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# Applications of gert to quality control

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# APPLICATIONS OF GERT TO QUALITY CONTROL

by

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CERTIFICATE OF APPROVAL

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# ABSTRACT

In this thesis, the characteristics of various sampling plans and control charts are modeled by GERT. Most of the sampling models provide information about the probabilities of accepting and rejecting a lot, the expected number and variance of units passed until the lot is accepted as well as rejected. The control chart models are used to compute the expected number of points under control until an out of control condition occurs.

For Dodge's continuous sampling plan CSP-1, a stopping rule and a cost model are discussed to attack the problem of shifts in quality level.

The areas have been studied are simple and double sampling plans, Military Standard 105 D, Bayes approach, Dodge's continuous sampling plan CSP-1 and control charts.

# I INTRODUCTION

## I-A Purpose and Scope

Quality control has become the first point of attack in methods improvement. A properly designed quality control system can reduce the losses from rejections, scrap, and reworked production to a very low percentage of total output and hold that level. Cost is reduced, and output is increased. Thus, quality control is one of the important tools for industrial engineers.

In probability theory and statistical theory there has been a growing interest in stochastic processes. GERT (Graphical Evaluation and Review Technique) is a technique that has been developed for the analysis of stochastic networks. Quality control system can be modeled in GERT. From the model, various characteristics of the system can be evaluated.

This thesis concerns the following areas:

(a) Lot acceptance sampling plans

(b) Bayes approach to a quality control model

(c) Dodge's continuous sampling plan CSP-1

(d) Control charts

## I-B Background

In recent years, the industrial engineer has been introduced the graph theory in the solution of network problems. Following uses of PERT and CPM, many other new graph theory techniques have been introduced as tools of the industrial engineer. Network and network analysis play an increasingly important role in the description and improvement of operational systems primarily because of the ease with which systems can be modeled in network form. Other reasons for using networks are:(1) the need for a communication mechanism to discuss the operational system in terms of significant features, (2) a means for specifying the data requirement for analysis of the system, and (3) a starting point for analysis of the scheduling of the operational systems.

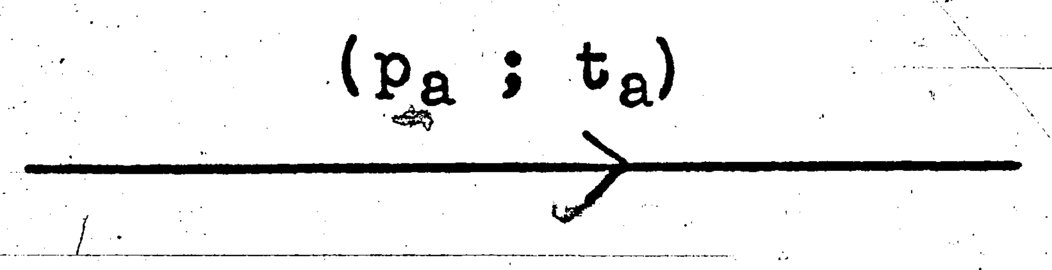
Pritsker and Happ (9) developed GERT to analyze the stochastic networks. Whitehouse (11) has shown this technique could be applicable to certain industrial engineering problems such as inventory, queue, and reliability, etc.

Fry (2), Powell (5), and Mullen (4) applied GERT directly in the area of quality control but to a limited extent and for a different purpose. They only concerned Dodge's continuous sampling plan CSP-1, and proposed some modifications to this plan.

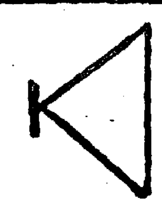
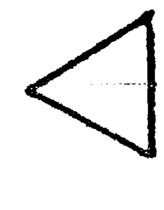
## II MATHEMATICAL BACKGROUND OF GERT 

### II-A The Elements of a GERT Network.

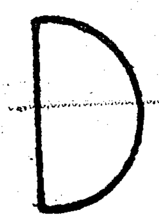
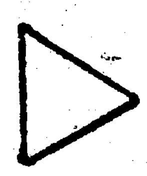
The elements of a GERT network are directed branches and logical nodes. A directed branch has associated with it one node from which it emanates and one node at which it terminates. Two parameters are associated with a branch: (1) the probability that a branch is taken, $p_a$, given that the node from which it emanated is realized; and (2) a time, $t_a$, required, if the branch is taken to accomplish the activity which the branch represents. The time can be random variable. The visual representation of a directed branch, without the nodes represented, is:



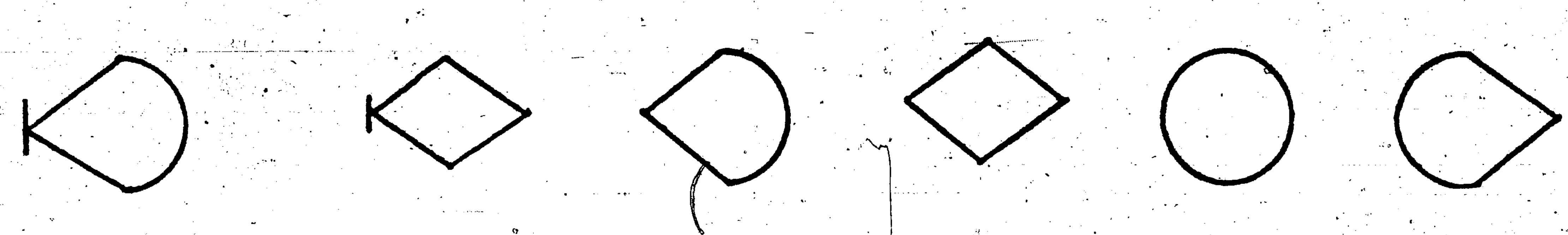

A node in GERT network consists of an input side and an output side. The three logical relations on the input side are:

| Name | Symbol | Characteristic |
|---|---|---|
| Exclusive - or |  | The realization of any branch leading into the node causes the node to be realized; however, one and only one of the branches leading into this node can be realized at a given time. |
| Inclusive - or |  | The realization of any branch leading into the node causes the node to be realized. |
| And |  | The node will be realized only if all the branches leading into the node are realized. |

On the output side, the two relations are defined as:

| Name | Symbol | Characteristic |
|---|---|---|
| Deterministic |  | All branches emanating from the node are taken if the node is realized. |
| Probabilistic |  | At most, one branch emanating from the node is taken if the node is realized. |

For notational convenience, the input and output symbols are combined into six possible types of nodes:



## II-B The W - Function

To include both parameters in a transformation function which could be treated uniformly throughout a network requires the creation of a new function. Pritsker's (6) approach is to make all parameters multiplicative. A W - function, W(s), is defined as the product of the conditional probability of selecting the branch, p, and the conditional moment generating function of the time to traverse the branch, M(s). The originators chose the moment generating function since the moment generating function of the sum of independent variables is the product of the individual generating functions. Thus,

$$W(s) = PM(s)$$

## II-C Network Analysis Employing a Topoligical Equation.

It has been shown that all other nodes can be reduced to combinations of Exclusive-or nodes (6). It also has been shown that with proper transformations a GERT network containing Exclusive-or node is essentially an open flowgraph (6). A topological equation of flowgraph theory will now be discussed. Some terminology are defined as follows:

Path - A path through a network is any sequence of nodes which will cause the terminal node to be realized without realizing any given node more than once.

1st Order Loop - A first order loop is a consecutive path of arrows leading from a node and returning to the same node.

n-Order Loop - A n-order loop is the product of n non-touching first order loops.

$$\sum \text{loops} = 1-L_1+L_2-L_3+\cdots+(-1)^i L_i$$

where $L_i$ is the sum of the i-order loops. The topological equation can be put in the following form:

$$W_E(s) = \left(\sum \left(\text{path} \times \sum \text{non-touching loops}\right)\right) / \sum \text{loops}$$

This equation, known as Mason's Rule, is stated as follows: Write down the product of transmittances along each path from the source to the sink node. Multiply its transmittance by the sum of the non-touching loops. Sum these modified path transmittances and divide by the sum of all the loops.

It can be observed that:

$$W_E(s) = P_E\, M_E(s)$$

The probability that the equivalent network is realized given that the starting node of the equivalent network is realized is given by

$$P_E = W_E(s)\Big|_{s=0}$$

Thus, the moment generating function of the time to traverse the equivalent network given the equivalent network is traversed is

$$M_E(s) = W_E(s)/P_E$$

Any number of moments of the distribution about the origin can be obtained by differentiating the moment generating function to the degree equivalent to the order of the moment and evaluating the derivative at s = o.

$$K_j = \frac{d^j}{ds^j} M_E(s)\Big|_{s=0}$$

II-D The W - Generating Function

A generating function is a power series in terms of an undefined variable Z. When used in conjunction with networks, the variable Z is used to multiply the W - function associated with a branch and, hence, the power of Z specifies the number of times branches were traversed whose values were multiplied by a Z. When a branch value is multiplied by Z, we say the branch is tagged. If we define $W(s|j)$ as the W-function associated with a network when the branches tagged with a Z are taken j times, then the equivalent function associated with the network can be written as a generating function as shown below:

$$W(s,Z) = W(s|0) + W(s|1)Z + W(s|2)Z^2 + \cdots + W(s|j)Z^j + \cdots$$
$$= \sum_{j=0}^{\infty} W(s|j)Z^j$$

The relationship between the conditional W-function and the conditional moment generating function is:

$$W(s|j) = P(j)M(s|j)$$

with

$$P(j) = W(0|j)$$

where $P(j)$ is the probability that the network is realized when branches tagged with a Z are traversed j times, and $M(s|j)$ is the conditional moment generating function associated with the network, given branches tagged with a Z are traversed j times.

The conditional W-function can be obtained from the W-function as follows:

$$W(s|j) = \frac{1}{j!} \left. \frac{\partial^j W(s,Z)}{\partial Z^j} \right|_{Z=0}$$

## II-E Counters



A counter is defined as a mechanism for determining the number of times a branch or node or set of branches and nodes is traversed during the realization of a network. The transmittance $e^c$ is placed on the branch being investigated. This can be employed in conjunction with the joint W - function.

$$W(s,c) = PM(s,c)$$

where M(s,c) is the joint moment generating function for a branch. If the number of counts is independent of the time to traverse the branch then

$$M(s,c) = M(s) \cdot e^c$$

If information concerning only the number counts is desired, then s can be set to zero and only $e^c$ need be used as discussed above.

## II-F Computer Programs

For large scale systems, it becomes extremely laborious to carry through the calculation necessary to obtaining the moment generating functions of the output. Thus a computer is mandatory to solve complex systems.

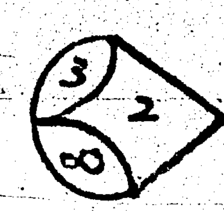
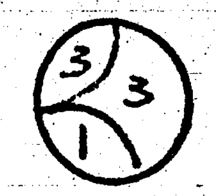
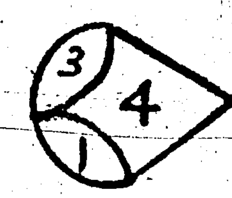
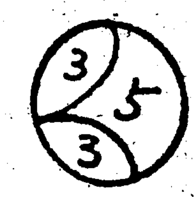
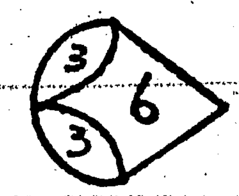
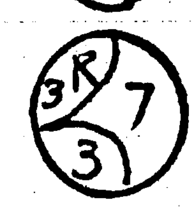
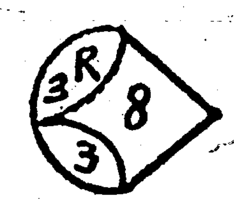
A program written by Ishmael and Pritsker (7) is available for analyzing GERT networks which contain nodes of the Exclusive — or type. The program calculates the probability, the expected time and the variance in the time to go from each source node of the GERT network to each sink node.

Another program, GERTS II, which is also written by Ishmael and Pritsker (8) can accommodate GERT networks which have OR and AND logical operations associated with the input side of a node and deterministic or probabilistic operations associated with the output side of a node. The result obtained from the program are:

1. The probability that a node is realized;
2. The average, standard deviation, minimum and maximum time to realize a node:
3. A histogram of the times to realize a node; and
4. The average, standard deviation, minimum, and maximum counts observed during the time to realize a node.

There are 8 node type that can be used in GERTS II. Figure 1 represents the 8 node types.

With GERTS II, two additional characteristics can be associated with an activity. These are a counter type and an activity number. The counter type number specifies the counter to be increased by 1 every time the activity is realized. Activity numbers are given to activities to permit network modifications based on the realization of the activity. Network modification involves the replacing of a node by another node. The node to be replaced is deleted

| Node Type | Scheduled Events Removed When Realized | Reset Number of Releases | Output Type (a) | Example of Symbol (b) |
|---|---|---|---|---|
| 1 | no | to | D |  |
| 2 | no | to | P |  |
| 3 | no | no | D |  |
| 4 | no | no | P |  |
| 5 | no | yes | D |  |
| 6 | no | yes | P |  |
| 7 | yes | yes | D |  |
| 8 | yes | yes | P |  |

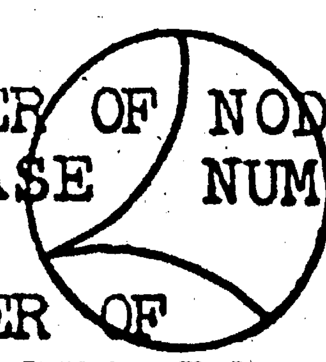
(a)
D = Deterministic

P = Probabilistic

(b)

NUMBER OF RELEASE

NODE NUMBER

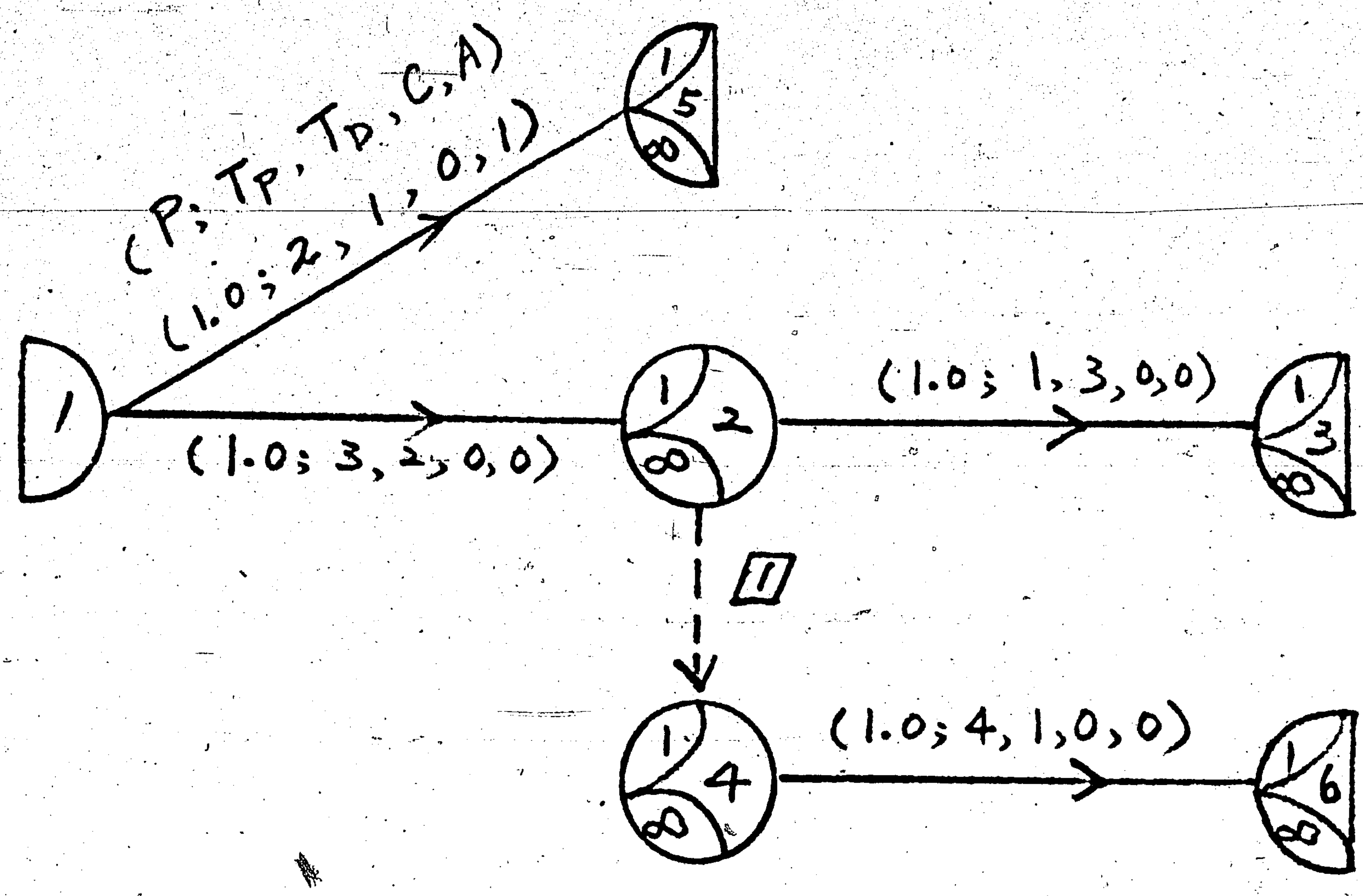


NUMBER OF RELEASES TO REPEAT

Figure 1. Node Type Definitions

from the network when and if it is realized. The activities then caused to occur are from the node that is inserted. In this way a node can be changed many times before it is actually realized.

Figure 2 illustrates the branch and node modification notation. Modifications will be shown by a dashed branch with the activity number attached in a square. Thus, when activity 1, a branch between node 1 and node 5 is realized, the network is modified by having node 2 replaced by node 4.




LEGEND

P = probability of realization
$T_P$ = parameter set for time
$T_D$ = distribution type
C = counter type
A = activity number

Figure 2. Illustration of Branch Descriptors and Network Modification Symbolism

# III LOT ACCEPTANCE SAMPLING PLANS

## III-A Single Sampling Plan

One of the major field of statistical quality control is acceptance sampling. The purpose of acceptance sampling is to determine a course of action, not to estimate lot quality. Acceptance sampling prescribes a procedure that will give a specified risk of accepting lots of given quality. In other words, acceptance sampling yields quality assurance.

A single sampling plan (1) is designated by two numbers n and c. A sample of size n is taken from a given lot. If it contains c or less defective unit, it is accepted. Otherwise, it is rejected.

An example is presented to show how GERT can be used in single sampling plan.

Example 1 Let $n = 100$, $c = 2$. Two kinds of information we can obtain from GERT. Let p be the lot fraction defective.

(a) The system is shown in Figure 3. Nodes 0, 1, 2, 3 represent the number of defective units. Node 3 also represents an absorbing state (i.e. the state in which the lot is rejected). Each branch of the network is multiplied by Z. From Mason's Rule, we get

$$W_{0,R} = \frac{p^3 Z^3}{1-3(1-p)Z+3(1-p)^2 Z^2-(1-p)^3 Z^3}$$

$$= a_3 Z^3 + a_4 Z^4 + \cdots + a_j Z^j + \cdots$$

The generating function can be obtained either by division or from the W-function as follows:

$$a_j = \frac{1}{j!} \cdot \frac{\partial^j W_{o,R}}{\partial Z^j} \Bigg|_{Z=0}$$

Thus, we can get

The probability of rejecting a lot = $P_r = \sum_{i=3}^{100} a_i$

The probability of accepting a lot = $1 - P_r$

This approach only illustrates GERT is applicable to this type of problem. However, it is not a practical way to get these probabilities from the above computation. Actually we can obtain these probabilities from the GERT Simulation as shown in Figure 4 for p = 0.03. Node 2 is the starting node. Node 3 and 7 are sink nodes. Node 3 represents the acceptance of a lot, and node 7 represents the rejection of a lot.

GERTS II program (8) can give us the following results immediately. The computer output is shown below:

** FINAL RESULTS FOR 200 SIMULATIONS **

| NODE | PROB./COUNT | MEAN | STD.DEV. | MIN. | MAX. |
|---|---|---|---|---|---|
| 3 | .4250 | 100.0000 | 0.0000 | 100.0000 | 100.0000 |
| 7 | .5750 | 59.9217 | 23.0884 | 14.0000 | 99.0000 |

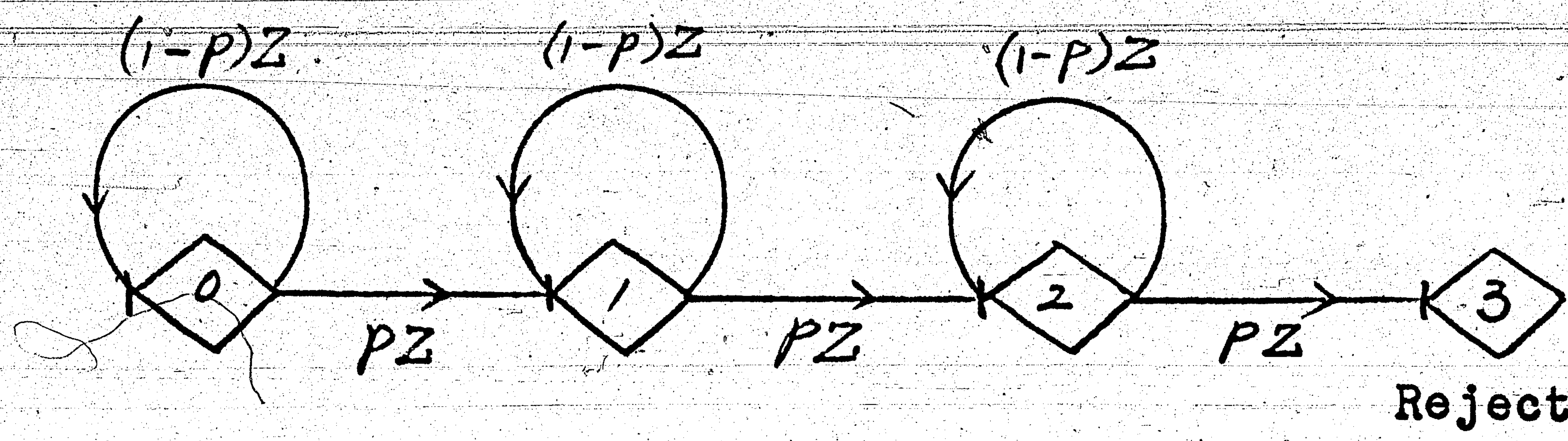



Figure 3. Single Sampling Plan (Exclusive-or type)

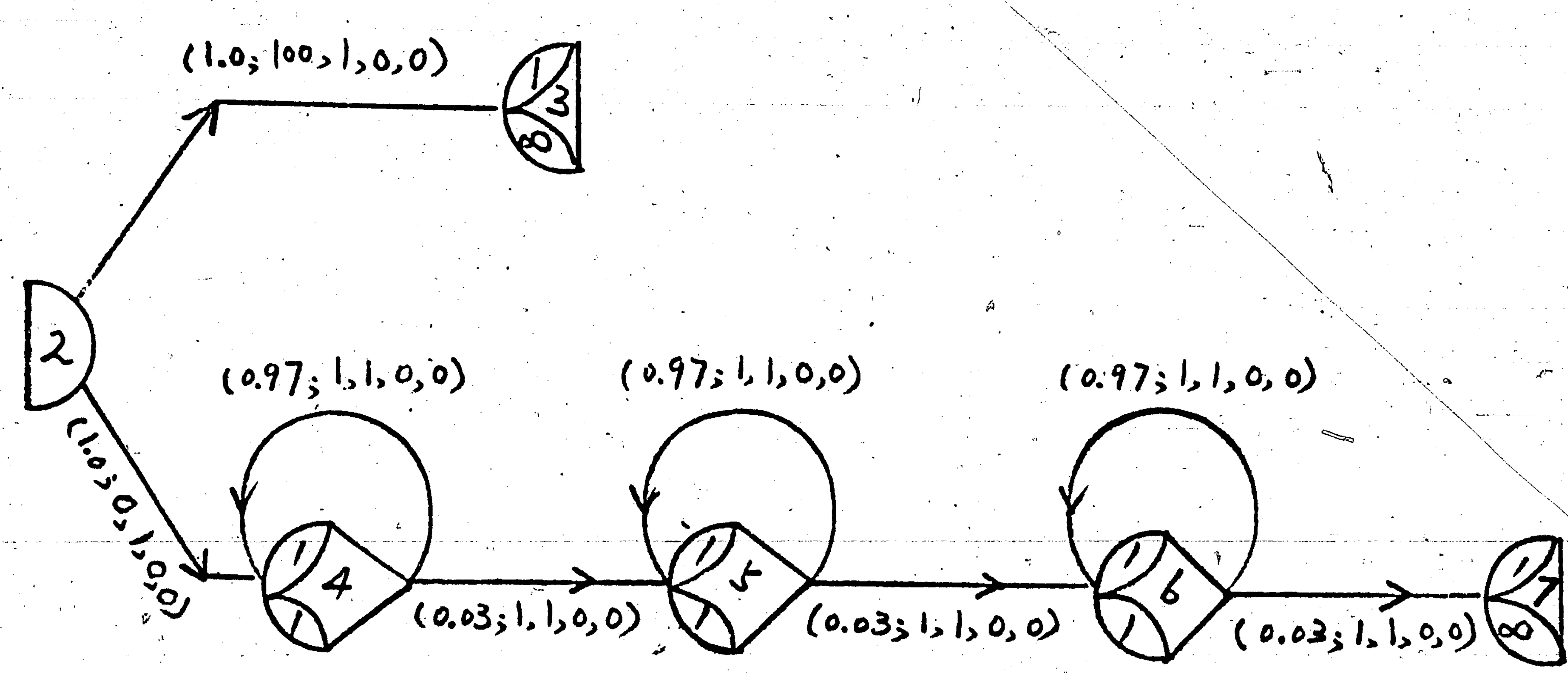



Figure 4. Single Sampling Plan (GERTS-II)

From the above results, we can see

The probability of accepting a lot = 0.425
The probability of rejecting a lot = 0.575
The expected number of units passed until the lot is accepted = 100.0
The expected number of units passed until the lot is rejected = 59.9217
The standard deviation of units passed until the lot is accepted = 0.0
The standard deviation of units passed until the lot is rejected = 23.0884
The minimum number of units passed until the lot is accepted = 100.0
The minimum number of units passed until the lot is rejected = 14.0
The maximum number of units passed until the lot is accepted = 100.0
The maximum number of units passed until the lot is rejected = 99.0

The results observed from Operating Characteristic Curve for the sampling inspection plan n = 100, c = 2, p = 0.03 (1) are as follows:

The probability of acceptance = 0.43
The probability of rejection = 0.57

Thus, we can see that the probabilities obtained from the above two methods are very close. However, GERTS II can give us the expected values and standard deviation of the distribution while OC curve cannot provide us such information.

(b) Suppose only c is known. We would like to know the expected number of sampling before rejecting the lot. For the loop system shown in Figure 5(a) standard flow-graph operation would reduce to the graph shown in Figure 5(b) (11).

Therefore for c = N, the system shown in Figure 6(a) can be reduced to the graph shown in Figure 6(b). Thus

$$W_{0,N+1} = \left(\frac{pe^c}{1-(1-p)e^c}\right)^{N+1}$$

$$\therefore \quad P_{0,N+1} = W_{0,N+1}\Big|_{c=0} = 1$$

$$\therefore \quad M_{0,N+1} = W_{0,N+1}/P_{0,N+1} = W_{0,N+1}$$

$$\frac{dM_{0,N+1}}{dc}\Bigg|_{c=0} = \frac{N+1}{p}$$

= expected number of sampling before rejection for c = N.

The second moment can be computed by

$$\frac{d^2M_{0,N+1}}{dc^2} = \frac{d}{dc}\left(\frac{(N+1)(pe^c)^{N+1}}{(1-(1-p)e^c)^{N+2}}\right)\Bigg|_{c=0}$$

$$= \frac{(N+1)(N-p+2)}{p^2}$$

The variance for the distribution may then be computed by

$$\sigma^2_{0,N+1} = \frac{(N+1)(N-p+2)}{p^2} - \left(\frac{N+1}{p}\right)^2$$

$$= \frac{(N+1)(1-p)}{p^2}$$

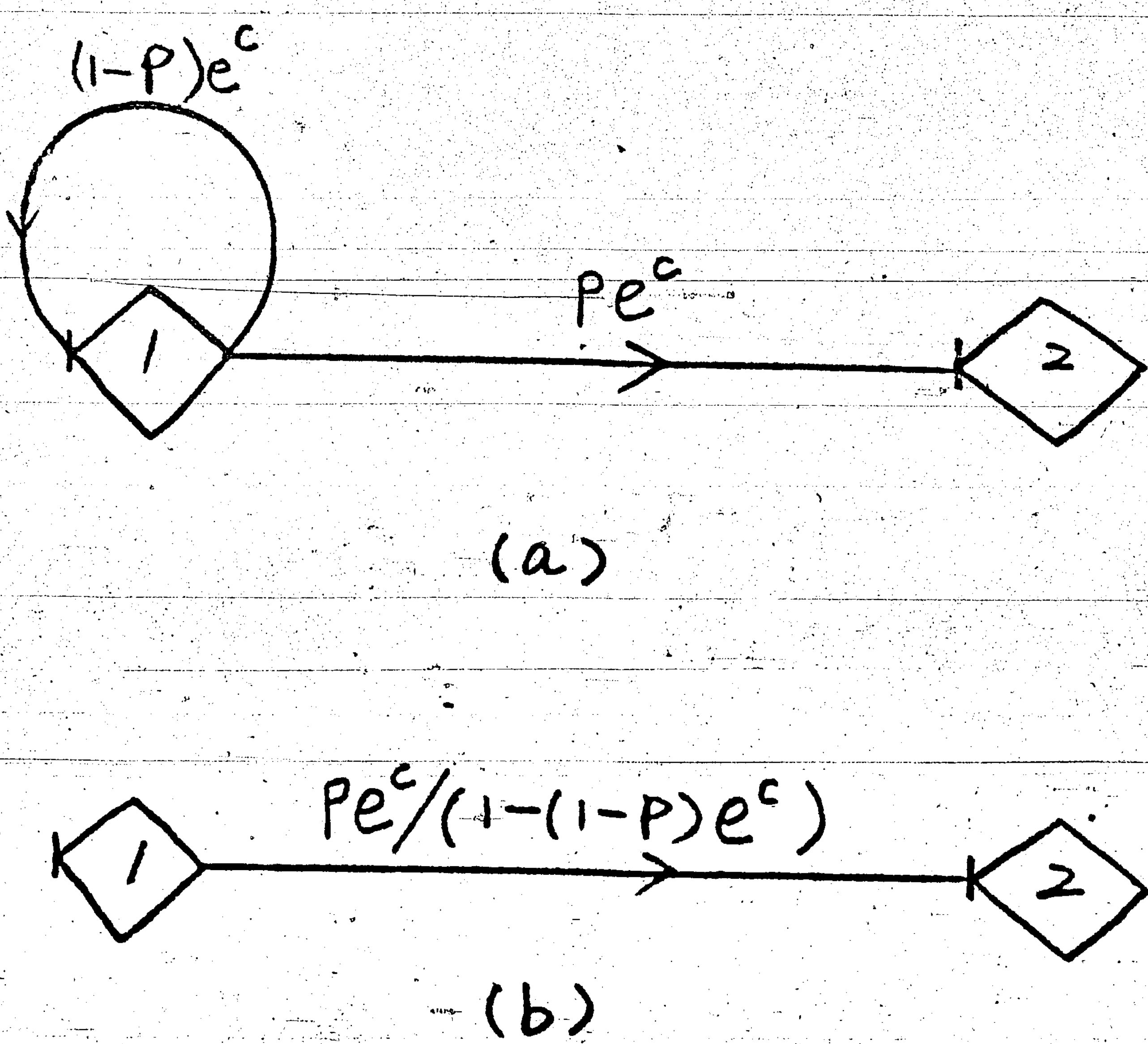



Figure 5. Basic loop GERT system

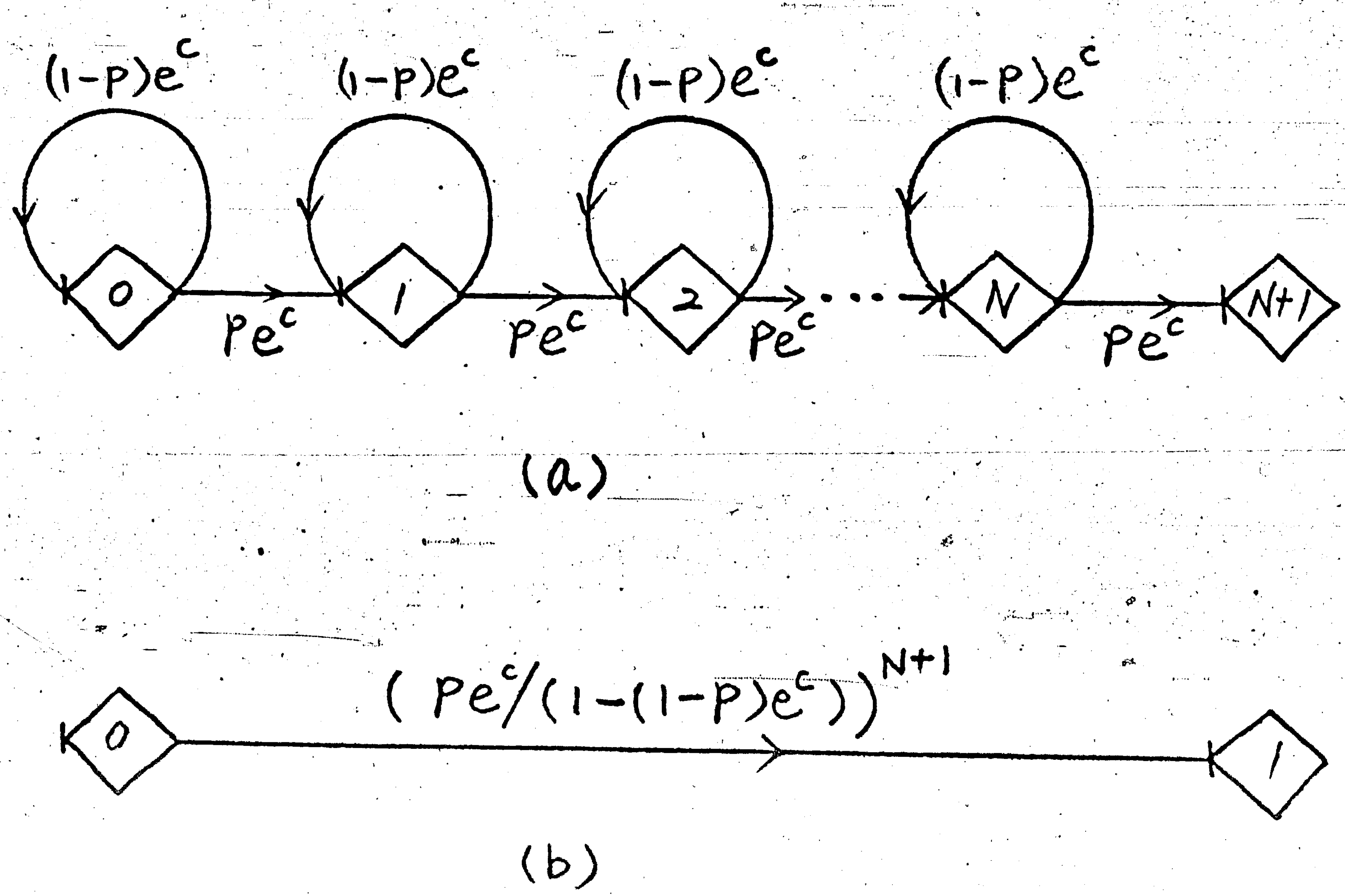



Figure 6. Single Sampling plan for c = N

## III-B Double Sampling Plan

Duncan (1) stated the plan as follows:

A double sampling plan is designated by five numbers $n_1$, $n_2$, $c_1$, $c_2$ and $c_3$, $c_1$ being less than $c_2$ and $c_2$ being less than or equal to $c_3$. A sample of size $n_1$ is taken from a given lot. If it contains $c_1$ or less defective units, it is immediately accepted. If it contains more than $c_2$ defec-defective units, it is immediately rejected. If the number of defective unit is greater than $c_1$ but not more than $c_2$, a second sample of size $n_2$ is taken. If there are $c_3$ or less defective units in the combined samples, the lot is accepted. If there are more than $c_3$ defective units, the lot is rejected. Frequently $c_2$ is taken equal to $c_3$.

The probability of either acceptance or rejection on first sampling can be obtained from the method proposed in previous section. To compute the probability of either acceptance or rejection on combined samples, an example is presented for this purpose as follows:

Example 2 Let $n_1 = 50$, $n_2 = 100$, $c_1 = 1$, $c_2 = c_3 = 3$, and p be the lot fraction defective. The probabilities of accepting and rejecting on combined samples can be obtained from the GERT network as shown in Figure 7. Nodes 0, 1, 2, 3, 4 represent the number of defective units. Node 4 represents an absorbing state (i.e. the state in which combined samples are rejected). The first three branches of the network are multiplied by $Z_1$ and the other branches are multiplied by $Z_2$ in order to distinguish the second sampling from the first sampling. From Mason's Rule, we get

$$W_{0,4} = p^4 Z_1 Z_2^3 / [1 - 2(1-p)Z_1 - 2(1-p)Z_2 + (1-p)^2 Z_1^2 + 4(1-p)^2 Z_1 Z_2^2 + (1-p)^2 Z_2^2 - 2(1-p) Z_1^2 Z_2 - 2(1-p) Z_1 Z_2^2 + (1-p)^4 Z_1^2 Z_2^2 ]$$

$$= c_1 Z_1 Z_2^3 + c_2 Z_1^2 Z_2^3 + c_3 Z_1 Z_2^4 + \cdots$$

$$= \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} c_i Z_1^j Z_2^k$$

The terms satisfy conditions $J<50$, and $j + k = 150$ are as follows:

$$c_1 Z_1 Z_2^{149},\ c_2 Z_1^2 Z_2^{148},\ c_3 Z_1^3 Z_2^{147},\ \cdots,\ c_{49} Z_1^{49} Z_2^{101}$$

Thus

The probability of rejection on combined samples $= P_r = \sum_{l=1}^{49} C_l$

The probability of acceptance on combined samples $= 1-P_r$

Since this is not a practical approach, we can easily obtain these probabilities from the GERT Simulation. In order to compare the results with Duncan's (1). Another example is presented as follows:

Example 3 Let $c_1 = 2$, $c_2 = c_3 = 6$, and $p = 0.08$. GERTS II model is formulated as shown in Figure 8. Node 2 is the starting node. Node 3 represent the first sampling and node 4 represent the second sampling. Nodes 6, 7, ----, 13 represent 0, 1, ----, 7 defective units respectively. Node 5 is an absorbing node in which the lot is accepted. Node 13 is also an absorbing node in which the lot is rejected.

Number shown in square on the figure indicate activity number and is used to illustrated potential modification to the network. Thus, when activity 1, a branch between node 8 and 9 is realized, the network is modified by having node 3 replaced by node 4. The dashed branch between node 3 and 4 labeled with a 1 in a square indicates the modification to occur when activity 1 is realized. In other words, a second sample of size 100 is taken if the number of defective units is greater than 2.

The computer output of GERTS II program (8) is shown below:

**FINAL RESULTS FOR 400 SIMULATIONS**

| NODE | PROB./COUNT | MEAN | STD.DEV. | MIN. | MAX. |
|---|---|---|---|---|---|
| 5 | .2125 | 52.3529 | 15.2477 | 50.0000 | 150.0000 |
| 13 | .7875 | 74.9651 | 23.3705 | 32.0000 | 145.0000 |

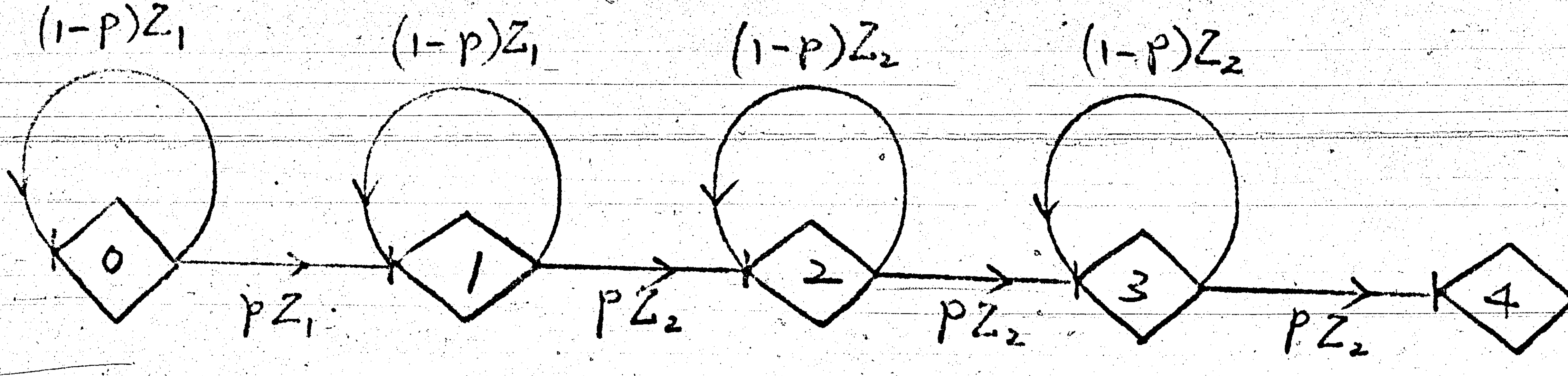



Figure 7. The GERT Network for Example 2

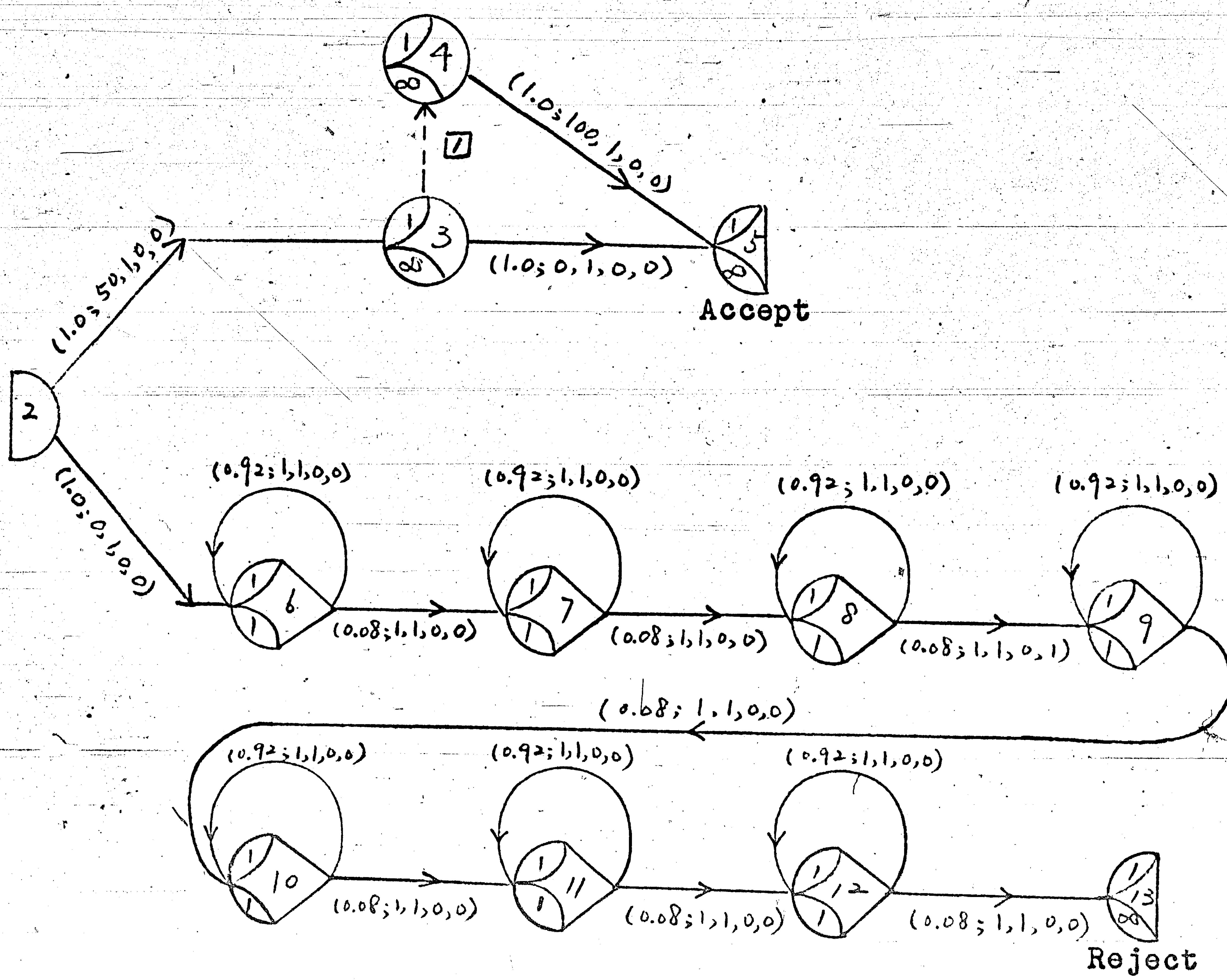



Figure 8. The GERT Network for Example 3

From the above results, we can see

The probability of acceptance on combined samples = 0.2125

The probability of rejection on combined samples = 0.7875

The results observed from OC curves (1) for double-sampling plan $n_1 = 50$, $n_2 = 100$, $c_1 = 2$, $c_2 = c_3 = 6$ are as follows:

The probability of acceptance on combined samples = 0.23

The probability of rejection on combined samples = 0.77

Thus, the results obtained from the above two methods are quite close. However, GERTS II can give us additional information such as mean and standard deviation of the distribution.

If $c_2 \neq c_3$, we can still use GERTS II to obtain the probabilities as well as mean and standard deviation.

III-C Military Standard 105 D

Military Standard 105 D is a combination of single sampling plan, double sampling plan, and multiple sampling plan. The purpose of this plan is to constrain the supplier that he will product of AQL (Acceptable Quality Level) quality. Mil. Std. 105 D is thus indexed with respect to a series of AQL's. It is also necessary in applying Mil. Std. 105 D to decide on the "inspection level." This determines the relationship between the lot size and the sample size. Three general levels of inspection are offered. Level II is designated as normal. Level I may be specified when less discrimination is needed, level III when more discrimination is needed. There are also four special levels: S-1, S-2, S-3, S-4, and may be used where relatively small sample sizes are necessary and large sampling risk can be tolerated.

For a specified AQL and inspection level, and a given lot size, Mil. Std. 105 D gives a normal sampling plan that is to be used as long as the supplier is apparently producing product of AQL quality. It also gives a tightened plan to which a shift is to be made if there is evidence that quality has deteriorated, and reduced plan if the quality is running especially good. Switching procedures are as follows:

1. Normal to tightened: when normal inspection is in effect, tightened inspection shall be instituted when 2 out of 5 consecutive lots or batches have been rejected on original inspection.

2. Tightened to normal: when tightened inspection is in effect, normal inspection shall be instituted when 5 consecutive lots or batches have been considered acceptable on original inspection.

3. Normal to reduced: when normal inspection is in effect, reduced inspection shall be instituted when the preceding 10 lots or batches have been considered all acceptable on original inspection.

4. Reduced to normal: when reduced inspection is in effect, normal inspection shall be instituted when a lot or batch is rejected.

In the event that 10 consecutive lots or batches remain on tightened inspection, inspection under provisions of this document should be discontinued pending action to improve the quality of submitted material.

A GERT network is formulated as shown in Figure 9. This model can provide us the mean and variance of the number of lots inspected. Node 5 represents the starting node. Nodes 1, 2, 3 represent a lot has been under normal, tightened, and reduced inspection respectively. Node 4 represents a absorbing state in which the inspection is discontinued.

Let $P_r$ be the probability that a lot is rejected. We can find out $P_r$ from the OC curve if p (lot fraction defective) is given.

Now let $P_r = 0.1$. The probabilities for each branch can be computed by:

$$P_1 = \binom{5}{2} P_r^2 (1-P_r)^3 = 10\, P_r^2 (1-P_r)^3 = 0.073$$

$$P_2 = (1-P_r)^{10} = 0.349$$

$$P_3 = 1 - P_1 - P_2 = 0.578$$

To compute $P_4$, a GERTS II network is formulated as shown in Figure 10.

$P_4$ can be interpreted as the probability of not having 5 consecutive accepted lots in 10 successive lots. Node 2 is the starting node. Nodes 3, 4, 5, 6, 7, 8 represent 0, 1, 2, 3, 4, 5 consecutive accepted lots respectively. Node 8 is an absorbing node in which having 5 consecutive accepted lots in 10 successive lots. Node 9 also represents an absorbing state in which there are no 5 consecutive accepted lots in 10 successive lots.

The computer output is shown below:

**FINAL RESULTS FOR 200 SIMULATIONS**

| NODE | PROB./COUNT | MEAN | STD.DEV. | MIN. | MAX. |
|---|---|---|---|---|---|
| 8 | .7950 | 5.6981 | 1.3015 | 5.0000 | 9.0000 |
| 9 | .2050 | 10.0000 | 0.0000 | 10.0000 | 10.0000 |

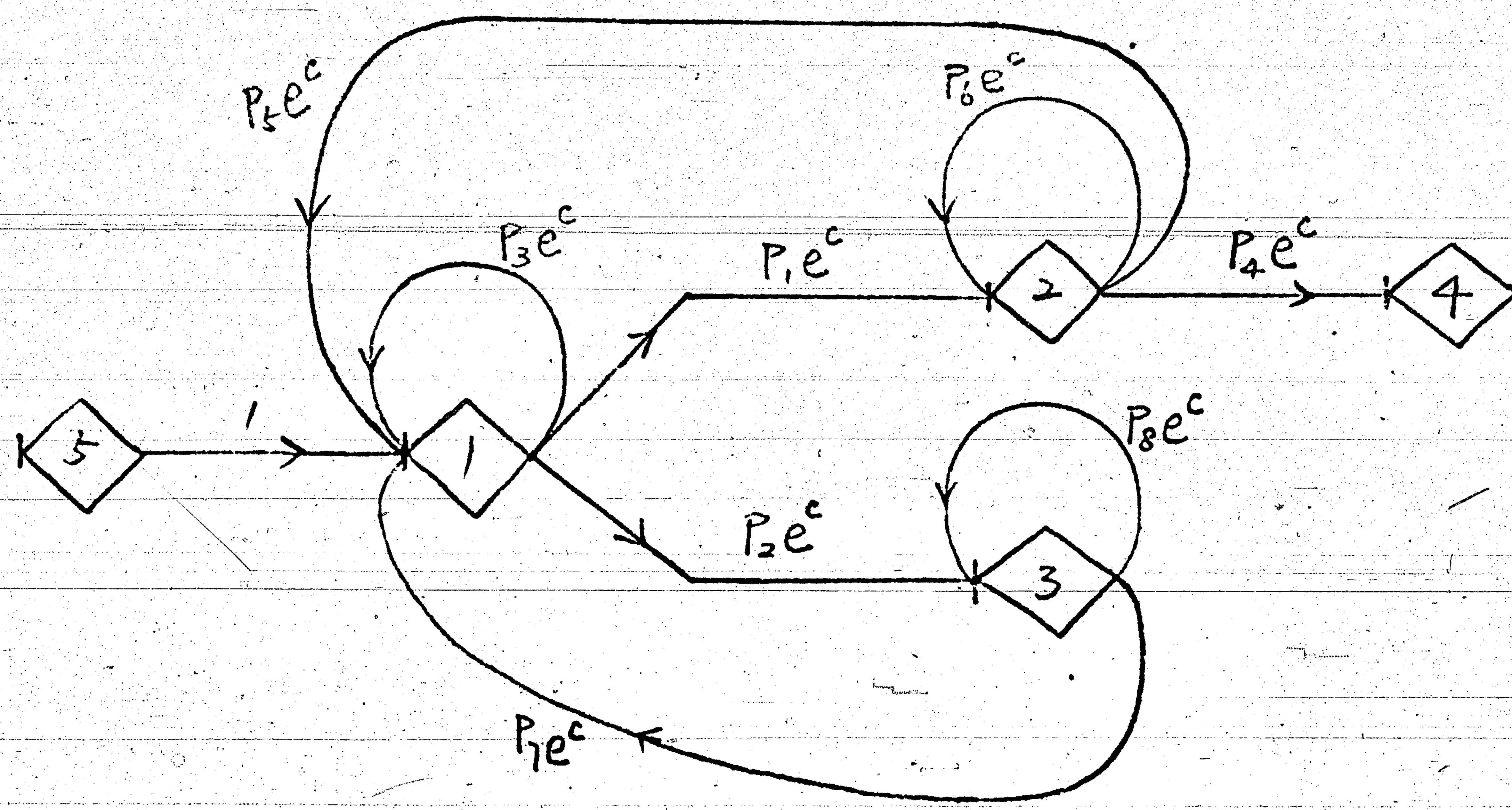



Figure 9. GERT network for Mil. Std. 105 D

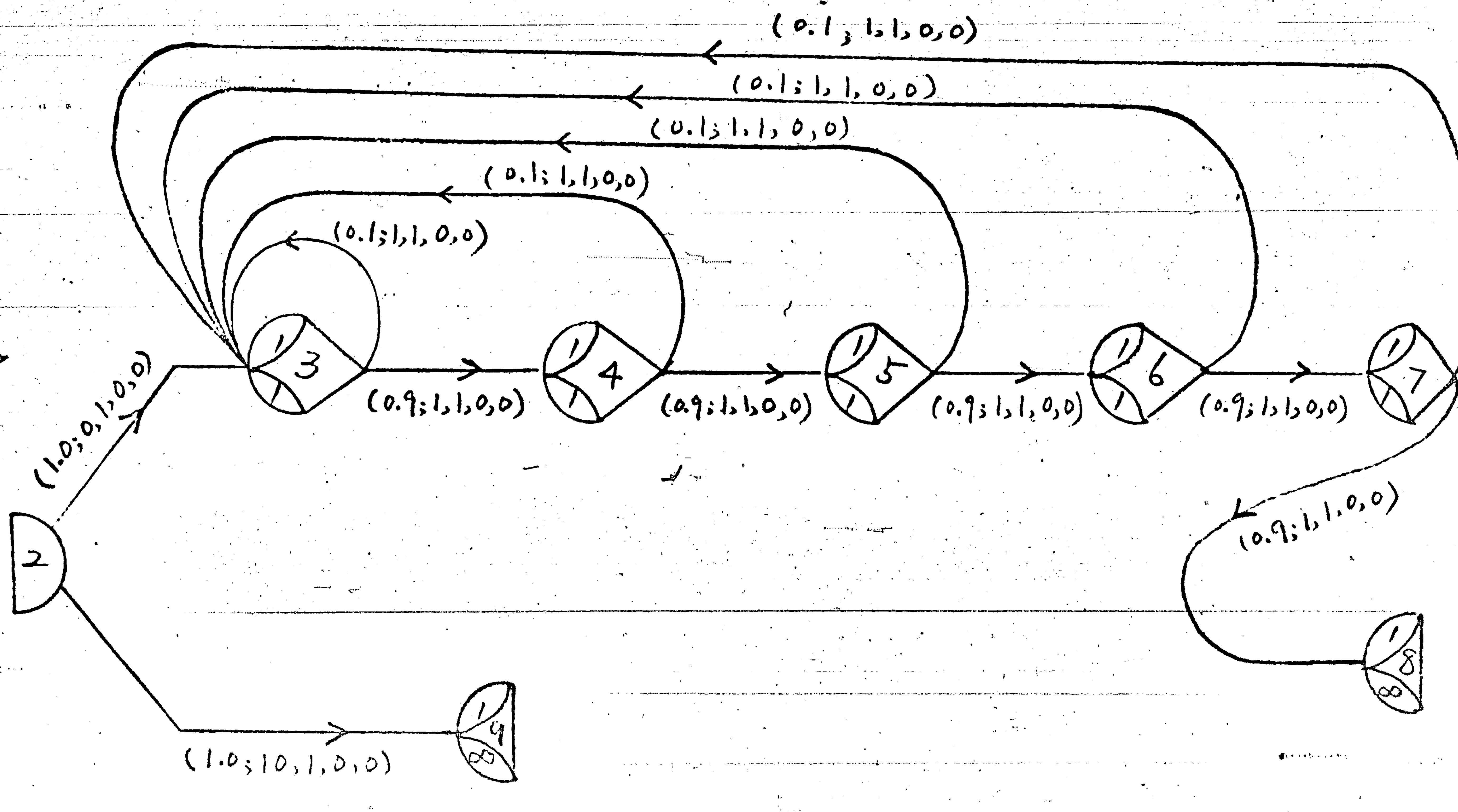



Figure 10. GERTS II network

From the above results, we can see

$P_4 = 0.205$

The rest of the probabilities can be computed by

$P_5 = (1-P_r)^5 = 0.59$

$P_6 = 1-P_4-P_5 = 0.205$

$P_7 = P_r = 0.1$

$P_8 = 1-P_7 = 0.9$

The computer output for Figure 9 is shown below:

**EQUIVALENT BRANCHES OF THE NETWORK**

| ENTRY | EXIT | PROBABILITY | | Variance |
|---|---|---|---|---|
| 5 | 4 | 1.000000 E + OD | 2.434046 E + 02 | 6.211039 E +4 |

From the above results, we can see:

The probability of discontinuing inspection = 1.0

The expected number of lots passed until inspection is discontinued = 243.4046

The variance of lots passed until inspection is discontinued = 62110.39

III-D Sampling Game

A lot of fraction defective p is ready for inspection. Suppose we take one item each time from the lot at a random manner. Good item counts 1 point, and bad item counts -1 point. We want to set up a sampling plan such that if the score reaches a specified points (M), the lot is accepted. If the score does not reach M after a specified time limit (K time units), the lot is rejected. A time unit is defined as the time that one item will be completed inspection.

An example is presented to show how GERT can be used in this type of problem.

Example 4 Let M = 4, K = 7, and p = 0.1. A GERTS II network is modeled as shown in Figure 11. The meaning of the nodes can be defined as follows:

11 - starting node
2 - the score is -2
3 - the score is -1
4 - the score is zero
5 - the score is 1
6 - the score is 2
7 - the score is 3
8 - the score is 4
9 - the lot is accepted
10 - the lot is rejected

We note that once the score reaches -2, the lot must be rejected. We want to compute the probability that a lot is accepted, the probability that a lot is rejected, the mean and variance of time before a lot is accepted, and the mean and variance of time before a lot is rejected.

The computer output is shown below:

**FINAL RESULTS FOR 400 SIMULATIONS**

| NODE | PROB./COUNT | MEAN | STD.DEV. | MIN. | MAX. |
|---|---|---|---|---|---|
| 9 | .9125 | 4.5370 | .8876 | 4.0000 | 6.0000 |
| 10 | .0875 | 6.8571 | .8452 | 2.0000 | 7.0000 |

From the computer results, we can see:

The probability that a lot is accepted = 0.9125
The probability that a lot is rejected = 0.0875
The expected time before a lot is accepted = 4.5370
The standard deviation of time before a lot is accepted = 0.8876
The expected time before a lot is rejected = 6.8571
The standard deviation of time before a lot is rejected = 0.8452

The GERTS II model can give us various probabilities for different values of M, K and p. One can thus set up a sampling plan to fit one's conditions. This sampling plan saves a lot of inspection time since K is much smaller than n (number of samples) in other sampling plans.

## IV BAYES APPROACH TO A QUALITY CONTROL MODEL

Girshick and Rubin (3) proposed a class of statistical quality control procedures and a production model. After simplifying their procedures, the model can be formulated in GERT.

A machine which is producing items can be in one of four states. In state $i = 1,2$ the machine is in production. In state $j = 3,4$ the machine is being repaired, having previously been in state $j-2$. When the machine is in state 1 which is assumed to be the desirable state there is a constant probability g that in the next time unit it will go into state 2. This probability is inherent in the production process and is assumed to be known. Once the machine enters state 2 it remains in this state until it is brought to repair (i.e. state 4). Let $P_1$ be the probability that the machine will go into state 3, and $P_2$ be the probability that the machine will go into state 4.

The GERT model is formulated as shown in Figure 12. Nodes 1, 2, 3 and 4 represent the four states. Node R represents the repaired state. The transmittance $e^c$ is placed on every branch of the network. From Mason's Rule, we get

$$W_{1,R} = \frac{P_1[1-(1-P_2)e^c]e^c + gP_2e^{2c}}{1-(1-g-P_1)e^c-(1-P_2)e^c+(1-g-P_1)(1-P_2)e^{2c}}$$

$$\because \quad P_{1,R} = W_{1,R}\Big|_{c=0} = 1$$

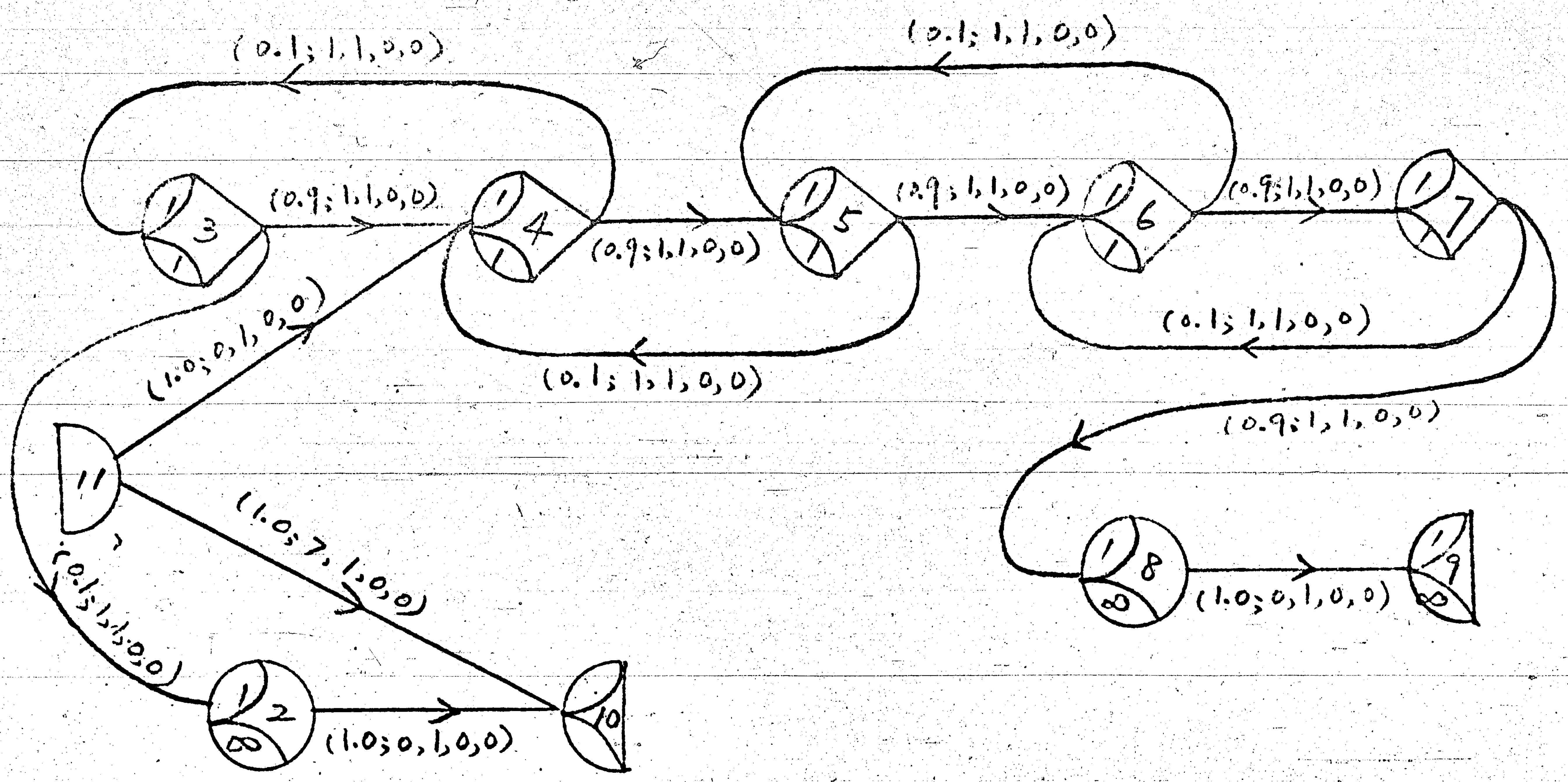



**Figure 11. GERT network for Sampling Game**

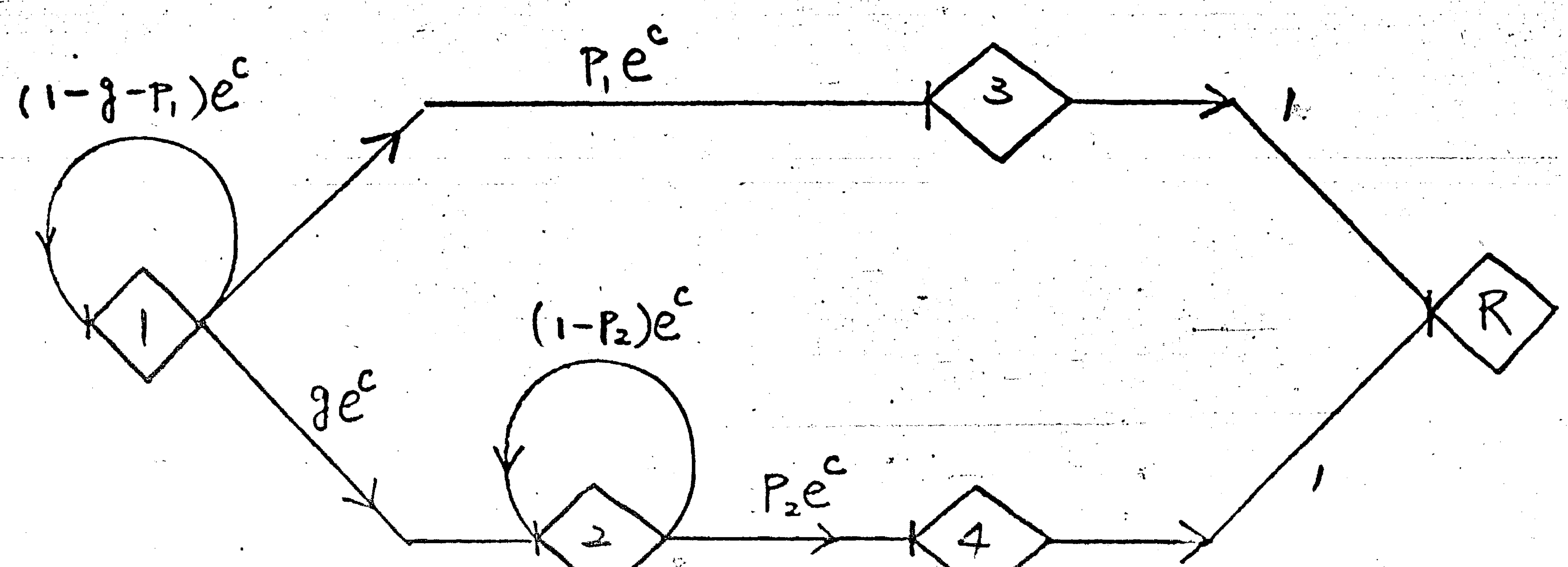



Figure 12. GERT network for Bayes approach

$$\therefore \quad M_{1,R} = W_{1,R} / P_{1,R} = W_{1,R}$$

$$\left.\frac{d M_{1,R}}{d c}\right|_{c=0} = \frac{(P_1 P_2 + g P_2)(2 P_1 P_2 + 2 g P_2 - P_1) + (P_1 P_2 + g P_2)(P_1 + P_2 + g - 2 g P_2 - 2 P_1 P_2)}{(P_1 P_2 + g P_2)^2}$$

$$= \frac{P_2 + g}{P_2 (P_1 + g)}$$

= expected number of items produced

## V DODGE'S CONTINUOUS SAMPLING PLAN CSP-1

Dodge's continuous sampling plan CSP-1 is a plan of sampling inspection for a product consisting of individual units (parts, subassemblies, finished articles, etc) manufactured in quality by an essentially continuous process.

The plan operates as follows:

1. An inspector selects a predetermined f percent (or fraction) of the product in such a manner as to assure an unbiased sample.

2. When a defect is found a predetermined clearing sequence of i subsequence and consecutive units of product must be found free of defects.

3. Upon find i units free of defects the inspector resumes sampling the fraction.

If during a period of clearing i units, a defective unit is found, the count must start over. This is a rectifying plan. i.e. all defective units found are to be corrected or replaced by good units.

The expected number of units passed under the sampling procedure before a defect is found can be obtained from a GERT model as shown in Figure 13(a). Suppose the process to be in statistical control so that the probability of any incoming unit being defective can be considered constant (p), and the probability of any unit being good is $1-p = q$. Node U represents an unit is not under inspection. Node I represents an unit is under inspection. Node D represents the detailing state. From Mason's Rule, we obtain

$$W_{U,D} = \frac{fpe^c}{1-(1-f)e^c - f(1-p)e^c}$$

$$P_{U,D} = W_{U,D}\Big|_{c=0} = 1$$

$$M_{U,D} = W_{U,D}/P_{U,D} = W_{U,D}$$

$$\frac{dM_{U,D}}{dc}\Big|_{c=0} = \frac{(fp)^2 + fp(1-fp)}{(fp)^2}$$

$$= \frac{1}{fp}$$ = expected number of units passed under the sampling procedure before a defect is found

To determine the expected number of units that will be inspected by the detailer while attempting to clear i units the GERT network shown in Figure 13(b) will be used. From Mason's Rule we get

$$W_{0,i} = \frac{(qe^c)^i}{1-[pe^c + pe^c qe^c + pe^c(qe^c)^2 + \cdots + pe^c(qe^c)^{i-1}]}$$

$$= \frac{(qe^c)^i}{1-pe^c[1+qe^c+(qe^c)^2+\cdots+(qe^c)^{i-1}]}$$

$$= \frac{(qe^c)^i}{1-pe^c\left[\frac{1-(qe^c)^i}{1-qe^c}\right]}$$

$$= \frac{(1-qe^c)(qe^c)^i}{1-qe^c-pe^c[1-(qe^c)^i]}$$

$$\therefore \quad P_{0,i} = W_{0,i}\Big|_{c=0} = 1$$

$$\therefore \quad M_{0,i} = W_{0,i}$$

$$\frac{dM_{0,i}}{dc}\Big|_{c=0} = \frac{[1-q-p(1-q^i)][iq^i-(i+1)q^{i+1}]+(1-q)q^i[1-(i+1)pq^i]}{(1-q-p(1-q^i))^2}$$

$$= \frac{q^i(1-q)(1-q^i)}{((1-q)q^i)^2}$$

$$= \frac{1-q^i}{pq^i}$$ = expected number of units that will be inspected during the detailing state

The result agree with that of Dodge (1).

Mullen (4) developed a model as follows: Suppose items on a conveyor line are inspected under a CSP –1 plan, where the M(M<i) items of concern are shown below

| | | | | | | |
|---|---|---|---|---|---|---|
| x | x | x | x | ---- | x | x |
| 0 | 1 | 2 | 3 | | M | M + 1 |

The GERT model is formulated as shown in Figure 13(c). After inspection the 0th item, there is a probability, $P_s$, of being in the sampling state and a probability, $P_d$, of being in the detailing state. Let A be the starting node. Node 1, 2, 3, ..., M + 1 represent actual items being inspected, and the B node represent an absorbing state (i.e. the state in which the detailing count equals zero).

The nodes in the upper half of the graph represent items being passed in the sampling state. Transmittance b is composed of three path.

$$b = p(1-f)e^{c} + q(1-f) + fq = p(1-f)e^{c} + q$$

where

$p(1-f)$ = probability that the item is defective and will not be sampled
$q(1-f)$ = probability that the item is nondefective and will not be sampled
$fq$ = probability that the item is nondefective and will be sampled

Any of the above three path will allow the inspection process to stay in the sampling state as it moves to the next item.

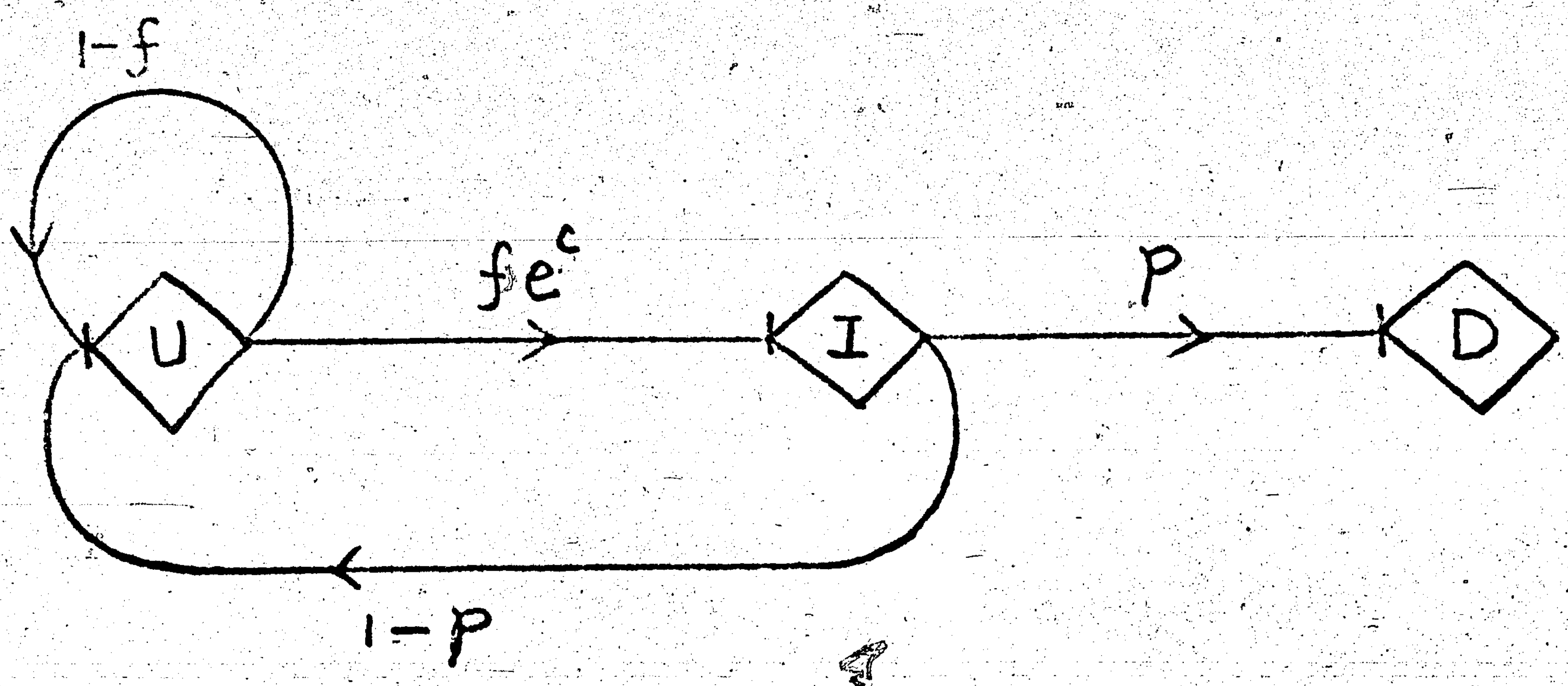
$fp$ = probability that the item is defective and will be sampled

The nodes in the lower half of the graph represent items being passed in the detailing state.

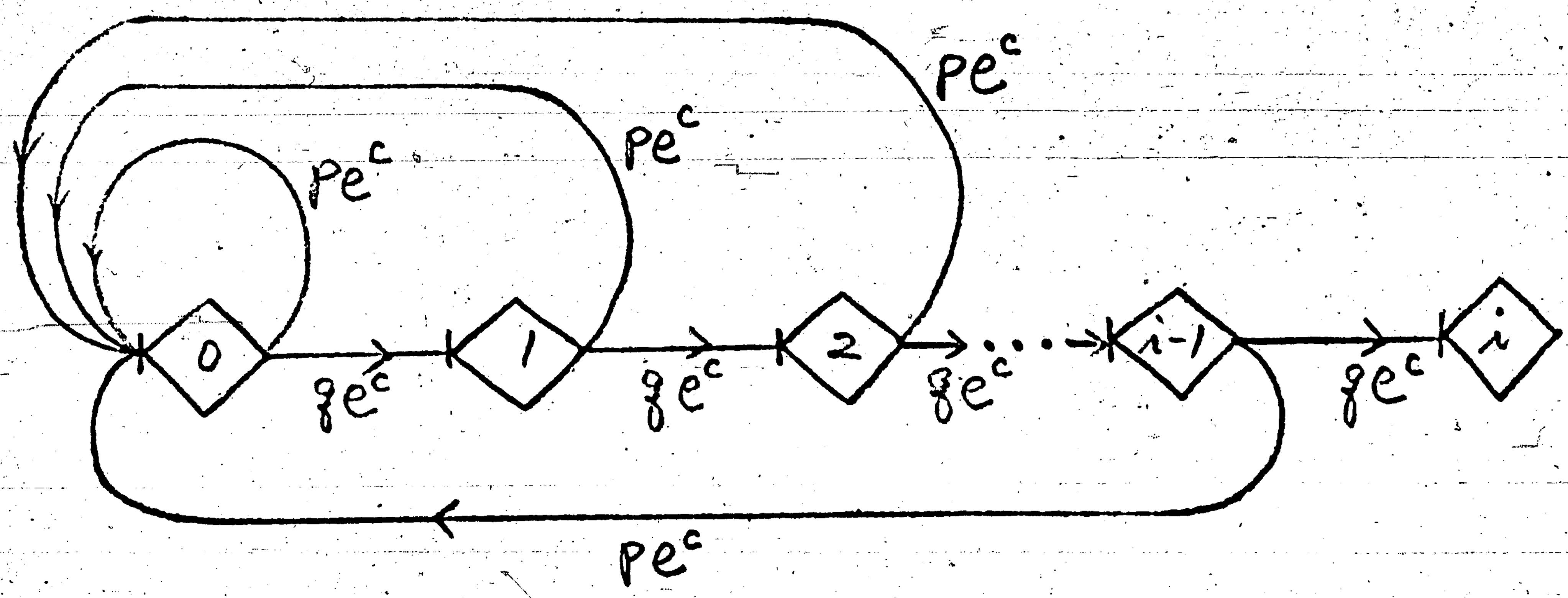
$P_{ds}$ = probability of going to the sampling state when the unit is detailed
p = fraction defective = probability that the item is defective
$h = 1 - p - P_{ds}$ = probability that the process will stay in the detailing state

All the quantities on the graph are known except $P_s$, $P_d$, and $P_{ds}$. These can be obtained by considering CSP-1 as a Markov chain. Thus,

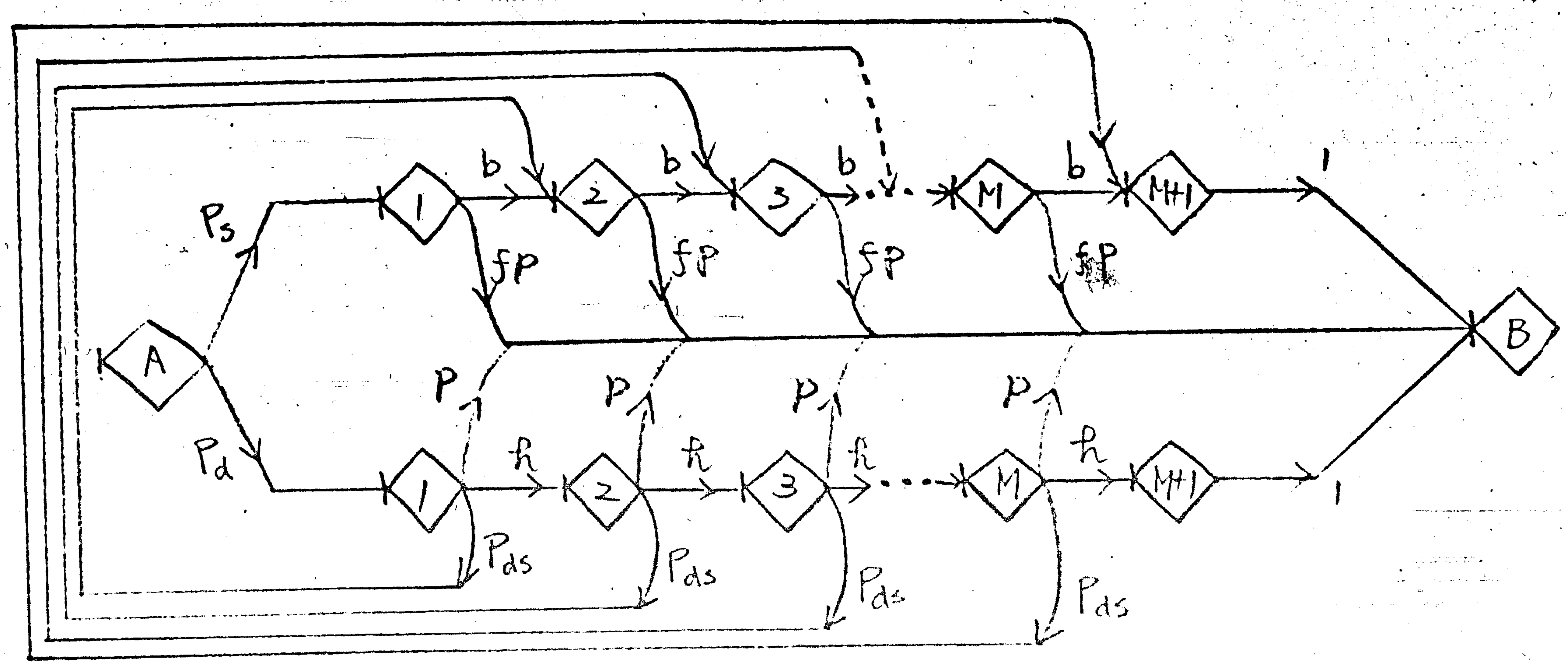
$$W_{A,B}(c) = P_s f p \frac{1-b^{M}}{1-b} + P_s b^{M} + P_d p \frac{1-h^{M}}{1-h} + P_d h^{M} + P_d P_{ds} f p \sum_{j=1}^{M-1} h^{M-j-1} \frac{1-b^{j}}{1-b} + P_d P_{ds} \sum_{j=0}^{M-1} h^{M-j-1} b^{j}$$




(a)




(b)




(c)

Figure 13, GERT networks for CSP -1

This function can be interpreted as the MGF for the distribution of defectives remaining in M units after inspection of the M unit is completed.

$$\left.\frac{d\,W_{A,B}(c)}{dc}\right|_{c=0} = \text{expected number of defectives remaining in M units after inspection}$$

Let

$$Y_j = (1-b^j)/(1-b)$$
$$= 1 + b + b^2 + \cdots + b^{j-1} \quad , \quad j = 1, 2, \cdots, M$$

and

$$Z_j = b^j$$

Letting $B = \frac{db}{dc} = p(1-f)e^c = b - q$ and take the first derivative we have

$$Y_j' = B + 2bB + 3b^2B + \cdots + (j-1)\,b^{j-2}B$$
$$= B\sum_{k=2}^{j}(k-2)\,b^{k-2}$$
$$Z_j' = j\,b^{j-1}B$$

Thus we have

$$\left.\frac{d\,W_{A,B}}{dc}\right|_{c=0} = \left( P_s f p Y_M' + P_s Z_M' + P_d P_s f p \sum_{j=1}^{M-1} h^{M-j-1} Y_j' + P_d P_{ds} \sum_{j=0}^{M-1} h^{M-j-1} Z_j' \right)\Bigg|_{c=0}$$

The moment generating function can be expanded as a power series in $e^c$ to obtain the discrete probability distribution.

$$W_{A,B}(c) = a_0 + a_1 e^c + a_2 e^{2c} + \cdots + a_M e^{Mc}$$

then,

$a_d$ = probability of having d defectives (d = 0,1,···,M)

$$\therefore\ W_{A,B}(c) = P_s f p (1+b+b^2+\cdots+b^{M-1}) + P_s b^M + P_d p \frac{1-h^M}{1-h}$$
$$+ P_d h^M + P_d P_{ds} f p \sum_{j=1}^{M-1} (1+b+b^2+\cdots+b^{j-1})$$
$$+ P_d P_{ds} \sum_{j=0}^{M-1} h^{M-j-1}$$

Recall that,

$$b = p(1-f)e^c + q$$

and
$$B = \frac{db}{dc} = p(1-f)e^c$$

thus
$$b = B + q$$

Note that the $a_d$ is simply the coefficient of $B^d$ multiplied by P(1-f). Thus, after simplication and collecting coefficients we obtain

$$a_d = p^d(1-f)^d \Big[ P_s f p \sum_{k=d}^{M-1} \binom{k}{d} q^{k-d} + P_s \binom{M}{d} q^{M-d}$$
$$+ P_d P_{ds} f p \sum_{j=d+1}^{M-1} h^{M-j-1} \Big( \sum_{k=d}^{j-1} \binom{k}{d} q^{k-d} \Big)$$
$$+ P_d P_{ds} \sum_{j=d}^{M-1} h^{M-j-1} \binom{j}{d} q^{j-d} \Big]$$

for $d = 1, 2, \cdots, M$.

## VI CONTROL CHARTS

A control chart is a device for describing in concrete terms what a state of statistical control is; second, a device for attaining control, and, third, a device for judging whether control has been attained.

Certain variations in the quality of product belong to the category of chance variations about which little can be done other than to revise the process. Besides chance variations, there are variations produced by "assignable causes." These are relatively large variations that are attributable to special causes. If a group of data is studied and it is found that their variation conforms to a statistical pattern that might reasonably be produced by chance causes, then it is assumed to be "under control." If the variations in the data do not conform to a pattern that might reasonably be produced by chance causes, then it is concluded that assignable causes are at work. In this case the conditions are said to be "out of control."

Now several out of control conditions are studied.

(a) A point falls out of the control limit. A GERT network is modeled for this condition as shown in Figure 14(a). Let P be the probability that a point falls out of the control limit. Under a normal curve the probability that a deviation from the mean will exceed $3\sigma$ in one direction is 0.00135 or in both direction is 0.0027. Node A represents the process is under control. Node B represents the process is out of control. From Mason's Rule, we get

$$W_{A,B} = \frac{Pe^c}{1-(1-P)e^c}$$

$$\because\ P_{A,B} = W_{A,B}\big|_{c=0} = 1$$

$$\therefore\ M_{A,B} = W_{A,B}$$

$$\frac{dM_{A,B}}{dc}\bigg|_{c=0} = \frac{P^2+P(1-P)}{P}$$

$$= \frac{1}{P}$$

= expected number of points within the control limit until a point falls out of the limit

(b) 5 points appear consecutively in one side of the central line. A GERT network is modeled for this condition as shown in Figure 14(b). Let P be the probability that a point is above (or below) the central line. From Mason's Rule, we get

$$W_{0,5} = \frac{P^5e^{5c}}{1-(1-P)e^c-P(1-P)e^{2c}-P^2(1-P)e^{3c}-P^3(1-P)e^{4c}-P^4(1-P)e^{5c}}$$

$$\because\ P_{0,5} = W_{0,5}\big|_{c=0} = 1$$

$$\therefore\ M_{0,5} = W_{0,5}$$

$$\frac{dM_{0,5}}{dc}\bigg|_{c=0} = \frac{1+P+P^2+P^3+P^4}{P^5}$$

= expected number of points under control until 5 points appear consecutively in one side of the central line

(c) 2 points out of 3 consecutive points are in the area between $2\sigma$ and $3\sigma$. A GERT network is modeled for this condition as shown in Figure 14(c). Node A is the starting node. Node B represents a point is in the area between $2\sigma$ and $3\sigma$. Node c represents a point is not in the area between $2\sigma$ and $3\sigma$. Node D represents a absorbing state in which the process is out of control. P is the probability that a point is in the area between $2\sigma$ and $3\sigma$. Under a normal curve, P = 0.0429. From Mason's Rule, we get

$$W_{A,D} = \frac{P^2(1-P)e^{3C} + P^2 e^{2C}}{1-(1-P)e^{C} - P(1-P)^2 e^{3C}}$$

$$\therefore P_{A,D} = W_{A,D}\Big|_{C=0} = 1$$

$$\therefore M_{A,D} = W_{A,D}$$

$$\frac{dM_{A,D}}{dc}\Big|_{C=0} = \frac{1+2P-P^2}{2P^2-P^3}$$

= expected number of points under control until 2 points out of 3 consecutive points are in the area between $2\sigma$ and $3\sigma$

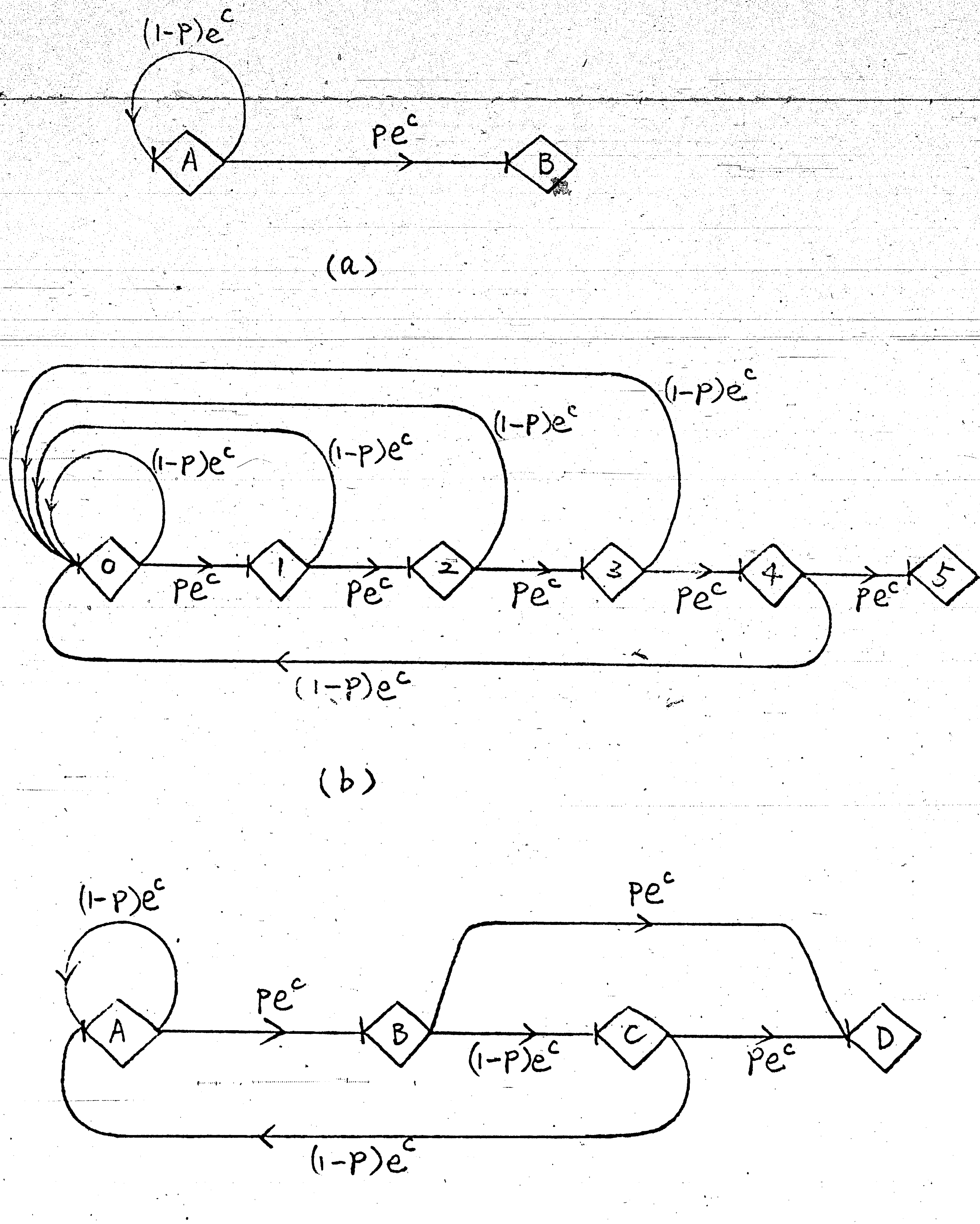



Figure 14. GERT networks for control chart

# VII AN EXAMPLE OF THE EFFECTIVE USE OF GERT MODELS

A number of GERT models have been developed in the previous chapters. Now the effective use of those models will be discussed. Since the purpose of this thesis is to apply GERT in various fields of quality control, the author does not intend to extend every model. Therefore, only one typical model will be discussed to show the effective use of the model.

Two fundamental questions arise in using Dodge's continuous sampling plans. These are 1) how to select an optimum plan to begin with, and 2) when to stop the inspection process if incoming quality deteriorates to an unacceptable level. Fry (2) proposed a stopping rule and a cost model for CSP-1 which will be discussed as follows:

A stopping rule is to permit stopping of the manufacturing process for analysis and repair when incoming quality deteriorate to an unacceptable level. A cost model is developed from which the optimal inspection plan can be selected under given input conditions. When the probability of a unit being defective changes from $P_0$ (normal process) to $P_1$ (out of control condition), it is said the quality is shifted. A GERT model for detailing state is formulated as shown in Figure 15(a). Let $q_0 = 1-P_0$. From Mason's Rule, we obtain

$$W_{0,i} = \frac{q_0^{\,i}}{1 - P_0 e^c (1 + q_0 + q_0^2 + \cdots + q^{i-1})}$$

$$P_{0,i} = W_{0,i}\Big|_{c=0} = 1$$

$$M_{0,i} = W_{0,i} = \frac{q_0^{\,i}}{1 - P_0 e^c (1 + q_0 + q_0^2 + \cdots + q_0^{i-1})}$$

$$= \frac{q_0^{\,i}}{1 - e^c (1 - q_0^{\,i})}$$

The moment generating function can be expanded further to a power series.

$$M_{0,i} = q_0^{\,i} + q_0^{\,i}(1 - q_0^{\,i}) e^c + q_0^{\,i}(1 - q_0^{\,i})^2 e^{2c} + \cdots$$

$P(x) = q_0^{\,i}(1 - q_0^{\,i})^x$ = the probability of x defects in clearing a sequence of i units

Let

$N_c$ = critical number of defects, which if found while attempting to clear i units, will cause the process to be stopped and investigated

Thus,

$$N_c = \min_{x=0,1,2,\cdots} \sum_{j=0}^{x} q_0^{\,i}(1 - q_0^{\,i})^j$$

This stopping rule can be interpreted as the process will be stopped as soon as the total number of defects found equals or exceeds $N_c$. Tables of $N_c$ for various $P_0$'s and $\alpha$'s (where $\alpha$ is the confidence level associated with stopping the process) can be found in Fry's paper (2). Once the shift in quality occurs the process is on a course to be stopped. The stopping cycle can be depicted as shown in Figure 15(b).

The expected number of units passed under the sampling procedure in the $P_1$ state before a defect is found is

$$E(N_s) = \frac{1}{fP_1}$$

It has been proved in Chapter V.

The stopping process in the detailing state can be modeled in GERT network as shown in Figure 15(c). Let $q_1 = 1-P_1$. From Mason's Rule, we obtain

$$W_{0,N_c} = \left(\frac{P_1 e^s}{1-q_1 e^s}\right)^{N_c}$$

$$\because \quad P_{0,N_c} = W_{0,N_c}\Big|_{s=0} = 1$$

$$\therefore \quad M_{0,N_c} = W_{0,N_c} = \left(\frac{P_1 e^s}{1-q_1 e^s}\right)^{N_c}$$

The expected number of units inspected until the process is stopped can be obtained by:

$$E(N_c) = \frac{d}{ds} M_{0,N_c}\Big|_{s=0}$$

$$= \frac{d}{ds}\left(\frac{P_1 e^s}{1-q_1 e^s}\right)^{N_c}\Bigg|_{s=0}$$

$$= \frac{N_c}{P_1^2}$$

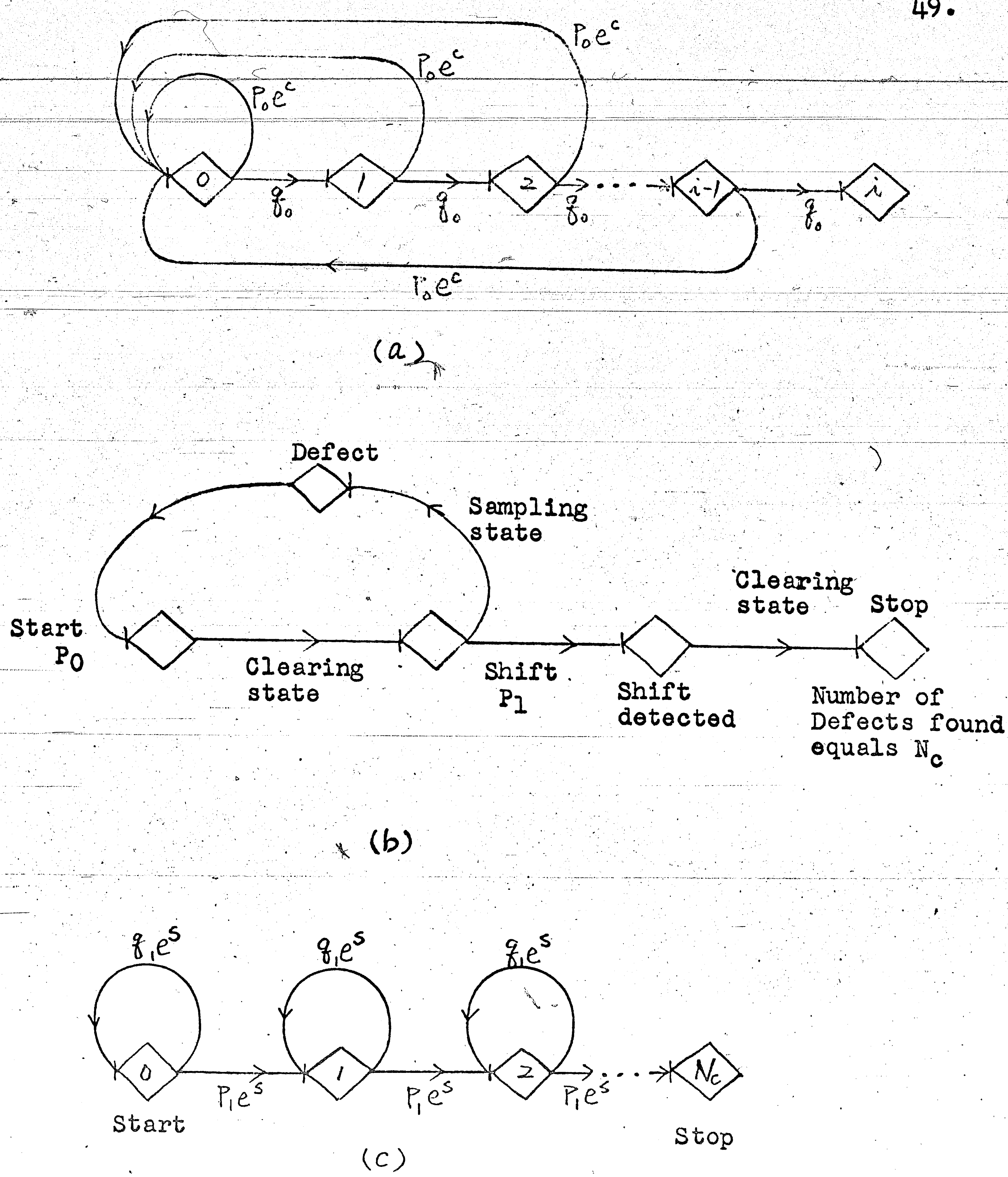



Figure 15. GERT networks for stopping rule in CSP—1

Thus, the total expected number of units produced in the $P_1$ state is:

$$E(N_{P_1}) = \frac{1}{fP_1} + \frac{N_c}{P_1^2}$$

Let N be the total number of units produced in a stopping cycle, and $\beta$ be the probability of entering the $P_1$ state. Then

$$E(N_{P_1}) = \beta N$$

$$\therefore \quad \beta N = \frac{1}{fP_1} + \frac{N_c}{P_1^2}$$

or

$$N = \frac{1}{\beta}\left(\frac{1}{fP_1} + \frac{N_c}{P_1^2}\right)$$

The expected number of units produced in the $P_o$ state can be found by:

$$E(N_{P_o}) = (1-\beta)N$$

$$= \frac{1-\beta}{\beta}\left(\frac{1}{fP_1} + \frac{N_c}{P_1^2}\right)$$

Thus, the expected number of units produced in a stopping cycle has been derived in both state $P_o$ and $P_1$.

The expected number of units inspected in the $P_1$

state can be derived by:

$$E(N_{P_1,\text{inspect}}) = f\left(\frac{1}{fP_1}\right) + \frac{N_c}{P_1^2}$$

$$= \frac{1}{P_1} + \frac{N_c}{P_1^2}$$

And then the expected number of units inspected in the $P_1$ state can be obtained from:

$$E(N_{P_1,\text{uninspect}}) = E(N_{P_1}) - E(N_{P_1,\text{inspect}})$$

$$= \frac{1}{fP_1} + \frac{N_c}{P_1^2} - \frac{1}{P_1} - \frac{N_c}{P_1^2}$$

$$= \frac{1-f}{fP_1}$$

These expected values are used to set up a cost model from which the optimal inspection plan can be selected under given input conditions.

Let

$c_1$ = the unit cost if inspecting an item
$c_2$ = the penalty cost of letting a defect slip through
$c_3$ = the cost of erroneously stopping process
$F_{P_0}$ = the total fraction of units inspected when process is in control

Dodge (1) formulated the following relations:

$$F_{P_0} = \frac{f}{f + q_0^i(1-f)}$$

Defining the expected total cost of a stopping cycle as ETC, we then have

ETC = cost of inspecting units in $P_0$ state
+ cost of uninspected defective units in $P_0$ state
+ cost of inspecting units in $P_1$ state
+ cost of uninspected defective units in $P_1$ state
+ cost of erroneously stopping job in $P_0$ state

$$= C_1\left(\frac{f}{f+q_0^{i}(1-f)}\right)\left(\frac{1-\beta}{\beta}\right)\left(\frac{1}{fP_1}+\frac{N_c}{P_1^2}\right)$$

$$+ C_2\left(\frac{q_0^{i}(1-f)}{f+q_0^{i}(1-f)}\right)P_0\left(\frac{1-\beta}{\beta}\right)\left(\frac{1}{fP_1}+\frac{N_c}{P_1^2}\right)$$

$$+ C_1\left(\frac{1}{P_1}+\frac{N_c}{P_1^2}\right)+C_2\left(\frac{1-f}{f}\right)+C_3(1-\alpha)(1-\beta)$$

This equation represents the cost model for a stopping cycle. It is by minimization of this cost that the optimal plan for a given set of conditions can be determined.

## VIII RESULTS

This thesis has investigated the application of GERT to quality control area. Simple and double sampling plans, Military Standard 105 D, Bayes approach, Dodge's continuous sampling plan CSP-1, and control charts were studied.

Single and double sampling plans were modeled from which the probabilities of accepting and rejecting a lot, the expected number and variance of units passed until the lot is accepted as well as rejected can be obtained. Military Standard 105 D was modeled from which we can obtain the probability of discontinuing inspection, and the expected number and variance of lots passed until inspection is discontinued. A sampling game was formulated from which a new sampling plan can be developed. Bayes approach to a quality control model (3) was simplified and formulated in GERT from which the expected number of items produced can be obtained.

Dodge's continuous sampling plan CSP-1 was modeled from which the expected number of units passed under the sampling procedure before a defect is found and the expected number of units that will be inspected during the detailing state can be obtained. In addition, the short-run characteristics of CSP-1 were modeled to define the probability distribution for the defectives in a short run of units.

The various characteristics of control charts were modeled to obtain the expected number of points under control until certain out of control conditions occurred.

A stopping rule and a cost model were discussed for Dodge's continuous sampling plan CSP-1 to show the effective use of GERT models. The stopping rule was presented to permit stopping of the manufacturing process for analysis and repair when incoming quality deteriorate to an unacceptable level, and the cost model was presented from which the optimal inspection plan can be selected under given input conditions.

Several problems have been solved by Pritsker and Ishmael's computer programs (7,8) successfully. The readers are thus encouraged to use these programs for analyzing GERT network containing either Exclusive-or or AND type nodes.

From the above results, we can see that GERT models provide the information of probabilities, and mean and variance of the distribution at the same time in an efficient way. In addition, the simplicity of formulation provides a starting point for analysis and scheduling of the operational system.

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# VITA

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