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Analysis of combined backward-forward extrusion

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ANALYSIS OF COMBINED
BACKWARD-FORWARD EXTRUSION

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NOMENCLATURE

a	A constant whose value is to be determined
b	A constant whose value is to be determined
$B(y)$	A function of y whose value is to be determined
h_1	See Fig. 1
h_2	Height of upper ram (see Fig. 1)
H_1	Height of lower die wall (see Fig. 1)
H_2	Height of upper die wall (see Fig. 1)
J^*	Upper bound on power
m	Friction factor
p_R	Average ram pressure
R_i	Lower ram radius
R_o	Upper die radius
R_1	Lower die radius (see Fig. 1)
R_2	Upper ram radius (see Fig. 1)
S_Γ	Surface of velocity discontinuity
T	Distance between ram and lower die
x	Substitution parameter
$\dot{U}$	Ram velocity
$\dot{U}_i$	Velocity components in the i direction
v_i	Velocities
V	Volume of deforming zone

$\dot{W}_f$	Power to overcome friction losses
$\dot{W}_i$	Power of internal deformation
$\dot{W}_S$	Power to overcome shear losses
z	Substitution parameter
Γ_i	Boundaries of velocity discontinuity
$\dot{\varepsilon}_{ij}$	Strain rate
σ_o	Yield strength in uniaxial tension
τ	Shear stresses

ABSTRACT

The upper bound approach is used to analyze combined backward - forward extrusion. The deformation region is divided into five zones separated by planer and cylindrical surfaces of velocity discontinuities. The internal power of deformation and shear and friction losses, are computed individually and summed. The pseudo-independent process parameter is the backward rate of flow with respect to which the total power of deformation is optimized. The optimal backward rate of flow is assumed to be the actual one. Thus, the backward rate of flow becomes a dependent parameter to be studied through this analysis. Conditions covering backward rates of flow from zero to maximum are demonstrated graphically. Examples are given for which combined flow results and for which either only forward flow or only backward flow occur.

INTRODUCTION

In recent papers (1,2) upper bound analyses (3) were presented for impact extrusion. The present study is an extension of that work to cover the case of backward - forward extrusion.

VELOCITY FIELD

A representation of backward - forward extrusion is shown in Fig. 1. The ram is shown moving forward (downward) with a velocity $\dot{U}$ (the downward direction is taken to be negative). Zones I and V, material which has already completed its deformation, are treated as rigid bodies. In Zones II, III, and IV the metal is undergoing deformation. The surfaces Γ_1 through Γ_{13} are surfaces of velocity discontinuity. The detailed analysis of the velocity field is given in the Appendix.

RESULTS

In the Appendix, the power for internal deformation, friction losses, and shear losses is calculated and equated to the external power provided through the ram. The resulting relationship is

$$\frac{p_R}{\sigma_o} = \frac{1}{\sqrt{3}\left(1 - \left(\frac{R_i}{R_2}\right)^2\right)} \left\{ \left(\frac{R_i}{R_2}\right)^2 \left| \frac{1 - \left(\frac{R_1}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2} \right| \right.$$

$$\times \left| \left(2 - \sqrt{1 + 3\left(\frac{R_1}{R_2}\right)^4 \left(\frac{R_2}{R_i}\right)^4} + \ln \left| \frac{1}{3}\left(\frac{R_i}{R_2}\right)^2 \left(\frac{R_2}{R_1}\right)^2 \right. \right. \right.$$

$$\left. \left. \left. \times \left(1 + \sqrt{1 + 3\left(\frac{R_1}{R_2}\right)^4 \left(\frac{R_2}{R_i}\right)^4} \right) \right| \right) \right| + \left| \left(1 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}} \right) \right.$$

$$\left. \times \left[\sqrt{1 + 3\left(\frac{\left(\frac{R_1}{R_2}\right)^2}{1 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}}} \right)^2} \right. \right. \qquad (1a)$$

$$- \sqrt{1 + \frac{3}{\left\{ 1 - \left[1 - \left(\frac{R_o}{R_2}\right)^2 \right] \frac{v_b}{\dot{U}} \right\}^2}} + \ln \left| \left(\frac{R_1}{R_2}\right)^2 \right.$$

$$\left. \times \frac{1 + \sqrt{1 + \frac{3}{\left\{ 1 - \left[1 - \left(\frac{R_o}{R_2}\right)^2 \right] \frac{v_b}{\dot{U}} \right\}^2}}}{1 + \sqrt{1 + 3 \left\{ \frac{\left(\frac{R_1}{R_2}\right)^2}{1 - \left[1 - \left(\frac{R_o}{R_2}\right)^2 \right] \frac{v_b}{\dot{U}}} \right\}^2}} \right] \Bigg| \Bigg| + \left| \frac{v_b}{\dot{U}} \left(\frac{R_o}{R_2}\right)^2 \right.$$

$$\left. \times \left(2 - \sqrt{1 + 3\left(\frac{R_2}{R_o}\right)^4} - \ln \left| \left(\frac{R_2}{R_o}\right)^2 \frac{3}{\sqrt{1 + 3 \left(\frac{R_2}{R_o}\right)^4}} \right| \right) \right|$$

$$+ m \left[\left| \frac{1 - \left(\frac{R_i}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2 \right] \frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2} \right| \right.$$

$$\times \left(2\frac{R_1}{R_2}\frac{H_1}{R_2} + 2\frac{R_i}{R_2}\frac{h_1}{R_2} + \frac{R_i}{R_2}\frac{T}{R_2} + \frac{R_2}{3T}\left(2\frac{R_i}{R_2} + \frac{R_1}{R_2}\right)\left(\frac{R_1}{R_2} - \frac{R_i}{R_2}\right)^2 \right)$$

$$+ \frac{2}{3}\frac{R_2}{T}\left|\left(1 - \frac{R_1}{R_2}\right)\left(2 - \frac{R_1}{R_2} - \left(\frac{R_1}{R_2}\right)^2 - 3\left[1 - \left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}}\right)\right| + \frac{R_2}{3T}\left|\frac{v_b}{\dot{U}}\right.$$

$$\times \left.\left(1 - 3\left(\frac{R_o}{R_2}\right)^2 + 2\left(\frac{R_o}{R_2}\right)^3\right)\right| + \frac{R_o}{R_2}\frac{T}{R_2}\left|\frac{v_b}{\dot{U}}\right| + 2\frac{h_2}{R_2}\left|1 - \frac{v_b}{\dot{U}}\right| + 2\left|\frac{v_b}{\dot{U}}\right|$$

$$\times \left.\frac{R_o}{R_2}\left|\frac{H_2}{R_2} - \frac{T}{R_2}\right|\right] + \left|1 - \left(\frac{R_1}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}}\right|$$

$$\times \left[\left|\frac{1}{3}\frac{R_2}{T}\left(2\frac{R_i}{R_2} + \frac{R_1}{R_2}\right)\frac{\frac{R_i}{R_2} - \frac{R_1}{R_2}}{\frac{R_i}{R_2} + \frac{R_1}{R_2}}\right| + \left|\frac{\frac{R_1}{R_2}\ \frac{T}{R_2}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2}\right|\right]$$

$$+ \left|1 - \frac{v_b}{\dot{U}}\right|\frac{T}{R_2} + \frac{1}{3}\left|\frac{v_b}{\dot{U}}\right|\left|\frac{R_2}{T}\right|1 - 3\left(\frac{R_o}{R_2}\right)^2 + 2\left(\frac{R_o}{R_2}\right)^3\left|\ \right\}$$

In symbolic form Eq. (1a) reads,

$$\frac{p_R}{\sigma_o} = f\left(\frac{R_i}{R_2}, \frac{R_1}{R_2}, \frac{R_o}{R_2}, \frac{H_1}{R_2}, \frac{H_2}{R_2}, \frac{h_1}{R_2}, \frac{h_2}{R_2}, \frac{T}{R_2}, m, \frac{v_b}{\dot{U}}\right) \qquad (1b)$$

The first eight terms in the dimensionless expression fully describe the geometry of the process and m denotes the friction conditions. The term $v_b/\dot{U}$ denotes the value of the exit backward velocity. The ratios of the relative backward exit velocity $(v_b/\dot{U})$ and the relative forward exit velocity $(v_f/\dot{U})$ in this study for Eq. (1) are dependent on each other. Here $(v_b/\dot{U})$ is considered a pseudo-independent parameter chosen by the process of minimization on the total power and the ram pressure. This admissible value of $(v_b/\dot{U})$ which minimizes Eq.(1) is considered to be the actual chosen value. According to the upper bound theorem metal flow will actually take place under conditions which minimize the relative ram pressure for a given geometry, ram velocity, and friction.

The relative ram pressure was calculated for several different type geometries and plotted versus relative backward velocity as shown in Figures 2 through 5.

In Figure 2, the relative ram pressure is shown for the case in which both the upper and lower gaps between the ram and the die are appreciable. Under these conditions both backward and forward extrusion take place. This can be seen from the fact that the minimum for relative ram pressure is located at a finite positive value of v_b less than (v_b) max.

If the lower gap is almost closed $(R_1 \approx R_i)$ the situation shown in Figure 3 results. Here it can be seen that the bulk of the metal flows backwards ($v_b \approx (v_b)$ max).

The case for which the upper gap is almost closed ($R_2 \approx R_o$) is shown in Figure 4. Under these circumstances the bulk of the metal flows forward ($v_b \approx 0$). An interesting variation of this case is in Figure 5 in which the upper gap is slightly smaller than in Figure 4. Under these conditions metal in Zone V actually flows forward ($v_b < 0$).

CONCLUSIONS

An upper bound solution has been given for backward-forward extrusion. Examples have been given for which backward-forward flow results and for which only forward flow and backward flow result.

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APPENDIX

1. Velocity Fields

When the ram moves forward with velocity $\dot{U}$, the metal flows backward with velocity v_b and forward with velocity v_f. To maintain volume constancy the following relationship must prevail. (See Figure 1.)

$$0 = \pi(R_2^2 - R_i^2)\,\dot{U} + \pi(R_o^2 - R_2^2)\,v_b - \pi(R_1^2 - R_i^2)\,v_f \qquad (2)$$

Solving equation (2) for the relative forward velocity

$$\frac{v_f}{\dot{U}} = \frac{1 - \left(\frac{R_i}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2} \qquad (3)$$

Also solving equation (2) for a maximum relative backward velocity

$$\left.\frac{v_b}{\dot{U}}\right|_{max} = \frac{1 - \left(\frac{R_i}{R_2}\right)^2}{1 - \left(\frac{R_o}{R_2}\right)^2} \qquad (4)$$

In Zone I, since the metal flows forward as a rigid body, with no rotation because of axial symmetry

$$\dot{U}_R = \dot{U}_\theta = 0 \qquad (5)$$

$$\dot{U}_y = v_f = \left[\frac{1 - \left(\frac{R_i}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2}\right]\dot{U} \qquad (6)$$

In Zone II, the same axial symmetry is assumed as in Zone I.

$$\dot{U}_\theta = 0 \tag{7}$$

$\dot{U}_y$ is assumed to change linearly.

$$\dot{U}_y = a + by \tag{8}$$

Obtaining the values of a and b by using the boundary conditions

$$\dot{U}_y\Big|_{y=0} = a = v_f = \left[\frac{1 - \left(\frac{R_i}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2 \right] \frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2} \right] \dot{U} \tag{9}$$

$$\dot{U}_y\Big|_{y=T} = a + bT = \dot{U} \tag{10}$$

Then

$$b = \frac{1}{T}(\dot{U} - a) \tag{11}$$

Substituting equation (9) and (11) into equation (8)

$$\dot{U}_y = -\frac{\dot{U}}{T} \left(\frac{1 - \left(\frac{R_i}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2 \right] \frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2} (y - T) - y \right) \tag{12}$$

The strain rates as function of the velocity components are:

$$\dot{\varepsilon}_{RR} = \frac{\partial \dot{U}_R}{\partial R} \tag{13}$$

$$\dot{\varepsilon}_{\theta\theta} = \frac{\dot{U}_R}{R} \tag{14}$$

$$\dot{\varepsilon}_{yy} = \frac{\partial \dot{U}_y}{\partial y} = -\frac{\dot{U}}{T}\left\{\frac{1-\left(\frac{R_1}{R_2}\right)^2-\left[1-\left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2-\left(\frac{R_i}{R_2}\right)^2}\right\} \tag{15}$$

$$\dot{\varepsilon}_{R\theta} = \dot{\varepsilon}_{\theta y} = \dot{\varepsilon}_{yR} = 0 \tag{16}$$

From the condition of volume constancy

$$\dot{\varepsilon}_{RR} + \dot{\varepsilon}_{\theta\theta} + \dot{\varepsilon}_{yy} = \frac{\partial \dot{U}_R}{\partial R} + \frac{\dot{U}_R}{R} + \dot{\varepsilon}_{yy} = 0 \tag{17}$$

Rewriting equation (17) in differential form

$$\frac{\partial}{\partial R}(R\,\dot{U}_R) = -R\,\dot{\varepsilon}_{yy} \tag{18}$$

Integrating equation (18) and solving for $\dot{U}_R$

$$\dot{U}_R = -\frac{R}{2}\dot{\varepsilon}_{yy} + \frac{B(y)}{R} \tag{19}$$

Substituting equation (15) into equation (19)

$$\dot{U}_R = \frac{1}{2}\,\frac{R}{T}\,\dot{U}\left(\frac{1 - \left(\frac{R_1}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2}\right) + \frac{B(y)}{R} \qquad (20)$$

Using the boundary condition

$$\dot{U}_R\Big|_{R=R_i} = 0 \qquad (21)$$

And solving equation (20) for B(y)

$$B(y) = -\frac{1}{2}\,\frac{R_i^2}{T}\,\dot{U}\left(\frac{1 - \left(\frac{R_1}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2}\right) \qquad (22)$$

Substituting equation (22) into equation (20)

$$\dot{U}_R = \frac{1}{2}\,\frac{R}{T}\,\dot{U}\left(\frac{1 - \left(\frac{R_1}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2}\right)\left[1 - \left(\frac{R_i}{R}\right)^2\right] \qquad (23)$$

In Zone III, the same procedure as described in Zone II is employed to determine $\dot{U}_y$ and $\dot{U}_R$. As in Zone I and II

$$\dot{U}_\theta = 0 \tag{24}$$

Obtaining the values of a and b by using the boundary conditions

$$\dot{U}_y\Big|_{y=0} = a = 0 \tag{25}$$

$$\dot{U}_y\Big|_{y=T} = a + bT = \dot{U} \tag{26}$$

Then

$$b = \frac{\dot{U}}{T} \tag{27}$$

Substituting equations (25) and (27) into the analog of equation (8)

$$\dot{U}_y = \frac{y}{T}\dot{U} \tag{28}$$

As the normal component of velocity across the boundary between two zones should be continuous because of volume constancy, the boundary condition is introduced for the radial velocity from Zone II.

$$\dot{U}_R\Big|_{R=R_1} = \frac{1}{2}\frac{R_1}{T}\dot{U}\left\{\frac{1-\left(\frac{R_1}{R_2}\right)^2-\left[1-\left(\frac{R_o}{R_2}\right)^2\right]\frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2-\left(\frac{R_i}{R_2}\right)^2}\right\}\left[1-\left(\frac{R_i}{R_1}\right)^2\right] \tag{29}$$

$$\dot{\varepsilon}_{yy} = \frac{\partial \dot{U}_y}{\partial y} = \frac{\dot{U}}{T} \qquad (30)$$

Substituting Equation (**30**) into the analog of Equation (19)

$$\dot{U}_R = -\frac{1}{2}\frac{R}{T}\dot{U} + \frac{B(y)}{R} \qquad (31)$$

Using Equation (29) as a boundary condition for Equation (31) and solving the equation for B(y)

$$B(y) = \frac{1}{2}\frac{(R_1)^2}{T}\dot{U}\left[\left\{\frac{1 - (\frac{R_1}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2]\frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2}\right\}[1 - (\frac{R_i}{R_1})^2] + 1\right] \qquad (32)$$

Substituting equation (32) into equation (31) and rearranging the obtained equation

$$\dot{U}_R = -\frac{1}{2}\frac{R}{T}\dot{U}\left[1 - \left\{1 - [1 - (\frac{R_o}{R_2})^2]\frac{v_b}{\dot{U}}\right\}(\frac{R_2}{R})^2\right] \qquad (33)$$

Equation (33) can also be obtained by the application of the boundary condition from Zone IV.

In Zone IV, the same procedure in Zone II is again used.

As in Zone I, II and III

$$\dot{U}_\theta = 0 \qquad (34)$$

Obtaining the values of a and b by using the boundary conditions

$$\dot{U}_y|_{y=0} = a = 0 \qquad (35)$$

$$\dot{U}_y|_{y=T} = a + bT = v_b \qquad (36)$$

Then

$$b = \frac{v_b}{T} \qquad (37)$$

Substituting equation (35) and (37) into the analog of equation (8)

$$\dot{U}_y = \frac{y}{T}\frac{v_b}{\dot{U}}\dot{U} \qquad (38)$$

$$\dot{\varepsilon}_{yy} = \frac{\partial \dot{U}_y}{\partial y} = \frac{v_b}{\dot{U}}\frac{\dot{U}}{T} \qquad (39)$$

Substituting equation (39) into the analog of equation (19)

$$\dot{U}_R = -\frac{1}{2}\frac{R}{T}\frac{v_b}{\dot{U}}\dot{U} + \frac{B(y)}{R} \qquad (40)$$

Using the boundary condition

$$\dot{U}_R|_{R=R_o} = 0 \qquad (41)$$

And solving equation (40) for B(y)

$$B(y) = \frac{1}{2}\frac{R_o^2}{T}\frac{v_b}{\dot{U}}\dot{U} \qquad (42)$$

Substituting equation (42) into equation (40)

$$\dot{U}_R = -\frac{1}{2}\frac{R}{T}\frac{v_b}{\dot{U}}\dot{U}\left[1 - \left(\frac{R_o}{R}\right)^2\right] \qquad (43)$$

In Zone V, the rigid body moves upward.

As in Zone I

$$\dot{U}_R = \dot{U}_\theta = 0 \tag{44}$$

$$\dot{U}_y = \frac{v_b}{\dot{U}} \dot{U} \tag{45}$$

2. Upper Bound

Prager and Hodge (3) formulated the upper bound theorem. If surfaces of velocity discontinuity are to be included and for the case of a Mises material, it reads: "Theorem 2. Among all kinematically admissible strain rate fields the actual one minimizes the expression:"

$$J^* = \frac{2}{\sqrt{3}} \sigma_o \int_V \sqrt{\frac{1}{2} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}}\, dV + \int_{S_\Gamma} \tau \cdot |\Delta v|\, dS \tag{46}$$

The individual terms of equation (46) are now computed.

a) Power for internal deformation

The general form to calculate the power of internal deformation is

$$\dot{W}_i = \frac{2}{\sqrt{3}} \sigma_o \int_V \sqrt{\frac{1}{2} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}}\, dV \tag{47}$$

Zones I and V with rigid body motion require no power of internal deformation. In Zone II (as well as III and IV)

$\dot{\varepsilon}_{R\theta} = \dot{\varepsilon}_{\theta y} = \dot{\varepsilon}_{yR} = 0$, thus equation (47) becomes

$$\dot{W}_{iII} = \frac{2}{\sqrt{3}} \sigma_o \int_V \sqrt{\frac{1}{2} (\dot{\varepsilon}_{RR}^{\,2} + \dot{\varepsilon}_{\theta\theta}^{\,2} + \dot{\varepsilon}_{yy}^{\,2})}\, dV \qquad (48)$$

The normal strain rates are

$$\dot{\varepsilon}_{RR} = \frac{\partial \dot{U}_R}{\partial R} = \frac{1}{2} \frac{\dot{U}}{T} \left[\frac{1 - (\frac{R_1}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right] [1 + (\frac{R_i}{R})^2]$$

(49)

$$\dot{\varepsilon}_{\theta\theta} = \frac{\dot{U}_R}{R} = \frac{1}{2} \frac{\dot{U}}{T} \left[\frac{1 - (\frac{R_1}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right] [1 - (\frac{R_i}{R})^2]$$

(50)

$$\dot{\varepsilon}_{yy} = \frac{\partial \dot{U}_y}{\partial y} = - \frac{\dot{U}}{T} \left[\frac{1 - (\frac{R_1}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right]$$

(51)

Substituting equations (49), (50) and (51) into equation (48)

$$\dot{W}_{iII} = \frac{2}{\sqrt{3}} \sigma_o \int_{R=R_i}^{R_1} \sqrt{\frac{1}{2}\frac{1}{2}\left(\frac{\dot{U}}{T}\right)^2 \left\{ \frac{1 - \left(\frac{R_1}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right] \frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2} \right\}^2 \left[3 + \left(\frac{R_i}{R}\right)^4\right]} \; 2\pi R dRT$$

(52)

Rearranging equation (52)

$$\dot{W}_{iII} = -2 \sigma_o \pi \dot{U} \left| \frac{1 - \left(\frac{R_1}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right] \frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2} \right| \int_{R=R_i}^{R_1} \sqrt{R^4 + \frac{R_i^4}{3}} \; \frac{dR}{R}$$

(53)

Considering the integral

$$z = \int_{R=R_i}^{R_1} \sqrt{R^4 + \frac{R_i^4}{3}} \; \frac{dR}{R} \qquad (54)$$

Substituting the values

$$R^2 = x \qquad \frac{dR}{R} = \frac{1}{2} \frac{dx}{x} \qquad (55)$$

into equation (54)

$$z = \frac{1}{2} \int \frac{\sqrt{x^2 + \frac{R_i^4}{3}}}{x} \; dx \qquad (60)$$

Integrating equation (60)

$$z = \frac{1}{2}\left[\sqrt{x^2 + \frac{R_i^{\,4}}{3}} - \frac{R_i^{\,2}}{\sqrt{3}} \ln \left| \frac{\sqrt{x^2 + \frac{R_i^{\,4}}{3}} + \frac{R_i^{\,2}}{\sqrt{3}}}{x} \right| \right] \tag{61}$$

Substituting equation (61) into equation (53) after replacing x by R^2

$$\dot{W}_{iII} = -\sigma_o \pi \dot{U} \left| \frac{1 - (\frac{R_1}{R_2})^2 - [\,1 - (\frac{R_o}{R_2})^2\,]\,\frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right|$$

$$X \left\{ \sqrt{R^4 + \frac{R_i^{\,4}}{3}} - \frac{R_i^{\,2}}{\sqrt{3}} \ln \left| \frac{\sqrt{R^4 + \frac{R_i^{\,4}}{3}} + \frac{R_i^{\,2}}{\sqrt{3}}}{R^2} \right| \right\}_{R=R_i}^{R_1} \tag{62}$$

Evaluating equation (62) in the specified range and rearranging the obtained equation

$$\dot{W}_{iII} = \frac{1}{\sqrt{3}} \sigma_0 \pi \left|\dot{U}\right| \left(\frac{R_i}{R_2}\right)^2 R_2^2 \left| \frac{1 - \left(\frac{R_1}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right] \frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2} \right|$$

$$X \left|\left\{ 2 - \sqrt{1 + 3\left(\frac{R_1}{R_2}\right)^4 \left(\frac{R_2}{R_i}\right)^4} + \ln \left| \frac{1}{3} \left(\frac{R_i}{R_2}\right)^2 \left(\frac{R_2}{R_1}\right)^2 \right.\right.\right.$$

$$X \left.\left.\left.\left[1 + \sqrt{1 + 3\left(\frac{R_1}{R_2}\right)^4 \left(\frac{R_2}{R_i}\right)^4} \right] \right| \right\} \right| \qquad (63)$$

In Zone III, the same steps are taken as in Zone II. The normal strain rates are

$$\dot{\varepsilon}_{RR} = -\frac{1}{2}\frac{\dot{U}}{T}\left\{1 + \left(1 - [1 - (\frac{R_o}{R_2})^2]\frac{v_b}{\dot{U}}\right)(\frac{R_2}{R})^2\right\} \qquad (64)$$

$$\dot{\varepsilon}_{\theta\theta} = -\frac{1}{2}\frac{\dot{U}}{T}\left\{1 - \left(1 - [1 - (\frac{R_o}{R_2})^2]\frac{v_b}{\dot{U}}\right)(\frac{R_2}{R})^2\right\} \qquad (65)$$

$$\dot{\varepsilon}_{yy} = \frac{\dot{U}}{T} \qquad (66)$$

Substituting equations (64), (65) and (66) into equation (47)

$$\dot{W}_{iIII} = \frac{2}{\sqrt{3}}\sigma_o \int_{R=R_1}^{R_2} \sqrt{\frac{1}{2}\frac{1}{2}(\frac{\dot{U}}{T})^2 [3 + \left(1 - [1 - (\frac{R_o}{R_2})^2]\frac{v_b}{\dot{U}}\right)^2 (\frac{R_2}{R})^4]}$$

$$X \qquad 2\pi R dR T \qquad (67)$$

Rearranging equation (67)

$$\dot{W}_{iIII} = -2\,\sigma_o \pi \dot{U} \int_{R=R_1}^{R_2} \sqrt{R^4 + \frac{1}{3}\left\{1 - [1 - (\frac{R_o}{R_2})^2]\frac{v_b}{\dot{U}}\right\}^2 R_2^4}\ \frac{dR}{R} \tag{68}$$

Integrating equation (68)

$$\dot{W}_{iIII} = -\sigma_o \pi \dot{U} \left[\sqrt{R^4 + \frac{1}{3}\left\{1 - [1 - (\frac{R_o}{R_2})^2]\frac{v_b}{\dot{U}}\right\}^2 R_2^4}\right.$$

$$- \frac{1}{\sqrt{3}}\left\{1 - [1 - (\frac{R_o}{R_2})^2]\frac{v_b}{\dot{U}}\right\} R_2^2$$

$$\left. X\ \ln\left|\frac{\sqrt{R^4 + \frac{1}{3}\left\{1 - [1 - (\frac{R_o}{R_2})^2]\frac{v_b}{\dot{U}}\right\}^2 R_2^4} + \frac{1}{\sqrt{3}}\left\{1 - [1 - (\frac{R_o}{R_2})^2]\frac{v_b}{\dot{U}}\right\} R_2^2}{R^2}\right|\ \right]_{R=R_1}^{R_2} \tag{69}$$

Evaluating equation (69) in the specified range and rearranging the obtained equation

$$\dot{W}_{iIII} = \frac{1}{\sqrt{3}}\sigma_o \pi |\dot{U}| R_2^{\,2} \left| \left\{ 1 - \left[1 - \left(\frac{R_o}{R_2}\right)^2 \right] \frac{v_b}{\dot{U}} \right\} \right.$$

$$\times \left\{ \sqrt{1 + 3\left\{ \frac{\left(\frac{R_1}{R_2}\right)^2}{1 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right] \frac{v_b}{\dot{U}}} \right\}^2} - \sqrt{1 + \frac{3}{\left(1 - \left[1 - \left(\frac{R_o}{R_2}\right)^2 \right] \frac{v_b}{\dot{U}} \right)^2}} \right.$$

$$\left. \left. + \ln \left| \left(\frac{R_1}{R_2}\right)^2 \frac{1 + \sqrt{1 + \frac{3}{\left\{ 1 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right] \frac{v_b}{\dot{U}} \right\}^2}}}{1 + \sqrt{1 + 3\left\{ \frac{\left(\frac{R_1}{R_2}\right)^2}{1 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right] \frac{v_b}{\dot{U}}} \right\}^2}} \right| \right\} \right| \qquad (70)$$

In Zone IV the normal strain rates are

$$\dot{\varepsilon}_{RR} = -\frac{1}{2}\frac{\dot{U}}{T}\frac{v_b}{\dot{U}}\left[1+\left(\frac{R_o}{R}\right)^2\right] \quad (71)$$

$$\dot{\varepsilon}_{\theta\theta} = -\frac{1}{2}\frac{\dot{U}}{T}\frac{v_b}{\dot{U}}\left[1-\left(\frac{R_o}{R}\right)^2\right] \quad (72)$$

$$\dot{\varepsilon}_{yy} = \frac{\dot{U}}{T}\frac{v_b}{\dot{U}} \quad (73)$$

Substituting equation (71), (72) and (73) into equation (47)

$$\dot{W}_{iIV} = \frac{2}{\sqrt{3}}\sigma_o \int_{R=R_2}^{R_o} \sqrt{\frac{1}{2}\frac{1}{2}\left(\frac{\dot{U}}{T}\frac{v_b}{\dot{U}}\right)^2\left[3+\left(\frac{R_o}{R}\right)^4\right]}\; 2\pi R dR T \quad (74)$$

Rearranging equation (74)

$$\dot{W}_{iIV} = 2\,\sigma_o\,\pi\,\frac{v_b}{\dot{U}}\,\dot{U}\int_{R=R_2}^{R_o}\sqrt{R^4+\frac{R_o^4}{3}}\;\frac{dR}{R} \quad (75)$$

Integrating equation (75)

$$\dot{W}_{iIV} = \sigma_o\pi\frac{v_b}{\dot{U}}\,\dot{U}\left[\sqrt{R^4+\frac{R_o^4}{3}} - \frac{R_o^2}{\sqrt{3}}\ln\left|\frac{\sqrt{R^4+\frac{R_o^4}{3}}+\frac{R_o^2}{\sqrt{3}}}{R^2}\right|\right]_{R=R_2}^{R_o} \quad (76)$$

Evaluating equation (76) in the specified range and rearranging the obtained equation

$$\dot{W}_{iIV} = \frac{1}{\sqrt{3}}\sigma_o\pi \left|\frac{v_b}{\dot{U}} \quad \dot{U}\right| \left(\frac{R_o}{R_2}\right)^2 R_2^2 \left| \left[2 - \sqrt{1+3\left(\frac{R_2}{R_o}\right)^4} - \ln\left|\left(\frac{R_2}{R_o}\right)^2 \frac{3}{1+\sqrt{1+3\left(\frac{R_2}{R_o}\right)^4}}\right| \right] \right| \tag{77}$$

b) Friction losses

Friction losses are determined by the relation

$$\dot{W}_f = \int_S \tau|\Delta v|\,dS \tag{78}$$

A constant shear stress is assumed as

$$\tau = m\frac{\sigma_o}{\sqrt{3}} \tag{79}$$

where $0 < m < 1$

The shear velocity is the velocity difference between the metal and the tool surface.

Along Γ_1, noting the velocity difference between the downward flowing metal in Zone I and the stationary die

$$\dot{W}_{f1} = \int_{y=-H_1}^{0} \frac{1}{\sqrt{3}} m\,\sigma_o \left| \left(\frac{1-\left(\frac{R_i}{R_2}\right)^2 - \left[1-\left(\frac{R_o}{R_2}\right)^2\right] b}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_1}{R_2}\right)^2} \right) \dot{U} \right| 2\pi R_1\, dy \tag{80}$$

Evaluating equation (80) in the specified range after integration

$$\dot{W}_{f1} = \frac{2}{\sqrt{3}} m \sigma_o \pi |\dot{U}| \frac{R_1}{R_2} \frac{H_1}{R_2} R_2^2 \left| \frac{1 - (\frac{R_i}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right| \tag{81}$$

Along Γ_2, noting the velocity difference between the downward flowing metal in Zone I and the downward moving ram

$$\dot{W}_{f2} = \int_{y=0}^{h_1} \frac{1}{\sqrt{3}} m \sigma_o \left| \frac{1 - (\frac{R_i}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \dot{U} - \dot{U} \right| 2\pi R_i dy \tag{82}$$

Evaluating equation (82) in the specified range after integration

$$\dot{W}_{f2} = \frac{2}{\sqrt{3}} m \sigma_o \pi |\dot{U}| \frac{R_i}{R_2} \frac{h_1}{R_2} R_2^2 \left| \frac{1 - (\frac{R_i}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right| \tag{83}$$

Along Γ_5, noting the velocity difference between the downward flowing metal in Zone II and the downward moving ram

$$\dot{W}_{f5} = \int_{y=0}^{T} \frac{1}{\sqrt{3}} m \sigma_o \left| \dot{U} + \frac{\dot{U}}{T} \left\{ \frac{1 - (\frac{R_i}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} (y - T) - y \right\} \right|$$

$$X \quad 2\pi R_i dy \qquad (84)$$

Rearranging equation (84)

$$\dot{W}_{f5} = -\frac{2}{\sqrt{3}} m \sigma_o \pi \dot{U} \frac{R_i}{T} \left| \frac{1 - (\frac{R_1}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right| \int_{y=0}^{T} \left| y - T \right| dy$$

(85)

Evaluating equation (85) in the specified range after integration

$$\dot{W}_{f5} = \frac{1}{\sqrt{3}} m \sigma_o \pi |\dot{U}| \frac{R_i}{R_2} \frac{T}{R_2} R_2^2 \left| \frac{1 - (\frac{R_1}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right|$$

(86)

Along Γ_6, noting the radial velocity difference between the metal in Zone II and the ram

$$\dot{W}_{f6} = \int_{R=R_i}^{R_1} \frac{1}{\sqrt{3}} m\, \sigma_o \left| \frac{1}{2}\frac{R}{T}\, \dot{U} \left\{ \frac{1 - (\frac{R_1}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2]\, \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right\} [1 - (\frac{R_i}{R})^2] \right|$$

$$X \qquad 2\pi R dR \qquad (87)$$

Rearranging equation (87)

$$\dot{W}_{f6} = -\frac{1}{\sqrt{3}}\, m\, \sigma_o \pi\, \frac{\dot{U}}{T} \left| \frac{1 - (\frac{R_1}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2]\, \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right| \int_{R=R_i}^{R_1} \left| R^2 - R_i^2 \right| dR \qquad (88)$$

Evaluating equation (88) in the specified range after integration

$$\dot{W}_{f6} = \frac{1}{3\sqrt{3}}\, m\, \sigma_o \pi \left| \dot{U} \right| \frac{R_2^3}{T} \left| \left\{ \frac{1 - (\frac{R_1}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2]\, \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right\} \right. \qquad (89)$$

$$X \qquad \left. [2\, (\frac{R_i}{R_2})^3 + (\frac{R_1}{R_2})^3 - 3\, (\frac{R_i}{R_2})^2\, \frac{R_1}{R_2}] \right|$$

Rearranging equation (89)

$$\dot{W}_{f6} = \frac{1}{3\sqrt{3}} m \sigma_o \pi |\dot{U}| \frac{R_2^3}{T} \left| \frac{1 - (\frac{R_1}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} (2 \frac{R_i}{R_2} + \frac{R_1}{R_2}) \right. \tag{90}$$

$$\left. \times \left(\frac{R_1}{R_2} - \frac{R_i}{R_2} \right)^2 \right|$$

Along Γ_7 and Γ_9, noting the radial velocity difference between the metal in Zone III and the ram,

$$\dot{W}_{f7,9} = \int_{R=R_1}^{R_2} 2 \frac{1}{\sqrt{3}} m \sigma_o \left| \frac{1}{2} \dot{U} \frac{R}{T} \left[\left\{ 1 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}} \right\} (\frac{R_2}{R})^2 - 1 \right] \right| 2\pi R dR \tag{91}$$

Rearranging equation (91)

$$\dot{W}_{f7,9} = - \frac{2}{\sqrt{3}} m \sigma_o \pi \frac{\dot{U}}{T} \int_{R=R_1}^{R_2} \left| \left\{ 1 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}} \right\} R_2^2 - R^2 \right| dR \tag{92}$$

Evaluating equation (92) in the specified range after integration and rearranging the obtained equation

$$\dot{W}_{f7,9} = \frac{2}{3\sqrt{3}} m \sigma_o \pi |\dot{U}| \frac{R_2^3}{T} \left| (1 - \frac{R_1}{R_2}) \left\{ 2 - \frac{R_1}{R_2} - (\frac{R_1}{R_2})^2 - 3 [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}} \right\} \right| \quad (93)$$

Along Γ_{10}, noting the radial velocity difference between the metal in Zone IV and the stationary die,

$$\dot{W}_{f10} = \int_{R=R_2}^{R_o} \frac{1}{\sqrt{3}} m \sigma_o \left| - \frac{1}{2} \frac{v_b}{\dot{U}} \dot{U} \frac{R}{T} [1 - (\frac{R_o}{R})^2] \right| 2\pi R dR \quad (94)$$

Rearranging equation (94)

$$\dot{W}_{f10} = \frac{1}{\sqrt{3}} m \sigma_o \pi \frac{v_b}{\dot{U}} \frac{\dot{U}}{T} \int_{R=R_2}^{R_o} \left| R^2 - R_o^2 \right| dR \quad (95)$$

Evaluating equation (95) in the specified range after integration

$$\dot{W}_{f10} = \frac{1}{3\sqrt{3}} m \sigma_o \pi \left| \frac{v_b}{\dot{U}} \dot{U} \right| \frac{R_2^3}{T} \left| [1 - 3 (\frac{R_o}{R_2})^2 + 2 (\frac{R_o}{R_2})^3] \right| \quad (96)$$

Along Γ_{11}, noting the velocity difference between the upward moving metal in Zone IV and the stationary die

$$\dot{W}_{f11} = \int_{y=0}^{T} \frac{1}{\sqrt{3}} m \, \sigma_o \left| \frac{y}{T} \frac{v_b}{\dot{U}} \dot{U} \right| 2\pi R_o dy \tag{97}$$

Evaluating equation (97) in the specified range after integration

$$\dot{W}_{f11} = \frac{1}{\sqrt{3}} m \, \sigma_o \pi \left| \frac{v_b}{\dot{U}} \dot{U} \right| \frac{R_o}{R_2} \frac{T}{R_2} R_2^{\,2} \tag{98}$$

Along Γ_{13}, noting the velocity difference between the upward flowing metal in Zone V and the downward moving ram

$$\dot{W}_{f13} = \int_{y=0}^{h_2} \frac{1}{\sqrt{3}} m \, \sigma_o \left| \frac{v_b}{\dot{U}} \dot{U} - \dot{U} \right| 2\pi R_2 dy \tag{99}$$

Evaluating equation (99) in the specified range after integration

$$\dot{W}_{f13} = \frac{2}{\sqrt{3}} m \, \sigma_o \pi |\dot{U}| \frac{h_2}{R_2} R_2^{\,2} \left| \left(1 - \frac{v_b}{\dot{U}} \right) \right| \tag{100}$$

Along Γ_{14}, noting the velocity difference between the upward flowing metal in Zone V and the stationary die

$$\dot{W}_{f14} = \int_{y=T}^{H_2} \frac{1}{\sqrt{3}} m \sigma_o \left| \frac{v_b}{\dot{U}} \dot{U} \right| 2\pi R_o dy \tag{101}$$

$$\dot{W}_{f14} = \frac{2}{\sqrt{3}} m \sigma_o \pi \left| \frac{v_b}{\dot{U}} \dot{U} \right| \frac{R_o}{R_2} R_2^2 \left| \left(\frac{H_2}{R_2} - \frac{T}{R_2} \right) \right| \tag{102}$$

c) Shear losses

Shear losses are also expressed by Equation (78)

$$\dot{W}_S = \int_S \tau |\Delta v| dS \tag{78}$$

The maximum shear the metal can stand is

$$\tau = \frac{\sigma_o}{\sqrt{3}} \tag{103}$$

The shear velocity here is the velocity difference along the boundary between two zones.

Along Γ_3, noting the radial velocity difference between the metal in Zone I and II, $\dot{W}_{s3}$ is obtained by replacing m with 1 in equation 90; thus

$$\dot{W}_{S3} = \frac{1}{3\sqrt{3}} \sigma_o \pi \left|\dot{U}\right| \left|\frac{R_2^3}{T}\right| \left\{ 1 - \left(\frac{R_1}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right] \frac{v_b}{\dot{U}} \left(2\frac{R_i}{R_2} + \frac{R_1}{R_2}\right) \right. \tag{104}$$

$$\left. \times \frac{\frac{R_i}{R_2} - \frac{R_1}{R_2}}{\frac{R_i}{R_2} + \frac{R_1}{R_2}} \right|$$

Along Γ_4, noting the velocity difference between the downward moving metal in Zone II and Zone III

$$\dot{W}_{S4} = \int_{y=0}^{T} \frac{\sigma_o}{\sqrt{3}} \left| \frac{\dot{U}}{T} \left\{ \frac{1 - \left(\frac{R_i}{R_2}\right)^2 - \left[1 - \left(\frac{R_o}{R_2}\right)^2\right] \frac{v_b}{\dot{U}}}{\left(\frac{R_1}{R_2}\right)^2 - \left(\frac{R_i}{R_2}\right)^2} (y - T) - y \right\} \right. \tag{105}$$

$$\left. + \frac{\dot{U}}{T} y \right| 2\pi R_1 dy$$

Rearranging equation (105)

$$\dot{W}_{S4} = -\frac{2}{\sqrt{3}}\sigma_o \pi \dot{U} \frac{R_1}{R_2} R_2 \left| \frac{1 - (\frac{R_i}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right| \int_{y=0}^{T} \left| \frac{y}{T} - 1 \right| dy$$

(106)

Evaluating equation (106) in the specified range after integration

$$\dot{W}_{S4} = \frac{1}{\sqrt{3}}\sigma_o \pi |\dot{U}| \frac{R_1}{R_2} \frac{T}{R_2} R_2^2 \left| \frac{1 - (\frac{R_i}{R_2})^2 - [1 - (\frac{R_o}{R_2})^2] \frac{v_b}{\dot{U}}}{(\frac{R_1}{R_2})^2 - (\frac{R_i}{R_2})^2} \right|$$

(107)

Along Γ_8, noting the velocity difference between the downward moving metal in Zone III and the upward moving metal in Zone IV

$$\dot{W}_{S8} = \int_{y=0}^{T} \frac{1}{\sqrt{3}} \sigma_o \left| \frac{y}{T} \frac{v_b}{\dot{U}} \dot{U} - \frac{y}{T} \dot{U} \right| 2\pi R_2 dy \qquad (108)$$

Rearranging equation (108)

$$\dot{W}_{S8} = \frac{2}{\sqrt{3}} \sigma_o \pi \frac{R_2}{T} \left(\frac{v_b}{\dot{U}} \dot{U} - \dot{U}\right) \int_{y=0}^{T} y \, dy \qquad (109)$$

Evaluating equation (109) in the specified range after integration

$$\dot{W}_{S8} = \frac{1}{\sqrt{3}} \sigma_o \pi \left| (1 - \frac{v_b}{\dot{U}}) \right| \left| \dot{U} \right| \frac{T}{R_2} R_2^2 \tag{110}$$

Along Γ_{12}, noting the radial velocity difference between the metal in Zone IV and V, $\dot{W}_{S12}$ is obtained by replacing m with 1 in equation (96); thus

$$\dot{W}_{S12} = \frac{1}{3\sqrt{3}} \sigma_o \pi \left| \frac{v_b}{\dot{U}} \right| \left| \dot{U} \right| \left| \frac{R_2^3}{T} \right| \left| \left[1 - 3 \left(\frac{R_o}{R_2}\right)^2 + 2 \left(\frac{R_o}{R_2}\right)^3 \right] \right| \tag{111}$$

d) Relative ram pressure

The external power supplied by the press through the ram is

$$J^* = \pi(R_2^2 - R_i^2)\, p_R |\dot{U}| \tag{112}$$

$$J^* = \pi R_2^2 \left[1 - \left(\frac{R_i}{R_2}\right)^2\right] p_R |\dot{U}| \tag{113}$$

Equating the external power to the required power and solving for the relative ram pressure for internal deformation, friction losses and shear losses equation (1) is obtained.

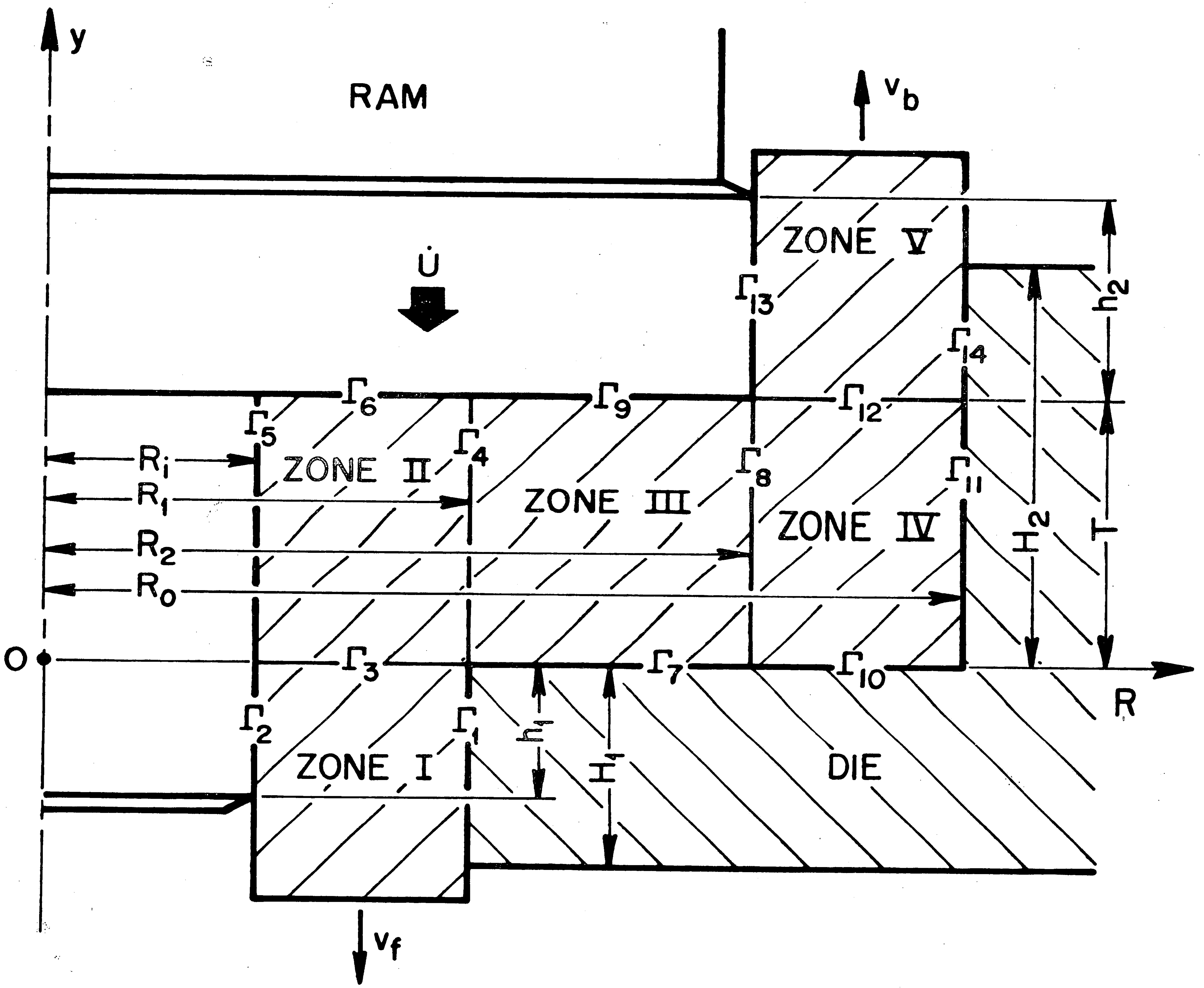

FIG. 1 GEOMETRY OF COMBINED BACKWARD - FORWARD EXTRUSION.

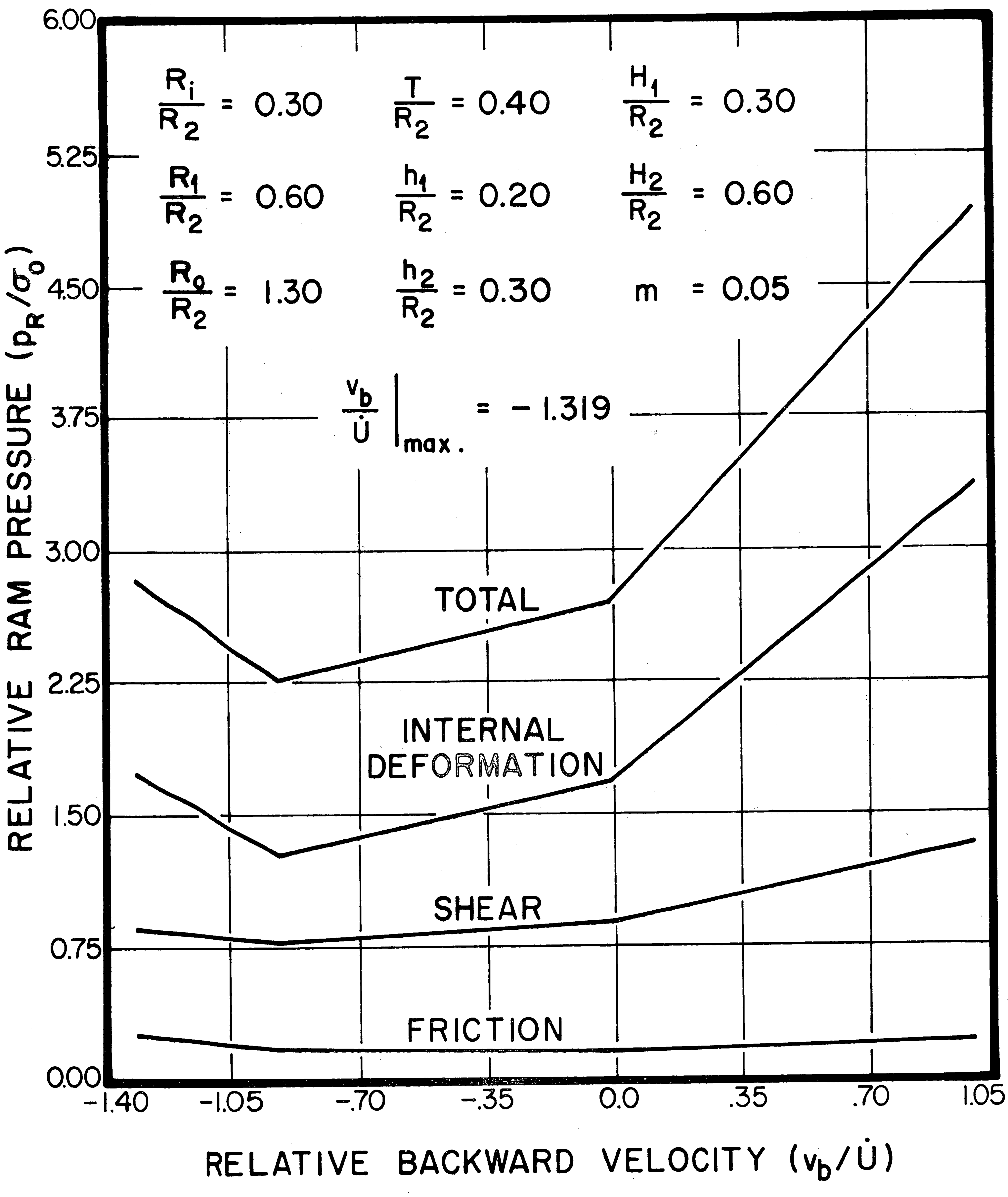

FIG.2 NORMAL GEOMETRY.

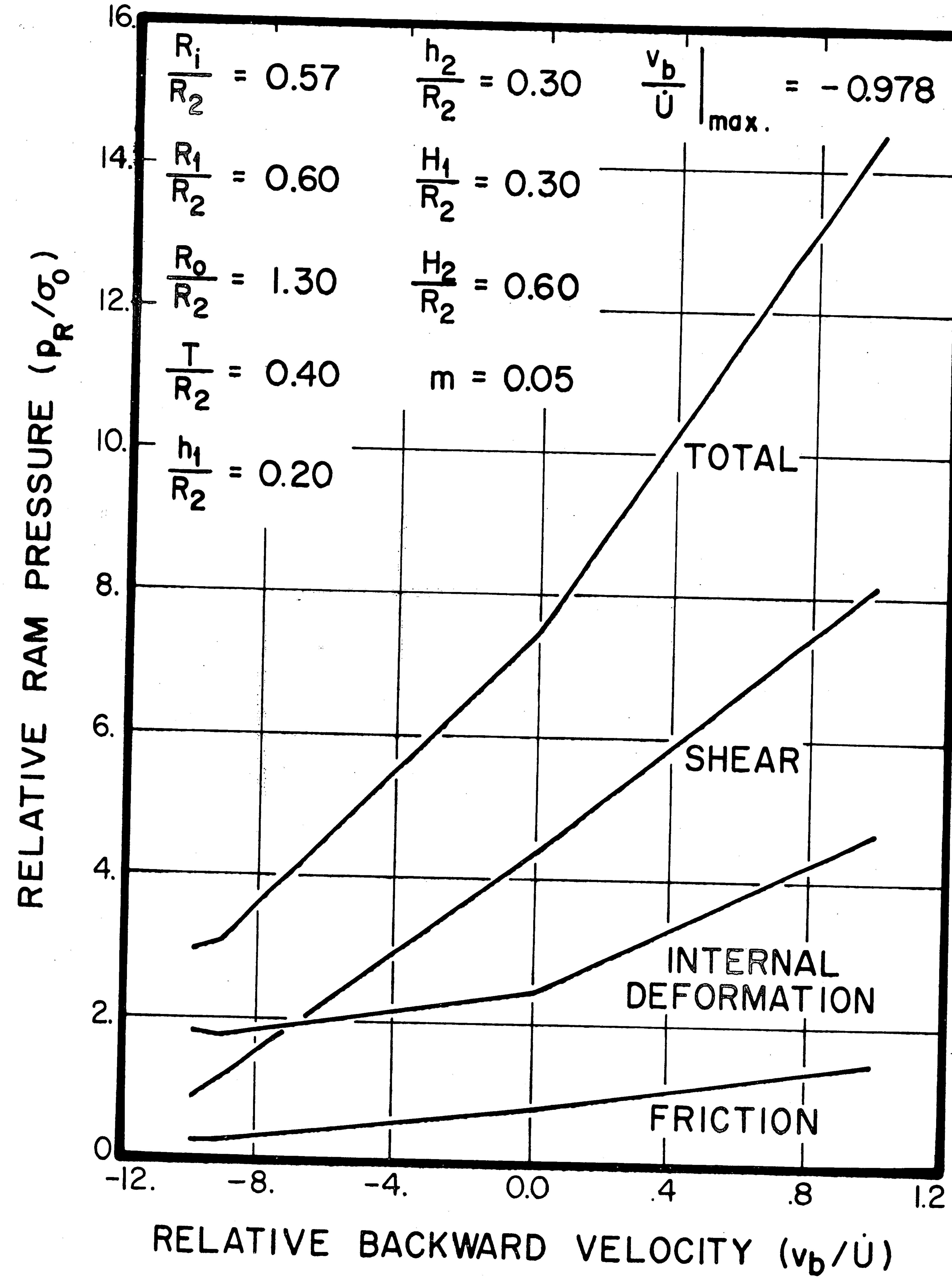

FIG.3 LOWER GAP IS ALMOST CLOSED.

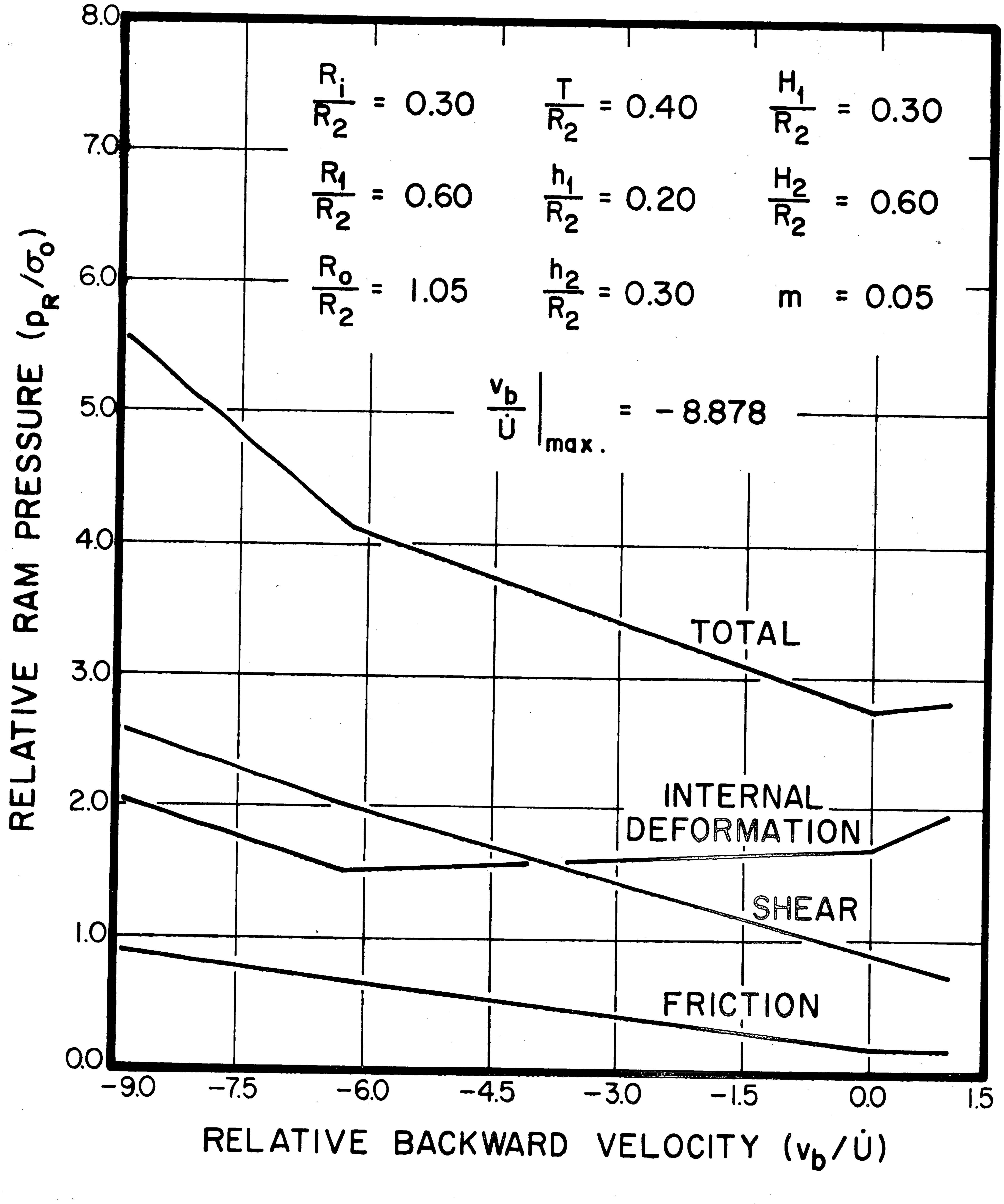

FIG.4 UPPER GAP IS ALMOST CLOSED.

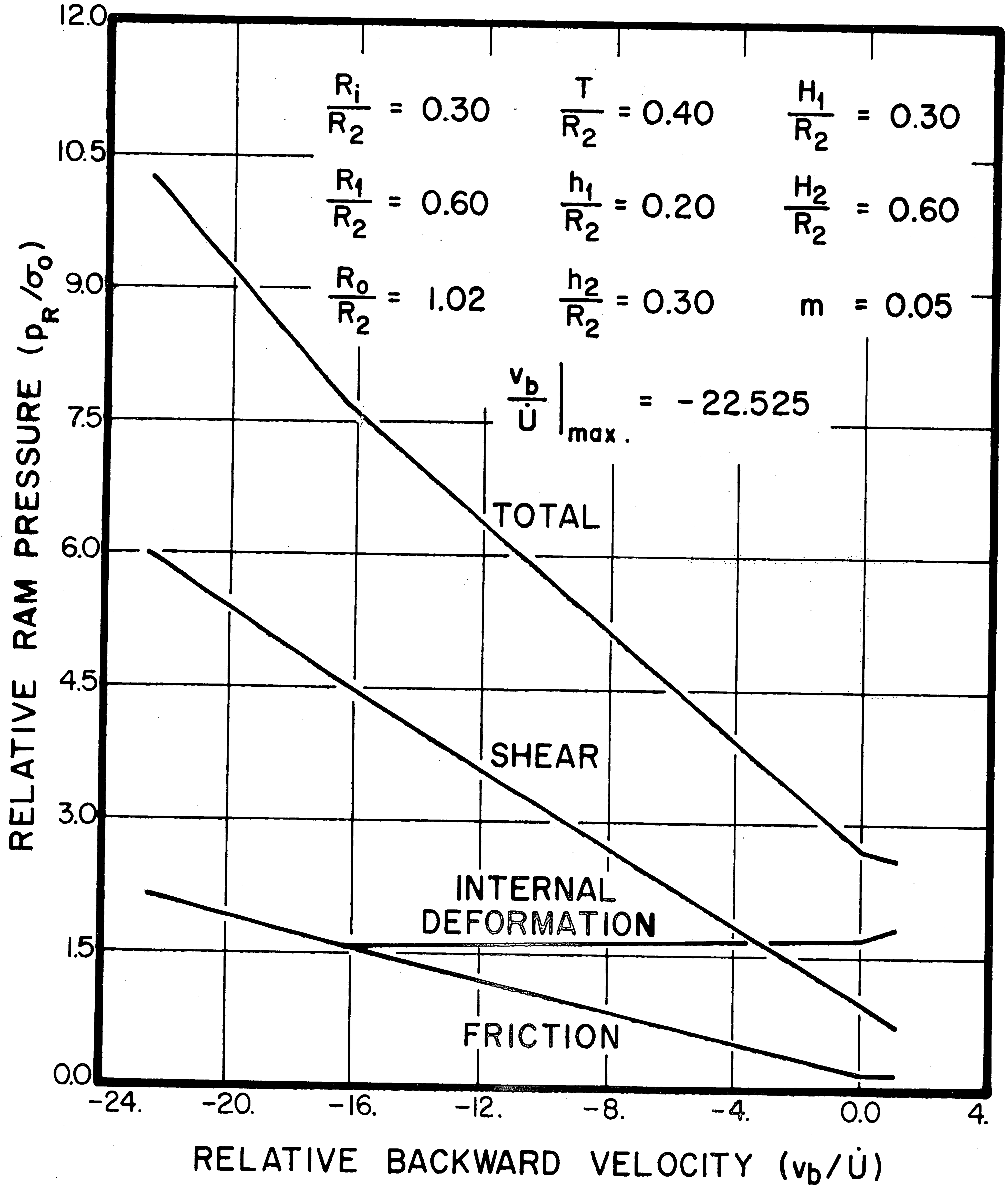

FIG.5 UPPER GAP IS ALMOST CLOSED.

Vita

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