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Evaluation of the risk when monthly auditing standards are imposed by sampling with weighed attributes

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EVALUATION OF THE RISK WHEN MONTHLY AUDITING STANDARDS ARE IMPOSED BY SAMPLING WITH WEIGHTED ATTRIBUTES

by

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ABSTRACT

A method is developed to assist the producer in controlling the product quality where monthly auditing standards are imposed after final inspection. The control involves monitoring the number of demerits accumulated by the auditing department and finding the risk that the product will be rated out of control at the end of the month. Thus the method is intended to be employed in addition to present control techniques.

Long term sample data is the basis for developing a mathematical model and then determining the parameters necessary to compute the risk from the producer's standpoint. Simulation of lot to lot changes in product quality is used to test the sensitivity of the model presented. Based on confidence limits placed on the product quality, constant parameters can be used in the model with little loss of accuracy.

Assuming that the risk is judged as too high at some point in the month, the effect of taking alternate action in final inspection or process control to improve the product quality is analyzed from the standpoint of reducing the risk. Thus the producer is able to evaluate various decisions by knowing how the outcome is effected.

I. INTRODUCTION

A. Background

A large amount of final inspection is accomplished by attributes, where articles are classified as conforming to specifications or failing to conform. In inspection for attributes, the important criteria is the number that fail to conform to a particular specification. This is in contrast to variables inspection, where the quality characteristic is measured. Attributes inspection may be thought as inspection using a go and no-go gage, whereas variables inspection is represented by a micrometer.

Usually a manufactured product is complicated enough to require inspection for many attributes. Each attribute is clearly defined as to whether the product being inspected has a defect (fails to conform to some criteria) under each attribute. In some inspection plans, a product unit is called a defective if it fails to conform to any one of a number of attributes. In other inspection plans, it is possible to have as many defects in each item as there are attributes. In the former case, a record is made of the number of defectives; while in the latter, the number of defects.

Acceptance procedures often divide the various possible defects into three or four classes, depending on the seriousness of the different defects. Here, seriousness is judged from the standpoint of the consumer (ultimate user). The ABC* standard calls for defects to be

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*Designation of international standards of attribute inspection. In the United States, these standards are known as MIL-STD-105D.

classified as critical, major, and minor. The definitions of these classifications are as follows:

Critical Defect: A critical defect is likely to result in hazardous or unsafe conditions; or is surely to cause product failure.

Major Defect: A major defect may result in reduced product performance as compared to design standards; or surely cause decreased life.

Minor Defect: A minor defect is not likely to reduce the usability of the product but detracts from intended design function or appearance.

If desired, minor defects can be divided further into two groups depending on their relative importance. The more important minors may be designated Minor A; the less important ones, Minor B. Minor B defects may be known as incidental.

B. Statement of the Problem

Dodge[1] described an inspection plan by attributes for determining product quality in which an arbitrary weight was assigned to each of four defect classes. Using the weight, the number of defects of each class found during inspection is combined into a single rating index. This rating scheme presently is used (with minor modifications) for auditing purposes assuming the customer's viewpoint. Such an auditing technique assigns demerit weights of 100, 50, 10 and 1 to defect classes and accumulates the number of demerits in a sample by the following relationship:

$$D = 100\ d_1 + 50\ d_2 + 10\ d_3 + 1\ d_4 \qquad (1)$$

where d_i is the number of defects found in the i^{th} classification.

For each product, a standard quality level in demerits per unit and standard variance level in demerits - squared per unit are established from prior sampling experience and from the quality needs of the customer. Control limits of $\pm$ 2 sigma are set to audit the product on a monthly basis according to the following relationship:

$$t = \frac{nU-D}{\sqrt{nC}} \qquad (2)$$

where t = the actual rating, with zero related to expected (normal) quality.

n = the monthly sample size.

U = standard quality level in demerits per unit.

C = standard variance level in demerits-squared per unit.

D = the number of demerits accumulated according to equation (1)

A major point to make here is that the auditing scheme described is applied only after final inspection has accepted the product. Also, the auditing procedures presume that adequate prior inspection and process controls are used by the producer.

In the above weighted attribute situation, the producer has a problem in controlling the product sufficiently within the monthly quality control limits. As can be seen by equation (1), the weighting causes D to be influenced mostly by the number of critical defects (d_1); and to a slightly lesser extent, the number of major defects (d_2). The

proportions of the defect classifications are relatively small, which forces very large samples in order to gain sensitivity and control the process. However, the proportion of minor defects is larger than critical or major defects, which indicates that smaller samples would be sensitive to changes in the process for minor defects. If inferences could be drawn between the occurrence of critical and major defects and the occurrence of minor defects, then the product could be controlled by relatively small samples. As shown in APPENDIX C for a typical manufactured product inspected under this scheme, the defect classifications are independent; therefore controlling minor defects does not assure that critical and major defects are controlled. From an economic standpoint, and in order to guarantee that sampling affords the protection necessary to avoid shipping undesirable product quality, it is imperative to accurately forecast the lot quality from some minimum sample size.

C. Objectives and Assumptions

The objective of this thesis is to develop a method for the producer to control the product quality where monthly auditing standards are imposed after final inspection. The control involves monitoring the accumulated demerits and finding the risk of the product being rated out of control based on these accumulated demerits. Thus the method presented is intended to be employed in addition to present final inspection and process control techniques.

It is assumed in this method that the monthly auditing sample is accumulated uniformly by taking intermediate samples and that each

intermediate sample is randomly dispersed among the product. In other words, the monthly sample is assumed to adequately represent the quality of the monthly production.

Prior sample data accumulated over a long term is the basis for selecting a mathematical model to describe the probability of finding various levels of demerits in the sample. The problem attacked is where there are three classifications of defects; namely, critical, major and minor. The number of defects found in each classification (d_i) is a random variable; therefore the joint distribution of three random variables is analyzed.

The parameters of the model are the sample size (n) and the proportions of defects (p_i), which are assumed constant. Since there appears no justification to assume that a manufacturing proces would result in constant defect proportions, it is necessary to test the sensitivity of the model to changes in p_i. Assuming that each p_i is a random variable, simulation is used to develop confidence interval estimates on the variation of p_i from lot to lot. Each defect proportion is assumed to vary according to a beta distribution, which is sufficiently general to take many distributional shapes.

Once a number of demerits have accumulated and the producer assesses the risk of having the product rated out of control is too high, a method is developed to analyze the effect of taking corrective action. This action could be correcting the process, reverting to tightened inspection or screening* the product.

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*Screening assumes all defective product found is discarded or repaired to an acceptable condition.

Screening normally denotes 100% inspection, but it may be impractical to inspect all the product. Therefore, screening will be used in the context of meaning inspection of some percentage of the product where the primary purpose is removing defective units. In this thesis, screening at various levels is evaluated from the standpoint of reducing the risk. Thus the producer is able to evaluate various decisions by knowing the effect each has on reducing the risk. Assuming that the risk is too high, the producer would select that decision that would reduce the risk to an acceptable level.

II. RISK OF GOING OUT OF CONTROL

After the scheduled monthly output of a product has been determined, the monthly sample size to audit that output is fixed. When the manufacturing department starts producing that month, the auditing department selects relatively small samples at periodic intervals (known as intermediate samples) and accumulates the number of demerits found since the start of the month.

This chapter presents a mathematical model to compute the probability that the accumulated total will be equal to or greater than the -2 sigma control limit at the end of the month, given that a number of demerits have accumulated and that a known quantity of intermediate samples have yet to be selected by the auditing department.

A. Independence of Random Variables

In a study of the joint relationship of two or more random variables, it is usually necessary to understand the statistical dependency between the random variables. In this situation, dependency would mean that occurrence of one defect classification predicts the presence of another defect classification. If such is the case, much expense is eliminated since sampling is restricted to only one class of defect.

The independence of the sample data is checked in APPENDIX C. The null hypothesis that the random variables are independent is accepted with a high level of significance. As a result, inspection for minor defects cannot be used to predict critical defects or

major defects, etc.

B. Mathematical Model

The mathematical model selected to represent the joint occurrences of the random variables is the multinomial distribution. The probability density function for this distribution is given by:

$$g(x_1,x_2,\ldots,x_k|n,p_1,p_2,\ldots,p_k) = \frac{n!}{x_1!x_2!\ldots x_k!} p_1^{x_1} p_2^{x_2}\ldots p_k^{x_k} \qquad (3)$$

which defines the probability that the discrete random variables X_1, $X_2,\ldots,X_k$ equal observations $x_1,x_2,\ldots x_k$, respectively. This distribution is subject to the following restrictions

$$\sum_{i=1}^{k} x_i = n, \qquad 0 \le x_i \le n \qquad (4)$$

$$\sum_{i=1}^{k} p_i = 1, \qquad 0 \le p_i \le 1 \qquad (5)$$

which say that the total number of occurrences must equal the total number of trials and that the probability of finding all outcomes must equal absolute certainty.

Equation (5) also says that the random variables are mutually exclusive, meaning here that each item produced contains only one defect or no defects. At first glance, this restriction would appear to rule out using the multinomial for attribute sampling where the number of defects in each classification is recorded. However, because of the relatively low probability of finding a defect in a single unit of product, the model appears to be reasonable. Therefore, defects of any class are assumed to occur in separate units of

product, even though there may be incidents where there are more than one defect actually present in a single unit.

The basis for estimating the p_i parameters for the model is the long term sample data accumulated for a typical manufactured product. Prior data becomes very valuable in that it can be used, with high assurance, to relate to the long term product quality. Maximum likelihood estimates are used to estimate these parameters from the data according to the equation below.

$$\tilde{p}_i = \frac{1}{n} \cdot \frac{1}{N} \sum_{j=1}^{N} r_{ij} \qquad i = 1,2,3 \qquad (6)$$

where $\tilde{p}_i$ designates an estimate of p_i.

r_{ij} is the outcome of the j^{th} sample for the i^{th} random variable.

N is the total number of observations (no. of samples).

n is the sample size.

Given the parameters, the null hypothesis that the sample data in APPENDIX B came from a multinomial parent population is tested in APPENDIX D. Assuming that the parameters are constant, the null hypothesis is rejected at the 5% level of significance. In Chapter III, the sensitivity of this model is determined assuming that the p_i are random variables, which should more nearly describe actual experience.

C. Probability of the Joint Occurrence of Defect Classifications

The probability or risk of having a product accumulate enough demerits for an out-of-control condition depends on a number of variables. The variables to be considered are: (1) the number of intermediate samples that the auditing department has yet to take at

some point in the month, (2) the difference between the number of demerits that have accumulated at that point in the month and the number of demerits associated with the monthly -2 sigma control limit, (3) the parameters relating to the probabilities of an occurrence of a critical, major, or minor defect, and (4) the intermediate sample size. In this chapter, the probability of an individual defect is constant and the intermediate samples are assumed to be the same size throughout the month.

Therefore, evaluating the risk involves combinatorial analysis of those joint occurrences of critical, major, and minor defects that will force the product to be rated out of control. An illustration will demonstrate the procedure used.

Assume that the -2 sigma control limit for a product is 740 demerits, that 720 demerits have been accumulated with one more intermediate sample yet to take in the month and that the intermediate sample size is 50. Then

$$
\begin{aligned}
\text{Risk} &= P_r\left\{ \text{no. demerits} \geq 20 \mid n, p_1, p_2, p_3 \right\} \\
&= 1 - P_r\left\{ \text{no. demerits} < 20 \mid n, p_1, p_2, p_3 \right\} \\
&= 1 - \Big[P_r\left\{ \text{no. demerits} = 0 \mid n, p_1, p_2, p_3 \right\} \\
&\qquad + P_r\left\{ \text{no. demerits} = 10 \mid n, p_1, p_2, p_3 \right\} \Big] \\
&= 1 - \Big[P_r\left\{ X_1=0, X_2=0, X_3=0 \mid n, p_1, p_2, p_3 \right\} \\
&\qquad + P_r\left\{ X_1=0, X_2=0, X_3=1 \mid n, p_1, p_2, p_3 \right\} \Big]
\end{aligned}
$$

$$= 1 - \left[\frac{50!}{0!\,0!\,0!} (.000428)^0 (.00360)^0 (.0454)^0 (1-.049428)^{50} \right.$$

$$\left. + \frac{50!}{0!\,0!\,1!} (.000428)^0 (.00360)^0 (.0454)^1 (1-.049428)^{49} \right]$$

$$= 1 - \left[.0795 + .1895 \right]$$

$$= .731$$

As the number of intermediate samples remaining in the month increases, the combinatorial analysis problem expands greatly. For example, assume for the above problem that 700 demerits have accumulated with two intermediate samples remaining.

Let D_2 = no. demerits found in the next to last intermediate sample in the month.

D_1 = no. demerits found in the last intermediate sample in the month.

Then

$$P_r\left\{\text{Product is within control limit at the end of the month}\right\} =$$

$$P_r\left\{ D_2 = 30 \right\}^* \cdot P_r\left\{ D_1 = 0 \right\}$$

$$+ P_r\left\{ D_2 = 20 \right\} \cdot P_r\left\{ D_1 = 0 \right\}$$

$$+ P_r\left\{ D_2 = 10 \right\} \cdot P_r\left\{ D_1 = 0 \right\}$$

$$+ P_r\left\{ D_2 = 0 \right\} \cdot P_r\left\{ D_1 = 0 \right\}$$

$$+ P_r\left\{ D_2 = 20 \right\} \cdot P_r\left\{ D_1 = 10 \right\}$$

*Parameters have been suppressed for brevity.

$$+ P_r\left\{D_2 = 10\right\} \cdot P_r\left\{D_1 = 10\right\}$$

$$+ P_r\left\{D_2 = 0\right\} \cdot P_r\left\{D_1 = 10\right\}$$

$$+ P_r\left\{D_2 = 10\right\} \cdot P_r\left\{D_1 = 20\right\}$$

$$+ P_r\left\{D_2 = 0\right\} \cdot P_r\left\{D_1 = 20\right\}$$

$$+ P_r\left\{D_2 = 0\right\} \cdot P_r\left\{D_1 = 30\right\}$$

But

$$\text{Risk} = 1 - P_r\left\{\text{Product will be rated within control limit}\right\}$$

Then

$$\text{Risk} = 1 - \Big[P_r\left\{D_2 \leq 30\right\} \cdot P_r\left\{D_1 = 0\right\}$$

$$+ P_r\left\{D_2 \leq 20\right\} \cdot P_r\left\{D_1 = 10\right\}$$

$$+ P_r\left\{D_2 \leq 10\right\} \cdot P_r\left\{D_1 = 20\right\}$$

$$+ P_r\left\{D_2 = 0\right\} \cdot P_r\left\{D_1 = 30\right\}\Big]$$

where the occurrence of demerits in one sample is assumed independent of another sample. Therefore, the probability of finding a number of demerits in the next to last intermediate sample (D_2) is multiplied times the probability of finding the remaining number of demerits in the last intermediate sample (D_1).

Extending the procedure for as many remaining samples and accumulated demerit levels as desired, results in corresponding probability points. A risk chart with six intermediate samples remaining are shown in Figure 2 (Chapter 5). The computer printout from a

FORTRAN IV program written to evaluate these risks, appears in APPENDIX A.

III. SENSITIVITY OF THE MODEL

A. Variability of P_i

In Chapter II a mathematical model is developed to describe the interrelationships and joint probabilities of occurrences of the random variables. As mentioned before, the model assumes constant parameters p_i. However, there is no justification that a manufacturing process would result in product with a constant proportion of defects. The variability of p_i from day to day or week to week is thought to be the major reason why a mathematical distribution cannot precisely describe the probability of obtaining various joint occurrences.

Borg[2] also encounters this problem and removes the variability of p by integrating it out of the joint probability density function of the two random variables p and x. The technique here is to also assume that p_i varies according to a beta distribution as done by Borg[2] and Skellam[28], but employing simulation to develop confidence intervals for p_i.

As a first step, the probability density function for the beta distribution is defined as

$$f(p|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)}\; p^{\alpha-1}\,(1-p)^{\beta-1} \qquad (7)$$

for $0<p<1$, $\alpha>0$, $\beta>0$ where $\Gamma(\alpha)$, $\Gamma(\beta)$ and $\Gamma(\alpha+\beta)$ are gamma functions of the α and β parameters. The beta distribution permits representation of a wide diversity of distributional shapes, depending on various values of its parameters. Some of these shapes

are summarized by Hahn and Shapiro (p. 91) and repeated here.

$\alpha > 1, \quad \beta > 1$ The distribution is single peaked with the peak at $(\alpha-1)/(\alpha+\beta-2)$

$\alpha < 1, \quad \beta \geq 1$ The distribution is reverse J shaped

$\alpha = \beta$ The distribution is symmetrical

$\alpha < 1, \beta < 1$ The distribution is U shaped

The method of moments is used to determine the values of α and β to describe the distribution of each p_i in the mathematical model. The following relationships are used[11].

$$\tilde{\beta}_i = \frac{(1-\bar{p}_i)}{S_i^2}\left[\bar{p}_i(1-\bar{p}_i) - S_i^2\right], \quad i = 1,2,3 \qquad (8)$$

$$\tilde{\alpha}_i = \frac{\bar{p}_i\,\tilde{\beta}_i}{1-\bar{p}_i}, \quad i = 1,2,3 \qquad (9)$$

where $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ are parameters of the i^{th} random variable

$\bar{p}_i$ sample mean of p_i

s_i^2 sample variance of p_i

For the sample data in Appendix B, estimates of the parameters are determined using equations (8) and (9) with the results given below

Critical Defects

$\tilde{\alpha}_1 = 0.893$

$\tilde{\beta}_1 = 2085.8$

Major Defects

$\tilde{\alpha}_2 = 0.632$

$\tilde{\beta}_2 = 174.8$

Minor Defects

$$\tilde{\alpha}_3 = 4.157$$

$$\tilde{\beta}_3 = 87.4$$

B. Simulation

In the absence of observational data or a mathematical relationship relating to the variability of an item of interest, the basis of using simulation is quite reasonable. For the problem in this thesis, enough information is not available to draw any conclusions about the variation of the proportion of defects.

In order to simulate from a beta distribution, it is necessary to generate beta variates using the cumulative distribution function. For the beta distribution, the cumulative distribution function can be expressed

$$F(y \mid \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^y p^{\alpha-1} (1-p)^{\beta-1} dp \qquad (10)$$

where y is the beta variate of the random variable p.

However, an explicit relationship for equation (10) does not presently exist and other means must be found to determine random variates. It is known that the beta distribution is the distribution of the ratio of two gamma variables t_1 and $(t_1 + t_2)$ where t_1 and t_2 are independent gamma variables[3,13].

Therefore

$$p = \frac{t_1}{t_1+t_2}, \qquad 0<p<1,\ t_1>0,\ t_2>0 \qquad (11)$$

has a beta distribution. The parameters of the gamma variables t_1 and t_2 for equation (11) are determined from the beta distribution where t_1 has parameter α and t_2 has parameter β. Therefore (t_1+t_2) has parameter $(\alpha+\beta)$ from the beta distribution.

Even though an explicit relationship does not exist for the cumulative gamma distribution function, an iterative procedure has been formulated[18,20]. However, another problem develops when applying the procedure to give gamma random variates for this thesis. The β parameters of the gamma distribution of t_2 for all defect classifications are too large for accommodation even on a large scale computer (as noted in reference 20). Therefore, an approximating procedure is necessary to generate t_2.

As the shape parameter of the gamma distribution (η) increases, it can be shown by the central limit theorem that the gamma distribution approaches a normal distribution[11,17,19]. Kendall and Stuart (p. 166) find a rather complicated polynomial approximation which they state is sufficiently accurate for $\eta \geq 9$. For this thesis, the approximation in Peizer and Pratt[19] is used to generate gamma variates for t_2. This equation takes the form

$$W = \eta\left(1 - \frac{1}{9\eta} + \frac{Z}{3\sqrt{\eta}}\right)^3 \qquad (12)$$

where W is a gamma variate

η is the gamma distribution shape parameter

Z is a standard normal variate

The procedure for generating normally distributed random variates is derived from the central limit theorem and can be expressed as[3]

$$Z = \frac{\sum_{i=1}^{k} r_i - k/2}{\sqrt{k/12}}, \quad 0 \leq r_i \leq 1, \; -\infty < Z < \infty \tag{13}$$

where Z is a standard normal variate (zero mean and unit variance)

$\sum r_i$ the sum of k uniformly distributed random variates.

Equation (13) is an approximation which can be improved by using Teichroew's technique[3,21]

$$Z' = a_1 Z + a_3 Z^3 + a_5 Z^5 + a_7 Z^7 + a_9 Z^9 \tag{14}$$

where Z is determined from equation (13)

$a_1 = 3.949846138$

$a_3 = 0.252408784$

$a_5 = 0.076542912$

$a_7 = 0.008355968$

$a_9 = 0.029899776$

In summary, to generate random variables for each p_i with parameters α_i and β_i, it is necessary to generate a t_1 with parameter α_i by the iterative procedure in references 18 and 20 and a t_2 with parameter β_i by the equations (12), (13), and (14). A FORTRAN IV program has been written for an IBM OS 360/50 computer to generate these random variates and analyze the output.

C. Confidence Interval Estimates

Monte Carlo simulation is used to generate the proportions of each defect classification at random, assuming that each p_i varies according to a beta distribution. Assuming that each of the random variates b_j are generated independently, the sequence $b_1, b_2, b_3, \ldots$ is formed for each defect classification. As the number of simulations increases, the quantity $\sum_{j=1}^{n} b_j$ is known to approach a normal distribution[3,22,23]. Also, if $E(b_j) = \theta$ and $V(b_j) = \sigma^2$ then

$$E\left(\sum_{j=1}^{n} b_j\right) = n\theta, \tag{15}$$

$$V\left(\sum_{j=1}^{n} b_j\right) = n\sigma^2, \tag{16}$$

and

$$Z = \frac{\sum_{j=1}^{n} b_j - n\theta}{\sigma\sqrt{n}} \tag{17}$$

in the sense that

$$\lim_{n\to\infty} p_r\left[u < \frac{\sum_{j=1}^{n} b_j - n\theta}{\sigma\sqrt{n}} < v\right] = \frac{1}{\sqrt{2\pi}} \int_u^v e^{-\frac{1}{2}z^2}\, dz \tag{18}$$

then z is a standard normal variate with distribution N(0,1). To develop a confidence interval for z, say 95%; then $Pr[u<z<v] = \Phi(v) - \Phi(u) = .95$, which says that the probability that z lies between some points u and v is .95. In order for z to be centrally located between u and v, $\Phi(v) = .9750$ and $\Phi(u) = .0250$.

Then

$$-1.96 < \frac{\sum b_j - n\theta}{\sigma\sqrt{n}} < 1.96$$

$$-1.96\,\sigma\sqrt{n} < \sum b_j - n\theta < 1.96\,\sigma\sqrt{n}$$

$$\frac{-\sum b_j - 1.96\,\sigma\sqrt{n}}{n} < -\theta < \frac{1.96\,\sigma\sqrt{n} - \sum b_j}{n}$$

$$\frac{\sum b_j + 1.96\,\sigma\sqrt{n}}{n} > \theta > \frac{\sum b_j - 1.96\,\sigma\sqrt{n}}{n} \qquad (19)$$

is a 95% confidence interval estimate for θ; which for large n is really $\tilde{p}_i$, an estimate of the proportion of defects in the i^{th} classification. An estimate of the sample variance $\tilde{\sigma}^2$ obtained from the Monte Carlo simulation of size n is found by

$$\tilde{\sigma}^2 = \frac{1}{n-1} \sum_{j=1}^{n} (b_j - \bar{b})^2 \qquad (20)$$

where the sample mean is

$$\bar{b} = \frac{1}{n} \sum_{j=1}^{n} b_j \qquad (21)$$

For this thesis, each b_j simulated corresponds to the proportion of one defect classification in a production lot. It is assumed that each defect proportion varies from lot to lot and that each follows a beta distribution. Figure 1 is a simplified representation of the simulation, where each "x" is the result of generating one defect proportion for a lot.

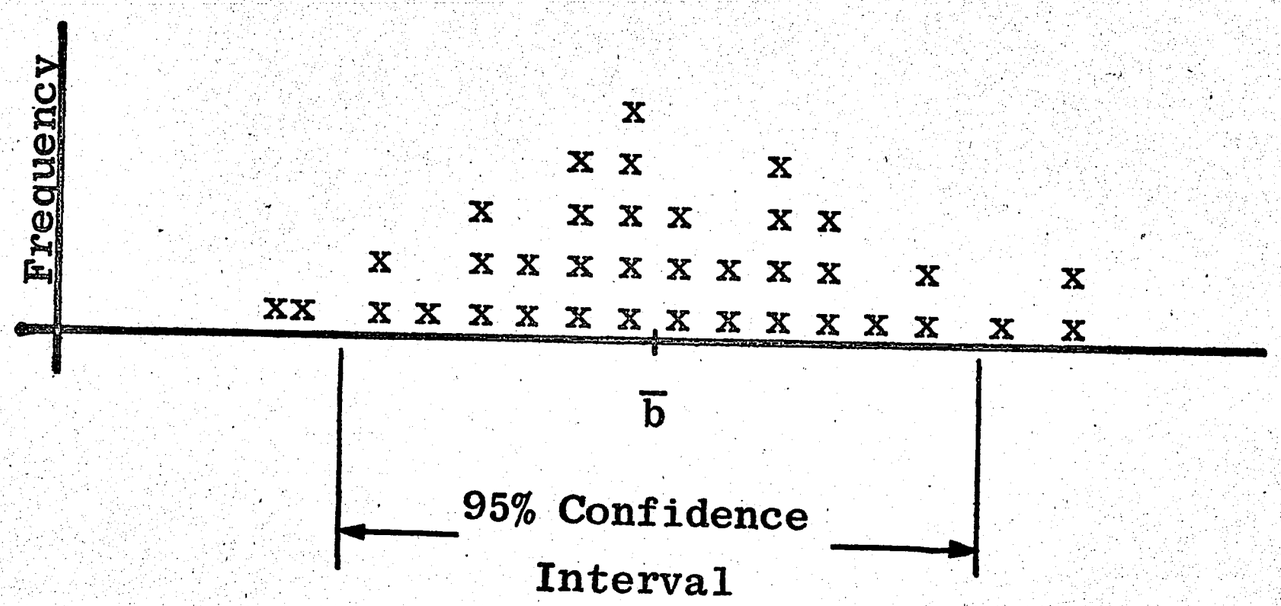

FIGURE 1

Simulation of a Defect Proportion

The stopping rule used to determine the Monte Carlo sample size or number of draws on the distribution is when each $\bar{b}$ stochastically converges to within .5% of p_i. Once this condition is met, then the confidence interval estimates are calculated from the sample mean and the sample variance. A number of trials, each starting with a unique initial "seed" and continuing until the stopping rule intercedes, were completed. The results of the various trials are not significantly different from one another when the sample values are tested in the mathematical model described in Chapter II. Table 1

contains the results for a typical trial which required 4933 random values of each defect classification before the stopping rule took effect.

$\tilde{p}_i$	$\bar{b}$	95% Confidence	Interval		Estimates
.000428	.0004272	.0004400	>	θ_1	> .0004145
.00360	.003620	.003745	>	θ_2	> .003496
.04540	.04553	.04614	>	θ_3	> .04492

TABLE 1

Simulation Results

The interpretation of the confidence interval estimate is that the p_i defect proportion is an actual lot quality which will fall between the stated intervals 95 out of 100 times (assuming that p_i varies according to a beta distribution). However, the defect proportion p_i will not lie in the interval 95% of the time unless enough b_j are sampled to have $\bar{b}$ an accurate estimate of $\tilde{p}_i$. Also, if a long term change is noted in $\tilde{p}_i$, either because the process changes or from some other reason, then the confidence intervals in Table 1 no longer hold.

Once confidence intervals have been set on each $\tilde{p}_i$, the sensitivity of the model can be determined. Table 2 summarizes the results comparing risk values with three intermediate samples remaining, when each $\tilde{p}_i$ is equal to the interval end points. These end points represent the best condition and the worst condition.

Number Demerits Accumulated	Risk at $\tilde{p}_1=.000428$ $\tilde{p}_2=.00360$ $\tilde{p}_3=.0454$	Risk at Lower Confidence Limit $\tilde{p}_1=.0004145$ $\tilde{p}_2=.003496$ $\tilde{p}_3=.04492$	Risk at Upper Confidence Limit $\tilde{p}_1=.0004400$ $\tilde{p}_2=.003745$ $\tilde{p}_3=.04614$
680	.827	.819	.840
660	.650	.639	.669
640	.480	.468	.501
620	.335	.323	.354
600	.224	.215	.239
580	.145	.138	.157
560	.090	.085	.098

TABLE 2

Sensitivity

Table 2 is typical of the sensitivity results for up to six intermediate samples remaining in the month. It is seen that assuming constant p_i parameters in the mathematical model yields values within 2% of the values found by assuming the p_i parameters are random values.

IV. REDUCING THE RISK

Demerits are accumulated by the auditing department in a sequential procedure, which the producer would monitor and evaluate the risk level by reference to risk curves as shown in Chapter V or by evaluation on a computer. If at some point in the month the risk is assessed by the producer as too high, a decision can then be made to (1) improve the process, (2) revert to tightened final inspection, (3) screen the product, or (4) do nothing (continue under existing final inspection practices) and take a chance that the produce may not be rated out of control.

Assuming that the penalty cost of going out of control outweighs the cost of improving the product quality, doing nothing is ruled out as an alternative. Improving or correcting the process is the most desirable decision from the long range quality effect and desirable because relying on inspection to clean up the fault of others does not give complete assurance that the quality is improved. However, dependence on tightened inspection and/or screening as a short term correction may be coupled with a long term objective of correcting process quality problems at the source. In fact, the $AOQL^{25}$ sampling plans are dependent on screening of rejected production lots in order to comply with the stated quality standard.

Screening is normally considered as 100% inspection. It may be impracticable to use 100% inspection because it is unduly costly or because of insufficient inspection facilities or personnel. In this thesis, screening is used to denote any level of inspection in which

the primary purpose is to remove product containing defects. The reason for this stand is that so many times 100% inspection is not reasonable and that some other technique is necessary since tightened inspection does not always give enough assurance. Moreover, screening at various levels, where some fraction of the product has defects removed, is readily evaluated from the standpoint of reducing the risk. By evaluating the effect of screening levels, the producer is aided in making a decision on the course of action.

Using the mathematical model developed in Chapter II, risk curves are generated assuming that screening is initiated at levels of 10%, 25% and 40%. The meaning of this is that 10%, 25% and 40% of all product yet to be manufactured in the month is inspected for all defects, removing those not conforming to specifications. In effect, the proportions of the defects are reduced by these levels, the resulting risk curves under screening are included in Chapter V as Figures 3, 4 and 5.

V. RESULTS

A. Risk Curves

Figure 2 shows various constant risk lines for up to six intermediate samples remaining to be taken by the auditing department. The mathematical model to generate these risks for a typical manufactured product has been programmed in FORTRAN IV language with a sample print out included in APPENDIX A. Only the results for up to six remaining samples are shown because the computer enumeration time expands greatly as the number of samples increases. However, information on the risk with more remaining samples may be obtained by extrapolating.

Figure 2 assumes that the producer continues under existing final inspection procedures. The figure then is useful for monitoring the accumulated demerits and determining the risk that the auditing department will rate the product out of control (with - 2 sigma limits) at the end of the month. From Chapter III, it is determined with 95% confidence limits that the risks in Figure 2 are within 2% of the true value.

Figures 3, 4, and 5 are generated assuming that the product quality is improved by the production organization screening the product at levels of 10%, 25% and 40%. Of course, any other procedure that would result in reducing the proportion of Critical, Major, and Minor defects by the same amounts would end in the same risk curves. Other procedures might entail correcting the process, tightened inspection, etc.

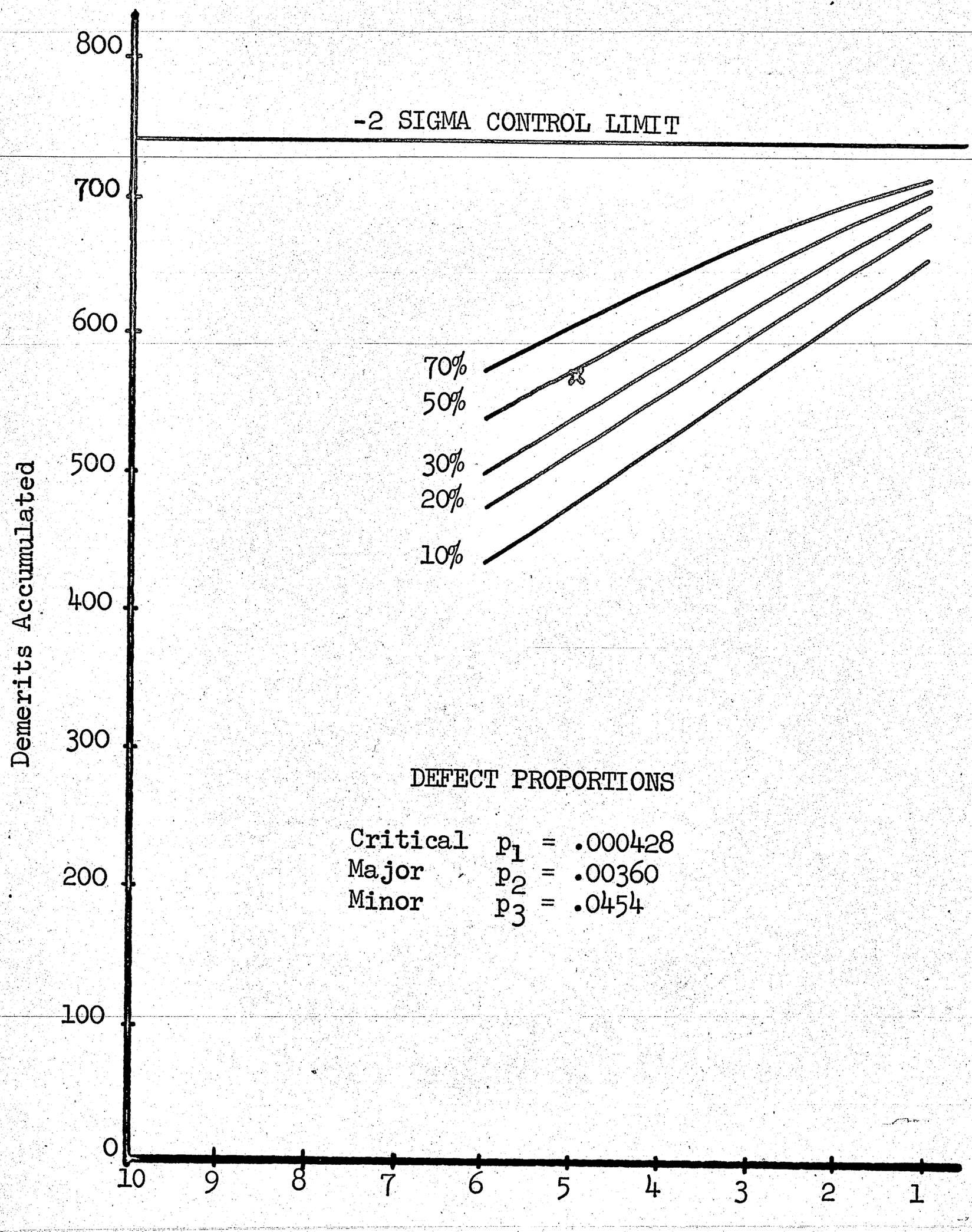

FIGURE 2 RISK CURVES

B. Use of the Risk Curves

An example will demonstrate the use of these risk curves. Assume that the auditing department has accumulated 570 demerits with five remaining intermediate samples. In Figure 2, this is shown as an "X" and is approximately a 50% probability that the product will be rated out of control at the end of the month, assuming the production process and final inspection continue under present procedures. This means that the probability is .50 that the auditing department will find at least enough demerits for the monthly total to be greater than the minus two sigma control limit.

If a risk of 50% is judged by the producer as too high, then Figures 3, 4, and 5 can be used to evaluate the effect of taking alternate action; that of screening at levels of 10%, 25% and 40%. As can be seen, with 10% screening of all future products made in the month, the risk drops to about 35%. At 25% screening, the risk drops to 25%; and at 40% screening, the risk falls to 10% (one chance in ten). Thus, the producer can evaluate the effect of various alternatives from an evaluation of the reduction in the risk.

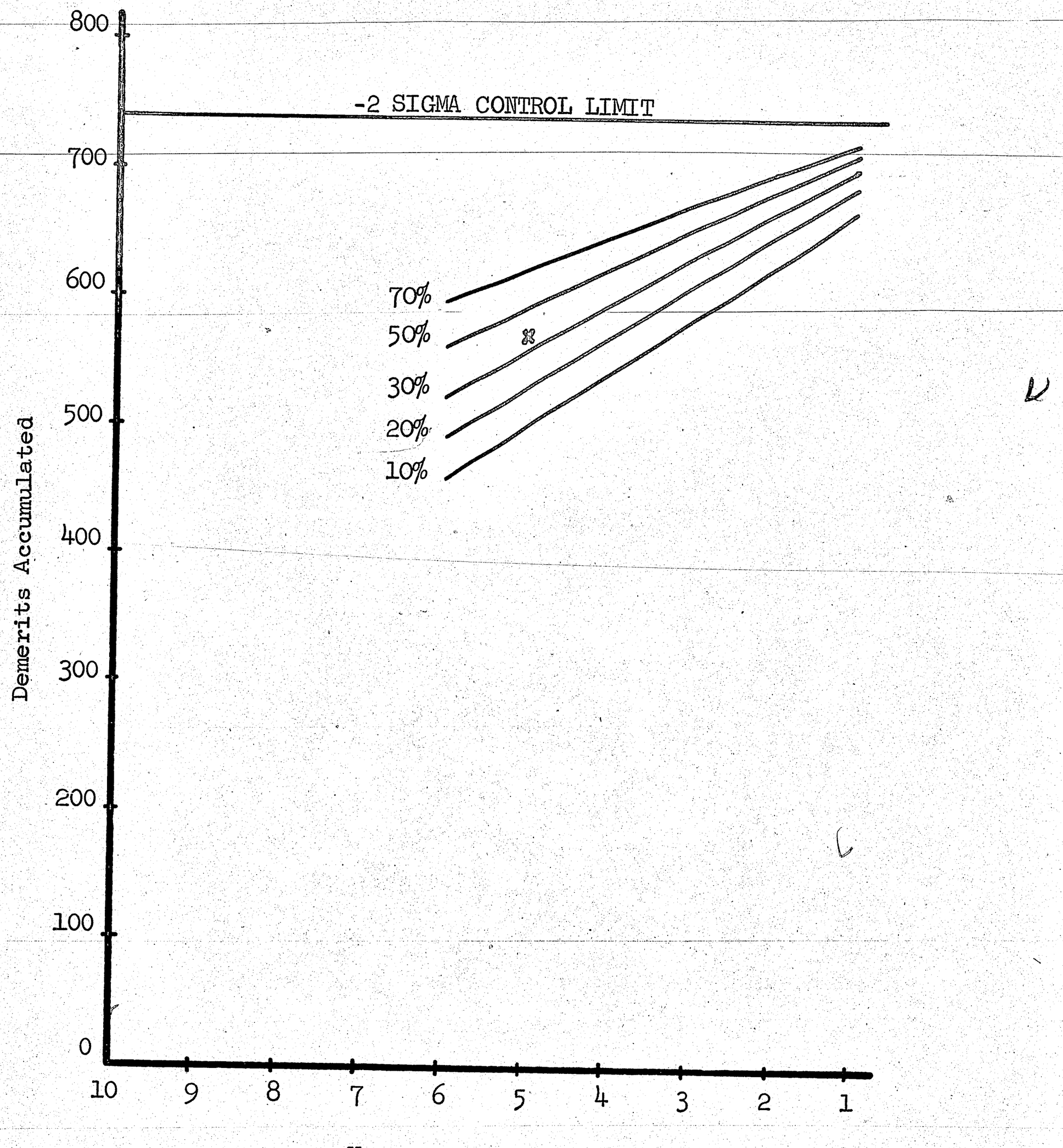

FIGURE 3 RISK CURVES WITH 10% SCREENING

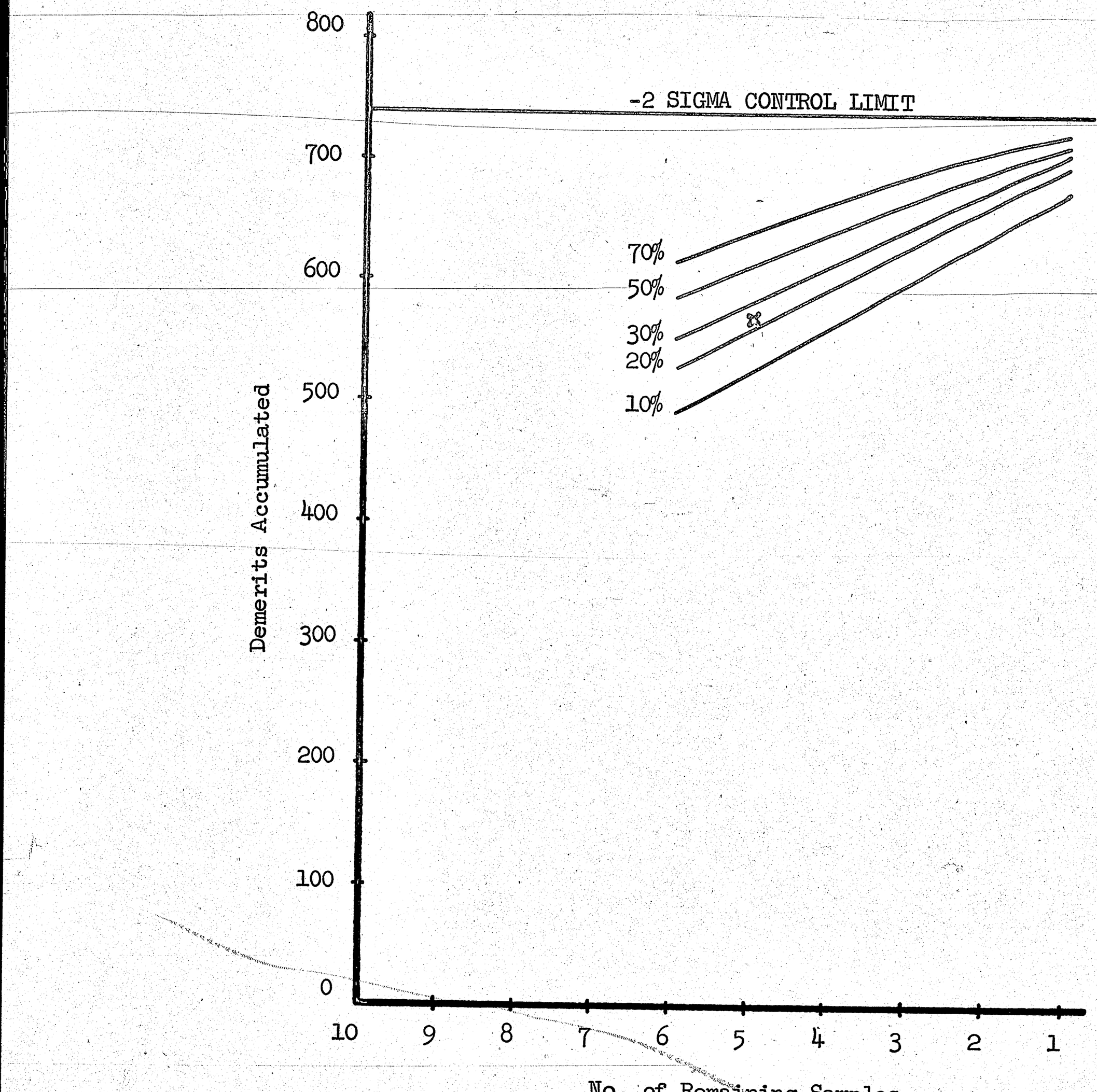

FIGURE 4 RISK CURVES WITH 25% SCREENING

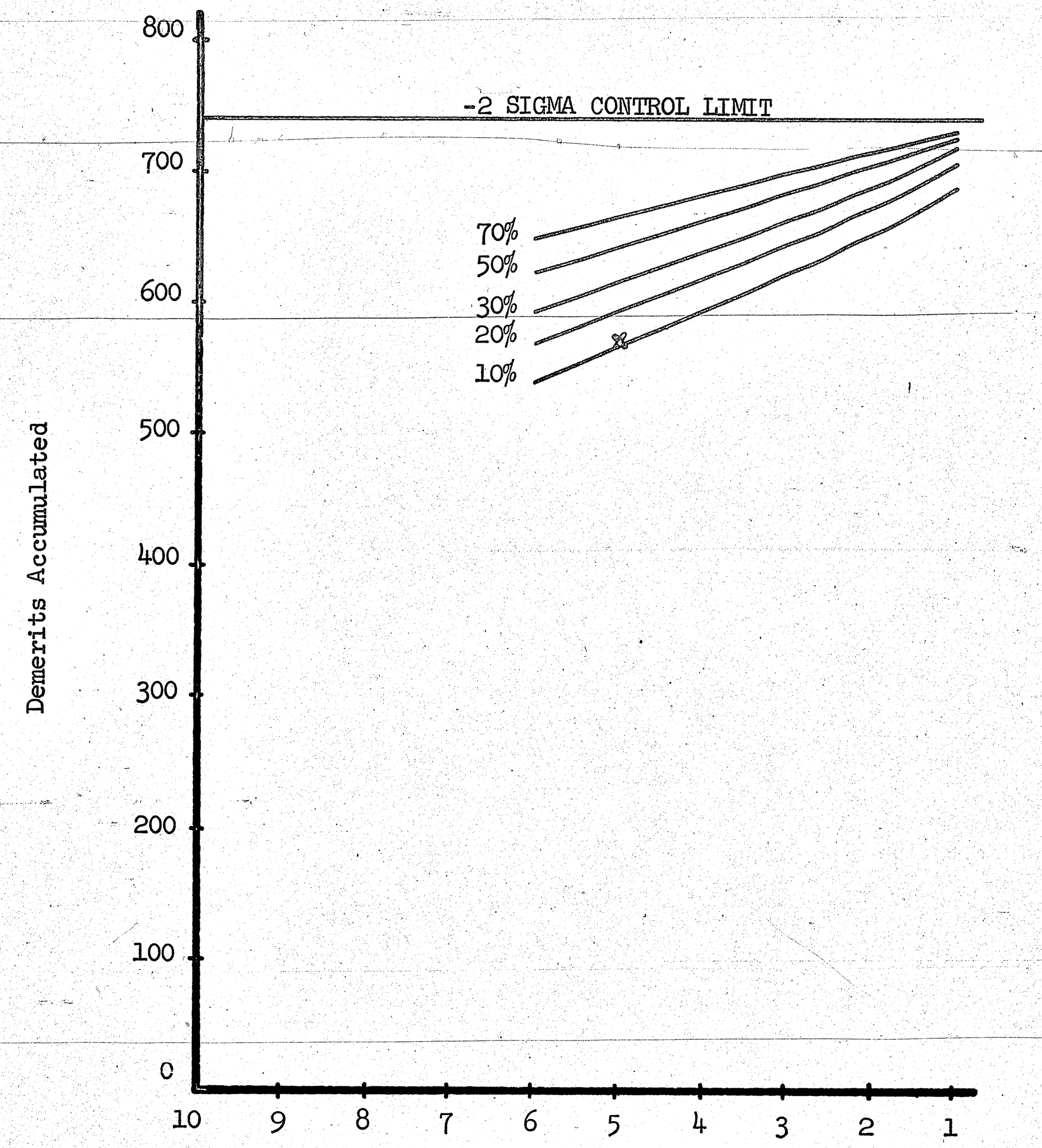

FIGURE 5 RISK CURVES WITH 40% SCREENING

VI. CONCLUSIONS

Assessing arbitrary weights to attributes, instead of considering all defects equally serious, severely complicates analyzing the situation in that the interrelationship is multivariate. Moreover, mixing the defect classifications into a single variable, namely demerits by equation (1), results in a multimodial frequency distribution. In the univariate scheme, the peaks of the frequency distribution occur near each of the demerit weights. Therefore, a step function results which is highly dependent on the arbitrary weights.

Attempting to use a mathematical model to describe the joint occurrences of the defect classifications is difficult for these reasons and is also found a problem from the variability of the proportions of defects (P_i) as mentioned in Chapter III. The model selected is the multinomial distribution with input parameters of the sample size and the proportions of defects. The multinomial distribution as a model is employed to compute the probability of various joint occurrences of the defects. Enumerating those joint occurrences that will cause the product to be rated out of control at the end of the month, given that a certain number of demerits have accumulated thus far in the month and that the auditing department has a certain number of samples remaining, determines the risk from the producer's standpoint.

The multinomial distribution requires the actual occurrences of the random variables to be mutually exclusive, which in sampling means

that each unit of product inspected can have only one outcome; namely, a critical defect or a major defect or a minor defect or no defects. This seems to preclude using the multinomial distribution here. However, the proportions of defects are relatively small, so a simplifying assumption is made that if a unit of product actually has more than one defect, then defects are assumed to occur in separate units.

Sample data accumulated under a weighted attribute inspection scheme for a typical manufactured product is shown in APPENDIX B. This data is used through out the thesis to develop, verify, and demonstrate using the models and procedures. In APPENDIX D, the chi-square goodness of fit test is used to check the null hypothesis that the data came from a multinomial parent population. Using constant parameters (sample size n and proportions of defects P_i), the null hypothesis is rejected at the 5% significance level. This means the probability is not more than .05 that the data does not come from a multinomial distribution. However, at the 1% significance level the null hypothesis could be accepted, which means the fit is not too bad at the 5% level.

As mentioned before, the mathematical model assumes constant parameters. The variability of P_i from lot to lot is one of the major reasons the data does not appear to come from a multinomial distribution and is likely to be a problem when checking other distributions.

The variability of P_i is attacked by Monte Carlo simulation. The assumption is made that each P_i follows a beta distribution.[5,28] The beta distribution is selected arbitrarily, but has two appealing

characteristics: 1) it is a continuous distribution where the random variable is only defined between O and 1, and 2) two parameters describe the distribution, which can result in many different shapes.

The quality of production lots is simulated until all the mean values stochastically converge to within .5% of the P_i parameters found from the data. When the mean values of the simulation stochastically converge very closely to each P_i, then the accumulated simulated values represent an accurate history of the variability of P_i. At this point 95% confidence limits are determined to describe how the P_i should vary in the future (assuming each P_i varies according to a beta distribution). These confidence limits are estimates which fix the maximum and minimum values of each P_i that an inspection department will find 95 times out of 100. Confidence limits are used in the mathematical model with the results very encouraging. The risk calculated by constant parameters in model is within $\pm$ 2% of the risk calculated by the maximum and minimum confidence limits. For the producer it means that constant parameters can be employed in the mathematical model with little loss of sensitivity.

One form of presenting the risk in a clear and straight forward manner is shown in Chapter V. In Figure 2, constant risk lines are plotted for up to six remaining samples in the month. Then it is necessary for the producer to monitor the number of demerits assessed, which then determines the risk at various stages in the month. However, what alternative does the producer have if the risk is determined as too

high? The effect of appropriate action, improving the product by degrees, is represented by Figures 3, 4, and 5, where the risk is reduced for a given level of demerits. It seems reasonable that the action under a situation with, say, five remaining samples in the month, could involve screening various levels of product (short term solution). The risk curves in Figures 3, 4, and 5 are for screening the product at levels of 10%, 25% and 40%, removing or repairing those units containing defects. Using the example mentioned in Chapter V with 570 demerits accumulated and five samples remaining, the risk is reduced from .50 to .10 by screening 40% of all units to be produced in the month. Thus the producer has a mechanism by which alternate action can be evaluated from its effect on reducing the risk.

Economically speaking, and because the producer should try to maintain the product quality at acceptable levels at all times, the conclusion should <u>not</u> be made that the short term process of screening at the end of the month can be relied upon month after month. Rather, the producer should concentrate on correcting the process which will hopefully have a long term effect, and use screening or tightened inspection as a last resort. The importance of the long term effect is underlined with the realization that next month (or the month after) the risk at the same point in the month should be lower, so that short term screening is unnecessary.

VII. AREAS FOR FURTHER STUDY

1. In Chapter IV, an example is mentioned where screening the product at various levels reduced the probability that the product will be rated out of control. An application of these probabilities could be in further evaluation of the economical outcomes of certain events. It would seem reasonable that the most rational decision could be determined by minimizing the expected penalty cost (cost associated with having the product rated out of control multiplied times the risk) plus the cost necessary to achieve the risk. Table 3 illustrates this procedure for a set of costs selected arbitrarily using the example from Chapter IV with five remaining samples. Then, for a penalty

	Action	Action Cost	Risk	Expected Penalty Cost	Total
a.	None	0	.50	3000 (.50) = $1500	$1500
b.	Screen 10%	$ 200	.35	3000 (.35) = $1050	$1250
c.	Screen 25%	$ 500	.25	3000 (.25) = $ 750	$1250
d.	Screen 40%	$1000	.10	3000 (.10) = $ 300	$1300

Table 3

cost and a cost to screen the product at each level, a unique total cost curve is defined which should be minimum at one or more points. The minimum cost point would descsribe the best decision for the producer based on the expected outcome. In Table 3, this would be when action b or c is undertaken. However, since costs are difficult to determine, a series of curves would be necessary for various costs

which may or may not show that one particular type of action would be the best decision most of the time.

2. As mentioned in Chapter I, imposing an auditing scheme employing weighted attributes causes a problem controlling the product quality. Sampling plans for final inspection with weighted attributes remains a complicated area. One common practice is to analyze each defect classification separately, resulting in a sampling plan for each classification.[27] However, only the risk of accepting a defect class is defined. Some overall measure of performance, taking account of the seriousness of the defect, is needed. Applying rejection procedures on each defect classification alone has the effect of changing drastically the probabilities, both of accepting what is bad and of rejecting what is good.[5] The answer to this problem may be in the computer, where simulation techniques employing accumulated sampling data in the long run could be used.

APPENDIX A

RISK OF GOING OUT OF CONTROL GIVEN THAT A NUMBER OF DEMERITS HAVE ACCUMULATED THUS FAR IN MONTH

MONTHLY SAMPLE SIZE = 1000
INTERMEDIATE SAMPLE SIZE = 50
STANDARD QUALITY LEVEL = 0.52 DEMERITS PER UNIT
STANDARD VARIANCE LEVEL = 13.0 DEMERITS-SQUARED PER UNIT
NO. OF DEMERITS FOR THE MONTH FOR -2 SIGMA IS 740

ESTIMATE OF PROPORTIONS OF CRITICAL MAJOR AND MINOR DEFECTS
0.000428 0.003600 0.045400

NO. OF INTERMEDIATE SAMPLES REMAINING	NO. DEMERITS ACCUMULATED	RISK
1	720	0.731
1	710	0.510
1	700	0.340
1	690	0.245
1	680	0.189
1	670	0.139
1	660	0.094
1	650	0.062
1	640	0.046
2	700	0.782
2	690	0.654
2	680	0.534
2	670	0.433
2	660	0.350
2	650	0.278
2	640	0.215
2	630	0.166
2	620	0.128

APPENDIX B

ATTRIBUTE SAMPLING DATA

n = 150

Critical	Major	Minor	Critical	Major	Minor	Critical	Major	Minor
		5		1	4			6
		8			5			6
	2	6			8		3	7
	1	6		1	9		1	4
		12			4		1	4
		6		1	7			4
		10			7	1		8
		4		1	5			3
		12	1		7			8
		12		2	9			4
		5		1	5			4
		3			5		1	6
		9			12			7
		2		1	4			6
		7			3			3
	1	2		1	15		2	3
		10			7			4
	1	7			9			4
1	1	6			4		1	8
	1	11		1	2		1	4
	1	7	1		9			5
	1	4		1	6			4
	1	10			6		3	8
	1	11			3		1	15
		5			2			14
	2	7		1	2			10
	1	13		1	3		1	10
	1	12	1	5	5			15
		9		2	6		1	9
		10			8			5
		2		1	9			11
		4		1	2			8
	3	20			2			8
		10		1	5		1	5
		3		2	2		1	4
		3			4			6
	1	12			6		1	7
		8	1	1	4		3	7
		6			6		1	5
1		5			5			9
		10			5			13

ATTRIBUTE SAMPLING DATA (Cont.)

n = 150

Critical	Major	Minor	Critical	Major	Minor	Critical	Major	Minor
		7		1	6		1	15
		9	1	1	11		3	6
		7	1	1	6			12
		12	1	1	3		1	12
		12			2			7
		10			4		1	6
		5			6		1	8
		6		2	8			14
		7		1	5			7

$\tilde{p}_1$ = estimate of the proportion of Critical defects = .000428

$\tilde{p}_2$ = estimate of the proportion of Major defects = .00360

$\tilde{p}_3$ = estimate of the proportion of Minor defects = .0454

APPENDIX C

The null hypothesis that the random variables are independent is checked using a chi-square test[9].

UNIVARIATE MARGINAL FREQUENCIES

Defects	Critical	Major	Minor
0	139	89	
1	11	48	
2		7	10
3		5	10
4			20
5		1	18
6			21
7			17
8			12
9			10
10			9
11			4
12			10
13			2
14			2
15			4
16			
17			
18			
19			
20			1
	150	150	150

CONTINGENCY TABLES

Observations:

≥ 0 Critical

Minor	0	≥1	
0-5	34	24	58
6-9	35	25	60
≥10	20	12	32
	89	61	150

Expectations:

		0	≥1
	0-5	34.4	23.6
Minor	0-5	29.6	20.3
	≥10	24.8	17.1

$$\tilde{\chi}^2 = \sum_{j=1}^{m} \frac{(O_j - E_j)^2}{E_j}$$

$$= \frac{(34.4-34)^2}{34.4} + \frac{(29.6-35)^2}{29.6} + \frac{(24.8-20)^2}{24.8} + \frac{(23.6-24)^2}{23.6}$$

$$+ \frac{(20.3-25)^2}{20.3} + \frac{(17.1-32)^2}{17.1} = 4.50$$

Compare this chi-square total with $\chi^2_{.90,4} = 7.8$ $\tilde{\chi}^2 < \chi^2_{.90,4}$ therefore, accept the null hypothesis that the random variables are independent.

APPENDIX D

The multinomial distribution is a multivariate extension to the binomial, where there are more than two outcomes possible in a finite number of trials. In fact, the possible outcomes in the multinomial are necessarily mutually exclusive. This means if one outcome occurs in one trial, no other outcome is allowable. In the sampling situation considered in this thesis for three random varialbes, the four possible outcomes are observance of a Critical defect, a Major defect, a Minor defect, or no defect in each of n total trials.

The joint probability density function

$$g(x_1,x_2,\ldots,x_k \mid n,p_1,p_2,\ldots,p_k) = \frac{n!}{x_1!x_2!\ldots x_k!} \quad p_1^{x_1} p_2^{x_1} \ldots p_k^{x_k}$$

is a relationship for the probability that the random variables X_1, X_2, ..., X_k equal observations x_1, x_2, ..., x_k (number of times the i^{th} outcome occurs). Parameters of the distribution are the number of trials in the experiment, n, and the proportion (probability) of the i^{th} outcome, p_i, i = 1, 2, ..., k.

Necessary and sufficient conditions for the multinomial are:

$$\sum_{i=1}^{k} x_i = n \qquad \text{(all outcomes exhaust the sample space n)}$$

and

$$\sum_{i=1}^{k} p_i = 1 \qquad \text{(k probabilities exhaust the probability space)}$$

A Chi-Square Goodness of Fit test is employed to check the null hypothesis that the sample data (APPENDIX B) came from a multinomial parent population. Maximum likelihood estimates of the proportions of the three defects are found with the relationship

$$\frac{1}{n} \cdot \frac{1}{N} \sum_{j=1}^{N} x_{ij} = \tilde{p}_i \qquad i = 1, 2, 3$$

where N = number of observations (no. of samples)

n = sample size

Estimates of the proportions are shown in APPENDIX B and are repeated here.

$$\tilde{p}_1 = .000428$$

$$\tilde{p}_2 = .00360$$

$$\tilde{p}_3 = .0454$$

These estimates are used to form the expected frequencies in the table below.

Expectations

≥ 0 Critical

Major

Minor	0	≥ 1
0 - 5	27.645	20.341
6 - 9	46.735	33.409
≥ 10	12.942	8.907

149.979

Another table is formed from the observed frequencies of the sample data.

		Observations: ≥ 0 Critical	
		0	≥ 1
Minor	0 - 5	34	24
	6 - 9	35	25
	≥ 10	20	12

The chi-square is then determined, with correction for continuity included (see Bryant, p. 113).

$$\tilde{\chi}^2 = \sum_{j=1}^{m} \frac{\left[|\tilde{O}_j - E_j| - .5\right]^2}{E_j}$$

$$= \frac{\left[|34-27.645|-.5\right]^2}{27.645} + \frac{\left[|35-46.735|-.5\right]^2}{46.735} + \frac{\left[|20-12.942|-.5\right]^2}{12.942}$$

$$+ \frac{\left[|24-20.341|-.5\right]^2}{20.341} + \frac{\left[|25-33.409|-.5\right]^2}{33.409} + \frac{\left[|12-8.907|-.5\right]^2}{8.907}$$

$$= 10.38$$

which is greater than $\chi^2_{.95,3} = 7.815$. Hence the null hypothesis is rejected. In other words, the multinomial distribution does not provide a good "fit" to the data at 5% level of significance.

APPENDIX E

The k variate normal probability density function

$$f(X \mid \mu, \Sigma) = (2\pi)^{-k/2} |\Sigma|^{-1/2} \exp\left[-\tfrac{1}{2}(X-\mu)' \Sigma^{-1}(X-\mu)\right]$$

where $X = (x_1\ x_2\ x_3\ \cdots\ x_k)$ a column vector containing the random variables.

$\mu = (\mu_1\ \mu_2\ \mu_3\ \cdots\ \mu_k)$ a column vector expressing the mean of each random variable.

Σ the k dimensional variance -covariance matrix is important in that no assumptions are necessary concerning the statistical dependency between the random variables.

When μ and Σ are unknown, they can be estimated from sample data using the principle of maximum liklihood with the following relationships[7,6]

$$\tilde{\mu}_i = \frac{1}{N}\sum_{j=1}^{N} x_{ij}, \qquad \text{for } i = 1,2,\ldots,k$$

where $\tilde{\Sigma} = \tilde{\sigma}_{ij}, \qquad \text{for } i,j = 1,2,\ldots,k$

$$\tilde{\sigma}_{ij} = \frac{1}{N-1}\sum_{v=1}^{N}(x_{iv} - \tilde{\mu}_i)(x_{jv} - \tilde{\mu}_j) \quad \text{for } i,j = 1,2,\ldots,k$$

It is known that the quadratic portion of the multivariate normal is distributed as a chi-square distribution with k degrees of freedom[6]. The quadratic portion is that relationship

$$Q = (X - \mu)' \Sigma^{-1} (X - \mu)$$

The method of Bates[8] is used to check the null hypothesis that the set of observational vectors came from a multivariate normal parent population. In this situation, each observational vector

$X_i = (x_{1i} x_{2i} x_{3i})$, for i = 1,2,...,N observations,

generates a unique $\tilde{Q}_i = (X_i - \tilde{\mu})' \sum^{-1} (X_i - \tilde{\mu})$ which falls into one cell j defined by Pearson's Chi-Square Closeness of Fit test. For each vector, $\tilde{Q}_i$ is compared with percentage points of the chi-square distribution for the p_j probability interval. $\tilde{Q}_i$ is an observation in the j^{th} interval if

$$\chi^2_{p_{j-1},k} \leq \tilde{Q}_i < \chi^2_{p_j,k}, \qquad i = 1,2,\ldots,N \quad j = 1,2,\ldots,m$$

For the set of data in Appendix B, the following table results

Cell j	p_j	$\chi^2_{p_j,3}$	E_j	$\tilde{O}_j$	$(\tilde{O}_j - E_j)^2/E_j$
1	.05	.3518	7.5	0	7.5
2	,10	.5844	7.5	31	73.3
3	.20	1.0052	15	31	17.1
4	.30	1.4237	15	29	13.1
5	.40	1.8692	15	8	3.27
6	.50	2.3660	15	3	9.75
7	.60	2.9462	15	19	1.07
8	.70	3.6649	15	5	6.67
9	.80	4.6416	15	2	11.27
10	.90	6.2514	15	6	5.4
11	.95	7.8147	7.5	1	5.63
12	1.00	∞	7.5	15	7.5
TOTAL			150	150	161.56 = $\tilde{\chi}^2$

When using sample data, the number of degrees of freedom is[8] (m-1) - (k/2)(k+3) for m cells and k dimensional multivariate normal distribution.

Then, compare the chi-square total with

$\chi^2_{.95,2} = 5.99$ But $\tilde{\chi}^2 > \chi^2_{.95,2}$

Therefore, reject the null hypothesis with great significance.

APPENDIX F

FORTRAN IV PROGRAM
IBM 360/50

Purpose: The computer program in this appendix calculates the following:

(a) Intermediate sample size to be taken by the auditing department based on the monthly sample size and the number of samples taken.

(b) Number of demerits which is the -2 sigma monthly control limit based on predetermined quality standards and the monthly sample size.

(c) Risk that the product will be rated out of control at the end of the month assuming various levels of demerits have been accumulated with up to six intermediate samples remaining in the month.

Input Card:

Data	Card Columns	Format
Standard Quality Level	1 - 4	F4.2
Standard Variance Level	5 - 8	F4.0
Monthly Sample Size	9 -12	I4
Proportion of Critical Defects	13 -22	F10.8
Proportion of Major Defects	23 -32	F10.8
Proportion of Minor Defects	33 -42	F10.8
Number of Samples Taken in the Month	43 -46	I4

```
C     ************************************************************************
C
C     MATHEMATICAL MODEL TO CALCULATE THE RISK WITH SIX REMAINING
C     INTERMEDIATE SAMPLES,  AS DESCRIBED IN CHAPTER II
C
C     ************************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON P1,P2,P3,N
      DIMENSION T1(50),T2(50),T3(50),T4(50),T5(50),T6(50),T7(50)
      READ(1,10)US,CS,MSS,P1,P2,P3,ISS
C
C
C     FIND THE NUMBER OF DEMERITS THAT MUST BE ACCUMULATED IN THE
C     MONTHLY SAMPLE FOR THE PRODUCT QUALITY TO BE OUT OF CONTROL
C
      SS=DFLOAT(MSS)
      STI=SS*CS
      XMD=SS*US + 2.*DSQRT(STI)
C
C     FIND THE INTERMEDIATE SAMPLE SIZE
C
      N=MSS/ISS
C
C     COMPUTE THE RISK OF BEING OUT OF CONTROL AT THE END OF THE
C     MONTH GIVEN THAT A CERTAIN NUMBER OF DEMERITS HAVE BEEN FOUND
C     IN THE X-TH INTERMEDIATE SAMPLE OF THE MONTH
C
C     START WITH THE EXPECTED DEMERITS AND REDUCE IN STEPS OF 10
C
      X=N
      ED=X*(P1*100.+P2*50.+P3*10.)
      NED=ED*.08
```

```
      NED=NED*10                                                        00360
      IF(NED .LT. 10)NED=10                                             00370
      ED=NED                                                            00380
      INT=XMD/10.                                                       00390
      XMD=INT*10                                                        00400
      MD=XMD                                                            00410
      FTN=FACT(X)                                                       00420
      WRITE(3,11)                                                       00430
      WRITE(3,12)MSS,N,US,CS,MD                                         00440
      WRITE(3,15)P1,P2,P3                                               00450
      WRITE(3,13)                                                       00460
C                                                                       00470
C        THE I1 LOOP CALCULATES THE RISK WITH ONE INTERMEDIATE SAMPLE    00480
C        YET TO TAKE IN THE MONTH AND THE I2 LOOP THE RISK WITH TWO      00490
C        INTERMEDIATE SAMPLES TO TAKE AND SO FORTH                       00500
C                                                                       00510
      NDAY=1                                                            00520
      CODE=1.                                                           00530
      IDM=XMD-ED                                                        00540
      DTG=ED                                                            00550
      DO 230 I1=1,10                                                    00560
      CALL PRNOM(DTG,PACC,CODE,FTN)                                     00570
      PACC=1.-PACC                                                      00580
      WRITE(3,14)NDAY,IDM,PACC                                          00590
      IF(PACC .LT. .05)GO TO 235                                        00600
      DTG = DTG+10.                                                     00610
      IDM=IDM-10                                                        00620
  230 CONTINUE                                                          00630
  235 CONTINUE                                                          00640
C                                                                       00650
C        FILL ARRAYS WITH THE PROBABILITY OF FINDING DT DEMERITS IN     00660
C        THE INTERMEDIATE SAMPLE          THEN USE THE ARRAYS INSTEAD   00670
C        OF RECALCULATING IN EACH LOOP                                  00680
C                                                                       00690
      CODE=-1.                                                          00700
```

```
      DT=0.0                                                            00710
      DO 50 K=1,50                                                      00720
      CALL PRNOM(DT,PAC,CODE,FTN)                                       00730
      T1(K)=PAC                                                         00740
      T2(K)=PAC                                                         00750
      T3(K)=PAC                                                         00760
      T4(K)=PAC                                                         00770
      T5(K)=PAC                                                         00780
      T6(K)=PAC                                                         00790
      T7(K)=PAC                                                         00800
      DT=DT+10.                                                         00810
   50 CONTINUE                                                          00820
      IDM=XMD-2.*ED                                                     00830
      DTG=2.*ED                                                         00840
      NDAY=NDAY+1                                                       00850
      DO 250 I2=1,12                                                    00860
      D2=DTG-10.                                                        00870
      PRT=0.                                                            00880
      K2=DTG/10.                                                        00890
      DO 240 I21=1,K2                                                   00900
      D1=0.                                                             00910
      DO 245 I22=1,K2                                                   00920
      IF((D1+D2) .GE. DTG)GO TO 240                                     00930
      J1=D1/10.+1                                                       00940
      J2=D2/10.+1                                                       00950
      PAC1=T1(J1)                                                       00960
      PAC2=T2(J2)                                                       00970
      PR2=PAC1*PAC2                                                     00980
      PRT=PRT+PR2                                                       00990
  245 D1=D1+10.                                                         01000
  240 D2=D2-10.                                                         01010
      PRT=1.-PRT                                                        01020
      WRITE(3,14)NDAY,IDM,PRT                                           01030
      IF(PRT .LT. .10)GO TO 255                                         01040
      DTG=DTG+10.                                                       01050
```

```
250  IDM=IDM-10                                                        01060
255  CONTINUE                                                          01070
      IDM=XMD-3.*ED                                                    01080
      DTG=3.*ED                                                        01090
      NDAY=NDAY+1                                                      01100
      DO 270 I3=1,12                                                   01110
      D3=DTG-10.                                                       01120
      PRT=0.                                                           01130
      K3=DTG/10.                                                       01140
      DO 265 I32=1,K3                                                  01150
      D2=0.                                                            01160
      K33=(DTG-D3)/10.                                                 01170
      DO 260 I31=1,K33                                                 01180
      D1=0.                                                            01190
      DO 280 I33=1,K33                                                 01200
      IF((D1+D2+D3) .GE. DTG)GO TO 260                                 01210
      J1=D1/10.+1                                                      01220
      J2=D2/10.+1                                                      01230
      J3=D3/10.+1                                                      01240
      PAC1=T1(J1)                                                      01250
      PAC2=T2(J2)                                                      01260
      PAC3=T3(J3)                                                      01270
      PR3=PAC1*PAC2*PAC3                                               01280
      PRT=PRT+PR3                                                      01290
280   D1=D1+10.                                                        01300
260   D2=D2+10.                                                        01310
265   D3=D3-10.                                                        01320
      PRT=1.-PRT                                                       01330
      WRITE(3,14)NDAY,IDM,PRT                                          01340
      IF(PRT .LT. .10)GO TO 290                                        01350
      DTG=DTG+20.                                                      01360
270   IDM=IDM-20                                                       01370
290   CONTINUE                                                         01380
      IDM=XMD-4.*ED                                                    01390
      DTG=4.*ED                                                        01400
```

```
      NDAY=NDAY+1                                                       01410
      DO 350 I4=1,14                                                    01420
      D4=DTG-10.                                                        01430
      PRT=0.                                                            01440
      K4=DTG/10.                                                        01450
      DO 340 I43=1,K4                                                   01460
      D3=0.                                                             01470
      K44=(DTG-D4)/10.                                                  01480
      DO 330 I42=1,K44                                                  01490
      D2=0.                                                             01500
      DO 320 I41=1,K44                                                  01510
      D1=0.                                                             01520
      DO 310 I44=1,K44                                                  01530
      IF((D1+D2+D3+D4) .GE. DTG)GO TO 320                               01540
      J1=D1/10.+1                                                       01550
      J2=D2/10.+1                                                       01560
      J3=D3/10.+1                                                       01570
      J4=D4/10.+1                                                       01580
      PAC1=T1(J1)                                                       01590
      PAC2=T2(J2)                                                       01600
      PAC3=T3(J3)                                                       01610
      PAC4=T4(J4)                                                       01620
      PR4=PAC1*PAC2*PAC3*PAC4                                           01630
      PRT=PRT+PR4                                                       01640
  310 D1=D1+10.                                                         01650
  320 D2=D2+10.                                                         01660
  330 D3=D3+10.                                                         01670
  340 D4=D4-10.                                                         01680
      PRT=1.-PRT                                                        01690
      WRITE(3,14)NDAY,IDM,PRT                                           01700
      IF(PRT .LT. .10)GO TO 360                                         01710
      DTG=DTG+20.                                                       01720
  350 IDM=IDM-20                                                        01730
  360 CONTINUE                                                          01740
      NDAY=NDAY+1                                                       01750
```

```
      IDM=XMD-5.*ED                                                     01760
      DTG=5.*ED                                                         01770
      DO 460 I5=1,14                                                    01780
      D5=DTG-10.                                                        01790
      PRT=0.                                                            01800
      K5=DTG/10.                                                        01810
      DO 450 I54=1,K5                                                   01820
      D4=0.                                                             01830
      K55=(DTG-D5)/10.                                                  01840
      DO 440 I53=1,K55                                                  01850
      D3=0.                                                             01860
      DO 430 I52=1,K55                                                  01870
      D2=0.                                                             01880
      DO 420 I51=1,K55                                                  01890
      D1=0.                                                             01900
      DO 410 I55=1,K55                                                  01910
      IF((D1+D2+D3+D4+D5) .GE. DTG)GO TO 420                            01920
      J1=D1/10.+1                                                       01930
      J2=D2/10.+1                                                       01940
      J3=D3/10.+1                                                       01950
      J4=D4/10.+1                                                       01960
      J5=D5/10.+1                                                       01970
      PAC1=T1(J1)                                                       01980
      PAC2=T2(J2)                                                       01990
      PAC3=T3(J3)                                                       02000
      PAC4=T4(J4)                                                       02010
      PAC5=T5(J5)                                                       02020
      PR5=PAC1*PAC2*PAC3*PAC4*PAC5                                      02030
      PRT=PRT+PR5                                                       02040
  410 D1=D1+10.                                                         02050
  420 D2=D2+10.                                                         02060
  430 D3=D3+10.                                                         02070
  440 D4=D4+10.                                                         02080
  450 D5=D5-10.                                                         02090
      PRT=1.-PRT                                                        02100
```

```
      WRITE(3,14)NDAY,IDM,PRT                                          02110
      IF(PRT .LT. .10)GO TO 470                                        02120
      DTG=DTG+20.                                                      02130
  460 IDM=IDM-20                                                       02140
  470 CONTINUE                                                         02150
      IDM=XMD-7.*ED                                                    02160
      DTG=7.*ED                                                        02170
      NDAY=NDAY+1                                                      02180
      DO 570 I6=1,12                                                   02190
      D6=DTG-10.                                                       02200
      PRT=0.                                                           02210
      K6=DTG/10.                                                       02220
      DO 560 I65=1,K6                                                  02230
      K66=(DTG-D6)/10.                                                 02240
      D5=0.                                                            02250
      DO 550 I64=1,K66                                                 02260
      D4=0.                                                            02270
      DO 540 I61=1,K66                                                 02280
      D3=0.                                                            02290
      DO 530 I63=1,K66                                                 02300
      D2=0.                                                            02310
      DO 520 I62=1,K66                                                 02320
      D1=0.                                                            02330
      DO 510 I66=1,K66                                                 02340
      IF((D1+D2+D3+D4+D5+D6) .GE. DTG)GO TO 520                         02350
      J1=D1/10.+1                                                      02360
      J2=D2/10.+1                                                      02370
      J3=D3/10.+1                                                      02380
      J4=D4/10.+1                                                      02390
      J5=D5/10.+1                                                      02400
      J6=D6/10.+1                                                      02410
      PAC1=T1(J1)                                                      02420
      PAC2=T2(J2)                                                      02430
      PAC3=T3(J3)                                                      02440
      PAC4=T4(J4)                                                      02450
```

```
      PAC5=T5(J5)                                                       02460
      PAC6=T6(J6)                                                       02470
      PR6=PAC2*PAC1*PAC3*PAC4*PAC5*PAC6                                 02480
      PRT=PRT+PR6                                                       02490
  510 D1=D1+10.                                                         02500
  520 D2=D2+10.                                                         02510
  530 D3=D3+10.                                                         02520
  540 D4=D4+10.                                                         02530
  550 D5=D5+10.                                                         02540
  560 D6=D6-10.                                                         02550
      PRT=1.-PRT                                                        02560
      WRITE(3,14)NDAY,IDM,PRT                                           02570
      IF(PRT .LT. .10)GO TO 580                                         02580
      DTG=DTG+20.                                                       02590
  570 IDM=IDM-20                                                        02600
  580 CONTINUE                                                          02610
   10 FORMAT(F4.2,F4.0,I4,3F10.8,I4)                                    02620
   11 FORMAT('1',////,14X,'RISK OF GOING OUT OF CONTROL GIVEN THAT A'   02630
     1' NUMBER OF',/,14X,' DEMERITS HAVE ACCUMULATED THUS FAR IN MONTH') 02640
   12 FORMAT(//14X,'MONTHLY SAMPLE SIZE = ',I4,/14X,'INTERMEDIATE'      02650
     1' SAMPLE SIZE = ',I4,/14X,'STANDARD QUALITY LEVEL = ',F6.2,       02660
     1'  DEMERITS PER UNIT',/,14X,'STANDARD VARIANCE LEVEL = ',F5.1,    02670
     1'  DEMERITS-SQUARED PER UNIT'/14X,'NO. OF DEMERITS FOR THE MONTH' 02680
     1' FOR -2 SIGMA IS ',I6)                                           02690
   13 FORMAT(///17X,'NO. OF INTERMEDIATE',8X,'NO. DEMERITS',/18X,       02700
     1'SAMPLES REMAINING',9X,'ACCUMULATED',10X,'RISK')                  02710
   14 FORMAT(/24X,I2,22X,I3,14X,E5.3)                                   02720
   15 FORMAT(//14X,'ESTIMATE OF PROPORTIONS OF CRITICAL  MAJOR AND '    02730
     1'MINOR DEFECTS',/5X,3F17.6)                                       02740
      STOP                                                              02750
      END                                                               02760
```

```
      SUBROUTINE PRNOM(DEM,PROB,CODE,FTX)                               02770
      IMPLICIT REAL*8(A-H,O-Z)                                          02780
      COMMON PA,PB,PC,NN                                                02790
      DA=0.                                                             02800
      DB=0.                                                             02810
      DC=0.                                                             02820
      PROB=0.                                                           02830
      PR= 1.-(PA+PB+PC)                                                 02840
      XN=NN                                                             02850
C                                                                       02860
C        CALCULATES THE PROBABILITY OF FINDING DEM DEMERITS OR LESS IN   02870
C        A SAMPLE OF SIZE N WITH PROPORTIONS OF PA PB AND PC OF FINDING  02880
C        CLASS A B AND C DEFECTS USING THE MULTINOMIAL DISTRIBUTION      02890
C                                                                       02900
C        FIRST FIND THE POSSIBLE COMBINATIONS OF CLASS A B AND C IN DEM  02910
C                                                                       02920
      I=DEM/100.+1.                                                     02930
      J=DEM/50.+1.                                                      02940
      K=DEM/10.+1.                                                      02950
      IF(CODE)105,108,108                                               02960
  105 DC=DEM/10.                                                        02970
  108 CONTINUE                                                          02980
C                                                                       02990
C        NEXT COMPUTE THE PROBABILITY THAT THE DISCRETE RANDOM VARIABLES 03000
C        ACTUALLY EQUAL THE VARIOUS COMBINATIONS ALLOWABLE IN DEM        03010
C                                                                       03020
      DO 140 IA=1,I                                                     03030
      DO 130 JB=1,J                                                     03040
      DO 120 KC=1,K                                                     03050
      DR=NN-(DA+DB+DC)                                                  03060
      XDEM= 100.*DA + 50.*DB + 10.*DC                                   03070
      IF(CODE)110,110,115                                               03080
  110 IF(XDEM - DEM)120,113,125                                         03090
  113 CONTINUE                                                          03100
      GO TO 117                                                         03110
```

```
  115 IF(XDEM .GE. DEM)GO TO 125                                          03120
  117 CONTINUE                                                            03130
      XLNPR= FTX + DA*DLOG(PA) + DB*DLOG(PB) + DC*DLOG(PC) +              03140
     1       DR*DLOG(PR) - FACT(DA) - FACT(DB) - FACT(DC) - FACT(DR)      03150
C                                                                         03160
C         XLNPR IS THE NATURAL LOG OF THE PROBABILITY                     03170
C                                                                         03180
      SUM = DEXP(XLNPR)                                                   03190
      PROB=PROB+SUM                                                       03200
      IF(CODE)125,120,120                                                 03210
  120 DC=DC+1.                                                            03220
  125 CONTINUE                                                            03230
      DC=0.                                                               03240
  130 DB=DB+1.                                                            03250
      DB=0.                                                               03260
  140 DA=DA+1.                                                            03270
      RETURN                                                              03280
      END                                                                 03290
```

```
      FUNCTION FACT(Q)                                                  03300
      IMPLICIT REAL*8(A-H,O-Z)                                          03310
C                                                                       03320
C         CALCULATES THE NATURAL LOG OF THE FACTORIAL OF Q              03330
C                                                                       03340
      Z=Q                                                               03350
      Z=Z+1.                                                            03360
      IF(Z-1)95,95,90                                                   03370
   90 A0=1./12.                                                         03380
      A1=1./30.                                                         03390
      A2=53./210.                                                       03400
      A3=195./371.                                                      03410
      A4=22999./22737.                                                  03420
      A5=29944523./19733142.                                            03430
      A6=109535241009./48264275462.                                     03440
      STEP1=Z+A6                                                        03450
      STEP2=A5/STEP1                                                    03460
      STEP3=Z+STEP2                                                     03470
      STEP4=A4/STEP3                                                    03480
      STEP5=Z+STEP4                                                     03490
      STEP6=A3/STEP5                                                    03500
      STEP7=Z+STEP6                                                     03510
      STEP8=A2/STEP7                                                    03520
      STEP9=Z+STEP8                                                     03530
      STEP10=A1/STEP9                                                   03540
      STEP11=Z+STEP10                                                   03550
      STEP12=A0/STEP11                                                  03560
      FACT=STEP12-Z+(Z-.5)*DLOG(Z)+.5*DLOG(2.*3.1415926536)             03570
      GO TO 96                                                          03580
   95 FACT = 0.                                                         03590
   96 RETURN                                                            03600
      END                                                               03610
```

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