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An investigation of the desired levels of investment and amplification of service levels in a multi-echelon new and reparable item inventory system

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AN INVESTIGATION OF THE DESIRED LEVELS OF INVESTMENT AND AMPLIFICATION OF SERVICE LEVELS IN A MULTI-ECHELON NEW AND REPARABLE ITEM INVENTORY SYSTEM

by

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ABSTRACT

The purpose of this thesis is to structure a periodic review model and investigate the dynamic response for a multi-echelon new and reparable inventory system. A reparable item inventory system deals with two classes of the same type of item. The first class consists of those items that have been used, returned and repaired; thereby returning them to the system. The second class consists of those new items that have been procured from another source. This two-echelon investigation consists of two levels of inventory, one at the distribution echelon (Service Center) and the other being the Merchandise echelon. The distribution echelon contains two sources of supply, they are: (1) Repair; and (2) Merchandise. The Merchandise echelon procures its inventory from a manufacturing organization.

The investigation consists of three objectives, they are:

(1) Determine the required investment level at each echelon to provide a desired service posture.

(2) Determine the impact on the system when the second echelon level of service is reduced.

(3) Investigate the response in terms of new investment required to a percentage change in repair or customer demand.

The inventory system is investigated in a cost-free structure. The measure of performance is the inventory required to provide a level of service.

CHAPTER I

INTRODUCTION

The existing reparable inventory system within the Western Electric Company, Incorporated supply pipeline is an exceedingly complex probabilistic system involving thousands of line items. The entire supply pipeline contains a network of stock points and repair points. To develop a model of a system with this complexity can only be a partial representation of reality. Despite this awareness, the fact that models have proven to be invaluable aids in managing complex systems is the primary motive behind efforts to model a reparable inventory system of such magnitude.

One aspect of the dynamic behavior of a multi-echelon supply pipeline seems to be characteristic of all such systems. This characteristic is the tendency for minor variations in the demand and repair at the lowest echelon to be amplified into larger variations in demand upon the highest echelon [1,2,3]. Statistically, the variance in demand increases with ascending levels in the supply pipeline reaching a maximum at the highest echelon. These large variations in demand result in more inventory required to provide a desired level of service.

Considerable research has taken place in the area of reparable-item inventory systems [4,5,6,7]. The literature has concentrated heavily upon the case of inventory control for an individual supplier. The individual supplier generally represents but one of several intermediate stock points in a much larger supply system, and it is assumed any inventory required can be obtained from another stocking point. Many models that have been developed will answer the question of how much inventory to ship to each stocking point and when to procure items such that shortages or total costs are minimized [8,9,10,11,15]. The optimum policy may or may not be the best operating policy depending on the accuracy of demand forecasts and the actual transmission of data [12]. Implicit in models is the assumption that the demands are accurately transmitted to the inventory stocking point. If this assumption is violated the main effect of errors is that the supply point may process a demand which differs considerably from the actual demand.

The application of feedback control theory to inventory control was first introduced by Simon [13] in 1952. The objective was to utilize engineering system design and stability criteria to achieve a controlled inventory position in a cost-free structure. Recently, Meyer [14] extended this idea considering analysis and synthesis of a multi-echelon inventory system, using the theory of feedback control. Multi-echelon inventory systems with continuous variables were formulated as linear, deterministic feedback control systems and mathematically analyzed and simulated.

In a multi-echlon inventory system consideration must be given to the total corporate structure as opposed to analysis of a single echelon. A change in management policy for one echelon may be of great benefit to that echelon, yet its impact on the total inventory system and the supply pipeline is undesirable. This thesis will add to a growing knowledge base by structuring a periodic review model for a two-echelon reparable item system experiencing normally distributed random demands. The model provides a means of understanding the parameter interactions, in a cost-free structure, that exists within the system. The approach taken is not to optimize the system, but to find stable operating points in terms of total pipeline investment and repair within the system so that a desired level of service is maintained.

Limited research has been devoted to the interactions between a reparable item and new stock investment required within a supply pipeline when there is a change in repair. Any time there is a percent change in repair, within a multi-echelon inventory system, the change is amplified at the higher echelons. A reduction in repair requires larger demand to the next echelon as well as increased inventory to maintain the same inventory level. The objective of this research is to investigate a two-echelon supply pipeline with a reparable item. The investigation will be divided into three divisions. They are:

(1) Investigate the required investment level at each echelon to provide a desired level of service. The stability of the system will be shown as a function of investment levels.

(2) Determine the impact on the system when the integrity of the second echelon is reduced. This will be a function of both the repair program and the desired investment level at the first echelon.

(3) Investigate the response in terms of new pipeline investment to a percentage change in the repair

function or customer demand.

The above objectives will be investigated with a large variability in demand and repair placed upon the system. Mathematical investment equations will be developed that determine the steady-state characteristics of such an inventory system.

Chapter II contains a description of the inventory system to be investigated. Chapter III contains a description of the periodic review model, called the "Logistics Environment Model", used to model the system. The model is based on the major response variables of the supply pipeline. Chapter IV contains simulation experiments and results including the development of new pipeline investment as a function of the repair program or customer demand. Chapter V summarizes the results and recommends areas for further study.

CHAPTER II

THE INVENTORY SYSTEM

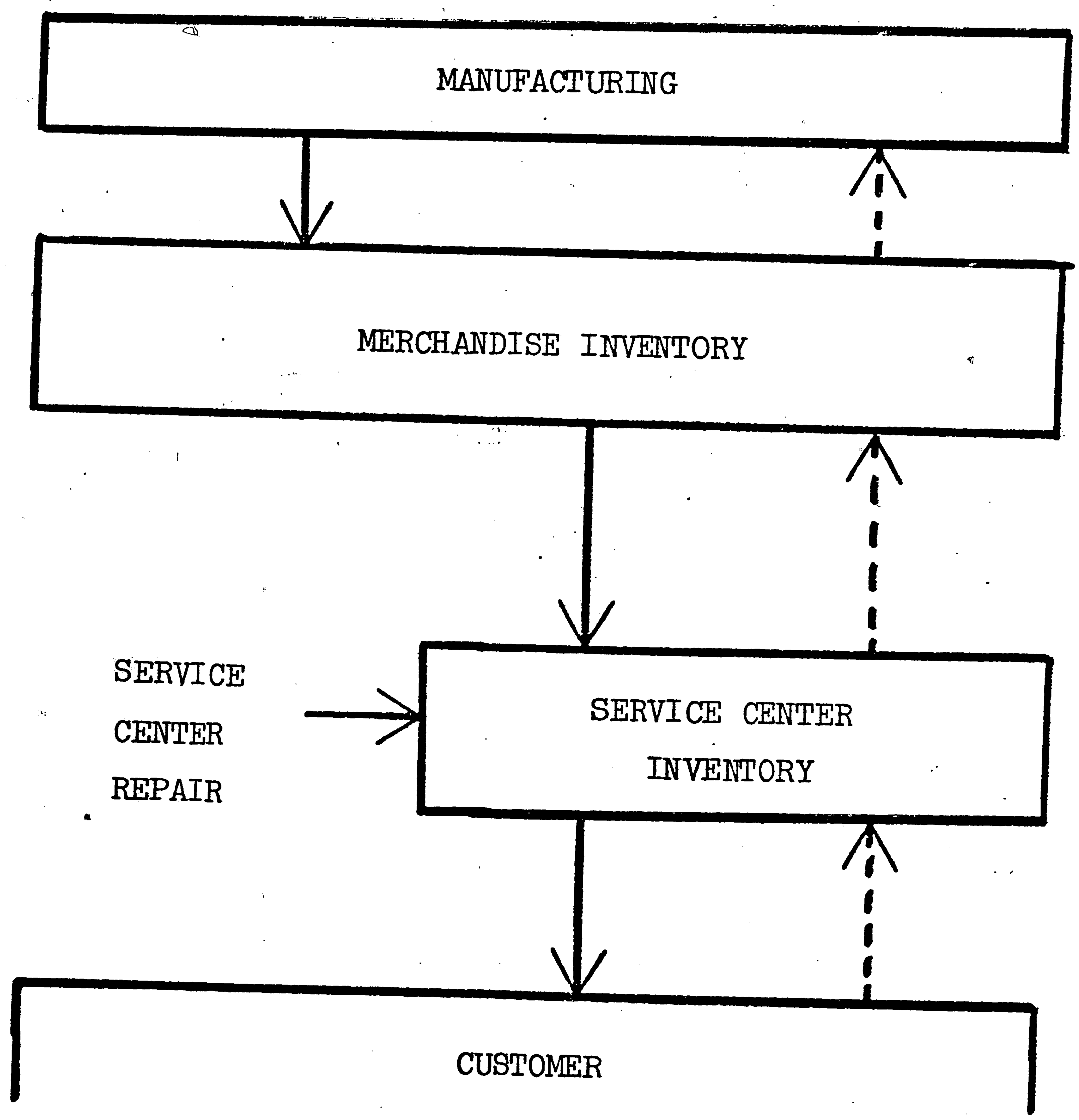
The purpose of this chapter is to explain the distribution system of the Service Centers (SC) and the Merchandise (MDSE) organization, on which the simulation model is developed. Figure 1 shows the two-echelon system boundary and flows of demand and merchandise. The dashed arrows show the flow of demands in the pipeline and the solid arrows show the flow of merchandise or stock. The first-echelon contains two sources of supply. They are:

(1) An item that has been repaired by the SC (repair function).

(2) New stock from MDSE.

2.1 SERVICE CENTER ORGANIZATION

Each week the SC receives orders from its customer, at which time they fill the demand from their inventory. If the SC does not have sufficient inventory in either repaired or new, the number of units short are back ordered and back order records are maintained. The SC loses no sale if it cannot supply an item demanded on time. It satisfies the demand when additional stock is received from either the repair shop or MDSE. Normally all orders on hand are filled from repair first whenever possible. The demand from the customer is only placed upon the SC and never at the MDSE echelon. When units fail or require repair of any kind the customer sends them back to the SC repair shop. After inspection the SC either scraps the unit or repairs it; thereby returning it to the system. The inventory level at the SC for the beginning of any week



TWO ECHELON INVENTORY SYSTEM

FIGURE 1

consists of both repaired stock from the repair shop and new stock from MDSE.

At the beginning of each week the repair shop determines a forecast of the number of units to be repaired. Given this forecast an order can be placed by the SC upon MDSE, once the desired level of inventory is known for the week. The SC uses a past moving average of the customer demand to estimate its average weekly demand. The past moving average is multiplied by some number of weeks of stock (WOS) to determine the weekly desired inventory level. After the desired inventory level and the forecasted shop repair are known, an order is placed upon MDSE. The order consists of this week's desired inventory level minus present inventory minus the forecasted repair. This assumes there are no back orders at either echelon. Several comments are in order here. If the SC has been back ordered from MDSE they add the number of units back ordered onto their present inventory level. Therefore, units desired in inventory are only accounted for once. If the SC has back ordered the customer, they add the amount of the back order onto their order to MDSE. The desired inventory level will not necessarily be the resulting SC inventory, since the order to MDSE contains the forecasted shop repair. By the end of a week their inventory may be higher or lower depending upon whether the actual repair was higher or lower respectively. The actual inventory equals the desired inventory adjusted by the difference between forecasted and actual repair. Westlake [7] has a detailed description of the repair shop scheduling and operation.

The distribution system contains at least thirty-five of these Service Centers each sending its orders to one MDSE organization. The functions performed by each individual SC are the same. However, repair shop scheduling and forecasting may vary slightly from one SC to another. Each SC maintains their investment level with the same decision rules. This thesis only considers a single-item for the system, whereas the total system contains thousands of line items.

Summarizing the SC operation they:

(1) Receive demand from the customer.

(2) Fill the demand from present inventory, consisting of repair and new stock. Priority is given to repair stock when filling orders.

(3) Back order the number of units not supplied.

(4) Determine a past moving average of customer demand.

(5) Determine desired inventory level based on past moving average and weeks of stock.

(6) Receive the forecasted repair rate from the shop.

(7) Determine the order to be placed upon MDSE.

(8) Determine actual inventory level based on stock received from MDSE and actual repair.

An important point to be considered is the level of investment maintained at the SC. To provide a certain level of service the level of investment must be sufficient to account for: (1) variation in demand, (2) variation in repair and (3) the integrity of MDSE.

2.2 MERCHANDISE ORGANIZATION

MDSE and manufacturing are two separate organizations having the primary objective to supply the pipeline with new merchandise. MDSE interfaces directly with the SC and maintains the finished stock to be sent to the SC. In process stock is considered to be manufacturing's inventory. After manufacturing and inspection is complete the finished stock becomes MDSE inventory. It is the purpose of MDSE to buffer the SC demand so that stable employment and production rates can be maintained in manufacturing. MDSE and SC use similar criteria to determine their inventory levels. MDSE desired inventory is expressed in terms of weeks of stock based on a future moving average of forecasted SC demand. The desired weeks of stock at the SC and MDSE are not necessarily the same number.

MDSE and manufacturing review the forecasted investment goals and determine a production plan to meet these goals. The investment level at MDSE varies considerably since it is their inventory that buffers the SC demand. Each week MDSE inventory equals last week's inventory minus SC demand plus this week's shop production. If MDSE inventory is not sufficient to fill SC demand the number of units short are back ordered. MDSE loses no sale if they cannot supply the demand on time. It satisfies the demand when additional units are received from manufacturing.

2.3 SCOPE AND LIMITATIONS

The quantitative description of such an inventory system at the microeconomic level would require many variables. With today's digital computers a large scale system can be modeled as a function of the major variables within the system. The Logistic Environment Model is a simulation model used to gain insight into the system's behavior and response to changes. The major variable of any inventory system is the investment maintained and, as a result, what level of service is provided to the customer. The investment in inventory is a cost and the level of service for this cost is the performance measure of the system. The opportunities for improvement in the system are large since inventory is the most maneuverable company asset. Inventory management is the prime role in integrating a Service and Manufacturing organization. For this reason the level of service provided by the system is investigated as a function of repair, customer demand and investment levels.

The Logistics Environment Model (LEM) considers all the Service Centers as one group. This assumes each SC operates independently so their net impact on the system is additive. Inherent in a model is the assumption that the inventory systems decision rules are always followed. LEM models the system on a weekly basis assuming the customer demand and SC repair are independent and normally distributed.

The investigation of the pipeline considers the impact on the system when the integrity of the second echelon is reduced. This is of great interest when considering the required investment for

MDSE to buffer SC demand. If MDSE level of service is reduced, and yet SC level of service is adequate, it is questionable as to what level of service MDSE should maintain.

A step function in SC repair or customer demand is considered to determine the impact on new stock investment within the supply pipeline. This analysis demonstrates how the system will attempt to respond as a result of a percentage change in either exogenous variable. These variables are regarded as acting upon the system but not being acted on by the system.

CHAPTER III

THE LOGISTICS ENVIRONMENT MODEL

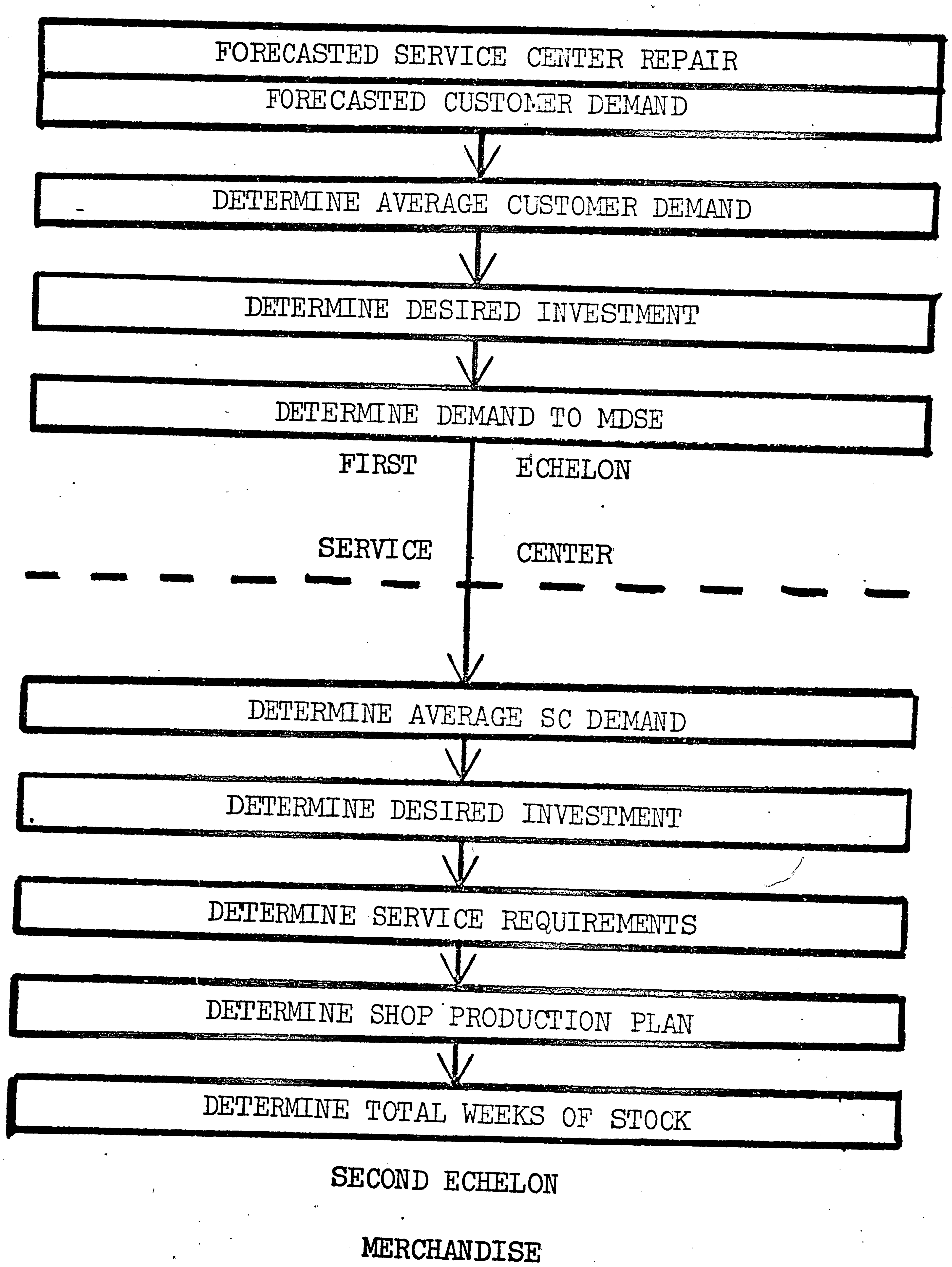
The purpose of this chapter is to explain the Logistics Environment Model (LEM). LEM is an interactive time-sharing model written in Fortran IV. The model is structured in such a way that, given the forecasted pipeline requirements, a production plan is determined. After the production plan is known the system can be tested against actual repair and demand data. LEM models both the flow of orders and merchandise within the pipeline maintaining records for echelon inventories, back orders, demands and shop production rates. The model consists of two major phases. They are: (1) planning phase; (2) transaction phase. The phases of LEM will each be described as a module. Each event within LEM is determined on a weekly basis.

3.1 PLANNING PHASE

The planning phase requires the forecasted weekly customer demand and forecasted SC repair. Given the forecasts, the model plans desired investment levels at both echelons as a function of desired weeks of stock (WOS), customer demand and repair. Once the pipeline requirements are known LEM determines a shop production plan to meet these requirements. Figure 2 is a block diagram showing the order of events within LEM for the planning phase.

3.1.1 FIRST ECHELON

The average customer demand $\overline{(\text{C DEM})}$ is a past moving average for some number of weeks. The length of time for this average is called



LEM PLANNING PHASE

FIGURE 2

"SC Length of Forecast" (SC LOF). Each week the past moving average is multiplied by the desired number of weeks of stock to be held in inventory (INV). Therefore, the desired SC INV at any week is:

$$\text{SC INV}(t) = \text{WOS} * \overline{\text{C DEM}(t)}. \tag{1}$$

To determine the SC demand (SC DEM) the forecasted repair rate, present desired inventory, last week's inventory and SC back orders (SC BO) are considered; that is,

$$\begin{aligned}\text{SC DEM}(t) = {} & \text{DESIRED SC INV}(t) - \text{SC INV}(t-1) \\ & - \text{FORECASTED SC REPAIR}(t) + \\ & \text{C DEM}(t) + \text{SC BO}(t).\end{aligned} \tag{2}$$

It is assumed here that all units demanded from MDSE will be supplied since the production plan will meet the forecasted requirements of the supply pipeline. Also, there are no negative demands to MDSE.

3.1.2 SECOND ECHELON

Knowing the forecasted SC DEM each week LEM determines a future moving average of this demand. The length of time for this average is called "MDSE Length of Forecast" (MDSE LOF). Each week the future moving average of SC DEM is multiplied by MDSE desired WOS. This investment is called "Service Investment" (SVC INV). It is the amount of inventory MDSE wants to have each week to buffer SC demand. Therefore, the desired SVC INV each week is

$$\text{SVC INV}(t) = \text{WOS} * \overline{\text{SC DEM}(t)}. \tag{3}$$

The model simulates the number of units MDSE would like to receive each week called "Service Requirements" (SVC RQTS). There actually are no orders between MDSE and manufacturing, but LEM uses SVC RQTS to base a production plan on and at the same time meet MDSE planned investment levels. To determine these requirements this week's desired inventory, last week's inventory and SC demand are considered; that is,

$$\text{SVC RQTS}(t) = \text{DESIRED SVC INV}(t) - \text{SVC INV}(t-1) + \text{SC DEM}(t). \tag{4}$$

After the SVC RQTS are known LEM determines the production plan for the length of run (LOR). The LOR is the number of weeks LEM is planning. In determining a production plan criterion can be given to LEM in terms of WOS called "Criterion for Dip." This criterion means MDSE is willing to allow their inventory to drop below that determined in equation (3) at any week during the run. For example, if MDSE WOS were three and the criterion for dip was one week, then LEM would allow MDSE inventory to drop to two WOS while planning for production rates. Another criterion LEM requires is called "Length of Subhorizon". It is this length of time in weeks LEM must maintain the same production rate. Given the above criteria and service requirements, LEM searches each week for the peak demand, using the average of SVC RQTS. The averages consist of week one, then weeks one and two, then weeks one, two and three, etc., until the length of run. After determining the weekly average of service requirements, LEM determines the shop production rates so that:

(1) SVC RQTS are filled.

(2) Load leveling for manufacturing is provided.

(3) Criterion for dip is not violated.

(4) Length of subhorizon is met.

(5) Inventory at the end of run equals the desired WOS for MDSE.

Excess inventory during the planning phase is placed into load leveling inventory. It is this inventory that must be held by MDSE in order for the peak demand to be met and to maintain a stable production rate. Therefore, during the planning phase load leveling plus MDSE inventory equals the total finished merchandise in any week. After the production has been mapped out LEM determines the total investment both load leveling and MDSE INV in terms of weeks of stock. This measurement of investment is used during the transaction phase to determine if replanning is required and also when the shop should work overtime.

3.2 TRANSACTION PHASE

The transaction phase requires the actual weekly customer demand, SC repair and shop production rates. The shop production rates each week are those determined in the planning phase of LEM. This phase of the model determines the actual weekly orders and inventories within the system. Each event is determined for each echelon before continuing to the next week. This phase determines the response of the system to actual data after a production plan is implemented.

3.2.1 FIRST ECHELON

At the service center level the forecasted and actual repair rates are required to determine SC INV each week. Since the SC bases its demand on a forecasted repair rate, a week's ending inventory may not be the same as that which is desired. The only time SC INV equals the desired INV is when the forecasted and actual repair are equal and there are no back orders. This assumes, of course, MDSE has not back ordered the SC. The transaction phase uses a past moving average of C DEM to determine desired inventory, as in the planning phase. However, this average is now actual C DEM instead of forecasted. This average ($\overline{\text{C DEM}}$) is multiplied by SC WOS to determine the desired inventory. Therefore, the SC INV at any time (t) is

$$\text{SC INV}(t) = (\text{WOS}) \cdot \overline{(\text{C DEM}(t))} + \text{SC ACTUAL REPAIR}(t) - \text{SC FORECASTED REPAIR}(t). \qquad (5)$$

Equation (5) assumes MDSE and the SC have not back ordered. If back orders are to be considered, we have:

$$\begin{aligned} \text{SC INV}(t) = (\text{WOS}) \cdot \overline{(\text{C DEM}(t))} &+ \text{SC ACTUAL REPAIR}(t) \qquad (6) \\ &- \text{SC FORECASTED REPAIR}(t) \\ &- \text{MDSE BACK ORDER}(t) \\ &+ \text{SC BACK ORDER}(t) \end{aligned}$$

If the SC back orders the customer any week, the number of units back ordered is added to SC INV, assuming MDSE has filled the order. The SC INV will never fall below a week's actual SC repair plus new stock received from MDSE in the case where the SC is back ordering every week. The actual timing of events in LEM will be discussed after the decision rules have been given.

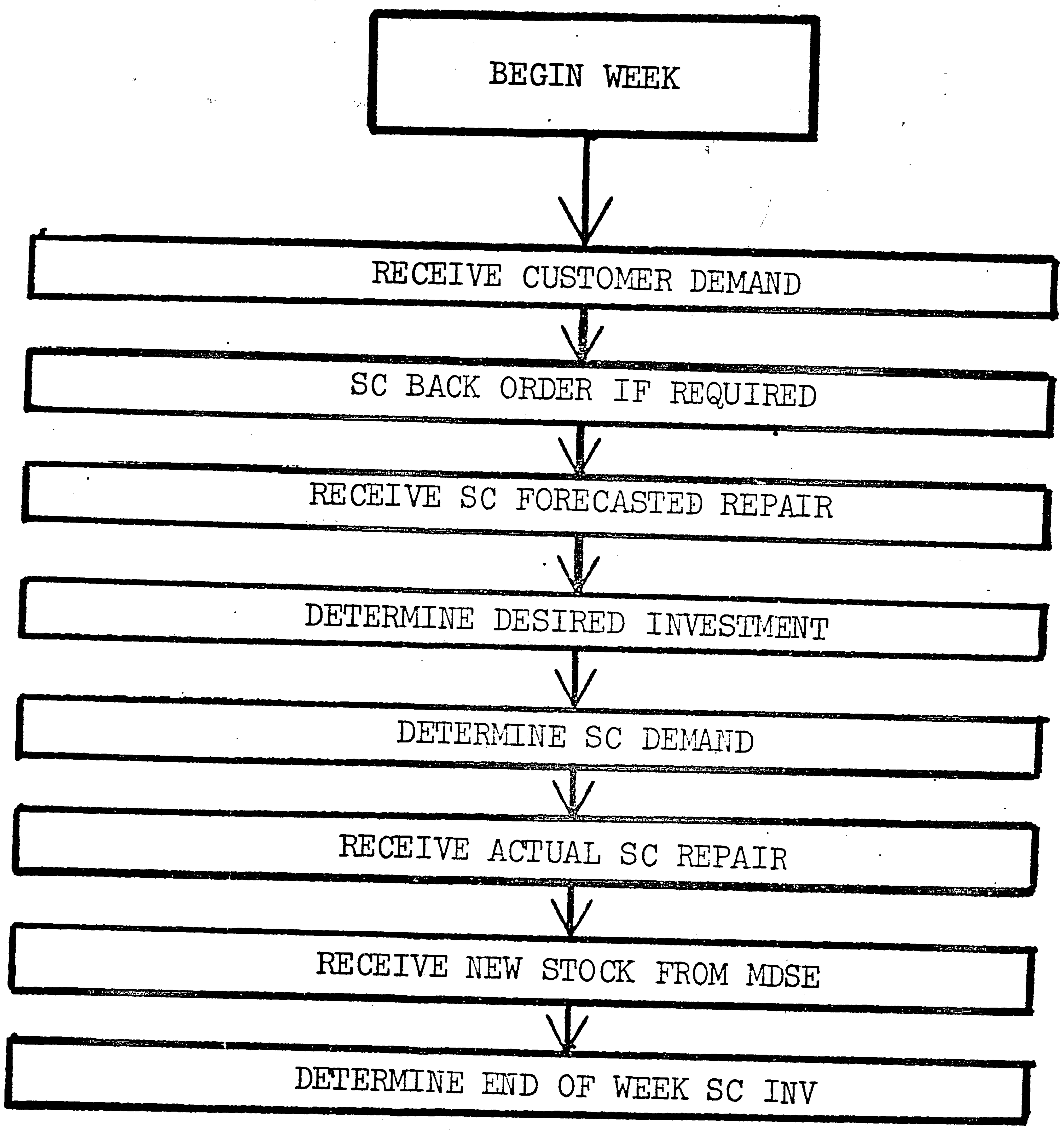
The SC demand each week is based on last week's ending inventory, SC back orders, MDSE back orders, this week's desired inventory, forecasted SC repair and customer demand. SC demand will be given here and is developed in Appendix A. Writing the SC DEM at any time (t), we have:

$$\begin{aligned} \text{SC DEM}(t) = \frac{(\text{WOS})}{(\text{LOF})} * (\text{C DEM}(t) - \text{C DEM}(t-\text{LOF})) \\ - \text{SC ACTUAL REPAIR } (t-1) \\ + \text{SC FORECASTED REPAIR } (t-1) \\ + \text{C DEM}(t) - \text{SC FORECASTED REPAIR } (t) \\ - \text{MDSE BACK ORDER } (t-1) \\ + \text{SC BACK ORDER } (t), \end{aligned} \tag{7}$$

where:

$(\frac{\text{WOS}}{\text{LOF}}) * (\text{C DEM}(t) - \text{C DEM}(t-\text{LOF}))$ = the inventory adjustment term and LOF is the length of SC forecast,

$-\text{SC ACTUAL REPAIR } (t-1) + \text{SC FORECASTED REPAIR } (t-1)$ = last week's error in forecasted SC repair.



FIRST ECHELON LEM EVENTS

FIGURE 3

Note from equation (7), the SC demand is a function of the forecasted SC repair and the accuracy of these forecasts. If the forecasted repair is in error the difference is not seen in demand until the next week. This is a result of the events within LEM.

Figure 3 is a diagram showing the timing within LEM. All events are determined on a weekly basis. At the beginning of the week an order is received from the customer. At that time the order is either filled completely from last week's ending inventory, or the number of units short are back ordered. Units demanded are not filled on a "total" order basis; that is, whatever units are available will be sent to the customer. The entire order is not held until sufficient units are obtained to fill the order. After the customer demand is filled and back order records are updated this week's forecasted shop repair is required. The desired investment is determined using the past moving average of customer demand times SC weeks of stock. The SC can now determine its demand to MDSE as in equation (7). At the end of the week closing inventory is determined as in equation (6). LEM does not account for the incoming repair shop inventory or the new stock inventory until the end of the week. At this time LEM continues to the next week. If the SC has back ordered the customer the number of units back ordered are added to customer demand.

3.2.2 SECOND ECHELON

The shop production rates used in this thesis will be those determined from the planning phase of LEM. If different production rates are desired they are given to LEM, in "shop lead time". The shop lead time is the number of weeks shop production rates are specified other than those determined from the planning phase. There are five different variables within LEM used for control in manufacturing, these are:

(1) Annual Overtime Percent of Production Plan.

(2) Weekly Overtime Minimum.

(3) Weekly Overtime Maximum.

(4) Criterion for Increasing Production.

(5) Criterion for Decreasing Production.

The annual overtime percent of production plan is the maximum yearly overtime percent the shop may work without replanning. At any week during the year if the percent of overtime exceeds the value given, LEM will go back to the forecasted requirements and determine a new production plan. Variables (2) and (3) set the amount of overtime allowed for any week. If MDSE INV is below that planned and the shop can work overtime without exceeding the planned INV, LEM will increase the production rate by either the maximum or minimum weekly overtime rate. For example, if LEM had planned for a production rate of twenty-five units per week and

(1) Annual Overtime Percent = 20%;

(2) Weekly Overtime Minimum = 20%;

(3) Weekly Overtime Maximum = 40%.

The transaction phase would start with a production rate of twenty units called "regular production". When MDSE INV falls below that planned the production rate can be increased to either twenty-four units or twenty-eight units. The regular production rate without overtime is equal to the planned minus planned times annual overtime percent. In order to determine the production rate to be used, LEM considers total planned INV (SVC INV + LOAD LEVELING INV) minus "mid-week inventory". Mid-week inventory is equal to last week's closing INV minus this week's SC demand minus last week's MDSE back orders; that is,

$$\text{MID-WEEK INV }(t) = \text{MDSE INV}(t-1) - \text{SC DEMAND }(t) - \text{MDSE B.O. }(t-1). \qquad (8)$$

LEM then takes the difference between total planned INV and mid-week INV and determines the week's shop production rate according to the following criteria. If the difference is:

(1) less than the weekly minimum overtime production rate, the shop production equals regular production.

(2) greater than or equal to the weekly minimum overtime production rate, consider the maximum overtime production rate.

(3) less than the maximum overtime rate, the shop will work the minimum overtime.

(4) greater than or equal to the maximum production rate, the shop will work the maximum overtime.

The criteria for increasing or decreasing production are in terms of weeks of stock, and used for a replanning phase. Each week

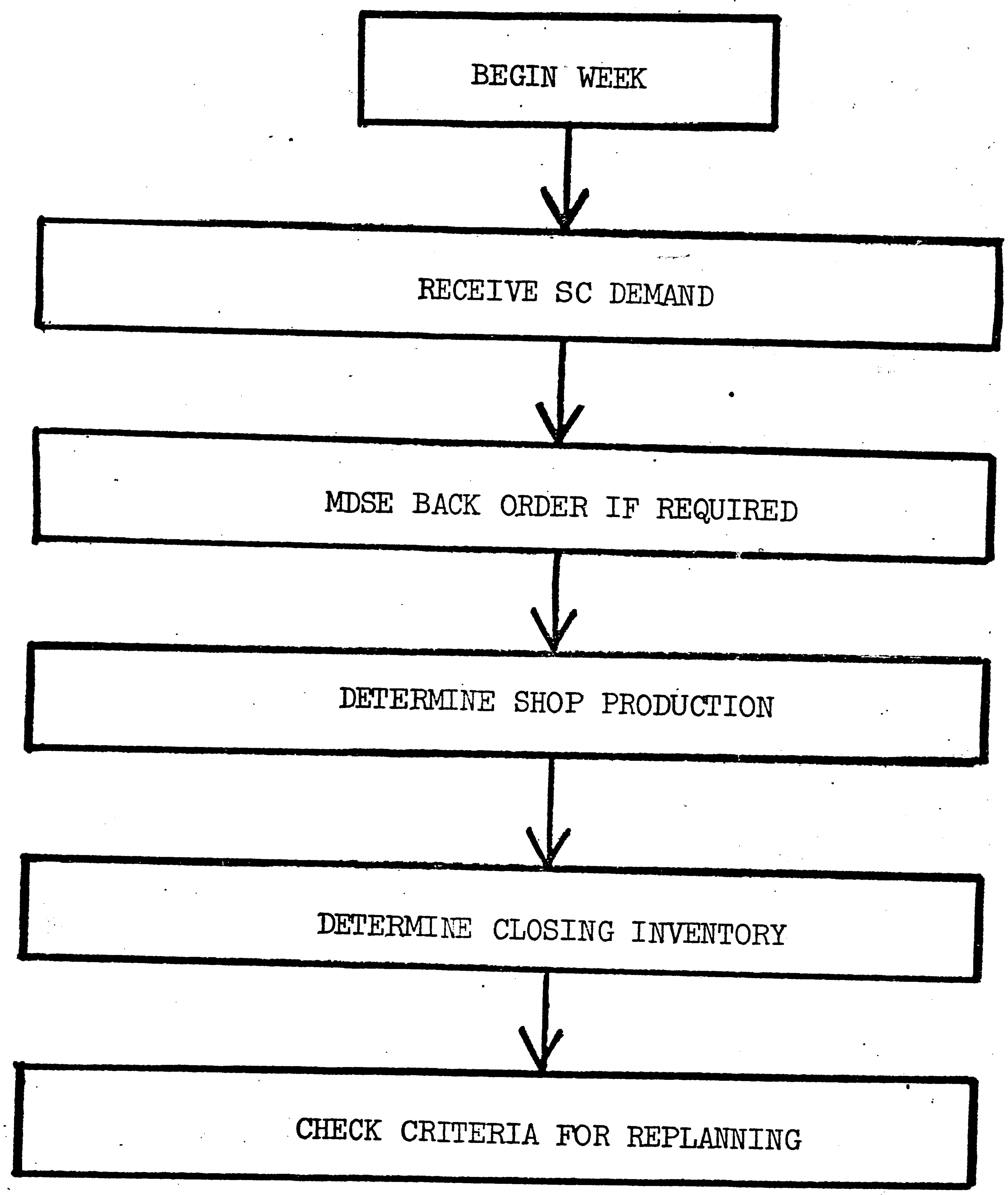
LEM compares the planned WOS with the actual MDSE WOS in the transaction phase. If either criterion is met LEM replans accordingly. For example, if MDSE presently had three WOS and the planned was five WOS, if criterion for increasing production was minus two WOS LEM would replan. Any time replanning is triggered LEM returns to the forecasted requirements and determines a new production plan for the remainder of the run as discussed under planning phase. Using the above five variables LEM can account for minor forecast errors with overtime or major forecast errors with replanning.

MDSE inventory each week equals last week's inventory minus SC demand plus shop production this week, assuming there are no back orders; that is,

$$\text{MDSE INV}(t) = \text{MDSE INV}(t-1) - \text{SC DEM}(t) + \text{SHOP PRODUCTION}(t). \quad (9)$$

Due to accounting procedures of LEM, MDSE INV will not fall below one week's production.

Figure 4 is a diagram showing the timing of events for MDSE and the shop production within LEM. At the beginning of the week an order is received from the SC. At that time the order is either filled completely from last week's ending inventory, or the number of units short are back ordered. Units demanded not filled are sent to the SC some future week when additional inventory is received from the shop. After the SC demand is filled and back order records are updated, this week's shop production rate is determined as



SECOND ECHELON LEM EVENTS

FIGURE 4

previously described. At the end of the week LEM determines MDSE closing INV. The closing INV is then divided by the forecasted SC demand to determine their inventory position in terms of weeks of stock. This measurement is used to compare with the planned weeks of stock to determine if replanning is required. If replanning is not required LEM continues to the next week.

CHAPTER IV

SIMULATION RESULTS AND ANALYSIS

The purpose of this chapter is to explain the simulation experiments, results and analysis. In simulation the attempt is made to use an abstraction of the real system for the purpose of prediction and the investigation of goals, policies and decision rules for inventory management. Since one objective is to determine the average long run amplification of levels of service as a function of the investment, the stationary behavior of the system is studied. Stationary analysis considered stochastic inputs, each with constant mean and variance. The SC forecasted as well as actual repair and customer demand are assumed to be stochastic inputs. They are distributed as normal random variables.

4.1 SIMULATION EXPERIMENTS

The major assumptions in the simulation experiments are:

(1) Customer demand is stochastic, stationary and normally distributed.

(2) Service Center repair is stochastic, stationary and normally distributed.

(3) The decision rules of the Logistics Environment Model.

(4) Service Center repair actual and forecasted are identically distributed.

Subrouting GAUSS is a subroutine given in the System/360 Scientific Subroutine Package. This subroutine was used in all simulation runs to generate normally distributed random repair and customer demand, each week, with a given mean and variance. The distribution of forecasted and actual SC repair are assumed to be identical. This was accomplished by using the same "seed" for each variable with GAUSS.

The length of each run was one-hundred seventeen weeks. The statistical counters were not started within LEM until the fourteenth week; thus all statistics within this thesis are based on one-hundred four weeks, or two years. Pilot runs were made to decide the best length of runs. In each pilot run the repair rate was constant and sufficient inventory from MDSE was available. The criterion used to judge the best length of run was the variation in the service center's level of service. The reason for sufficient inventory at MDSE for the pilot runs was to make the level of service provided to the customer only a function of the demand itself. Since the repair rates were constant the level of service was also independent of repair. The only effect repair has in this case is to shift the level of demand to MDSE. Several runs were made and averaged using the same "seeds" for GAUSS. In each pilot run the service center variables were:

(1) WOS = 1.3 weeks.

(2) Length of Forecast = 13 weeks.

(3) Customer Demand = 100 $\pm$ 20.

(4) SC Repair = 75.

Table 1 contains the results of the pilot runs.

The level of service in Table 1 is one minus the sum of the number of times the service center back ordered the customer divided by the

LENGTH OF RUN	52	104	104	208
LEVELS OF SERVICE IN PERCENT	92.3 97.4 100.0 89.7 94.9	93.0 92.0 98.0 93.0 91.0	92.3 98.1 91.3 93.3 90.3 92.3 95.2 88.4 91.3 93.3 92.3	91.8 94.4 95.9 90.8 90.3 93.3
GRAND MEANS	94.8%	93.4%	92.6%	92.6%
STANDARD DEVIATIONS	4.1%	2.7%	2.5%	2.4%

PILOT RUN RESULTS

TABLE 1

number of orders; that is,

$$\text{LEVEL OF SERVICE} = 1 - \frac{\sum^{\text{LOR}} \text{NO. BACK ORDERED}}{\text{NO. ORDERS}}. \qquad (1)$$

This measure was used for the pilot runs so the level of service could be compared with that predicted from probability theory and also it tends to have a larger variance than level of service based on the number of units. Stocking 1.3 weeks of stock and the true mean demand is one hundred with a standard deviation of twenty; the probability of not having a back order is 93.32%. As seen from Table 1, the grand mean percent level of service for all pilot runs is within about one percent of that predicted. Since the standard deviation of levels of service did not change significantly, and considering the computer requirements from two to four years, the length of run was left at two years.

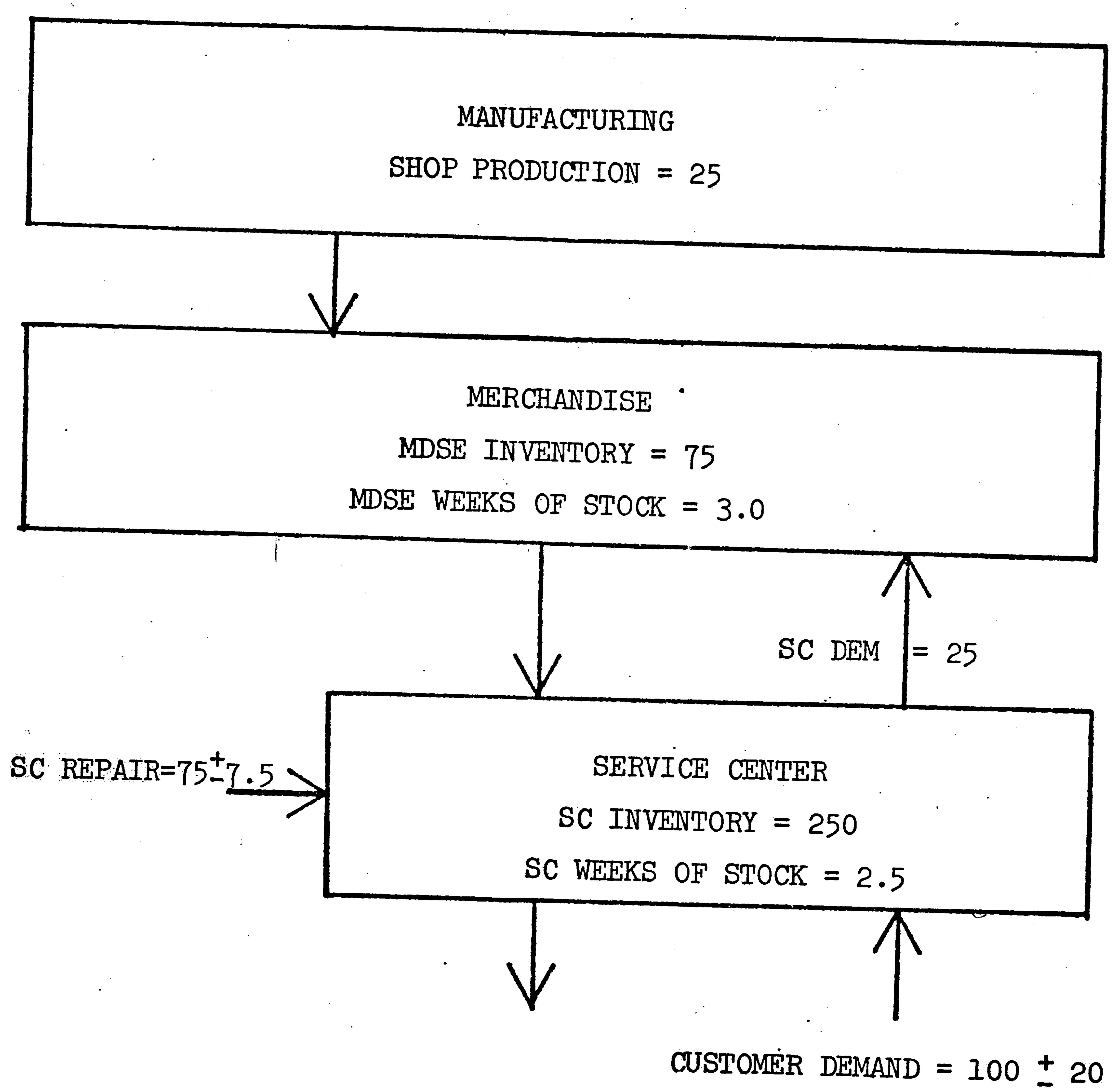
After the length of run was determined a different measurement of level of service was used. This new performance measure is used throughout the remainder of this thesis. The new service measure is used for both echelons within the supply pipeline. The level of service is one minus the sum of the number of units back ordered divided by the total number of units demanded; that is,

$$\text{LEVEL OF SERVICE} = 1 - \frac{\sum^{\text{LOR}} \text{NO. UNITS BACK ORDERED}}{\sum^{\text{LOR}} \text{NO. UNITS DEMANDED}} \qquad (2)$$

At the beginning of each week the demand is compared with last week's ending inventory. If the demand is greater, the difference is added to the sum of back ordered units. This measure of service gives the response in terms of units back ordered instead of the number of back orders. When evaluating the response of the pipeline we will be more concerned with the number of units back ordered in the long run, for a specified investment level.

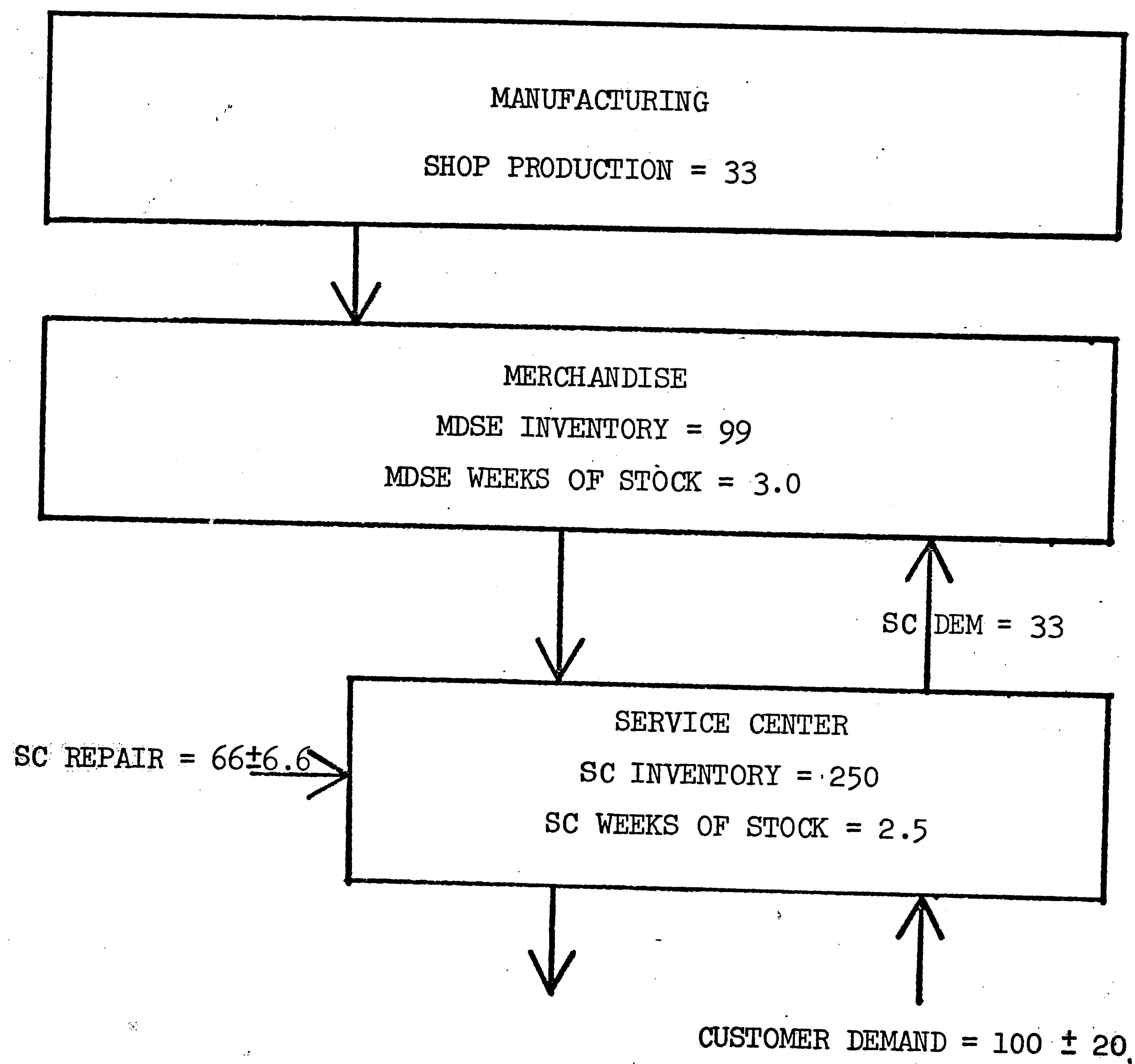
After the length of SC forecast LEM will be at steady-state independent of the initial conditions. This assumes the demand and repair inputs are stationary and the means of the time-series are approximately the same as in the planning phase. Figures five, six and seven show the steady-state conditions for the inventory system with the repair to new (R/N) ratios equal to three, two and one respectively. The repair/new ratio is the average SC shop repair divided by the average SC demand. This ratio will be referred to as the repair function or program. It may be that ratio of average weekly customer demand that is filled from repair. This is not to imply the ratio is constant from week to week but the long run average ratio is equal to R/N.

The true population mean of customer demand is one hundred with a standard deviation of twenty for all simulation runs. The actual mean and standard deviation of the time-series will differ from run to run, depending on the "seed" used for GAUSS. The SC repair true population mean and standard deviation vary from one figure to another depending on the repair/new ratio. In all simulation runs the standard deviation of SC repair equals ten percent of the mean.



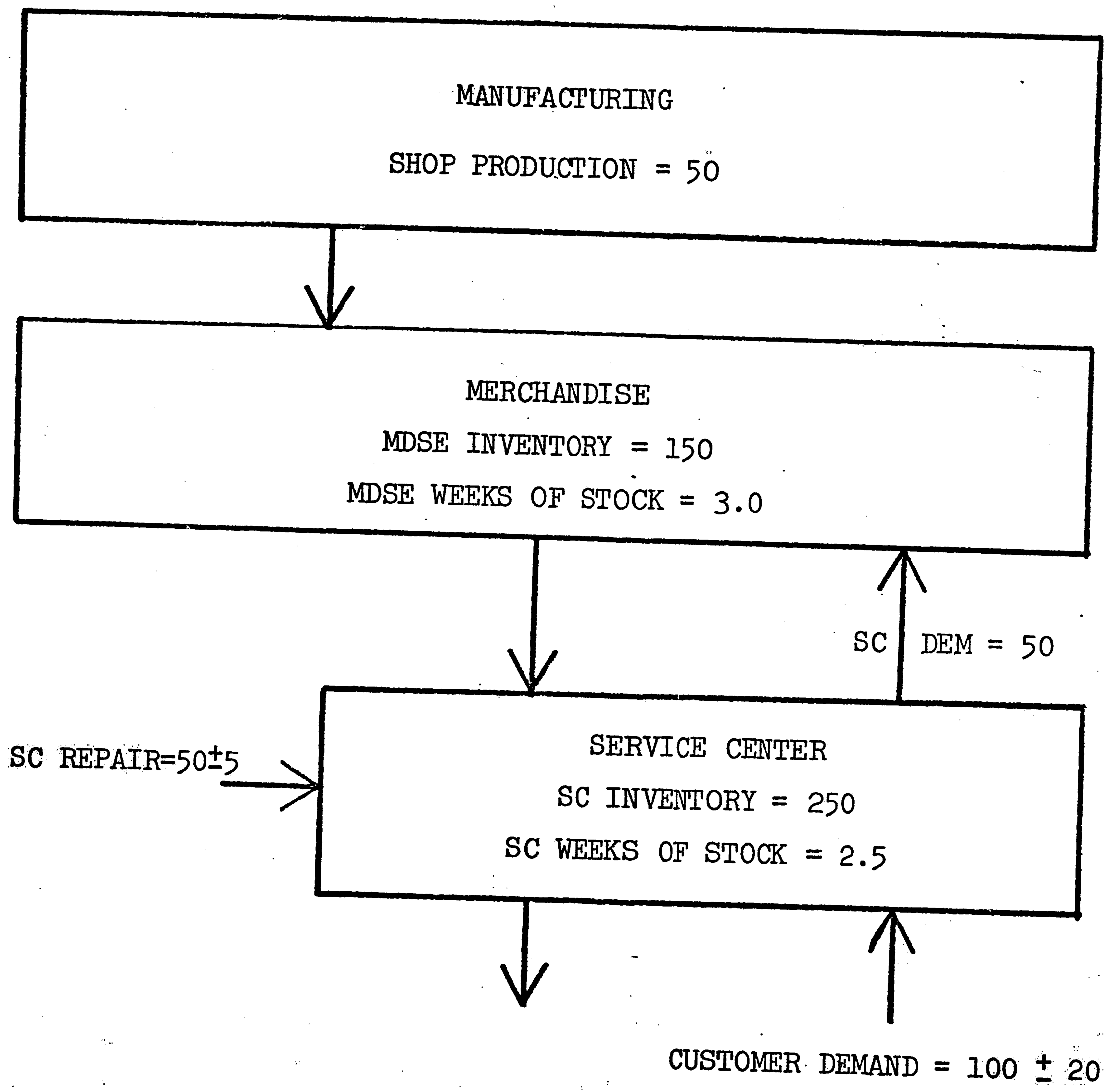
STEADY-STATE CONDITIONS

FIGURE 5



STEADY-STATE CONDITIONS

FIGURE 6



STEADY-STATE CONDITIONS

FIGURE 7

The SC INV is 250 units since 2.5 weeks of stock are desired. MDSE inventory and manufacturing shop production rate vary depending on the level of SC demand. These three figures demonstrate how LEM plans for the three different repair-new ratios under investigation. In the planning phase of LEM there is no variance in the forecasted SC repair or customer demand; thus the planned manufacturing shop production rates are 25, 33 and 50, for repair/new ratios of three, two and one respectively. The actual production rates are set as discussed in Chapter III. The variables used for all simulation runs are:

(1) Length of Run = 117 weeks.

(2) Length of SC Forecast = 13 weeks.

(3) Length of MDSE Forecast = 13 weeks.

(4) Length of Subhorizon = 13 weeks.

(5) Annual Overtime Percent of Production Plan = 20%.

(6) Weekly Overtime Minimum = 20%.

(7) Weekly Overtime Maximum = 40%.

(8) Criterion for Increasing Production = -12 WOS.

(9) Criterion for Decreasing Production = +12 WOS.

Since the mean customer demand will be approximately equal to the planned, overtime was used to account for minor shifts in demand and repair. Also the criteria for replanning (variables 8 and 9) was set for a large number of weeks of stock, so LEM would not replan every time there was a shift in demand or repair. The reason weeks of stock for the SC and MDSE are not listed above is because these are the controllable variables in the experiments. The level of service provided at each echelon is the response variable, for a given desired investment level (weeks of stock).

For each run the means and standard deviations were determined for each echelon in the following manner.

$$\text{MEAN} = \frac{\sum^{N} X(t)}{N}, \qquad (3)$$

and

$$\text{STANDARD DEVIATION} = \frac{\sum^{N} (X(t) - \overline{X})^2}{N - 1}, \qquad (4)$$

where:

N is the number of weeks considered.

To determine the actual average weeks of stock maintained to provide a level of service, the average inventory was divided by the average demand at each echelon; that is,

$$\text{SC ACTUAL WOS} = \frac{\sum^{N} \text{SC INV}(t)}{\sum^{N} \text{C DEM}(t)}, \qquad (5)$$

and

$$\text{MDSE ACTUAL WOS} = \frac{\sum^{N} \text{MDSE INV}(t)}{\sum^{N} \text{SC DEM}(t)}, \qquad (6)$$

where:

N is the number of weeks considered.

In the simulation results the level of service performance measure (equation (2)) has been plotted versus the actual weeks of stock maintained at each echelon. Equations (5) and (6) are necessary since, as the service integrity of MDSE is reduced, the actual inventory maintained at the SC may no longer be equal to that desired from their decision rule. For example, if the SC wanted to stock 2.5 weeks of stock and MDSE was only providing a service level of 65%, the SC can only provide approximately 90% (one WOS) even though they desire to stock 2.5 weeks of stock. This will be discussed and demonstrated quantitatively in detail under simulation results. Equations (5) and (6) will also account for shifts in the means from GAUSS. If the mean customer demand from GAUSS is lower than one hundred, the actual weeks of stock maintained should be greater for a fixed inventory level. For example, if the SC maintains approximately 250 units and the mean customer demand is 100 they have maintained 2.5 weeks of stock. If the mean demand was 96 the SC has maintained 2.6 weeks of stock. The same analysis applies to the SC and MDSE interface.

4.2 SIMULATION RESULTS

The simulation results are divided into two sections. These are:

(1) Level of Service versus Weeks of Stock.

(2) MDSE Level of Service versus SC Level of Service.

The level of service versus weeks of stock section determines the service level for MDSE and the SC as a function of the repair/new ratio and investment level. The section demonstrates the stability of each echelon for specified investment levels. The second section demonstrates the amplification of levels of service within the pipeline as a function of the repair function and SC investment level.

4.2.1 LEVEL OF SERVICE VERSUS WEEKS OF STOCK

Figures eight, nine and ten are plots of levels of service versus weeks of stock maintained for both MDSE and SC echelons, with repair/new ratios equal to three, two and one respectively. The results have not been averaged since it is believed the variation from run to run as a result of using different "seeds" gives an insight into the characteristics of the system. The plots show the stability of such a system for a given investment level. Stability of the system or echelon means, given an investment level, what confidence we can have in providing a level of service to the customer. In considering MDSE we are concerned with the level of service provided to the SC. Note the stability of the SC for a predetermined number of WOS compared to MDSE. If the SC stocks 1.5 to 2.0 weeks of stock it is almost sure of providing at least 98%, independent of the repair function. This is not the case for MDSE. If MDSE stocks 4 WOS it may provide a considerably different level of service depending upon the repair function at the SC. Of course the measurement of investment in terms of weeks of stock varies from one figure to another. Four weeks of stock for a repair/new ratio of three is approximately 100 units. Whereas four weeks of stock for a repair/new ratio of one is approximately 200 units.

From the figures it is seen that stocking an average of 200 units per week almost assures at least a 99% service level to the SC. From this investigation it is questionable as to whether MDSE should

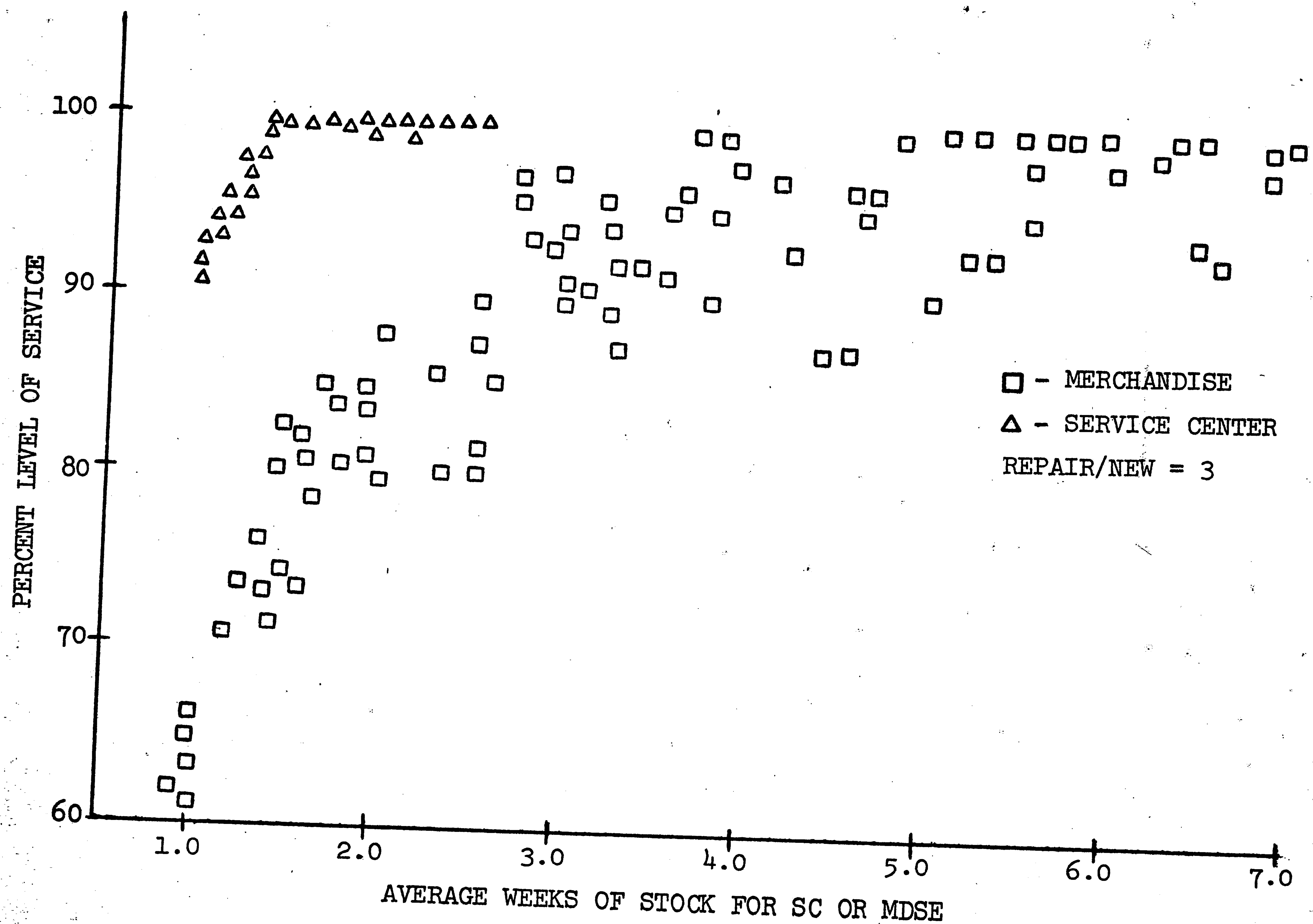


FIGURE 8 SERVICE-INVESTMENT CHARACTERISTICS

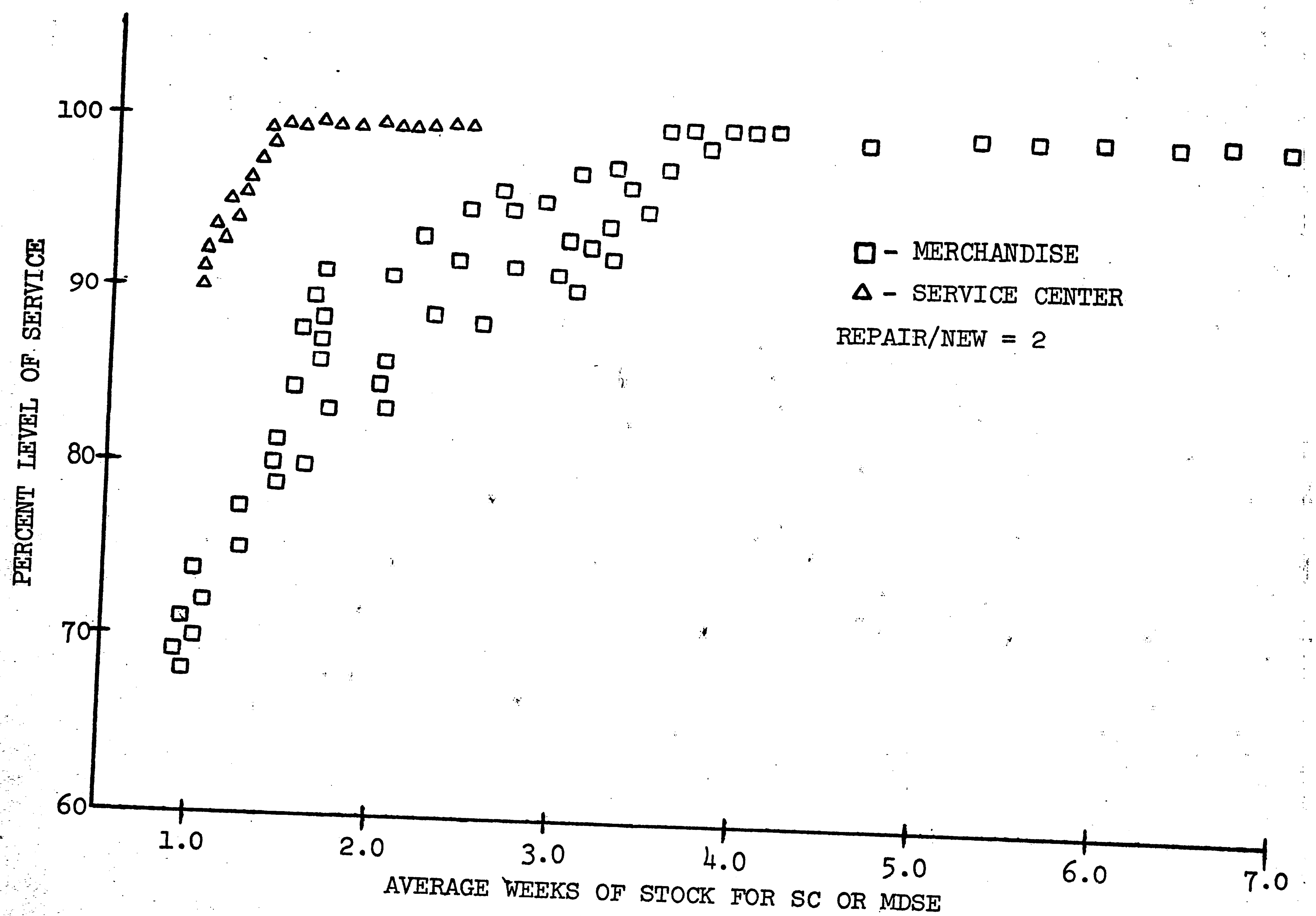


FIGURE 9 SERVICE-INVESTMENT CHARACTERISTICS

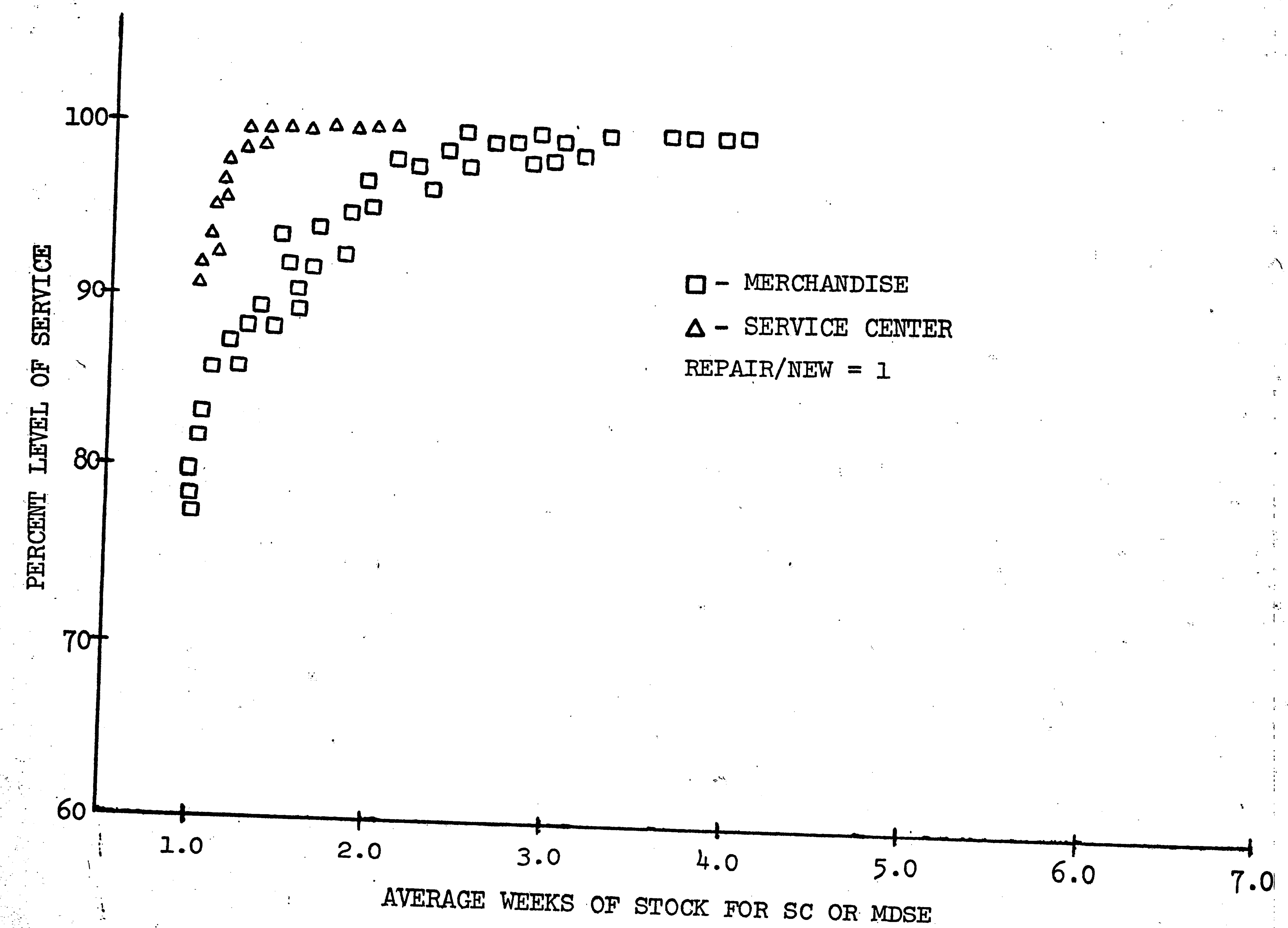


FIGURE 10 SERVICE-INVESTMENT CHARACTERISTICS

provide such a high service level. When MDSE back orders the SC there isn't necessarily any impact on the customer. As long as the SC can maintain a reasonable investment level, as the service integrity of MDSE is reduced, there is no loss of service to the customer.

For the SC to reduce its investment level below 1.5 weeks of stock moves their operating point toward the knee of the curve. This would be undesirable from a service viewpoint. This investment level (1.5 weeks of stock) may seem to be large; however, the SC must buffer the variations in customer demand as well as repair. The curves for the SC tend to shift slightly to the left as the repair/new ratio is reduced. This is a result of the reduced variance in SC repair. As the SC repair variance decreases the SC can provide a better level of service for the same average investment level. The decreased variance in SC repair results in a more stable inventory.

The stability of the SC compared to that of MDSE can be explained by analyzing the demand data. Appendix B contains data chosen at random from the simulation runs for different SC investment levels and repair/new ratios. These data show the means and standard deviations of C DEM and SC repair from GAUSS using different "seeds" and also the SC demand determined from the decision rules in LEM. Several comments are in order as to the characteristics of this inventory system. In all runs the coefficient of variation for SC demand is greater than the coefficient of variation for C DEM. This is a common characteristic of most multi-echelon systems. As the R/N ratio decreases the coefficient of variation of SC demand decreases. The coefficient of variation is the ratio of the standard deviation to its mean. Changing the R/N ratio shifts the means of SC demand but the standard deviations remain reasonably constant. This implies that as the SC demand mean increases the tail on the distribution of demand decreases; as a result MDSE can provide a more stable level of service to the SC with a fixed investment level. Also, as the SC WOS decision variable is increased there is no significant change in the standard deviation of their demand.

When the R/N ratio is three, the mean and standard deviation of SC demand are of the same order of magnitude. As seen in Figure eight, the variation in level of service (stability) provided for a fixed investment level is considerable. From Figure nine to Figure ten the increased stability of MDSE can be seen compared to Figure seven. This does not imply that a lower repair/new ratio is desirable for the best operation of the pipeline. As the repair/new ratio is decreased the SC becomes more dependent upon the service integrity of MDSE, as discussed in the next section. Also, new pipeline investment increases. Note the knee of the curves for the MDSE echelon are easier to specify as the repair/new ratio is decreased. Again, this is a result of the decreased tail in the distribution of SC demand.

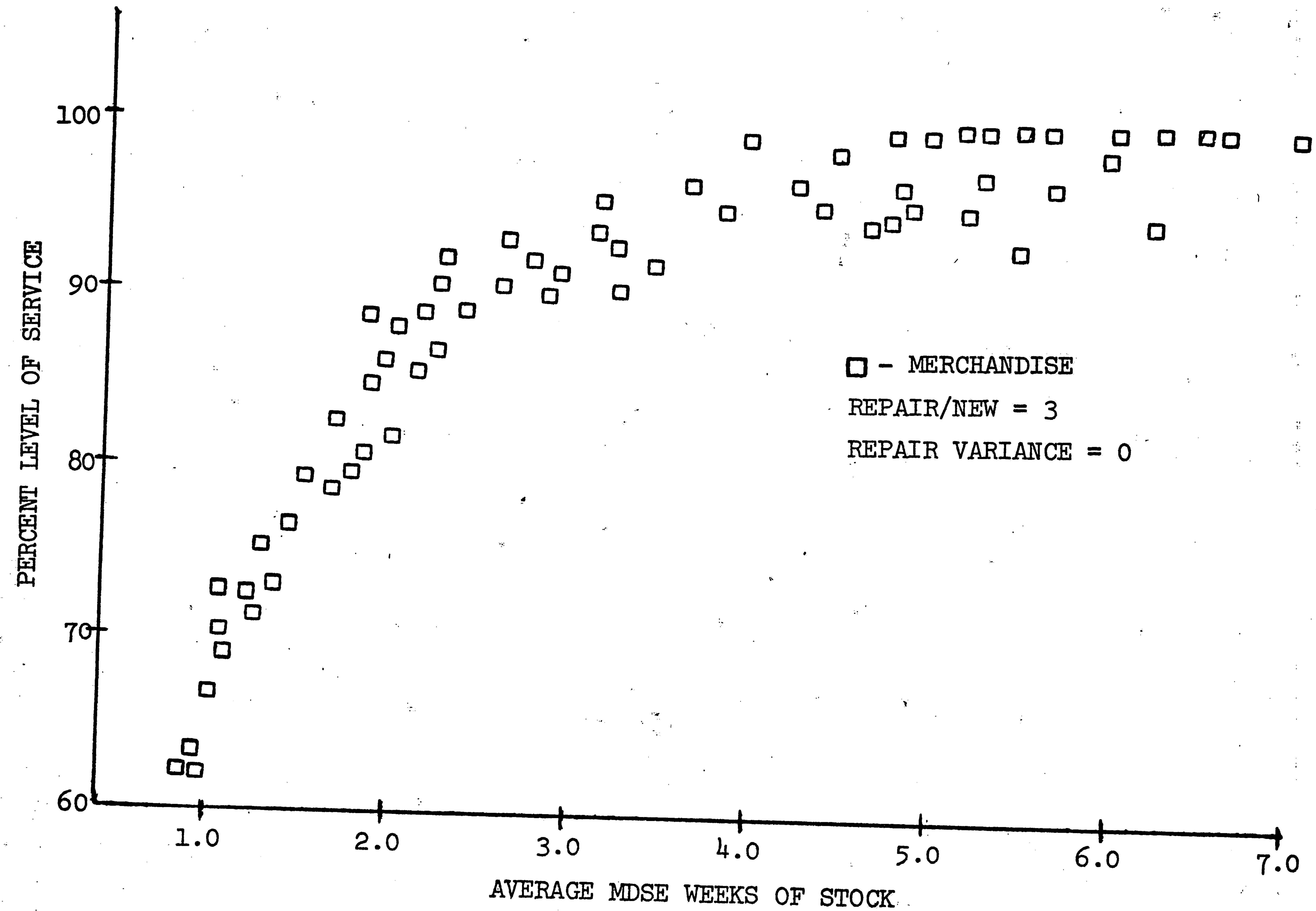


FIGURE 11 SERVICE-INVESTMENT CHARACTERISTICS

In order to consider the operation of the pipeline under ideal conditions, simulation runs were made with a constant SC repair rate. Figure 11 is a plot of the weeks of stock versus level of service for MDSE, with the repair/new ratio equal to three and SC repair rate a constant 75 units per week. Figure 11 demonstrates the random variation at the MDSE echelon due to decision rules and customer demand alone. Note the band of level of service for a fixed investment level has decreased somewhat, as a result of the SC repair rate being constant, thus resulting in increased stability. For example, if MDSE wanted to provide at least 90% service to the SC they would stock approximately five weeks of stock (from Figure eight). This assumes repair/new ratio equal to three and the SC repair standard deviation is about 10% of the mean. Whereas from Figure 11 MDSE could provide at least 90% level of service if they stocked three weeks of stock. Both cases are based on the customer demand standard deviation being 20% of the mean. The reason MDSE can provide the same level of service with less inventory is the reduced variance in SC demand. The component in SC demand each week for SC repair is constant for Figure 11, thus their demand variance is less. Since the SC demand has less variance MDSE can provide a more stable level of service for a given investment level.

4.2.2 AMPLIFICATION OF LEVELS OF SERVICE

The purpose of this section is to demonstrate the amplification of levels of service down the pipeline, as a function of the repair/new ratio and investment level at the SC. As in the previous section the results have not been averaged since it is believed the random variation from run to run gives an insight into the characteristics of the system. Figures 12, 13 and 14 are plots of the level of service for MDSE versus the level of service for SC with repair/new ratios equal to three, two and one respectively. On each graph there are two plots. One plot is for the desired SC investment level at 2.5 WOS and the other for 1.5 WOS. As MDSE level of service decreases the two curves would eventually meet, since the SC would only receive a certain amount of inventory, regardless of what they desired. For each plot the SC started the run with its desired investment level (1.5 WOS or 2.5 WOS). MDSE weeks of stock were set at various values to obtain the different levels of service. As mentioned previously,it is questionable as to what level of service MDSE should provide. From Figure 12 we see MDSE level of service can be reduced to 65% and yet the SC will still be providing at least 90% service to the customer (SC WOS = 1.5). If the SC stocks 2.5 WOS they would provide at least 93% service. The same situation can be seen for Figures 13 and 14 only the levels of service are different. There are two significant factors to note from these figures. They are:

(1) The difference in the amplification of levels of service as the repair/new ratio is decreased.

(2) The increased service provided to the customer when the investment level at the SC is increased by one week of stock.

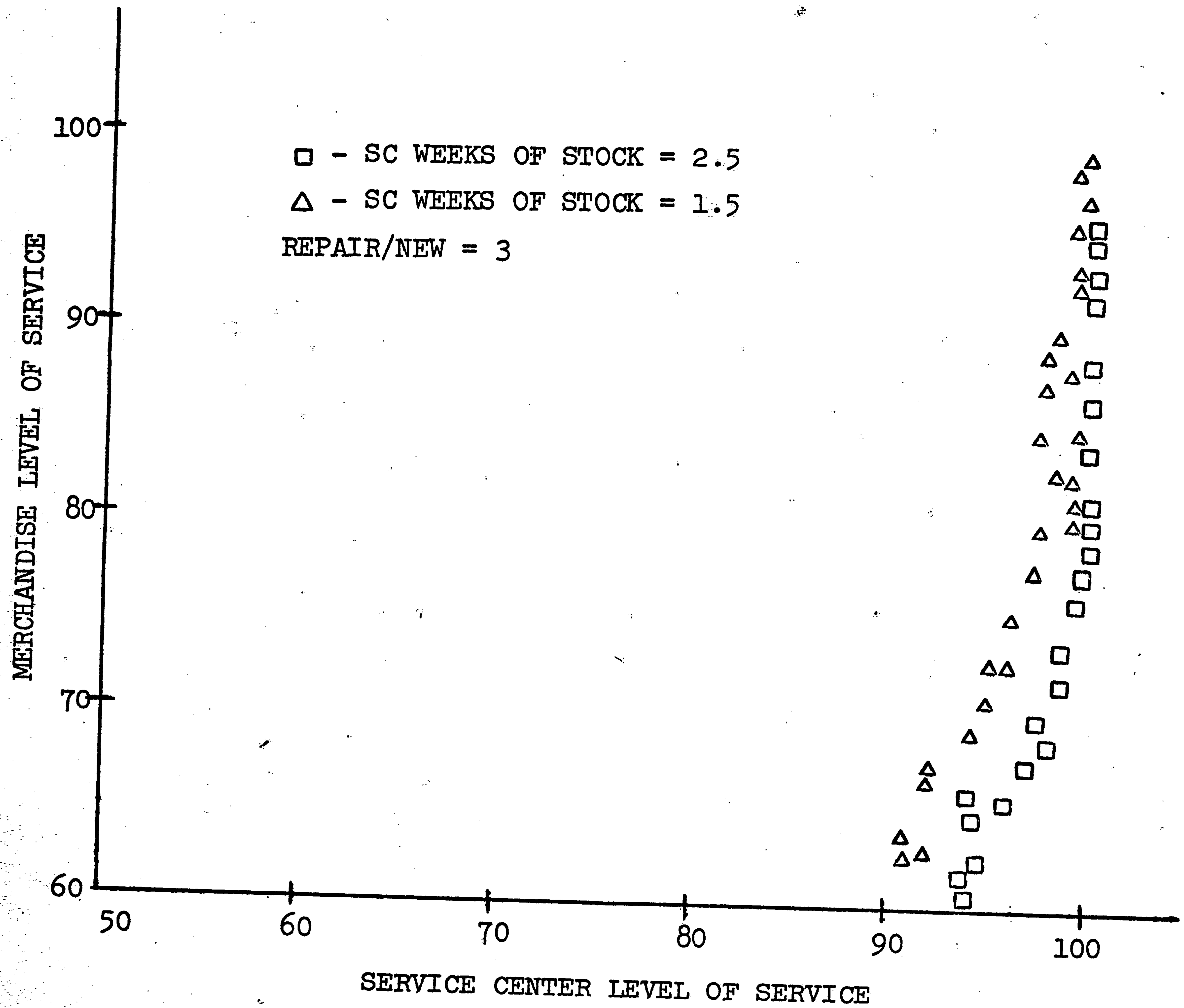


FIGURE 12 MDSE-SC SERVICE CHARACTERISTICS

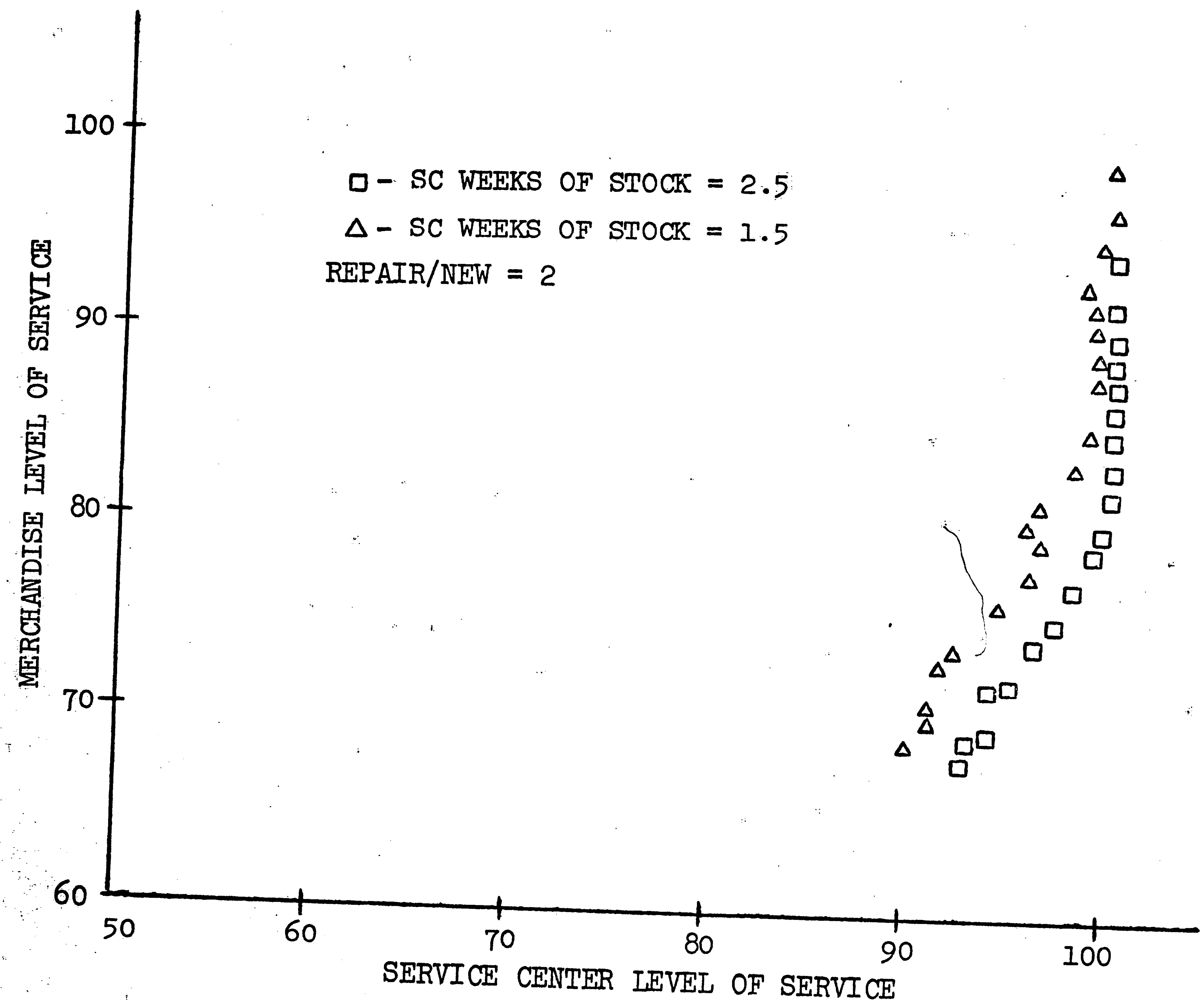


FIGURE 13 MDSE-SC SERVICE CHARACTERISTICS

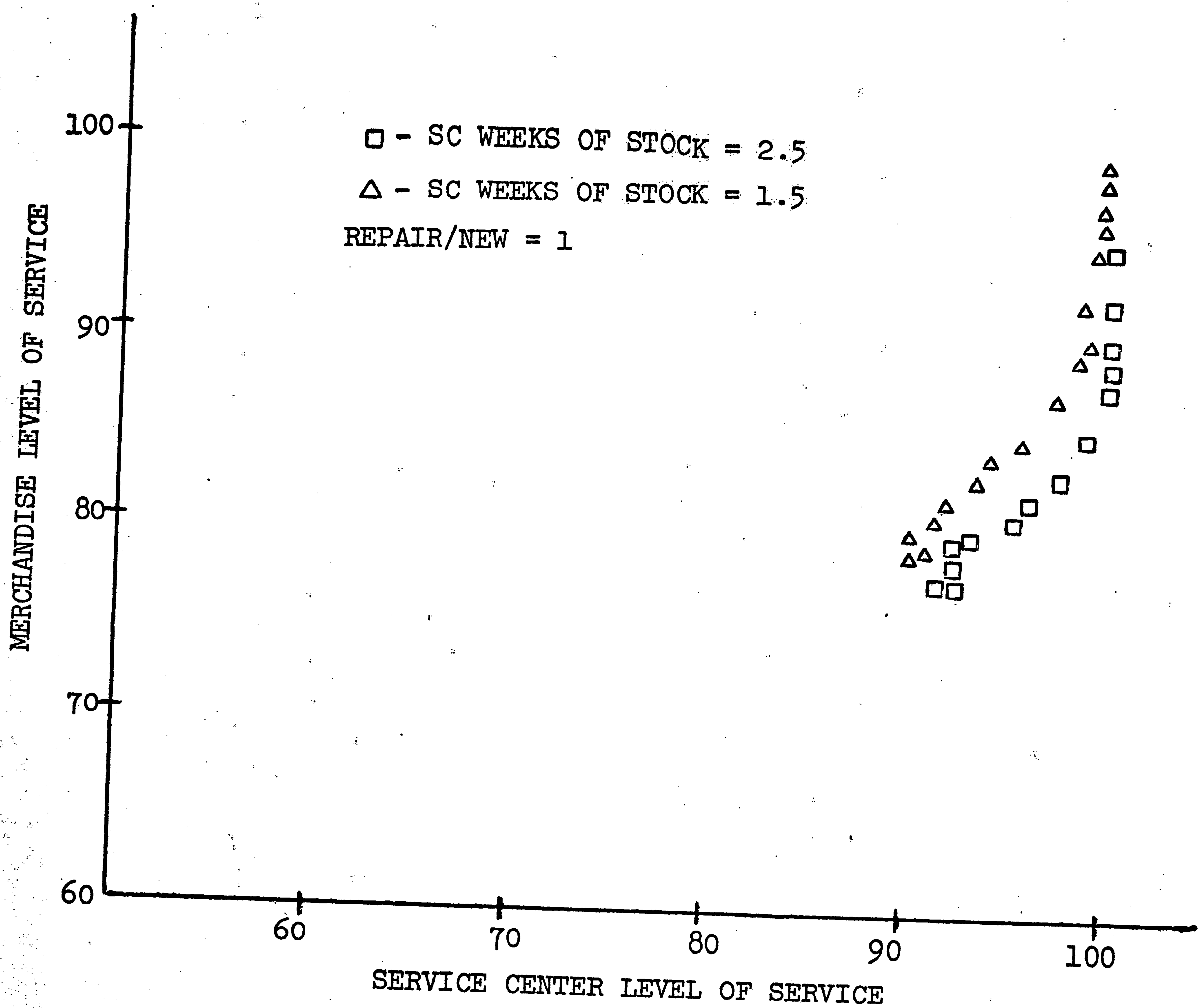


FIGURE 14 MDSE-SC SERVICE CHARACTERISTICS

As the repair/new ratio is decreased the amplification of service is larger. This result is straightforward since as the SC repairs less they are more dependent upon the level of service provided by MDSE. A quantitative measure can be seen from the three plots. If the SC investment level is 2.5 WOS they can provide 99% service or better even if MDSE service level drops to 90%. Whereas if the SC investment level is 1.5 WOS their service level may drop to 97%. The reason for stating SC service may drop to 97% is because this measure depends on the repair/new ratio.

For 90% service or better from MDSE the increased service to the customer, as a result of SC investment increasing one week of stock, is not very significant. It is at most 2% in the case where the repair/new ratio is three. In the other two cases the increased service is at most 1%.

To answer the question of how many weeks of stock each echelon should maintain would depend upon what management has set as their criteria for level of service and investment. In general, there appears to be no reason why MDSE should not plan for a service level of 90% since at this service level the impact on the SC is not significant. In all cases under investigation the SC could still provide at least 97% service to the customer.

4.3 INVESTMENT AMPLIFICATION ANALYSIS

The purpose of this section is to develop the response of the pipeline to a percentage change in SC repair or customer demand.

The response measure will be the new stock investment required in the pipeline for a given percentage change. The analysis considers the pipeline changing from one steady-state level to a new steady-state level. Thus, the percentage change must be observed for at least the length of the SC forecast when analyzing the response as a function of customer demand. This minimum length of time is not necessarily required when analyzing the response as a function of SC repair. Once the SC repair function changes by some percentage the SC inventory will adjust accordingly this week. Since the SC uses a past moving average of customer demand, they require at least the length of this moving average before their inventory has changed to a new steady-state level.

It is assumed in this analysis that the capital investment in equipment does not change when the SC repair rate changes. The SC repair rate can be changed by 20% by simply working a full day on Saturday or with overtime during the week. The total percentage change in the investment is that pipeline inventory which will be required from manufacturing, as a result of a percentage change in repair or customer demand. The analysis assumes each echelon desires to maintain the same investment level in terms of weeks of stock after a change is observed. The development will be in two sections. These are:

(1) New investment required as a function of a percent change in customer demand.

(2) New investment required as a function of a percent change in SC repair.

Note the above is required new investment. The pipeline may or may not be able to respond to certain changes due to the limitations at manufacturing and merchandise. Any time a percentage change in customer demand or SC repair is observed both merchandise and the SC will desire some number of weeks of stock on hand. The actual number of units in inventory will not change at the SC echelon due to a change in SC repair; however, the amount of new stock investment will change. The actual number of units in inventory will change at the MDSE echelon due to a change in either input variable.

Throughout the development it is assumed the SC has essentially no investment in the repair stock since the merchandise has already been sold to the customer. Once it has been sold it is still the customer's investment and has only been sent to the SC to be repaired. The new stock from MDSE contains the investment in materials as well as capital and labor.

The terms "new investment" and "new pipeline investment" both refer to the required investment within the system. They are a quantitative measure of how the system will attempt to respond to changes. The investment level will always refer to that quantity of inventory maintained in terms of weeks of stock.

4.3.1 INVESTMENT AS A FUNCTION OF CUSTOMER DEMAND

This section assumes the SC repair rate is constant. A step percentage change in customer demand amplifies up the pipeline in terms of new investment. This analysis will define the required new pipeline investment as the sum of the SC plus MDSE echelons. The amplification is a function of two variables defined to be:

$$(1) \quad \lambda = \frac{\text{CUSTOMER DEMAND}}{\text{SC DEMAND}} \Big/ \text{ AT STEADY-STATE.}$$

$$(2) \quad \alpha = \frac{\text{SC INVENTORY}}{\text{NEW SC INVENTORY}} \Big/ \text{ AT STEADY-STATE.}$$

Lambda is the amplification factor that determines the percentage change in new MDSE investment and SC demand as a result of a step change in customer demand. Alpha is the amplification factor that determines the percentage change in new investment at the SC echelon. Tables two, three and four give alpha as a function of the repair/new ratios and weeks of stock. These assume the customer demand is one hundred units per week. As the repair/new ratio decreases the amplification factor alpha decreases for a fixed investment level. Also, as the investment level increases, alpha decreases for a fixed repair/new ratio.

Since the SC repair rate is assumed to be constant any increase in SC inventory is new investment. The percent change in SC new investment (NI) is determined by multiplying alpha times the change in customer demand; that is,

$$\text{SC} - \%\Delta\text{ NI} = (\alpha) * (\%\Delta\text{C DEM}). \qquad (7)$$

INITIAL STEADY-STATE VALUES

WOS	1.0	1.5	2.0	3.0
NEW STOCK	25	75	125	225
REPAIR STOCK	75	75	75	75
α	4	2	1.6	1.33

CUSTOMER DEMAND = 100

ALPHA WITH REPAIR/NEW = 3

TABLE 2

INITIAL STEADY-STATE VALUES

WOS	1.0	1.5	2.0	3.0
NEW STOCK	33.33	83.33	133.33	233.33
REPAIR STOCK	66.67	66.67	66.67	66.67
α	3	1.8	1.5	1.28

CUSTOMER DEMAND = 100

ALPHA WITH REPAIR/NEW = 2

TABLE 3

INITIAL STEADY-STATE VALUES

WOS	1.0	1.5	2.0	3.0
NEW STOCK	50	100	150	250
REPAIR STOCK	50	50	50	50
α	2	1.5	1.33	1.2

CUSTOMER DEMAND = 100

ALPHA WITH REPAIR/NEW = 1

TABLE 4

If MDSE is to maintain the same investment level the percent change in customer demand is multiplied by lambda; that is,

$$\text{MDSE} - \%\,\Delta\,\text{NI} = (\lambda) * (\%\,\Delta\,\text{C DEM}). \tag{8}$$

Therefore, adding equations (7) and (8) and multiplying each component by its respective new stock steady-state inventory, we have the change in new pipeline investment; that is,

$$\text{PIPELINE} - \Delta\,\text{NI} = (\%\,\Delta\,\text{C DEM}) * (\alpha) * (\text{NEW SC INV/S.S.}) + (\%\,\Delta\,\text{C DEM}) * (\lambda) * (\text{MDSE INV/S.S.}) \tag{9}$$

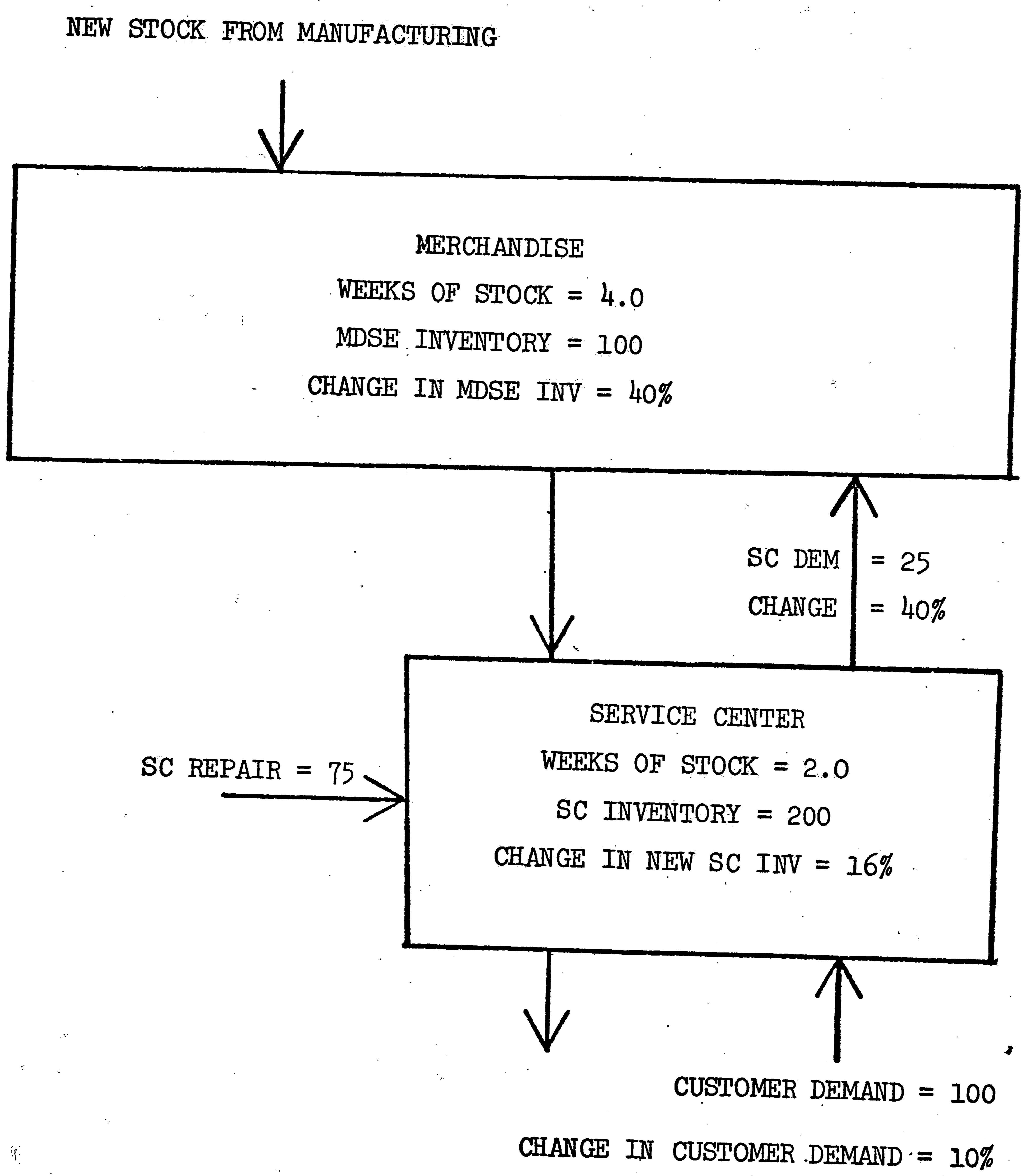
where:

/S.S. means at steady-state.

The first term is for the percent change in SC inventory and the second term is the percent change in MDSE inventory. Figure 15 is a diagram demonstrating the new pipeline investment changing from one steady-state level to another as a function of a percent change in customer demand. The initial steady-state conditions are given in terms of units. The new steady-state levels of inventory are given in percentages. Lambda is equal to four and alpha equals 1.6; thus from equation (9) the

$$\text{PIPELINE} - \Delta\,\text{NI} = (.10) * (1.6) * (125) + (.10) * (4) * (100)$$

$$= 20 + 40 = 60 \text{ UNITS.}$$



PERCENT CHANGE IN NEW INVESTMENT

FIGURE 15

This is a result of a ten unit increase in customer demand or ten percent. This change in new pipeline investment would require at least the length of the SC forecast. This analysis has made several assumptions so the approximate response of an exceedingly complex inventory system could be made. It enables one to investigate the interactions of the major parameters within the system. The actual response of such a system is very dependent upon the SC repair rates since changing the repair also changes lambda and alpha. Given a constant repair the amplification of new investment, within the pipeline, as a result of a percentage change in customer demand, is significant. A ten percent change in customer demand could result in MDSE level of service being reduced significantly.

The percent change in new pipeline investment is equal to the change in new investment divided by the steady-state pipeline investment; that is,

$$\text{PIPELINE} - \%\,\Delta\,\text{NI} = \frac{(\text{PIPELINE} - \Delta\,\text{NI}) * (100)}{\text{PIPELINE NI/S.S.}} \qquad (10)$$

From the example in Figure 15 we have,

$$\text{PIPELINE} - \%\,\Delta\,\text{NI} = \frac{(60) * (100)}{225} = 26.7\%.$$

This demonstrates how the inventory system will attempt to respond to a given percentage change, providing manufacturing is capable of responding. The same analysis applies when customer demand is decreased; if manufacturing cannot decrease its production rapidly

MDSE inventory would increase. As in the previous example, if customer demand decreased by 10% SC INV would decrease by 16%; also, SC demand would decrease by 40%, resulting in an attempt by MDSE to reduce its inventory by 40%.

Figure 16 is a graph showing the percent change in new SC investment as a function of weeks of stock maintained for a repair/new ratio of three. As seen from the graph a percent increase in customer demand results in significant increase in new SC investment.

Proper planning for such an inventory system would require significant inventory at the MDSE echelon if there is a possibility of a step change in customer demand. One way of introducing a step into the system would be a promotional sale on a product. From a control point of view, any time a promotion on a product is planned MDSE should be notified so production plans can be reviewed.

4.3.2 INVESTMENT AS A FUNCTION OF SERVICE CENTER REPAIR

This section assumes customer demand is constant. A percentage change in SC repair (SCR) amplifies up the pipeline in terms of new investment. This amplification is a function of two variables defined to be:

(1) Repair/new ratio.

(2) Investment ratio (IR).

The second variable is the ratio of steady-state repair to that portion of SC INV that is new investment. Since the SC fills

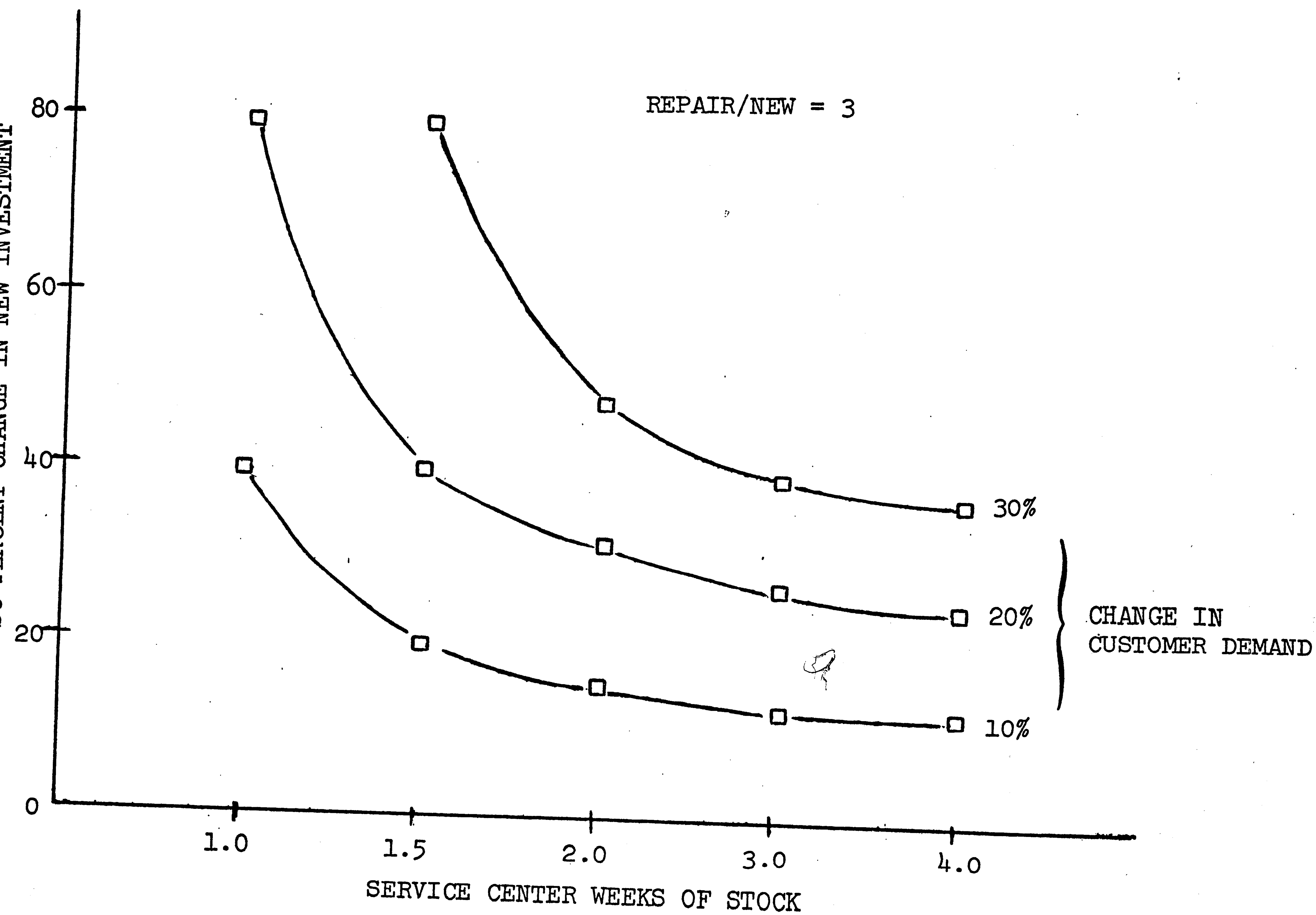


FIGURE 16 INVESTMENT-WEEKS OF STOCK CHARACTERISTICS

customer demand with repair stock first it is assumed the inventory on hand minus the SC repair is the new investment inventory. For example, if the SC is repairing 75 units per week and it stocks 200 units per week, then

$$IR = \frac{\text{SC REPAIR}}{\text{NEW SC INV}} \quad / \text{ AT STEADY-STATE}$$

$$= \frac{75}{125} = 0.6. \tag{11}$$

The investment ratio is a function of the weeks of stock at the SC. Therefore they cannot be given as a function of only the repair/new ratio. Tables five, six and seven give the IR's as a function of repair/new ratios and weeks of stock. These assume the C DEM is one-hundred units per week.

This analysis defines the response of the pipeline from one steady-state level to another as a result of a percent change in SC repair. The percent change in new SC investment is equal to minus the investment ratio times the percent change in SC repair; that is,

$$\text{SC} - \%\Delta\ \text{NI} = -(\text{IR}) * (\%\Delta\ \text{SC REPAIR}). \tag{12}$$

The percent change in MDSE investment equals minus the repair/new ratio times the percent change in SC repair; that is,

$$\text{MDSE} - \%\Delta\ \text{NI} = -(\text{R/N}) * (\%\Delta\ \text{SC REPAIR}). \tag{13}$$

INITIAL STEADY-STATE VALUES

WOS	1.0	1.5	2.0	3.0
NEW STOCK	25	75	125	225
REPAIR STOCK	75	75	75	75
INVESTMENT RATIO	3	1	0.6	0.33

CUSTOMER DEMAND = 100

REPAIR/NEW = 3

SERVICE CENTER INVESTMENT RATIOS

TABLE 5

INITIAL STEADY-STATE VALUES

WOS	1.0	1.5	2.0	3.0
NEW STOCK	33.33	83.33	133.33	233.33
REPAIR STOCK	66.67	66.67	66.67	66.67
INVESTMENT RATIO	2.0	0.8	0.5	0.286

CUSTOMER DEMAND = 100

REPAIR/NEW = 2

SERVICE CENTER INVESTMENT RATIOS

TABLE 6

INITIAL STEADY-STATE VALUES

WOS	1.0	1.5	2.0	3.0
NEW STOCK	50	100	150	250
REPAIR STOCK	50	50	50	50
INVESTMENT RATIO	1.0	0.5	0.33	0.2

CUSTOMER DEMAND = 100

REPAIR/NEW = 1

SERVICE CENTER INVESTMENT RATIOS

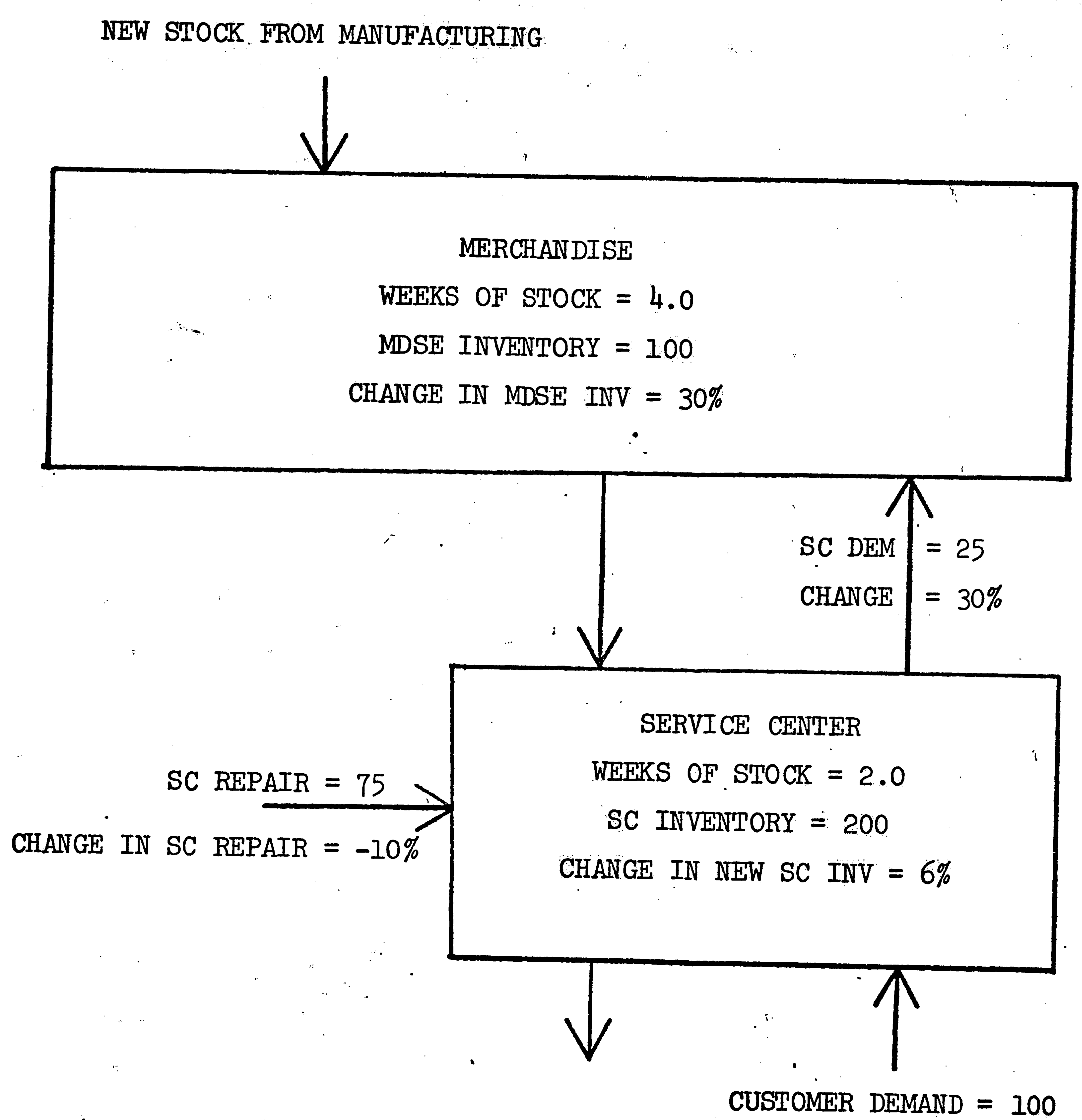
TABLE 7

All inventory at the MDSE echelon is new investment. Equation (13) is independent of the investment level (WOS) for MDSE, whereas the investment ratio for the SC is a function of its investment level. Adding equations (12) and (13) and multiplying each component by its respective new stock steady-state inventory, we have the change in new pipeline investment (Δ NI); that is,

$$\text{PIPELINE} - \Delta\,\text{NI} = -(\text{IR}) * (\text{NEW SC INV/S.S.}) * (\%\,\Delta\,\text{SC REPAIR}) - (\text{R/N}) * (\text{MDSE INV/S.S.}) * (\%\,\Delta\,\text{SC REPAIR}). \quad (14)$$

The first term is for the percent change in new SC inventory and the second term is the percent change in MDSE inventory. The component for SC inventory is not a change in the total number of units stocked but the change in new investment required to maintain the same investment level. Figure 17 is a diagram demonstrating the new pipeline investment changing from one steady-state level to another as a function of a change in SC repair. The initial steady-state values are given in terms of units. New steady-state levels of investment are given in percentages. The repair/new ratio equals three and the investment ratio equals 0.6 from Table five. SC repair is decreased by ten percent. Therefore, using equation (14) we have,

$$\begin{aligned}\text{PIPELINE} - \Delta\,\text{NI} &= -(0.6) * (125) * (-0.10) \\ &\quad - (3) * (100) * (-0.10) \\ &= 7.5 + 30 = 37.5 \text{ UNITS.}\end{aligned}$$



PERCENT CHANGE IN NEW INVESTMENT

FIGURE 17

The required new pipeline investment is 37.5, as a result of 7.5 units per week decrease in SC repair.

The proper operation of the pipeline is very dependent upon the SC repair rates as seen from the example given. When the repair rate is changed the pipeline requires new investment at both echelons to maintain the same investment levels, in terms of weeks of stock. For the pipeline to reach this new steady-state level would require only one week if manufacturing could respond. Since manufacturing may fail to respond the result would be a loss in MDSE inventory. But sometime in the future MDSE would desire the increased new investment in order to buffer SC demand.

Figures 18, 19 and 20 are graphs showing the percent change in new investment as a function of the weeks of stock maintained at the SC for repair/new ratios of three, two and one respectively. Each figure is given for a 30, 20 or 10 percent decrease in SC repair. As the repair/new ratio increases the investment ratio increases; thus the amplification factor increases. This results in a greater impact on the pipeline for two reasons. When the repair/new ratio increases, this also increases the amplification factor for MDSE inventory as well as increasing the investment ratio for SC inventory. The net result has a very significant impact on the operation of the pipeline.

Figure 21 is a graph showing the amplification of a change in SC repair to the MDSE echelon for repair/new ratios of three, two and one. As seen from Figure 21, if MDSE is to maintain a certain

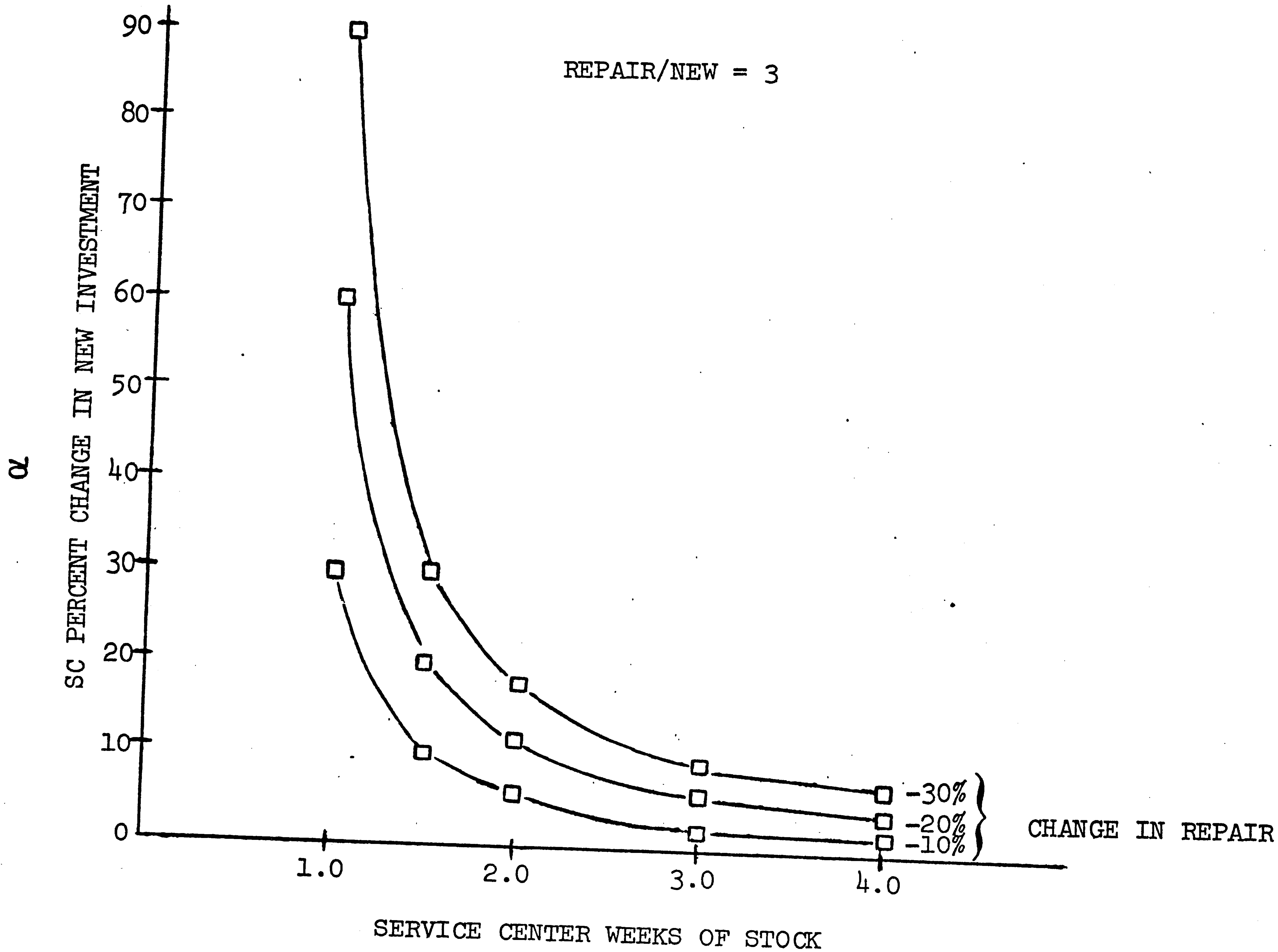


FIGURE 18 INVESTMENT-WEEKS OF STOCK CHARACTERISTICS

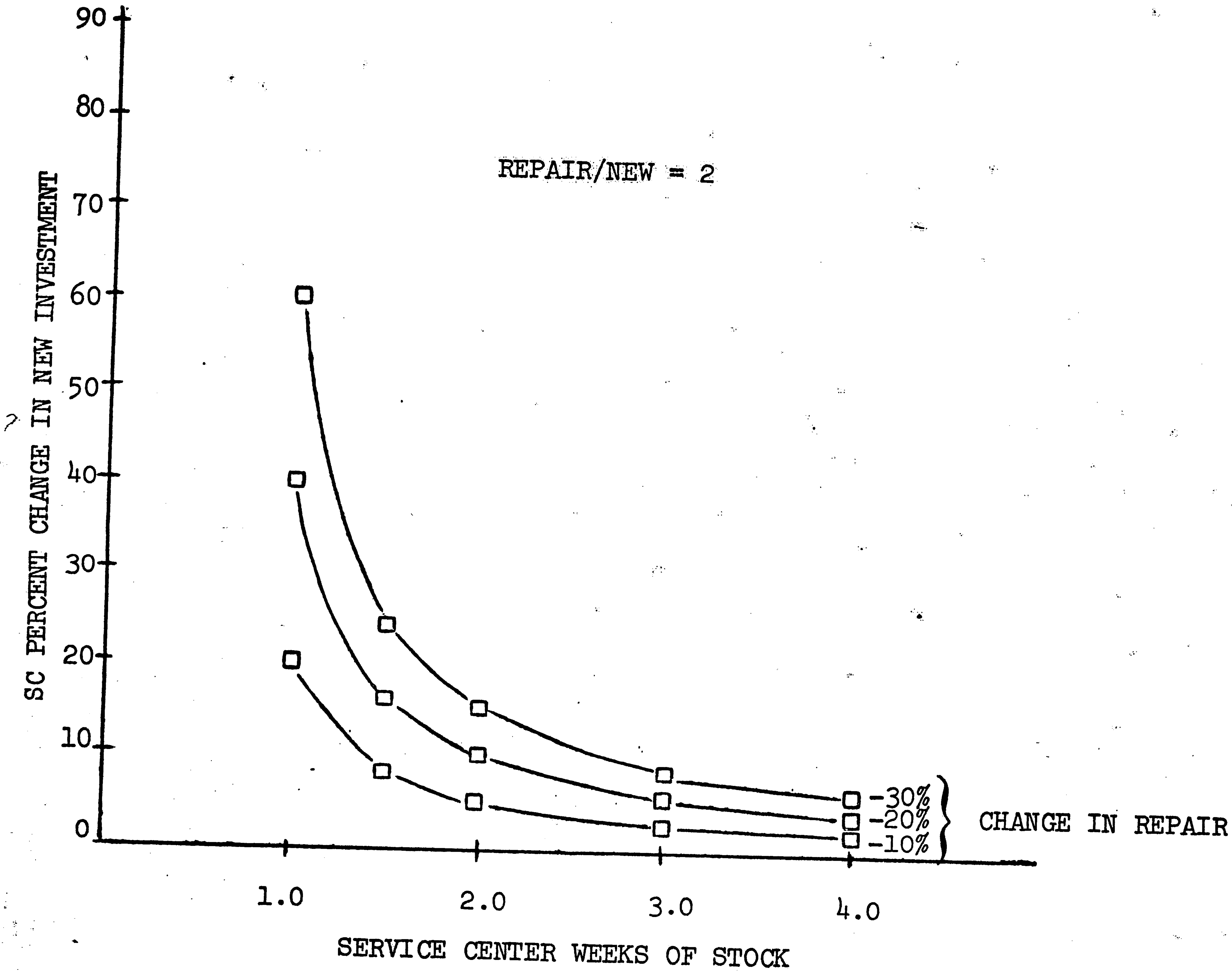


FIGURE 19 INVESTMENT-WEEKS OF STOCK CHARACTERISTICS

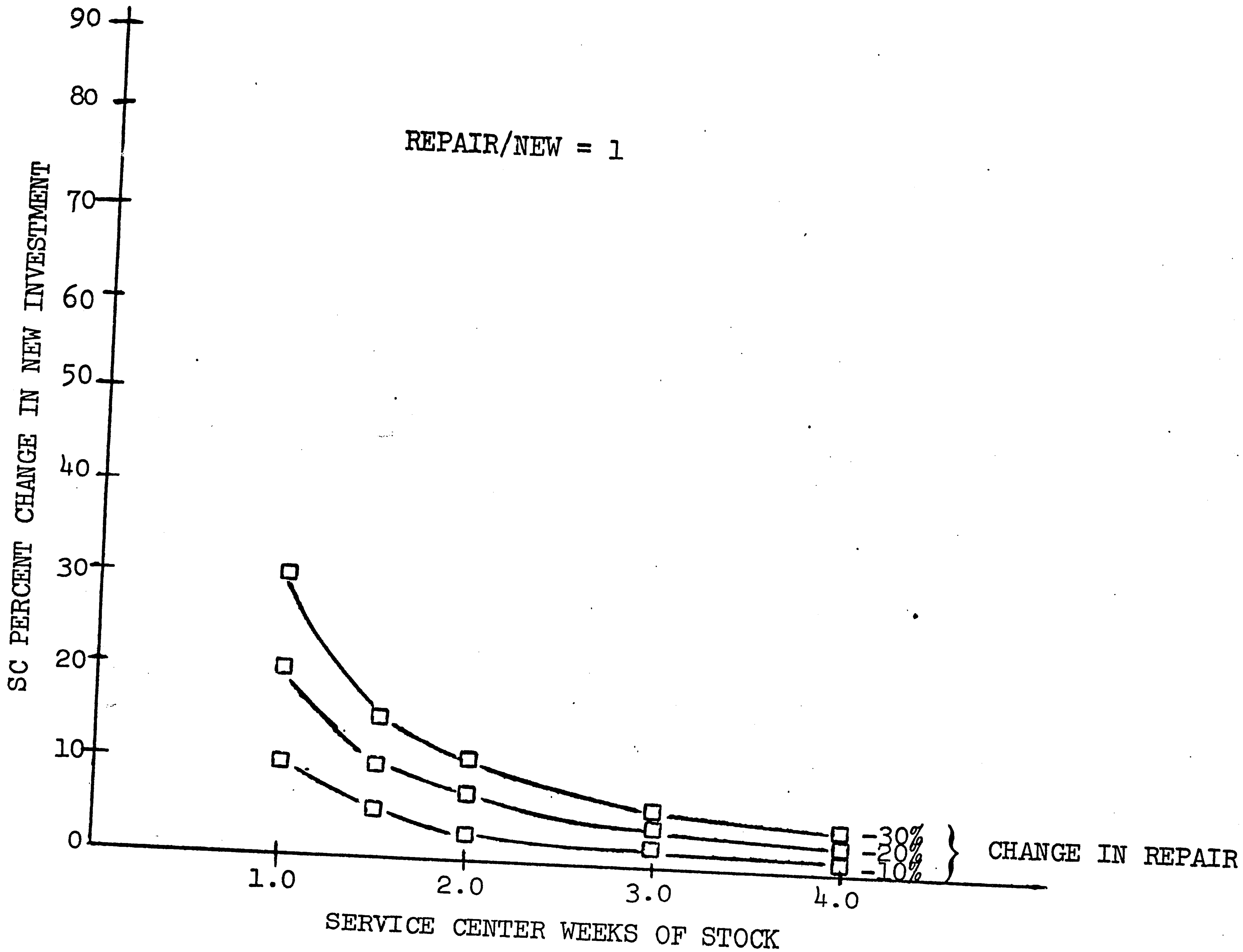


FIGURE 20 INVESTMENT-WEEKS OF STOCK CHARACTERISTICS

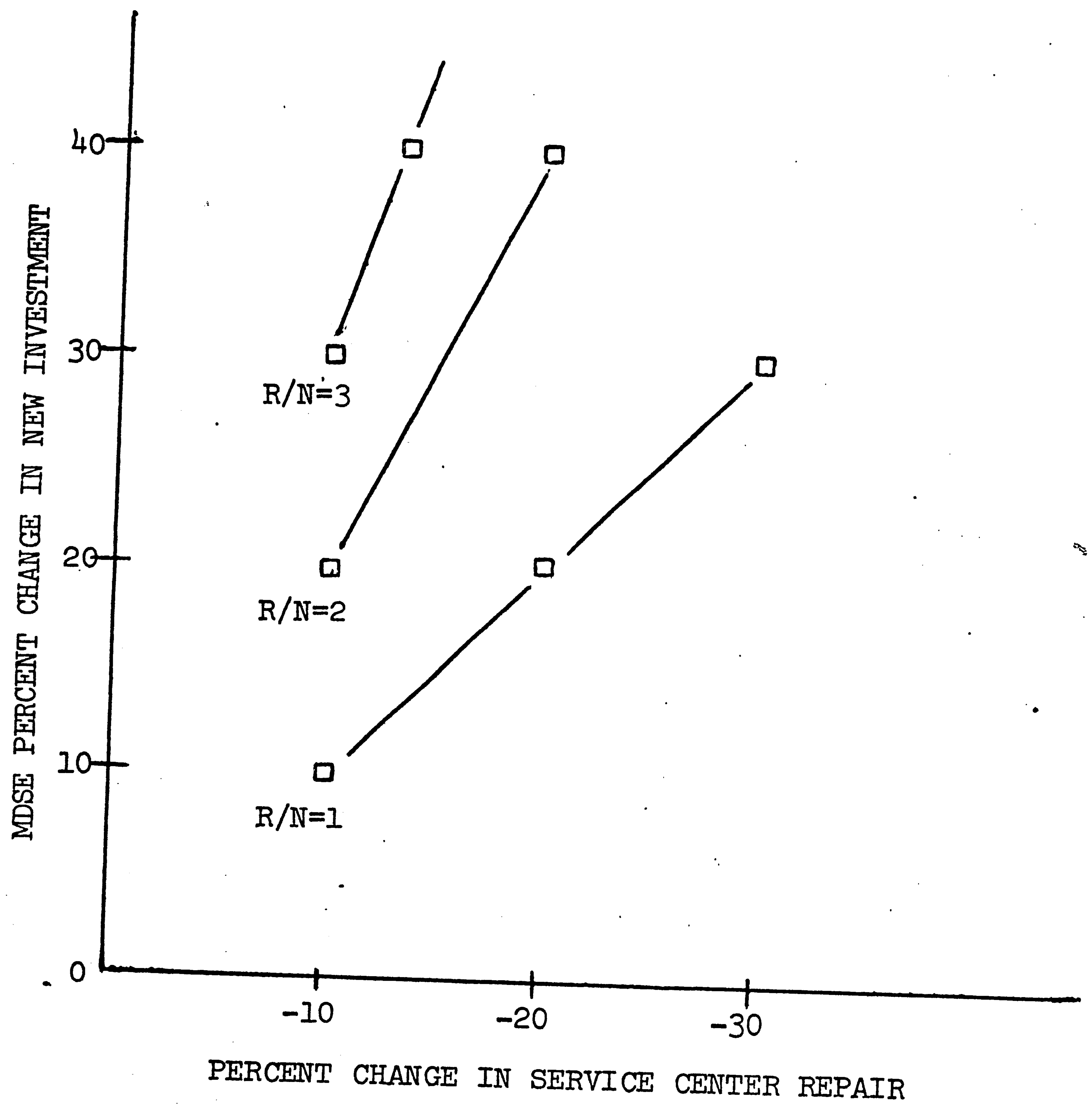


FIGURE 21 INVESTMENT-REPAIR CHARACTERISTICS

investment level the impact of changing the SC repair is significant. These results again give reasons for the SC repair rates to be as constant as possible.

To analyze the pipeline consider the following steady-state conditions.

(1) SC Weeks of Stock = 1.5

(2) MDSE Weeks of Stock = 4

(3) Customer Demand = 100

(4) SC Repair = 75

(5) SC INV = 150

(6) SC Demand = 25

(7) MDSE INV = 100

For some reason the SC decreases the repair by 20%. From the initial pipeline conditions we have,

$$IR = 1$$

and

$$R/N = 3.$$

Therefore we have,

$$SC - \Delta NI = -(IR) * (NEW\ SC\ INV/S.S.) * (\%\ \Delta\ SC\ REPAIR)$$

$$= -(1) * (75) * (-0.20) = 15\ UNITS,$$

and

$$MDSE - \Delta\ NI = -(R/N) * (MDSE\ INV/S.S.) * (\%\ \Delta\ SC\ REPAIR)$$

$$= -(3) * (100) * (-.020) = 60\ UNITS.$$

Thus, if the repair rate is decreased by 15 units the pipeline would require a total of 75 new units to maintain the same investment levels in terms of weeks of stock. In percentage we have,

$$\text{PIPELINE} - \% \Delta \text{ NI} = \frac{(75) * (100)}{175} = 42.8\%.$$

Note in this example both MDSE INV and SC demand increase by 60%

$(-(R/N) * (\% \Delta \text{ SC Repair}))$.

By the same analysis an increase in SC repair can be detrimental from a cost point of view. If the SC repair rates were to increase by 20% for a quarter of a year, MDSE could be left with very large inventories, unless manufacturing could respond. Even if manufacturing was capable of responding the cost of idle lines may be significant.

This analysis shows that proper control of such an inventory system is not a trivial task. There should be some type of feedback from the SC to MDSE as to their plans for repair. If the changes in repair have been in the plan for manufacturing, the requirements for the pipeline can be met by increasing (or decreasing) inventory at the MDSE echelon prior to the change. This analysis demonstrates the dependence of the pipelines operation upon SC repair.

CHAPTER V

SUMMARY AND RECOMMENDATIONS

This thesis has considered a multi-echelon inventory system with a reparable item. The first echelon was a Service Center whose purpose is to supply the customer demand from repair or new stock. Repair stock is obtained from the repair shop at the Service Center. New stock is obtained from the second echelon (Merchandise). Merchandise inventory is obtained from a manufacturing organization.

A periodic review model called "The Logistics Environment Model" was used to evaluate two of the three objectives these were:

(1) Determine the required investment level at each echelon to provide a desired service posture.

(2) Determine the impact on the system when the second echelon level of service is reduced.

To evaluate these two objectives simulation experiments were designed. For the experiments both customer demand and Service Center repair were stationary time-series. They were stochastic inputs approximated by the normal distribution.

In addition to the previously stated assumptions, the model was developed for a specfic high demand rate item. However, the model could be used to evaluate a system with a low volume as well as a system with no repair. The major limitation being the decision rules upon which the Logistics Environment Model was developed.

Based on the decision rules of the Logistics Environment Model the expected level of service for both echelons has been given as a function of their investment level. Also, given a specified investment level the stability of each echelon's service posture has been demonstrated. Stability meaning given an investment level what confidence we can have in providing a desired level of service to the customer. The service becomes more unstable at the Merchandise echelon as the repair/new ratio increases. This was a result of variations in customer demand and Service Center repair being amplified to the Merchandise echelon. The Service Center echelon was found to be stable for all repair/new ratios. Based on the results of the simulation experiments a minimum of 1.5 weeks of stock should be maintained at the first echelon if management has set a desired service level of 99%. The objective being to operate off the knee of the level of service curve.

The dependence of the Service Center on the service integrity from the second echelon has been demonstrated as a function of the repair program and investment level. From this investigation it is not necessary for Merchandise to provide a high level of service, since as the level of service is reduced there isn't necessarily a significant impact on the customer. The service posture from Merchandise could be reduced by 10% with a small impact (2%) on the customer for all repair programs. As the service level of the Merchandise echelon was reduced the impact on service to the customer increased as the repair/new ratio decreased. In the case where the repair/new ratio was three Merchandise service could be reduced to 60% and the Service Center could still provide 90% service to the customer.

The third objective of this thesis was to investigate the response of the pipeline in terms of required new investment to a percentage change in the repair program or customer demand. To accomplish this objective new investment equations were developed to analyze the system changing from one steady-state level to another. The analysis demonstrates the amplification of a disturbance or change in a multi-echelon inventory system. The analysis assumed each echelon would maintain the same investment level, in terms of weeks of stock, after a percentage change was placed on the system. The method of analysis provides a means of obtaining an understanding of the interactions between the major parameters. It also gives insight into how the interactions affect the inventory levels and the operation of such a system.

For the system to buffer a positive step change in customer demand requires significant changes in new investment at both echelons. When the step change is seen at the Service Center echelon it orders to build inventory as well as to meet increased customer demand. This results in an amplification of the original percentage change. The results of this investigation point out the necessity for control requirements in such an environment.

In order to accurately control a system of this complexity a knowledge of the following elements is required by Merchandise.

(1) Trends or changes in Service Center repair.

(2) Changes in the parameters of the stock maintainer's decision rules.

(3) Customer product promotional plans and activities.

In future research, control limits should be investigated for the Merchandise echelon. These limits would determine the time at which manufacturing production plans require replanning due to Merchandise investment position and forecasts. From the results of this investigation the control limits should be large; thereby the amplification in a multi-echelon environment can be buffered by Merchandise inventory such that a temporary change in environment does not disrupt manufacturing plans.

BIBLIOGRAPHY

1. Forrester, J. W., Industrial Dynamics, The M.I.T. Press, Cambridge, Mass., 1961.

2. Gerson, M. L., "A Military Inventory Control System," Master's Thesis, The Massachusetts Institute of Technology, 1959.

3. Zymelman, M., "A Stabilization Policy For The Cotton Textile Cycle," Management Science, Vol. 11 (March 1965), pp. 572-580.

4. Freiheit, J. W., "A Continuous Review Model For The Reparable Item Inventory System", Master's Thesis, Naval Post Graduate School, 1967.

5. Hastings, D. A., "A Constrained Periodic Review Model For A Probabilistic Reparable-Item Inventory System", Master's Thesis, Naval Post Graduate School, 1967.

6. Hatchett, J. W., "A Reparable Item Inventory Model", Master's Thesis, Naval Post Graduate School, 1966.

7. Westlake, J. J., "An Industrial Dynamics Study Of A Distributing House Operation For Reparable Items", Master's Thesis, Lehigh University, 1969.

8. Iglehart, D. L., and R. C. Morey, "Optimal Policies For A Multi-Echelon Inventory System With Demand Forecasts", Naval Research Logistics Quarterly, Vol. 18, No. 1, (March 1971) pp. 115-118.

9. Kruse, K. W., "An Algorithm For Computing Optimum Stock Levels In A Two-Level Maintenance System", Technical Report AMC Inventory Research Office, United States Army Logistics Management Center, Fort Lee, Virginia, January, 1970.

10. Clark, A., and H. Scarf, "Optimal Policies For A Multi-Echelon Inventory Problem", Management Science, Vol. 6, (1960), pp. 475-490.

11. Scarf, H., Mathematical Methods In The Social Sciences, Stanford University Press, Stanford, California, 1960.

12. Morey, R. C., "Inventory Systems With Imperfect Demand Information", Naval Research Logistics Quarterly, Vol. 18, No. 1 (March 1971) pp. 287-295.

13. Simon, H.A., "On The Application Of Servomechanism Theory In The Study Of Production Control", Econometrica, Vol. 20 (1952), pp. 247-268.

BIBLIOGRAPHY (Cont'd)

14. Meyer, V., "Analysis And Synthesis of Multi-Echelon Inventory Systems Using The Theory of Feedback Control Systems", Ph.D. Dissertation, Lehigh University, 1971.

15. Scarf, H., D. Gilford, and M. Shelly, Multistage Inventory Models And Techniques, Stanford Press, Stanford, California, 1963.

APPENDIX A

SERVICE CENTER DEMAND

The purpose of this appendix is to develop the SC demand for any time (t). The SC demand is based on desired inventory, last week's ending inventory, repair rates and back orders. The mathematical devlopment is given in summation notation.

The SC INV at any time is:

$$\text{SC INV}(t) = \frac{(\text{WOS})}{(\text{LOF})} * \sum_{T = t-\text{LOF}+1}^{T = t} \text{C DEM}(T)$$

$$-\text{SC FORECASTED REPAIR }(t)$$
$$+\text{SC ACTUAL REPAIR }(t)$$

where,

WOS - weeks of stock for the SC,
LOF = length of forecast for the SC. (2)

The first term is the past moving average of customer demand times the weeks of stock to be held in inventory. It also equals desired SC INV for any week. Writing the SC demand at any time (t), we have:

$$\text{SC DEM}(t) = \text{DESIRED SC INV}(t) - \text{SC INV }(t-1)$$
$$+ \text{C DEM}(t) - \text{SC FORECASTED REPAIR }(t).$$

Define:

$$\text{REPAIR ERROR }(t) = \text{SC ACTUAL REPAIR }(t) - \text{SC FORECASTED REPAIR }(t).$$

Substituting into SC DEM the SC INV and REPAIR ERROR, we have:

$$\text{SC DEM}(t) = \frac{(\text{WOS})}{(\text{LOF})} * \sum_{T=t-\text{LOF}+1}^{T=t} \text{C DEM}(T)$$

$$- \frac{(\text{WOS})}{(\text{LOF})} * \sum_{T=t-\text{LOF}+1}^{T=t} \text{C DEM}(T-1) - \text{REPAIR ERROR }(t-1)$$

$$+ \text{C DEM}(t) - \text{SC FORECASTED REPAIR}(t).$$

After subtracting the two summations we are left with two terms; therefore:

$$\text{SC DEM}(t) = \frac{(\text{WOS})}{(\text{LOF})} * (\text{C DEM }(t) - \text{C DEM}(t-\text{LOF}))$$

$$- \text{REPAIR ERROR }(t-1) + \text{C DEM}(t)$$

$$- \text{SC FORECASTED REPAIR }(t).$$

This is equation (7) in Chapter III not including back orders. If back orders are considered last week's MDSE back orders are added into last week's inventory. Therefore, the units desired will only be ordered once from MDSE. This is the same as subtracting last week's MDSE back order from SC demand for this week. If the SC has back ordered the customer, the number of units are added to SC DEM. Note that this week's demand to MDSE is a function of this week's forecasted SC repair as well as last week's error in forecasted repair.

APPENDIX B

RANDOM DATA

This appendix contains data chosen at random from the simulation runs for different SC investment levels and repair/new ratios. The purpose of the data is to give some insight into the characteristics of the SC demand as a result of the investment decision rules within the Logistics Environment Model.

Customer Demand	Standard Deviation	Service Center Repairs	Standard Deviation	Service Center Demand	Standard Deviation
99.6	20.1	75.0	7.4	24.7	20.8
101.4	19.4	74.8	7.9	26.5	22.8
100.6	21.7	75.7	7.7	25.1	21.8
100.0	21.7	75.0	7.3	24.9	23.1
100.2	19.6	74.6	7.5	25.6	20.8
102.8	17.7	74.5	7.2	28.2	19.7
102.8	22.6	75.9	7.0	26.9	21.9
98.8	20.3	74.7	7.7	24.1	22.0
100.2	21.7	75.0	6.7	25.2	20.9

Service Center Weeks of Stock = 1.5

Repair/New = 3

Customer Demand	Standard Deviation	Service Center Repair	Standard Deviation	Service Center Demand	Standard Deviation
99.6	20.1	75.0	7.4	24.8	22.0
101.4	19.4	74.8	7.9	26.4	23.7
99.9	24.0	75.0	7.4	24.7	24.8
100.2	22.8	75.7	7.7	24.7	24.5
100.3	18.9	74.5	7.2	26.0	21.8
100.2	19.6	74.6	7.5	25.6	22.0
100.3	18.9	73.8	7.5	26.6	21.9
101.7	19.6	75.4	7.8	26.4	22.4
97.9	20.1	75.0	7.8	23.2	22.2

Service Center Weeks of Stock = 2.5

Repair/New = 3

Customer Demand	Standard Deviation	Service Center Repairs	Standard Deviation	Service Center Demand	Standard Deviation
100.2	19.6	66.4	7.1	33.8	22.2
99.6	19.8	67.2	6.8	32.3	22.5
100.4	21.7	67.2	6.8	33.4	22.6
99.3	19.6	66.9	6.2	32.4	20.9
99.1	18.9	67.0	6.4	31.9	20.7
99.6	20.1	66.6	6.5	33.1	22.1
101.4	19.4	66.5	7.1	34.8	23.7
99.9	24.0	66.6	6.5	33.1	25.7
100.2	22.8	67.3	6.8	33.0	25.3

Service Center Weeks of Stock = 1.5

Repair/New = 2

Customer Demand	Standard Deviation	Service Center Repair	Standard Deviation	Service Center Demand	Standard Deviation
100.3	18.9	66.2	6.4	34.1	22.9
100.2	19.6	66.3	6.7	33.9	23.1
100.3	18.9	65.6	6.7	34.8	22.4
101.7	19.6	67.0	6.9	34.9	23.4
97.9	20.1	66.6	6.9	31.5	23.6
99.6	20.1	66.6	6.5	33.2	23.3
101.4	19.4	66.5	7.1	34.7	24.9
99.9	24.0	66.6	6.5	33.0	27.2
100.2	22.8	67.3	6.8	33.1	26.7

Service Center Weeks of Stock = 2.5

Repair/New = 2

Customer Demand	Standard Deviation	Service Center Repairs	Standard Deviation	Service Center Demand	Standard Deviation
102.8	17.7	49.7	4.8	53.1	20.3
102.8	22.6	50.6	4.7	52.2	24.9
98.8	20.3	49.8	5.1	49.0	23.3
100.2	21.7	50.0	4.4	50.2	24.0
100.0	17.4	49.8	4.7	50.4	20.0
99.6	20.1	50.0	4.9	49.7	23.8
101.4	19.4	49.9	5.3	51.4	23.2
99.9	24.0	50.0	4.9	49.8	27.3
100.2	22.8	50.5	5.1	49.9	26.2

Service Center Weeks of Stock = 1.5

Repair/New = 1

Customer Demand	Standard Deviation	Service Center Repair	Standard Deviation	Service Center Demand	Standard Deviation
99.6	20.1	50.0	4.9	49.8	25.6
101.4	19.4	49.9	5.3	51.4	24.7
99.9	24.0	50.0	4.9	49.6	29.0
100.2	22.8	50.5	5.1	50.0	27.9
97.3	18.6	50.0	4.8	47.4	22.7
100.2	19.6	49.8	5.3	50.3	24.1
99.6	19.8	50.4	5.1	49.0	24.8
100.4	21.7	50.4	5.1	50.0	25.5
99.3	19.6	50.1	4.6	49.1	23.8

Service Center Weeks of Stock = 2.5

Repair/New = 1

APPENDIX C

ABREVIATIONS

BO - Back Orders

C - Customer

$\overline{\text{CDEM}}$ - Average Customer Demand

DEM - Demand

INV - Inventory

IR - Investment Ratio

LOF - Length of Forecast

LOR - Length of Run

MDSE - Merchandise

NI - New Investment

R/N - Repair to New

RQTS - Requirements

SC - Service Center

SS - Steady-State

SVC - Service

t - Time in Weeks

WOS - Weeks of Stock

Δ - Change

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