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THIRD ORDER INTERMODULATION DISTORTION AND HARMONIC DISTORTION IN L-5 COAXIAL REPEATER SYSTEM TRANSISTORS

by
K. D. Lesh

A Thesis

Presented to the Graduate Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

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Nov. 14 72.

Date

Professor in Charge

Chairman of Department of
Electrical Engineering

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DEFINITION OF SYMBOLS

a - Transfer function amplitude coefficient

A - Input frequency amplitude for frequency f_1 (volts)

b - Product frequency combination coefficient

B - Input frequency amplitude for frequency f_2 (volts)

c - Model parameter

C - Input frequency amplitude for frequency f_3 (volts)

d - Model parameter

e - Model parameter

E_{in} - Input voltage (volts)

E_{out} - Output voltage (volts)

f_1 - Fundamental frequency (Hz)

f_2 - Fundamental frequency (Hz)

f_3 - Fundamental frequency (Hz)

f_{prod} - Product frequency (Hz)

P_{f1} - Power of fundamental frequency (dbm)

P_{f2} - Power of fundamental frequency (dbm)

P_{f3} - Power of fundamental frequency (dbm)

P_{3f1} - Power of third harmonic frequency (dbm)

$P_{f1 \pm f2 \pm f3}$ - Power of intermodulation product (dbm)

ABSTRACT

The thesis presents an analysis of the relation between intermodulation distortion and harmonic distortion for a bipolar transistor used in an L-5 coaxial repeater of the Bell Telephone System. Its aim is to develop a simple and reliable method of characterizing the distortion properties of the transistor that can be advantageously used on the production line. While present testing methods employ two time consuming, three tone intermodulation tests, in this work a simpler, third harmonic measurement is introduced, enabling a reduction in test time by 75%.

The theoretical analysis compares harmonic and intermodulation distortion first on an elementary basis and then by a more rigorous method which takes into account the transistor equivalent circuit parameters as a function of frequency. Experimental work comparing three tone intermodulation distortion to third harmonic distortion confirms the validity of the theoretical analysis.

1. Introduction

Coaxial cable telephone repeater systems form the backbone of all long distance telephone communications. The L-5 coaxial repeater of the Bell Telephone System is the latest in a family of many long distance communications networks.(1) The L-5 system has an operating bandwidth from a few hertz to approximately fifty megahertz which allows about ten thousand simultaneous telephone conversations to be multiplexed on one coaxial cable. A typical L-5 system repeater route is about three thousand miles long. A repeater is used every mile to compensate frequency dependent line losses in order to amplify the signal and to keep a high signal to noise ratio. To minimize interference among the various phone conversations, a high degree of repeater linearity is of utmost importance. Since any repeater nonlinearity can lead to interference and nonlinearities in successive repeaters are additive in phase, distortion requirements for each repeater are extremely critical.

Distortion and nonlinearity in a repeater are primarily due to the active devices, and especially to the transistors, that it contains. The most critical device is the output amplifier as it operates at the highest power level and hence, is more prone to distortion and nonlinearity. This thesis deals with the method of testing this output device. Currently, a third order intermodulation test is required at two frequencies for each device; however, a test requiring fewer measurements as a third harmonic test is desirable from the standpoint of increased production and decreased test time. This paper contains both a theoretical and experimental comparison of third harmonic and third order intermodulation distortion which allows the currently used intermodulation tests to be replaced by a harmonic test with assurance that the device will perform adequately.

This paper is concerned with the method of testing a transistor for the amount of distortion it produces. It does not deal with device optimization as it applies to distortion, nor does it apply to the repeater circuitry involved with distortion. These topics have been treated adequately by others.(2,3,4) However, the results of Narayanan and Poon using a charge control model(5) for frequency dependent nonlinear distortion will be used in deriving the relation between harmonic and intermodulation distortion. Another recently developed transistor model(6) optimizes circuit parameters such as source and load impedances, biasing, etc., for minimum distortion. It should be noted that it is assumed the particular L-5 devices used in this analysis have already been designed with optimum distortion properties in mind and that the repeater circuitry has also been optimized. Therefore, this paper is aimed at finding the easiest method of testing production line devices for their distortion properties.

The test equipment is computer controlled featuring an amplitude resolution of one hundredth of a db with accuracy of five hundredths of a db. One device can be tested in about four seconds. A third order intermodulation distortion measurement requires the setting and reading of three fundamental frequency amplitudes and then the measurement of the product frequency amplitude. A third harmonic test requires only the setting and measurement of one fundamental and then the third harmonic level measurement. A reduction in test time from four seconds per device to one second per device can be realized through third harmonic testing. A detailed description of the experimental apparatus appears in Section 4.

The remaining parts of the paper are arranged as follows: Section 2 reviews the derivation of harmonic and intermodulation distortion equations. Section 3 computes the frequency dependent harmonic and intermodulation distortion for the L-5 repeater system output transistors in particular. Section 5 covers the experimental results obtained. Section 6 summarizes the paper.

2. Harmonic and Intermodulation Distortion Equations

A nonlinear two port device can be characterized by a transfer function,

$$E_{out} = E_{in}a_1 + E_{in}^2 a_2 + E_{in}^3 a_3 + \ldots\ldots \qquad (1)$$

where E_{in} and E_{out} are the input and output voltages. The above device, when driven by a single input frequency, f_1, given by:

$$E_{in} = A \cos f_1 t \qquad (2)$$

has an output consisting of the following terms:

$$E_{out} = a_1 A \cos f_1 t + a_2 A^2 \cos^2 f_1 t + a_3 A^3 \cos^3 f_1 t + \ldots \quad (3)$$

By considering only the first three terms of the above expression and applying suitable trigonometric identities, O'Neil(7) has derived the following expression:

$$\begin{aligned} E_{out} = {} & 1/2\ a_2 A^2 + (a_1 A + 3/4\ a_3 A^3) \cos f_1 t + \\ & 1/2\ a_2 \cancel{A^2} \cos^2 f_1 t + 1/4\ a_3 A^3 \cos^3 f_1 t \end{aligned} \quad (4)$$

Examination of Eq. (4) shows that a second and third harmonic term are apparent as part of the output voltage of the device.

Intermodulation is the creation of combination (often called product) frequencies resulting from the nonlinear transfer characteristic, Eq. (1), when the input comprises several frequencies. These product frequencies can be derived by a linear combination of n input frequencies, ie:

$$f_{prod} = b_1 f_1 + b_2 f_2 + b_3 f_3 + \ldots + b_n f_n \quad (5)$$

where b_1, b_2, ... are positive or negative integers or zero. For the case of three inputs frequencies, the input voltage is,

$$E_{in} = A \cos f_1 t + B \cos f_2 t + C \cos f_3 t \quad (6)$$

The output of an amplifier with a transfer function given by Eq. (1) and an input signal given by Eq. (6) contains many different frequency terms of various amplitudes (see Table 1). Considering only terms of the $f_1 \pm f_2 \pm f_3$ variety, the output becomes:

$$E_{out} = 3/2\ a_3\ ABC\ (\cos\ (f_1 + f_2 + f_3)t + \cos\ (f_1 + f_2 - f_3)t + \cos\ (f_1 - f_2 + f_3) + \cos\ (f_1 - f_2 - f_3)t) \qquad (7)$$

The magnitudes of the $f_1 \pm f_2 \pm f_3$ terms are $3/2\ a_3ABC$. The magnitude of the $3f_1$ product, Eq. (4), is given by $1/4\ a_3A^3$. Thus the third order intermodulation products are higher than the third harmonic by the factor of $6BC/A^2$. Expressing the above factor in dbm, remembering that all the above terms are in voltage, with each of the input frequencies having the same amplitude, the power of the $f_1 \pm f_2 \pm f_3$ products will be 15.6 db greater than that of the $3f_1$ harmonic.

For continuity in distortion measurements, (7) all measurements are referred to a 0 dbm fundamental level, and an arbitrary measure of distortion, M, is used. For a third harmonic measurement, M, is

$$M_3 = 3P_{f1} - P_{3f1}\ \text{dbm}. \qquad (8)$$

where M_3 is the distortion measure, Pf_1 is the power in dbm of the fundamental frequency, and P_{3f1} is the power in dbm of the harmonic. The distortion measure of a third order intermodulation product of the type is given by:

$$M_3 = P_{f1} + P_{f2} + P_{f3} + 15.6 - P_{f1 \pm f2 \pm f3}\ \text{dbm}. \qquad (9)$$

The above analysis indicates that a 15.6 db difference is predicted between third harmonic and intermodulation distortion measurements at the same fundamental level; however, the model presented in this section is a simplified one and does not predict real device behavior very accurately. The major factor not taken into account is the frequency dependence of distortion. This will be covered in the next section.

3. Frequency Dependent Distortion

The frequency dependent distortion analysis uses the charge control model of Narayanan and Poon mentioned previously. Their model is based upon a general Volterra series expansion.(8) The transistor characteristics are considered as the only source of nonlinearities. The results of their work are a number of very simple equations relating the distortion coefficient M to device parameters, and in particular:

$$M_3 = 10 \log \frac{d(1 + e\, f_{prod}^2)}{(1 + c\, f_{prod}^2)} \qquad (10)$$

where $f_{prod} = f_1 \pm f_2 \pm f_3$, similar to Eq. (5). The variables c, d, and e were evaluated by a computer program for the L-5 repeater output device following the derivation of Narayanan and Poon. Appendix B describes this derivation in greater detail. The important device parameters which were measured for use in the above derivation are presented in Appendix A.

Equation 10 can also be used to predict third harmonic generation where $f_{prod} = 3f_1$. The derivation of the variables is slightly modified to take into account the transistor's higher gain at f_1. In third order intermodulation tests, the three fundamental frequencies are in the vicinity of the product frequency which is usually on the h_{fe} knee. For a harmonic test, the product frequency remains the same but the fundamental frequency is one third lower.

The calculated results for harmonic and intermodulation distortion are shown in Figure 3. The calculations were based on four different product frequencies, chosen to be the same as those available for experimental verification.

4. Automatic Testing of Distortion Parameters in Transistors

A computer controlled spectrum analyzer system was developed to measure second and third order intermodulation as well as harmonic distortion in transistors for the L-5 coaxial repeater system. The system was designed with high accuracy, on the order of a tenth of a db, and repeatability on the order of a hundredth of a db. While the basic design is simply a computer controlled spectrum analyzer, the system is capable of performing many different testing functions through simple program changes. The frequency range of the system is from one hundred kilohertz to one hundred megahertz, with a dynamic range of one hundred forty db.

System Description

Figure 2 is a block diagram of the test system. The source subsystem contains crystal oscillators for generating the desired fundamental frequencies. A switching matrix chooses any combination of oscillators, up to three at a time, to be fed to the device under test. Each oscillator is followed by a bandpass filter to remove any harmonics or spurious frequencies. The level of the selected oscillators is controlled by a series of three step attenuators to an accuracy of a tenth of a db. The resultant signals are combined on a single coaxial line through the use of a hybrid power combiner. Bias to the input side of the transistor is added onto this line through a bias tee.

The signal from the output side of the transistor after going through another bias tee goes to the input control section. This section includes a seventy db step attenuator (under program control) to set a rough input level. Also included in this section is a switch matrix to insert various band reject filters in series with the signal. From this point the signal goes to the spectrum analyzer proper where the signal is detected and digitized amplitude and frequency information is passed to the computer.

System Calibration

The basic spectrum analyzer system has an absolute accuracy of one db at thirty megahertz, with an additional half db of frequency sensitivity throughout its frequency range. Obviously, this is more error than could be tolerated. A calibration technique was developed in which an oscillator is tuned to one of the frequencies of interest, its power read on a very accurate power meter, and then also read by the system. A correction table is set up in memory so that every time a reading is taken at a particular frequency, a correction factor is added to that particular measurement. Also, the insertion loss of the band reject filters is measured and stored in the correction table, which are then added to a particular reading when necessary.

With the calibration technique described, an accuracy of better than one tenth of a db has been achieved with repeatibility on the order of a hundredth of a db. This specification applies for signals over the entire frequency range of the instrument for signals as far down as minus eighty dbm. System stability is sufficient so that recalibration is necessary only once per day.

Device Measurement

The method of making a third order intermodulation test will be discussed here as second order tests and harmonics tests are similar except for the number of fundamental frequencies and their power levels. The L-5 devices require three input frequencies each at a level of nine plus or minus a half dbm at the collector for a third order intermodulation test. A second order intermodulation test requires two fundamental frequencies, each at a power of 10.7 dbm, to realize the same total fundamental frequency power output of 13.7 dbm as in a third order test. A third harmonic test requires a single fundamental frequency at a power level of 13.7 dbm.

For a third order test, the analyzer is instructed to measure the power of the three fundamentals for the first device. These three levels must be within one tenth of a db of nine dbm. If any of the powers are out of range, an operator instruction appears on the face of a large screen oscilloscope telling the operator how to reset the input attenuators to reach the desired levels. Once these levels have been set for the first device, subsequent devices from the same lot fall within the broader nine plus or minus one half dbm input specification without the necessity of readjusting the input attenuators for the following devices. It should be noted that all fundamental levels are stored for later use in calculation of the distortion coefficient.

Once the input levels have been set, a band notch filter at each of the fundamental frequencies is inserted into the output line to avoid overload due to the high fundamental levels. The analyzer is then instructed to measure the amplitude of the product frequencies of interest. All measurements have automatically been corrected for absolute amplitude and frequency dependent errors through use of the error correction table mentioned previously. The insertion loss of the fundamental reject filters at the product frequencies has also been taken into account.

The computer then calculates the distortion coefficient using the previously obtained fundamental power levels and the product frequency power level. The complete process, as outlined above, takes approximately four seconds per device. A second order intermodulation distortion test requires approximately three seconds, while a third harmonic distortion is completed in less than one second.

5. Experimental Results

The device is biased in a common base configuration with V_{CB} at 14.2 volts and base current, i_B, at a constant 110 milliamperes. The a.c. signals, however, see a common emitter configuration as shown in Figure 1.

The fundamental frequency amplitudes at the output of the device are adjusted individually to a nine dbm level for the three tone intermodulation tests. Third harmonic measurements were run at two different levels. The fundamental was first set to a nine dbm output level to verify the 15.6 db difference in distortion products as predicted in Section 2. The level was then raised to the equivalent of three 9.0 dbm signals or 13.7 dbm to compare with the calculated results of Section 3.

The h_{fe} knee for these devices, defined as the frequency at which the transistor gain falls to .707 times the low frequency gain, typically occurs between 45 and 55 megahertz. Fundamental frequencies, f_1, f_2, f_3, were at or slightly below the h_{fe} knee for the intermodulation tests.

$$\begin{aligned} f_1 &= 40.1 \text{ Mhz} \\ f_2 &= 43.1 \text{ Mhz} \\ f_3 &= 50.0 \text{ Mhz} \end{aligned} \qquad (11)$$

Three third order products are generated by these fundamentals:

$$\begin{aligned} f_{prod_1} &= 40.1 + 43.1 = 50.0 = 33.2 \text{ Mhz} \\ f_{prod_2} &= 50.0 + 40.1 - 43.1 = 47.0 \text{ Mhz} \\ f_{prod_3} &= 50.0 + 43.1 - 40.1 = 53.0 \text{ Mhz} \end{aligned} \qquad (12)$$

These three product frequencies are below, near, and above the h_{fe} knee, respectively.

6. Conclusions

Table 2 tabulates the difference between measured and calculated distortion levels. Table 3 tabulates for ten different transistors the difference between harmonic and intermodulation experimental results. From this data it appears that there is a constant two db offset between calculated and measured results. However, the difference between harmonic and intermodulation distortion for both the calculated and experimental results is nearly the same for each product frequency. It should be noted that, at low frequencies, the difference between the distortion coefficients is nearly zero indicating the predicted 15.6 db difference in level of the actual products as in Section 2. At higher frequencies the frequency dependent model of Section 3 predicts the actual device behavior more accurately.

Present specifications call for the measurement of intermodulation distortion at the two product frequencies 33.2 megahertz and 53.0 megahertz. From the results developed here, these two tests can be replaced by two third harmonic tests at the same product frequencies with the M_3 test limits as specified in Table 3.

The reduction in testing time using a harmonic test rather than an intermodulation test is on the order of seventy five percent. This could lead to a possible reduction in device cost as well as providing the extra testing capacity to meet future programs with no increase in test equipment cost. As a verification of the above results, one hundred devices were tested; and all those which failed the intermodulation distortion requirement also failed the harmonic distortion requirement.

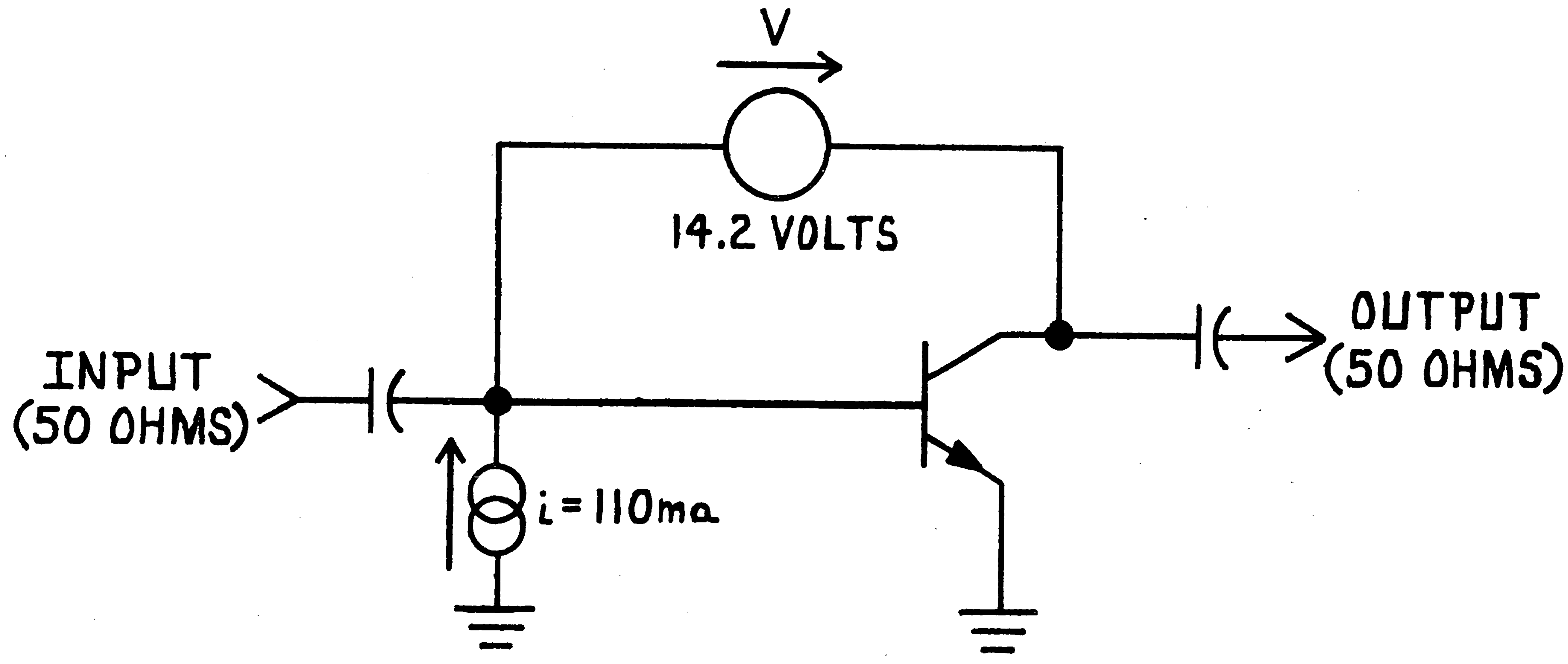
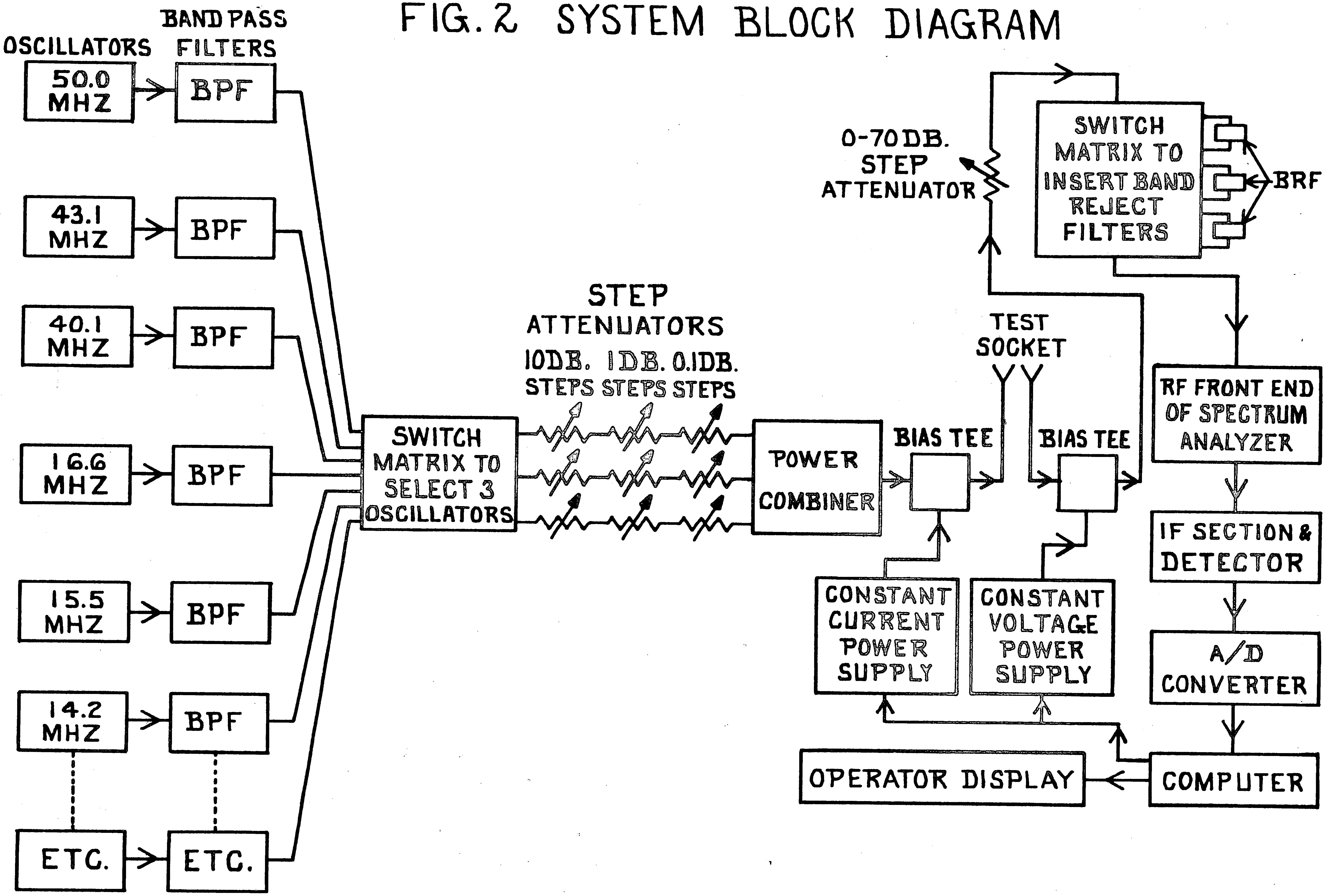


FIGURE 1 - TEST CIRCUIT

FIG. 2 SYSTEM BLOCK DIAGRAM



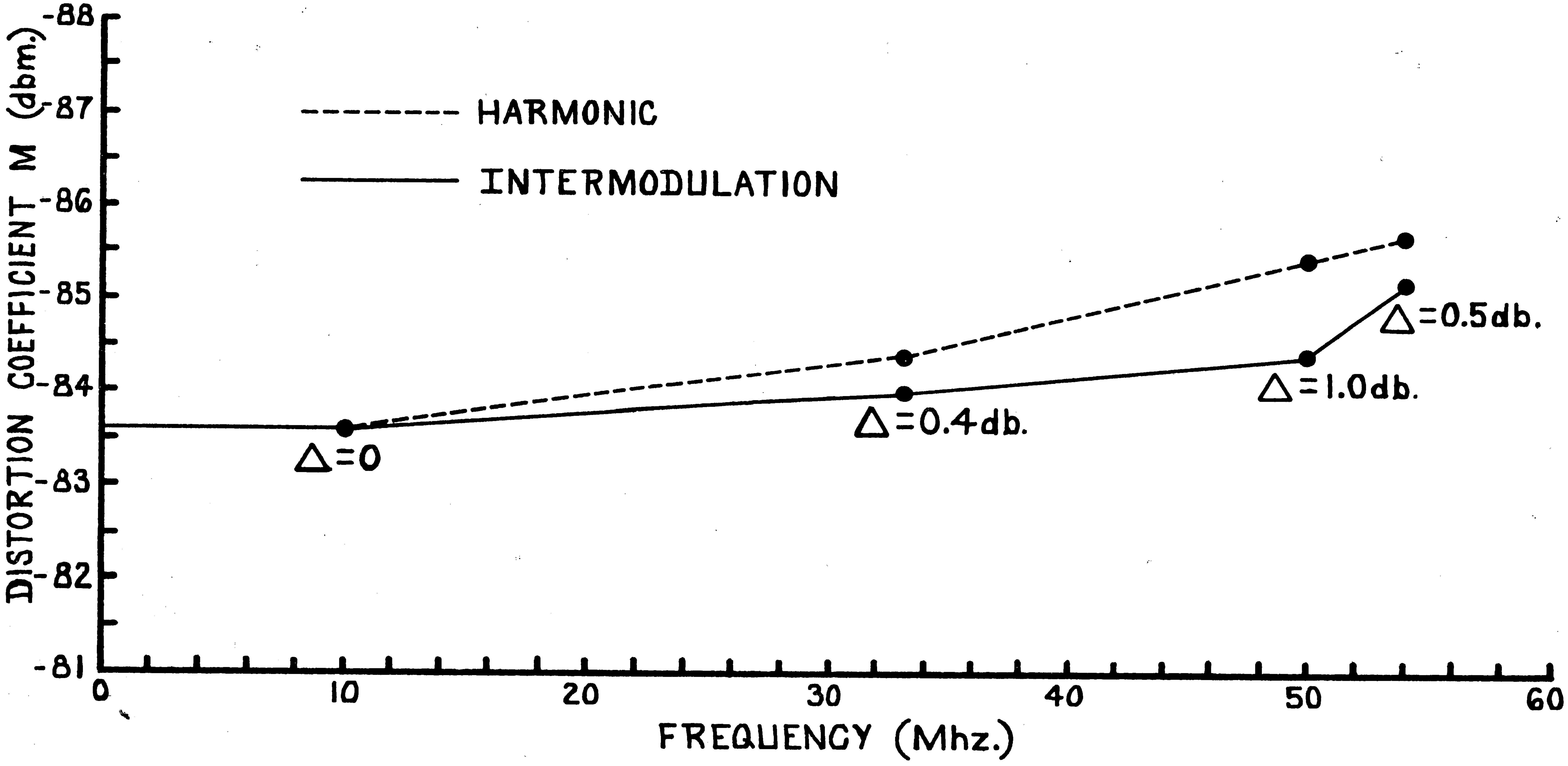


FIGURE 3 - CALCULATED DISTORTION LEVELS

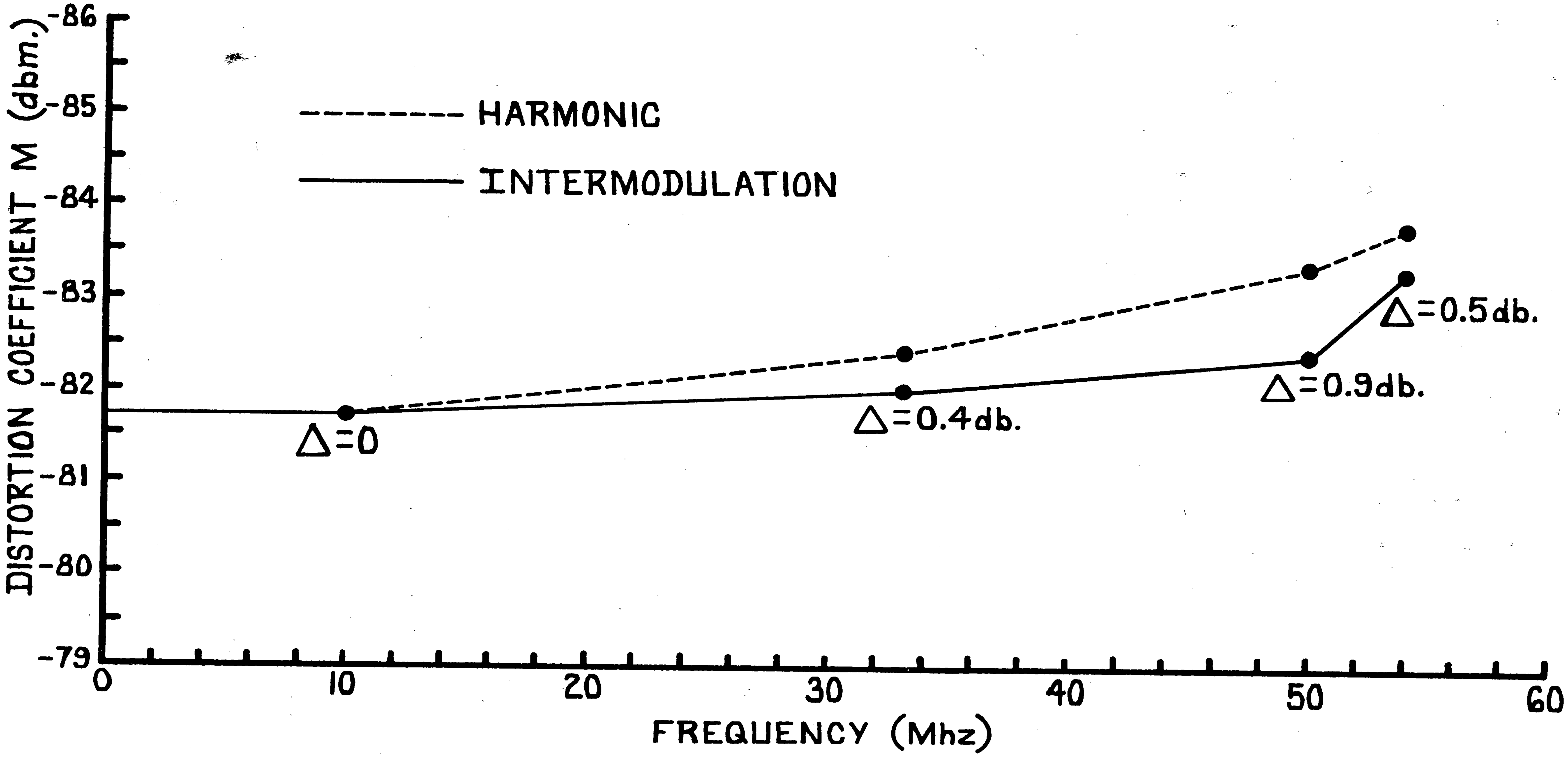


FIGURE 4- MEASURED DISTORTION LEVELS

TABLE 1

Third Order Intermodulation Products

Input Frequency	Output Frequency	Amplitude
$f_1 \pm f_2 \pm f_3$	$3f_1$	$1/4\ a_3A_1^3$
for all cases	$3f_2$	$1/4\ a_3A_2^3$
	$3f_3$	$1/4\ a_3A_3^3$
	$2f_1 \pm f_2$	$3/4\ a_3A_1^2A_3$
	$2f_1 \pm f_3$	$3/4\ a_3A_1^2A_3$
	$2f_2 \pm f_1$	$3/4\ a_3A_2^2A_1$
	$2f_2 \pm f_3$	$3/4\ a_3A_2^2A_3$
	$2f_3 \pm f_1$	$3/4\ a_3A_3^2A_1$
	$2f_3 \pm f_2$	$3/4\ a_3A_3^2A_2$
	$f_1 \pm f_2 \pm f_3$	$3/2\ a_3A_1A_2A_3$

TABLE 2

Tabulated Difference of Calculated and Experimental Distortion Levels

Product Frequency (Mhz.)	Harmonic M Difference Measured - Calculated	Intermodulation M Difference Measured - Calculated
10.0	1.9 db.	1.8 db.
33.2	2.0 db.	2.0 db.
47.0	2.1 db.	2.0 db.
53.0	1.9 db.	1.9 db.

TABLE 3

Tabulated Difference of Experimental Intermodulation and Harmonic Distortion Levels

Device	Frequency (Mhz.)	Measured Harmonic M (dbm.)	Measured Intermodulation M (dbm.)	Difference (db.)	Harmonic Test Limit*
1	33.2	-87.4	-87.0	-0.4	-80.4
	47.0	-88.0	-87.0	-1.0	-81.0
	53.0	-87.5	-87.0	-0.5	-80.5
2	33.2	-83.6	-83.1	-0.5	-80.5
	47.0	-84.3	-83.3	-1.0	-81.0
	53.0	-83.6	-83.1	-.05	-80.5
3	33.2	-79.8	-79.3	-0.5	-80.5
	47.0	-80.3	-79.4	-0.9	-80.9
	53.0	-80.1	-79.7	-0.4	-80.4
4	33.2	-83.0	-82.6	-0.4	-80.4
	47.0	-83.6	-82.6	-1.0	-81.0
	53.0	-83.5	-83.0	-0.5	-80.5
5	33.2	-85.6	-85.2	-0.4	-80.4
	47.0	-84.7	-83.6	-1.1	-81.1
	53.0	-85.6	-85.0	-0.6	-80.6
6	33.2	-88.9	-88.5	-0.4	-80.4
	47.0	-89.0	-88.0	-1.0	-81.0
	53.0	-88.5	-88.0	-0.5	-80.5
7	33.2	-87.2	-86.9	-0.3	-80.3
	47.0	-87.5	-86.6	-0.9	-80.9
	53.0	-85.4	-84.9	-0.5	-80.5
8	33.2	-83.0	-82.6	-0.4	-80.4
	47.0	-83.6	-82.6	-1.0	-81.0
	53.0	-82.7	-82.0	-0.7	-80.7
9	33.2	-79.8	-79.5	-0.3	-80.3
	47.0	-81.2	-80.0	-1.2	-81.2
	53.0	-80.6	-80.1	-0.5	-80.5
10	33.2	-83.4	-83.0	-0.4	-80.4
	47.0	-84.2	-83.2	-1.0	-81.0
	53.0	-82.1	-81.7	-0.5	-80.5

* If the intermodulation distortion coefficient requirement is -80 dbm. for all three product frequencies, then this number is the equivalent third harmonic distortion test limit. Note that this limit is almost the same for each product frequency independent of the device.

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APPENDIX A

Typical Device Parameters

Test	Condition	Symbol	Value
1. Emitter Current	$V_{be} = 0.85$ volts	I_e	2.8 ma.
2. Emitter Cutoff Current, Collector Open	$V_{eb} = 4.0$ volts	I_{ebo}	76 ua.
3. Static Forward Current Transfer Ratio	$V_{ce} = 10$ volts $I_c = 100$ ma.	h_{fe}	102
4. Collector Emitter Saturation Voltage	$I_c = 100$ ma. $I_b = 5.0$ ma.	$V_{ce}(sat)$	.24 v.
5. Base Emitter Saturation Voltage	$I_c = 100$ ma. $I_b = 5$ ma.	$V_{be}(sat)$	.75 v.
6. Collector Base Cutoff Current	$V_{cb} = 5$ volts	I_{cbo}	0.5 ua.
7. Emitter Base Cutoff Current	$V_{eb} = 1.5$ volts	I_{ebo}	0.7 ua.
8. Collector Cutoff Current - Short Circuited Base Emitter	$V_{ce} = 5$ volts	I_{ces}	6.5 ua.

APPENDIX B - Derivation of Distortion Variables

The derivation of the distortion parameters c, d, and e in Eq. (10) follow directly the method used by Narayanan and Poon$^{(5)}$. The major results of their work used in the derivation are presented below. Both the experimental work and the equations used assume a fifty ohm source and load impedance. The equation relating distortion coefficient to device parameters is,

$$M_{3E} = 10 \log \frac{d(1 + ef_t^{2})}{(1 + cf_t^{2})} + h(f_t, f_s) \qquad \text{(B-1)}$$

The first term in Eq. (B-1) is due to third order nonlinearities only, and it depends only on the third order product frequency, f_t. (10 log d) and (10 log de/c) give the low frequency and high frequency values of distortion directly. Distortion due to this term will be decreased with increase in frequency if c is greater than e. The exact expression for the second term is involved as it depends on second order distortion as well as third order distortion. If the second order distortion is very small, this term can be neglected. In the particular devices used in this work, second order distortion is negligible; so this term will not be considered further.

The distortion parameters c, d, and e are the result of three equations developed by Narayanan and Poon (5).

$$\frac{V_{eb}}{R_g} = i_{ex1}\, q_b + i_{ex2}\, q_b^{2} + i_{ex3}\, q_b^{3} \qquad \text{(B-2)}$$

$$\frac{V_{ce}}{R_L} = i_c = i_{o1}q_b + i_{o2}q_b^2 + i_{o3}q_b^3 \qquad \text{(B-3)}$$

$$i_b = i_{r1}q_b + i_{r2}q_b^2 + i_{r3}q_b^3 \qquad \text{(B-4)}$$

Equation (B-2) relates emitter to base voltage V_{eb} to base charge q_b, (B-3) relates collector current i_c to base charge q_b, and (B-4) relates the recombination current i_b to base charge q_b.

These equations represent exponential current, frequency cutoff, and recombination nonlinearities, respectively. The variables c, d, and e are the result of a simultaneous solution to the above equations as noted below:

$$c = \frac{4\pi^2}{(i_{r1} - i_{ex1})} \qquad \text{(B-5)}$$

$$d = \frac{10^{-3}}{2R_L} \; \frac{R^3_g}{i^3_{o1}} \; \frac{i_{o1}(i_{r3} - i_{ex3}) + i_{o3}(i_{r1} - i_{ex1})^2}{R^3_g\,(i_{r1} - i_{ex1})} \qquad \text{(B-6)}$$

$$e = \frac{2\pi\, i_{03}}{i_{o1}(i_{r3} - i_{ex3}) + (i_{r1} - i_{ex1})i_{o3}} \qquad \text{(B-7)}$$

Equations (B-2), (B-3), and (B-4) are first solved using known device parameters and a computer program to develop the Taylor expansion for the unknown currents i_{ex}, i_o, and i_r, respectively. Once these currents are known, the variables c, d, and e are solved for assuming a source and load impedance, R_g and R_L, respectively, of fifty ohms.

APPENDIX C - Bibliography

Kevin D. Lesh was born in Allentown, Pennsylvania on January 10, 1947 to R. Fred Lesh and Vera T. Lesh. He received the B.S.E.E. from Lehigh University in 1968. In 1968 he joined the Western Electric Company, Allentown, Pa. doing microwave device and radio frequency transistor test set design and development. At the same time Mr. Lesh was employed by the Lehigh County Community College as an instructor in radio frequency theory.

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