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A study of job-shop sequencing problems through simulation

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A STUDY OF JOB-SHOP SEQUENCING
PROBLEMS THROUGH SIMULATION

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ABSTRACT

The objective of this study was to investigate the applicability of certain scheduling and dispatching procedures to the job-shop sequencing problem. The Job-Shop Simulator and the IBM-7040 computer were used as necessary tools for performing the experiments. The job-shop model used in the simulations was general in the sense that the product mix was such that wide variations were present among processing times, numbers of operations, routings, and dollar-values of jobs. However, the simulations were conducted under controlled, idealized conditions in order to study the effects of each dispatching rule upon several selected measures of performance in the job-shop.

The scheduling rule used was one in which the higher valued jobs were scheduled to be processed in a shorter interval than the lower valued jobs. The objective was to reduce the average cost of carrying in-process inventory. The dispatching rules tested were MINSOP, FCFSV, and MINDD. MINSOP (Minimum Slack Time Remaining per Operation) attempts to reduce the deviations of actual job deliveries from their due-dates without regard to value. FCFSV (First Come, First Served within Value Class) reduces the level of in-process inventory carrying costs by processing all higher valued jobs in the order of arrival without regard to schedule. MINDD (Minimum Due-Date) attempts to minimize the lateness of jobs by processing the jobs in order of proximity of due-dates.

In order to study the relative sensitivity of the scheduling and dispatching procedures to fluctuations in loading and to changes in

percentage of jobs which were given special preference, each dispatching rule was tested at two different loads - 100 percent and 85 percent - and at four different percentages of high-valued jobs - 10, 15, 20 and 25 percent.

The costs associated with carrying in-process inventory, delivering completed jobs after their due-dates, and having idle capacity should be balanced for an efficiently operated job-shop. Since the actual costs associated with each of these factors depend upon the specific job-shop, a balance among these costs could not be obtained. However, the relative effects upon the variance of the completion distribution, the machine utilization, and the average inventory carrying cost ratio were compared for each dispatching rule under different loadings and various percentages of high-valued jobs.

Chapter I

INTRODUCTION

Production activities in a job-shop involve the processing of a wide-variety of products on general-purpose machines, which are grouped into a number of work centers according to their homogeneity. The product is grouped into lots or jobs for processing through the shop. The sequence of operations required to complete each job and the estimated processing time for each machine operation are known when the job arrives in the shop. The primary problem in the job-shop is to determine the most economical procedure for satisfying the production demands placed upon the shop.

There are three major phases associated with planning the job shop production program. They are:

Loading: The comparison of total hours of demand with total hours of capacity in order to maintain a reasonable balance in load on the shop.

Scheduling: The establishment of the overall manufacturing cycle which results in assignment of demand to specific increments of the time period.

Dispatching: The procedure which determines the sequence in which the jobs at a specific machine group will be selected for processing.

Each of these three phases must be performed in such a manner that certain specified objectives are attained. The goals or objectives may vary widely among different job-shops and are, in general, difficult to define. Some of the more desirable objectives which may be associated with any production shop are:

1. Minimum production costs
2. Maximum profits
3. Minimum number of overdue orders

4. Minimum idle time
5. Maximum man and machine utilization
6. Minimum in-process inventories
7. Minimum number of early order completions
8. Minimum in-process interval

One can easily show that the simultaneous accomplishment of the above objectives is not possible, since the attainment of any single objective can only be accomplished at the expense of at least one of the others.

The development of either a loading, scheduling or dispatching procedure is an extremely complex task because of the many unpredictable factors such as: unexpected delays, variations in workers' efficiency, machine breakdowns, changes in customer's specification and due-dates, variation in raw material and product quality, and absenteeism.

The task is further complicated because the interdependencies among the three phases prevent their being studied separately. For example, a good system for dispatching might prove infeasible if a scheduling method fails to provide the proper balance of orders for each machine group. Similarly, attempts to manufacture economic lot sizes may conflict with meeting due dates.

Chapter II

REVIEW OF THE LITERATURE

There have been numerous studies of the job-shop problem reported in the literature. The models which have been used fall into two general categories: deterministic and stochastic. For the deterministic models, no random or stochastic variation is allowed in the parameters of the shop. Stochastic models permit random variation in one or more of the parameters and usually are tested through simulation.

Combinatorial Approach

A general formulation of the job-shop problem has been stated as follows: [36]

Given n jobs and m machines, each job having a specified sequence of operations on some or all machines, with the time for each operation known, what job sequence optimizes certain desired objectives? There are $(n!)^m$ possible sequence in this problem. Some of these are not feasible because they conflict with prescribed routings, but the number of feasible sequences grows rapidly as m and n increase.

Consider the case of 10 machines and 10 jobs, each job having one operation on each machine, for which the objective is to minimize the total time to complete all jobs. The number of possible routings is: $(10!)^{10} \approx 4 \times 10^{65}$. Even if only .1% are feasible, the number of sequences from which the optimum is to be selected is greater than 10^{62}. This approach is called enumeration and is only feasible when both m and n are small.

The combinatorial problem has been solved for $m \leq 3$. Johnson [27] has developed an algorithm which provides optimum sequences for the case of two machines and n jobs where each job is processed first, on machine 1 and then, on machine 2. The objective is to minimize total processing time for all n jobs when the processing times are known. Johnson proved that the optimum solution has the same sequence of jobs on machine 2 as occurred on machine 1. Attempts to extend this approach to cases having more than 2 machines, have failed except that Johnson was able to obtain optimal solutions for m = 3 for the special cases when the minimum processing time on machine 1 is greater than the maximum processing time on the second machine, or the minimum processing time on the third machine is greater than the maximum on the second.

Mitten [33] has extended the 2 machine, n job problem to include specified arbitrary time lags between processing of each job. Mitten specifies certain prescribed time lags as start-lag, the minimum time between the start of a job on machine 1 and machine 2, and stop-lag, the minimum time between completions of a job on machine 1 and machine 2. Johnson [26] points out that this problem has the same solution as his 3-machine case.

Giffler and Thompson [14] have approached the problem with the view that the set of all feasible solutions includes a subset of optimum solutions. They have developed algorithms for generating one, or all, schedule(s) of a particular subset of all possible schedules, called "active" schedules. This subset contains a subset of the optimal schedules. Giffler [16] describes the method used for obtaining an "active" schedule as essentially one of generating one possible array of jobs on a Gantt Chart. From this array the length of time is easily determined. Two cases may arise:

1. If the problem is small enough (relative to the size of computer available) it can be solved by enumerating all "active" schedules and choosing the shortest.

2. If total enumeration is impractical, generate as many "active" schedules as is practical and choose the best.

The second approach is predicated on the fact that there is a finite probability that the "sample" generated contains an optimum schedule. This probability can be made as near to unity as desired by increasing the number of feasible schedules generated. Even if an optimum schedule is not in the generated sample, one which is near-optimum (almost as good) is very likely.

Giffler, Thompson, and Van Ness [15] have programmed the algorithms for the IBM-704 computer. The complete enumeration and the Monte Carlo techniques were tested under two cases:

1. The non-numerical case in which the processing time is unity for each job on each machine.

2. The numerical case in which processing times may be other than unity.

Complete enumeration of the feasible schedules for a shop having 6 machines, 6 jobs and 5 operations per job with all processing times equal to unity was started, but the generation was halted after 70 minutes, since only 84,802 feasible schedules of the total number of approximately eight million had been generated. The results confirm our expectation that only very small problems can be solved economically.

Tests of the Monte Carlo Algorithm indicated that the technique is promising if the probability of observing an optimum schedule is .02 or greater. In this case, the Monte Carlo process is 98 percent certain of generating an optimum schedule in a sample of only 200. The time to generate 200 feasible schedules is only a few minutes for practical size problems. As the probability of finding an optimum schedule decreases, the number of samples required increases rapidly. Unfortunately, optimum schedules do not appear with a frequency of two percent in practical problems.

Integer Linear Programming Approach

Bowman [2], Wagner [47], and Manne [29] have reported separate formulations of the job-shop problem which give optimum solutions using integer linear programming techniques. The objective in each case is to minimize the total time to process n jobs on m machines subject to certain constraints. The constraints for each of these formulations differ widely and depend upon the assumptions included in the model. The constraints used by Manne in his formulation will serve to illustrate.

Manne's formulation includes non-interference, sequencing, and due-date restrictions. Non-interference restrictions prevent the processing of more than one job on a particular machine at a time. These restrictions take the form that the difference in time between the start of processing of one job on a machine and the start of the next job on that machine must be greater than or equal to the processing time for the first job. Sequencing restrictions occur when there is some precedence relation which requires that job j precedes job k on the machine. This restriction has the form that the start time for job k is at least A_j time units after the start of job j, where A_j is the processing time for job j. Due-date restrictions are necessary to assure that individual delivery requirements are satisfied for each job and have the form that the start date for job i on its last operation plus its processing time must be equal to or less than its due-date.

Since each of the three formulations mentioned above requires that some solution variables be restricted to integer values, an all-integer linear programming algorithm, which was developed by Gomory [17], is used. Manne points out that his formulation is not computationally feasible for solving problems of practical size, but his formulation does lead to optimum solutions. For a shop having 5 machines and 10 jobs, each job having one operation on each machine, Manne's formulation leads to a total of 275 unknowns, excluding slack variables. Wagner's formulation for the same problem results in 600 unknown variables, again excluding slack variables. If difficulty of solution is related to the number of constraints, Manne's formulation is computationally more feasible than Wagner's. Further tests of the computational aspects of the integer linear programming problem by Wagner and Story [45] have led to the conclusion that no integer programming method exists which can solve practical job shop problems rapidly. Bowman's formulation is also computationally inadequate for practical size problems.

Queuing Approach

In the queuing approach the job-shop is treated as a network of service centers (machine groups), each consisting of one or more homogeneous channels (machines). Jackson [21] has shown that the job-shop behaves like a collection of independent elementary waiting-lines under the following conditions:

1. Jobs are selected for processing on a first come, first served priority system.

2. Job arrivals into the shop are distributed as a Poisson process.

3. A job leaving one machine group goes to another or is finished.

4. Processing times are exponentially distributed.

The basic advantage of the queuing approach is that the effects of job arrival rates, job processing rates, and priority rules upon certain measures of shop performance can be anticipated from queuing theory.

Jackson [24] has demonstrated that, in the one machine case, the maximum lateness of jobs, which have due-dates assigned, is minimized by a priority-rule which selects the jobs for processing in the order of their due-dates.

Many other studies [7, 18, 19, 30, 31, 39] have been made for one machine group cases to determine the behavioral patterns under various conditions. No attempt will be made to itemize the results of these studies here.

Although the simple one-machine queuing approaches to the sequencing problem provide some basis for predicting the effects of arrival rates, processing rates and dispatching rules in the job-shop, they do not supply sufficient information for loading, scheduling, and dispatching in a job-shop of practical size so that a proper balance among in-process inventory cost, costs of late delivery and cost of idle capacity is obtained.

Holt [18] has taken the approach that a "global" analysis of the job-shop when viewed as a network of queues can be set up in such a way that an optimal allocation of queue delays for individual products is obtained. The problem is that m jobs are to be processed on n machines with the operations to be performed in a specified sequence. Each job has a given due-date, T_i. Holt formulates a total expected cost equation which includes holding costs for all jobs in-process and penalty costs for late deliveries as follows:

$$EC = \sum_{i=1}^{n} \sum_{s=1}^{j} v_i (S) \left[Q_i (S) \right] + \sum_{i=1}^{n} f_i \left\{ t_i + \sum_{s=1}^{j} \left[Q_i(S) + M_i (S) \right] - T_i \right\}$$

where,

i is the job index.

s is the operation index.

$V_i(S)$ is the holding cost factor for the i^{th} job waiting for its S^{th} operation.

$f_i \{\}$ is the function of penalty cost for lateness of the i^{th} job.

t_i is the time of entry for the i^{th} job into the shop.

$M_i(S)$ is the processing time for the i^{th} job at operation S.

T_i is the due-date for the i^{th} job.

For a specific set of jobs from a particular job-shop, the costs can be maximized with respect to Q_i and t_i subject to certain restrictions. The restrictions are of the following form:

$Q_i (S) \geq 0$,

$t_i \geq t$,

and $$\frac{\sum_{i=1}^{k} Q_{ij}}{K} = g_j \left[1 - \frac{1}{M_j} \sum_{i=1}^{k} M_{ij} \right]$$

where

t = earliest possible start date.

Q_{ij} = processing time for the i^{th} job at j^{th} machine group.

K = no. of jobshaving operations at j^{th} machine group.

M_j = mean processing time for j^{th} machine group.

The latter restriction assumes that a stable relation exists between the average queue delay at a machine group and the expected idle capacity at that machine group. This restriction is necessary to assure that the average idle capacity at any machine does not fall below a specified minimum. Holt assumes that an economic balance between the cost of the idle capacity and the costs of additional waiting can be obtained experimentally.

The solution of the global problem does not appear to be computationally feasible for a large job-shop. One further problem exists, even when the global problem is solved, in that a procedure for enforcing the adherence to the optimum queue delays is required. Holt proposes three dispatch-priority rules which are possible means for achieving this goal:

1. The Time Schedule Priority Rule.
2. The First Queue Cost Priority Rule.
3. The Second Queue Cost Priority Rule.

The Time Schedule rule is designed to select jobs in a sequence so that start-date schedules for each operation are met. The start-date schedules are generated using the optimum $Q_i(S)$ from the global solution. The two Queue Cost Rules attempt local optimization through minimization of queue time at each machine group subject to delivery restrictions.

Computationally, Holt's global solution appears to offer more promise than the Combinatorial Approach but no experimental results have been reported.

Reinitz [38] has developed an approach to the total job-shop problem in which a job is looked upon as a member of a population of jobs so that the statistical properties of the population can be evaluated in terms of the influence of the job-shop parameters. Such parameters may be labor and machine capacities, capability of the personnel, storage facilities, etc. In the model developed, the basic assumptions are that the job-shop system is a Markov Process and that no attempt need be made to know the location of a job at any time, but only the probabilities that it will be in various locations at a specific time.

A sequencing method has been proposed by Ackerman [1] for a job-shop in which the total time a job spends in the shop is much greater than the total processing time. When this is true, the job-shop can be treated as an assembly line in which the machine groups are the stations, and the work to be done at each station consists of all the operations which must be performed if the jobs are to flow evenly from machine group to machine group. When the above is true, the job-shop interval required to complete a job is a function of the number of operations only; therefore, the scheduling rule could be to allow one time unit (e.g. one week) for completing each operation. Then, each machine group could be loaded with all of the jobs scheduled for completion on that machine during the particular time period. The dispatching rule would simply be to transmit each job to its next operation at the end of each week. Jobs which fall behind schedule could be transmitted upon completion instead of waiting until the end of the week.

Thus, it would be possible for a job which is delayed because of material shortages, engineering changes, etc. to process through two or more operations within one time period.

Ackerman proposes that the difficulties of maintaining a balance between machine groups, caused by minor variations in product mix, could be overcome by expending overtime in order to assure that all work scheduled for a particular time period is completed during that period. Results of tests in which this method is compared to other dispatching procedures are given, but no attempt is made to show that job-shops actually exist in which the best economic policy is to make the job interval a function of the number of operations.

General Simulation Approach

The failure to find computationally feasible methods of solution of job-shop sequencing problems of practical size has led researchers to develop numerous decision rules, which can be compared through computer simulation. The most general usage of simulation is obtained by simulating an actual job-shop under current operating conditions and then, testing the effects of "proposed" changes in the system through subsequent simulations. Simulation is also useful in evaluating the relative merits of different scheduling and dispatching rules using empirical data.

The majority of the simulations in connection with job-shop sequencing has been reported by two groups: a group at U.C.L.A. (Jackson, Kuratani, McKenney, Nelson to name a few) and a group at Cornell University (Conway, Maxwell, Johnson, etc.).

Sisson [43] has given the following statement of the work at U.C.L.A. (obtained from a letter from R. T. Nelson, June 2, 1959.):

> "We have a simulation model of a general job shop production process (general, in the sense that it is meant to include processes with no attempt made to simulate the details of any particular shop).
>
> The model is a simulation model which takes into account the following basic factors:
>
> 1. Mean arrival rate of jobs in shop.
> 2. Mean service time at each machine center.
> 3. Shop size.
> 4. Form of distribution of job arrivals in shop.
> 5. Form of service time distribution.
> 6. Job routing probability distribution.
> 7. Lot size variation vs. operation complexity variation.
> 8. Priority rule for job assignment (queue discipline).
>
> The actual simulation of production deals with a continuous statistical input of jobs to the shop. The factors above will be assigned different levels with each combination of parameter values constituting one run. Output such as flow time distributions, tardiness, etc. will be recorded for each run. Experimentation will include:
>
> 1. Analysis of variance to measure effects of the factors in the model on certain output quantities.
> 2. Evaluation of the decision parameter (priority rules) over a range of parameter values and relative to various output quantities."

In general, a dispatching rule of the following form has been used in the tests reported [19]:

$$\varphi = D_i - a \sum_{j=K+1}^{n} P(i,j) - b \sum_{K}^{n} W(k,j) - C\,P(k,k)$$

where,

D_i = due-date of the i^{th} job.

$P(i,j)$ = processing time for the i^{th} job on the j^{th} machine.

$W(i,j)$ = expected waiting time for the i^{th} job at the j^{th} machine.

a, b and c are weighting-constants.

The job with the least φ is selected from queue for processing. The criterion function used was to minimize the maximum lateness of all jobs.

In a letter to Sisson [43] on June 15, 1959, Conway described the activities of his group as follows:

> "With the assistance of several graduate students I am presently working on an investigation of the properties and behavior of networks of queues. We are concerned with three measures of performance: system inventory, throughput, and the distribution of unit completion times. We are investigating the effect upon these measures of different precedence (dispatching) rules; different disciplines (flexibility in routing, in specification of servor); different arrival and service distribution; and different load characteristics (intensity, balance, routing). We are interested in both steady-state and transient behavior.
>
> Most of this investigation is experimental and is being conducted by means of digital simulation."

References [4, 5, 6, 8, 9, 10] describe some of the studies reported by the research group at Cornell University. These studies, in general, have investigated the effects of "static" priority rules. Static rules consider only the local properties - processing time on the current operation, scheduled start date, due-date, dollar-value and time of arrival - in determining the order of selection of jobs for processing. Some static rules which have been compared are [8]:

1. First Come, First Served.
2. First Come, First Served within Dollar-value Class.
3. Shortest Processing Time for Present Operation.
4. Longest Processing Time for Present Operation.
5. Earliest Planned Start Date.
6. Earliest Due Date.
7. Random.

Conway and Maxwell [10] have reported that the Shortest Processing Time for Present Operation rule is optimal with respect to aggregate measures of performance for each of the following conditions:

1. In simple n job, 1 machine sequencing problems.
2. In simple queuing systems with exponentially distributed inter-arrival times.
3. In a system consisting of a network of queues when compared to other static priority rules.

Results of attempts to reduce some of the disadvantages of the Shortest-Operation Rule by alternating its use with the First Come, First Served Rule have also been reported [11].

While the work of the research group at Cornell has been oriented toward simulations using static priority rules, Rowe [39] has reported studies which use "dynamic" dispatching rules. "Dynamic" rules take into account factors such as: the remaining number of operations, the remaining expected waiting-time per operation and the remaining processing time for each job.

Rowe's approach is to break the job-shop sequencing problem into two phases - scheduling and dispatching. In the scheduling phase, start dates are generated for each operation. Flow allowances, which are related to the expected waiting time, are used in conjunction with the processing times and the due-dates to establish start dates. The dispatching phase includes application of a priority rule which will give the best aggregate performance with respect to all measures of performance. Simulation experiments are used to test various dispatching rules and to determine better flow allowances.

Rowe [40] summarizes his appraisal of the problem as follows:

"The behavior of a job lot production system is extremely complex and determination of optimal decision rules is a difficult problem. The present study was concerned with evaluating the applicability of Sequential Decision Rules (flow allowances) to the scheduling problem. Decision rules which are based on the value of parts being processed appear to provide reduced costs while still assuring a desired completion level......To insure that the planned flow rates would be carried out, a priority queue discipline was established based on correcting for deviations from the planned flow. In this way, decisions were made sequentially rather than attempting to predict the precise job assignment permutation. Monte Carlo simulation was used to evaluate the sequential rules under various shop conditions. This approach appears to provide an extremely flexible means for studying the behavior of complex systems where analytical formulations are not available. Statistical experiments can be carried out, including replication, which would otherwise be impossible directly in the factory. Computer simulation also provides a means for evaluating some of the interdependencies in a production system."

We have seen that the combinatorial approach to the job-shop problem will have little practical value, even when sampling is used, until much faster computational facilities are available.

Integer linear programming solutions are better, but practical size problems cannot be solved using presently available computers. An additional restriction to the linear programming approach exists. A practical job-shop formulation requires that some solution variables be integers and some be non-integers. Techniques for the solution of the "mixed" problem have not been developed.

For the present, at least, it seems that better sequencing techniques can only be obtained through logical development of decision rules which can be subsequently tested and improved upon through simulation. Of course, analytical studies of simple, deterministic models will give researchers some basis for developing logical decision rules.

Chapter III

OBSERVATIONS AND CONSIDERATIONS

Characteristics of the Job-Shop

A job-shop can be looked upon as a set of multi-channel queuing centers. Each queuing center is composed of a number of homogeneous machines, which are treated as parallel service channels. Each machine within a machine group must be capable of processing any job which arrives at that group for service. The jobs which arrive at a machine group and are required to wait for service must form in a single queue. Queues are not allowed to form at individual machines within a group.

Elementary queuing theory shows that the queue length will tend toward infinity if the mean arrival rate of jobs exceeds the effective service rate over a continuous period of time. The effective service rate is the mean service rate which has been adjusted to take into account the time that a service facility is expected to be idle while waiting for additional jobs to arrive. Thus, when the arrival rate and the service rate are not deterministic, an overload condition cannot be avoided unless the mean arrival rate is less than the mean service rate at each machine group.

For an entire job-shop the problem of balancing arrival rates with effective service rates is compounded because of the interactions which might cause deviations from the expected arrivals of jobs from other machine groups. In general, a job-shop having a specified arrival rate must have an effective service rate that is, at least, equal to the arrival rate. In addition, the product mix of the arriving jobs must be distributed in such a manner that the effective capacity of each machine group is not exceeded.

Consider a job-shop for which the product mix of the arriving jobs are distributed so that none of the machine groups are overloaded. If the shop is allowed to operate over a continuous period of time, the mean rate of departure of completed jobs tends to become equal to the mean arrival rate and the number of jobs in-process will tend toward a fixed level. Of course, if the number of jobs arriving or the product mix tends to cause overload conditions, the shop will not tend toward an equilibrium level, but the number of jobs in process will increase without bound.

The steady-state or equilibrium level of the shop has a direct influence upon the efficiency of the job-shop operations. Consider a shop having an equilibrium level of 150 jobs when the arrival rate is 30 jobs per day. The average processing interval (i.e. the expected time between the release of a job to the shop and its completion) will be $\frac{150}{30} = 5$ days. If the shop had an equilibrium level of 180 jobs, the average interval would be 6 days. Thus, it is obvious that the average number of jobs in the shop affects the average amount of time that a job spends in the shop and therefore, influences the cost of carrying in-process inventory. Also, the average cycle time, which is important to prospective customers, is increased by higher steady-state numbers of jobs.

Although a high level of in-process inventory is undesirable from the standpoint of cycle time and inventory carrying costs, it is a desirable factor in maintaining a high utilization of facilities and manpower. In order to maintain utilization at a relatively high level, jobs must be waiting when facilities become available. When a facility must wait for the arrival of a job, there is a loss in actual production capacity because of the idle time. Thus, a procedure which maintains a balance between the cost of carrying in-process inventory and the cost of idle capacity is required.

The delivery of the completed product to the customer on time is another important consideration. The due-date, which is assigned prior to the arrival of each job in the shop, has a great deal of significance in most job-shops since there is a penalty associated with failures to complete jobs on time. There is also a penalty for completing a job early since the product must be carried in inventory until its due-date.

The equilibrium level of the shop also has important effects upon the distribution of deliveries. If the level of the shop is relatively low, a job should tend to flow through the shop with relatively little competition with other jobs for machine capacity. Thus, the expected total processing time could be predicted with accuracy, and the performance of actual completions versus due-date should be improved. However, if the equilibrium level is sufficiently high to give high utilization of facilities, complex interactions develop among the competing jobs and prediction of total processing time becomes more difficult. The resulting performance of actual deliveries versus due-dates becomes less consistent at higher levels of in-process inventory.

It is reasonably obvious at this point that the primary objective in the operation of a job-shop must be to perform the functions of loading, scheduling and dispatching in such a manner that a balance among the costs of carrying in-process inventories, the costs of late deliveries, and the cost of idle capacity is obtained.

The determination of the actual costs which are associated with these factors is a very difficult problem. Consider the cost of late deliveries. For a particular job the penalty may be a fixed amount for each day of lateness (e.g. fifty dollars per day). However, suppose that a second product, which is required for assembly with the first product, is also late. There is no additional loss, over that loss associated with the lateness of the first job, until the first job has been delivered. Similar problems arise in determination of the other costs. The problem of determining costs will not be considered further.

Many unpredictable problems arise in the operation of a job-shop which grossly affect the steady-state conditions and the consistency with which performance can be predicted. Some of the more troublesome of these problems include:

1. Variability of product mix
2. Variation in processing times
3. Unexpected machine breakdown
4. Employee absenteeism
5. Special or expedited jobs
6. Material shortages
7. Engineering difficulties

The long-range average effects of some of these factors can usually be minimized by adjusting the effective capacity. Others can only be handled as they occur through special management action (i.e. work overtime, change shop parameters, etc.). The short-range effects in some cases must be accepted as random fluctuations which are inherent in the process.

Even when a problem arises, determining the necessary corrective action to be specified is not as simple as it might appear on the surface. For example, an unanticipated machine breakdown will not only delay those jobs waiting for service at that particular machine group but may cause other facilities to become idle because certain jobs failed to arrive for processing as anticipated. It may seem that the specification of overtime hours equal to the total down time of the machine on which the breakdown occurred would solve the problem; however, this action would not alleviate the bottlenecks created at the other machine groups.

Proposal

The sequencing problem in the job-shop can be divided into two inter-related phases, scheduling and dispatching. The scheduling phase is applied prior to production of the jobs in order to establish either the start dates when due-dates are known or the due-dates when start-dates are given. The dispatching phase is used for determining the order in which jobs will be processed to satisfy, economically, the requirements specified by the scheduling phase. These phases depend upon the loading phase to determine that the number or the product mix of jobs will not cause any overload conditions.

There are two scheduling procedures, forward scheduling and backward scheduling, which are applicable to the scheduling problem. The choice between the use of the two rules depends upon whether a due-date is associated with the incoming jobs. If a due-date is given, backward scheduling is used to generate an expected start-date; however, in the absence of a due-date, the given start-date is used in conjunction with forward scheduling to generate a due-date.

For either rule, the most important parameter is the amount of time that a job will be delayed during its processing through the shop. Since the delay time for a job may vary widely from one machine group to another, the expected waiting time at each operation must be specified. Using the expected delay time and the expected processing time for each operation, a total expected cycle time, T, can be computed for each job in the following manner:

$$E(T_i) = \sum_{j=1}^{L_i} \left[E(Q_{ij}) + E(P_{ij}) \right]$$

where,

i is the job index,

j is the operation index,

L_i is the number of operations for the i^{th} job,

$E(\ )$ is the mean or the expected value of the variable,

Q_{ij} is the delay of the i^{th} job at its j^{th} operation,

P_{ij} is the processing time for the i^{th} job at the j^{th} operation,

and T_i is the total processing interval for the i^{th} job.

If the due-date is given, the start-date (the latest date that the job can be released to the shop and expect to be completed on time with normal processing) can be determined by subtracting the expected cycle time from the due-date. Due-dates are determined by adding the expected cycle time to the given start-date when forward scheduling is used.

The expected waiting-time for a job at each of its operations is a function of the dispatching rule used and the load on the machine group as well as the factors which contribute to the complex interactions that exist in the shop. It is often advantageous to assign different waiting-time allowances to jobs according to a plan which classifies jobs into categories determined by some property of the job.

One important classification plan is to segregate jobs into value classes and assign waiting-time allowances according to value class such that high-valued jobs are assigned low waiting-time allowances. The result is that the cycle time for the high valued jobs is reduced at the expense of increased cycle times for the low valued jobs. This will affect a reduction in the total value of in-process inventory. However, the performance of deliveries versus due-date may be upset by this procedure.

Dispatch rules, in general, are used to either enforce the plans specified by the scheduling procedure or to optimize some pre-determined objective. For example, the average waiting time of jobs in queue is minimized by the dispatch rule which selects jobs for processing according to the shortest operation time for the present operation [10].

The objective of this thesis is to study the effects of three different dispatching rules upon the job-shop when used in conjunction with a scheduling rule which divides the jobs into three value classes, high, medium and low, and assigns delay allowances according to the value class to which the job belongs at each operation. The dispatching rules are:

1. First-come, first served within dollar value class (FCFSV)
2. Earliest due-dates (MINDD)

3. Minimum slack time remaining per operation (MINSOP)

Rule 1 selects jobs for processing in the order in which they arrived. Rule 2 selects the job from queue which has the most imminent due-date. Rule 3 computes priority numbers for each job in queue according to the amount of slack time per remaining operation in the following manner:

$$N_i = \frac{(P.D.) - (D.D.) - \sum_{j=S_i}^{L_i} \overline{P}_{ij} - \sum_{j=S_i+1}^{L_i} \overline{Q}_{ij}}{L_i - S_i}$$

where,

i is the job index,

j is the operation,

(P.D.) is the present date,

(D.D.) is the due-date for the i^{th} job,

$\overline{P}_{ij}$ is the expected processing time for the i^{th} job, at the j^{th} operation,

$\overline{Q}_{ij}$ is the expected delay time for the i^{th} job at the j^{th} operation,

L_i is the total number of operations required to complete the i^{th} job, and

S_i is the operation number of the current operation for the i^{th} job.

This dispatch rule then selects from the queue the job which has the minimum priority number.

In order to test the effects of the scheduling and dispatching rules a model of a job-shop was formulated. The following conditions and restrictions were used to partially describe the model:

1. Inter-arrival times for the input jobs are distributed according to a negative exponential distribution.

2. Processing times for all jobs at each machine group are distributed according to a negative exponential distribution.

3. A machine can process only one operation at a time.

4. Each operation, once started, must be performed to completion.

5. Jobs will not be split into two or more groups for expediting processing.

6. The routing and processing times are known for each job.

The study of the scheduling and dispatching rules was made using a Job Shop Simulator[1] [47] for the IBM-7040 computer.

1. The original IBM-704 Job Shop Simulator was reprogrammed for the IBM-7040 computer by Wayne R. Maple, IBM Data Center, Chicago, Ill.

Chapter IV

THE JOB SHOP SIMULATOR

The Job Shop Simulator utilizes an IBM-7040 computer with an input sequence of jobs, each of which is to be released to the shop at random intervals of time. Each job is represented by certain information such as the job identification number, the sequence of operations to be performed, the processing time for each operation, and the initial material cost for each job.

Associated with each machine group is a single waiting line of jobs to be processed. By progressing step by step through short intervals of simulated time and examining the status of each job and each machine at the end of each interval, the computer is able to apply specified decision rules for each situation in which an operation is completed or a new job arrives in the shop.

During the simulation process, the computer gathers statistics concerning machine utilization, idle capacity, average waiting times and average queue lengths by value classes for each machine group. The number of job completions and the lateness or earliness of each job with respect to its due-date are also tabulated.

The computer program package for the Simulator is divided into four sections. They are:

1. Order Generation
2. Scheduling
3. Simulation
4. Output

There is also an auxiliary program for use in studying the distributions of processing times and dollar-values for each machine group; it is called the Order Analyzer.

Order Generation

The Order Generator is provided to generate a number of synthetic orders for input to the Simulator; however, if real or actual orders are already available, the order generation section can be by-passed. The input requirements include a mean machining time for each machine group, a mean initial material cost and a transition matrix of probabilities of making transitions from any machine group to any other machine group. The transition matrix also includes for each machine group, the probability that an operation just completed is the last operation for that job and the probability that a job entering the shop will have its first operation at a certain machine group.

The transition matrix for the model used in this study is shown in Table 1. Row 1 of the matrix gives the probabilities that a job entering the shop will have its first operation on each of the five machine groups. The probabilities that a job leaving machine group 1 will have its next operation on any of the other four machine groups or will leave the shop as a completed product are given in Row 2. Notice that the probability of a job arriving in the shop and being completed without having at least one operation is zero and that a job cannot remain at the same machine group for two successive operations. The sum of the probabilities in each row must be 1, since the matrix includes all possible states to which a job can belong.

A Monte Carlo sampling method is used to determine the sequence of operations for each order generated. The Monte Carlo sampling begins by

TABLE 1

Transition Probability Matrix

	1	2	3	4	5	Completion
Arrival	.123	.236	.207	.198	.236	.000
1	.000	.230	.206	.192	.230	.142
2	.130	.000	.222	.210	.250	.188
3	.125	.240	.000	.202	.240	.193
4	.119	.238	.202	.000	.224	.217
5	.118	.236	.198	.188	.000	.260

generating a random number between 0 and 1. The random number is then applied against the cumulative frequency histogram of the first row of the transition matrix to determine the machine group for the first operation of the job. Another random number is generated and used to determine the state to which the job will go after the first operation by applying the random number to the cumulative frequency histogram for the row of the matrix corresponding to the machine group used for the first operation. This Monte Carlo method is continued until the job leaves the shop after the current operation or until a maximum allowable number of operations has been reached. The maximum number of operations allowed is specified as an input parameter to the Order Generator.

Once the routing has been determined, machining time for each operation is generated by Monte Carlo sampling from negative exponential distributions. A negative exponential distribution of processing times is associated with each of the five machine groups in the shop. The means of these distributions are based upon the average machining time for all jobs which are processed by each machine group. The set-up time is computed by taking a fixed percentage of the machining time for each operation. The set-up time plus machining time constitutes the processing time for an operation. For this thesis, the set-up time for each operation is twenty percent of the machining time.

An additional random number is required in order to generate the initial raw material cost of the job. These costs are generated by sampling from a negative exponential distribution having a mean which is given as an input parameter to the computer program. In this case, a mean material cost of three-hundred dollars was used.

The primary deficiency in the order generation program is that no provision is made for randomly generating due-dates.

Scheduling

The scheduling phase of the Simulator calculates the start-date of each job when the due-date is given or the due-date of each job when the start-date is specified. Since the Order Generator does not assign due-dates to the jobs, only the forward scheduling routine can be used unless due-dates are assigned externally; however, random assignment of due-dates can be accomplished by assigning the actual date of arrival of the job as the start-date for that job and then, computing the due-date using the forward scheduling routine. This procedure is equivalent to assuming that each job will be released to the shop on the start-date generated from given due-dates by the backward scheduling routine. The delay-allowances for each value class must be supplied for all machine groups as input parameters.

Simulation

The simulation phase actually carries out the step-by-step processing of each job according to the conditions specified in the model. Some of the input parameters which must be supplied for the simulation phase are:

1. Available work force by labor class
2. Number of operating shifts and the hours in each shift
3. Mean arrival rate of jobs
4. Dispatch Rule to be used
5. Initial shop load
6. Number of days per reporting period and the number of periods of simulation desired

7. Mean transition times

8. Value-class limits

9. Number of operating machines per machine group per shift

The simulation begins by releasing to the shop the number of jobs specified by the initial shop load parameter. The jobs are randomly distributed through various stages of completion which are determined from random numbers. A random number is generated for each job in the initial load and is used to determine the percent of each job's processing requirements which were completed prior to the beginning of the simulation. Since only one job at a time may be processed on a machine, the initialized jobs which arrive after a particular machine group has been loaded must be assigned to the queue for that machine group. This initialization routine permits the shop to reach its equilibrium level within a short time after simulation begins.

The release of jobs to the shop is based upon an arrival rate R, and a negative exponential distribution. The time of the arrival of the next job is determined by Monte Carlo sampling from the negative exponential distribution which has a mean inter-arrival time of $\frac{1}{R}$. Since the inter-arrival times are exponentially distributed, the number of jobs arriving per day follows a Poisson distribution [42].

In the simulation, a new job will be placed at the machine group specified in the routing for its first operation. If all machines at that group are busy, the job must wait in queue. When any machine completes an operation, the dispatch rule is applied to determine which of the jobs waiting in queue will be selected next for processing.

The Simulator keeps track of the status of every job so that the transaction which will occur next can be processed next. A transaction can be a new job arriving in the shop, a job completing an operation, or a job completing its transition from one machine group to another.

Tabulation of Results

As was previously mentioned, certain statistics are kept by the program during the simulation run. The output section of the Simulator tabulates the results of the simulation into several reports. These reports are printed for each period of simulation giving the results for all periods simulated. These reports are:

1. Load Analysis
2. Shop Performance
3. Labor Utilization
4. Analysis of Queues - Current Period
5. Analysis of Queues - Year-to-Date
6. Inventory Carrying Cost
7. Tabulation of Completions

The Load Analysis report shown in Table 2 gives a break-down of the total load on each machine group within the three value-classes, high, medium and low. The total load on each machine group is compared to the capacity of the machine group by taking their ratio. For example, the first row in Table 2 gives the load analysis for machine group 1 during period 2 and also for year-to-date, which includes period 1, period 2 and initialization. The numbers, 91, 390 and 419, indicate the number of scheduled hours of load for period 2 for the high, medium and low value-classes, respectively. The total load is the sum of these numbers

TABLE 2

LOAD ANALYSIS

PERIOD 2

MACHINE GROUP	MACHINE GROUP DESCRIPTION	THIS PERIOD LOAD H HRS	LOAD M HRS	LOAD L HRS	LOAD T HRS	AVAIL CAPAC HRS	LOAD CAP	YEAR TO DATE LOAD H HRS	LOAD M HRS	LOAD L HRS	LOAD T HRS	AVAIL CAPAC HRS	LOAD CAP
1	111111	91	390	419	900	810	1.112	256	782	776	1814	1620	1.119
2	222222	60	200	251	512	540	.948	153	410	505	1069	1080	.990
3	333333	87	364	531	983	1170	.840	230	749	1067	2047	2340	.875
4	444444	108	272	363	744	810	.919	231	553	823	1608	1620	.992
5	555555	81	284	326	692	720	.961	208	572	696	1477	1440	1.026
TOTAL		429	1512	1891	3833	4050	.946	1079	3068	3869	8017	8100	.989

TABLE 3

SHOP PERFORMANCE

PERIOD 2

LABOR CLASS	MACH GROUP	MACHINE GROUP DESCRIPTION	THIS PERIOD AVAIL CAPAC HRS	UTIL CAPAC HRS	IDLE TOTAL HRS	UTIL CAPAC	YEAR TO DATE AVAIL CAPAC HRS	UTIL CAPAC HRS	IDLE TOTAL HRS	UTIL CAPAC
1	1	111111	810	802	7	.990	1620	1610	9	.994
1	2	222222	540	511	28	.947	1080	1035	44	.958
2	3	333333	1170	1007	162	.861	2340	1966	373	.840
2	4	444444	810	726	83	.896	1620	1520	99	.938
3	5	555555	720	653	66	.907	1440	1372	67	.953
TOTAL			4050	3701	348	.913	8100	7506	594	.926

and is equal to 900 hours. The actual capacity for period 2 is only 810 hours; therefore, a slight overload exists. The remaining numbers in Row 1 give the same types of information for the "year-to-date" period.

The Shop Performance and Labor Utilization reports shown in Tables 3 and 4, respectively, give load versus capacity information which is the results of the actual performance during the simulation. The Shop Performance report gives results based upon machine group utilization while the Labor Utilization report shows the utilization of manpower within each labor class.

The Analysis of Queues report shown in Table 5 includes information concerning the performance of the queue at each machine group. Row 1 gives the number of job arrivals at machine group 1 (high-valued-17, medium-valued-55, low-valued-62, total-134) and the number of job departures by value-class from machine group 1 (16, 53, 52, 121) for period 2. The average waiting-time per job by value-class and the average number of jobs waiting in queue by value-class is also tabulated for each machine group. The Analysis of Queues-Year-to-Date report shown in Table 6 gives the same types of information except that it includes cumulative results of period 1, period 2 and the initial load.

The Inventory Carrying Cost report shown in Table 7 includes the costs of carrying jobs in inventory both while waiting and while processing for each value-class by machine groups. Also, the cost of carrying in inventory those jobs which were completed early is tabulated. The costs are determined by multiplying the average instantaneous value of the jobs at each machine group by the annual interest rate

TABLE 4

Labor Utilization

PERIOD 2	THIS PERIOD			YEAR TO DATE		
Class	Avail Man Hrs.	Utiliz Man Hrs.	Utiliz	Avail Man Hrs.	Utiliz Man Hrs.	Utiliz
1	1350	1314	.973	2700	2645	.979
2	1980	1734	.875	3960	3487	.880
3	720	653	.907	1440	1372	.953
TOTAL	4050	3701	.913	8100	7506	.926

TABLE 5

ANALYSIS OF QUEUES

PERIOD 2

THIS PERIOD

MACH GROUP	ARRIV H NO.	ARRIV M NO.	ARRIV L NO.	ARRIV T NO.	DEPART H NO.	DEPART M NO.	DEPART L NO.	DEPART T NO.	Q-TIME H DAYS	Q-TIME M DAYS	Q-TIME L DAYS	Q-TIME T DAYS	AVER Q H NO.	AVER Q M NO.	AVER Q L NO.	AVER Q T NO.
1	17	55	62	134	16	53	52	121	.031	.141	.290	.191	.0	2.0	5.4	7.6
2	29	83	95	207	29	88	97	214	.019	.055	.080	.062	.1	1.1	1.9	3.1
3	22	80	99	201	25	82	98	205	.007	.026	.027	.024	.0	.4	.6	1.1
4	24	72	86	182	23	66	89	178	.013	.025	.045	.033	.0	.4	.9	1.4
5	28	86	105	219	32	83	95	210	.015	.031	.057	.040	.0	.7	1.7	2.5
TOTAL	120	376	447	943	125	372	431	928	.016	.050	.081	.060	.4	4.8	10.7	16.0

TABLE 6

ANALYSIS OF QUEUES

PERIOD 2

YEAR TO DATE

MACH GROUP	ARRIV H NO.	ARRIV M NO.	ARRIV L NO.	ARRIV T NO.	DEPART H NO.	DEPART M NO.	DEPART L NO.	DEPART T NO.	Q-TIME H DAYS	Q-TIME M DAYS	Q-TIME L DAYS	Q-TIME T DAYS	AVER Q H NO.	AVER Q M NO.	AVER Q L NO.	AVER Q T NO.
1	30	99	119	248	28	93	104	225	.046	.231	.260	.222	.1	2.9	4.4	7.5
2	64	168	207	439	64	167	205	436	.041	.051	.065	.056	.2	1.0	1.6	3.0
3	51	147	199	397	51	145	192	388	.007	.019	.020	.018	.0	.3	.4	.8
4	45	133	170	348	44	123	167	334	.050	.082	.063	.068	.2	1.2	1.3	2.8
5	58	163	214	435	56	157	201	414	.059	.113	.090	.094	.3	2.3	2.4	5.2
TOTAL	248	710	909	1867	243	685	869	1797	.040	.088	.084	.080	1.1	7.9	10.4	19.5

TABLE 7

INVENTORY CARRYING COST IN $ PER ANNUM

INTEREST RATE 20.0 PERCENT

PERIOD 2	THIS PERIOD						YEAR TO DATE					
GROUP	Q(H)	Q(M)	Q(L)	Q(T)	M(T)	RATIO	Q(H)	Q(M)	Q(L)	Q(T)	M(T)	RATIO
1	25	244	192	462	994	.465	40	301	176	518	989	.523
2	40	121	77	240	552	.434	72	116	71	259	574	.451
3	10	57	25	93	1132	.082	12	37	20	70	1087	.064
4	19	49	41	110	796	.138	60	157	55	273	838	.325
5	27	80	74	182	739	.246	94	258	105	458	761	.601
TOTAL	123	553	412	1089	4216	.258	280	871	428	1579	4252	.371
EARLY JOBS		54	38	73	4216	.017	57	165	52	275	4252	.064
GRAND TOTAL	123	588	450	1162	4216	.275	338	1036	480	1855	4252	.436

(20 percent in this case). Both current period and year-to-date results are tabulated; however, the year-to-date results are not simply the averages of the cumulative results for all previous periods of simulation but are computed from the average instantaneous values taken over a longer period of time.

The final report is the Tabulation of Completions. This report, which is shown in Table 8, indicates the number of jobs in each value-class which were completed during period 2 as well as the relative lateness of each delivery with respect to its due-date. The deliveries are further broken-down into categories corresponding to late-arrival or on-time arrival into the shop. For this simulation all jobs were forced to arrive on-time. From Table 8, it can be seen that 31 high-valued, 73 medium-valued and 61 low-valued jobs were completed on their due-dates; however, 2 medium-valued and 4 low-valued jobs were delivered one day early. A total of 10 jobs were delivered one day late during period 2. The year-to-date tabulation includes completion data for periods 1 and 2.

Order Analysis

Although the Order Analyzer is not an integral phase of the Simulator, it is extremely helpful as an auxiliary program in the analysis of processing time and dollar-value distributions. The input to this program is the synthetic or actual orders to be used in the simulation. The output is a tabulation of the number of jobs which fall within specified categories of processing time and dollar-value for each machine group. Table 9 shows the output of the Order Analyzer for machine group 1. The first row of the report gives the mean value

TABLE 8
TABULATION OF COMPLETIONS

PERIOD 2

THIS PERIOD

EARLY OR LATE	ON TIME RELEASED H	M	L	T	LATE RELEASED H	M	L	T	ALL ORDERS H	M	L	T
E31+UP												
E26-30												
E21-25												
E16-20												
E11-15												
E 6-10												
E 5												
E 4												
E 3												
E 2												
E 1		2	4	6						2	4	6
0	31	73	61	165					31	73	61	165
L 1	4	2	4	10					4	2	4	10
L 2												
L 3												
L 4												
L 5												
L 6-10												
L11-15												
L16-20												
L21-25												
L26-30												
L31+UP												

YEAR TO DATE

EARLY OR LATE	ON TIME RELEASED H	M	L	T	LATE RELEASED H	M	L	T	ALL ORDERS H	M	L	T
E31+UP												
E26-30												
E21-25												
E16-20												
E11-15												
E 6-10												
E 5												
E 4												
E 3												
E 2	1	1		2					1	1		2
E 1	1	13	9	23					1	13	9	23
0	50	142	161	353					50	142	161	353
L 1	8	2	6	16					8	2	6	16
L 2												
L 3												
L 4												
L 5												
L 6-10												
L11-15												
L16-20												
L21-25												
L26-30												
L31+UP												

TABLE

ORDER ANALYSIS - MACHINE GROUP I

MACHINE 111111 MEAN VALUE 465.32 MEAN TIME 6.56

DEC.		VALUE	1	2	3	4	5	6	7	8	9	10	TOT.	PCT.	CPCT.
	LRAN.		70.00	259.26	448.52	637.78	827.04	1016.29	1205.55	1394.81	1584.07	1773.33			
TIME		LR/AV.	0.15	0.56	0.96	1.37	1.78	2.18	2.59	3.00	3.40	3.81			
1	2.20	0.34	111.	77.	80.	41.	25.	10.	8.	1.	0.	3.	356.	61.59	100.00
2	5.82	0.89	29.	29.	19.	7.	3.	0.	2.	1.	0.	2.	92.	15.92	38.41
3	9.44	1.44	23.	18.	7.	6.	4.	4.	1.	0.	0.	1.	64.	11.07	22.49
4	13.06	1.99	10.	5.	4.	5.	1.	1.	1.	0.	0.	0.	27.	4.67	11.42
5	16.68	2.54	7.	5.	2.	1.	1.	2.	0.	0.	0.	0.	18.	3.11	6.75
6	20.31	3.10	2.	2.	2.	1.	0.	0.	1.	1.	0.	0.	9.	1.56	3.63
7	23.93	3.65	3.	0.	3.	0.	0.	0.	0.	0.	0.	0.	6.	1.04	2.08
8	27.55	4.20	1.	1.	0.	0.	1.	1.	0.	0.	0.	0.	4.	0.69	1.04
9	31.17	4.75	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.17	0.35
10	34.79	5.30	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	1.	0.17	0.17
TOTAL			186.	138.	118.	61.	35.	18.	13.	3.	0.	6.	578.		
PCT.			32.18	23.88	20.42	10.55	6.06	3.11	2.25	0.52	0.	1.04			
CPCT.			100.00	67.82	43.94	23.53	12.98	6.92	3.81	1.56	1.04	1.04			

TABLE 10

ORDER ANALYSIS - TOTAL

FINAL VALUE VS. TOTAL TIME MEAN VALUE 538.80 MEAN TIME 21.60

DEC.		VALUE	1	2	3	4	5	6	7	8	9	10	TOT.	PCT.	CPCT.
	LRAN.		92.00	347.92	603.84	859.75	1115.67	1371.59	1627.51	1883.42	2139.34	2395.26			
TIME		LR/AV.	0.17	0.65	1.12	1.60	2.07	2.55	3.02	3.50	3.97	4.45			
1	2.20	0.10	248.	93.	33.	10.	13.	3.	0.	2.	0.	0.	402.	46.15	100.00
2	13.52	0.63	75.	67.	29.	10.	8.	3.	1.	0.	0.	0.	193.	22.16	53.85
3	24.83	1.15	1.	64.	25.	10.	4.	2.	1.	2.	0.	0.	109.	12.51	31.69
4	36.15	1.67	0.	26.	29.	8.	3.	2.	0.	0.	0.	0.	68.	7.81	19.17
5	47.47	2.20	0.	1.	32.	7.	2.	1.	0.	0.	0.	0.	43.	4.94	11.37
6	58.79	2.72	0.	0.	7.	11.	4.	1.	2.	0.	0.	0.	25.	2.87	6.43
7	70.10	3.25	0.	0.	0.	9.	2.	0.	1.	0.	0.	0.	12.	1.38	3.56
8	81.42	3.77	0.	0.	0.	3.	6.	1.	0.	0.	0.	1.	11.	1.26	2.18
9	92.74	4.29	0.	0.	0.	0.	1.	3.	0.	0.	0.	0.	4.	0.46	0.92
10	104.05	4.82	0.	0.	0.	0.	1.	2.	1.	0.	0.	0.	4.	0.46	0.46
TOTAL			324.	251.	155.	68.	44.	18.	6.	4.	0.	1.	871.		
PCT.			37.20	28.82	17.80	7.81	5.05	2.07	0.69	0.46	0.	0.11			
CPCT.			100.00	62.80	33.98	16.19	[illegible].38	3.33	1.26	0.57	0.11	0.11			

(465.32) of all orders when they arrive at machine group 1, and the mean processing time (6.56 hours) for all jobs which are processed by machine group 1. The second row contains ten numbers which represent the deciles of dollar-value into which the jobs may fall. The third row ("LRAN") gives the lower limit in dollars for each decile (70.00 to 259.25 is the value range for decile 1). The "LR/AV" row is the result of dividing each value in the "LRAN" row by the mean value. The column under "TIME" contains headings for the deciles of processing time into which the jobs may fall. The second column (LRAN) gives the lower limits for each decile (2.20 hours for decile 1). The "LR/AV" column is the result of dividing the "LRAN" time in column 2 by the mean time.

The encircled number, 18, indicates the number of jobs which will have a value of at least $259.26 but not more than $448.51 when they arrive at machine group 1; and which will require at least 9.44 hours but not more than 13.05 hours of processing time on machine group 1. The "TOT" row gives the total number of jobs which fall into each decile of value, and the "TOT" column gives the number of jobs which fall into each decile of processing time. The grand total (578) is the number of jobs which will arrive at machine group 1 if all jobs are completed during the simulation. The "PCT" row and the "PCT" column indicate the percent of the arriving jobs which fall into each value decile and each time decile, respectively. Cumulative percentages are given by the "CPCT" row and the "CPCT" column.

The output of the Order Analyzer also includes similar information about the final values of all completed jobs and the total processing time required to complete each job. This report is shown in Table 10 and its format is identical to the format of Table 9.

Limitations of the Simulator

Although the "Job Shop Simulator" is quite flexible in ability to simulate many of the important details of the shop, certain restrictions do exist which limit its ability to duplicate actual conditions. Some of these limitations are:

1. A job must be completely finished at one machine group before it is moved to the next machine group; therefore, "lap phasing" operations are not permitted.

2. Each job must have a fixed routing through the shop; alternative routes are not allowed.

3. A machine cannot be held open in anticipation of the arrival of a job in the future when an assignable job is already available.

4. A job may not be split into two or more groups for processing on two or more machines within a machine group.

5. A job, once started, cannot be bumped from a machine by a higher priority job.

Even with these restrictions, the Simulator is certainly indispensable in studies of this type.

Chapter V

EXPERIMENTAL PROCEDURE

The general conditions under which the model was used in the simulations were described previously. The job-shop chosen for simulation consisted of five machine groups, having a combined total of forty-five individual machines. The shop operated three, six-hour shifts per day and five days per week. The duration of each simulation run was four periods, each period being one week in length. Output results were obtained for each period of simulation.

Generation and Analysis of Synthetic Orders

The primary task in preparation for the actual simulation was to obtain a set of jobs whose "product mix" would constitute a balanced load upon the shop. There originally was some hope that actual shop orders could be used; however, it became necessary to generate synthetic orders for input to the simulator. The "transition matrix" which was used for randomly generating routing sequences was shown in Table 1. Mean processing times for the five machine groups were specified to be 6.0, 1.25, 5.0, 3.75 and 2.5 hours, respectively.

After the orders were generated, the Order Analyzer program was used to interpret the distributions of processing times and dollar-values for each machine group. The results of this Analysis are tabulated in Table 11. The initial problem was that the mean processing times of all jobs on a machine group did not agree with those specified in the input to the Order Generator. The cause of these deviations was determined, after extensive investigation, to be the result of a restriction in the program. One of the requirements of the

TABLE 11

ORDER ANALYSIS

MACHINE GROUP	PROCESSING TIME (HOURS) SPECIFIED	PROCESSING TIME (HOURS) ACTUAL	NUMBER OF OPERATIONS	PROCESSING TIME (1 MACHINE) (HOURS)	NUMBER OF MACHINES REQUIRED CALCULATED	NUMBER OF MACHINES REQUIRED ACTUAL	MEAN VALUE (DOLLARS)
1	6.0	6.38	1137	7254.0	9.26	9	472.58
2	1.25	2.12	1968	1763.5	6.08	6	482.38
3	5.0	5.49	1742	9563.6	12.23	13	476.63
4	3.75	4.32	1673	7227.4	9.22	9	483.08
5	2.5	3.21	1984	6128.2	8.21	8	465.27
TOTAL	-	20.32	8504	-	45.0	45	539.11

Simulator was that the lowest allowable processing time be specified as an input parameter. Any processing time generated by the Monte Carlo sampling technique that was less than this minimum time was set equal to the minimum. This restriction alone does not present any particular problems since the lowest processing time parameter could be set very near zero (e.g. .001 hours). However, the simulation phase of the program has a restriction that the largest processing time for any operation cannot be more than 32 times as large as the lowest processing time. For a minimum processing time of .001 hours, the maximum would be .032 hours.

This restriction arises because of the limited size of the "field" assigned for storing processing times in the computer. Since the individual values of processing time were determined by sampling from a negative exponential distribution, a processing time as high as 40 hours is possible when the mean is 6.0 hours; therefore, the limit was specified to be 1.2 hours. Fortunately, the actual processing times generated did not cause "overflow" in the computer during the simulations.

Another effect of this restriction on processing times was to slightly change the shape of the distributions of processing times. In order to analyze the extent of the changes, the Chi-Squared "goodness of fit" test shown in Table 12 was conducted for the total processing time per job. The result of the test was that the null hypothesis - that the data came from a negative exponential distribution - could not be rejected at the 5 percent level of significance.

The distribution of initial material costs was affected by a similar restriction upon the ratio of the highest and lowest allowable

TABLE 12

"CHI-SQUARED GOODNESS OF FIT" TEST - NEGATIVE EXPONENTIAL

PROCESSING TIME Xi	NEGATIVE EXPONENTIAL THEORETICAL FREQUENCY Fi	OBSERVED FREQUENCY fi	$\frac{(fi-Fi)^2}{Fi}$
2.20	846	813	1.287
13.38	368	393	1.698
24.55	221	225	.072
35.73	127	124	.149
46.91	75	80	.333
58.09	42	48	.857
69.26	24	32	2.666
80.44	16	11	.312
91.62	8 }	5 }	3.267
102.79	7 }	3 }	
	1734	1734	10.631

$\bar{X} = 20.32$ $\qquad \chi^2_{.95,9} = 16.919$ $\qquad \chi^2 = 10.631$

material costs; however, the restriction was less stringent than that on the processing times and its effects were negligible. The "analysis" of the orders provided the basis for determining the number of machines in each machine group required for assuring balanced operations for simulation.

Table 11 shows how the 8,504 operations of all the generated jobs were distributed among the machine groups. Since the mean processing times were known, the total expected number of hours required at each machine group to process all operations, if only one machine were used, was computed for each machine group. These times are shown in column 5 of Table 11. If two machines were used, the total processing times at each machine group would be reduced by one-half. It was necessary to determine how the 45 machines should be divided among the five machine groups in order to make the time to process all operations at each machine group approximately equal. This result was accomplished by solving the following set of linear equations:

$$\sum_{i=1}^{5} N_i = 45, \text{ and } \frac{T_1}{N_1} = \frac{T_2}{N_2} = \ldots = \frac{T_5}{N_5}$$

where,

N_i is the number of machines in the i^{th} group,

and

T_i is the total processing time for all operations on the i^{th} machine group if only one machine were used.

The values obtained from the solution of these equations are given in column 6 of Table 11, and the actual values used in the simulation are given in column 7.

Distribution of Dollar-Value Among Jobs

The decision to study the effects of giving preferential treatment to high-valued jobs was based upon the expectation that a relatively high percentage of the total dollar-value would be reflected in a small percentage of the jobs. The distribution of the dollar-value among all jobs was obtained from the order analysis and is shown in Figure 1. The curve shows that 50 percent of the jobs constitute 77 percent of the total value and 10 percent of the jobs constitute 25 percent of the total value. From this analysis it was decided to test the dispatching rules at four different percentages of jobs in the high value class - 10, 15, 20 and 25 percent.

In order to determine the limits of the high value class required for each percentage, the distribution of dollar-values for all jobs processed at each machine group which was obtained from the order analysis was plotted as shown in Figure 2. From this graph, the lower limits for the high-value class to include approximately 10, 15, 20 and 25 percent of the jobs were chosen to be $935.00, $820.00, $725.00 and $650.00, respectively. Approximately forty-five percent of all jobs were placed in the low-value class for each simulation run by establishing its upper limit at $350.00.

Establishing Delay Allowances

The expected queue delays at each machine group depend upon such factors as arrival rate of jobs, the dispatching rule used, and the load on the shop. As a result, the waiting time allowances, which are required for the scheduling phase of the simulation, vary with conditions from one simulation run to another. Since these delays could not be

FIGURE 1

DISTRIBUTION OF DOLLAR-VALUE AMONG ALL JOBS

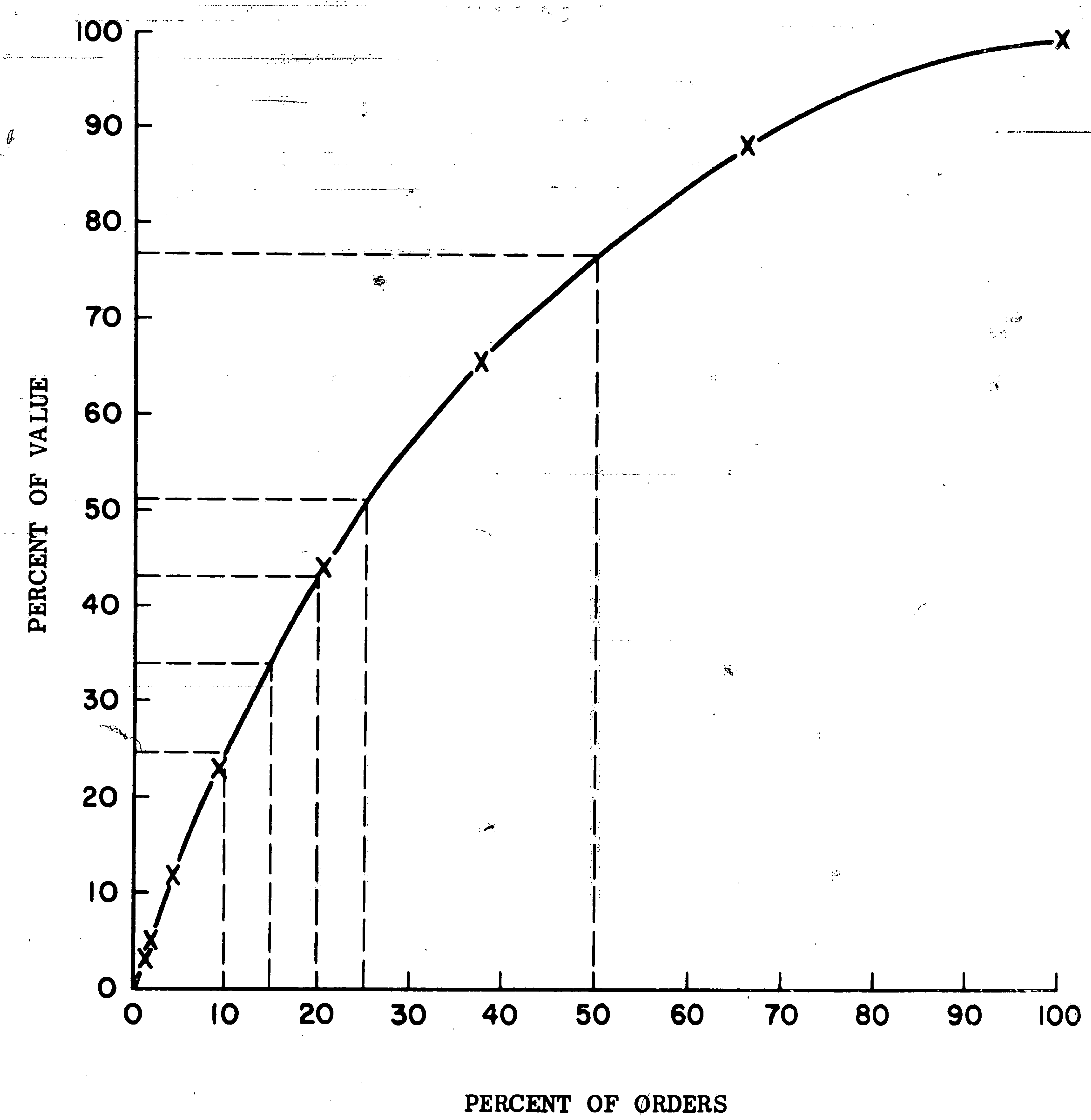
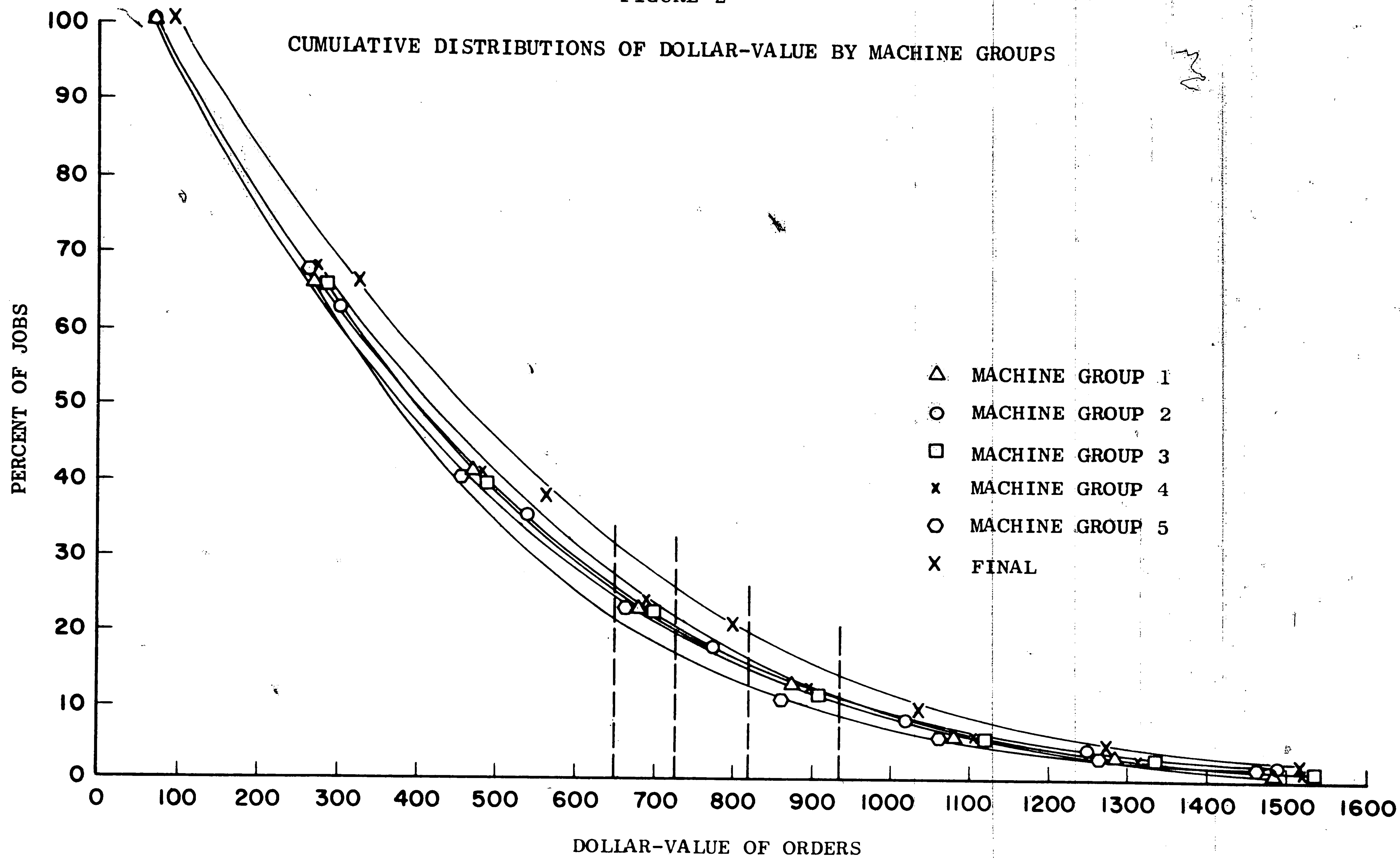


FIGURE 2

CUMULATIVE DISTRIBUTIONS OF DOLLAR-VALUE BY MACHINE GROUPS



determined analytically, six preliminary simulation runs were set-up to empirically establish the waiting-time allowances to be used in testing each of the three dispatching rules at two different levels of loading. The two levels of loading chosen for use in the simulations were 85 and 100 percent of capacity.

Initial delay allowances were assumed for each value class in order to begin the preliminary simulations. The actual average delays by value class for each machine group were obtained from the Analysis of Queues reports and used as the delay allowances in the succeeding simulations. This iterative procedure can be used to obtain accurate delay allowances by continuing until the differences between the average delay allowances and the actual average delays are negligible. Only two iterations for each of the six simulations were required in order to obtain delay allowances which were sufficiently accurate for this study. However, other simulations which preceded the iterative ones had provided a basis for choosing relatively good starting delay values.

Determination of Arrival Rate and Initial Load

The arrival rate of jobs has an important effect upon the equilibrium level of the shop, and the initial load is important in minimizing the start-up effects at the beginning of the simulations by allowing the shop to reach its equilibrium level quickly. In order to determine the arrival rate and initial load required to assure a load of approximately 100 percent, iterative simulations were required. Since the scheduled load depends upon the delay allowances, which are in turn dependent upon the shop load, the iterative procedure must simultaneously determine the initialization quantity, the arrival rates and the delay allowances for a given load. Using an arrival rate of 40 jobs per day and an initial load of 100 jobs, the Load Analysis reports indicated that some of the machine groups were overloaded. When the arrival rate and initial load were reduced to 36 and 91, respectively, the average scheduled load on some of the machine groups was approximately 100 percent. Further simulation indicated that an average scheduled load of approximately 85 percent was obtained with an arrival rate of 31 jobs per day and an initialization of 78 jobs. The corresponding delay values by value class obtained for each machine group are shown in Table 13 for each of the six sets of conditions.

Design of the Simulation Experiments

The objective of this study was to study the three dispatching rules under constant conditions in order to compare their relative effectiveness. Further, it was desired to obtain an indication of the effects of changing the loading and the percentage of the jobs in the high-value class. In order to pursue this course, a set of 24 simulation experiments were established as shown in Table 14. Each dispatching rule was tested at two different levels of loading - 100 and 85 percent - and at four different levels of percentage of high-valued jobs - 10, 15, 20 and 25 percent. The entries in the body of Table 14 correspond to the experiment number assigned to that particular set of conditions. The input parameters to the simulator were held constant for each separate simulation, except as specified in Table 14. The waiting-time allowances were different for all simulation experiments in any single row of Table 14; however, the allowances were identical

TABLE 13

DELAY ALLOWANCES FOR EACH SIMULATION RUN

DISPATCHING RULE NAME

MACHINE GROUP	MINSOP HIGH	MINSOP MED.	MINSOP LOW	FCFSV HIGH	FCFSV MED.	FCFSV LOW	MINDD HIGH	MINDD MED.	MINDD LOW
100 PERCENT LOAD									
1	.020	.266	.330	.026	.055	.873	.060	.196	.412
2	.010	.064	.075	.012	.020	.135	.032	.085	.124
3	.007	.036	.055	.011	.020	.152	.022	.041	.044
4	.010	.059	.070	.015	.032	.155	.026	.081	.099
5	.015	.094	.120	.012	.025	.188	.041	.084	.130
85 PERCENT LOAD									
1	.038	.082	.120	.017	.028	.220	.031	.043	.065
2	.011	.017	.032	.009	.010	.038	.012	.021	.021
3	.012	.018	.033	.004	.005	.015	.023	.015	.010
4	.011	.020	.025	.007	.011	.030	.014	.012	.015
5	.018	.026	.041	.008	.009	.053	.017	.027	.039

TABLE 14

TABULATION OF SIMULATION EXPERIMENTS

	A1		A2		A3	
	B1	B2	B1	B2	B1	B2
C1	13	14	15	16	17	18
C2	19	20	21	22	23	24
C3	25	26	27	28	29	30
C4	31	32	33	34	35	36

A. DISPATCH RULE
  1. MINDD
  2. FCFSV
  3. MINOD
B. SHOP LOAD
  1. 100 PERCENT
  2. 85 PERCENT
C. HIGH-VALUED JOBS
  1. 10 PERCENT
  2. 15 PERCENT
  3. 20 PERCENT
  4. 25 PERCENT

for the experiments listed in each column. For example, the delay allowances were different for experiments 13, 14, 15, 16, 17 and 18 but were identical for experiments 13, 19, 25 and 31.

Since the mean arrival rate for 100 percent load was 36 jobs per day, the mean time between job arrivals in the shop was $\frac{18}{36} = 0.5$ hours. However, since the actual inter-arrival times were determined in the simulations by a Monte Carlo sampling method, the number of jobs arriving in the shop per unit of time was a random variable. This was the only random variation which was allowed during simulation; all other parameters were constant.

Simulation Results

The results of the 24 simulation experiments, are summarized and tabulated in the following tables.

Table 15 compares the job arrivals and job completions for each dispatch rule by periods for the four different percentages of high-valued orders, when the load is 100 percent. YTD is an abbreviation for year-to-date and shows the cumulative results for the four periods of simulation. The job arrivals for period 1 include the initialization quantity. Table 16 provides similar information for a load of 85 percent. A comparison of the scheduled load and the performance of the shop is given for each simulation run in Tables 17 and 18. Table 17 reflects the performance under 100 percent load while Table 18 is for a load of 85 percent.

The cost of carrying inventory consists of the following three components:

TABLE 15

NUMBER OF JOB ARRIVALS VS. NUMBER OF COMPLETIONS

100 PERCENT LOAD

DISPATCH RULE NAME	Period 1 ARRV.	Period 1 COMPL.	Period 2 ARRV.	Period 2 COMPL.	Period 3 ARRV.	Period 3 COMPL.	Period 4 ARRV.	Period 4 COMPL.	YTD ARRV.	YTD COMPL.
10 PERCENT IN HIGH-VALUE CLASS										
MINSOP	279	215	173	172	172	189	208	186	832	762
FCFSV	268	205	175	181	189	184	184	190	816	760
MINDD	293	207	170	186	181	201	179	170	823	764
15 PERCENT IN HIGH-VALUE CLASS										
MINSOP	268	213	196	181	195	210	183	178	842	782
FCFSV	259	194	177	184	187	191	169	184	792	753
MINDD	261	200	176	182	181	190	195	186	813	758
20 PERCENT IN HIGH-VALUE CLASS										
MINSOP	253	206	174	168	194	188	188	191	809	753
FCFSV	273	204	180	186	195	190	175	184	823	764
MINDD	262	204	147	157	196	195	205	192	810	748
25 PERCENT IN HIGH-VALUE CLASS										
MINSOP	289	220	191	190	178	195	192	181	850	786
FCFSV	269	202	178	184	176	180	191	192	814	758
MINDD	268	210	190	184	196	207	184	176	838	777

TABLE 16

NUMBER OF JOB ARRIVALS VS. NUMBER OF COMPLETIONS

85 PERCENT LOAD

DISPATCH RULE NAME	PERIODS 1 ARRV.	1 COMPL.	2 ARRV.	2 COMPL.	3 ARRV.	3 COMPL.	4 ARRV.	4 COMPL.	YTD ARRV.	YTD COMPL.
10 PERCENT IN HIGH-VALUE CLASS										
MINSOP	264	206	167	173	149	161	164	168	744	708
FCFSV	236	189	153	160	167	159	164	179	720	687
MINDD	217	182	159	154	165	163	154	164	695	663
15 PERCENT IN HIGH-VALUE CLASS										
MINSOP	236	199	194	178	142	156	143	151	715	684
FCFSV	225	184	151	153	183	168	148	165	707	670
MINDD	217	182	159	154	135	145	148	144	659	625
20 PERCENT IN HIGH-VALUE CLASS										
MINSOP	261	204	171	179	173	179	144	151	749	713
FCFSV	241	189	146	157	148	144	155	159	690	649
MINDD	217	182	159	154	135	145	148	144	659	625
25 PERCENT IN HIGH-VALUE CLASS										
MINSOP	261	204	171	179	173	179	151	157	756	719
FCFSV	244	188	148	168	163	152	162	180	717	688
MINDD	217	182	159	154	135	145	148	144	659	625

TABLE 17

SCHEDULED LOAD VS. MACHINE UTILIZATION

100 PERCENT LOAD

DISPATCH RULE NAME	1 SCHED. LOAD	1 MACH. UTIL.	2 SCHED LOAD	2 MACH. UTIL.	3 SCHED. LOAD	3 MACH. UTIL.	4 SCHED. LOAD	4 MACH. UTIL.	YTD SCHED. LOAD	YTD MACH. UTIL.
10 PERCENT IN HIGH-VALUE CLASS										
MINSOP	1.057	.926	.876	.916	.890	.885	.910	.916	.933	.911
FCFSV	1.017	.931	.850	.915	.944	.914	.891	.900	.925	.915
MINDD	1.087	.909	.909	.935	.874	.913	.865	.906	.934	.915
15 PERCENT IN HIGH-VALUE CLASS										
MINSOP	1.033	.939	.946	.913	.980	.959	.889	.911	.962	.931
FCFSV	.962	.893	.863	.913	.964	.937	.824	.853	.903	.899
MINDD	.980	.886	.876	.923	.915	.886	.932	.909	.926	.901
20 PERCENT IN HIGH-VALUE CLASS										
MINSOP	.966	.899	.862	.872	.948	.886	.900	.910	.919	.892
FCFSV	1.047	.939	.873	.917	.941	.928	.878	.905	.935	.922
MINDD	.997	.908	.742	.807	.981	.907	.941	.928	.915	.887
25 PERCENT IN HIGH-VALUE CLASS										
MINSOP	1.107	.944	.949	.934	.912	.941	.921	.911	.972	.932
FCFSV	1.024	.930	.851	.905	.909	.915	.910	.894	.923	.911
MINDD	.995	.910	.988	.934	.929	.935	.896	.913	.952	.923

TABLE 18

SCHEDULED LOAD VS. MACHINE UTILIZATION

85 PERCENT LOAD

DISPATCH RULE NAME	PERIODS 1 SCHED. LOAD	1 MACH. UTIL.	2 SCHED. LOAD	2 MACH. UTIL.	3 SCHED. LOAD	3 MACH. UTIL.	4 SCHED. LOAD	4 MACH. UTIL.	YTD SCHED. LOAD	YTD MACH. UTIL.
10 PERCENT IN HIGH-VALUE CLASS										
MINSOP	1.060	.918	.844	.865	.819	.841	.726	.761	.862	.846
FCFSV	.941	.860	.765	.799	.885	.853	.755	.786	.837	.825
MINDD	.872	.816	.780	.777	.875	.843	.714	.761	.810	.800
15 PERCENT IN HIGH-VALUE CLASS										
MINSOP	.946	.874	.980	.888	.756	.843	.641	.667	.831	.820
FCFSV	.910	.840	.734	.762	.934	.877	.700	.748	.820	.807
MINDD	.872	.816	.780	.778	.727	.738	.663	.667	.761	.750
20 PERCENT IN HIGH-VALUE CLASS										
MINSOP	1.056	.917	.890	.877	.854	.883	.678	.753	.869	.857
FCFSV	.964	.869	.712	.768	.790	.770	.706	.718	.793	.781
MINDD	.872	.816	.780	.778	.732	.738	.658	.667	.761	.750
25 PERCENT IN HIGH-VALUE CLASS										
MINSOP	1.057	.917	.889	.877	.854	.883	.708	.777	.877	.864
FCFSV	.968	.869	.779	.836	.849	.825	.740	.764	.834	.824
MINDD	.872	.816	.780	.778	.727	.738	.663	.667	.761	.750

1. cost while machining

2. cost while waiting in queue

3. cost while awaiting delivery

"Cost while awaiting delivery" is a function of the scheduling parameters, which can be adjusted. This cost is usually treated as a cost of failure to meet due-dates generated by the input to the scheduling routine. "Cost while machining" is a function of the processing times and the number of job arrivals. "Cost while waiting in queue" is a function of the number of job arrivals and the dispatching rule. Table 19 gives the latter two components of inventory carrying cost for each simulation run at 100 percent load. Table 20 gives the same results for the simulation run at 85 percent load. Q (T) is the annual inventory carrying costs while waiting based upon the average of many instantaneous valuesof jobs in queue and M (T) is the carrying cost while machining converted to annual base. The ratio of Q (T) to M (T) can be used to compare the results of the different simulation runs.

A summary of queue discipline for all simulation runs associated with 100 percent and 85 percent loads is given in Tables 21 and 22. The average waiting time for each job processed and the mean number of jobs in queue for each period is given for the 24 simulation runs. The completion results are tabulated for each of the simulation runs in Tables 23 and 24. Class intervals of lateness (i.e. deviations of actual completion date from due date) were established and each completed job was associated with a class interval according to its

TABLE 19

INVENTORY CARRYING COST WHILE WAITING VS. WHILE MACHINING

100 PERCENT LOAD

PERIODS

DISPATCH RULE NAME	1			2			3			4			YTD		
	Q(T)	M(T)	RATIO	Q(T)	M(T)	RATIO	Q(T)	M(T)	RATIO	Q(T)	M(T)	RATIO	Q(T)	M(T)	RATIO
10 PERCENT IN HIGH-VALUE CLASS															
MINSOP	2432	4235	.574	1003	4226	.237	1211	4215	.287	1212	4213	.287	1464	4222	.346
FCFSV	1457	4428	.329	940	4109	.288	999	4422	.226	710	4102	.280	1026	4265	.240
MINDD	2003	3958	.506	2144	4454	.481	1587	4365	.363	1079	4143	.260	1703	4230	.402
15 PERCENT IN HIGH-VALUE CLASS															
MINSOP	2070	4287	.482	1089	4216	.258	1644	4506	.364	1258	4349	.289	1515	4340	.349
FCFSV	1145	4242	.269	840	4151	.202	1017	4483	.226	372	3844	.097	843	4140	.201
MINDD	1610	4005	.402	1137	4288	.265	1285	4184	.307	1621	4137	.392	1413	4153	.284
20 PERCENT IN HIGH-VALUE CLASS															
MINSOP	1839	4173	.440	727	4050	.179	1205	4174	.288	1048	4150	.252	1205	4137	.291
FCFSV	1529	4490	.340	1356	4132	.328	1263	4462	.283	870	4178	.208	1255	4315	.290
MINDD	1572	4087	.384	735	3875	.189	1024	4203	.243	1213	4148	.171	1136	4078	.278
25 PERCENT IN HIGH-VALUE CLASS															
MINSOP	2490	4334	.574	1308	4326	.302	1693	4365	.387	1577	4324	.364	1767	4337	.407
FCFSV	1472	4437	.331	884	4069	.217	891	4453	.200	579	3999	.144	957	4239	.225
MINDD	1283	4077	.314	1725	4361	.397	1497	4443	.336	1605	4190	.383	1527	4263	.358

TABLE 20

INVENTORY CARRYING COST WHILE WAITING VS. WHILE MACHINING

85 PERCENT LOAD

PERIODS

DISPATCH RULE NAME	1			2			3			4			YTD		
	Q(T)	M(T)	RATIO	Q(T)	M(T)	RATIO	Q(T)	M(T)	RATIO	Q(T)	M(T)	RATIO	Q(T)	M(T)	RATIO
10 PERCENT IN HIGH-VALUE CLASS															
MINSOP	1703	4060	.419	791	4117	.192	675	4062	.166	258	3444	.075	857	3921	.218
FCFSV	808	3996	.202	195	3784	.051	416	3926	.106	248	3614	.068	417	3830	.108
MINDD	681	3703	.184	253	3748	.067	529	3781	.140	288	3630	.079	438	3715	.117
15 PERCENT IN HIGH-VALUE CLASS															
MINSOP	1124	3938	.285	1070	4057	.263	559	4134	.135	100	3134	.032	713	3816	.187
FCFSV	805	3918	.205	159	3557	.044	532	4023	.132	278	3518	.079	443	3754	.118
MINDD	681	3703	.184	253	3748	.067	192	3236	.059	150	3315	.045	319	3500	.091
20 PERCENT IN HIGH-VALUE CLASS															
MINSOP	1684	4062	.414	982	4108	.239	997	4247	.231	173	3491	.049	959	3977	.241
FCFSV	913	4052	.225	230	3643	.063	200	3385	.059	197	3454	.057	385	3634	.106
MINDD	681	3703	.184	253	3748	.067	192	3236	.059	150	3315	.045	319	3500	.091
25 PERCENT IN HIGH-VALUE CLASS															
MINSOP	1684	4062	.414	982	4108	.239	997	4247	.231	194	3583	.054	964	4000	.241
FCFSV	914	4044	.225	321	3935	.081	306	3851	.079	171	3478	.050	429	3825	.112
MINDD	681	3703	.184	253	3748	.067	192	3236	.059	150	3315	.045	319	3500	.091

TABLE 21

SUMMARY OF QUEUE ANALYSIS

100 PERCENT LOAD

PERIODS

DISPATCHING RULE NAME	1 Mean Waiting Time	1 Mean Number in Queue	2 Mean Waiting Time	2 Mean Number in Queue	3 Mean Waiting Time	3 Mean Number in Queue	4 Mean Waiting Time	4 Mean Number in Queue	YTD Mean Waiting Time	YTD Mean Number in Queue
10 PERCENT IN HIGH-VALUE CLASS										
MINSOP	.116	27.4	.060	14.0	.073	15.2	.050	13.2	.074	17.4
FCFSV	.119	29.7	.074	17.1	.078	17.3	.056	12.4	.081	19.1
MINDD	.096	25.8	.113	27.8	.100	18.4	.053	11.7	.091	20.9
15 PERCENT IN HIGH-VALUE CLASS										
MINSOP	.101	23.0	.060	16.0	.096	22.1	.062	15.8	.079	19.2
FCFSV	.100	23.4	.066	14.9	.076	17.9	.032	5.7	.068	15.5
MINDD	.080	18.1	.055	12.8	.060	12.9	.080	17.6	.068	15.4
20 PERCENT IN HIGH-VALUE CLASS										
MINSOP	.094	20.3	.037	9.0	.053	11.0	.067	13.6	.063	14.2
FCFSV	.121	31.7	.117	27.2	.103	21.4	.077	16.2	.105	24.9
MINDD	.079	18.5	.016	8.1	.052	12.1	.058	13.2	.059	13.1
25 PERCENT IN HIGH-VALUE CLASS										
MINSOP	.124	29.2	.071	21.5	.120	24.3	.085	21.5	.100	24.1
FCFSV	.120	29.8	.096	23.0	.090	20.7	.039	9.1	.077	17.8
MINDD	.061	15.2	.071	20.8	.096	19.9	.071	17.2	.076	18.2

TABLE 22

SUMMARY OF QUEUE ANALYSIS

85 PERCENT LOAD

PERIODS

DISPATCHING RULE NAME	1 Mean Waiting Time	1 Mean Number in Queue	2 Mean Waiting Time	2 Mean Number in Queue	3 Mean Waiting Time	3 Mean Number in Queue	4 Mean Waiting Time	4 Mean Number in Queue	YTD Mean Waiting Time	YTD Mean Number in Queue
				10 PERCENT IN HIGH-VALUE CLASS						
MINSOP	.078	18.3	.049	10.6	.040	7.1	.016	2.6	.046	9.7
FCFSV	.081	17.1	.014	2.6	.028	6.3	.021	3.5	.036	7.4
MINDD	.043	8.2	.013	2.8	.031	6.1	.012	2.3	.025	4.8
				15 PERCENT IN HIGH-VALUE CLASS						
MINSOP	.056	12.2	.047	12.9	.010	6.0	.011	1.0	.040	8.0
FCFSV	.081	16.8	.012	1.8	.033	8.3	.029	1.3	.039	7.8
MINDD	.043	8.2	.013	2.7	.014	2.1	.006	.9	.019	3.5
				20 PERCENT IN HIGH-VALUE CLASS						
MINSOP	.077	18.1	.056	13.1	.056	10.8	.011	1.6	.052	10.9
FCFSV	.086	18.9	.020	3.0	.019	3.5	.015	2.7	.036	7.0
MINDD	.013	8.3	.013	2.7	.011	2.1	.006	.9	.019	3.5
				25 PERCENT IN HIGH-VALUE CLASS						
MINSOP	.077	18.1	.056	13.1	.056	10.8	.016	2.0	.052	11.0
FCFSV	.085	18.6	.027	1.9	.023	1.5	.013	2.1	.037	7.6
MINDD	.013	8.2	.013	2.7	.011	2.1	.006	.9	.019	3.5

TABLE 23

TABULATION OF COMPLETIONS

100 PERCENT LOAD

DISPATCH RULE

PERCENT IN HIGH-VALUE CLASS	MINSOP 10%	MINSOP 15%	MINSOP 20%	MINSOP 25%	FCFSV 10%	FCFSV 15%	FCFSV 20%	FCFSV 25%	MINDD 10%	MINDD 15%	MINDD 20%	MINDD 25%
LATENESS (DAYS)						HIGH-VALUED ORDERS						
-5												1
-4					1	3	3	3		1	1	1
-3					0	2	1	3	1	3	1	0
-2		1	2		0	3	3	1	0	0	3	4
-1	2	1	6	4	9	11	9	29	8	9	14	22
0	68	97	125	110	69	91	117	142	44	83	109	104
1	13	25	19	68	6	12	24	9	20	19	20	35
2		1		11					10	3	2	19
3										1	1	2
4												0
5												1
TOTALS	83	125	152	193	85	122	157	187	83	119	151	189
						ALL ORDERS						
-5												1
-4					5	5	1	4	1	1	1	1
-3	1				8	9	7	11	3	5	3	2
-2	1	2	3	1	17	39	14	25	18	18	18	18
-1	46	26	53	14	173	179	127	161	151	185	177	160
0	677	708	659	640	486	466	504	501	483	481	501	500
1	37	45	38	119	70	54	104	54	84	62	12	64
2		1		12	1	1	4	2	19	5	4	27
3									1	1	2	3
4									1			0
5												1
TOTALS	762	782	753	786	760	753	761	758	761	758	718	777

TABLE 24

TABULATIONS OF COMPLETIONS

85 PERCENT LOAD

DISPATCH RULE

PERCENT IN HIGH VALUE CLASS	MINSOP				FCFSV				MINDD			
LATENESS (DAYS)	10%	15%	20%	25%	10%	15%	20%	25%	10%	15%	20%	25%
HIGH VALUED ORDERS												
-5												
-4		1	1	1	1	2	2	2				
-3		1	0	0	0	0	0	0	1	2	2	2
-2	1	1	2	2	0	2	2	2	0	1	1	1
-1	0	2	2	3	3	5	8	10	3	4	8	12
0	49	84	85	102	66	86	116	144	58	88	112	135
1	26	21	52	65	8	13	11	19	11	7	9	10
2	2	2	5	6					1	1	1	1
3												
4												
5												
TOTALS	78	112	147	179	78	108	139	177	74	103	133	161
ALL ORDERS												
-5												
-4	1	1	1	1	2	2	2	2				
-3	0	1	0	0	0	0	0	0	2	2	2	2
-2	3	2	3	3	7	7	7	7	2	2	2	2
-1	11	29	18	15	42	45	33	41	37	34	33	33
0	601	566	556	563	530	504	506	527	547	544	545	545
1	89	83	129	131	96	104	94	102	72	40	40	40
2	3	2	6	6	9	8	7	9	3	3	3	3
3					1							
4												
5												
TOTALS	708	684	713	719	687	670	649	688	663	625	625	625

lateness. The entries in Tables 23 and 24 indicate the number of jobs which fell into each class interval. A separate tabulation of high-valued jobs completed is given in addition to the tabulation of all jobs completed. Negative values of lateness indicate the jobs were delivered early.

Chapter VI

ANALYSIS OF RESULTS

The results of specific interest were the distributions of completions, the relative costs of carrying in-process inventory, and the utilization of machines and labor under the various conditions of load, dispatching rule, and percentage of jobs in the high-value class. The effects of the dispatching rule upon these measures of effectiveness was of primary importance.

Distribution of Completions

A tabulation of job completions by days of lateness was given in Tables 23 and 24 for all simulation runs. In order to study these distributions, the means and standard deviations were computed for each simulation run. Tables 25 and 26 give the computed means and standard deviations with the degrees of freedom (D.F.) for each simulation at loads of 100 and 85 percent, respectively. The means of these lateness distributions were all near-zero and could possibly have been made even closer to zero by repeated iterations of each simulation run in order to determine more accurate delay allowances. The maximum deviation from zero among all computed means was 0.4455 days.

The deviations for the simulations were taken about the actual means instead of about zero in order to remove any biases resulting from the differences among the means. The variance of the lateness distribution is a function of the dispatching rule as well as the load on the shop and measures the consistency with which the dispatching rule enforces the plan specified by the scheduling phase.

In order to compare the lateness distributions obtained under the

TABLE 25

DISTRIBUTION OF COMPLETIONS

100 PERCENT LOAD

10 PERCENT IN HIGH-VALUE CLASS

DISPATCHING RULE NAME	HIGH MEAN	HIGH STD.DEV.	HIGH D. F.	TOTAL MEAN	TOTAL STD.DEV.	TOTAL D. F.
MINSOP	.1325	.4063	82	-.0183	.3546	761
FCFSV	-.0823	.6018	84	-.2355	.7481	759
MINDD	.3493	.9030	82	-.0811	.7873	763

15 PERCENT IN HIGH-VALUE CLASS

DISPATCHING RULE NAME	HIGH MEAN	HIGH STD.DEV.	HIGH D. F.	TOTAL MEAN	TOTAL STD.DEV.	TOTAL D. F.
MINSOP	.1920	.4868	124	.0217	.3252	781
FCFSV	-.1885	.8937	121	-.3293	.7925	752
MINDD	.0504	.8815	118	-.2176	.7020	757

20 PERCENT IN HIGH-VALUE CLASS

DISPATCHING RULE NAME	HIGH MEAN	HIGH STD.DEV.	HIGH D. F.	TOTAL MEAN	TOTAL STD.DEV.	TOTAL D. F.
MINSOP	.0592	.4636	151	-.0278	.3690	752
FCFSV	-.0382	.8076	156	-.1047	.7433	763
MINDD	.0000	.7659	150	-.2272	.6640	747

25 PERCENT IN HIGH-VALUE CLASS

DISPATCHING RULE NAME	HIGH MEAN	HIGH STD.DEV.	HIGH D. F.	TOTAL MEAN	TOTAL STD.DEV.	TOTAL D. F.
MINSOP	.4455	.6360	192	.1615	.4577	785
FCFSV	-.2299	.7588	186	-.2664	.7555	757
MINDD	.2380	1.0873	188	-.1016	.8082	776

TABLE 26

DISTRIBUTION OF COMPLETIONS

85 PERCENT LOAD

10 PERCENT IN HIGH-VALUE CLASS

DISPATCHING RULE NAME	MEAN	HIGH STD.DEV.	D. F.	MEAN	TOTAL STD. DEV.	D. F.
MINSOP	.3589	.6024	77	.1045	.4325	707
FCFSV	.0128	.5920	77	.0771	.5901	686
MINDD	.0945	.6006	73	.0467	.4688	662

15 PERCENT IN HIGH-VALUE CLASS

DISPATCHING RULE NAME	MEAN	HIGH STD.DEV.	D. F.	MEAN	TOTAL STD. DEV.	D. F.
MINSOP	.1250	.7244	111	.0687	.4682	683
FCFSV	-.0370	.7353	107	.0791	.5949	669
MINDD	-.0291	.6015	102	.0032	.4236	624

20 PERCENT IN HIGH-VALUE CLASS

DISPATCHING RULE NAME	MEAN	HIGH STD.DEV.	D. F.	MEAN	TOTAL STD. DEV.	D. F.
MINSOP	.3537	.7384	146	.1584	.5043	712
FCFSV	-.0647	.6506	138	.0816	.5701	648
MINDD	-.0375	.5695	132	.0048	.4217	624

25 PERCENT IN HIGH-VALUE CLASS

DISPATCHING RULE NAME	MEAN	HIGH STD.DEV.	D. F.	MEAN	TOTAL STD. DEV.	D. F.
MINSOP	.3687	.7176	178	.1641	.4987	718
FCFSV	-.0169	.6259	176	.0828	.5839	687
MINDD	-.0496	.5454	160	.0048	.4217	624

various sets of conditions, an attempt was made to determine if the data was normally distributed. A Chi-squared test for normality was run on several sets of completions data; however, these tests were unsuccessful because of the manner in which the output data were grouped by the Simulator. One of the requirements of the Chi-squared "goodness of fit" tests is that the frequency within each class must be at least five. When a class has a frequency which is less than five, it is combined with an adjacent class. In the tests upon the completions data, the number of separate classes which contained frequencies that were greater than five was always three. The result was that the degrees of freedom associated with the tests was zero in each case. Since the Chi-squared distribution is undefined for zero degrees of freedom, the tests could not be concluded.

The degrees of freedom for the Chi-squared "goodness of fit" test are determined as follows:

$$D.F. = K - 1 - P$$

where,

K is the number of classes into which the data are grouped;

P is the number of parameters which were estimated from the data.

For the tests of the distributions of lateness, K = 3 and P = 2 (since both the mean and standard deviations were estimated). Hence, D.F. = 0. Although the validity could not be verified from the data, the lateness distributions were assumed to be normal in order to apply the F test.

The F test was used to compare the variances of the lateness distributions for each dispatching rule operating under the same overall conditions of loading and percentage of high-valued jobs. The results, which are tabulated in Table 27, show that for the simulations at 100 percent load, the variances for the MINSOP dispatching rule are smaller than those for the FCFSV and MINDD dispatching rules for all percentages of jobs in the high-value class tested. However, the results of the F test comparing the variances from the MINDD and FCFSV rules are somewhat inconclusive since the hypothesis that the data came from distributions having equal variances cannot be rejected for every percentage of high-valued orders tested. The results of the F tests for the 85 percent load, shown in Table 28, indicate that the MINSOP dispatching rule produces a lateness distribution having a smaller variance than that of the FCFSV rule; however, all other results of the F tests of the differences between the variances produced by the three dispatching rules are inconclusive at the 10% level of significance.

The results of further F tests comparing the variances of the lateness distributions are given in Tables 29 and 30 for 100 percent and 85 percent loads, respectively. In these cases, the comparison was made between the different percentages of high-valued orders within the same dispatching rule. The F tests for high valued orders in both Table 29 and Table 30 show significant differences between some of the variances obtained for different percentages of high-valued jobs while there is not a significant difference for the other percentages. However, when all of the job completions are included, the tests for the FCFSV rule shows no significant difference at the 10 percent level. On the other hand, the F-Ratios for the other rules are inconclusive in that a

TABLE 27

F RATIOS
BETWEEN DISPATCH RULES

100 PERCENT LOAD

	FCFSV:MINSOP			MINDD:MINSOP			MINDD:FCFSV		
VALUE CLASS	F(.05)	F	F(.95)	F(.05)	F	F(.95)	F(.05)	F	F(.95)
10 PERCENT IN HIGH-VALUE CLASS									
HIGH	.695	2.193	1.438	.693	4.937	1.441	.695	2.251	1.437
TOTAL	.887	4.449	1.126	.887	4.927	1.126	.887	1.107	1.126
15 PERCENT IN HIGH-VALUE CLASS									
HIGH	.741	3.370	1.347	.740	3.278	1.349	.738	.972	1.352
TOTAL	.887	5.936	1.126	.888	4.658	1.125	.887	.784	1.127
20 PERCENT IN HIGH-VALUE CLASS									
HIGH	.766	3.034	1.305	.764	2.728	1.308	.765	.899	1.305
TOTAL	.887	4.057	1.127	.886	3.237	1.127	.887	.797	1.127
25 PERCENT IN HIGH-VALUE CLASS									
HIGH	.786	1.423	1.270	.787	2.922	1.270	.785	2.053	1.272
TOTAL	.888	2.724	1.125	.888	3.118	1.125	.887	1.144	1.126

TABLE 28

F-RATIOS
BETWEEN DISPATCH RULES

85 PERCENT LOAD

VALUE CLASS	FCFSV:MINSOP F(.05)	FCFSV:MINSOP F	FCFSV:MINSOP F(.95)	MINDD:MINSOP F(.05)	MINDD:MINSOP F	MINDD:MINSOP F(.95)	MINDD:FCFSV F(.05)	MINDD:FCFSV F	MINDD:FCFSV F(.95)
10 PERCENT IN HIGH-VALUE CLASS									
HIGH	.685	.965	1.458	.681	.993	1.464	.681	1.029	1.464
TOTAL	.882	1.861	1.132	.881	1.175	1.133	.880	.631	1.135
15 PERCENT IN HIGH-VALUE CLASS									
HIGH	.728	1.030	1.371	.724	.689	1.376	.723	.669	1.380
TOTAL	.881	1.614	1.134	.878	.818	1.137	.878	.507	1.138
20 PERCENT IN HIGH-VALUE CLASS									
HIGH	.757	.776	1.318	.754	.594	1.322	.752	.766	1.328
TOTAL	.881	1.278	1.134	.879	.699	1.135	.877	.547	1.139
25 PERCENT IN HIGH-VALUE CLASS									
HIGH	.780	.760	1.281	.774	.577	1.288	.774	.759	1.289
TOTAL	.883	1.370	1.132	.880	.714	1.135	.879	.521	1.137

TABLE 29

F-RATIOS
WITHIN DISPATCH RULES

100 PERCENT LOAD

	DISPATCHING RULES								
	MINSOP			FCFSV			MINDD		
VALUE CLASS	F(.05)	F	F(.95)	F(.05)	F	F(.95)	F(.05)	F	F(.95)
10 VS. 15 PERCENT IN HIGH-VALUE CLASS									
HIGH	.721	1.435	1.403	.721	2.205	1.401	.718	.952	1.407
TOTAL	.888	.840	1.125	.887	1.122	1.127	.887	.795	1.126
10 VS. 20 PERCENT IN HIGH-VALUE CLASS									
HIGH	.732	1.301	1.389	.735	1.801	1.383	.731	.719	1.390
TOTAL	.887	1.082	1.127	.887	.987	1.126	.887	.711	1.127
10 VS. 25 PERCENT IN HIGH-VALUE CLASS									
HIGH	.739	2.449	1.375	.740	1.589	1.373	.738	1.449	1.377
TOTAL	.888	1.665	1.125	.887	1.019	1.126	.887	1.053	1.126
15 VS. 20 PERCENT IN HIGH-VALUE CLASS									
HIGH	.755	.907	1.329	.755	.816	1.330	.752	.754	1.336
TOTAL	.887	1.287	1.126	.887	.879	1.127	.886	.894	1.127
15 VS. 25 PERCENT IN HIGH-VALUE CLASS									
HIGH	.767	1.706	1.314	.764	.720	1.319	.764	1.521	1.321
TOTAL	.889	1.980	1.124	.887	.908	1.127	.887	1.325	1.126
20 VS. 25 PERCENT IN HIGH-VALUE CLASS									
HIGH	.777	1.881	1.291	.777	.882	1.290	.776	2.015	1.293
TOTAL	.888	1.538	1.126	.887	1.033	1.126	.887	1.481	1.126

TABLE 30

F - RATIOS

WITHIN DISPATCH RULES

85 PERCENT LOAD

DISPATCHING RULES

VALUE CLASS	MINSOP F(.05)	MINSOP F	MINSOP F(.95)	FCFSV F(.05)	FCFSV F	FCFSV F(.95)	MINDD F(.05)	MINDD F	MINDD F(.95)
10 VS. 15 PERCENT IN HIGH-VALUE CLASS									
HIGH	.711	1.445	1.423	.709	1.542	1.426	.702	1.003	1.440
TOTAL	.882	1.172	1.132	.881	1.016	1.134	.878	.816	1.138
10 VS. 20 PERCENT IN HIGH-VALUE CLASS									
HIGH	.726	1.502	1.403	.723	1.207	1.407	.717	.898	1.419
TOTAL	.883	1.359	1.131	.880	.933	1.135	.878	.808	1.138
10 VS. 25 PERCENT IN HIGH-VALUE CLASS									
HIGH	.735	1.418	1.391	.735	1.117	1.391	.713	.824	1.407
TOTAL	.883	1.329	1.131	.882	.979	1.133	.878	.808	1.138
15 VS. 20 PERCENT IN HIGH-VALUE CLASS									
HIGH	.747	1.039	1.346	.742	.782	1.355	.737	.896	1.365
TOTAL	.882	1.159	1.132	.879	.918	1.136	.876	.990	1.140
15 VS. 25 PERCENT IN HIGH-VALUE CLASS									
HIGH	.758	.981	1.333	.755	.724	1.339	.748	.822	1.351
TOTAL	.883	1.134	1.132	.881	.963	1.134	.876	.990	1.140
20 VS. 25 PERCENT IN HIGH-VALUE CLASS									
HIGH	.772	.944	1.300	.768	.925	1.307	.761	.917	1.318
TOTAL	.884	.978	1.131	.880	1.048	1.136	.876	1.000	1.140

definite pattern of significant changes does not exist.

The F tests in Tables 27 through 30 can be interpreted as two-tailed tests at the 10 percent level of significance or one-tailed tests at the 5 percent level. The upper limit, F_{95}, was computed from the following equation [12]:

$$\log_{10} F_{1-\alpha}(v_1, v_2) = \left(\frac{a}{\sqrt{h - b}}\right) - c\,g$$

where,

$$h = \frac{2v_1 v_2}{v_1 + v_2} \text{ and } g = \frac{v_2 - v_1}{v_1 v_2}$$

The constants a, b, and c are functions of the level of significance and their values for $\alpha = .05$ are 1.4287, 0.95, and 0.681, respectively and v_1 and v_2 are the degrees of freedom associated with the variances in the numerator and denominator of the F Ratio, respectively. The lower limit for the F test is determined by taking the reciprocal of the $F_{1-\alpha}$ obtained from the above equation with the degrees of freedom reversed. For example,

$$F_{\alpha}(v_1, v_2) = \frac{1}{F_{1-\alpha}(v_2, v_1)}$$

Inventory Carrying Costs

Comparisons of in-process inventory carrying costs were made in Tables 19 and 20. Examination of these results revealed that the ratio of carrying costs while waiting to carrying costs while machining for the first period of simulation for all sets of conditions seemed excessively high when compared with the results of the other periods. It was also observed that these ratios fluctuated quite widely from period to period. The cause of these wide discrepancies was determined, after extensive analysis and observation, to be related to the number of job arrivals during the period. Since the number of job arrivals during period 1 for each simulation run includes the initialization quantity, this explains the reason for higher carrying-costs during the first period of simulation. A further observation was that the percentage of jobs in the high-value class did not appear to affect the inventory carrying-cost.

Since comparison of the inventory carrying-costs among the dispatching rules was impossible because of the variation in numbers of job arrivals, it was decided to attempt to normalize these ratios to a common basis for comparison. The relationship between the number of arrivals and the inventory carrying cost ratio within each dispatching rule did not appear to be linear; however, investigation showed that a linear relationship appeared to exist between the number of arrivals and the cost of carrying in-process inventory both while waiting and while machining when the results of the first period of simulation were omitted.

Linear equations were fitted to the data for all simulation runs within each dispatching rule by the least-squares method and an attempt was made to predict the values of Q (T) and M (T) for given numbers of job arrivals with the actual range obtained. The least-squares equations were of the form:

$$M'(T) = \beta N + \alpha \quad \text{and} \quad Q'(T) = \beta' N + \alpha'$$

where,

N is the number of job arrivals, and α, β, α' and β' are constants determined from the data.

The ratio of $\frac{Q'(T)}{M'(T)}$ was then used for comparison with the actual ratios obtained during the simulations. The coefficients of the resulting least-squares equations obtained for the three dispatching rules at loads of 100 and 85 percent are shown in Table 31. However, these equations should not be used to extrapolate results outside the range of the actual data. The equations were used within the range of the data to compute least-squares ratios for comparison with the actual ratios obtained during the simulations. The results are shown in Table 32.

In order to compare the inventory-carrying cost ratios among the different dispatching rules, normalized ratios were computed from the least-squares equations for the range of numbers of job arrivals which actually occurred during simulation. The resulting curves are plotted in Figures 3 and 4 for loads of 100 and 85 percent, respectively. The analysis shows that the FCFSV rule tends to give the lowest inventory carrying costs and that the MINSOP rule provides consistently lower inventory carrying costs than the MINDD rule for 100 percent load. However, at the 85 percent load, the ratios in Figure 4 indicate that the inventory carrying costs are less under MINDD rule than for the MINSOP rule. The FCFSV rule again gives the lowest in-process inventory carrying costs.

Utilization of Machines and Labor

The model used in the simulations was for a machine limited job-shop. A machine-limited shop is one in which the machine capacity is critical and the labor supply is essentially unlimited. This condition was accomplished by assigning one man per shift to each machine in the

TABLE 31

COEFFICIENTS OF LEAST-SQUARES EQUATIONS

DISPATCHING RULE	100 PERCENT LOAD				85 PERCENT LOAD			
	α	β	α'	β'	α	β	α'	β'
	INVENTORY CARRYING COSTS							
MINSOP	-1011.597	8.505	-9339.748	17.084	801.274	5.752	1661.589	4.295
FCFSV	1643.557	4.191	-9725.746	17.409	-1159.663	9.289	-1069.752	2.574
MINDD	-2694.071	11.249	-15020.877	26.648	-369.739	7.972	-1908.718	4.417
	MACHINE UTILIZATION							
MINSOP	-.023440	.001511	- -	- -	.119646	.001314	- -	- -
FCFSV	.481250	.000700	- -	- -	-.259128	.002018	- -	- -
MINDD	-.318574	.001992	- -	- -	-.146338	.001833	- -	- -

TABLE 32

INVENTORY CARRYING COSTS AND MACHINE UTILIZATION
(LEAST-SQUARES VALUES VS. ACTUAL VALUES)

100 PERCENT LOAD

	Least-Squares				Actual		
DISPATCHING RULE NAME	NUMBER OF JOB ARRIVALS	MACHINE UTILIZATION	M′(T)	RATIO	MACHINE UTILIZATION	M (T)	RATIO
10 PERCENT HIGH-VALUED ORDERS							
MINSOP	617	.909	4236	.283	.906	4218	.271
FCFSV	611	.909	4204	.217	.910	4211	.209
MINDD	616	.909	4235	.329	.918	4321	.370
15 PERCENT HIGH-VALUED ORDERS							
MINSOP	629	.927	4338	.324	.928	4357	.305
FCFSV	598	.890	4150	.165	.901	4159	.179
MINDD	613	.903	4202	.313	.906	4203	.321
20 PERCENT HIGH-VALUED ORDERS							
MINSOP	603	.888	4117	.233	.889	4125	.241
FCFSV	619	.915	4238	.248	.917	4257	.273
MINDD	606	.889	4123	.274	.881	4075	.243
25 PERCENT HIGH-VALUED ORDERS							
MINSOP	603	.929	4347	.327	.929	4338	.352
FCFSV	612	.910	4209	.220	.905	4174	.186
MINDD	628	.927	4370	.392	.927	4331	.372

85 PERCENT LOAD

	Least-Squares				Actual		
DISPATCHING RULE NAME	NUMBER OF JOB ARRIVALS	MACHINE UTILIZATION	M′(T)	RATIO	MACHINE UTILIZATION	M (T)	RATIO
10 PERCENT HIGH-VALUED ORDERS							
MINSOP	538	.826	3896	.166	.822	3874	.148
FCFSV	531	.812	3773	.079	.813	3775	.076
MINDD	513	.794	3718	.096	.794	3720	.096
15 PERCENT HIGH-VALUED ORDERS							
MINSOP	516	.798	3769	.147	.799	3775	.153
FCFSV	523	.796	3698	.075	.796	3699	.087
MINDD	477	.728	3433	.058	.728	3433	.058
20 PERCENT HIGH-VALUED ORDERS							
MINSOP	545	.836	3936	.173	.838	3949	.182
FCFSV	501	.752	3494	.063	.752	3494	.060
MINDD	477	.728	3433	.058	.728	3433	.058
25 PERCENT HIGH-VALUED ORDERS							
MINSOP	552	.845	3976	.178	.846	3979	.182
FCFSV	529	.808	3754	.078	.808	3751	.071
MINDD	477	.728	3433	.058	.728	3433	.058

FIGURE 3

LEAST-SQUARES EQUATIONS FOR INVENTORY CARRYING-COST RATIOS
100 PERCENT LOAD

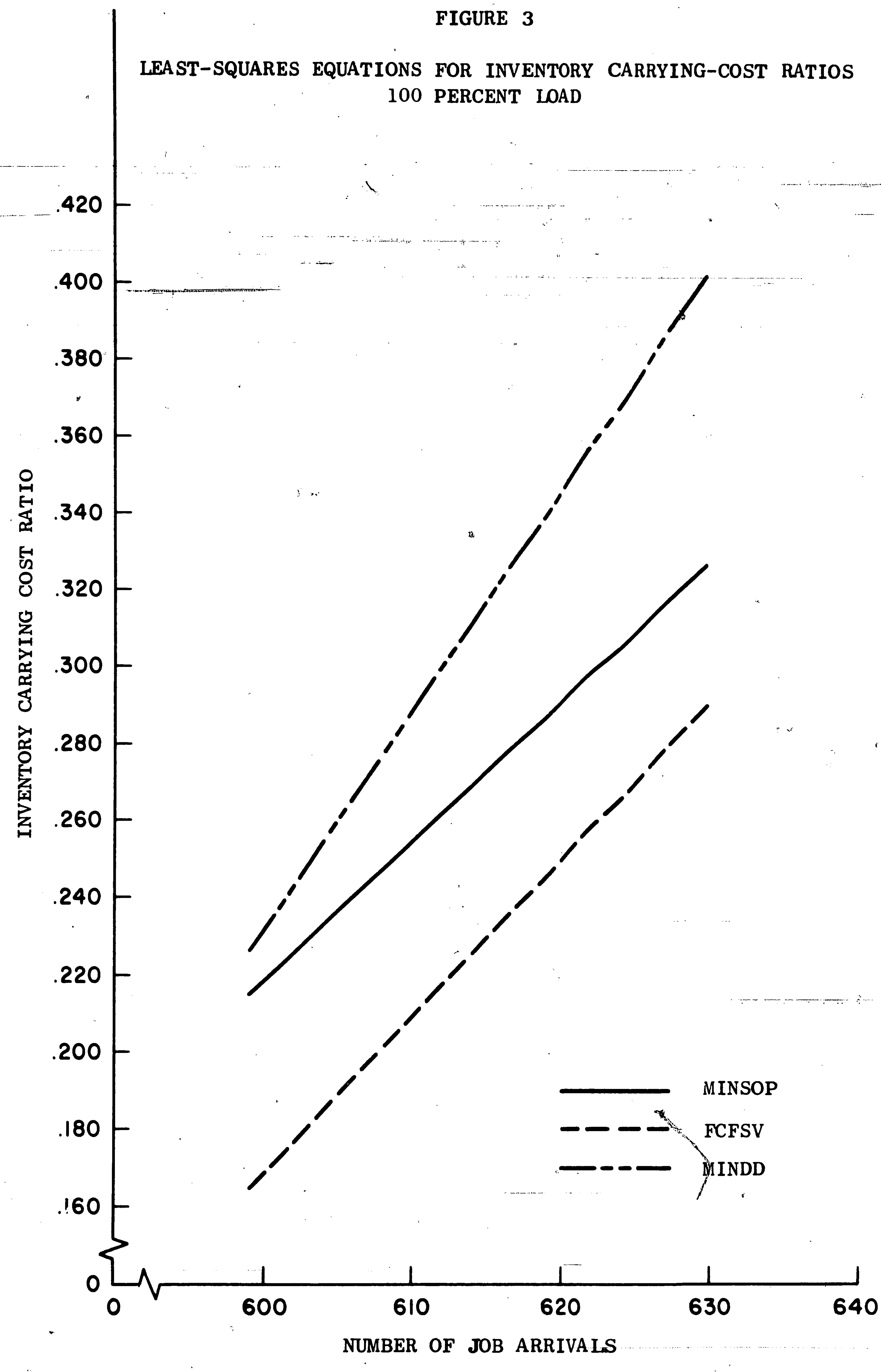
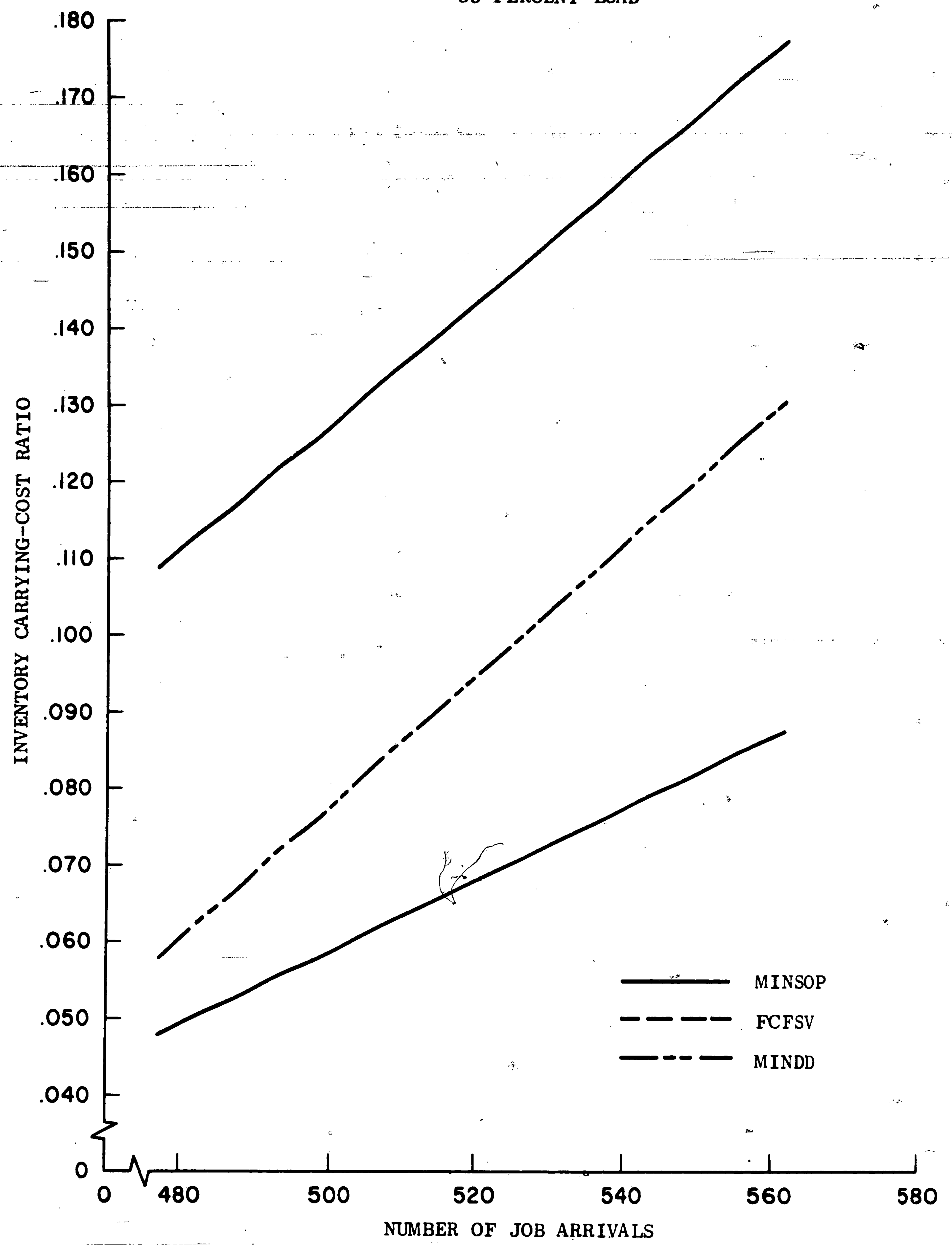


FIGURE 4

LEAST-SQUARES EQUATIONS FOR INVENTORY-CARRYING COST RATIOS
85 PERCENT LOAD



shop. As a result, the utilization of labor will be identical to the utilization of machines in all cases. Therefore, it will not be necessary to consider labor performance further.

The results of machine utilization was given for all simulations runs in Tables 17 and 18. The relationship between the machine utilization and the number of job arrivals was studied. This investigation indicated that a linear correlation appeared to exist when the data for the first period of simulation was omitted. Least-squares equations were fitted to the data in order to normalize the results for comparison between the dispatching rules. The machine utilization did not appear to be appreciably affected by the changes in the percentages of jobs in the high-value class.

The resulting coefficients for the least-squares equations for machine utilization are shown in Table 31. Comparisons of the normalized machine utilization for 100 and 85 percent loads are shown in Figures 5 and 6. For 100 percent load, Figure 5 shows that the lowest utilization is given by the MINDD rule. However, the results for MINSOP and FCFSV rules are inconclusive in that the MINSOP rule gave higher machine utilizations when the number of job arrivals was large and the FCFSV rule gave the highest machine utilizations when the number of job arrivals was small. The comparisons at the 85 percent load gave no clear-cut indication of the dispatching rule which might be expected to provide the highest machine utilizations. Precautions were taken to make comparisons only within the range of the data in order to avoid the pitfalls of extrapolation.

FIGURE 5

LEAST-SQUARES EQUATIONS FOR MACHINE UTILIZATION
100 PERCENT LOAD

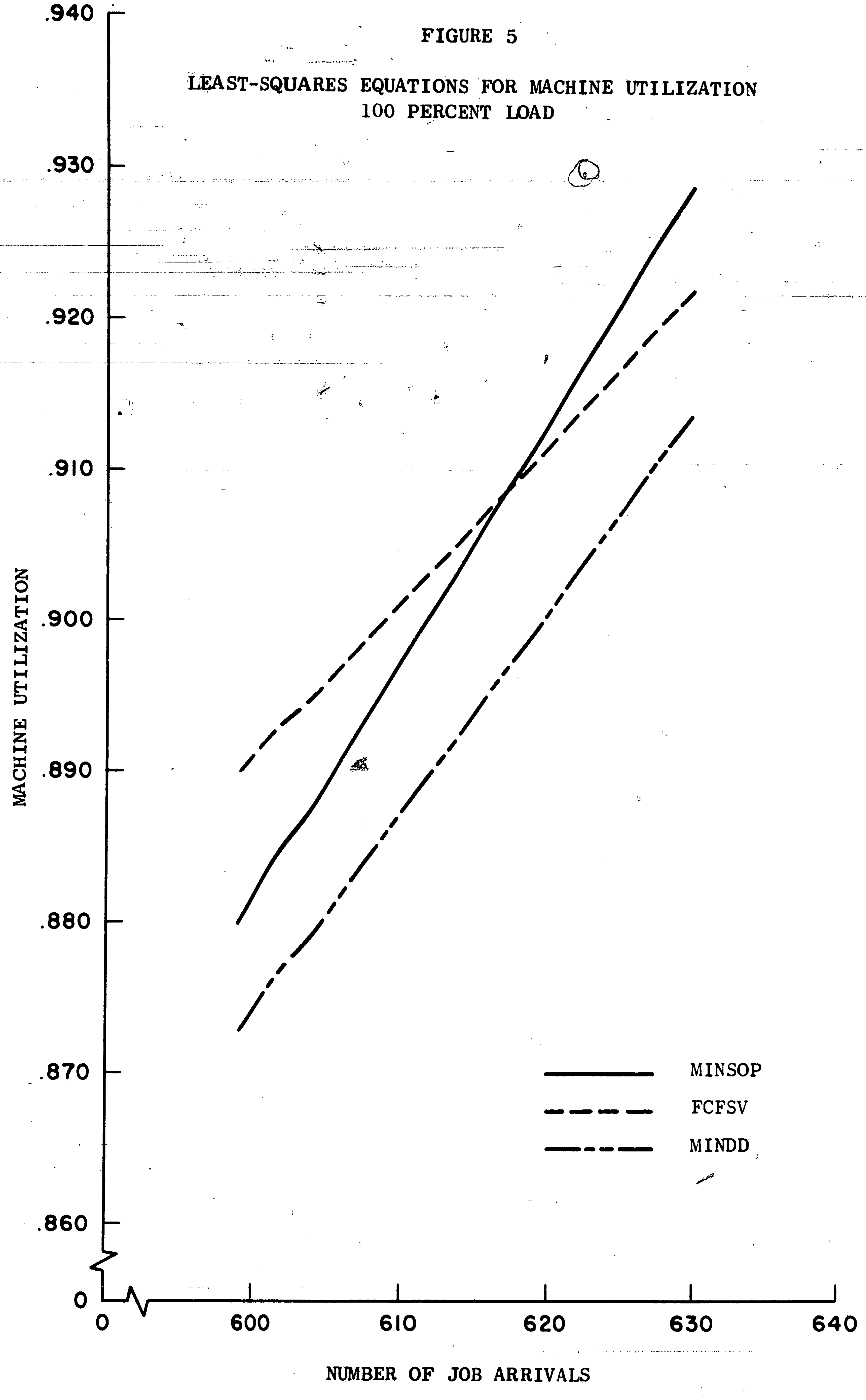
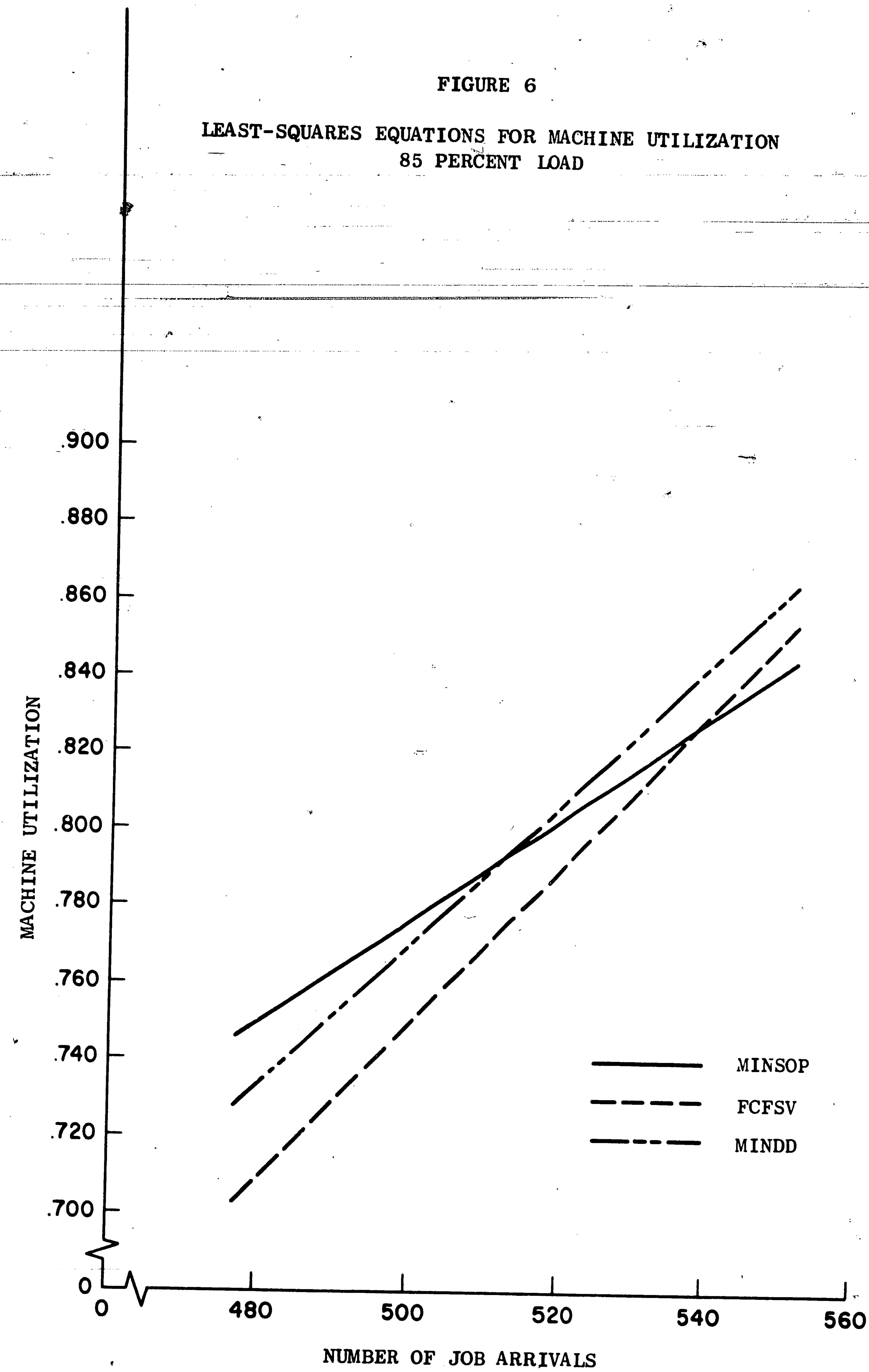


FIGURE 6

LEAST-SQUARES EQUATIONS FOR MACHINE UTILIZATION
85 PERCENT LOAD



Chapter VII

SUMMARY

Conclusions

1. The loading of the shop has an important effect upon the performance of the dispatching rule with respect to the variance of the lateness distribution, the in-process inventory carrying costs, and the machine utilization. For a load of 100 percent, the MINSOP rule is significantly "better" with respect to these measures of performance than the MINDD rule. However, the MINDD rule gives lower in-process inventory costs than the MINSOP rule while the variances of the lateness distribution and the machine utilizations for these rules do not appear to be significantly different at 85 percent load. The FCFSV provides minimum in-process inventory carrying cost at both 100 percent and 85 percent loads and provides machine utilizations which are not consistently different from the MINSOP rule at 100 percent load. Also, the machine utilizations for FCFSV are not consistently different from those of either the MINSOP or MINDD rules at 85 percent load.

2. The use of a scheduling procedure which assigns shorter processing intervals to high-valued jobs appears to produce a reduction in in-process inventory costs. The effectiveness of this procedure appears only to be limited by the ability of the dispatching rule to enforce the schedules.

3. The percentage of jobs assigned to the high-value class does not appear to be critical over the range tested (10 to 25 percent). The machine utilizations and inventory carrying costs are not affected by changes in the percentages of jobs in the high-value class for the

same dispatching rule. However, significant differences were encountered in the variances obtained under different percentages of high-valued jobs for both the MINSOP and MINDD rules. The variances were not significantly different for the FCFSV rule. Further tests would be necessary to determine if these differences in variances can be wholly attributed to the differences in percentages of high-valued jobs.

4. Each dispatching rule affects the measures of performance differently than the other dispatching rules even under identical conditions. FCFSV is significantly "better" for reducing in-process inventory costs. MINSOP provides minimum variances of lateness distributions when the load is heavy, and MINDD minimizes these variances when the load is light. Neither of the dispatching rules tested appears to consistently provide "optimum" machine utilizations.

The results of this study were obtained under controlled, idealized conditions for which many factors which normally occur as random variables in actual job-shops were treated as fixed parameters; therefore, it would be extremely hazardous to attempt to extrapolate these results to include more general job-shop conditions. If the costs associated with late deliveries, idle capacity, and carrying in-process inventory were available, the "best" dispatching rule could be selected for each set of conditions.

Areas of Further Investigation

A natural continuation of this study would be to investigate the effects when certain percentages of the jobs arrive too late for normal processing; when the processing times cannot be predicted with accuracy;

or when the transition times from one machine group to another are random variables. However, each addition of another factor to be included in the study greatly increases the number of simulation runs required.

The ultimate limit to the study of the job-shop through simulation would be to develop a model which includes all factors of importance in the job-shop operations and design a complete experiment for testing each combination of factors at all practical levels of the factors. Replications of each simulation would be necessary for statistical analysis. A project of this magnitude would be extremely ambitious in that many hours of computer simulation and many man-months of effort would be required.

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