shelly (Eds.), Multi-Stage Inventory Models and Techniques, Stanford, Calif.: Stanford University Press, 1963.

25. Scarf, H., A Survey of Analytical Techniques in Inventory Theory, Ch. 7 in Multistage Inventory Models and Techniques, H. Scarf, D. Gilford, and M. Shelly (Eds.), Stanford, Calif.: Stanford Press, 1963.

26. Siecienski, E. A., "Simulation of the Inventory Space Factor for Various Operating Policies," Proceedings of the Fifth Conference on Applications of Simulation, December 1971.

27. Starr, M. K. and D. W. Miller, Inventory Control: Theory and Practice, London: Prentice-Hall, Inc., 1962.

28. Veinott, A. F., Jr., "The Status of Mathematical Inventory Theory," Management Science, XXII (1966), pp. 745-777.

29. Whitin, T. M., The Theory of Inventory Management, Princeton, N.J.: Princeton University Press, 1953.

Private Communication

30. Monroe, S. Private Communication (with permission).

VITA

PERSONAL HISTORY

Name:	James L. Kingsley
Date of Birth:	November 21, 1946
Place of Birth:	El Dorado, Kansas
Wife:	Georgia A. (Farr) Kingsley
Parents:	Mitchel L. and Hattie I. Kingsley

EDUCATIONAL BACKGROUND

Auguata High School — Graduated 1964
Auguata, Kansas

Lamar University — 1964-1968
Beaumont, Texas
Bachelor of Science in Mathematics

University of Missouri — Attended 1969
Kansas City, Missouri

Lehigh University — 1970-1972
Bethlehem, Pennsylvania
Candidate for Master of Science in Industrial Engineering

HONORS

Alpha Pi Mu, Industrial Engineering Honor Society

Phi Kappa Phi, Honor Society

Pi Mu Epsilon, Mathematics Honor Society

PROFESSIONAL EXPERIENCE

Western Electric Company, Inc.
Kansas City Works
Lee's Summit, Missouri

Designer, Information Systems Staff	1968-1969
Member, Information Systems Staff	1969-1970

Western Electric Company, Inc.
Engineering Research Center
Princeton, New Jersey

Member, Information Systems Staff	1970-1972