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# Comparison of a linear programming formulation of a production smoothing model and a combined linear and quadratic model

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**COMPARISON OF A LINEAR PROGRAMMING  
FORMULATION OF A PRODUCTION SMOOTHING MODEL  
AND A COMBINED LINEAR AND QUADRATIC MODEL**

by  
**Roy L. Epp**

**A THESIS**

**Presented to the Graduate Faculty  
of Lehigh University  
in Candidacy for the Degree of  
Master of Science**

**Lehigh University**

**1965**

This thesis is accepted and approved in partial fulfillment of  
the requirements for the degree of Master of Science.

16 May 18, 1965  
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## ABSTRACT

Two existing production smoothing and inventory control models have been compared. One is a linear programming model (simplex technique) and the second model combines a linear inventory and manpower adjustment rule with a quadratic production cost function.

The model comparison is based on the results obtained by simulation from a  $2^4$  factorial experiment. The factors varied are (1) a variable market, (2) market trend, (3) hiring and firing costs and (4) inventory storage cost.

The results indicate there is no significant cost difference in the two models in relation to the measurement variables (1) total cost and (2) the cumulative premium costs. However, significant differences exist with three of the variables which are a part of premium costs. The variables, overtime and idletime, were significantly less for the linear program model and the costs of manpower adjustments were significantly less for the quadratic model. The quadratic model requires 1/120 of the computer time required by the linear program to perform the production smoothing for the same period.



## CHAPTER I

### INTRODUCTION

#### Statement of the Problem

This paper presents a comparison of the performance of two production smoothing models under identical shop parameters and production requirements to determine if any significant differences in costs exist. One model is a linear program developed and applied to "shop X" by Elmaghraby, Jeske, and O'Malley<sup>(1)</sup>. The second model, which is based on the quadratic production costs concept developed by Modigliani and Hohn<sup>(2)</sup>, is applied to "shop X" by Atkins<sup>(3)</sup>. Both models are specifically concerned with the problem of how much to produce in each period of a multi-period planning horizon so as to minimize the combined costs of inventory, overtime, idletime and manpower adjustments.

The shop under consideration is termed "labor limited" since the production rate is limited by the number of persons employed rather than by machine capacity. It is assumed that the equipment can handle all peak loads within practical limits and that the over-all equipment costs are dependent on the total volume. There is no limitation on the amount of inventory or changes in the production rate per month.

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1. Elmaghraby, S. E., J. W. Jeske, and R. L. O'Malley, "An Operational System for Smoothing Batch-Type Production," Presented at the 11th Annual Meeting of the Institute of Management Science, Pittsburgh, Pa., March, 1964.
  2. Modigliani, F. and F. Hohn, "Production Planning Overtime and the Nature of the Expectation and Planning Horizon," Econometrica, Vol. 23, No. 1 (Jan. 1955), pp. 46-66.
  3. Atkins, H., "A Study of a Proposed Production and Inventory Control Model Formulated by Combining a Linear Cost Manpower Adjustment Rule with Modigliani and Hohn's Quadratic Production Cost Model," Lehigh University, May, 1964.

Overtime is restricted to 20% of the regular working hours. The forecasts are given in labor hours since several different products are produced in the shop. The costs to be accounted for are regular time, overtime, idletime, inventory, hiring and firing. The actual shop costs associated with these variables are given in Appendix B.

Because the cost of backorders is extremely high, the restriction that the planned inventory plus production must equal or exceed the forecasts for each period in the horizon is placed on the models. In actual application backorders can occur because of poor forecasts or failure to meet the production schedule. When the demand is less than the forecast and an inventory is created, each model is allowed to distribute the inventory through-out the planning horizon in scheduling the production. If the demand is greater than production and present inventory, a backorder is created. The backorder is assigned to the first and each succeeding periods of the horizon based on a percentage (20%) of the production capacity of the shop until the backorder is depleted.

The sensitivity of each model is tested by changes in the cost structure within the shop and environmental factors which affect demand. The experimental factors selected are (1) variability of the forecast, (2) market trend, (3) manpower adjustment costs and (4) inventory storage costs. The evaluation of the performance of the two smoothing models will be based on the measurement variables (1) total cost, (2) premium costs, (3) overtime, (4) idletime, (5) inventory, (6) manpower adjustments and (7) backorders. The specific models and the design of



the experiment are described in detail in Chapters II and III respectively.

#### Amplification of the Production Scheduling Problem

Mathematically the problem may be stated as the scheduling of a portion,  $x_{ij}$ , of the production in month  $i$  to use in period  $j$ , (where  $i \leq j$ ), with a cost  $c_{ij}$  which minimizes the total costs.

$$\text{minimize } Z = \sum_{ij} c_{ij} x_{ij}$$

$$x_{ij} \geq 0$$

$$x_{ij} = 0 \text{ when } i > j$$

subject to the restriction

$$P_i + h_0 \geq S_i$$

$P_i$  = cumulative production through the  $i$ th period of the horizon

$h_0$  = initial inventory

$S_i$  = cumulative requirements through the  $i$ th period of the horizon

Since backorders are not allowed, the cost of allocating production from period  $i$  to period  $j$  is equal to the sum of the production cost plus the number of periods stored times the storage cost.

$$C_p + (j-i)C_s$$

where,

$C_p$  = production cost

$C_s$  = storing cost

The amount of production scheduled for the first period of the horizon by each model may include some production to inventory for use in the succeeding periods of the horizon.

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The optimum production schedule is recalculated at the end of each period with a new inventory. The new inventory level is different from the planned inventory since demand is probabilistic.

If the cost for a change in production capacity is zero then the minimum-cost schedule is to produce in period  $i$  for use in period  $i$ . Thus the inventory charges are avoided and the minimum cost solution for the  $N$  periods is determined by the product of the unit production costs and the difference in total requirements and initial inventory. This type of schedule is not feasible since the cost associated with the change in production rate is not considered.

If the forecast for period  $t$  of the horizon is less than the production capacity, the production ( $p_t$ ) supplies the entire requirement for period  $t$ . As the forecast becomes larger than  $p_t$ , requirements for period  $t$  are first produced in period  $t-1$  if slack is available. As the forecasts are increased, allocations for period  $t$  are stored for additional time periods. The models optimize the cost of storage, overtime, idletime and the change in production rate in determining the optimum schedule for each period in the horizon.

#### Review of the Advances in Inventory and Production Control

The linear programming model uses a method for finding a solution to a system of simultaneous linear equations and linear inequalities. A solution to this type of problem was provided by Dantzig<sup>(4)</sup> in 1947, with the development of the simplex method. Linear programming has

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4. Dantzig, G. B., Linear Programming and Extensions, Princeton, New Jersey; Princeton University Press, 1963.

provided a new insight into business operations through the mathematical analysis of the various operating systems within an organization. As a result it has become an important managerial tool with which to improve the efficiency of operations. Linear programming techniques have been applied to scheduling problems in the petroleum, food, railroad and many other industries. Production smoothing models<sup>(5)</sup> have been formulated to meet stipulated requirements in a manner that minimize storage costs.

Modigliani and Hohn<sup>(6)</sup> developed a model which schedules production over the planning horizon to meet the given requirements at the lowest cost for a deterministic demand and a cost of production that can be expressed as a quadratic function. Holt, Modigliani, Muth and Simon<sup>(7)</sup> extended the quadratic cost concept and developed quadratic cost functions for (1) regular payroll, (2) hiring and layoff, (3) overtime and (4) deviations from optimal inventory level. Based on the quadratic approximations, a minimum cost function was developed which provided linear decision rules for scheduling of aggregate production, employment and inventories.

The work of Holt et. al. was extended by Schild<sup>(7)</sup> to include factors such as inventories of individual products, interest factors to discount future costs and the cost of additional temporary storage when inventory exceeds a specified level. The simplicity of the linear

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5. See Bibliography references (3), (9), (16).

6. Modigliani, F. and F. E. Hohn, op. cit., pp. 46-66.

7. See Bibliography references (11), (15).



decision rules was maintained.

The cost structure of the Modigliani and Hohn (M-H) model consisted of a convex production cost function and a linear inventory cost function. Hoffman and Jacobs (H-J)<sup>(8)</sup> treated a similar problem by assuming the production cost function to be linear but added a charge for positive changes in production levels during successive periods. Klein<sup>(8)</sup> combined the results of the M-H and H-J models and obtained a convex production cost function, a linear inventory cost function and a piece-wise linear smoothing charge for production rate changes.

Bellman and Dreyfus<sup>(8)</sup> present dynamic programming solutions to a number of scheduling problems where it is expedient to follow a middle path, balancing one type of cost against another so as to minimize the total cost. One of the mathematical models formulated is applicable to the production scheduling problem considered. The basic feature of dynamic programming is that an optimal decision is reached by proceeding from one stage to the next. An optimal solution is determined for the given conditions in the first stage. The optimal solution from the first set is integrated with the next stage. At each stage an optimal solution from all previous stages is carried forward to the following stage. The technique is defined by Bellman and Dreyfus<sup>(8)</sup> as:

"The Principle of Optimality. An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

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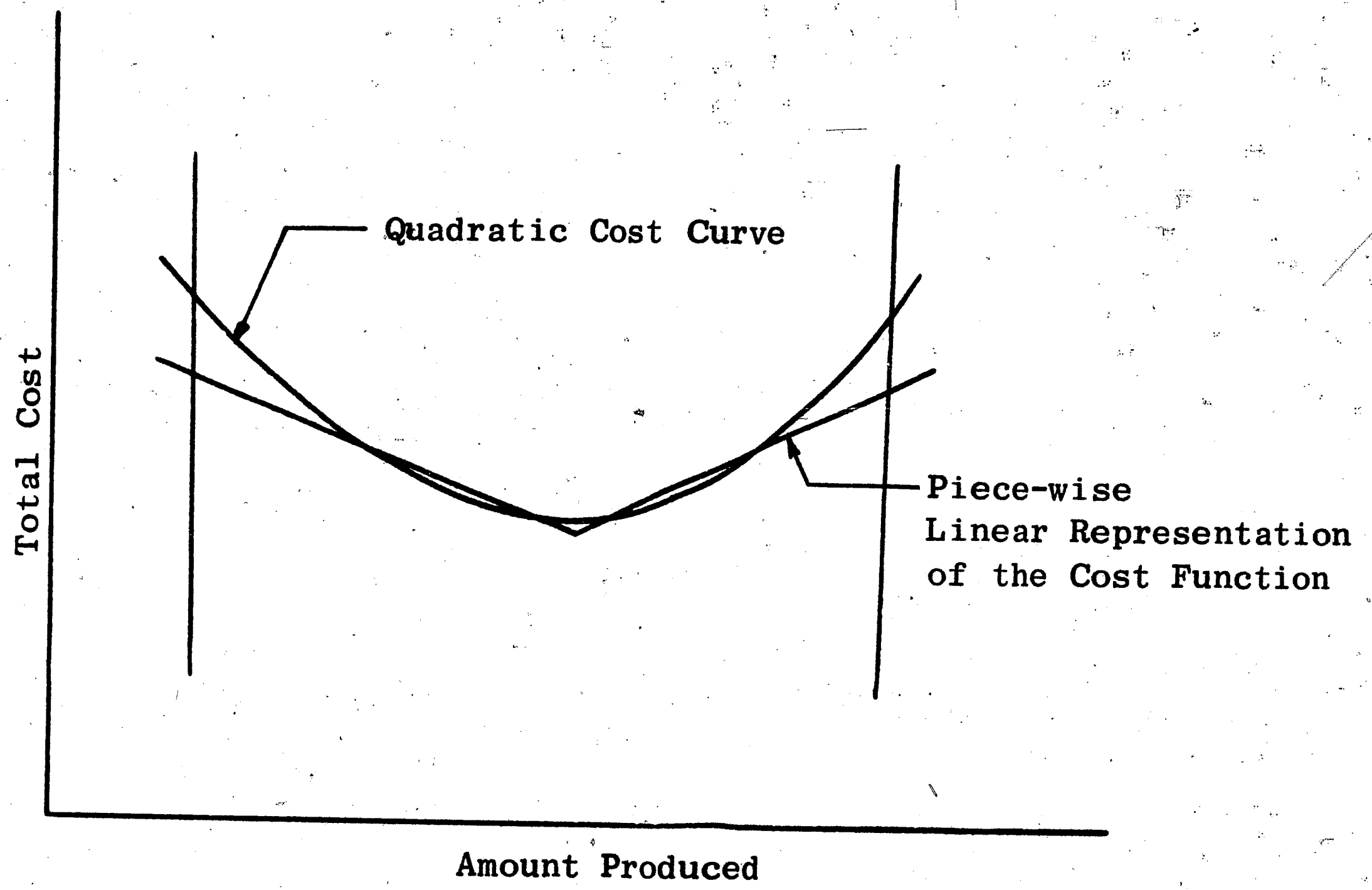
8. See Bibliography references (2), (10), (12).

### Comparison of the Two Methods of Representing Production Costs

In order to compare the linear programming model with the combined linear and quadratic model, it is important to note the costs and restrictions that are associated with each model. The linear program includes linear costs with upper and lower bound restrictions on the variables. Typical restrictions are (1) production must be non-negative, (2) the number of regular production hours must be equal to or less than the regular hours available and (3) a restriction on the maximum amount of overtime available per period. Linear programs can also include a piece-wise linear convex cost curve as shown in Figure 1.1.

The piece-wise linear convex cost curve can be replaced by a continuous quadratic cost curve which is continuous over the interval from minus infinity to plus infinity. No boundaries are required to be examined numerically since all points on the curve are from a continuous function and can be found analytically. The distinction between the quadratic and linear curve in Figure 1.1 should be noted. If the model operates in the area of the vertex of the two linear functions, the quadratic approximation is a very good one. If the operation is close to the upper and lower boundaries then the error can become quite large. The hypothesis is that a piece-wise linear approximation may be a more accurate description of the inventory and production costs than the quadratic model shown, but the quadratic curves describe the physical costs accurately enough to warrant the use of a quadratic approximation in a production smoothing problem. The relatively simple approximation to the production costs as proposed by Modigliani and Hohn





**FIGURE 1.1** The Comparison of a Piece-wise Linear and Quadratic Cost Curves

is very appealing when it meets the objectives of a production smoothing system within the approximate cost range of the linear program.

## CHAPTER II

### THE MODELS

#### The Quadratic Model

Modigliani and Hohn<sup>(9)</sup> developed a production smoothing model for the problem of scheduling production of an item over  $N$  periods of the horizon (1) to satisfy known requirements and (2) to minimize cost.

For a feasible solution the inventory restraints are

$$h_k = h_0 + X_k - S_k \geq 0 \quad k = 1, 2, \dots, t \quad (1)$$

$$h_N = h_0 + X_N - S_N = 0 \quad (2)$$

$h_0$  = initial inventory

$h_k$  = inventory in the  $k$ th period

$X_k$  = cumulative production through period  $K$

$S_k$  = cumulative requirements through period  $K$

$t$  = number of production periods

The production constraints are

$$X_k \geq 0 \quad k = 1, 2, 3, \dots, t$$

$$h_0 < S_N$$

$N$  = periods in the planning horizon

The cost structure of the model which is based on linear inventory costs and a quadratic function to represent the production costs is formulated as follows:

$$C_N = C_p + C_s + C_o$$

$C_N$  = total cost of producing and storing the given requirements for the planning period

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9. Modigliani, F., and F. Hohn, op. cit., pp. 46-66.

$C_p$  = production costs

$C_s$  = inventory storage costs

$C_o$  = costs which are independent of the production schedule

The assumptions of the model are

1. The cost of production is the same function in each period of the horizon and can be represented by:

$$C_p = \sum_{t=1}^N F(x_t)$$

2. The marginal cost function

$$f(x) = \frac{dF(x)}{dx}$$

is non-negative, monotonically increasing and continuous.

3. The storage costs are proportional to the length of time stored and can be represented by:

$$C_s = \alpha \left[ \frac{h_0}{2} + \sum_{t=1}^N h_t \right]$$

$h_t$  = average change in inventory per period

$\alpha$  = cost of storing one unit for one period

$h_0$  = initial inventory

The optimal solution derived by Modigliani and Hohn for a quadratic cost function represented by

$$C_p = Ax^2 + Bx + C \quad (3)$$

is as follows:

$$P(\bar{x}_1, t) = t\bar{x}_1 + \frac{t(t-1)\alpha}{2a} \quad (4)$$

$P(\bar{x}_1, t)$  = cumulative production requirements through period  $t$

$t$  = time base

$a = 2A$ , where  $A$  is the coefficient of the second order term in

equation (3).

The optimal production quantity for the first period and the length of the sub-horizon is obtained by the following equation:

$$\bar{x}_1 = \text{Max}_t \left[ \frac{P_{r@t}}{t} - \frac{t(t-1)\alpha}{2a} \right] \quad t = 1, 2, \dots, N \quad (5)$$

$P_{r@t}$  = cumulative requirements less initial inventory

The value  $\bar{x}_i$  is calculated for each period within the planning horizon by equation (5). The maximum  $\bar{x}_i$  is selected as the optimal production quantity for the first period. The value of  $t$  corresponding to the maximum  $\bar{x}_i$  is the end of the first sub-horizon. The optimum production quantity for each period of the sub-horizon is calculated by equation (4). The process is repeated for the remaining  $t+1$  to  $N$  periods; however, the cumulative production requirements are reduced by the amount produced in the first  $t$  periods.

#### Application of the Quadratic Model

To apply the model(1) the production costs were represented by a quadratic function and (2) a manpower adjustment rule was developed to provide for changes in the production level. Atkins<sup>(10)</sup> developed an optimal quadratic approximation of production costs and a linear manpower adjustment rule for the Modigliani and Hohn model to apply to a labor limited shop. The linear manpower adjustment rule requires that the cumulative savings in overtime (idletime) in the planning horizon by adding (firing) a worker, be equal to or greater than the cumulative cost of hiring and firing. It is not required that the savings occur in consecutive months but they must occur within the same horizon.

The inventory costs are represented by a linear function in the

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10. Atkins, H., op. cit., pp. 26-38.



model and the change in manpower is determined by the overtime (idle-time) schedule for the horizon. The remaining costs to be approximated by the quadratic production cost function are regular time, idletime and overtime. The production costs are represented by

$$C_p = C_{pb} + C_{pp}$$

$C_p$  = production costs

$C_{pb}$  = base labor costs

$C_{pp}$  = premium labor costs

The base labor is assumed to be a linear function and the premium labor cost a quadratic function.

$$C_{pb} = Hx$$

H = average hourly pay rate

x = required production hours

$$C_{pp} = A' (x-k)^2 + B' (x-k) + C'$$

k = shop labor capacity in man hours

By combining the two equations the cost of production is

$$C_p = A' x^2 + (H - 2A'k + B')x + A'k^2 = B'k + C'$$

As previously noted a of the proposed model is equal to twice the value of the coefficient of the second order term (A') in the quadratic cost equation. Therefore by determining the quadratic approximation to the linear premium costs of idletime and overtime the model can be applied to the particular shop. By simulation, Atkins<sup>(11)</sup> determined a = .5 was the "best value" for the defined shop costs.

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11. Atkins, H., op. cit., pp. 31-38.



## The Linear Programming Model

Elmaghraby, Jeske, and O'Malley<sup>(12)</sup> developed a production smoothing model employing the technique of linear programming for scheduling production in an optimal manner for the planning horizon. This model is applied to the same conditions as described for the quadratic model.

The restraints on the model are stated as follows:

1. Regular time

$$X_t \leq B_t W_t \quad (1)$$

$B_t$  = number of regular hours available per man

$W_t$  = size of work force in period  $t$

$X_t$  = hours of regular time production in period  $t$

2. Overtime

The overtime is restricted to 20% of regular time.

$$Y_t = B'_t W_t \quad (2)$$

$B'_t$  = overtime hours per employee in period  $t$

$Y_t$  = overtime hours of production in period  $t$

Converting the regular time and overtime restraints to the cononical form:

$$X_t + S_{1t} = B_t W_t$$

$$Y_t + S_{2t} = B'_t W_t$$

$S_{1t}$  = idletime of regular production hours

$S_{2t}$  = idletime of overtime hours which is zero

3. Inventory

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12. Elmaghraby, S.E., J.W. Jesky, and R.L. O'Malley, op. cit.

$$I_t = I_{t-1} + X_t + Y_t - R_t \geq 0 \quad (3)$$

$I_{t-1}$  = inventory in the previous period

$R_t$  = forecast of requirements for period t

$I_t$  = inventory in period t

4. Changes in the size of the workforce

$$W_t - W_{t-1} = U_t - V_t \quad (4)$$

$W_t - W_{t-1}$  = change in work force during period t

$U_t$  = represents an increase in the work force

$V_t$  = represents a decrease in the work force

The objective function of the model is

$$C_t = hU_t + lV_t + aX_t + bY_t + eS_{1t} + iI_t$$

h = hiring costs of one employee

l = firing costs of one employee

a = regular time production cost per hour

b = overtime production cost per hour (1.5a)

e = idletime cost per hour

i = storage cost per hour production per month

The number of original variables was reduced by using the following equation in place of equation (3).

$$I_t = I_0 + \sum_{j=1}^t X_j + \sum_{j=1}^t Y_j - \sum_{j=1}^t R_j \geq 0$$

The cononical form is as follows:

$$\sum_{j=1}^t X_j + \sum_{j=1}^t Y_j - S_{3t} = \sum_{j=1}^t R_j - I_0 \quad t = 1, 2, \dots, N$$

$S_{3t}$  = production in excess of the requirements

$I_0$  = initial inventory

$R_j$  = cumulative requirements through period  $t$

$N$  = periods in the horizon

The revised objective function for period  $t$  is written as:

$$C_t = hU_t + lV_t + aX_t + bY_t + eS_{1t} + iS_{3t}$$

The cumulative objective function to be minimized is as follows:

$$TC = \sum_{t=1}^N C_t$$

TC = total cost for the planning horizon

The necessity of bringing the manpower output of the linear program to an integer value is reduced by (1) the lack of accuracy in the forecast and cost data and (2) the size of the work force. However, Elmaghraby et. al. developed the following rule for a systematic method of bringing the manpower to an integer value.

Any rounding of manpower is reflected in changes of overtime, idletime, and/or inventory during the horizon period. The guidelines for rounding are

1. If the manpower figures obtained from the linear program are integral, then the manpower level is not changed.
2. The value of manpower for the last period of the horizon is the largest integer value obtained by

$$m_s = m_{lp} + .5$$

where,

$m_s$  = smoothed manpower

$m_{lp}$  = manpower level obtained from the linear program

3. All remaining values with a mantissa  $\geq .85$  or  $\leq .15$  are rounded up or down respectively.

4. If the mantissa of the manpower level of a single element or a series of elements is between 0.15 and 0.85, the series of values are rounded to the nearest integer based on the values of the integral endpoints of the series.

The rules are

- a. If both end points are larger than the elements in the series, all values are rounded up to the nearest integer.
- b. If both end points are smaller than the elements in the series, all values are rounded down to the nearest integer.
- c. If the integral endpoints are different, the series is rounded half up and half down to the nearest integer based on one of the following:

When

$$m_{k-1} < m_i < [m_{j+1}]$$

$$m'_i = \begin{cases} [m_i] & \text{for } i = k, k+1, \dots, k + \left[ \frac{j-k}{2} \right] \\ [m_i] + 1 & \text{for } i = k + \left[ \frac{j-k}{2} \right] + 1, \dots, j \end{cases}$$

When

$$m_{k-1} > m_i > [m_{j+1}]$$

$$m'_i = \begin{cases} [m_i] + 1 & \text{for } i = k, k+1, \dots, k + \left[ \frac{j-k}{2} \right] \\ [m_i] & \text{for } i = k + \left[ \frac{j-k}{2} \right] + 1, \dots, j \end{cases}$$

$m_k$  = beginning element of the series

$m_j$  = last element of the series

$j = k$  if only one element is under consideration

$j - k + 1$  = length of the series

[ ] = largest integer value

All adjustments to the manpower are converted to an intermediate value of inventory which is calculated by the equation:

$$\hat{I}_i = I_i - I_{i-1} + I'_{i-1} + M_i \quad i = 1, 2, \dots, N$$

Where,

$$I_0 = I'_0 = 0$$

$\hat{I}_i$  = intermediate inventory value

$I_i$  = inventory level obtained from the linear program

$I'_{i-1}$  = new inventory level for the previous period

$M_i$  = change in the production capacity

Since the inventory figures represent cumulative quantities, more than one period in the horizon is affected by a change in the manpower. If the intermediate value of inventory ( $\hat{I}_i$ ) is negative, the overtime for period  $i$  is increased.

$$O'_i = O_i - \hat{I}_i$$

Where  $I'_i = 0$

$O_i$  = overtime scheduled by the linear program

$O'_i$  = adjusted overtime

When inventory and overtime are scheduled in the same period and the production rate is decreased, the overtime is adjusted for the required change. Tests are performed to assure overtime and idletime do not



occur in the same period. Any inventory created in the Nth period of the horizon as a result of rounding manpower, is converted to idletime.



## CHAPTER III

### DESIGN OF THE EXPERIMENT

#### Variables in the Experimental Model

The operating conditions under which the models were compared involved two classes of variables: (1) changes in the external environment which are not under the control of the shop and (2) cost parameters which are controlled by the shop. The experimental variables (factors) selected to evaluate each model were (1) variability of the forecast, (2) market trend, (3) hiring and firing costs and (4) inventory storage cost. Each variable was assigned two levels of value. This represented sixteen different conditions under which to test each model. The number of variables that could have been specified was quite large, some were selected for study and others ignored. In effect the factors selected represent a sample from all possible changes.

The variability of the forecast is a measure of the stability of the demand for the product. As the fluctuation in demand increases the variability increases. This was controlled in the experiment by varying the standard deviation of the forecast distribution. The low and high variability of the forecast were represented by a standard deviation of two and five units respectively. Figure 3.1 illustrates the two cases considered.

A second factor external to the shop is the growth and decline in the demand for the product. In the standard model the environment was represented by a market growth of 5% per year upon which was superimposed a moderate cyclic pattern with an amplitude of  $\pm 10\%$  of the

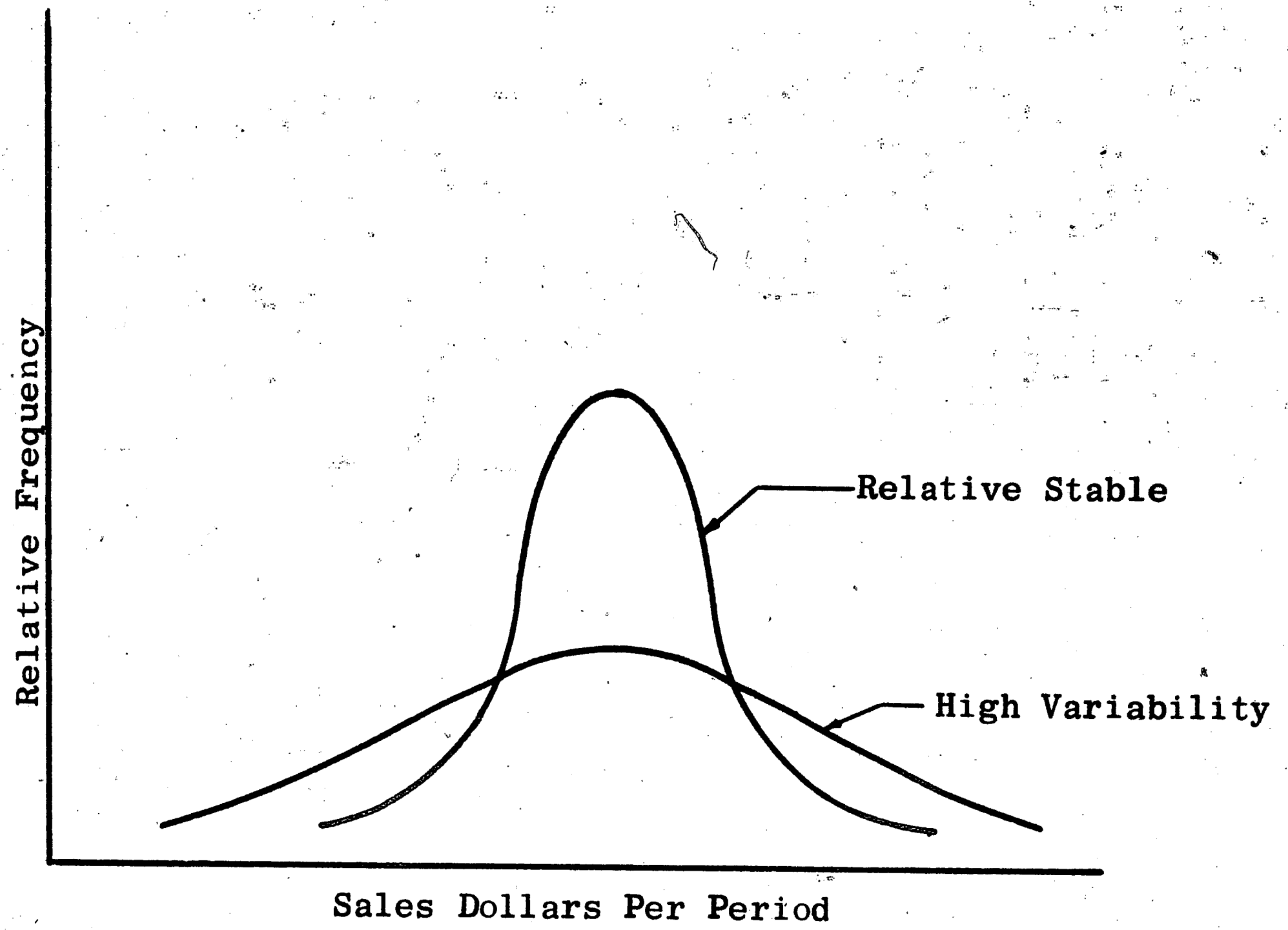


FIGURE 3.1 Probability Distribution of Sales

mean and a time period of three years. The alternate level of the factor was a declining market with the same cyclic pattern described above. To allow the models in the experiment to operate over the same range of forecast values of growth and declining market conditions, starting means of 62 and 50 man hours per hour were selected for the declining and growth markets respectively.

The cost associated with hiring or firing a man in "shop X" is \$425 per man. The alternate value selected for the experiment was \$625 per man. The cost associated with inventory storage in "shop X" is \$0.00212557 per man hour per hour. The alternate value was twice the standard cost. By changing these two cost parameters in the experiment noticeable changes in the amount of overtime, idletime, inventory and changes in the work force occurred. The sensitivity of each model was evaluated by comparing the changes in the costs associated with each measurement variable.

#### Experimental Design and Analysis

Techniques considered in designing the experiment included making a series of independent alterations of the experimental variables in the standard model and noting the effect of each change. Disadvantages of this method are that (1) the interactions are ignored and (2) little generality in the results are obtained. It is possible that a change in one of the factors of the experiment may affect the model only if some other factor is varied. By making single changes in the model, the effect can only be applied to similar shops. However, by evaluating a change in relation to all the other changes some degree of generality is obtained. The factorial design of an experiment

enables a study of the main effects and higher order interactions.

The four experimental variables and the two levels associated with each variable represent a  $2^4$  factorial design. The following notation is used to note the level of each factor.

1 = all factors in the standard model

a = all factors in the standard model except A

b = all factors in the standard model except B

ab = all factors in the standard model except A and B

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. .  
. . .  
. . . .

abcd = all factors in the alternative case. Figure 1 of the Appendix A gives a composite list of the experimental factors. The equation<sup>(14)</sup> of estimation for the experiment is written as:

$$Y_{abcd} = \mu + A_a + B_b + AB_{ab} + C_c + AC_{ac} + BC_{bc} + ABC_{abc} + D_d + \\ AD_{ad} + BD_{bd} + ABD_{abd} + CD_{cd} + ACD_{acd} + BCD_{bcd} + ABCD_{abcd} + \\ E_{abcd}$$

where,

$Y_{abcd}$  = the observed response for each cell

$\mu$  = mean of all trials

$A_a$  = main effect of factor A

$B_b$  = main effect of factor B

$AB_{ab}$  = interaction between factors A and B. When the interaction is different from zero it means the combination of A and B has some effect over and above

14. Davies, O. L., The Design and Analysis of Industrial Experiments, New York: Hafner Publishing Co., 1963, p. 255.



the individual effects of A and B.

$E_{abcd}$  = the experimental error term.

The analysis of variance is normally used in the evaluation of the data obtained from a factorial experiment. The estimate of the error term can usually be obtained (1) by assuming the interactions higher than the first order are zero, (2) by previous data or (3) by replication of the experiment. Preliminary investigation indicated the assumption that the second and third order interactions were zero did not appear reasonable. Previous data was not available to estimate the error term for either model, and the allocation of computer time did not allow replication of the linear program. Therefore, a method presented by Cuthbert Daniel<sup>(15)</sup> for inspection and criticism of data from factorial experiments was used in the evaluation of the linear programming model.

The half-normal plot analysis of the data from a  $2^n$  factorial experiment reveals problems such as (1) the error variance is not the same for all observations, or (2) the effects are not normally distributed, or (3) the existence of extreme values in the data. To evaluate the data, the absolute value of the effects are ordered and plotted on half-normal grid paper. The half-normal grid can be constructed on a sheet of arithmetic (normal) probability paper by deleting the printed probability scale  $P$  for the range less than 50%. For the range greater than 50% replace each value of  $P$  by the corresponding value  $P'$  where:

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15. Daniel, C., "Use of Half-Normal Plots in Interpreting Factorial Two-Level Experiments," Technometrics, Vol. 1, No. 4, (Nov. 1959) pp. 311-341.



$$P' = (2P - 100)/100, \quad i = 1, 2, 3, \dots, n \quad (1)$$

The ordinate of the effect to be plotted is obtained by:

$$P' = (i - 1/2)/n, \quad i = 1, 2, \dots, n \quad (2)$$

where,

$n$  = number of effects to be plotted.

The absolute value of the contrast is plotted on the abscissa.

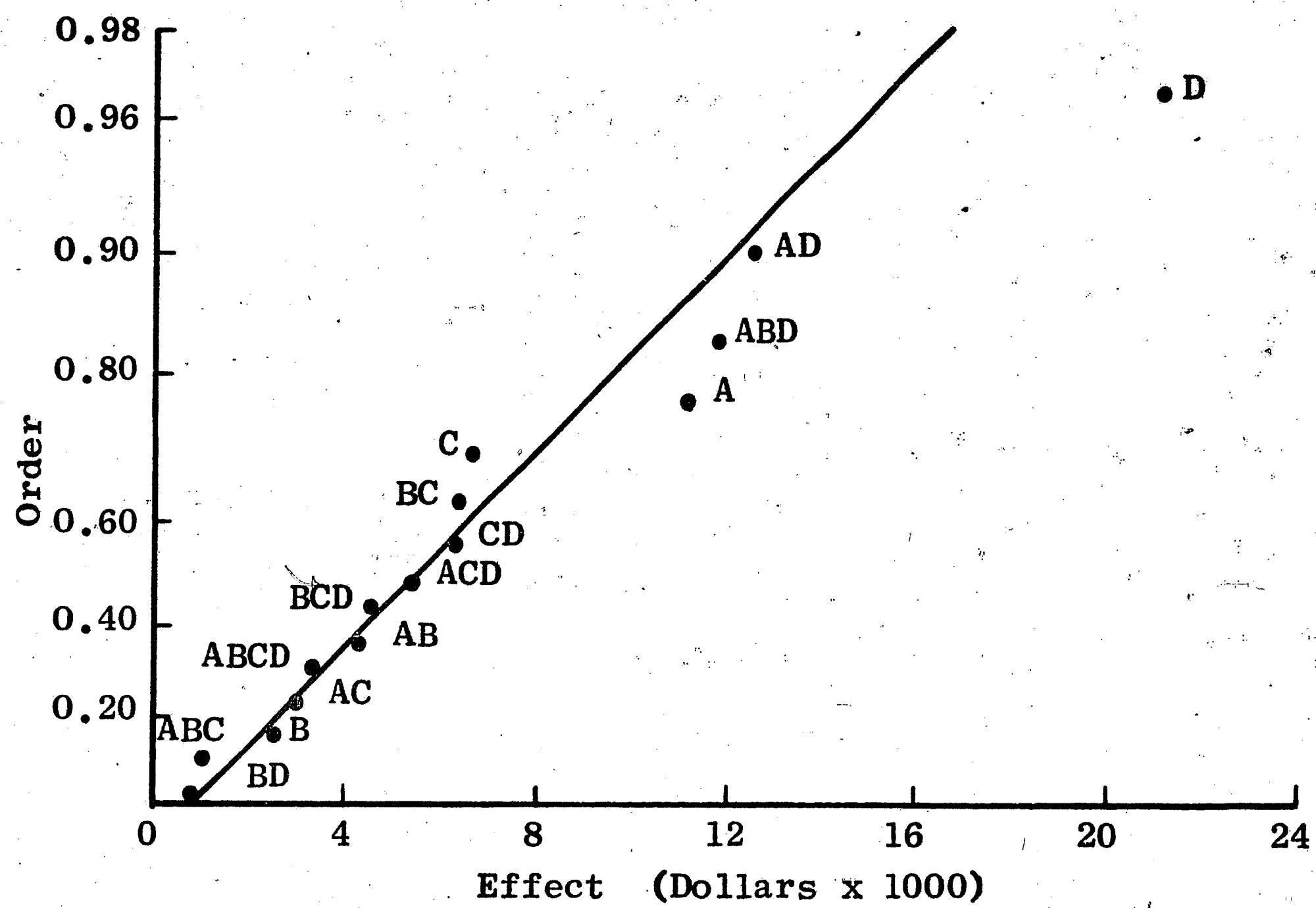
As an illustration of the half-normal plot technique, the analysis was applied to the measurement variables total cost and overtime of the quadratic model. The absolute values of the effects were ranked by taking the square root of the mean square associated with each effect. The mean square values were obtained by an analysis of variance computer program. The results are given in the first column of Figure 3.2. The ordinate values which were obtained by equation 2 are given in column 3. Figure 3.3 presents the plot of the data. The data is assumed to be normally distributed and the points should lie on a straight line. The variable D indicates an extreme effect since it is not in line with the other data. The main effect and interactions A, ABD and AD also appear to affect the normality of the data but do not indicate extreme effects. The graph of the data should point to the origin however as indicated a bias of approximately 900 units was introduced by the extreme effect D.

A standardized half-normal plot of the data allows the use of a fixed line for comparison rather than an arbitrary line drawn through the points. Guardrails are available for the 5% level of significance. To convert the effects to a scale-free set of ordered statistics, the absolute values are divided by the estimate of the standard deviation.

FACTOR COMBINATIONS	TOTAL COST DATA	HALF NORMAL PLOT COORDINATE	STANDARD HALF NORMAL PLOT DATA
D	21301	.967	3.66
AD	12648	.900	1.89
ABD	11983	.834	1.79
A	11259	.767	1.68
C	6708	.700	1.00
BC	6526	.633	.972
CD	6478	.567	.965
ACD	5506	.500	.821
BCD	4594	.433	.685
AB	4456	.367	.664
ABCD	3450	.300	.514
AC	2974	.233	.443
B	2509	.167	.386
ABC	1062	.100	.158
BD	810	.033	.121

FIGURE 3.2

Data for Half-normal Plot of Variable  
Total Cost (Quadratic Model)



**FIGURE 3.3** Half-normal Plot of the Variable Total Cost (Quadratic Model)

The value of the standard deviation is roughly estimated by the contrast for which the ordinate is most nearly 0.683 or 68.3 percent. For the 15 cells considered in the experiment this value corresponds to the 11th ordered statistic. The standard half-normal plot is given in Figure 3.4. The results indicate the effects D, ABD and A are significant at the 5 percent level. However, the interaction AD should not be discarded until additional investigation is performed. Also note the bias is indicated by the failure of the data to be distributed about the standard line.

To illustrate a different type of relation between the variables, the absolute ordered effects obtained from the variable overtime are given in Figure 3.5. The results indicate the data is from two separate distributions. The failure of the contrasts to point toward the origin indicate the three extreme effects AB, ABC and C act together to inflate the remaining effects in the experiment. The amount of bias introduced is approximately 2400 units. To convert the data to a standard half-normal plot only the first twelve ordered values excluding the three extreme effects, were considered in the estimation of the standard deviation. The ninth ordered value was selected since it is nearest 68.3%. Figure 3.6 is a standardized half-normal plot of the variable overtime with the 2400 units of bias removed. The graph of the combined replications of the quadratic model (Figure 3.7) indicate the extreme values and the bias introduced into the remaining effects were removed from the measurement variable.

#### Simulation

A simulation model is not used continuously, therefore allowances



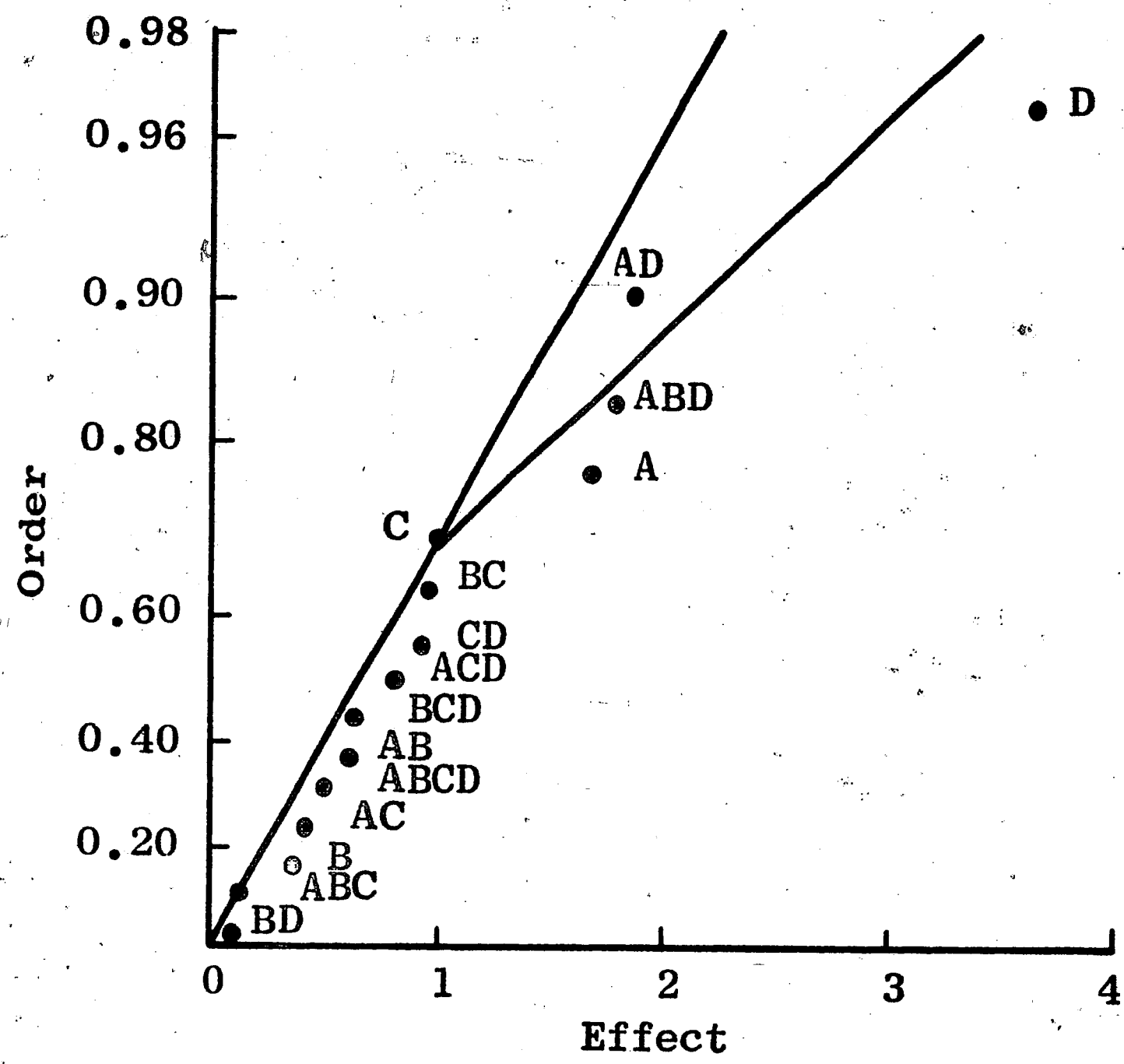


FIGURE 3.4 Standardized Half-normal Plot of the Variable Total Cost (Quadratic Model)

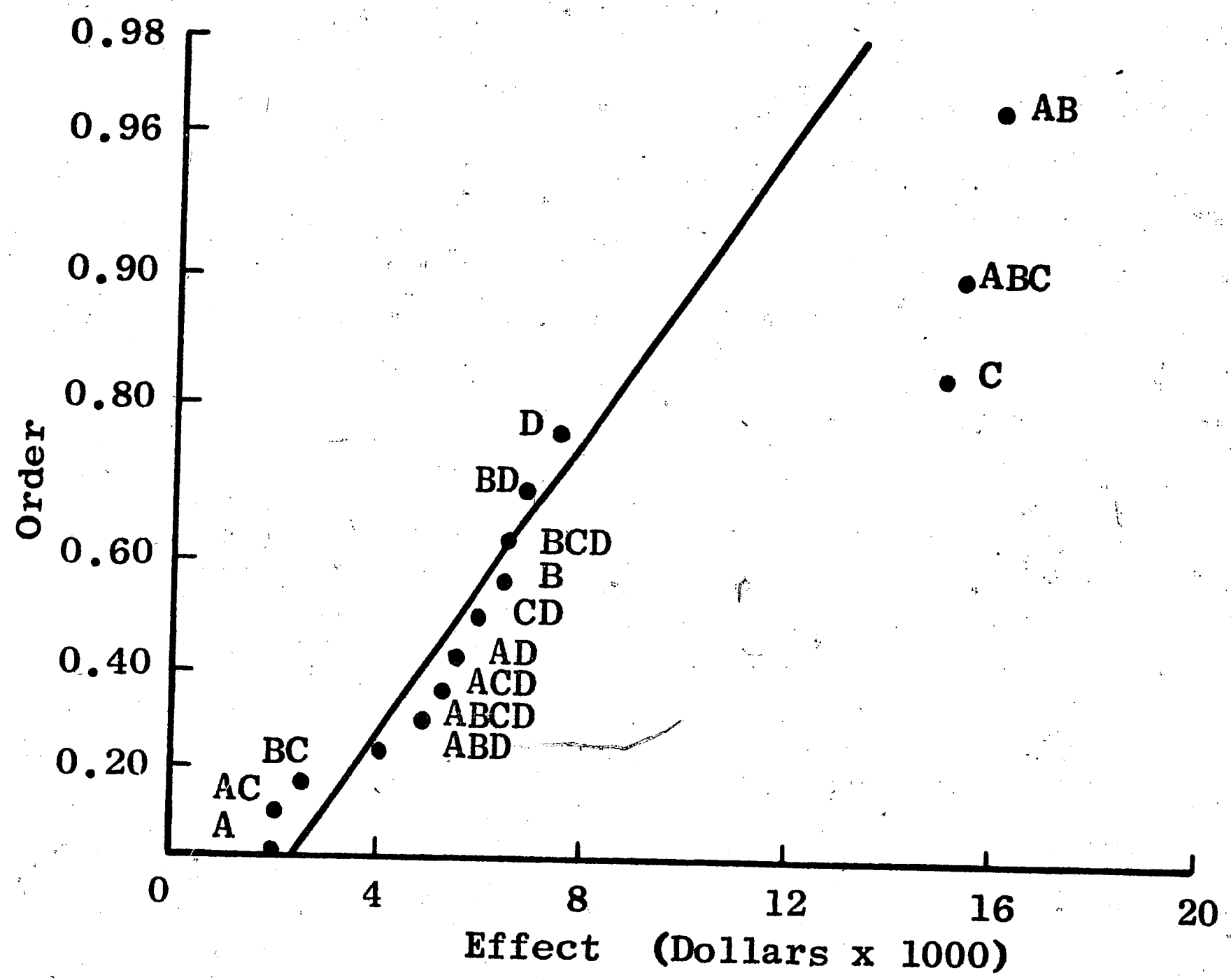


FIGURE 3.5 Half-normal Plot of the Variable Overtime (Quadratic Model)

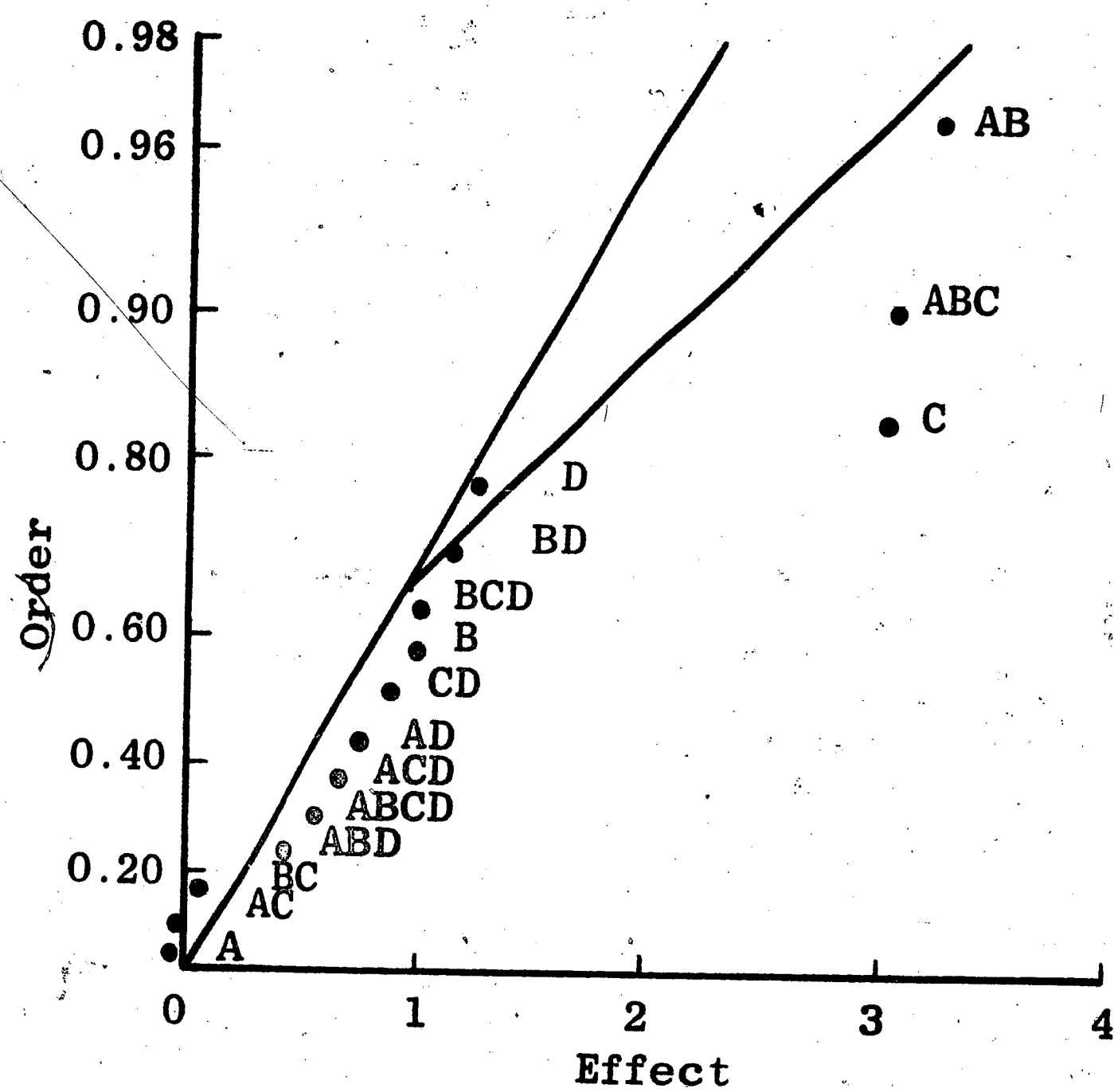


FIGURE 3.6. Standardized Half-normal Plot of Variable Overtime (Quadratic Model)

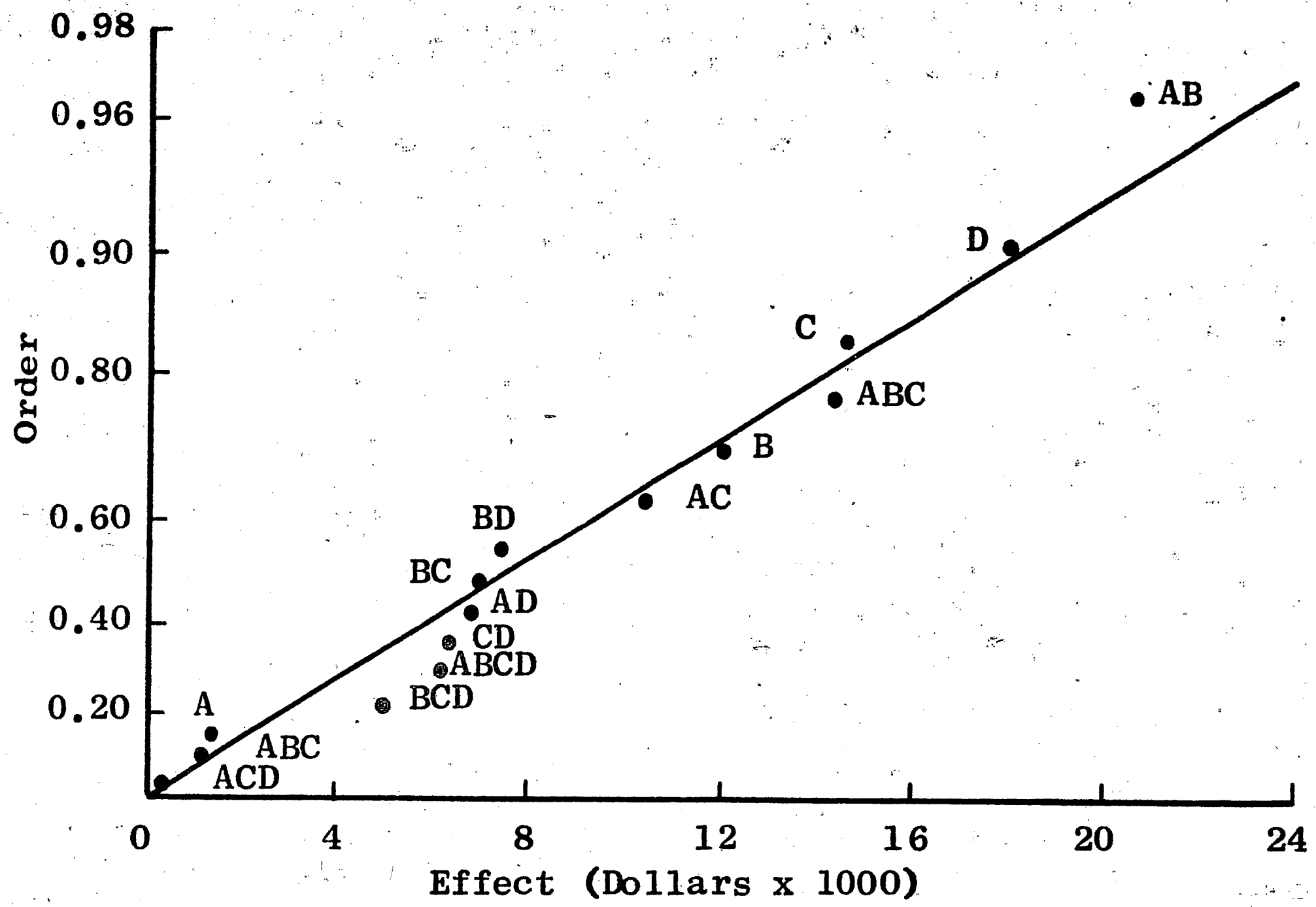


FIGURE 3.7 Half-normal Plot of the Variable Overtime Data Obtained by Replication of the Quadratic Model.

are made to enable the model to adjust to the abrupt beginning of operations. This problem was adjusted for by discarding initial data from consideration and the choosing of starting conditions to reduce initial transients. The two starting conditions specified for each model were (1) starting manpower and (2) inventory. The manpower was based on the starting production capacity of the shop. The estimation of the beginning inventory was difficult since each model determines the optimal inventory based on the shop parameters. The best estimate of the starting inventory was obtained by simulating each model for a period of five years under the standard conditions of "shop X" without trend. The average inventory was 900 hours for the quadratic model and 1100 hours for the linear programming model.

Each model was tested with a starting inventory of 0 and 900 hours. The quadratic model reached identical production, overtime, idletime, inventory and manpower level after the 9th period regardless of the starting inventory. The linear program did not obtain the same production figures; however, the cost of production for the first nine periods was reduced by 4.2% with a starting inventory of 900 hours.

Obtaining a good compromise set of starting conditions is certainly not a trivial task. From the limited analysis 900 hours was selected as the starting inventory level. This level was certainly more efficient than the zero starting conditions. However, the question may be asked as to the use of different reasonable starting conditions for each model. This presumably reduces the initial time required to allow the starting transient to "die out", but the problem



of possible biasing the results was considered.<sup>(16)</sup> If it is desirable to make statements about two alternatives, the models should be tested under as close to identical conditions as possible.

For every combination of the experimental variables, each model was simulated for a period of 48 months. After the first 12 months, cumulative costs were maintained for each of the measurement variables.

The production schedule generated by each model was based on the forecasts for each month of the horizon. Thus another problem to consider is the source of the simulation data. The forecast is a prediction of the demand that will occur during the period. The actual demand cannot be determined until approximately the last day of the month. As previously analyzed<sup>(17)</sup> the forecast data in man hours per hour was normally distributed. The experimental factor trend includes a cyclic variation of the forecast data. Therefore, for each month in the simulation period, a normal distribution with a different mean which moves along the trend and cyclic path but with the same standard deviation was used to generate the forecast. The forecast for each month in the simulation period was determined by multiplying the standard deviation,  $\sigma = 2$  or  $5$ , by a random normal number, ( $\mu = 0$ ,  $\sigma = 1$ ), and adding the value to the mean. To convert the forecasted demand in man hours per hour to man hours per month, multiply by the number of productive hours for the corresponding month. The productive hours per month varies depending on vacations, schedule down time and

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16. Conway, R. W., "Some Tactical Problems in Digital Simulation," Management Science, Vol. 10, No. 1 (October 1963), pp. 47-61.

17. Atkins, H., op. cit., p. 33.

the number of working days per month.

Demand data was generated to vary within an approximate range of 20% of the forecast data. To achieve this variation, a coefficient of variation (C.V. = .05), was selected for the demand distribution. The forecast for period  $t$  of the horizon was used as the mean of the demand distribution for the  $t$  period. The mean was multiplied by the coefficient of variation to determine the standard deviation of the particular demand distribution. To determine the actual value of the demand the standard deviation was multiplied by a random normal number ( $\mu = 0, \sigma = 1$ ) and added to the mean of the demand distribution. Different forecast and demand data was generated for each cell of the  $2^4$  factorial experiment. However, the same data for each cell in the experiment was used with each model.

## CHAPTER IV

### RESULTS

Each model was simulated with every combination of the experimental variables. For each simulation seven measures were obtained to estimate the main and interacting effects. The analysis of variance was used to analyze the quadratic model results and the half-normal plot technique presented by Cuthbert Daniel<sup>(18)</sup> was employed in estimating the extreme effects in the linear program model. Typical inputs for the two models are given in Appendix C. The cumulative costs for the measurement variables of each model are presented in Appendix D. For each cell of the experiment a different set of random numbers was generated, however, the same set of random numbers was used with each model.

Replication of the quadratic model was possible since the calculations per month required approximately 10 seconds on the IBM 1410 computer. The linear program model required approximately 20 minutes per month on the IBM 1410 computer. The IBM 7094 computer was less expensive to perform the simulation of the linear program, however, the budget allowed only one replication of the factorial experiment.

#### Behavior of the Typical Model

The results for the "average" shop are obtained by using the arithmetic average over the 16 runs of the corresponding measure for each cell. These averages are presented in Table 4.1.

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18. Daniel, op. cit., pp. 311-341.

<u>Measurement Variables</u>	<u>L.P. Averages</u>	<u>Q.P. Averages</u>	<u>Difference L.P. - Q.P.</u>
Total Cost	664,779	665,822	-1,043
Overtime	14,366	18,717	-4,351
Idle time	1,241	3,118	-1,877
Change in Work Force	13,601	9,709	3,892
Inventory	12,537	13,829	-1,292
Premium	32,171	32,890	-719
Backorders	3,055	2,866	169

Table 4.1

Over-All Averages of the 16 Experimental Runs

By using total cost as a criterion for comparison, the linear program averaged 0.16% less than the quadratic. This difference is insignificant cost-wise; however, the difference in the utilization of the variables overtime, idle time and change in work force is worth noting. (1) The linear program adjusted manpower to adapt to the changing environment while the quadratic model utilized the variables of overtime and idle time. This variance in the two models reflects the difference in the decision rules related to hiring and firing. (2) The linear program will hire or fire an employee if the savings in overtime or idle time exceed the cost of hiring or firing respectively. (3) The quadratic model requires the savings in overtime or idle time to be equal to or greater than the cost of hiring and firing. (4) Both models base the decision of changing production capacity on the proposed production schedule for the horizon period. Thus, one model



may vary the work force or overtime more than the other model but on the average the costs of production are the same.

The difference in the premium time is \$719. Premium time represents the costs of idletime, inventory, change in work force and the premium charges of overtime (\$1.00 per hour). One-third of the regular overtime changes are included in the premium variable.

The costs associated with overtime, idletime, inventory and change in work force indicate the difference in the models to be \$3628 while the difference in total cost indicates a difference of \$1043. The difference of \$2585 is the additional amount expended by the linear program for regular time production during the 36 month simulation period.

The typical model is a description of the average performance of each model under the various conditions defined by the different combinations of the experimental variables. In particular it is the object of this study to determine which conditions resulted in behavior by the model significantly different from the average. Hence, the average is the standard of comparison by which to evaluate the effects of the various alterations in the model.

#### Main Effects of High Variability

The increase in the variability of demand in the environment was expected to increase the variation in overtime and idletime. These increases would then be reflected in changes in the work force if the length of the subhorizons remained unchanged.

The effects of the change in the variability of the data are given in Table 4.2. The values given represent the deviation of the measurement variables from the respective average model.



<u>Measurement Variables</u>	<u>L.P. Averages</u>	<u>Q.P. Averages</u>	<u>Differences L.P. - Q.P.</u>
Total Cost	1,240	5,628	-4,388
Overtime	12	1,042	-1,030
Idle time	-238	-662	424
Change in Work Force	-1160	750	-1,910
Inventory	632	2,200	-1,568
Premium	-772	2,646	-3,418
Hours Short	282	210	72

Table 4.2

Effects of a Change in the Variability of the Environment

The environmental change of increased market variability significantly increased the premium costs of the quadratic program. The increase in inventory was not expected since the variability tends to shorten the subhorizon planning period. If the subhorizon is shorter, the length of time for carrying inventory is decreased. The comparison of the two models indicates the linear program model performed more efficiently than the quadratic model under the changed parameter.

The results obtained by performing the analysis of variance on the replicated data of the quadratic program indicate the increase in the variability of the market is significant at the 5% level with respect to the variables of total cost and inventory (See Appendix E). The analysis of the measurement variables associated with the linear program did not indicate any main effect from the alteration in the market conditions (See Appendix E).

The backorders for the linear program are 241 hours greater than the quadratic model. Since the cost of backorders is extremely high for shop X, a larger safety stock may be required for the linear program.

Main Effects of Market Trend

The market growth and decline occurred at a 5% rate. The growth and decline were irregular since there were cyclic swings above and below the trend. The effect of a declining market on the cumulative costs are given in Table 4.3.

<u>Measurement Variables</u>	<u>L.P. Averages</u>	<u>Q.P. Averages</u>	<u>Difference L.P. - Q.P.</u>
Total Cost	-578	-1,254	676
Overtime	2,542	3,290	-748
Idle time	172	350	-178
Change in Work Force	-1,090	-724	-366
Inventory	340	-2,222	2,562
Premium Time	266	-212	478
Hours Short	1,120	1,428	-300

Table 4.3

Effects of a Declining Market

Considering the measurement variables total cost and premium time and the differences in the average results of the typical models, both models scheduled the production with approximately the same costs. The differences in the total costs and premium costs associated with each model are \$367 and \$241 respectively.

The decrease in the changes of the work force is related to the

length of the subhorizon period. As the subhorizon is decreased, the changes in the manpower are decreased. This is reflected by both models under the declining market conditions. The quadratic program requires twice the time period as the linear model to justify a change in the work force. The effect of the decrease in manpower adjustments is indicated by the increase in the variables overtime and idletime. The analysis of the experimental data related to the quadratic and linear programs indicate the measurement variables are not significantly affected by the change in the market trend.

Main Effects of Increasing Hiring and Firing Costs

The hiring or firing cost for one man in shop X was approximately \$425. The alternate experimental value selected was \$625. As a result of the increased manpower adjustment cost, it was expected that the total cost of performance should not change significantly; however, the costs of overtime and idletime associated with each model should increase. The changes in the measurement variables associated with each model are given in Table 4.4.

<u>Measurement Variables</u>	<u>L.P. Averages</u>	<u>Q.P. Averages</u>	<u>Differences L.P. - Q.P.</u>
Total Cost	4,380	3,354	1,026
Overtime	2,880	7,510	-4,630
Idletime	-352	-252	-100
Change in Work Force	4,358	1,674	2,634
Inventory	-346	356	-702
Premium Time	4,624	4,294	330
Hours Short	98	190	-92

Table 4.4

Effects of a Change in the Hiring and Firing Costs

The total costs of performance by the two models are similar.

The increase in the manpower reflects the additional cost of hiring and firing and does not reflect an increase in the number of manpower changes. The overtime is increased to adjust for the decrease in the fluctuation of the manpower. The idletime did not increase as predicted. However, the indicated effects are related to the average performance of each model over all combinations of the experimental variables and are not related directly to the performance of each model under the standard shop conditions.

The analysis of the experimental data related to the linear program indicates the measurement variable change in work force is affected by the increase in the cost parameter. The quadratic model variables of overtime and change in work force are significantly affected (5% level) by the increase in the hiring and firing costs.

#### Main Effects of Increasing Inventory Storage Costs

The increase in storage costs was expected to create a positive effect on the measurement variables of overtime and idletime because of a decrease in the length of time for profitable carrying of inventory. The models would then depend upon the variables of overtime and idletime in order to adapt to the fluctuating forecast. At this juncture the linear program has not scheduled any idletime in meeting the production requirements with the three previously discussed variables.

Equation (4) of the quadratic model is:

$$\bar{x}_1 = \max_t \left[ \frac{P_{r@t}}{t} - \frac{t(t-1)\alpha}{2a} \right] \quad t = 1, 2, \dots, N$$



$\alpha$  = storage cost of inventory

$a$  = twice the coefficient of the second order term of the quadratic production cost equation.

Effectively the value of  $a$  which was obtained by simulation is being reduced from 0.5 to 0.25 by the increase in the storage costs. This reduces the value of  $a$  to the lower end of the optimal range as determined by Atkins<sup>(19)</sup>. When the boundaries of the optimum range are exceeded, the value of  $a$  should be re-evaluated for the quadratic model. The effects created as a result of changing the storage costs are given in Table 4.5.

<u>Measurement Variables</u>	<u>L.P. Averages</u>	<u>Q.P. Averages</u>	<u>Differences L.P. - Q.P.</u>
Total Cost	12,862	10,650	2,212
Overtime	1,760	3,794	-2,034
Idle time	2,482	1,898	584
Change in Work Force	3,260	1,230	2,030
Inventory	5,044	4,676	368
Premium Time	11,376	9,088	2,288
Hours Short	502	428	74

Table 4.5

Effects of Increasing Inventory Storage Cost

The results indicate the additional charge for storage increased the cost of every measurement variable in both models. The analysis of the data from the quadratic model indicates the measurement variables total cost, idle time, premium time, overtime and inventory are significantly increased. All variables listed are significant at the

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19. Atkins, op. cit., pp. 86-89.



1% level with the exception of overtime which is significant at the 5% level. The analysis of the linear program data indicates the measurement variables of total cost, premium time, idletime, inventory and change in work force are affected by the increase in storage costs. The increase in the storage costs created extreme effects on the measurement variables of premium time, idletime and inventory.

#### Effects of Various Interactions on the Measurement Variables

To simplify the reference to the interactions, the following letters are employed to represent the higher level of each factor:

- A = increased variability
- B = declining market
- C = increased cost of hiring or firing
- D = increased cost of inventory storage

The measurement variable total cost of the quadratic model is not affected significantly by any of the interactions. The interactions of ABD and CD indicate possible effects on the total cost of the linear program. The interaction ABD is dependent on the level of D since the interaction of AB did not appear significant in any of the measurement variables. ABD is also a significant factor in creating additional use of overtime by the linear model. The additional overtime is in part caused by the increased variability of the forecast and the declining market. Both factors tend to decrease the length of the subhorizon. The increase in the cost of storage decreases the length of time it is profitable to carry inventory. The reduced inventory level in conjunction with the reduced subhorizon forces the model to vary the production rate by the use of overtime to meet the given requirements

rather than smoothing production through the use of inventory and manpower changes. The CD interaction decreases the amount of inventory and the fluctuation in the work force. To adjust for these increased costs the overtime and idletime are increased to meet the production requirements. The CD interaction is a significant factor in creating a larger number of backorders in the linear program. The increase in the backorders is a result of a decreased inventory level.

The measurement variables premium time, backorder and inventory of the quadratic model are sensitive at the 5% level to the interaction AD. The increase in premium time reflects the additional storage costs. The backorders increased since the actual level of the inventory decreased with the increased variability and storage costs.

The quadratic model measurement variables of overtime and backorders are affected by the AB interaction at the 1% and 5% level respectively. Both variables tend to shorten the subhorizon planning period which in turn decreases the inventory level and increases the emphasis on changes in the production rate to meet the given requirements. The decrease in the inventory level increases the backorders. The second order interaction ABC also affects the overtime costs. The additional interaction of an increased cost of hiring further restricts the changes in the work force by increasing the subhorizon period required to adjust the employment. The interactions AB and ABC do not affect the premium time which indicates the measurement variables idletime, inventory and changes in the work force are adjusted to absorb the increased cost of overtime. The results are presented in Appendix E.

### Analysis of the Combined Experimental Data

The results obtained from each  $2^4$  factorial experiment were combined into a  $2^5$  factorial experiment for analysis. The experiment was analyzed by the half-normal plot technique as previously described. The results of the analysis are presented in Appendix E.

The measurement variables total cost, premium time, idletime and inventory are affected by the increase in storage costs. This was expected since both models were affected by the experimental variable D.

The quadratic program affects the measurement variables change in work force and idletime. As previously discussed the work force changes by the quadratic program are significantly less than an idletime is significantly greater than the linear program. The analysis indicates the manpower changes are affected by the increased cost of hiring and firing.

Overtime is possibly affected by the second order interaction ABC and first order interaction AB. These effects are not extreme and should be considered in future investigation of the changes that affect overtime.

In addition to the experimental factor D, the measurement variable inventory is affected by ACD and BD. All three conditions created extreme effects. The results of the analysis related to the number of backorders indicate possible effects from CD and B. Further analysis is necessary before a definite experimental variable can be identified that affects the number of backorders.

### Comparison of the Two Models

The sign test<sup>(20)</sup> was performed on the two models to determine if they differ in average performance. The experimental results obtained from the two models are in pairs since the observations from each model were obtained under similar conditions. The sign test determines whether the median difference between A and B is considered to be zero. If the sign test was not significant, the  $t$  test was performed for paired observations<sup>(21)</sup> to determine whether the two models differ. The results of the tests are presented in Appendix E.

The tests indicate there are no differences between the two models based on the measurement variables of total cost, premium time, inventory and backorders. The difference in the two models is based on the variables overtime, idletime and change in work force. The tests indicate at the 99% confidence level, the costs of overtime and idletime are less for the linear program. The quadratic program with the same 99% confidence performed with less changes in the work force.

The difference in the two models is again directed toward the two different manpower adjustment rules. In the selection of a manpower adjustment criteria, Atkins<sup>(22)</sup> tested the rule of only considering the respective costs when hiring or firing an employee. The results obtained by simulating this rule indicate the premium costs were increased approximately 3.6% and the changes in the employment level were increased 74% in relation to the results obtained by using the

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20. Natrella, M. G., Experimental Statistics, Washington, D.C.: U. S. Government Printing Office, August 1963, Section 16.5.

21. Ibid, Section 3.31.4.

22. Atkins, H., op. cit., p. 81.



present employment policy under the same conditions.

The average data from the experiments, Table 4.1, was modified by these percentages to estimate the results if the manpower adjustment criteria was changed.

The results are:

	<u>Q.P.</u> <u>Rule Change</u>		<u>L.P.</u>	<u>Cost Difference</u> <u>L.P. - Q.P.</u>	
	Before	After			
Inventory	}				
Overtime		23,219	17,181	18,566	1,385
Idle time					
Change in Work Force	9,709	16,889	13,601	<u>-3,288</u>	
Premium time difference				-1,903	

If the manpower adjustment criteria of the quadratic model is modified to consider only the respective costs of hiring or firing, the approximation of the results indicate the premium charges are increased approximately \$1184 in relation to the linear program. This indicates the manpower adjustment rule can be modified to meet the needs of a particular shop without a significant change in the total cost of operation.



## CHAPTER V

### CONCLUSIONS AND RECOMMENDATIONS

Two production smoothing models designed for the same shop conditions have been compared. The results obtained indicate the following conclusions:

1. There is no significant difference in the total cost of performance by the two models.
2. There is no significant difference in the premium costs of the two models. However, because of the difference in the manpower adjustment rules, the linear program adjusts manpower while the quadratic model varies overtime and idletime but maintains a relatively stable work force.
3. Both models are sensitive to a change in inventory storage costs.
4. The change in the storage cost is the only experimental variable to create idletime in the linear program production schedule. This difference in the two models reflects (1) the different manpower adjustment rules for the respective models and (2) the differences in the approximation of the linear premium costs by a quadratic function.
5. The manpower adjustment rules for the respective models can be adjusted to vary the relation of the premium variables.

To minimize the premium costs of production, a work pool from which additional employees are selected should be considered with a production smoothing system especially where more than

one production area of similar work is involved. This would enable the production rate to be fluctuated with less costs.

6. The computer time required to perform the linear program is 120 times more than the quadratic model. (20 minutes for the linear program and 10 seconds for the quadratic to perform the scheduling for one period). The cost difference can be significant as the frequency in calculating the production schedule is increased and the system is applied to the other shops.

#### Recommendations for Future Investigation

1. Simulation of both models based on different industrial conditions to further evaluate the quadratic cost concept.
2. Development of the model presented by Modigliani, Holt, Muth and Simon<sup>(25)</sup> and compare with the linear and quadratic models to determine if the approximation of each cost by a quadratic function improves the production schedule.
3. Development of a nomograph for selecting the best value of a which represents the quadratic approximation of the production costs for the quadratic model, under different shop parameters.

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23. See Bibliography Reference (11).

APPENDICES

APPENDIX A

A. Variability of the Forecasted Demand.

1. Standard deviation = 2
2. Standard deviation = 5

B. Trend of Forecasted Demand.

1. Ascending trend with cyclic variation
2. Descending trend with cyclic variation

C. Cost of Hiring or Firing.

1. Cost of hiring = firing = \$425 per man
2. Cost of hiring = firing = \$625 per man

D. Inventory Storage Cost

1. Storage Cost = 3.90 per man hour per year
2. Storage Cost = 7.80 per man hour per year

Figure 1. The Factors and Levels in the Experiment.

APPENDIX B

Data From Shop X

Average Regular Time	\$2.00/hr.
Average Premium Overtime Rate	1.00/hr.
Average Hiring Cost	425.00/man
Average Firing Cost	425.00/man
Average Inventory Holding Cost	3.90/hr/yr.
Working Hours per Day	7.55/hr/day
Working Days per Month	
January	19 days
February	20 days
March	24 days
April	20 days
May	20 days
June	24 days
July	9 days
August	20 days
September	24 days
October	20 days
November	20 days
December	23 days



APPENDIX C

Typical Inputs to Models

Quadratic Model

00050	Starting Manpower
00425	Cost of Hiring
00425	Cost of Firing
0.00212557	Inventory Holding Cost
001	Experiment Number
.4512793	Random Number Seed
0050.00	Starting Forecast Mean
00.2083	Trend Factor
2.00	Standard Deviation
016	Experiments to be Performed

Linear Program

001	Experiment Number
036	Simulation Period
009	Periods in Horizon
001	Starting Month
009	Number of Cost Factors
000425.000	Hiring Cost
000000.325	Inventory Storage Cost
000002.000	Regular Time
000003.000	Overtime
000000.00	Work Force
0050	Starting Manpower
000002.000	Idletime Cost
000000.000	Idle Overtime

00425.000

Firing Cost

010000.00

Artificial Slack

00007210.

Forecast Data (First of 9 Months)

00007518.

Forecast Data (9th Month)

00007163.

Demand Data (First of 9 Months)

00008767.

Demand Data (9th Month)

00019

Working Days Per Month (January)

00023

" " " " (December)

APPENDIX D

Model Outputs

Table 1 - Linear Program Measurement Variables

<u>Factor Levels</u>	<u>Total Cost</u>	<u>Overtime</u>	<u>Idle time</u>	<u>Manpower Adjustments</u>	<u>Inventory</u>	<u>Back- Orders</u>	<u>Premium Costs</u>
(1)	646794	8389	000	9350	12613	363	24759
a	656757	15912	000	13175	10261	2544	28740
b	661721	8102	000	9350	11731	2178	23781
ab	648150	12586	000	7650	7621	3226	19467
c	661934	2033	000	16875	9461	1101	27013
ac	663940	21916	000	11250	9802	3284	28357
bc	670875	29603	000	17500	8009	6036	35377
abc	656614	9348	000	10625	10625	3700	24366
d	666153	15723	3087	11900	12983	4985	33211
ad	671655	6631	1067	10625	15971	1978	29874
bd	672333	18571	2742	17850	10878	4869	37661
abd	677149	17492	4446	11475	19630	3903	41381
cd	674408	15391	3208	17500	15697	1426	41563
acd	678915	18763	1879	22500	12150	4283	42784
bcd	659057	17168	1845	13125	16399	2362	37091
abcd	670023	12232	1590	16875	16772	2654	39314

TABLE 2

Quadratic Model - Measurement Variables Run 2

<u>Factor Levels</u>	<u>Total Cost</u>	<u>Overtime</u>	<u>Idle time</u>	<u>Manpower Adjustments</u>	<u>Inventory</u>	<u>Back-Orders</u>	<u>Premium Costs</u>
(1)	656502	5295	2606	8500	21567	693	34441
a	658593	12941	2762	10200	12380	1901	29658
b	662834	10194	2871	10200	8392	2121	24861
ab	650874	17737	1765	5100	7426	3696	20179
c	658627	7465	1798	10000	9061	548	23348
ac	671586	27820	372	9375	16801	3169	35822
bc	665415	36583	2693	9375	6200	5533	30464
abc	659547	16529	2487	10000	10100	3717	28097
d	662879	17537	4210	8500	11355	4608	29911
ad	674591	14870	2528	7650	16028	1986	31165
bd	666082	22593	4791	9350	10237	5297	31911
abd	680809	18528	4423	11475	21830	3452	43919
cd	672076	19843	5643	10000	17803	479	40060
acd	676743	30806	3627	11250	14530	3972	39677
bcd	659647	26064	2984	8750	17224	2875	37647
abcd	676354	14678	4336	15625	20337	2044	45192

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TABLE 3

Quadratic Model - Measurement Variables Run No.1

<u>Factor Levels</u>	<u>Total Cost</u>	<u>Overtime</u>	<u>Idle time</u>	<u>Manpower Adjustments</u>	<u>Inventory</u>	<u>Back-Orders</u>	<u>Premium Cost</u>
(1)	658962	10097	2601	11475	9698	1725	26033
a	662541	18041	1273	9975	12843	3441	29907
b	654686	21518	2024	7650	8350	2755	24198
ab	675203	27783	2599	10625	8302	1781	30903
c	661603	9423	2805	14375	16334	1173	36658
ac	660845	16527	2565	13125	8243	3849	29440
bc	649658	22827	1232	10000	9479	2288	28318
abc	645421	9922	3538	10000	10732	2704	27721
d	664996	9079	4762	11900	13033	2773	32725
ad	681516	23491	2809	11050	23933	1888	45635
bd	653982	23801	2875	9350	11107	3333	31269
abd	669795	23777	4565	6375	16329	1783	35495
cd	678762	32132	2239	11250	8835	4110	33058
acd	694361	29244	4490	15000	19723	4048	48969
bcd	672072	40260	5262	11875	9082	7706	39679
abcd	670976	20540	4392	10625	22235	1337	44111

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APPENDIX E

Results of Analysis

<u>Measurement Variables</u>	<u>Quadratic Program*</u>	<u>Linear Program**</u>	<u>Combined Linear** Program and Quadratic Model</u>	<u>Comparison of the Two Models***</u>
Total Cost	D <sup>1</sup> , A	D, ABD, CD	D, ABD, AD	No Difference
Premium Time	D <sup>1</sup> , C, A, AD	D, C, BCD, ABD	D, C, BCD, ABD, BD	No Difference
Manpower Adjustment	C	C, ACD, D, BCD	E(Quadratic Model) C	Quadratic Less Than Linear Program (99% Confident)
Overtime	AB <sup>1</sup> , D, C, ABC	ABC, ABD, AB, BCD	ABC, AB, C, E <sup>2</sup>	Linear Model Less Than Quadratic (99% Confident)
Idle time	D <sup>1</sup>	D	D, E <sup>2</sup>	Linear Model Less Than Quadratic (99% Confident)
Inventory	D <sup>1</sup> , AD, A	D, ACD	D, ACD, BD	No Difference
Backorders	AB, AD	CD, ACD	CD, B, ACD, ABC	No Difference

\*Results of Analysis of Variance of Two Replications.

\*\*Results From Half Normal Plots of the Data

\*\*\*Results Based on a Sign and t Tests.

1. Significant at the 1% level. All others 5% level.

2. Quadratic Model

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