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# Analysis of Power Network Defense Under Intentional Attacks

By

Weiming Lei

A Thesis  
Presented to the Graduate and Research Committee  
of Lehigh University  
in Candidacy for the Degree of  
Master of Science

in  
Industrial and Systems Engineering

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Jan 2018

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This thesis is accepted and approved in partial fulfillment of the requirements for the Master of Science.

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Date

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Lawrence V. Snyder, Thesis Advisor

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Ted Ralphs, Chairperson of Department

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# Abstract

In this thesis, we introduce a method to identify the most critical components (e.g., generators, transformers, transmission lines) in an existing electric power grid, that contains renewable (wind) generators. We assume the power system is under threat of intentional attacks. By learning the potentially best attacking plan, the system operator can have a better understanding of the most important components in the system. We use a bilevel optimization model to describe the problem and a decomposition approach to solve the bilevel model by finding maximally disruptive attack plans for attackers who have limited attacking resources. The testing data are based on standard reliability test networks and we formalized the original data with real data collected from Texas by the Electric Reliability Council of Texas (ERCOT). Our results show that the method in this thesis can be used by the operator of the power system to find out critical components and make better defensive plans to improve system security.

# 1. Introduction

Electric power systems are of great importance to every country's security and economy. In the United States, the system's vulnerability to physical disruptions from various causes has been recognized for a long time [1]. The system's unreliability is continuously becoming more severe in recent years since infrastructure has not expanded as quickly as demand has grown, so the system can fail or be damaged more easily [2]. What's more, as The Committee on Science and Technology for Countering Terrorism states [3], the threat of human attacks on the system has become more serious, too.

Wind power is environmental friendly with many benefits [4]. Those advantages have led to the rapid increase of wind energy in power systems in recent years all over the world. When considering models with wind energy, however, some characteristics of it are very different from traditional generators because of high stochasticity and intermittency in production output. Due to that, considering wind power in a power system will bring uncertainty to its short-term and real-time operation. These factors call for new models for power systems operation.

Considering the problems above, this thesis presents a bilevel interdiction optimization model and solution techniques to analyze the vulnerability of an electricity power system that contains renewable generators against intentional terrorist attacks.

We determine important power grid components by locating maximally disruptive,

simultaneous attacks on a grid. By studying how the attackers would attack the power system, we will eventually have a clear understanding of which components are more important to be protected. We search for optimal attacks, i.e., a set of attacks that cause the largest extra operational cost given certain offensive resources. And we only consider physical attacks on the power system. We report results for our techniques applied to reliability-benchmark networks.

## 2. Literature Review

In this chapter, we present an overview of the studies most related to this thesis. We will review the interdiction problem, characteristics of electric power system and the characteristics of renewable generators in electric power systems.

### 2.1 Interdiction problem

A dictionary definition of interdict, in the military sense, is:” to destroy, cut or damage by ground or aerial firepower (enemy lines of reinforcement, supply, or communication) in order to stop or hamper enemy movement and to destroy or limit enemy effectiveness [6].”

It is obvious that the definition of interdiction can be expanded in different ways. Examples of what we call network interdiction are throughout human war history, from antiquity to modern war [7]. In these kinds of interdiction, the actions of the interdictors, also known as the attackers, are usually modeled using network optimization. In this model, they attack the components of the network to disrupt the network’s function. The target could be a bridge, road, critical facilities, power transmission line and so on.

The simplest network interdiction problem arises from the well-known max flow-min cut theorem [8]. In the 1950’s, Harris [9] did research about evaluating railway

capacity and this method aimed to cut off the function of the railway system. This can be regarded as a “single-level” network interdiction problem. After a decade, the earliest research about “bi-level” network interdiction problem (BNI) was presented by Wollmer [10]. His study focused on a flow capacitated network and give a method to find the most critical path. Almost at the same time, Danskin [11] introduced “max-min models” and that could be regarded as a generalization of BIN.

In 1993 Wood [12] considered a problem in which the network operator attempts to maximize flow through a known and capacitated network while an interdictor tries to minimize this maximum flow by interdicting network arcs using limited resources. Then in 1998, Cormican et al. [13] investigated a stochastic version of the previous network interdiction problem, and a two-stage stochastic integer program model is made to handle uncertain arc capacities.

In 2002, the National Strategy for Homeland Security deemed 13 infrastructure sectors critical to the United States [14], such as Energy, Transportation, Information and Telecommunications. After that, a great deal of research has been done to analyze the vulnerability of critical infrastructure and their defense plan under intentional attacks in various fields. Among these studies, the interdiction model is widely considered. Brown et al [15] applied bilevel and trilevel optimization models and methodology accordingly to improve the resilience of critical infrastructure against terrorist attacks in general. In these models, information is transparent for attacker and defender and the actions are

taken alternately by both sides. In the field of supply chain management, Snyder et al. [16] presented a broad range of models for designing supply chains resilient to disruptions, and Scaparra et al. [17] present a bilevel formulation of the  $r$ -interdiction median problem with fortification.

Also, there are papers relevant to interdiction in electric power systems. Yao et al. [18] presented a trilevel optimization model of resource allocation and a decomposition approach to find optimal solutions in electric power network defense. The most relevant literature to this thesis is presented by Salmeron et al. [19]. In their paper, they describe a method to analyze and increase the security of a known electric power system facing terrorist attacks by identifying the most critical components. The purpose of the attacking plan is to cause maximal disruptions to the power system under limited offensive resources. The author used a bilevel optimization model and a heuristic to solve the model. In this thesis, we use the bilevel model and solution methodology by Salmeron et al. [19] as a base. As we take renewable energy in to account, we make changes to their model accordingly.

## **2.2 Optimal power flow problem**

Among the interdiction and defense problems we talked above, the network operator is always intending to find a way to maximize the system's capacity (e.g. max-flow problem) or minimize a certain kind of "price" (e.g. transportation time, transmission cost) while the interdictor is trying to do the opposite. When considering electric power

systems, in general, the system operator wants to have better control and minimize the total operation cost. This problem is often known as the optimal power flow problem (OPF).

In general, the purpose of OPF is to optimize a specific cost, planning, or reliability objective by controlling parameters in power generation, transmission, and distribution networks within an electrical network without violating network power flow constraints and equipment operating limits. OPF was first introduced by Carpentier in 1962. After that, this optimization method has been widely used in power system operation, analysis, and planning. The general structure of OPF given in the survey of Rebennack et al. [20] is as follows:

$$\begin{aligned} & \min f(u, x) \\ \text{s.t.} & \\ & g(u, x) = 0 \\ & h(u, x) \leq 0, \end{aligned}$$

where  $u$  represents controllable system variables while  $x$  are dependent or state variables. The objective function  $f(u, x)$  represents the system's optimization goal (most commonly the generation cost). Vector functions  $g(u, x)$  and  $h(u, x)$  represent system equality and inequality constraints, respectively. In the survey of Rebennack et al., they show in detail that depending on the selection of  $f$ ,  $g$ , and  $h$ , the OPF problem has many different formulation variations, e.g. linear, mixed integer-linear, nonlinear, or mixed integer-nonlinear programming problem. Here, we would like to introduce the

most classical formulation and the linearized version which is most closely related to the multi-period OPF model in this thesis. The classic formulation can be written exclusively in terms of voltage and this formulation given by [21] is as follows:

$$\begin{aligned}
& \min_{v,p,q} f(v,p,q) \\
& \text{s.t.} \\
& p_{ij} + iq_{ij} = v_i(v_i^* - v_j^*)y_{ij}^* \quad (\text{OPF1}) \\
& \sum_j p_{ij} = p_i \quad (\text{OPF2}) \\
& \sum_j q_{ij} = q_i \quad (\text{OPF3}) \\
& \underline{p}_i \leq p_i \leq \bar{p}_i \quad (\text{OPF4}) \\
& \underline{q}_i \leq q_i \leq \bar{q}_i \quad (\text{OPF5}) \\
& p_{ij}^2 + q_{ij}^2 \leq \bar{s}_{ij}^2 \quad (\text{OPF6}) \\
& \underline{v}_i \leq |v_i| \leq \bar{v}_i \quad (\text{OPF7})
\end{aligned}$$

In this formulation,  $f$  is the objective function, which is a function of  $v,p,q$ . The variables  $v,p,q$  represent voltage, real power flow and reactive power flow respectively. In the power system, if node  $i$  is connected to node  $j$  by a transmission line, the real and reactive power flows between the two nodes are  $p_{ij}$  and  $q_{ij}$ . If real power goes from node  $i$  to  $j$ ,  $p_{ij} > 0$  and  $p_{ji} < 0$ .  $y_{ij}$  is the admittance and  $*$  denotes complex conjugate. Constraint (OPF1) determines the relationship between power flows and voltages. Constraints (OPF2) and (OPF3) mean that the real and reactive powers into or out of node  $i$  are the sums of the flows through the transmission lines connected to node  $i$ . And the powers into or out of node  $i$  have lower and upper bounds  $\underline{p}_i, \bar{p}_i, \underline{q}_i, \bar{q}_i$  and the relationship is shown by (OPF4) and (OPF5). In (OPF6),  $\bar{s}_{ij}$  denotes the



transmission line's apparent power capacity and this constraint shows that the complex power flow magnitude must be below the transmission line's capacity. (OPF7) ensures that the voltage has lower and upper bounds.

The above classic formulation of OPF is of great advantage when considering the accuracy of the system's behavior. However, constraints (OPF1) are nonlinear and make this formulation very hard to solve. From a practical aspect, we usually need the OPF problem to be solved in relatively short time (say every 15 minutes), and also the requirement on accuracy is relative lower. Thus, it is reasonable to make some approximations and relaxations to this optimization problem to make the formulation more practical. Based on some observations of the power system features, we could linearize the constraints to get a linear feasible set which is known as linearized optimal power flow or DC-OPF. The DC-OPF introduced in [21] is as follows:

$$\begin{aligned}
 & \min_{\theta, p} f(\theta, p) \\
 \text{s.t.} \quad & p_{ij} = b_{ij}(\theta_i - \theta_j) && \text{(DC-OPF1)} \\
 & \sum_j p_{ij} = p_i && \text{(DC-OPF2)} \\
 & p_i \leq p_i \leq \bar{p}_i && \text{(DC-OPF3)} \\
 & p_{ij}^2 \leq \bar{s}_{ij}^2 && \text{(DC-OPF4)}
 \end{aligned}$$

$\theta_i$  is the phase angle in node  $i$ .  $b_{ij}$  is the susceptance between node  $i$  and  $j$ . In this formulation, we get rid of all the reactive power variables and constraints. Now it is much easier to solve.

## **2.3 Wind energy, energy storage and multiperiod OPF**

In this section, we discuss the characteristics of wind energy and its effect on electric power systems in practice and modeling.

### **2.3.1 Characteristics of wind energy and its effect on power system operation**

Currently, there is increasing concern over the environmental impact and sustainability of conventional fossil-fueled power plants. As a result, renewable energy sources are becoming an important part of the generation mix worldwide. In particular, wind energy is one of the fastest growing renewable energy technologies in the world [22] and will play a crucial role in the future energy supply. According to the forecasts of the Global Wind Energy Council, wind energy will supply around 16% worldwide in 2020 [23]. In the state of California, peak demand for power in the year 2030 will exceed 80 GW [24], [25]. As fossil fuel and nuclear plants retire in the next few decades, a required 15% reserve margin, which means 10GW of new generation capacity, will be needed by 2030. That capacity will be supplied by wind and solar energy.

Though wind energy is appealing due to its environmentally friendly nature, high potential and low generating cost, it also has some characteristics that bring us technical difficulties. The notorious drawback of wind generation is the difficulty in its output

prediction [26]. This is because wind power plants generate power when the wind is blowing, and the plant output depends on the wind speed. Since wind fluctuates from minute to minute and hour to hour, we cannot get a high-accuracy prediction of its speed or the energy output.

The poor prediction of wind energy output leads to difficulty in power system operation. In a power system, the total power generated by all power generators must equal the aggregate demand for electric power at every moment. In the case that both wind generators and conventional fossil generators are in the same power system, total output given by all generators should be equal to the real-time demand. Since the wind power fluctuates a lot, the system operator needs to make the output of conventional generators change accordingly to compensate the wind power fluctuation and keep the system in balance. That would not only be expensive but also nearly impossible to execute in practice, especially when the power generator takes up a large portion of the total generation amount. (This kind of characteristic of wind generation is referred to as non-dispatchable.) Thus, new tools, technologies and additional grid services are needed to provide the required level of system resiliency. [27-28]

### **2.3.2 Grid-scale energy storage systems (ESS)**

From the last subsection, we know that technologies that help increase power system flexibility are critical in implementing renewable generation without decreasing system

efficiency and reliability. Grid-scale energy storage is widely believed to have the potential to provide this added flexibility. There is a great deal of research and multiple types of technology related to this topic [29]. The basic idea of energy storage systems is to absorb short-term fluctuations and transmission capacity that not only transports power from generation to load, but can also provide spatial diversity in generation to mitigate intermittency of renewable sources [30]. In other words, the function of an energy storage system is to save extra power when the power demand is low and the generated power is high and release that power when the generators in the system cannot provide enough power.

### 2.3.3 Multiperiod optimal power flow

For the classic OPF problem, we are solving a static system. However, when considering a storage system, it requires us to model charge/discharge dynamics. Also, if the renewable generator takes up a large portion of the whole generation system, we need to consider restrictions in the magnitude of changes in conventional generation (ramping). In the above case, it is better so model the optimal power flow problem in a multiperiod way. Here, we introduce a linearized finite-horizon multiperiod OPF model (FOPF) given by Chandy et al. [30]. Later, we use this model as a base and modify the objective function in our interdiction OPF model and add the cost for unmet demand. Chandy's model is as follows:

$$\begin{aligned}
& \min_{\theta, q, r, g, b} \sum_{i \in G} \sum_{t=1}^T \left( c_i(g_i(t), t) + h_i(b_i(t), r_i(t)) \right) + \sum_{i \in G} h_i^T(b_i(T)) \\
& \text{s.t.} \quad V_i V_j Y_{ij} (\theta_i(t) - \theta_j(t)) \leq \bar{q}_{ij}(t), i \neq j \in N \quad \text{(FOPF1)} \\
& \quad \quad q_i(t) = \sum_{j \in N} V_i V_j Y_{ij} (\theta_i(t) - \theta_j(t)), i \in N \quad \text{(FOPF2)} \\
& \quad \quad q_i(t) = -d_i(t), i \in D \quad \text{(FOPF3)} \\
& \quad \quad q_i(t) = g_i(t) + r_i(t), i \in G \quad \text{(FOPF4)} \\
& \quad \quad g_i(t) \geq 0, i \in G \quad \text{(FOPF5)} \\
& \quad \quad b_i(t) = b_i(t-1) - r_i(t), i \in G \quad \text{(FOPF6)} \\
& \quad \quad 0 \leq b_i(t) \leq B_i, i \in G \quad \text{(FOPF7)}
\end{aligned}$$

In the objective function, the decision variables are  $\theta, q, r, g, b$ , and they represent for phase angle, power, charging amount, generated power amount and battery level,

respectively. Sets  $G, D$  represent generation nodes and demand nodes, respectively.  $N = G \cup D$  is the set of all nodes.  $t = 1, \dots, T$  represents for the time horizon.  $c_i(g_i(t), t)$  is the generation cost.  $h_i(b_i(t), r_i(t))$  is the storage cost at the end of each time period.  $h_i^T(b_i(T))$  is the cost for leftover power in the storage system at the end of  $T$ .

In (FOPF1),  $V_i$  and  $V_j$  are the voltage at node  $i$  and  $j$ , respectively.  $Y_{ij}$  is the admittance between nodes  $i$  and  $j$ .  $\bar{q}_{ij}(t)$  represents the line capacity from nodes  $i$  to  $j$ .

In (FOPF2),  $q_i(t)$  represents the net power export from node  $i$  at time  $t$ .

In (FOPF3),  $d_i(t)$  is the demand in node  $i$  at time period  $t$ . This constraint means that demand must be met by supply from the generation.

(FOPF4) means that the net power export from a generator node consists of power from the node's generator and battery.

(FOPF5) means generation power is non-negative.

(FOPF6) means the battery energy level in a certain time period is equal to its energy level in the last time period minus the energy it releases in this time period.

(FOPF7) sets the battery energy level to be non-negative and has an upper bound.

## 2.4 Novelty of this thesis

With the implementation of ESS, there is a trend that the proportion of renewable energy will grow in power system. Then it is necessary to take renewables into account

when considering system security. However, the interdiction-related literature above does not consider renewables when modeling the power system. This thesis considers a bilevel interdiction problem of a known power system where we use FOPF instead of OPF to get a better model of the system containing wind generator. In the next chapters, we will define our problem, describe our model and present an algorithm to solve it.

# 3. Problem Definition and mathematical Model

## 3.1 Overview

The object of our study is a given electric power system. We consider the situation in which the operator of the system (defender) tries to minimize the cost of meeting electric power demand while the interdictor (attacker) who is aware of the whole information intends to destroy the most critical components of the system under limited resources in order to maximize the defender's subsequent operational cost.

We assume that the defender can satisfy the system's power demand in two ways. One is using power generated in the system, which has a generation cost. The other is when the system cannot meet its own demand, the defender will use methods from outside of the system (e.g. buying from another company or using emergency power storage), which has a much higher cost. So, by minimizing the total cost of meeting demand, the defender will avoid using methods out of the system since they cost much more. On the other hand, the attacker will first aim at destroying the components which will lead to the most unmet demand. By solving this problem, we will get to know the best attack plan, which can help us to identify the most important components in a deterministic system. We formulate this problem as a bilevel interdiction problem in an



electric power system containing wind generation.

This problem can be represented by a bilevel optimization model as follows:

$$\begin{aligned} & \max_{\delta \in \Delta} \min_{\mathbf{p}} \mathbf{c}^T \mathbf{p} \\ & \text{s. t. } \mathbf{g}(\mathbf{p}, \boldsymbol{\delta}) \leq \mathbf{b} \\ & \quad \mathbf{p} \geq \mathbf{0}. \end{aligned}$$

The inner minimization problem is a MOPF problem, we have mentioned the mathematical model in the last section and will give a detailed description in the following subsection. Here,  $\mathbf{p}$  represents variables in the power system related to the cost of meeting demand, including generated power, unmet demand and storage related variables and  $\mathbf{c}^T$  is the corresponding coefficients. Therefore  $\mathbf{c}^T \mathbf{p}$  is the total cost of meeting demand. The outer maximization problem chooses the best attack plan for the attacker which would cause the most severe disruption to the power system and increase the operational cost. Here,  $\Delta$  is a discrete set representing attack plans that the attacker might be able to choose, and  $\boldsymbol{\delta} \in \Delta$  is a certain attack plan. Here,  $\boldsymbol{\delta}$  is a binary vector, whose  $k$ th entry  $\delta_k$  is 1 if the corresponding  $k$ th component of the system is attacked and is 0 otherwise. The constraints  $\mathbf{g}(\mathbf{p}, \boldsymbol{\delta}) \leq \mathbf{b}$  include all physical constraints in the power system for the defender and resource constraints for the attacker, and those constraints will be introduced in detail in the following subsections.

The rest of this section presents the detailed mathematical model and algorithms used in this thesis. We first explain our original inner MOPF model without considering

the interdicting plan, and then explain our interdiction assumptions. After that, we combine the elements in the interdiction problem and MOPF together and get our bilevel interdiction model. Finally, we give a heuristic algorithm for solving the bilevel interdiction model.

### 3.2 MOPF (Inner problem)

In section 2.3.3, we introduced the FOPF by Chandy et al. In this subsection, we will talk in detail about the model, especially our modification to Chandy's model.

In FOPF, we consider the situation in which every wind generator is related to a corresponding energy storage. We assume that each generation cost is constant and the demands are changing over time in a deterministic manner.

Consider a set  $\mathbf{G}$  of generator. Instead of using the set  $\mathbf{N}$  to denote nodes, we use a set  $\mathbf{B}$  to denote buses, which can be both loads and generators at the same time. The transmission network is modeled by the susceptance matrix  $\mathbf{B}_{ij}$ . The element in matrix  $\mathbf{B}_{ij}$  is the susceptance  $b_{ij}$  between bus  $i$  and  $j$ . If two buses are not directly connected then  $b_{ij} = 0$ , where  $i \in \mathbf{B}, j \in \mathbf{B}$ . Then according to [21, FEASIBLE SET 3.3] we can reformulate (FOPF1) as:

$$-\overline{p_{ij}} \leq p_{ij}(t) = b_{ij} (\theta_i(t) - \theta_j(t)) \leq \overline{p_{ij}}, \quad (\text{MOPF1})$$

where  $p_{ij}(t)$  represents the power flow on transmission line  $ij$  in period  $t$  (positive if

power flow is from  $i$  to  $j$ ) with line capacity  $\overline{p}_{ij} \geq 0$ , and  $\theta_i(t)$  is still the phase angle on bus  $i$  in period  $t$ .

For any bus  $i$ , the power flow through it is balanced, so we have:

$$\sum_j p_{ij}(t) = g_i(t) + r_i(t) + u_i(t) - d_i(t),$$

where:

- $g_i(t)$  is the power generated at bus  $i$ , period  $t$  with lower bound  $\underline{g}_i(t)$  and upper bound  $\overline{g}_i(t)$ , which can be represented by:

$$\underline{g}_i(t) \leq g_i(t) \leq \overline{g}_i(t); \quad (\text{MOPF2})$$

Specially, we make the wind generation in this model deterministic but non-dispatchable, e.g.  $\underline{g}_i(t) = g_i(t) = \overline{g}_i(t)$  for wind generator.

- $r_i(t)$  is the discharged power from storage at bus  $i$ , period  $t$ .  $r_i(t)$  can be positive, which means the ESS is discharging, or negative when the ESS is charging;
- $u_i(t)$  is the unmet demand at bus  $i$ , period  $t$ ;
- $d_i(t)$  is the demand at bus  $i$ , period  $t$ .

Here we have a big change from (FOPF3-FOPF5). First, we set both lower and upper bounds for the generator to make it more practical. Then, for the power flow balance constraint, we allow unmet demand to exist and to be satisfied from outside the power system with a high cost. We are doing this to make the problem still feasible even if many of the system's generators are destroyed by the attacker and the demand cannot

be satisfied from the inside of the system.

As in (FOPF2), we can similarly define the power flow in bus  $i$  as:

$$p_i(t) = \sum_j p_{ij}(t). \quad (\text{MOPF3})$$

Then we can write the power flow balance constraint as follows:

$$p_i(t) = g_i(t) + r_i(t) + u_i(t) - d_i(t) \quad (\text{MOPF4})$$

Combining (FOPF6) and (FOPF7), and using  $b_i(t)$  to denote the battery energy level at bus  $i$  period  $t$  we have:

$$0 \leq b_i(t) = b_i(t-1) - r_i(t) \leq B_i. \quad (\text{MOPF5})$$

Let  $c_i(g_i)$  be the generation cost related to the amount of power generated in  $i \in \mathbf{G}$ . Also, there is a battery storage cost  $h_i(b_i)$  when the energy level is  $b_i$ . Finally, there is a terminal cost  $h_i^T(b_i(T))$  on the final battery energy level  $b_i(T)$  and unmet demand incurs cost  $s_i$  per unit.

The formulation of the multi-period OPF problem (**MOPF**) with energy storage is:

$$\min_{\theta, p, r, g, b} \sum_{i \in B} \sum_{t=1}^T (c_i(g_i(t)) + h_i(b_i(t), r_i(t)) + u_i(t)s_i) + \sum_{i \in B} h_i^T(b_i(T)) \quad (\text{MOPF6})$$

s.t. (MOPF1)- (MOPF5)

Since the constraints and the rest of the terms of the objective function are linear, when  $c_i(\cdot)$  is convex, the problem can be solved using a convex optimization solver such as CPLEX.

### **3.3 Interdiction assumptions**

In this subsection, we give assumptions about the vulnerability of the electric power system and explain how the attacking plans work.

In our electric power system, the attackable components include buses, generators, and transmission lines. We make the following assumptions on the effect of each interdiction:

1. Bus interdiction: All lines and generators connected to the bus are disconnected from the system. The load of that bus must be met from outside of the system.
2. Generator interdiction: The generator is disconnected from the grid. Also, for wind generator, we assume its storage is connected with the generator and if the generator is attacked, the storage will disconnect from the grid.
3. Line interdiction: If a transmission line is attacked, it is disconnected from the system.

The Attacker makes an attacking plan, where they are going to interdict some of the components of the grid and for each interdiction in this plan, there will be some required resources for the attacker. Here we assume the resource to be money.

### **3.4 Bilevel interdiction model**

In this subsection, introduce our bilevel interdiction model. We first introduce the interdiction elements (i.e., new variables and parameters) related to the attacking plan.

Then we modify the MOPF model by adding the interdiction elements. Finally, we give our bilevel interdiction model.

To indicate the attacking plan and introduction resources, we introduce the corresponding variables and parameters as follows:

1. Interdiction Variables:

$\delta_i^{Bus}, \delta_g^{Gen}, \delta_{ij}^{Line}$ : Attacking decision on bus ( $i \in B$ ), generator ( $g \in G$ ), transmission line ( $i, j \in B$ ), respectively. There are binary variables that take the value 1 if the corresponding component is attacked and 0 otherwise.

2. Resource parameters:

$M_B, M_G, M_L$ : Resources required to interdict a certain bus, generator, line, respectively;

$M$ : total interdiction resource available to attackers.

We define our interdiction MOPF (IPF) as:

$$\begin{aligned} & \gamma(\boldsymbol{\delta}^{Gen}, \boldsymbol{\delta}^{Line}, \boldsymbol{\delta}^{Bus}, \boldsymbol{\delta}^{Sub}) \\ &= \min_{\theta, p, r, g, b} \sum_{i \in B} \sum_{t=1}^T (c_i(g_i(t)) + h_i(b_i(t), r_i(t)) + u_i(t)s_i) \\ &+ \sum_{i \in B} h_i^T(b_i(T)) \end{aligned} \quad (IPF1)$$

s.t.:

$$-\overline{p}_{ij} \leq p_{ij}(t) = b_{ij}(\theta_i(t) - \theta_j(t))(1 - \delta_{ij}^{Line})(1 - \delta_i^{Bus})(1 - \delta_j^{Bus}) \leq \overline{p}_{ij} \quad \forall i, j \in B \quad (IPF2)$$

$$p_i(t) = g_i(t) + r_i(t) + u_i(t) - d_i(t) \quad \forall i \in B \quad (IPF3)$$

$$p_i(t) = \sum_j p_{ij}(t) \quad \forall i, j \in B \quad (\text{IPF4})$$

$$0 \leq g_i(t) \leq \bar{g}_i(t)(1 - \delta_g^{Bus}) \quad \forall i \in B \quad (\text{IPF5})$$

$$0 \leq b_i(t) = b_i(t-1) - r_i(t) \leq B_i(1 - \delta_i^{Bus}) \quad \forall i \in B \quad (\text{IPF6})$$

$$\text{all } \delta \in \{0,1\} \quad (\text{IPF7})$$

Note: Nonlinear constraints in  $\delta$  are used for ease of understanding; linear replacements of those constraints can be written by splitting the above constraints into constraints that only contain a single  $\delta$ .

Then, we can formulate of the bilevel interdiction problem:

$$\max_{\delta_B, \delta_G, \delta_L} \gamma(\delta_B, \delta_G, \delta_L) \quad (\text{I1})$$

s.t.:

$$\sum_{\text{all buses}} M_B \delta_B + \sum_{\text{all generators}} M_G \delta_G + \sum_{\text{all lines}} M_L \delta_L \leq M \quad (\text{I2})$$

(IPF2)- (IPF7)

Equations (IPF2)- (IPF6) are analogues of (MOPF1)- (MOPF5). Here, however, the components that have been interdicted are removed from the equations through the binary interdiction variables.

# 4. Solution Methodology

## 4.1 Overview

In this thesis, we use a decomposition-based heuristic based on the algorithm of Salmerson, et al. [19] to solve our multiperiod interdiction model. This algorithm may be viewed as a heuristic version of Benders decomposition [31]. There are three steps in each iteration of the algorithm: first solving MOPF under a certain attacking plan (IPF); then using the power flow as input to evaluate the “relative value” of grid components; finally using the components’ value to make a new attacking plan and go back to the first step. In this section, we describe the algorithm in detail both in words and in mathematics.

## 4.2 Subproblem

We start the algorithm by solving IPF under the original power network conditions. Later, we refer to solving IPF under a certain interdiction plan as “the subproblem”. However, at the beginning, there is no attack and the power system remains untouched. In either condition, by solving the subproblem, we get an optimal power flow in finite horizon; these results are parameters of the power grid which minimize generation and storage cost.



Assume that at iteration  $t$  of the algorithm, the interdiction plan is  $\widehat{\delta}^t = (\widehat{\delta}^{Bus,t}, \widehat{\delta}^{Gen,t}, \widehat{\delta}^{Line,t})$ . We plug these  $\widehat{\delta}^t$  into (IPF1)-(IPF6), which gives us the subproblem for iteration  $t$ . Its solution yields the optimal power flows, generation, discharge amount and unmet demand under the attacking plan represented by  $\widehat{\delta}^t$ . We write the result as  $\widehat{\mathbf{P}}^t(t) = (\widehat{\mathbf{P}}^{Line,t}(t), \widehat{\mathbf{G}}^{Gen,t}(t), \widehat{\boldsymbol{\theta}}^{Bus,t}(t), \widehat{\mathbf{r}}^{Bus}(t), \widehat{\mathbf{b}}^{Bus}(t), \widehat{\mathbf{u}}^{Bus}(t))$ . In this formulation, the superscript  $t$  represents iteration  $t$ , and  $(t)$  represent the time period.

### 4.3 Value estimates

The next step is to use the  $\widehat{\mathbf{P}}^t(t)$  to determine the “values” of all the components of the power grid. It worth mentioning that when we evaluate the value of each component, we are using the average power flow of all previous iterations. This is because if we did not use an average and if the power flow through a currently interdicted component is zero, which means this component should be important as it was in the attacking plan, this component would be regarded as not valuable in the next iteration. which is undesirable.

To estimate the value, we define some parameters which will be used in the master problem of the same iteration. For every bus  $i$  we compute

$$\begin{aligned} & \sum_{\text{time period}} \hat{\mathbf{P}}^t(t) \\ = & \left( \sum \hat{\mathbf{P}}^{Line,t}(t), \sum \hat{\mathbf{G}}^{Gen,t}(t), \sum \hat{\boldsymbol{\theta}}^{Bus,t}(t), \sum \hat{\mathbf{r}}^{Gen,t}(t), \sum \hat{\mathbf{b}}^{Gen,t}(t), \sum \hat{\mathbf{u}}^{Bus}(t) \right) \end{aligned}$$

We write the summation as:

$$\hat{\mathbf{P}}^t = (\hat{\mathbf{P}}^{Line,t}, \hat{\mathbf{G}}^{Gen,t}, \hat{\boldsymbol{\theta}}^{Bus,t}, \hat{\mathbf{r}}^{Gen,t}, \hat{\mathbf{b}}^{Gen,t}, \hat{\mathbf{u}}^{Bus})$$

Then we compute:

$$F_i^{Out,t} = \sum_{l|o(l)=i \wedge \hat{p}_l^{Line} > 0} \hat{p}_l^{Line,t} + \sum_{l|d(l)=i \wedge \hat{p}_l^{Line} < 0} |\hat{p}_l^{Line,t}|, \text{ flow out of bus } i$$

$$F_i^{Met,t} = \sum_c (d_{ic} - \hat{u}_{ic}^t), \text{ load supplied to bus } i.$$

These totals are then used to compute:

$$V_g^{Gen,t} = w^{Gen} \hat{P}_g^{Gen,t} + w^{Sto} \hat{r}_g^{Gen,t} + w^{Sto} \hat{b}_g^{Gen,t}, \quad \forall g \in \mathbf{G}$$

$$V_l^{Line,t} = w^{Line} |\hat{p}_{ij}^{Line,t}|, \quad \forall i, j \in \mathbf{B}$$

$$V_i^{Bus,t} = w^{Bus} (F_i^{Out,t} + F_i^{Met,t}), \quad \forall i \in \mathbf{B}$$

The weights  $w^{Gen}$ ,  $w^{Sto}$ ,  $w^{Bus}$ ,  $w^{Line}$  and  $w^{Sub}$  are weight coefficients which are defined to reflect the relative importance of each type of component. Here we set  $w^{Gen} = 1$ ,  $w^{Sto} = 1$ ,  $w^{Bus} = 5$ ,  $w^{Line} = 1$  and  $w^{Sub} = 5$ .

#### 4.4 Master problem

After we have the estimated value of each component in the grid, we make a new interdiction plan by solving a problem that maximizes the value of the interdicted components under limited attacking resources. We call this part the “master problem”.

Assume that the estimated values of the grid components can be represented by

$\mathbf{V}^t = (\mathbf{V}^{Gen,t}, \mathbf{V}^{Line,t}, \mathbf{V}^{Bus,t})$ , which is calculated at iteration  $t$ . Also assume the vector of interdiction plans generated in all previous iterations is  $\hat{\Delta}^t = (\hat{\delta}^1, \dots, \hat{\delta}^t)$ . The interdiction master problem can be given by:

$$\max_{\delta^{Gen}, \delta^{Line}, \delta^{Bus}} \sum_g V_g^{Gen,t} \delta_g^{Gen} + \sum_l V_l^{Line,t} \delta_l^{Line} + \sum_i V_i^{Bus,t} \delta_i^{Bus} \quad (\text{MP0})$$

s.t.

$$\sum_{\text{all buses}} M_B \delta_B + \sum_{\text{all generators}} M_G \delta_G + \sum_{\text{all lines}} M_L \delta_L \leq M \quad (\text{MP1})$$

$$\delta_g^{Gen} + \delta_i^{Bus} \leq 1, \forall g \in \mathbf{G}_i, i \in \mathbf{B} \quad (\text{MP2})$$

$$\delta_l^{Line} + \delta_i^{Bus} \leq 1, \forall l \in \mathbf{L}_i, i \in \mathbf{B} \quad (\text{MP3})$$

$$\sum_{\hat{\delta}_g^{Gen,t'}=1} (\hat{\delta}_g^{Gen,t'} - \delta_g^{Gen}) + \sum_{\hat{\delta}_l^{Line,t'}=1} (\hat{\delta}_l^{Line,t'} - \delta_l^{Line}) + \sum_{\hat{\delta}_i^{Bus,t'}=1} (\hat{\delta}_i^{Bus,t'} - \delta_i^{Bus}) \geq 1, \quad t' \leq t \quad (\text{MP4})$$

The objective function (MP0) maximizes the estimated value of the interdicted grid components under limited resources without waste. (MP1) is the same as (I2) which limits the resources for interdiction. Constraints (MP2) mean that at a certain bus  $i$ , if the bus itself is interdicted, then none of its generators can be attacked since they would not work at all if the bus is interdicted, so there is no sense attacking them; also, if a generator in bus  $i$  is interdicted, then we do not allow the bus to be attacked, otherwise it would be a waste of resource to interdict the generator. Similarly, (MP3) restricts the attacking option among a certain bus  $i$  and all transmission lines connected to it.

Constraints (MP4) compare the new interdiction plan for this iteration with every previous plan and ensures that the new interdiction plan has at least one attacking decision which is different from each previous plan from previous iterations.

Let  $\widehat{\delta}^{t+1} = (\widehat{\delta}^{Gen,t+1}, \widehat{\delta}^{Line,t+1}, \widehat{\delta}^{Bus,t+1})$  denote the solution to the master problem in the current iteration. The vector  $\widehat{\delta}^{t+1}$  will be used as a parameter in the subproblem to start a new iteration of the algorithm which constructs a complete loop.

#### 4.5 Pseudocode of the algorithm

In this subsection, we summarize the whole algorithm described above with pseudocode.

##### **Input Data:**

Grid data (susceptance of transmission lines, generation cost, generation bound etc.); interdiction data (resource limit); iteration limit (T).

##### **Output Results:**

$\widehat{\delta}^*$  is a feasible interdiction plan and the cost to meet the system demand  $\gamma^*$ . If the algorithm stops because the master problem is infeasible, which means there is no new interdiction plan, then  $\widehat{\delta}^*$  is therefore the best interdiction plan and  $\gamma^*$  is the highest operational cost by this algorithm.

**Initialization:**

1. Set  $\widehat{\boldsymbol{\delta}}^1 \equiv (\widehat{\boldsymbol{\delta}}^{Gen,1}, \widehat{\boldsymbol{\delta}}^{Line,1}, \widehat{\boldsymbol{\delta}}^{Bus,1}) \leftarrow (0,0,0)$  (initial attacking plan).
2. Set  $\widehat{\boldsymbol{\delta}}^* \leftarrow \widehat{\boldsymbol{\delta}}^1$  (best plan so far) and  $\widehat{\Delta}^1 \leftarrow \{\widehat{\boldsymbol{\delta}}^1\}$  (all attacking plans so far).
3. Set  $\gamma^* \leftarrow 0$  (initial best operational cost).
4. Set  $t \leftarrow 1$  (iteration number).

**Subproblem:**

1. Solve IPF with  $\widehat{\boldsymbol{\delta}}^t$ , get solution  $\widehat{\boldsymbol{P}}^t = (\widehat{\boldsymbol{P}}^{Line,t}, \widehat{\boldsymbol{G}}^{Gen,t}, \widehat{\boldsymbol{\theta}}^{Bus,t}, \widehat{\boldsymbol{r}}^{Gen,t}, \widehat{\boldsymbol{b}}^{Gen,t}, \widehat{\boldsymbol{u}}^{Bus})$  and objective value  $\gamma(\widehat{\boldsymbol{\delta}}^t)$ .
2. If  $\gamma(\widehat{\boldsymbol{\delta}}^t) > \gamma^*$  then  $\gamma^* \leftarrow \gamma(\widehat{\boldsymbol{\delta}}^t)$  and  $\widehat{\boldsymbol{\delta}}^* \leftarrow \widehat{\boldsymbol{\delta}}^t$ .
3. If  $t=T$ , then Print  $(\widehat{\boldsymbol{\delta}}^*, \gamma^*)$  and stop.

**Value Estimates**

1. Compute “relative value” using  $\widehat{\boldsymbol{P}}^{t'}$ ,  $t'=1, \dots, t$ ,

$$\boldsymbol{V}^t = (\boldsymbol{V}^{Gen,t}, \boldsymbol{V}^{Line,t}, \boldsymbol{V}^{Bus,t}) = \frac{1}{t} \sum_{t'=1}^t (\boldsymbol{V}^{Gen,t'}, \boldsymbol{V}^{Line,t'}, \boldsymbol{V}^{Bus,t'}).$$

**Master problem:**

1. Solve the master problem and get  $\widehat{\boldsymbol{\delta}}^{t+1}$ .
2. If the master problem is infeasible, then print  $(\gamma^*, \widehat{\boldsymbol{\delta}}^*)$  and stop.

Otherwise, update  $\hat{\Delta}^{t+1} \leftarrow \hat{\Delta}^t \cup \{\hat{\delta}^{t+1}\}$ .

3. Set  $t \leftarrow t + 1$ .
4. Return to Subproblem

# 5. Computational Results

## 5.1 Test-case description

In this thesis, we used the IEEE 1996 Reliability Test System [32] (24 buses), IEEE 57-bus system and IEEE 118-bus system to test our model.

Since the generators in these systems are all conventional generators, they do not fit our multi-period model with renewables very well. For this reason, we changed some original data; for RTS 1996, we assume the generators at bus #18 and #21 are wind generators. The reason we choose these generators is mainly because of their max generation capacity. The whole system's generate on capacity is 3401MW and usually the renewable capacity generators take less than 30% of the power in whole system. The max generate on capacity of #18 and #21 are both 400MW. Thus, it would be suitable to have them modified. We assume their ESS have a storage capacity of 400MWh and their generating costs are set to zero. Similarly, we assume the generator at bus #1 in IEEE 57-BS to be a wind generator with generation capacity 600MW and a corresponding storage capacity of 600MWh. For IEEE 118-BS, we assume the generators at bus #10, #26, #69, #80 and #89 to be wind generators, with generation capacity of 550MW, 420MW, 800MW, 600MW and 700MW, respectively; and corresponding storage capacity of 550MWh, 420MWh, 800MWh, 600MWh and 700MWh, respectively.

To make the original demand data for static OPF fit our finite-horizon OPF model, we also modified the demand data of all testing instances by combining them with real demand data in Texas. We first randomly take 24 consecutive hourly demand data (5000-5023) from the database of Electric Reliability Council of Texas (ERCOT) [33] in 2014 (shown in Figure 1). When we implement a instance, we scale the demands from the instance in each period according to the proportional changes in the ERCOT data.

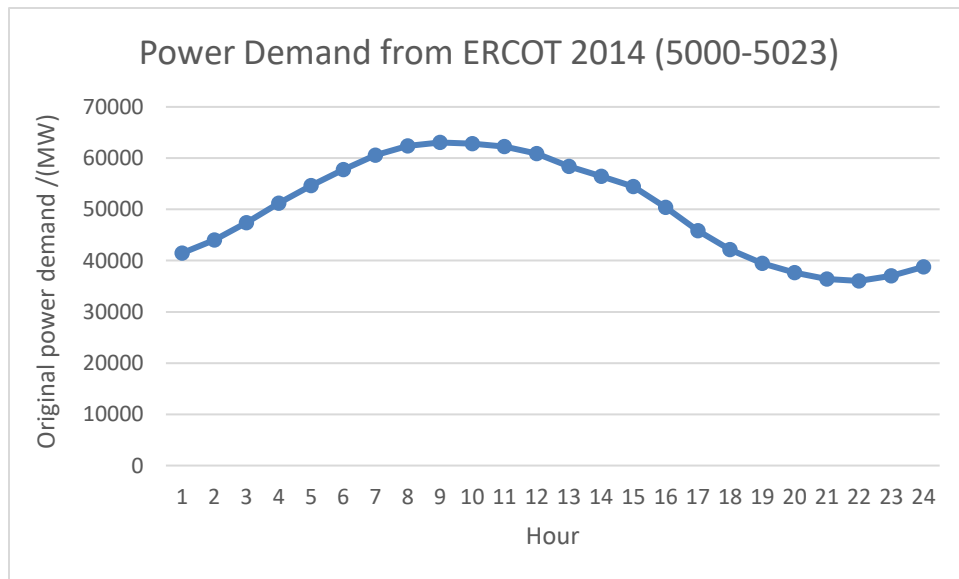


Figure 5-1 Power Demand

Then we distribute the demand according to the percentage of each bus taken in the whole system and add 10% random disturbance.

For interdiction data, we set  $M_B = 5$ ,  $M_G = 3$ , and  $M_L = 1$ . For the total resource, we will change it to see how it affects load shedding.



## **5.2 Implementation**

The model and algorithm are implemented in AIMMS, which is an algebraic modeling language for numerical optimization problems. AIMMS enables easy generation and manipulation of the subproblems and master problems, which are solved with CPLEX 12.7.0.0. Tests were carried out on a 2GHz personal computer with 8GB of RAM.

In the tests, we assume a fixed per-unit penalty for unmet load of 1000 unit price per MW. This shedding cost is much higher than generation costs.

## **5.3 Testing result**

In this subsection, we show the testing results from three aspects: 1. The relationship between interdiction resources given and the total load shedding in all instances; 2. The comparison of total time to solve the problem between all instances with different amount of interdiction resources; 3. Analyses of the changes between the static OPF model with conventional generators and the finite horizon OPF model with renewable generators.

### 5.3.1 Load shed-interdiction resource relationship

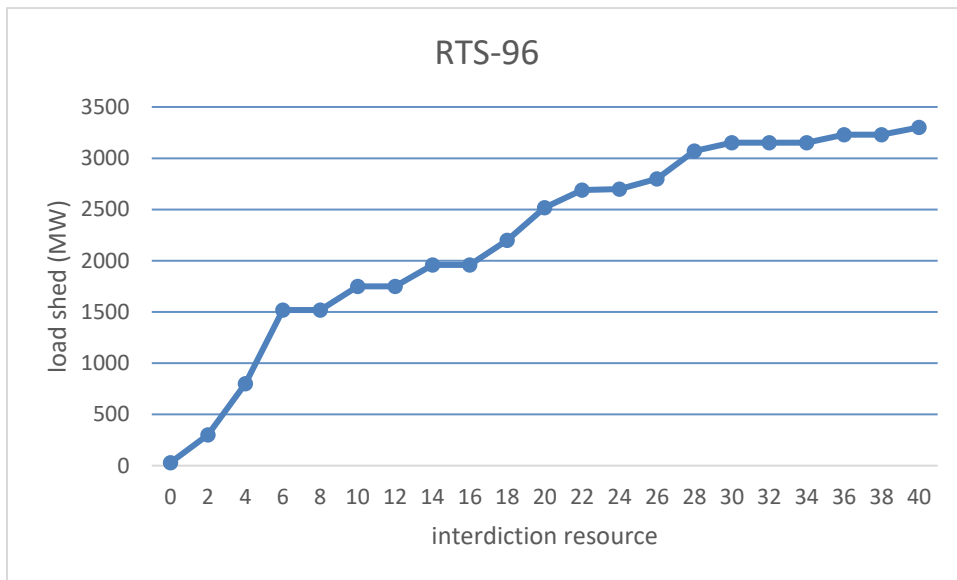


Figure 5-2 RTS-96 Testing Result

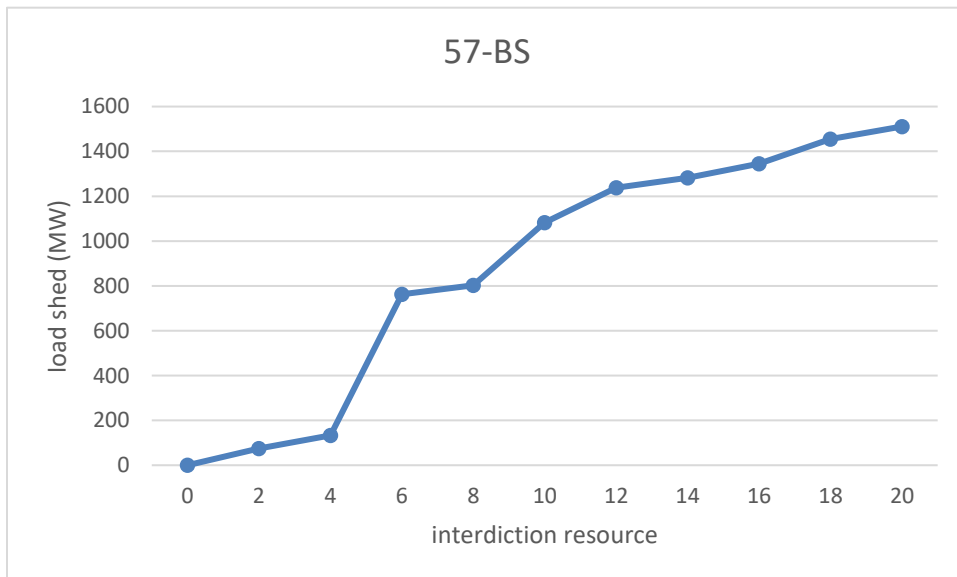


Figure 5-3 57-BS Testing Result

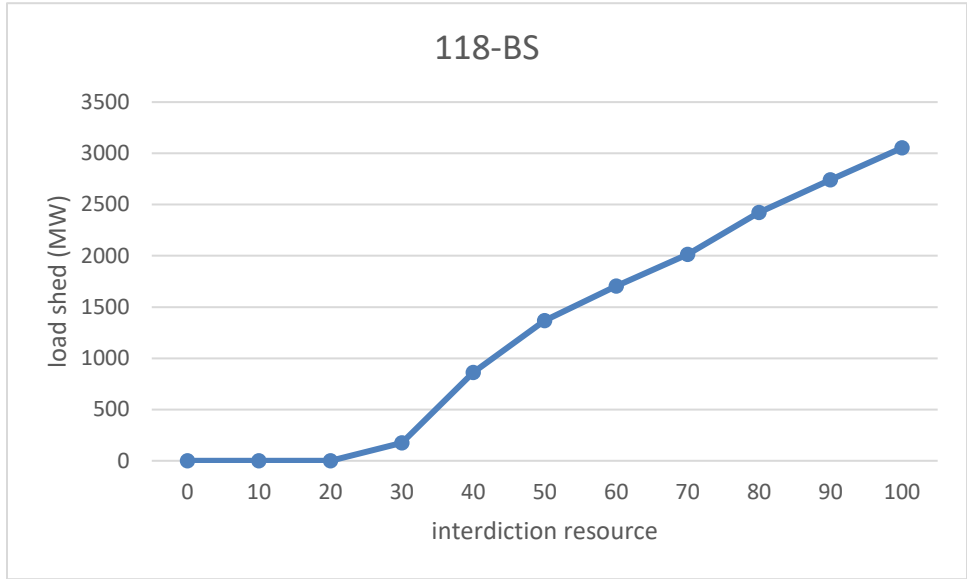


Figure 5-4 118-BS Testing Result

Figures 4-2, 4-3 and 4-4 show the relationship between average amount of load shed in the grid and the total interdiction resource  $M$ . For all the three figures, the amount of load shed is nondecreasing as the interdiction resource  $M$  increases. Also, there is a tendency of the function toward concavity as  $M$  increases.

For both RTS-96 and 57-BS, as  $M$  increases to the end of the figure, 95% of the load is unmet, which takes 40 and 20 units of resource, respectively. However, for 118-BS, though the interdiction resource limit (up to 100 units) is much higher than the other two systems, the load shed at the end is only around 60% of the total load, and further testing shows that it would cost around 150 units of resource to make the load shed over 95%. This observation can be explained by several reasons. The first factor is the relative amount of demand and the generation capacity. For RTS-96, the average demand in the whole system is over 90% of the generation capacity, so that once the interdiction is made,

even if the resource limit is low, there would be obvious load shed. On the other hand, for 57-BS and 118-BS, the demand is around 50% of the total generation capacity, so that at the beginning, the load shed is small. The second factor is about the number of generators in the system or the distribution of power generation. For 57-BS, although there are 57 buses, the total number of generators is only 7 and each of them has a large generation capacity. This means it only takes 21 units of resource for the attacker to interdict all the generators, which would cause the most load shed. As for RTS-96, though there are only 24 buses in the system, the number of generators is 33, which would cost the attacker more resources to interdict either more generators or higher cost buses. And the situation is analogous for 118-BS.

The other observation is that for each figure, there is a relative big “jump” at the beginning of load shed increasing. The reason for that is some of the components in the power system carry much power load, for example the low cost renewable generator with large generation capacity, and those components are of great value when solving the master problem and would be attacked prior. Once these components are interdicted, there would be a huge amount of unmet demand, which explains the “jump”.

### 5.3.2 CPU time

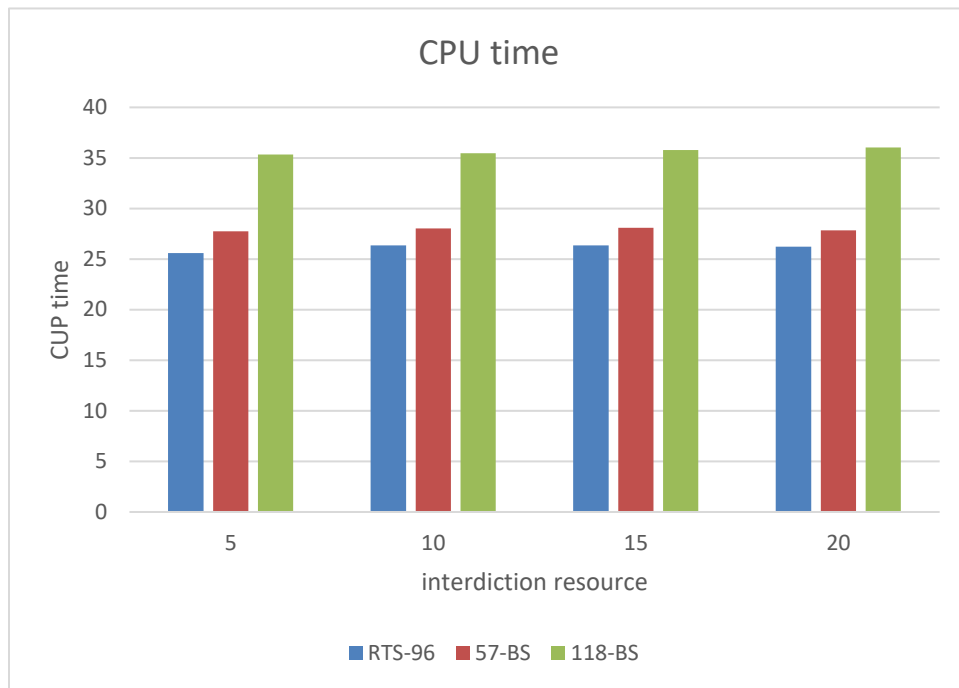


Figure 5-5 CPU time comparison

The Figure shows that the 118-BS takes the most CPU time among these three instances, with around 35 seconds when the interdiction resource is from 5-20.

### 5.3.3 Attacking plan comparison between static OPF and finite horizon MOPF

In this thesis, we also use the static OPF model in the subproblem, which is analogous to Salmeron's paper et al. [19] to get a result for comparison. We use RTS-96 as our testing example, keeping all generators as conventional generator.

By comparing results from the two models, we find that the main change in attacking plan is that when the interdiction resource is relative little (e.g. 5 units), in the

MOPF model, the interdiction plan includes the renewable generator while the static OPF model includes other components (transmission lines) instead of the generator. This is sensible since the renewable generator has no generation cost, so when solving the MOPF, the renewable generator would provide relatively larger generation, which makes it more “valuable” when solving the master problem and more likely to be chosen prior.

## 6. Conclusion

In this thesis we formulate a bilevel interdiction model for electric power systems containing wind generators and use a decomposition based heuristic algorithm to solve the problem. By solving the problem, we will have a better understanding of the vulnerability of the system and find out the most critical components. We test our model with 3 popular IEEE instances and get some general conclusions for electric power systems containing renewables.

1. From our test, we can see that if we have more generators with less generation capacity instead of fewer generators with larger generation capacity, and also distributed in more places in the system instead of gathered together, it is harder for attackers to cause huge unmet loads.
2. If our power system has large generation capacity, it would have better tolerance towards disturbance from interdiction.

Also, if we assume some of the components, say generators, cannot be interdicted, then by solving our model we can acquire the knowledge of what is the most critical component among the infrastructure in a certain system, since we will get the attacking plan on buses, transmission lines and so on. This allows us to analyze a system by changing conditions and solving our model.

Numerous issues remain for future work, and they include:

1. Considering restoration over time: though we have modeled the OPF as a multi-period problem, we do not consider the recovery of the power system's infrastructure. Different kinds of components have different recovery times and that would be an important factor for power network security.
2. Considering defensive action: By analyzing the attacker's potential behavior, the defender would have a better understanding of which component is the most vulnerable in the system. After fortification, the attacker would make a new attacking plan. If the defender has limited resources on fortification, what would be the optimal defensive action? This could be formulated as a trilevel problem.
3. Considering other ways of solving the original bilevel problem: In this thesis, we use a heuristic to find an acceptable attacking plan. However, it might not be the optimal solution. We may find some other ways to solve this problem to find the optimal solution or reduce the gap. For example, we may find some way to linearize the form

$$\begin{aligned}
 (Mm) \quad & \max_{\delta \in \Delta} \min_{\mathbf{p}} \mathbf{c}^T \mathbf{p} \\
 \text{s. t.} \quad & \mathbf{g}(\mathbf{p}, \delta) \leq \mathbf{b} \\
 & \mathbf{p} \geq \mathbf{0}.
 \end{aligned}$$



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# Vita

Weiming Lei was born in Leshan, Sichuan Province, China, in 1992. After graduating from Beijing 101 Middle School, he went to Beihang University (BUAA, Beijing University of Aeronautics and Astronautics) and got his Bachelor of Engineering Degree in July 2014. After that, he worked as a planning specialist in CAXA Beijing office for one year. In August 2015, he went to Lehigh University, pursuing his Master of Science in Engineering Degree under Dr. Lawrence V. Snyder's supervisor in the Industrial and Systems Engineering Department.