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Diode reverse recovery time

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DIODE REVERSE RECOVERY TIME

by

TED YOUNG NICKEL

A Thesis

Presented to the Graduate Faculty

of Lehigh University

in Candidacy for the Degree of

Master of Science in

Electrical Engineering

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

May 11, 1962

Date

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ABSTRACT

The objective of this paper is to fully describe the reverse recovery parameter of the P-N junction diode. The reverse recovery phenomenon is discussed quantitatively and the mathematical treatments of some previous authors are analyzed and compared.

A circuit is proposed for measuring the parameter in question.

Methods of reducing the reverse recovery time are discussed and an application of the recovery parameter is shown.

II INTRODUCTION

The reverse recovery time of a diode will be defined in terms of the time required for a specified reverse current transient in the diode circuit to decay to a particular value. This transient is important to the designers of fast switching circuits, such as those used in high speed computers, because it represents an upper limit to the switching speed of the diode.

A discussion follows of the ways in which this transient can be treated mathematically. The approximations used in these treatments are also discussed. The various circuit parameters which affect reverse recovery time are shown. A comparison of some previous authors' works is shown in the discussion.

The problems in accurately measuring this phenomena are then explained. A circuit is shown which is the author's attempt at solving these problems. The reasons for adopting a standard circuit to measure this diode parameter are then presented.

Finally, some conclusions about how to reduce this transient time are discussed and a basic application for this parameter is shown.

III ANALYSIS

Consider the familiar voltage-current characteristic curve for a P-N diode as shown in Fig. 1

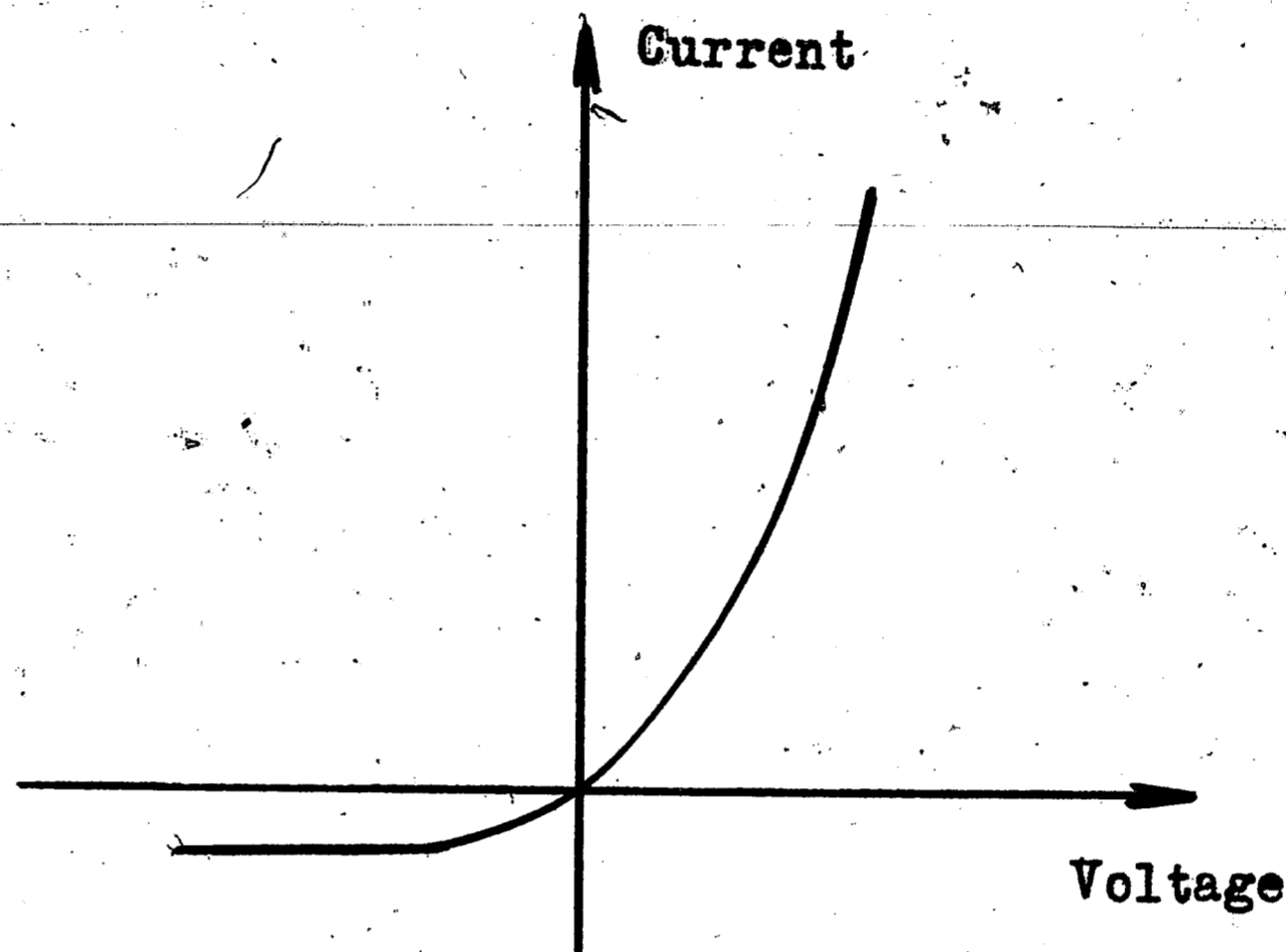


Figure 1.

It is seen that, in the forward conduction region, a large current may result from a small applied voltage. Alternately, it may be said that there is a small impedance in the forward bias direction. Likewise, in the reverse direction, there is a very high impedance and the current through the diode is very small.

However, when the bias on the diode is switched from the forward direction to the reverse direction, the impedance condition presented by the diode to the bias circuit, in general, does not change instantaneously. The usual change of current with time is as given in Fig. 2. There is a time delay before the reverse current decays to its final value, that is, the high reverse impedance does not appear immediately.

Reverse recovery time, t_{RR} , is defined as the time taken for the reverse current in the diode, I_R , to reach 1/10 its initial

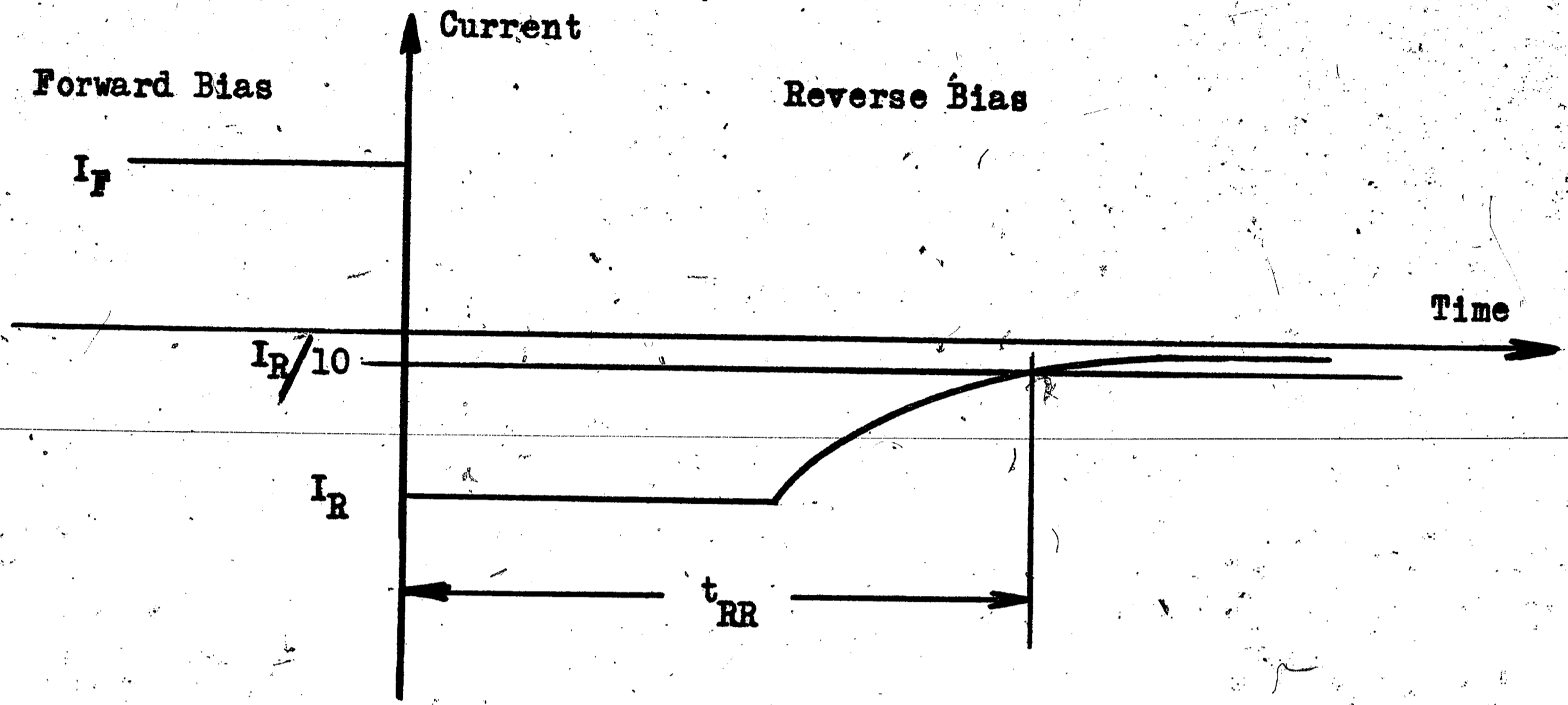


Figure 2.

value. (1) This time is measured from the instant the bias voltage is switched from the forward direction to the reverse direction. The diode must have been conducting in the forward direction for a time much greater than t_{RR} prior to the application of the reverse bias.

A quantitative description of the reverse recovery parameter will be given under the following assumptions (2):

- (1) The diode to be studied is a planar diode, as shown in Fig. 3, and a one dimensional analysis is sufficient.

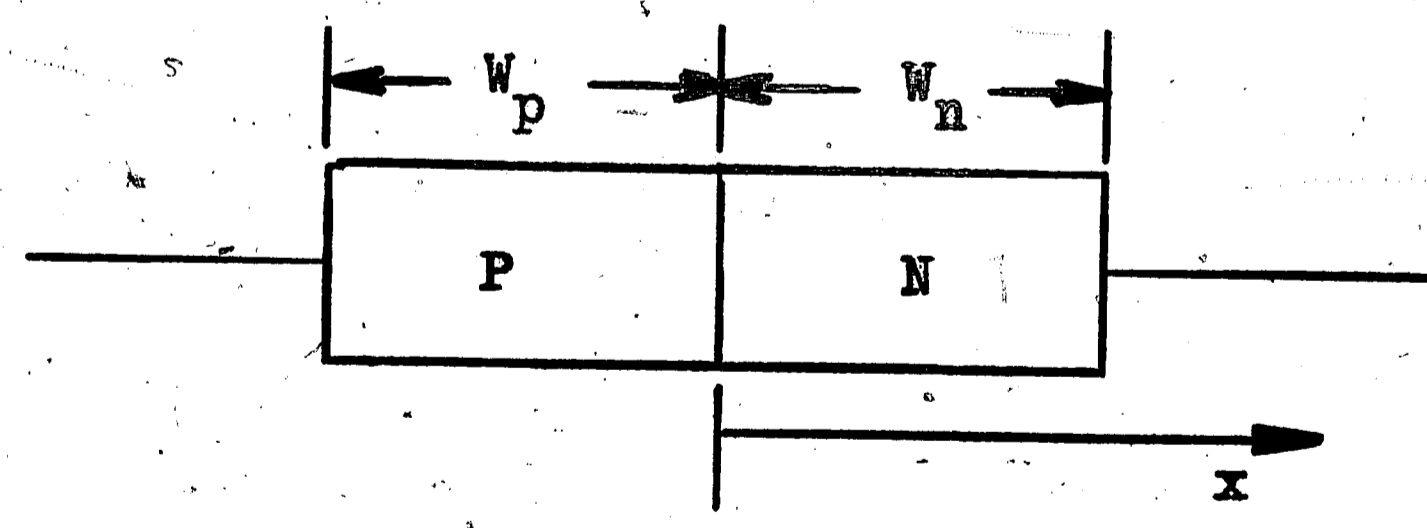


Figure 3

- (2) The length of the N-region, W_N , is much greater than L_p , the diffusion length of the holes in the N-region.
- (3) The transition region, or junction region, is narrow and

abrupt, as in an alloy junction diode.

(4) The conductivity of the P-region, σ_p , is much greater than that of the N-region, σ_n , thus only the minority carriers in the N-region must be examined.

(5) The minority carrier density, $p(x, t)$, is much smaller than the majority carrier density, n_{no} .

(6) The diffusion current is much greater than the drift current.

When there is no bias on the diode the thermal equilibrium carrier density in the diode may be shown as in Fig. 4. (3)

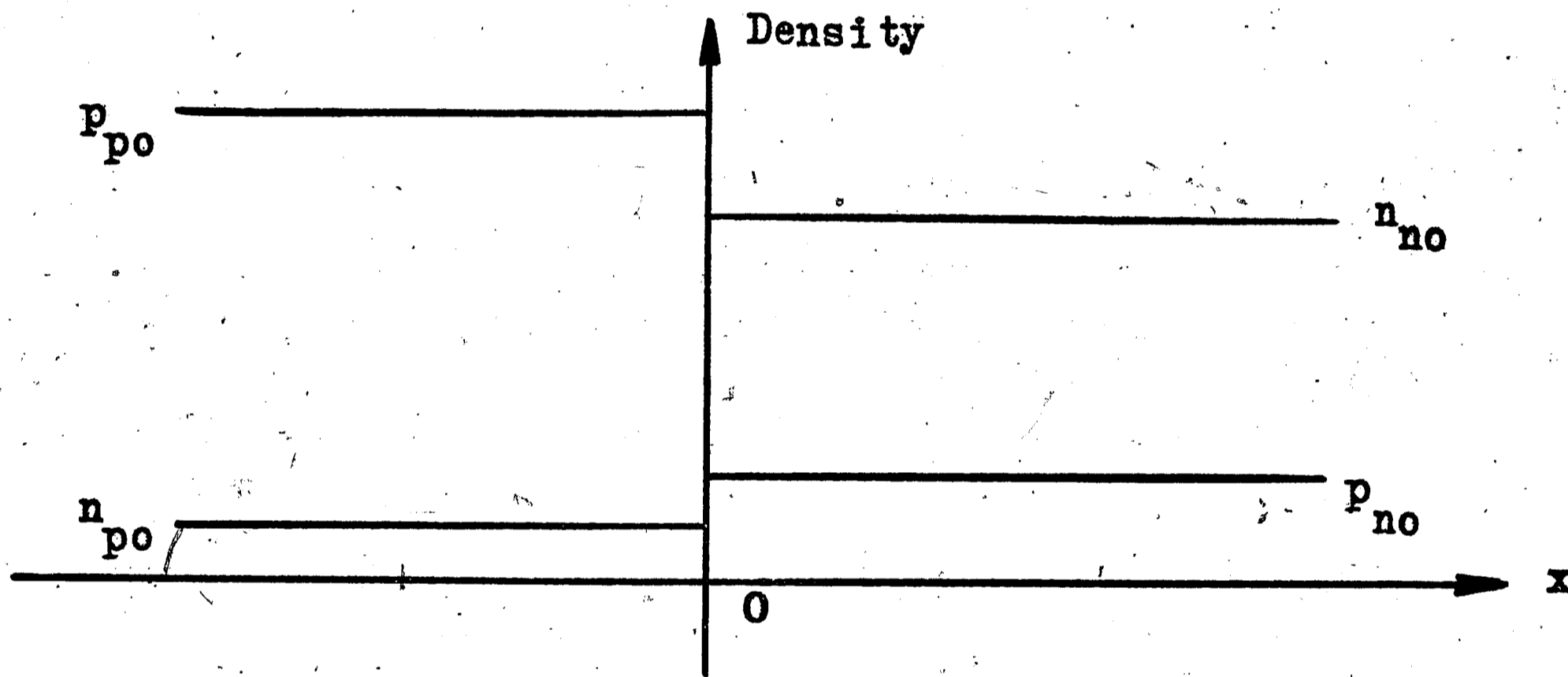


Figure 4.

In Fig. 4, p_{po} and n_{no} are majority carriers, n_{po} and p_{no} are minority carriers.

If the same diode is biased in the forward direction the carrier densities are changed to those in Fig. 5. (4)

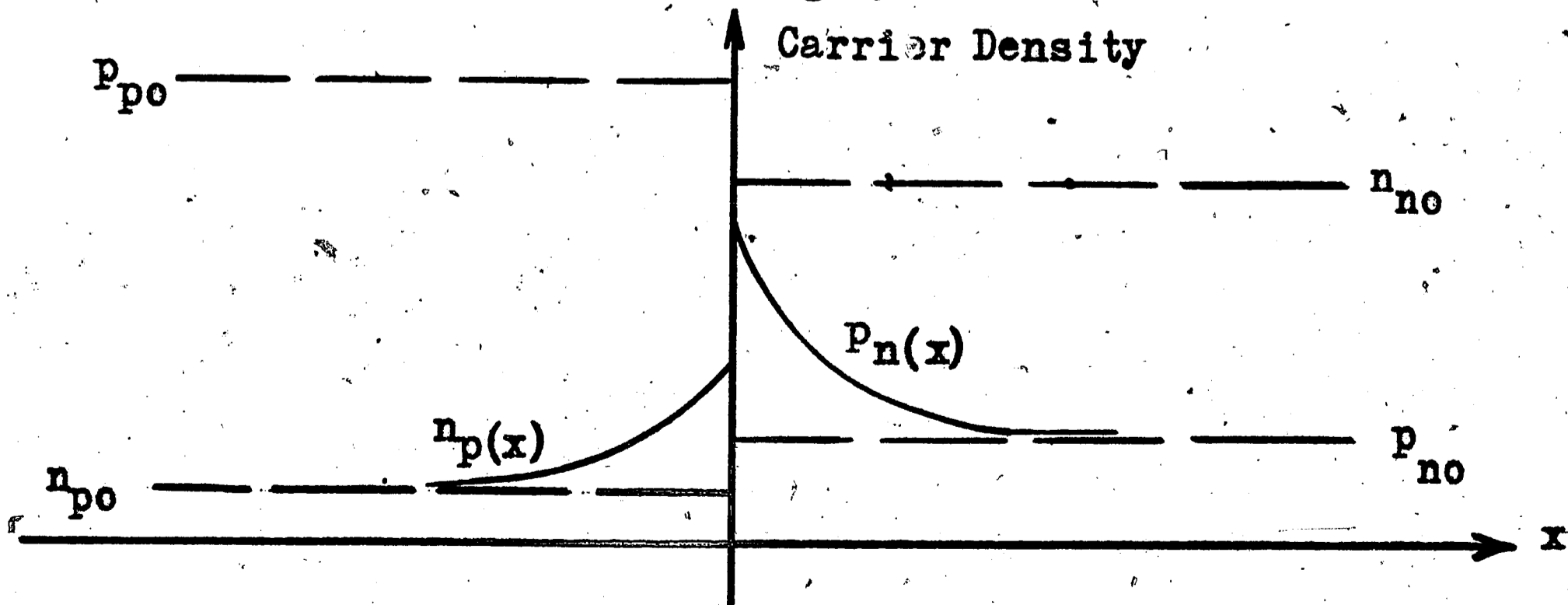


Figure 5.

The densities are given by (5):

$$p_n(x) = p_{no} + p_{no} (e^{qV/kK} - 1) e^{-x/L_p} \quad (1)$$

and:

$$n_p(x) = n_{po} + n_{po} (e^{qV/kK} - 1) e^{-x/L_n} \quad (2)$$

where:

$p_{no} (e^{qV/kK} - 1) e^{-x/L_p}$ is the injected hole density.

p_{no} is the thermal equilibrium density of holes in the N-region.

K is the absolute temperature.

The quantities in the equation for $n_p(x)$ are defined in a similar manner.

Another relationship which will be used frequently is (6):

$$L_p^2 = D_p \tau_p$$

where:

L_p = Diffusion length of holes in the N-region.

τ_p = Mean lifetime of holes in the N-region.

D_p = Diffusion constant for holes.

It is further assumed that space charge neutrality must always be maintained. When the diode is conducting therefore, equal amounts of majority and minority carriers must flow. The current is proportional to the change of the carrier density with distance (7):

$$I \propto \left[\frac{\partial(\text{carrier density})}{\partial x} \right]$$

Since the majority carrier density is much greater than the minority carrier density, equal currents represent a much greater percentage change of minority carriers than of majority carriers. Thus, the change of majority carriers with distance, to produce equal currents, will be small compared with the change necessary in the minority carriers. Therefore any transient behavior is dominated by the minority carriers and the changes in these carriers (8).

Consider the diode to be conducting in the forward direction until, at time $t = 0$, the bias voltage is switched from the forward direction to the reverse direction. The transients caused by this switching of bias voltage will be studied in two different cases. (9) In the first case, it will be assumed that there is no resistance, R , in the circuit to limit the current; in the second, and more practical case, the effects of a current limiting resistance are taken into account.

The initial hole distribution at $t = 0$, due to the forward conduction period, is:

$$p_n(x) = p_{no} + p_{no} (e^{qV/kK} - 1) e^{-x/L_p} \quad (1)$$

and can be plotted as in Fig. 6 for both cases.

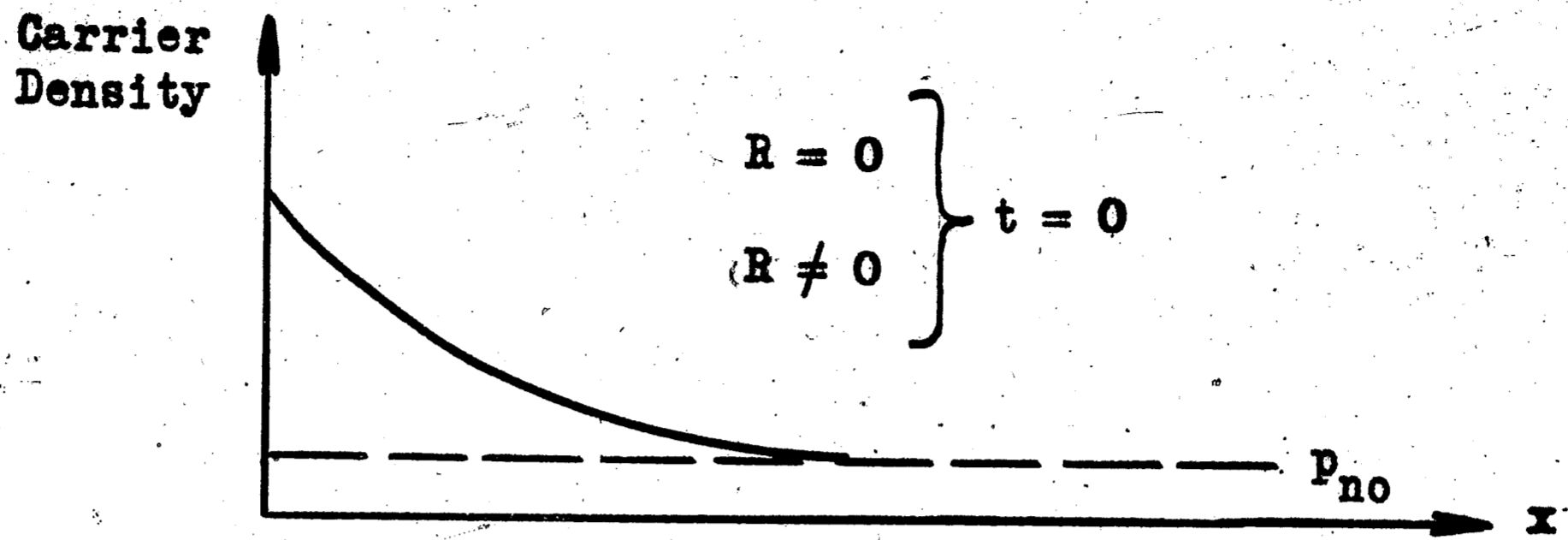


Figure 6.

The following circuit may be used to illustrate the type of switching circuit employed:

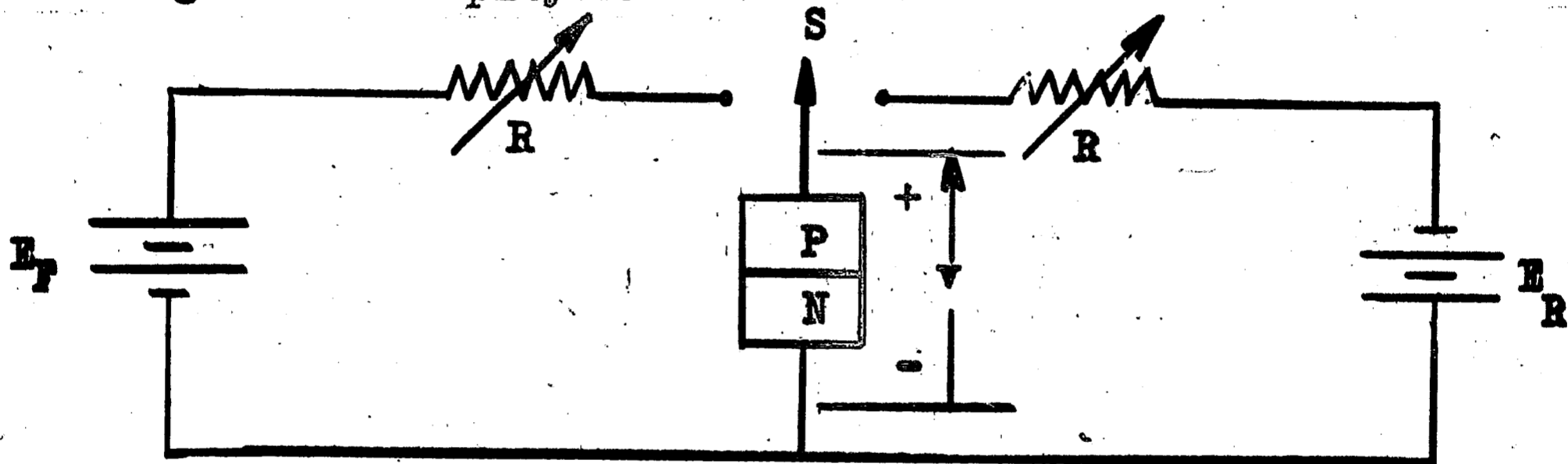


Figure 7.

Upon switching to the reverse bias the minority carriers in the N-region are swept back across the junction to the P-side. There are enough holes present in the N-region so that the initial current is only limited by the external resistance R. This initial period is called the recovery phase. The equations for the voltage and current are: ⁽¹⁰⁾

$$i(t) = - \left(\frac{E_R + v}{R} \right) \quad (4)$$

$$v(t) = \frac{kK}{q} \ln \left(\frac{P_{\text{injected}} + P_{no}}{P_{no}} \right) \quad (5)$$

As the holes are being swept back across the junction, during recovery, there is a constant current equal to $-\frac{E}{R}$ provided E_R is much larger than the positive voltage, v , across the junction. From Eq. 5, the junction voltage, v , is seen to remain positive since a finite time is required to remove sufficient carriers to reduce p_{injected} to zero. The plot of hole density with resistance R in the circuit is:*

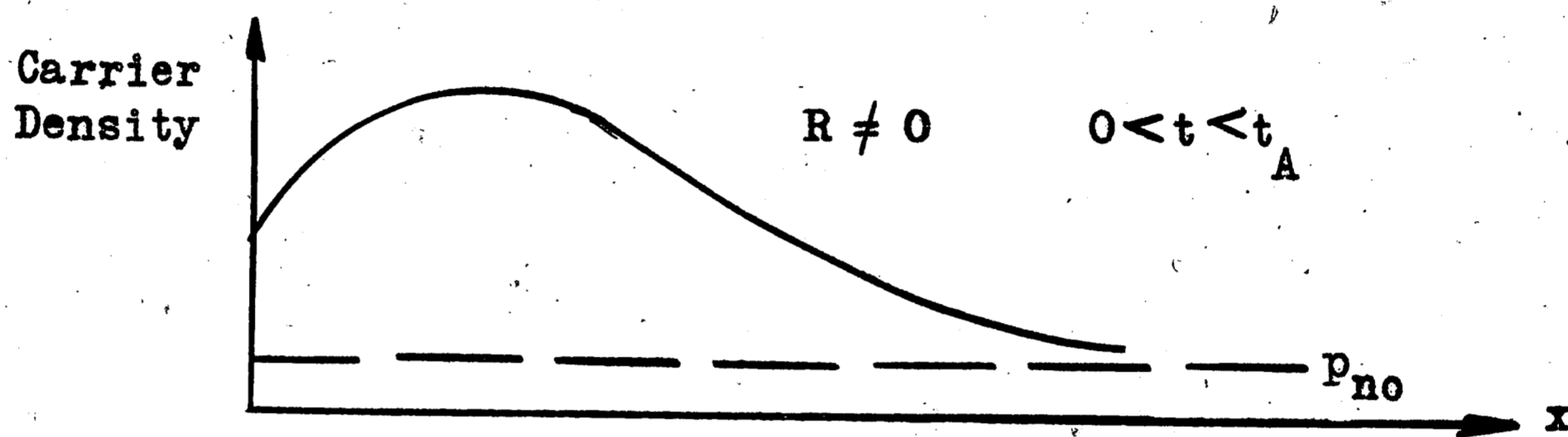


Figure 8

If, on the other hand, $R = 0$, then it would seem that the injected hole density, at the junction, should immediately drop to zero, leaving the plot of hole density, in the N-region, as in Figure 9.*

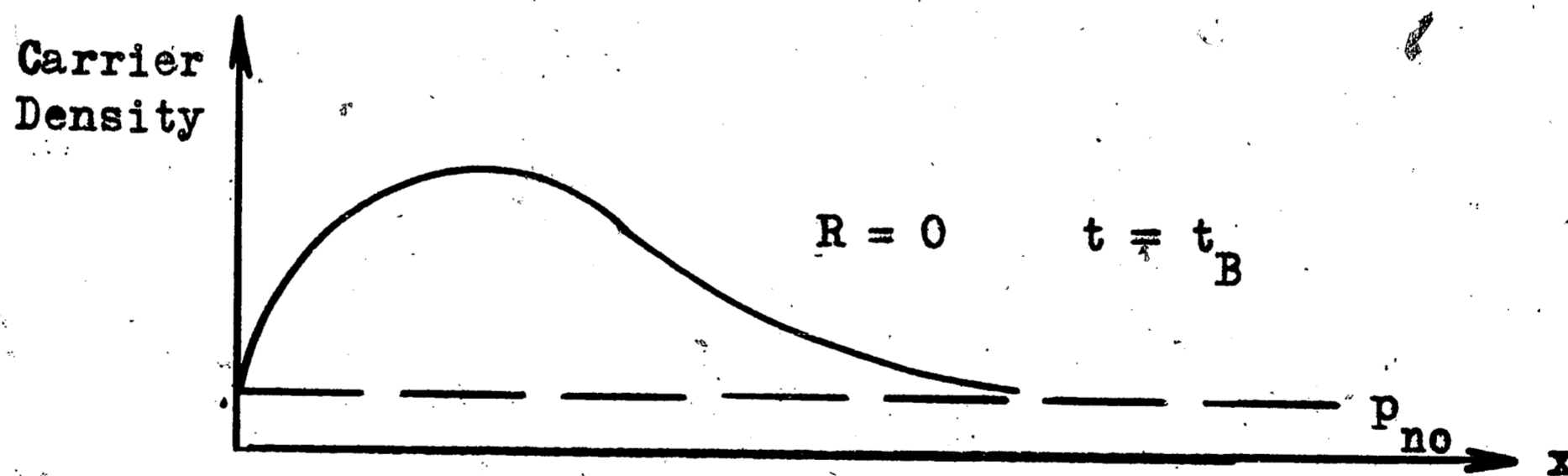


Figure 9

* t_A and t_B are defined as the time when the excess density reaches zero for $R \neq 0$ and $R = 0$, respectively.

However, since the junction exhibits a diffusion capacitance, the voltage across the junction, with $R = 0$, does not immediately drop to zero when the bias is changed. A finite time, t_B , is taken by the hole density at the junction, to drop to zero. Thus the plot in Fig. 9 shows the hole density at $t = t_B$ not at $t = 0$.

When $R \neq 0$ the excess hole density drops to zero at time $t = t_A$. At t_A the total hole density at the junction is p_{no} . At this point in time the junction voltage also drops to zero as seen from Eq. 5. The plot of hole density is shown in Fig. 10.

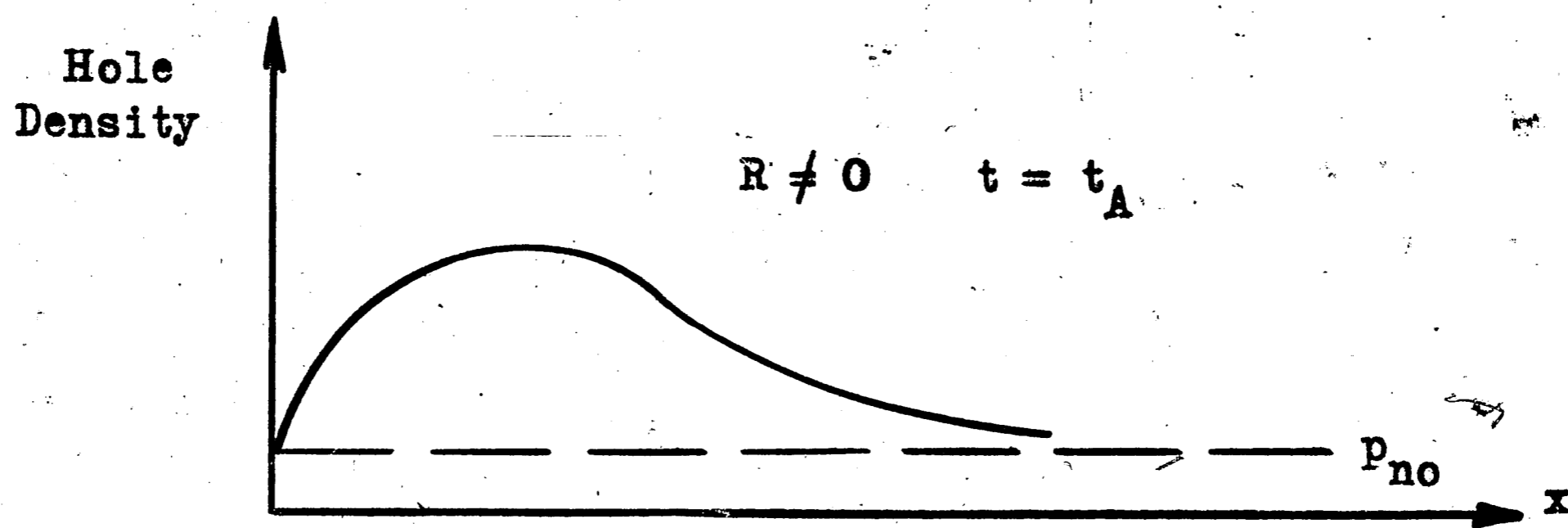


Figure 10.

After the recovery phase, the voltage across the junction goes negative and the reverse phase begins. The hole density in both resistance cases is shown in Fig. 11.

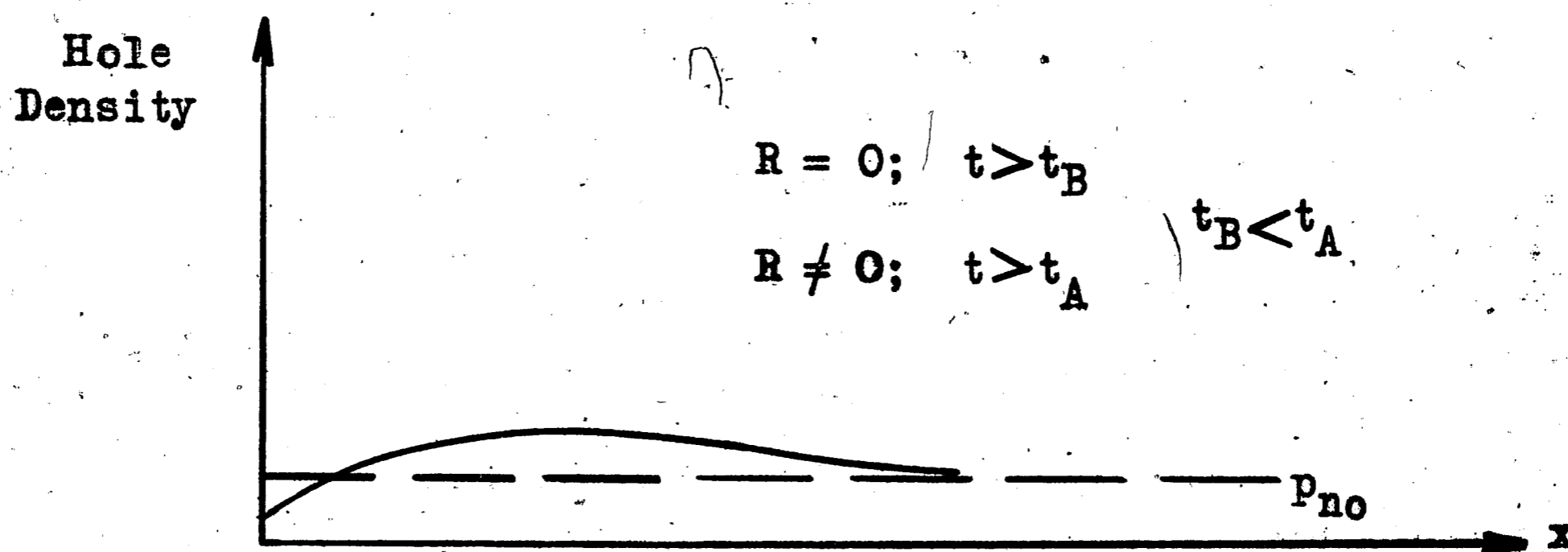


Figure 11.

The current decreases rapidly as the reverse junction voltage rises in magnitude. The steady state condition is reached and the electric field produced by the reverse bias acts to remove minority carriers from the junction. Thus, at some large value of time, the hole density at the junction drops to zero. In both resistance cases the hole density plot is as in Fig. 12.

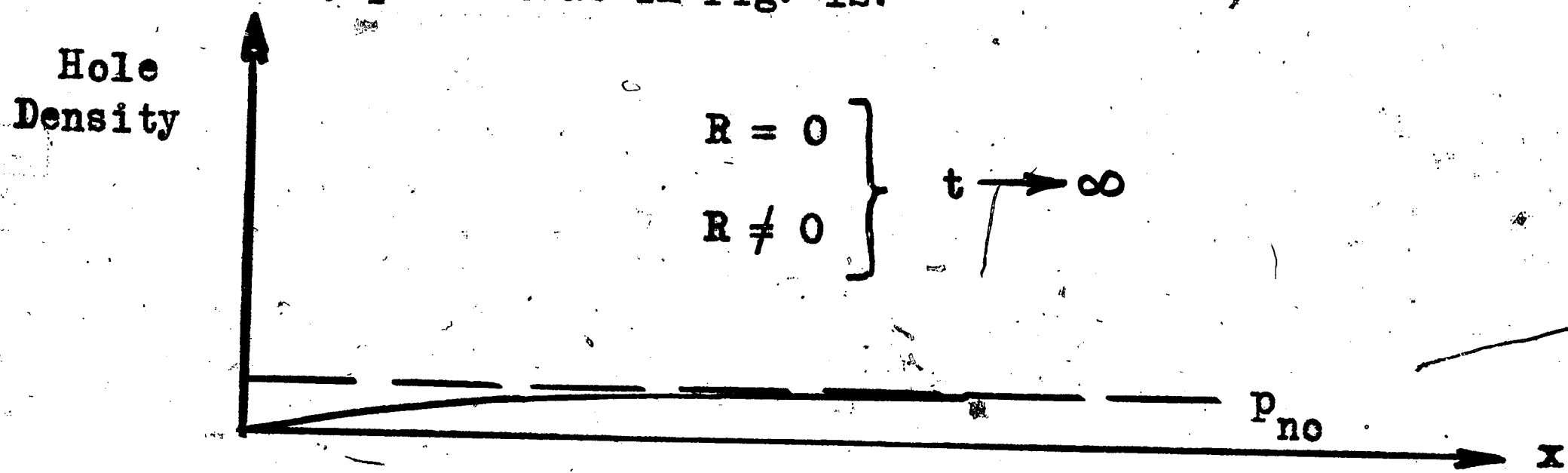


Figure 12.

Fig. 13 shows the voltage and current transients assumed by Ko. (11) The dotted line is for $R = 0$, and the solid line represents the transient when $R \neq 0$.

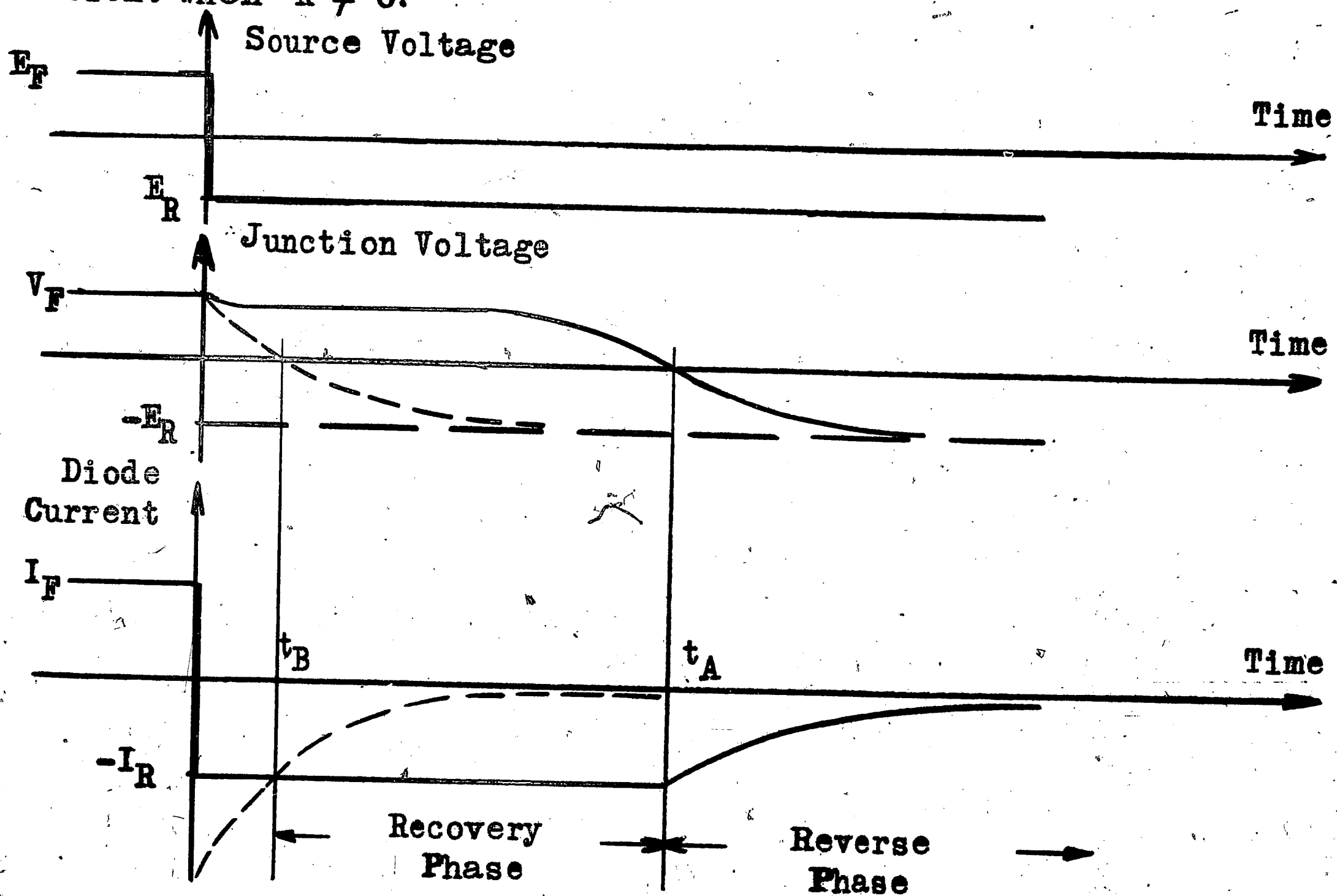


Figure 13.

Consider now the calculation of these transients. Henderson and Tillman⁽¹²⁾ have shown that when $R = 0$, the hole density in the N-region, $p(x, t)$, is given by:

$$p(x, t) = p_{no} (1 - e^{-x/L_p}) + \frac{p_0}{2} \left\{ e^{-x/L_p} \operatorname{erfc} \left[\sqrt{\frac{t}{\tau_p}} - \frac{x}{2\sqrt{D_p t}} \right] - e^{x/L_p} \operatorname{erfc} \left[\sqrt{\frac{t}{\tau_p}} + \frac{x}{2\sqrt{D_p t}} \right] \right\} \quad (6)$$

and the reverse current density at the junction is:

$$-J_R = \frac{-eD_p p_{no}}{L_p} - \frac{eD_p p_0}{L_p} \left[\frac{e^{-t/\tau_p}}{\sqrt{\frac{\pi t}{\tau_p}}} - \operatorname{erfc} \sqrt{\frac{t}{\tau_p}} \right] \quad (7)$$

where p_0 is the hole density at the junction. The derivation of these equations, as well as the boundary conditions used, are shown in Appendix I. It is seen that, as t increases from zero, the current density falls rapidly from minus infinity. The equation for the current in this case, with $R = 0$, will be used later as an approximation for the current in the reverse phase in the circuit with $R \neq 0$. Of course, a time shift must be applied from t_B to t_A .

Consider now the circuit with $R \neq 0$. During the recovery period the current, $-I_R$, is a constant approximately equal to $-E_R/R$. The current density, $-J_R$, is also a constant equal to $-E_R/AR$, A being the junction area. The current can be considered as composed of two parts.⁽¹³⁾ One part is the steady state forward current, I_F , the

other part the transient current in the reverse direction, $-(I_F + I_R)$.

By superposition the total current is thus $I_F - (I_F + I_R) = -I_R$.

The injected hole density during the recovery period is given by Kingston⁽¹⁴⁾ as:

$$p(X, T) = p_0 \left[e^{-X} - \frac{J_F + J_R}{2J_F} \left\{ e^{-X} \operatorname{erfc}\left(\frac{X}{2\sqrt{T}} - \sqrt{T}\right) - e^X \operatorname{erfc}\left(\frac{X}{2\sqrt{T}} + \sqrt{T}\right) \right\} \right] \quad (8)$$

where: $X = \frac{x}{L_p}$; $T = t/\tau_p$.

Kingston's derivation is shown in Appendix II. Ko⁽¹⁵⁾ has derived the same quantity (see Appendix III) and obtained:

$$p(X, T) = \frac{J_F L_p}{qD_p} \left[S(X, T + T_F) - \left(\frac{J_F + J_R}{J_F} \right) S(X, T) \right] \quad (9)$$

where:

$$S(X, T) = \frac{1}{2} \left[e^{-X} \operatorname{erfc}\left(\frac{X}{2\sqrt{T}} - \sqrt{T}\right) - e^X \operatorname{erfc}\left(\frac{X}{2\sqrt{T}} + \sqrt{T}\right) \right] \quad (10)$$

Appendix III also contains a proof that Eqs. (8) and (9) are equal.

Henderson and Tillman⁽⁴⁾ have adopted Kingston's solution for the hole density during this period.

During this recovery phase the injected hole density at the junction, p_0 , is related to the junction voltage, v , in the following manner:

$$p_0 = p_{no} \left(e^{\frac{qv}{kK}} - 1 \right) \quad (11)$$

Combining Eqs. (11) and (9) the expression for the instantaneous junction is:

$$v(T) = \frac{kK}{q} \ln \left[1 + \frac{I_F}{I_S} - \frac{I_F + I_R}{I_S} \operatorname{erf} \sqrt{T} \right] \quad (12)$$

where I_S is the reverse saturation current. (See Appendix III). Eq. 12, due to Ko,⁽¹⁶⁾ is in much better agreement with the experimental circuits than the expressions derived by Kingston and Henderson and Tillman. In particular, at $t = t_A$, the voltage calculated from Eq. (12) drops to zero while the voltage expression of the other authors approaches minus infinity, primarily due to their neglect of p_{no} . Infinite voltage is a violation of Kirchoff's second law.

The time T_A , (t_A/τ_p), can now be calculated from Eqs. (8) and (9). At $T = T_A$ the injected hole density, $p(X, T)$, has dropped to zero, consequently $p(0, T_A) = 0$ at the junction. Thus from Eq. (8):

$$0 = p_{no} \left[1 - \frac{J_F + J_R}{2J_F} \left\{ \operatorname{erfc}(-\sqrt{T_A}) - \operatorname{erfc}(\sqrt{T_A}) \right\} \right] \quad (13)$$

and therefore:

$$\operatorname{erf} \sqrt{T_A} = \frac{I_F}{I_R + I_F} \quad (14)$$

This is an important result showing the method of calculating T_A , the end of the recovery phase.

During the reverse phase, the exact solution for the current, which Henderson and Tillman⁽¹⁷⁾ have shown to be an infinite series of

arctan functions, is cumbersome and not simple enough to be of practical value. Thus, approximations must be made in order to obtain any useful information from the derivations. Kingston has assumed that: (18)

$$\left[\frac{\partial p}{\partial x} \Big|_{x=0} \right]_{R \neq 0}^{T \geq T_A} = \left[\frac{\partial p}{\partial x} \Big|_{x=0} \right]_{R = 0}^{T \geq T_B} \quad (15)$$

That is, when $R \neq 0$, the reverse phase of the current, after T_A , is equal to the current in the circuit with $R = 0$ after time T_B . The time must be shifted ($T_A - T_B$) since $T_B < T_A$. This assumption of Kingston is a useful one because it gives an estimate of current decay during the reverse phase. The similarity between the current transient after T_B , with $R = 0$, and the transient after T_A , with $R \neq 0$, is the basis for this approximation.

The equation Kingston (19) derived for the hole density transient with $R = 0$ is :

$$p(x, T) = p_0 e^{-x} - \frac{p_0}{2} \left[e^{-x} \operatorname{erfc} \left(\frac{x}{2\sqrt{T}} - \sqrt{T} \right) + e^x \operatorname{erfc} \left(\frac{x}{2\sqrt{T}} + \sqrt{T} \right) \right] \quad (16)$$

and the corresponding current equation is:

$$-I_R' = I_F - I_F \left[\operatorname{erf} \sqrt{T} + \frac{e^{-T}}{\sqrt{\pi T}} \right] \quad (17)$$

(See Appendix II).

Equation (16), derived by Kingston, equals Eq. (6) of Henderson and Tillman at $X = 0$ only, since Kingston ignored p_{no} , the thermal equilibrium density, in his calculations. However, Kingston's solution is still reasonable. As seen from Fig. 13, the only part of Eq. (17) which is of use in approximating the current in the reverse phase is that which occurs after $t = t_B$. Again the curve must be shifted in time by an amount $(t_A - t_B)$.

Still another approach was adopted by Ko. He obtained an integral solution and then made an approximation for the integral. The following is the equation obtained for the current in the reverse phase:

$$-i(T') = I_F \left[\frac{e^{-T'}}{\sqrt{\pi T'}} - \operatorname{erfc} \sqrt{T'} \right] \quad (18)$$

where $T' = T - T_A$ is used to shift the time reference to the end of the recovery phase. (Eq. (18) is derived in Appendix IV.) From the definition of $\operatorname{erfc} \sqrt{X} = 1 - \operatorname{erf} \sqrt{X}$ it is seen that Eqs. (17) and (18) are identical except for the arbitrary shifting of the time base. The approximation seems a reasonable attempt at arriving at a useful equation for the current during the reverse phase.

As seen from the previous calculations, one of the most influential parameters affecting the recovery time is the lifetime of the minority carriers, τ_p . Consider again Eq. (14).

$$\operatorname{erf} \sqrt{\frac{t_A}{\tau_p}} = \frac{I_F}{I_F + I_R} \quad (14)$$

From Carslaw and Jaeger: ⁽²⁰⁾

$$\operatorname{erf} \sqrt{y} = \frac{2}{\sqrt{\pi}} \sqrt{y} \left[1 - \frac{y}{3} \dots \right] \quad (19)$$

For small y :

$$\operatorname{erf} \sqrt{y} \approx \frac{2\sqrt{y}}{\sqrt{\pi}} \quad (20)$$

Thus:

$$\operatorname{erf} \sqrt{\frac{t_A}{\tau_p}} = 2 \sqrt{\frac{t_A}{\pi \tau_p}} \quad (21)$$

Substituting Eq. (21) into Eq. (14) and squaring:

$$t_A = \left(\frac{I_F}{I_R + I_F} \right)^2 \frac{4 \tau_p}{\pi} \quad (22)$$

The duration of the recovery phase of the reverse current transient is directly controlled by the lifetime of the holes, and increases with increasing lifetime. Since lifetime is functionally related to temperature by: ⁽²¹⁾

$$\tau_p = \tau_0 + A e^{-E_t/kK} \quad (23)$$

where τ_0 , A , E_t , and k are constants and K is absolute temperature, the recovery time also depends on the temperature of the diode.

IV TESTING FOR REVERSE RECOVERY TIME

Figure 7 illustrates the basic type of circuit used for the measurement of reverse recovery time. That is, after a period of time, $t_F \gg t_{RR}$, during which the diode has been conducting in the forward direction, the bias on the diode is switched to the reverse direction so that the diode current is also reversed. A measurement must now be made of the length of time, t_{RR} , taken by the current to drop to 1/10 its initial value at the instant the bias was switched.

The particular type of diode for which a reverse recovery test set was constructed was the Western Electric 426 series. The test specifications for this diode require that the recovery time be less than 100 nanoseconds when $|I_R| = |I_F| = 10$ milliamperes.*

A mechanical switch will not function satisfactorily in a circuit in which such small time measurements are to be made, and therefore, an electronic switch in the form of a pulse generator must be used. The rise time of the pulse generator must be about 1 nanosecond for best results.

The circuit of Fig. 14 is an example of one which could be used for measuring t_{RR} . In this particular circuit the forward bias is supplied by the battery, E_F , and the reverse bias by a pulse of magnitude $2E_F$ from the pulse generator. Thus, when a pulse is applied,

* The test set design was done by the author while employed by the Western Electric Company.

the voltage between points A and B is of magnitude E_F and of reverse polarity. The current transient can be observed by placing a fast rise oscilloscope across the output.

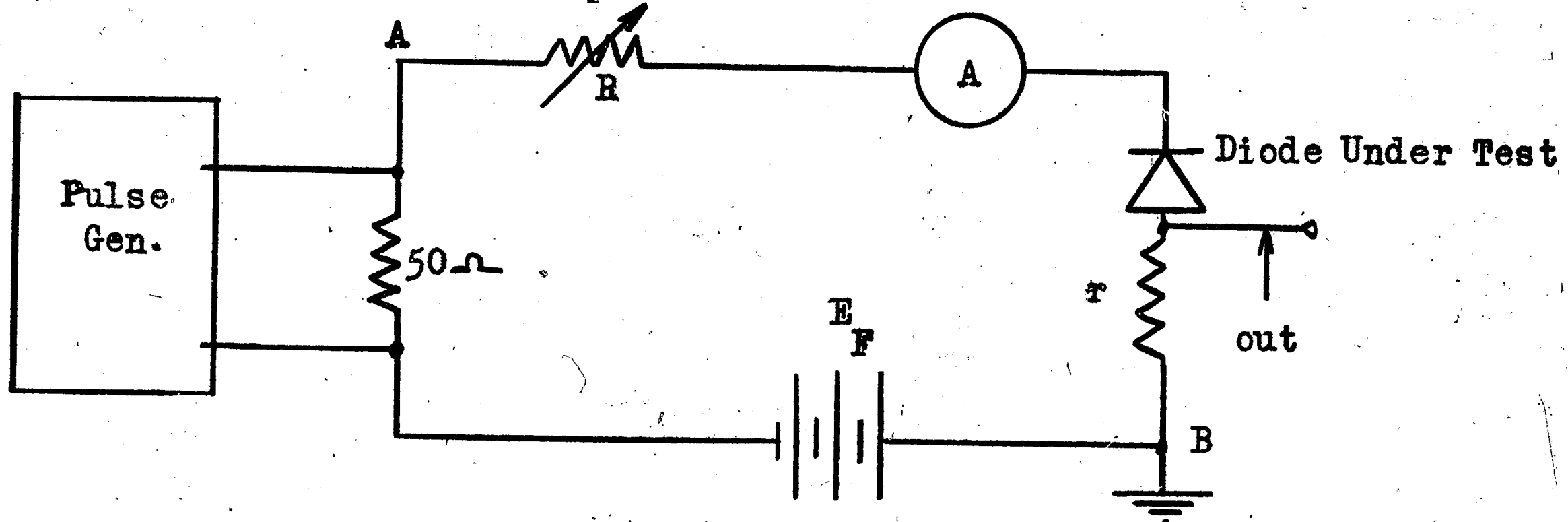


Figure 14.

There are three objections to this circuit. The first is the fact that the actual measurement of t_{RR} is done by a human being, observing the oscilloscope pattern, and is subject to human error. The reverse phase of the current transient decays in such a manner as to make the determination of $1/10 I_R$ difficult. Second, the voltage comparison must be made on an oscilloscope since the pulse amplitude cannot be measured readily on a voltmeter. Finally, the forward impedance, as seen by the battery E_F , is 50 ohms greater than the initial reverse impedance seen by the pulse generator. The 50 ohm load on the pulse generator does not enter into the measurement of the pulse generator output, but does affect the forward bias current. Thus the current immediately after switching is slightly greater than the current through the diode in the forward direction. Although the $|I_F| = |I_R|$ condition could be obtained by lowering the pulse output slightly, an alternate method was sought which would permit the battery to be used to standardize

the pulse amplitude.

Ignoring for the moment the standardization problem, the circuit of Fig. 15 provides a method for reducing the possibility of error by eliminating the oscilloscope.* The essential feature of this circuit is the use of the sample-and-hold circuit comprised of the diode D, the capacitor C, and the vacuum-tube.

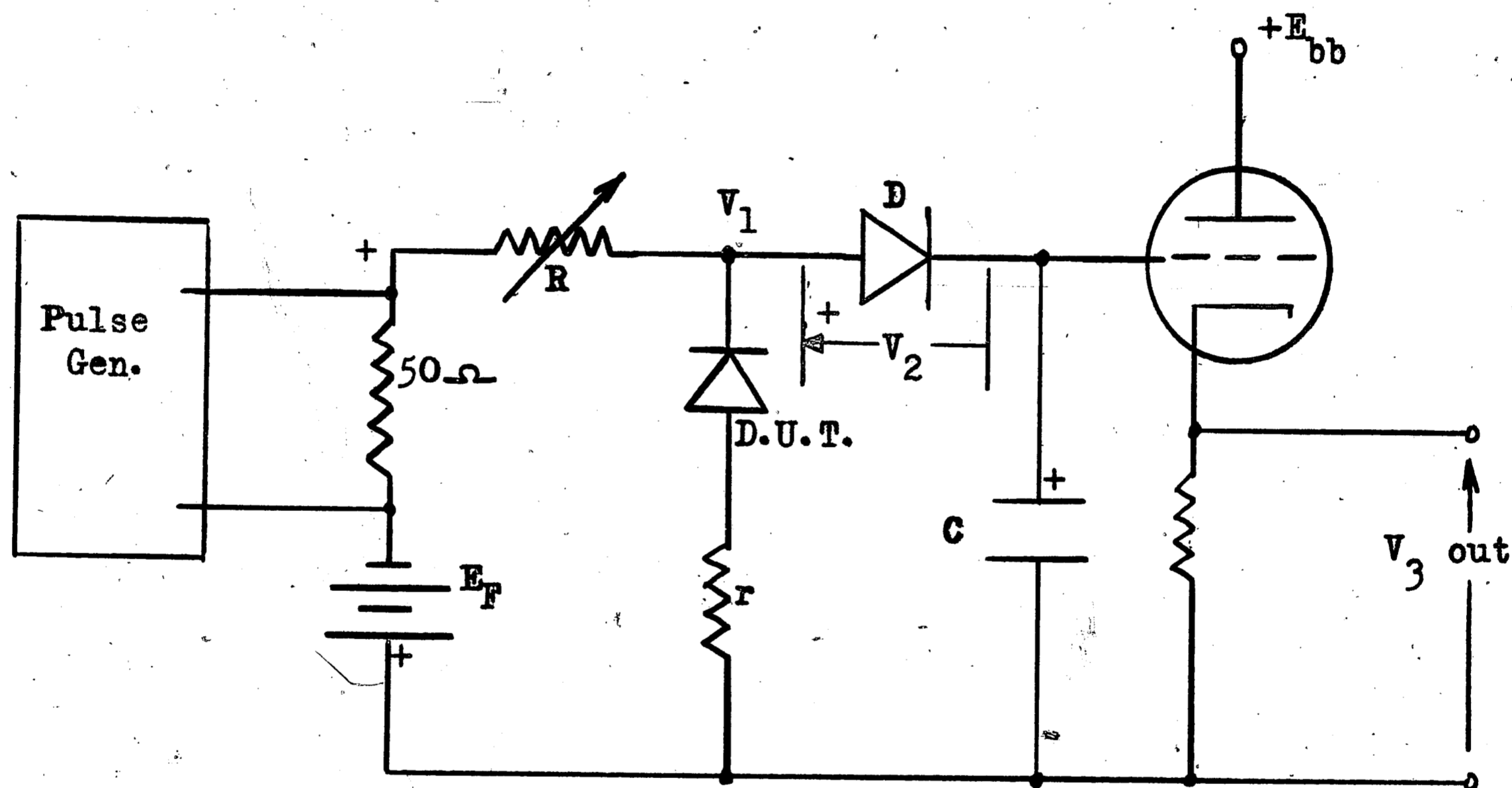


Figure 15.

In this circuit the reverse recovery time, t_{RR} , is obtained by measuring the voltage transient, V_1 in Fig. 15, rather than the current transient. The same time, t_{RR} , is required both for the current to decay from I_R to $1/10 I_R$ and for the voltage, V_1 , to rise to a value $0.9V_{1 \text{ Final}}$.

* This circuit is the author's improvement upon one suggested by Western Electric. These improvements involve changes necessary for accurate measurement.

This relationship is shown in Fig. 16

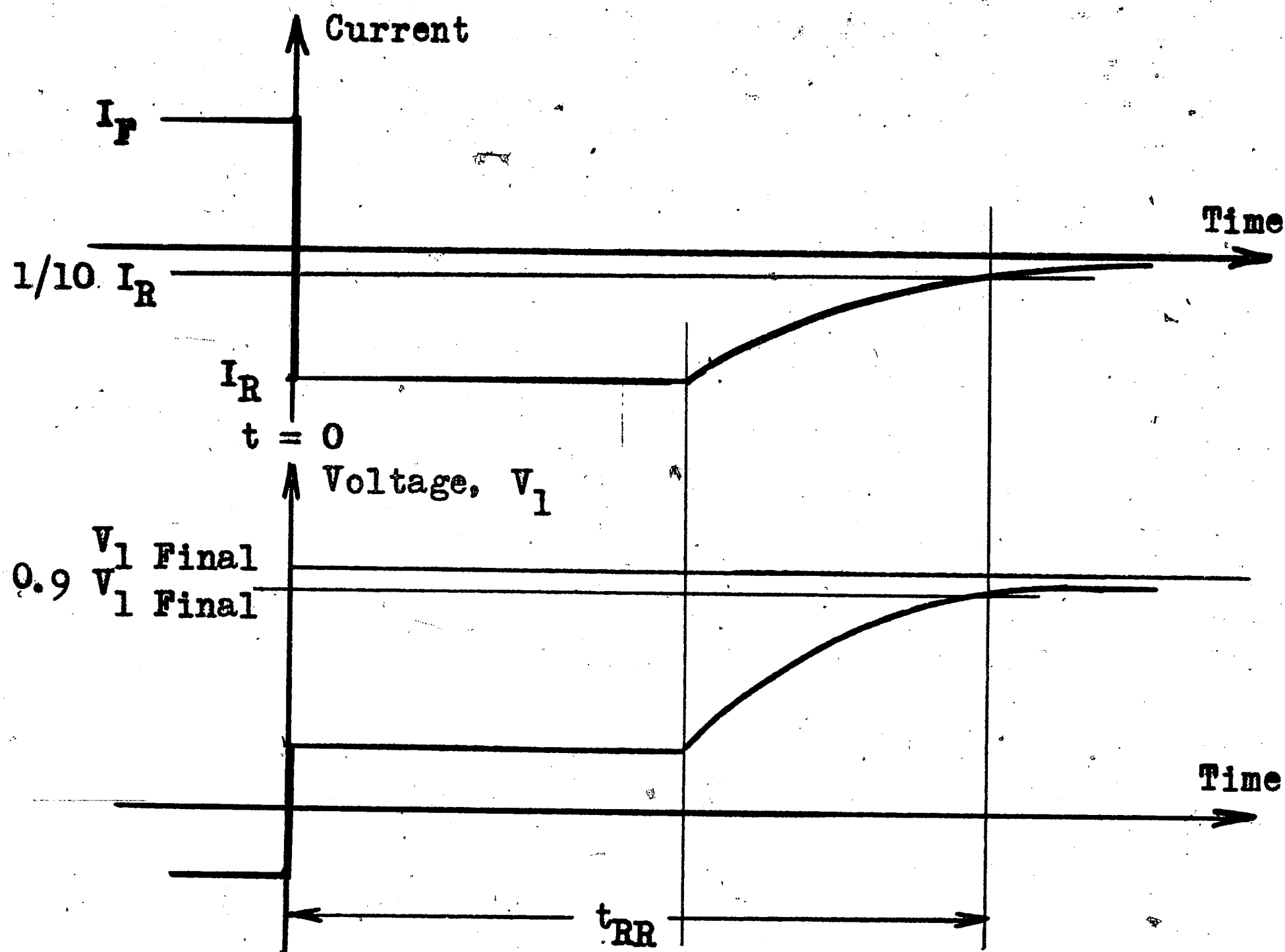


Figure 16.

This voltage, V_1 , minus a small drop, V_2 , across diode D, will be impressed upon C. Thus, C will charge to a value $(V_1 - V_2)$ volts. Since diode D was chosen for its low reverse saturation current, and the input impedance to the vacuum-tube is extremely high, capacitor C will store this voltage during the off period of the pulse generator. As explained below, the stored voltage can be used as a measure of the reverse recovery time of the diode. The vacuum-tube serves as a read-out device for the stored charge.

The voltage, V_1 , is used in the following manner as a means of measuring the reverse recovery time of the diode. A very long pulse, much longer than t_{RR} , is applied to the circuit. The voltage is as shown in Fig. 16 and this causes a change in the output of the cathode

follower of value ΔV_{final} . Then the pulse is shortened to a value of 100 nanoseconds, the maximum allowable value of t_{RR} . Since the pulse is shortened the voltage V_1 may not reach its final value as shown in Fig. 17.

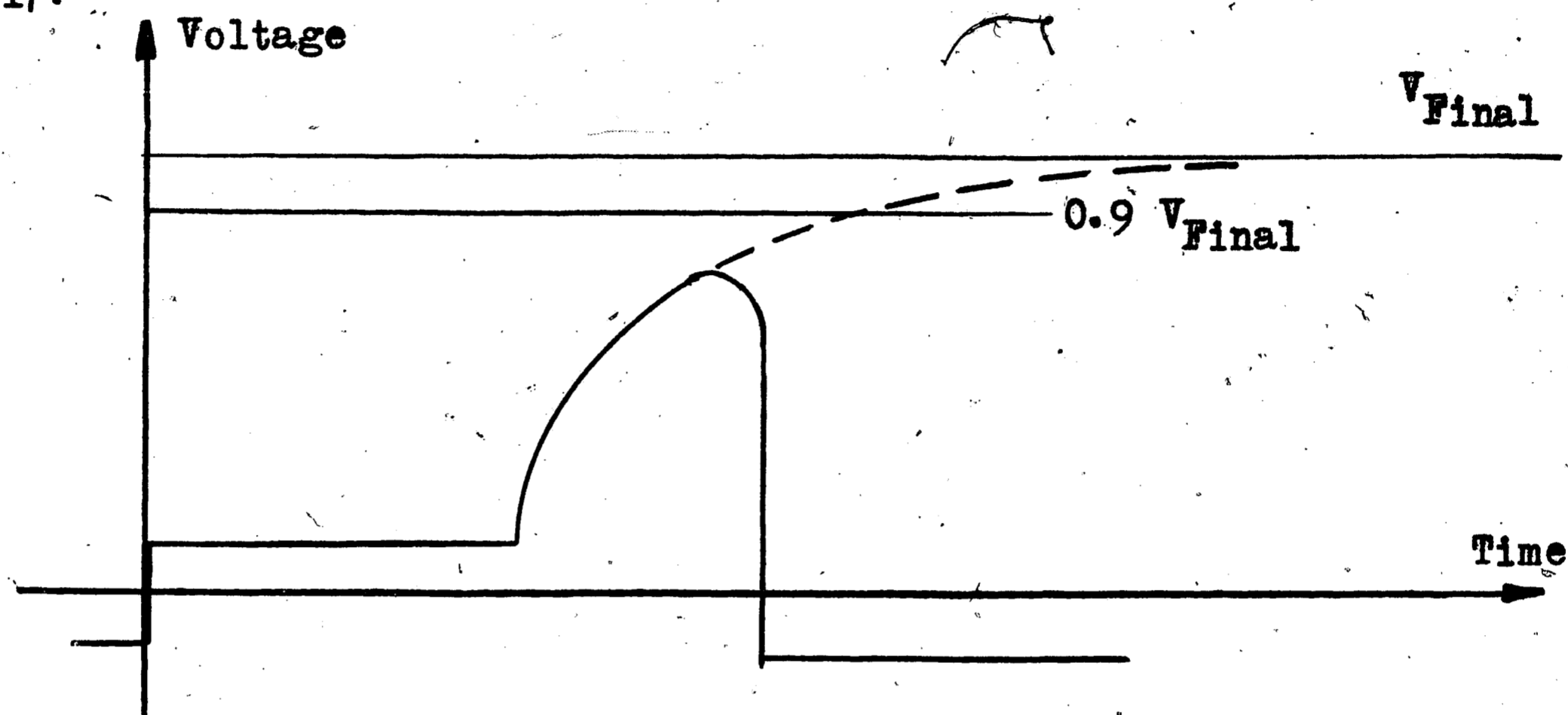


Figure 17.

If the new change in the output is $0.9\Delta V_{\text{Final}}$, or greater, the reverse recovery time of the diode is equal to, or shorter than, 100 nanoseconds and the diode is acceptable. Between pulses the output of the cathode follower is a constant since the voltage on capacitor C decays slowly.

An accurate method is needed to determine if the output voltage rises to $0.9\Delta V_{\text{final}}$ with the required pulse width. One method of measuring this change in output voltage is by use of a trigger circuit. A Schmidt trigger circuit was tested and found to be unacceptable due to the low input impedance and the wide dead band, both characteristic of the Schmidt circuit. However, the required system can be obtained by inserting a second cathode follower in the circuit, the output of which is fed into a D.C. amplifier. The dead band of the D.C. amplifier can be made extremely small.

The complete circuit for measuring the voltage, V_1 , is shown in Fig. 18. This voltage is a measure of the reverse recovery time, t_{RR} .

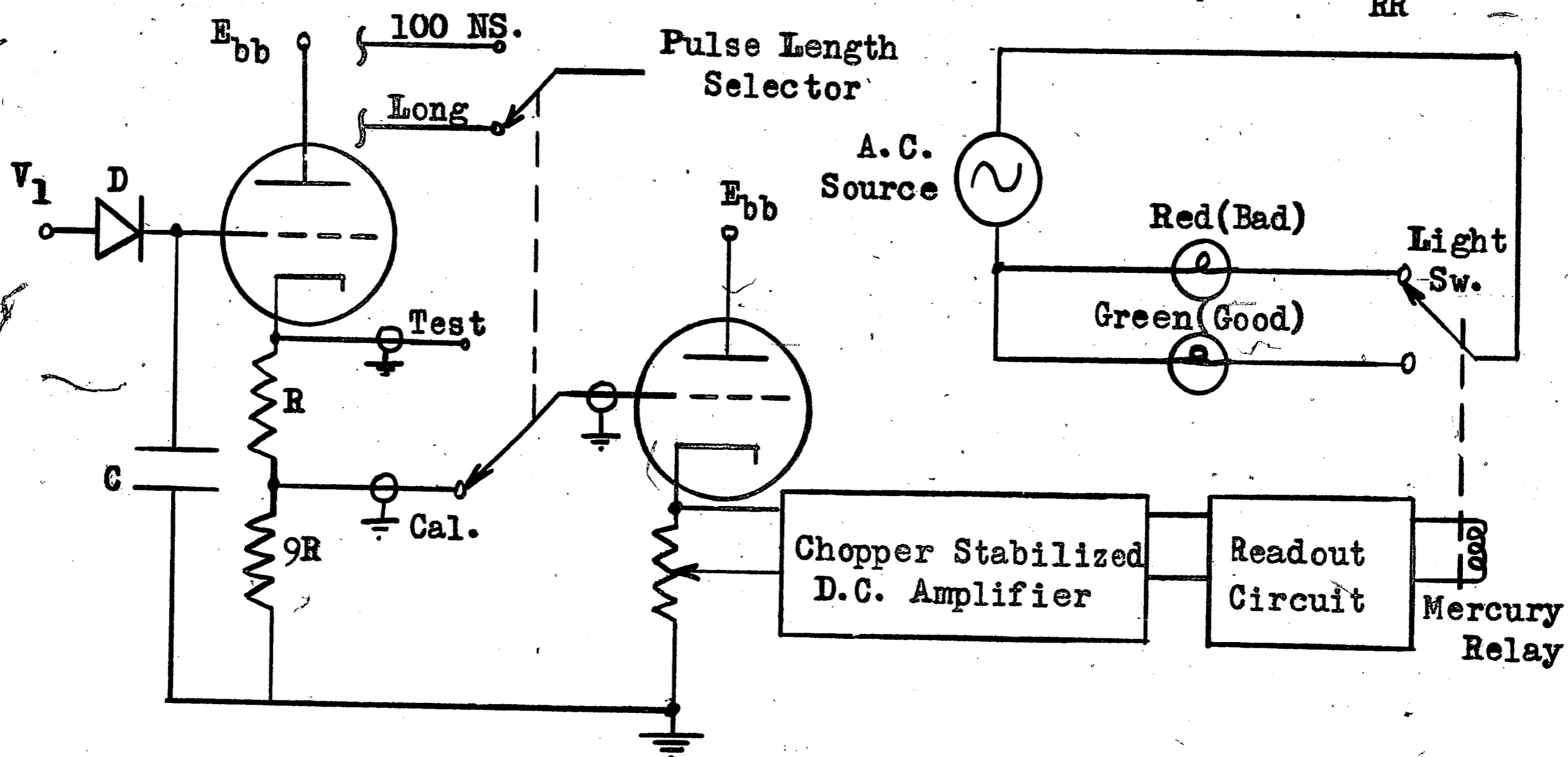


Figure 18.

In the 'calibrate' position the pulse is set to be much longer than t_{RR} . The output of the first cathode follower is $0.9 (= 9R / 9R + R)$ of the total value of the output voltage. Therefore, the 'calibrate' position automatically biases the second cathode follower at $0.9 \Delta V_{final}$. The output potentiometer on the second cathode follower can be adjusted to just activate the read-out circuit and the mercury relay. This will change the light switch from the 'bad' to the 'good' light. The input to the second cathode follower is then repositioned to 'test', the total output resistance of the first cathode follower now being used as a bias for the grid of the second cathode follower. In the 'test' position, the pulse length is decreased to 100 nanoseconds. A test may now be made and if the voltage across the diode rises to $0.9 V_{final}$ the readout circuit will trip the light switch from the 'bad' (red) position to

the 'good' (green) position.

A circuit was also devised to compare forward and reverse voltage. This calibration was accomplished by first changing the basic circuit to that in Fig. 19.

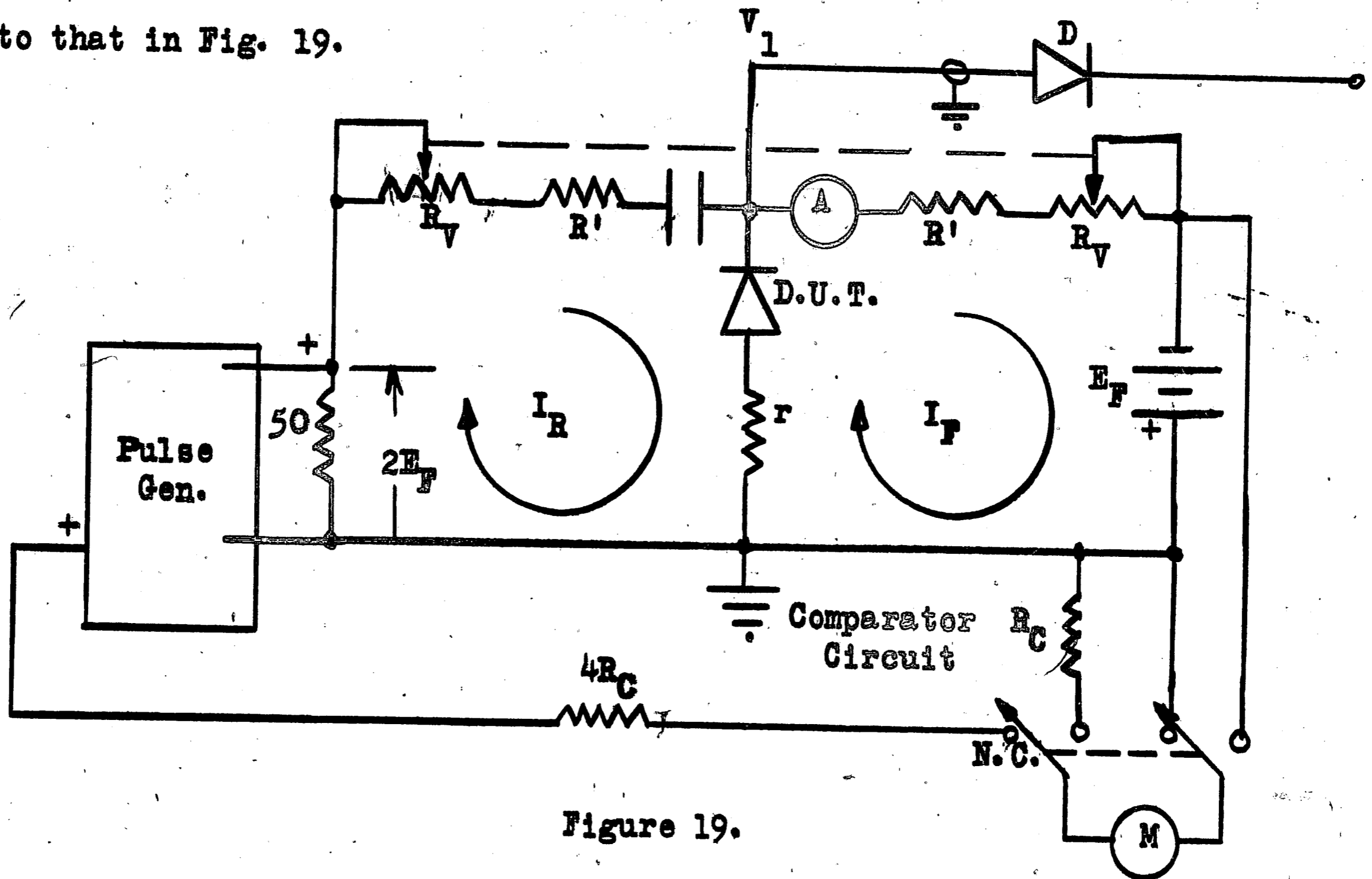


Figure 19.

In this circuit the impedance presented to both the forward and reverse bias supplies is the same. The pulse generator, an S. K. L. model 503, produces a pulse output which must be adjusted to a magnitude of $2E_F$. Also available to the rear of the pulse generator is a D.C. voltage, the magnitude of which is always twice the pulse voltage, and hence should be $4E_F$. The D.C. voltage from the pulse generator produces a current through the resistor $4R_C$, which is measured by the meter in the comparator circuit. This current is compared with that produced in resistor R_C by the forward bias, E_F . Thus, the pulse voltage can be adjusted to a magnitude $2E_F$.

In the test circuit itself care was taken to avoid stray capacitance by short lead lengths and by shielding critical wires. The two potentiometers, R_V , are mounted on the same nylon shaft and track each other within 1/2%. Both R' resistors are precision and are to prevent overloading the diode if R_V is set to zero. The resistor, r , is kept in the circuit to permit a check of conditions with an oscilloscope. The capacitor blocks the diode forward current from the pulse generator side of the circuit and the milliammeter measures I_F .

The author's contributions were: (1) the use of the fast rise pulse generator with a circuit to calibrate forward and reverse voltages, (2) the high input impedance circuit to measure the magnitude of the charge on capacitor C , (3) the use of a D. C. amplifier to allow a fine adjustment of the trip point of the read-out circuit, and (4) the overall circuit concept which minimizes error in the measurement of the reverse recovery time.

V CONCLUSIONS

The accurate measurement of the reverse recovery time of diodes is a problem which has not been thoroughly solved. However, the problem is not without solution, and when an accurate and standard circuit is devised and adopted for this purpose much useful information will be available to the designer of fast switching circuits.

It is clear that the circuit used for measuring the t_{RR} time must be included in any specification sheet. The resistance external to the diode, voltages and currents used, and circuit used to obtain the results must be known for the results to be of any value. Thus a standard circuit should be adopted for the measurements concerned. The author considers the circuit shown in Figs. 17 and 18 acceptable as a standard when the reverse recovery time is near 100 nanoseconds. The circuit is easily built and no visual measurements are required. Also all components are readily available and extreme caution against stray capacitance is not necessary.

The following methods are available to reduce t_{RR} :⁽²²⁾

- (1) Decrease τ_p , the lifetime of the holes.
- (2) Decrease the widths of the P and N- regions.
- (3) Decrease the forward current used. (This eliminates using

$$|I_R| = |I_F| .)$$

- (4) Increase the reverse current I_R .

The method shown for the measurement of t_{RR} uses relatively inexpensive components and provides accurate results. It is hoped that

this basic design will be useful for diodes with reverse recovery times much less than 100 nanoseconds.

The reverse recovery phenomena can, in some instances, be put to use and there have been devised a number of applications⁽²³⁾ of the phenomenon. One such use is in the determination of the lifetime of minority carriers. Using Eq. (14) and an oscilloscope to measure t_A , the value of τ_p can be determined since t_A , I_R , and I_F are all known constants.

VI APPENDIX I

The following is a complete solution of the equations for hole density, $p(x, t)$, in the N-region and the current density, $-J_R$, at $x = 0$ as derived by Henderson and Tillman. This is the case in which there is no resistance in the reverse bias circuit and the voltage at the junction is suddenly reversed.

The boundary conditions on $p(x, t)$ are:

$$p(x, 0) = (p_0 - p_{no}) e^{-x/L_p} + p_{no}$$

$$p(0, 0) = 0 \quad \text{at} \quad x = 0$$

$$p(x, \infty) = p_{no} (1 - e^{-x/L_p})$$

where:

$p_0 = p_{no} e^{qV/kK}$ is the hole density at the junction, $x = 0$.

p_{no} = thermal equilibrium value of hole density.

The hole density distributions corresponding to the above 3 boundary conditions are shown in Fig. 20.

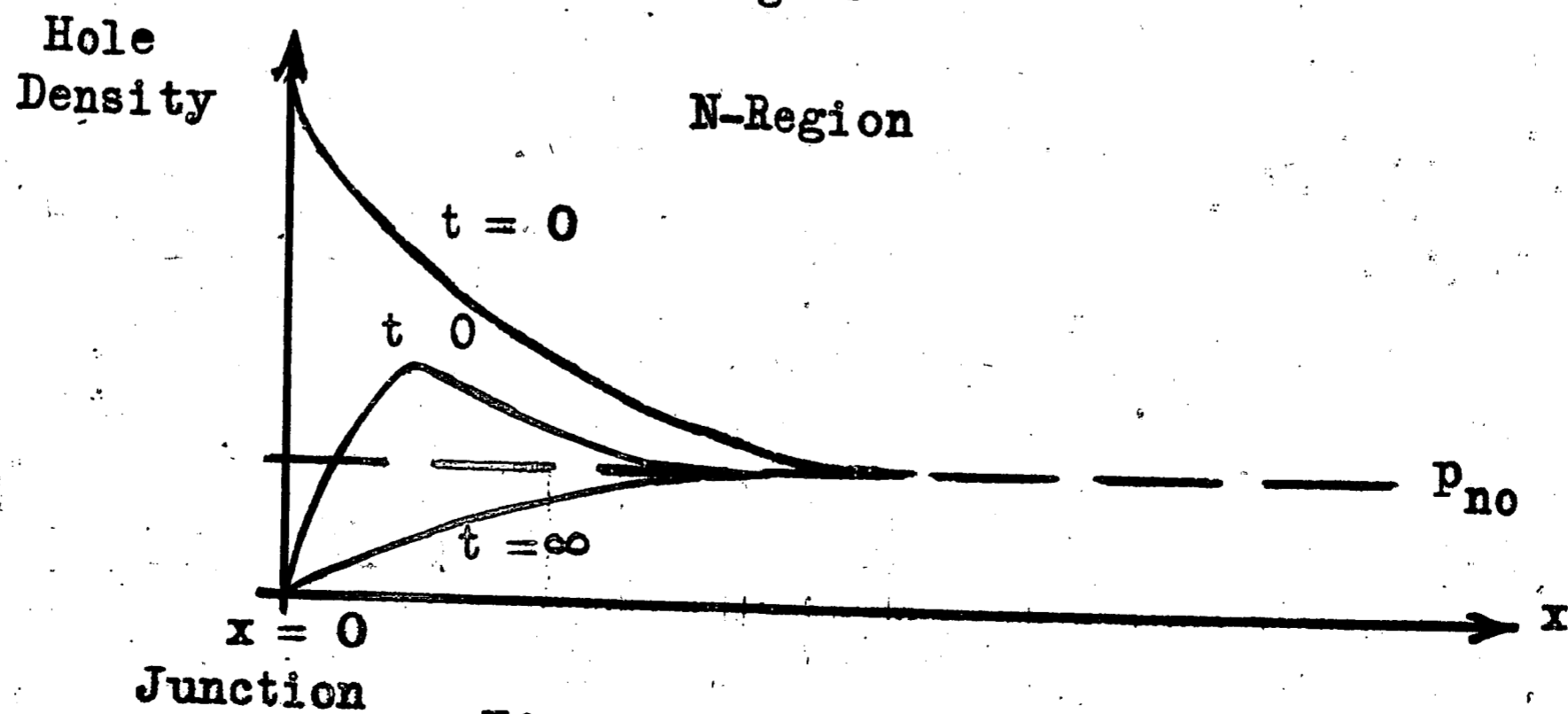


Figure 20

The continuity equation for the holes is:

$$D_p \frac{\partial^2 p}{\partial x^2} = \frac{\partial p}{\partial t} + \frac{p - p_{no}}{\tau_p}$$

where:

p = hole density at any time and point in the N-region.

t = time.

Now substitute $y(x, t) = (p(x, t) - p_{no})$:

$$D_p \frac{\partial^2 y}{\partial x^2} = \frac{\partial y}{\partial t} + \frac{y}{\tau_p}$$

Rearranging the constants:

$$\frac{\partial^2 y}{\partial x^2} - \frac{y}{L_p^2} = \frac{1}{D_p} \frac{\partial y}{\partial t}$$

The boundary conditions on $y(x, t)$ are:

$$y = (p_{o_0} - p_{no}) e^{-x/L_p} = y_0 e^{-x/L_p} \quad t = 0$$

$$y = -p_{no} \quad t > 0$$

$$y = -p_{no} e^{-x/L_p} \quad t = \infty$$

Now transform the continuity equation by Laplace transforms:

$$L \left[y(x, t) \right] = Y(x, s)$$

$$L \left(\frac{\partial^2 y}{\partial x^2} \right) = \frac{d^2 Y}{dx^2}$$

$$\rightarrow L\left(\frac{\partial y}{\partial t}\right) = sY(x, s) - y(x, 0) = sY - y_0 e^{-x/L_p}$$

The continuity equation is:

$$Y'' - \frac{Y}{L_p^2} = \frac{1}{D_p}(sY - y_0 e^{-x/L_p})$$

$$Y'' - Y\left(\frac{1}{L_p^2} + \frac{s}{D_p}\right) = \frac{-y_0}{D_p} e^{-x/L_p}$$

$$\text{Let: } k^2 = \left(\frac{1}{L_p^2} + \frac{s}{D_p}\right)$$

$$\text{Then: } Y'' - k^2 Y = \frac{-y_0}{D} e^{-x/L_p}$$

The general solution of this equation is :

$$Y = Ae^{-kx} + Be^{+kx} + \frac{y_0}{s} e^{-x/L_p}$$

where A and B are constants. Since Y is finite at $x = \infty$, $B = 0$.

Solve for A using the second boundary condition, namely:

$$y(0, t) = -p_{no} \qquad Y(0, s) = -\frac{P_{no}}{s}$$

$$\text{Thus: } A = -\frac{P_0}{s}$$

$$\text{and: } Y = \frac{y_0}{s} e^{-x/L_p} - \frac{p_0}{s} e^{-kx}$$

From Carslaw and Jaeger⁽²⁴⁾ the inverse transforms are:

$$\mathcal{L}^{-1} \left(\frac{y_0}{s} e^{-x/L_p} \right) = y_0 e^{-x/L_p}$$

and

$$\mathcal{L}^{-1} \left(\frac{e^{-kx}}{s} \right) = \frac{1}{2} \left[e^{x/L_p} \operatorname{erfc} \left(\frac{x}{2\sqrt{D_p t}} + \sqrt{\frac{D_p t}{L_p^2}} \right) + e^{-x/L_p} \operatorname{erfc} \left(\frac{x}{2\sqrt{D_p t}} - \sqrt{\frac{D_p t}{L_p^2}} \right) \right]$$

Also from Carslaw and Jaeger the following definitions are obtained:

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\epsilon^2} d\epsilon \quad \text{The error function.}$$

$$\operatorname{erfc} x = 1 - \operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-\epsilon^2} d\epsilon \quad \text{Complimentary error function.}$$

$$\operatorname{erf} \infty = 1 \quad \operatorname{erf}(-x) = -\operatorname{erf}(x)$$

For small x:

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)n!}$$

For large x:

$$\operatorname{erfc} x = \frac{e^{-x^2}}{\sqrt{\pi}} \left(\frac{1}{x} - \frac{1}{2x^3} + \frac{3}{4x^5} - \dots \right)$$

Therefore from the inverse transforms:

$$y(x,t) = y_0 e^{-x/L_p} - \frac{p_0}{2} \left[e^{x/L_p} \operatorname{erfc} \left(\frac{x}{2\sqrt{D_p t}} + \sqrt{\frac{D_p t}{L_p^2}} \right) + e^{-x/L_p} \operatorname{erfc} \left(\frac{x}{2\sqrt{D_p t}} - \sqrt{\frac{D_p t}{L_p^2}} \right) \right]$$

And since:

$$p(x, t) = y(x, t) + p_{no}$$

Then:

$$p(x, t) = p_{no} (1 - e^{-x/L_p}) + p_0 e^{-x/L_p} - \frac{p_0}{2} \left[e^{x/L_p} \operatorname{erfc} \left(\frac{x}{2\sqrt{D_p t}} + \sqrt{\frac{t}{\tau_p}} \right) + e^{-x/L_p} \operatorname{erfc} \left(\frac{x}{2\sqrt{D_p t}} - \sqrt{\frac{t}{\tau_p}} \right) \right]$$

But:

$$p_0 e^{-x/L_p} - \frac{p_0}{2} e^{-x/L_p} \operatorname{erfc} \left(\frac{x}{2\sqrt{D_p t}} - \sqrt{\frac{t}{\tau_p}} \right) = \frac{p_0}{2} e^{-x/L_p} \operatorname{erfc} \left(\sqrt{\frac{t}{\tau_p}} - \frac{x}{2\sqrt{D_p t}} \right)$$

So the hole density equation is now in the form of Henderson and Tillman:

$$p(x, t) = p_{no} (1 - e^{-x/L_p}) + \frac{p_0}{2} \left[e^{-x/L_p} \operatorname{erfc} \left(\sqrt{\frac{t}{\tau_p}} - \frac{x}{2\sqrt{D_p t}} \right) - e^{x/L_p} \operatorname{erfc} \left(\sqrt{\frac{t}{\tau_p}} + \frac{x}{2\sqrt{D_p t}} \right) \right]$$

The current density at the junction is:

$$-J_R = -qD_p \left(\frac{\partial p}{\partial x} \right) \Big|_{x=0}$$

$$\left. \frac{\partial p(x, t)}{\partial x} \right|_{x=0} = \frac{p_{no}}{L_p} + \frac{p_0}{2} \left[-\frac{2}{L_p} \operatorname{erfc} \sqrt{\frac{t}{\tau_p}} + \frac{2}{\sqrt{\pi D_p t}} e^{-t/\tau_p} \right]$$

Therefore the current density is:

$$-J_R = -\frac{qD_p p_{no}}{L_p} - \frac{qD_p p_0}{L_p} \left[\frac{e^{-t/\tau_p}}{\sqrt{\frac{\pi t}{\tau_p}}} - \operatorname{erfc} \sqrt{\frac{t}{\tau_p}} \right]$$

It is seen that at $t = 0$, $-J_R = -\infty$, and there is no recovery phase.

APPENDIX II

For the circuit with limiting resistance, $R \neq 0$, Kingston neglected the p_{no} term in the continuity equation by assuming that $p \gg p_{no}$. He used the following equation to solve for the hole density during the recovery phase.

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \frac{p}{\tau_p} \quad p = p(x, t)$$

It is noteworthy that the same results can be obtained by a substitution of variables, $y = p - p_{no}$, as in Appendix I.

Upon substituting $T = t/\tau_p$, and $X = x/L_p$, the continuity equation becomes:

$$\frac{\partial p}{\partial T} = \frac{\partial^2 p}{\partial X^2} - p$$

The initial hole distribution at $t = 0$ has a steady state solution producing a current density, $+J_F$. After the bias has been switched, the total solution can be considered as:

$$p_{total} = p_{steady\ state} + p_{transient}$$

If the current due to the transient solution is, $-(J_F + J_R)$, then the condition is satisfied that $J_{total} = -J_R$ since:

$$-J_R = J_F - (J_F + J_R)$$

At $t = 0$, the hole density, ignoring the p_{no} term, is $p = p_0 e^{-X}$

anywhere in the N-region.

Using LaPlace transforms, the continuity equation becomes:

$$P''(X, s) - (1 + s)P(X, s) = -p_0 e^{-X}$$

Solving again for $P(X, s)$:

$$P = A e^{-kX} + \frac{p_0}{s} e^{-X}$$

with: $k^2 = (1 + s)$

The definition of the diffusion hole current density is:

$$J(s) = - \frac{qD_p s}{L_p} \frac{\partial P}{\partial X}$$

The transient current density is $(J_F + J_R)$, and the transient part of $P(X, s)$ is $A e^{-kX}$, so that:

$$A = \frac{(J_F + J_R)L_p}{D_p s k q}$$

and:

$$P = \frac{p_0}{s} e^{-X} + \left(\frac{(J_R + J_F)L_p}{D s k q} \right) e^{-X \sqrt{s+1}}$$

Using the same type of inverse LaPlace transformations as in Appendix I, and also using the following integral: (25)

$$\int_0^z e^{-\mu^2} e^{-\frac{X^2}{4\mu^2}} d\mu = \frac{\sqrt{\pi}}{4} \left[e^{-X} \operatorname{erfc}\left(\frac{X}{2z} - z\right) - e^X \operatorname{erfc}\left(\frac{X}{2z} + z\right) \right]$$

The solution for $P(X, T)$ is obtained as:

$$p = p_0 \left[e^{-X} - \left(\frac{J_R + J_F}{2J_F} \right) \left\{ e^{-X} \operatorname{erfc} \left(\frac{X}{2\sqrt{T}} - \sqrt{T} \right) - e^X \operatorname{erfc} \left(\frac{X}{2\sqrt{T}} + \sqrt{T} \right) \right\} \right]$$

with: $J_F \approx qD_p p_0 / L_p$. Note that the above equation applies to the circuit with limiting resistance, $R \neq 0$.

For the reverse phase Kingston used a solution similar to the Henderson and Tillman solution for the current density in the case of $R = 0$.

Kingston assumed $p_{X=0} = 0$ for all $T > 0$.

Thus:

$$p_{\text{total}} = p_{\text{steady state}} + p_{\text{transient}}$$

$$0 = p_0 - p_0$$

Using the same continuity equation, a solution for the hole density is:

$$P = A e^{-X\sqrt{s+1}} + \frac{p_0}{s} e^{-X}$$

where, from the transient condition that $p_{\text{transient}} = -p_0$:

$$A = \frac{p_0}{s}$$

Therefore:

$$p(X, T) = p_0 e^{-X} - \frac{p_0}{2} \left[e^{-X} \operatorname{erfc} \left(\frac{X}{2\sqrt{T}} - \sqrt{T} \right) + e^X \operatorname{erfc} \left(\frac{X}{2\sqrt{T}} + \sqrt{T} \right) \right]$$

The transient current density is:

$$-(J_F + J_R) = - \frac{qD_p}{L_p} \left(\frac{\partial p_{\text{transient}}}{\partial x} \right) \Big|_{x=0}$$

And since:

$$\frac{\partial \text{erfc} Y}{\partial X} \approx - \frac{2}{\sqrt{\pi}} e^{-Y^2} \frac{\partial Y}{\partial X}$$

Then:

$$-(J_F + J_R) = - J_F \left[\text{erf} \sqrt{T} + \frac{e^{-T}}{\sqrt{\pi T}} \right]$$

The total current density is:

$$J_F - (J_F + J_R)$$

Multiply by the junction area to give the reverse current:

$$-I_R = I_F - I_F \left[\text{erf} \sqrt{T} + \frac{e^{-T}}{\sqrt{\pi T}} \right]$$

This is the current, during the reverse phase, in the circuit with a limiting resistance. However, it is an approximation derived from the case of $R = 0$.

APPENDIX III

Ko, like Kingston, chose to consider the reverse current, $-I_R$, as a sum of two currents I_F , and $-(I_F + I_R)$, with I_F starting at $-t_f$, and time set equal to zero at the instant of switching. Thus the forward current, I_F , starts during negative time.

To obtain the initial condition at time $t = 0$, the continuity equation must first be solved for the forward bias conditions. (The following solution, incidently, gives some idea of the forward transients.)

The continuity equation, used for the injected holes only, is:

$$\frac{\partial p(X, T)}{\partial T} = \frac{\partial^2 p(X, T)}{\partial X^2} - p(X, T)$$

With: $X = x/L_p$, $T = t/\tau_p$.

When the forward bias is turned on, $t = 0$, and the boundary conditions on the injected holes are:

$$p(X, 0) = 0$$

$$p(\infty, T) = 0$$

$$p(0, T) = p_{no} \left(e^{\frac{qV}{kK}} - 1 \right)$$

The steady state condition is:

$$p(0, \infty) = \frac{J_F L_p}{qD_p}$$

(The $t = 0$ point will later be shifted.)

Using Laplace transforms the solution of the continuity equation is:

$$p(X, T) = \frac{p(0, \infty)}{2} \left[e^{-X} \operatorname{erfc}\left(\frac{X}{2\sqrt{T}} - \sqrt{T}\right) - e^X \operatorname{erfc}\left(\frac{X}{2\sqrt{T}} + \sqrt{T}\right) \right]$$

Define:

$$S(X, T) = \frac{1}{2} \left[e^{-X} \operatorname{erfc}\left(\frac{X}{2\sqrt{T}} - \sqrt{T}\right) - e^X \operatorname{erfc}\left(\frac{X}{2\sqrt{T}} + \sqrt{T}\right) \right]$$

During the recovery phase the same continuity equation is used. The time is now set to zero at the instant of switching of the bias from the forward to the reverse direction. The switching follows the application of the forward bias by a length of time equal to T_F .

Therefore the initial condition is:

$$p(X, 0) = p(0, \infty) S(X, T_F)$$

The boundary conditions are:

$$p(\infty, T) = 0$$

$$p(0, T_A) = 0$$

The second boundary condition states that at the end of the recovery phase the injected hole density at $X = 0$ is reduced to zero. The hole density due to the forward current is:

$$p_F(X, T) = p(0, \infty) S(X, T + T_F)$$

Also, due to the initial condition, and the fact that $-I_R = I_F - (I_F + I_R)$, the hole density starting at $t = 0$, and due to $-(I_F + I_R)$, is:

$$p_R(X, T) = - \frac{(J_F + J_R)}{qD_p} L_p S(X, T)$$

$$p_R(X, T) = - \frac{J_F L_p}{q D_p} \left(1 + \frac{J_R}{J_F} \right) S(X, T)$$

The total hole density during the recovery period is:

$$p(X, T) = p_F - p_R$$

$$p(X, T) = p(0, \infty) \left[S(X, T + T_F) - \frac{I_F + I_R}{I_F} S(X, T) \right]$$

This is Eq. (9) in the analysis. When $T_F \rightarrow \infty$, as in Kingston's work, and in the practical case:

$$p(X, T) = \frac{J_F L_p}{q D_p} \left[S(X, \infty) - \left(\frac{J_F + J_R}{J_F} \right) S(X, T) \right]$$

And:

$$S(X, \infty) = e^{-X}$$

Therefore:

$$p(X, T) = \frac{J_F L_p e^{-X}}{q D_p} - \frac{(J_F + J_R)}{q D_p} L_p S(X, T)$$

And: $J_F = \frac{q D_p p_0}{L_p}$

Thus:

$$p(X, T) = p_0 e^{-X} - \frac{(J_F + J_R)}{J_F} p_0 S(X, T)$$

This is Kingston's solution in equation (8).

APPENDIX IV

To solve for the current function in the reverse phase Ko set a new time coordinate $T' = T - T_A$. Thus $T' = 0$ at, T_A , the start of the reverse phase.

The initial condition is now:

$$p(X, 0') = \frac{J_F L_p}{q D_p} \left[S(X, T_F + T_A) - \left(\frac{J_F + J_R}{J_F} \right) S(X, T_A) \right]$$

The boundary condition is:

$$p(0, T') = p_{no} \left(e^{\frac{qV(T')}{kK}} - 1 \right)$$

Since $p(0, T')$ is always less than p_{no} , and decreases with time, the following approximation is made:

$$p(0, T') \cong 0.$$

$$p(\infty, T') = 0.$$

$$-i(T') = \frac{q D_p}{L_p} \left(\frac{\partial p(X, T')}{\partial X} \right) \Big|_{X=0}$$

The diffusion equation may be solved by Duhamel's theorem as shown in Lax and Neustadter⁽²⁶⁾ or in Carslaw and Jaeger.⁽²⁷⁾ The variables are first changed to:

$$z = e^{T'} p(X, T')$$

The continuity equation is thus:

$$\frac{\partial z}{\partial T} = \frac{\partial^2 z}{\partial X^2}$$

The initial conditions of z are:

$$z(X, 0') = p(X, 0')$$

$$z(0, T') \approx 0$$

$$z(\infty, T') = 0$$

Thus:

$$z(X, T') = \frac{1}{2\sqrt{\pi T'}} \int_0^{\infty} z(\mu, 0) \left[e^{-\frac{(X-\mu)^2}{4T'}} - e^{-\frac{(X+\mu)^2}{4T'}} \right] d\mu$$

Therefore:

$$p(X, T') = \frac{e^{-T'}}{2\sqrt{\pi T'}} \int_0^{\infty} \left[\frac{J_F L}{qD_p} \left[S(\mu, T_A + T_F) - \left(\frac{J_F + J_R}{J_F} \right) S(\mu, T_A) \right] \left[e^{-\frac{(X-\mu)^2}{4T'}} - e^{-\frac{(X+\mu)^2}{4T'}} \right] d\mu \right]$$

By differentiating $p(X, T')$ with respect to X and setting $X = 0$, the following is obtained:

$$i(T') = \frac{I_F e^{-T'}}{2\sqrt{\pi T'^3}} \int_0^{\infty} \left[\mu \left[S(\mu, T_A + T_F) - \left(\frac{I_F + I_R}{I_F} \right) S(\mu, T_A) \right] e^{-\frac{\mu^2}{4T'}} \right] d\mu$$

This is an integral whose exact solution is not very practical. Thus an approximation must be made for the S functions.

Since: $S(0, T) = \text{erf} \sqrt{T}$

$S(X, \infty) = e^{-X}$

An approximation would be:

$S(X, T) = \text{erf} \sqrt{T} e^{-X}$

Of course other approximations can be made.

Since, at $T = T_A$, $T' = 0$, the new zero of time is T_A , and since,

$S(\mu, 0) = 0$:

$$\frac{-i(T')}{I_F} = \frac{\text{erf} \sqrt{T_F}}{\sqrt{\pi T'}} \int_0^{\infty} \frac{\mu}{2T'} \left[e^{-(T' + \frac{\mu^2}{4T'} + \mu)} \right] d\mu$$

Define: $R^2 = \frac{\mu^2}{4T'}$ and $\mu = 2R\sqrt{T'}$

Then: $2RdR = \frac{\mu d\mu}{2T'}$

$$\begin{aligned} \int_0^{\infty} e^{-(T' + \frac{\mu^2}{4T'} + \mu)} \frac{\mu}{2T'} d\mu &= \int_0^{\infty} e^{-(T' + R^2 + 2R\sqrt{T'})} 2RdR \\ &= \int_0^{\infty} e^{-(R + \sqrt{T'})^2} 2RdR \\ &= \int_0^{\infty} e^{-(R + \sqrt{T'})^2} 2\sqrt{T'} dR - \int_0^{\infty} e^{-(R + \sqrt{T'})^2} 2\sqrt{T'} dR \\ &= \int_0^{\infty} e^{-(R + \sqrt{T'})^2} 2(R + \sqrt{T'}) dR - \int_0^{\infty} e^{-(R + \sqrt{T'})^2} 2\sqrt{T'} dR \\ &= e^{-T'} - \sqrt{\pi T'} \text{erfc} \sqrt{T'} \end{aligned}$$

$$\frac{-i(T')}{I_F} = \operatorname{erf} \sqrt{T_F} \left[\frac{e^{-T'}}{\sqrt{\pi T'}} - \operatorname{erfc} \sqrt{T'} \right]$$

Allowing $T_F \rightarrow \infty$:

$$-i(T') = I_F \left[\frac{e^{-T'}}{\sqrt{\pi T'}} - \operatorname{erfc} \sqrt{T'} \right]$$

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BIOGRAPHY

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