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Development, evaluation, and selection of a dodge continuous sampling plan when the recitifying operation is not perfect

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DEVELOPMENT, EVALUATION, AND SELECTION OF A DODGE CONTINUOUS SAMPLING PLAN WHEN THE RECTIFYING OPERATION IS NOT PERFECT

by
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CERTIFICATE OF APPROVAL

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

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Date

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This thesis could not have been written without the continued encouragement of my wife, Sharron.

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ABSTRACT

H. F. Dodge developed a widely used sampling plan for the rectifying inspection of a continuous output. The theoretical predictions of the average fraction inspected (AFI) and the average outgoing quality (AOQ) for this basic plan are valid only when the rectifying operation of the plan is perfectly performed:

> "It is now assumed for purposes of solution that the inspection operation itself never overlooks a defect and that all defective units found during the inspection of f [sampling] and i [100%] will be corrected or replaced by good units." - H. F. Dodge

There are many instances, however, when this assumption will not be valid, and as a result various forms of reinspection are used to insure such values as the AOQ. In particular, inspecting i units refers to 100% inspection of all units until i consecutive units are found free of defects. It is known, however, that during 100% inspection operations defects may be overlooked because of the magnitude of the incoming process average, the nature of the defects themselves, or because of various forms of fatigue. This paper considers the development, evaluation and selection of a Dodge continuous sampling plan when this portion of the rectifying operation is not perfect.

A method is developed whereby the detrimental effect of violating Dodge's assumption can be investigated analytically. Two basic forms of reinspection frequently used with this plan are given formal development and evaluated as to their effectiveness. The selection

of the plan under this condition is also considered. Plan selection is investigated from the viewpoint of physical requirements, and from the viewpoint of optimizing the costs involved.

I - INTRODUCTION

I-A The Sampling Plan Considered

Sampling plans fall into two major categories depending upon the manner in which product is presented at the place of inspection. This will in turn be determined by the nature of the manufacturing process. When product can be conveniently gathered into lots, some form of a lot by lot sampling plan is usually applied. When it is not feasible to group product into lots, some form of a continuous sampling plan becomes necessary. The latter type of plan, which is the plan to be discussed, has found widespread use in areas such as conveyorized production.

Sampling plans are usually further classified as being either rectifying or non-rectifying. In more common lot by lot plans the word non-rectifying implies "pure" acceptance sampling. An acceptance sampling plan "prescribes a procedure that, if applied to a series of lots, will give a specified risk of accepting lots of a given quality" (9). It is not in itself an attempt to control quality, as it merely accepts or rejects product which has been grouped into lots.

The simplest lot by lot sampling plan using this definition is called a single-sampling plan. The procedure is as follows:

1. Select a sample of size n from a lot of size N.
2. If c or less defectives are found accept the lot.
3. If more than c defectives are found reject the lot.

The word "rectifying" will apply to the plan when some specified procedure is given as to the disposition (correction, replacement, etc.) of the defectives in the lots rejected. It is when this correction of defectives is present, that it becomes possible to make statements concerning outgoing quality.

The sampling plan discussed in this thesis is a continuous sampling plan. Further, it is a rectifying single level plan to be used under the assumption that the process under examination is in control. The basic plan to be used as a starting point will be the Dodge type CSP-1 continuous sampling plan.*

The difference between rectifying and non-rectifying as it applies to a continuous sampling plan will now be discussed. This difference, however, is not as clear as in the previous situation.

A continuous sampling plan might be considered non-rectifying when the procedure is to look at a fraction of product with no resulting action other than to remove the defectives found (19), (26). However, this is in reality a form of rectifying sampling, but not to the degree as in the case to be illustrated below. Further, the case just illustrated is a partial screening plan. The word partial is used because the fraction looked at, "f", satisfies $0 \leq f \leq 1$. If the fraction is 0 or 1 it is considered to mean no sampling or total screening, respectively.

A continuous sampling plan is called rectifying when some action is required such as detailing (100% sorting of good from bad) a

*The exact description of this plan and its parameters will be discussed in section I-B.

portion of the product flow upon finding too many defectives. There are many variations of this plan (7), (8), (12), (24), (27), (30) and, because of this, there are many degrees of rectification.* The continuous sampling plan discussed in this thesis will be of the rectifying type just defined.

In many situations where sampling plans are used it becomes advantageous to consider different levels of sampling. Among those applying levels to continuous sampling plans are Derman, Littauer, and Solomon (5), Lieberman and Solomon (18), Resnikoff (23), and Guthrie and Johns (15). A multi-level sampling plan is one in which different degrees of tightness is employed. Words such as tight, normal, and reduced are often used, and refer to either the fraction of the product that must be looked at, or to the specified portion of product to be detailed. Different levels are appropriate under conditions of changing process quality. However, for the purposes of this thesis it will not be necessary to consider such level changes as it has been stated that the process is in control (i.e., the process average is constant).

That the process is assumed to be in control, is possibly the most important assumption. This is due to the fact that the assumption will also negate the necessity for including in this thesis a discussion on optimal stopping characteristics.

*See also the Bibliography provided by Ascombe (1).

Basic continuous sampling plans, including those proposed by Dodge, have no explicit means by which the inspection process will terminate upon harmful deterioration of incoming quality. This topic has been treated in depth by several authors. See Ascombe (1), Murphy (20), Gregory (14), and Fry (11). Some plans developed by the military also consider this (24), (27). The results from these studies vary from simple warning rules (20) to complicated equations (14) based upon economic considerations. These results are certainly valid and their application would be a necessity in many situations. However, from the standpoint of this thesis the above results would require many modifications in order to remain valid. Therefore, it will be assumed for purposes of this thesis, that deteriorations in quality will be sufficiently handled without the aid of an analytically developed rule.

It is hoped, however, that the results of this thesis can be merged with ideas such as multi-level sampling and optimal stopping rules. This would provide for a very complete sampling system consisting of a general sampling model applicable to many situations.

I-B Structure of Dodge's CSP-1

Dodge's continuous sampling plan (CSP-1) is a "plan of sampling inspection for a product consisting of individual units (parts, sub-assemblies, finished articles, etc.) manufactured in quantity by an essentially continuous process" (6), (7), (8). The plan is applicable only to the following situation:

a. Product with characteristics subject to non-destructive testing.

b. The characteristics are to be examined on a "Go-No Go" basis.

c. Continuous flow of consecutive parts or articles.

d. The product is to be offered to the inspector in the order of production.

The plan was primarily intended by Dodge for use in process inspection of parts or final inspection of finished articles. Further, it was intended to be used where it is desired to have assurance that the percentage of defective units in accepted product will be held to some prescribed low figure.

The plan operates as follows:

1. An inspector selects a predetermined f percent (or fraction) of the product in such a manner as to assure an unbiased sample.
2. When a defect is found a predetermined clearing sequence of i subsequent and consecutive units of product must be found free of defects.

3. Upon finding i units free of defects the inspector resumes sampling the fraction f.

It is important to note that, if during a period of clearing i units, a defective unit is found, the count must start over. This is, as stated before, to be a rectifying plan and all defective units found are to be corrected or replaced by good units.

The use of an f and i combination will, for a given incoming fraction defective, result in a long run average fraction inspected (AFI) quantity. The protection provided by the plan is described in the following quote (6). "For given values of f, i, and p (incoming fraction defective), there will result for product of statistically controlled quality a definite average outgoing fraction defective (average outgoing quality, AOQ). For given values of f and i, the AOQ will have a maximum for some particular fraction defective p_1 of incoming quality." This maximum is referred to as the average outgoing quality limit (AOQL). Many combinations of f and i yield the same AOQL. The relationship is illustrated below in Figure 1.

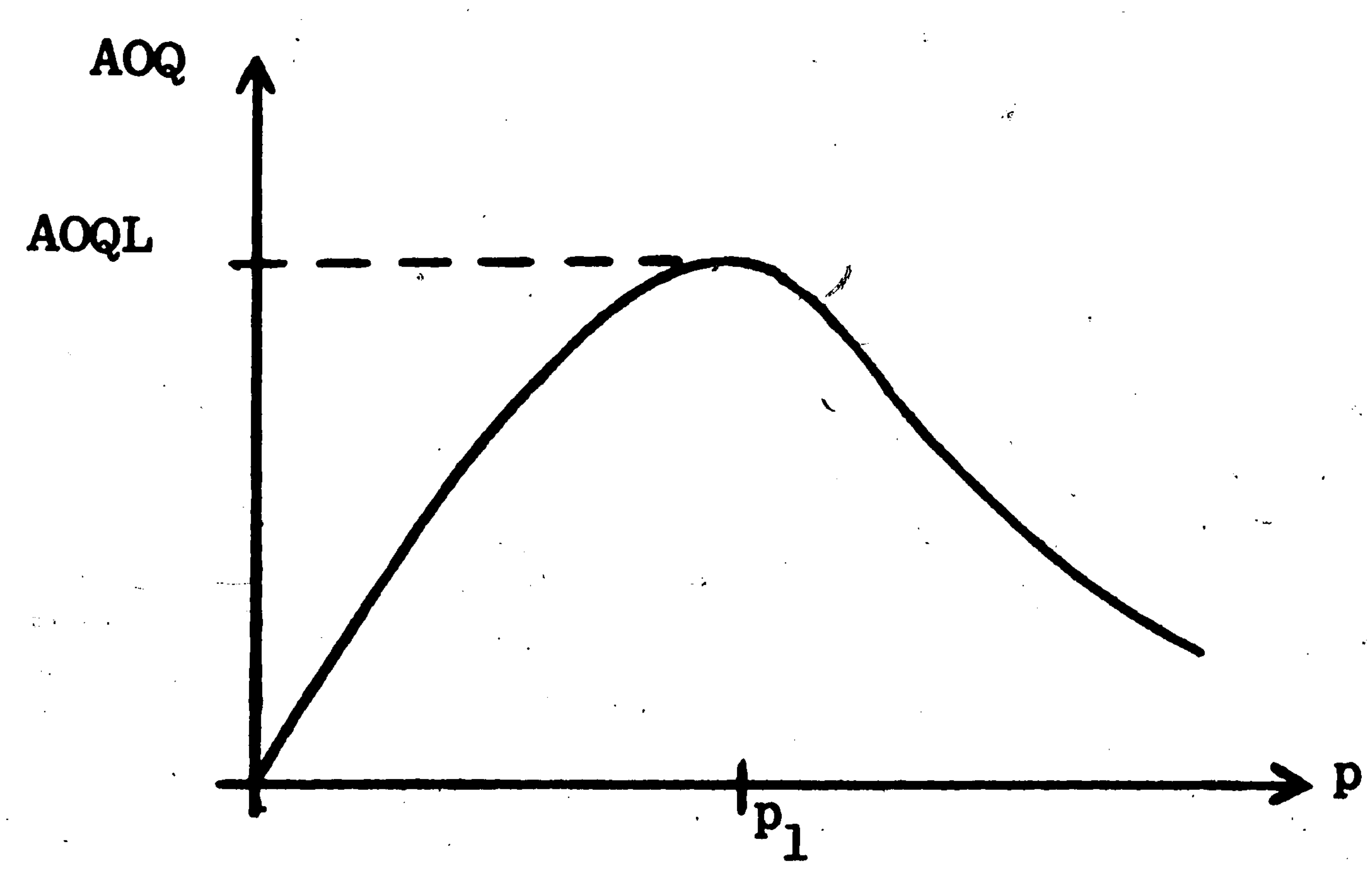


Figure 1. The Average Outgoing Quality as a Function of Fraction Defective in Submitted Product

Note that for incoming quality $p > p_1$ the average outgoing quality will be detailed into the product.

In the theoretical development of his plan Dodge makes the assumption that all phases of inspection are 100% accurate. This enables him to develop the outgoing quality relations on the basis that sampling inspection (obtaining f) and detailing (clearing i) can be considered as performed by the same person. He does state that these functions can be physically performed by two separate parties, but by the above assumption this will have no bearing on his development. This assumption is one of the reasons for this thesis.

Dodge concludes his first paper (6) by stating that his general plan provides a structure, which with possible variations in procedure, may be useful in designing addition sampling inspection procedures.

The basic structure referred to, the assumption regarding inspection accuracy, and the economic selection of the plan form the basis for this thesis. The following section will discuss these items further and relate the objectives and scope of this thesis.

I-C Purpose and Scope

I-C-a Imperfect Rectifying Operation

Between manufacturing companies one can expect the actual administration of a continuous sampling plan to differ. This is also true within a given manufacturing company. The differences exist because of varying inspection policies, which are established to account for inaccuracies on the part of the personnel responsible for a given plan.

The variations that are under examination in this paper reside in the procedure specifying the inspector's duties upon finding a defect during sampling inspection. In addition to being individually different, the specified plans may deviate in some manner from the original continuous sampling plan as described by Dodge. This is because the values, AFI, AOQ, and AOQL, originally derived by Dodge to characterize a given plan, apply only to those conditions where the detailer and inspector can be considered as the same person. The values were derived without considering reinspection, or the need for it. Thus, Dodge made the assumption throughout his derivations that inspecting and detailing were both 100% accurate.

The thesis will explore the results when this assumption fails. The investigation will encompass the use of this type of sampling plan under the aforementioned variations, and the application of the plan as Dodge originally derived it.

This will require an investigation of the presence of 100% accuracy (or the lack of it). It has been the author's experience that the inspector is often very accurate. The detailer, however, is more apt to miss defects. Among the reasons for the latter, two of the most prevalent are: that this is non-productive work on the part of the detailer and there may be a tendency to do a hurried job, and that often the detailer will be a person who is not familiar with the particular defect in question. The inspector, on the other hand, is performing the job he has been trained to do, is directly accountable for any defects found later on inspected units, and has less reason for doing a hurried and inaccurate job. Therefore, in this paper the inspector's function will be assumed to be performed perfectly.

I-C-b Economic Selection

The remaining part of the thesis will be devoted to the question of economical plan selection. The majority of sampling plans are selected on the basis of consumer risk, producer risk, average outgoing quality, or some combination of these three criteria. When this is done costs are certainly imputed. However, a more economical approach would seem to be to select the plan from the start on the basis of an economical trade-off. The trade-off will be between the cost of inspection and the cost of a defect. There have been two basic approaches to this problem, and these will be presented later. Neither approach, however, considers that detailing and inspecting are performed by different persons, nor that there may be errors on the part of the detailer. The fact that these functions are performed by different persons will lead to a different cost formulation than has yet been presented.

II The Derivation of a Dodge CSP-1

The mathematical aspects of this plan will now be derived. The derivation will use the Graphical Evaluation and Review Technique (GERT), which Whitehouse (32), (22) has shown to be applicable to this type of situation. Fry (11) appears to be the first to apply GERT directly in this area, but to a limited extent and for a different purpose. The results for the basic plan may be compared with those obtained by Dodge following a different procedure.

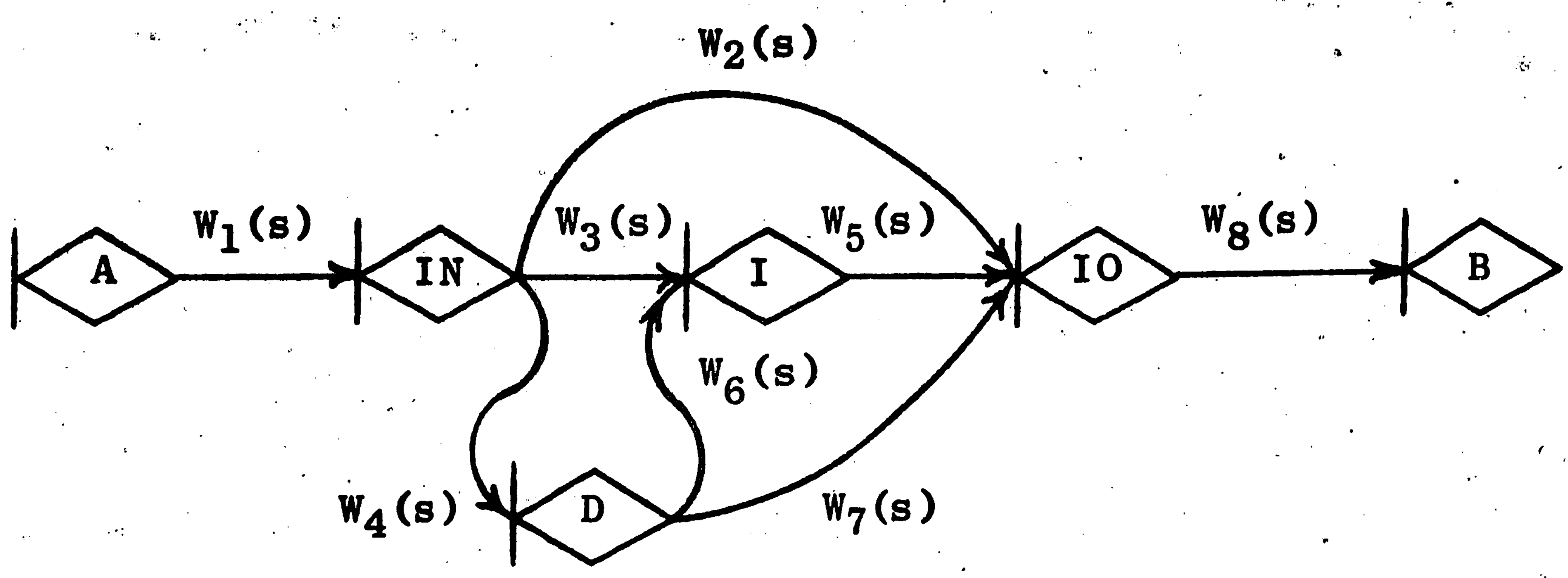
This technique is used here for two main reasons. The first reason is that it can provide information not available using the Dodge procedure. Specific advantages are given at the end of this section. Secondly, the technique is used here to provide an introduction to the theory behind it, and to maintain a consistent approach throughout the thesis, since its use makes possible later developments.

II-A Mathematical Development

For the basic plan it will be desired to know:

1. The Average Fraction Inspected - AFI
2. The Average Outgoing Quality - AOQ
3. The Average Outgoing Quality Limit - AOQL

The AFI will consist of the expected fraction of product looked at during inspection and the expected fraction looked at during periods of detailing. The network illustrating the continuous sampling plan in general would be as follows:



Events or Nodes

A - A unit has left a previous manufacturing operation

B - A unit has entered the next operation

IN - A unit has entered the place of inspection

IO - A unit has left the place of inspection

I - Inspection was performed on a unit

D - A unit was detailed

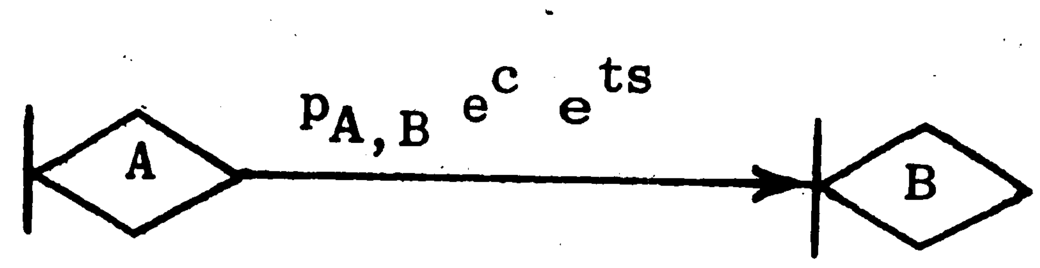
Transmittances

$W_j(s)$ = Probability of traversing a path times the moment generating function of the time to traverse the path

$= p_j MGF_j(s)$

The Symbol representing the nodes designate that they are all of the exclusive - or input type with probabilistic output. The realization of any branch leading into the node causes the node to be realized, but only one can occur at a given time. Upon realization of the node, at most one path emanating from the node can be taken (21).

If one were interested in the time to traverse a path, and this time was constant, then the $MGF_j(s) = e^{ts}$, where t would represent the constant time. The first derivative of $MGF_j(s)$ for $s = 0$ would be t as required. In this thesis, however, the interest is in the mean number of times an element is traversed. This value will be independent of the time involved. To represent this count an e^c is placed on the path being investigated. e^c is then equivalent to the MGF of a constant equal to one. This development can be represented as follows:



$$W_{A,B}(s,c) = p_{A,B}\ MGF_{A,B}(s,c) = p_{A,B}\ e^c\ e^{ts}$$

Since time is not of immediate interest in this thesis, t will be set equal to zero. Therefore, $W_{A,B}(c) = p_{A,B}e^c$.

The transmittances between any two nodes of an open flowgraph such as those used here can be obtained by Mason's rule (32). This is an extension of the topological equation for a closed flowgraph and is represented by:

$$T = \frac{\Sigma\ (\text{path between two nodes } (\Sigma \text{ non-touching loops}))}{\Sigma \text{ loops}}$$

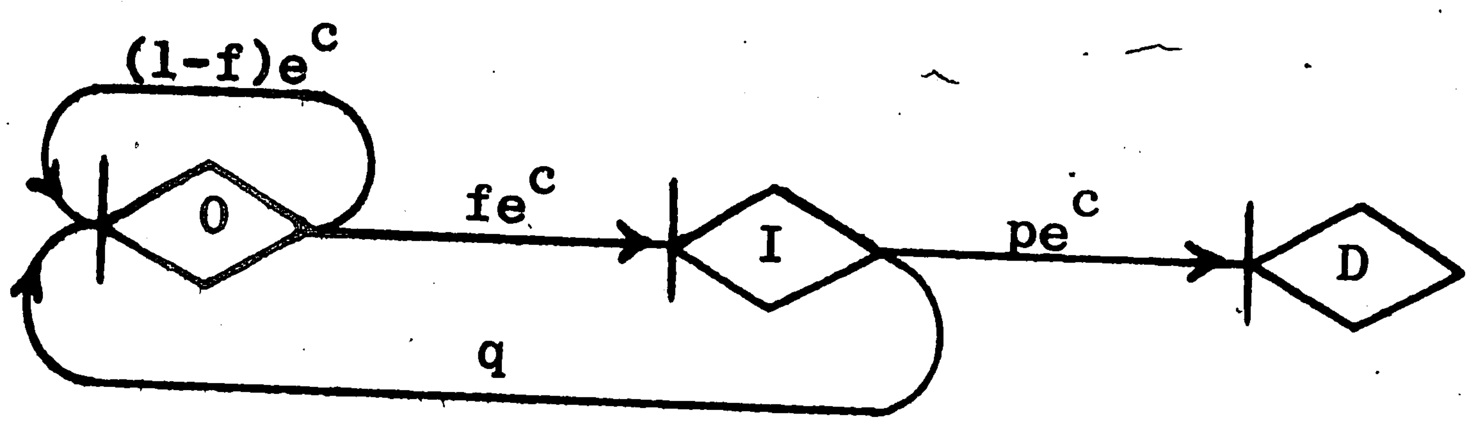
where: Σ loops $= 1 - L_1 + L_2 - L_3 + \ldots + (-1)^i\ L_i$, and

L_i is the sum of the i^{th} order loops

A first order loop is a consecutive path of arrows leaving a node and returning to the same node. A second order loop is a combination of two non-touching first order loops, etc. The value of a loop is the product of the transmittances associated with the loop. The ideas presented here will be made clearer through their use in the remaining work in this section of the thesis.

The original network is too general, as it presently exists, for finding the values of the AFI and AOQ. The basic plan will be developed first. The development will consist of determining the mean number of units that flow through the system during the various stages of the plan.

The expected number of units passing through the system during a period of inspection, including inspected and uninspected units and the defective unit causing detailing, can be obtained from:



The transmission function is: $W_{0,D}(c) = p_{0,D}MGF_{0,D}(c)$

or

1. $$W_{0,D}(c) = \frac{fpe^{2c}}{1 - (1-f)e^c - fqe^c}$$ where $q = 1-p$

2. $W_{0,D}(0) = P_{0,D} \cdot 1 =$ probability of reaching D from 0

$$= \frac{fp}{1 - (1-f) - fq} = \frac{fp}{f(1-q)}$$

$$= \frac{p}{(1-q)} = 1;$$ a defect will be found with certainty

therefore:

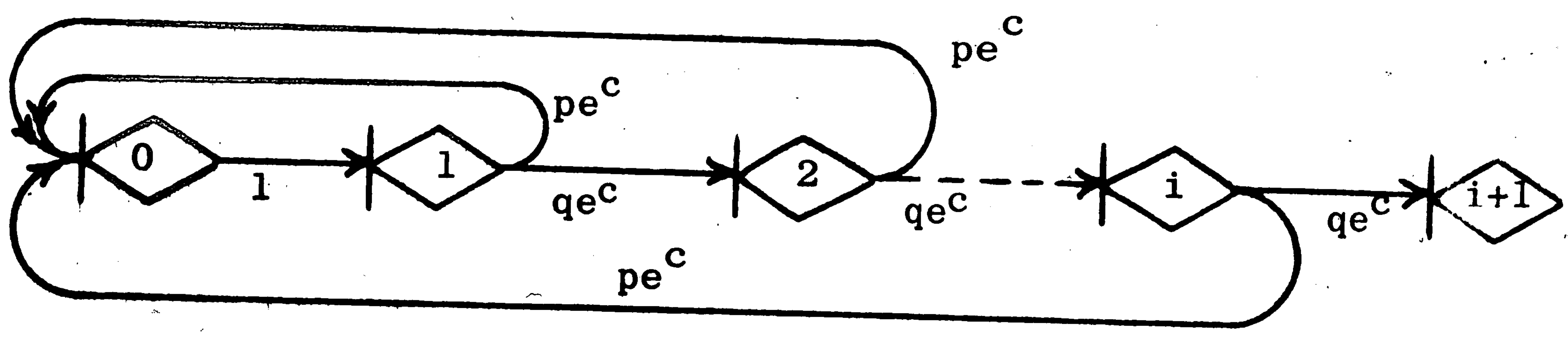
3. $$M_{0,D}(c) = \frac{fpe^{2c}}{1 - (1-f)e^c - fqe^c}$$

4. $$\left.\frac{\partial M_{0,D}(c)}{\partial c}\right|_{c=0} = \text{expected number of units passing through the system during a period of sampling inspection}$$

$$= \frac{fp + 1}{fp} = 1 + \frac{1}{fp}$$ (See Appendix A for complete derivation)

The 1 in equation 4 represents the defect found which ends the sampling inspection period. This value (one) could be eliminated by dropping the e^c term on the path from I to D.

To determine the expected number of units that will be looked at by the detailer while attempting to clear i units the following network will be used:



5. $W_{0,i+1}(c) = P_{0,i+1}MGF_{0,i+1}(c)$

$$= \frac{(qe^c)^i}{1 - (pe^c + pe^c qe^c + pe^c(qe^c)^2 + \ldots + pe^c(qe^c)^{i-1})}$$

$$= \frac{(qe^c)^i}{1 - pe^c(1 + qe^c + (qe^c)^2 + \ldots + (qe^c)^{i-1})}$$

The term in parentheses in the denominator is a geometric series which can be summed by $Sum_k = \frac{a(1-r^k)}{1-r}$, where $k = i$; $r = qe^c$; $a = 1$. Thus this term can be represented by:

$$Sum_i = \frac{(1 - (qe^c)^i)}{1 - qe^c}$$

and

6. $$W_{0,i+1}(c) = \frac{(qe^c)^i}{1 - \frac{pe^c(1-(qe^c)^i)}{1-qe^c}}$$

$$= \frac{(qe^c)^i \quad (1-qe^c)}{1 - qe^c - pe^c(1 - (qe^c)^i)}$$

7. $$P_{0,i+1} = W_{0,i+1}(0) = \frac{q^i \ (1-q)}{1-q - p(1-q^i)} = \frac{q^i p}{p(1-1+q^i)} = 1$$

Which should be the result, i.e., detailing will eventually cease provided enough units are supplied to the detailer, and a sequence of at least i good units exist.

8. $$MGF_{0,i+1}(c) = W_{0,i+1}(c)/1$$

9. $$\left.\frac{\partial MGF_{0,i+1}(c)}{\partial c}\right|_{c=0} = \frac{1 - q^i}{pq^i}$$ (See Appendix A for complete derivation)

Therefore, we now have the total expected number of units encountered at both detailing and inspection. The total average fraction looked at is:

10. $$AFI = \frac{\text{Equation 9} + f(\text{Equation 4})}{\text{Equation 9} + \text{Equation 4}}$$

$$= \frac{(1 - q^i)/pq^i + f(fp + 1)/fp}{(1 - q^i)/pq^i + (fp + 1)/fp}$$

This result agrees with that of Dodge except for the 1 in equation 4. For any reasonable values of $1/fp$, $\frac{1}{fp} \gg 1$. Therefore, to simplify future work, equation 10 will be expressed as:

10. $$AFI = \frac{(1 - q^i)/pq^i + 1/p}{(1 - q^i)/pq^i + 1/fp}$$

and upon simplifying

$$= \frac{f}{f + (1-f)(1-p)^i}$$

Before proceeding with this development, it would seem worthwhile to evaluate this method of approach. There are at least three areas where it appears to be more powerful and more useful than the approach taken by Dodge.

First would be the fact that a pictorial representation of the process is provided. The second advantage is the ability to use time information if it is available. If there were a $t \neq 0$ in the term e^{ts}, or for that matter, if there were any other form of $MGF_j(s)$ present for path times, then processing times would be available. For example, the mean and variance of the time for the detailer to detail i units could be obtained.

The third area concerns the ability to go beyond the mean of the fraction inspected, etc., and give an expression for the variance. This can be done, for example, by differentiating equation 8 twice with respect to c, evaluating the result at $c = 0$, and subtracting the square of equation 9.

This latter information would be useful (9), and it could possibly lead to a sound basis for comparing an entire class of sampling plans. Further, the second area, regarding time information, would be valuable to industrial engineers in time study problems. In fact, the application of GERT to sampling plans and the information thus available would appear to be an area for further investigation.

Sufficient information is now available to determine the average outgoing quality. This quantity can be determined from:

11. $$AOQ = p(1 - AFI)$$

The upper limit of this value will occur for some value of $p = p_1$ as indicated in Figure 1. The AOQ is differentiated with respect to p, set equal to zero, and solved for the value of $p = p_1$ which gives the limit. The above procedure will yield the following results (6):

12. $$AOQL = \frac{(1-f)(1-p_1)^{i+1}}{fi}$$

$$\text{where } p_1 = \frac{1 + i\ AOQL}{i + 1}$$

These are the immediate values of interest, and the effect of the presence of an inaccurate detailing function upon these values will now be investigated.

III The Accuracy of the Detailing Function

As previously indicated, the basic plan assumes, that during inspection and detailing, the rectifying portion of the plan is perfect. That is, all defects will be detected and corrected or replaced when they are in the inspector's sample or in the units being detailed. Further, it was indicated in the introduction that, with respect to the inspection function, this assumption will also be used in this thesis.

The question of inspection (or detailing) accuracy is certainly not new. A review of the literature on quality control and especially the work of J. M. Juran (16), (17) and E. L. Grant (13) indicates that the error in 100% inspection ranges between 70 and 95%. In fact, the presence of inaccuracy during a period of 100% inspection, whether it be called sorting or detailing, was one of the prime reasons for going to sampling inspection. The following quotes appear in the literature supporting this statement:

> "In still other cases, because of the effect of inspection fatigue involved in 100% inspection, a good sampling inspection plan may actually give better quality assurance than 100% inspection."--E. L. Grant (13)

> "It may result in accepting some defective material. A number of independent checks on the reliability of 100 per cent inspection in sorting out <u>all</u> bad parts from good have cast considerable doubt upon its complete effectiveness in every instance."--A. V. Feigenbaum (10)

> "As noted above, 100 per cent inspection may not mean 100 per cent perfect quality, and the percentage of defective items passed may be higher than under a scientifically designed sampling plan."--A. J. Duncan (9)

The above quotes are presented here for a dual purpose. The Dodge CSP-1 is a scientifically developed plan, but it has both an acceptance and a rectifying portion. Therefore, the inspection function, since it involves sampling, will be assumed perfect. However, since detailing (100 per cent inspection) is required upon finding a defect in the sampling portion, the effect of detailer accuracy will be investigated.

Before discussing the effect of accuracy, it is necessary to develop a method to describe this accuracy, and to be able to work with it analytically. As indicated by the previous quotes, there have been studies on this subject, which illustrate that the effect is not always negligible. Further, fatigue is often found to be an important factor. The following list should be adequate to cover the many reasons for inaccuracies during detailing:

1. Fatigue - it can be both physically and mentally tiring.
2. Monotony and boredom - especially when the incoming fraction defective p is low and there is little "action".
3. The detailer is often not familiar with the defect in question - the consequences being worse for a high p.
4. This is non-productive work on the part of the detailer (an operating employee) - there may be a tendency toward overlooking a defect (especially if it is marginal) because the count starts over upon recognizing a defect. For this reason it is further assumed that a good product will not be called defective.

As far as can be determined from the literature, little has been done to express the accuracy in terms general enough to include all of these factors. This is, therefore, a possible area for future investigation. The approach usually taken is to determine an average value for the probability of detecting a bad unit (13). This value would be used under the same assumption as the process average, i.e., the probability of detecting a defect remains constant from unit to unit.

This value would have to be derived in such a manner as to be representative of the situation. Sufficient study, prior to implementing the plan, would be required so that the value reflected the difficulty in detecting the defect, the expected number of units looked at, and the process average.

A specific procedure for determining this value, call it A, will now be discussed. The approach is similar to that taken by Albert Beck, Jr. (3) for determining his value E. He defined E to be the probability that a defective unit will be recognized as such during an inspection. Note that $A \equiv E$.

The first step would be to determine a representative sample size, N. The mean number detailed under perfect conditions could be used as representative. Of course, if $A \neq 0$ this number will not be as high.

The procedure following this depends on whether or not the number of good units in the sample is to be a known quantity from the start. When it is desired and possible to know this quantity, estimate the process average, p, and place Np defectives in the sample. The following quantities and procedure will then be applicable to estimate A:

G = number of good units in the sample (a known constant quantity).

x = number of test or trial.

y_x = fraction defective detected and removed by the detailer on trial x.

A_x = an estimate of A obtained from the x^{th} test.

N_x = the total number in the sample at the start of trial x. $N = N_1$

Then:

(a) $A_x = N_x y_x / (N_x - G)$

= defects found on test x/defects in N_x

or

(b) $G = N_x(1 - y_x/A_x)$

Since G is known here, all that is needed is one trial to estimate A. However, it would be appropriate to perform repeated trials to achieve a more accurate estimate of A.

When it is not practical to rig the sample with a known number of defects in advance, a recurrence relationship can be established. The procedure is to perform repeated detailing on the sample. G is still defined as before, but its value is not known. The estimate of A can be determined from the following:

$$N_1 = N$$

$$N_2 = N_1 (1 - y_1)$$

$$N_3 = N_2 (1 - y_2)$$

$$= N_1 (1 - y_1)(1 - y_2)$$

or

$$\text{(c)} \quad N_{x+1} = N_x (1 - y_x)$$

and from (b)

$$G = N_x (1 - y_x/A)$$

G is independent of the number of the test x, and remains constant. Therefore:

$$N_x (1 - y_x/A) = N_{x+1} (1 - y_{x+1}/A)$$

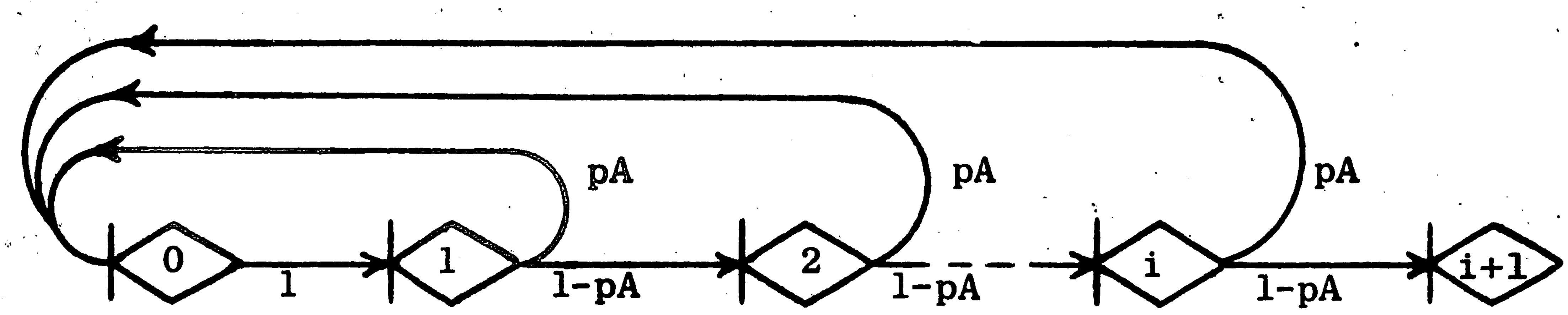
$$= N_x (1 - y_x)(1 - y_{x+1}/A)$$

solving for A

$$\text{(d)} \quad A = 1 - (y_{x+1}/y_x)(1 - y_x)$$

Thus, an estimate of the constant A can be obtained from only two test runs. However, as Beck indicates, it would be better to determine an average A based upon repeats of the above procedure. This is especially true if some $y_x = 0$.

Now assuming we have our estimate A, and that it will remain constant, we can proceed to determine the effect of its presence on the average outgoing quality. The new GERT network would appear as:



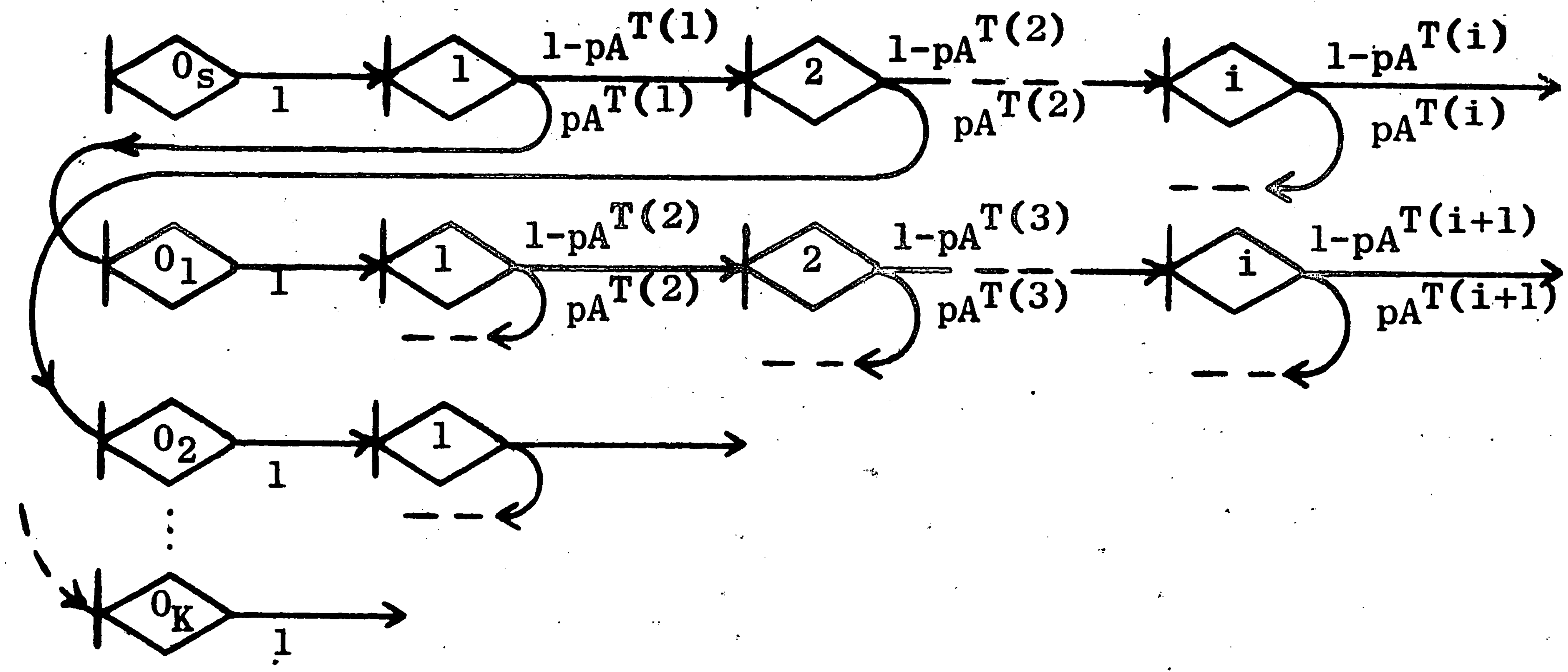
The probability that the detailer will increase the count is the probability that the present unit is good, or that the present unit is bad and it goes undetected. This would equal $q + p(1 - A) = 1 - pA$. Note that A is actually the probability that a unit will be detected given that it is defective.

The mean number looked at can be determined from the previous result of equation 9, replacing p by pA and q by 1 - pA. This generalization is shown in Appendix A. Thus, the mean number detailed becomes, after placing e^c in the above paths:

$$13. \quad \left. \frac{\partial MGF_{0,i+1}(c)}{\partial c} \right|_{c=0} = \frac{1 - (1-pA)^i}{pA\ (1-pA)^i}$$

Before investigating this result numerically, however, it will be of interest to consider a more general situation.

Assume that $A^{T(t)}$ represents the probability of detecting a defective unit, when the particular unit is the t^{th} unit in a detailing sequence. T is to be a function of t, representing the increase in fatigue, monotony, etc., as t increases. Further, assume that the value of A reflects the present process average and the difficulty of detecting the defect. This treatment is similar to that used by Savage (25) to represent a process with degenerating quality. The GERT network for determining the new average fraction looked at by the detailer would be as follows:

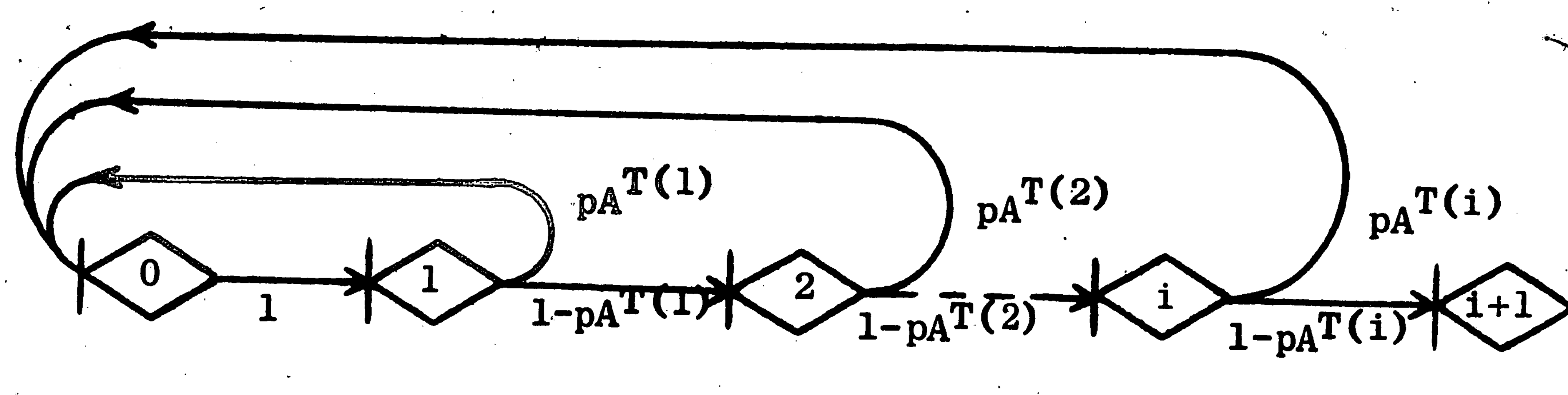


$pA^{T(t)}$ = p (probability of a defective unit) · $A^{T(t)}$ (probability of detecting the t^{th} unit if it is defective).

K ≡ the number of times the count would start over.

Computations using this network would be extremely difficult. It would also be difficult to determine the proper T(t) so that $A^{T(t)}$ will actually represent the detailing process. As it now stands, it is very likely that the detailer would never detect a defect for large t.

A development more computationally feasible would be to assume that upon finding a defect, the detailer's accuracy factor starts over at $A^{T(1)}$. This is still a very realistic situation because the build-up of fatigue and monotony is broken, and the detailer would be alerted to a possible run of defects. The GERT network would be simplified to:



after placing an e^c in each path as before:

14. $$W_{0,i+1}(c) = \frac{(1-pA^{T(1)})(1-pA^{T(2)})\dots(1-pA^{T(i)})e^{ci}}{1-\left[pA^{T(1)}e^{c} + pA^{T(2)}e^{2c}(1-pA^{T(1)}) + \dots + pA^{T(i)}e^{ic}(1-pA^{T(1)})\dots(1-pA^{T(i-1)})\right]}$$

$$P_{0,i+1} = W_{0,i+1}(c)\Big|_{c=0} = 1$$

The detailer must clear faster than when A = 1, and for the case A = 1, $P_{0,i+1}$ has been proven to equal 1.

Therefore:

15. $$MGF_{0,i+1}(c) = \frac{e^{ci}\prod_{N=1}^{i}(1-pA^{T(N)})}{1-pe^{c}\left[A^{T(1)} + \sum_{N=1}^{i-1} A^{T(N+1)}e^{NC}\prod_{j=1}^{N}(1-pA^{T(j)})\right]}$$

The MGF must now be differentiated with respect to c, and evaluated for c = 0. Upon doing this, and also recalling that the numerator and denominator of the above expressions are equal at c = 0, the mean number detailed will become:

$$16.\quad \left.\frac{\partial\, MGF_{0,i+1}(c)}{\partial c}\right|_{c=0} = i + \frac{p\left[A^{T(1)} + \sum_{N=1}^{i-1}(N+1)A^{T(N+1)}\prod_{j=1}^{N}(1-pA^{T(j)})\right]}{1-p\left[A^{T(1)} + \sum_{N=1}^{i-1}A^{T(N+1)}\prod_{j=1}^{N}(1-pA^{T(j)})\right]}$$

The remaining problem is to determine the form of $A^{T(t)}$. As indicated previously, no studies to date have revealed a specific form for this. It is probably true that the form of $A^{T(t)}$ will be highly dependent upon the particular set of conditions. Therefore, for the purpose of evaluation a form will be assumed.

Consider the situation where the probability is one for detecting a defect in the first unit of detailing, or in the first unit after finding a defect. This probability then will be assumed to decrease to a value equal to A, which will be the value after a sequence of i units. Thus, the value of A could be determined in the same manner as before, viz., following the approach taken by Beck. The value of A by itself is assumed to be constant. The desired form will be as in Figure 2.

An analytical representation for this could be:

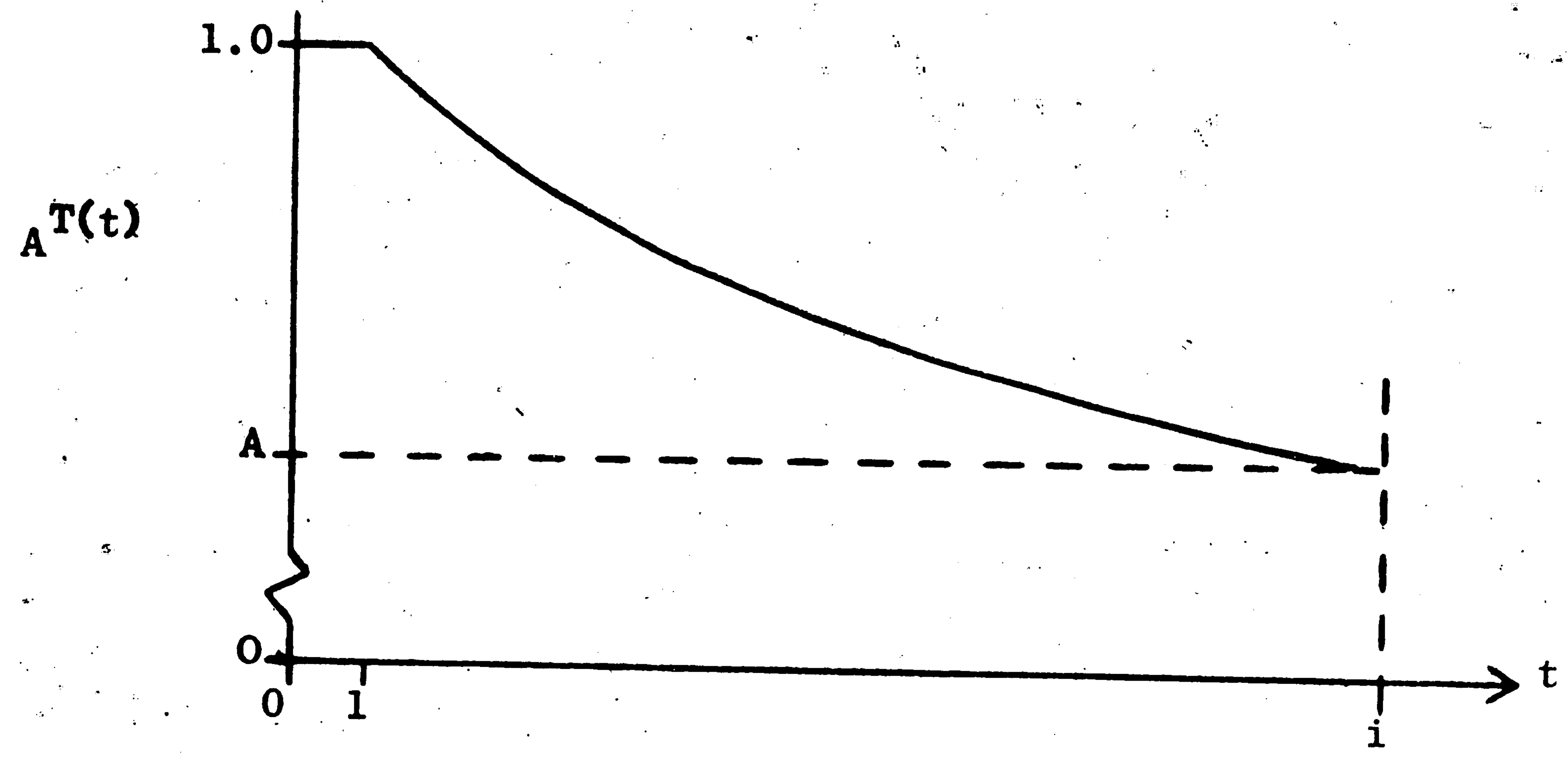
$$T(t) = (t-1)/(i-1);\ t = 1, 2, 3, \ldots, i$$

$$0\ ;\ t=0$$

and therefore:

$$A \le A^{T(t)} \le 1$$

The investigation of the effect of this decreasing function will follow a similar investigation for the constant case.



t ≡ the number in detailing sequence since start or last defect detected.

Figure 2. Increasing Error of Non-Detection

III-A Numerical Effect of Detailing Accuracy

III-A-a Constant Case

In the previous section it was determined that the mean number detailed in the presence of an accuracy factor of $A < 1.0$ can be computed from equation 9 by replacing p by pA and q by 1-pA. Since the inspector is assumed to be 100% accurate, the mean number in appearing in the sampling state will remain the same.

To keep the following equations and discussion clearer, the various values for the sampling plan will be given the following notations:

U = the mean number detailed when A = 1.0 (i.e., equation 9).

V = the mean number passing through the system before a defect is found.

U_A = the mean number detailed when $A < 1.0$.

AFI_A = the average fraction inspected when $A < 1.0$.

AOQ_A = the average outgoing quality when $A < 1.0$.

with this notation:

$$AFI = (U + fV)/(U + V)$$

First it must be determined what the new average fraction inspected will be. This can be obtained in the same manner as before. The detailer looks at U_A, and the inspector will look at fV. Therefore the new AFI_A will be:

17. $$AFI_A = (U_A + fV)/(U_A + V)$$

The determination of the average outgoing quality, however, will be more complicated, since not all of what is looked at by the detailer is rectified. This derivation can be developed in the following manner:

$U_A + V$ = total number of units

$[(U_A + V) - (U_A + fV)]$ = number of units not examined by either the detailer or inspector.

$p[(U_A + V) - (U_A + fV)]$ = number of defectives in those not examined.

fV = number of units inspected by the inspector, of which no defectives remain.

U_A = number of units detailed by the detailer.

pU_A = number of defectives presented to the detailer.

$pU_A(1-A)$ = number of defectives not removed during detailing.

$p[(U_A + V) - (U_A + fV)] + pU_A(1-A)$ = total number of defectives.

$[p[(U_A + V) - (U_A + fV)] + pU_A(1-A)]/(U_A + V)$ = average outgoing fraction defective.

Dividing through gives:

18. $$AOQ_A = p(1-AFI_A) + p(U_A/(U_A + V))(1-A)$$

or this can be written as:

18a. $$AOQ_A = p\left(1 - \frac{AU_A}{U_A+V} - \frac{fV}{U_A+V}\right)$$

A FORTRAN program was written for an IBM 1130 computer to aid in evaluating the effect of different accuracy factors for varying values of f, i, and p.

It can be seen from Figures 3, 4, and 5, that as f increases, for a fixed i, the value of the increase in the AOQ over that expected when A = 1.0 also increases.* This value is denoted Δ(AOQ). Figure 6 illustrates the same general relationship when holding f constant and varying i. The former case is to be expected, since as f increases, the number of units appearing in the sampling state, V = 1/fp, must decrease. This would increase the number of times that detailing starts, and the detailing state is where the inaccuracy appears.

However, this rate is greater in the case where f is held fixed and i is varied. This is to be expected also as the major contributor to the discrepancy is the new expected number detailed, which is only a function of i, p, and A. A further illustration of this result follows:

A = .5 and p = .075

i = 15 , f = 5% ; Δ(AOQ) = .0027

i = 30 , f = 5% ; Δ(AOQ) = .0101

f = 5% , i = 10 ; Δ(AOQ) = .0025

f = 10% , i = 10 ; Δ(AOQ) = .0046

For the values indicated, doubling i has a much greater effect on Δ(AOQ) than that obtained by doubling f.

Another noticeable effect is that, as f increases, the range over values of A for the Δ(AOQ) values also increases. The presence of this fact produces the following result for moderate values of i;

*An increasing AOQ represents a worsening in average outgoing quality.

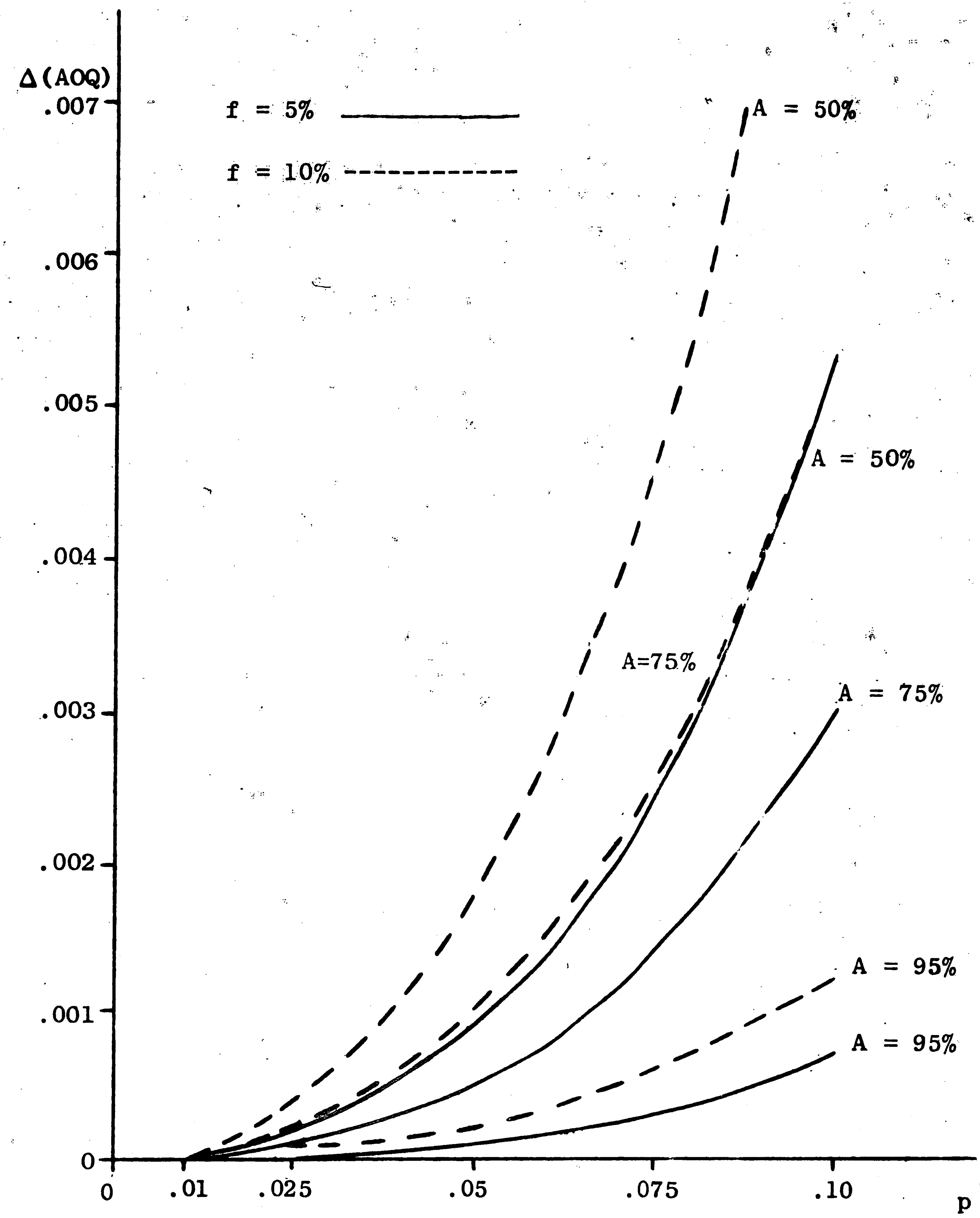


Figure 3. Deviation from Expected AOQ vs. Process Average: i Fixed at 10 Units, f Varied (5&10%), and A Varied (50,75,&95%).

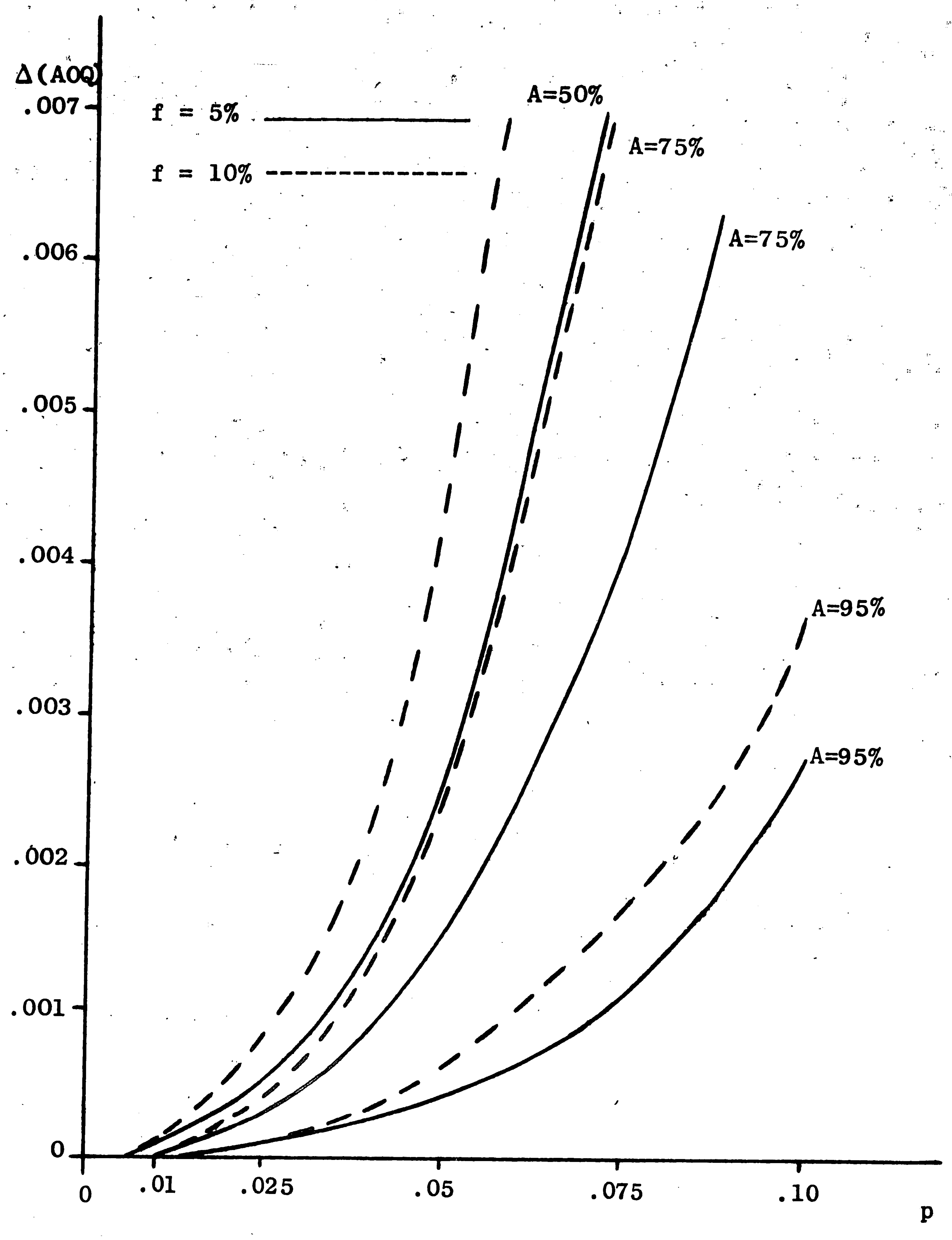


Figure 4. Deviation from Expected AOQ vs. Process Average: i Fixed at 20 Units, f Varied (5&10%), and A Varied (50,75,&95%)

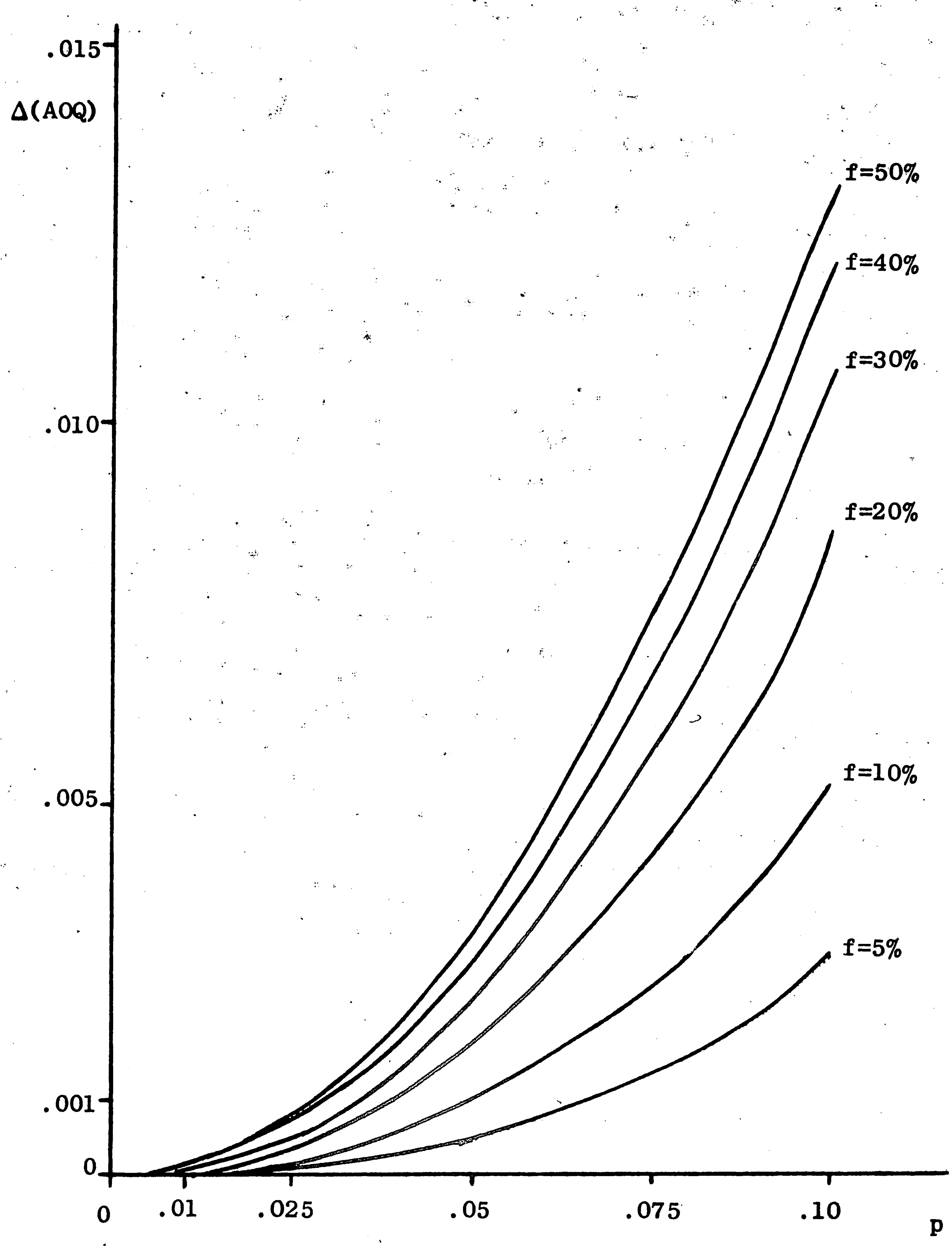


Figure 5. Deviation from Expected AOQ vs. Process Average: i Equal 10 Units, A Equal 75%, and f Varied (5-50%)

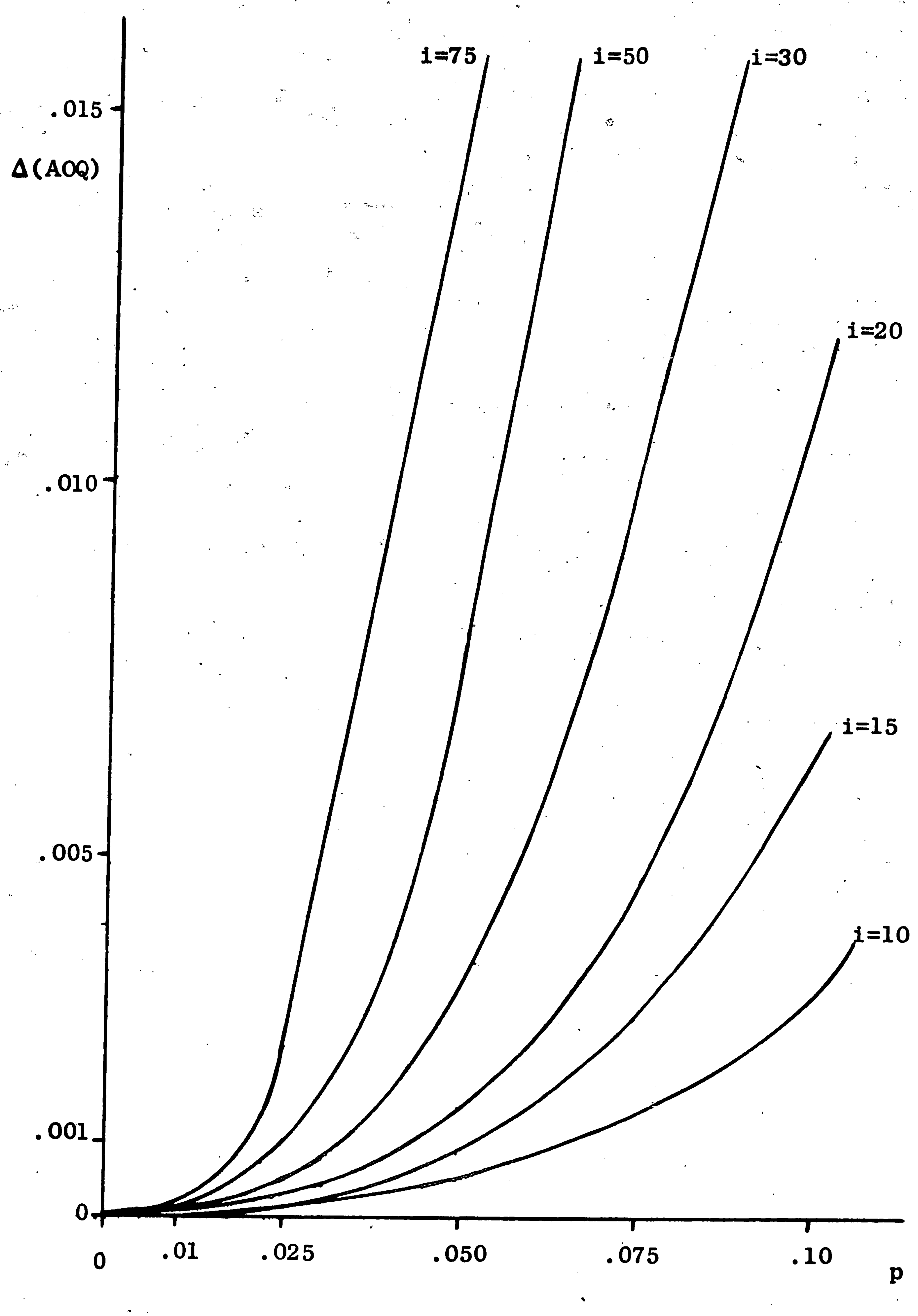


Figure 6. Deviation from Expected AOQ vs. Process Average: f Equal 5%, A Equal 75%, and i Varied (10-75 Units)

for a fixed i and p , doubling f and raising A from .5 to .75 yields approximately the same effect on the Δ(AOQ).

A generalization of the results up to this point would be, that for fixed operating conditions, i.e., a fixed $A < 1.0$ and p, increasing either i or f will not directly reduce the detrimental effect of A. This is because, when $A = 1.0$ and f is increased, the AOQ value will be lowered, however, when $A < 1.0$ and f is increased, the AOQ value increases or worsens. Thus Δ(AOQ) increases because detailing occurs more frequently, and the inaccuracy is in the detailing state. The importance of this fact would be that a plan user should be discouraged from simply increasing f, or i, or both to overcome the effect of an inaccurate detailer. The proper use of this type of approach to achieve improvement is discussed in section IV. Further, the effect increases as the process average increases. Actually the effect in many cases will be slight when p is low.

An example will now be presented to illustrate the above results for a specific sampling plan. This plan will be used as a standard in the remaining portions of the thesis.

In keeping with the usual way in which a continuous sampling plan is chosen, assume it is desired to meet an AOQL or approximately 1.5%. Assume further that for reasons external to the plan, that f must be no greater than 10%. This will require an i of approximately 75 units. The actual AOQL as shown in Figure 7 is approximately 1.44%. Finally, assume that the process average is

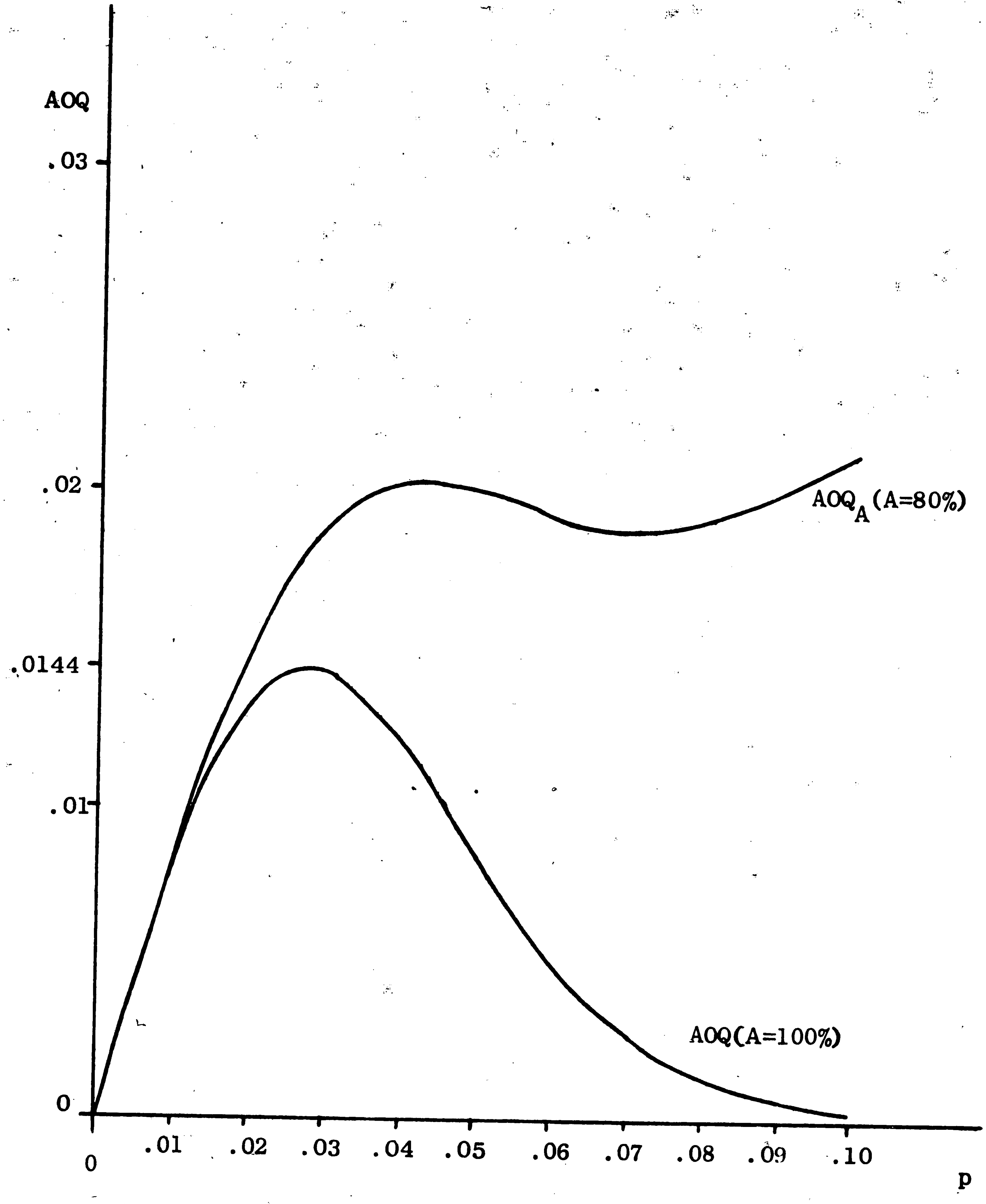


Figure 7. AOQ Curves: f Equal 10%, i Equal 75 Units, and A Equal 100% and 80%

estimated to be 2% defective, and that the detailer is 80% accurate.

Figure 7 illustrates clearly the difference between what is expected if the detailer is 100% accurate and what will actually happen if he is only 80% accurate. For the expected process average of p = 2% the expected AOQ = 1.32%, but the AOQ is actually 1.48%. Therefore, the actual AOQ is already exceeding the expected limit.

This limit or AOQL was not expected to be reached until p shifted to 2.75% defective.

The most important result of an $A < 1.0$ is that the actual AOQL, as p covers its range, is going to be much greater than the 1.44% expected. That this is so can be readily seen from Figure 7 and also from the actual AOQ formulation, equation 18:

$$AOQ = p\left(1 - \frac{U_A + fV}{U_A + V}\right) + p(1 - A)\frac{U_A}{U_A + V}$$

As p is increased, U_A becomes very large, and the first term will approach zero just as it would in the case where $A = 1.0$. However, the second term will approach 1-A as p approaches 1. Thus the true AOQL for an $A < 1.0$ becomes 1-A, or 20% defective for this set of sampling plan parameters.

This situation would not be expected to be this bad, however, as daily sample estimates of p will probably not deviate so radically from its expected value of 2% defective. To get a feel for the possible variation of p, assume that the sampling distribution of p can be approximated by the normal, i.e., $np > 5$, where n is the number of units used to estimate p, say a day's production.

Since the term U_A for p = 2% is approximately 200 it would not be unreasonable to assume that the day's production is greater than 200, and that np > 5.* Therefore, with an n of at least 300, the standard deviation of the fraction defective would be no greater than:

$$\sigma_p = \sqrt{(.02 \times .08)/300}$$

$$\simeq .00225$$

and

$$3\sigma_p \simeq .00675$$

This being the case, then $0 \leq p \leq .027$, which would put a practical limit of .018 on the AOQ over a short term.

III-A-b Decreasing Function Case

The effect of a decreasing function, $A^{(t-1)/(i-1)}$, representing detailer accuracy will be investigated next. The investigation will be centered around the previous plan.

The average fraction inspected will remain in the same form as in all previous cases, except that it will be computed from equation 15. The new average outgoing quality, however, can no longer be determined exactly. As in the previous case, the AOQ can be expressed as a sum of two terms. The first term will represent the percent defective in product not sampled, nor detailed:

$$p(1-AFI_A)$$

*It seems the exception rather than the rule to have a detailing sequence continue throughout a large portion of a day's production.

The second term will be p times the average fraction detailed times the fraction of this that remains defective. However, this latter quantity will no longer be simply (1-A), as it was in the constant case.

The information available does not provide a means to determine exactly the "over-all" probability of not catching a defect, given that a defect is present. This probability is continually starting over at zero each time a defect if found, and increasing up to (1-A).

For the aforementioned reasons, an approximation will be used that will in all cases by an upper limit to this value for any specific process average. The limiting value just mentioned is not to be confused with an AOQL. The formulation for the AOQ_A:

19. $$AOQ_A = p\left(1 - \frac{U_A + fV}{U_A + V}\right) + p(1 - A_L)\left(\frac{U_A}{U_A + V}\right)$$

where:

$$A_L = A^{(t-1)/(i-1)} \quad \text{at} \quad t=i$$

The number of times a run of units of length $j \leq i$ appears is greater for short lengths. That is, runs of two occur with a frequency equal to or greater than runs of three, and this continues until there is only one run of i with an accuracy as low as A_L. Therefore by placing $A^{(t-1)/(i-1)}$ constant at t=i, the contribution to the AOQ will always give a higher AOQ than is actually present.

The previously mentioned computer program was adapted to this decreasing function case.

Figure 8 shows the difference for the standard plan between a constant A of 80% and an $A^{T(t)}$ decreasing from 100% to either $A_{t=i}$ = 50%, or $A_{t=i}$ = 80%.

It is apparent that one would not want to estimate outgoing quality for an accuracy function that decreased from A = 100% to A = 50% with a constant A of 80%. On the other hand, one would generally be pessimistic if he approximated an accuracy that decreased to a lower value with a constant equal to this lower value. This is illustrated in Figure 8 by the curve for A = 80% and the curve for A decreasing to 80%. The extent of the pessimistic attitude is even greater than is indicated by the graph. It should be remembered that an AOQ value for a decreasing function includes in it an overstatement of bad quality because of the approximation to the second term using A at its lowest value.

A breakdown of the two portions making up the curve for an A decreasing to 80% is given in Figure 9. AOQ_1 represents the outgoing quality in the product not examined. AOQ_2 represents the approximated outgoing quality in the product detailed. The true average outgoing quality must lie somewhere between AOQ_1 and AOQ_A. As p increases, this area in question also increases, thus the approximation becomes more critical.

However, for the specific plan under examination the value of AOQ_2 is not critical. p was assumed to be estimated at 2% defective with a 3σ limit no greater than 2.7% defective. Thus the estimate of AOQ_A by the method used is satisfactory for this situation.

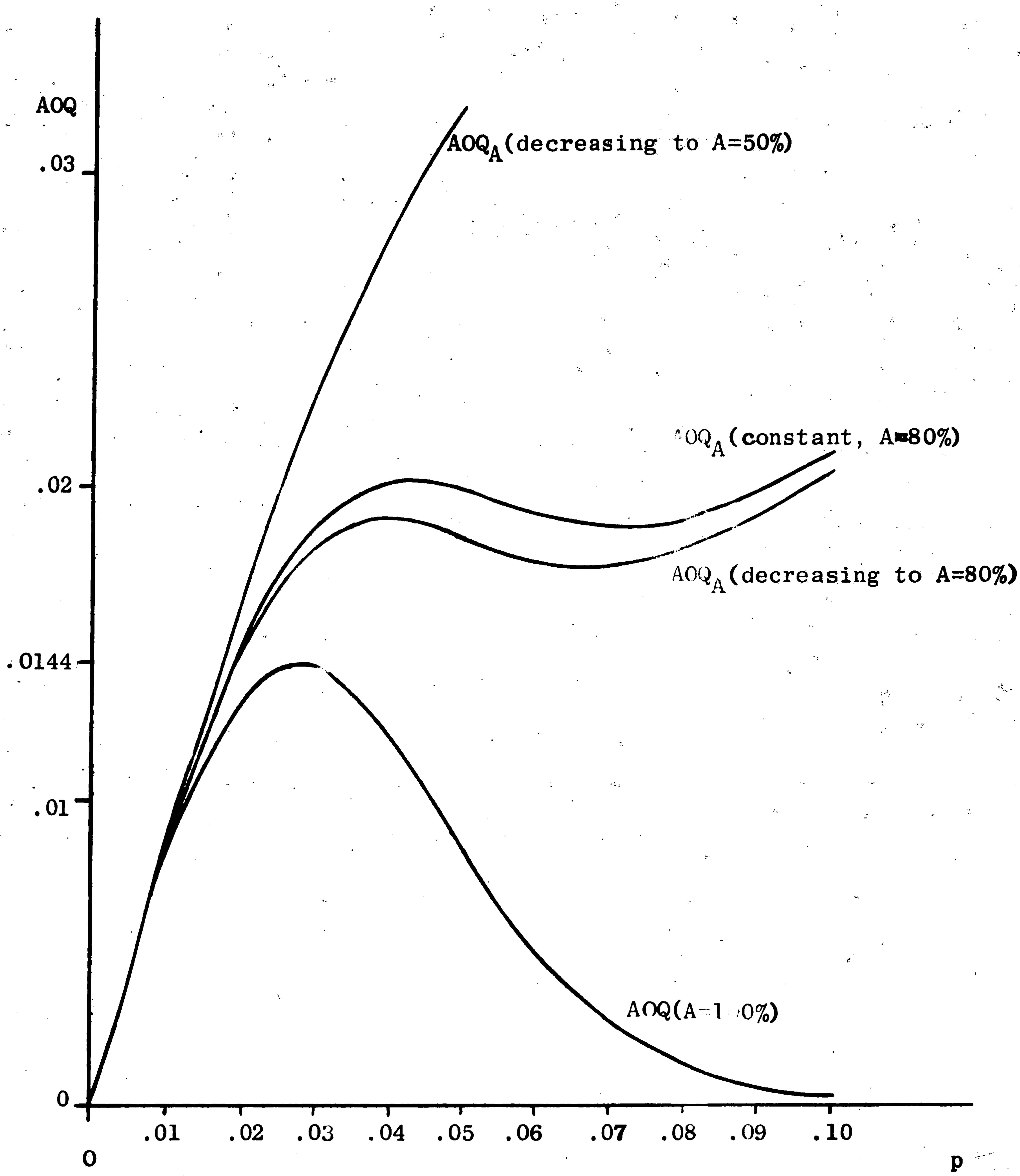


Figure 8. AOQ Curves: f Equal 10%, i Equal 75 Units, A Constant at 100 and 80%, and A Decreasing to 80 and 50%.

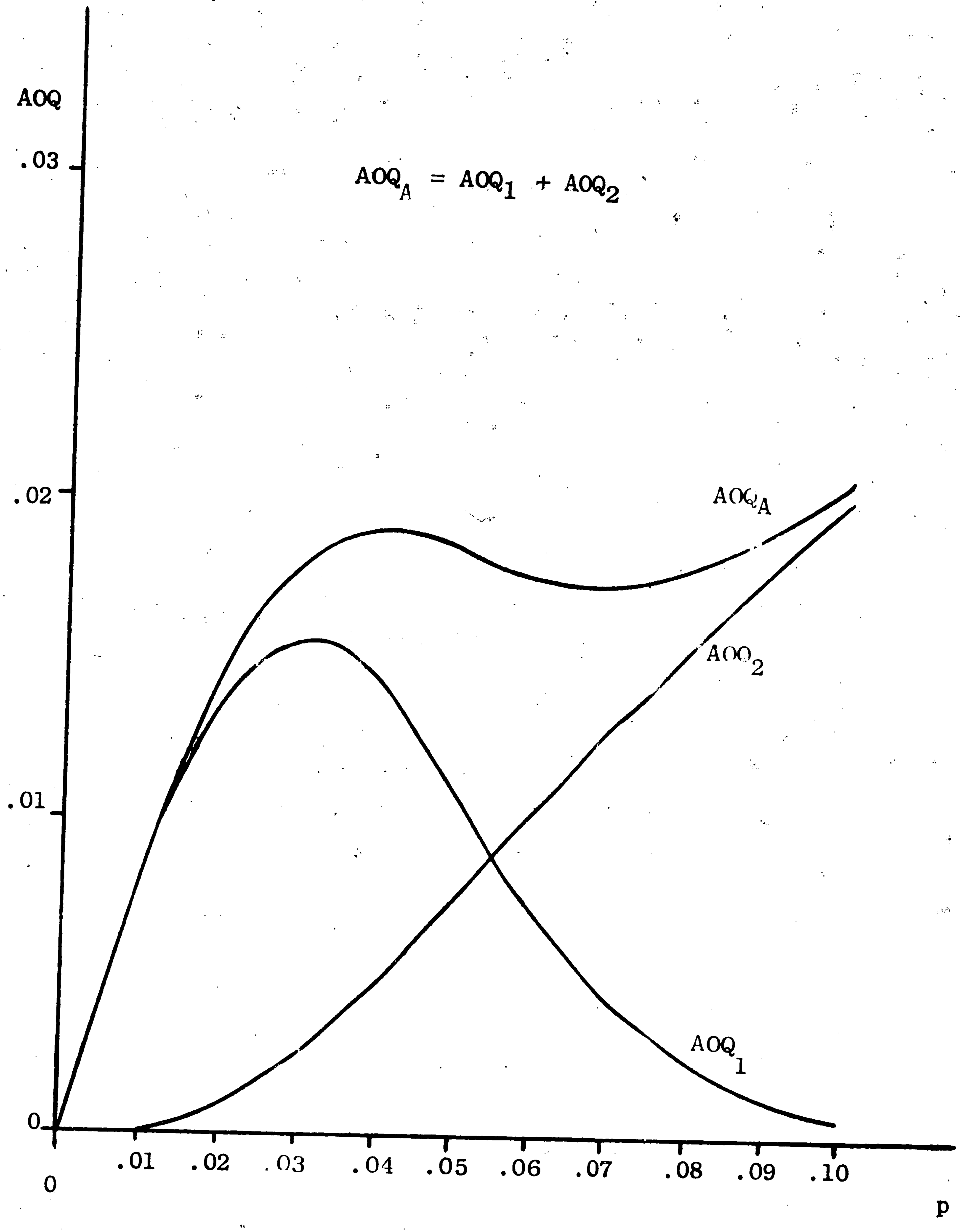


Figure 9. AOQ Curves: f Equal 10%, i Equal 75 Units, and A Decreasing to 80%. AOQ of Examined and Unexamined product

This specific example illustrates a difference between actual and expected results when the detailer is only 80% accurate, or when his accuracy falls off to 80% in a run of i units. Depending upon how critical the AOQ or AOQL really is to the user, this discrepancy may or may not be large enough for concern. However, from the standpoint of this thesis the presence of a difference and its determination, not the actual numerical result, is of greatest importance.

In the next section of the thesis methods will be developed to overcome the effect of detailer inaccuracy.

IV Development of a Continuous Sampling Plan to Account for Detailer Inaccuracy

IV-A Approaches Presently Used

It was indicated in the purpose and scope section of this thesis that different variations of Dodge's basic plan are in actual use. Further, it was indicated that there are different variations of the plan because of inaccuracies on the part of the people involved. In the previous section it was shown that actual results due to this inaccuracy can deviate significantly from the expected results. Thus, it would generally be unsound to use Dodge's basic formulas and nomograph to determine the characteristics of a specific plan, if it is known that the detailer is not accurate.

The fact that there are attempts to account for inaccuracy would tend to imply some improvement in average outgoing quality. However, as far as the author has been able to determine, all of the variations of the basic plan simply specify a different procedure to follow upon finding a defect in the sampling state. There has been no mathematical or formal development to determine the results of such procedures.

The plans mentioned above can actually be classified as one of two main variations, depending upon the specification of procedure upon finding a defect in the sampling state. For ease of reference these two classes will be denoted Variation I and Variation II. Both variations are used under the following policy statements:

1. The inspection function is to cover only that part of the

plan that concerns sampling. That is, it will usually not be in an inspector's job description that he should detail the product.

2. The detailer will be a person from the operating department responsible for the defect.

Further, in both variations the inspector selects a predetermined f percent (or fraction) of the product in such a manner as to assure an unbiased sample. Upon the event of a defect appearing in the sample, however, the procedures deviate.

IV-A-a Variation I

In addition to the basic plan, which is the foundation for his theory, Dodge prescribes a manner of plan administration. The plan specifies what procedure is followed when a defect is found, and is as follows:

1. A detailer is notified and begins detailing i units.
2. The inspector continues inspecting the fraction f.
3. The detailer details i units <u>except</u> those required by the inspector for his sample.
4. If a defect is found by either the inspector or detailer, the i count starts over.

This obviously is different from the basic plan, and these differences will be discussed.

IV-A-b Variation II

The other main deviation from the basic plan (28), (27) is based on the same policy as the variation above, however, it provides for a more direct reinspection. The procedure upon finding a defect is as follows:

1. A detailer is notified and begins detailing i units.
2. The inspector continues to select the fraction f after the product has passed the detailer.
3. If a defect is found by either the inspector or detailer, the i count starts over.

and/or

4. The inspector may be provided with the option to stop inspecting (thus, stop product shipment) if he feels it is necessary due to improper detailing.

Note that condition 4 is an and/or condition. In the procedure specified in the Statistical Quality Control Handbook put out by the Western Electric Co. (28), this is in addition to condition 3. However, in the continuous sampling plan handbook published by the Department of Defense (27) this condition is the only one specified. They specify that, if a critical defect is found during reinspection (they actually call reinspection "verifying inspection"), such inspection will cease until:

(a) action has been taken to improve the process average, and

(b) detailing has been improved through the provision of better supervision and/or by retraining the detailer.

The differences between the basic plan and the two main variations are, therefore, in the average fraction of the total product looked at and the manner in which the check on detailed product is handled. In the basic plan the inspector stops inspecting while detailing is performed. In both variations the inspector will sample a fraction f of all units sent to the place of inspection.

The basic plan provides no reinspection. The first variation provides a check on the period of detailing, but not on the actual performance of the detailer. The inspector designates a particular unit to be inspected and the detailer doesn't look at it. Therefore, the inspector never reinspects, nor samples a detailed unit. The level of protection of this plan compared to that of the basic plan will be higher. The degree of improvement will depend on whether or not the units sampled and found good are to be used to clear the i. This is discussed further in section IV-B.

In the second variation a direct check is provided on the detailer since his results are directly sampled. Thus, there will be an improvement in average outgoing quality because it is assumed that the inspector will detect a bad unit whenever it is in his sample.

As just indicated, the first variation is not a reinspection, nor is it a direct check on the detailer, but there will be an improvement in outgoing quality. The inspector will be present during a period of detailing, and this will permit the following improvements of possible improvements:

1. The AOQ is directly improved since the inspector substitutes for the detailer on f of the units present in a detailing sequence.
2. The inspector will be present to detect changes in the process average.
3. The inspector will have a constant work load.
4. The inspector will be available to "keep an eye" on the detailer, and to aid in making proper decisions with respect to marginal defects.

Thus, for the above reasons, Variation I as well as Variation II will be given formal development in the next section.

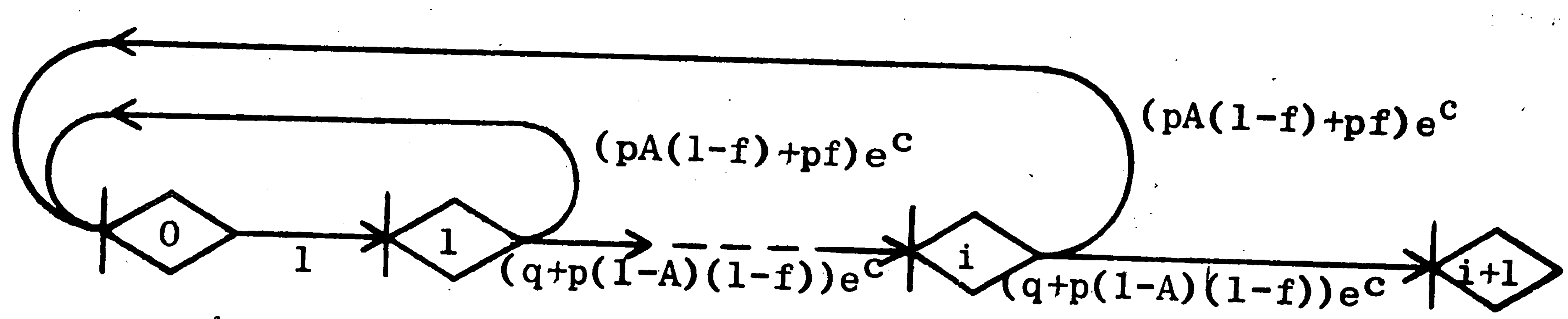
The development will be carried out only for the case when A is constant because of the difficulties in finding a true expression for the average outgoing quality when A is raised to the T(t). If necessary, however, the following development could be adapted to determine a true average fraction inspected quantity for this latter case.

IV-B Mathematical Development

IV-B-a Variation I

For the first variation, the values will depend upon whether or not good units, found by the inspector during a period of detailing, are to be used to clear the i. The situation where they are included will be developed first. The subscript Ia will be used to denote the appropriate values for this case. For example, U_{Ia} is the mean number detailed, i.e., the mean number detailed and inspected in the detailing state. V will have the same meaning as before.

The GERT network for this plan, accounting for an $A<1.0$ is as follows:



The probability of the leaving path, i.e., the probability of increasing the count is the sum of the following probabilities:

1. the probability that the inspector inspects the next unit and it is good = fq
2. the probability that the detailer details the next unit and it is good = (1-f)q
3. the probability that the detailer details the next unit and it is bad, but he does not detect it = (1-f)(1-A)p

$$\text{Sum} = fq + (1-f)q + (1-f)(1-A)p = q + (1-f)(1-A)p$$

The probability that the count starts over is the sum of:

1. the probability that the inspector inspects a bad unit = fq
2. the probability that the detailer details and detects a bad unit = (1-f)Ap

Sum = pA(1-f) + pf

Note that pA(1-f) + pf + q + (1-f)(1-A)p = 1

The mean number of units examined in this state can be computed directly from equation 9. viz.:

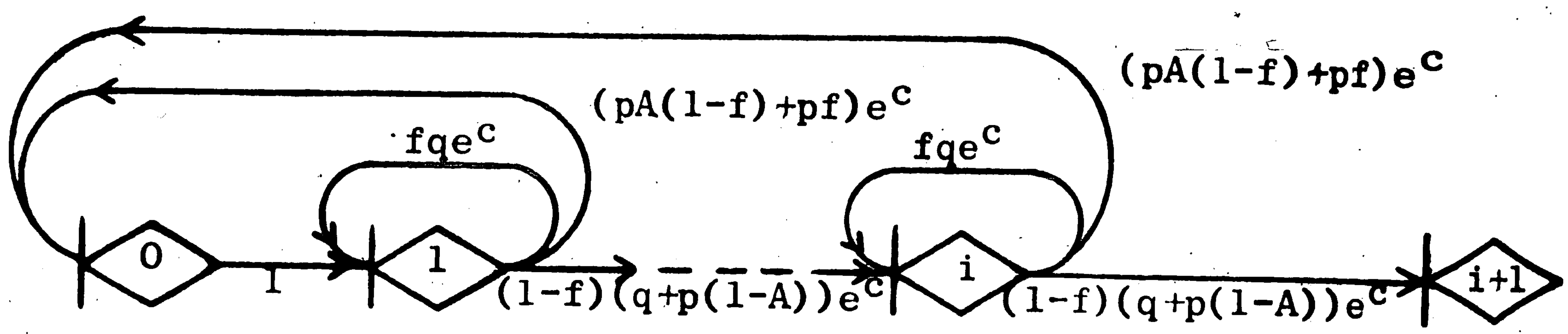
20. $$U_{Ia} = \frac{1-(q+p(1-A)(1-f))^{i}}{(pA(1-f)+pf)(q+p(1-A)(1-f))^{i}}$$

The average outgoing quality is computed in the same manner as in Section III. The exception here is that, of the total fraction in the detailing stage $(U_{Ia}/(U_{Ia}+V))$, the detailer details only (1-f). Therefore, the average outgoing quality is:

21. $$AOQ_{Ia} = p(1-AFI_{Ia})+p(1-A)(1-f)(U_{Ia}/(U_{Ia}+V))$$

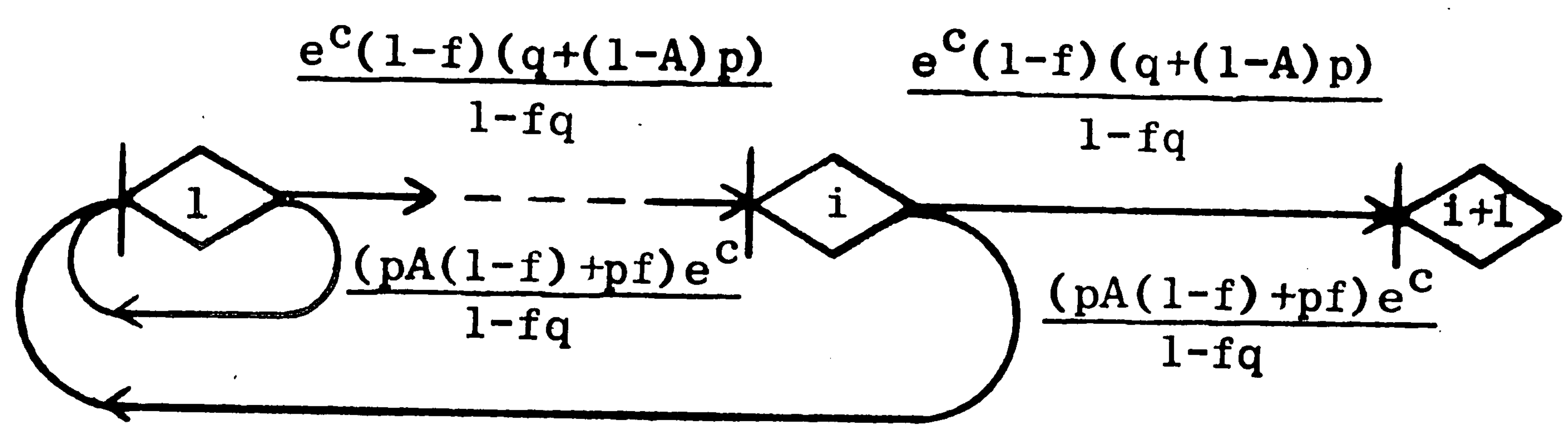
The situation, where the units inspected by the inspector are not used to clear the i, will be considered next. Either method, including or excluding the inspectors sample, is certainly feasible. Including them should tend to release the detailer sooner, but it would also aggravate the problem of keeping account of the number of units detailed. The subscript Ib will be used for this case.

U_{Ib} and AFI_{Ib} can be found with the solution of the following GERT network:



The change in this network with respect to the previous one is the self-loop created when the inspector finds a good unit. In other words, the count remains the same.

The solution of this network would be very cumbersome if it was not for the fact that self-loops can be reduced from the network. An explanation of this is given in Appendix B. The resulting network was obtained by multiplying leaving paths, excluding the self-loop, by $\frac{1}{1-fq}$. The network is as follows:



Therefore, from equation 9:

22. $$U_{IB} = \frac{1 - \left[\frac{(1-f)(q+(1-A)p)}{1-fq}\right]^i}{\frac{(pA(1-f)+pf)}{1-fq}\left[\frac{(1-f)(q+(1-A)p)}{1-fq}\right]^i}$$

The average outgoing quality equation will be in precisely the same form as before, but containing the new expression for the mean number in the detailing sequence:

$$AOQ_{Ib} = p(1-AFI_{Ib})+p(1-A)(1-f)(U_{Ib}/(U_{Ib}+V)) \tag{23}$$

Thus, formulas have been developed for Variation I to determine the average fraction inspected and average outgoing quality. Variation II will be considered next.

IV-B-b Variation II

In this plan, the inspector reinspects detailed product at the fraction f.

This plan is similar to the basic plan in that every unit in the detailing state is looked at by the detailer. It is assumed that the inspector is located after the detailer, and that a detailed unit is immediately sampled with probability f. With this situation, the detailer will not detail any more than necessary.

The probability that the count is increased is the sum of:

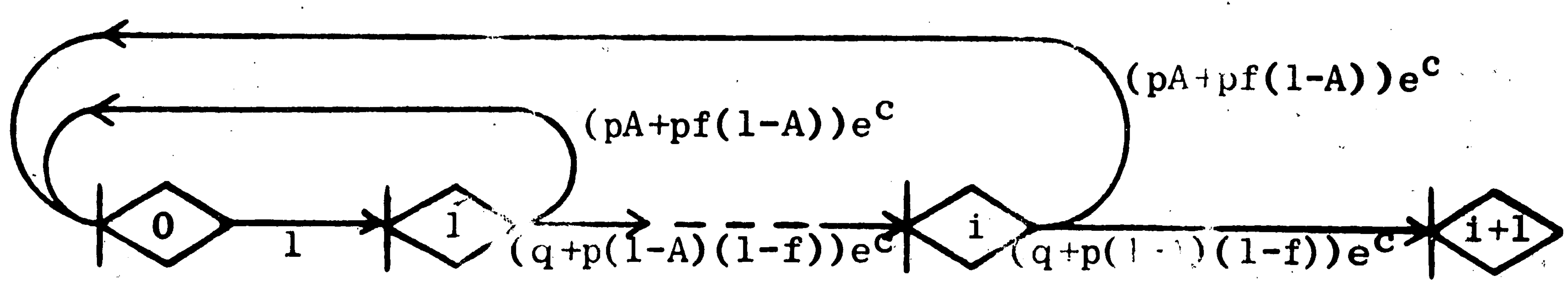
1. The probability that the unit is good = q, and
2. The probability that the unit is bad and it was not detected by the detailer, nor was it sampled by the inspector = p(1-A)(1-f).

The probability that the count starts over is the sum of:

1. The probability that the unit is bad and the detailer detects this fact = pA, and
2. The probability that the inspector samples a bad unit

passed by the detailer = fp(1-A)

The GERT representation for this is as follows:



The interesting result of this plan and its network is that, mathematically, it has the same probabilities as the network for Variation I, when the inspectors sample was used to clear the i. Thus, using the subscript II for the values in this case:

24. $U_{II} = U_{Ia}$ and $AFI_{II} = AFI_{Ia}$

However, AFI_{II} is not the true or total average fraction inspected in this plan. In Variation I the detailer details $(1-f)U_{Ia}$ units and the inspector inspects fU_{Ia} units. In Variation II the detailer details every unit in the detailing sequence, or U_{II}, and in addition, the inspector reinspects fU_{II}. Therefore, the total average fraction inspected and detailed under this plan is:

$$\frac{fV+U_{II}(1+f)}{U_{II}+V}$$

AFI_{II} is, however, the appropriate value to use in determining the AOQ_{II} because this will determine the fraction of product not looked at. viz.:

$$1-fV/(U_{II}+V) - U_{II}/(U_{II}+V)$$

Therefore, the following result is true:

25. $$AOQ_{II} = AOQ_{Ia} = p(1-AFI_{II}) + p(1-A)(1-f)(U_{II}/(U_{II}+V))$$

The implication of this, from the standpoint of the thesis, is that the expected AOQ's are the same, but it takes more inspection to attain this quality level under Variation II. Therefore, Variation II will no longer be considered. This leaves Variation I, including or excluding the inspectors sample in the i count, to be evaluated from the standpoint of improvement over the situation when no inspector is present during detailing.

IV-C Improvement in the AOQ with Variation I

Graphs similar to those in the previous section were plotted to illustrate the expected improvement under this variation. Figures 10 and 11 illustrate the same general effect as did the basic plan with an inaccurate detailer. That is, for a fixed f, the effect increases radically with increasing i, and the spread in Δ(AOQ), as A decreases, remains about the same.

These two plots are for Variation Ia, and the improvement over the basic plan would be as indicated. The amount of improvement increases as p increases. A similar investigation, which was carried out for Variation Ib (the results are not included), illustrated the same general relationships, but with some added improvement.

Figures 12 and 13, however, show a greater deviation in the change in Δ(AOQ) for a fixed i, A, and p and an increasing f. Figures 12 and 13 are for Variations Ia and Ib, respectively. As f increases under either variation, the corresponding Δ(AOQ) increases at a decreasing rate until an f is finally reached where the Δ(AOQ) starts to decrease. This is not true for the basic plan in the face of detailer inaccuracy. Further, by comparing Figures 12 and 13, it can be seen that the degree of improvement and the extent of the result just mentioned is greater for Variation Ib.

The results with these variations indicate that, for a fixed i, A, and p, the greatest degree of improvement lies with the choice of f. That this is so is due to the following reasons:

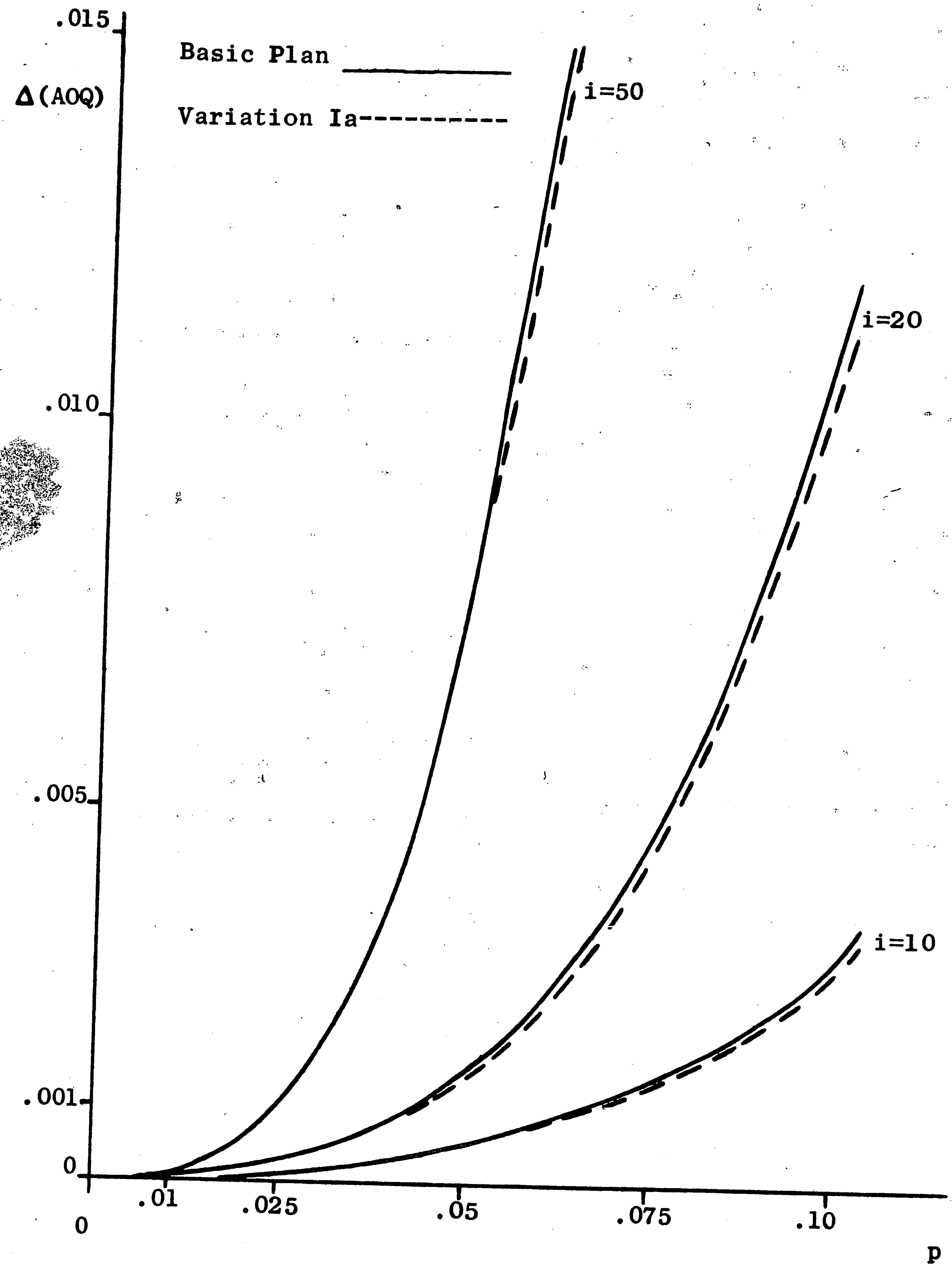


Figure 10. Improvement in Δ(AOQ) With Variation Ia: f Equal 5%, A Equal 75%, and i Varied (10,20,&50 Units)

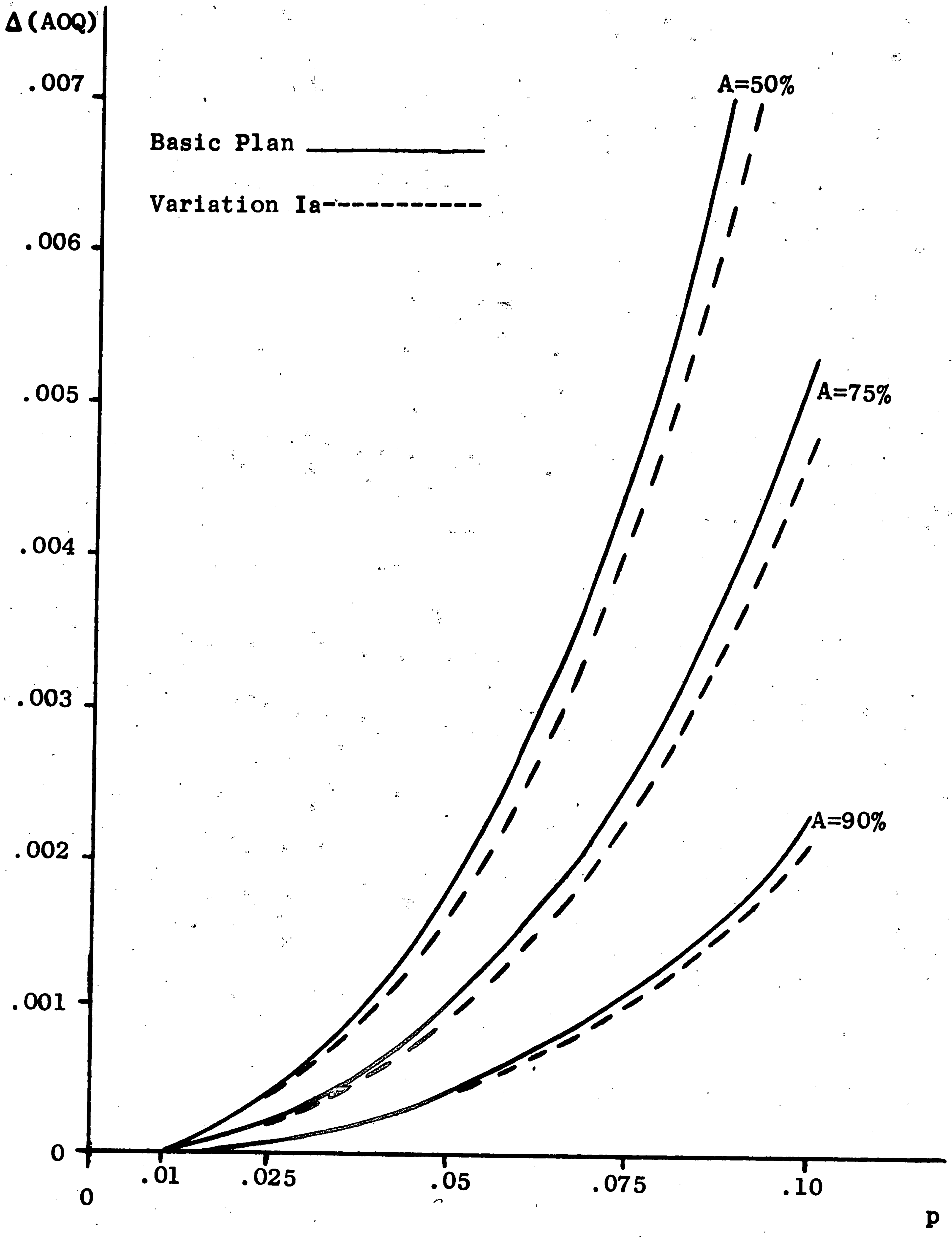


Figure 11. Improvement in Δ(AOQ) With Variation Ia: f Equal 10%, i Equal 10 Units, and A Varied (50,75,&90%)

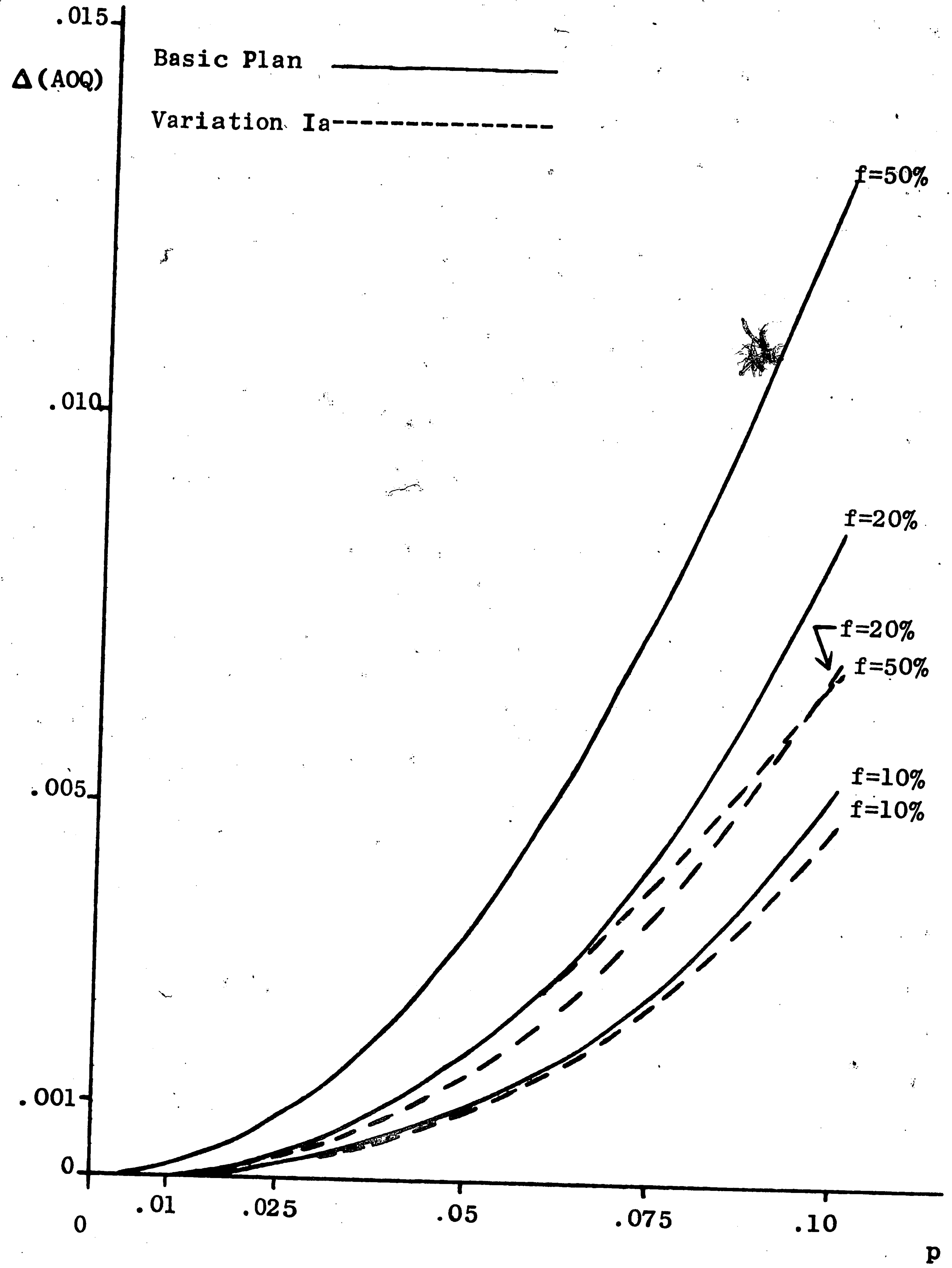


Figure 12. Improvement in Δ(AOQ) With Variation Ia: i Equal 10 Units A Equal 75%, and f Varied (10,20,&50%)

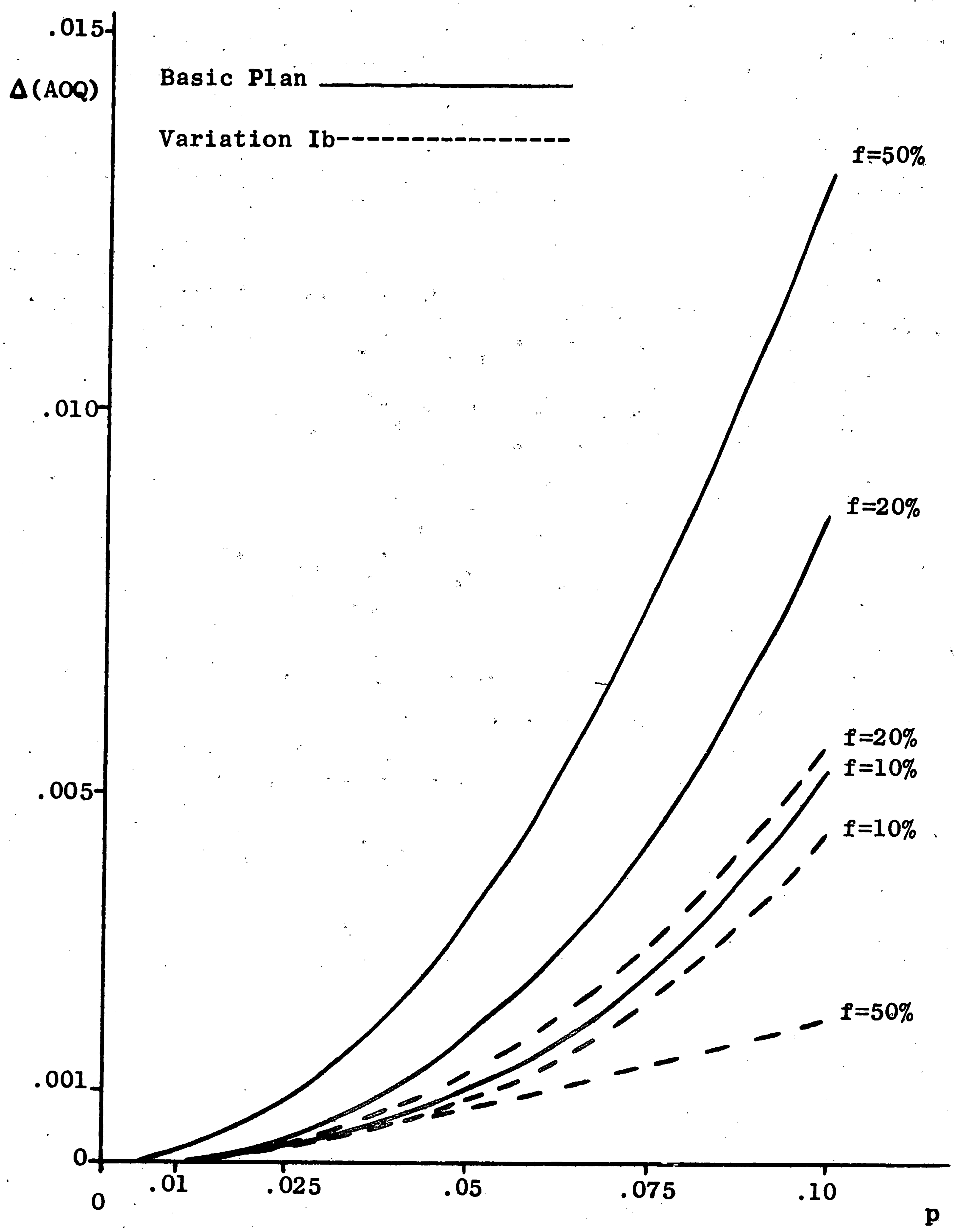


Figure 13. Improvement in Δ(AOQ) With Variation Ib; i Equal 10 Units, A Equal 75%, and f Varied (10,20,&50%)

1. The term $p(1-A)(1-f)(U_I/(U_I+V))$ for both variations will approach zero as f approaches 1.
2. As f is increased under the second variation, the quantity U_{Ib} must also increase because fewer good units are being used to clear the i. In Variation Ia, however, increasing the f also increases the probability of clearing.

The results with respect to the standard plan are illustrated in Figures 14 and 15. From Figure 14 it is evident that the both plans give the same slight improvement in AOQ for the expected process average of 2% defective. For p = 2%, AOQ_{Ia} = .0146, and AOQ_{Ib} = .0144. This difference increases as p increases until p approaches a value such that U_{Ia} and U_{Ib} both become very large. At this point both values approach the same quantity, viz., $(1-A)(1-f)p$.

The degree of improvement in the AOQ by using Variation Ib, as opposed to Variation Ia, would depend upon the specific situation. For the standard plan a decision, as to which variation to use, would depend upon the trade-off between the amount of improvement, and the additional effort needed to attain the improvement. Figure 15 indicates the difference in the total average fraction inspected curves for the standard plan. There appears to be little difference, however, and the decision would probably be based on more intangible reasons.

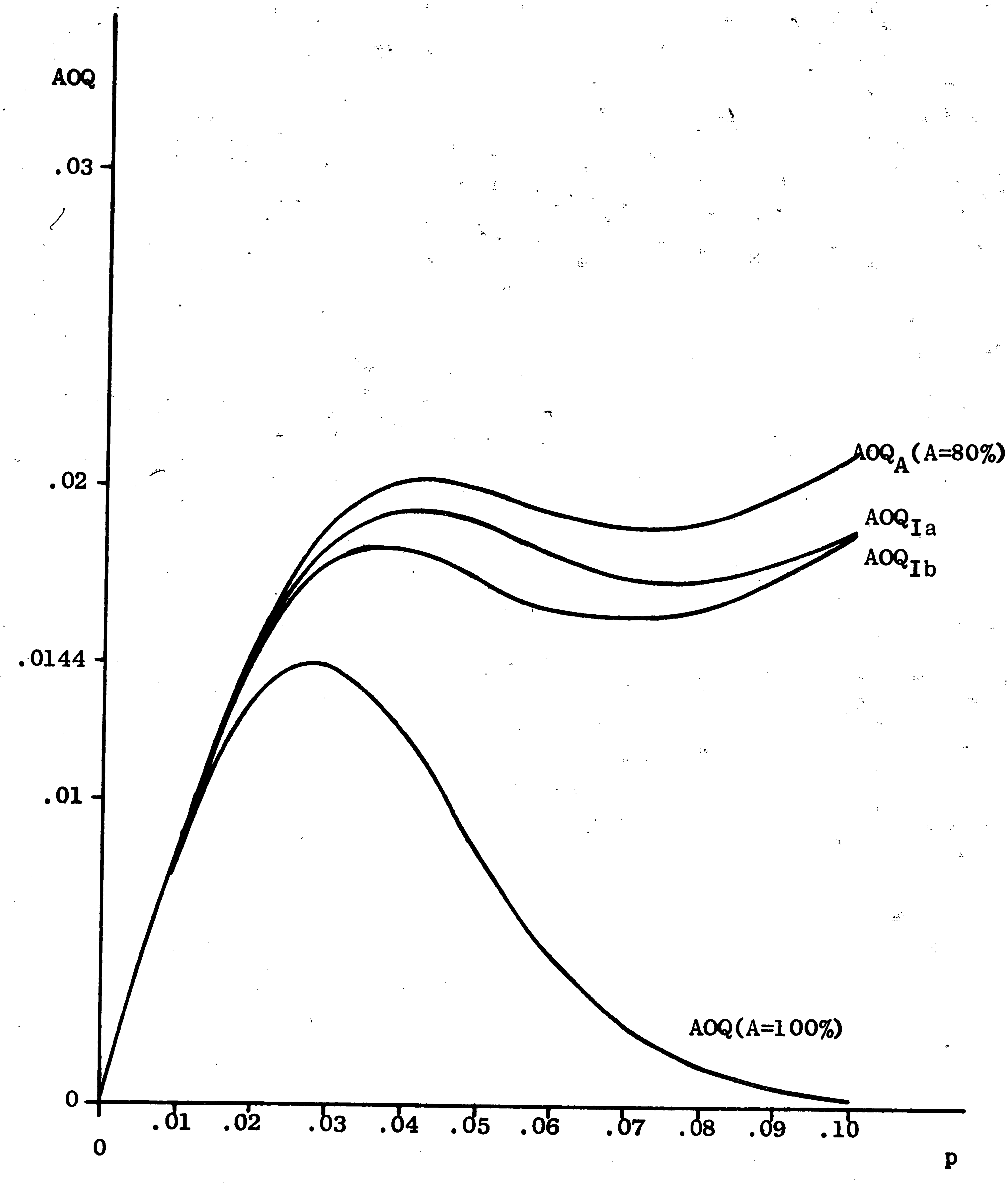


Figure 14. AOQ Curves: f Equal 10%, i Equal 75 Units, and A Equal 80%. Variations Compared.

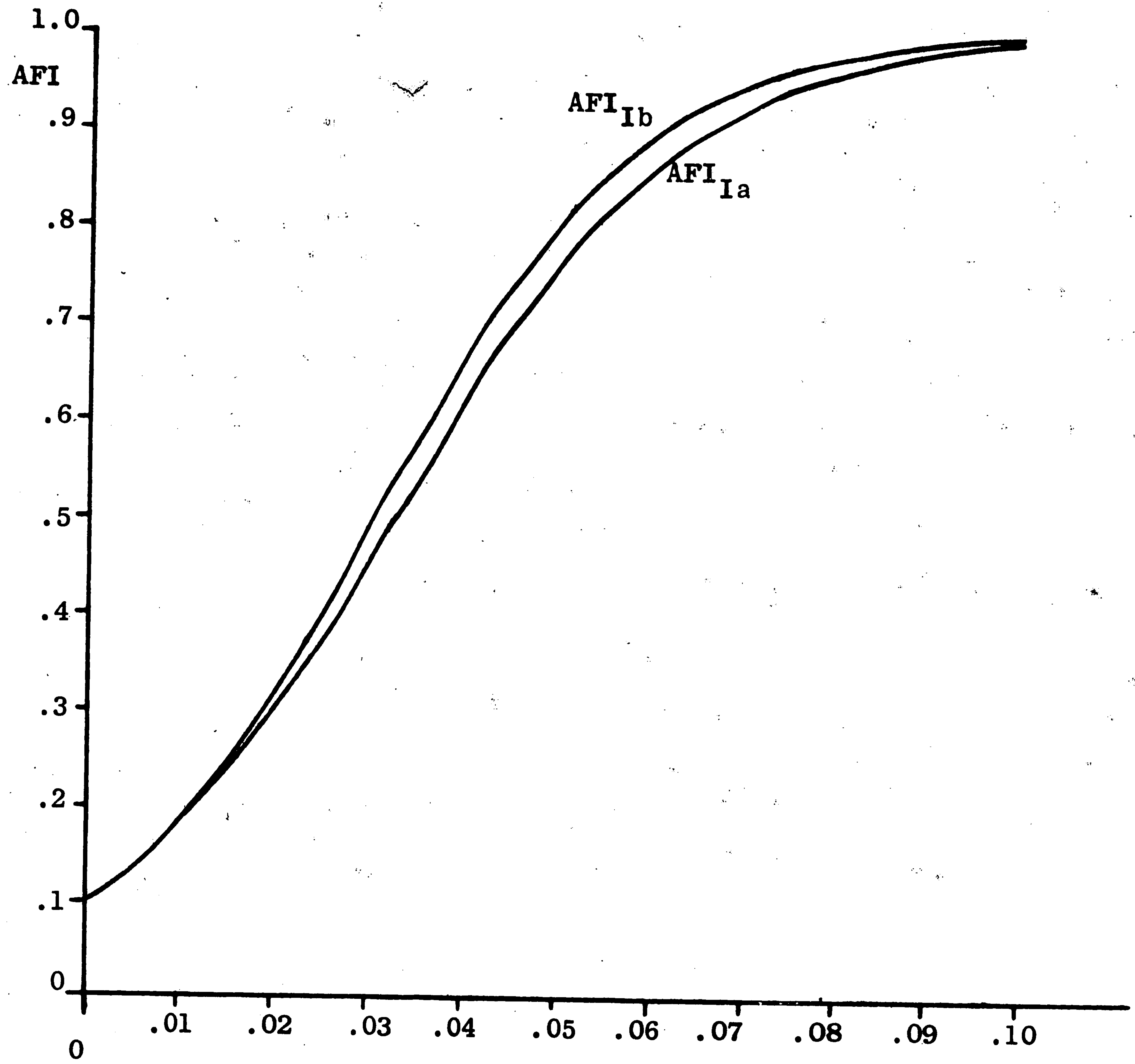


Figure 15. AFI Curves: f Equal 10%, i Equal 75 Units, and A Equal 80%. Variations Compared

IV-D Methods for Further Improvement

The possibility for further improvement by increasing f will now be investigated. This possibility was mentioned in the previous section. The investigation will be carried out for Variaion Ib only as it yields a lower AOQ than Variation Ia, and also represents the more difficult case.

It is important to note that f cannot be increased indiscriminantly. It has been stated that the policy governing these plans require that inspectors sample only, that is, the inspector will not perform 100% inspection.

A further mention as to the value of f is appropriate here. If f is raised too high, the inspector may be subject to errors for the same reasons as the detailer. This cannot be tolerated under the assumptions of this thesis. Also, increasing the value of f during the sampling state is no direct solution to the problem of inaccuracies during the detailing state.

Most importantly, the value of f during the sampling state is physically limited by the nature of the requirements being examined. During the sampling state an inspector may be inspecting a unit for more than one grouping of defects. He must select a unit in an unbiased manner and examine it for all requirements. The detailing state on the other hand is concerned only with the defect or group of defects which caused the rejection.

For these reasons, two approaches for increasing f to attain the desired AOQ will be investigated. The first of these approaches is to determine the equivalent new f required to yield the specified AOQ. In this case f is to remain the same in both states. Secondly, the situation will be investigated where the f during sampling remains as specified, but the f during detailing can be increased to yield the specified AOQ. The following investigations were aided by further modification of the previously used computer program.

The equivalent f, where f is to remain the same in both states, can be determined by solving the expressions for the AOQ simultaneously. That is, for what f_{Ib} is:

$$p(1-AFI)=p(1-AFI_{Ib})+p(1-A)(1-f_{Ib})(U_{Ib}/(U_{Ib}+V_{Ib}))$$

or simplifying

$$\frac{V(1-f)}{U+V} = \frac{V_{Ib}(1-f_{Ib})+(1-A)(1-f_{Ib})(U_{Ib})}{U_{Ib}+V_{Ib}}$$

Note, f in the term on the left must remain as originally specified.

Table I shows the results for various combinations of i, f, and p for an A of 80%. The values in the table represent the ratio; f_{Ib}/f. The ratios indicate that f must be increased as p increases; and for larger values of p, f must be further increased as i increases. For the standard plan, see Table I, an increase in f of only 3% would be required.

This method, however, would have to be used with caution. For example, assume that i = 50 and p = 2.5%. The table indicates a ratio of 1.4 for both values of an original f of 5 and 20%. For the first f this is an increase of only 2%, but for f = 20, f_{Ib} must be

i	10			25			50			75		
f%	5	10	20	5	10	20	5	10	20	5	10	20
p												
.005	1.1	1.1	1.05	1.1	1.1	1.05	1.2	1.1	1.05	1.2	1.1	1.1
.010	1.1	1.1	1.05	1.1	1.1	1.05	1.2	1.1	1.1	1.2	1.2	1.1
.020	AOQ = .0132 AOQ_{Ib} = .0144 (f=.1)										1.3	
.025	1.1	1.1	1.05	1.1	1.2	1.1	1.4	1.3	1.4	1.6	1.4	1.3
.050	1.1	1.1	1.1	1.2	1.3	1.2	1.8	1.6	1.45	2.8	3.0	3.05
.075	1.1	1.2	1.15	1.6	1.4	1.3	2.8	3.1	3.15	14.8	8.8	4.75
.100	1.2	1.2	1.15	1.8	1.7	1.55	11.2	7.8	4.5	20.0	10.0	5.0

TABLE I. Ratio of f_{Ib}/f, Where f_{Ib} is to be Used in Inspection and Detailing Under an A of 80 Per Cent

28%, or an increase of 8%. This increase of 8% might seriously restrict the inspector's ability to perform an adequate job during the sampling state.

The second approach of only raising f during the detailing state will be investigated next. The previous equality becomes:

$$\frac{V(1-f)}{U+V} = \frac{V(1-f)+(1-A)(1-f_{Ib})U_{Ib}}{U_{Ib}+V}$$

where: f represents the fraction inspected in the sampling state and remains fixed.

f_{Ib} represents the fraction inspected in the detailing state and will be increased.

The results are shown in Table II, and values in the table give f_{Ib}/f. For low values of p a considerable increase in the fraction inspected during detailing is required for an exact equality of AOQ and AOQ_{Ib}. This is due to the fact that the denominators in the previous equations are approximately equal. Thus $(1-A)(1-f_{Ib})U_{Ib}$ must approach zero, or f_{Ib} must become large. Because of this, the computer program was rerun to yield an f_{Ib} in the detailing state such that:

$$AOQ_{Ib} \leq AOQ + .00001$$

The new f_{Ib}/f appears in parentheses below the original ratio in Table II, whenever the above constraint is active.

The adjusted entries in Table II vary in the same manner with i, f, and p as did the entries in Table I, but are generally of higher magnitude. For large values of p and i, the entries become the same in both tables, and the present method of only increasing f_{Ib} in the detailing state would become superior in this region from the standpoint of less overall inspection effort.

The new f_{Ib} required under the specific plan is 31% during the detailing state. Depending upon the actual operating conditions this may or may not be realistic. The effect of both approaches on the specific plan have been plotted in Figure 16. Increasing f_{Ib} during the detailing state from 10 to 31% yields an AOQ curve very close to the desired for $p \leq 4\%$ defective. If p had a reasonable probability of shifting from the expected 2% to 4%, this latter approach would be more desirable provided the increased f_{Ib} is reasonable.

The question of selecting a continuous sampling plan on an economical basis is taken up in the next section.

i	10			25			50			75		
f%	5	10	20	5	10	20	5	10	20	5	10	20
p												
.005	14.4	7.2	3.7	12.2	6.2	3.2	10.2	5.2	2.7	9.0	4.7	2.5
	(1.0)	(1.0)	(1.0)	(1.0)	(2.3)	(2.8)	(4.8)	(3.9)	(2.5)	(6.6)	(4.1)	(2.3)
.010	12.6	6.4	3.3	10.2	5.2	2.7	8.2	4.3	2.3	7.2	3.8	2.1
	(1.6)	(4.2)	(2.8)	(8.0)	(4.7)	(2.5)	(7.6)	(4.1)	(2.2)	(7.0)	(3.7)	(2.0)
.020	AOQ = .0132				AOQ_{Ib} = .0144 (f=.1)						3.1	
.025	10.0	5.1	2.7	7.6	4.0	2.1	6.2	3.3	1.8	5.6	3.1	1.8
	(9.2)	(4.9)	(2.2)		(3.9)							
.050	8.2	4.2	2.2	6.2	3.3	1.9	5.6	3.1	1.9	6.2	4.1	3.1
	(8.0)											
.075	7.2	3.8	2.0	5.8	3.1	1.8	6.4	4.2	3.2	14.8	8.8	4.8
		(3.7)										
.100	6.6	3.5	1.9	5.6	3.2	2.0	11.4	7.8	4.5	20.0	10.0	5.0

TABLE II. Ratio of f_{Ib}/f, Where f_{Ib} is to be Used Only in Detailing Under an A of 80 Per Cent

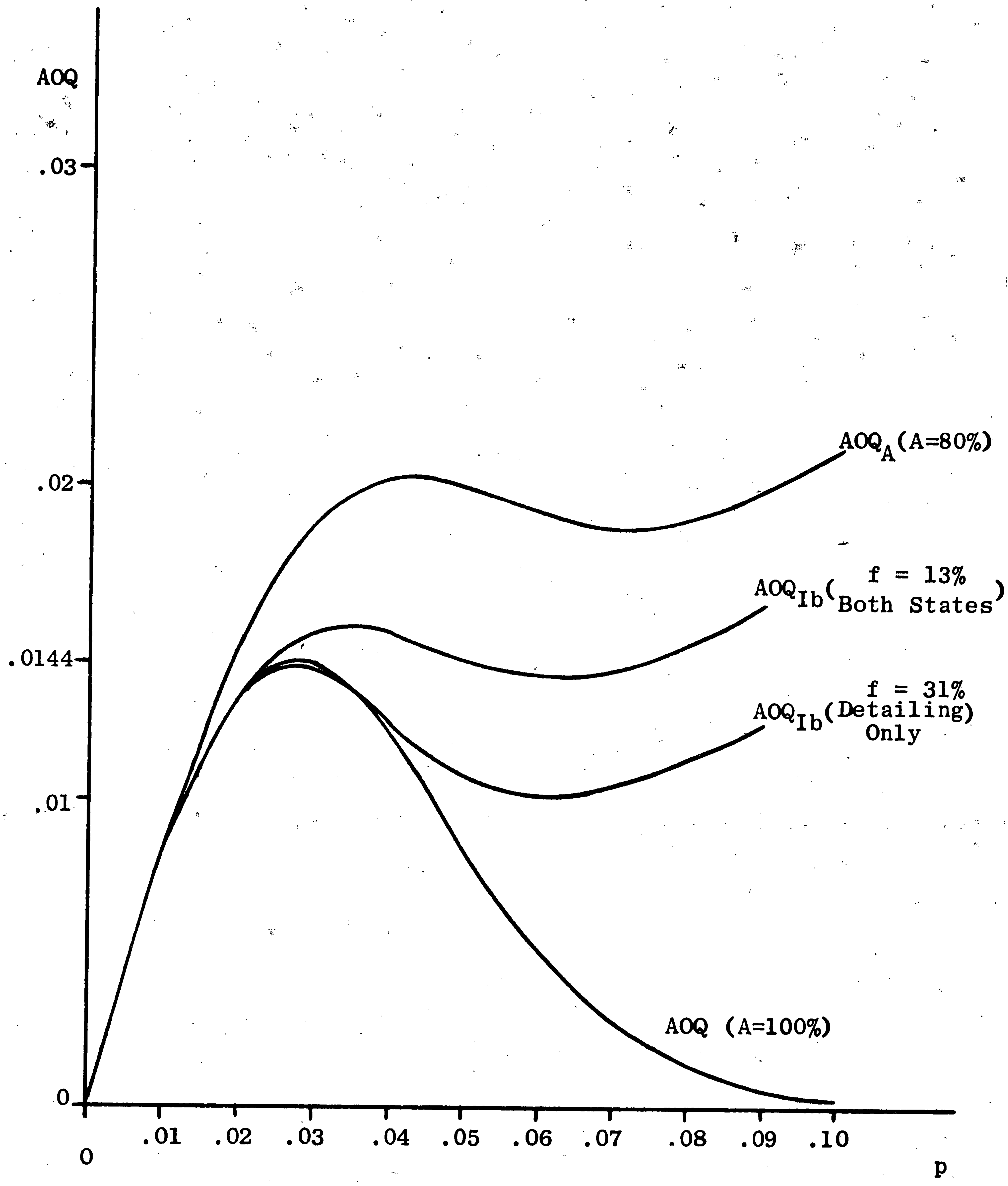


Figure 16. AOQ Curves: f Equal 10%, i Equal 75 Units, and A Equal 80%. Increasing f During Both States vs. During Detailing Only

V The Economic Selection of a Continuous Sampling Plan When the Detailer is Inaccurate

In the previous sections of the thesis the effect of an error on the part of the detailer was investigated, and various methods were developed by which this effect might be overcome. Further, the methods of plan selection that were indicated were based on physical requirements. Namely an AOQL was chosen based on the process average, and an f and i combination were chosen to fit physical limitations. However, this method of selection is not always optimal from the standpoint of trading off costs between detailing and inspection, and the resultant cost of a defect.

This section of the thesis will consider the development of such a model for use when the detailer has a probability A that a defect is detected. As indicated in the purpose and scope section of the thesis, there have been two different approaches taken for the economic selection of a continuous sampling plan. The first approach is that taken by Anscombe (1), (2), where an attempt is made to find an optimal f and i combination without regard to any resulting AOQL. Anscombe states the following:

> "When one fairly considers the matter, it is not clear what bearing the AOQL has on rectifying inspection. The AOQL is a statistician's guarantee, quoted because it can be calculated easily, not a <u>user's</u> requirement. No user of inspection not corrupted by contact with statisticians, would ever think of setting himself on AOQL as a target."

While this is a strong point, it is often the case that an AOQL is specified. For example, this is true when a large company has a quality organization separate from the inspection department. This former organization represents the customer, and establishes quality standards which can be translated into AOQLs (29). Thus the second approach is to find the optimal f and i combination for a specified AOQL. This approach was used by Fry (11). Other economic formulations using one of these two approaches can be found in the bibliography given by Anscombe (1).

In both approaches it is desired to minimize the expected total cost of an average cycle, that is, the cost is per a state of detailing and a state of inspecting. Neither of the previous approaches differentiated between the inspector and detailer, as they considered the cost of inspection versus the cost of a defect. In the following development a third cost, the cost of detailing, will be considered separate because of the recognition that the detailer is doing non-productive work, and is only A per cent accurate. The three costs are defined as follows:

C_1 = unit cost of detailing

C_2 = unit cost of inspection

C_3 = penalty cost of passing a defect.

V-A Development Without Regard to an AOQL

The expected total cost of a cycle using the first approach would be:

$$26. \qquad ETC = C_1 \frac{U_A}{U_A+V} + C_2 \frac{fV}{U_A+V} + C_3 p \left[1 - \frac{U_A+fV}{U_A+V} + (1-A) \frac{U_A}{U_A+V} \right]$$

This equation is the same as Anscombe's when $C_1=C_2$ and A=1. Upon simplifying, the equation can be rewritten as:

$$26a. \qquad ETC = \frac{C_1U_A+C_2/p+C_3/f-C_3+C_3pU_A(1-A)}{U_A+1/fp}$$

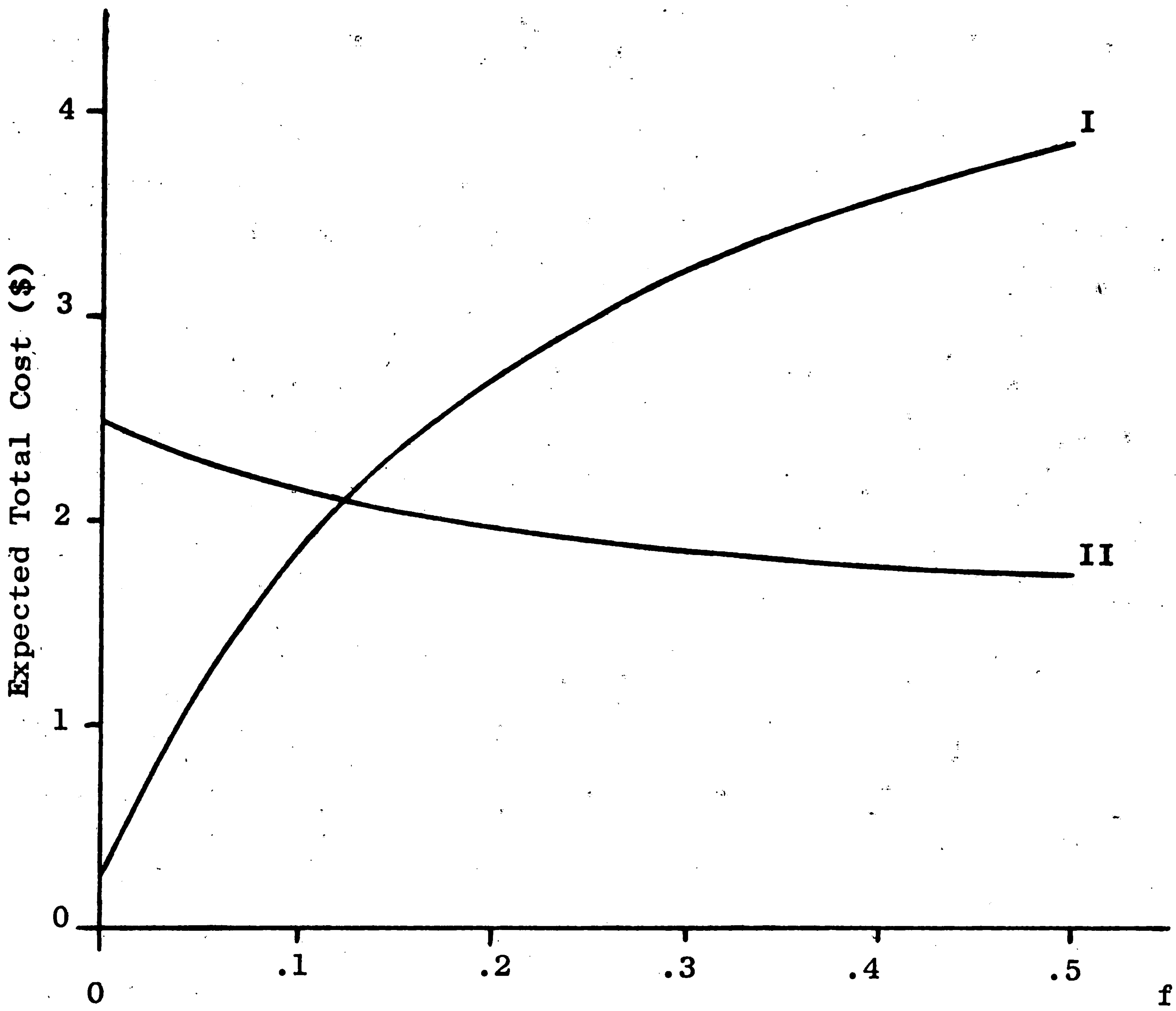
or

$$= \frac{fX+C_3}{fU_A+1/p}; \text{ where: } X=C_1U_A+C_2/p-C_3+C_3pU_A(1-A)$$

Differentiating this equation with respect to f yields:

$$\frac{\partial ETC}{\partial f} = \frac{(fU_A+1/p)X-(fX+C_3)U_A}{(fU_A+1/p)^2} = \frac{X/p-C_3U_A}{(fU_A+1/p)^2}$$

The minimum for this equation must occur at either f = 0, or f = 1 for any fixed set of variables, C_1, C_2, C_3, i, p, and A. This must be the case since the denominator is always positive, and for any fixed set of other variables the numerator is always positive or always negative. Thus the curve must always be rising or failing, and the concavity for a given set of other variables could be determined from the sign of the second derivative. Examples of this result is shown graphically in Figure 17 for $0 \leq f \leq .5$ only.



Cost	Curve I	Curve II
C_1	5	1
C_2	2	2
C_3	10	100

Figure 17. Total Expected Cost vs. Fraction Inspected: i Equal 75 Units, p Equal 2.5%, and A Equal 80%.

The remaining question lies with the value of i. If the optimal value of f is 0, then it means there should not be a sampling plan. Therefore, i has no meaning because a detailer will never be present. The expected cost per cycle is C_3p. On the other hand if f = 1, then the question as to what i should be must be answered. Note that if f = 1, a defect will be found in the sampling state for any p > 0. For f = 1 the cost equation can be written as:

27. $$ETC = \frac{U_A(C_1+C_3p(1-A))+C_2/p}{U_A+1/p}$$

U_A is a function of i, p, and A, which are fixed for any situation. Further, U_A increases as i increases, and we can determine the minimum cost by differentiating with respect to U_A only. However, by inspection, it is seen that the equation is of the same form as before, and therefore, the optimal mean number to be detailed is either 0 or ∞. Likewise i is either 0 or ∞ (a more rigorous treatment is provided in Appendix C).

Thus the results with this equation are "all or none", that is, there should be no inspection of any form, or the inspector inspects everything, until he finds a defect and the detailer details the remaining. The equation itself is useful, however, as it will give the cost of any plan, and could also be used as a tool to decide whether or not to have a plan in the first place.

This result is not uncommon in this area of economic analysis (4), (19), (31), and Anscombe arrived at similar conclusions when

$C_1=C_2$ and A=1. To avoid these two extremes, Anscombe developed a working rule, employing a "logistic" function of p. This was done so that the average fraction inspection (AFI) would be approximately 1/2 when $p=C_2$. His cost equation was for $C_1=C_2$, A=1, and C_3=cost of a defect = one cost unit. The cost equation, which follows, shows that the AFI should be close to 0 if $p < C_2$, or close to 1 if $p > C_2$:

$$ETC = C_2(AFI) + p(1-AFI) = p+AFI(C_2-p)$$

The logistic function for the working rule is:

$$AFI \simeq \frac{1}{1 + e^{-i(p-C_2)}}$$

Investigation of this possibility for $C_i \neq C_2$, $C_3 \neq 1$, and $A \neq 1$, is beyond the scope of this thesis, however, and will be left as an area for future investigation.

V-B Development For a Specified AOQL

The second approach proposed by Fry (11) specifys an AOQL and then selects an optimal f and i combination under this constraint. Fry's cost equation is the same as the previous equation except for terms used to represent his stopping conditions. These latter terms can be disregarded here because it is assumed that the process average is not going to change.

Upon careful examination of this approach, however, it is not clear just what relationship there is between an implicit cost of a defect (the specified AOQL), and a corresponding explicitly stated cost of a defect (C_3 - Fry denotes this C_2 in his paper). It would seem more logical to either impute the cost of a defect, or state it implicitly as in the last section, than to attempt to require both at the same time.

For this reason, the remaining investigation of this approach considers only the imputed cost resulting from a specified AOQL. The remaining explicit trade-off will be between the cost of inspecting and the cost of detailing:

28. $$ETC = \frac{C_1U_A+C_2fV}{U_A+V} = \frac{f(C_1(1-(1-p)^i)+C_2(1-p)^i)}{f(1-(1-p)^i)+(1-p)^i}$$

This equation must be optimized with respect to i and f, subject to the constraint that the f and i combination gives a specified AOQL.

Assuming for the moment that A = 1.0, the relationship relating f, i and the AOQL, was stated in Section II of the thesis, viz.:

12. $$AOQL = \frac{(1-f)(1-p_1)^{i+1}}{fi}$$

where $p_1 = \frac{1+iAOQL}{i+1}$ = the value of p at which the AOQL occurs

Thus an iterative approach can be easily programmed for a computer to search out the optimal cost. This is done by using equation 12 to get a relationship for f in terms of i and the AOQL, substituting this f into the above cost equation with A=1, iterating over possible values of i, and seeking the minimum cost.

The solutions that will be obtained from this method, however, will not be practical in many cases. The above cost equation is very similar to the previous cost formulation which has been proven to be optimal only at f=0, or f=1. This essentially holds here also, and can be seen by setting $C_3=0$ in the previous optimization procedure. However, under the constraining AOQL, f=0 is not a feasible solution. Thus under the constraint f must be some small positive quantity, and i, therefore must be quite large. This fact can be seen from the following set of results obtained with the aid of a computer program for $C_2=4$ cost units, p=.03 and i iterated from 1 to ∞ :

AOQL	C_1	Optimal F	Optimal i	Cost/Cycle
.045	0-21	$\longrightarrow 0$	$\longrightarrow \infty$	$\longrightarrow 0$
	22-100	83.52%	1	3.812-5.774
.025	0	$\longrightarrow 0$	$\longrightarrow \infty$	$\longrightarrow 0$
	1-13	.0004-.059%	203-191	.168-2.162
	14-100	90.48%	1	3.902-6.243

Note that the "all or none" situation is not quite true either when the AOQL < P. This can probably be explained from examination of p_1 in equation 12. For a $C_1=C_2=4$, and an AOQL = .025, the optimal i is 194. Thus:

$$P_1 = \frac{1 + i\ AOQL}{1 + i} = .03$$

Apparently then, when the AOQL is less than the process average (p=.03) and the costs are relatively equal, the optimal situation is to have the AOQL occur at the process average.

The only useful conclusion from this investigation is that the entire method offers little over the unconstrained cost equation presented in the last section. Further, from the standpoint of the objectives of this thesis, it would not serve any purpose to investigate detailing error when using this formulation.

For those situations where some constraining AOQL is required, Duncan (9) provides the following:

> "In a letter to the author Dodge suggests that the process average p should be about two thirds of the AOQL for a continuous-sampling plan to be economical."

Thus by knowing the process average an f and i combination can be selected from Dodge's nomograph to give this AOQL. This can be done whenever the detailer is 100% accurate. However, as pointed out in a previous section of the thesis, this AOQL value will not be achieved for the same f and i combination when A < 1.0.

The remaining portion of this section is devoted to this problem.

Recalling Figure 7 and the accompanying discussion, it was noted that the shape of the AOQ and AOQ_A curves were similar in the proximity of the process average. Thus it will be assumed that an adequate condition will be reached by selecting the first maximum of the AOQ_A curve, when it exists, (i.e., the first hump as p is increased) to be equal to the AOQL. The desired relation will be similar to that illustrated in Figure 16 for the AOQ curve and the lower AOQ_{Ib} curve.

In essence then it is desired to find a relationship similar to equation 12 in order to relate the f and i values to this particular limit on the AOQ_A curve denoted AOQ_{AL}. This can be done by finding the value of p which forces first derivative of the AOQ_A curve to vanish. Since the shape of this curve indicates that there are two values of $0 < p \leq 1$, where this occurs, it will be necessary to select the first value of p. This p will be denoted p_a to distinguish it from Dodge's p_1. The resulting equations are presented below.

From equation 18a:

$$AOQ_A = p\left(1 - \frac{AU_A - fV}{U_A + V}\right)$$

Simplifying:

$$AOQ_A = p\left(1 - \frac{fA}{f + (1-pA)^i(A-f)}\right)$$

therefore:

$$AOQ_{AL} = p_a\left(1 - \frac{fA}{f + (1-p_aA)^i(A-f)}\right)$$

and:

$$\frac{\partial AOQ_A}{\partial p} = 1-p\left[\frac{fA^2 i(1-pA)^{i-1}(A-f)}{(f+(1-pA)^i(A-f))^2}\right] - \frac{fA}{f+(1-pA)^i(A-f)}$$

setting the above equation = 0, and simplifying further:

29. $$\left(\frac{A-f}{f}\right)(1-pA)^{2i} + (2-A)(1-pA)^i - pA^2 i(1-pA)^{i-1} + f\left(\frac{1-A}{A-f}\right) = 0$$

and p_a is the first p as p is increased which provides this equality.

Note that no explicit relationship is available to equate p_a, f, i and AOQ_{AL} as in the case of equation 12. This is due to the fact that the first derivative set equal to zero is a general polynomial, and that there are two possible real roots between $0 < p \leq 1$.

This method must be applied with caution and its results interpreted with care. A computer program was written with the goal in mind to provide a nomograph similar to Dodge's to illustrate combinations of f and i which give the same AOQ_{AL}. This attempt was not successful because the first derivative does not always go to zero. It is possible that there are no real roots to the equation of the first derivative, and all that exists is an inflection point. An example of this is provided below for an A = 80%, and an f = 20%, and in Figure 18 the AOQ_A curve is plotted for i = 15.

i	AOQ_{AL}
10	.099
11	.091
12	.084
13	.078
14	.073
15	NON-EXISTENT

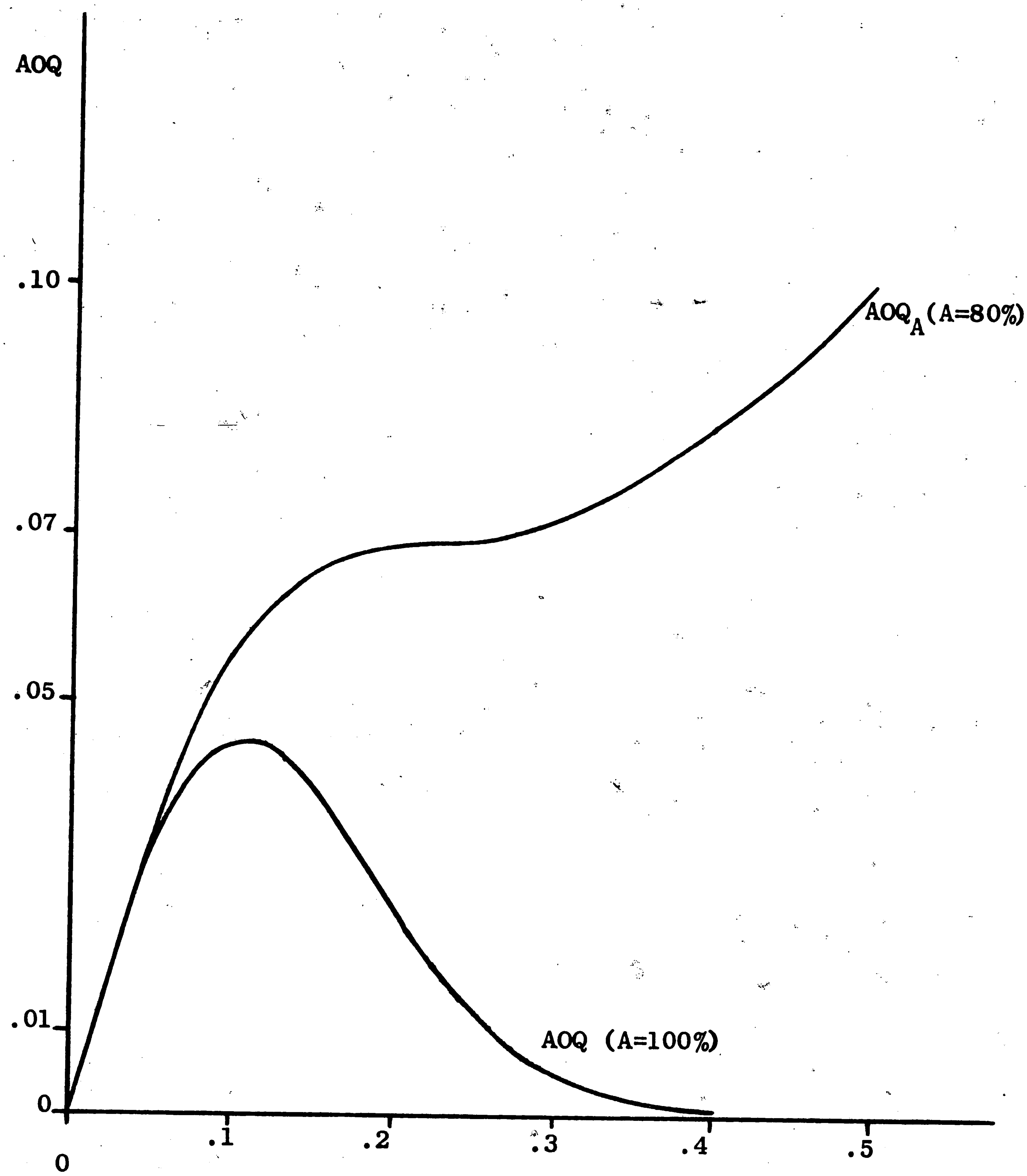


Figure 18. AOQ Curves: f Equal 20%, A Equal 80%, and i Equal 15 Units Inflection Point Only

VI SUMMARY AND CONCLUSIONS

This thesis has investigated the effect of inaccuracies present during the operating of a Dodge continuous sampling plan (CSP-I). In particular, Dodge assumed that all defects appearing in the f (the sample) and the i (the 100 per cent inspection following a defect) will be detected and corrected or replaced.

Emphasis was placed on the latter part of this assumption as it is well known that 100 per cent inspection or detailing as it is referred to in this paper, seldom provides 100 per cent detection and rectification of defects (9),(10),(13). This problem was quantified by establishing a value A to represent the probability of a detailer detecting, and thus rectifying a defect. A sampling procedure was proposed by which the value of A can be determined.

A method for the analytical investigation of A was then developed. This method utilizes the Graphical Evaluation and Review Technique (GERT - 32) as its foundation. Equations were derived using this method which yield the true average fraction inspected (AFI) and average outgoing quality (AOQ) values for a constant A, a specified process average p, and a chosen f and i combination. Equations were also derived for the fatigue case, when the probability of detecting a defect varies with the number of units detailed. The equation for the $\overline{AOQ}$ under this situation could only be approximated, however, as no expression could be found to represent the "overall" probability of detecting a defect.

Results obtained in the investigation of these equations provided the following conclusions:

1. The true AOQ becomes significantly worse as the value of A decreases from 100 percent to 50 percent. For good accuracy, that is for a high A, there is little deviation from an AOQ given by Dodge's equations. For very low process averages, the value of A has little or no effect on the AOQ.
2. As either f or i is increased for fixed A and p values, the deviation of the true AOQ value from the value expected from Dodge's equations will increase in a worsening direction. The importance of this conclusion is that a user of a basic Dodge plan should be discouraged from simply increasing f or i to overcome the effect of an inaccurate detailer.
3. Most importantly, a primary attribute of the rectifying aspects of Dodge's plan will no longer exist. Under perfect rectification the AOQ must approach zero as p approaches one. When A is less than 100 percent, the AOQ must approach the value of (1-A) as p approaches one.

Two procedures frequently used as an attempt to insure the AOQ value were analytically developed by this method and evaluated. The first procedure, denoted Variation I requires that the inspector essentially substitute for the detailer on a fraction f of the units to be detailed. Variation II requires that the inspector reinspect a fraction f of the units detailed. Under the assumptions of this thesis the two variations give identical AOQ values, but Variation II requires a higher total AFI for a given A, p, f and i.

Therefore, the possibility of improving the AOQ was only investigated for Variation I. This variation can actually be administrated in one of two ways, depending upon the use made of the good units found by the inspector during a detailing state. These units may or may not be used in the detailer's i count. Excluding these units from the count gave greater improvement than that obtained by including them, but this method results in a higher AFI value.

The investigation revealed that no generalized statement can be made as to the amount of improvement possible, or as to which method is the best. These decisions, however, can be made by applying the equations derived herein to the specific situation being considered (the desired AOQ, p, A, and feasible f and i combinations).

Two cost models previously developed for the economic selection of f and i were extended to include the ideas presented herein. The important conclusion drawn from the derivation and investigation of the cost models is that, with very few exceptions, the analytically optimal solution from both models is to inspect nothing, or to inspect everything 100 percent.

This dichotomy holds without exception in the extension of Anscombe's original model (1). The resulting cost equations are monotone. That is, they are either always increasing, or always decreasing functions. The presence of an increasing or a decreasing function depends upon the relationship between the sampling plan parameters and their associated costs. The few exceptions are present in the extension of Fry's model (11), which includes the constraint that an AOQL must be met. The analytical results indicate that there should always be a plan. That is, the optimal f will never be exactly zero or one, but will approach one or the other of these values.

The models are useful in this respect for making a decision concerning whether or not a continuous sampling plan is economically justified; even though it could not be considered optimal. The cost formulations are also useful for comparing the cost of one plan with another, and to determine the cost associated with an inaccurate detailer.

VII Areas for Further Study

During the preparation of this thesis it became apparent that there were several areas for further study. These areas can be considered as direct extensions of this thesis, or as research independent of the main objectives of the present thesis.

In applying GERT as a means to determine the AOQ and AFI it became apparent that the technique might also be used to determine the variance in these quantities. This information could then be used as a basis to compare Dodge's CSP-1 plan with other plans for continuous sampling (9).

The technique of selecting continuous sampling plans on an optimal basis would also appear as an area for further research. A Ph.D. dissertation by C. S. Beightler (4), has considered optimization over a series of inspection stages, each using a single-sampling plan. This work could be extended to consider continuous sampling plans when single-sampling plans are not appropriate.

Finally, the various forms taken by detailing errors could be researched further, with the objective being to classify these forms according to the nature of the process being inspected.

APPENDIX A

I. Derivation of Equation 4.

The path labeled q, which represents a good unit does not have an e^c because a good unit has already been counted if it has been inspected.

From Equation 3:

$$M_{O,D}(c) = \frac{fpe^{2c}}{1-(1-f)e^c-fqe^c}$$

$$\frac{\partial M_{O,D}(c)}{\partial c} = \frac{(1-(1-f)e^c-fqe^c)2fpe^{2c}-fpe^{2c}(-(1-f)e^c-fqe^c)}{(1-(1-f)e^c-fqe^c)^2}$$

$$\left.\frac{\partial M_{O,D}(c)}{\partial c}\right|_{c=0} = \frac{(f-fq)2fp+fp((1-f)+fq)}{(f-fq)^2}$$

$$= \frac{(fp)2fp+fp(1-fp)}{f^2p^2}$$

since $p = 1-q$

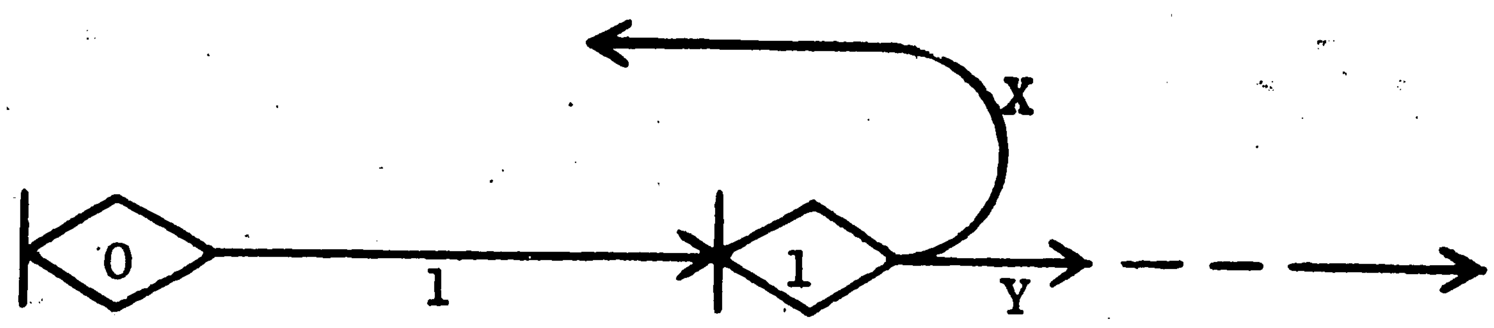
Therefore:

$$4. \quad \left.\frac{\partial M_{O,D}(c)}{\partial c}\right|_{c=0} = 2 + \frac{1}{fp} - 1 = \frac{1+fp}{fp}$$

Note: Leaving e^c off the path from I to D will make the 2 in the above equation be 1, such that the mean number passed excluding the defect is $\frac{1}{fp}$.

II. Derivation of Equation 9.

This will be solved in general for any leaving and returning path probabilities, viz.:



$X + Y = 1$

This will provide for repeated use of the derivation throughout the thesis.

From equations 6 and 8:

$$M_{0,i+1}(c) = \frac{(Ye^c)^i(1-Ye^c)}{1-Ye^c-Xe^c(1-(Ye^c)^i)} = \frac{(Ye^c)^i-(Ye^c)^{i+1}}{1-e^c+Xe^c(Ye^c)^i}$$

$$\frac{\partial M_{0,i+1}(c)}{\partial c} = \Big[\left[1-e^c+Xe^c(Ye^c)^i\right]\left[i(Ye^c)^{i-1}Ye^c-(i+1)(Ye^c)^iYe^c\right]$$
$$-\left[(Ye^c)^i-(Ye^c)^{i+1}\right]\left[-e^c+Xe^ci(Ye^c)^{i-1}Ye^c+(Ye^c)^iXe^c\right]\Big]$$
$$\div \left[1-e^c+Xe^c(Ye^c)^i\right]^2$$

$$\left.\frac{\partial M_{0,i+1}(c)}{\partial c}\right|_{c=0} = \frac{XY^i(iY^i-(i+1)Y^{i+1})-(Y^i-Y^{i+1})(-1+XiY^i+XY^i)}{(XY^i)^2}$$

$$= \frac{XiY^{2i}-XiY^{2i+1}-XY^{2i+1}+Y^i-XiY^{2i}-XY^{2i}-Y^{i+1}+XiY^{2i+1}+XY^{2i+1}}{(XY^i)^2}$$

$$= \frac{Y^i(1-Y-XY^i)}{X^2Y^{2i}} = \frac{X(1-Y^i)}{X^2Y^i} = \frac{1-Y^i}{XY^i}$$

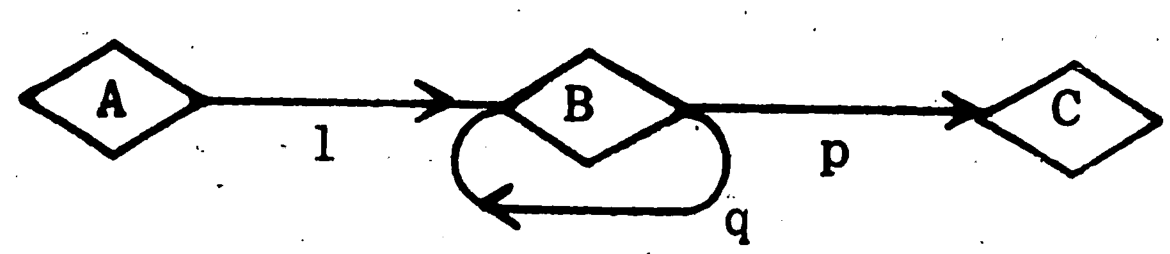
Therefore, by letting $X = p$ and $Y = q$

9. $$\left.\frac{\partial M_{0,i+1}(c)}{\partial c}\right|_{c=0} = \frac{1-q^i}{pq^i}$$

APPENDIX B

Reduction of Self-Loops

Consider the following network:

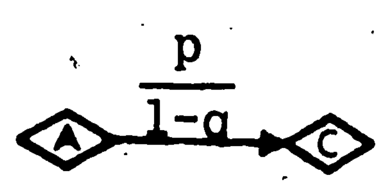


p+q=1

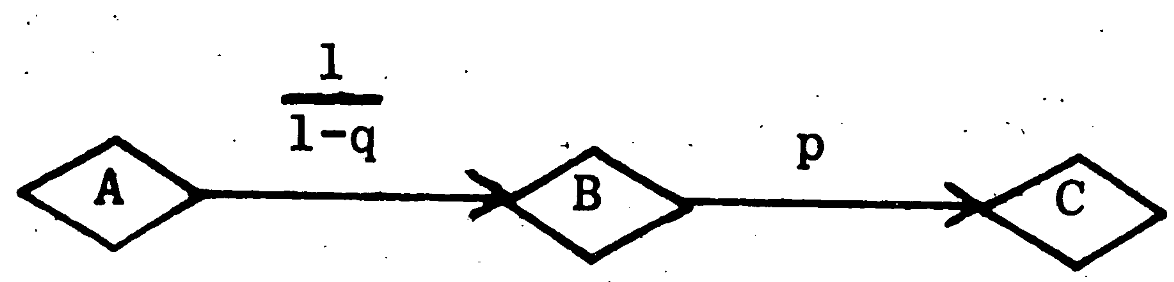
Let W = probability of going from A to C, i.e., the transmittance

$$W = p + pq + pq^2 + pq^3 \ldots = p(1+q+q^2+q^3+\ldots)$$

$$= \frac{p}{1-q}, \text{ since this is a geometric series with } q < 1$$

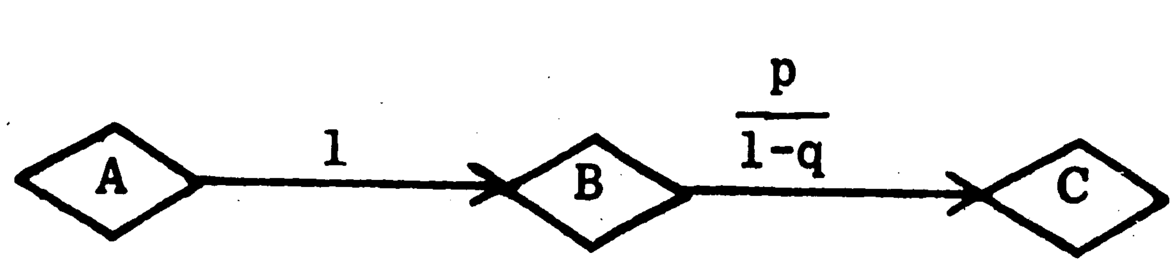
Therefore, one could have used 

directly, which is the same result that would have been obtained by Mason's rule.. Thus an equivalent network would be:



In words, multiply the entering path by the reciprocal of one minus the value of the self-loop.

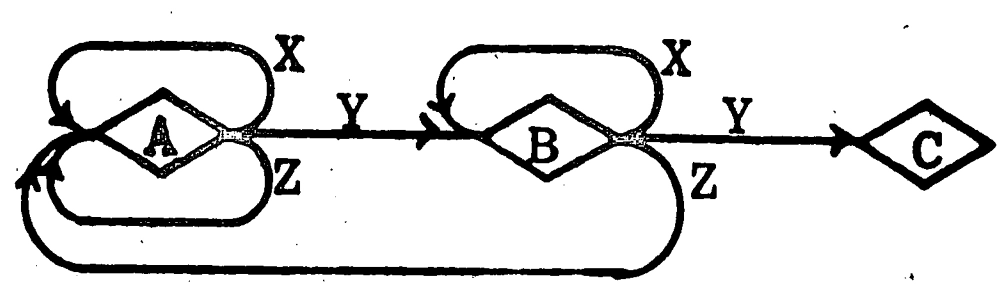
In the problem at hand the same result can be obtained by multiplying all leaving paths by 1/(1 - self-loop value). This is valid here only because the total transmittance is all that is desired. For example the previous network would have the same total transmittance if it was designated as:



Notice, however, the probability of going from A to B is no longer correct, but the total transmittance is correct, and this is all that is required.

An example specifically related to the present problem follows:

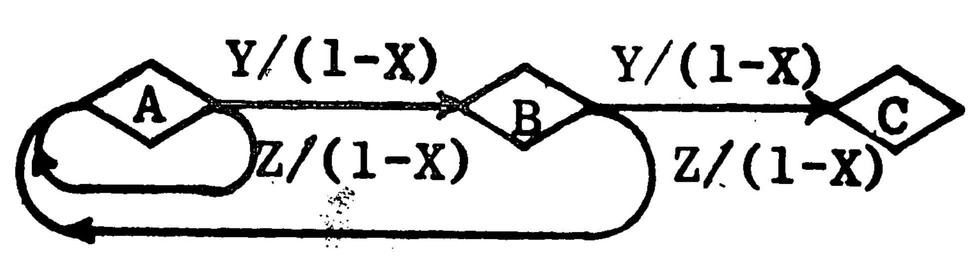
<u>No Self-loop Reduction</u>



$$W_1 = \frac{Y^2}{1-X-Z-YZ-X+X^2+XZ}$$

$$W_1 = \frac{Y^2}{1-(2X+YZ+Z)+X^2+XZ}$$

<u>Self-loop Reduction</u>



$$W_2 = \frac{Y^2/(1-X)^2}{1-Z/(1-X)-YZ/(1-X)^2}$$

$$= \frac{Y^2}{(1-X)^2-Z(1-X)-YZ}$$

$$W_2 = \frac{Y^2}{1-(2X+YZ+Z)+X^2+XZ}$$

APPENDIX C

The Derivative of Equation 27. with Respect to i

$$27. \qquad ETC = \frac{U_A(C_1+C_3p(1-A))+C_2/p}{U_A+1/p}$$

$$= \frac{\frac{(1-(1-pA)^i)}{pA(1-pA)^i}(C_1+C_3p(1-A))+C_2/p}{\frac{1-(1-pA)^i}{A(1-pA)^i} + \frac{1}{p}}$$

Let $Z = C_1+C_3p(1-A)$, since we are differentiating with respect to i. Upon further simplification:

$$ETC = \frac{Z-(1-pA)^i Z+C_2A(1-pA)^i}{1+(1-pA)^i(A-1)}$$

And:

$$\Delta ETC(i) = ETC(i+1)-ETC(i)$$

$$= \frac{Z-(1-pA)^{i+1}Z+C_2A(1-pA)^{i+1}}{1+(1-pA)^{i+1}(A-1)} - \frac{Z-(1-pA)^i Z+C_2A(1-pA)^i}{1+(1-pA)^i(A-1)}$$

For a minimum:

$$\Delta ETC(i^*-1) < 0 < \Delta ETC(i^*)$$

What would be required as proof that $i^*=0$, or ∞, is to demonstrate that the above expression is impossible, or that $\Delta ETC(i)$ is either always +, or always −. That is, to show that the optimal solution cannot lie on the open interval. This would be similar to the approach taken by Beightler (4). The author could find no reasonable means other than by computer programming to do this; and this would not provide conclusive proof due to the infinity of values involved. What follows then is conclusive proof only for the special case of A=1.

Setting A=1:

$$\Delta ETC(i^*) = C_1 - (1-p)^{i^*+1} C_1 + C_2(1-p)^{i^*+1} - C_1 + (1-p)^{i^*} C_1 - C_2(1-p)^{i^*}$$

For a minimum:

$$-(1-p)^{i^*} C_1 + C_2(1-p)^{i^*} + (1-p)^{i^*-1} C_1 - C_2(1-p)^{i^*-1} < 0 < \Delta ETC(i)$$

Divide by $(1-p)^{i^*-1}$:

$$-(1-p)C_1 + C_2(1-p) + C_1 - C_2 < 0 < -(1-p)^2 C_1 + C_2(1-p)^2 + (1-p)C_1 - (1-p)C_2$$

$$-(1-p)(C_1-C_2) + C_1 - C_2 < 0 < -(1-p)^2(C_1-C_2) + (1-p)(C_1-C_2)$$

Divide by (C_1-C_2), assume it is +:

$$-(1-p)+1 < 0 < -1+2p-p^2+1-p$$

$$1 < 0 < 1-p$$

$$0 > 1 > p$$

or, $0 < 1 < p$; if $(C_1-C_2) < 0$

which is clearly impossible

BIBLIOGRAPHY

1. Anscombe, F. J., "Rectifying Inspection of a Continuous Output," Journal of the American Statistical Association, Vol. 53, 1958, pp. 702-719.

2. Anscombe, F. J., "The Cost of Inspection," Royal Statistical Society, Statistical Method in Industrial Production, A Conference at Sheffield, 1950, pp. 61-67.

3. Beck, A., Jr., "Determining Testing Efficiency," Industrial Quality Control, Vol. 15, No. 3, September, 1958, pp. 4-7.

4. Beightler, C. S., "Systems of Interrelated Quality Control Stations Structured as Multistage Decision Processes," Unpublished Ph.D. Dissertation, Northwestern University, Dept. of Industrial Engineering, June, 1961, Chapter 4.

5. Derman, C., S. Littauer, and H. Solomon, "Tightened Multi-Level Continuous Sampling Plans," Annals of Mathematical Statistics, Vol. 28, 1957, pp. 395-404.

6. Dodge, H. F., "A Sampling Inspection Plan for Continuous Production," Annals of Mathematical Statistics, Vol. 14, No. 3, September, 1943, pp. 264-279.

7. Dodge, H. F., "Sampling Plans for Continuous Production," Industrial Quality Control, Vol. 14, 1947, pp. 5-9.

8. Dodge, H. F. and M. N. Torrey, "Additional Continuous Sampling Inspection Plans," Industrial Quality Control, Vol. 7, No. 5, March 1951, pp. 7-12.

9. Duncan, A. J., Quality Control and Industrial Statistics, Homewood, Illinois, Richard D. Irwin, Inc., 1959, Chap. 7, 17.

10. Feigenbaum, A. V., Total Quality Control, New York, McGraw-Hill Book Company, Inc., 1961, pp. 309-310.

11. Fry, J. H., "Selection, Stopping Conditions, and Response Characteristics of Dodge's Continuous Sampling Plans When Incoming Quality Deteriorates to an Unacceptable Level," Masters Thesis, Lehigh University, June, 1966.

12. Girshick, M. A., "A Sequential Inspection Plan for Quality Control," Technical Report No. 16, Applied Mathematics and Statistics Laboratory, Stanford University, July, 1954.

BIBLIOGRAPHY (cont'd)

13. Grant, E. L., Statistical Quality Control, New York, McGraw-Hill Book Company, Inc., 1952, Chap. 13, 17.

14. Gregory, G., "An Economic Approach to the Choice of Continuous Sampling Plans," Technical Report No. 30, Applied Mathematics and Statistics Laboratory, Stanford University, 1956.

15. Guthrie, D., and M. V. Johns, Jr., "Alternative Sequences of Sampling Rates for Tightened Multi-Level Continuous Sampling Plans," Technical Report No. 36, Applied Mathematics and Statistics Laboratory, Stanford University, 1958.

16. Juran, J. M., Management of Inspection and Quality Control, New York, Harper & Brothers Publishers, 1945, Chap. 3.

17. Juran, J. M., (ed), Quality Control Handbook, New York, McGraw-Hill Book Company, Inc., 1962, pp. 216-233.

18. Lieberman, G. J. and H. Solomon, "Multi-Level Continuous Sampling Plans," Annals of Mathematical Statistics, Vol. 26, 1955, pp. 686-704.

19. Lindsay, G. F., and A. B. Bishop, "Allocation of Screening Effort - A Dynamic Programming Approach," Management Science, Vol. 10, No. 2, January 1964, pp. 342-352.

20. Murphy, R. B., "A Criterion To Limit Inspection Effort in Continuous Sampling Plans," Bell System Technical Journal, Vol. 37, January 1958, pp. 115-134.

21. Pritsker, B. A. A., and W. W. Happ, "Graphical Evaluation and Review Technique," The Journal of Industrial Engineering, Vol. 17, No. 5, May 1966, pp. 267-274.

22. Pritsker, B. A. A., and G. E. Whitehouse, "Graphical Evaluation and Review Technique," The Journal of Industrial Engineering, Vol. 17, No. 6, June 1966, pp. 293-301.

23. Resnikoff, G. J., "Some Modifications of the Lieberman-Solomon Multi-Level Continuous Sampling Plan," Technical Report No. 26, Applied Mathematics and Statistics Laboratory, Stanford University, 1956.

24. Sampling Procedures and Tables for Inspection by Attributes on a Moving Line (Continuous Sampling Plans), NAVORD OSTD 81, U. S. Navy Bureau of Ordnance, 15 August 1952.

BIBLIOGRAPHY (cont'd)

25. Savage, I. R., "Statistical Production Models and Continuous Sampling Plans," Technical Report No. 29, Applied Mathematics and Statistics Laboratory, Stanford University, 1956.

26. Schmidt, A. B., and G. E. Sorber, "Economic Point for Screening Defective Material From a Process," The Western Electric Engineer, Vol. 6, No. 2, April 1962, pp. 44-48.

27. Single Level Continuous Sampling Procedures and Tables for Inspection by Attributes, H 107, Office of the Assistant Secretary of Defense, 30 April 1959.

28. Small, B. B., et.al., Statistical Quality Control Handbook, Quality Assurance, Western Electric Co. Inc., New York, September 1956, Part B.

29. Stephens, K. S., "A Multi-Level AOQL Sampling Plan," The Western Electric Engineer, Vol. 6, October 1962, pp. 39-49.

30. Wald, A. and J. Wolfowitz, "Sampling Inspection Plans For Continuous Production Which Insure a Prescribed Limit on the Outgoing Quality," Annals of Mathematical Statistics, Vol. 16, 1945, pp. 30-49.

31. White, L. S., "The Analysis of a Simple Class of Multistage Inspection Plans," Management Science, Vol. 12, No. 9, May 1966, pp. 685-693.

32. Whitehouse, G. E., "Extensions, New Developments, and Applications of GERT: Graphical Evaluation and Review Technique," Ph.D. Dissertation, Arizona State University, June, 1966, Chap. 3, 6.

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