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A water table for the study of unsteady flow in turbomachinery

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A WATER TABLE FOR THE STUDY OF UNSTEADY FLOW IN TURBOMACHINERY

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CERTIFICATE OF APPROVAL

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

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SYMBOLS

c	speed of sound
d	water depth
M	Mach number
P	pressure
Q/ρ	volume rate of flow
T	temperature
u_i	velocity component in the i direction
V	velocity
w	width of the flow section between blades or between nozzles
x_i	length in the i direction
γ	gas constant
ρ	density

ABSTRACT

This thesis is concerned with the development of a water table to study certain unsteady flow phenomena in turbomachinery using the hydraulic analogy. The theoretical basis for the analogue is discussed and set down in the appendix, but certain necessary features are expanded in the second section. Due, in part, to the physically restricting nature of the analogy, components of the water table were constructed so as to meet these limits.

The amount of water required was based on the size of the table plus the water depths necessary so that there was a method available for determining an adequate circulation system. On the basis of knowing the different heads of water necessary to produce certain analogous Mach numbers, the design of the plastic components as well as the nozzles and blades was determined. Since the analogy calls for water depths on the order of one half to one quarter of an inch in the flow area, the range of RPM's through which testing could be accomplished was effectively limited, thus requiring a speed control and timing mechanism which would give sensitive readings in the zero to sixty RPM range. In

addition, other equipment was necessary in the construction of the analogue for various purposes. Depth measuring devices were required as well as a means of visualizing the extremely small pressure pulses which were propagated in the flow field. A surface acting agent was used to reduce the surface tension in the water which helped to enhance one of the assumptions made in the analogy.

The water table has been separated into a number of components, some of which are mentioned in the previous paragraph, so that a more detailed discussion of the various parts could be realized. Design and construction of the different segments are explained, with a summary of some preliminary findings also included.

INTRODUCTION

In order to study certain unsteady flow phenomena occurring in turbomachinery, the hydraulic analogue was turned to for obtaining qualitative results which may lead the way to more advanced work. This analogy between the pressure waves in a compressible gas and gravity waves on the free surface of a liquid allows the study of two-dimensional gas flow to be developed under conditions whereby certain elements may be more easily visualized, and possibly quantitative data may be obtained. Although this particular water table does not adhere completely to the vigorous tenets set down in the analogy for flow in a rectangular channel with a free surface and horizontal bottom, the conditions of flow are still similar enough to warrant its use as a qualitative means of determining whether certain pressure waves can be seen to exist, and, if they can be predicted to occur under certain predetermined conditions.

The correspondence between the two-dimensional gas flow and the analogous liquid flow is founded on the similarity of their respective continuity and modified Euler equations. The relationships which are derived as well as the assumptions involved are clearly developed in a number of references (10, 11, 16) and, thus, are not repeated in their entirety here. The suppositions and the mathematical development may, however, be found in appendix A. A portion of the assumptions and some of the mathematics used to derive a number of relations for this water table are, nevertheless, submitted in the following section to indicate that certain premises are met within this analogue.

Despite the fact that each segment of the water table cannot be entirely divorced from the whole project, an attempt has been made to break the table down into a number of component parts and present each as a separate entity. The design and construction of each area is, therefore, explained in a more detailed manner which gives a better understanding of the project.

ANALYSIS OF THE FLOW

Before moving on to the actual construction of the water table, a few words on the theoretical analysis involved might be in order. From the general analogy, the assumptions that the flow is two-dimensional, the water is incompressible, and viscosity, surface tension, and vertical accelerations are negligible are chosen to be such in this table. If a steady, inviscid fluid motion is assumed, the premise that the flow in the test section is irrotational can be shown to be true. That is, if the flow outside of the boundary layer is all that is considered. Under these conditions, it must be shown that the Bernoulli equation has the same constant throughout the flow field. That is,

$$\int \frac{dP}{\rho} + \frac{1}{2}V^2 = \text{constant throughout the flow field} \qquad (1)$$

or, for incompressible flow,

$$P + \frac{1}{2}\rho V^2 = \text{constant}$$

To show this equation is valid, we can take any arbitrary streamline and say that

$$\frac{P_1}{\rho} + \frac{1}{2}V_1^2 = \frac{P_2}{\rho} + \frac{1}{2}V_2^2 \qquad (2)$$

Substituting for the pressure term $\frac{p}{\rho} = gd$, we obtain

$$gd_1 + \frac{1}{2}V_1^2 = gd_2 + \frac{1}{2}V_2^2 \qquad (3)$$

If we take $d_2 = d_0$, then $V_2 = 0$, thus

$$gd_1 + \frac{1}{2}V_1^2 = gd_0 \qquad (4)$$

However, d_0 is kept constant throughout the periphery of the circular sluice gate, which means that equation (4) is the equation of every streamline. Thus, under the assumption that only the flow outside the boundary layer is examined, the flow can be considered irrotational in the test region.

We can now use the Bernoulli's equation to get the velocity in any area we are testing by

$$gd_1 + \frac{1}{2}V_1^2 = gd_0 \qquad (4)$$

$$V_1 = \sqrt{2g(d_0-d_1)} \qquad (5)$$

Due to the circular symmetry of the table, the water depth d_1 will be the same at any point on the circumference of any particular radius circle we choose in the test area. This constant depth means that the flow velocity will also be uniform for any given radius.

In the theory, the analogous speed of sound in the water may be taken to be

$$c = \sqrt{gd} \tag{6}$$

if the water depth is on the order of one quarter to one half inch. The Mach number is then

$$M = \frac{V}{C} = \sqrt{2g(d_0 - d_1)} / \sqrt{gd_1} \tag{7}$$

or $$M = \sqrt{2(d_0 - d_1)/d_1} \tag{8}$$

If d_1 is one quarter of an inch, then M is only a function of d_0. To create low Mach numbers, d_0 must be small, thereby making the corresponding change in depth small. A chart is inserted to indicate the various changes in fluid velocity, mass flow, and back depth for different Mach numbers at flow depths of one quarter and one half inches.

Flow Depth 1/4"

M	d_0 (inches)	V_1 (ft/sec)	c (ft/sec)	$\frac{Q}{\rho}$ (cu.ft/sec)
0.707	5/16	0.580	0.821	0.0172
1.000	3/8	0.821	0.821	0.0244

Flow Depth 1/2"

M	d_0 (inches)	V_1 (ft/sec)	c (ft/sec)	$\frac{Q}{\rho}$ (cu.ft/sec)
0.707	5/8	0.819	1.16	0.0486
1.000	3/4	1.16	1.16	0.0688

The rate of flow Q/ρ is determined by the flow of water at a point where the cross-sectional area, the water depth, and flow velocity are known or can be calculated. Section (2) contains a sample calculation of Q/ρ. Other related calculations were necessary, and they are shown in subsequent sections.

NECESSARY EQUIPMENT

(1) Framework and Overhead Support

To study the unsteady flow phenomena as related to turbomachinery, it was first necessary to develop a table which would have a radial inlet and an axial sink so that a continuous motion throughout the periphery of the model could be observed. To produce a constant radial velocity around the perimeter, a circular sluice gate with a constant head of water behind it was required. A table with a square configuration having a circular gate concentric with a sink in the middle was decided upon as the means of producing such a flow in the radial direction. In a preliminary design, a rough outline of the extremities of the model turbine was developed and the dimensions of the supporting frame were determined.

It was decided that the outside measurements of the table should be roughly forty-four inches square, thereby allowing access to any part of the model from the extremities of the frame. A height of thirty-six inches was considered adequate enough to allow for a collecting tank under the sink while at the same time being conveniently high for easy observation. In making as sturdy a structure as possible, to eliminate vibrations, 2" x 4" construction fir was used for the legs and main frame. Supporting 1" x 2" members were attached diagonally to the legs to give them more rigidity. A diamond shaped brace was fabricated from 1" x 3" wood to reinforce the main frame as well as to sustain some of the load. A top and front view drawing of this framework used to support the water table itself is shown in figure 2 for clarification.

In addition to the supporting frame discussed previously, a means by which the proposed circular gate could be raised, a place for a driving motor, and a location for shaft bearings used to sustain rotation of the model were required. These three functions were incorporated in an overhead support designed to have adequate strength to carry any required loads and stiffness enough to eliminate transmission of vibrations to the table. Since the projected loading was of minor consequence, the main consideration was the stiffness of the structure. Two steel 3" x 1 1/2" channels, 43 5/8" long each, were interlocked and welded at their centers, and drilled (slightly oversized) to accept a one half inch diameter shaft through the middle and two SKF, FYTP-8, No. 478203-008 bearings to secure the shaft. The vertical supports used to hold the interlocked pieces above the table were sixteen inch long standard 3" x 3" equal leg angles welded to the four ends of the crossed channels. These legs were then bolted to the main frame, as shown in figure 1, to provide the necessary skeleton for the water table. As can be seen from figure 1, these angles were not mounted flush with the channel ends, but were offset 5/8" on each one so that the bolts used to mount the angles would not interfere with the braces already in place in the main frame.

In addition, four 1/4" x 2" steel struts were welded to the crossed channel sections, two on each one, so that they extended down seven inches from the plane of the interlocked members. These were each secured at a distance of 14 3/4" from the centerline of the channel intersection. At the end of each strut was welded a 2" x 2" angle, 1 1/2" wide with a 3/8" hole provided to accept a bolt to be used as a mechanism for adjusting the height of the sluice gate. Reference to the front view of figure 1 should help clarify the previous description.

(2) Pump and Circulation System

As mentioned previously, a constant head of water was required behind the circular gate so that a constant radial flow towards the sink could be realized. In order to maintain such an equilibrium state, an inflow of water equal to that leaving through the sink exit was needed. It was decided that a circulation system was the best answer, thus, a fifty-five gallon drum was placed directly under the table to collect the water which flowed out of the sink. After preliminary calculations, a pump which could discharge the maximum requirements of the table was utilized to draw the water from the collecting tank and redistribute it into the constant head area.

To determine what the maximum rate of discharge needed would be, the extreme case of flow was assumed. That turned out to be a one-half inch depth of water in the flow field produced from a constant head of three quarters of an inch. From Bernoulli's equation,

$$gd_1 + \frac{1}{2}V_1^2 = gd_0 \tag{4}$$

thus $$V_1 = \sqrt{2g(d_0 - d_1)} \tag{5}$$

for $d_0 = 3/4''$, $d_1 = 1/2''$,

$$V_1 = \sqrt{2g(3/4 - 1/2) \times 1/12}$$

$V_1 = 1.16$ ft/sec.

From the design of the nozzles used for the model, the total cross-sectional area through which the water flowed at one half inch depth was calculated to be 17.1 inches. For the rate of flow, then

$$Q = \rho \times V \times A \quad \text{or} \quad \frac{Q}{\rho} = V \times A$$

$$\frac{Q}{\rho} = V_1 \times d_1 \times 17.1 \quad (A = 17.1 \times d_1)$$

$$\frac{Q}{\rho} = 1.16 \times 1/2 \times 1/12 \times 17.1 \times 1/12$$

$$\frac{Q}{\rho} = 0.0688 \text{ cu.ft./sec.}$$

This figure was then the maximum discharge requirement for keeping the three quarter inch level behind the sluice gate. A three horsepower Worthington pump was tested to see if it would operate under the conditions desired. To determine the discharge rate at different pressures, a setup was established whereby the time needed to pump a certain amount of water was tabulated. This information was converted to cubic feet per second for the different pressures at which the pump was operated. Although the test was quite crude, it did indicate that the pump in question was more than sufficient to handle the needs of the water table.

In the first attempts at discharging the water into the constant head area, it was found that the pressure developed by the pump caused an extreme amount of waviness which greatly impaired the analogy. An attempt was made to reduce this turbulence through the use of long copper tubing with numerous holes along its length to distribute the water more evenly in the area behind the sluice gate. This method also proved fruitless as the discharge was still too violent to insure any possibility of a smooth, constant water depth. It appeared that the major problem was to reduce this pressure head of the pump; therefore, a settling basin was incorporated.

The water was pumped into an intermediate tank made of plastic, roughly 11 1/2" x 15 1/2" x 8" with numerous holes in the bottom. Plastic and rubber hosing was extended from these holes and the water was transferred, by gravity, from this intermediate basin into the water table. With the tubing judiciously placed about so that the water flowed into the area evenly, the problem of a smooth water source was solved. The use of aluminum meshing at the water inlet sections was also instrumental in decelerating the fluid so that a veritably constant head source of water was produced behind the sluice gate.

The gravity tank not only helped eliminate the pressure head, but also provided a place for filtering the water to keep rust particles and other debris from entering the test section. Absorbent gauze placed over the pump outlet and plexiglass insulation in the settling basin itself helped to greatly reduce the surface contamination problem.

(3) Plastic Components

The table itself was constructed from plexiglass of one half inch thickness for the bottom, and one quarter inch thickness for the sidewalls. Figure 3 shows the dimensioned plastic parts only and should be referred to for clarification. The outside dimensions are the same as the supporting frame, 43 5/8" square. In the center of the table, a nozzle was fitted to allow a smooth exit at the sink. This nozzle was made from three inch thick plexiglass with an outside diameter of ten inches and an inside diameter of six inches. A quarter arc of two inch radius was turned on the inside of the cylinder to produce a smooth, continuous change from the radial to the axial direction.

The sluice gate used to regulate the flow in the test area was formed into a circle from one half inch plexiglass, four and one half inches high with an outside diameter of twenty nine and one half inches. At four ninety degree intervals, plastic hinges were attached to provide a means of suspending the gate from the overhead support. These four hinges were connected to the four struts described in section (1) by means of a three inch long 1/4" bolt and nut. These nut and bolt assemblies provided the support for and the screw adjustment of the sluice gate. By tightening or loosening these four bolts, the gate height could be changed, hence the water flow in the test area could be regulated.

As mentioned previously, a constant water depth behind the sluice gate was desired in order to insure a constant water flow in the test section. The first attempt was a plexiglass ring behind the gate, the height of this ring being the desired constant head. The water was pumped into the area between the gate and the ring with any excess water flowing over into the area between the ring and the sidewalls. Unfortunately, this method proved unsatisfactory because the incoming water created turbulence which did not dampen itself out. To overcome this unsteady condition, the area into which the water would enter was increased. Four four-inch squares made from 1/8" plastic strips, machined to a height equal to the desired back head, were built. These pens were placed in the corners, as shown in figure 3, surrounding the drain holes drilled in the plastic base. If too much water was being pumped in, the excess merely flowed over into the enclosed area and was removed via the drain there. If greater or lesser depths of water were desired, the squares were merely replaced with others having the proper height.

In the previous section, it was stated that the water inlet problem was solved through the use of a gravity tank and numerous hose connections from the tank to the water table. These hoses were placed around the perimeter of the table and attached to the sidewalls with a downward exit. The entering water was thus at a maximum distance from the sluice gate but near the overflow enclosures. Aluminum meshing (the kind used for ordinary window screening) was cut into long strips and placed on the table lengthwise to act as baffles thus decelerating the inflow and reducing the turbulent nature of the water at the sidewall. The inlet section of the table was restricted therefore to a thin area roughly one inch wide along the perimeter of the base. Consequently, the turbulence was dampened out in that small section away from the sluice gate. The assumption, therefore, that all streamlines originated from the same constant head of depth d_0 can be considered valid.

(4) Blades and Nozzles

The design of the model used in the water table analogy was based on a number of factors which contributed to the ultimate configuration of the nozzles and blades. Of primary importance was the need for uniform motion throughout the test area so that no extraneous disturbances would be propagated in the flow field. Also, the symmetry of the model was a necessity since certain test results depended on the accurate spacing of the nozzle and blades. Then, too, the surfaces of the model in contact with the fluid were required to be smooth so that no secondary disturbances would be produced in the flow field.

Among the factors in the nozzle design was that a seven degree inlet angle to the blades was desired. The incoming water had to be diverted from a radial inflow to the seven degree angle smoothly throughout the circumference of the model. In order that the flow would be uniform, nozzles had to be symmetrically placed about the perimeter to produce the desired change in flow without causing any turbulence. Eighteen of these spaced every twenty degrees were used to divert the flow the desired amount. Figures 1, 4, and 8 show the dimensions and relative positioning of this part of the model, while figure 6 presents a full-sized drawing of the nozzles themselves. Their general shape was designed with the flow directions in mind, and have no particular similarity with any standard nozzle.

In the design, the change in the cross-sectional flow area was kept as gradual as possible so that any tendency towards turbulence would be reduced. Referring to figure 6, the section AB along the side of the nozzle was made a plane surface rather than a curved one to keep the streamlines as nearly parallel as possible in the short distance from the nozzle to the blades, and the angle at A was also made small.

The design of the blade assembly, shown in figures 5 and 9, has a number of advantageous features. The forty blades, spaced every nine degrees, can be changed to a twenty blade wheel with provisions also made to alter the distance of the leading edge of the buckets to one quarter inch from the nozzles instead of the one half inch distance if that is so desired. Thus four different blade configurations are possible with this arrangement. In getting the shape of the blade shown in figure 7, there were four factors to consider.

A smooth entrance for the approaching fluid had to be made, the leading edge of the bucket had to be round, the curvature of the blades had to be such that no separation would occur, and the cross-sectional flow area at the trailing edge had to be slightly greater than that at the leading edge. In addition, the direction of the absolute motion of the water leaving the buckets was chosen to be radial.

To obtain the shape of the blades, sketches were drawn until a feasible design was obtained. The leading edge was an arc of a one quarter inch diameter circle, and the curvature of the blades was small enough so that the change in the flow direction relative to the blades was only 36.7 degrees. Also, the cross-sectional area between the blades was such that the flow was slightly accelerated, the inlet width being 0.8 inches and the exit width, 0.88 inches.

At the leading edge, the relative angle of the water flow with respect to the tangential velocity of the blades was seventy-one degrees so that, for a particular back head and flow depth, the speed of the rotor could be obtained. A sample vector diagram in appendix C indicates how the design speed of rotation was determined. An off-angle entrance of plus or minus twelve degrees was allowed, producing a small range of RPM's at which the blades could rotate. Knowing the speed or speed range at which the buckets could operate, a check was made on the exit of the water from the blades. From the mass rate of flow and the total cross-sectional area of the exit, a cubic equation was set up to determine the exit velocity. To illustrate, a sample calculation is shown below.

assuming $d_0 = 5/8"$, $d_1 = 1/2"$

$$\frac{1}{2}V_e^2 + gd_e = gd_0$$

$$d_e = d_0 - \frac{1}{2}gV_e^2$$

The volume rate of flow, for $d_0 = 5/8"$ and $d_1 = 1/2"$ is 0.0486 cu. ft./sec. At the nozzle exit,

$$\frac{Q}{\rho} = V_e \times A_e$$

$$0.0486 = V_e \times w_e \times d_e$$

$$0.0486 = V_e \times (d_0 - \frac{1}{2}gV_e^2) \times \frac{0.88}{12} \times 40$$

thus we get the cubic equation

$$V_e \left(\frac{5}{96} - \frac{1}{64.4} V_e^2\right) \times \frac{8.8}{3} = 0.0486$$

or $$V_e \left(\frac{5}{96} - \frac{1}{64.4} V_e^2\right) = 0.0154$$

Solving this equation by trial and error, we get a value for V_e, the velocity relative to the blades. Since the RPM range was calculated previously, an average radius at the exit can be used to get the tangential velocity there. That plus the magnitude and direction of the relative velocity will then give the true exit velocity of the water from the blades. Appendix C shows a sample of such a velocity diagram which indicates that the flow leaving the buckets is approximately radial.

Both the blade and nozzle assemblies were made to the specifications shown in the drawings by a commercial firm. The material used was aluminum with the surfaces polished to give them a smooth finish. The top view of figure 1 indicates their relative positioning on the table itself. Also, the affixing of the nozzles and buckets to their respective rings are shown in figures 8 and 9.

(5) Speed Control and Timing Device

Althougn the amount of torque needed to turn the blaces was small, the water flow through the nozzles proved insufficient so that an outside source of power was required. From the velocity diagrams described in the previous section, it was evident that the range of

RPM's and the actual operating speeds of this equipment were small, the former on the order of 0.5 RPM's and the latter about eight RPM's. Hence, not only was an accurate means of measuring the speed required, but also a method of sensitive speed regulation. To produce the desired control, a 1/20 horsepower D.C. gearmotor with a 40:1 gearhead ratio was purchased. The power output proved sufficient and the speed reduction unit helped twofold. It not only provided a means of getting a low speed output while allowing the motor itself to operate at more favorable speeds, but also provided a better control of that output. That is, a change in the armature speed was reduced by the 40:1 gear ratio. A speed control unit made for this motor was equipped with both coarse and fine adjustments for changing the motor speed. The latter allowed a variation of only two-hundred RPM's over its entire range or five RPM's at the output. Thus the unit was capable of changing the model speed on the order of 0.1 RPM's.

To measure these velocities, a clock mechanism was built to operate on every half a revolution. There were two contacts spaced 180 degrees apart with a disk mounted on the model shaft, rotating between them. A small screw was inserted on the circumference of the disk and, as it turned, the screwhead would alternately close the contacts. The closing of the first one would activate a relay circuit to start a clock. The second contact, on closing, would open the aforementioned circuit, thus stopping the clock. Measurements of one-half a revolution could therefore be read to 1/100 of a second. A table of values was made to indicate time readings versus their corresponding RPM's for ease of conversion. For the range of speeds in question, a 1/100 of a second change was roughly the equivalent of 0.03 RPM variation. Although control of the motor was still difficult, the speed for a particular control setting stayed reasonably constant (within 0.09 to 0.12 RPM's). Figure 10 shows the dimensions of the contact housing and its position on the water table is indicated in figure 1.

(6) Auxiliary Equipment

In addition to the equipment necessary for producing the test conditions in the water table, some auxiliary apparatus was required. Paramount among these was a depth measuring mechanism to accurately determine the water height behind the sluice gate and in the flow area. A threaded rod was screwed into a cantilever section attached to the overhead support. A disk was secured to the rod with markings on its circumference every fifteen degrees. Since the rod had twenty-four threads per inch, each revolution was 0.04167" in vertical motion. Thus the depth could be measured by counting the number of turns of the disk required to bring the rod from the table surface to the water surface. A similar arrangement was made using a micrometer head suspended over the flow area. Zero readings from the table surface were subtracted from the water surface reading to get the flow depth in that section.

An area where subsidiary equipment was also necessary was in the visualization of the flow phenomena. The pressure waves expected were of such a small magnitude that they were not discernible with the naked eye. Therefore, small bits of cork were suspended from the top of the nozzles with sewing thread to just touch the water surface. The disturbance would then cause a motion of the cork and string which would be more readily recognized.

Due to the small heads of water developed, the meniscus formed at the model surface was fairly large. To reduce this effect, a surface acting agent, Pluronic F68 was used. Small quantities were added with little or no sudsing occurring except in the extremely agitated collecting tank. The chemical was effective, however, in reducing the surface tension without affecting the water table equipment.

SOME PRELIMINARY FINDINGS

The use of the cork float suspended on the water surface greatly enhanced the visualization of the small disturbances produced in the analogue. Their use, however, was only valuable in the main flow as they tended to adhere to the nozzle surface when placed too close to that area. In determining a proper size piece of cork to use, it was found that too small a chunk tended to reproduce too many small extraneous disturbances while too large a cork disturbed the flow pattern. Attempts were made to determine a frequency of motion of the string and cork with a Strobotac. However, the vibrations were of such a low magnitude that watching them under the strobe proved difficult.

A row of cork floats on one nozzle was tried to see whether the major disturbance would change its relative position with a change in speed of the blades. This method proved somewhat difficult since the corks tended to adhere to each other due to their proximity. Also, the number of floats in one nozzle passage had a disrupting affect on the flow pattern which showed up in large motions of the corks.

The use of Pluronic F68 definitely reduced the amount of surface tension in the fluid. This change could be readily seen in the reduction of the meniscus at the nozzle surface and where the corks touches the water surface. Little or no sudsing has occurred in the test area, and only a slightly slick residue, much like that of a detergent, has been left.

APPENDIX A

The following assumptions and the development of the ratios obtained from the hydraulic analogy are taken from reference 16. Included in this appendix is a table of corresponding values between the gas and its analogous fluid flow.

The analogy is established under the following assumptions:

1. (a) The water channel is of constant cross-section

   (b) The flow of gas is two-dimensional

2. (a) The flow of water and

   (b) The flow of gas is steady and irrotational

3. (a) The viscosity and surface tension of the water and

   (b) The viscosity of the gas is neglected

4. (a) The water is incompressible and

   (b) The pressure is a given function of the density for the gas

5. (a) The vertical acceleration of the water is negligible when compared to the gravitational acceleration and

   (b) The extraneous forces acting on the gas are neglected.

The cross-section of the channel may be represented by function $y = f(z)$ where y and z are rectangular coordinates. If $f(z)$ is a power function, $y = az^n$ where a = size factor, n = shape factor.

The water height measured from the origin (along the z-axis) $y = y_0(z/d)^n$

y_0 and d are corresponding coordinates

$n = 0$	rectangular cross-section
$n = 1$	triangular cross-section
$n = \frac{1}{2}$	parabolic cross-section
$n = 2$	parabolic cross-section
$n \quad 0$	hyperbolic, no application

The velocity of propagation of long gravity waves for the case of small depths is given by P. Kelland to be

$$c_w = \sqrt{g\frac{A}{2y_0}} \quad \text{or} \quad c_w = \sqrt{\frac{gd}{n+1}} \tag{3}$$

A is the cross-sectional area given by $A = \frac{2a}{n+1}d^{n+1}$

Comparing the continuity equations for a gas

$$\frac{\partial(\rho u_i)}{\partial x_i} = 0 \quad (1 = 1,2) \tag{4}$$

$\left(u_1, u_2\right.$ velocity components in the x_1, x_2 directions$\left.\right)$ with that for water,

$$\frac{\partial(Au_i)}{\partial x_i} = 0, \quad \frac{\partial(d^{n+1}u_i)}{\partial x_i} = 0 \tag{5}$$

we see that

$$\underline{\underline{\frac{\rho}{\rho_0} = \left(\frac{d}{d_0}\right)^{n+1}}} \tag{6}$$

The Euler equation (considering assumptions two and three)

$$\frac{1}{2}\rho\frac{\partial u_j u_j}{\partial x_i} = -\frac{\partial P}{\partial x_i} + \rho f_i \tag{7}$$

$i, j = 1,2,3$ for water $\quad$ $i, j = 1,2$ for gas

f_i = extraneous forces

plus assumption five (b) says

$f_1 = f_2 = 0$ for the gas

$f_1 = f_2 = 0,\ f_3 = -g$ for the water

From assumption four (a) and five (a),

$$P_a - P_z = (d-z)\rho g \tag{8}$$

P_a - pressure at the free surface

Substituting into the Euler equation yields

$$\frac{1}{2}\frac{\partial u_j u_j}{\partial x_i} = -\frac{1}{\rho}\frac{\partial}{\partial z}(d-z)\rho g - \frac{1}{\rho}\rho g$$

$$\frac{1}{2}\frac{\partial u_j u_j}{\partial x_i} = - g\frac{\partial d}{\partial z} + g - g$$

or $$\frac{1}{2}\frac{\partial u_j u_j}{\partial x_i} = - g\frac{\partial d}{\partial z}$$

multiplying by u_i

$$\frac{1}{2}\, u_i \frac{\partial u_j u_j}{\partial x_i} = - g\, u_i \frac{\partial d}{\partial z} \tag{9}$$

Expanding the continuity equation, we have,

$$(n+1)\, u_i \frac{\partial d}{\partial x_i} + d\frac{\partial u_i}{\partial x_i} = 0$$

$$-g\, u_i \frac{\partial d}{\partial x_i} = \frac{gd}{n+1}\frac{\partial u_i}{\partial x_i}$$

or, $$-g\, u_i \frac{\partial d}{\partial x} = c_w^2 \frac{\partial u_i}{\partial x_i} \tag{10}$$

combining with the equation 9,

$$\frac{1}{2}\, u_i \frac{\partial u_j u_j}{\partial x_i} = c_w^2 \frac{\partial u_i}{\partial x_i} \tag{11}$$

for the gas flow,

$$\frac{\partial P}{\partial x_i} = C_g^2 \frac{\partial \rho}{\partial x_i} \tag{12}$$

c_g = local sound velocity

Equation (7) plus assumption five (b) becomes

$$\frac{1}{2}\rho\frac{\partial u_i u_j}{\partial x_i} = -\frac{\partial P}{\partial x_i} \tag{12}$$

Multiplying by u_i/ρ and substituting in equation (12), gives

$$\frac{1}{2}u_i\frac{\partial u_j u_j}{\partial x_i} = -c_g^2\frac{u_i}{\rho}\frac{\partial P}{\partial x_i} \tag{13}$$

Expanding the continuity equation for the gas gives

$$\rho\frac{\partial u_i}{\partial x_i} + u_i\frac{\partial \rho}{\partial x_i} = 0 \qquad \text{or,}$$

$$-\frac{u_i}{\rho}\frac{\partial \rho}{\partial x_i} = \frac{\partial u_i}{\partial x_i} \tag{14}$$

combining equations (13) and (14)

$$\frac{1}{2}u_i\frac{\partial u_j u_j}{\partial x_i} = c_g^2\frac{\partial u_i}{\partial x_i} \tag{15}$$

comparing equations (11) and (15), we see that

$$c_g/c_{g_0} = c_w/c_{w_0} \tag{16}$$

0 subscript refers to some refenence conditions

If assumption four (b) is restricted to the adiabatic law

$$c_g = \sqrt{P/\rho} \tag{17}$$

substituting equations (3) and (17) into (16) gives

$$\frac{P}{P_0} = \frac{\rho}{\rho_0}\frac{d}{d_0}$$ using equation (6) gives

$$\frac{P}{P_0} = \left(\frac{d}{d_0}\right)^{n+2} \qquad (18)$$

Again, using the adiabatic law and equation (6),

$$\frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right) = \left(\frac{d}{d_0}\right)^{(n+1)} \qquad (19)$$

Comparing the powers of (d/d_0) in equations (18) and (19) result in

$$\gamma = \frac{n+2}{n+1} \quad \text{or} \quad n = \frac{2-\gamma}{\gamma-1} \qquad (20)$$

Referring again to equation (16), we also get

$$\frac{T}{T_0} = \frac{d}{d_0} \qquad (21)$$

Thus, for a - rectangular cross-section, $n = 0$, $\gamma = 2$

triangular cross-section, $n = 1$, $\gamma = 1.5$

parabolic cross-section, $n = \frac{1}{2}$, $\gamma = 1.67$

$n = 2$, $\gamma = 1.33$

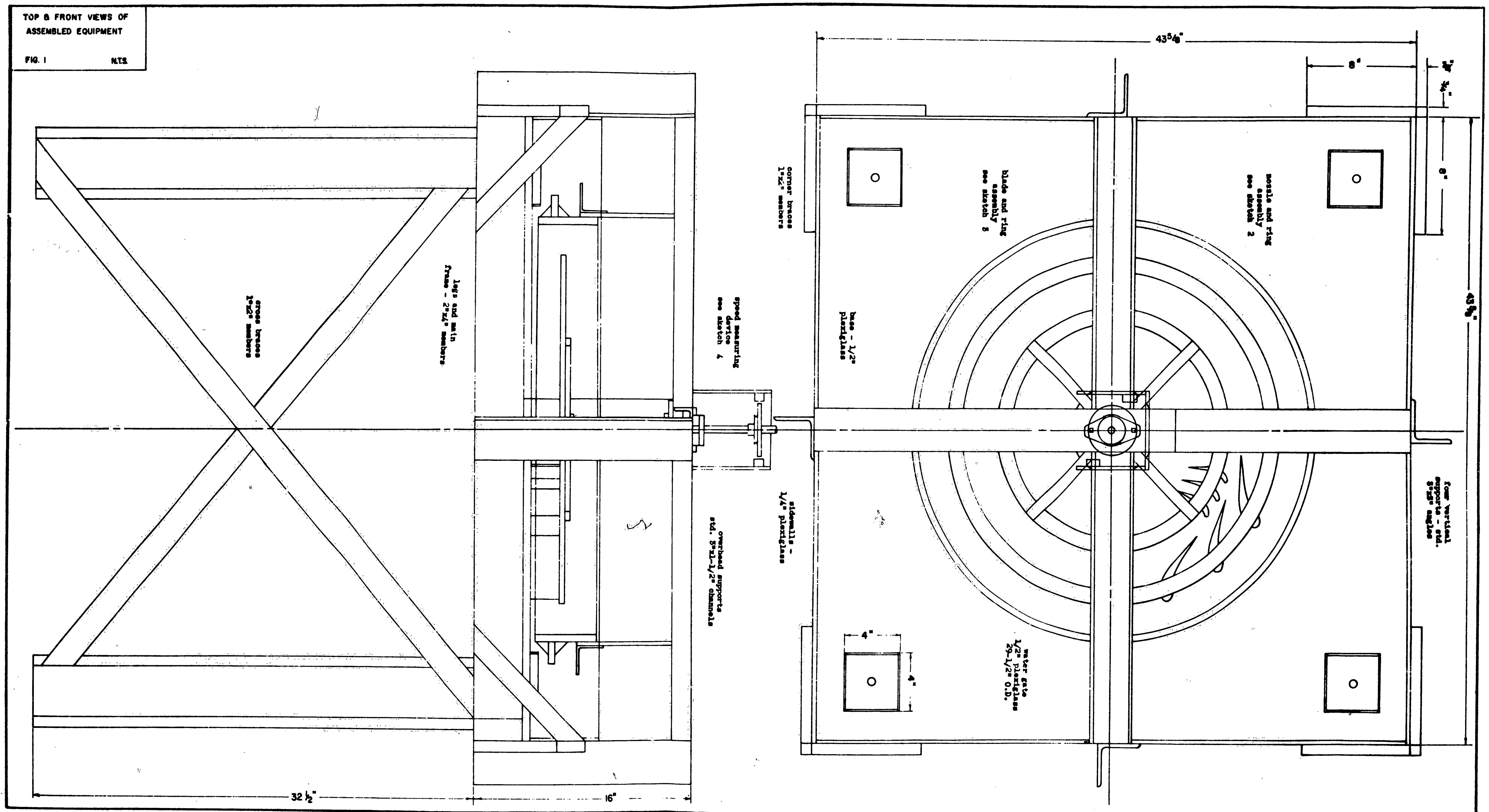
Some analogous values obtained are

<u>Two-Dimensional Gas Flow</u>	<u>General Channel</u>
Adiabatic Gas Constant	$\gamma = \frac{n+2}{n+1}$

Two-Dimensional Gas Flow (continued)	General Channel
Temperature Ratio T/T_0	Water Depth Ratio d/d_0
Density Ratio ρ/ρ_0	(n+1) power of water depth ratio $(d/d_0)^{n+1}$
Pressure Ratio P/P_0	(n+2) power of water depth ratio $(d/d_0)^{n+2}$
Velocity of Sound $c_g = \sqrt{P/\rho}$	Wave Velocity $c_w = \sqrt{\dfrac{gd}{n+1}}$
Mach Number $M = u_g/c_g$	Froude Number $F = u_w/c_w$
Subsonic Flow	Streaming Water
Supersonic Flow	Shooting Water
Shock Wave	Hydraulic Jump

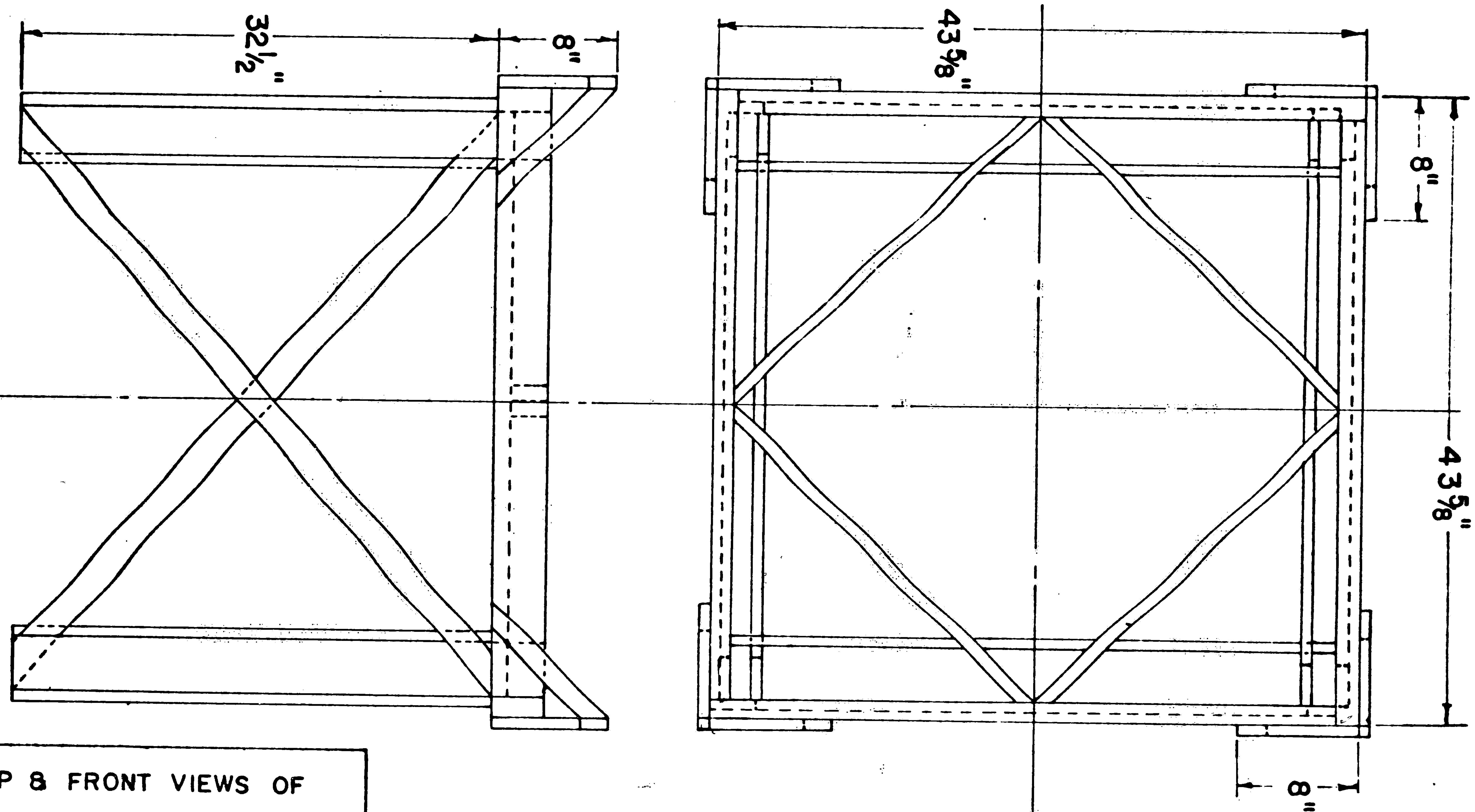
APPENDIX B

This section contains drawings of the various components of the water table discussed in the thesis. These figures are numbered one to ten and a list of them is given on page (v).



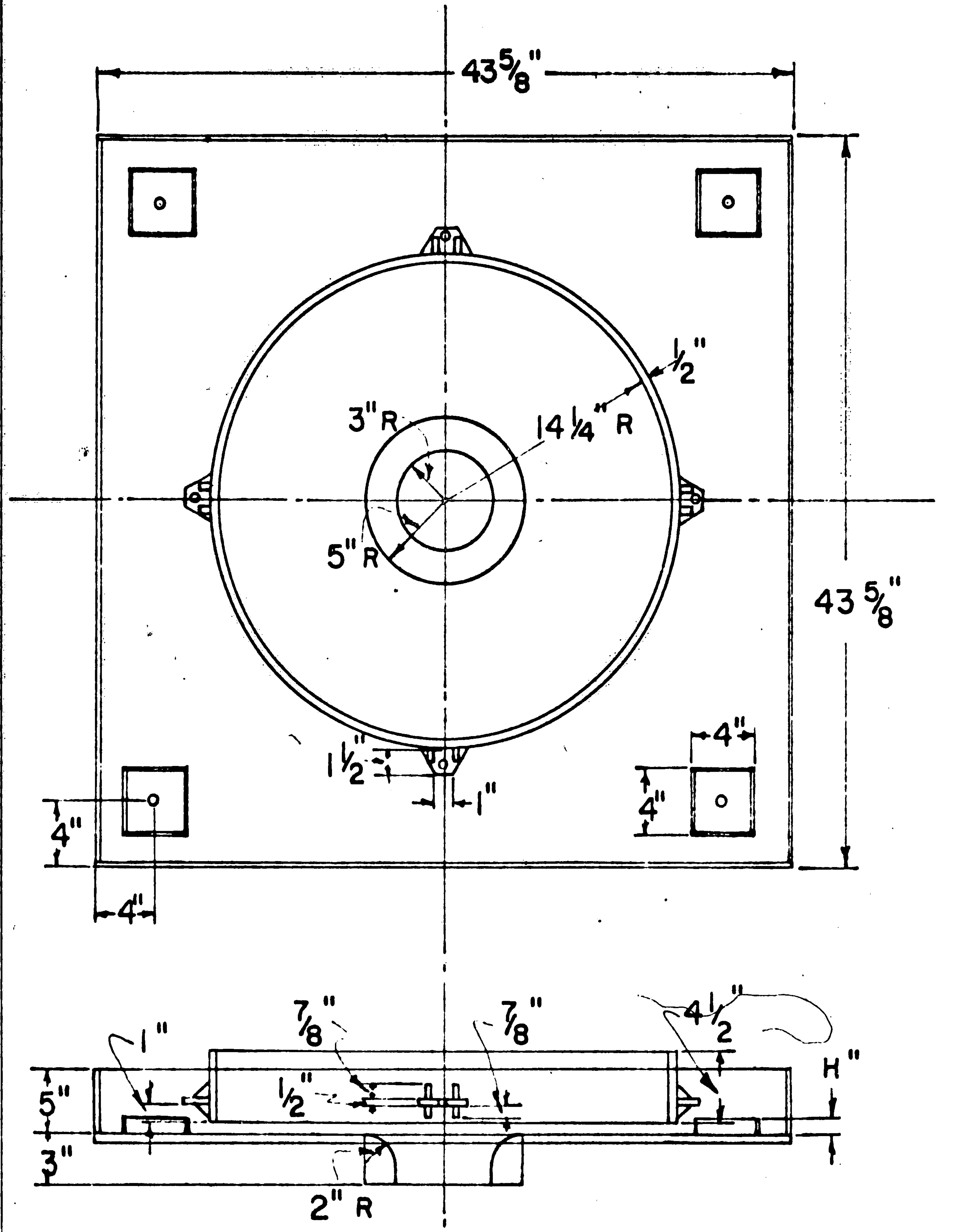
TOP & FRONT VIEWS OF ASSEMBLED EQUIPMENT

FIG. 1 N.T.S.



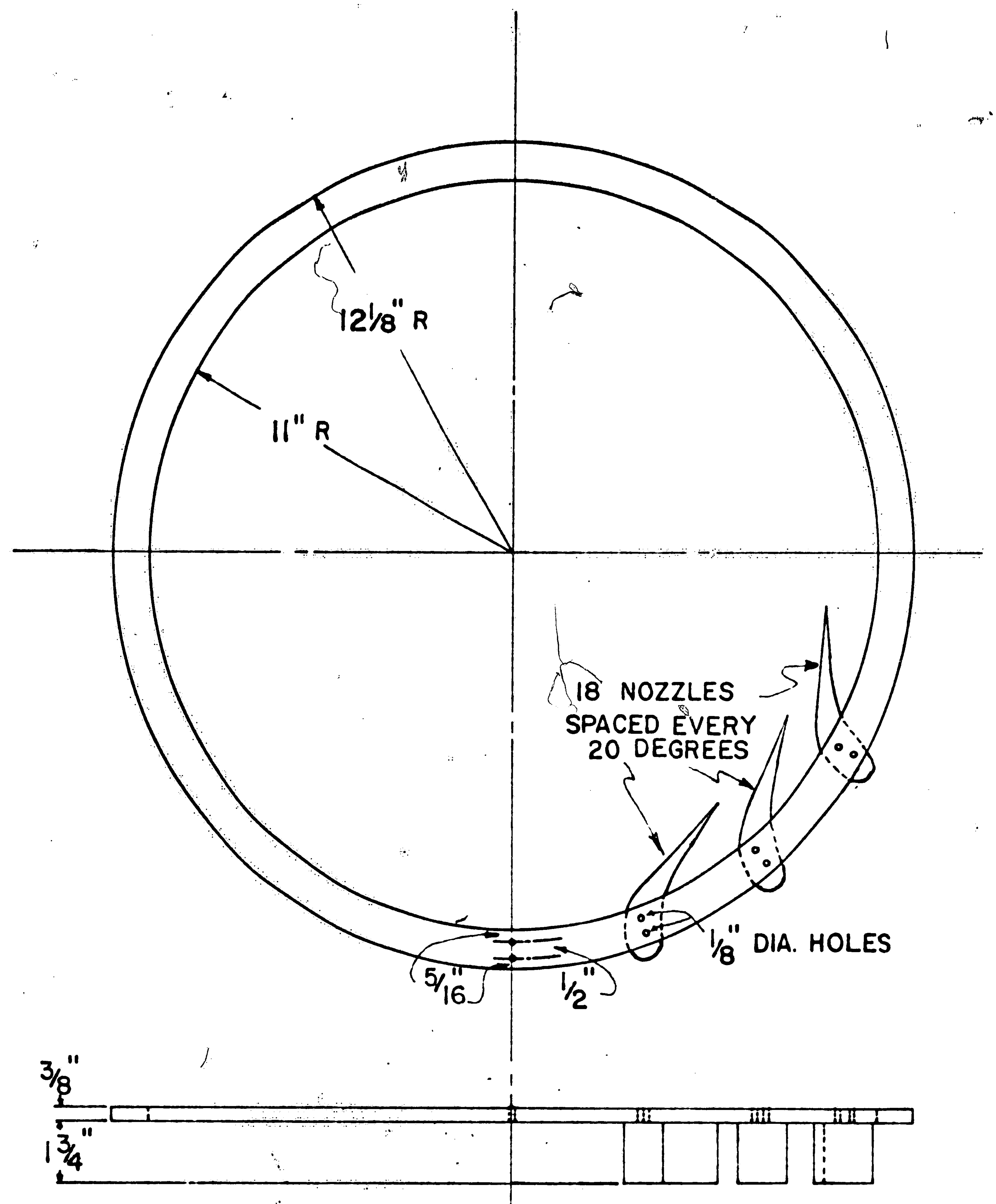
TOP & FRONT VIEWS OF
WOOD FRAMEWORK

FIG. 2 N.T.S.



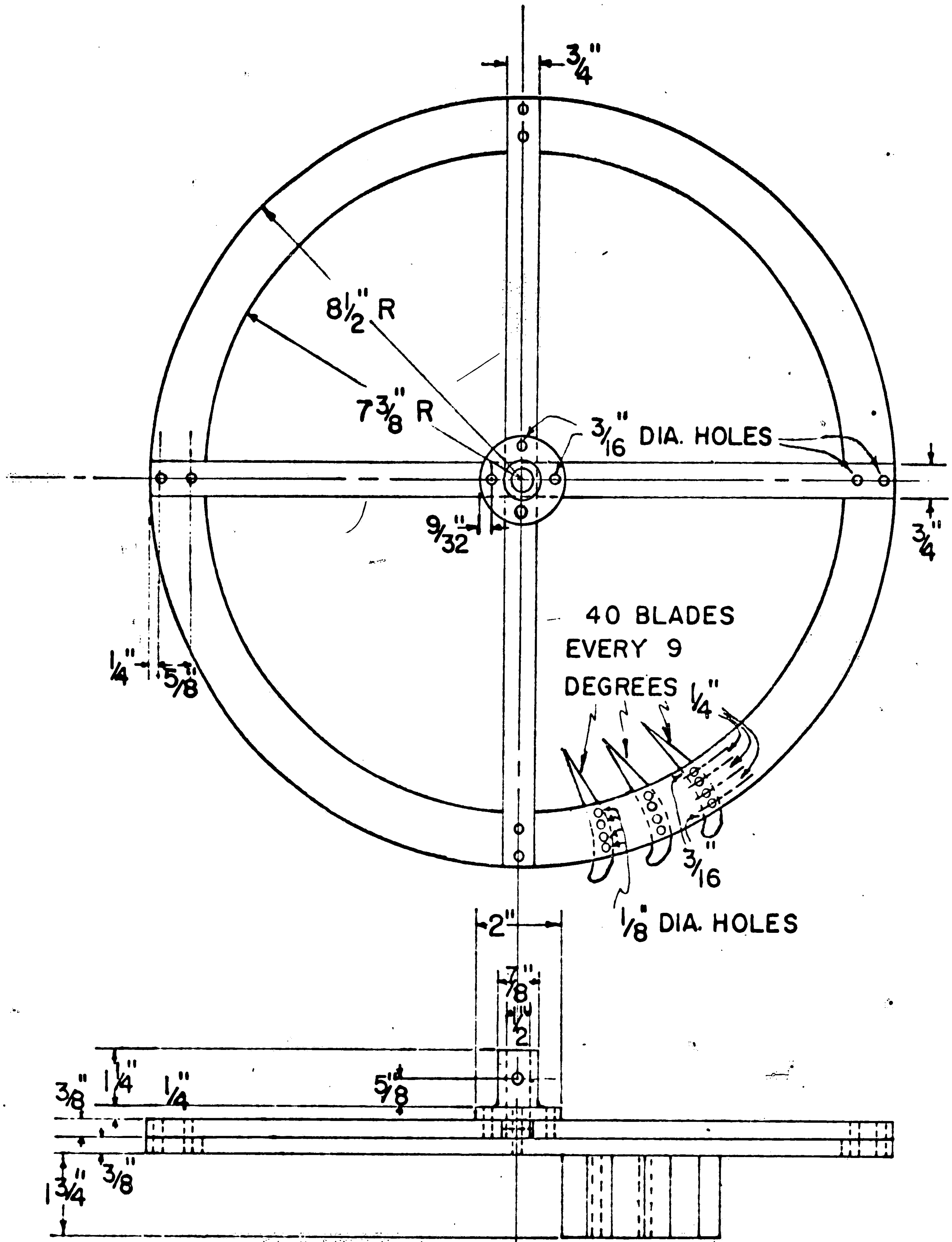
TOP & FRONT VIEWS OF
PLASTIC COMPONENTS

FIG. 3 N.T.S.



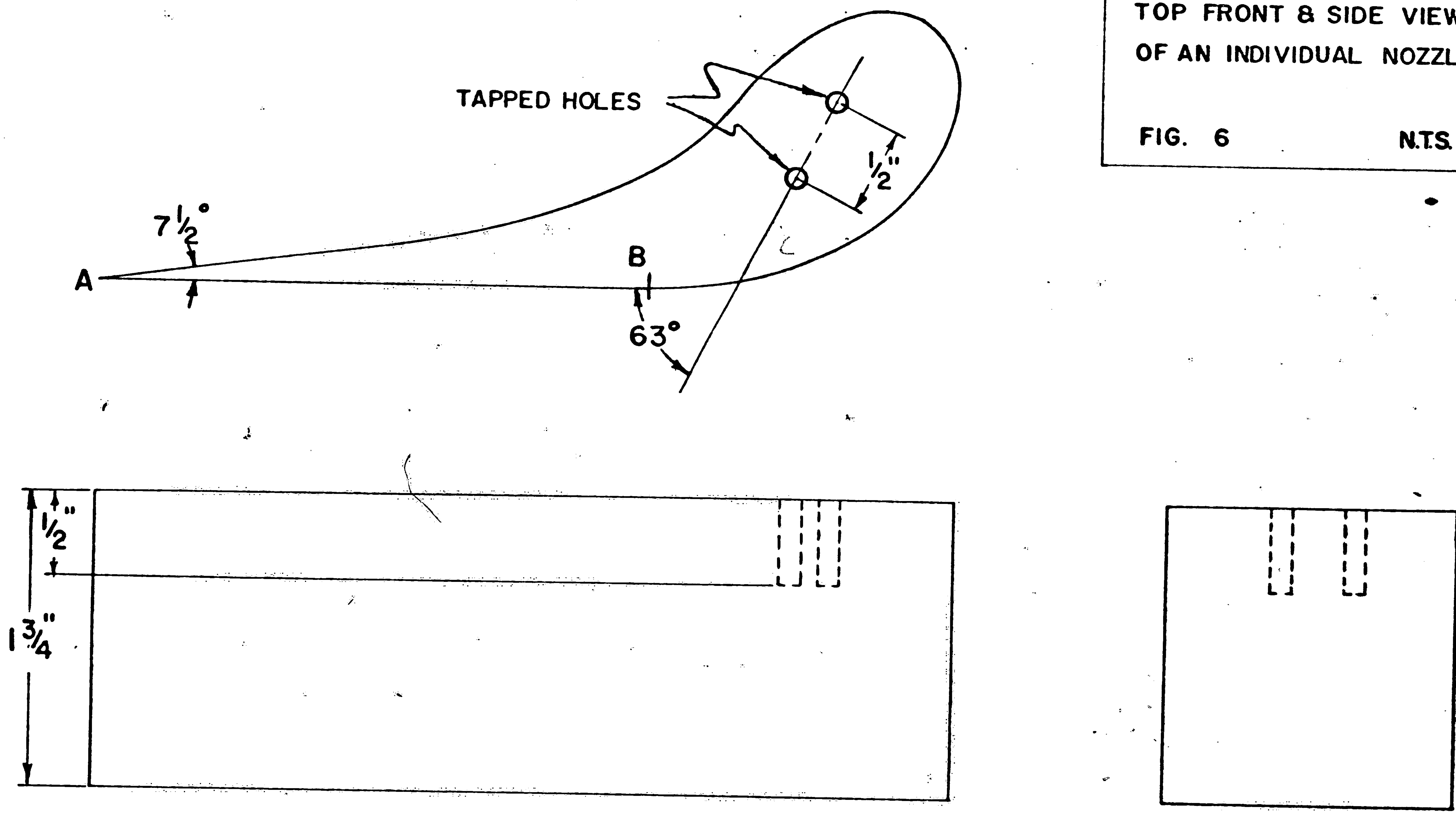
TOP & SIDE VIEWS OF
NOZZLE RING ASSEMBLY

FIG. 4 N.T.S.



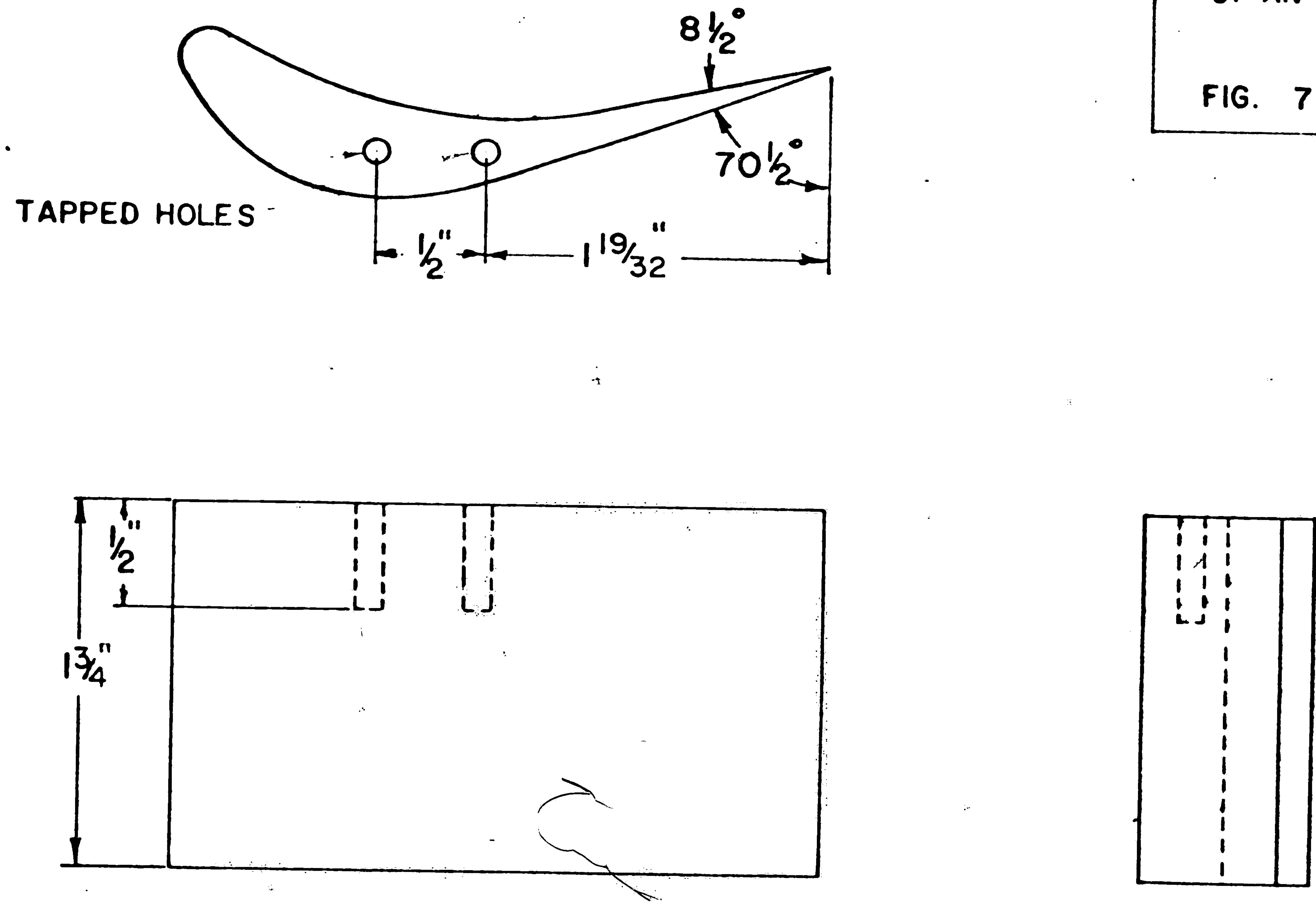
TOP & SIDE VIEWS OF BLADE RING ASSEMBLY

FIG. 5 N.T.S.



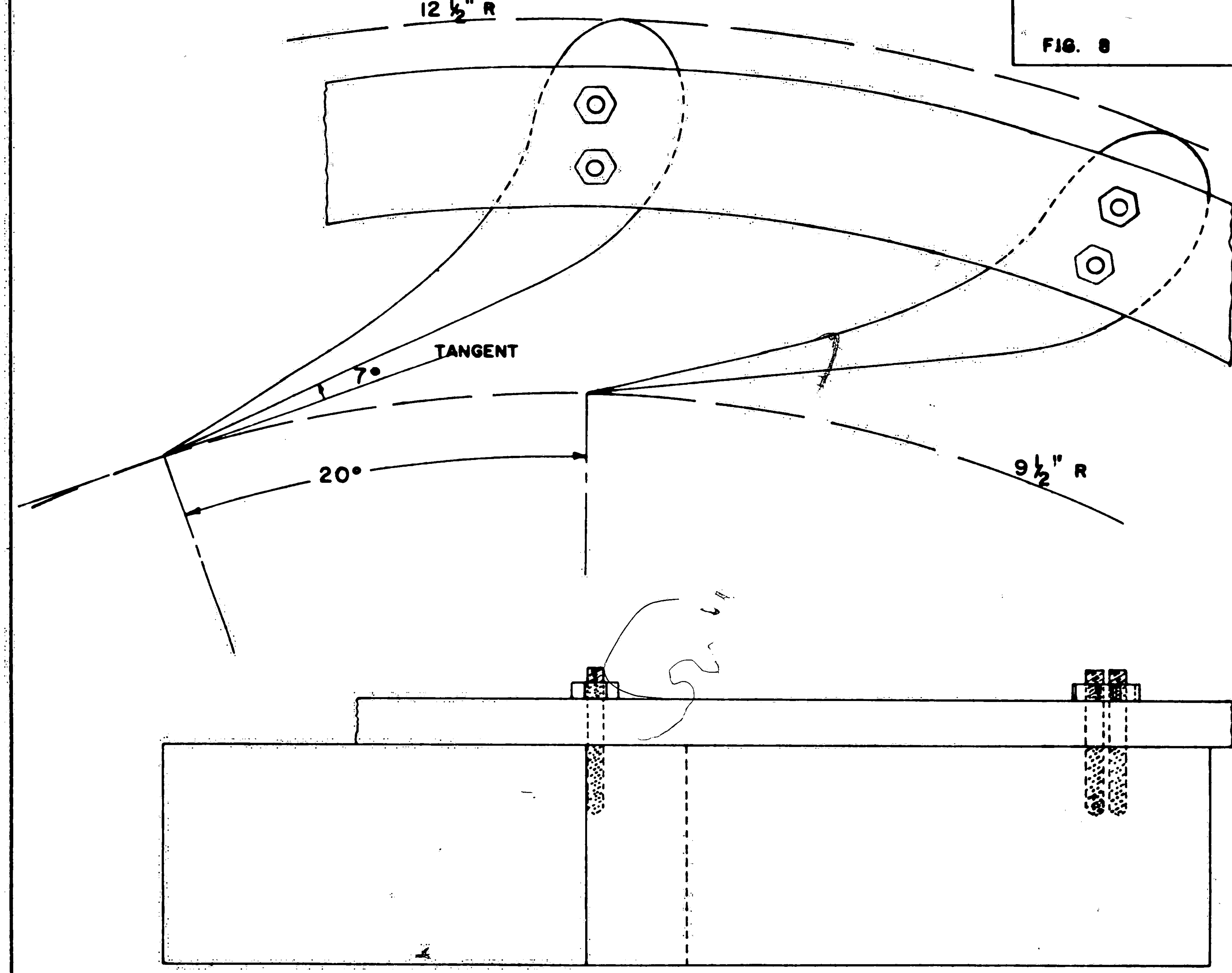
TOP FRONT & SIDE VIEWS OF AN INDIVIDUAL NOZZLE

FIG. 6 N.T.S.



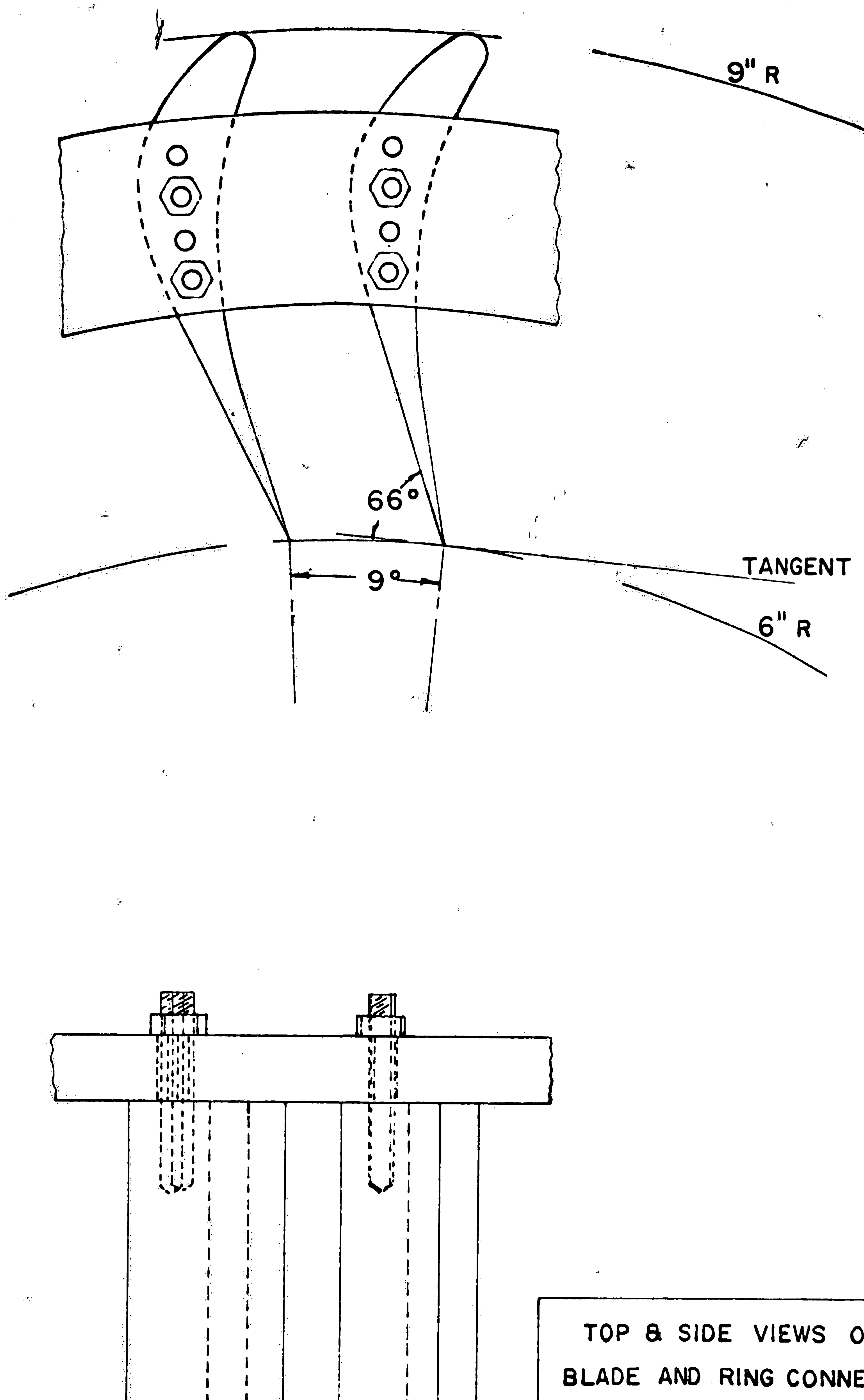
TOP FRONT & SIDE VIEWS
OF AN INDIVIDUAL BLADE

FIG. 7 NTS.



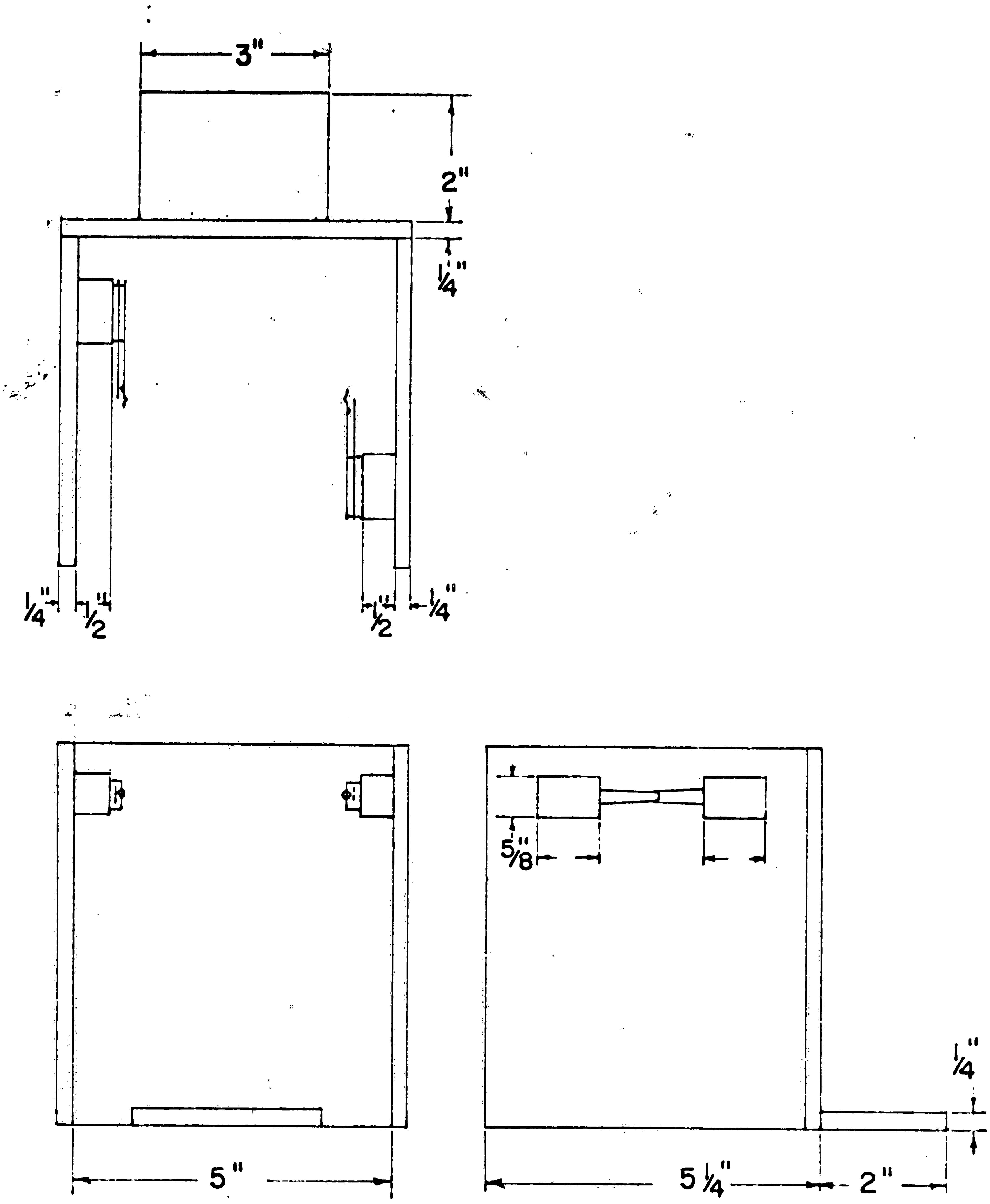
TOP & SIDE VIEWS OF
NOZZLE AND RING CONNECTION

FIG. 8 N.T.S.



TOP & SIDE VIEWS OF
BLADE AND RING CONNECTION

FIG. 9 N.T.S.



TOP FRONT & SIDE VIEWS OF TIMING MECHANISM HOUSING

FIG. 10 N.T.S.

APPENDIX C

The following data applies to the velocity diagram for the inlet to the blades, figure 11. The back head of water is 5/16", and the flow depth is 1/4" for the sample drawing.

AB - absolute velocity to the blades, 0.580 ft/sec.

CB, DB, EB - velocity relative to the blades

AC, AD, AE - tangential velocity of the blades

Triangle ADB represents the optimum conditions of flow for the nozzle and blade configuration, while triangles ACB and AEB are the extreme velocity conditions if an off angle entrance of plus or minus twelve degrees is allowed.

The tangential velocities are

AC = 0.569 ft/sec.

AD = 0.552 ft/sec.

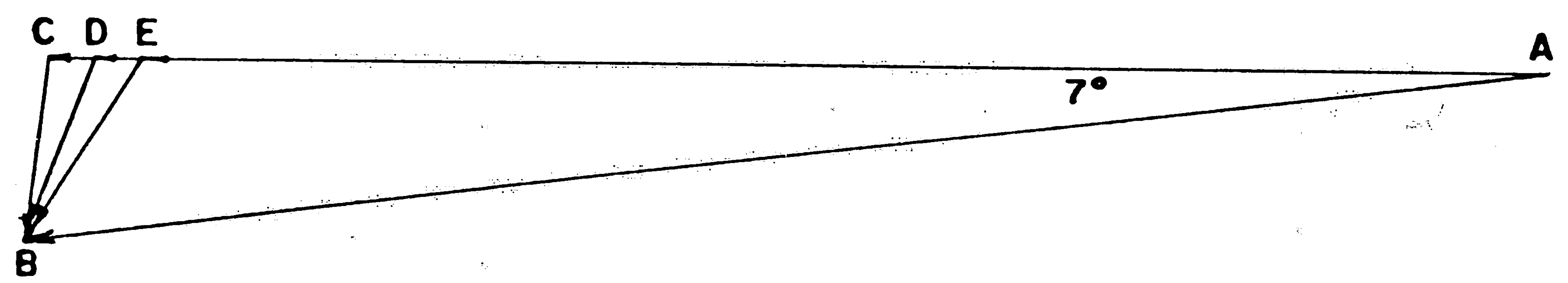
AE = 0.534 ft/sec.

These, in turn, represent angular speeds of

AC = 7.23 RPM

AD = 7.00 RPM

AE = 6.78 RPM



VELOCITY DIAGRAM
FOR INLET TO BLADES

FIG. 11 SCALE: 1"=0.1 FT/SEC.

The following data applies to the velocity diagram for the exit from the blades, figure 12. The back head of water is 5/16" and the flow depth at the nozzle is 1/4" for the sample drawing.

AC, AD, AE - tangential velocity of the blades

CC', DD', EE' - velocity relative to the blades

AC', AD', AE' - absolute velocity leaving the blades

From the velocity diagram for the blade inlet, the speeds (RPM's) at which the equipment will operate are known so that AC, AD, and AE can be calculated.

AC = 0.386 ft/sec.

AD = 0.374 ft/sec.

AE = 0.364 ft/sec.

The relative velocity has already been calculated

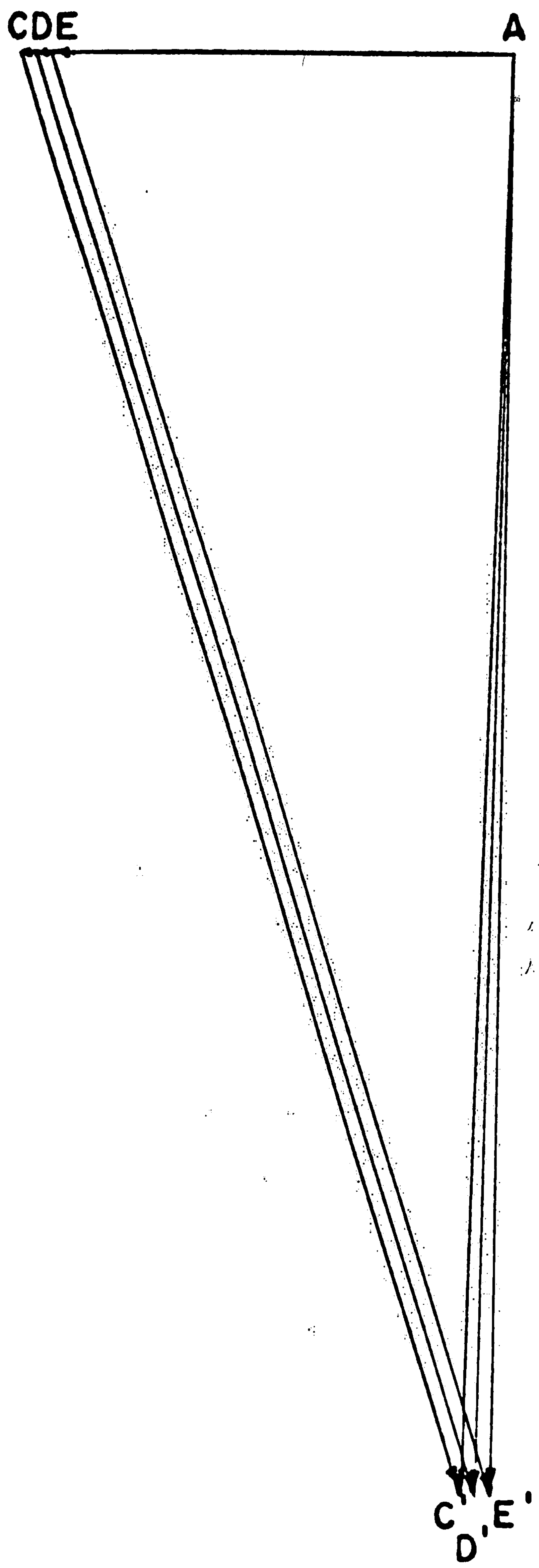
CC' = DD' = EE' = 1.161 ft/sec.

Knowing the exit angle to be 72.3°, the direction of the absolute velocity can be determined. The differences from radial is

AC = 1.8°

AD = 1.0°

AE = 0.5°



VELOCITY DIAGRAM
FOR EXIT FROM BLADES

FIG. 12

SCALE:
1" = 0.2 FT/SEC

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VITA

This writer was born in Delano, California, on March 27, 1941, the son of Masao and Bess Yano. In 1943, my family moved to Baltimore, Maryland, where I attended the Boys' Latin School for my primary and Junior High school training from 1947 to 1955. From 1955-1959, I studied at the Baltimore Polytechnic Institute where I received a diploma from the "A" course in June, 1959. From 1959-1962, I attended Lehigh University and was graduated with a Bachelor of Science degree in Mechanical Engineering in June, 1962. While an undergraduate there, I was a member of Pi Mu Epsilon and Pi Tau Sigma honorary fraternities.