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AN EXPERIMENTAL METHOD
FOR DETERMINING CUTTING FORCES

by

KIRAN M. GANDHI

A Thesis
Presented to the Graduate Faculty
of Lehigh University
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Master of Science

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This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

5/20/69
(date)

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ABSTRACT

The prediction of the force required to cut a material has been one of the central problems of interest in metal cutting. Many theories have been presented, but because of their predictive inaccuracies, they have rendered themselves wholly unacceptable for practical use. In this thesis an empirical method of predicting the forces produced during cutting is developed for SAE 1117 CRS, SAE 4145 HRS and SAE 6150 HRS materials. The same type of empirical method is also presented to predict cutting forces for any type of steel by knowing its ultimate shear strength. This can be used to predict the cutting force and feed force, once the cutting conditions have been selected and without the necessity of conducting a trial cut.

The prediction of the cutting forces generated is extremely useful for machineability studies, cutting tool design and machine tool design.

INTRODUCTION

When a chip is formed in the cutting of a metal, a force is produced on the tool which is performing the cutting. This force is caused by the plastic deformation of the grain structure that takes place in a shear plane which extends from the edge of the cutting tool to the surface of the workpiece ahead of the tool, and shear which causes the chip to separate from the metal and from the friction which occurs when the chip slides across the face of the tool.

This study will be limited to the orthogonal cutting. Orthogonal cutting is "the case where the cutting tool generates a plane surface parallel to the original plane of the surface being cut and is set with its cutting edge perpendicular to the direction of relative motion of the tool and the workpiece."[1]

The forces acting on the chip in orthogonal cutting are illustrated in Figure 1.[2] In this figure the line OC represents the shear plane and d represents the depth of cut. The angle α is the back rake angle of the tool.

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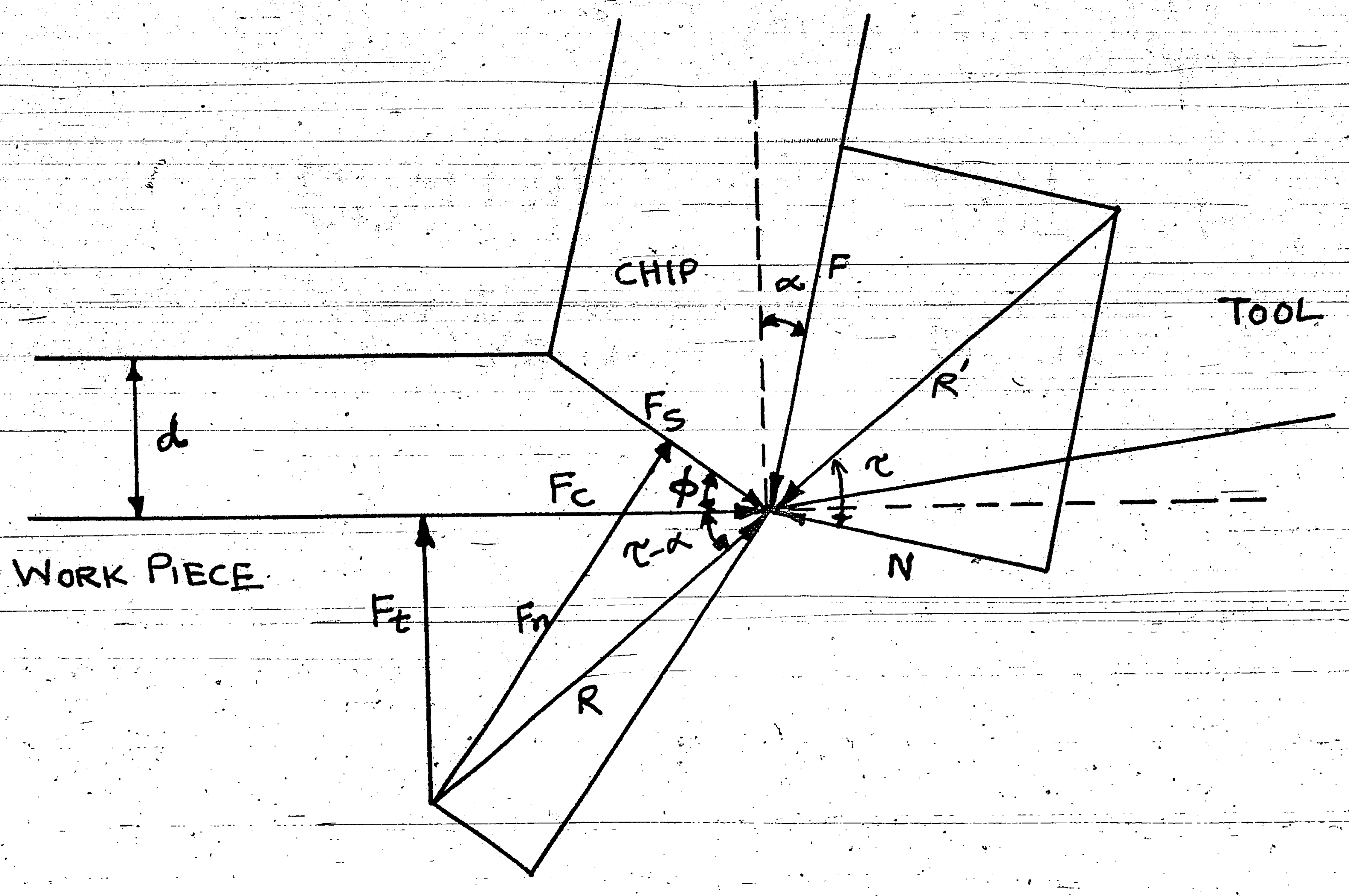
[1] Merchant, M. Eugene, "Mechanics of the Metal Cutting Process. I. Orthogonal Cutting and a Type 2 Chip," Journal of Applied Physics, Volume 16, No. 5, May 1945, p. 268.

[2] Ibid., p. 273.



Forces Acting on the Chip in Orthogonal Cutting

Figure 1

The force F is the friction force of the tool acting on the chip. It acts in a direction which is opposite to the motion of the chip as it slides along the face of the tool. The force N is normal to the cutting face of the tool and is provided by the tool. The forces F and N can be combined into a resultant force R' which is the force that the tool exerts on the chip. The angle between the normal force N and the resultant force R' is called the friction angle τ.

The force F_s, is the force required to plastically deform and shear the material, and it is generally assumed that the plastic deformation is caused by pure shear; hence the force is called the shear force. The force F_n, is normal to the force F_s, and the vector sum of the two is called the resultant force R. The resultant forces R and R' represent two equal and opposite forces which hold the chip in equilibrium.

The resultant force R can be resolved in two component forces F_c and F_t. The force F_c acts along the direction of motion of the tool relative to the workpiece and is known as the vertical or cutting force. The force F_t acts perpendicular to the cutting force and is known as the feed or horizontal force. These forces can most easily be measured by the use of a force dynamometer.

This force diagram cannot be solved by knowing only depth of cut and rake angle of the tool. For the solution, we must know the shear plane angle (ϕ), the friction angle (τ), and one of the force magnitudes.

There are presently only five theories of any significance in the field of predicting the force in metal cutting. All of these theories deal with the orthogonal cutting and try to find a relationship between the shear plane angle (ϕ), the friction angle (τ), and rake angle (α). These theories will not predict the forces directly which occur during a cut. Before predicting the forces, either shear plane angle or friction angle must be known. Usually the shear plane angle is determined empirically.

Ernst and Merchant

The solution presented by Ernst and Merchant[3] in 1941, was derived using the principle of minimum energy. The assumption made is that a material when being cut would fail by pure shear in a shear plane which will be located such that the minimum amount of energy will be required to shear the material. They also assumed that the shear stress S, is maximum in the direction of shear plane and that the shear stress on the shear plane was uniformly distributed.

With this assumption, it is evident from the force diagram

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[3] Ernst, H. and Merchant, M.E., "Chip Formation, Friction, and High Quality Machined Surfaces," Surface Treatment of Metals (American Society of Metals, 1941), pp. 299-398.

$$S = \frac{F_s}{A_s} = \frac{R' \cos(\phi + \tau - \alpha) \sin\phi}{A} \qquad (1)$$

where A_s is the area of the shear plane, A is the cross sectional area being cut and is equal to the feed times the depth of cut. Ernst and Merchant reasoned that ϕ should be an angle such that S would be maximum, and a relationship for ϕ was obtained by differentiating Equation (1) with respect to ϕ, holding R' and τ constant, and equating the resulting expression to zero. This led to the result

$$\phi = 45 - \frac{\tau}{2} + \frac{\alpha}{2} \qquad (2)$$

In order to predict the cutting force, they still needed a way to either predict τ or ϕ, so from the geometrical change which the material must go through when it is being deformed in the shear plane, they found that

$$\tan\phi = \left(\frac{r \cos\alpha}{1-r \sin\alpha}\right) \qquad (3)$$

where r is the cutting ratio which is the ratio of the depth of cut to the chip thickness after cutting.

This theory has been questioned by many theorists since its development. One of the biggest criticisms is that this theory does not predict the forces accurately because the cutting ratio must be known in order to calculate the shear plane angle. To find the cutting ratio we have to take the actual cut at the same conditions for which the force is to be predicted.

The assumptions made by Ernst and Merchant have been questioned by Shaw, Cook, and Finnie.[4] They found that a tool operating with a fixed resultant force at an angle $(\tau - \alpha)$ must have a shear plane at a fixed position, but it does not have to be in the direction of maximum shear stress. Experimental data has shown that the resultant force R and the friction angle τ , are dependent on the shear plane angle, and therefore cannot be assumed to be independent of the shear plane angle. The assumption that the material fails in pure shear and in a shear plane has been questioned by Oxley.[5]

Merchant modified the theory in 1945 and came to the same result as Equation (2). In this modification,

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[4] Shaw, M. C., Cook, N. H., and Finnie, I., "The Shear Angle Relationship in Metal Cutting," Transactions of the A.S.M.E., Vol. 75, No. 2, February, 1953, pp. 273-288.

[5] Oxley, P.L.B., "Mechanics of Metal Cutting," Proceedings of the International Production Engineering Research Conference, 1963, pp. 50-60

he assumed that the shear plane angle would be such that the total power consumed in the cutting process would be a minimum. He expressed the total power as

$$P = F_c V = SAV \frac{\cos(\tau - \alpha)}{\sin \phi \cos(\phi + \tau - \alpha)} \qquad (4)$$

In this derivation Merchant made the following assumptions:

1. The shear plane angle will be such that the total power consumed in cutting will be minimum.

2. The friction angle is independent of the shear plane angle.

3. The shear stress on the shear plane is independent of the shear plane angle.

When Equation (4) is differentiated with respect to ϕ, considering S and τ to be independent of the shear plane angle as assumed above, the result is

$$\phi = 45 - \frac{\tau}{2} + \frac{\alpha}{2} \qquad (5)$$

which is the same as Equation (2). Since many examples can be found where a process occurs in such a manner that the energy consumed is not a minimum, there is no clear reason why assumption 1 should be true. Assumption 2 was previously discussed and assumption 3 is only approximately correct as Shaw, Cook, and Finnie have observed that the shear stress on the shear plane is not independent of the shear angle.

Later, Merchant changed his theory and introduced the concept that the shear strength would be dependent on the normal stress on the shear plane according to the following relationship:[6]

$$S'_s = S_s + KS_n \qquad (6)$$

where S_s is the shear strength of the material under zero compressive stress, S'_s is the shear strength of the material under load, S_n is the normal stress on the shear plane, and K is a constant of the material and represents the slope of the curve of S'_s vs. S_n.

Using his minimum energy theory, he derived the following expression for shear plane angle:

$$\phi = \frac{\cot^{-1}(K)}{2} - \frac{\tau}{2} + \frac{\alpha}{2} \qquad (7)$$

Merchant called the term $\cot^{-1}(K)$, the machining constant C, and expressed his equation in more common form

$$2\phi + \tau - \alpha = C \qquad (8)$$

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[6] Merchant, M. E., "Mechanics of the Metal Cutting Process. II. Plasticity Conditions in Orthogonal Cutting," Journal of Applied Physics, Vol. 16, No. 6, June, 1945, pp. 320-322.

Unfortunately, the dependence of the shear strength on the normal compressive stress on the shear plane has not been supported by independent experimental evidence.[7] In this case, C along with the cutting ratio must be known to predict the forces.

Lee and Shaffer

In 1951, Lee and Shaffer[8] presented their solution to the problem of determining the shear plane angle. They introduced the concept of the ideal theory of plasticity or ideal slip line theory. They made the following assumptions:

1. The material being cut behaves as an ideal plastic which does not strain harden.
2. The shear plane represents the direction of maximum stress in the material being cut.

With these assumptions they concluded that the shear stress and the normal stress on the shear plane should be equal. Using Mohr's circle, they concluded the following relationship for the shear plane angle:

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[7] Oxley, P.L.B., "Shear Angle Solution in Orthogonal Cutting," International Journal of Machine Tool Design and Research, Vol. 2, No. 3, July-September, 1962, p. 221.

[8] Lee, E. H. and Shaffer, R. W., "The Theory of Plasticity Applied to a Problem of Machining," Journal of Applied Mechanics, Trans. of the A.S.M.E., Vol. 73, (1951), pp. 405-413.

$$\phi = 45 - \tau + \alpha \qquad (9)$$

Generally the shear stress on the shear plane T and the normal stress on the shear plane σ, are not equal and hence Equation (9) cannot be regarded as a general solution. They modified their solution to include the effect of the built up edge. Then they arrived at the following expression:

$$\phi = 45 + \theta - \tau + \alpha \qquad (10)$$

which differs from Equation (9) only by the angle θ which is a measure of the built up edge, and it is given by

$$\theta = \frac{\frac{\sigma}{T} - 1}{2} \qquad (11)$$

where σ is the normal stress on the shear plane, and T is the shear stress on the shear plane. They claimed good agreement between the actual results and theoretical prediction, but they did not present a method for calculating the normal stress σ, or the shear stress T without first knowing the forces existing on the shear plane.

Hill[9] in particular challenged the theory because Lee and Shaffer did not allow for a stress singularity which must occur at each end of the shear plane.

Hucks

Hucks presented a solution to the problem of determining shear plane angle that is similar to Lee and Shaffer's solution. In his solution, Hucks made the following assumptions:

1. The stress between the chip and tool along the face of the tool is evenly distributed.

2. The shear plane is in the direction of maximum stress.

3. The resistance to flow in the direction of chip motion is relatively small and thus the normal stress on a plane perpendicular to the face of the tool can be assumed to be zero.

With these assumptions, Hucks presented the following solution:

$$\phi = 45 - \frac{\tan^{-1}(2\mu)}{2} + \alpha \tag{12}$$

where μ is the coefficient of friction of the tool.

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[9] Hill, R., "The Mechanics of Machining: A New Approach," Journal of the Mechanics and Physics of Solids, Vol. 3, No. 10, October, 1954, pp. 47-53.

Hucks' assumptions were criticized by Shaw, Cook, and Finnie.[10] In particular, the third assumption was criticized more because there is no reason to assume that the normal stress is zero, and if this is done, it leads to the conclusion that the normal stress on the shear plane must be less than shear stress which has been shown experimentally to be untrue.

Hucks also modified his results to include the normal stress on the shear plane. Equation (12) then became:

$$\phi = \frac{\cot^{-1}(K)}{2} - \frac{\tan^{-1}(2\mu)}{2} + \alpha \qquad (13)$$

where $\cot^{-1}(K)$ is Merchant's machining constant. Unfortunately, Hucks' theory is not supported very well experimentally.

Shaw, Cook, and Finnie

In 1953, Shaw, Cook, and Finnie modified the Lee and Shaffer solution by making a change in assumption that the plane of maximum stress will not be the shear plane, but will make an angle N' with the shear plane.

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[10] Shaw, M. C., Cook, N. H., and Finnie, I., "The Shear Angle Relationship in Metal Cutting," Transactions of the A.S.M.E., Vol. 75, (1953), p. 276.

They presented the relationship

$$\phi = 45 - \tau + \alpha + N' \qquad (14)$$

Their results did not agree with experimental results. Lee[11] has criticized the results and commented that only a positive N' has been reported, and that deformation may occur ahead of the shear plane.

In their research work, Shaw, Cook, and Finnie did find that the following statements could be made about metal cutting:

1. Friction encountered in the cutting process is basically different from that for ordinary sliding contacts, owing to a variation of the effective indentation hardness of the metal at the point of the tool under different cutting conditions.

2. There is significant interrelationship between the shear and friction processes in cutting brought about by the close proximity of the processes and the fact that they are connected by a common stress field.

3. The effective hardness of the chip metal increases as a result of the increased restraint associated with a decrease in the angle between shear plane and tool face.

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[11] *Ibid.*, p. 285.

4. The effective hardness of the chip thus increases with a decrease in rake angle, which in turn gives rise to a significant decrease in the coefficient of friction with decreased rake angle.

5. The interconnection between the shear and friction process prevents the shear plane from being in the direction of maximum shear stress in the general use.

6. The assumption of a uniform state of stress in the vicinity of the tool point is a good first approximation.

7. Normal stress on a shear plane does play a significant role in determining whether a given chip will be continuous or discontinuous.[12]

Oxley

Oxley[13] found that the shear plane stress was greatly influenced by the relative motion of the cutting tool and the workpiece. The shear zone was thick at slower speeds, but at higher speed this zone reduces to a single shear plane. Using his slip-line theory for materials, he developed the following relationship:

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[12] Ibid., pp. 282, 283.

[13] Oxley, P., "Shear Angle Solutions in Orthogonal Machining," International Journal of Machine Tool Design and Research, Vol. 2, pp. 219-229.

$$\Theta = \phi + \tau - \alpha \qquad (15)$$

$$\tan\Theta = \frac{1}{2}\left[1 + \frac{\pi}{2} - 2\phi + \frac{\cos 2(\phi - \alpha)}{\tan\tau} - \sin 2(\phi - \alpha)\right] \qquad (16)$$

Given τ and α or ϕ and α, the two equations could be solved simultaneously for Θ and ϕ or τ and Θ. Experimental agreement is fair; however, certain assumptions have been questioned.

In 1963, Alfred Burfeind[14] proved that friction angle is dependent on the cutting conditions. He developed experimental methods to predict the friction angle for materials AISI 1020 HRS and SAE 4340 HRS, knowing cutting conditions. In the same way, Sachse[15] developed an experimental method to predict shear plane angle for material AISI 1020 HRS. He proved tha[illegible] Merchant's force relationship is invalid for SAE 4340 HRS when the shear plane angle is greater than 35 to 40 degrees.

In the preceding discussion, a brief summary of the previous work has been presented in the field of metal cutting in order to predict the forces by predicting the shear plane angle empirically. No work is

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[14] Burfeind, A. F., "The Effect of Cutting Conditions on the Friction Angle," Unpublished thesis, Lehigh University, (1963), p. 18.

[15] Sachse, C. D., "An Experimental Method for Determining the Shear Plane Angle," Unpublished thesis, Lehigh University, (1964), pp. 47, 48.

done until now to predict the cutting forces by knowing the cutting conditions and without conducting a test at the same conditions.

In this thesis, a relationship will be developed which can be used to predict the cutting forces once the workpiece material and cutting conditions have been selected and without conducting a test at those conditions or without predicting the shear plane or friction angles. This type of information is extremely useful for the design of cutting operations, cutting tools and the machines which use them.

EXPERIMENTAL PROCEDURE

As previously indicated, the objective of this study is to determine an experimental method of calculating cutting forces. To obtain data for the study, the following workpiece materials, tool geometries and cutting conditions were selected:

A. Workpiece Material - SAE 1117 CRS, SAE 4145 HRS,
SAE 6150 HRS.

B. Rake Angle - -5, 0, +5 degrees

C. Feed Rate - 0.0051, 0.0102, 0.0203 in./rev.

D. Depth of Cut - 0.020, 0.040, 0.060 in.

All cuts were made at a cutting speed of 450 SFPM. This speed is in range of normal speeds for carbides. In this normal range of speeds, the speed has negligible effect on the cutting forces.[16]

The tool material was carbide of KENDEX KSH TPU 322 grade. The results should hold true for all grades of carbides, although very little investigation has been done in this area.

The data was not taken in a random order point by point, due to the setup difficulties. The cuts were

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[16] Boston, O. W., Metal Processing, John Wiley and Sons, Inc., New York, Second Edition, p. 162.

randomized in feed rate and rake angle given the material and depth of cut. The reasons for not varying randomly point to point are

1. Impractical to change work material each time.
2. Setting the tool in the dynamometer is critical for proper force readings and therefore data was taken for each depth of cut separately.

In order to have a wide range of materials, the materials are chosen as low carbon steel, medium carbon steel with low alloy content and high alloy steel. Two cuts were made at each cutting condition to eliminate the experimental error. This gave a grand total of 162 cuts taken. The cuts were taken in a random order, as listed in Appendix B.

Data was collected using the LeBlond 16-inch Heavy Duty Engine Lathe with the Varidyne Control unit to maintain constant surface speed regardless of workpiece diameter. A three component lathe dynamometer and a two channel strain gage amplifier-recorder were used to measure the horizontal and vertical forces. As the amount of tool overhang affects the recorded forces due to the magnitude of the strain measured by the dynamometer, the measured forces were corrected for the overhang of the tool holder. This was done by multiplying the measured force by the overhang factor K, as described in the instruction manual for the dynamometer.

The duration of each cut was about 10 secs. One edge was used for two cuts of small duration. The wear of the tool was not significant. The force values were read after each cut. A listing of the corrected cutting forces for all runs can be found in Appendix C.

To determine the ultimate shear strength for the materials used in the study, a torsion test was conducted using specimens machined from material from the same heat and bar as that used to collect the force data. Test specimens were machined according to the American Society for Testing Materials Standard A 260-47. The specimens were tested on a Tinius Olsen & Co. torsion test machine. The results of the tests are shown below.

Material	Ultimate torque
SAE 1117 CRS	5,762 in-lbs.
SAE 4145 HRS	9,200 in-lbs.
SAE 6150 HRS	10,240 in-lbs.

The ultimate shear strength was then calculated using the relationship:

$$S = \frac{TC}{J}$$

where S = Ultimate shear strength (psi)

T = Ultimate torque (in-lbs.)

C = Distance from the axis of the specimen to the outermost fiber or one-half of the diameter (in.)

J = Polar moment of inertia = $\frac{\pi}{32} d^4$ (in.^{4})

The results of these calculations are listed below.

Material	Shear strength
SAE 1117 CRS	69,700 psi
SAE 4145 HRS	111,600 psi
SAE 6150 HRS	124,000 psi

While performing the experiment, it was found out that it is not possible to take the cut at 0.060 in. depth of cut and 0.0203 in./rev. feed rate for any rake angle, for material SAE 6150 HRS only. Therefore, the forces at that condition were estimated from the graph and then the data was analysed. But for the regression analysis, those forces were not used.

RESULTS

In Appendix D, the graphs are presented to show the relationship between the cutting forces and the independent variables depth of cut and feed rate. An examination of the graphs reveals that depth of cut, feed rate and back rake angle had an effect on the cutting forces. This conclusion was later confirmed by analysis of variance.

A multiple linear regression technique is used for developing the empirical equation. For each material, and for both the forces, multiple linear regression equations are developed. The results are shown in Appendix F.

A common multiple regression equation is developed for cutting force which combines all material. The first equations will have fifteen factors, but for practical cases a second equation can also be used.

$$\begin{aligned} F_c = {} & 18.998 - 1.403A - 2.828B - 26.813C - 1.604D \\ & + 0.828AB + 2.566AC + 18.476BC + 0.261AD \\ & - 0.430BD + 4.006CD + 0.634ABC + 0.020ABD \\ & - 0.414ACD - 0.940BCD + 0.076ABCD \end{aligned}$$

$$\begin{aligned} F_c = {} & 18.998 - 1.403A - 2.828B - 26.813C + 0.828AB \\ & + 2.566AC + 18.476BC + 0.634ABC \end{aligned}$$

where A = Shear strength of material in psi/10,000

B = Depth of cut in inch x 100

C = Feed Rate in in./rev. x 100

D = Rake Angle in degrees

The same types of relationships are developed for feed force, as follows:

$$\begin{aligned} F_t = {} & 123.416 - 12.140A - 20.080B - 137.496C + 2.254D \\ & + 2.787AB + 12.848AC + 33.694BC - 0.320AD \\ & - 2.833BD - 2.541CD - 2.546ABC + 0.265ABD \\ & + 0.287ACD + 1.187BCD - 0.163ABCD \end{aligned}$$

$$\begin{aligned} F_t = {} & 123.416 - 12.140A - 20.080B - 137.496C + 2.787AB \\ & + 12.848AC + 33.694BC - 2.546ABC \end{aligned}$$

where A = Shear strength of material in psi/10,000

B = Depth of cut in inch x 100

C = Feed Rate in in./rev. x 100

D = Rake Angle in degrees

ANALYSIS OF RESULTS

The cutting forces were analyzed using the statistical technique of analysis of variance to test the significance of the independent variables and their interaction terms. The data was analyzed for each material separately. Then the complete data of cutting force was analyzed by adding one more independent variable as shear strength of the material. The same technique was used for the feed force. The results of the analysis of variance are shown in Appendix E. The depth of cut, feed rate, rake angle and their interaction terms were found to be significant at 99.9% confidence level for materials SAE 1117 CRS and SAE 4145 HRS and for both the forces. But in the case of SAE 6150 HRS material, except the independent variable rake angle, all other terms were found to be significant at 99.9% confidence level, for both the forces.

When the combined data of all three materials was analyzed, the shear strength of the material, depth of cut, feed rate, rake angle and their interaction terms were found to be significant at 99.9% confidence level. This was found true in case of both forces.

Having determined the significant variables, the next step was to develop an equation to predict the cutting forces in terms of independent variables. The graphs (Appendix D) showed that the effects of independent variables on the cutting forces were linear and the technique of multiple linear regression was used to develop the relationship.

The multiple linear regression equations are developed for cutting force and feed force separately for each material. Then one multiple linear regression equation is developed to predict the cutting force developed during cutting of any material. The same type of equation is also developed for the feed force. These equations contain all the fifteen significant factors. But for practical use this will be a complicated equation. By the close examination of analysis of variance for cutting force and feed force, it is clear that the sums of squares contributed by the rake angle and the terms of interactions of rake angle with other variables are very small. This is due to the smaller range of rake angle. Therefore, we can eliminate those factors for the regression considered. By this way, another multiple linear regression equation was developed for cutting force and feed force containing seven variables. The results of the multiple linear regression are shown in Appendix F.

When using the developed equations, the actual value of the back rake angle must be used, not the effective back rake angle which is a combination of back rake angle and side rake angle.

The results of the multiple linear regression equation for cutting force show the excellent agreement between the predicted value and the measured value. But in the case of feed force, poor agreement is obtained between the predicted force, calculated from the developed multiple linear regression equation, and the measured force. This is due to some other factors affecting the results and the effect of those factors has not been determined in this study.

To estimate the horsepower requirement, the unit horsepower ratios are given for different materials at different feed rates.[17] The results of the unit power calculations are presented in Appendix G. Those are shown for different materials and at different cutting conditions. The results are obtained by using both the measured and estimated cutting forces and those results

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[17] Vidosic, J. P., "Metal Machining and Forming Technology," Table 17-2, p. 365, Ronald Press, New York, 1964.

show the close agreement between the calculated unit power ratios and that given for a rough estimate of the horsepower.

CONCLUSIONS

As stated previously, the purpose of this study was to develop an empirical relationship for determination of cutting forces for any work material. Based on the results of this study, the following conclusions were drawn:

1. Any one of the following relationships can be used to predict the cutting force. For more accurate estimate, the first relationship must be used. But for practical use, a second relationship can be used.

$$\begin{aligned} F_c = {} & 18.998 - 1.403A - 2.828B - 26.813C - 1.604D \\ & + 0.828AB + 2.566AC + 18.476BC + 0.261AD \\ & - 0.430BD + 4.006CD + 0.634ABC + 0.020ABD \\ & - 0.414ACD - 0.940BCD + 0.076ABCD \end{aligned}$$

$$\begin{aligned} F_c = {} & 18.998 - 1.403A - 2.828B - 26.813C + 0.828AB \\ & + 2.566AC + 18.476BC + 0.634ABC \end{aligned}$$

where A = Shear strength of material in psi/10,000
B = Depth of cut in inch x 100
C = Feed Rate in in./rev. x 100
D = Rake Angle in degrees

2. From the above relationship, it is confirmed that the cutting force can be predicted by an empirical method which is valid for all materials.

3. The same type of relationships are obtained for the feed force, but the results are in poor agreement with the measured values. This is due to some other factors affecting the results indirectly. Those factors were not determined in this study.

AREA FOR FUTURE STUDY

The technique used and the problem encountered in this study have brought to mind the following areas in which further study should be conducted. These areas include:

1. An expansion of this study for feed force, by greater variety of nose radius and side cutting edge angle.

2. A careful study and evaluation of the existing feed force relationship to determine why it gives poor agreement with the measured values.

3. Theoretical work should be done to predict the cutting forces by the mechanism of plastic metal failures during cutting.

APPENDIX A

EQUIPMENT AND MATERIALS

List of Equipment

1. LeBlond 16-inch Heavy Duty Engine Lathe
2. U.S. Electrical Motors, Inc. Varidyne Control Unit
3. Cook, Smith and Associates Model L3-2, Three Component Lathe Dynamometer
4. Sanborn Company Strain Gage Amplifier and Twin Visco Recorder

Tool Holders and Inserts

Tool Holders -

Positive tool holders

Inserts -

KENDEX K5H TPU 322

Tool Geometry

1. (0, +5, 10, 5, 30, 0, 2/64)
2. (0, 0, 10, 10, 30, 0, 2/64)
3. (0, -5, 10, 15, 30, 0, 2/64)

Workpiece Materials

1. SAE 1117 CRS
   - A. Diameter 3.5 inches

EQUIPMENT AND MATERIALS (Cont'd.)

B. Hardness 137 BHN

C. Shear Strength 69,700 psi

D. Chemical Composition

C	M_n	P	S
0.17	1.15	0.040	0.10

2. SAE 4145 HRS - Bethlehem Steel Co., Heat No. 136P254-5

A. Diameter 4.5 inches

B. Hardness 320 BHN

C. Shear Strength 111,600 psi

D. Chemical Composition

C	M_n	P	S	Si	Cr	Mo
0.45	0.85	0.04	0.04	0.30	0.95	0.20

3. SAE 6150 HRS - Bethlehem Steel Co., Heat No. 124K505-AA

A. Diameter 4.5 inches

B. Hardness 330 BHN

C. Shear Strength 124,000 BHN

D. Chemical Composition

C	M_n	P	S	Si	Cr	V
0.51	0.80	0.04	0.04	0.30	0.95	0.15

APPENDIX B

LIST OF ORDER IN WHICH DATA WAS COLLECTED

Work Piece Material	Depth of Cut (inch)	Feed Rate (in/rev.)	Rake Angle (Degrees)
1117 CRS	0.020	0.0203	+5
1117 CRS	0.020	0.0051	0
1117 CRS	0.020	0.0051	+5
1117 CRS	0.020	0.0051	+5
1117 CRS	0.020	0.0203	+5
1117 CRS	0.020	0.0203	0
1117 CRS	0.020	0.0102	+5
1117 CRS	0.020	0.0203	-5
1117 CRS	0.020	0.0203	-5
1117 CRS	0.020	0.0203	0
1117 CRS	0.020	0.0051	0
1117 CRS	0.020	0.0051	-5
1117 CRS	0.020	0.0102	0
1117 CRS	0.020	0.0102	+5
1117 CRS	0.020	0.0102	0
1117 CRS	0.020	0.0102	-5
1117 CRS	0.020	0.0051	-5
1117 CRS	0.020	0.0102	-5
1117 CRS	0.040	0.0051	+5
1117 CRS	0.040	0.0203	+5
1117 CRS	0.040	0.0051	-5

LIST OF ORDER IN WHICH DATA WAS COLLECTED (cont'd.)

Work Piece Material	Depth of Cut (inch)	Feed Rate (in/rev.)	Rake Angle (Degrees)
1117 CRS	0.040	0.0203	+5
1117 CRS	0.040	0.0051	0
1117 CRS	0.040	0.0203	0
1117 CRS	0.040	0.0051	+5
1117 CRS	0.040	0.0203	0
1117 CRS	0.040	0.0102	0
1117 CRS	0.040	0.0102	0
1117 CRS	0.040	0.0102	+5
1117 CRS	0.040	0.0051	-5
1117 CRS	0.040	0.0051	0
1117 CRS	0.040	0.0102	+5
1117 CRS	0.040	0.0203	-5
1117 CRS	0.040	0.0203	-5
1117 CRS	0.040	0.0102	-5
1117 CRS	0.040	0.0102	-5
1117 CRS	0.060	0.0102	-5
1117 CRS	0.060	0.0051	+5
1117 CRS	0.060	0.0203	+5
1117 CRS	0.060	0.0051	+5
1117 CRS	0.060	0.0102	0
1117 CRS	0.060	0.0203	0

LIST OF ORDER IN WHICH DATA WAS COLLECTED (cont'd.)

Work Piece Material	Depth of Cut (inch)	Feed Rate (in/rev.)	Rake Angle (Degrees)
1117 CRS	0.060	0.0102	-5
1117 CRS	0.060	0.0102	+5
1117 CRS	0.060	0.0102	0
1117 CRS	0.060	0.0203	+5
1117 CRS	0.060	0.0203	0
1117 CRS	0.060	0.0203	-5
1117 CRS	0.060	0.0051	0
1117 CRS	0.060	0.0051	0
1117 CRS	0.060	0.0051	-5
1117 CRS	0.060	0.0102	+5
1117 CRS	0.060	0.0051	-5
1117 CRS	0.060	0.0203	-5
4145 HRS	0.020	0.0203	0
4145 HRS	0.020	0.0203	-5
4145 HRS	0.020	0.0051	0
4145 HRS	0.020	0.0102	-5
4145 HRS	0.020	0.0102	0
4145 HRS	0.020	0.0203	0
4145 HRS	0.020	0.0203	-5
4145 HRS	0.020	0.0102	0
4145 HRS	0.020	0.0203	+5

LIST OF ORDER IN WHICH DATA WAS COLLECTED (cont'd.)

Work Piece Material	Depth of Cut (inch)	Feed Rate (in/rev.)	Rake Angle (Degrees)
4145 HRS	0.020	0.0051	0
4145 HRS	0.020	0.0203	+5
4145 HRS	0.020	0.0051	-5
4145 HRS	0.020	0.0051	-5
4145 HRS	0.020	0.0102	-5
4145 HRS	0.020	0.0102	+5
4145 HRS	0.020	0.0102	+5
4145 HRS	0.020	0.0051	+5
4145 HRS	0.020	0.0051	+5
4145 HRS	0.040	0.0051	+5
4145 HRS	0.040	0.0102	0
4145 HRS	0.040	0.0051	-5
4145 HRS	0.040	0.0203	-5
4145 HRS	0.040	0.0051	0
4145 HRS	0.040	0.0102	-5
4145 HRS	0.040	0.0102	-5
4145 HRS	0.040	0.0203	0
4145 HRS	0.040	0.0051	+5
4145 HRS	0.040	0.0102	+5
4145 HRS	0.040	0.0203	0
4145 HRS	0.040	0.0051	-5

LIST OF ORDER IN WHICH DATA WAS COLLECTED (cont'd.)

Work Piece Material	Depth of Cut (inch)	Feed Rate (in/rev.)	Rake Angle (Degrees)
4145 HRS	0.040	0.0051	0
4145 HRS	0.040	0.0203	-5
4145 HRS	0.040	0.0102	+5
4145 HRS	0.040	0.0203	+5
4145 HRS	0.040	0.0102	0
4145 HRS	0.040	0.0203	+5
4145 HRS	0.060	0.0203	+5
4145 HRS	0.060	0.0051	0
4145 HRS	0.060	0.0051	+5
4145 HRS	0.060	0.0051	+5
4145 HRS	0.060	0.0203	+5
4145 HRS	0.060	0.0203	0
4145 HRS	0.060	0.0102	+5
4145 HRS	0.060	0.0203	-5
4145 HRS	0.060	0.0203	-5
4145 HRS	0.060	0.0203	0
4145 HRS	0.060	0.0051	0
4145 HRS	0.060	0.0051	-5
4145 HRS	0.060	0.0102	0
4145 HRS	0.060	0.0102	+5
4145 HRS	0.060	0.0102	0

LIST OF ORDER IN WHICH DATA WAS COLLECTED (cont'd.)

Work Piece Material	Depth of Cut (inch)	Feed Rate (in/rev.)	Rake Angle (Degrees)
4145 HRS	0.060	0.0102	-5
4145 HRS	0.060	0.0051	-5
4145 HRS	0.060	0.0102	-5
6150 HRS	0.020	0.0203	+5
6150 HRS	0.020	0.0203	-5
6150 HRS	0.020	0.0203	+5
6150 HRS	0.020	0.0051	+5
6150 HRS	0.020	0.0203	0
6150 HRS	0.020	0.0051	+5
6150 HRS	0.020	0.0051	-5
6150 HRS	0.020	0.0051	0
6150 HRS	0.020	0.0102	0
6150 HRS	0.020	0.0051	-5
6150 HRS	0.020	0.0102	+5
6150 HRS	0.020	0.0102	+5
6150 HRS	0.020	0.0203	-5
6150 HRS	0.020	0.0051	0
6150 HRS	0.020	0.0102	0
6150 HRS	0.020	0.0203	0
6150 HRS	0.020	0.0102	-5
6150 HRS	0.020	0.0102	-5

LIST OF ORDER IN WHICH DATA WAS COLLECTED (cont'd.)

Work Piece Material	Depth of Cut (inch)	Feed Rate (in/rev.)	Rake Angle (Degrees)
6150 HRS	0.040	0.0102	+5
6150 HRS	0.040	0.0203	-5
6150 HRS	0.040	0.0051	+5
6150 HRS	0.040	0.0051	-5
6150 HRS	0.040	0.0203	0
6150 HRS	0.040	0.0203	0
6150 HRS	0.040	0.0051	-5
6150 HRS	0.040	0.0203	-5
6150 HRS	0.040	0.0102	+5
6150 HRS	0.040	0.0102	0
6150 HRS	0.040	0.0102	0
6150 HRS	0.040	0.0051	0
6150 HRS	0.040	0.0203	+5
6150 HRS	0.040	0.0102	-5
6150 HRS	0.040	0.0102	-5
6150 HRS	0.040	0.0051	+5
6150 HRS	0.040	0.0203	+5
6150 HRS	0.040	0.0051	0
6150 HRS	0.060	0.0051	-5
6150 HRS	0.060	0.0102	0

LIST OF ORDER IN WHICH DATA WAS COLLECTED (cont'd.)

Work Piece Material	Depth of Cut (inch)	Feed Rate (in/rev.)	Rake Angle (Degrees)
6150 HRS	0.060	0.0102	0
6150 HRS	0.060	0.0203	-5
6150 HRS	0.060	0.0203	0
6150 HRS	0.060	0.0102	-5
6150 HRS	0.060	0.0102	+5
6150 HRS	0.060	0.0051	-5
6150 HRS	0.060	0.0203	0
6150 HRS	0.060	0.0051	+5
6150 HRS	0.060	0.0203	-5
6150 HRS	0.060	0.0051	+5
6150 HRS	0.060	0.0102	-5
6150 HRS	0.060	0.0203	+5
6150 HRS	0.060	0.0051	0
6150 HRS	0.060	0.0203	+5
6150 HRS	0.060	0.0051	0
6150 HRS	0.060	0.0102	+5

APPENDIX C

CORRECTED FORCE DATA

WORKPIECE MATERIAL SAE 1117 CRS
CUTTING SPEED 450 SFPM

INSERT K5H TPU 322

Depth of Cut (in.)	Feed Rate (in/rev.)	Rake Angle (Degrees)	Vertical Force Run 1	Vertical Force Run 2	F_c(lbs) Average	Horizontal Force Run 1	Horizontal Force Run 2	F_t(lbs) Average
0.020	0.0051	-5	28	28	28	13	12	12.5
0.020	0.0051	0	28	28	28	15	10	12.5
0.020	0.0051	+5	26	29	27.5	10	12	11
0.020	0.0102	-5	55	56	55.5	24	25	24.5
0.020	0.0102	0	53	54	53.5	20	20	20
0.020	0.0102	+5	52	54	53	17	20	18.5
0.020	0.0203	-5	96	98	97	30	30	30
0.020	0.0203	0	93	93	93	25	25	25
0.020	0.0203	+5	92	92	92	20	22	21

CORRECTED FORCE DATA

WORKPIECE MATERIAL SAE 1117 CRS
CUTTING SPEED 450 SFPM

INSERT K5H TPU 322

Depth of Cut (in.)	Feed Rate (in/rev.)	Rake Angle (Degrees)	Vertical Force F_c(lbs)			Horizontal Force F_t(lbs)		
			Run 1	Run 2	Average	Run 1	Run 2	Average
0.040	0.0051	-5	70	68	69	48	49	48.5
0.040	0.0051	0	56	60	58	30	35	32.5
0.040	0.0051	+5	52	54	53	25	24	24.5
0.040	0.0102	-5	118	118	118	73	73	73
0.040	0.0102	0	114	113	113.5	61	62	61.5
0.040	0.0102	+5	112	110	111	57	55	56
0.040	0.0203	-5	188	186	187	85	83	84
0.040	0.0203	0	188	186	187	70	73	71.5
0.040	0.0203	+5	180	178	179	65	63	64

CORRECTED FORCE DATA

WORKPIECE MATERIAL SAE 1117 CRS
CUTTING SPEED 450 SFPM

INSERT K5H TPU 322

Depth of Cut (in.)	Feed Rate (in/rev.)	Rake Angle (Degrees)	Vertical Force F_c(lbs)			Horizontal Force F_t(lbs)		
			Run 1	Run 2	Average	Run 1	Run 2	Average
0.060	0.0051	-5	102	104	103	82	84	83
0.060	0.0051	0	82	82	82	50	50	50
0.060	0.0051	+5	76	77	76.5	38	35	36.5
0.060	0.0102	-5	176	176	176	115	115	115
0.060	0.0102	0	168	168	168	100	105	102.5
0.060	0.0102	+5	152	154	153	80	80	80
0.060	0.0203	-5	320	318	319	150	145	147.5
0.060	0.0203	0	272	272	272	120	120	120
0.060	0.0203	+5	268	268	268	100	100	100

CORRECTED FORCE DATA

WORKPIECE MATERIAL 4145 HRS
CUTTING SPEED 450 SFPM

INSERT K5H TPU 322

Depth of Cut (in.)	Feed Rate (in/rev.)	Rake Angle (Degrees)	Vertical Force F_c(lbs) Run 1	Run 2	Average	Horizontal Force F_t(lbs) Run 1	Run 2	Average
0.020	0.0051	-5	44	44	44	20	20	20
0.020	0.0051	0	44	44	44	20	20	20
0.020	0.0051	+5	40	45	42.5	20	26	23
0.020	0.0102	-5	74	72	73	24	25	24.5
0.020	0.0102	0	62	70	66	20	22	21
0.020	0.0102	+5	74	78	76	27	31	29
0.020	0.0203	-5	124	129	126.5	30	32	31
0.020	0.0203	0	122	123	122.5	22	22	22
0.020	0.0203	+5	118	122	120	20	16	18

CORRECTED FORCE DATA

WORKPIECE MATERIAL 4145 HRS
CUTTING SPEED 450 SFPM

INSERT K5H TPU 322

Depth of Cut (in.)	Feed Rate (in/rev.)	Rake Angle (Degrees)	Vertical Force F_c(lbs) Run 1	Run 2	Average	Horizontal Force F_t(lbs) Run 1	Run 2	Average
0.040	0.0051	-5	82	84	83	44	45	44.5
0.040	0.0051	0	77	79	78	42	40	41
0.040	0.0051	+5	88	85	86.5	58	60	59
0.040	0.0102	-5	144	144	144	62	64	63
0.040	0.0102	0	136	136	136	55	52	53.5
0.040	0.0102	+5	128	134	131	50	60	55
0.040	0.0203	-5	246	240	243	85	80	82.5
0.040	0.0203	0	235	232	233.5	70	66	68
0.040	0.0203	+5	232	230	231	58	50	54

CORRECTED FORCE DATA

WORKPIECE MATERIAL SAE 4145 HRS
CUTTING SPEED 450 SFPM

INSERT K5H TPU 322

Depth of Cut (in.)	Feed Rate (in/rev.)	Rake Angle (Degrees)	Vertical Force F_c(lbs) Run 1	Run 2	Average	Horizontal Force F_t(lbs) Run 1	Run 2	Average
0.060	0.0051	-5	122	122	122	72	70	71
0.060	0.0051	0	118	114	116	65	62	63.5
0.060	0.0051	+5	118	112	115	72	70	71
0.060	0.0102	-5	210	208	209	110	100	105
0.060	0.0102	0	200	202	201	85	85	85
0.060	0.0102	+5	200	194	197	86	92	89
0.060	0.0203	-5	368	378	373	142	146	144
0.060	0.0203	0	336	333	334.5	110	104	107
0.060	0.0203	+5	344	343	343.5	84	90	87

CORRECTED FORCE DATA

WORKPIECE MATERIAL 6150 HRS
CUTTING SPEED 450 SFPM

INSERT K5H TPU 322

Depth of Cut (in.)	Feed Rate (in/rev.)	Rake Angle (Degrees)	Vertical Force F_c(lbs)			Horizontal Force F_t(lbs)		
			Run 1	Run 2	Average	Run 1	Run 2	Average
0.020	0.0051	-5	42	42	42	20	20	20
0.020	0.0051	0	46	44	45	22	24	23
0.020	0.0051	+5	48	49	48.5	32	31	31.5
0.020	0.0102	-5	78	74	76	30	35	32.5
0.020	0.0102	0	72	70	71	25	22	23.5
0.020	0.0102	+5	80	86	83	35	33	34
0.020	0.0203	-5	134	136	135	40	34	37
0.020	0.0203	0	127	124	125.5	25	25	25
0.020	0.0203	+5	112	122	117	40	34	37

CORRECTED FORCE DATA

WORKPIECE MATERIAL 6150 HRS
CUTTING SPEED 450 SFPM

INSERT K5H TPU 322

Depth of Cut (in.)	Feed Rate (in/rev.)	Rake Angle (Degrees)	Vertical Force Run 1	Run 2	F_c(lbs) Average	Horizontal Force Run 1	Run 2	F_t(lbs) Average
0.040	0.0051	-5	86	82	84	45	43	44
0.040	0.0051	0	88	83	85.5	47	58	52.5
0.040	0.0051	+5	94	90	92	53	50	51.5
0.040	0.0102	-5	148	154	151	75	83	79
0.040	0.0102	0	146	160	153	67	80	73.5
0.040	0.0102	+5	146	140	143	70	70	70
0.040	0.0203	-5	264	262	263	162	148	155
0.040	0.0203	0	280	294	287	200	195	197.5
0.040	0.0203	+5	254	264	259	142	155	148.5

CORRECTED FORCE DATA

WORKPIECE MATERIAL SAE 6150 HRS
CUTTING SPEED 450 SFPM

INSERT K5H TPU 322

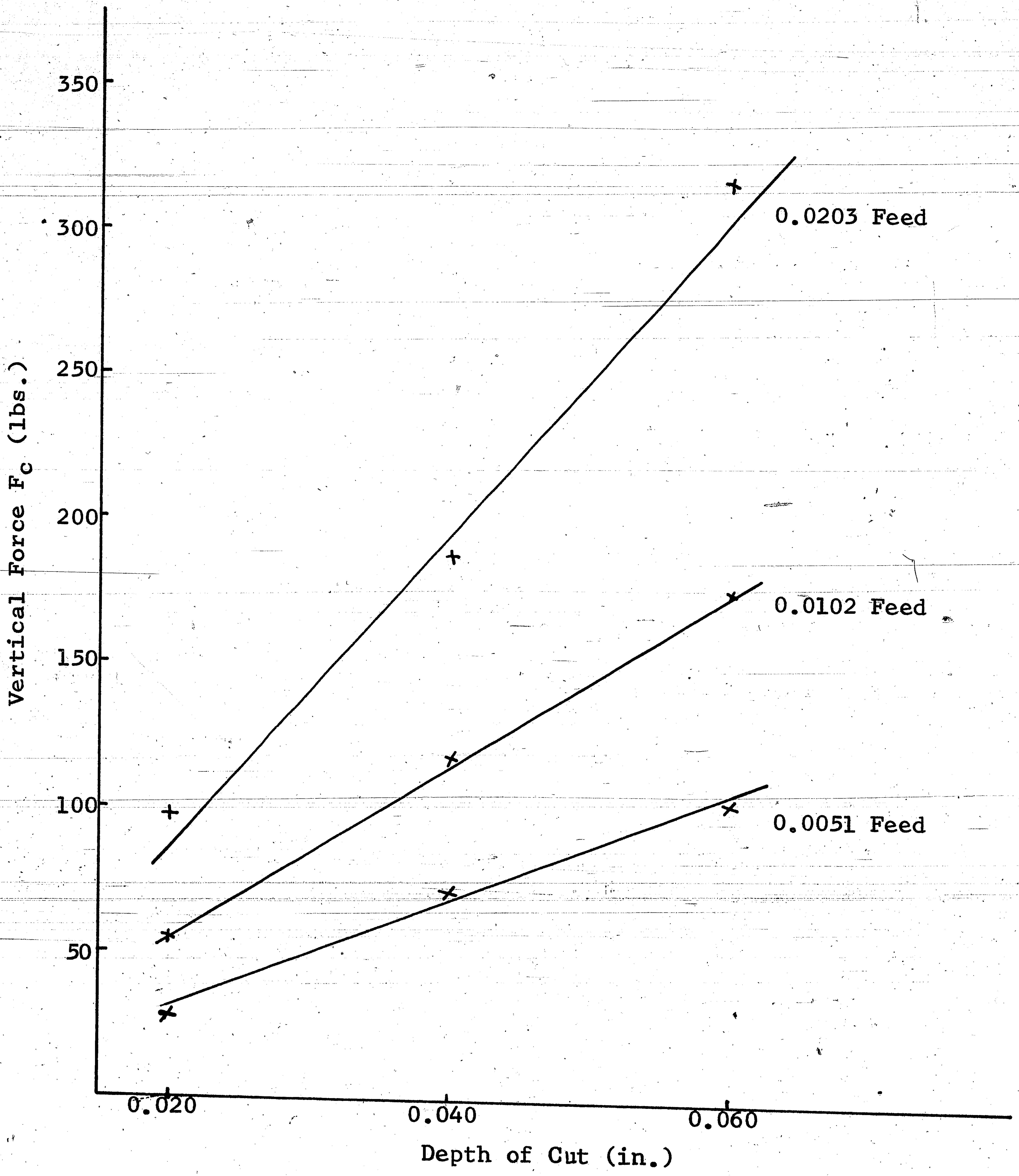
Depth of Cut (in.)	Feed Rate (in/rev.)	Rake Angle (Degrees)	Vertical Force F_c(lbs)			Horizontal Force F_t(lbs)		
			Run 1	Run 2	Average	Run 1	Run 2	Average
0.060	0.0051	-5	120	126	123	65	66	65.5
0.060	0.0051	0	128	128	128	63	65	64
0.060	0.0051	+5	124	122	123	78	72	75
0.060	0.0102	-5	228	224	226	108	110	109
0.060	0.0102	0	201	206	203.5	82	88	85
0.060	0.0102	+5	218	224	221	92	95	93.5
0.060	0.0203	-5	*	*	*	*	*	*
0.060	0.0203	0	*	*	*	*	*	*
0.060	0.0203	+5	*	*	*	*	*	*

APPENDIX D

VERTICAL FORCE (F_c) vs. DEPTH OF CUT

MATERIAL SAE 1117 CRS

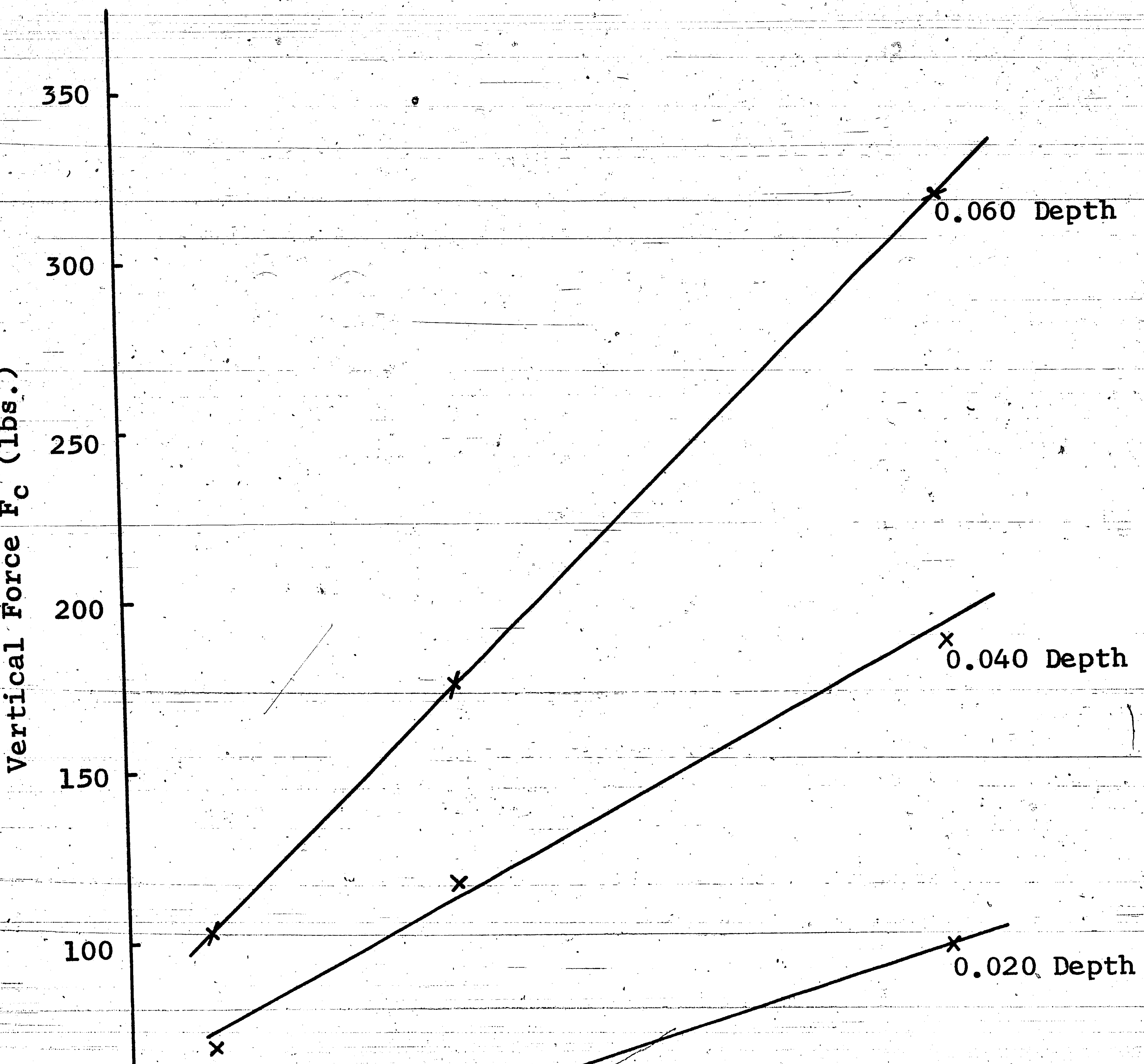
RAKE ANGLE -5 DEGREES



VERTICAL FORCE (F_c) vs. FEED RATE

MATERIAL SAE 1117 CRS

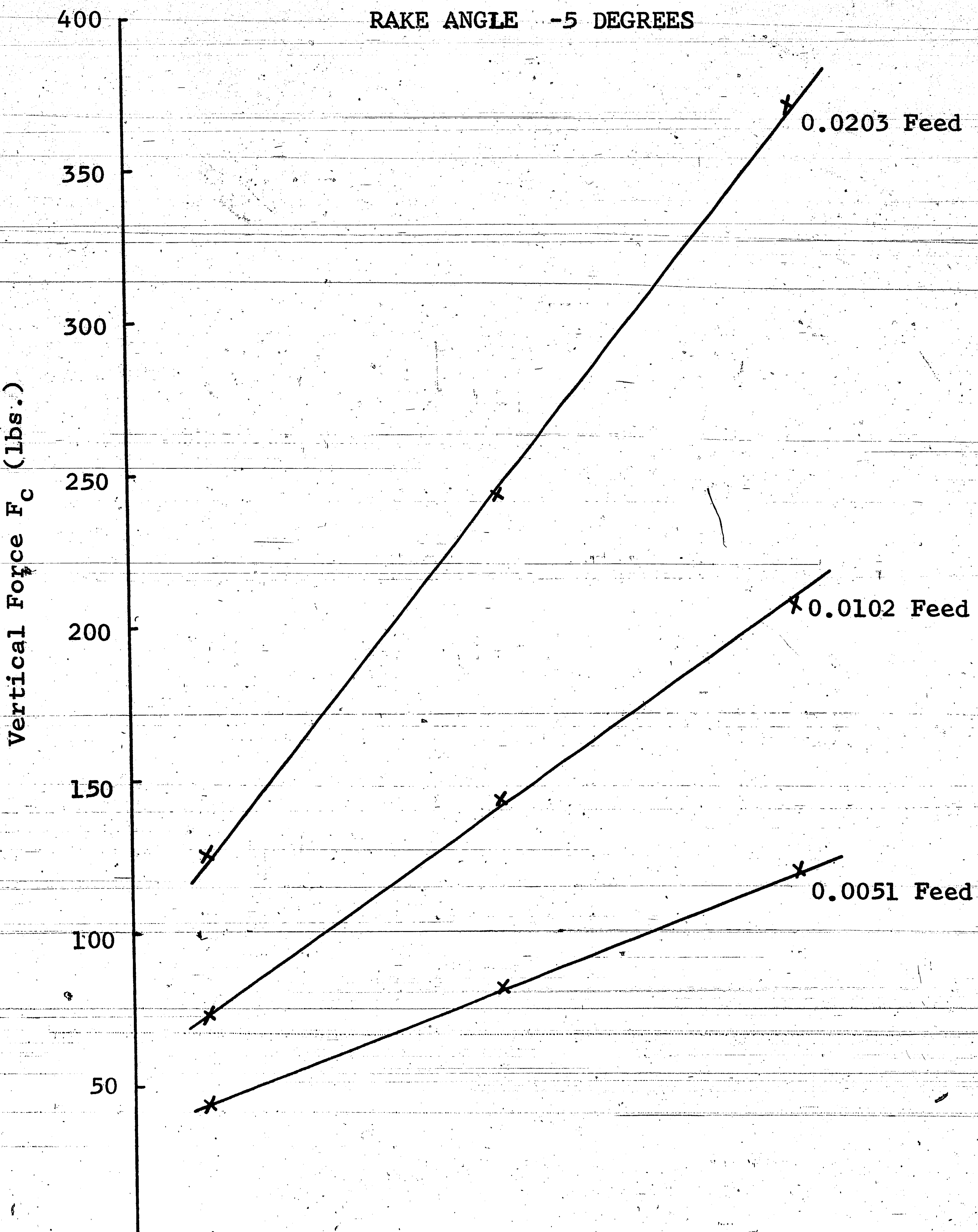
RAKE ANGLE -5 DEGREES



VERTICAL FORCE (F_c) vs. DEPTH OF CUT

MATERIAL SAE 4145 HRS

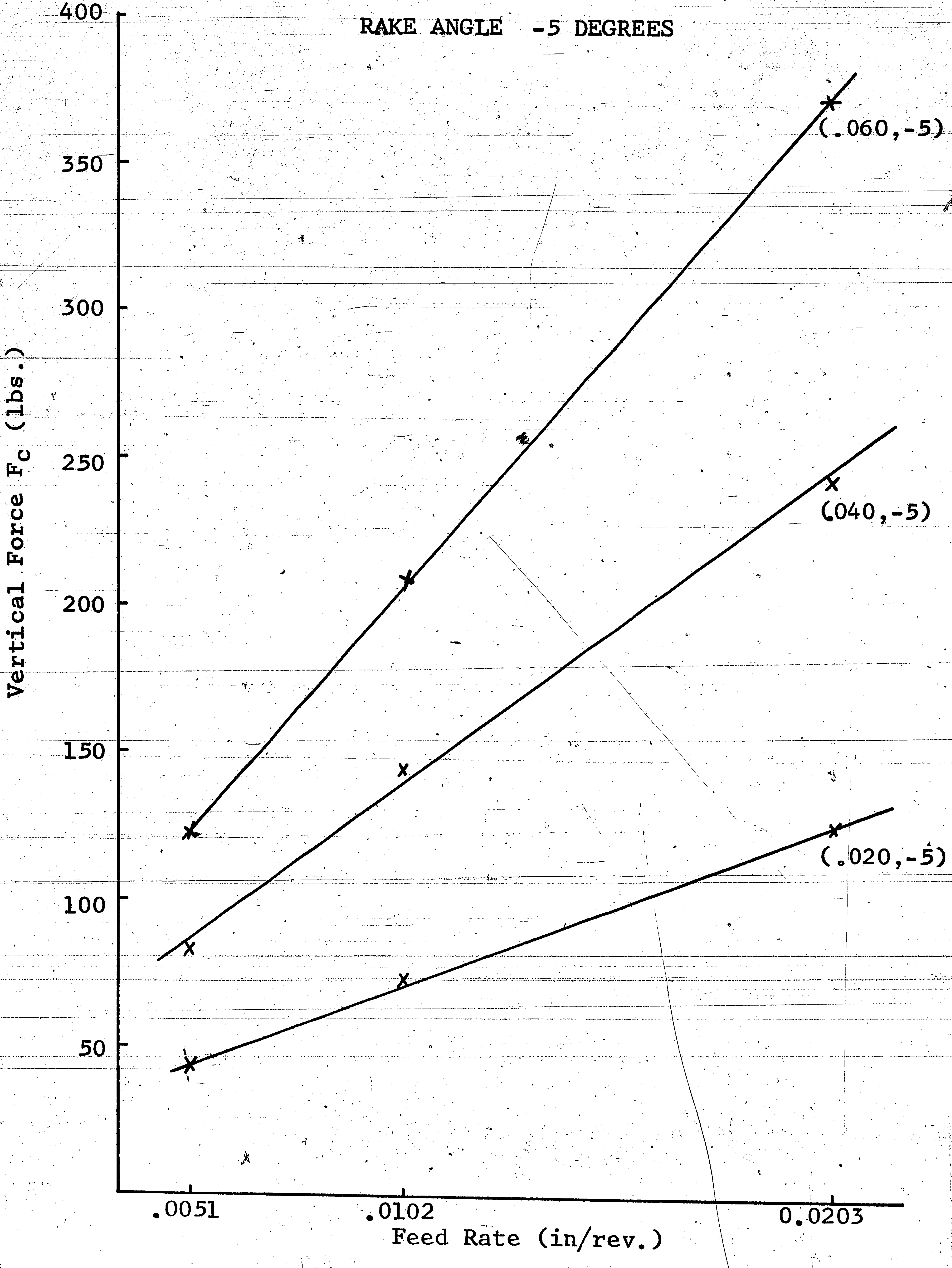
RAKE ANGLE -5 DEGREES



VERTICAL FORCE (F_c) vs. FEED RATE

MATERIAL SAE 4145 HRS

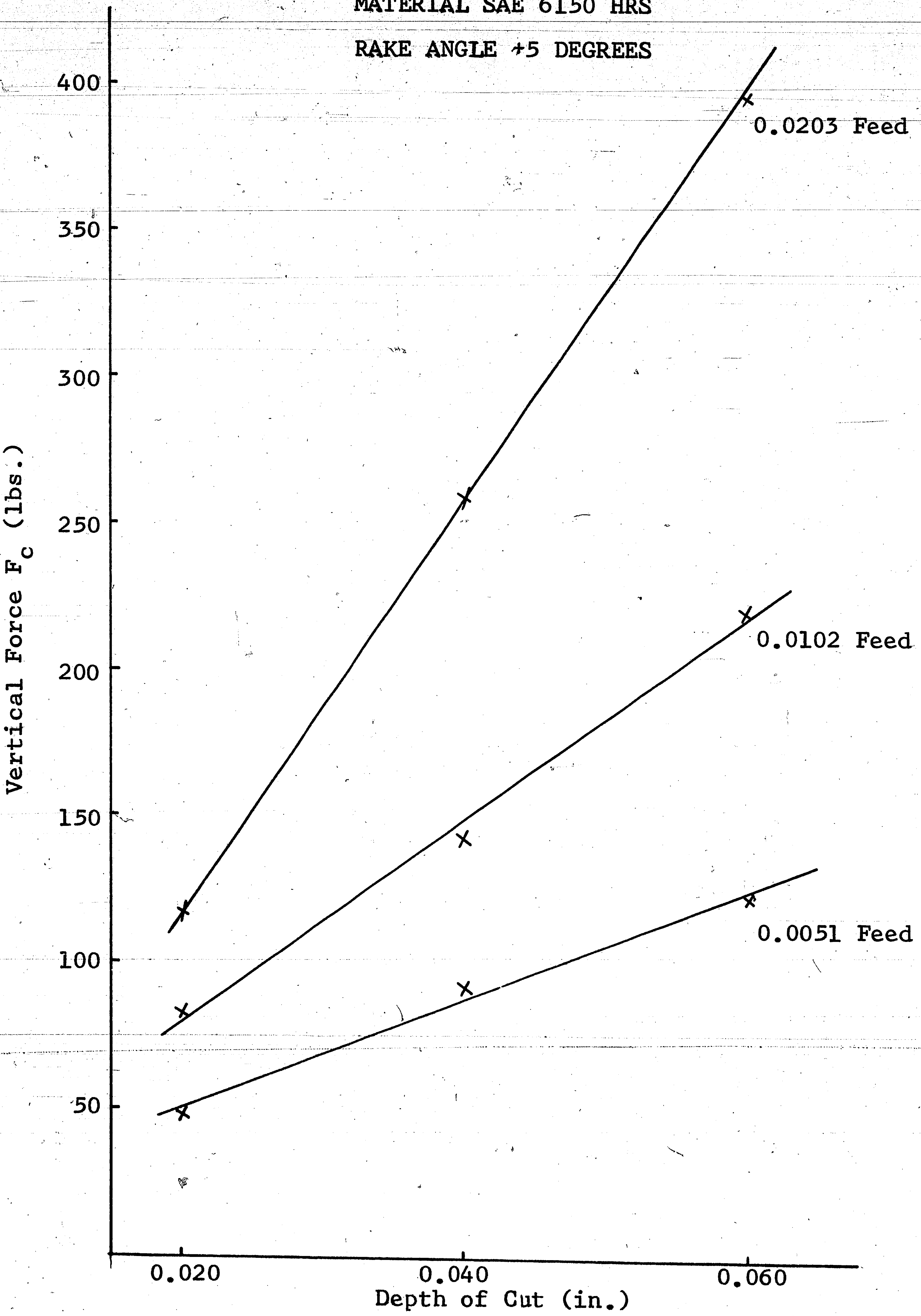
RAKE ANGLE -5 DEGREES

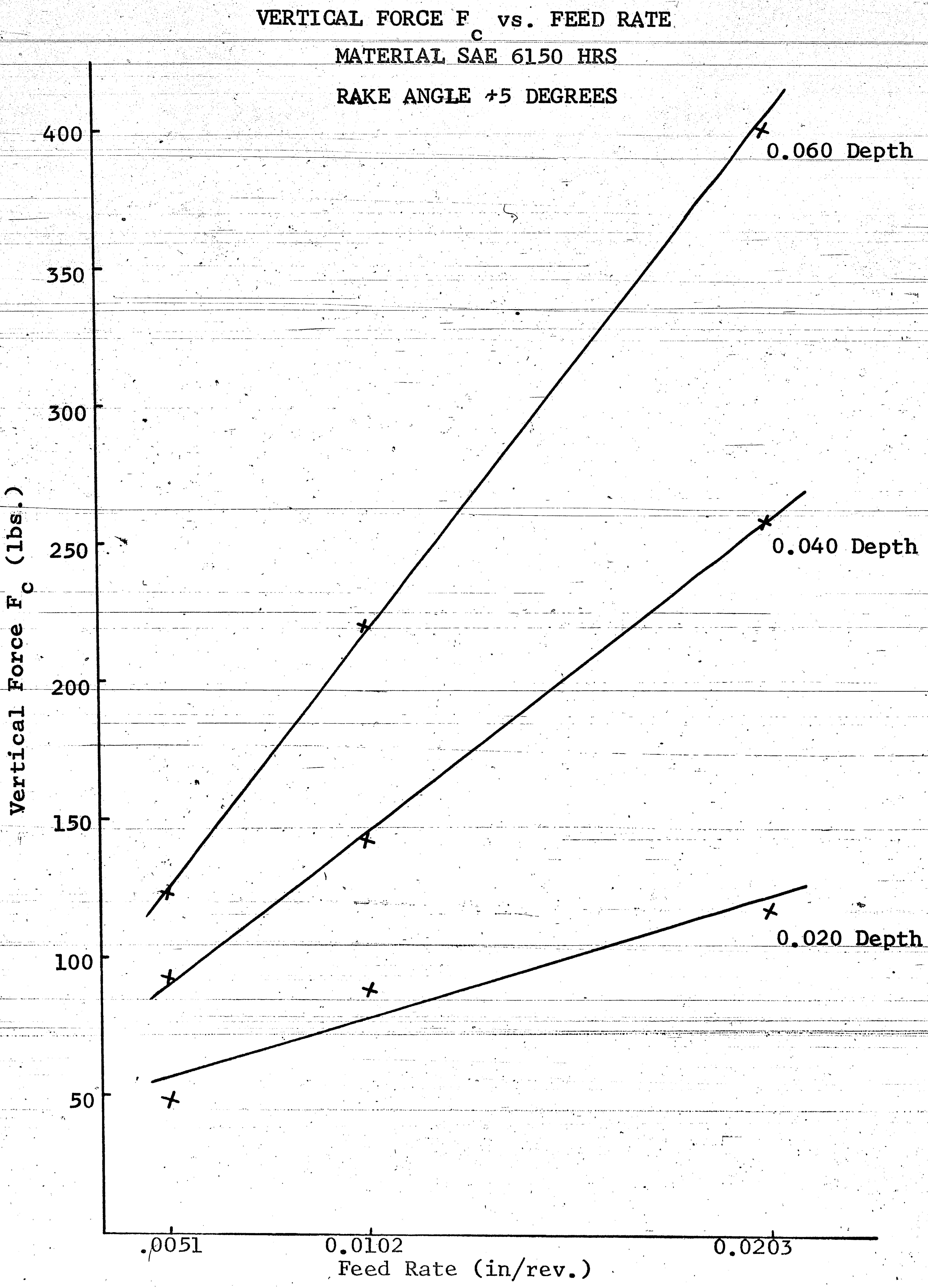


VERTICAL FORCE F_c vs. DEPTH OF CUT

MATERIAL SAE 6150 HRS

RAKE ANGLE +5 DEGREES



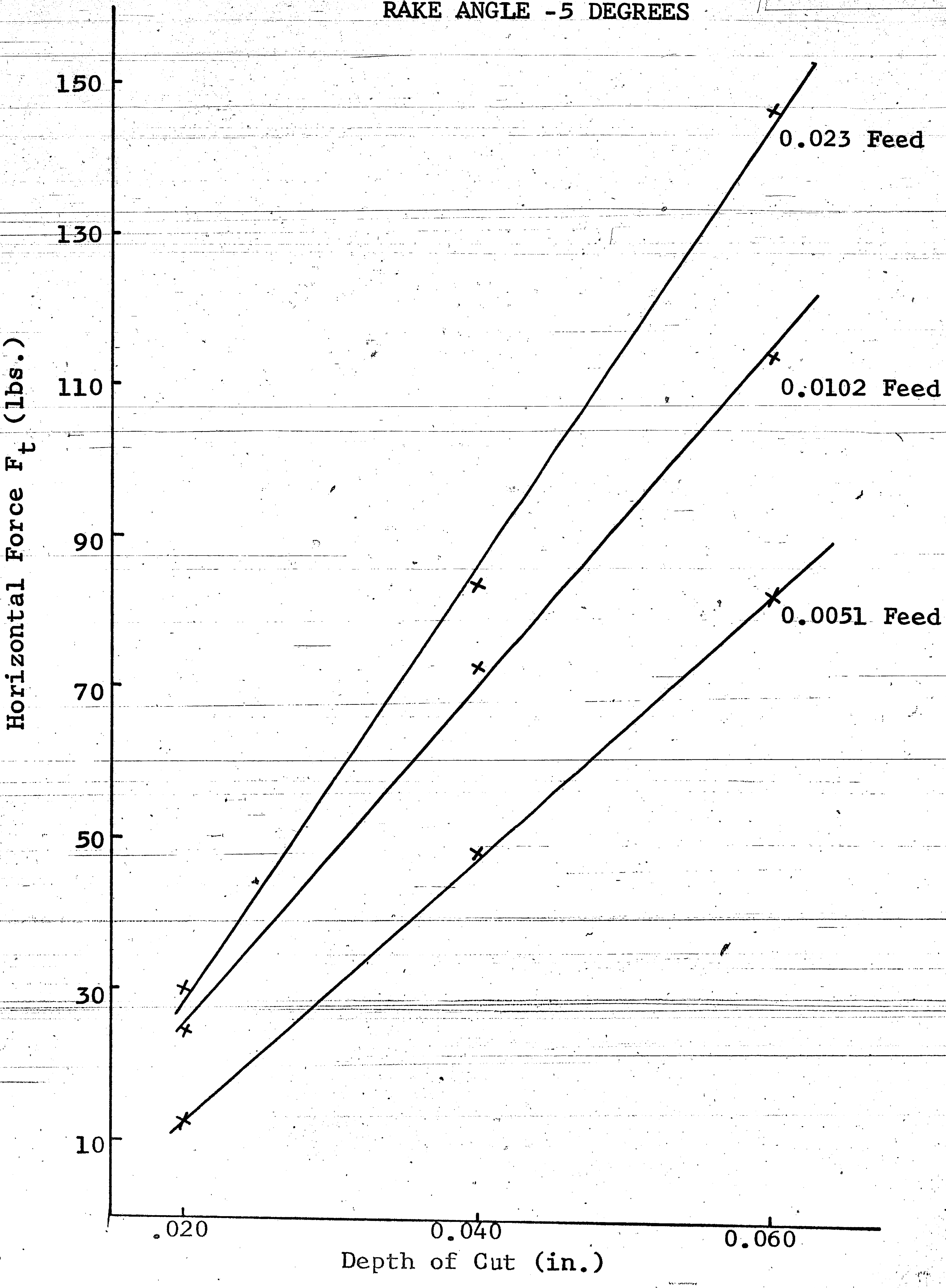


VERTICAL FORCE F_c vs. FEED RATE
MATERIAL SAE 6150 HRS
RAKE ANGLE +5 DEGREES

HORIZONTAL FORCE (F_t) vs. DEPTH OF CUT

MATERIAL SAE 1117 CRS

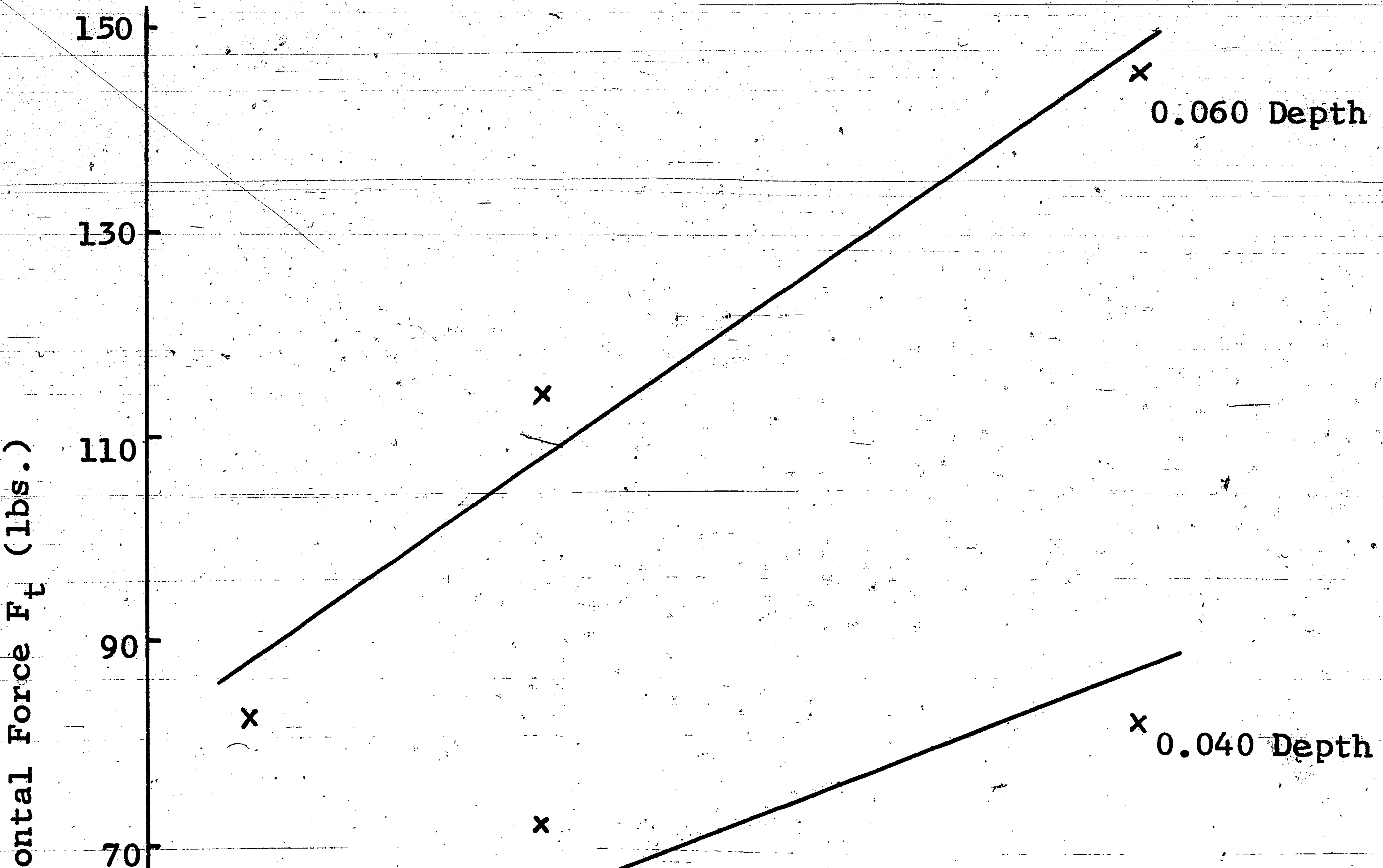
RAKE ANGLE -5 DEGREES



HORIZONTAL FORCE (F_t) vs. FEED RATE

MATERIAL SAE 1117 CRS

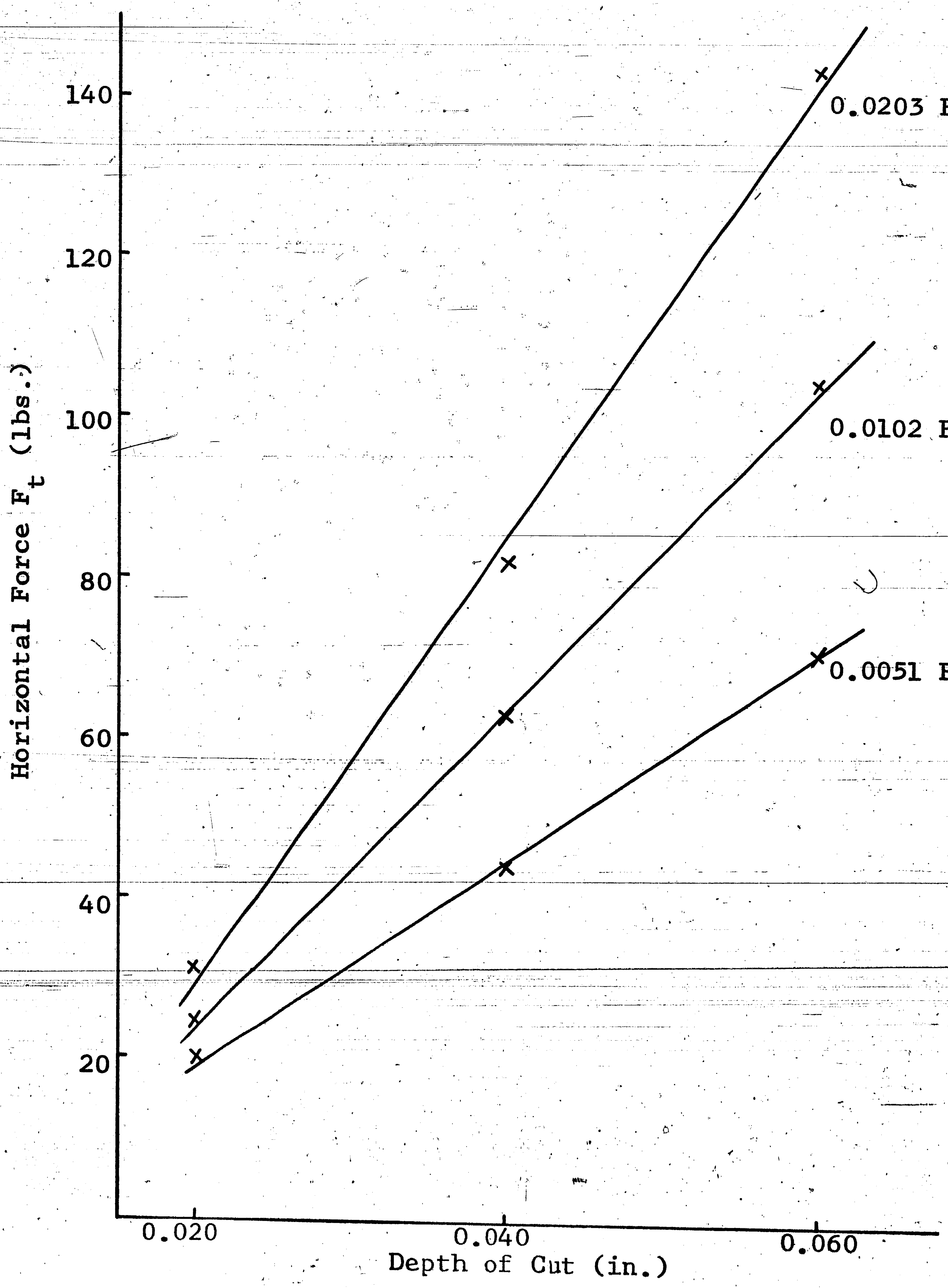
RAKE ANGLE -5 DEGREES



HORIZONTAL FORCE (F_t) vs. DEPTH OF CUT

MATERIAL SAE 4145 HRS

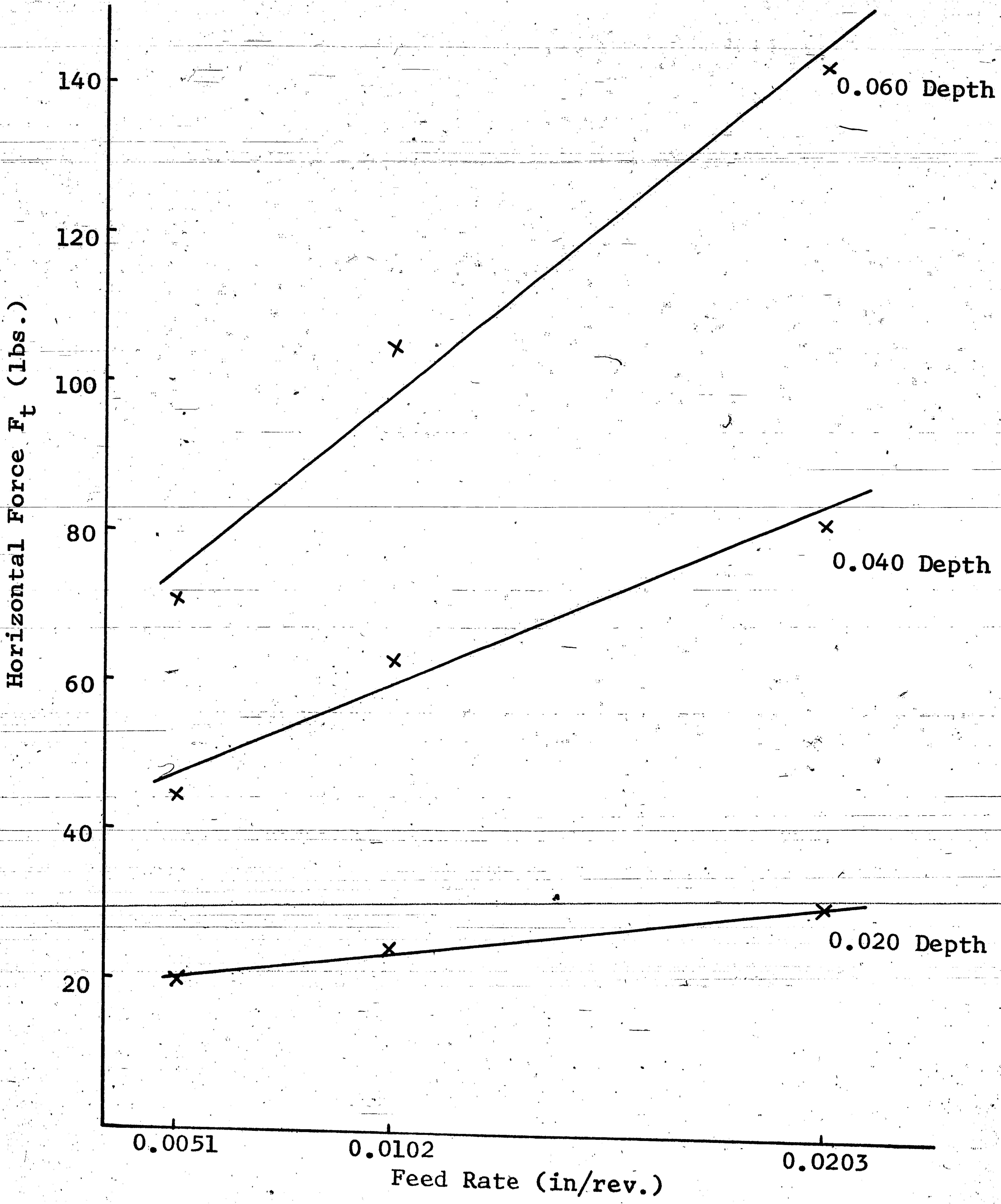
RAKE ANGLE -5 DEGREES



HORIZONTAL FORCE (F_t) vs. FEED RATE

MATERIAL SAE 4145 HRS

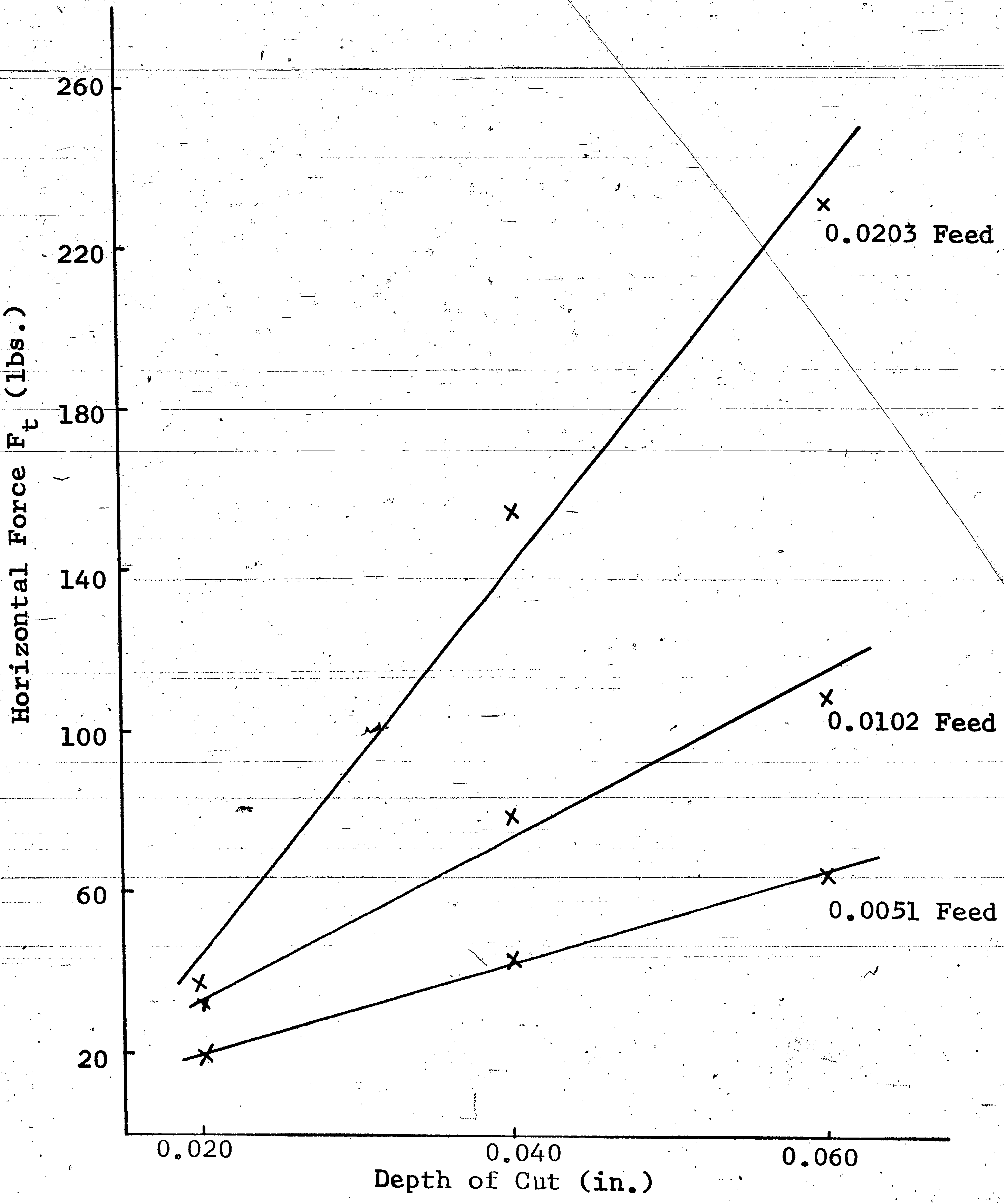
RAKE ANGLE -5 DEGREES



HORIZONTAL FORCE (F_t) vs. DEPTH OF CUT

MATERIAL SAE 6150 HRS

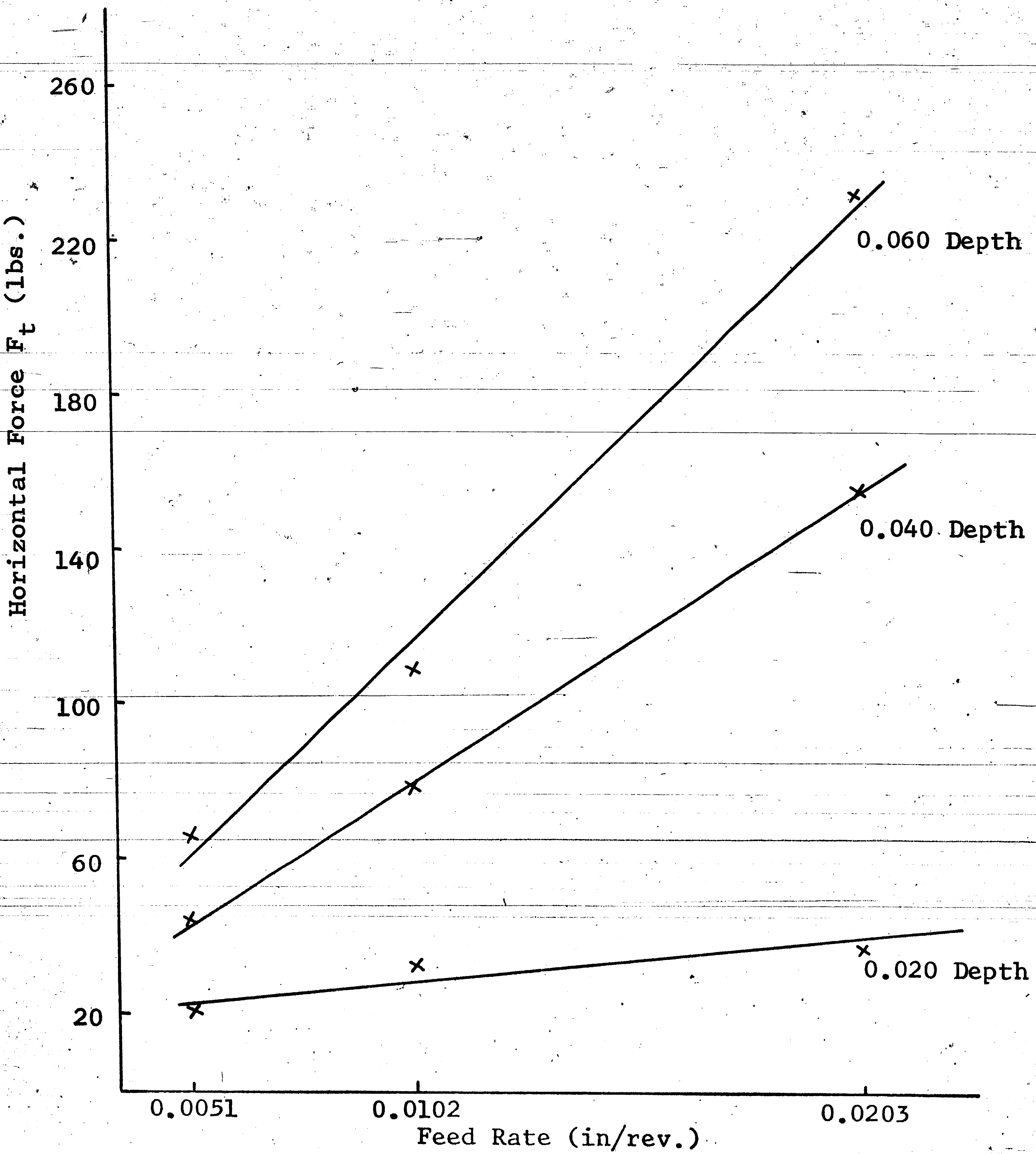
RAKE ANGLE -5 DEGREES



HORIZONTAL FORCE (F_t) vs. FEED RATE

MATERIAL SAE 6150 HRS

RAKE ANGLE -5 DEGREES



APPENDIX E

RESULTS OF ANALYSIS OF VARIANCE

VERTICAL FORCE $-F_c$

Number of Factors	4
Number of Replicates	2
Levels of A (Shear Strength)	3
Levels of B (Depth of Cut)	3
Levels of C (Feed Rate)	3
Levels of D (Rake Angle)	3
Total No. of Observations	162

Effect	Degrees of Freedom	Mean Square	"F" Ratio
A	2	31,762.7	3,550.0 *
B	2	279,671.8	31,250.0 *
C	2	334,379.3	37,450.0 *
D	2	1,155.9	129.5 *
AB	4	2,340.6	262.0 *
AC	4	3,429.9	384.0 *
BC	4	30,556.7	3,420.0 *
AD	4	248.4	27.4 *
BD	4	450.7	50.4 *
CD	4	182.3	20.4 *
ABC	8	563.7	63.0 *
ABD	8	110.6	12.4 *

* Highly Significant Effects

RESULTS OF ANALYSIS OF VARIANCE (cont'd.)

Effect	Degrees of Freedom	Mean Square	"F" Ratio
ACD	8	98.9	11.0 *
BCD	8	131.6	14.7 *
ABCD	16	67.1	7.6 *
R(ABCD) Residual	81	8.94	

* Highly Significant Effects

"F" Ratio	99% Level	95% Level
2/81	4.79	3.07
4/81	3.48	2.45
8/81	2.66	2.02
16/81	2.19	1.75

RESULTS OF ANALYSIS OF VARIANCE

VERTICAL FORCE F_c

MATERIAL SAE 1117 CRS

Number of Factors	3
Number of Replications	2
Levels of A (Depth of Cut)	3
Levels of B (Feed Rate)	3
Levels of C (Rake Angle)	3
Total No. of Observations	54

Effect	Degrees of Freedom	Mean Square	"F" Ratio
A	2	66,006.2	48,700.0 *
B	2	76,784.1	56,600.0 *
C	2	1,138.2	840.0 *
AB	4	6,792.5	5,020.0 *
AC	4	432.1	319.0 *
BC	4	67.1	49.5 *
ABC	8	88.9	65.6 *
R(ABC) Residual	27	1.35	

* Highly Significant Effects

"F" Ratio	99% Level	95% Level
2/27	5.39	3.32
4/27	4.02	2.69
8/27	3.17	2.27

RESULTS OF ANALYSIS OF VARIANCE

VERTICAL FORCE F_c

MATERIAL: SAE 4145 HRS

Number of Factors	3
Number of Replications	2
Levels of A (Depth of Cut)	3
Levels of B (Feed Rate)	3
Levels of C (Rake Angle)	3
Total No. of Observations	54

Effect	Degrees of Freedom	Mean Square	"F" Ratio
A	2	93,384.8	10,960.0 *
B	2	111,198.0	13,100.0 *
C	2	486.7	57.3 *
AB	4	9,003.2	1,047.0 *
AC	4	105.3	12.4 *
BC	4	102.2	12.0 *
ABC	8	72.8	8.5 *
R(ABC) Residual	27	8.5	

* Highly Significant Effects

"F" Ratio	99% Level	95% Level
2/27	5.39	3.32
4/27	4.02	2.69
8/27	3.17	2.27

RESULTS OF ANALYSIS OF VARIANCE

VERTICAL FORCE F_c

MATERIAL SAE 6150 HRS

1

Number of Factors	3
Number of Replications	2
Levels of A (Depth of Cut)	3
Levels of B (Feed Rate)	3
Levels of C (Rake Angle)	3
Total No. of Observations	54

Effect	Degrees of Freedom	Mean Square	"F" Ratio
A	2	124,962.1	7,370.0 *
B	2	153,257.2	9,050.0 *
C	2	27.7	1.64
AB	4	15,888.4	935.0 *
AC	4	134.5	7.93*
BC	4	210.7	12.42*
ABC	8	104.2	6.15*
R(ABC) Residual	27	16.9	

* Highly Significant Effects

"F" Ratio	99% Level	95% Level
2/27	5.39	3.32
4/27	4.02	2.69
8/27	3.17	2.27

RESULTS OF ANALYSIS OF VARIANCE

HORIZONTAL FORCE F_t

Number of Factors	4
Number of Replications	2
Levels of A (Shear Strength)	3
Levels of B (Depth of Cut)	3
Levels of C (Feed Rate)	3
Levels of D (Rake Angle	3
Total No. of Observations	162

Effect	Degrees of Freedom	Mean Square	"F" Ratio
A	2	14,453.6	1,410.0 *
B	2	88,572.2	8,640.0 *
C	2	37,136.8	3,610.0 *
D	2	2,041.1	198.0 *
AB	4	1,896.9	185.0 *
AC	4	8,511.8	803.0 *
BC	4	7,893.0	770.0 *
AD	4	573.9	55.8 *
BD	4	772.5	75.3 *
CD	4	493.3	48.0 *
ABC	8	2,093.7	204.0 *
ABD	8	214.4	20.8 *

* Highly Significant Effects

RESULTS OF ANALYSIS OF VARIANCE (cont'd.)

Effect	Degrees of Freedom	Mean Square	"F" Ratio
ACD	8	241.6	23.5 *
BCD	8	95.1	9.28 *
ABCD	16	75.2	7.34 *
R(ABCD) Residual	81	10.3	

* Highly Significant Effects

"F" Ratio	99% Level	95% Level
2/81	4.79	3.07
4/81	3.48	2.45
8/81	2.66	2.02
16/81	2.19	1.75

RESULTS OF ANALYSIS OF VARIANCE

HORIZONTAL FORCE F_T

MATERIAL SAE 1117 CRS

Number of Factors	3
Number of Replications	2
Levels of A (Depth of Cut)	3
Levels of B (Feed Rate)	3
Levels of C (Rake Angle)	3
Total No. of Observations	54

Effect	Degrees of Freedom	Mean Square	"F" Ratio
A	2	24,171.9	8,340.0 *
B	2	7,186.9	2,480.0 *
C	2	2,396.5	826.0 *
AB	4	1,075.8	372.0 *
AC	4	539.5	186.5 *
BC	4	25.7	8.9 *
ABC	8	24.9	8.62*
R(ABC) Residual	27	2.9	

*Highly Significant Effects

"F" Ratio	99% Level	95% Level
2/27	5.39	3.32
4/27	4.02	2.69
8/27	3.17	2.27

RESULTS OF ANALYSIS OF VARIANCE

HORIZONTAL FORCE F_t

MATERIAL SAE 4145 HRS

Number of Factors	3
Number of Replications	2
Levels of A (Depth of Cut)	3
Levels of B (Feed Rate)	3
Levels of C (Rake Angle)	3
Total No. of Observations	54

Effect	Degrees of Freedom	Mean Square	"F" Ratio
A	2	20,946.1	2,080.0 *
B	2	2,243.6	223.0 *
C	2	779.1	77.2 *
AB	4	659.6	65.3 *
AC	4	225.3	22.3 *
BC	4	593.6	58.9 *
ABC	8	71.2	7.06*
R(ABC) Residual	27	10.1	

* Highly Significant Effects

"F" Ratio	99% Level	95% Level
2/27	5.39	3.32
4/27	4.02	2.69
8/27	3.17	2.27

RESULTS OF ANALYSIS OF VARIANCE

HORIZONTAL FORCE F_t

MATERIAL SAE 6150 HRS

Number of Factors	3
Number of Replications	2
Levels of A (Depth of Cut)	3
Levels of B (Feed Rate)	3
Levels of C (Rake Angle)	3
Total No. of Observations	54

Effect	Degrees of Freedom	Mean Square	"F" Ratio
A	2	47,248.0	2,650.0 *
B	2	44,729.9	2,510.0 *
C	2	13.4	0.753
AB	4	10,344.9	580.0 *
AC	4	436.4	24.5 *
BC	4	357.3	20.0 *
ABC	8	149.5	8.4 *
R(ABC) Residual	27	17.8	

* Highly Significant Effects

"F" Ratio	99% Level	95% Level
2/27	5.39	3.32
4/27	4.02	2.69
8/27	3.17	2.27

APPENDIX F

RESULTS OF THE MULTIPLE LINEAR REGRESSION

VERTICAL FORCE F_c

$$F_c = 18.998 - 1.403A - 2.828B - 26.813C - 1.604D$$
$$+ 0.828AB + 2.566AC + 18.476BC + 0.261AD$$
$$- 0.430BC + 4.006CD + 0.634ABC + 0.020ABD$$
$$- 0.414ACD - 0.940BCD + 0.076ABCD$$

Where:

A = Shear Strength in psi/10,000

B = Depth of Cut in inch x 100

C = Feed Rate in inch/rev. x 100

D = Rake Angle in Degrees

Analysis of Variance

Effect	Degrees of Freedom	Mean Square	"F" Ratio
Regression	15	68,129.998	1169.625
Residual	140	58.249	

Multiple Correlation Coefficient = 0.996

Standard Deviation of Residual = ±7.65 lbs.

"F" Ratio Significance = 99.9%

RESULTS OF THE MULTIPLE LINEAR REGRESSION

VERTICAL FORCE F_c

$$F_c = 18.998 - 1.403A - 2.828B - 26.813C + 0.828AB + 2.566AC + 18.476BC + 0.634ABC$$

Where:

A = Shear Strength in psi/10,000

B = Depth of Cut in inch x 100

C = Feed Rate in inch/rev. x 100

Analysis of Variance

Effect	Degrees of Freedom	Mean Square	"F" Ratio
Regression	7	145,217.014	1581.955
Residual	148	91.796	

Multiple Correlation Coefficient = 0.9934

Standard Deviation of Residual = ±9.54 lbs.

"F" Ratio Significance = 99.9%

RESULTS OF THE MULTIPLE LINEAR REGRESSION

VERTICAL FORCE F_c

MATERIAL SAE 1117 CRS

$$F_c = 0.87 + 4.64A - 2.28B + 0.41C + 21.61AB - 0.33AC + 0.94BC - 0.37ABC$$

Where:

A = Depth of Cut in inch x 100

B = Feed Rate in inch/rev. x 100

C = Rake Angle in degrees

Analysis of Variance

Effect	Degrees of Freedom	Mean Square	"F" Ratio
Regression	7	45,029.979	808.823
Residual	46	55.673	

Multiple Correlation Coefficient = 0.9960

Standard Deviation of Residual = ±7.45 lbs.

"F" Ratio Significance = 99.9%

RESULTS OF THE MULTIPLE LINEAR REGRESSION

VERTICAL FORCE F_c

MATERIAL SAE 4145 HRS

$$F_c = 5.69 + 6.33A + 1.51B + 0.42C + 25.01AB - 0.036AC + 0.17BC - 0.275ABC$$

Where:

A = Depth of Cut in inch x 100

B = Feed Rate in inch/rev. x 100

C = Rake Angle in Degrees

Analysis of Variance

Effect	Degrees of Freedom	Mean Square	"F" Ratio
Regression	7	63,753.525	1930.968
Residual	46	33.016	

Multiple Correlation Coefficient = 0.9983

Standard Deviation of Residual = ±5.75 lbs.

"F" Ratio Significance = 99.9%

RESULTS OF THE MULTIPLE LINEAR REGRESSION

VERTICAL FORCE F_c

MATERIAL SAE 6150 HRS

$$F_c = 17.467 + 1.807A - 15.184B + 33.805AB + 0.151AC - 0.347BC - 0.117ABC$$

Where:

A = Depth of Cut in inch x 100

B = Feed Rate in inch/rev. x 100

C = Rake Angle in Degrees

Analysis of Variance

Effect	Degrees of Freedom	Mean Square	"F" Ratio
Regression	6	38,958.577	609.706
Residual	41	63.897	

Multiple Correlation Coefficient = 0.9944

Standard Deviation of Residual = ±7.98 lbs.

"F" Ratio Significance = 99.9%

RESULTS OF THE MULTIPLE LINEAR REGRESSION

HORIZONTAL FORCE F_t

$$
\begin{aligned}
F_t = {} & 123.416 - 12.140A - 20.080B - 137.496C + 4.254D \\
& + 2.787AB + 12.848AC + 33.694BC - 0.320AD \\
& - 2.833BD - 2.541CD - 2.546ABC + 0.265ABD \\
& + 0.287ACD + 1.187BCD - 0.163ABCD
\end{aligned}
$$

Where:

A = Shear Strength in psi/10,000

B = Depth of Cut in inch x 100

C = Feed Rate in inch/rev. x 100

D = Rake Angle in Degrees

Analysis of Variance

Effect	Degrees of Freedom	Mean Square	"F" Ratio
Regression	15	12,721.078	44.758
Residual	140	284.219	

Multiple Regression Coefficient = 0.9096

Standard Deviation of Residual = ±16.8 lbs.

"F" Ratio Significance = 99.9%

RESULTS OF THE MULTIPLE LINEAR REGRESSION

HORIZONTAL FORCE F_t

$$F_t = 123.416 - 12.410A - 20.080B - 137.496C + 2.787AB + 12.848AC + 33.694BC - 2.546ABC$$

Where:

A = Shear Strength in psi/10,000

B = Depth of Cut in inch x 100

C = Feed Rate in inch/rev. x 100

D = Rake Angle in Degrees

Analysis of Variance

Effect	Degrees of Freedom	Mean Square	"F" Ratio
Regression	7	25,620.024	73.962
Residual	148	346.396	

Multiple Correlation Coefficient = 0.8819

Standard Deviation of Residual = ±18.6 lbs.

"F" Ratio Significance = 99.9%

RESULTS OF THE MULTIPLE LINEAR REGRESSION

HORIZONTAL FORCE $-F_t$

MATERIAL SAE 1117 CRS

$$F_t = -6.619 + 8.701A - 8.577B + 1.937C + 8.105AB - 1.007AC - 0.406BC + 0.059ABC$$

Where:

A = Depth of Cut in inch x 100

B = Feed Rate in inch/rev. x 100

C = Rake Angle in Degrees

Analysis of Variance

Effect	Degrees of Freedom	Mean Square	"F" Ratio
Regression	7	10,173.073	149.034
Residual	46	68.260	

Multiple Correlation Coefficient = 0.9787

Standard Deviation of Residual = ±8.26 lbs.

"F" Ratio Significance = 99.9%

RESULTS OF THE MULTIPLE LINEAR REGRESSION

HORIZONTAL FORCE $-F_t$

MATERIAL SAE 4145 HRS

$$F_t = 3.955 + 9.234A - 12.4B + 1.024C + 6.591AB + 0.222AC + 0.092BC - 0.661ABC$$

Where:

A = Depth of Cut in inch x 100

B = Feed Rate in inch/rev. x 100

C = Rake Angle in Degrees

Analysis of Variance

Effect	Degrees of Freedom	Mean Square	"F" Ratio
Regression	7	7,603.631	238.260
Residual	46	31.913	

Multiple Correlation Coefficient = 0.9865

Standard Deviation of Residual = ±5.65 lbs.

"F" Ratio Significance = 99.9%

RESULTS OF THE MULTIPLE LINEAR REGRESSION

HORIZONTAL FORCE F_t

MATERIAL SAE 6150 HRS

$$F_t = 40.240 - 8.945A - 52.463B + 30.864AB + 0.283AC + 1.009BC - 0.573ABC$$

Where:

A = Depth of Cut in inch x 100
B = Feed Rate in inch/rev. x 100
C = Rake Angle in Degrees

Analysis of Variance

Effect	Degrees of Freedom	Mean Square	"F" Ratio
Regression	6	14,184.698	46.386
Residual	41	305.798	

Multiple Correlation Coefficient = 0.9336
Standard Deviation of Residual = ±17.45 lbs.
"F" Ratio Significance = 99.9%

APPENDIX G

RESULTS OF UNIT POWER CALCULATIONS

Cutting Speed 450 SFPM

Material	Hardness	Depth of Cut	Feed Rate	Cutting Force (lbs)		H.P.		Metal Removal in Cu.in/min.	H.P.-min/in^3	
				Measured	Estimated	Measured	Estimated		Measured	Estimated
SAE 1117 CRS	137 BHN	0.060	0.0051	82.0	92.38	1.120	1.260	1.550	0.720	0.810
		0.020	0.0102	53.0	52.70	0.724	0.720	1.100	0.657	0.655
		0.040	0.0203	187.0	188.78	2.560	2.575	4.200	0.610	0.614
SAE 4145 HRS	320 BHN	0.020	0.0051	44.0	43.16	0.600	0.588	0.584	1.030	1.010
		0.040	0.0102	136.0	135.10	1.860	1.845	1.780	1.045	1.038
		0.060	0.0203	373.0	356.73	5.080	4.870	6.440	0.790	0.760
SAE 6150 HRS	330 BHN	0.040	0.0051	85.5	87.64	1.168	1.198	0.890	1.310	1.345
		0.060	0.0102	203.5	212.53	2.775	2.900	3.240	0.857	0.895
		0.020	0.0203	135.0	133.57	1.842	1.825	2.320	0.795	0.780

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VITA

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