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An investigation of heuristic decision rules for allocating limited resources in stochastic activity networks

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AN INVESTIGATION OF HEURISTIC DECISION RULES FOR ALLOCATING LIMITED RESOURCES IN STOCHASTIC ACTIVITY NETWORKS

by

Gerald Stephen Chupik

ABSTRACT

The effort of this thesis has been directed toward the examination of decision rules concerning resource allocation in stochastic activity networks whose resources are constrained. Specifically, the objective was to determine if any one decision rule serves as an effective one for producing consistently low completion times in any network. The approach taken was to simulate the performance of each rule in a given network using a simulation program called GERTS III R. Four networks were chosen to provide the situations for exercising each of eleven decision rules selected. Since resources are limited, it becomes important to decide which activities within the network are to be undertaken at a given point in time, in order to achieve minimum project completion time.

The results of the experiment showed that - (1) there is a definite effect on project completion time resulting from the use of different decision rules, (2) the resulting project completion times are dependent upon the conjunctive use of network configurations and rules, and (3) the application of no one particular rule will guarantee a minimum completion time in all networks.

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by

Gerald Stephen Chupik

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CERTIFICATE OF APPROVAL

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The results of the experiment showed that - (1) there is a definite effect on project completion time resulting from the use of different decision rules, (2) the resulting project completion times are dependent upon the conjunctive use of network configurations and rules, and (3) the application of no one particular rule will guarantee a minimum completion time in all networks.

I. INTRODUCTION AND OBJECTIVE

Introduction

Many large scale projects such as those found in a research and development environment contain one of a kind activities whose completion times are difficult to estimate. While similar activities may have been undertaken in the past, sufficient information concerning their completion times is inconclusive. At best, activity durations can be estimated over a range of time distributions. Furthermore, the project completion time is dependent upon the availability of resources necessary to complete the individual activities. The proper sequencing of activities under limited resources will have a definite effect on minimizing the project completion. As one may suspect, the problem of resource allocation is not limited to R & D projects, but may also include other projects such as large scale construction, production planning, and maintenance activities.

One of the most common methods of representing project activities is by means of a PERT (Program Evaluation and Review Technique) chart which depicts the project as a network of precedence-constrained activities and events. If the resources available to the project are constrained, the scheduling of the available resources to meet these constraints could become a major problem in meeting the objectives.

As pointed out, activity completion times are stochastic in nature, i.e., their times cannot be determined with exact certainty. Each activity in the network has associated with it three time estimates of completion, optimistic, pessimistic, and most likely. The model of the distribution of an activity time is normally represented by the beta distribution whose mode is the likely time, whose range is the interval between the optimistic and pessimistic times and whose standard deviation is approximated as one-sixth of the range.

Numerous allocation plans and rules mainly heuristic in nature have been proposed for projects having limited resources and known or deterministic activity completion times. Heuristic rules and procedures are those whose methods are unproved or incapable of proof. Most often they are based on intuitive logic and experimental observations. Of the various procedures known, most do not offer optimum solutions which is characteristic of heuristics. The best that can be hoped for is a sub-optimal solution satisfying the constraints and needs that management has imposed on the objectives.

Objectives

The objective of this thesis is to compare the effects of several of the more popular allocation rules on several networks of different configurations using a simulation program known as GERTS (Graphical Evaluation and Review Technique Simulator) III R. Variations to some of the rules will also be tested in an effort to arrive at some general procedure for allocating resources. The networks themselves are structured as a series of precedence constrained activities, i.e., certain activities must be completed before succeeding ones are started. An example of this would be the pouring of a foundation for a house before the frame is erected. In addition to the precedence constraints, further restrictions are placed on the availability of resources required by activities which may be scheduled, i.e., whose predecessor activities have been completed.

It is recognized that there are many different resource-time relationships in existence, some being complex. Among these relationships are concave, convex and linear functions. Different combinations of these relationships may occur among the activities in the same network. For this thesis, however, it will be assumed that a linear relationship exists between the amount of resources allocated to an activity and its most likely completion time. The completion times are not known for certain, but follow a beta distribution about the most likely time.

II. DEVELOPMENTS IN RESOURCE ALLOCATION

This chapter discusses some of the heuristic allocation methods currently being used.

J. E. Kelley, Jr.[14] presents two algorithms, serial and parallel, to handle the problems of limited resources where duration of the project is not necessarily minimized but is within a reasonable neighborhood of the minimum. The input to the serial method is a list of all the activities in order of precedence. The activity with the earliest start time is scheduled first. Earliest start time for an activity is the latest time any of its predecessors are completed. If the resources required to complete an activity are unavailable, the start time is delayed to the point where adequate resources are available. It is assumed that activities can be split into parts which are integral number of time units long so that available resources may be applied to these parts rather than waiting for sufficient resources to complete the entire activity. If activities cannot be split, schedule the activity from its earliest start. If this is not possible because of resource availability, schedule it to start at the earliest possible time resources are available. The process is continued until all activities have been completed. In addition to early start time ordering, some attribute should reflect the relative importance of the activities in the project. One way of doing this is to allow total slack to be a measure of criticalness. The activities with the least amount of slack may be given precedence. Other priorities such as the number of men required and the duration of the job may be given consideration.

Parallel methods schedule several jobs at a time. At a given time, there may exist a set of activities which may be undertaken because their predecessors have been completed. Some of the activities in this set may be scheduled to start while the remaining must be delayed due to resource constraints. It may occur that no activity in the set meets the resource constraints and all the activities must be delayed. In order to determine a set of activities which may be scheduled, a decision rule must be applied. Kelley presents the following rule: of the activities which can be scheduled, order the activities in ascending sequence of total slack. Starting at the top of the order, an activity may be selected if its resource requirements, when added to those previously selected, do not violate the resource constraint.

Kelley offers no preference to either the serial or parallel method but feels that serial methods are more practical.

As a first step in the technique, a critical-path analysis is performed, activity durations determined and schedule information computed. By applying the algorithm to the project assuming unlimited resource availability, an estimate of resource requirements can be obtained. The need for additional resources at a given time may then be appropriately determined.

A computer program called SPAR-1 (Scheduling Program for Allocation of Resources) incorporates the characteristics and features of a general heuristic model proposed by J. W. Wiest[24]. The heuristic rules bear a striking similarity to those proposed by Kelley. Available resources are allocated period by period to activities listed in order of their early start times. Along with their early start times, activities are ordered by magnitude of slack. Critical activities are determined by their effect on total project length, i.e., if delaying the start time of an activity will result in a longer critical path than now exists, the activity is critical. The most critical jobs are scheduled first along with as many jobs which can be completed within the constraints of available resources. An attractive feature of this program is the reschedule routine. It may occur that a critical activity could be scheduled if activities previously scheduled were postponed without increasing the critical path or creating a longer path than currently exists. If this is so, the resources which were allocated to the previously scheduled activities are now made available to the critical activity under consideration. In effect, the scheduling and allocation "errors" are corrected as activities are encountered with a more critical need of resources than activities scheduled earlier.

Moder and Phillips[13] describes an algorithm adapted from an unpublished paper by George H. Brooks, Professor of Industrial Engineering, Purdue University. Although the algorithm assumes a maximum resource availability which is constant in time, it points out that this assumption may be modified to deal with the case of a maximum resource availability which is changing in time. In proceeding through the algorithm, the first step is to arrange the activities such that the maximum remaining path length is decreasing in magnitude. Next, form a set defined as a "decision set" of all unscheduled activities whose preceding activities have all occurred. Choose from the current decision set, the activity with the largest maximum remaining path length. Compare the resouces required to complete this activity with the resources available. If there are insufficient resources to complete this activity, select the activity with the next largest maximum remaining path length. If, however, there are sufficient resources to complete the activity originally selected, schedule this activity and subtract the resources required from the resources available. Scan the decision set for the next activity candidate, that is, the activity with the next largest maximum remaining path length. Determine if this activity can also be scheduled without exceeding the resource constaints. This procedure is continued until all activities in the decision set have been scheduled or the available resources are exceeded. Those activities not scheduled must wait until the necessary resources are made available. As each activity is completed, its suceeding activities are added to the decision set. The process is continued until all the activities in the project have been scheduled.

P. M. Ghare[8] describes an activity network model based on the assumption that the duration of an activity can be controlled by modifying the resources allocated to the activity. Adding resources to an activity will generally result in a decrease in expected activity completion time. This statement is valid within certain resource limitations. As an example, two men working on an activity may complete their task in time t, and doubling the number of men assigned may reduce the time of completion to t/2. However, adding a fifth man to the crew may create confusion and interference to the point where no additional time advantage is gained. Conversely, assigning one man to an activity that normally requires two could result in the activity not being completed at all. The resource time function may now be described in terms of the preceeding idea. For every activity in the project, there exists a smallest possible resource allocation such that for any resource allocation less than the minimum it is physically impossible to complete the activity. There also exists a maximum resource allocation resulting in a minimum completion time. In addition, there exists a region of feasibility defined as an area bounding these two extremes. The resource time function may assume a number of different curves including concave, convex and linear, but all are considered as piecewise linear over a finite period of time. Ghare then states that the resource time function leads to certain principles of optimum allocation which he states in the form of theorems and omitting their proofs as trivial. A literal interpretation of the theorems are as follows:

1) If two activities have the same starting and ending times, the resource allocation will be optimum if it is made such that the completion times for each activity are equal provided that the sum of the resource requirements do not exceed the resource constraints.
2) If the maximum completion time for activity one is less than that for activity two and if the resource allocation for any given completion time of activity two within its feasible region is such that the difference between the resources available and the minimum required for activity one creates a completion time for activity two which is greater than the maximum time required for activity one, the optimum allocation would be to allocate the minimum amount to activity one and the remainder (difference between that allocated for activity one and the amount available) to activity two.
3) Slack exists following an activity k in a project with an optimum resource allocation only if the difference in

earliest occurrence time of activities j and i exceeds the maximum time to complete activity k.

4) If two activities are scheduled to start at different times, the resource allocation will be optimum when the sum of the earliest occurrence time and expected completion time of activity one are equal to that of activity two.

5) Assume that the earliest occurrence time plus the maximum completion time of activity one is less than that of activity two. Also, the earliest occurrence time of the succeeding activity corresponds to an allocation to activity two which is the difference between that available and the minimum amount required for activity one. If the early occurrence time for the succeeding activity then is greater than the sum of the early occurrence time and maximum completion time for activity one but less than the sum for activity two the optimum allocation will be to allocate the minimum required for activity one and the remainder to activity two.

The theorems are applied successively to all events in a project starting with the sink node and working backwards through the network to the source node. Having defined the through variable as the resource allocation for an activity and the across variable as the completion time required for an activity, the techniques of system theory are used to express the network relations in equation form. With the objective of minimizing the total time between project start and end, the equations are used to optimize the allocation of limited resources.

Linus Schrage[20] poses an implicit enumerative procedure for generating all active schedules for a problem that has both precedence constraints and resource constraints. The precedence constraint states that the start time of an activity i must be greater than the sum of all start plus completion times of all the activities preceding i.

$$s\ (i) = s\ (k) + t\ (k) \text{ for all } k \in P\ (i)$$

where

$s\ (i)$ is the start time of activity i

$s\ (k) + t\ (k)$ is the start plus completion times of activity k preceding i

and

$P\ (i)$ is the set of activities preceding i

The resource constraint states that a resource can be assigned to only one activity at a time. In addition to the two constraints mentioned, a third constraint states that once an activity has been scheduled, the required resources for that activity are assigned for the duration of the activity time interval. A schedule is defined as the assignment of resources over time to activities satisfying the constraints and a partial schedule is defined as one which contains the start times of a subset of the activities in the project which do not violate the constraints. Essentially, the procedure generates complete schedules from partial schedules in a tree-generating procedure. A partial schedule or subset of activities is determined from the succession of activiites from an activity under consideration for scheduling, i.e., one whose predecessors have been completed. The succession of activities and the candidate for scheduling must lie within the bounds of the resource availability. The activity with the earliest start time whose sum of activity time plus the early start times in the partial schedule are the greatest is scheduled first. This procedure is followed through the network to its completion. For each partial schedule, the lower bounds on all complete schedules generated from the partial schedule are calculated. Thus, a branch and bound technique is used to enumerate all active schedules implicitly.

Verhines[23] claims a decision rule for allocating resources in activity networks which will result in the shortest overall project completion time. Simply stated, it is proposed that if two activities compete for the same resource and the resource availability is such that only one of these activities may be started, the one with the longest remaining series of activities should be given priority.

III. EXPERIMENTAL PROCEDURE

To facilitate testing the various decision rules selected, use was made of a simulation program known as GERTS III R. GERTS III R is a FORTRAN IV program designed to simulate GERT type networks which involve resource allocation decisions. Detailed operations, and user instructions may be found in the Bibliography (ref. 16, 17, 18).

One of the most attractive features of this simulation program is the minimum amount of programming required on the part of the user. The user is required to write two programs, Subroutine SCHDL and Function CALAT. The scheduling rules to be used during the simulation of a network are defined in SCHDL while CALAT is used for calculating an attribute value on which activities can be ranked.

After making the necessary modifications to the standard program deck, the simulation program was loaded on a disk file in the Engineering Research Center's time sharing system. In order to facilitate the testing of several different networks, the input and output devices were specified as separate storage files. When it was desired to test a certain network, the network's input data was renamed to correspond to the input device specified. The results of a network simulation were stored in the output file as specified. Once a network had been simulated, a hard copy of the output data was obtained in order to make room for output data concerning other networks in the file. Also, in order to ease the method of specifying network data, the read statements in the program were changed to A and G formats. Format A allows alphanumeric data to be transmitted in a manner similar to numeric data while G format is used to transmit real, double precision, integer, logical, or complex data. Using these formats, the meticulous care regarding spacing required in F, and I formats need not be observed. Although the formats were changed, the relative data fields remained the same as specified in the GERTS III R user's manual.

In order to perform a statistical analysis of the results, it was desired to observe the time to realize the sink node in each simulation run. The present version of the program has been modified to allow for this. Also, in testing the various decision rules proposed, certain activity attributes had to be added to the program. The event file in the program contains the set of activities emanating from a node and is divided into two parts: a fixed point array called NSET and a floating point array called QSET. Each activity entry in the event file has associated with it a set of three floating point attributes (ATRIBs) and eight fixed point attributes (JTRIBs). The attributes associated with an activity stored in the file are the following:

ATRIB (1) - the expected time required to perform the activity

ATRIB (2) - the time the activity was marked

ATRIB (3) - specified by the user in Function CALAT

JTRIB (1) - the end node for the activity

JTRIB (2) - the parameter set number for the activity (the parameter set number is specified in the input data)

JTRIB (3) - the distribution type for the time required to perform the activity

JTRIB (4) - the counter type

JTRIB (5) - the activity number

JTRIB (6) - the number of resources of type one required to perform the activity

JTRIB (7) - the number of resources of type two required to perform the activity

JTRIB (8) - the number of resources of type three required to perform the activity

It was intended to modify the program to allow the addition of three other floating point attributes. In so doing, the attribute currently specified as ATRIB (3) had to be changed to ATRIB (6). This was so because of the method used in reading in and filing the data. As the files are set up within the program, the next integer following the last real attribute is specified as the real or floating point attribute used to call Function CALAT. The following floating point attributes were added to the event file:

ATRIB (3) - number of activities bursting from the activity end node, i.e., the number of activities immediately following the end node

ATRIB (4) - the number of activities following the end node to project completion

ATRIB (5) - the slope of the resource-time relationship

The program modifications required to add these attributes to the file are the following:

(1) IMM (the number of attribute rows) in subroutine DATAN was increased from 3 to 6.

(2) Each common statement containing the term ATRIB (3) was changed to ATRIB (6).

(3) Data card type six was added in order to file ATRIBs (3) through (5).

Since GERTS III R is a "next event simulator", and the program itself is in a preliminary form, several difficulties in its use exist. One of the difficulties encountered is the inability to determine the remaining path length succeeding an activity under consideration for scheduling. Part of the problem stems from the fact that stochastic type networks are being tested. Path lengths in stochastic networks vary in each succeeding simulation run. As a result, decision rules concerning maximum remaining path length were not considered.

As mentioned earlier in this section, three additional attributes were added to the program. The purpose in adding these attributes was to provide some measure for ranking the activities released for scheduling in the event file. Unfortunately, some difficulty was experienced in trying to rank on some of the atrributes. According to Pritsker[16,17,18], ranking on any attribute may be accomplished by specifying the attribute of interest in field 9 of data card type 2. When this was attempted, identical completion times were noted for attributes ATRIB (4) and ATRIB (5). In attempting to trace this prblem, a number of different procedures were followed. First, it had to be determined if the real attributes were being read properly and filed. This was verified by having each attribute printed on the terminal as it was entered in Subroutine FILEM. To determine if the attributes can be called from the files, Function CALAT was used. It was indeed verified that each attribute had appeared in Function CALAT when specified. Furthermore, proper ranking was achieved when the functions in CALAT were completed. Based on these findings and after several lengthy discussions with Pritsker, it was decided to pursue the experimentation by specifying most of the decision rules in CALAT.

In all, eleven decision rules were selected for testing. Among the rules, some were either direct applications or variations of those proposed in Chapter 2. Additional rules were based upon intuitive or heuristic reasoning.

Specifically, the following rules were tested:

Rule #1 - Allocate the available resources to the activities whose completion times are the longest. This is a direct application of a rule proposed by Kelly and is implemented simply by specifying that the ranking be done on ATRIB (1), HVF (highest value first). The ranking is accomplished by specifying a 1 for ATRIB (1) in field 9 and a 2 for HVF in field 10 of data line type 2.

Rule #2 - Allocate the available resources to the activities whose completion times are the shortest. This is the complement of Rule #1 and is accomplished by specifying 1 and 1 in fields 9 and 10 respectively of data line type 2.

Rule #3 - Allocate the available resources to the activities which have the largest number of remaining activities to project completion. Rule #3 is a variation of that proposed by Verhines and Molder which specifies the application of resources to the activities with the longest remaining path length first. The rule is exercised by specifying a 3 for ATRIB (3) and 2 for HVF in fields 9 and 10 respectively of data line 2.

Rule #4 - Allocate the available resources to the activities with the largest resource requirements. The rationale here may be thought of as similar to priority dispatching in a job shop. By assigning a 106 for JTRIB (6) and a 2 for HVF in data line 2, Rule #4 will be enacted.

Rule #5 - Allocate the available resources to the activities with the smallest resource requirements first. Obviously this rule is the complement of Rule #4 and similar inputs are required with the exception of specifying a 1 in field 10 of data line 2 for LVF (low value first).

Rule #6 - Allocate the available resources to the activities whose slopes of the resource-time function are the largest. According to Fulkerson[13], a reduction in critical path length and hence project completion may be accomplished by a time-cost trade-off procedure. Logically, the activity to receive additional resources would be the one on the critical path with the smallest slope since its time reduction will be the greatest for an incremental increase in resources. The negative aspects of not following this procedure are examined in this rule and the positive aspects are hopefully brought out by the application of Rule #7. In order to test Rule #6, Function CALAT must be used. Function CALAT may be called by inserting a 6 for ATRIB (6) in field 9 of data line 2. Ranking is based on HVF by inserting a 2 in field 10 of data line 2. The steps to be followed in CALAT are as follows:

```
      JOT = 0
      DO 20  J = 1, NRESC
   20 JOT = JOT + JTRIB (J + 5)
      CALAT = JOT/ATRIB (1)
      RETURN
      END
```

where

NRESC = number of different resource types

Rule #7 - Allocate the available resources to the activities whose slopes of the resource-time function are the smallest. Rule #7, of course, is the complement of Rule #6 and the only change to be made is to replace 2 with 1 in field 10 of data line 2 for LVF.

Rule #8 - Allocate the available resources to the activities whose product of resource requirements and completion times are the smallest. Rules 8 and 9 are a direct application of a rule used by Pritsker in his description of the use of GERTS III R. Again, CALAT must be used and the steps within the subroutine are:

```
    JOT = 0
    DO 20  J = 1, NRESC
20  JOT = JOT + JTRIB (J + 5)
    CALAT = JOT * ATRIB (1)
    RETURN
    END
```

Fields 9 and 10 of data line 2 must contain 6 and 1 respectively.

Rule #9 - Allocate the available resources to the activities whose product of resource requirements and completion times are the largest. This rule, of course, is the complement of Rule #8 and the only change required is to replace 1 with 2 in field 10 of data line 2.

Rule #10 - Allocate the available resources to the activities whose sum of early start times plus completion times are the largest. This rule is a direct application of that proposed by Schrage and described in Chapter 2. Once again, CALAT is used to calculate the attribute value.

```
    CALAT = T NOW + ATRIB (1)
    RETURN
    END
```

where

T NOW - current time in the simulation

Rule #10 may be thought of as processing the activity with the smallest slope first. The requirements for data line 2 are a 6 and 2 in fields 9 and 10 respectively.

Rule #11 - Allocate the available resources to the activities whose slope-slack product are the smallest. Extending Fulkerson's time-cost trade-off procedure one step further, the urgency of processing the activity with the smallest slope has been coupled with the attractiveness of smallest slope. Function CALAT appears as follows:

```
      JOT = 0
      DO 20  J = 1, NRESC
20    JOT = JOT + JTRIB (J + 5)
      CALAT = (T NOW + ATRIB (1) *
              (FLOAT (JOT)/ATRIB (1))
      RETURN
      END
```

Data line 2 will contain a 6 and 1 in fields 9 and 10 respectively to test this rule.

Network Selection and Description

Four networks were chosen at random to provide the situations for exercising the decision rules selected. Details of each network may be found in Tables 1 through 4. In stating that the selection was random, it is meant that no particular attention was paid to characteristics such as the number of nodes, activities, or paths in the network. Selectivity, however, was based on the diverse project representation of each network.

An examination of Table 1 will reveal the pertinent characteristics required as input data to perform the experiments desired. Columns 1 through 3 require little or no explanation since they represent the activity number (which is arbitrarily assigned), and the starting and end nodes for each activity. Columns 4 through 6 represent the most likely (m), optimistic (a), and pessimistic (b) completion times, respectively, for each activity. The number of burst activities listed in column 7 represents the number of activities immediately following the end node, i.e., the number of activities emanating from the end node. Column 8 lists the total number of activities following the end node to project completion regardless of the paths on which they appear. Column 9 lists the slope of the resource-completion time function of the activity in question. As mentioned in Chapter 1, the resource-time functions of all the activities are assumed to be linear. Finally, column 10 represents the total amount of resources required to complete the activity for the most likely time specified.

Network 1
(S. E. Elmaghraby, ref. 6)

Activity No.	Start Node	End Node	m	a	b	No. Burst Activities	No. Succeeding Activities	Slope	Resources
1	1	2	20	15	40	1	36	.750	20
2	1	2	10	5	15	3	35	.667	10
3	1	6	10	0	50	4	30	.625	30
4	1	8	15	15	15	2	14	0	15
5	2	3	75	30	90	3	35	.750	50
6	3	4	50	25	100	3	24	.800	60
7	3	9	35	15	50	3	12	.735	20
8	3	16	40	5	70	1	2	1.0	40
9	4	7	15	5	40	4	26	0.5	20
10	4	11	55	50	70	3	10	4.0	80
11	4	16	25	10	45	1	2	1.0	25
12	5	7	60	50	85	4	24	.678	65
13	5	10	30	10	40	3	9	1.0	20
14	6	7	20	15	25	4	24	2.0	20
15	6	8	30	10	40	2	14	0.5	30
16	6	16	15	5	40	1	2	0.4	20
17	6	20	15	0	50	0	0	0.5	20
18	7	8	10	5	15	2	14	0.5	10
19	7	11	50	20	105	3	9	.615	40
20	7	12	45	15	80	1	1	1.04	50
21	7	15	45	20	95	4	7	1.25	50
22	8	9	30	10	45	3	12	0.5	30
23	8	18	35	5	65	1	2	1.0	60
24	9	10	10	5	35	3	9	1.0	30
25	9	13	55	45	75	2	5	0.5	55
26	9	19	20	5	50	1	1	0.5	25

TABLE 1
NETWORK 1 CHARACTERISTICS

Network 1 (Cont'd)

Activity No.	Start Node	End Node	m	a	b	No. Burst Activities	No. Succeeding Activities	Slope	Resources
27	10	12	50	40	75	1	1	0.5	60
28	10	13	25	5	35	2	5	1.28	20
29	10	20	15	5	20	0	0	0.532	5
30	11	13	30	10	40	2	5	1.0	20
31	11	16	20	5	40	1	2	1.0	30
32	11	19	15	5	50	1	1	.850	35
33	12	20	20	10	25	0	0	.5	5
34	13	17	55	25	110	1	1	1.64	125
35	13	18	50	20	85	1	2	1.0	50
36	14	15	55	25	105	3	7	2.5	150
37	14	19	35	15	50	1	1	.715	20
38	15	17	40	10	75	1	1	.645	40
39	15	18	25	10	30	1	2	1.33	20
40	15	19	55	40	60	1	1	1.0	60
41	15	20	15	0	55	0	0	0.86	40
42	16	19	55	35	85	1	1	1.28	55
43	17	20	30	10	40	0	0	0.74	30
44	18	19	25	5	45	1	1	1.0	25
45	19	20	25	5	50	0	0	.75	25

Network 2
Pipeline Renewal Project

Activity No.	Start Node	End Node	m	a	b	No. Burst Activities	No. Succeeding Activities	Slope	Resources
1	1	3	10	8	12	1	15	.25	1
2	2	8	26.5	26	36	1	9	1.2	28
3	3	4	2	1	3	1	13	.5	2
4	4	5	1	0.5	1.5	3	12	1	1
5	5	6	42.5	40	60	1	6	.05	2
6	5	7	28	28	40	1	8	.835	30
7	5	9	1.63	1.5	4	1	8	4.3	45
8	6	12	0.9	0.5	2	2	5	.5	1
9	7	10	4.5	4	8	1	7	1	6
10	8	9	1	1	1	1	8	0	5
11	9	10	6	4	8	1	7	0	1
12	10	11	5.25	5	10	1	6	.4	6
13	11	12	2	1	3	2	5	.5	2
14	12	13	3.75	3	6	1	2	.835	1
15	12	14	1	0.5	1.5	1	2	2	4
16	13	15	1	1	1	1	1	0	1
17	14	15	3.75	3	6	1	1	.835	1
18	15	16	1	1	1	0	0	0	1

TABLE 2

NETWORK 2 CHARACTERISTICS

Network 3
Computer Installation Project

Activity No.	Start Node	End Node	m	a	b	No. Burst Activities	No. Succeeding Activities	Slope	Resources
1	1	2	3	2	6	3	29	.667	11
2	2	3	4	2	6	5	20	5	20
3	2	4	10	5	15	5	17	.6	10
4	2	5	5	3	7	1	11	7.5	55
5	3	5	2	1	3	1	11	4	20
6	3	6	10	8	12	1	10	2.5	25
7	3	7	10	8	15	1	8	1.43	25
8	3	8	7	5	10	1	7	.6	11
9	3	10	50	30	70	1	7	.1	15
10	4	9	7	5	9	1	10	1	15
11	4	12	4	2	5	1	6	3.3	20
12	5	15	5	3	6	1	3	1.33	10
13	4	16	2	2	3	1	3	3	15
14	4	19	5	2	8	0	0	.75	12
15	5	9	15	10	20	1	10	.4	20
16	6	11	20	15	35	2	9	.5	25
17	7	10	0.2	0.1	0.3	1	7	5	5
18	8	13	1	1	1	1	6	0	10
19	9	11	4	2	6	2	9	2.5	30
20	10	13	4	2	8	1	6	1	16
21	11	12	2	1	4	1	6	.667	10
22	11	13	4	2	8	1	6	2.5	25
23	12	14	0.2	0.2	0.2	2	5	0	5
24	13	14	2	1	3	2	5	5	20
25	14	15	1	1	1	1	1	3	5

TABLE 3
NETWORK 3 CHARACTERISTICS

Network 3 (Cont'd)

Activity No.	Start Node	End Node	m	a	b	No. Burst Activities	No. Succeeding Activities	Slope	Resources
26	14	17	1	0.5	2	1	2	2.67	10
27	15	17	0	0	0	1	2	0	0
28	16	17	1	0.5	2	1	2	.667	5
29	17	18	4	2	8	1	1	.5	10
30	18	19	0.2	0.1	0.5	0	0	7.5	10

Network 4
Ground Tracking Complex

Activity No.	Start Node	End Node	m	a	b	No. Burst Activities	No. Succeeding Activities	Slope	Resources
1	1	2	8	6	12	2	57	10	80
2	2	3	3	2	6	1	1	6.67	30
3	2	5	0	0	0	5	56	0	0
4	3	4	6	5	7	0	0	5	30
5	5	6	3	2	4	1	13	15	50
6	5	10	3	2	4	2	19	5	15
7	5	13	2	1	3	2	18	2	10
8	5	16	4	3	6	2	18	1.33	5
9	5	19	4	3	5	2	23	1	8
10	6	7	2	1	3	1	12	1	2
11	7	8	35	30	50	1	11	.184	2
12	8	9	10	8	12	1	10	2.5	25
13	9	36	0	0	0	1	8	0	0
14	10	11	7	5	12	1	17	.286	5
15	10	12	10	8	15	2	16	8.56	90
16	11	12	0	0	0	2	16	0	0
17	12	22	10	8	12	1	14	5	60
18	12	26	0	0	0	1	13	0	0
19	13	14	5	4	7	1	17	.667	5
20	13	15	9	8	10	2	16	1	7
21	14	15	0	0	0	2	16	0	0
22	15	24	7	5	10	1	13	3	25
23	15	26	0	0	0	1	12	0	0
24	16	17	9	7	11	1	17	.5	7
25	16	18	12	10	15	2	16	.4	8

TABLE 4
NETWORK 4 CHARACTERISTICS

Network 4 (Cont'd)

Activity No.	Start Node	End Node	m	a	b	No. Burst Activities	No. Succeeding Activities	Slope	Resources
26	17	18	0	0	0	2	16	0	0
27	18	26	0	0	0	1	14	0	0
28	18	28	6	5	7	1	13	2	9
29	19	20	9	7	12	1	21	.4	6
30	19	21	13	11	16	3	20	.45	4
31	20	21	0	0	0	3	20	0	0
32	21	26	0	0	0	1	12	0	0
33	21	30	6	4	10	1	13	.166	3
34	21	31	8	6	12	1	12	.667	11
35	22	23	5	4	7	1	9	.667	4
36	23	36	0	0	0	1	8	0	0
37	24	25	5	3	7	1	9	.5	6
38	25	36	0	0	0	1	8	0	0
39	26	27	15	12	18	2	2	.333	12
40	27	36	0	0	0	1	8	0	0
41	27	41	10	9	15	1	1	1.67	20
42	28	29	3	2	6	1	10	.5	8
43	29	36	0	0	0	1	9	0	0
44	30	32	1	1	1	1	13	0	2
45	31	33	4	3	5	1	12	3	10
46	32	33	7	5	12	1	12	.143	1
47	33	34	5	4	7	1	11	.667	7
48	34	35	3	2	5	1	11	.667	6
49	35	36	0	0	0	1	9	0	0
50	36	37	10	8	12	3	8	5	40

Network 4 (Cont'd)

Activity No.	Start Node	End Node	m	a	b	No. Burst Activities	No. Succeeding Activities	Slope	Resources
51	37	38	6	4	10	1	5	2.5	30
52	37	39	0	0	0	1	4	0	0
53	37	40	12	9	15	2	3	5	80
54	38	40	0	0	0	2	3	0	0
55	39	40	2	1	3	2	3	1.5	9
56	40	41	0	0	0	1	1	0	0
57	40	42	3	2	4	0	0	15	40
58	41	42	0	0	0	0	0	0	0

The resources may represent any type desired, i.e., money, manpower, equipment, etc. However, for the networks presented, the resources represent labor per unit time.

Input Data

A sample of the input data required may be found in Appendix 1. The field description and their formats for each line are described in Appendix 2.

Output Data

Each network was simulated 100 times for each of the 11 different decision rules. As mentioned earlier, the completion time for each simulation was recorded. The amount of resources allotted to each network depended upon the highest resource required by any one activity. The only exception to the resource allocation occurred when any one of the rules could not be exercised due to the resource limitation. In these cases, the allocation for that particular network was increased by approximately 10% and held at this level for each of the 11 rules. The purpose in keeping the resource allocation to a minimum was to eliminate the possibility of introducing a variable which could be attributed to resource level variation.

A sample output of a network simulation is shown in Appendix 3. The information appearing under the headings "Network Descriptions," "Activity Parameters" and "Activity Description" is simply an echo check of the input data. The data of prime interest appears under "Final Results for 100 Simulations". In this section, the mean value of all the simulations as well as the standard deviation, minimum observed value and maximum observed value are presented. Immediately following this information is a section called "Histograms". By specifying a lower limit and the cell width, the distribution of observations over 32 cells may readily be examined. The final section of the output data is called "Resource Utilization" and is defined as the resource-hours used divided by the project completion time.

IV. ANALYSIS OF RESULTS

General Discussion

The main objective of the experiment was to determine if any one decision rule will serve as an effective one for producing consistently low completion times in any network. In effect, then, it was desired to investigate the effect of k different kinds of rules and n different kinds of networks on the completion times of a certain project. The description of the objective, therefore, implies a Two-Way Analysis of Variance. In addition, it was desired to investigate the possibility of joint effects or interactions of the two variables. Interactions are examined to determine if a particular rule will yield a low completion time if and only if it is used in conjunction with a particular network. In order to test hypotheses concerning the effects of rules, networks and interactions, it was necessary to replicate, i.e., to take more than one observation of each combination of rules and networks.

A summary for the mean values of project completion times is shown in Table 5. The rank column indicates the relative position of the corresponding rule to minimum completion time. As an example, Rule 1 of Network 1 has a rank of 4 which indicates that its mean value of completion time is the fourth lowest of the eleven values observed. Columns 1 through 5 under Observations are the mean values of 20 simulations and provide the necessary replicates for the analysis. Notice that the mean completion times

TABLE 5
PROJECT COMPLETION TIMES

			Observations					
Network	Rule	Rank	1	2	3	4	5	Mean
1	1	4	628.61	644.37	642.69	631.07	633.37	636.02
	2	9	668.19	671.16	670.00	660.44	662.38	666.43
	3	1	621.01	625.96	622.16	630.04	632.00	624.24
	4	8	666.51	664.93	665.00	657.21	655.40	661.81
	5	7	650.36	645.72	652.20	640.36	673.38	652.40
	6	11	661.56	688.11	671.98	680.09	680.44	676.44
	7	2	636.14	622.95	625.35	633.29	617.53	627.05
	8	10	666.50	683.34	673.48	670.16	664.24	671.55
	9	6	653.75	651.29	647.14	646.24	633.39	646.36
	10	3	638.97	636.14	635.10	627.76	640.76	635.70
	11	5	641.21	632.82	640.24	642.65	630.97	637.58
2	1	3	78.35	78.69	78.87	76.99	76.82	77.94
	2	10	81.42	81.23	80.76	80.19	79.73	80.93
	3	7.5	79.05	80.67	81.15	78.93	78.88	80.01
	4	6	80.36	80.00	81.15	79.13	78.70	79.86
	5	3	78.35	78.74	78.87	76.99	76.82	77.94
	6	10	81.42	81.23	82.11	80.19	79.73	80.93
	7	3	78.35	78.69	78.87	76.99	76.82	77.94
	8	10	81.42	81.23	82.11	80.19	79.73	80.93
	9	7.5	80.40	80.68	81.15	78.93	78.88	80.01
	10	3	78.35	78.70	78.86	76.99	76.82	77.94
	11	3	78.35	78.69	78.87	76.99	76.82	77.94

			Observations					
Network	Rule	Rank	1	2	3	4	5	Mean
3	1	1	76.06	76.08	76.94	77.18	77.81	76.81
	2	7	84.35	83.67	82.31	85.34	82.24	83.58
	3	9	89.48	87.44	88.00	87.78	88.24	88.19
	4	10	89.08	89.10	88.33	89.80	88.64	88.99
	5	6	83.85	81.90	82.99	80.84	81.02	82.12
	6	11	99.87	94.25	99.97	104.44	99.42	99.59
	7	3	77.58	79.17	78.99	78.78	81.14	79.23
	8	5	82.26	81.33	82.19	80.17	81.92	81.57
	9	8	85.98	86.49	86.04	83.67	85.11	85.38
	10	2	76.26	77.89	78.26	76.50	78.26	77.43
	11	4	80.92	81.47	79.92	76.94	81.07	80.06
4	1	8.5	122.47	122.42	121.90	124.03	121.78	122.52
	2	5	114.61	113.68	115.04	113.30	112.08	113.74
	3	6	114.87	113.07	114.26	114.01	114.52	114.15
	4	3	111.61	110.66	112.32	113.88	110.04	111.70
	5	4	125.38	126.48	123.26	126.87	124.70	112.34
	6	2	110.60	110.97	111.98	112.21	111.95	111.54
	7	10	124.36	126.09	123.54	126.55	124.74	125.16
	8	7	119.95	116.88	116.57	117.77	119.22	118.18
	9	1	110.33	110.73	109.59	110.95	111.30	110.61
	10	8.5	122.47	122.42	121.90	124.03	121.78	122.52
	11	11	124.35	127.25	124.05	126.63	124.77	125.21

			Observations					
Network	Rule	Rank	1	2	3	4	5	Mean
1 (Transformed Data)	1	4	104.77	107.40	107.12	105.18	105.56	105.00
	2	9	111.37	111.86	111.67	110.67	110.40	111.07
	3	1	103.50	104.33	103.69	105.01	105.33	104.04
	4	8	111.09	110.82	110.83	109.54	109.23	110.30
	5	7	108.39	107.62	108.70	106.73	112.23	108.73
	6	11	110.26	114.69	112.00	113.35	113.41	112.74
	7	2	106.02	103.83	104.23	105.55	102.92	104.51
	8	10	111.08	113.89	112.25	111.69	110.71	111.93
	9	6	108.96	108.55	107.86	107.71	105.57	107.73
	10	3	106.50	106.02	105.85	104.63	106.72	105.95
	11	5	106.87	105.47	106.71	107.11	105.16	106.26

for Network 1 are considerably higher than the remaining three networks. In order not to bias the analysis and to create a data population closer to normal, a linear transformation was performed on the data of Network 1.

In applying an analysis of variance, it is assumed that the k groups of independent observations $x_{11}, \ldots, x_{1n_1}, x_{21}, \ldots, x_{2n_2}, \ldots, x_{k1}, \ldots, x_{kn_k}$ all come from normally distributed populations with means $\mu_1, \ldots, \mu_k$, and with the same variance σ^2. According to Brownlee[2], the assumption of normality is not very restrictive since a variable may be transformed, if the general form of its distribution is known, into a variable which is approximately normally distributed. Also, it can be arranged to deal with means, and the central limit theorem roughly assures, that if a population has a finite variance then the distribution of the sample mean approaches normality as n increases. Even faced with some doubt, the deviations from normality cause fewer gross errors than the lack of constancy of variance or the lack of independence. Since the nature of the experiment itself assures independence, the only remaining element of doubt is the constancy of variance.

Levene's Test

Levene[5] suggests a test for equality of variances of several equal sized groups of observations and has shown through sampling studies that the test possesses "almost unbelievable robustness" against departures fron normality of the underlying distribution of observations. A robust procedure is one which is affected only slightly by appreciable departures from the assumptions involved. Implicit in the concept is the feeling that conclusions in error by a certain amount will not prove to be embarrassing.

Details of calculations for Levene's test may be found in Appendix 4. The results of the test indicated that there was no significant difference among the variances.

Two-Way Analysis of Variance

Having established homoscedasticity (constant variance) as a result of Levene's test, the analysis of variance was conducted. Again, the objective was to determine any variations in the data which may be attributed to the differences among the rules, differences among the networks, and interactions. The detailed computations for the analysis may be found in Appendix 5. As a result of the analysis, it was found that there was a significant difference in the rules used, a significant difference among networks, and interactions, i.e., certain rules produced low completion times only if used with a particular network.

Duncan's Multiple Range Test

The results of the analysis of variance indicated that there are significant differences among the decision rules, but did not show which mean completion time (or group of means) resulting from the use of a particular rule, differed significantly from another mean completion time (or group of means). Specifically, having determined that the mean completion times obtained by using the 11 decision rules differed significantly, it is desired to establish which rules differ from the others. Again, the objective was to determine which rule or rules produces the lowest project completion time. One method proposed to handle multiple comparisons is the Duncan Multiple-Range Test which will be used here. The test compares the range of any set of p means with an appropriate least significant range R_p, given by

$$R_p = (S_{\bar{x}})\ (r_p)$$

$$S_{\bar{x}} = \sqrt{\frac{MSE}{n}}$$

where

r_p = A value depending upon the desired level of significance α and the number of degrees of freedom corresponding to MSE. The table from which r_p is obtained is called "Critical Values for Duncan's New Multiple Range Test".

MSE = The error mean square in the analysis of variance for the individual network considered.

n = Number of different rules

and $S_{\bar{x}}$ = An estimate of $\sigma_{\bar{x}} = \sigma / \sqrt{n}$

The details of computation for Duncan's Test may be found in Appendix 6. For Network 1, the results showed that rules 3 and 7 produce the lowest completion time and there is no significant difference between the use of either rule. Further results showed that there is no significant difference between the use of rules 10, 1 and 11, between the use of rules 9 and 5, 4 and 2, 2 and 8 or 8 and 6. There was, however, a significant difference in using rules 3 and 7, and the remaining 9 rules. Therefore, rules 3 and 7 produce the lowest completion times and are the best ones to use for Network 1.

Network 2's results indicate that there is no significant difference between rules 1, 5, 7, 10 or 11 and a significant difference between the use of these rules and the remaining rules. In order to obtain a minimum completion time for Network 2, then, any one of rules 1, 5, 7, 10 or 11 may be used.

The results of the Duncan Test on Network 3 indicate that there is no significant difference between the use of rules 1 and 10 to obtain minimum completion time. There is, however, a significant difference in using rules 1 or 10 and the remaining rules.

Finally, the results for Network 4 indicate that rule 9 is significantly different from the remaining rules and is the best one to use in order to obtain minimum completion time.

Rank Correlation

From the results of the Duncan Test, it appears that there is no one rule which will produce a minimum completion time in every network considered. However, by calculating the coefficient of concordance[10], it can be determined if some general agreement as to rule rankings among the networks exists. If a reasonable agreement as to rankings does exist, it can be concluded that one or more rules perform consistently better (or worse) than the rest. Appendix 7 contains the calculations necessary to perform this test. The results indicate that there is a significant difference in the rankings of the rules for each network. The fact that there is a significant difference in the rankings indicates that the rules react differently in each network used. A rule which produces the lowest completion time or any other rank of completion time in one network will not necessarily produce the same ranking in another network. Furthermore, the relative rankings among the rules differs in each network used.

V. CONCLUSIONS

The main objective of this thesis has been to determine if any of the eleven decision rules selected serves as an effective one for producing consistently low completion times in any network.

From the results of the experiment, the following conclusions may be drawn:

1. The results of the Two-Way Analysis of Variance showed that; (a) there is a definite effect on completion time resulting from the use of different decision rules, (b) a difference does exist among the networks used, and (c) there is a dependency of completion times upon the conjunctive use of networks and rules, i.e., interactions do exist and a particular rule might yield a minimum completion time if and only if it is used in conjunction with a particular network.
2. The results of Duncan's Multiple range test and the rule(s) producing lowest completion time are summarized below.

Network	Rule(s) Producing Lowest Completion Time
1	3,7
2	1,5,7,10,11
3	1,10
4	9

Generally speaking, it is evident that the application of no one particular rule will guarantee a minimum completion time in all networks. Furthermore, it was found that there were rules which consistently performed the same as others. For instance, it was found that no significant difference resulted from the use of rules 1 and 10 in each of the four networks. Also, there was no significant difference between the performance of rules 7 and 11 in networks 2, 3 and 4.

3. The Duncan Test showed that there was no one rule which produces a minimum completion time. It did not, however, indicate the general agreement as to rule rankings among the networks. In other words, regardless of each rule's performance, did it perform consistently in each network? If a reasonable agreement as to rule rankings among the networks does exist, it can be concluded that one or more rules perform consistently better (or worse) than the rest. By calculating the Coefficient of Concordance, it was found that a significant difference in the rankings of the rules for each network does exist, and there is no consistent performance among the rules.

VI. RECOMMENDATIONS FOR FURTHER STUDY

Several aspects of the allocation of limited resources are open for further investigation. Some of these areas are discussed below.

1. The fact that no one general rule was found to produce minimum completion time does not necessarily indicate that none exists. It should be pointed out that every possible rule in existence has not been tried. Also, combinations of existing rules may be used to form other heuristic rules which may achieve the objective.
2. The number of networks used in the experiment is a relatively small sample of a virtually infinite population. By expanding the number of networks and reapplying the principles of this thesis, it may be possible that one of the rules used does indeed prove to be a valid general rule.
3. From the results of the present experiment, it is strongly suspected that there may be some network characteristic(s) which dictate the use of a particular decision rule for a given network. One such characteristic would be the ratio of the number of nodes to the number of activities in a network. As this ratio approaches a value of one, it is fairly obvious that the application of almost any rule will produce the same results as any other. However, as the ratio assumes values much less than one, the selection of a particular decision rule becomes more critical. Further experimentation with network characteristics would then seem to be highly justified.

4. The criteria used in this experiment regarding resource availability, was based upon the largest need of any one activity in the network. It may be interesting to study the effects of increasing the resources. Obviously, as resource availability becomes unlimited, the precedence constraints of the network dominate.

APPENDIX 1
SAMPLE INPUT DATA

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21,3,1,1,1,0,0,1,6,2, 150                               (Line Type 2)
2,0,1                                                   (Line Type 3)
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4,0,1
5,1
6,0,1
7,0,3
8,0,3
9,0,2
10,0,2
11,0,2
12,0,2
13,0,3
14,1
15,0,2
16,0,4
17,0,2
18,0,3
19,0,6
20,2,6,0,   340.,2.,A
21,1
0
20.,15.,40.
10.,5.,15.                                              (Line Type 4)
10.,0.,50.
15.,0,100.,.15
75.,30.,90.
50.,25.,100.
35.,15.,50.
40.,5.,70.
15.,5.,40.
55.,50.,70.
25.,10.,45.
60.,50.,85.
30.,10.,40.
20.,15.,25.
30.,10.,40.
15.,5.,40.
15.,0.,50.
10.,5.,15.
50.,20.,105.
45.,15.,80.
45.,20.,95.
30.,10.,45.
```

```
35.,5.,65.                                              (Line Type 4)
10.,5.,35.
55.,45.,75.
20.,5.,50.
50.,40.,75.
25.,5.,35.
15.,5.,20.
30.,10.,40.
20.,5.,40.
15.,5.,50.
20.,10.,25.
55.,25.,110.
50.,20.,85.
55.,25.,105.
35.,15.,50.
40.,10.,75.
25.,10.,30.
55.,40.,60.
15.,0.,55.
55.,35.,85.
30.,10.,40.
25.,5.,45.
25.,5.,50.
1.,21,2,1,9,0,0,20                                      (Line Type 5)
1.,36.,.750                                             (Line Type 6)
1.,21,3,2,9,0,0,10                                      (Line Type 5)
3.,35.,.667                                             (Line Type 6)
1.,21,6,3,9,0,0,30
4.,30.,.625
1.,21,8,4,2,0,0,15
2.,14.,0.
1.,2,3,5,9,0,0,50
3.,35.,.750
1.,3,4,6,9,0,0,60
3.,24.,.8
1.,3,9,7,9,0,0,20
3.,12.,.735
1.,3,16,8,9,0,0,40
1.,2.,1.
1.,4,7,9,9,0,0,20
4.,26.,.5
1.,4,11,10,9,0,0,80
3.,10.,4.
```

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1.,4,16,11,9,0,0,25                              (Line Type 5)
1.,2.,1.                                         (Line Type 6)
1.,5,7,12,9,0,0,65
4.,24.,.678
1.,5,10,13,9,0,0,20
3.,9.,1.
1.,6,7,14,9,0,0,20
4.,24.,2.
1.,6,8,15,9,0,0,30
2.,14.,.5
1.,6,16,16,9,0,0,20
1.,2.,.4
1.,6,20,17,9,0,0,20
0.,0.,.5
1.,7,8,18,9,0,0,10
2.,14.,.5
1.,7,11,19,9,0,0,40
3.,9.,.615
1.,7,12,20,9,0,0,50
1.,1.,1.04
1.,7,15,21,9,0,0,50
4.,7.,1.24
1.,8,9,22,9,0,0,30
3.,12.,.5
1.,8,18,23,9,0,0,60
1.,2.,1.
1.,9,10,24,9,0,0,30
3.,9.,1.
1.,9,13,25,9,0,0,55
2.,5.,.5
1.,9,19,26,9,0,0,25
1.,1.,.5
1.,10,12,27,9,0,0,60
1.,1.,.5
1.,10,13,28,9,0,0,20
2.,5.,1.28
1.,10,20,29,9,0,0,5
0.,0.,.532
1.,11,13,30,9,0,0,20
2.,5.,1.
1.,11,16,31,9,0,0,30
1.,2.,1.
1.,11,19,32,9,0,0,35
1.,1.,.85
```

```
1.,12,20,33,9,0,0,5                                  (Line Type 5)
0.,0.,.5                                             (Line Type 6)
1.,13,17,34,9,0,0,125
1.,1.,1.64
1.,13,18,35,9,0,0,50
1.,2.,1.
1.,14,15,36,9,0,0,150
3.,7.,2.5
1.,14,19,37,9,0,0,20
1.,1.,.715
1.,15,17,38,9,0,0,40
1.,1.,.645
1.,15,18,39,9,0,0,20
1.,2.,1.33
1.,15,19,40,9,0,0,60
1.,1.,1.
1.,15,20,41,9,0,0,40
0.,0.,.86
1.,16,19,42,9,0,0,55
1.,1.,1.28
1.,17,20,43,9,0,0,30
0.,0.,.74
1.,18,19,44,9,0,0,25
1.,1.,1.
1.,19,20,45,9,0,0,25
0.,0.,.75
0
0
```

APPENDIX 2

DESCRIPTION OF DATA INPUT FOR GERTS III R

Line Type 1

Field 1	The analyst's name (6A2)
Field 2	The project number (G)
Field 3	The month number (G)
Field 4	The day number (G)
Field 5	The year (G)
Field 6	The number of times the network is to be simulated (G)
Field 7	The number of activities with different time characteristics (G)
Field 8	The number of branches in the network plus an estimate of the maximum number of activities which can occur simultaneously (G)
Field 9	An integer random number seed (1267 was used) (G)
Field 10	A floating point random number seed (0.0 was used) (G)

Line Type 2

All fields are integer type numbers

Field 1	The largest node of the network (G). The smallest node number permitted is 2 (G)
Field 2	Number of source nodes (G)
Field 3	Number of sink nodes (G)
Field 4	Number of sink nodes that must be realized before the network is realized (G)
Field 5	Number of nodes which statistics are to be collected on, including all sink nodes (G)

APPENDIX 2 (Cont'd)

Field 6	Number of types of counts (G)
Field 7	A 1 if network modifications exist; a 0 otherwise (G)
Field 8	Number of different resource types (G)
Field 9	The attribute on which ranking is to be done for files NOQ and (NOQ-1). Add 100 to attribute number if a JTRIB value is to be ranked on (G)
Field 10	The priority system to be used forfiles NOQ and (NOQ-1). A 1 indicates low-value first. A 2 indicates high-values first (G)
Field 11	Number of available resources of Type 1 (G)
Field 12	Number of available resources of Type 2 (G)
Field 13	Number of available resources of Type 3 (G)

Line Type 3

One line required for each node

Field 1	The node number (descriptor) associated with the node characteristics given on this card (G)
Field 2	Special characteristic of the node. Codes for special characteristics are: 1. Source node 2. Sink node 3. Node on which statistics are collected 4. A mark node If Field 2 is left blank, no special characteristic is associated with the node (G)
Field 3	The number of releases required to realize the node for the first time (G)
Field 4	The number of releases required to realize the node after the first realization (G)
Field 5	Output characteristic of the node. Codes for input are: P for PROBABILISTIC; and D for DETERMINISTIC (A1)

APPENDIX 2 (Cont'd)

Field 6	If events that have been scheduled to end on this node are to be removed (cancelled) when this node is realized, an "R" should be put in this field. If removal is not desired, leave blank (A1)

Fields 7, 8 and 9 are used only if

The node is a sink node or a statistics node (code 2 or 3 in field 2)

Field 7	The lower limit of the second cell for the histogram to be obtained for this node. The first cell of the histogram will contain the number of times the node was realized in a time less than the value given in this field (G - floating point)
Field 8	The width of each cell of the histogram. Each histogram contains 32 cells. The last cell will contain the number of times the node was realized in a time greater than or equal to the lower limit (specified in Field 7) + 30 * (cell width (Specified by Field 8)) (G - floating point)
Field 9	Statistical quantities to be collected (A1) F. The time of *first* realizations of the node. A. The time of *all* realizations of the node. B. The time *between* realizations of the node. I. The time *interval* required to go between two nodes. D. The time *delay* from first activity completion on the node until the node is realized.

The last line of this type must have a zero in Field 1.

Line Type 4

The parameters associated with the distribution of the time to perform each activity. One line is required for each activity with a different time characterization. The number of lines is specified by Line Type 1, Field 7. A maximum of 300 is permitted. The lines must be arranged by ascending parameter number and the parameters must be numbered consecutively or blank lines appropriately placed. Nine distribution types are available which are:

APPENDIX 2 (Cont'd)

1. Constant
2. Normal
3. Uniform
4. Erlang
5. Lognormal
6. Poisson
7. Beta
8. Gamma
9. Beta fitted to three parameters as in PERT

The fields required are dependent on the distribution type of the activity. For distribution type 9 (Beta fitted to 3 values as in PERT), the fields are as follows:

Field 1	The most likely value, m (G - floating point)
Field 2	The optimistic value, a (G - floating point)
Field 3	The pessimistic value, b (G - floating point)
Field 4	Not used

Parameters for the remaining types of distribution may be found in the reference to the user's manual.

Line Type 5

One line required for each activity associated with the network

Field 1	Probability of realization (G - floating point)
Field 2	Start node (G)
Field 3	End node (G)
Field 4	Parameter number (G)
Field 5	The distribution type (G)
Field 6	Count type (G)
Field 7	Activity number (G)

APPENDIX 2 (Cont'd)

Line Type 6

One line required for each activity associated with the network. All fields are floating point numbers.

Field 1	Number of burst activities (G)
Field 2	Number of activities remaining to project completion (G)
Field 3	Slope of resource-time function (G)

Note - line types 5 and 6 must be interleaved, i.e., line type 6 for activity 1 must immediately follow line 5 for activity 1 and line 5 for activity 2 must immediately follow line 6 for activity 1.

Two lines each containing a zero in Field 1 must follow the last line of type 6.

APPENDIX 3
OUTPUT DATA

```
1                GERT SIMULATION PROJECT 1 BY GS CHUPIK
                           DATE 3/2/1971

                      **NETWORK DESCRIPTION**

                        NODE CHARACTERISTICS

           HIGHEST NODE NUMBER IS     21
           NUMBER OF SOURCE NODES IS     3
           NUMBER OF SINK NODES IS     1
           NUMBER OF NODES TO REALIZE THE NETWORK IS      1
           STATISTICS COLLECTED ON     1 NODES
           NUMBER OF PARAMETER SETS IS    45
           INITIAL RANDOM NUMBER IS    1267    0.0000

              NUMBER     NUMBER OF RELEASES    OUTPUT    STATISTICS BASED
   NODE      RELEASES        FOR REPEAT         TYPE     ON REALIZATIONS

     2          1               9999              D
     3          2               9999              D
     4          1               9999              D
     5          0               9999              D
     6          1               9999              D
     7          3               9999              D
     8          3               9999              D
     9          2               9999              D
    10          2               9999              D
    11          2               9999              D
    12          2               9999              D
    13          3               9999              D
    14          0               9999              D
    15          2               9999              D
    16          4               9999              D
    17          2               9999              D
    18          3               9999              D
    19          6               9999              D
    20          6               9999              D               A
    21          0               9999              D

                        SOURCE NODE NUMBERS
                          5    14    21

                         SINK NODE NUMBERS
                                20
1
```

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	20.0000	15.0000	40.0000	0.0000
2	10.0000	5.0000	15.0000	0.0000
3	10.0000	0.0000	50.0000	0.0000
4	15.0000	0.0000	100.0000	0.1500
5	75.0000	30.0000	90.0000	0.0000
6	50.0000	25.0000	100.0000	0.0000
7	35.0000	15.0000	50.0000	0.0000
8	40.0000	5.0000	70.0000	0.0000
9	15.0000	5.0000	40.0000	0.0000
10	55.0000	50.0000	70.0000	0.0000
11	25.0000	10.0000	45.0000	0.0000
12	60.0000	50.0000	85.0000	0.0000
13	30.0000	10.0000	40.0000	0.0000
14	20.0000	15.0000	25.0000	0.0000
15	30.0000	10.0000	40.0000	0.0000
16	15.0000	5.0000	40.0000	0.0000
17	15.0000	0.0000	50.0000	0.0000
18	10.0000	5.0000	15.0000	0.0000
19	50.0000	20.0000	105.0000	0.0000
20	45.0000	15.0000	80.0000	0.0000
21	45.0000	20.0000	95.0000	0.0000
22	30.0000	10.0000	45.0000	0.0000
23	35.0000	5.0000	65.0000	0.0000
24	10.0000	5.0000	35.0000	0.0000
25	55.0000	45.0000	75.0000	0.0000
26	20.0000	5.0000	50.0000	0.0000
27	50.0000	40.0000	75.0000	0.0000
28	25.0000	5.0000	35.0000	0.0000
29	15.0000	5.0000	20.0000	0.0000
30	30.0000	10.0000	40.0000	0.0000
31	20.0000	5.0000	40.0000	0.0000
32	15.0000	5.0000	50.0000	0.0000
33	20.0000	10.0000	25.0000	0.0000
34	55.0000	25.0000	110.0000	0.0000
35	50.0000	20.0000	85.0000	0.0000
36	55.0000	25.0000	105.0000	0.0000
37	35.0000	15.0000	50.0000	0.0000
38	40.0000	10.0000	75.0000	0.0000
39	25.0000	10.0000	30.0000	0.0000
40	55.0000	40.0000	60.0000	0.0000
41	15.0000	0.0000	55.0000	0.0000
42	55.0000	35.0000	85.0000	0.0000
43	30.0000	10.0000	40.0000	0.0000
44	25.0000	5.0000	45.0000	0.0000
45	25.0000	5.0000	50.0000	0.0000

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	ACTIVITY NUMBER	PROB.	FIRST RESOURCE
2	3	5	9	5	1.000	50
3	4	6	9	6	1.000	60
3	9	7	9	7	1.000	20
3	16	8	9	8	1.000	40
4	7	9	9	9	1.000	20
4	11	10	9	10	1.000	80
4	16	11	9	11	1.000	25
5	7	12	9	12	1.000	65
5	10	13	9	13	1.000	20
6	7	14	9	14	1.000	20
6	8	15	9	15	1.000	30
6	16	16	9	16	1.000	20
6	20	17	9	17	1.000	20
7	8	18	9	18	1.000	10
7	11	19	9	19	1.000	40
7	12	20	9	20	1.000	50
7	15	21	9	21	1.000	50
8	9	22	9	22	1.000	30
8	18	23	9	23	1.000	60
9	10	24	9	24	1.000	30
9	13	25	9	25	1.000	55
9	19	26	9	26	1.000	25
10	12	27	9	27	1.000	60
10	13	28	9	28	1.000	20
10	20	29	9	29	1.000	5
11	13	30	9	30	1.000	20
11	16	31	9	31	1.000	30
11	19	32	9	32	1.000	35
12	20	33	9	33	1.000	5
13	17	34	9	34	1.000	125
13	18	35	9	35	1.000	50
14	15	36	9	36	1.000	150
14	19	37	9	37	1.000	20
15	17	38	9	38	1.000	40
15	18	39	9	39	1.000	20
15	19	40	9	40	1.000	60
15	20	41	9	41	1.000	40
16	19	42	9	42	1.000	55
17	20	43	9	43	1.000	30
18	19	44	9	44	1.000	25

```
                    **ACTIVITY DESCRIPTION**

START   END    PARAMETER   DISTRIBUTION   ACTIVITY            FIRST
 NODE   NODE    NUMBER         TYPE        NUMBER    PROB.   RESOURCE

 19      20       45             9           45      1.000      25
 21       2        1             9            1      1.000      20
 21       3        2             9            2      1.000      10
 21       6        3             9            3      1.000      30
 21       8        4             2            4      1.000      15

                 RESOURCE AVAILABILITY      150

         RANKING FOR FILE NOQ IS BASED ON ATTRIBUTE     1 WITH HVF
1
```

```
1                    GERT SIMULATION PROJECT   1 BY GS CHUPIK
                                  DATE 3/2/1971

                     **FINAL RESULTS FOR  100 SIMULATIONS**

         PROB./                                # OF                                  NODE
NODE     COUNT        MEAN       STD. DEV.     OBS.        MIN.         MAX.          TYPE

 20     1.0000      636.0200     38.0252       100.      539.7414     761.0704        A


                                **HISTOGRAMS**

         LOWER      CELL
NODE     LIMIT      WIDTH                    FREQUENCIES

 20     550.00      5.00    3   0   1   0   1   0   0   2   1   4   5
                            1   4   6   6   3   7   5   4   4   8   7
                            2   5   3   2   3   3   6   1   0   3

                    FINAL RESULTS FOR RESOURCE UTILIZATION

                                           # OF
RESOURCE      AVERAGE       STD. DEV.      OBS.        MIN.          MAX.

   1          114.5123      4.6864         100.      98.4405       128.0006
```

APPENDIX 4
LEVENE'S TEST

The detailed procedure of Levene's Test for equality of variances of several groups of observations is as follows:

Let

p = number of different rules = 11

n = number of different networks = 4

and x_{pn} = observed mean completion time resulting from rule p and network n.

From the p groups means x_{1n}, the following matrix may be formed:

$$\begin{array}{ll} x_{11}, x_{12}, \ldots, x_{1n}, & \text{average } \bar{x}_1,\ V(x_{1n}) = \sigma_1^2 \\ x_{21}, x_{22}, \ldots, x_{2n}, & \text{average } \bar{x}_2,\ V(x_{2n}) = \sigma_2^2 \\ \vdots & \\ x_{p1}, x_{p2}, \ldots, x_{pn}, & \text{average } \bar{x}_2,\ V(x_{2n}) = \sigma_p^2 \end{array}$$

Define

$$z_{ij} = \left| x_{ij} - \bar{x}_i \right| \qquad j = 1,2,\ldots, n;\ i = 1,2,\ldots, p$$

and

$$\sum_{j=1}^{n} z_{ij} = \mathbf{Z}_i \qquad i = 1,2,\ldots, p$$

A standard analysis of variance on the z_{ij} may be performed as shown below.

Source	Sum of Squares	Degrees of Freedom	Mean Square
Between Groups	$\sum_{i=1}^{p} \frac{Z_1^2}{n_i} - \frac{G^2}{\sum n_i}$	$p-1$	$s_1^2 \quad F_1 = s_1^2/s^2$
Within Groups	by difference	$\sum_{i=1}^{p} (n_i-1)$	s^2
Mean	$\frac{G^2}{\sum n_i}$	1	
Total	$\sum_{i=1}^{p} \sum_{j=1}^{n} z_{ij}^2$	$\sum n_i$	

where $G = Z_1 + Z_2 + \dots Z_p$

If $F_1 \geq F \left\{ (p-1), \sum(n_i-1), 1-\alpha \right\}$ there is evidence that differences exist between $\sigma_1^2, \sigma_2^2, \dots, \sigma_p^2$. If F_1 is significantly less, accept the hypothesis that the variances are all equal. Choosing a significance level of $\alpha = 0.05$, the value of $F_{10,6,.95}$ is 3.22. The results of the calculations produced an F_1 value of 0.9227. Since $F_1 << F_{10,6,.95}$ the hypothesis $\sigma_1^2 = \sigma_2^2 = \dots = \sigma_p^2$ must be accepted.

APPENDIX 5
TWO-WAY ANALYSIS OF VARIANCE

Having established homoscedasticity (constant variance), the analysis of variance may now be undertaken.

The following variables are defined:

k = number of different rules = 11

n = number of different networks = 4

m = number of replicates = 5

A_i = rules for $i = 1,2,\ldots k$

B_j = networks for $j = 1,2,3,4$

x_{ij} = completion time obtained with ith rule and jth network

α_i = effect of the ith rule

β_j = effect of the jth network

γ_{ij} = interaction effect of the ith rule and jth network

Accounting for possible interactions of the rules and networks, the completion time obtained with rule A_i and network B_j may be viewed as a value assumed by a random variable having a normal distribution with the mean

$$\mu_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij}$$

where

$$\sum_{i=1}^{k} \alpha_i = 0, \qquad \sum_{j=1}^{n} \beta_j = 0, \qquad \sum_{i=1}^{k} \gamma_{ij} = 0$$

for each j and

$$\sum_{j=1}^{n} \gamma_{ij} = 0 \quad \text{for each } i.$$

Now defining x_{ijr} as the rth value obtained with rule A_i and network B_j and taking m observation of each kind, the appropriate model becomes

$$x_{ijr} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijr}$$

for

$$i = 1, 2, \ldots, k = 11$$

$$j = 1, 2, \ldots, n = 4$$

$$\text{and} \quad r = 1, 2, \ldots, m = 5$$

The error term ϵ_{ijr} is usually considered a normally and independently distributed random effect whose mean value is zero and whose variance is the same for all treatments. To summarize the objectives of the analysis, it is desired to test the null hypothesis

$$H_o: \quad \alpha_i = 0, \quad \beta_j = 0, \text{ and } \gamma_{ij} = 0$$

for $i = 1, 2, \ldots, k$

and $j = 1, 2, \ldots, n$

against the alternate hypothesis

$$H_1: \quad \alpha_i \neq 0 \quad \beta_j \neq 0 \text{ and } \gamma_{ij} \neq 0$$

These tests are aimed at isolating any variations in the data which may be attributed to the differences among the rules (As), differences among the networks (Bs), interactions, and chance or experimental error.

The equation of the two-way analysis of variance is given as

$$\sum_{i=1}^{k} \sum_{j=1}^{n} \sum_{r=1}^{m} (x_{ijr} - \bar{x})^2 = nm \sum_{i=1}^{k} (\bar{x}_{i..} - \bar{x})^2$$

$$+ km \sum_{j=1}^{n} (\bar{x}_{.j.} - \bar{x})^2$$

$$+ m \sum_{i=1}^{k} \sum_{j=1}^{n} (\bar{x}_{ij.} - \bar{x}_{i..} - \bar{x}_{.j.} + \bar{x})^2$$

$$+ \sum_{i=1}^{k} \sum_{j=1}^{n} \sum_{r=1}^{m} (x_{ijr} - \bar{x}_{ij.})^2$$

where

$\bar{x}$ = mean of all the data or grand mean

$\bar{x}_{i..}$ = mean of all the data for rule A_i

$\bar{x}_{.j.}$ = mean of all the data for network B_j

and $\bar{x}_{ij}$ = mean of all the data for rule A_i used in combination with network B_j

The left-hand side of the analysis equation is a measure of the total variability of the data and is represented by SST. The components of the right-hand side of the equation are:

$SSA = nm \sum_{i=1}^{k} (\bar{x}_{i..} - \bar{x})^2$ = A measure of the variation of the data concerning the rules.

$SSB = km \sum_{j=1}^{n} (\bar{x}_{.j.} - \bar{x})^2$ = A measure of the variation of the data concerning networks.

$SSI = m \sum_{i=1}^{k} \sum_{j=1}^{n} (\bar{x}_{ij.} - \bar{x}_{i..} - \bar{x}_{.j.} + \bar{x})^2$ = A measure of the variation of the data concerning interactions between rules and networks.

and $SSE = \sum_{i=1}^{k} \sum_{j=1}^{n} \sum_{r=1}^{m} (\bar{x}_{ijr} - \bar{x}_{ij.})^2$ = A measure of the variation due to chance or experimental error.

Therefore, the analysis equation may be written as

$$SST = SSA + SSB + SSI + SSE$$

A summary of the analysis of variance may be represented by the following table:

Source of Variation	df	Sum of Squares	Mean Square	F
Between A's	k-1	SSA	$MSA = \frac{SSA}{df}$	$\frac{MSA}{MSE}$
Between B's	n-1	SSB	$MSB = \frac{SSB}{df}$	$\frac{MSB}{MSE}$
Interaction	(k-1)(n-1)	SSI	$MSI = \frac{SSI}{df}$	$\frac{MSI}{MSE}$
Error	kn(m-1)	SSE	$MSE = \frac{SSE}{df}$	
Total	mkn-1	SST		

The null hypothesis concerning the α_i is based on the statistic F_A. If

$$F_A \geq F_{\alpha,\ k-1,\ kn(m-1)}, \qquad \text{reject } H_o$$

The null hypothesis concerning the β_j is based on the statistic F_B. If

$$F_B \geq F_{\alpha,\ n-1,\ kn(m-1)}, \qquad \text{reject } H_o$$

And finally, the null hypothesis concerning γ_{ij} is based on the statistic F_I. If

$$F_I \geq F_{\alpha,\ (k-1)\ (n-1),\ kn(m-1)}, \qquad \text{reject } H_o$$

The following analytical results were obtained:

Source of Variation	df	Sum of Squares	Mean Square	F
Between A's	10	470.10	47.01	30.93
Between B's	3	58364.79	19454.93	12799.29
Interaction	30	4125.25	137.51	90.47
Error	176	268.28	1.52	
Total	219	63228.42		

Selecting a significance level of $\alpha = 0.05$, the following statistics were obtained:

$$F_{0.5,\ 10,\ 176} = 1.83$$

$$F_{0.5,\ 3,\ 176} = 2.60$$

and $$F_{0.5,\ 30,\ 176} = 1.46$$

Since $$F_A = 30.93 > 1.83$$

$$F_B = 12799.29 > 2.60$$

and $$F_I = 90.46 > 1.46$$

the null hypotheses concerning α_i, β_j, and γ_{ij} must be rejected. The underlying conclusion is that there are significant differences among the rules, between the networks and interactions of both.

APPENDIX 6
DUNCAN'S MULTIPLE RANGE TEST

This test compares the range of any set of p means with an appropriate least significant range R_p, given by

$$R_p = (s_{\bar{x}})\ (r_p)$$

$$s_{\bar{x}} = \sqrt{\frac{MSE}{n}}$$

where

r_p = A value depending upon the desired level of significance α and the number of degrees of freedom corresponding to MSE. The table from which r_p is obtained is called "Critical Values for Duncan's New Multiple Range Test".

MSE = The error mean square in the analysis of variance for the individual network considered.

n = Number of different rules.

and $s_{\bar{x}}$ = An estimate of $\sigma_{\bar{x}} = \sigma / \sqrt{n}$

<u>Network 1</u>

$$MSE = \frac{SSE}{(k-1)(n-1)} = \frac{64.06}{40} = 1.601$$

$$s_{\bar{x}} = \sqrt{\frac{1.601}{11}} = 0.382$$

$$\alpha = 0.05$$

The values of r_p for $p = 2,3,\ldots 11$, and $(k-1)(n-1) = 40$ degrees of freedome are:

p	2	3	4	5	6	7	8	9	10	11
r_p	2.858	3.006	3.102	3.171	3.224	3.266	3.300	3.328	3.352	3.373
R_p	1.092	1.148	1.185	1.211	1.232	1.248	1.261	1.271	1.280	1.288

Next, rank the rules according to their ascending completion times starting with lowest value first.

Rule #	3	7	10	1	11	9	5	4	2	8	6
Mean	104.04	104.51	105.95	106.00	106.26	107.73	108.73	110.30	111.07	111.93	112.74

The range of all the means is 8.70 which exceeds $R_{11} = 1.288$, the least significant range. A line is drawn under any set of adjacent means for which the range is less than the appropriate value of R_p, that is, under any set of adjacent means for which differences are not significant. From the results of the test, it may be concluded that there is no significant difference between rules 3 and 7, and a significant difference between the use of either 3 or 7 and the remaining rules. Obviously, rules 3 and 7 produce the lowest completion times and are therefore the best ones to use for Network 1.

Network 2

$$MSE = \frac{SSE}{(k-1)(n-1)} = \frac{3.537}{40} = 0.088$$

$$s_{\bar{x}} = \sqrt{\frac{0.088}{11}} = 0.090$$

$$\alpha = 0.05$$

p	2	3	4	5	6	7	8	9	10	11
r_p	2.858	3.006	3.102	3.171	3.224	3.266	3.300	3.328	3.352	3.373
R_p	.257	.271	.279	.285	.290	.294	.297	.300	.302	.304

Rule #	1	5	7	10	11
Mean	77.94	77.94	77.94	77.94	77.94

4	3	9	2	6	8
79.86	80.01	80.01	80.93	80.93	80.93

The range of all means is 2.99 which exceeds $R_{11} = .304$, the least significant range. Network 2's results indicate that there is no significant difference between rules 1, 5, 7, 10 or 11 and a significant difference between the use of these rules and the remaining rules. In order to obtain a minimum completion time for Network 2, then, any one of rules 1, 5, 7, 10 or 11 may be used.

Network 3

$$MSE = \frac{SSE}{(k-1)(n-1)} = \frac{100.9}{40} = 2.524$$

$$s_{\bar{x}} = \sqrt{\frac{2.524}{11}} = 0.477$$

$$\alpha = 0.05$$

p	2	3	4	5	6	7	8	9	10	11
r_p	2.858	3.006	3.102	3.171	3.224	3.266	3.300	3.328	3.352	3.373
R_p	1.363	1.434	1.480	1.513	1.538	1.558	1.514	1.507	1.599	1.609

Rule #	1	10	7	11	8	5	2	9	3	4	6
Mean	76.81	77.43	79.23	80.06	81.57	82.12	83.58	85.38	88.19	88.99	99.59

The range of all means is 22.78 which exceeds $R_{11} = 1.609$ the least significant range. The results of the Duncan Test on Network 4 indicates that there is no significant difference between the use of rules 1 and 10, 7 and 11, 8 and 5, and 3 and 4. Therefore, either rules 1 or 10 may be used to obtain minimum project completion time.

Network 4

$$MSE = \frac{SSE}{(k-1)(n-1)} = \frac{44.66}{40} = 1.116$$

$$s_{\bar{x}} = \sqrt{\frac{1.116}{11}} = 0.318$$

$$\alpha = 0.05$$

p	2	3	4	5	6	7	8	9	10	11
r_p	2.858	3.006	3.102	3.171	3.224	3.266	3.300	3.328	3.352	3.373
R_p	.909	.956	.986	1.008	1.024	1.039	1.049	1.058	1.066	1.073

Rule # 9 6 4 5

Mean 110.61 $\underline{111.54 \qquad 111.70}$ 112.34

2 3 8 1 10

$\underline{113.74 \qquad 114.15}$ 118.18 $\underline{122.52 \qquad 122.52}$

7 11

$\underline{125.16 \qquad 125.21}$

It is evident from the results that rule 9 is significantly different from the rest and would be the best one to use in order to obtain minimum project completion time.

APPENDIX 7
RANK CORRELATION

The following table of information is used in calculating the coefficient of concordance (W).

	Rule Rankings										
Network	1	2	3	4	5	6	7	8	9	10	11
1	4	9	1	8	7	11	2	10	6	3	5
2	3	10	7.5	6	3	10	3	10	7.5	3	3
3	1	7	9	10	6	11	3	5	8	2	4
4	8.5	5	6	3	4	2	10	7	1	8.5	11
Total	16.5	31	23.5	27	20	34	18	32	22.5	16.5	23
Dev. from mean	-7.5	7	-0.5	3	-4	10	-6	8	-1.5	-7.5	-1

An inspection of the table reveals that rule 1 for network 1 produced the fourth lowest completion time, rule 2 the ninth lowest completion time, rule 3 the first lowest completion time, and so forth. In the case of ties, the average position number was assigned to each rule involved in the tie. As an example, rules 1, 5, 7, 10, and 11 applied to network 2 all produced the lowest completion time and would therefore occupy the first five positions of completion. Summing the positions and dividing by the number of rules involved in the tie produces

$\frac{1 + 2 + 3 + 4 + 5}{5} = 3$. Therefore, rules 1, 5, 7, 10 and 11 are each assigned the rank of 3. By summing the totals for each column, and dividing by the number of rules, a mean value of the ranking is obtained. Next, the deviation of each rule from the mean is determined.

Defining the following variables,

μ = mean of rule rankings = 24

S = sum of squares of deviations = 390

m = number of networks = 4

n = number of rules = 11

W = coefficient of concordance

W may be thought of as the communality of rankings for the m networks. If the rankings all agree, it can be shown that W = 1. If the rankings differ very much, W will assume a value which approaches zero as the disagreement increases.

For ties,

$$T_i' = \frac{1}{12} \sum_j (t^3 - t)$$

where

t = number of rules involved in a tie

i = network number

j = sets of ties

$T_1' = T_3' = 0$

$T_2' = \frac{1}{12}(5^3 - 5) + \frac{1}{12}(3^3 - 3) + \frac{1}{12}(2^3 - 2) = 12.5$

$T_4' = \frac{1}{12}(2^3 - 2) = 0.5$

$$W = \frac{S}{\frac{1}{12} m^2 (n^3 - n) - m \sum_{i=1}^{m} T_i'} = 0.1698$$

Now it is desired to test the significance of the observed value of W. If all the networks are independent in their rankings, then any set of rankings is just as probable as any other set. The null hypothesis to be tested then, is that all the rankings are the same. Using Fisher's z - distribution, if

$$z_1 \geq Z_{\gamma_1, \gamma_2, \alpha} \quad \text{accept } H_0$$

where

$$z_1 = 1/2 \log_\epsilon \frac{(m-1)\,W}{1-W} = .246$$

$$\gamma_1 = n-1- \frac{2}{m} = 9.5$$

$$\gamma_2 = (n-1)\,\gamma_1 = 28.5$$

$$\alpha = 0.05$$

$$z_{9.5,\ 28.5,\ 0.05} \approx .388$$

Since $z_1 < Z_{\gamma_1, \gamma_2\ \alpha}$ the null hypothesis is rejected and it may be concluded that there is a significant difference in the rankings of the rules for each network. The significance of the difference in rankings indicates that the rules react differently in each network used. A rule which produces the lowest completion time or any other rank of completion time in one network will not necessarily produce the same ranking in another network.

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