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# Optimum design of unbraced multi-story frames by plastic theory

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# OPTIMUM DESIGN OF UNBRACED MULTI-STORY FRAMES BY PLASTIC THEORY

by

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# A B S T R A C T

A computer program is described which performs weight optimization of three-bay multi-story frames using plastic theory. The input consists of the frame geometry and the load intensity. The maximum number of stories depends only on the computer storage capacity. Three loading cases with corresponding load factors are considered: vertical loads only, vertical loads combined with lateral loads from the left, and vertical loads combined with lateral loads from the right. Loads are modified to include pertinent load reduction criteria. Where combined loads govern the size of members, the ratio of the capacities of individual members is determined by an iterative procedure using frame weight as the minimized function. Provision is made for consideration of second-order effects (P-Δ effects). The output of the program consists of the member forces needed for design of connections, and of member sizes. Refinement of the member sizes to account for frame instability and constructional details should be performed to complete the design.

Weight comparisons for a frame designed for allowable stresses and plastic theory without optimization are presented.

## INTRODUCTION

At present, methods are not available to determine the ratios of individual member capacities in multi-story frames for greatest economy. When a structure is designed by plastic theory, redistribution of moments due to the successive formation of hinges is assumed.(1) This assumption permits the designer-analyst, when using plastic moment balancing, to specify within certain limits the member moments at any location in the frame without prior knowledge of the member capacities.(2) This approach opens a way for optimum weight design of multi-story frames.

The adaptation of computers for optimization in the field of structural design has been limited. Attempts to apply linear programming have met with some success.(3) However, the applications have been for very simple frames with only one set of loads.

The purpose of this paper is to describe a computer procedure developed for optimum weight design of unbraced three-bay, regular rectangular plane building frames using plastic theory. A regular rectangular plane frame is defined as one composed exclusively of horizontal girders and vertical columns, all column lines extending continuously from top to base without offset, and all girders lines extending continuously from one side of the frame to the other. The frame is assumed to be unbraced so that all resistance to lateral loads, sway, and frame instability is provided by rigid frame action, and all connections are assumed capable of developing plastic hinge moments in the members. The maximum number of stories depends only on the computer storage capacity; bay widths and story heights may vary arbitrarily. A story is defined as a portion of a multi-story frame consisting of all girders on a level and the columns immediately below. Acceptable load systems may consist of area loadings, concentrated column loads, and lateral loads from either direction. Load-deflection (P-$\Delta$) effects may be included in the load system by increasing the lateral loads. The output gives member sizes and member forces needed for design of connections.

A brief description of the sequence of operations which the computer program performs is as follows: Vertical loads are applied to the structure and preliminary member capacities are determined. Combined lateral and vertical loads are then applied and, using the previous loading case as a parameter, a weight function, related to the cost of the frame, is minimized. The sequence of the application of lateral loads is reversed and the design operation repeated. All members are then designed for the most economical arrangement of beam and column capacities.

# DISTRIBUTION OF LOAD SYSTEMS TO THE FRAME

The distribution of the load system to the frame is assumed as follows:

1. Floor dead and live loads are distributed through the floor system to the girders as uniformly distributed line loads or as concentrated loads at the column lines.
2. Exterior wall dead loads are distributed to each bent as concentrated loads at connection points of spandral beams.
3. Lateral loads of wind, earthquake, and as a measure of load-deflection moments are distributed to each bent as concentrated loads at the level of the girders.

The complete loading system consists of three loading cases: vertical loads only, vertical loads combined with lateral loads from the left, and vertical loads combined with lateral loads from the right. In each case live load reduction, is applied to the working loads before multiplication by the appropriate load factor. Load factors may vary arbitrarily at the discretion of the designer. (It has been suggested that a load factor of 1.7 be used for vertical loads and 1.3 for vertical loads combined with wind and earthquake).[4]

Live load reduction percentages as permitted by the American Standards Association Specification[5] are calculated for each girder after the uniform dead and live floor loads are converted to equivalent line loads per foot on the girder. These reductions are limited to the smallest of the three percentages:

1. 0.08 times the floor area served in square feet,
2. a formula based on the ratios of dead and live load,
3. 60 percent.

No reduction is permitted for roof girders, for areas where the live load is greater than 100 psf or where the tributary area of a floor associated with a girder is less than 150 square feet.

Column thrusts due to floor loads are calculated as if the floor spans were simply supported at the column center lines. Live load reduction percentages for each column are computed with the same limitations as stated for girders, except that a 20 percent reduction is allowed when the live load is greater than 100 psf. The sum of the weight of the exterior walls, and of the column and fireproofing in addition to the dead load of the floor spans is assumed to constitute the column dead load. Thus, the maximum reduction of 60 percent is applied to all but the uppermost column tiers.

## ASSUMED MEMBER BEHAVIOR

For the vertical load case a beam mechanism is assumed to form under a uniformly distributed design ultimate load (factored working load). The span is taken equal to the distance between column center lines as shown in Fig. 1. The required plastic moment capacity is then given by:

$$M_p = F_1 wL^2/16 \tag{1}$$

where $F_1$ = load factor for vertical load case,

$w$ = uniform working load (k/ft.),

$L$ = girder span (c-c columns) (ft.).

Static equilibrium requires that the column be able to resist the girder end moment $M_p$. This moment is assumed to be distributed equally above and below the girder line, except in the case of roof girders where it must be applied to the column below. The column end moments obtained in this manner together with the factored column thrust are used to select a preliminary column. The reduced moment capacity, $M_{pc}$, of this section is used as a parameter in future loading cases.

When lateral loads are applied to the frame, the sum of the end moments of all columns in a story may be determined from an equilibrium equation based on the horizontal shear in the story. Figure 2 is a free body sketch of the columns in a story of a three bay frame. The total horizontal shear, $\Sigma H$, which must be resisted by the columns is given by:

$$\Sigma H = \Sigma H_e + \Sigma H_w + \frac{\Sigma P\Delta}{h} \tag{2}$$

where

$\Sigma H_e$ = a concentrated load equal to the sum of the external horizontal loads above the story due to earthquake loading.*

$\Sigma H_w$ = a concentrated load equal to the sum of the external horizontal loads above the story due to wind loading,

$\frac{\Sigma P\Delta}{h}$ = a concentrated load equal to the horizontal component exterted by the sum of the column loads on the story acting in the displaced position resulting from the relative story sway,

$\Delta$ = relative story sway,

$h$ = story height.

For preliminary design purposes the sum of the horizontal shears, $\Sigma H$, can be assumed to be applied at the girder line and the equilibrium imposed on undeformed columns. With all forces having a positive direction as shown in Fig. 2, the equilibrium equation is as follows:

$$\Sigma H + \Sigma M_c/h = 0 \tag{3a}$$

or $$\Sigma M_c = - h\,(\Sigma H) \tag{3b}$$

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* It is not intended here to assume that plastic design is advanced sufficiently (1966) to include earthquake effects, but merely to indicate an approach for preliminary design purposes.

For equilibrium, the summation of the girder end moments at any level, must be equal to the summation of the column end moments joining these girders. It may be assumed that half of the moments in an average story occur at the top and half at the bottom of the columns. The total moment to be resisted by the girders at a particular level is then equal to the average of the sums of the column moments of the story above and the story below.

$$\Sigma M_g + \tfrac{1}{2} \left( (\Sigma M_c)_{n-1} + (\Sigma M_c)_n \right) = 0 \tag{4}$$

where $\Sigma M_g$ = the sum of the girder end moments at level n,

$(\Sigma M_c)_{n-1}$ = the sum of the column end moments above level n,

$(\Sigma M_c)_n$ = the sum of the column end moments below level n.

Substituting Eq. (3b) in Eq. (4) an expression for $M_g$ is obtained in terms of story heights and horizontal shears

$$\Sigma M_g = \tfrac{1}{2} \left( ((\Sigma H)h)_{n-1} + ((\Sigma H)h)_n \right) \tag{5}$$

If the girder end moments are known, the proportion of the story moment resisted by the column above can be determined from

$$\tfrac{1}{2} (\Sigma M_c)_{n-1} = DF\, \Sigma M_g \tag{6}$$

where $DF = ((\Sigma H)h)_{n-1} \Big/ \left( ((\Sigma H)h)_n + ((\Sigma H)h)_{n-1} \right)$

When lateral loads are applied to the structure, each girder must resist moments caused by gravity loads as well as the lateral load moments given in Eq. (5). Usually the load factor for the vertical load case is greater than the load factor used for the case of combined vertical and lateral loads. The excess moment capacity of the girder can be used to resist the lateral girder moments. Figure 3a shows the moment diagram for a uniformly loaded girder. $M_{pm}$ is defined as the minimum plastic moment required to resist the working gravity loads multiplied by load factor $F_2$, used for the combined load case:

$$M_{pm} = \frac{F_2 w L^2}{16} \tag{7}$$

The dashed lines in Fig. 3a represent the girder moment capacity, $M_p$, as obtained from the vertical load case. The unused bending capacity makes it possible for the girder to redistribute moments so that it can resist lateral load moments. Figure 3b shows the maximum redistribution possible without exceeding $M_p$. The maximum lateral moment which the girder can resist is then the algebraic sum of the clockwise girder moments.

$$M_g \quad M_p + M_{min} \tag{8}$$

If the girder capacity obtained in this manner is not sufficient to resist the sway moments, additional sway capacity can be realized if an effective load factor $F_{1E}$, greater than $F_1$, is assumed. Figures 3c and d show two moment diagrams possible with larger girders. In Fig. 3d the second plastic hinge forms at the left hand end and therefore is the limiting case. For all values of $M_g$ greater than that shown in Fig. 3d the moment diagram will appear to be the same but with larger values of actual $M_p$.

It should be noted in the sequence of moment diagrams in Fig. 3 that 1) the parabolic portion of the moment diagram due to the uniformly distributed load, $F_2w$, always has the same relative configuration and size, 2) the location of the second plastic hinge is variable and moves from the center of the span to the left hand end, 3) the moment $M_{min}$ gradually changes from a maximum counterclockwise moment to a maximum clockwise moment.

Figure 4 shows two free body diagrams for a uniformly loaded girder.[6]

In Figure 4a:

$$\Sigma M_A + \circlearrowright = 0$$

$$V_B = \frac{F_2wL}{2} + \frac{(A+1)M_p}{L} \tag{9}$$

In Figure 4b:

$$\Sigma F_y + \downarrow = 0$$

$$V_B = F_2wx \tag{10}$$

$$\Sigma M_o + \circlearrowright = 0$$

$$V_B = \frac{F_2wx}{2} + \frac{2M_p}{x} \tag{11}$$

Equating Eq. (10) and Eq. (11),

$$x = \sqrt{\frac{4M_p}{F_2w}} \tag{12}$$

Substituting into Eq. (10),

$$V_B = F_2 w \sqrt{\frac{4M_p}{F_2 w}} \tag{13}$$

From Eq. (9)

$$(A+1)\ M_p = 2L\sqrt{F_2 w\ M_p} - \frac{F_2 wL^2}{2} \tag{14}$$

Substituting the value of $M_p$ from Fig. 3c or 3d,

$$(A+1) = \frac{8F_2}{F_1}\left(\sqrt{\frac{F_{1E}}{F_2}} - 1\right) \tag{15}$$

Since

$$(A+1)\ M_p = M_{min} + M_p, \tag{16}$$

Equation (8) becomes,

$$M_g = \frac{8F_2}{F_{1E}}\left(\sqrt{\frac{F_{1E}}{F_2}} - 1\right) M_p \tag{17}$$

If $F_{1E} = 4F_2$ then, $M_g = 2M_p$ which is the limiting case shown in Fig. 3d. The sway resistance of a loaded girder in terms of the load factors can be expressed by two equations:

$$M_g = 8F_2\left(\sqrt{\frac{F_{1E}}{F_2}} - 1\right)\frac{wL^2}{16}, \qquad F_2 \leq F_{1E} \leq 4F_2 \tag{18a}$$

$$M_g = 2\ F_{1E}\frac{wL^2}{16} = F_{1E}\frac{wL^2}{8}, \qquad 4F_2 \leq F_{1E} \tag{18b}$$

The total capacity of the girders on a particular level to resist sway moments is the summation of the sway capacity of each individual girder. It is not necessary for the effective load factor, $F_{1E}$, to be the same for all girders.

For a certain number of levels below the roof, the girder capacity will be controlled by gravity load alone at load factor $F_1$. The total moment capacity of gravity controlled girders on a level is given by

$$\Sigma M_g = 8F_2\left(\sqrt{\frac{F_1}{F_2}} - 1\right) \Sigma \frac{wL^2}{16} \tag{19}$$

Comparison of Eq. 19 with the required moment capacity given by Eq. 5 will determine for which level gravity load no longer controls the girder design.

Consideration of a symmetric three-bay frame symmetrically loaded shows that under gravity loads the interior columns theoretically are not required to resist any moment. However, with lateral loads applied the interior columns must exhibit some moment capacity. Thus, sufficient girder moment capacity does not insure sufficient frame capacity to resist lateral loads. The distribution of the sway moments at a level for economy is dependent on the column moment capacities from the vertical load case.

At levels where the excessive girder capacity from the vertical load case is not sufficient to resist sway moments an effective load factor, $F_{1E}$, can be assumed for two bays and the load factor required for the third bay calculated.

Using the notation of Fig. 5 and assuming effective load factors for spans AB and BC the sway resistance girder capacity of these bays at level n is given by:

$$M_gA - C = K_{AB} \frac{w_{AB}L_{AB}^2}{16} + K_{BC} \frac{w_{BC}L_{BC}^2}{16} \tag{20}$$

where $K = 8F_2 (\sqrt{F_{1E}/F_2} - 1) \qquad 0 \leq F_{1E} \leq 4F_2$

or $\quad K = 2F_{1E}, \qquad 4F_2 \leq F_{1E}$

The required sway moment capacity of span CD is

$$M_{gCD} = (F_2 \Sigma M_c)_n - M_{g\,A-C} \tag{21}$$

The load factor for span CD is obtained by solving Eqs. 18a and 18b for $F_{1E}$.

$$(F_{1E}) = (1.0 + \frac{K_{CD}}{8F_2})\, 2F_2, \qquad 0 \leq K_{CD} \leq 8F_2 \tag{22a}$$

$$(F_{1E})_{CD} = \frac{K_{CD}}{2}, \qquad 8F_2 < K_{CD} \tag{22b}$$

where

$$K_{CD} = \frac{M_{gCD}}{\frac{wL_{CD}^2}{16}}$$

The column end moments can then be found by using Eq. 6 and equations of equilibrium at each joint.

As a result of the assumption that half of the moments in an average story occur at the top and half at the bottom of the columns, each level can be designed independently and various combinations of girder capacities can be investigated readily without complete reanalysis of the entire frame. It is then possible to calculate the weight, as a measure of the cost of the frame for each combination and select the case with minimum weight for the final design.

## THE WEIGHT FUNCTION

It is assumed here that the weight of a member is proportional to its moment capacity. Thus, a weight function is defined in terms of the maximum required moment capacity and the length of each column and girder in a story.*

$$Wt = h\Sigma\ (M_{cmax}) + \Sigma\ (M_{gmax}\ L_g) \tag{23}$$

where $M_{cmax}$ = the maximum required moment capacity of a column,

$h$ = story height,

$M_{gmax}$ = the maximum required moment capacity of a girder,

$L_g$ = girder length.

Substituting Eq. 6, 18, and 22 into Eq. 23 the weight function is expressed in terms of $(F_{1E})_{AB}$ and $(F_{1E})_{BC}$. Figure 6 is a sketch of the general shape of the resulting function.

If the moment capacity for any previous loading of any member is greater than the moment capacity for the distribution being considered its value is substituted in Eq. 23. This substitution tends to flatten the shape of the weight function as shown in Fig. 7.

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* A more accurate expression for the weight function is

$$Wt = h\Sigma\ \frac{M_{cmax}}{d_c} + \Sigma\ \frac{M_{gmax}}{d_g}\ L_g$$

where $d_c$ and $d_g$ are the depths of columns and girders, respectively.

It can be concluded from Fig. 6 and 7 that a unique minimum value exists. Minimization of the weight function in Fig. 6 will lead to the most economical arrangement of girder and column moment capacities for a particular loading case. Minimization of the weight function in Fig. 7 will lead to the most economical arrangement for a sequence of loadings since member capacities required in previous loading cases are considered.

The dashed lines in Fig. 8 represent the moment capacities of the members obtained from a previous loading case. The minimum value of the weight function will have the tendency to distribute moments in a manner which will utilize available capacity. The solid lines represent the moment diagram for an applied sway moment just equal to the given girder capacities. As the sway moment is increased the additional moment will be distributed in the following order of preference:

1. To the shortest girder span if the adjacent columns have excess moment capacity.
2. To the girder span where excess column moment capacity exists.
3. To the shortest span.

If additional sway moment is applied to the story in Fig. 8, analysis of the weight function shows that the additional moment will be taken by the girder in span BC until the moment capacities of columns B and C are exceeded. Any additional moment will be taken by span AB until the capacity of column A is exceeded. Further moment will be distributed to span BC only, for economy. For tall frames it can be seen that the required girder size for span BC of Fig. 8 can be quite large and that the distribution of moments for a practical design may not be the most economical as determined from the weight function. The assumption of maximum values for the effective load factors will not affect the shape of the weight function, but it will reduce the validity of the weight function to the portion bounded by these maximum load factor values. This is shown graphically in Fig. 9.

It is possible then for the designer to analyze the frame under vertical loads only, and using the maximum moment capacities for all members as parameters, apply lateral loads from the left and determine the most economical arrangement of the member capacities from the weight function. Lateral loads from the right can then be applied, and using the previous loading cases as a parameter a new distribution determined. A different sequence of loadings--vertical, lateral loads from the right, lateral loads from the left--may result in a different distribution of moment capacities. The two designs can be compared and the most economical arrangement used for the final preliminary design. This approach may not result in the most economical arrangement possible, but it is judged to be fully adequate for design purposes.

## DESIGN OF MEMBERS

Member sizes are chosen for member forces obtained by the procedure described above. Members are assumed to be adequately braced to preclude out-of-plane and lateral-torsional instability.

When the interaction of bending moment and axial force is considered, the approximate end moment capacity of a column is taken to be

$$M_{pc} = M_p \qquad \text{when } P/P_y \leq 0.15 \tag{24a}$$

$$M_{pc} = 1.18\,(1-P/P_y)M_p \qquad \text{when } 0.15 \leq P/P_y \leq 1.0 \tag{24b}$$

where $M_{pc}$ = plastic hinge moment modified to include the effect of axial compression,

$P$ = concentrated axial load,

$P_y$ = axial load corresponding to yield stress level: $P = A \cdot F_y$,

$M_p$ = $Z \cdot F_y$ = plastic moment capacity of the section,

$Z$ = plastic section modulus.

This is in accordance with the present AISC Specification[7] for plastic design and is valid for columns subjected to double curvature bending. The approximation is acceptable for the purpose of preliminary design since, except for a few upper stories, the critical column design condition is when the frame is subjected to combined vertical and lateral loads and columns are in double curvature bending.

Girder sizes are selected from section economy tables using the required member end capacities with consideration given to maximum depths for architectural reasons.

## DIGITAL COMPUTER PROGRAM

A digital computer program for a General Electric 225 computer has been developed to handle the extensive calculations involved in the method described in the previous sections. The program is written in the FORTRAN II language and consists of a main routine and five sub-routines. Running time on the GE 225 can be estimated at one half minute per story excluding the start up time. A brief description of the program is as follows.

Input data:

1. Frame Geometry:

   Bent spacing

   Span lengths

   Parapet wall and story heights

   Number of stories

2. Load System

   Area dead and live loads

   Weight of exterior walls

   Estimated weight of columns plus fireproofing

   Area lateral loads

   Number of stories for which the magnitude of each load is applicable

3. Miscellaneous:

Load factors for each loading case

Yield stresses of the steel to be used

Maximum effective load factors

Maximum depth of girders

Section properties needed for girder and column design.

Area loads are converted to uniform girder loadings and to column thrusts. Live load reduction allowed by the ASA 58.1[5] specification is applied to all loadings. Girder working load end moments and shears are computed and printed for reference. Lateral loads are converted to concentrated loads at the girder lines.

Using the load factor for the vertical load case, factored girder and column end moments, girder end shears, and column thrusts are calculated and printed. Equations (24a) and (24b) together with the factored column moments and thrusts are used to select preliminary column sizes from a list of suitable wide flange sections.

Next, the sum of the girder end moments, $\Sigma M_g$, required to resist the factored overturning moments of the lateral loads from the left are determined at the roof level (Eq. 3). An effective load factor $F_{1E} = F_1$ is assumed for span AB. With one variable, $(F_{1E})_{AB}$, equal to a constant the weight function becomes a two dimensional curve shown in Fig. 11. Since the effective load factor for span CD can be expressed as a function of the effective load factors for spans AB and BC (Eq. 22) any point on the curve can be determined by assuming a value for $(F_{1E})_{BC}$.

The maximum moment capacities required for the columns and girders from either load case are used in evaluating the weight function.

The minimum point on the curve is found using the procedure shown in the flow diagram in Fig. 11. The largest effective load factor, $(F_{1E})_{max}$, may be selected by the designer to limit the size of girders. The trial load factor F is assumed equal to the average of $F_{min}$ and $F_{max}$ and the slope of the curve in the region of F is determined by evaluating the weight function at F and at F + 0.001. If the slope is negative, $F_{min} = F$. If the slope is positive, $F_{max} = F$. The procedure is then repeated using the new value of $F_{min}$ or $F_{max}$. When the values of $F_{min}$ and $F_{max}$ are within 0.1, Wt (1) is assumed to be the minimum value.

$(F_{1E})_{AB}$ is then incremented and a new minimum value for the weight function is found. This value is then compared to the previous value. If the new value is larger the procedure stops and a complete moment balance is performed using the load factors from the previous trial. If the new value is smaller, the procedure is continued until a larger value is found.

Using column moments obtained from the moment balance and the appropriately factored column thrusts, the column sections selected from the vertical load case are checked and if found inadequate new sections chosen.

The same procedure is then carried out for each successive story, and member end moments, girder shears, and column thrusts are printed for reference.

The direction of the lateral load is reversed and the weight function is again minimized for each story successively beginning at the roof. The maximum moment capacity of each member, as determined from previous loading cases or as required by the case under consideration, is used for determining the value of the weight function at each point along the curve for each story. Factored member end moments, girder shears and column thrusts are printed at each level.

Using the largest required moment capacity from any of the loadings, final girder sections are selected from a complete list of available shapes. Final column and girder sizes and the total weight of the frame are are printed.

The procedure is repeated with the second loading sequence: vertical loads, vertical loads combined with lateral loads from the right, and finally vertical loads combined with lateral loads from the left. The weights of the two frames are compared and the configuration having the minimum weight is selected as the final preliminary design.

A simplified flow diagram of the program is shown in Fig. 12. This diagram essentially follows the outline given above and no additional explanation is deemed necessary. An example design of a twenty-four story three-bay frame is included in Appendix I. This is the same frame as was designed in Refs. 4, 8, and 9. Comparison of this design with preliminary and final designs of the frame using allowable-stress design and plastic theory without optimization are presented in Appendix II.

Comparison of the three designs in Appendices I and II shows that the weight optimization procedure will result in a significant reduction in the cost of unbraced multi-story frames. The computer program can be used not only to speed up the design, but also to produce a design more economical than by any other method available.

## CONCLUSIONS

A computer program was devised for the optimum design of unbraced multi-story frames using plastic theory. This program is applicable to three-bay frames of any number of stories, but without missing members. The basic theory of the method is valid for any number of bays. However, the approach used to determine the minimum value of the weight function must be modified since each additional bay adds an additional dimension to the function. Operations Research or non-linear programming may be used for this purpose.

Comparison of the design of an example frame using the optimization procedure shows weight savings of 12.8% over the allowable stress method and 6.7% over the plastic method without optimization.

Further improvements of the program which are desirable and feasible include: 1) use of clear span for girder design, 2) more accurate column design method for upper stories where columns are not in double curvature bending, 3) inclusion of the depth of members in the weight function, 4) refinement of the weight function to include the already selected girders on a level when determining the third girder, 5) inclusion of P-$\Delta$ effect, 6) methods for handling multi-bay frames, 7) consideration of working load deflections.

When the sway subassemblage method of column design* described in Ref. 10 was used to check the member sizes shown in Table 1. It was found that member sizes in all but the uppermost stories are suitable for a final design. Deflection checks show that the addition of a horizontal force equal to two percent of the girder dead and live loads at a level is too conservative for estimating the effect of P-$\Delta$ moments.[11] A one percent horizontal force would produce deflections which more closely approximate those predicted by the subassemblage method of design.

The results of the program are to be considered as preliminary and could be used as input data for programs which make deflection estimates or check the stability of the frame as needed for final design.

* This semi-graphical method is considered at present (1966) to be the most rigorous for plastic design of multi-story frames. P-$\Delta$ effects are included and the maximum strength of the structure, even though it may occur before the formation of a mechanism, is considered.

# APPENDIX I

## EXAMPLE FRAME

The twenty-four story three-bay frame in Fig. 13 is used to illustrate the method. This is the same frame as was designed in Ref. 4 and 9.

Dead loads are based on assumed weights for 5-in. reinforced concrete slabs, plaster ceilings, fireproofing, and the weight of steel system members. A constant wind pressure was assumed. A horizontal force, equal to two percent of the girder dead and live loads at a level, was added to the wind force at each level to include the effect of P-$\Delta$ moments. (The total wind pressure in the design was 37.9 psf, which included P-$\Delta$ effects).

For the sake of comparison with designs in Ref. 4 and 9, A-36 steel was used for all girders and for all columns above the eleventh level. A-441 steel was used for the remaining column sections.

Maximum effective load factors of 5.5, 19.5, 5.0 were assumed for Bay AB, Bay BC, and Bay CD, respectively. Additional reduction in the weight of the girders could be obtained if larger effective load factors had been assumed in Bays AB and BC.

Table 1 shows final girder end column sections together with weight tabulations.

Columns

| Tier | Col A | Col B | Col C | Col D |
|---|---|---|---|---|
| 1-3 | 12WF 40 | 10WF 39 | 14WF 53 | 14WF 61 |
| 3-5 | 14WF 61 | 14WF 53 | 14WF 78 | 14WF 84 |
| 5-7 | 14WF 84 | 14WF 78 | 12WF106 | 14WF111 |
| 7-9 | 12WF106 | 12WF106 | 14WF136 | 14WF136 |
| 9-11 | 14WF127 | 14WF127 | 14WF158 | 14WF167 |
| 11-13 | (14WF119) | (14WF136) | (14WF150) | (14WF150) |
| 13-15 | (14WF142) | (14WF176) | (14WF176) | (14WF176) |
| 15-17 | (14WF167) | (14WF211) | (14WF202) | (14WF193) |
| 17-19 | (14WF190) | (14WF264) | (14WF264) | (14WF211) |
| 19-21 | (14WF211) | (14WF314) | (14WF314) | (14WF246) |
| 21-23 | (14WF246) | (14WF314) | (14WF370) | (14WF264) |
| 23-25 | (14WF287) | (14WF342) | (14WF398) | (14WF287) |

Girders

| Level | Bay AB | Bay BC | Bay CD |
|---|---|---|---|
| 1 | 16B26 | 10B15 | 18WF45 |
| 2 | 16WF36 | 14B17.5 | 18WF55 |
| 3 | do | do | do |
| 4 | do | 12B22 | 21WF55 |
| 5 | 16WF40 | do | do |
| 6 | 16WF45 | 14B26 | do |
| 7 | do | 14WF30 | 12WF62 |
| 8 | 18WF45 | 16WF36 | do |
| 9 | do | 16WF40 | do |
| 10 | 16WF50 | 18WF45 | 21WF68 |
| 11 | do | 21WF55 | do |
| 12 | do | 21WF62 | do |
| 13 | do | 24WF68 | do |
| 14 | 21WF55 | 24WF76 | 21WF62 |
| 15 | 21WF68 | do | 21WF55 |
| 16 | do | 27WF84 | do |
| 17 | 24WF68 | do | do |
| 18 | do | 27WF94 | do |
| 19 | 24WF76 | do | do |
| 20 | do | do | 21WF55 |
| 21 | do | do | 21WF62 |
| 22 | do | do | 24WF68 |
| 23 | do | do | 24WF94 |
| 24 | do | do | 27WF94 |

Weight
A36 and A441 Steel

| | A36 | A441 (Columns in parenthesis) |
|---|---|---|
| Girder Wt. | 40.3 T | -- |
| Column Wt. | 22.9 T | 76.0 T |
| Total Wt. | 63.2 T | 76.0 T |

Table 1 Member Sizes of Example Frame Designed by Plastic Method With Optimization

## APPENDIX II

## COMPARISON WITH OTHER METHODS

Tables 2 and 3 show the final column and girder sizes for the example frame of Appendix I designed by allowable-stress method and by plastic method without optimization.(9) The load assumptions, including the horizontal force to produce the effect of P-Δ moments, and the choice of steel were the same as for the frame of Appendix I.

Figure 14 shows graphically a comparison of girder weight, column weight and total weight of the frame by the three design methods. A savings of 12.8% can be realized using the proposed method over allowable-stress design and 6.7% over the plastic design method without optimization.

It is to be noted that the comparison of the design example of Appendix I and the design using plastic method without optimization (Table 3) would be more favorable if clear span were used in the design example. This assumption was used to obtain the girder sections in Table 3, and it reduces the required moment capacity (and weight) for girders with a slight increase in the required moment capacity of the columns.

## COLUMNS

| Tier | Col A | Col B | Col C | Col D |
|---|---|---|---|---|
| 1-3 | 14WF 48 | 14WF 40 | 12WF 58 | 14WF 61 |
| 3-5 | 14WF 68 | 14WF 61 | 14WF 87 | 14WF 95 |
| 5-7 | 14WF 87 | 14WF 84 | 14WF119 | 14WF119 |
| 7-9 | 14WF127 | 14WF119 | 14WF150 | 14WF158 |
| 9-11 | 14WF150 | 14WF142 | 14WF176 | 14WF184 |
| 11-13 | (14WF150) | (14WF142) | (14WF176) | (14WF193) |
| 13-15 | (14WF184) | (14WF158) | (14WF193) | (14WF246) |
| 15-17 | (14WF211) | (14WF176) | (14WF211) | (14WF287) |
| 17-19 | (14WF264) | (14WF193) | (14WF264) | (14WF314) |
| 19-21 | (14WF287) | (14WF237) | (14WF287) | (14WF370) |
| 21-23 | (14WF314) | (14WF246) | (14WF314) | (14WF298) |
| 23-25 | (14WF370) | (14WF287) | (14WF342) | (14WF458) |

## GIRDERS

| Level | Bay AB | Bay BC | Bay CD |
|---|---|---|---|
| 1 | 14WF34 | 12B16.5 | 18WF55 |
| 2 | 16WF45 | 16B26 | 21WF62 |
| 3 | do | 14WF30 | do |
| 4 | do | 14WF34 | do |
| 5 | do | 16WF36 | do |
| 6 | 16WF50 | do | 21WF68 |
| 7 | 18WF50 | 16WF40 | 24WF68 |
| 8 | 18WF55 | 16WF45 | 24WF68 |
| 9 | 21WF55 | 18WF45 | 24WF76 |
| 10 | 21WF62 | 16WF50 | 24WF84 |
| 11 | do | 18WF50 | do |
| 12 | 21WF68 | do | 27WF84 |
| 13 | 24WF68 | 18WF55 | 24WF94 |
| 14 | do | 21WF55 | 27WF94 |
| 15 | 24WF76 | 21WF62 | do |
| 16 | do | do | 30WF99 |
| 17 | 24WF84 | 21WF68 | do |
| 18 | do | do | 30WF108 |
| 19 | 27WF84 | do | do |
| 20 | 24WF94 | 24WF68 | 30WF116 |
| 21 | 27WF94 | do | do |
| 22 | do | do | 33WF118 |
| 23 | do | 24WF76 | do |
| 24 | 30WF99 | do | do |

## WEIGHT
A36 and A441 Steel

| | | A36 | A441 (Columns in Parenthesis) |
|---|---|---|---|
| Girder Wt. | = | 50.2 T | -- |
| Column Wt. | = | 24.5 T | 85.0 T |
| Total Wt. | = | 74.7 T | 85.0 T |

Table 2 Member Sizes of Example Frame Designed by Allowable-Stress Method

## COLUMNS

| Tier | Col A | Col B | Col C | Col D |
|---|---|---|---|---|
| 1-3 | 12WF 40 | 12WF 40 | 12WF 58 | 12WF 58 |
| 3-5 | 12WF 58 | 12WF 58 | 12WF 79 | 12WF 79 |
| 5-7 | 14WF 78 | 14WF 78 | 14WF111 | 14WF111 |
| 7-9 | 14WF111 | 14WF111 | 14WF136 | 14WF136 |
| 9-11 | 14WF127 | 14WF127 | 14WF158 | 14WF158 |
| 11-13 | (14WF136) | (14WF142) | (14WF193) | (14WF158) |
| 13-15 | (14WF142) | (14WF167) | (14WF211) | (14WF184) |
| 15-17 | (14WF167) | (14WF193) | (14WF246) | (14WF202) |
| 17-19 | (14WF211) | (14WF237) | (14WF314) | (14WF246) |
| 19-21 | (14WF246) | (14WF264) | (14WF342) | (14WF287) |
| 21-23 | (14WF287) | (14WF314) | (14WF370) | (14WF314) |
| 23-25 | (14WF314) | (14WF342) | (14WF398) | (14WF320) |

## GIRDERS

| Level | Bay AB | Bay BC | Bay CD |
|---|---|---|---|
| 1 | 14B26 | 12Jr11.8 | 16WF45 |
| 2 | 16WF36 | 12B16.5 | 18WF55 |
| 3 | do | do | do |
| 4 | do | do | do |
| 5 | do | 16B31 | do |
| 6 | 16WF45 | 16WF40 | do |
| 7 | do | do | do |
| 8 | 18WF50 | 18WF50 | do |
| 9 | do | do | do |
| 10 | 21WF55 | 21WF55 | 21WF55 |
| 11 | do | do | do |
| 12 | 21WF62 | 21WF62 | 21WF62 |
| 13 | do | do | do |
| 14 | 21WF68 | 21WF68 | 21WF68 |
| 15 | do | do | do |
| 16 | 24WF68 | 24WF68 | 24WF68 |
| 17 | do | do | do |
| 18 | 24WF76 | 24WF76 | 24WF76 |
| 19 | do | do | do |
| 20 | do | do | do |
| 21 | 24WF84 | 24WF84 | 24WF84 |
| 22 | do | do | do |
| 23 | do | do | do |
| 24 | 27WF84 | 27WF84 | 27WF84 |

## WEIGHT
A36 and A441 Steel

| | | A36 | A441 (Columns in Parenthesis) |
|---|---|---|---|
| Girder Wt. | = | 42.8 T | -- |
| Column Wt. | = | 23.0 T | 83.4 T |
| Total Wt. | = | 65.8 T | 83.4 T |

Table 3 Member Sizes of Example Frame Designed by Plastic Method Without Optimization

# APPENDIX III

## COMPUTER PROGRAM DESCRIPTION

### FORTRAN Statements

The complete listing of the FORTRAN II program described in this paper is shown in Display A. Sufficient explanation has been added to enable the designer, who is expected to be familiar with the FORTRAN language, to follow the logic of the program.

The core storage capacity of the GE 225, the computer for which the program was developed, was not sufficient to permit the storage of the entire program and the necessary data. A system called CHAIN was used to alleviate this deficiency. CHAIN permits the storage of sections of a program on magnetic tape. Each section or "link" is placed in core storage as required for execution of the program. Values of the variables assigned to COMMON storage are not affected by the execution of individual links. Several changes in the program statements are necessary if sufficient core capacity is available to store the program in its entirety (about 10,000 locations). These revisions have been noted in Display A.

The program can also be modified for designing a frame of any number of stories by changing the array specifications following the COMMON and DIMENSION statements.

## Data Preparation

The input data is arranged in the following manner:

1. Frame number (must be positive if the program is to be executed).
2. Frame geometry
3. Area loadings
4. Weight of exterior walls and columns plus fireproofing
5. Wind pressure
6. Load factors
7. Column section properties (83 sections are required in order of increasing weight).
8. Yield points of steel to be used for column design (not more than three).
9. Maximum effective load factors for girder design.
10. Girder section properties (221 sections required in order of increasing weight.
11. Yield points of steel to be used for girder design (not more than three).
12. Maximum girder depths.
13. Frame number (negative to stop the program).

The number of stories for which each piece of data is applicable is also included as input. Linear measurements are assumed to be in feet; area loadings, weight of exterior walls, and wind pressure in pounds per square foot; weight of columns plus fireproofing in pounds per foot. Column and girder section properties required are: nominal depth, in., weight, lb/ft.; plastic section modulus, in.$^3$

The program is organized in such a manner as to allow the designer, at his discretion, to vary the story height and items 3, 4, 5, 9, and 12 at each story. The maximum number of steel yield points (items 8 and 11) which may be used in a given design is three for columns and three for girders. However, if the input data for the yield point of columns is 441.0, the program will consider the first 56 columns at a yield point of 50.0 ksi, the next 12 at 46.0 ksi, and the last 15 at 42.0 ksi. These stresses correspond to the yield points allowed by the AISC Specification for A441 steels.[7]

Display B lists the order of the input data and gives the acceptable FORMAT. Display C shows a set of data cards for a ten story frame. Display D shows part of the output for the data of Display C.

```
C         PRELIMINARY PLASTIC DESIGN OF MULTI-STORY-THREE-BAY FRAMES
C                                 BY T.M. MURRAY, LEHIGH UNIVERSITY
C              SPECIFICATION OF ARRAY DIMENSIONS
      COMMON  A[3],B[3],ALL[4],ALLOW[4],D[3],E[3],F[3],FW[3],G[14]
      COMMON  CM[25,3],H[25],P[25,3],R[4],RC[4],SUMA[4],SUMLL[4]
      COMMON  SUMRL[4],ST[4],XL[3],T[25,7],V[6],WDL[3],WL[25],WLL[3]
      COMMON  AA,BS,CCL,DF,F1,F2,FY,F1MAX,GIRCAP,GIRREQ,GMC,GMC1,I,J,JJ
      COMMON  K,NN,NS,PWH,W,WALL,WEIGHT,WINC,WIND,WTMN1,WTMN2,X,XCOL
      COMMON IP[92],W[92],Z[92],ICOL[25,4],KK,LC
      COMMON SUMWT,SPEC[3],NSPEC[3],TEMPWT
      EQUIVALENCE [W[1],PY[1]],[Z[1],XMP[1]],[T[10]],SV[1]]
      EQUIVALENCE [WL[1],WLM[1]]
      DIMENSION  PY[92],XMP[92],WST[3],SV[25,3],WLM[25]
C              INPUT FRAME NUMBER AND EXIT IF NEGATIVE
    1 READ 100 ,NN
  100 FORMAT [I10]
      IF [NN]2,3,3
    2 CALL EXIT
C              INPUT/OUTPUT FRAME GEOMETRY [ALL DIMENSIONS IN FT.]
    3 READ 101,NS,BS,[XL[K],K=1,3],PWH
  101 FORMAT [I10,4F10.2/F10.2]
      PRINT 102,NN,NS,BS
  102 FORMAT [63H1 PRELIMINARY PLASTIC DESIGN OF MULTI-STORY-THREE BAY F
     1RAME NO.,I5///7X,14HFRAME GEOMETRY//
     220H  NUMBER OF STORIES.,I4// 15H  BENT SPACING,,F 6.2,4H FT.//]
      PRINT 135,[XL[K],K=1,3],PWH
  135 FORMAT [23H  SPAN LENGTHS,  BAY AB,F6.2,4H FT./
     1          17X,                6HBAY BC,F6.2,4H FT./
     2    17X,                      6HBAY CD,F6.2,4H FT.//
     3          30H  STORY HEIGHTS,  PARAPET WALL,F15.2,4H FT.]
      KJ=0
      IJ=0
      DO 90 K=1,4
      SUMRL[K]=0.0
   90 SUMA[K]=0.0
      DO 48 I=1,NS
      IF [I-JJ]49,49,50
   50 READ 133,H[I],J
  133 FORMAT [F10.2,I10]
      IJ=JJ+J
      PRINT 134,I,JJ,H[I]
  134 FORMAT [17X,                  8H STORIES,I6,4H TO ,I3,F7.2,4H FT.]
      GO TO 48
   49 H[I]=H[I-1]
   48 CONTINUE
C            INPUT WORKING DEAD AND LIVE LOADS, NO. OF LEVELS APPLICABLE
      PRINT 136
  136 FORMAT [20H1          WORKING LOADS//7X,     16HA.  GIRDER LOADS/
     111X,    38H[PER CENT LIVE REDUCTION BY ASA A58.1]///
     2  13H  ROOF GIRDER.  27X,22HBAY AB  BAY BC  BAY CD]
      DO 45 K=1,3
      A[K]=BS*XL[K]
   45 ALL[K]=A[K]/20.
```

**Display A FORTRAN Program Statements**

```
      IJ=0
      AA=RS/10.
      DO 21  I=1,NS
      IF (I-JJ)17,17,16
   16 READ 121,(WLL(K),WDL(K),K=1,3),J
  121 FORMAT (6F10.2,I10)
      IJ=JJ+J
C            OUTPUT WORKING LOADS
      IF (I-1)4,4,5
    5 PRINT 110,I,IJ
  110 FORMAT (    9H  LEVELS ,I3,4H TO ,I3)
    4 PRINT 111,(WLL(K),K=1,3) ,(WDL(K),K=1,3)
  111 FORMAT (31H0 WORKING LIVE LOAD, LB./SQ.FT.,7X     ,3F8.2/
     1          31H  WORKING DEAD LOAD, LB./SQ.FT.,7X     ,3F8.2)
      DO 7 K=1,3
      WLL(K)=WLL(K)*RS/1000.
    7 WDL(K)=WDL(K)*RS/1000.
      PRINT 113,(WLL(K),K=1,3)
  113 FORMAT (26H  WORKING LIVE LOAD, K/FT.,12X           ,3F8.2)
C            COMPUTE LIVE LOAD REDUCTION
      DO 8 K=1,3
      IF (I-1)9,9,10
   10 IF (WLL(K)-AA)11,11,9
   11 IF (A(K)-150.) 9,9,12
   12 R(K)=.08*A(K)
      ALLOW(K)=100.*(WDL(K)+WLL(K))/4.33/WLL(K)
      IF (R(K)-ALLOW(K))13,13,14
   14 R(K)=ALLOW(K)
   13 IF (R(K)-60.)15,15,18
   18 R(K)=60.
      GO TO 15
    9 R(K)=0.00
   15 P(I,K)=(1.0-R(K)/100.)*WLL(K)
    8 CONTINUE
C            OUTPUT GIRDER LOADS
      PRINT 114,(R(K),K=1,3),(P(I,K),K=1,3),(WDL(K),K=1,3)
  114 FORMAT (38H  PER CENT REDUCTION OF LIVE LOAD          ,3F8.2/
     1          30H  NET WORKING LIVE LOAD, K/FT.,8X      ,3F8.2/
     2          26H  WORKING DEAD LOAD, K/FT.  ,12X       ,3F8.2)
      DO 19 K=1,3
   19 P(I,K)=P(I,K)+WDL(K)
      PRINT 117,(P(I,K),K=1,3)
  117 FORMAT (27H  TOTAL WORKING LOAD, K/FT.   ,11X      ,3F8.2 //)
      GO TO 46
   17 DO 20 K=1,3
   20 P(I,K)=P(I-1,K)
   46 DO 47 K=1,3
      T(I,K)=WDL(K)*XL(K)/2.
   47 T(I,K+4)=WLL(K)*XL(K)/2.
   21 CONTINUE
C            OUTPUT WORKING LOAD GIRDER MOMENTS AND SHEARS
      PRINT 118
  118 FORMAT (36H1          B.  GIRDER MOMENTS AND SHEARS//
     1 7H  LEVEL,9X,7HMOMENTS,40X,    6HSHEARS/
     2 9X,22HBAY AB  BAY BC  BAY CD, 14X,6HBAY AB,12X,6HBAY BC,
```

Display A (Continued)

```
      3 1X. 6HRAY C)]
        DO 22 I=1,NS
        DO 23 K=1,3
        V[2*K]=P[I,K]*XL[K]/2.
        V[2*K-1]=-V[2*K]
   23 P[I,K]=V[2*K]*XL[K]/8.
   22 PRINT 120,I,[P[I,K],K=1,3],[V[K],K=1,6]
  120 FORMAT [I7,3F8.1,5X,3[F10.1,F8.1]]
C          INPUT WEIGHT OF EXTERIOR WALLS, COLUMNS, AND FIREPROOFING
C          OUTPUT SAME
        PRINT 129
  129 FORMAT [25H1         C.   COLUMN THRUSTS///
     1 8H  LEVELS,7X,9HDEAD LOAD,11X,19HESTIMATED DEAD LOAD/
     2 12X,14HEXTERIOR WALLS,7X,24HOF COLUMN + FIREPROOFING//]
C          CALCULATE DEAD LOAD COLUMN THRUSTS
        JJ=0
        DO 41 I=1,NS
        IF [I-JJ] 43,43,42
   42 READ 127, WALL,COL,J
  127 FORMAT [2F10.2,I10]
        JJ=JJ+J
        PRINT 130,I,JJ,WALL,COL
  130 FORMAT [I4,4H TO ,I3,F9.2,4H PSF,F20.2,6H LB/FT]
   43 T[I,4]=T[I,3]
        XCOL=H[I]*COL/1000.
        IF [I-1] 52,52,53
   52 T[1,3]=T[1,3]+T[1,2]+XCOL
        T[1,2]=T[1,2]+T[1,1]+XCOL
        T[1,1]=T[1,1]+XCOL+PWH*WALL*BS/1000.
        T[1,4]=T[1,4]+XCOL+PWH*WALL*BS/1000.
        GO TO 41
   53 T[I,3]=T[I-1,3]+T[I,3]+T[I,2]+XCOL
        T[I,2]=T[I-1,2]+T[I,1]+XCOL+T[I,2]
        DO 54 K=1,4,3
   54 T[I,K]=T[I-1,K]+T[I,K]+XCOL+H[I-1]*XWALL
   41 XWALL=WALL*BS/1000.
C          CALCULATE TOTAL COLUMN LOAD INCLUDE LIVE LOAD REDUCTION
        DO 60 I=1,NS
        IF [I-1] 61,61,62
   61 T[1,1]=T[1,1]+T[1,5]
        SUMLL[1]=T[1,5]
        T[1,4]=T[1,4]+T[1,7]
        SUMLL[4]=T[1,7]
        DO 63 K=2,3
        T[1,K]=T[1,K]+T[1,K+3]+T[1,K+4]
   63 SUMLL[K]=T[1,K+3]+T[1,K+4]
        GO TO 60
   62 DO 64 K=1,4
        GO TO [65,66,66,67],K
   65 IF [T[I,5]-ALL[1]]68,68,69
   68 SUMRL[1]=SUMRL[1]+T[I,5]
        SUMA[1]=SUMA[1]+A[1]/2.
        GO TO 75
   69 SUMLL[1]=SUMLL[1]+.80*T[I,5]
        GO TO 72
```

Display A (Continued)

```
   67 IF (T(I,7)-ALL(3))73,73,74
   73 SUMRL(4)=SUMRL(4)+T(I,7)
      SUMA(4)=SUMA(4)+X(3)/2.
      GO TO 75
   74 SUMLL(4)=SUMLL(4)+.80*T(I,7)
      GO TO 72
   66 IF (T(I,K+3)-ALL(K-1)) 79,79,80
   80 SUMLL(K)=SUMLL(K)+.80*T(I,K+3)
      GO TO 87
   79 SUMRL(K)=SUMRL(K)+T(I,K+3)
   87 IF (T(I,K+4)-ALL(K)) 81,81,86
   86 SUMLL(K)=SUMLL(K)+.80*T(I,K+4)
      GO TO 85
   81 SUMRL(K)=SUMRL(K)+T(I,K+4)
   85 IF (RC(K)-.60) 84,72,72
   84 SUMA(K)=SUMA(K)+(A(K-1)+A(K))/2.
      GO TO 82
   75 IF(RC(K)-.60) 82,72,72
   82 IF (SUMA(K)-150.) 70,70,71
   70 RC(K)=0.0
      GO TO 72
   71 RC(K)=.0008*SUMA(K)
      ALLOW(K)=(SUMRL(K)+T(I,K))/4.33/SUMRL(K)
      IF (RC(K)-ALLOW(K))  76,76,77
   77 RC(K)=ALLOW(K)
   76 IF(RC(K)-.60) 72,72,78
   78 RC(K)=.60
   72 T(I,K)=T(I,K)+SUMLL(K)+(1.0-RC(K) )*SUMRL(K)
   64 CONTINUE
   60 CONTINUE
C         OUTPUT COLUMN THRUSTS
      PRINT 128
  128 FORMAT (///41H0 GRAVITY LOADS IN COLUMNS OF FRAME BASED/
     1    8X,          27HON TRIBUTARY AREA OF FLOORS/
     2 10X, 23H(WORKING LOADS IN KIPS)//
     3          44H  STORY     COL A    COL B    COL C    COL D//)
      PRINT 140, (I,(T(I,K),K=1,4),I=1,NS)
  140 FORMAT (I7,F10.1,3F9.1)
C         INPUT WIND PRESSURE/OUTPUT WIND SHEARS
      PRINT122
  122 FORMAT (32H1       D.  WIND LOADS AND SHEARS//
     115H   WIND PRESSURE)
      JJ=0
      DO 24 I=1,NS
      IF (I-JJ)25,25,26
   26 READ 126, WIND,J
  126 FORMAT (F10.2,I10)
      JJ=JJ+J
      PRINT 125,I,JJ,WIND
  125 FORMAT (15H          LEVELS,I4,4H TO ,I3,3H - ,F4.1,4H PSF)
      WIND=WIND*BS
   25 IF (I-1) 27,27,28
   27 WL(1)=(PH+H(1)/2.)*WIND/1000.
      GO TO 24
```

Display A (Continued)

```
 28 WL[I]=WL[I-1]+(H[I]+H[I-1])*WIND/2000.
 24 CONTINUE
    PRINT 123
123 FORMAT (29H0 CUMMULATIVE HORIZONTAL SHEAR / 33H        AT STORY LEVEL
  1    SHEAR,KIPS)
    DO 29 I=1,NS
 29 PRINT 124,I,WL[I]
124 FORMAT (I20,F13.2)
    CALL CHAIN (3) ——> CALL CASE1
    END                GO TO 1
                       END
```

C LOADING CASE - GRAVITY LOADS

```
    COMMON        (SAME AS ABOVE)
    EQUIVALENCE   (SAME AS ABOVE)
C         INPUT LOAD FACTORS
    READ 106,F1,F2
106 FORMAT (2F10.2)
    LC=1
    X=F1
C         COMPUTE FACTORED MEMBER END MOMENTS
    PRINT 103,LC,F1
103 FORMAT (19H1        LOADING CASE,I2,16H - GRAVITY LOADS/
  1  24X,14HLOAD FACTOR = ,F4.2///
  27X  ,  42HA.  FACTORED MEMBER END MOMENTS, FT.-KIPS.//
  3 9H AT LEVEL,4X,5HCOL A,6X,9HGIRDER AB,7X,5HCOL B,6X,9HGIRDER BC,7
  4X,5HCOL C,6X,9HGIRDER CD,6X,5HCOL D/
  523X,4HLEFT,4X,5HRIGHT,14X,4HLEFT,4X,5HRIGHT,14X,4HLEFT,4X,5HRIGHT)
  2 LC=LC+1
    DO 39 I=1,NS
    DO 29 K=1,3
    GM[I,K]=0.0
 29 SV[I,K]=0.0
C         COMPUTE FACTORED COLUMN THRUSTS
    DO 36 K=1,4
    ICOL[I,K]=1
 36 T[I,K]=X *T[I,K]
    J=0
    DO 31 K=4,12,4
    J=J+1
    G[K]=F1*P[I, J]
    G[K-1]=-G[K]
 31 GM[I,J]=G[K]
    G[1]=-.5*G[3]
    G[5]=-.5*[G[4]+G[7]]
    G[9]=-.5*[G[8]+G[11]]
    G[13]=-.5*G[12]
    IF (I-1) 32,32,33
 32 DO 34 K=2,14,4
 34 G[K]=2.*G[K-1]
    IF (LC-5) 30,39,30
```

Display A (Continued)

```
C           DESIGN COLUMNS FOR VERTICAL LOADS
   33 CALL COLUMN
      DO 35 K=2,14,4
   35 G(K)=G(K-1)
      IF (LC-5) 40,39,40
C           OUTPUT MEMBER END MOMENTS
   40 PRINT 105,(G(K),K=1,13,4)
  105 FORMAT (9X,4(F9.1,18X))
   30 PRINT 104,I,(G(K),G(K+1),K=3,11,4),(G(K),K=2,14,4)
  104 FORMAT (I9,3(9X,2F9.1)/9X,4(F9.1,18X)//)
   39 CONTINUE
      I=NS+1
      CALL COLUMN
      IF (LC-5) 3,4,3
      PRINT 105, (G(K),K=2,14,4)
C           COMPUTE FACTORED GIRDER SHEARS AND OUTPUT
    3 PRINT 107
  107 FORMAT (1H1,6X,37HB.  FACTORED GIRDER END SHEARS, KIPS.//
     1 7H  LEVEL,9X,6HBAY AB,12X,6HBAY BC,12X,6HBAY CD)
      DO 37 I=1,NS
      DO 38 K=1,3
      V(2*K)=8.*P(I,K)/XL(K)*F1
   38 V(2*K-1)=-V(2*K)
   37 PRINT 108,I,(V(K),K=1,6)
  108 FORMAT (I7,3(F19.1,F8.1))
C           OUTPUT FACTORED COLUMN THRUSTS
      PRINT 112,(I,(T(I,K),K=1,4),I=1,NS)
  112 FORMAT (1H1,5X,28H C.  FACTORED COLUMN THRUSTS//2X,5HSTORY,5X,5HCO
     1L A,5X,5HCOL B,5X,5HCOL C,5X,5HCOL D/(I7,4F10.1))
    4 X=F2/F1
      DO 20 I=1,NS
      DO 20 K=1,4
   20 T(I,K)=T(I,K)*X
      CALL CASE2
      X=F1/F2
      GO TO 2 ————→ IF (LC-4) 1,2,1
      END         1 RETURN
                    END


C           LOADING CASE - COMBINED LOADS

      SUBROUTINE CASE2
      COMMON        (SAME AS ABOVE)
      EQUIVALENCE   (SAME AS ABOVE)
      DIMENSION  PY(92),XMP(92),WST(4),SV(25,3),WLM(25)
      DIMENSION UT(2),COLM(4)
   44 GO TO (65,65,63,64,63,62,64),LC
   65 FORF2=4.0*F2
      DELTA=0.01
      BB=8.0*(SQRTF(F1/F2)-1.0)*F2
      DO 37 I=1,NS
   37 WLM(I)=WL(I)*H(I)*F2
   62 PRINT 101,LC
  101 FORMAT (19H1         LOADING CASE,I2,17H - WIND FROM LEFT)
      KK=0
      GO TO 36
```

Display A (Continued)

```
   63 PRINT 100,LC
  100 FORMAT (19H1    .  LOADING CASE,I2,18H - WIND FROM RIGHT)
      XK=-1
   36 PRINT 102, F2
  102 FORMAT (24X,14HLOAD FACTOR = ,F4.2///7X,42HA.  FACTORED MEMBER END
     1 MOMENTS, FT.-KIPS.///9H AT LEVEL,4X,5HCOL A,6X,9HGIRDER AB,7X,
     25HCOL B,6X,9HGIRDER BC,7X,5HCOL C,6X,9HGIRDER CD,7X,5HCOL D/23X,
     313HLEFT    RIGHT,14X,13HLEFT    RIGHT,14X,13HLEFT    RIGHT)
      DO 8 K=1,4
      WST(K)=0.0
    8 ST(K)=0.0
      SUMWT=0.0
      JX=0
      LL=0
      JJ=0
      LC=LC+1
      DO 1 I=1,NS
      IF(I-JJ)2,2,3
C             INPUT MAXIMUM EFFECTIVE LOAD FACTORS FOR GIRDERS
    3 READ 103, F1MAX,F2MAX,F3MAX,J
  103 FORMAT (3F10.2,I10)
      JJ=JJ+J
    2 IF (I-1) 11,11,16
   16 WST(1)=WST(1)-SV(I-1,1)
      DO 57 L=2,3
   57 WST(L)=WST(L)+SV(I-1,L-1)-SV(I-1,L)
      WST(4)=WST(4)+SV(I-1,3)
   11 IF (I-JX) 70,70,71
   71 LL=LL+1
      JX=JX+NSPEC(LL)
      FY=SPEC(LL)
C             COMPUTE REDUCED COLUMN MOMENT CAPACITIES
   70 DO 17 K=1,4
      NCOL=ICOL(I,K)
      IF (SPEC(LL)-441.0) 72,73,72
   73 IF (NCOL-56) 74,74,75
   74 FY=50.0
      GO TO 72
   75 IF (NCOL-68) 76,76,77
   76 FY=46.0
      GO TO 72
   77 FY=42.0
   72 PPY=(T(I,K)+WST(K))/PY(NCOL)/FY
   25 IF (PPY-0.15) 9,9,10
    9 COLM(K)=Z(NCOL)*FY
      GO TO 17
   10 COLM(K)=1.18*(1.0-MIN1F(PPY,1.0))*Z(NCOL)*FY
   17 CONTINUE
C             COMPUTE WEIGHT FUNCTION AND OPTIMIZE
      WTMN=1000000.
      IF(I-1) 4,4,5
    4 GIRREQ=.5*WLM(1)
      DF=-1.0
      GO TO 40
    5 GIRREQ=.5*(WLM(I)+WLM(I-1))
```

Display A (Continued)

```
      DF=-ALM[I]/GIRREQ/2.
      IF [GIRREQ-HD*[P[I,1]+P[I,2]+P[I,3]]] 40,41,41
   40 F[1]=F2
      GO TO 42
   41 DO 14 K=1,3
      IF [H[I]-H[I-1]]12,13,13
   13 IF [P[I,K]-P[I-1,K]] 12,14,14
   12 F[1]=F1
   14 CONTINUE
   42 N=1
   18 M=4*N+KK
      G[M]=F[N]*P[I,N]
      M=4*N-1-KK
      IF [F[N]-FORF2] 15,19,19
   15 G[M]=P[I,N]*[8.0*F2*[SQRTF[F[N]/F2]-1.0]-F[N]]
      GO TO 20
   19 G[M]=F[N]*P[I,N]
   20 GO TO [21,22,23],N
   21 WT1=H[I]*MAX1F[COLM[1],ABSF[DF*G[3]]]+XL[1]*MAX1F[G[KK+4],GM[I,1]]
   38 FMIN=F2
      FMAX=F2MAX
   35 F[2]=[FMIN+FMAX]/2.0-2.0*DELTA
      DO 23 K=1,2
      F[2]=F[2]+DELTA
      N=2
      GO TO 18
   22 B[3]=[GIRREQ-ABSF[G[3]+G[4]+G[7]+G[8]]]/P[I,3]
      IF [B[3]-8.0*F2] 27,28,28
   28 F[3]=B[3]/2.0
      GO TO 29
   27 F[3]=[1.0+B[3] /[F2*8.0]]**2*F2
   29 IF [F[3]-F3MAX] 32,32,34
   32 N=3
      GO TO 18
   23 WT[K]=WT1+MAX1F[G[KK+8],GM[I,2]]*XL[2]+MAX1F[G[KK+12],GM[I,3]]*
     2 XL[3]+H[I]*[MAX1F[ABSF[DF*[G[4]+G[7]]],COLM[2]]+MAX1F[ABSF[DF*[G[
     38]+G[11]]],COLM[3]]+MAX1F[ABSF[DF*G[12]],COLM[4]]]
      IF [WT[1]-WT[2]] 33,34,34
   33 FMAX=F[2]
      GO TO 26
   34 FMIN=F[2]
   26 IF [ABSF[FMIN-FMAX]-0.05] 31,31,35
   31 IF [WTMN-WT[2]] 45,45,46
   46 DO 39 K=1,3
   39 FF[K]=F[K]
      WTMN=WT[2]
      IF [F[1]+DELTA-F1MAX] 24,24,45
   24 F[1]=F[1]+0.1
      GO TO 42
   45 DO 43 K=1,3
   43 F[K]=FF[K]
      SUMWT=SUMWT+WTMN
      CALL MD
      DO 30 K=1,3
   30 GM[I,K]=MAX1F[ABSF[G[4*K]],ABSF[G[4*K-1]],GM[I,K]]
      DO 52 K=1,3
```

Display A (Continued)

```
   52 SV[I,K]=[G[4*K]+G[4*K-1]]/XL[K]
      IF [I-1] 60,1,60
C             OUTPUT FACTORED MEMBER MOMENTS
   60 PRINT 105, [G[K],K=1,13,4]
  105 FORMAT [9X,4[F9.1,18X]]
    1 PRINT 106,I,[G[K],G[K+1],K=3,11,4],[G[K],K=2,14,4]
  106 FORMAT [I9,3[9X,2F9.1]/ 9X,4[F9.1,18X]//]
      I=NS+1
      DO 61 K=1,13,4
   61 G[K]=G[K+1]
      CALL COLUMN
      PRINT 105, [G[K],K=1,13,4]
C             COMPUTE FACTORED GIRDER SHEARS AND OUTPUT
      PRINT 107
  107 FORMAT [44H1        B.  FACTORED GIRDER END SHEARS, KIPS.//
     1 7H   LEVEL,9X,6HBAY AB,12X,6HBAY BC,12X,6HBAY CD]
      DO 49 I=1,NS
      DO 48 K=1,3
      V[2*K]=8.0*F2*P[I,K]/XL[K]
      V[2*K-1]=-V[2*K]+SV[I,K]
      V[2*K]=V[2*K]+SV[I,K]
      IF [I-1] 48,48,56
   56 SV[I,K]=SV[I-1,K]+SV[I,K]
   48 CONTINUE
   49 PRINT 108, I,[V[K],K=1,6]
  108 FORMAT [I7,3[F10.1,F8.1]]
C             COMPUTE FACTORED COLUMN THRUSTS AND OUTPUT
      PRINT 112
  112 FORMAT [34H1        C.  FACTORED COLUMN THRUSTS//
     1 7H   STORY,5X,5HCOL A,5X,5HCOL B,5X,5HCOL C,5X,5HCOL D]
      DO 51 I=1,NS
      ST[1]=T[I,1]   -SV[I,1]
      DO 53 K=2,3
   53 ST[K]=T[I,K]+SV[I,K-1]-SV[I,K]
      ST[4]=T[I,4]+SV[I,3]
      DO 54 K=1,3
   54 SV[I,K]=0.0
   51 PRINT 104, I,[ST[K], K=1,4]
  104 FORMAT [I7,4F10.1]
      GO TO 44
   64 CALL GIRDER
      RETURN
      END PROGRAM
```

```
C                SUBROUTINE TO DESIGN COLUMNS USING MPC

      SUBROUTINE COLUMN
      COMMON         [SAME AS ABOVE]
      EQUIVALENCE    [SAME AS ABOVE]
      DIMENSION   PY[92],XMP[92],WST[3],SV[25,3],WLM[25]
      IF [KJ] 1,1,2
C             INPUT COLUMN SECTION DATA
    1 READ 100, [ID[J],W[J],Z[J],J=1,83]
```

Display A (Continued)

```
  100 FORMAT (4(I3,F5.1,F7.1))
C              INPUT COLUMN DESIGN YIELD POINTS
      READ 102, (SPEC(J), NSPEC(J), J=1,3)
  102 FORMAT (3(F10.2,I10))
      DO  10 J=1,83
      PY(J)=W(J)    /490.0*144.0
   10 XMP(J)=Z(J)    /12.0
      KJ=1
    2 IF (I-2) 8,8,9
    8 DO 12 K=1,3
   12 WST(K)=0.0
      I=0
      IX=0
    9 I=I-1
C              COMPUTE COLUMN THRUSTS
      DO 13 K=1,3
   13 WST(K)=WST(K)+SV(J,K)
      ST(1)=T(J,1)-WST(1)
      DO 22 K=2,3
   22 ST(K)=T(J,K)+WST(K-1)-WST(K)
      ST(4)=T(J,4)+WST(3)
      IF(J-JX) 19,19,15
   15 I=L+1
      IX=JX+NSPEC(I)
      FY=SPEC(I)
C              CHECK AND/OR SELECT COLUMN SIZES
   19 DO 23 K=1,4
      NCOL=ICOL(J,K)
      DO 24 N=NCOL,83
      IF (SPEC(L)-441.0) 33,17,33
   17 IF (N-56) 31,31,32
   31 FY=50.0
      GO TO 33
   32 IF (N-68) 34,34,35
   34 FY=46.0
      GO TO 33
   35 FY=42.0
   33 IF (ABSF(ST(K))-PY(N)*FY) 7,7,24
    7 PPY=ST(K)/PY(N)/FY
      IF (PPY-.15)  25,25,26
   25 XMPC=XMP(N)*FY
      GO TO 27
   26 XMPC=1.18*(1.0-PPY)*XMP(N) *FY
   27 IF (XMPC-MAX1F(ABSF(G(4*K-2)),ABSF(G(4*K-3)))) 24,24,28
   24 CONTINUE
C              ERROR MESSAGE IF INSUFFICIENT COLUMN SECTIONS
      PRINT 101
  101 FORMAT (23H COLUMNS JOB TERMINATED)
      CALL EXIT
   28 ICOL(J,K)=N
   23 CONTINUE
   11 RETURN
      END
```

Display A (Continued)

```
C          SUBROUTINE TO DISTRIBUTE MOMENTS FOR COMBINED LOADS

      SUBROUTINE MD
      COMMON         [SAME AS ABOVE]
      EQUIVALENCE    [SAME AS ABOVE]
      DIMENSION   PY[92],XMP[92],WST[3],SV[25,3],WLM[25]
      DF=1.0+DF
      DO 3 K=1,3
      M=4*K+KK
      G[M]=F[K]*P[I,K]
      M=4*K-KK-1
      IF [F[K]-4.*F2]5,4,4
    4 G[M]=F[K]*P[I,K]
      GO TO 3
    5 G[M]     =P[I,K]*[8.0*F2*[SQRTF[F[K]/F2]-1.0]-F[K]]
    3 CONTINUE
      G[1]=-DF*G[3]
      G[5]=-DF*[G[4]+G[7]]
      G[9]=-DF*[G[8]+G[11]]
      G[13]=-DF*G[12]
      IF [I-1] 1,1,2
C          CHECK AND REDESIGN COLUMNS FOR COMBINED LOADS
    2 CALL COLUMN
    1 G[2]=-G[1]-G[3]
      G[6]=-G[4]-G[5]-G[7]
      G[10]=-G[8]-G[9]-G[11]
      G[14]=-G[12]-G[13]
      IF [KK] 6,7,7
    6 DO 8 K=1,14
    8 G[K]=-G[K]
    7 RETURN
      END


C          SUBROUTINE TO SELECT LOAD SEQUENCE

      SUBROUTINE GIRDER
      COMMON         [SAME AS ABOVE]
      EQUIVALENCE    [SAME AS ABOVE]
      DIMENSION   PY[92],XMP[92],WST[3],SV[25,3],WLM[25]
      DIMENSION TGM[25,3], ITCOL[25,4]
      NN=LC/3
      GO TO [1,2],NN
    1 TEMPWT=SUMWT
      DO 4 I=1,NS
      DO 5 K=1,3
    5 TGM[I,K]=GM[I,K]
      DO 4 K=1,4
    4 ITCOL[I,K]=ICOL[I,K]
      GO TO 21
    2 IF [TEMPWT-SUMWT] 6,6,7
    6 DO 8 I=1,NS
      DO 9 K=1,3
    9 GM[I,K]=TGM[I,K]
```

**Display A (Continued)**

```
      DO 8 K=1,4
    8 ICOL[I,K]=ITCOL[I,K]
C            OUTPUT LOAD SEQUENCE USED FOR FINAL DESIGN
    7 PRINT 101,NN
  101 FORMAT [1H0////13HLOAD SEQUENCE,I2]
      CALL CHAIN [2]
   21 RETURN
      END
```

*Insert statement : 2 PRINT 100, and all following statements from next subroutine. & Combine DIMENSION statements*

```
C            SUBROUTINE TO OTUPUT COLUMN SECTIONS AND
C            SELECT AND OUTPUT GIRDER SECTIONS

      COMMON        [SAME AS ABOVE]
      EQUIVALENCE   [SAME AS ABOVE]
      DIMENSION  PY[92],XMP[92],WST[3],SV[25,3],WLM[25]
      DIMENSION STEEL[3],IDEPTH[225],WEIGHT[225],PM[225]
    2 PRINT 100
  100 FORMAT [19H1        MEMBER SIZES///7X,11HA.  COLUMNS//
     1 2X,5HSTORY,8X,5HCOL A,8X,5HCOL B,8X,5HCOL C,8X,5HCOL D]
      DO 3 I=1,83
    3 W[I]=PY[I]*490.0/144.0
      DO 4 I=1,NS
      DO 4 K=1,4
      J=ICOL[I,K]
      ICOL[I,K]=ID[J]
    4 T[I,K]=W[J]
      MM=4
   15 JJ=0
      NN=1
      SUM=0.0
      DO 9 K=1,3
    9 STEEL[K]=0.0
      DO 5 I=1,3
      PRINT 101,SPEC[L]
  101 FORMAT [16H0SPECIFICATION A,F4.0//]
      JJ=JJ+NSPEC[I]
C            OUTPUT COLUMN OR GIRDER SECTIONS
      DO 6 I=NN,JJ
      PRINT 102,  I,[ICOL[I,K],T[I,K],K=1,MM]
  102 FORMAT [5X,I2,4[4X,I2,2HWF,F5.1]]
C            COMPUTE WEIGHT OF SECTIONS IN FRAME
      DO 6 K=1,MM
      IF [MM-4] 23,22,23
   22 STEEL[L]=STEEL[L]+H[I]*T[I,K]
      GO TO 6
   23 STEEL[L]=STEEL[L]+[XL[K]-1.0]*T[I,K]
    6 CONTINUE
      SUM=SUM+STEEL[L]
      IF [NS-JJ] 5,8,5
    5 NN=JJ+1
    8 PRINT 103, [SPEC[K],STEEL[K],K=1,3],SUM
  103 FORMAT [1H0//6HWEIGHT//3[3H  A,F4.0,F10.1/],7X,F10.1,4H LB.]
```

Display A (Continued)

```
      IF [I-M-4] 21,20,21
C            INPUT GIRDER SECTION PROPERTIES
   20 READ 104, [IDEPTH[I],WEIGHT[I],PM[I],I=1,221]
  104 FORMAT [4[I3,F5.1,F7.1]]
C            INPUT GIRDER DESIGN YIELD POINTS
      READ 106, [SPEC[K],NSPEC[K],K=1,3]
  106 FORMAT [3[F10.2,I10]]
      JJ=0
      NN=0
      L=1
      PRINT 105
  105 FORMAT [1H1,5X,11HB.  GIRDERS//7H  LEVEL,7X,6HBAY AB,7X,6HBAY BC,
     1 7X,6HBAY CD]
C            SELECTION OF GIRDER SIZES
      DO 10 I=1,NS
      IF [I-JJ] 11,11,12
   12 JJ=JJ+NSPEC[L]
      FY=SPEC[L]
      L=L+1
   11 IF [I-NN] 13,13,14
C            INPUT MAXIMUM DEPTH OF GIRDERS
   14 READ 107, [ID[K],K=1,3],J
  107 FORMAT [4I10]
      NN=NN+J
   13 DO 10 K=1,3
      GM[I,K]=GM[I,K]*12.0/FY
      DO 18 M=1,221
      IF [GM[I,K]-PM[M]] 17,17,18
   17 IF [ID[K]-IDEPTH[M]] 18,19,19
   18 CONTINUE
      M=1
   19 ICOL[I,K]=IDEPTH[M]
   10 T[I,K]=WEIGHT[M]
      MM=3
      GO TO 15
   21 CALL CHAIN [1]  → Eliminate when combining subroutines
      END
```

Display A (Continued)

| Card No. | Data | Format |
|---|---|---|
| 1 | Frame number | I10 |
| 2 | Number of stories, bent spacing, bay spacings | I10, 4F10.2 |
| 3 | Parapet Wall Height | F10.2 |
| 4F* | Story Height, No. of stories for which this height is applicable | F10.2, I10 |
| 5F | Area live loads & area dead loads for each bay, No. of stories applicable | 6F10.2, I10 |
| 6F | Wt. of exterior walls, wt. of columns plus fireproofing, No. of stories applicable | 2F10.2, I10 |
| 7F | Wind pressure, No. of stories applicable | F10.2, I10 |
| 8 | Vertical loads only load factor, combined loads load factor | 2F10.2 |
| 9F | Column section properties, depth, weight plastic section modulus | 4(I3, F5.1, F7.1) |
| 10 | Yield point of columns, number of stories applicable | 3(F10.2, I10) |
| 11F | Maximum effective load factors for girder design for each bay, No. of stories applicable | 3F10.2, I10 |
| 12,13, 14 | Three sets of data exactly as 11F | |
| 15F | Girder section properties, depth, weight plastic section modulus | 4(I3, F5.1, F7.1) |
| 16 | Yield point of girders, number of stories applicable | 3(F10.2, I10) |
| 17F | Maximum girder depth, No. of stories applicable | 4I10 |
| 18 | Frame Number | I10 |

* F refers to " and following data cards as required "

Display B Data Specification

```
     -11
     24          18          24          1
     18          18          18          9
24.0             10
  0  0.       0.    6  4.4     2.8  7  5.5     4.0  3  5.7     1.9    Girders
2.2          2.6         3.4             1
1.75         2.6         2.6             9
2.2          2.6         3.4             1
1.75         2.6         2.6             9
2.2          2.6         3.4             1
1.75         2.6         2.6             9
2.2          2.6         3.4             1
1.75         2.6         2.6             9
24.0                  9   441.0          1
 0  0.       0.    4 12.0     2.5  4 13.0     6.2  4 13.0     6.1    Columns
1.7          1.0
29.0               15
45.0         350.0          15
150.0        120.0       80.0        80.0        150.0       120.0       1
80.0         80.0        80.0        80.0        80.0        80.0        8
20.0         60.0        30.0        60.0        30.0        60.0        1
15.0               1
11.0               5
4.0
       10  20.0        30.0        24.0        24.0
     11
```

Display C  Sample Input Data

PRELIMINARY PLASTIC DESIGN OF MULTI-STORY-THREE BAY FRAME NO. 11

FRAME GEOMETRY

NUMBER OF STORIES, 11

BENT SPACING, 20.00 FT.

```
SPAN LENGTHS,  BAY AB 30.00 FT.
               BAY BC 24.00 FT.
               BAY CD 24.00 FT.

STORY HEIGHTS,  PARAPET WALL               4.00 FT.
                STORIES      1 TO   9  11.00 FT.
                STORIES     10 TO  11  15.00 FT.
```

WORKING LOADS

A. GIRDER LOADS
(PER CENT LIVE REDUCTION BY ASA A58.1)

| ROOF GIRDER | BAY AB | BAY BC | BAY CD |
|---|---|---|---|
| WORKING LIVE LOAD, LB./SQ.FT. | 30.00 | 30.00 | 30.00 |
| WORKING DEAD LOAD, LB./SQ.FT. | 60.00 | 60.00 | 60.00 |
| WORKING LIVE LOAD, K/FT. | 0.60 | 0.60 | 0.60 |
| PER CENT REDUCTION OF LIVE LOAD | 0. | 0. | 0. |
| NET WORKING LIVE LOAD, K/FT. | 0.60 | 0.60 | 0.60 |
| WORKING DEAD LOAD, K/FT. | 1.20 | 1.20 | 1.20 |
| TOTAL WORKING LOAD, K/FT. | 1.80 | 1.80 | 1.80 |

| LEVELS 2 TO 9 | | | |
|---|---|---|---|
| WORKING LIVE LOAD, LB./SQ.FT. | 80.00 | 80.00 | 80.00 |
| WORKING DEAD LOAD, LB./SQ.FT. | 80.00 | 80.00 | 80.00 |
| WORKING LIVE LOAD, K/FT. | 1.60 | 1.60 | 1.60 |
| PER CENT REDUCTION OF LIVE LOAD | 46.19 | 38.40 | 38.40 |
| NET WORKING LIVE LOAD, K/FT. | 0.86 | 0.99 | 0.99 |
| WORKING DEAD LOAD, K/FT. | 1.60 | 1.60 | 1.60 |
| TOTAL WORKING LOAD, K/FT. | 2.46 | 2.59 | 2.59 |

| LEVELS 10 TO 11 | | | |
|---|---|---|---|
| WORKING LIVE LOAD, LB./SQ.FT. | 150.00 | 80.00 | 150.00 |
| WORKING DEAD LOAD, LB./SQ.FT. | 120.00 | 80.00 | 120.00 |
| WORKING LIVE LOAD, K/FT. | 3.00 | 1.60 | 3.00 |
| PER CENT REDUCTION OF LIVE LOAD | 0. | 38.40 | 0. |
| NET WORKING LIVE LOAD, K/FT. | 3.00 | 0.99 | 3.00 |
| WORKING DEAD LOAD, K/FT. | 2.40 | 1.60 | 2.40 |
| TOTAL WORKING LOAD, K/FT. | 5.40 | 2.59 | 5.40 |

Display D Sample Output Data

B. GIRDER MOMENTS AND SHEARS

| LEVEL | MOMENTS BAY AB | MOMENTS BAY BC | MOMENTS BAY CD | SHEARS BAY AB | | SHEARS BAY BC | | SHEARS BAY CD | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 201.2 | 64.8 | 64.8 | -27.0 | 27.0 | -21.6 | 21.6 | -21.6 | 21.6 |
| 2 | 138.4 | 93.1 | 93.1 | -36.9 | 36.9 | -31.0 | 31.0 | -31.0 | 31.0 |
| 3 | 138.4 | 93.1 | 93.1 | -36.9 | 36.9 | -31.0 | 31.0 | -31.0 | 31.0 |
| 4 | 138.4 | 93.1 | 93.1 | -36.9 | 36.9 | -31.0 | 31.0 | -31.0 | 31.0 |
| 5 | 138.4 | 93.1 | 93.1 | -36.9 | 36.9 | -31.0 | 31.0 | -31.0 | 31.0 |
| 6 | 138.4 | 93.1 | 93.1 | -36.9 | 36.9 | -31.0 | 31.0 | -31.0 | 31.0 |
| 7 | 138.4 | 93.1 | 93.1 | -36.9 | 36.9 | -31.0 | 31.0 | -31.0 | 31.0 |
| 8 | 138.4 | 93.1 | 93.1 | -36.9 | 36.9 | -31.0 | 31.0 | -31.0 | 31.0 |
| 9 | 138.4 | 93.1 | 93.1 | -36.9 | 36.9 | -31.0 | 31.0 | -31.0 | 31.0 |
| 10 | 303.7 | 93.1 | 194.4 | -81.0 | 81.0 | -31.0 | 31.0 | -64.8 | 64.8 |

C. COLUMN THRUSTS

| LEVELS | DEAD LOAD EXTERIOR WALLS | ESTIMATED DEAD LOAD OF COLUMN + FIREPROOFING |
|---|---|---|
| 1 TO 15 | 45.00 PSF | 351.00 LB/FT |

GRAVITY LOADS IN COLUMNS OF FRAME BASED
ON TRIBUTARY AREA OF FLOORS
(WORKING LOADS IN KIPS)

| STORY | COL A | COL B | COL C | COL D |
|---|---|---|---|---|
| 1 | 34.4 | 52.4 | 47.0 | 29.0 |
| 2 | 90.4 | 124.0 | 113.0 | 77.5 |
| 3 | 134.9 | 182.9 | 163.6 | 118.6 |
| 4 | 176.5 | 252.3 | 225.6 | 152.3 |
| 5 | 223.8 | 321.7 | 287.6 | 191.6 |
| 6 | 271.2 | 391.1 | 349.6 | 232.2 |
| 7 | 318.5 | 460.5 | 411.7 | 272.8 |
| 8 | 365.9 | 529.9 | 473.7 | 313.5 |
| 9 | 413.2 | 599.3 | 535.7 | 354.1 |
| 10 | 500.4 | 626.6 | 620.2 | 426.8 |

D. WIND LOADS AND SHEARS

WIND PRESSURE
LEVELS 1 TO 15 - 20.0 PSF

CUMMULATIVE HORIZONTAL SHEAR

| AT STORY LEVEL | SHEAR, KIPS |
|---|---|
| 1 | 5.51 |
| 2 | 11.89 |
| 3 | 18.27 |
| 4 | 24.65 |
| 5 | 31.03 |
| 6 | 37.41 |
| 7 | 43.79 |
| 8 | 50.17 |
| 9 | 56.55 |
| 10 | 64.09 |

Display D (Continued)

LOADING CASE 1 - GRAVITY LOADS
LOAD FACTOR = 1.70

A. FACTORED MEMBER END MOMENTS, FT.-KIPS.

| AT LEVEL | COL A | GIRDER AB LEFT | GIRDER AB RIGHT | COL B | GIRDER BC LEFT | GIRDER BC RIGHT | COL C | GIRDER CD LEFT | GIRDER CD RIGHT | COL D |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | -172.1 | 172.1 | | -110.2 | 110.2 | | -110.2 | 110.2 | |
| | 172.1 | | | -62.0 | | | 0. | | | -110.2 |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| 2 | | -235.3 | 235.3 | | -158.2 | 158.2 | | -158.2 | 158.2 | |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| 3 | | -235.3 | 235.3 | | -158.2 | 158.2 | | -158.2 | 158.2 | |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| 4 | | -235.3 | 235.3 | | -158.2 | 158.2 | | -158.2 | 158.2 | |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| 5 | | -235.3 | 235.3 | | -158.2 | 158.2 | | -158.2 | 158.2 | |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| 6 | | -235.3 | 235.3 | | -158.2 | 158.2 | | -158.2 | 158.2 | |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| 7 | | -235.3 | 235.3 | | -158.2 | 158.2 | | -158.2 | 158.2 | |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| 8 | | -235.3 | 235.3 | | -158.2 | 158.2 | | -158.2 | 158.2 | |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| 9 | | -235.3 | 235.3 | | -158.2 | 158.2 | | -158.2 | 158.2 | |
| | 117.7 | | | -38.5 | | | 0. | | | -79.1 |
| | 258.2 | | | -179.1 | | | 86.1 | | | -165.2 |
| 10 | | -516.4 | 516.4 | | -158.2 | 158.2 | | -330.5 | 330.5 | |
| | 258.2 | | | -179.1 | | | 86.1 | | | -165.2 |

B. FACTORED GIRDER END SHEARS, KIPS.

| LEVEL | BAY AB | | BAY BC | | BAY CD | |
|---|---|---|---|---|---|---|
| 1 | -45.9 | 45.9 | -36.7 | 36.7 | -36.7 | 36.7 |
| 2 | -62.8 | 62.8 | -52.7 | 52.7 | -52.7 | 52.7 |
| 3 | -62.8 | 62.8 | -52.7 | 52.7 | -52.7 | 52.7 |
| 4 | -62.8 | 62.8 | -52.7 | 52.7 | -52.7 | 52.7 |
| 5 | -62.8 | 62.8 | -52.7 | 52.7 | -52.7 | 52.7 |
| 6 | -62.8 | 62.8 | -52.7 | 52.7 | -52.7 | 52.7 |
| 7 | -62.8 | 62.8 | -52.7 | 52.7 | -52.7 | 52.7 |
| 8 | -62.8 | 62.8 | -52.7 | 52.7 | -52.7 | 52.7 |
| 9 | -62.8 | 62.8 | -52.7 | 52.7 | -52.7 | 52.7 |
| 10 | -137.7 | 137.7 | -52.7 | 52.7 | -110.2 | 110.2 |

C. FACTORED COLUMN THRUSTS

| STORY | COL A | COL B | COL C | COL D |
|---|---|---|---|---|
| 1 | 59.6 | 80.2 | 30.0 | 40.4 |
| 2 | 153.7 | 213.0 | 192.0 | 131.8 |
| 3 | 220.3 | 317.0 | 278.0 | 201.6 |
| 4 | 300.3 | 423.0 | 383.5 | 258.9 |
| 5 | 380.5 | 546.0 | 488.0 | 325.7 |
| 6 | 461.0 | 664.0 | 594.4 | 394.7 |
| 7 | 541.5 | 782.0 | 699.8 | 463.8 |
| 8 | 622.0 | 900.9 | 805.3 | 532.9 |
| 9 | 702.5 | 1018.0 | [illegible] | 602.0 |
| 10 | 850.7 | 1184.2 | 1354.4 | 725.6 |

Display D (Continued)

LOADING CASE 2 - WIND FROM LEFT
LOAD FACTOR = 1.30

A. FACTORED GIRDER END MOMENTS, FT.-KIPS.

| AT LEVEL | COL A | GIRDER AB LEFT | GIRDER AB RIGHT | COL B | GIRDER BC LEFT | GIRDER BC RIGHT | COL C | GIRDER CD LEFT | GIRDER CD RIGHT | COL D |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | -102.[illegible] | 1[illegible].7 | | -76.4 | 86.9 | | -92.4 | 81.6 | |
| | 1[illegible]2.[illegible] | | | -65.3 | | | 5.5 | | | -81.6 |
| | 31.9 | | | -31.0 | | | -1.8 | | | -38.5 |
| 2 | | -101.0 | 217.6 | | -100.8 | 124.9 | | -119.0 | 121.7 | |
| | 68.9 | | | -66.9 | | | -3.9 | | | -83.1 |
| | 25.2 | | | -44.0 | | | -14.1 | | | -52.1 |
| 3 | | -64.1 | 22[illegible].5 | | -109.8 | 124.8 | | -88.9 | 132.0 | |
| | 38.8 | | | -67.7 | | | -21.7 | | | -80.0 |
| | 27.2 | | | -65.7 | | | -32.1 | | | -60.0 |
| 4 | | -64.0 | 221.5 | | -67.0 | 139.0 | | -64.4 | 140.9 | |
| | 36.7 | | | -88.7 | | | -43.4 | | | -80.9 |
| | 12.7 | | | -92.1 | | | -36.0 | | | -60.9 |
| 5 | | -28.7 | 235.5 | | -27.3 | 155.0 | | -73.7 | 137.5 | |
| | 16.0 | | | -115.0 | | | -45.3 | | | -76.6 |
| | 13.0 | | | -102.8 | | | -61.6 | | | -70.4 |
| 6 | | -28.7 | 235.3 | | -8.5 | 162.6 | | -26.7 | 155.3 | |
| | 15.7 | | | -124.0 | | | -74.3 | | | -84.9 |
| | -2.4 | | | -119.3 | | | -71.0 | | | -73.9 |
| 7 | | 5.2 | 249.2 | | 9.8 | 170.2 | | -14.1 | 160.3 | |
| | -2.8 | | | -139.6 | | | -84.2 | | | -86.5 |
| | -2.4 | | | -152.2 | | | -84.6 | | | -73.8 |
| 8 | | 5.2 | 249.2 | | 77.4 | 200.4 | | -18.8 | 158.4 | |
| | -2.8 | | | -174.4 | | | -97.1 | | | -84.6 |
| | -2.4 | | | -145.0 | | | -119.6 | | | -90.7 |
| 9 | | 5.2 | 249.2 | | 61.2 | 192.9 | | 61.6 | 193.0 | |
| | -2.8 | | | -164.5 | | | -134.8 | | | -102.3 |
| | -4.5 | | | -211.5 | | | -55.5 | | | -132.8 |
| 10 | | 11.4 | *546.7 | | -8.5 | 162.6 | | -21.2 | 338.2 | |
| | -6.9 | | | -326.8 | | | -85.9 | | | -205.3 |
| | -6.9 | | | -326.8 | | | -85.9 | | | -205.3 |

B. FACTORED GIRDER END SHEARS, KIPS.

| LEVEL | BAY AB | | BAY BC | | BAY CD | |
|---|---|---|---|---|---|---|
| 1 | -33.8 | 36.4 | -27.6 | 28.5 | -28.5 | 27.6 |
| 2 | -44.4 | 51.5 | -30.7 | 41.0 | -40.2 | 40.4 |
| 3 | -42.7 | 53.2 | -39.7 | 41.0 | -38.5 | 42.1 |
| 4 | -42.7 | 53.2 | -37.3 | 43.4 | -37.1 | 43.5 |
| 5 | -41.1 | 54.0 | -35.0 | 45.7 | -37.7 | 43.0 |
| 6 | -41.1 | 54.9 | -33.9 | 46.9 | -35.0 | 45.7 |
| 7 | -39.5 | 56.5 | -32.9 | 47.8 | -34.2 | 46.4 |
| 8 | -39.5 | 56.5 | -28.6 | 51.9 | -34.5 | 46.2 |
| 9 | -39.5 | 56.5 | -29.7 | 50.0 | -29.7 | 50.0 |
| 10 | -86.7 | 123.9 | -33.9 | 46.8 | -71.0 | 97.4 |

C. FACTORED COLUMN THRUSTS

| STORY | COL A | COL B | COL C | COL D |
|---|---|---|---|---|
| 1 | 43.5 | 69.1 | 62.1 | 37.3 |
| 2 | 112.7 | 165.1 | 148.2 | 100.4 |
| 3 | 165.2 | 246.2 | 212.0 | 155.6 |
| 4 | 214.1 | 330.7 | 293.3 | 202.7 |
| 5 | 268.7 | 430.4 | 376.6 | 256.3 |
| 6 | 323.4 | 521.1 | 455.3 | 314.5 |
| 7 | 376.5 | 612.4 | 540.4 | 373.4 |
| 8 | 429.5 | 698.5 | 626.7 | 432.1 |
| 9 | 482.6 | 787.6 | 717.4 | 495.5 |
| 10 | 577.3 | 926.2 | 810.4 | 603.3 |

Display D (Continued)

LOADING CASE 3 - WIND FROM RIGHT
LOAD FACTOR = 1.30

Analogous to LOADING CASE 2

**LOADING CASE 4 - GRAVITY LOADS**

Same as LOADING CASE 1
Not reprinted

LOADING CASE 5 - WIND FROM RIGHT
LOAD FACTOR = 1.30

Analogous to LOADING CASE 2

LOADING CASE 6 - WIND FROM LEFT
LOAD FACTOR = 1.30

Analogous to LOADING CASE 2

LOAD SEQUENCE 2

Indicates second load sequence
(vertical--wind right--wind left)
was used for design of members.

MEMBER SIZES

A. COLUMNS

| STORY | COL A | COL B | COL C | COL D |
|---|---|---|---|---|
| SPECIFICATION A 36. | | | | |
| 1 | 12WF 40.0 | 8WF 28.0 | 5WF 16.0 | 10WF 30.0 |
| 2 | 12WF 40.0 | 8WF 35.0 | 8WF 20.0 | 8WF 35.0 |
| 3 | 14WF 43.0 | 12WF 40.0 | 8WF 32.6 | 10WF 30.0 |
| 4 | 12WF 53.0 | 14WF 48.0 | 12WF 40.0 | 12WF 40.0 |
| 5 | 14WF 61.0 | 14WF 61.0 | 14WF 48.0 | 14WF 48.0 |
| 6 | 14WF 68.0 | 14WF 74.0 | 14WF 61.0 | 12WF 53.0 |
| 7 | 14WF 74.0 | 14WF 84.0 | 14WF 74.0 | 14WF 61.0 |
| 8 | 14WF 84.0 | 12WF 99.0 | 14WF 94.0 | 14WF 68.0 |
| 9 | 14WF111.0 | 14WF127.0 | 12WF106.0 | 14WF 84.0 |
| SPECIFICATION A441. | | | | |
| 10 | 12WF 99.0 | 14WF111.0 | 14WF 84.0 | 14WF 74.0 |

WEIGHT

A 36. 23304.6
A441. 5520.0
A 0. 0.
28824.6 LB.

B. GIRDERS

| LEVEL | BAY AB | BAY BC | BAY CD |
|---|---|---|---|
| SPECIFICATION A 36. | | | |
| 1 | 16WF 35.0 | 14WF 26.0 | 14WF 26.0 |
| 2 | 18WF 45.0 | 16WF 31.0 | 16WF 31.0 |
| 3 | 18WF 45.0 | 16WF 31.0 | 16WF 31.0 |
| 4 | 18WF 45.0 | 16WF 31.0 | 16WF 31.0 |
| 5 | 18WF 45.0 | 16WF 31.0 | 16WF 31.0 |
| 6 | 18WF 45.0 | 14WF 34.0 | 16WF 31.0 |
| 7 | 18WF 45.0 | 14WF 34.0 | 16WF 36.0 |
| 8 | 18WF 45.0 | 14WF 34.0 | 16WF 40.0 |
| 9 | 18WF 45.0 | 16WF 40.0 | 16WF 40.0 |
| 10 | 24WF 76.0 | 14WF 31.0 | 21WF 55.0 |

WEIGHT

A 36. 29282.0
A 0. 0.
A 0. 0.
29282.0 LB.

Display D (Continued)

# APPENDIX IV

## NOMENCLATURE

| | | |
|---|---|---|
| $A$ | = | ratio of girder end moments, area of section |
| $DF$ | = | distribution factor |
| $F_1$ | = | load factor for vertical loads |
| $F_2$ | = | load factor for vertical loads combined with lateral loads |
| $F_{1E}$ | = | effective load factor |
| $F_y$ | = | yield stress |
| $h$ | = | story height |
| $\Sigma H$ | = | a concentrated load equal to the sum of the external horizontal loads above a story |
| $\Sigma H_e$ | = | a concentrated load equal to the sum of the external horizontal loads above a story due to earthquake loading |
| $\Sigma H_w$ | = | a concentrated load equal to the sum of the external horizontal loads above a story due to wind loading |
| $K$ | = | ratio of lateral load moment capacity of a uniformly loaded girder to the working load moment ($wL^2/16$). |
| $L$ | = | span length |
| $M_g$ | = | girder end moment |
| $M_{min}$ | = | minimum girder end moment |
| $M_p$ | = | plastic moment |
| $M_{pc}$ | = | plastic moment modified to include the effect of axial compression |

$M_{pm}$ = minimum plastic moment

$\Sigma M_c$ = summation column end moments at a level

$\Sigma M_g$ = summation girder end moments on a level

n = number of stories from top

P = axial load

$P_y$ = axial force corresponding to yield stress level, $F_yA$

V = vertical reaction

W = distributed load per unit length

Wt = weight function

x = location of second plastic hinge of girder subjected to gravity plus sway moments

$\Delta$ = relative lateral deflection of a story

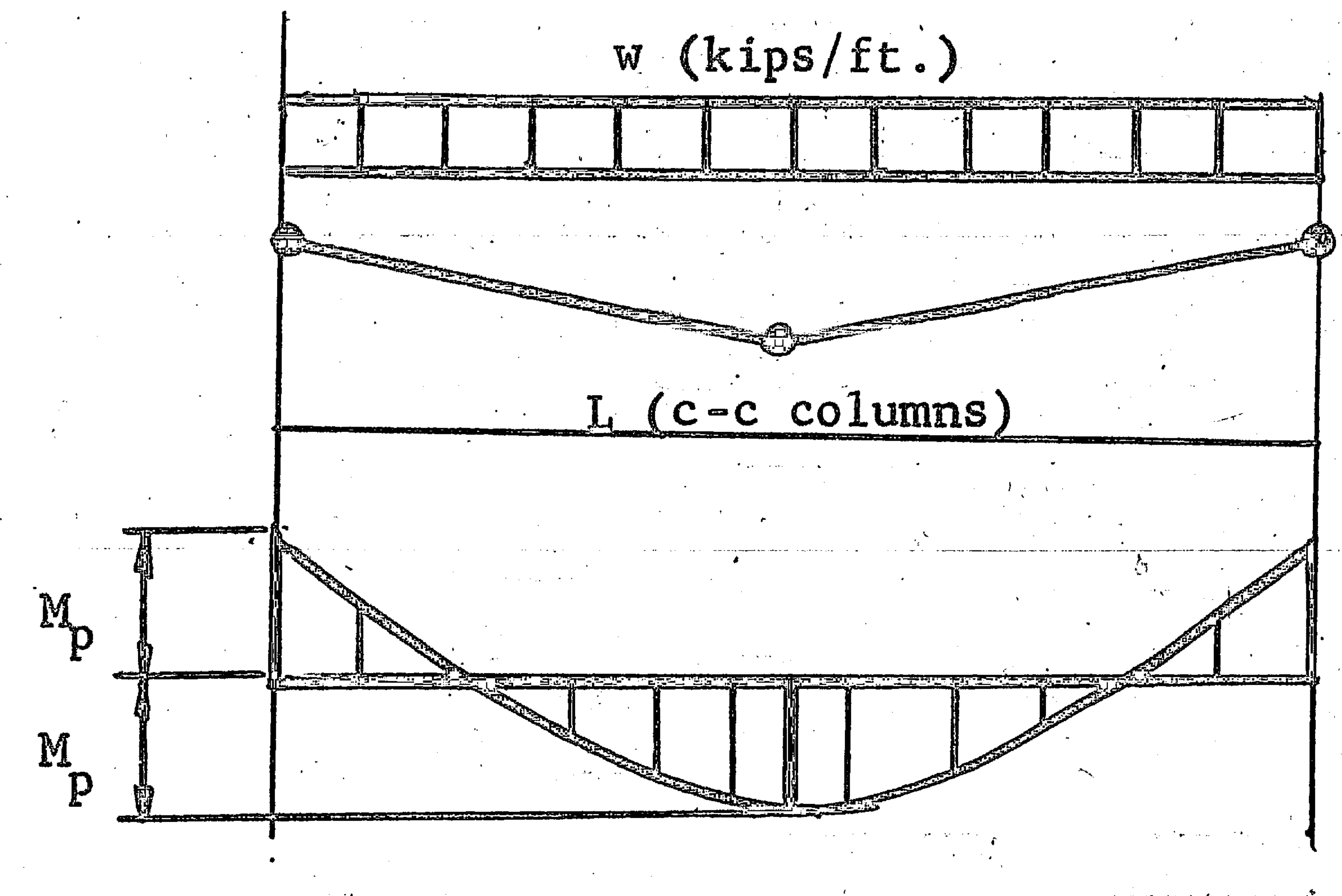



Figure 1 Beam Mechanism

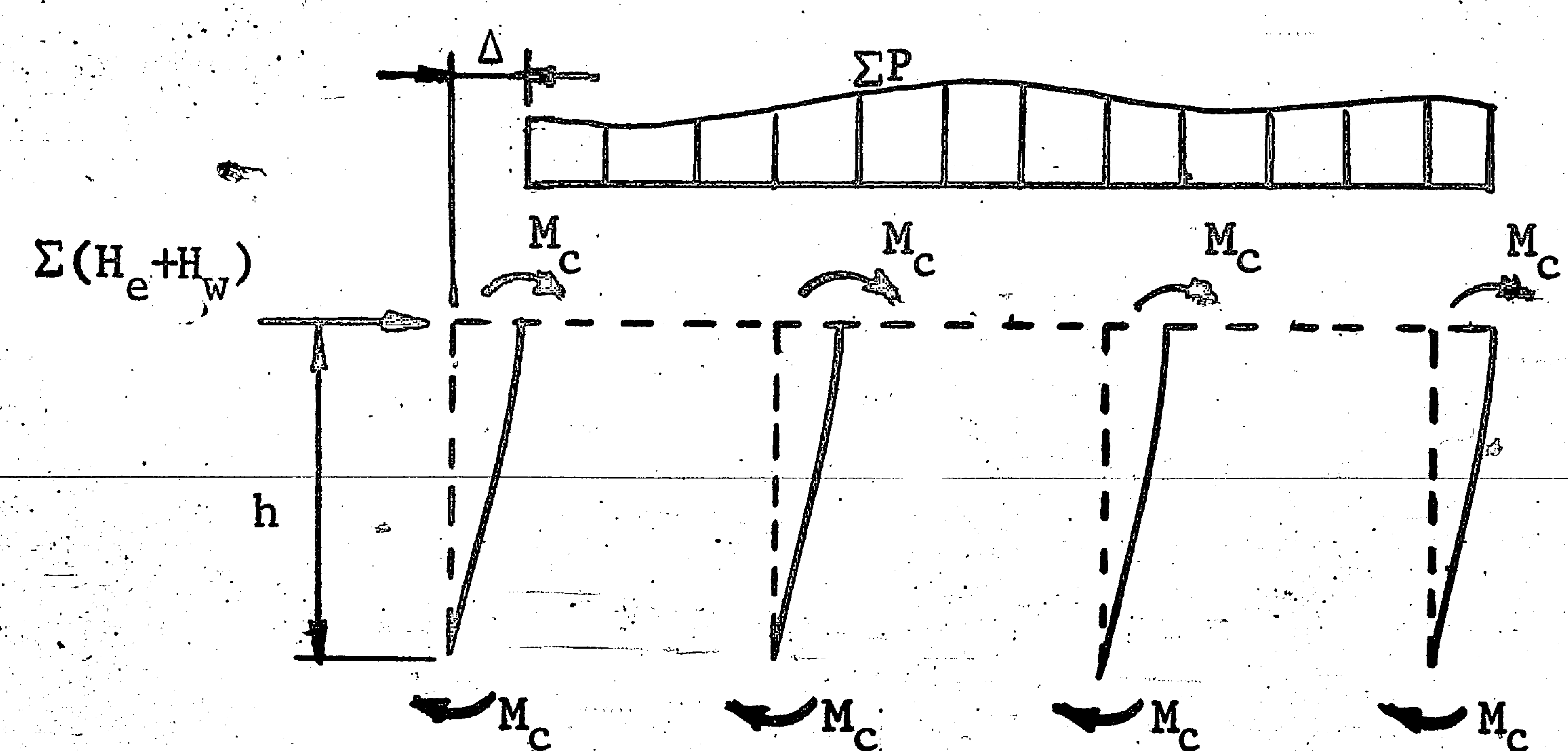



Figure 2 Horizontal Shear Equilibrium in a Story

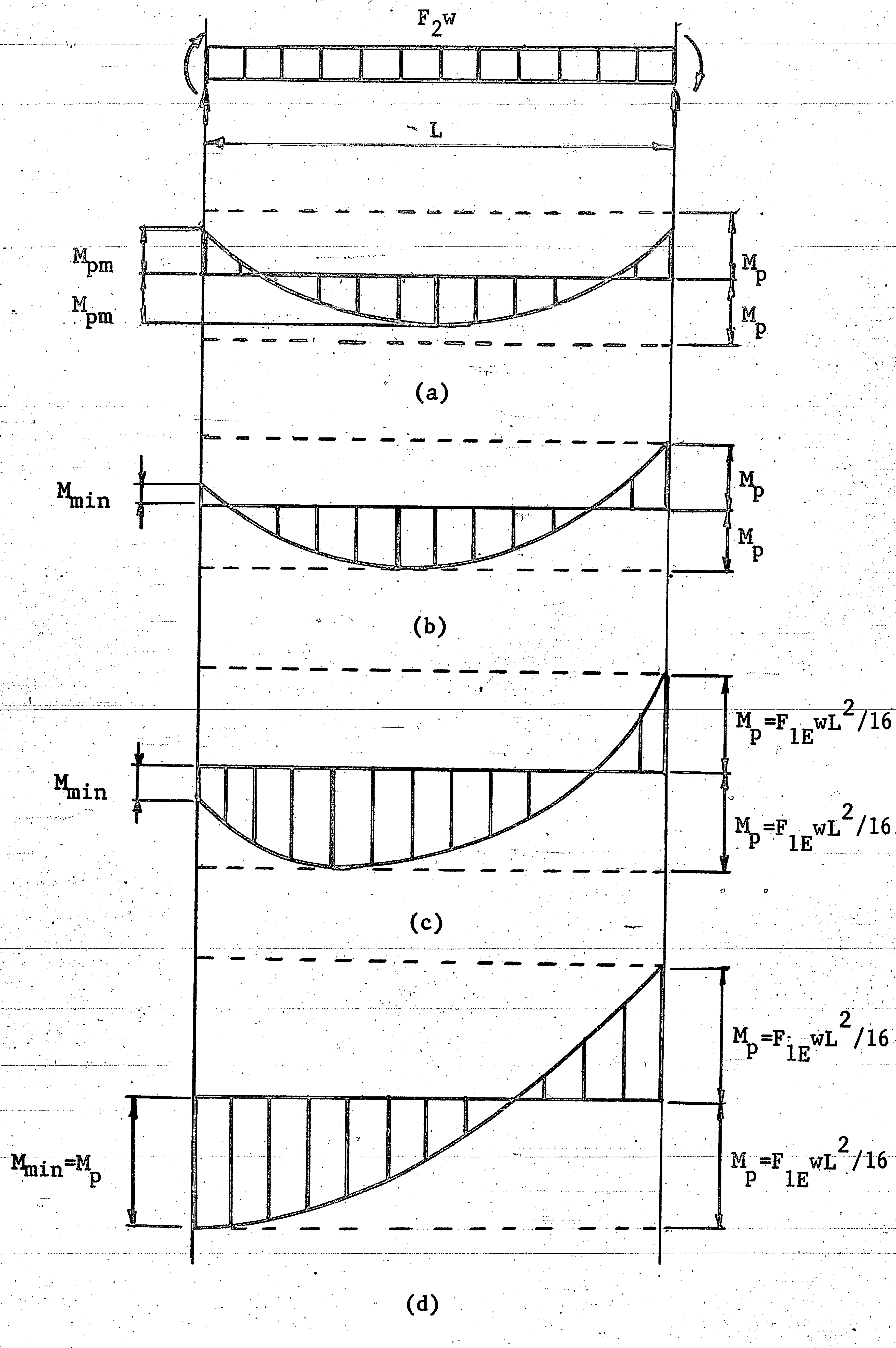



Figure 3 Girders Subjected to Gravity Plus Sway Moments

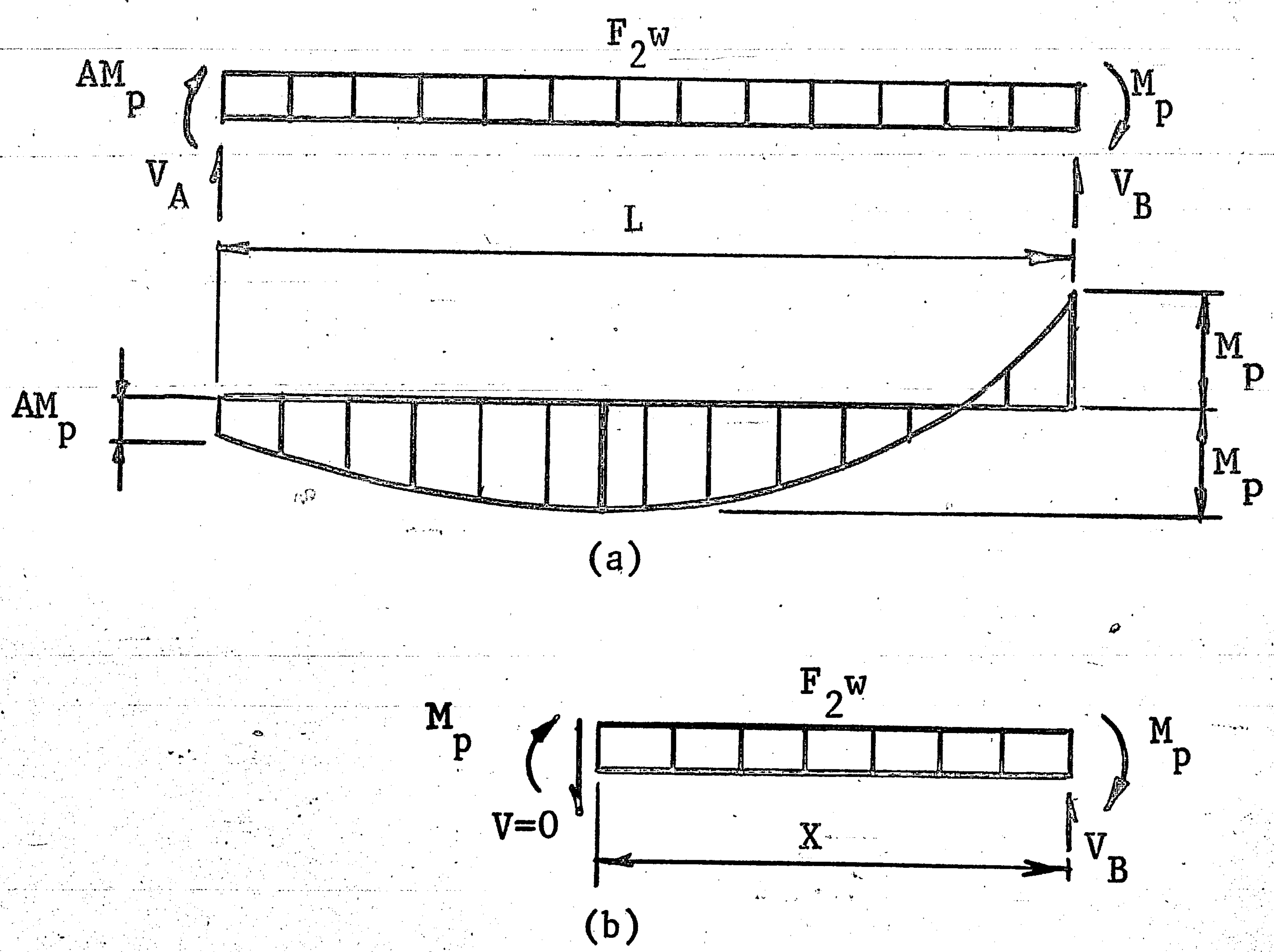



Figure 4 Girders Having Plastic Moments in Clear Span

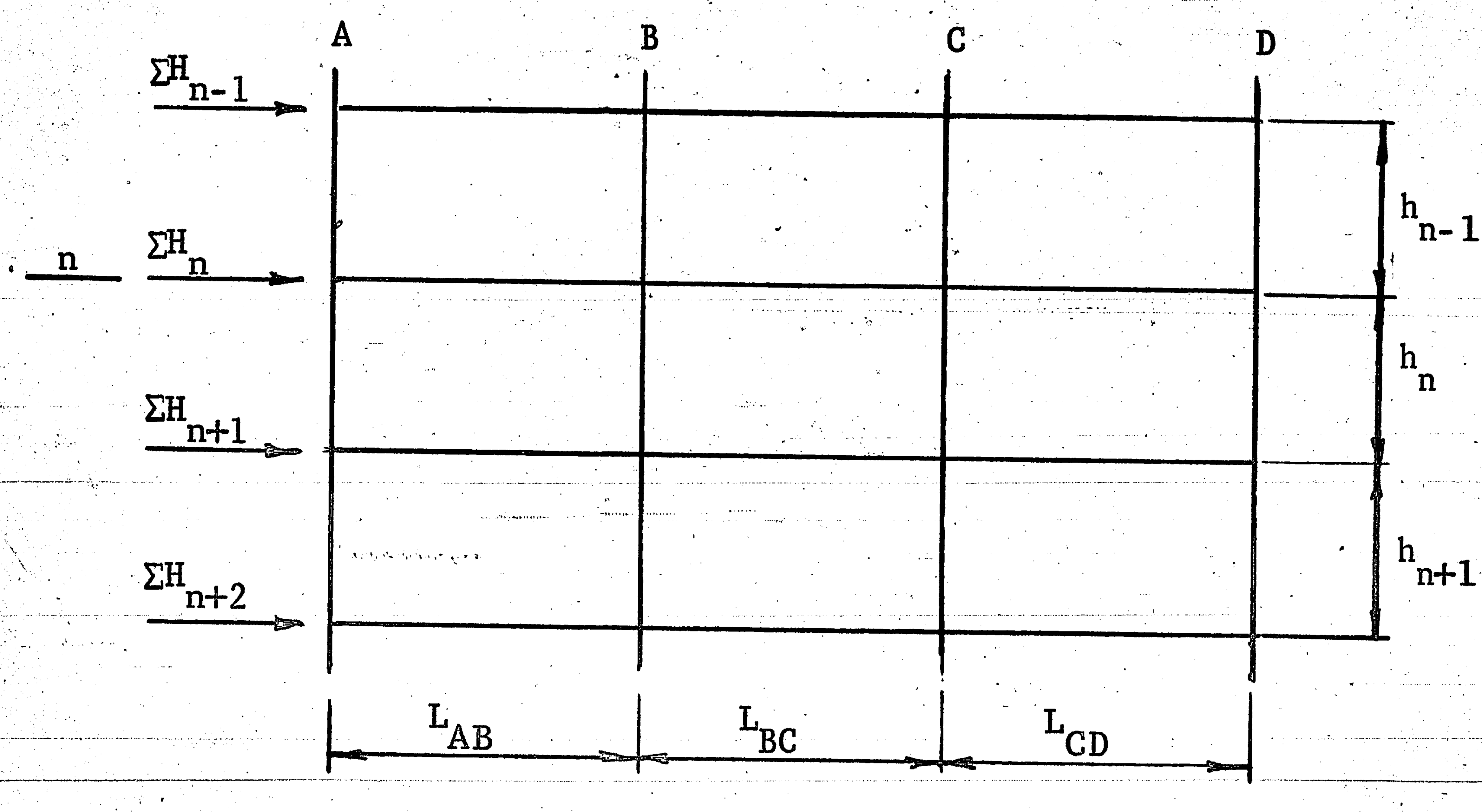



Figure 5 Frame Notation

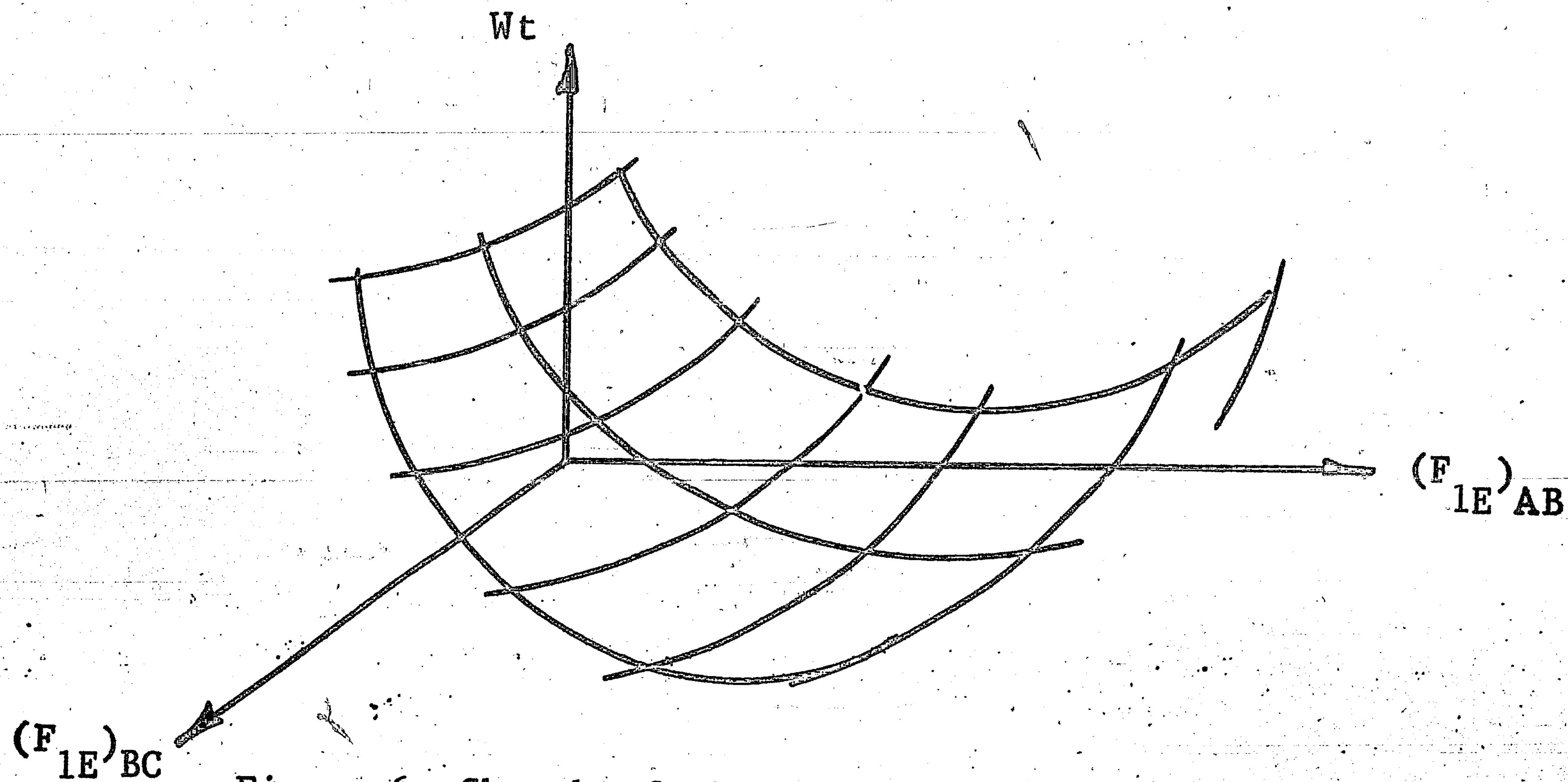



Figure 6 Sketch of the Weight Function

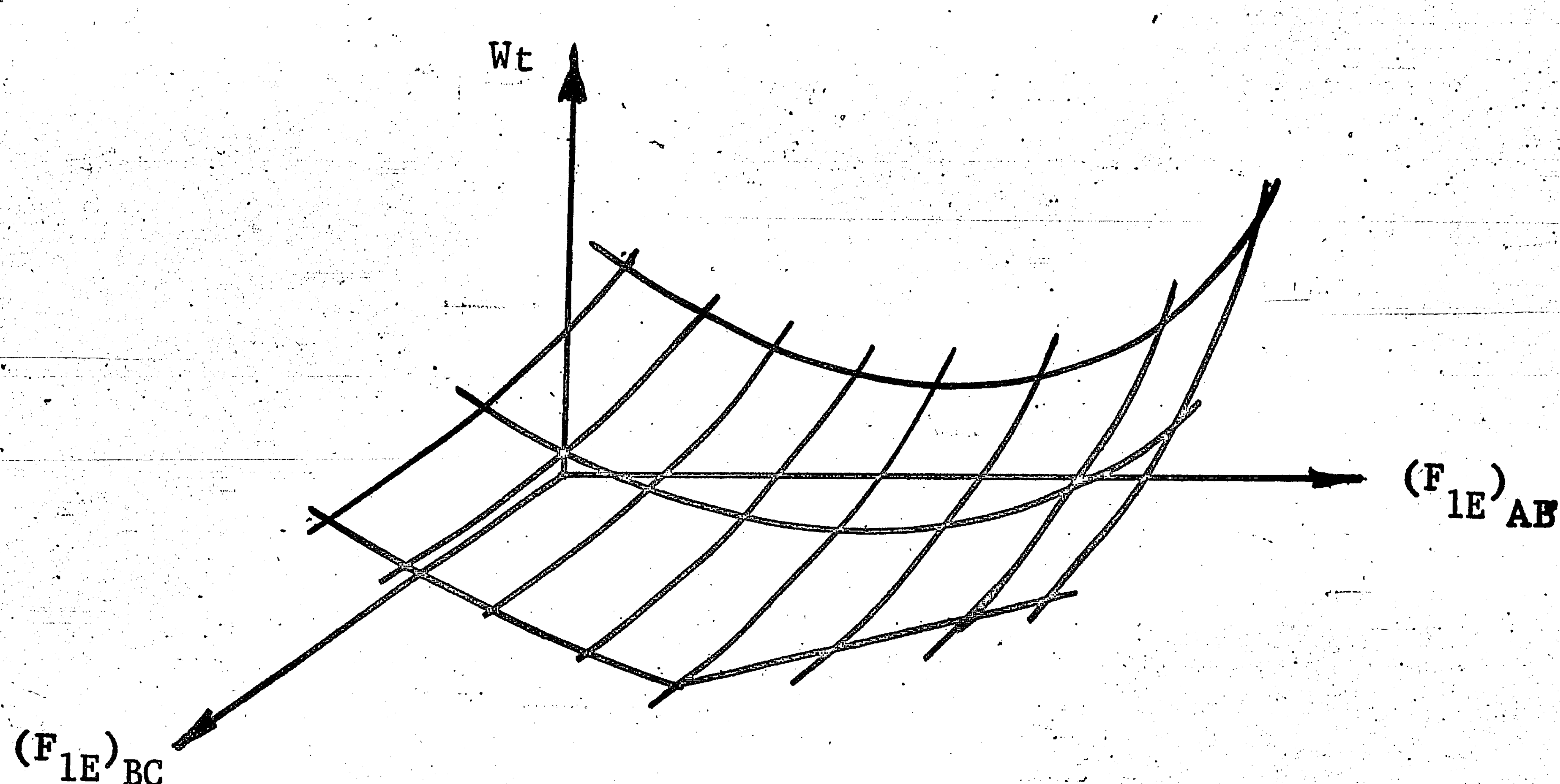



Figure 7 Modified Weight Function

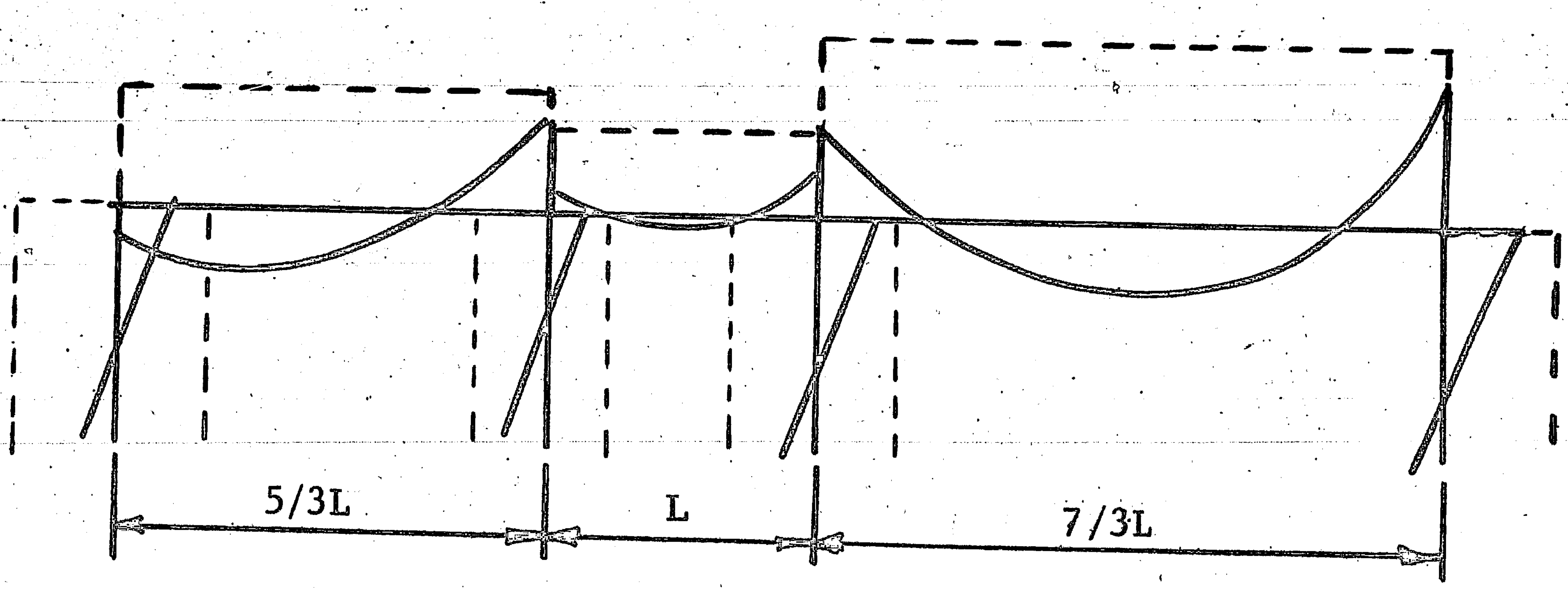



Figure 8 Moment Diagram for Combined Load

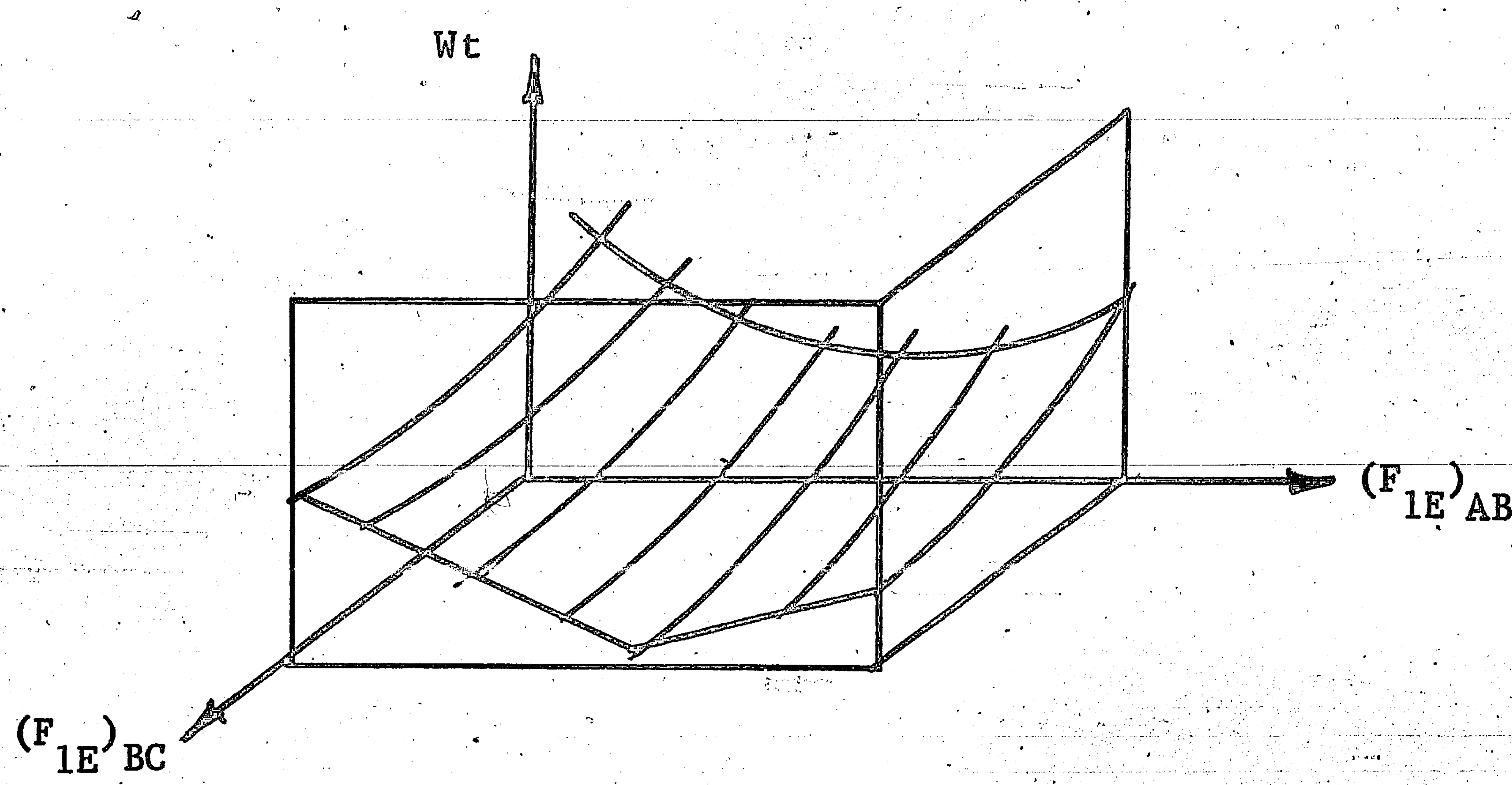



Figure 9 Bounded Weight Function

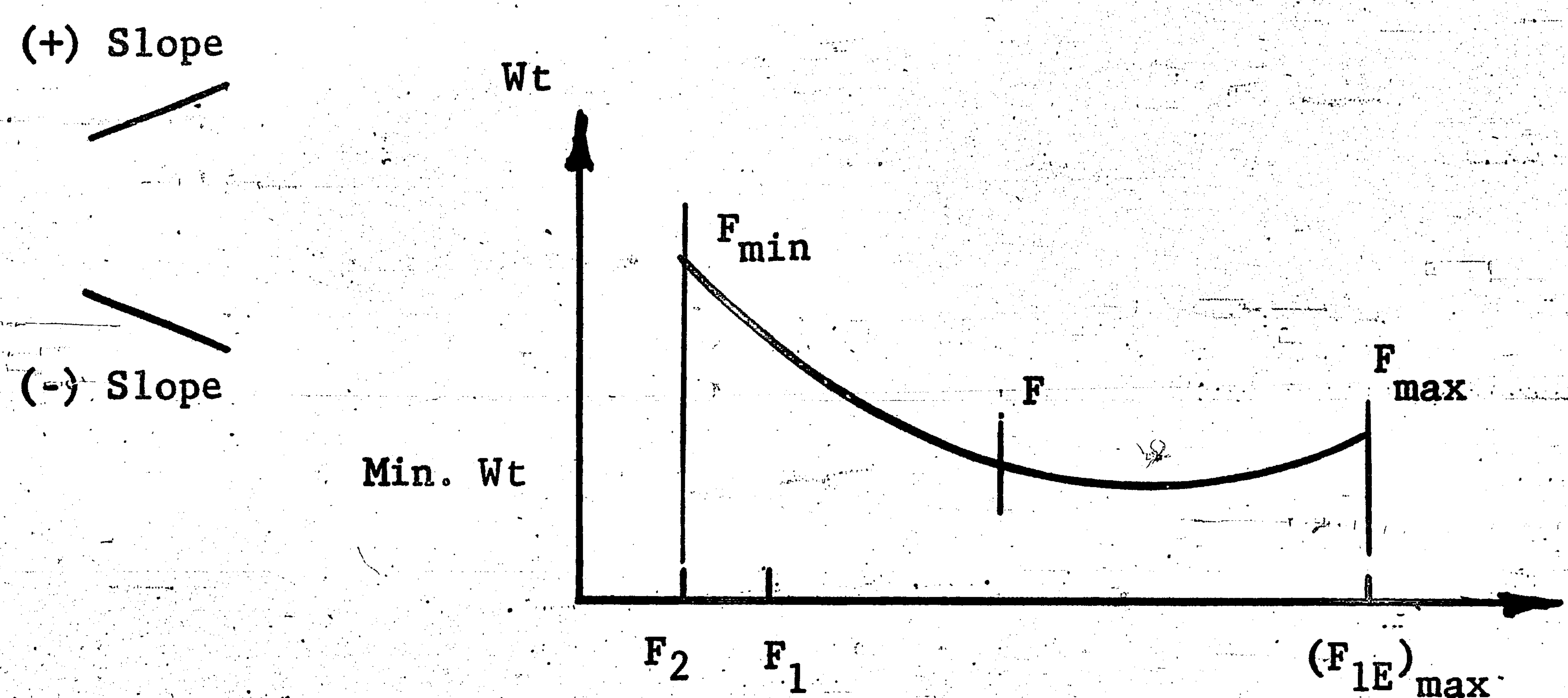



Figure 10 Two-Dimensional Weight Function

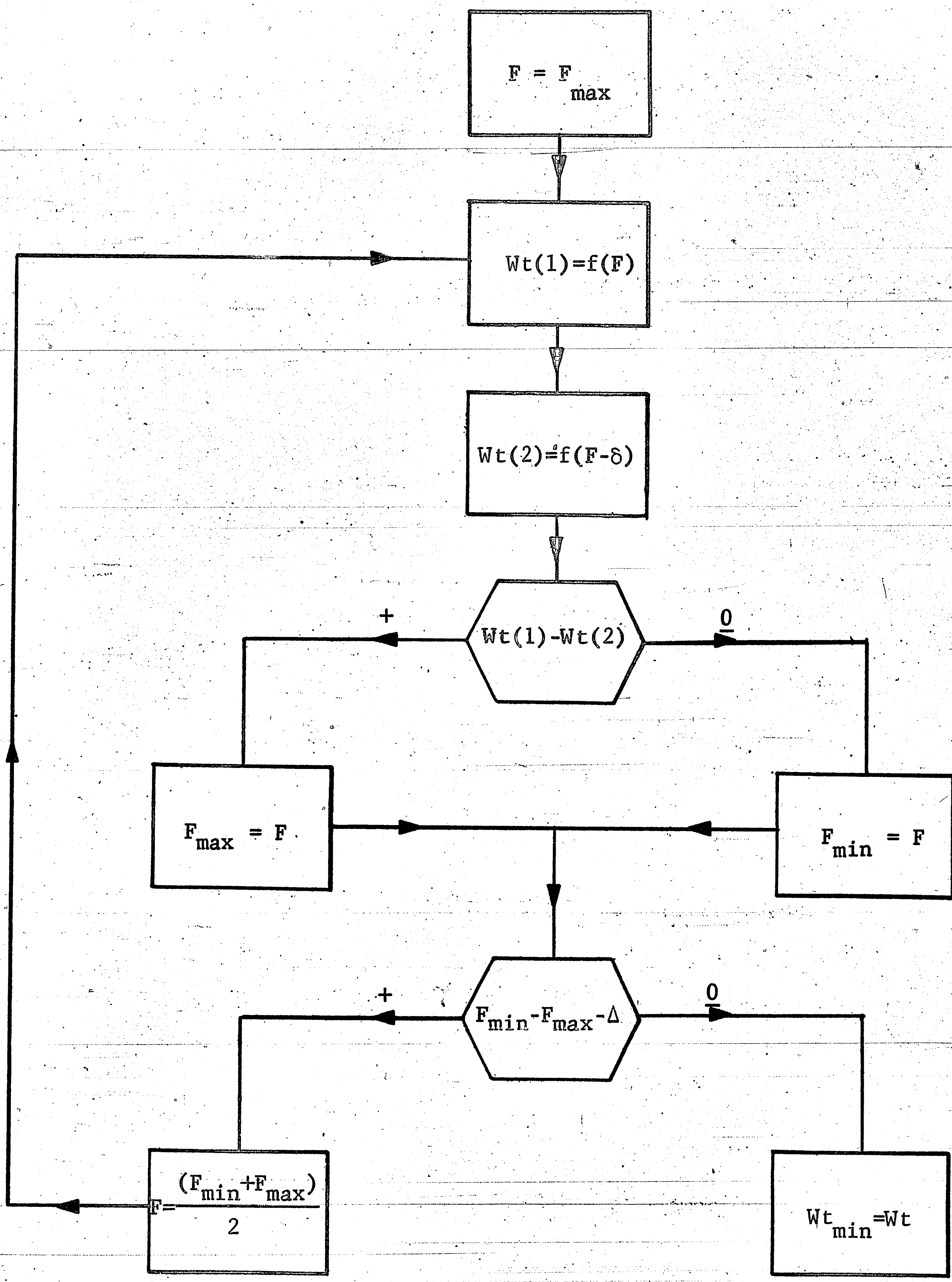



Figure 11 Flow Diagram for Determining Minimum Value of the Weight Function

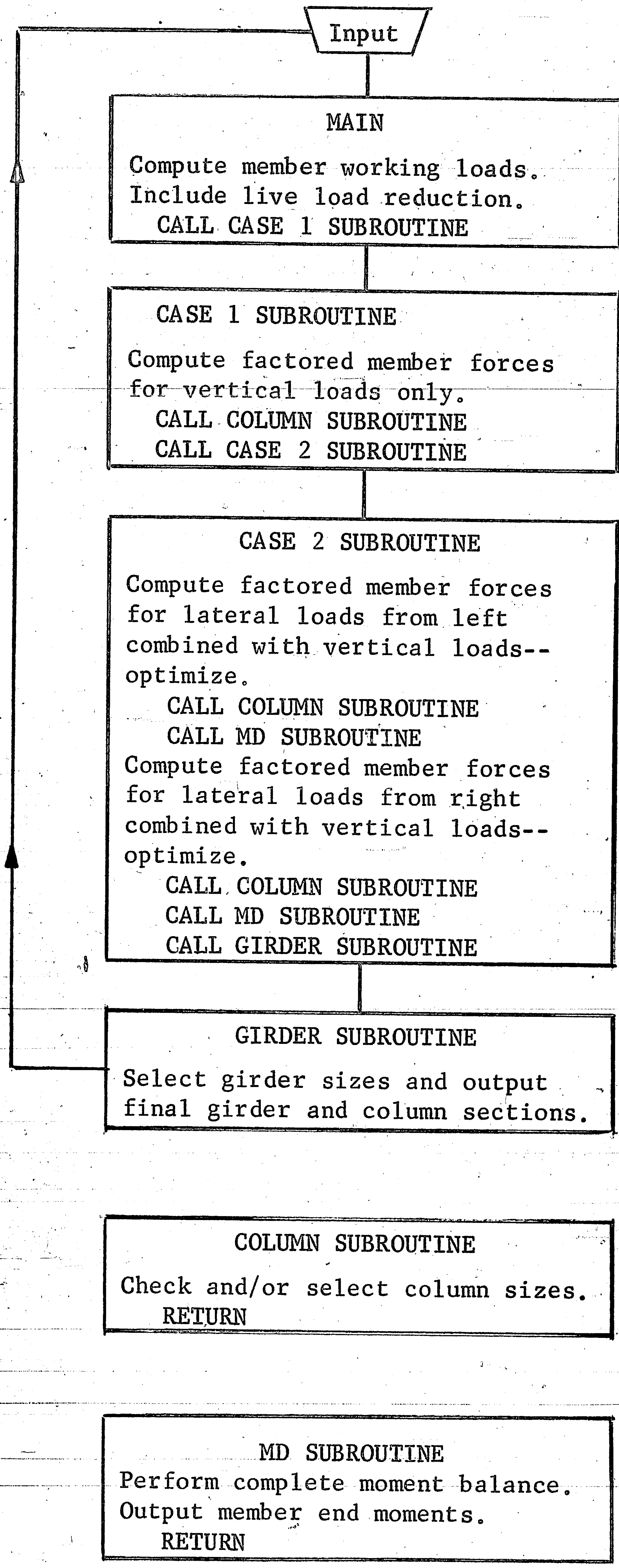
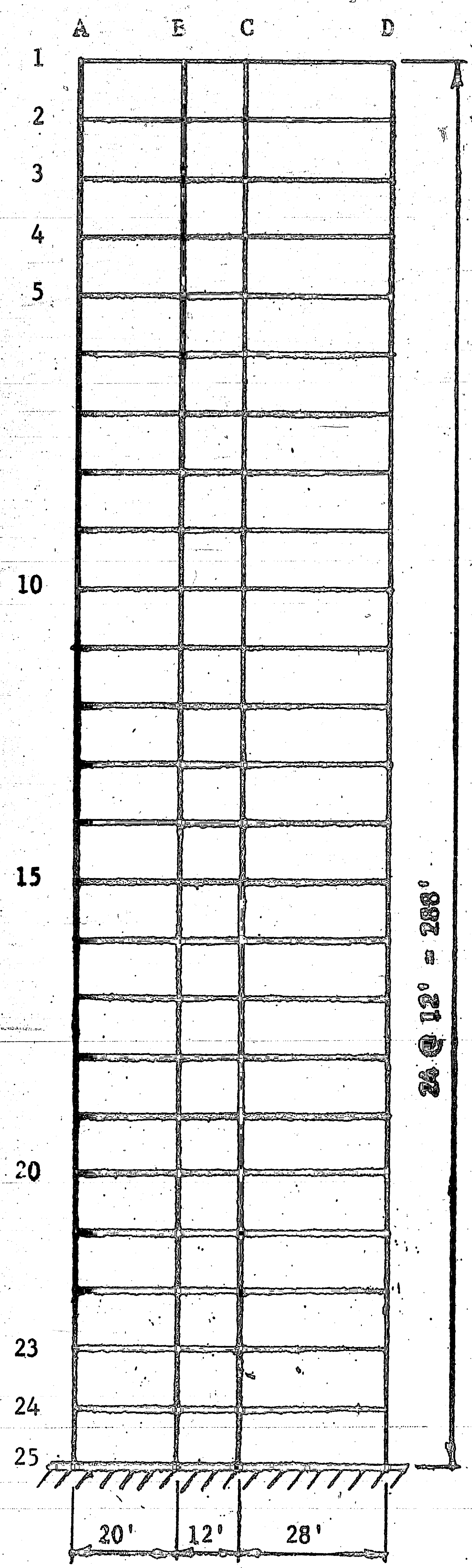



Figure 12 Simplified Flow Diagram




Bent Spacing = 24 ft.

Working Loads:

Roof: $w_L$ = 30 psf

$w_D$ = 95 psf

Floors: $w_L$ = 100 psf

$w_D$ = 120 psf

Exterior Walls: $w_D$ = 85 psf

Wind: 20 psf

Column + Fireproofing: 625 lb/ft

Figure 13 Example Frame

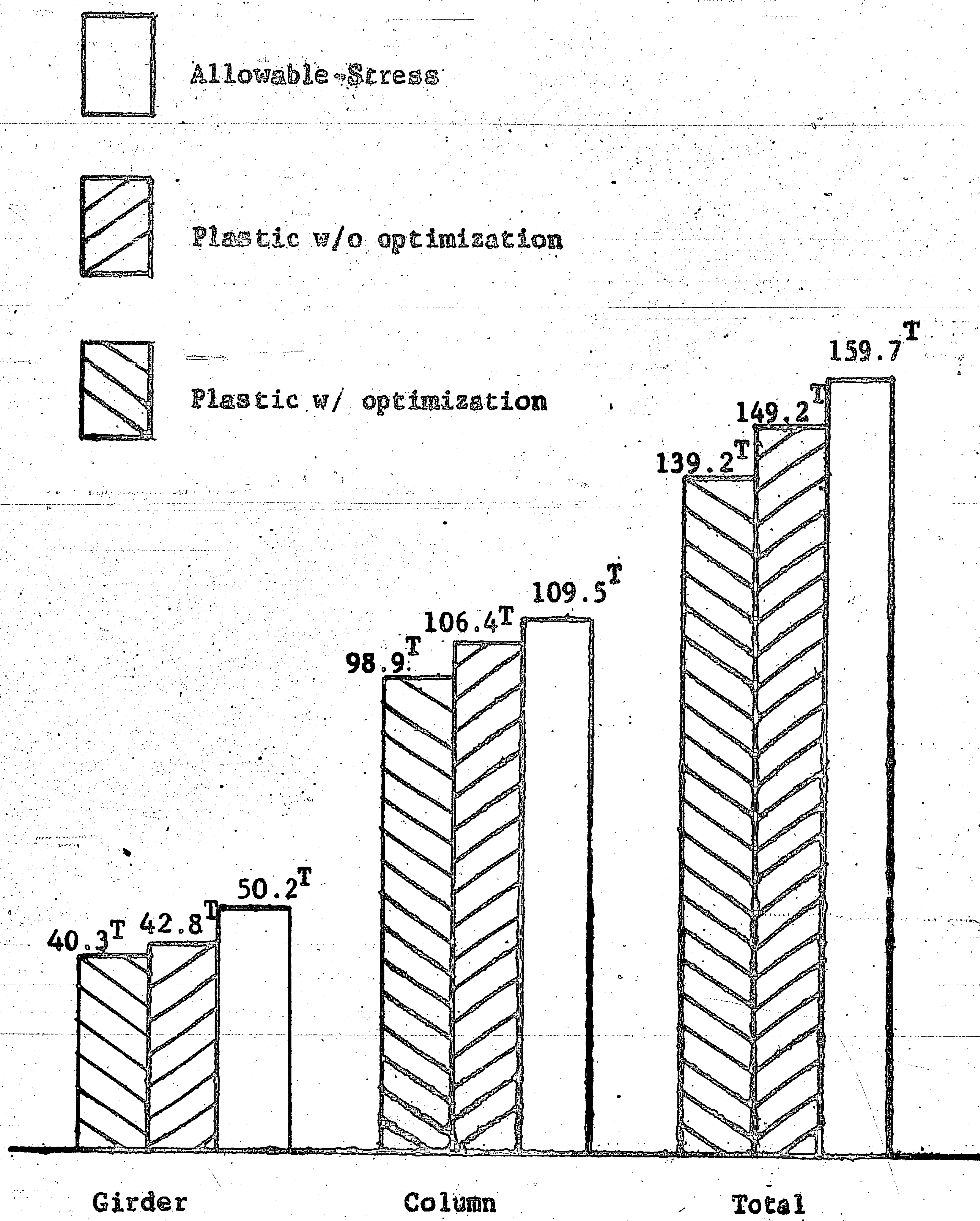



Figure 14 Weight Comparison for Example Frame

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# V I T A

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After graduating from St. Mary's High School, Waterloo, Iowa, in May, 1958, the author enrolled in the Engineering College, Iowa State University, Ames, Iowa. He completed the requirements of the Department of Civil Engineering and received the degree of Bachelor of Science in Civil Engineering in May, 1962. He was then employed by Pittsburgh-Des Moines Steel Company, Des Moines, Iowa. In September, 1964 he joined the staff of the Department of Civil Engineering, Lehigh University as a teaching assistant. During the summer of 1965 he was employed in the Erection Department of Bethlehem Steel Corporation.