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The determination of technology coefficients in applications of leontief input-output to accounts of a single firm

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THE DETERMINATION OF TECHNOLOGY COEFFICIENTS IN APPLICATIONS OF LEONTIEF INPUT-OUTPUT TO ACCOUNTS OF A SINGLE FIRM

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ABSTRACT

Leontief input-output models having utility in studies of national and regional economics are reviewed for application to accounting and resource allocation in a single firm. In particular, the derivation from industrial accounts of technology coefficients which are essential to such models is considered from the viewpoints of (1) the appropriateness of underlying assumptions in the single industry environment, and (2) the means of acquiring these coefficients from industrial accounts.

Fundamental accounts reasonably satisfying the homogeneous requirements are identified. A translation from the industrial accounting basis to model format is given including partial treatment for multiple outputs. The basic assumption that inputs equal outputs for a given period is relieved by formulation of a dynamic model treating process delays. The dynamic model formulated is a structural lag model as contrasted with the Leontief Dynamic Input-Output model containing a stock-flow ratio, but no lags. The significance of this model is that industrial resource allocation might be accommodated utilizing available accounting and production control measures.

Applications involving the replacement of traditional double entry, T-Account representations by a transactions matrix are conceptualized. Conclusions and recommendations for continued study are given.

CHAPTER I

INTRODUCTION

Statement of the Problem

Increasing emphasis is being placed on centralized resource management within the industrial firm. Such emphasis is a logical consequence of (1) larger, more complex product with associated rapid obsolescence, and (2) current feasibility of systems optimization planning made possible by advances in information processing and communication. The means of accomplishing optimum (or near optimum) resource allocation are suggested in the recent development of linear programming, and more general mathematical programming models.

Two problems seem to have limited the application of mathematical programming to large and complex structures such as industrial firms - (1) the number of variables and equations or inequations required for an adequate model are excessive with respect to computational capabilities, and (2) the acquisition of process measurements or parameters has been inadequately resolved. The problem of acquiring process measurements and parameters which might be used in resource allocation models is the central problem for this thesis. The central problem is developed within the context of Leontief input-output concepts which include considerations for aggregative or consolidated treatment of accounts suggesting relief for the number of variables and equations or inequations normally encountered in mathematical programming formulations. The search for an approach to the problem of resource allocation within a large industrial firm has led the author to a review of the earliest developments of linear programming--developments undertaken by the U. S. Air Force in the logistics planning area, 1942-1953 (33), which fostered the development of linear programming by George B. Dantzig, such development including the earlier work of Wassily Leontief on input-output analysis (26). The fundamental Leontief models are of particular interest though they are developed in the broader view of national economy because their basis in accounting theory suggests their application to accounts of a firm. Furthermore, these models include a basic consideration of both of the problems inhibiting application of mathematical programming through principles of:

(1) An accounting transaction matrix

(2) An account consolidation scheme.

(3) A set of process flow parameters known as "technology coefficients."[1]

These principles hold for the Leontief statical model. The Leontief dynamic model extends this list by addition of a stock-flow ratio. Dorfman, Samuelson, and Solow in (9) have shown that the Leontief statical model is a special case of a more general linear programming model such that there is only one activity per commodity, such that input plus consumption equals total output (rather than equal or less than), and such that capacities are assumed infinitely

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[1]"Technology coefficients" infer an economic production function relating outputs of a technical process to its inputs through fixed coefficients. One such function is the Leontief function in (26). See Appendix.

adjustible, the latter preventing objective maximization. The situation changes in the dynamic Leontief formulation where the stock-flow consideration implies a capacity restraint.

A development leading from Leontief's formulations and principles is believed to be significant in that it does not require the LP formulation with objective maximization, and, most importantly it treats the resource allocation problem through use of industrial cost accounting measurements and production control variables.

Thesis Objectives

A review of the Leontief statical input-output model outlined herein as Chapter II revealed that technology coefficients of the model are critically dependent upon five basic assumptions. This review developed the general objective of this thesis--the determination of technology coefficients in applications of Leontief input-output models to accounts of a single firm. The development of a resource allocation model within an industrial firm on the basis of the Leontief models depends on justification of these assumptions, or the development of means to extend the statical model to the particular accounting practices of a firm.

The model assumes the availability of a set of fundamental accounts--fundamental in the sense that diverse products measured in each such account are reasonably homogeneous, and such that the total set is representative of the total economy. By reasonably homogeneous we mean that the products included in one of these accounts have reasonably identical input requirements per dollar of output. In addition the number of such accounts must be within the bounds of feasible computer processing. The availability of such a set is not to be taken for granted. In Chapter III, review of a particular firm, the Western Electric Company suggests that sub-classifications to the general ledger work-in-process account do reasonably represent the total economy based on the practice of allocating burden (assignment of overhead as non-base and allowance on base hours within standard costing, plus inclusions in hourly rate), and that a specialized class of K-Orders accounts meet the needs of homogeneous product groupings, and that the number of such accounts is not excessive.

The model further assumes what the economist refers to as constant returns to scale, that is, homogeneity of degree one of a production function (See Appendix). Since the firm under consideration utilizes a standard cost accounting system in which set-up charges are included in the standard cost computation it appears reasonable to accept this assumption as it is one used effectively in current planning. Under homogeneity of degree one, if we double output we double all inputs. If we reduce output to any fractional level, we reduce all inputs by like fraction. Set-up costs are included in standard costs for a given production rate. Provided this rate is maintained, assumption of a linear relation through zero is satisfactory. If the rate changes the non-linear portions shows up as a volume variance in an associated variance account. When a volume variance becomes significant it is common practice to recompute the standard at the new rate (add average variance). Provided that standard costs are recomputed with sufficient frequency or, that transactions measured are standard cost plus variation the assumption will hold. This is not to say that this may not be a problem for further consideration but more critical problems exist in the following areas which represent the primary contribution of this thesis.

The third assumption is that of a single form of output for a given account. In Chapter IV a formulation based on the set of K-Orders accounts selected reveals the multiple nature of outputs occurring, and proposes a means by which one of these, scrap outputs, might be resolved; and the second of three such outputs, inventory variation, might be minimized in its effect. The effect of investment variation (which equates to inventory variation in industrial accounting) is well known and is the subject of Leontief's more general dynamic model contained in (27), and Hubble and Ekey's model in (22). Adaptation of these models to the model evolved herein is left for further study.

The two remaining assumptions--that inputs equal outputs for a given period, and that no process delays occur--are related assumptions which are justified by Walrasian general equilibrium in Leontief models. A status of equilibrium is not evident in industrial accounts which involve shorter periods than those utilized by Leontief. The objective in Chapter V is the development of a dynamic formulation for acquiring technology coefficients under varying demands and where production intervals are longer than accounting periods. Such development has a similarity to developments of both Vazsonyi in (32), and Homer in (20) but with the stronger requirement for periods longer than a single day and additional need for a cost distribution.

The overall objective is to develop an approach to the problem of central resource allocation in an industrial firm, that is, to distribute a future periods demand for multiple product over a set of accounts representing resources of labor, material, and intermediate products in such a manner as to take into consideration the limited capacity of individual activities represented by t he accounts involving a displacement in time. In Chapter VI, systems application extended to an accounting system design, accounting language development, and report generation is considered.

In Chapter VII conclusions of the study are enumerated with suggested areas for continuing study.

CHAPTER II

THE LEONTIEF STATICAL INPUT-OUTPUT MODEL

Accounting Basis

In study of the American economy during the period 1919-1939, Professor Wassily Leontief, the Henry Lee Professor of Economics, Harvard University, utilized an accounting basis for construction of a model permitting empirical analysis of national or regional economics (26). In his words:

> "The conceptual basis of the subsequent statistical analysis is rather simple. The economic activity of the whole country is visualized as if covered by one huge accounting system. Not only all branches of industry, agriculture, and transportation, but also the individual budgets of all private persons are supposed to be included within the system. Each business enterprise as well as each individual household is treated as a separate accounting unit. A complete bookkeeping system consists of a large number of different types of accounts. For our particular purpose, however, only one is important: the expenditure and revenue account. It registers on the credit side the outflow of goods and services from the enterprise or household (which corresponds to total receipts or sales) and on the debit side the acquisition of goods or services by the particular enterprise or household (corresponding to its total outlays). In other words, such an account describes the flow of commodities and services as it enters the given enterprise or household at one end and leaves it at the other. In contrast to a balance sheet, this type of account is not related to a single instant but rather to a period of time, say a year, a month, a week."

Leontief describes a matrix representation of cumulative accounting transactions for one period of time. Unlike the conventional T-account representation with debits on the left of the T and credits on its right, and where for each credit there is an associated debit (double entry) in another T-account; this is a single entry record where the debited account is read from the row, the credited account read from the column. Each account from the total set of accounts is represented by a row and a column. The matrix which he describes is theoretical in the sense that accounts are considered for each individual household and each business enterprise. Since his conception was that all accounts were to be represented with no external demands to be made on this system of accounts, this model was said to be closed. In subsequent development, he represented the household as a demand sector also supplying labor input, giving rise to an open, or exogenously determined model which will be considered in this article.

Since such a theoretical model is totally infeasible due to the enormous number of accounts to be represented, he proceeds with the development of a consolidation scheme by which two or more accounts might be summed by addition of their corresponding row and column entries which would yield for the first time entries on the matrix diagonal (representing internal transactions within the consolidated accounts; there would be no internal transactions on the fundamental accounts theorized). He proceeds to show that internal transactions of a consolidated account might be eliminated by reduction of corresponding row and column totals yielding a net basis for transactions.

Given a consolidated transactions matrix of this type, and under the assumption that the total outputs of a given account are equal to its total inputs for the period considered, Leontief divides the inputs to an account represented by the transactions in its column by its total output, yielding a matrix of entries defined to be technology coefficients. The tacit assumption of the model is that this technology matrix becomes useful in estimating the transactions which would occur in the set of accounts under alternate external demand requirements.

Related Models and Studies

In addition to the original statical closed model formulated in (26), various adaptations including the statical open model, and dynamic models appear in (4), (5), (9), (13), (14), (15), (21), (22), (27). These adaptations have in common with the original model the determination of a matrix of technology coefficients as flow parameters. Of particular significance are Rosenblatt's "On Some Aspects of Models of Complex Behavioral Systems" (30), Hubble and Ekey's consideration of "The Application of Input-Output Theory to Industrial Planning and Forecasting" (22), Charnes, Cooper, and Ijiri's "Breakeven Budgeting and Programming to Goals" (5), and the critical treatment given to the model by Dorfman, Samuelson, and Solow in "Linear Programming and Economic Analysis" in (9).

Hubble and Ekey note the significant problem of translating a set of industrial accounts into Leontief equivalents (while treating burden distribution via the model). Dorfman, Samuelson, and Solow review the problems of the statical assumption. Charnes, Cooper, and Ijiri review possible industrial extensions of the model under goal programming.

We are concerned here with the assumptions underlying the development of technology coefficients. These may be brought out by review of the development of the statical Leontief model.

Formulation

We consider a set of n fundamental Leontief accounts representing a closed economy (no external inputs or outputs). The cumulative transactions in such a economy over a given period of time might be recorded in a transactions tableau as illustrated in Figure 1.

Outputs (Credits)

		1	2	3	4	.	.	n	Total
	1	0	x_{12}	x_{13}	x_{14}	.	.	x_{1n}	$\sum_{j=1}^{n} x_{1j}$
	2	x_{21}	0	x_{23}	x_{24}	.	.	x_{2n}	$\sum_{j=1}^{n} x_{2j}$
Inputs (Debits)	3	x_{31}	x_{32}	0	x_{34}	.	.	x_{3n}	$\sum_{j=1}^{n} x_{3j}$
	4	x_{41}	x_{42}	x_{43}	0	.	.	x_{4n}	$\sum_{j=1}^{n} x_{4j}$
	.	.	.	.	.	.	.	x	.
	.	.	.	.	.	.	.	.	.
	n	x_{n1}	x_{n2}	x_{n3}	.	.	.	0	$\sum_{j=1}^{n} x_{nj}$
Total		$\sum_{i=1}^{n} x_{i1}$	$\sum_{i=1}^{n} x_{i2}$	$\sum_{i=1}^{n} x_{i3}$				$\sum_{i=1}^{n} x_{in}$	

Figure 1. Fundamental Closed Accounting Transactions Tableau

Since this set of accounts is considered to be fundamental, no transactions are recorded for x_{ij}, $i=j$. Since a fundamental level of accounts exists by definition only, we may consider the effect of a lesser number of accounts obtained by consolidation, that is, we may hypothesize a fundamental set, then successively consolidate such a set to consider the effect of measurements where the true fundamental set is not subject to measurement. Consider the case were accounts 2 and 3 are to be consolidated from the transactions data. The new tableau would be that of Figure 2, where corresponding row and column entries are simply added.

		Outputs (Credits)					
		1	2&3	4	. .	n	Total
	1	0	$x_{12}+x_{13}$	x_{14}		x_{1n}	$\sum_{j=1}^{n} x_{1j}$
	2&3	$x_{21}+x_{31}$	$x_{32}+x_{23}$	$x_{24}+x_{34}$		$x_{2n}+x_{3n}$	$\sum_{j=1}^{n} x_{2j}+x_{3j}$
Inputs (Debits)	4	x_{41}	$x_{42}+x_{43}$	0		x_{4n}	$\sum_{j=1}^{n} x_{4j}$
	.	.		.			
	.	.		.			
	n	x_{n1}	$x_{n2}+x_{n3}$	x_{n4}		0	$\sum_{i=1}^{n} x_{nj}$
Total		$\sum_{i=1}^{n} x_{i1}$	$\sum_{i=1}^{n} x_{i2}+x_{i3}$	$\sum_{i=1}^{n} x_{i4}$		$\sum_{i=1}^{n} x_{in}$	

Figure 2. Consolidated Gross Tableau

In this tableau we measure the internal transactions of the combined account as a gross amount and we consider the tableau a gross transaction representation.

A net transaction representation might be obtained by eliminating the diagonal entry, subtracting it from its row and its column.

By successive consolidations one might group the initial set of fundamental accounts into sectors which might represent various industrial sectors, household sectors, or government sectors as in Leontief studies. Limitations on accounts which may be consolidated in this manner will appear in the subsequent development of the model, however.

Consider, however, that we have obtained a netted out consolidated array in which one of the sectors represents households, such households also supplying labor inputs into this system. We may represent households as an external demand on the system, returning an equivalent amount of labor as shown in Figure 3, in which the economy is represented by two internal sectors and the households as an external demand sector.

Inputs (Debits)

	1	2	Households	Total Outputs
1	0	x_{12}	Y_1	$x_{12}+Y_1 = X_1$
2	x_{21}	0	Y_2	$x_{21}+Y_2 = X_2$
Labor	x_{01}	x_{02}	0	$x_{01}+x_{02}= X_0$
Total	$x_{01}+x_{21}$	$x_{02}+x_{12}$	Y_1+Y_2	

Figure 3. Demand Sector Representation

It is clear that total outputs are equal to total inputs, and that we assume that Households make no demands on labor. We may write a system of linear equations on this structure, replacing the Total Outputs with the equivalent notation X_i:

$$x_{12} + Y_1 = X_1$$

$$x_{21} + Y_2 = X_2$$

$$x_{01} + x_{02} = X_0$$

First we wish to show that the third equation is dependent We have under assumption that:

$$x_{01} + x_{21} = x_{12} + Y_1$$

$$x_{12} + x_{02} = x_{21} + Y_2$$

By addition, $x_{01} + x_{02} + x_{12} + x_{21} = x_{12} + x_{21} + Y_1 + Y_2$

or, $x_{01} + x_{02} = Y_1 + Y_2$

That is, the last expression is a linear combination of the first two equation and therefore dependent. Thus, the first two equations represent the system and this gives rise to the Leontief open model with the last equation eliminated. The development is as follows.

We may consider a system of accounts in which the transactions for a given period are recorded as in Figure 4, where, it is convenient to consider producing departments and consuming departments in view of ultimate industrial use possibilities.

		Consuming Departments 1 2 3 . . j . . n	External Demand	Total Output
Producing	1		Y_1	X_1
Departments	2		Y_2	X_2
	3		Y_3	X_3
	i	x_{ij} (under j)	Y_i	X_i
	n		Y_n	X_n

Figure 4. Accounting Transactions Tableau

Given that transactions might be cumulatively recorded in this manner, including the external transactions Y_i (X_i is simply their sum) we wish to develop a model permitting determination of the X_i for various demand sets, Y. The recorded transactions may be written in linear equations:

$$X_1 - x_{12} - x_{13} - \cdots - x_{1n} = Y_1$$

$$-x_{21} + X_2 - x_{23} - \cdots - x_{2n} = Y_2$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$-x_{n1} - x_{n2} - x_{n3} \cdots + X_n = Y_n$$

For convenience the total outputs, X_i, have been placed in the diagonal openings (made possible by the fact that $x_{ij} = 0$ for $i = j$) retaining a square matrix.

We may define an input-output coefficient, a_{ij}, as follows:

$$a_{ij} = \frac{x_{ij}}{X_j}$$

The a_{ij} represent the input of account i for one unit of output of account j, X_j being the total output of account j. Then:

$$x_{ij} = a_{ij} X_j$$

Substituting the expression for the x_{ij} we obtain:

$$\begin{aligned} X_1 - a_{12}X_2 - \ldots\ldots - a_{1n}X_n &= Y_1 \\ -a_{21}X_1 + X_2 - \ldots\ldots - a_{2n}X_n &= Y_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ -a_{n1}X_1 - a_{n2}X_2 - \ldots\ldots + X_n &= Y_n \end{aligned}$$

This last system of equations may be written in matrix algebra form as:

$$\begin{bmatrix} 1 & -a_{12} & \ldots\ldots & -a_{1n} \\ -a_{21} & +1 & \ldots\ldots & -a_{2n} \\ \cdot & & & \\ \cdot & & & \\ -a_{n1} & -a_{n2} & \ldots\ldots & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_n \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ Y_n \end{bmatrix}$$

Letting I equal the n x n identity matrix, and A represent the n x n matrix of input-output coefficients, the equivalent matrix representation is: $(I - A) X = Y$

Provided that the Leontief matrix, (I - A) has an inverse:

$$X = (I - A)^{-1} Y$$

This last equation might be used for determining the set of transactions X, resulting from a given demand set Y, provided that the assumptions of the model hold.

Assumptions Relevant to Coefficient Determination

We now wish to review these assumptions as they are implicit in use of the model, and constitute its limitations.

(1) We have assumed that the total inputs to an account are equal to its total output, thus:

$$\sum_{i=1}^{n} x_{ij} = \sum_{k=1}^{n} x_{ij} + Y_i$$

and $$Y_i = X_i - \sum_{j=1}^{n} x_{ij}$$

therefore, $$\sum_{i=1}^{n} x_{ij} = x_j$$

or, total inputs = total outputs

This requires that investment or inventory remains stable during the period treated or that it be incorporated within elements of demand vector.

(2) We have assumed that all inputs will vary linearly with output, since: $x_{ij} = a_{ij} X_j$

If $X_j = 0$, then $x_{ij} = 0$.

This is inconsistent with any fixed charges of production such as set-up charges. The economist notes this as constant returns to scale, or equivalently, homogeneity of degree one.

(3) We have assumed that the x_{ij} for $i = j$ are indeed zero, that is, are capable of elimination.

(4) We have assumed a single form of external demand (households in Leontief's application, which return an equivalent amount of labor).

(5) We have assumed no process delays.

Consider a process where two intermediate products supply a single final product as shown by Figure 5, where activity intervals are equal multiples of accounting intervals.

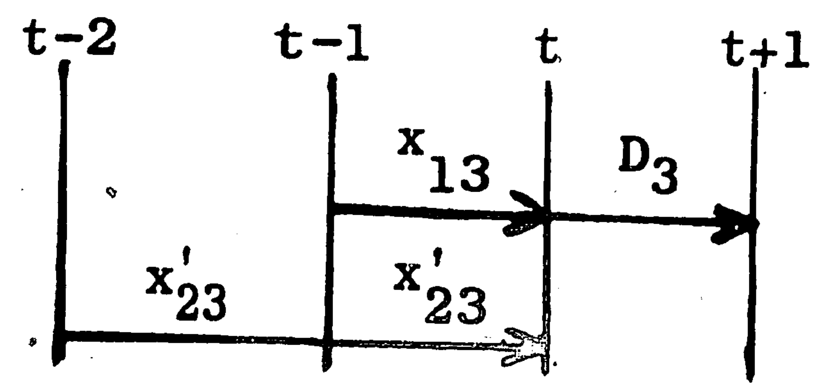


Figure 5 Process Delay

The demand, D_3, is that demand for a single period. Inputs from the first intermediate process occur over one interval immediately preceeding demand. Inputs from the second intermediate process are accumulated over two periods in this instance. Provided that D_3 is stable over several months it is clear that the sum of x'_{23}'s for a single period (there will be two of them) will yield a correct measurement. If the demand is not stable we are constrained to use a dynamic model treating the associated delays.

The first assumption with the resultant assumption of stable inventory has been approached by the Leontief dynamic model (26), (9), which creates a separate matrix for investment variation, and by Hubble and Ekey in (22). We retain this assumption through discussion contained in subsequent chapters III, IV, and V suggesting its inclusion by extension of the dynamic model given in Chapter V in further studies. The assumption of linear dependance (homogeneity of degree one of a production function) is a normal one for standard cost accounting processes and will be accepted on this basis.

The assumption that x_{ij} for $i = j$ is zero constitutes the first major aspect of the problem of determining appropriate technology coefficients from industrial accounts. It may be resolved by selection of a set of reasonably homogeneous fundamental industrial accounts. For such accounts the elimination process of Leontief is automatically incorporated. This is the subject of Chapter III.

The assumption of a single form of external demand is related to the problem of translating a set of industrial accounts into equivalent Leontief accounts and tableau. It constitutes the second problem of determining appropriate technology coefficients from industrial accounts. A means of performing this translation and reconciling difficulty is reviewed in Chapter IV.

The assumption of no process delay is a third aspect of the problem of determining appropriate technology coefficients from industrial accounts. It is a major problem which requires dynamic adaptation of the model. Such adaptation is reviewed in Chapter V under the specialized condition of unilateral flow (that is, for x_{ij} and given i and j, $x_{ji} = 0$).

CHAPTER III

ESTABLISHMENT OF FUNDAMENTAL ACCOUNTS

Structive Analysis of One Firm

In application of Leontief's models, the economy to be represented is studied structurally by sector so that consolidations, or aggregations as they are called, represent reasonably homogeneous groupings having a like "technology", or set of input-output coefficients. Aggregations within a firm might be expected to occur around logical groupings within the management structure or product structure heirarchy. A set of accounts might be defined at various levels of this heirarchy in accord with the analysis sought, and furthermore, the number of such accounts should be within the feasible bounds of efficient computer processing. Finally, it is essential to determine a level for such accounts within the heirarchy in which the accounts are sufficiently homogeneous to permit a check of the reasonableness of the assumptions. For these purposes, a review of the organizational structure of the Western Electric Company was undertaken to determine in what manner a model might be developed.

The firm under consideration is a wholly owned subsidiary of the AT&T Corporation, Western Electric Co., Inc. - the manufacturing and supply arm of the Bell System, with additional responsibilities for services to telephone companies. Its organizational structure is undergoing change around regions, and around "product centers", however, the prior status organization is believed to be adequate for the immediate study.

The company has been managed through an executive policy committee inclusive of nine vice-presidents representing the nine divisions of the company, the thence through some 100 administrative officers and assistants including the managers at various locations.

The levels of supervision have been Chief Executive Officer, Executive Officer, Administrative Officer, Assistant Administrative Officer, Superintendent, Assistant Superintendent, Department Chief, and Section Chief, embracing both line and staff functions. The Superintendent level is the fourth level of management and includes managers of distribution houses, installation areas, manufacturing shops, as well as certain staff managers. The Superintendent is the chief manager except at various major works.

The major divisions have been:

Manufacturing	Legal & Patent
Service	Finance
Administration	Personnel and Public Relations
Engineering	Organizational Planning
Defense Activities	

While aggregate sectors, as accounts, under the process of reorganization might be expected to encompass all or most of these divisions, the primary concern of this article is for that area known as Manufacturing, representing 63% (approximately) of the people in the firm. Subsequent consideration of Service and Defence Activities would represent 95% of the people in this system.

The Manufacturing Division, as the first subset under consideration, manufactures roughly 50,000 items ranging from semi-conductors (transistors, diodes, etc.), switches, and relays at the Allentown Works, Allentown, Pennsylvania to central office switching equipment at the Hawthorne Works, Chicago, Illinois. There are 11 of these major works locations plus associated plants in three manufacturing regional areas.

A cursory review of organization numbers and functions as contained in the company's telephone directory suggests that products might be aggregated (within the manufacturing division) at departmental level within the line organization yielding approximately 250 account sectors, with an approximate 30% addition for non-line sectors. The lower level of aggregation is significant to the eventual use of a resulting matrix. Matrices up to 450 x 450 have been treated in the federal governments PARM system (33), where the matrices are predominantly right upper triangular, and any entries below the diagonal are treated by an approximation technique.

For purposes of this study a selective study has been made of one subset to the manufacturing division, that is one works location. It is anticipated that if a satisfactory model can be generated for this works location, one might assume that a similar procedure might be used for other works.

Accounting Structure Relationship

From an accounting viewpoint, each works location within the manufacturing division maintains on approval of headquarters a set of general ledger accounts. These accounts include the normal subsets of plant accounts, current assets, deferred assets, accounts payable, labor, cash recievals, and work-in-process, profit and loss, and operating variation. Standard and variance cost accounts are maintained almost exclusively. Within the accounting system we are primarily concerned with the work-in-process account, and cost subclassifications thereof which may represent an operating or line department. Such accounts are believed representative of the total economy since service or non-line operations are loaded onto standard costs accounted for in such accounts.

The works location selected for study produces a most fundamental product set (diodes, transistors-components) and therefore should permit a reasonable check on the model. There are at this location roughly 27 operating or line departments of interest. It has been found convenient at this location to generate as subclasses to the work-in-process account approximately 41 merchandise or K-Orders representing developed product groupings. Incoming orders are assigned to these K-Orders under the criterion:

1. Like configuration.
2. Similar stage of development.
3. Similar process requirements around work centers.

These criteria should adequately provide for the homogeneous requirements of Leontief aggregation groups. It should be noted that in addition to the K-Orders, there are a group of some 200 so-called direct orders which represent for the most part work which is continuing to undergo development in conjunction with the design group. Since these orders do not constitute a large amount of the net shop output, they are excluded from consideration here.

The existence of K-Orders provides in this instance an empirical basis for study and development. They represent the total economy under consideration based on the practices of allocating burden or load. Provided that they exist we may reasonably assume that x_{ij} for $i = j$ are equal to zero by prior elimination, by definition in this instance, thus resolving the first problem of establishing coefficients.

CHAPTER IV

TRANSLATION OF INDUSTRIAL ACCOUNTS INTO MODEL FORM

We now consider the problem of translating industrial accounts into Leontief statical open form, including the problem of multiple outputs. The problem may be clarified by considering the set of K-Order subclasses to the work-in-process account previously isolated as homogeneous groupings. We will defer the problem of process delay until the next chapter. Here we assume that a statical model applies.

It is proposed that the monthly closings of the K-Order subclasses to the work-in-process account might be utilized to construct a transaction tableau from which a technology matrix could be generated.

The K-Order, when represented as a T-Account, includes the class of entries shown in Figure 6, below:

K-Order Number

Debit	Credit
1. Labor	5. Good Product
2. Load	6. Scrap
3. Other K-Orders	7. Inventory (Balancing Entry)
4. Raw Materials	

Figure 6. K-Order Representation as a T-Account

We note that output in this instance includes three items - good output, scrap, and inventory. Similarly, there are three external inputs to the system - labor, load, and raw materials.

The entries are clarified as follows:

1. Labor - This represents the number of direct hours worked multiplied

by the current standard hourly rate inclusive of a non-basic or allowance factor.

2. Load - This represents the number of hours worked times a current load rate consisting of what is variously called burden, expense, or overhead as specially defined for this firm.

3. Other K-Orders - This represents the debit corresponding to a credit on some other K-Order when product is transferred between K-Orders.

4. Raw Materials - This represents a debit when raw or, exceptionally, outside sourced components are recieved into stock against the K-Order.

5. Good Product - This represents the quantity of delivered product times their standard cost value at present, however, separately accounted variance could be included with this entry if desired.

6. Scrap - This represents the credit taken when scrap output is accounted against the K-Order.

7. Inventory - This represents a balancing entry which may either be positive or negative for the period. (Separate inventory estimates are determined each month physically however.)

We may consider a hypothetical example of three K-Order accounts to outline a procedure for generating technology coefficients. Let these K-Orders have period book closure as represented in Figure 7.

	K-X			K-Y		K-Z	
Labor	36.2	85.9 Good		22.7	76.0	28.1	156.5
Load	31.3	11.7 Good to K-Y		26.3	0.0	47.5	0.0
Other K.	0.0	15.0 Scrap		11.7	12.4	0.0	22.3
Raw Mt.	149.6	4.5 Inventory		12.4	(15.3)	111.8	8.6
	117.1	117.1					
	-11.7	-11.7					
	105.4	105.4		73.1	73.1	187.4	187.4

Figure 7 Hypothetical K-Order Closings

The internal transaction has been subtracted from the first account to aid the following discussion. A transactions matrix might be constructed from these closing statements, as follows:

We construct an equivalent tableau representation.

	Labor	Load	Raw Mtl	K-X	K-Y	K-Z	External Demand	Good	Scrap	Inv Adj
Labor	0	0	0	36.2	22.7	28.1	0	0	0	0
Load	0	0	0	31.3	26.3	47.5	0	0	0	0
Raw Mtl	0	0	0	49.6	12.4	111.8	0	0	0	0
K-X	0	0	0	0	11.7	0	105.4	97.6	15.0	4.5
K-Y	0	0	0	0	0	0	73.1	76.0	12.4	(15.3)
K-Z	0	0	0	0	0	0	187.4	156.5	22.3	8.6

In this table the Good Output, Scrap, and Inventory Balancing entry sums to a T-Account output which we shall represent as External demand. Multiple outputs of this type require that external demands on the shop be factored by a scrap allowance factor developed from previous months data before use as External Demand in any resultant model. Inventory adjustment creates an error in derived coefficients when future demands will not sustain a similar rate of change. To obtain coefficients one might summarize several months data to reduce perturbations in inventory.

By adding the external demand to the intermal transactions one may develop a total output and rearrange data as follows:

	Total Output	Labor	Load	Raw Mtl	K-X	K-Y	K-Z	External Demand
Labor	87.0	0	0	0	36.2	22.7	28.1	0
Load	105.1	0	0	0	31.3	26.3	47.5	0
Raw Mtl	173.8	0	0	0	49.6	12.4	111.8	0
K-X	117.1	0	0	0	0	11.7	0	105.4
K-Y	73.1	0	0	0	0	0	0	73.1
K-Z	187.4	0	0	0	0	0	0	187.4

In this form we may now recognize the desired set of linear equations.

	Labor	Load	Raw Mtl	K-X	K-Y	K-Z	External Demand
Labor	87.0 -	0 -	0	- 36.2 -	22.7 -	28.1 =	0
Load	- 0	+105.1 -	0	- 31.3	-26.3 -	47.5 =	0
Raw Mtl	- 0	- 0 +	173.8	- 49.6	- 12.4 -	111.8 =	0
K-X	- 0	- 0 -	0	+117.1	-11.7 -	0 =	105.4
K-Y	- 0	- 0 -	0	- 0	+73.1 -	0 =	73.1
K-Z	- 0	- 0 -	0	- 0	- 0	+187.4 =	187.4

By dividing each column by its diagonal entry, and placing this entry in a vector for multiplication, we may convert this system into an equivalent matrix multiplication form:

$$\begin{bmatrix} 1 & 0 & 0 & -.3091 & -.3105 & -.1499 \\ 0 & 1 & 0 & -.2673 & -.3598 & -.2535 \\ 0 & 0 & 1 & -.4236 & -.1696 & -.5966 \\ 0 & 0 & 0 & 1 & -.1600 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 87.0 \\ 105.1 \\ 173.8 \\ 117.1 \\ 73.1 \\ 187.4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 105.4 \\ 73.1 \\ 87.4 \end{bmatrix}$$

This last form is equivalent to the Leontief expression:

$$(I - A)\ X = Y$$

where I is the 6 x 6 identity matrix, and A is the 6 x 6 technology matrix of input-output coefficients (the left hand matrix above with 1's replaced by zeros).

Provided that (I - A) has an inverse, then:

$$X = (I - A)^{-1}\ Y$$

The matrix will have an inverse based on diagonal ones, therefore,

$$X = \begin{bmatrix} 1 & 0 & 0 & .3091 & .3600 & .1499 \\ 0 & 1 & 0 & .2673 & .4026 & .2535 \\ 0 & 0 & 1 & .4236 & .2374 & .5966 \\ 0 & 0 & 0 & 1 & .1600 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} Y$$

In the above development we have described how accounting transactions of a particular class of work-in-process accounts might be translated into the Leontief statical open model. The coefficients derived depend on factoring of real external demand by a scrap factor to obtain an External Demand for use in the model-the factors used should be derived from the same data on which the coefficients are determined, that is, derived from the accounting system. A similar factor might be derived for inventory change only if inventory change is expected to continue at existing rate - normally the inventory adjustment does not follow similar rate, and this constitutes an error in the method which has been treated in dynamic Leontief models. One might reduce the amount of this error by accumulating transactions over several accounting periods to average out temporary perturbations. Inventory will be reviewed in Chapter VI after consideration in the next chapter of error in statically determined coefficients based on time lags.

CHAPTER V

THE DYNAMIC PROBLEM INVOLVING TIME DELAYS

Introduction

To this point we have restricted our attention to the open form of the Leontief statical model, and to four of five assumptions concerning the statical model. The fifth assumption concerned time lags. To consider time lags we must consider a dynamic or time representative model.

Leontief considered the time element through what he considered to be its equivalent - a stock flow ratio - in his dynamic model (9). He mentions the use of turn-over ratio by industrial managers. Given the inventory in an account and its rate of output, one might determine a turn-over period by dividing this inventory by the rate of output. In this sense an inventory measure would be an equivalent means of acquiring a time interval since the flow rate is measured. This assumes, however, that there is no dead stock or stock held for "safety" or leveling. The relationship for stock and flow becomes more complex in the model of Hubble and Ekey in (22).

Leontief recognized that a treatment for inventory might be (1) a structural lag model, (2) a stock-flow model, or (3) a combination of both. His choice was dictated by the macro level of study where detail was not available. In this chapter we revert to the structural lag approach because of the availability of data from the firms production control departments (delivery intervals), and because of the problems associated with an inventory approach as mentioned above.

Our objective remains the same as previous - the acquisition of "technology coefficients", the fractional dollar inputs per dollar of output for a given account but now the formal definition must be relieved - in order to develope a structural lag model. The coefficient, a_{ij} defined:

$$a_{ij} = \frac{x_{ij}}{X_j}$$

where x_{ij} = transactions for a single period

X_j = total output of account j for the period

will not hold. We will drop the period restrictions retaining, however, the verbal meaning, "the fractional dollar inputs required per dollar output." Our reason for doing this is the difficulty of acquiring the true relationship through the accounting system when the period of measurement is shorter than the overall intervals for the system. This will be clarified by illustration.

The Problem of Time Lags

The problem of time lag can be demonstrated in one special case of the general model - one involving unilateral flow. A unilateral flow is one in which for any given accounts i and j, and transaction x_{ij}, x_{ji} is zero. It's transaction matrix is nilpotent (a matrix A^p such that $A^p = 0$ for some positive integer p is nilpotent) and may be arranged to be triangular.

Consider the system of accounts whose transactions tableau is as shown in Figure 8.

	1	2	3	4	D	X_i
1	0	0	x_{13}	x_{14}	0	$X_1 = x_{13} + x_{14}$
2		0	x_{23}	x_{24}	0	$X_2 = x_{23} + x_{24}$
3			0	0	D_3	$X_3 = D_3$
4				0	D_4	$X_4 = D_4$

Figure 8. Unilateral Flow Transactions Tableau

The technology coefficients for this tableau are:

$$a_{13} = \frac{x_{13}}{X_3} = \frac{x_{13}}{D_3} = \frac{x_{13}}{x_{13} + x_{23}}$$

$$a_{23} = \frac{x_{23}}{X_3} = \frac{x_{23}}{D_3} = \frac{x_{23}}{x_{13} + x_{23}} = 1 - a_{13}$$

$$a_{14} = \frac{x_{14}}{X_4} = \frac{x_{14}}{D_4} = \frac{x_{14}}{x_{14} + x_{24}}$$

$$a_{24} = \frac{x_{24}}{X_4} = \frac{x_{24}}{D_4} = \frac{x_{24}}{x_{14} + x_{24}} = 1 - a_{14}$$

The transactions matrix might have been obtained from a process having real but unknown coefficients of 1/2, 1/2, 1/3, 2/3 respectfully; and costs distributed linearly over activity intervals. For clarity in the discussion which follows, we define the following time measurements:

Period - The period of time between successive closures of the books of account - usually monthly.

Unit interval - The interval of time that a single unit may be expected to spend in a process represented by the associated account.

Activity interval - The interval of time that a quantity of units representing a future period demand may be expected to spend in a process represented by the associated account. This interval may represent a batch produced quantity or a sequentually produced quantity.

Set-back - The interval of time that the last unit of a quantity of units representing a future period demand must precede the completion of a supported activity.

With variable period demands for period t, t+1, t+2, the distribution of transactions from successive period demand might have obtained as shown in Figure 9.

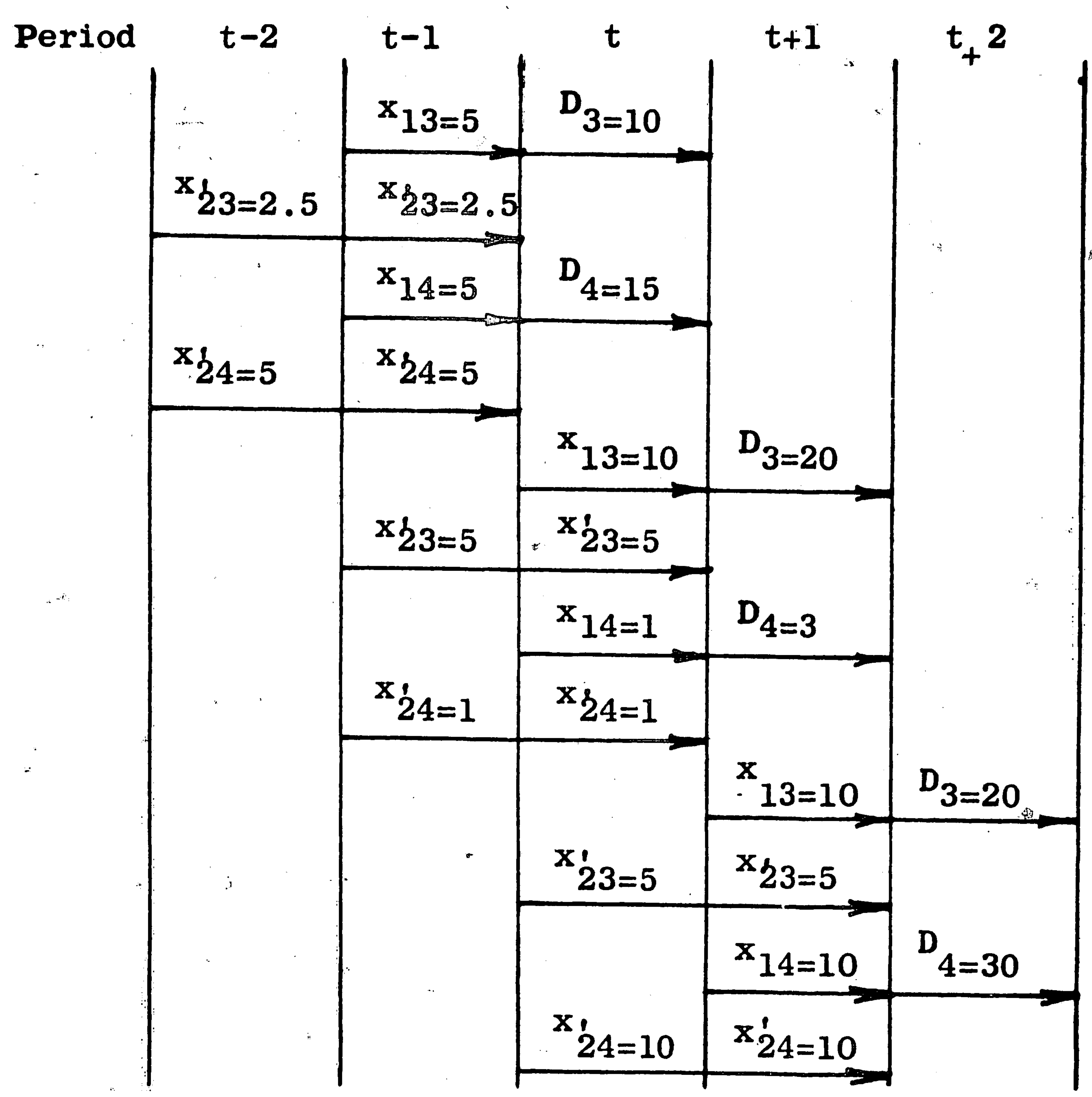


Figure 9 Cost Distribution Under Dynamic Demand

Consider that transactions are measured in period t-1 where a complete set of overlapped demands will have occurred. Since the accounting system does not recognize period demands it obtains:

$$x_{13} = 5 \qquad a_{13} = \frac{5}{12.5} \neq 1/2$$

$$x_{23} = 7.5 \qquad a_{23} = \frac{7.5}{12.5} \neq 1/2$$

and

$$x_{14} = 5 \qquad a_{14} = \frac{5}{11} \neq 1/3$$

$$x_{24} = 6 \qquad a_{24} = \frac{6}{11} \neq 2/3$$

The coefficients measured are, thus, not the real but unknown coefficients distributing costs. Appropriate coefficients would only be obtained if, (1) the demand was constant, or (2) the intervals were equal. Inputs equal outputs for the first case in the period measured. Inputs equal outputs for the period in the latter case only if the intervals are equal or less than the period, otherwise there is a variation in inventory.

We may observe an additional fact from the illustration relative to an input or output orientation. This was not significant in the static model because no time was associated with the model. We have assumed that costs occur and accumulate in a linear fashion over some interval and that they are recorded in accordance with this interval (whether as debits or credits). We have assumed that their intervals are backed off from the demand month, that is, scheduled as late as possible based on these intervals. We must, therefore, say that they are output oriented.

The intervals are dependent on demands and capacity, therefore, we may recognize a problem not unlike the problem of stock-flow coefficients - if coefficients determined from one period are used under a new set of demands for a new period, are intervals changed by some function? Our experience has been that planned and therefore executed intervals do not change so drastically, and that intervals updated in a previous period need not be changed for an additional period forecast.

Formulation of Dynamic Model Resolving Process Delays

How then might these redefined coefficients be determined in dynamic processes with variable activity intervals, and activity intervals longer than the accounting period? In the much simplified example given where intervals are multiples of accounting periods, where activity intervals of preceding and succeeding activities do not overlap, and where a relatively simple cost distribution applies - we may write the following equations. Superscripts denote the period, and the activity interval is dependent only on the supporting shop or account.

$$(1) \quad x_{13}^{t} = \frac{1}{T_1} a_{13} D_3^{t}$$

$$(2) \quad x_{23}^{t} = \frac{1}{T_2} a_{23} D_3^{t} + \frac{1}{T_{23}} a_{23} D_3^{t+1}$$

$$(3) \quad x_{14}^{t} = \frac{1}{T_1} a_{14} D_4^{t}$$

$$(4) \quad x_{24}^{t} = \frac{1}{T_2} a_{24} D_4^{t} + \frac{1}{T_2} a_{24} D_4^{t+1}$$

But, $a_{23} = 1 - a_{13}$; $a_{24} = 1 - a_{14}$

Therefore, equations (1) and (2) are sufficient yielding:

$$a_{13} = \frac{x_{13}^{t} T_1}{D_3^t}$$

$$a_{23} = 1 - a_{13}$$

$$a_{14} = \frac{x_{14}^{t} T_1}{D_4^t}$$

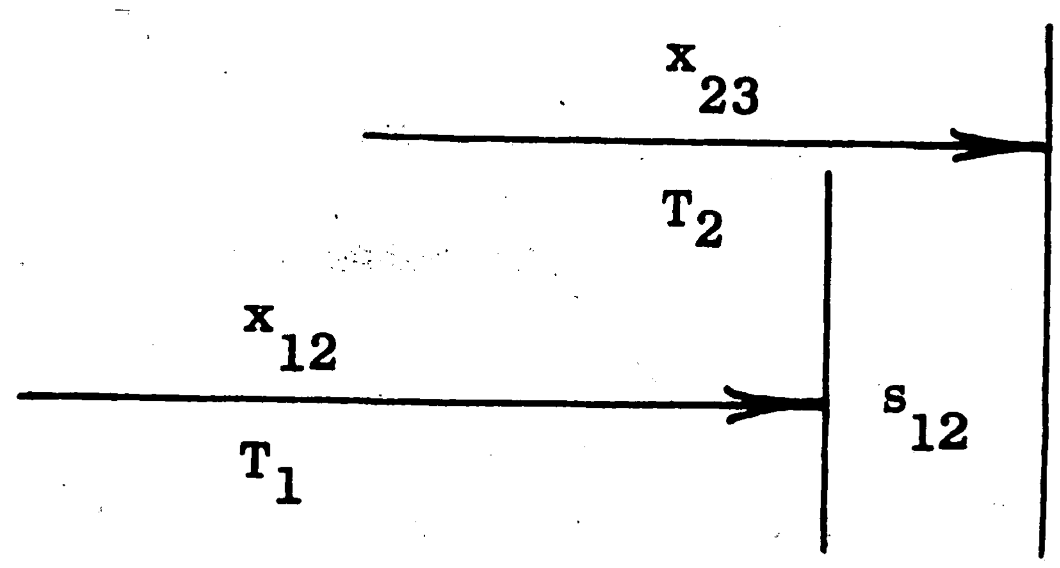
$$a_{24} = 1 - a_{14}$$

The problem of determining technology coefficients in the general case is similar to the classical Gozinto scheduling problem approached by Vazsonyi in (32) and by Homer in (20). Unlike these approaches, the problem here is the inverse one of determining the "quantity going into", under more stringent restrictions with respect to periods and activity intervals, and with the additional need for a cost distribution.

An Approach

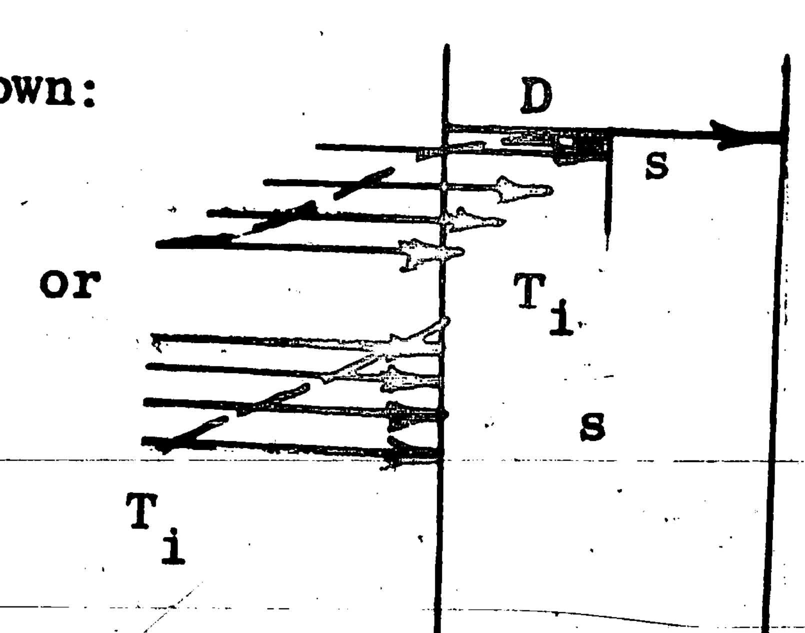
Vazsonyi approaches the scheduling problem by utilizing a period of one day, intervals multiples thereof, and batched production for periods longer than one day. Homer's solution, similarly, requires that intervals be multiples of the periods when greater than one period. Neither approach is sufficient for the accounting problem since the periods are normally monthly and since any lesser period would introduce excessive noise from perturbations in inventory. Longer periods require an alternate activity time relationship.

The time relationship of any two activities may be totally defined by two measures - activity interval and setback, which may be further clarified through use of the following diagram:



The transaction, x_{ij}, is a given amount of activity of account or process i in support of account or process j. The support activity will terminate at some point in time equal to or less than the termination of the supported activity measured by a setback, s_{12}, in the example. The setback is normally the leadtime of a single subassembly item over completion of an assembly and is normally obtainable from shop records. In the case of labor or load, the setback is assumed to be zero (although this is not imperitive).

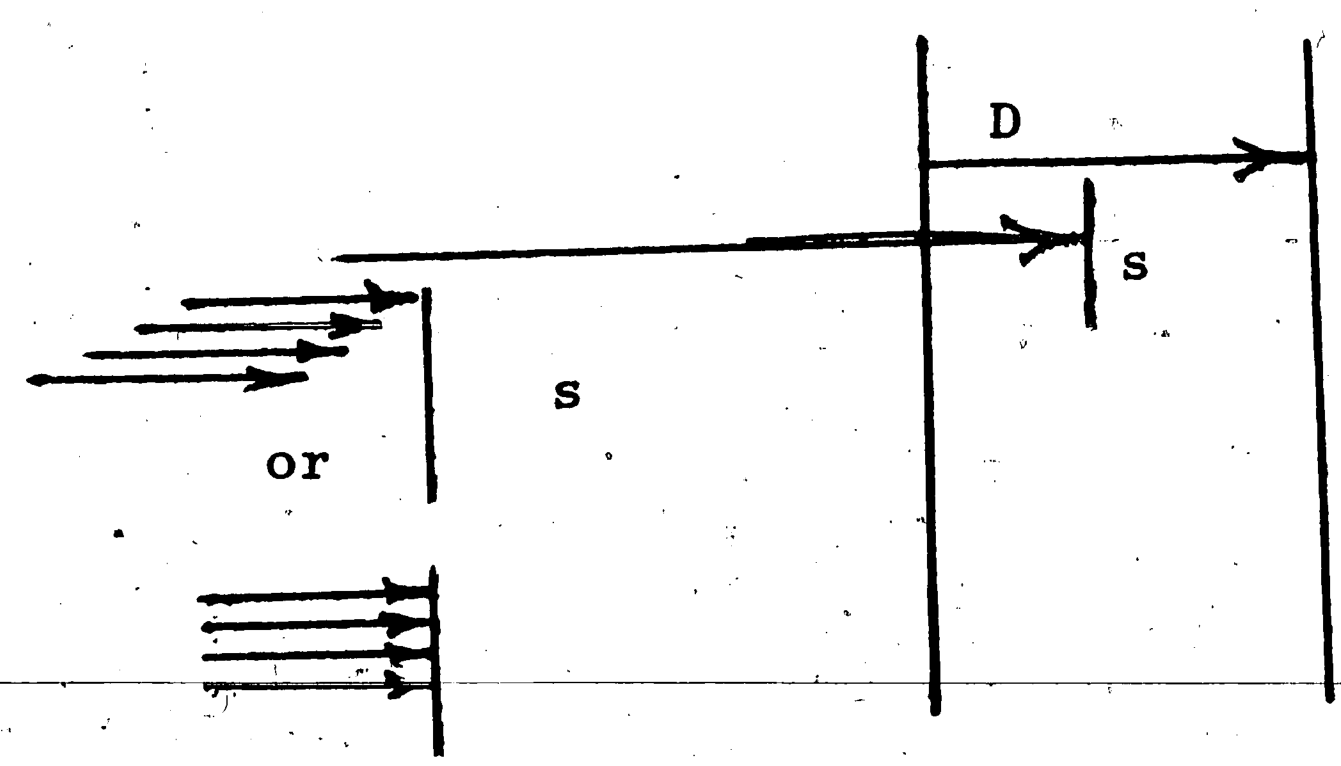
The activity interval is more complex. The duration of a final demand is one accounting period, by definition, and it is reasonable to assume that product is delivered on the first to last day of this period. Incoming subassemblies may be expected to have a constant time in their own shop, but we neither know the quantities required nor whether they are batched or sequentally produced. The alternate extremes might be as shown:



The upper illustration depicts sequentially produced product. The lower illustration depicts batch produced product. We are interested in a distribution of costs. If we assume a linear buildup of costs on individual units, the dotted lines in the above figure show the cumulative costs in the first case follow as S curve distribution, a linear distribution in the latter. If the costs on individual units followed an S curve, the variance of the cost distribution would be less, very little cost would occur in the initial interval. The point is that a unit interval may be taken as an activity interval, and costs allowed to distribute linearly over this interval as an approximation to the variety of conditions which may prevail.

We anticipate that in practice it will be quite difficult to distinguish between a unit, a batch, or a sequentually produced order interval. The intent here is to show that if a unit or batch interval is used, error in the exceptional sequentually produced order will be reduced by the cost distribution in most instances. Production controls interval input will be assumed to be representative of planning intervals for monthly increments as batch intervals (or equivalent unit intervals) in the following derivation.

For the case of intermediate activities, a similar case prevails. Two such activities are shown in the following figure with a period demand.



The end point is fixed by the sum of set-backs, and the unit interval is known. The use of unit interval and linear cost build-up is taken to be a reasonable approximation of the cost distribution in either intermediate or final support activities.

These simplyfying assumptions will be made in the following analysis of the problem, and means of acquiring a more accurate distribution of costs will be discussed subsequently. Unlike the approach of Vazsonyi and Morris, the set-back and the interval may be given in fractions of accounting intervals in the approach taken. In the special case of labor and load, the duration T_i will be taken as the duration of the activity supported with no associated setback.

With this background, we may consider a multistage process represented by the transactions tableau of Figure 10.

		1	2	3	4	5	6	7	8	D_i
Labor	1	0	0	0	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	0
Load	2		0	0	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	0
Mtl	3			0	x_{34}	x_{35}	x_{36}	x_{37}	x_{38}	0
	4				0	x_{45}	x_{46}	x_{47}	0	0
	5					0	0	x_{57}	x_{58}	0
	6						0	x_{67}	x_{68}	0
	7							0	0	D_7
	8								0	D_8

Figure 10. Multistage Unilateral Transactions Tableau.

The associated set-back, and intervals may be given in a setback matrix, S, and a column vector, T, as shown in Figure 11. Note that external support activities - labor, load, and material - take on the interval of the supported activity, as indicated by T_j.

$$
\begin{array}{ll}
\text{Labor} & 1 \\
\text{Load} & 2 \\
 & 3 \\
 & 4 \\
 & 5 \\
 & 6 \\
 & 7 \\
 & 8
\end{array}
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & 0 & s_{45} & s_{46} & s_{47} & s_{48} \\
 & & & & 0 & 0 & s_{57} & s_{58} \\
 & & & & & 0 & s_{67} & s_{68} \\
 & & & & & & 0 & 0 \\
 & & & & & & & 0
\end{bmatrix}
\begin{bmatrix}
T_j \\ T_j \\ T_j \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8
\end{bmatrix}
$$

Figure 11. Setback Matrix and Duration Vector

The timeframed dollar flow for this system for a single period demand might be as shown in Figure 12, where the external input activities (labor, load, and material) have been omitted for clarity.

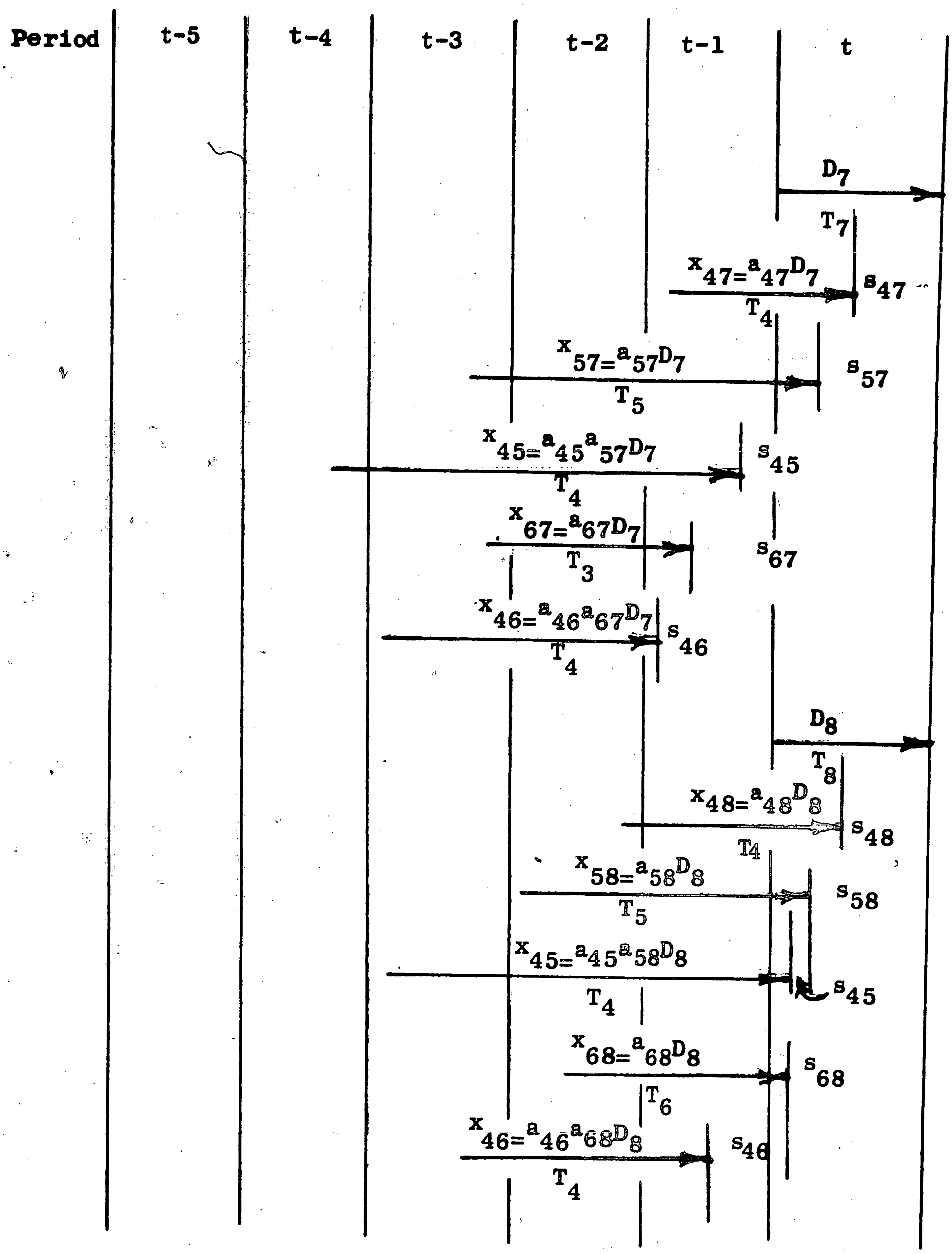


Figure 12 Multistage Activity Distribution in Time

If we now assume that costs accumulate in a linear manner over these intervals we have a means of timeframing costs for successive period demands.

Consider some equations which may be written for the system given:

(1) $$x_{47}^{t} = a_{47}D_{7}^{t}\,\frac{1 - s_{47}}{T_4} \quad + \quad a_{47}D_{7}^{t+1}\,\frac{s_{47} + T_4 - 1}{T_4}$$

(2) $$x_{45}^{t} = a_{45}a_{57}D_{7}^{t+1}\,\frac{2-s_{45}-s_{47}}{T_4} + a_{45}{}^{2}{}_{57}D_{7}^{t+2}\,\frac{1}{T_4} + a_{45}a_{57}D_{7}^{t+3}\,\frac{1}{T_4}$$
$$+ a_{45}a_{57}D_{7}^{t+3}\,\frac{s_{45} + s_{57} + T_{4}-3}{T_4} + a_{45}a_{58}D_{8}^{t}\,\frac{1-s_{45}-s_{58}}{T_4}$$
$$+ a_{45}a_{58}D_{8}^{t+1}\,\frac{1}{T_4} + a_{45}a_{58}D_{8}^{t+2}\,\frac{1}{T_4} + a_{45}a_{58}D_{8}^{t+3}\,\frac{s_{45}+s_{58}+T_{4}-3}{T_4}$$

These equations are obtained in the following manner. Consider equation (1). The transaction x_{47} is seen to extend over two periods as determined by T_4 and s_{47}, and in any given period t, denoted x_{47}^{t}, will be a proportional part of the demand during period t plus a proportional part of the demand at a future period t+1 (not shown-simply consider the period t-1 as the time displaced portion). The proportion of the demand at t is simply $1-s_{47}$ divided by T_4 provided that intervals and setbacks are measured in accounting periods, and that we assume a linear accumulation of cost over the activity interval. The proportion of the demand at t+1 is equal to $s_{47} + T_4$ minus one period, all divided by T_4 (for the extremety).

A similar development applies for the transaction x_{45}^{t}, but in this case the transaction is generated from two external demands - D_7 and D_8. Furthermore, the interval T_4 extends over three periods creating the intermediate terms whose proportional amount is simply $1/T_4$.

In order to generalize these considerations, review what we have done. We have summed over all external demands D_i which have a connecting path, and over each connecting path for each such demand. We have multiplied the coefficients a_{ij} in each path to obtain the resultant proportion of successive proportions.

Identical coefficient proportions are taken for successive period demands determined by the sum of setbacks and the activity interval. That is, the first superscript of demand, denoting period, is the period t incremented by a next largest integer for the sum of setbacks minus one. Note that if the total setback is less than one period we take the first demand in period t; if the total setback is greater than one period we take the first demand for a period one less than the next largest integer for the summed setbacks. Having obtained the first period, additional periods are dependent on the activity interval. If the next largest integer for the activity interval is equal or less than one there is only one period. If the next largest integer for the activity interval is greater than one then several periods must be considered. Having determined the first period we simply increment the first period by one, two, three - up to the next largest integer for the activity interval less one (the one we took in the first term). The linear proportion of each of these additional demands is simply $1/T_i$ up to the last demand. The extremity of the last demands generated activity interval is equal to the sum of setbacks plus the activity interval minus the number of periods by which the demand leads the period t. The linear proportion of the last demand is this extremity over T_i.

Therefore, the generalized expression of the above is obtained by defining terms as follows:

$\bar{T}_i$ = next largest integer for T_i

$s_{ik(p)}$ = sum of the setbacks in the pth path from account i to k.

$\bar{s}_{ik(p)}$ = next largest integer for the pth path from account i to k.

$a_{ik(p)}$ = product of the coefficients in the pth path from i to k.

k = the set of externally demanded product

X = number of preceding periods prior to determination period for a given activity.

The generalized expression is:

For $T_i \le 1$

$$x_{ij}^{t} = \sum_{k}\sum_{p} a_{ik(p)} D_k^{t + \bar{s}_{ik(p)} - 1} \; \frac{\bar{s}_{ik(p)} - s_{ik(p)}}{T_i}$$

For $T_i > 1$

$$x_{ij}^{t} = \sum_{k}\sum_{p} a_{ik(p)} D_k^{t + \bar{s}_{ik(p)} - 1} \; \frac{\bar{s}_{ik(p)} - s_{ik(p)}}{T_i}$$

$$+ \; \frac{a_{ik(p)}}{T_i} \sum_{x=1}^{\bar{T} - 1} D_k^{t + \bar{s}_{ik(p)} - 1 + X} \underset{\text{Minimum}}{\begin{Bmatrix} 1 \\ s_{ik(p)} + T_i - X \end{Bmatrix}}$$

The equations given envolve summation over all possible combinations of products and paths. An algorithm which sums over just those paths which are feasible based on the large number of zeros anticipated in the triangular form will be shown. The setback summations and the coefficient products can be generated for just these paths, and finally the a_{ij} can be solved in simpler expressions first and successively entered in the higher powered expressions in the algorithm.

Clearly, many of the $a_{ik(p)}$ terms will be zero eliminating the associated expression. It is highly unlikely that even in large matrices, 450 x 450, that more than 6 stage processes will be encountered. Even the triangular form will be non-dense. An a_{ij} term is zero whenever the associated x_{ij} term is zero. The setback summation, $s_{ik}(p)$ is not required in these cases. A means of generating the restrictive set of combinations and their associated terms for storage in a table (since this need normally be done only once) will now be reviewed.

Calculating the number of paths from an account j to an account k is a combinatorial problem. It is in fact the summation of the combinations of the intervening accounts taken zero at a time, one at a time, up to 'all at a time', that is, a binomial expansion.

Example: How many possible paths from account 3 to account 8?

There are 4 intervening accounts. The number of possible paths is: $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4}$

But this is a binomial expansion, $(1 + 1)^4$.

The number of paths equals $2^4 = 16$

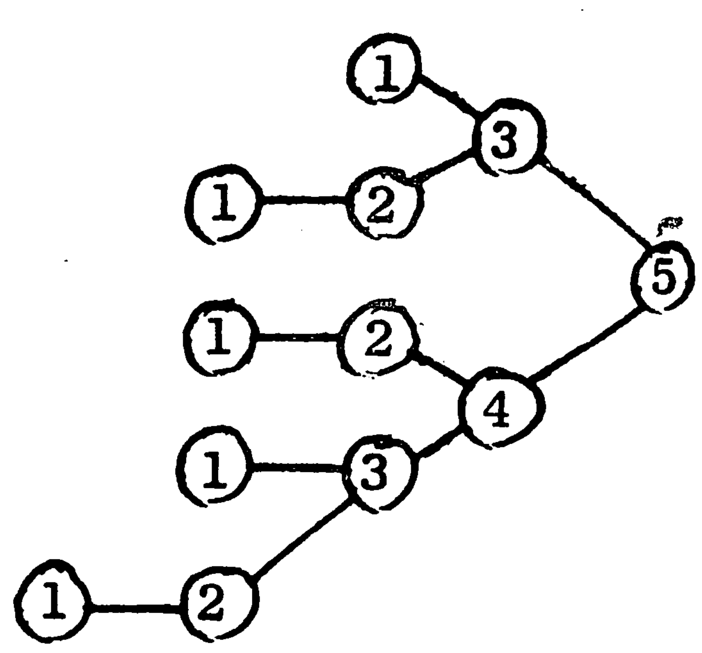
This type of expansion is common to scheduling models in general. It is in large part the basis for failure to find practical models for scheduling application. One may appreciate the magnitude of the problem by considering the number of possible paths from account 4 to account 405 in a 450 x 450 matrix, there are 2^{400} of them!

Fortunately, in our own case we can expect a relatively non-dense upper triangular matrix, and it is highly unlikely that any process combination will exceed 6 or 7 stages. The approach advocated is to generate the paths from a tree diagram representation taken from the transactions matrix, and pick off the combinations for storage in tables since this need only be done once.

Given the matrix:

	1	2	3	4	5	D
1	0	40	20	0	0	0
2		0	60	20	0	0
3			0	50	40	0
4				0	80	0
5					0	200
						34

We may draw the flow diagram from 1 to 5 (all other paths are obtained from this):



The combinations are:

Path 1	1, 3, 5
Path 2	1, 2, 3, 5
Path 3	1, 2, 4, 5
Path 4	1, 3, 4, 5
Path 5	1, 2, 3, 4, 5

Thus we are sufficient to analyze 5 paths rather than $2^3 = 8$. Much greater economies are obtained in larger, far less dense matrices anticipated. Algorithms such as those given in reference (8) or (17) might be utilized for programming of this step.

The set-backs $s_{ij(p)}$, and coefficient products, $a_{ij(p)}$ are directly obtained from the combinations:

$$a_{15(1)} = a_{13}a_{35} \qquad s_{15(1)} = s_{13} + s_{35}$$

$$a_{15(2)} = a_{12}a_{23}a_{35} \qquad s_{15(2)} = s_{12} + s_{23} + s_{35}$$

$$a_{15(3)} = a_{12}a_{24}a_{45} \qquad s_{15(3)} = s_{12} + s_{24} + s_{45}$$

$$a_{15(4)} = a_{13}a_{34}a_{45} \qquad s_{15(4)} = s_{13} + s_{34} + s_{45}$$

$$a_{15(5)} = a_{12}a_{23}a_{34}a_{45} \qquad s_{15(5)} = s_{12} + s_{23} + s_{34} + s_{45}$$

The set of terms are easily arranged in tables for varying i within a k:

<u>k = 5</u>

(1)	$a_{12}a_{23}a_{34}a_{45}$	$a_{23}a_{34}a_{45}$	$a_{34}a_{45}$	a_{45}
(2)	$a_{13}a_{34}a_{45}$			
(3)	$a_{12}a_{24}a_{45}$	$a_{24}a_{45}$		
(4)	$a_{12}a_{23}a_{35}$	$a_{23}a_{35}$	a_{35}	
(5)	$a_{13}a_{35}$			

From such a table the pattern for iterative solution of the a_{ij} becomes clear. Based on the transactions matrix, Figure 13, we solve for the largest j (last intermediate product column), and largest i for which a transaction is given, proceeding by decrementing the i and using the previously determined a_{ij}'s for successive solution.

With the a_{ij}'s thus determined we obtain a system of linear equations for determining the transactions anticipated in any given period from a given demand schedule or forecast.

$$\text{Let } A_{ik(p)} = a_{ik(p)} \frac{\bar{s}_{ik(p)} - s_{ik(p)}}{T_i}$$

$$B_{ik(p)} = \frac{a_{ik(p)}}{T_i}$$

$$C_{ik(p)} = s_{ik(p)} + T_i$$

$$D_{ik(p)} = \bar{s}_{ik(p)} - 1$$

The system of equations are then:

For $T_i \leq 1$

$$x_{ij}^{t} = \sum_{k}\sum_{p} A_{ik(p)} D_k^{t+D_{ik(p)}}$$

For $T_i > 1$

$$x_{ij}^{t} = \sum_{k}\sum_{p} A_{ik(p)} D_k^{t+D_{ik(p)}} + B_{ik(p)} \sum_{x=1}^{\bar{T}_i - 1} D_k^{t+D_{ik(p)}+x} \cdot \min\begin{Bmatrix} 1 \\ C_{ik(p)} - x \end{Bmatrix}$$

A means of prereviewing the number of paths required in application of the model is suggested by Hoffman in (17) which makes use of theorems proven by Luce and Perry in (28).

We define a precidence matrix as the matrix identical to the transactions matrix except that the $x_{ij} > 0$ are replaced by ones. Only immediate precidence relationship is described by such a matrix (none of the implied relations, that is, if 1 precedes 2, 2 precedes 3, the implication that 1 precedes 3 is not described) therefore the set of relations are said to be minimal. Further if 1 immediately precedes 2, 2 immediately precedes 3, then the chain 1-2-3 is said to be a 2-chain since there are two precedence relations. In general such chains are defined as n-chains inlike manner. The matrix is said to be consistent if there are no relations such as 1 precedes 2, 2 precedes 3, 3 precedes 1. A precidence matrix formed from a unilateral transactions matrix will be minimal and consistent under these definitions.

Theorem: If the minimal set of precidence relationships in a precidence matrix, P, are given, then the entry P_{ij} in the nth power of P will equal C if the only if there are c distinct p-chains between element i and j.

Corollary: For a consistent matrix having a finite number of elements, there exists some N for which all $m > N$, $P^m=0$.

From the theorem, P^n will determine all n chains. It follows that a summation matrix, $S = \sum_{m=1}^{n} P^m$ will count all chains up to chains n long. Since the precidence matrix is nilpotent, by raising it to a finite power (its dimension) one has a means of determining a count of all possible combinatorial chains for all ij pairs. Furthermore, a simple addition approach may be taken to determine the summation matrix as shown by Hoffman.

The summation matrix may be obtained from the precidence matrix by proceeding from the lowest column indice to the highest. The jth column of S is equal to the summation of those preceding columns of S as indicated by the row indices of P having ones in the jth column of P, plus the jth column of P.

Example:

$$P=\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad S=\begin{bmatrix} 0 & 1 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In this chapter the effect of time delay has been introduced via a dynamic model since time delay cannot be treated in a static representation. The effect of time delays on determination of technology coefficients having the essential meaning of "required fractional dollar inputs per dollar of output" (without period restriction) has been demonstrated with the conclusion that such coefficient could only be obtained if demand was constant over periods, or the intervals of supporting activities were equal - or offsetting adjustments were made in the demands (the Walrasian concept). The correlated input-output relationship suggests that when there is a significant change in investment or inventory during the period under review one is constrained to utilize a dynamic model. For these reasons formulation of a dynamic model for acquiring the newly defined coefficients was undertaken.

Choice of a structural lag model was dictated by measurement availability through the production control operation, and because of problems associated with the alternate turn-over or stock-flow relationships. The structural lag model has its own problems - the need for cost distribution assumption, the lack of a direct correlation between demand and interval. Production control's estimates of interval, which may be based on conditions such as machine breakdown, epidemics, etc., are believed to be superior to derived intervals. Further, the one month lag in interval update is not believed to be excessive and pospibly more in keeping with real situations of master loading or scheduling where the plan dictates the action. This may be the price of master planning in centrallized systems.

Under the assumptions of output orientation, linear unit or batch cost distributions, and nominal demand variation over periods;aformula has been developed for transactions of a given period as a function of period demands, intervals, and set-backs utilizing coefficients or flow parameters derived from a previous period. The intervals are taken to represent capacity restraints. The formula permits an algorithmic solution of the coefficients from previous actual and planned data in which demands and transactions are known and such that new planning may proceed on a month to month basis, the primary objective of the thesis. When the period is long with respect to intervals and investment variation during the period is nominal, the statical model may prove to be adequate. When this is not the case, the complexities of the dynamic model are envolved, and the model presented in this chapter is the suggested approach.

CHAPTER VI

SYSTEMS APPLICATIONS

This article has been primarily concerned with the determination of "technology" or, perhaps more in context, input-output coefficients. In this chapter, however, their ultimate utility in systems accounting designs will be reviewed.

The equations obtained after deriving coefficients and combining terms were as follows:

$$x_{ij}^{t} = \sum_{k}\sum_{p} A_{ik(p)}\, D_{k}^{t+D_{ik(p)}} \;; \qquad \text{for } \bar{T}_i \leq 1$$

$$x_{ij}^{t} = \sum_{k}\sum_{p} A_{ik(p)}\, D_{k}^{t+D_{ik(p)}} + B_{ik(p)} \sum_{x=1}^{\bar{T}_i - 1} D_{k}^{t+D_{ik(p)}+x} \cdot \text{minimum} \begin{Bmatrix} 1 \\ C_{ik(p)} \end{Bmatrix} ; \qquad \text{for } \bar{T}_i > 1$$

Transactions derived in this manner are dollar transactions. The particular point is that the consolidated accounts with dollar measure permit a feasible approach to the resource allocation function of a central group. While such scheduling is not definitive, it permits a variety of simulations on the system, using coefficients gathered through the accounting system and subject to periodic or continuous revision from actual measured results.

In addition, the transactions matrix has a general utility in an overall accounting systems design. Its logical form is adaptive to computer files organization, to accounting programming language, to report generation, and to integrated production-financial accounting in an on-line computer system.

Consider first the possibility of a rudimentary accounting programming language for updating accounts (files) in accordance with a set of debit/credit decision rules. Let T be a set of independent transactions, t_1, t_2, ..., t_n. Let B be a set of accounts, b_1, b_2, ..., b_m. The general conditional might be stated:

If t(i) then credit b(j,..,k,..l) and debit b(g,..,h,..)

Let an m x m Boolean switching matrix, S, be defined where the identical m accounts are ordered on the rows and on the columns such that the rows represent credit entries, the columns represent debit entries; e.g., for each credit there is a corresponding debit. Let a matrix entry of 1 represent a debit/credit pair, a 0 represent no transaction on accounts.

It is proposed that such a switching matrix be stored in core as a Logic Unit. Each transaction would carry a code (1,.....,m). There would be a one -many correspondence between the transaction code and credit accounts, thence to debit accounts.

If there is one-to-one correspondence between the switching matrix and transaction types, then a simplified macro instruction might be written. Let this macro be called TRANS. The instruction TRANS(I) where I is the transaction storage location would credit and debit the transaction at I in accordance with the logic unit which was part of the subroutine, examining first its code, referring to the logic unit to determine the appropriate accounts and debit and credit action on the amounts of I. The plural is used for it is anticipated that dollars, manhours, and quantities are subject to simultaneous processing. The transactions processed in this manner would update the transaction matrix based on random entries. We might define a macro, LOGIC, for the entry of the switching matrix. We might also define a macro, ORG (a,b,c) as a files organization generator allocating the appropriate memory space for the accounting system based on the tree parameters, a,b,c, which define the number of stages, the number of levels, and the number of accounting periods treated. Other macros which might be constructed are BAL (a,b,c) - to create a balance sheet as described by Charnes, Cooper, and Ijiri in (5), CASH - to create a cash flow analysis, STAT - to create an income statement as suggested by the same article. These and other reports would be generated for that section of the tree defined by the input parameters. Their data would be derived from the transactions matrix.

Rosenblatt in (30) suggests a means for automatic trial balance based on the transactions matrix. Let X be the transactions matrix. He defines an effective closure of X as the square matrix, $\bar{X}$, of order m+1.

$$\bar{X} = \begin{bmatrix} X & A \\ B & 0 \end{bmatrix}$$

$$A_i = \max(r_i, c_j) - r_i$$

$$B_j = \max(r_i, c_j) - c_j$$

$$r_i = x_{ij} \text{ for } j = 1 \text{ to } m$$

$$c_j = x_{ij} \text{ for } i = 1 \text{ to } m$$

The A_i and B_j are usual balance sheet account entries, and for closed systems (financial accounting):

$$A_i = B_j$$

is the usual debit-credit identity of double entry accounting.

The above are suggestive of the utility of an accounting system design based on a transactions matrix concept permitting on-line update of integral files, accounting language development, report generation, and system simulation.

CHAPTER VII

CONCLUSIONS AND SUGGESTED CONTINUING STUDY

The objective of this thesis has been the determination of technology coefficents in applications of Leontief input-output models to accounts of a single firm - within the context of a resource allocation model, and with specific attention to the assumptions inherent in Leontief models. Means have been sought for resolution of assumptions which are held to be excessive, and for which solutions are not provided by available literature.

We conclude that the Leontief statical model provides an appropriate approach to resource allocation based on the accounting system - a matrix form for transaction representation, a consolidation scheme, a defined flow parameter or technology coefficient, and an inverse relationship. We conclude qualitatively that those assumptions relative to process time delay, single output, and work-in-process inventory are excessive and require dynamic extension of the model.

Leontief dynamic models given in (27) consider in-process inventories as a function of demand, but without specific attention to process time lags. We conclude that in single industry consideration the process time lag is the more relevant consideration and that in-process inventory may be less a function of demand than of rate of change of demand or overall fill or balancing. We conclude that the dynamic iterative procedure outlined in Chapter V represents a feasible approach to both coefficient determination and their subsequent use, recognizing that variation in inventory not based on demand is not included in the expressions given.

It is recognized that technology coefficients derived as outlined in Chapter V will include at least four classes of error:

$$a_{ij} + \beta_{ij} + \alpha_{ij} + \gamma_{ij} + \epsilon_{ij}$$

Here, β_{ij} is the error due to consolidating non-homogeneous product. The availability of K-Order subclassifications to Work-In-Process accounts is believed to be the means of minimizing this error in an industry model without excessive dimension.

The second error term, α_{ij}, is the error due to cost distribution assumption in the dynamic model given in Chapter V. The assumption presumes a linear distribution over a single unit interval irrespective of batch or sequential production. The degree of this error depends on subsequent empirical and sensitivity study. Piecewise linear distributions fitting hypothetical distributions or tracer lot determinations might be incorporated if the error is significant.

The third error term, γ_{ij}, is the error due to inventory lags - transactions on inventory not occasioning fixed interval intermediate account, labor, material, transactions. The criticalness of this term depends on the frequency of coefficient redetermination, the period over which costs are accumulated, and the degree to which process inventories are accumulated on speculative basis not dependant on recorded demand. Since period inventories are subject to physical determinations, adaption including such determination might be included if warranted.

The last error term, ϵ_{ij}, represents the error due to changing technology over time. Again the frequency of coefficient redetermination is critical - and if it is rapidly changing, some form of exponential smoothing might be incorporated.

It is clear that further refinements require an increasingly complex model formulation, one possibly exceeding the data capacity of current or evolving system. It is recommended that continuing studies concentrate attention on model sensitivity to varying parameters under empirical trial conditions, retaining the simplicity characteristic of Leontief models and the model contained herein where it is warranted.

APPENDIX

ECONOMIC PRODUCTION FUNCTION

Economic models have historically dealt with broad, aggregate theoretical considerations - total economies with emphasis on pricing theory. More recently distinction has been made between macroeconomics of this sort, and microeconomics dealing with economic actions of well defined individuals or groups of individuals. In microeconomics this individualized study has resulted in continuing development of a theory of consumer behavior, and a theory of the firm. In building a theory of the firm efforts have been directed to formalizing a production function, the firms output as a function of its input variables. Let X be the total output of indistry 1. Let x_{11}, x_{21}, x_{31} be all inputs to industry 1 from itself and industries 2 and 3. Then:

$$X_1 = F\ (x_{11},\ x_{21},\ x_{31})$$

The economist notes the relationship to represent "constant returns to scale" if the functional relation is homogeneous of degree one.

Consider proportionate increases in input variables of a production function satisfying the following relation:

$$x_j' = F^i\ (tx_{11},\ tx_{21},\ tx_{31}) = t^k F^i\ (x_{11},\ x_{21},\ x_{31})$$

Provided this is true for all values t and a constant k, the relation is said to be homogeneous of degree k by definition. If k > 1 the economist notes a condition of increasing returns to scale. If k = 1 the relation is one of constant returns to scale.

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