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Stiffener requirements for plate girders

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STIFFENER REQUIREMENTS FOR PLATE GIRDERS

by

Joseph Juin-shyong Huang

A Thesis

Presented to the Graduate Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Department of Civil Engineering

Lehigh University

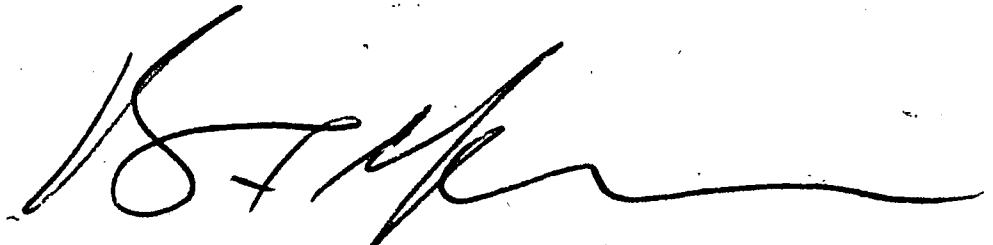
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CERTIFICATE OF APPROVAL

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

May 21, 1968

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ABSTRACT

Both the static and fatigue behavior of plate girders are influenced by the stiffeners. Girder strength could be substantially increased by the proper use of stiffeners. The objective of this thesis is to review the requirements for stiffeners of welded plate girders.

Two types of stiffeners are commonly used, namely, transverse (vertical) stiffeners and longitudinal (horizontal) stiffeners. In order to effectively fulfill their role in reinforcing the web of a plate girder, the stiffeners must meet rigidity and strength requirements.

The stiffeners must have sufficient rigidity to ensure the formation of a nodal line during web buckling, and be strong enough to help the web-panel framing.

These requirements are presented in a form which can be used for design specifications.

1. INTRODUCTION

The objectives of the study presented herein were to examine the requirements for stiffeners of plate girders. A plate girder is a deep flexural member subject to web instability. The buckling and ultimate strength of plate girders are greatly influenced by the behavior of the principal component parts, namely, the flanges, the web, and the stiffeners. There are two types of stiffeners which are commonly used, that is, transverse (vertical) stiffeners and longitudinal (horizontal) stiffeners.

Transverse stiffeners are to provide rigidity for the web-panel framing and can increase the resistance of the web to buckling in shear but are not efficient in increasing resistance to buckling in bending unless they are very closely spaced.^(1,2,3) Longitudinal stiffeners located in the compression zone of the web can effectively control the lateral web deflection and prevent the stress redistribution from the web to the compression flange, therefore, the resistance of the web to buckling due to bending can be increased substantially.^(4,5)

In order to serve their functions, the stiffeners have to meet several requirements which were established based upon both analytical and experimental studies.

This thesis deals with the analytical and experimental studies on stiffener requirements in detail. Finally, realistic design recommendations for the stiffeners of plate girders are given for practical use.

2. BRIEF REVIEW ON THE BUCKLING ANALYSIS OF PLATES

2.1 Elastic Buckling of Plates

The linear buckling theory of plates was initiated by Bryan in 1890 when he studied the problem of a simply supported flat plate under compression⁽⁶⁾. The impetus for the analysis of the stability of plate was provided by the solution of problems pertaining to the ship plating which was encountered by many early investigators. In the early 1900's, Timoshenko⁽⁷⁾ and Reissner⁽⁸⁾ made extensive studies of the buckling problems of rectangular plates under various boundary conditions.

In comparison with the theory of stability of columns, the problem of the stability of plates is more complicated due to the fact that the critical buckling load may deviate substantially from the ultimate load which the plate can sustain. Whereas the buckling load for practical purposes may define the strength of a column^(9,10,11), plates may be able to sustain external loads in the buckled state noticeably which is due to the contribution of the post-buckling strength of web plate as shown in Fig. 1.⁽²⁾ The differences between buckling and ultimate loads become substantial especially for very thin plates and for materials with low modulus of elasticity, such as aluminum alloys. The determination of the ultimate load of a plate girder is

not a stability problem. The web buckling behavior is influenced by the boundary conditions which are furnished by the stiffeners and flanges. In order to examine the stiffener requirements, it is of interest here to describe the concept of treating the problem of stability of plates. Because of its technical importance and simplicity, the buckling of a simply supported, rectangular plate under pure bending will be illustrated on the basis of the following assumptions:

- a. The plate is initially perfectly flat.
- b. The plate is made of an elastic and homogeneous material.
- c. The bending moments are applied in the plane of the middle surface of the plate.
- d. The transverse deflections are small compared to the thickness of the plate.

As shown in Fig. 2, a rectangular plate, with dimensions D , d_0 , and a thickness, t_w , is subjected to pure bending in the x -direction. The differential equation for the plate subject to small lateral deflection can be expressed as follows: (7,12)

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D'} \left(N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right) \quad (2.1)$$

Where

D' = flexural rigidity of the plate per unit width, $D' = \frac{Et^3}{12(1-\nu^2)}$

w = z displacement

x, y, z = Coordinates

N_x, N_y, N_{xy} = forces per unit length acting in the middle plane of the plate (Fig. 3).

For a plate subjected to pure bending (Fig. 2),

$N_y = N_{xy} = 0$, and $N_x = \sigma t_w \frac{2y}{D}$, where σ is the tensile stress at $y = \frac{D}{2}$. The boundary conditions can be shown as

$$w = 0 \quad \text{and} \quad \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{for} \quad x = \pm d_0/2 \quad (2.2a)$$

$$w = 0 \quad \text{and} \quad \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} = 0 \quad \text{for} \quad y = \pm D/2 \quad (2.2b)$$

The deflection of the buckled plate simply supported on all sides can be taken in the form of the double trigonometric series

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{2m\pi x}{d_0} \sin \frac{2n\pi y}{D} \quad (2.3)$$

By substituting membrane forces, deflection equation Eq. 2.3 into Eq. 2.1, the general solution of Eq. 2.1 can be determined. Again, by imposing the boundary conditions, Eq. 2.2, the lowest characteristic value of σ which corresponds to the buckling stress σ_{cr} of the plate can be expressed in terms of the so-called Euler's reference stress which is defined as⁽¹³⁾

$$\sigma_e = \frac{\pi^2 D'}{D^2 t_w} = \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t_w}{D}\right)^2 \quad (2.4)$$

where E is the modulus of elasticity and ν is Poisson's ratio, t_w is the plate thickness, and D is the web depth or clear distance between flanges. The critical buckling stress σ_{cr} can therefore be written as

$$\sigma_{cr} = k_b \sigma_e \quad (2.5)$$

where the quantity k_b is commonly referred to as the buckling coefficient which is a function of the plate geometry, loading conditions, and the boundary conditions.

Figure 4 gives values of the buckling coefficient k_b for plates subjected to pure bending.⁽¹¹⁾ Similarly, the critical buckling stress τ_{cr} of a plate subjected to pure shear in its plane can be expressed as

$$\tau_{cr} = k_s \sigma_e \quad (2.6)$$

The buckling coefficient for a simply supported plate subjected to pure shear is shown in Fig. 14. (15) If the plate is subjected to combined action of external forces such as bending and shear, compression and shear, etc, the buckling stresses will be defined by the interaction formula (7,14,15) :

$$f \left(\frac{\sigma_{cr}}{\sigma_{cr}^0}, \frac{\tau_{cr}}{\tau_{cr}^0} \right) = 1 \quad (2.7)$$

where σ_{cr} and τ_{cr} are the direct stresses and shearing stresses which cause plate buckling when applied simultaneously; and σ_{cr}^0 , τ_{cr}^0 are the critical buckling stresses of the plate subjected to direct stresses or shearing stresses alone. For example, in the case of a simply supported plate under combined action of pure bending and shear, the interaction curve, which is derived from Timoshenko's solution of this problem (16), can be represented by the equation which is part of a circle (Fig. 5).

$$\left(\frac{\sigma_{cr}}{\sigma_{cr}^0} \right)^2 + \left(\frac{\tau_{cr}}{\tau_{cr}^0} \right)^2 = 1 \quad (2.8)$$

The descriptions developed so far have been given to the problem of buckling of plate in the elastic range only. In other words, the intensity of stress, σ_i , defined by the critical buckling stresses σ_{cr} and τ_{cr} can be determined by the plasticity hypothesis of Huber, Von Mises, and Hencky, the so-called energy of distortion theory.

$$\sigma_i = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} \quad (2.9)$$

Now, we substitute $\sigma_x = \sigma_{cr}$, $\sigma_y = 0$, $\tau_{xy} = \tau_{cr}$ into Eq. 2.9, the yield criterion can be expressed as

$$\sigma_i = \sqrt{\sigma_{cr}^2 + 3\tau_{cr}^2} \quad (2.10a)$$

However, if the Tresca yield criterion is used, the equivalent stress will be

$$\sigma_i = \sqrt{\sigma_{cr}^2 + 4\tau_{cr}^2} \quad (2.10b)$$

σ_i may be considered as an equivalent tensile stress producing the same strain as the combined stresses σ_{cr} and τ_{cr} . When the buckling of plate occurs elastically, the equivalent stress σ_i must be less than the proportional limit of the material which for practical purposes is taken to be equal to σ_y , yield point of the material.

$$\sigma_i < \sigma_y \quad (2.11)$$

2.2 Inelastic Buckling of Plates

Analogous to the findings in the column theory, it is possible that the critical buckling stress of plates exceeds the proportional limit of the material. In other

words, if the Eq. 2.11 is violated, the buckling of plates occurs in the inelastic range and the phenomena become more complicated. The theory of inelastic buckling of plates was developed during the 1940's by Bijlaard, Illyushin, and Stowell. The following presentation of inelastic stability theory of plates is based upon Stowell's theory. (17,18,19)

In considering the buckling of plates in the elastic range, the stress and strain are linearly related by the modulus of elasticity, E . Beyond the proportional limit, the basic assumption of plasticity theory suggests the following plastic stress-strain relation which may be written as

$$\sigma = E_s \epsilon \quad (2.12)$$

In Eq. 2.12, E_s is the secant modulus which is a function of stress or strain. It is assumed that when the plate is stressed beyond the proportional limit, buckling and increase in load proceed simultaneously so that no strain reversal occurs in any part of the plate. In such a case, the Poisson's ratio is taken to be equal to 0.5 which implies that the material is incompressible in the plastic state. The effects of inelastic behavior are incorporated into a single parameter η which is referred to as the plasticity reduction factor.

By definition,

$$\eta = \frac{\sigma_{cr} \text{ (plastic)}}{\sigma_{cr} \text{ (elastic)}} \quad (2.13)$$

The critical buckling stress for the inelastic case then can be obtained by multiplying the critical stress for elastic buckling by the plasticity reduction factor η .

3. PLATE GIRDERS WEB STIFFENED TRANSVERSELY

3.1 Theoretical Requirements for Transverse Stiffeners

3.1.1 Rigidity Requirement

If a plate girder web is stiffened transversely, the transverse stiffeners serve two purposes: to provide rigidity for keeping the cross-section of girder in shape and to insure post-buckling strength. When the plate girder is subjected to external loads, the web panel boundary is assumed not to deflect laterally perpendicular to the plane of the web. This requires that all transverse stiffeners have proper rigidity in that direction. If the transverse stiffener is not rigid enough, it will deflect laterally with the web. The deflected web panel will have a horizontal cross section as shown in Fig. 6a. However, when the transverse stiffener provides sufficient rigidity, the web plate will now deflect on each side of the stiffener and the stiffener will remain straight, forming a nodal line (Fig. 6b). The relationship between the rigidity of transverse stiffeners and the buckling of the web described herein can be depicted by Fig. 7.⁽¹³⁾ The value γ for the abscissa is the relative rigidity of the stiffener which is defined as the ratio of the flexural rigidity (EI) of the stiffener to the product of panel length (d_0) and the flexural rigidity (D') of the corresponding web portion, $\gamma = EI/D'd_0$.

The ordinate is the buckling stress of the plate, σ_{cr} or τ_{cr} . Point A in the figure corresponds to the buckling stress of the plate without stiffener at all, namely, $\gamma = 0$. The portion AB of this curve corresponds to the case of stiffener deflected with the web. When the rigidity of a stiffener is just sufficient for it to remain straight during the web buckling (point B) this rigidity value is considered the optimum. With larger value of stiffener rigidity, the critical buckling stress will remain the same value, σ_{cr}^0 or τ_{cr}^0 .

The optimum rigidity γ_0 needed to produce a nodal line can be obtained by means of the energy method. (15)
The expression for the potential energy of a plate with transverse stiffeners (Fig. 8) can be written as

$$I = V + U_w + V_s \quad (3.1)$$

in which

I = total potential energy

V = strain energy of bending of the plate,

$$V = \frac{D'}{2} \iint \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dx dy$$

U_w = the change of the potential energy of the external forces when the plate deflects from its original position to the deformed shape.

V_s = strain energy of stiffeners,

$$V_s = \frac{1}{2} \int [EI \left(\frac{d^2 w}{dy^2}\right)^2 + EI_\omega \left(\frac{d^2 \theta}{dy^2}\right)^2 + GK_T \left(\frac{d\theta}{dy}\right)^2] dy$$

Usually, in calculating the strain energy contributed by transverse stiffeners, it is assumed that both the St. Venant torsion stiffness (GK_T) and the warping stiffness (EI_ω) of the stiffeners can be neglected, and the strain energy includes the bending energy of each stiffener only.

$$\text{Thus, } V_s = \frac{EI}{2} \int \left(\frac{d^2 w}{dy^2}\right)^2 dy$$

The critical buckling stress (σ_{cr}^o or γ_{cr}^o) can be determined from the theorem of stationary potential energy. The optimum rigidity γ_o then is the relative rigidity of the stiffener which is required to ensure this critical buckling stress for the web plate. Various investigators obtained different results through the Rayleigh-Ritz and Lagrangian multiplier methods. (15,20,21,22,23) For the case of a plate subjected to pure bending, the optimum rigidity of the transverse stiffener is, according to Refs. 20 and 24:

$$\gamma_o = 6.2 - 12.7 \alpha + 6.5 \alpha^2$$

$$\text{for } 0.6 \leq \alpha \leq 0.935 \quad (3.2)$$

where, α = aspect ratio = $\frac{\text{panel length}}{\text{panel depth}} = \frac{d_o}{D}$

For $\alpha > 0.935$, the transverse stiffeners have practically no effect on the plate buckling by bending. When the plate is under pure shear, the optimum rigidity is^(20,24)

$$\gamma_o = \frac{5.4}{\alpha} \left(\frac{2}{\alpha} + \frac{2.5}{\alpha^2} - \frac{1}{\alpha^3} - 1 \right) \text{ for } 0.5 \leq \alpha \leq 2.0 \quad (3.3)$$

A comparison is made on the different results from various investigators of the optimum rigidity for transverse stiffeners which is shown in Fig. 9.

It was pointed out in Ref. 24 that for the transverse stiffeners to remain practically straight up to the rupture load of the girder, the optimum rigidity γ_o should be multiplied by a factor of 20 which was based on the experimental results, whereas a multiplying factor of only 3 was reported sufficient.⁽²⁵⁾

3.1.2 Width-to-Thickness Ratio Requirement

The strength of transverse stiffeners may be affected by the buckling of the stiffener plate itself in two ways: the buckling may cause an overall failure by making the stiffener plate element fully ineffective in providing rigidity along web plate boundary, or it may produce a redistribution of stresses and thus influence the function of stiffener as to insure post-buckling strength of web panel.

The local buckling is prevented usually by limiting the width-thickness ratio such that the stiffener plate does not buckle at a stress below the yield point of the material. (26)

$$(\sigma_{cr})_{\text{plate}} \geq \sigma_y \quad (3.4)$$

$$\sigma_{cr} = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b'}\right)^2 \quad (3.5)$$

$$\frac{b'}{t} \leq 0.951 \sqrt{\frac{kE}{\sigma_y}} \quad (3.6)$$

where

b' = the projecting width of the stiffener plate

t = the thickness of the stiffener

3.1.3 Area Requirement

The area requirement for transverse stiffeners is determined when a plate girder is subjected to shear force. When a plate girder is under bending moment, there is no such area requirement.

The carrying capacity of plate girders in shear is generally described into two parts, namely, simple beam action up to critical buckling stress and the tension field action in the post-buckling range up to yielding in the web. (2)

The behavior of a plate girder panel resisting external shear forces is similar to a Pratt truss as illustrated in Fig. 10. When the tension field is developed, the diagonal

strip of the web acts as a tension member, while the transverse stiffeners act as compression struts. Consequently, the transverse stiffeners must resist the vertical component of the tension field force. The compressive force on a transverse stiffener due to the tension field force is derived in Ref. 2 which can be expressed as

$$F_s = \sigma_t t_w D \left(\frac{\alpha}{2} - \frac{\alpha^2}{2\sqrt{1+\alpha^2}} \right) \quad (3.7)$$

At ultimate load,

$$\frac{\sigma_t}{\sigma_y} = 1 - \frac{\gamma_{cr}}{\gamma_y} \quad (3.8)$$

Then Eq. 3.7 becomes

$$F_s = \frac{1-C}{2} \left(\alpha - \frac{\alpha^2}{\sqrt{1+\alpha^2}} \right) D t_w \sigma_y \quad (3.9)$$

where $C = \frac{\gamma_{cr}}{\gamma_y}$

If local buckling is prevented, the ultimate axial stress in the stiffener is practically equal to the yield stress σ_y . Thus the required area for transverse stiffener to carry the vertical component of the tension field force will be

$$A_s = \frac{F_s}{\sigma_y} = \frac{1-C}{2} \left(\alpha - \frac{\alpha^2}{\sqrt{1+\alpha^2}} \right) Y D t_w \quad (3.10)$$

The factor Y is defined as the ratio of yield point of web steel to the yield point of stiffener steel. Equation 3.10 is adopted as Formula (10) in AISC Specification. (27)

In Eq. 3.10, the constant C is the ratio of critical shear buckling stress, according to the linear buckling theory, to the shear yield point of web material.

$$C = \frac{\tau_{cr}}{\tau_y} = \frac{\tau_{cr}}{\sigma_y/\sqrt{3}} = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t_w}{D}\right)^2 \frac{\sqrt{3}}{\sigma_y} \quad (3.11)$$

Where $\sigma_y/\sqrt{3}$ is the yield stress in shear by Mises' Yield condition, for webs with simply supported edges, the shear buckling coefficient is

$$k = 4.00 + 5.34 \left(\frac{D}{d_o}\right)^2 \quad \text{for } \frac{d_o}{D} \leq 1 \quad (3.12a)$$

$$k = 5.34 + 4.00 \left(\frac{D}{d_o}\right)^2 \quad \text{for } \frac{d_o}{D} \geq 1 \quad (3.12b)$$

For webs with transverse edges simply supported and the longitudinal edges clamped,

$$k = 5.34 \left(\frac{D}{d_o}\right)^2 + 6.55 \left(\frac{D}{d_o}\right) - 13.71 + 14.10 \left(\frac{d_o}{D}\right) \quad \text{for } \frac{d_o}{D} \leq 1 \quad (3.13a)$$

$$k = 8.98 + 6.18 \left(\frac{D}{d_o}\right)^2 - 2.88 \left(\frac{D}{d_o}\right)^3 \quad \text{for } \frac{d_o}{D} \geq 1 \quad (3.13b)$$

Equations 3.12 and 3.13 are plotted in Fig. 11.

3.2 Design Recommendations for Transverse Stiffeners

3.2.1 Rigidity Requirement

The optimum rigidity for transverse stiffeners to form a nodal line during web buckling was suggested to be an equation given in Ref. 15 and Ref. 23 with slight modifications. The optimum rigidity was obtained for an infinitely long plate with identical, equally spaced, transverse stiffeners. The plate was assumed to be simply supported at all four edges and subjected to shear forces as shown in Fig. 12. The transverse stiffeners were assumed to have bending stiffness but no torsional stiffness and were assumed to be concentrated along transverse lines in the middle plane of the plate.

The results were found by means of the Lagrangian multiplier method and were presented in the form of plate buckling coefficient k versus relative rigidity γ curves for three different stiffener spacings, namely D , $0.5D$, $0.2D$.⁽²³⁾ Theoretically, k continues to increase as γ increases. However, for practical purposes there is an optimum value γ_0 beyond which the increase in k is small. The three curves obtained by Stein and Fralich⁽²³⁾ were studied by Bleich⁽¹⁵⁾ and came out with an approximate equation for the buckling coefficient k . The optimum

rigidity can be expressed as follows: (Fig. 9)

$$\gamma_o = 28 \frac{D^2}{d_o^2} - 20 \quad (3.14)$$

This equation has been adopted by AASHO⁽²⁸⁾ but with a slightly modified value of

$$\gamma_o = 25 \frac{D^2}{d_o^2} - 20 \quad (3.15)$$

and not to be less than 5. By definition, $\gamma_o = EI/D'd_o$.

The moment of inertia for the transverse stiffener is

$I = (d_o D'/E) \gamma_o$. The flexural rigidity of the plate D' is $Et_w^3/12(1-\nu^2)$. By assuming Poisson's ratio is 0.3, the

following equation is obtained.

$$I = \frac{d_o t_w^3}{10.92} \gamma_o \quad (3.16)$$

Equation 3.15 has been plotted in Fig. 13 as a comparison with the specifications of European countries. It is of interest to note that except for the range of aspect ratio between 0.5 and 0.7, both the British⁽²⁹⁾ and the German⁽³⁰⁾ specifications are more conservative than AASHO.

3.2.2 Width-to-Thickness Ratio Requirement

In the design of transverse stiffeners, it is expected that the stiffener plate can develop yield point stresses

without premature local elastic buckling. It is reported that for a value of $\lambda = 0.6$ in the non-dimensional plate buckling curve, where $\lambda = \sqrt{\sigma_y / \sigma_{cr}}$, the failure of stiffener plate element by local buckling can be prevented. (26)

Then by substituting Eq. 3.5 into the equation,

$$\lambda = \sqrt{\frac{\sigma_y}{\sigma_{cr}}} = 0.6 \quad (3.17)$$

and taking for steel $\nu = 0.3$, $E = 29 \times 10^6$ psi. The expression for limiting the width-to-thickness ratio will be

$$\frac{b'}{t} \leq \frac{3070 \sqrt{k}}{\sqrt{\sigma_y}} \quad (3.18)$$

For the case of stiffeners on plate girders, the lowest value for buckling coefficient k is considered to be equal to 0.72. (15,26) The Equation 3.18 becomes

$$\frac{b'}{t} \leq \frac{2600}{\sqrt{\sigma_y}} \quad (3.19)$$

The AISC⁽²⁷⁾ and AASHO⁽²⁸⁾ Specifications adopt the following equation for limiting the width-to-thickness ratio which is derived from the assumption that $\lambda = 0.7$.

$$\frac{b'}{t} \leq \frac{3000}{\sqrt{\sigma_y}} \quad (3.20)$$

The AASHO Specification specifies that the transverse stiffener may be A36 steel which implies that $b'/t \leq 16$.

3.2.3 Area Requirement

In deriving the required area of transverse stiffeners for plate girders (Eq. 3.10), it was assumed that the vertical component of tension field force was resisted by the transverse stiffeners alone which acted as compression struts in a Pratt truss. This assumption was pointed out to be too conservative.

The experimental results obtained previously at Lehigh⁽³⁾ is summarized in Table 1. Where F_s is the theoretical stiffener force computed by Eq. 3.9, A_s is the actual area of transverse stiffener designed, and σ_s is the stress of transverse stiffener measured at ultimate load. It is worthwhile to note that all values of the ratio of existing stresses to the hypothetical stresses are much less than unity. The first and second largest percentages of stiffener forces compared to the theoretical values are equal to 54% and 34%, respectively. The similar results have been reported in Ref. 31.

For the practical range of α , the expression, $\alpha - \alpha^2 / \sqrt{1+\alpha^2}$, varies between 0.2 and 0.3. By arbitrarily choosing an average value of 0.26, the Eq. 3.9 becomes

$$F_s = 0.13 (1-C) D t_w \sigma_y \quad (3.21)$$

Taking into consideration the material property, the ratio of applied load to the shear capacity V/V_u , the required gross area of the stiffener will be

$$0.13 Y D t_w (1-C) \frac{V}{V_u}$$

Furthermore, similar to the practice of bearing stiffeners, a small portion of the web strip with $12t_w$ in width is assumed to participate in resisting the compression force coming from the tension field action. Thus the required area of the stiffener plate is

$$A_s = [0.13 D t_w (1-C) \frac{V}{V_u} - 12 t_w^2] Y \quad (3.22)$$

Table 2 substantiates that the latter assumption is also conservative.

To find the factor C in Eq. 3.22 is an involved procedure⁽²⁾. In an attempt to serve the practical design purposes, the simplification and approximation have been made. These equations for k (Eqs. 3.12) are combined into one by approximation:

$$k = 5.34 + 5.00 (D/d_o)^2 \quad (3.23)$$

which is shown in Figs. 11 and 14.

The formula for C is rather complicated. (2) An approximation is made as follows:

$$C = 8000 \left(\frac{t_w}{D} \right) \sqrt{\frac{5.34 + 5(D/d_o)^2}{\sigma_y}} - 0.3 \leq 1.0 \quad (3.24)$$

which is plotted in Fig. 15 as compared to the corresponding values obtained from Ref. 2. For a given material and panel geometry, the value of C varies linearly with the reciprocals of slenderness ratio, t_w/D . The value of C then can be easily obtained with the aid of plots such as the one shown in Fig. 16.

4. PLATE GIRDERS WEB STIFFENED LONGITUDINALLY

4.1 Theoretical Requirements for Longitudinal Stiffeners

The most effective type of stiffener for web plates subjected to bending is the longitudinal or horizontal stiffener. It was pointed out previously^(1,3) that when a plate girder was subjected to bending, the compressed portion of the web did not carry the stress (MC/I) predicted by beam theory because of the gradual lateral deflection of the web. In other words, some of the compressive force supposedly to be carried by the web was redistributed to the compression flange. The stress in the compression flange, therefore, exceeded the value obtained by using beam theory, (Fig. 17).⁽³²⁾

The main purposes of using the longitudinal stiffener are to control the lateral web deflections and prevent stress redistribution from the web to the compression flange, and to increase the web buckling strength. In order to fulfill its purposes, there are several requirements which must be met by a longitudinal stiffener.

4.1.1 Position Requirement

For the case of web buckling by pure bending, in order to be able to control the lateral web deflection effectively, the longitudinal stiffener has to be located in the compression portion of the web. As long as shear

is present, the stiffener usually is placed in a lower position to eliminate the bulging out of web due to the diagonal tension field action. The location of a longitudinal stiffener was suggested to be determined by means of an auxiliary chart. For each given value of shearing stress to bending stress ratio (τ/σ) and aspect ratio (d_o/D) of a web plate panel, the position of a stiffener can be obtained by a set of interaction curves as shown in Fig. 13 of Ref. 13.

4.1.2 Rigidity Requirement

In order to insure the formation of a nodal line in the stiffened panel, the longitudinal stiffener must provide sufficient rigidity. The optimum rigidity of a longitudinal stiffener can be obtained by the same method used for transverse stiffeners. For the case of a web plate under pure bending, it was pointed out that the optimum rigidity for a longitudinal stiffener located at 1/5 of the depth from the compression flange was ⁽²⁰⁾

$$\gamma_o = 3.87 + 5.1 \alpha + (8.82 + 77.6 \delta) \alpha^2 \quad (4.1)$$

$$\text{for } 0.5 \leq \alpha \leq 1.5$$

where

γ_o = the optimum rigidity of a longitudinal stiffener, $\gamma_o = \frac{EI}{D'D}$ (D' = flexural rigidity of web plate, D = depth of plate girder).

$$\alpha = \text{aspect ratio} = \frac{\text{panel length } (d_o)}{\text{panel depth } (D)}$$

$$\delta = \frac{\text{area of stiffener}}{\text{area of web}}$$

When the longitudinal stiffener was placed at one-quarter depth from the compression flange, the following expression was suggested⁽¹⁵⁾ for $\alpha \leq 1.6$:

$$\gamma_o = (12.6 + 50\delta) \alpha^2 - 3.4 \alpha^3 \quad (4.2)$$

For the case of a web plate subjected to pure shear and reinforced by one longitudinal stiffener at mid-height of the depth, the expression for optimum rigidity of the stiffener will be

$$\gamma_o = 5.4 \alpha^2 (2\alpha + 2.5 \alpha^2 - \alpha^3 - 1)$$

$$\text{for } 0.5 \leq \alpha \leq 2.0 \quad (4.3)$$

At static tests of welded plate girders, it was observed that the theoretical web buckling phenomenon did not cause immediate failure of the girder. The experimental investigation showed that the post-buckling strength of plate girder was substantial⁽³⁾. The stiffeners used to reinforce the plate girder practically remain straight up to the ultimate load of the girder. The required rigidity of an actual stiffener then will

be larger than the optimum rigidity which is determined based upon the theoretical web buckling analysis. The following equation was suggested to relate those two quantities. (13)

$$\gamma_{\text{req}} = n \gamma_0 \quad (4.4)$$

The factor n depends mainly on the location of the stiffener and is suggested to be of the value as follows:

	Value of
Distance between Longitudinal Stiffener and Compression Flange	n
D/2	3
D/3	4
D/4	6
D/5	7

4.1.3 Width-to-Thickness Ratio Requirement

The longitudinal stiffener as a flat plate element is susceptible to the failure by local buckling if the proper width-to-thickness ratio is not selected. The theoretical analysis of the local buckling problem was discussed in Article 3.1.2 which could also be applied to the longitudinal stiffener.

4.1.4 Strength Requirement

As far as the buckling of the web is concerned, the longitudinal stiffener serves as a column, just as the

compression flange of a plate girder does. The stiffener must have enough rigidity to maintain a linear distribution of bending stress along a cross section of the girder. In other words, the stiffener column consisting of the stiffener and a part of the web must be strong enough to withstand this without lateral buckling. The detailed derivation of a requirement for the longitudinal stiffener column to sustain the lateral buckling stress will be given later on in Article 4.2.4.

4.2 Design Recommendations for Longitudinal Stiffeners

4.2.1 Position Requirement

From the viewpoint of web buckling, the increase of web buckling strength as well as the reduction of lateral web deflection can be best achieved so long as the longitudinal stiffener is placed in the most effective position, the so-called optimum position.

The optimum position of a longitudinal stiffener, when the web panel is subjected to pure bending, has been shown theoretically to be at one-fifth of the depth from the compression flange⁽³³⁾. This conclusion was also confirmed by test results^(4,5). Since the shear is always accompanied by bending moment, and the longitudinal stiffener at any location still controls the lateral web deflections in shear,⁽⁴⁾ it is recommended here that the one-fifth depth location be adopted for all panels as

long as a longitudinal stiffener is used. This conforms to the current bridge design practices.

4.2.2 Rigidity Requirement

Those equations (Eqs. 4.1, 4.2, and 4.3) for computing the optimum rigidity of longitudinal stiffener, are too lengthy to meet the design purpose. The equation presented here⁽³⁴⁾ is the same one which is adopted for use in current AASHO Specifications⁽²⁸⁾.

$$I \geq Dt_w^3 \left[2.4 \left(\frac{d_o}{D} \right)^2 - 0.13 \right] \quad (4.5)$$

The ratio of required moment of inertia for longitudinal stiffener to web plate rigidity has been plotted against panel aspect ratio for the AASHO⁽²⁸⁾, British⁽²⁹⁾, German⁽³⁰⁾ Specifications (Fig. 18). From these curves it reveals that both the British and the German Specifications are more conservative than AASHO in the range of $d_o/D < 1$, which is the upper limit of aspect ratio permitted by AASHO Specifications.

4.2.3 Width-to-Thickness Ratio Requirement

The provision for preventing the failure of longitudinal stiffeners by premature local buckling is proposed to be the same one for the transverse stiffeners (Eq. 3.19). The yield point should be that of the stiffener if it

differs from the girder in yield point. Equation 3.19, based on the yield point of the stiffener, is more conservative than what is specified in Article 1.7.73 of the AASHTO Specification⁽²⁸⁾ which can be shown here for reference.

$$t \geq \frac{b' \sqrt{f_b}}{2250} \quad (4.6)$$

where t = the thickness of the longitudinal stiffener

b' = width of stiffeners

f_b = calculated compressive bending stress in the flange, $f_b \leq 0.55 \sigma_y$

Longitudinal stiffeners are usually placed on only one side of the web plate. In order to avoid the problem of residual stresses due to welding as well as to prevent the reduction of fatigue strength, they need not be continuous and may be cut at their intersections with the transverse stiffeners.

4.2.4 Strength Requirement

Beyond buckling of the web, if the longitudinal stiffener is properly proportioned, a linear distribution of bending stress is maintained along a cross section of the plate girder. The longitudinal stiffener at one-fifth depth is then subjected to a compressive stress of 0.6 times that of the compression flange as shown in

Fig. 19. If the critical buckling stress is taken to be equal to the yield point of the material used in the flange, the stiffener column must be rigid enough to withstand a stress of $0.6 \sigma_y$ without lateral buckling.

Making use of the Column Research Council basic column strength formula and taking into account the imperfections, such as the eccentricity of load, initial out-of-straightness and a partially restrained end condition for the stiffener, a formula to evaluate the required radius of gyration of the longitudinal stiffener is established.

$$\left(\frac{0.6 \sigma_y}{\sigma_y}\right) (1.2) = 1 - \frac{\sigma_y}{4\pi^2 E} \left(\frac{0.7 d_o}{r}\right)^2 \quad (4.7)$$

and

$$r = \frac{d_o \sqrt{\sigma_y}}{23000} \quad (4.8)$$

To anchor the tension field force, the longitudinal stiffener must have sufficient area. With the requirements, Eq. 3.19, 4.5, 4.8, shown above, a sufficient stiffener area is provided for; thus no additional provision is needed.

5. PLATE GIRDER WEB STIFFENED
TRANSVERSELY AND LONGITUDINALLY

In order to effectively increase the strength of a plate girder and to obtain an economical design, a combination of both transverse and longitudinal stiffeners is often used. Little study has been made on a plate girder reinforced by both transverse and longitudinal stiffeners. The following discussion gives some results of study on this problem.

5.1 Theoretical Requirements for Stiffeners

5.1.1 Rigidity Requirement

The case of a web plate under pure shear and reinforced by both transverse stiffeners and a central longitudinal stiffener was reported in Ref. 35. It was pointed out that if the transverse stiffeners possess the optimum rigidity, then the optimum rigidity for the longitudinal stiffener is

$$\gamma_{ol} = 11.25 \left(\frac{D}{d_o} \right)^2 \quad (5.1)$$

It was also found that the total weight of transverse and longitudinal stiffeners required to achieve a given web buckling stress can be one half of the stiffener weight required when only transverse stiffeners are used. (11,35)

The ultimate strength of a plate girder stiffened with transverse stiffeners can be substantially improved by using one or more longitudinal stiffeners. This phenomenon is the same as that which occurs in the field of aircraft structures investigated at the N.A.C.A. Structures Laboratory. In such cases the rigidity of the transverse stiffeners must be greater than that specified previously, so that they will remain straight until the increased ultimate strength is reached. The concept of an equivalent web with a thickness t_e was proposed^(11,25) which was based on the assumption that the critical buckling stress of the equivalent web without longitudinal stiffeners will be equal to the critical buckling stress of the given web plate of thickness t_w with longitudinal stiffeners. The design of the transverse stiffeners then will be based on the equivalent web thickness t_e , and this results in greater rigidity.

5.1.2 Strength Requirement

A longitudinal stiffener is used to form a nodal line in the deformed pattern of the web; to control lateral web deflections and to prevent the stress redistribution from the web to the compression flange. Those actions will subject the longitudinal stiffener to lateral load and the adjacent transverse stiffeners to concentrated forces at the intersection of the two stiffeners.⁽⁴⁾

By considering the possible deflections and the location of the longitudinal stiffener, a relationship between the section moduli of the transverse and longitudinal stiffener can be derived as shown in the following to make sure that the former does not fail under the concentrated forces.

If the longitudinal stiffener were removed from the web, the deformed shape of the web between transverse stiffeners could be approximated by a sinusoidal curve. Hence, it is reasonable to assume that the web subjects the longitudinal stiffener to a sinusoidal lateral load as shown in Fig. 20(a). The resultant of the sinusoidal load is

$$\text{Resultant} = \int_0^{d_o} P_o \sin \frac{\pi x}{d_o} dx = \frac{2}{\pi} P_o d_o \quad (5.2)$$

and the reactions at the ends of the stiffener will be $R = (P_o d_o) / \pi$. The moment at midspan can be found as follows:

$$M_L = R \left(\frac{d_o}{2} \right) - \int_0^{d_o/2} (P_o \sin \frac{\pi x}{d_o}) \left(\frac{d_o}{2} - x \right) dx \quad (5.3)$$

After performing the integration, this expression is obtained:

$$M_L = \frac{P_o d_o^2}{\pi^2} \quad (5.4)$$

By assuming the normal bending stress in the longitudinal stiffener to be as high as the yield point, the following derivation for the concentrated force R to be applied to the transverse stiffener can be performed.

$$M_L = \sigma_y S_L \quad (5.5)$$

Equating Eqs. 5.5 to 5.4, we end up with

$$P_o = \frac{\sigma_y S_L \pi^2}{d_o^2} \quad (5.6)$$

and

$$R = \frac{\sigma_y S_L \pi}{d_o} \quad (5.7)$$

At its intersection with the longitudinal stiffener, a transverse stiffener is subjected to a concentrated force $2R$ from the two adjacent web panels, as indicated in Fig. 20(b). For a welded plate girder, the flanges are relatively rigid when compared to the web, flange rotations are generally very small.⁽³⁶⁾ It is reasonable to assume that a transverse stiffener is clamped at both ends by flanges.

By considering the partial restrained end condition of transverse stiffeners and the common practice of making a discontinuity for longitudinal stiffeners at their intersections with the transverse stiffeners, the moment under

concentrated load is considered to be essential for determining the required section modulus S_T of the transverse stiffener. The moment at intersection is found to be:

$$M_T = 0.102 RD \quad (5.8)$$

Substituting Eqs. 5.7 into 5.8, the moment can be written as

$$M_T = 0.321 \sigma_y S_L \left(\frac{D}{d_o}\right) \quad (5.9)$$

The required section modulus of the transverse stiffener S_T is obtained based on the assumption that the bending stress in the transverse stiffener is permitted to reach yield point.

$$S_T = \frac{M_T}{\sigma_y} \quad (5.10)$$

The relationship between the section moduli of the transverse stiffener and the longitudinal stiffener is then established by substituting the expression for M_T , Eq. 5.9, into Eq. 5.10.

$$S_T = 0.321 \left(\frac{D}{d_o}\right) S_L \quad (5.11)$$

5.2 Design Recommendations for Stiffeners

5.2.1 Rigidity Requirement

When a plate girder is stiffened transversely and longitudinally, the web panel is separated into subpanels, each subpanel behaves in the manner as that of an individual panel. Consequently, the subpanels may be treated independently with the depth of the subpanel as the panel depth and the design recommendations presented in Chapters 3 and 4 can be applied accordingly.

When a web panel is reinforced by several longitudinal stiffeners, it is suggested and examined by tests that each stiffener may be designed as if it were alone^(13,24).

5.2.2 Area Requirement

For the case of a web panel stiffened with transverse stiffeners and one longitudinal stiffener at a distance $D/5$ from the compression flange, the effect of longitudinal stiffener on the shear strength is relatively small⁽⁴⁾, and it is suggested that the longitudinal stiffener be neglected in computing the shear strength of the plate girder. The required area for the transverse stiffeners will then be computed based on the overall panel depth.

5.2.3 Strength Requirement

When a plate girder panel is stiffened transversely and longitudinally, the transverse stiffener must provide

enough section modulus S_T to avoid the failure at its intersection with the longitudinal stiffeners. The expression for the required section modulus of the transverse stiffener, for a plate girder reinforced with longitudinal stiffeners at one-fifth depth from compression flange, is proposed as in the following.

$$S_T \geq \frac{N}{6} \left(\frac{D}{d_o} \right) S_L \quad (5.12)$$

where S_T = section modulus of transverse stiffeners

S_L = section modulus of longitudinal stiffeners
at $D/5$ from inner surface of compression
flange.

$N = 1$ or 2 corresponding to the cases of the
transverse stiffeners intersect with the
longitudinal stiffener on one side or two
sides, respectively.

6. PLATE GIRDER WEB STIFFENED
BY ONE-SIDED STIFFENERS

6.1 Theoretical Requirements for Stiffeners

6.1.1 Rigidity Requirement

For the design of stiffeners of a plate girder, the requirement which generally governs the stiffener sizes is the rigidity, or the moment-of-inertia criterion, that is, the stiffeners are designed to maintain the shape of girder cross section. It is of interest to compare the effect of different arrangements by using two-sided and one-sided stiffeners.

The moment-of-inertia of double stiffeners is taken about an axis passing through the centerline of the web plane, and that of one-sided stiffeners is usually taken about the axis at the interface between stiffener and web.

With reference to Fig. 21(a), neglecting the web thickness, the moment of inertia of the two-sided stiffener is

$$I = \frac{t(2b)^3}{12} = \frac{2tb^3}{3} \quad (6.1)$$

The moment of inertia of the one-sided stiffener about the stiffener-web interface is

$$I' = \frac{t'(b')^3}{3} \tag{6.2}$$

By assuming these two cases provide the same moment of inertia, $I = I'$, and the same thickness, $t = t'$, we have

$$b' = 1.26 b \tag{6.3}$$

It is readily seen that the same moment-of-inertia is provided by these two different arrangements with the outstanding leg of a one-sided stiffener being only 26% greater than the width of one half of a stiffener pair.

The area of the two-sided arrangement is $2bt$ and that of the one-sided arrangement is $1.26bt$. It shows that the use of a one-sided stiffener requires only 63% of the total area of a two-sided stiffener when only stiffener moment-of-inertia is the basis of design. This favors the use of one-sided stiffeners. For this reason transverse stiffeners are often placed on one side of the web and the longitudinal stiffeners on the other. It also saves on fabricating time and cuts down production costs.

The method for determining the optimum rigidity still holds for one-sided stiffeners. The following formula for the optimum value γ_o of one-sided stiffeners was obtained from an experimental investigation. (22)

$$\gamma_o = 21.5 \left(\frac{D}{d_o} \right)^2 - 7.5 \quad (6.4)$$

which is plotted in Fig. 9 for a comparison with various results. It was recommended that this formula could be used only when the thickness of the stiffener leg is equal to or greater than the thickness of the web plate.

6.1.2 Area Requirement

In the post-buckling range, the stiffener axial force resulting from the tension field action is applied in the plane of the web. Thus the one-sided stiffeners, like a beam-column, will be subjected to bending moment as well as axial compression force since they will be loaded eccentrically. For this reason, a one-sided stiffener will be less efficient in carrying the compression load, and it would need to have larger cross-sectional area than the stiffener pairs.

6.2 Design Recommendations for Stiffeners

6.2.1 Rigidity Requirement

The design recommendations presented in Sections 3.2 and 4.2 are applicable for one-sided stiffeners. It was

recommended that the moment-of-inertia of the one-sided stiffener taken about the neutral axis of the cross section composed of the stiffener and a portion of the web of $20 t_w$, as shown in Fig. 21(e).⁽¹³⁾ For this purpose, an effective web width of $30 t_w$ was also suggested.⁽¹⁵⁾

To conform to the current rules for bridge design, it is recommended here that a web strip of $18 t_w$ is to be included as a part of the stiffener column. (Fig. 19)

6.2.2 Area Requirement

By allowing the one-sided stiffener to become fully yielded under the combined bending moment and axial force, (Fig. 22), and using the case of a two-sided stiffener as a reference, the expression for the required area of transverse stiffeners, Eq. 3.22, will become

$$A_s = [0.13 B D t_w (1-C) \frac{V}{V_u} - 12 t_w^2] Y \quad (6.5)$$

where $B = 1.0$ for stiffener pairs, Fig. 21(a).

$= 1.8$ for single angle stiffeners, Fig. 21(d).

$= 2.4$ for single plate stiffeners, Fig. 21(c).

6.2.3 Stiffener Details Requirement

In previous tests conducted at Lehigh, no movement of the tension flange with respect to the transverse

stiffeners was observed until after the ultimate load was reached. (3) It was concluded that transverse stiffeners could be stopped short of the tension flange a distance not to exceed 4 times the web thickness. However, the transverse stiffeners should always be fitted to the compression flange. When one-sided stiffeners are used they should be welded to the compression flange as to resist any flange torsion.

In order to provide for possible nonuniformity in shear flow, the stiffener force F_s is to be developed over a distance of one-third of the web depth. The maximum value of the stiffener force is found to be (2)

$$F_s = 0.015 D^2 \sqrt{\frac{\sigma_y^3}{E}} \quad (6.6)$$

The connectors then are proportioned to count for a total shear transfer of $3 F_s / D$. The shear flow per unit length of transverse stiffeners is

$$q_u = 0.045 D \sqrt{\frac{\sigma_y^3}{E}} \quad (6.7)$$

By assuming the factor of safety of 1.65, and the modulus of elasticity of steel of 29,000,000 psi, the Eq. 6.7 can be written as (27)

$$q \geq D \sqrt{\left(\frac{\sigma_y}{3400}\right)^3} \quad (6.8)$$

where q = shear flow between girder web and transverse stiffeners, in pounds per linear inch of stiffeners.

D = depth of girder panel, in inch

σ_y = yield point of steel, in psi

Equation 6.8 specifies the required shear flow for which the connectors (fillet welds or rivets) must be designed to insure an adequate shear transfer between stiffener-web interface.

Because of their relatively high torsional rigidity, the tubular stiffeners are greatly superior to the stiffener plate in increasing the web frame stability, namely, the strength of a plate girder.⁽³⁷⁾ In the meanwhile, however, the theoretical basis for the design of the web plates reinforced by tubular stiffeners has not yet been well developed. For instance, the values of the buckling coefficient k , and the optimum rigidity of the stiffeners are still unknown. The design recommendations for this type of stiffeners, Fig. 21 (f) and (g), are not available at present.

7. BEARING STIFFENERS

7.1 Stability Considerations

The external loads or reactions in direct bearing on the flanges of a plate girder can cause the following detrimental effect in cases where proper bearing stiffeners are absent. The resulting bearing pressure on the web can cause local web yielding resulted in web crippling, also the web may collapse as a result of overall buckling. Therefore, bearing stiffeners shall be used over the end bearings and along the length of the girder where concentrated loads must be carried.

7.2 Design Recommendations for Bearing Stiffeners

The bearing stiffener is designed like a column. The effective width of a centrally located web strip to be included as a part of the column is equal to $25 t_w$ at interior stiffeners and $12 t_w$ for the stiffeners at the end of the web, ⁽²⁷⁾ or $18 t_w$ for both cases. ⁽²⁸⁾ The effective length is to be taken as not less than $3/4$ of the length of the stiffeners in computing the slenderness ratio l/r . ⁽²⁷⁾ The radius of gyration is to be computed about the axis through the center line of the web plate. Their connection to the web shall be designed to transmit the entire end reaction to the bearings. Such stiffeners usually consists of two plates, shall have a close bearing

against the flange, or flanges, through which they receive their loads or reactions, and shall extend approximately to the outer edges of the flange plates.

The AASHO Specifications also gives the following equation for the required thickness of the bearing stiffener plates. (28)

$$t \geq \frac{b'}{12} \sqrt{\frac{\sigma_y}{33,000}} \quad (7.1)$$

where t = the thickness of the bearing stiffeners

b' = width of stiffeners

σ_y = yield point of stiffener material

8. SUMMARY AND CONCLUSION

The following is a summary of the design recommendations presented heretofore on stiffener requirements for plate girders:

1. Transverse Stiffeners Requirements

1.1 Rigidity

The moment of inertia of a transverse stiffener shall not be less than:

$$I = \frac{d_o t_w^3}{10.92} J$$

where $J = 25 \left(\frac{D}{d_o}\right)^2 - 20$, but not less than 5.0.

d_o = distance between transverse stiffeners.

When stiffeners are in pairs, the moment of inertia shall be taken about the center line of the web plate. When one-sided stiffeners are used, the moment of inertia shall be taken about the neutral axis of the cross section comprising a web strip of $18 t_w$ and the stiffener.

1.2 Width-to-Thickness Ratio

The width-to-thickness ratio of transverse stiffeners shall be such that

$$\frac{b'}{t} \leq \frac{2600}{\sqrt{\sigma_y}}$$

where b' = the projecting width of the stiffeners,
in inch

σ_y = yield point of stiffener material, in
pounds per square inches.

1.3 Area

The gross cross-sectional area, in square inches, of transverse stiffeners shall be not less than:

$$A_s = [0.13 B D t_w (1-C) \left(\frac{V}{V_u}\right) - 12 t_w^2] Y$$

where $Y = \frac{\text{yield point of web steel}}{\text{yield point of stiffener steel}}$

$B = 1.0$ for stiffener pairs

$= 1.8$ for single angle stiffeners

$= 2.4$ for single plate stiffeners

$V =$ applied shear, in pounds per square inches

$V_u =$ shear capacity in pounds per square inches

$$C = 8000 \left(\frac{t_w}{D} \right) \sqrt{\frac{5.34 + 5 (D/d_o)^2}{\sigma_y}} - 0.3 \leq 1.0$$

D = clear, unsupported distance between flanges

1.4 Strength

When a web panel is stiffened both transversely and longitudinally, the transverse stiffener shall be designed according to Sections 1.1 to 1.3 in this chapter, except that the depth of subpanels shall be used instead of the total panel depth, D.

In addition, the section modulus of the transverse stiffener shall be such that:

$$S_T \geq \frac{N}{6} \left(\frac{D}{d_o} \right) S_L$$

where S_T = section modulus of transverse stiffeners

S_L = section modulus of longitudinal stiffeners
at D/5 from inner surface of compression
flange.

N = 1 for the transverse stiffeners intersect
with the longitudinal stiffener on one side

= 2 for the transverse stiffeners intersect
with the longitudinal stiffener on two
sides.

1.5 Stiffener Detail

The transverse stiffeners may be stopped short of the tension flange a distance not to exceed 4 times the web thickness. The one-sided stiffeners must be attached to the compression flange.

The transverse stiffeners shall be connected for a shear transfer, in pounds per linear inch of stiffener, such that

$$q \geq D \sqrt{\left(\frac{\sigma_y}{3400}\right)^3}$$

where σ_y = yield point of web steel

2. Longitudinal Stiffeners Requirements

2.1 Position

The longitudinal stiffeners shall be placed at a distance $D/5$ from the inner surface of the compression flange component.

2.2 Rigidity

The longitudinal stiffener shall be proportioned so that:

$$I \geq D t_w^3 \left[2.4 \left(\frac{d_o}{D}\right)^2 - 0.13 \right]$$

The moment of inertia I of the longitudinal stiffener shall be taken the same way as that of the transverse stiffeners.

2.3 Width-to-Thickness Ratio

The width-to-thickness ratio of the longitudinal stiffener shall be controlled by the same equation as that for the transverse stiffener.

2.4 Strength

The radius of gyration of the longitudinal stiffener is not less than:

$$r \geq \frac{d \sqrt{\sigma_y}}{23000}$$

In computing the value of r , a centrally located web strip not more than $18 t_w$ in width shall be considered as a part of the stiffener column.

In conclusion, the design recommendations presented in this paper may now be applied to the plate girder design.

However, a further study on the rigidity requirement for transverse stiffeners can be made. The investigation on the problem of an infinitely long plate reinforced by equidistant transverse stiffeners was made in Ref. 38.

The plate was assumed to be either clamped or simply-supported along the longitudinal edges and subjected to shear forces. Since the flanges of plate girders do provide some restraints along the boundary, it may be of interest to digest the results presented⁽³⁸⁾ and to see if there is any conclusion which will add to the proposed design recommendations. This task is being undertaken as one of the current research efforts.

Table 1. FORCES ON TRANSVERSE STIFFENERS

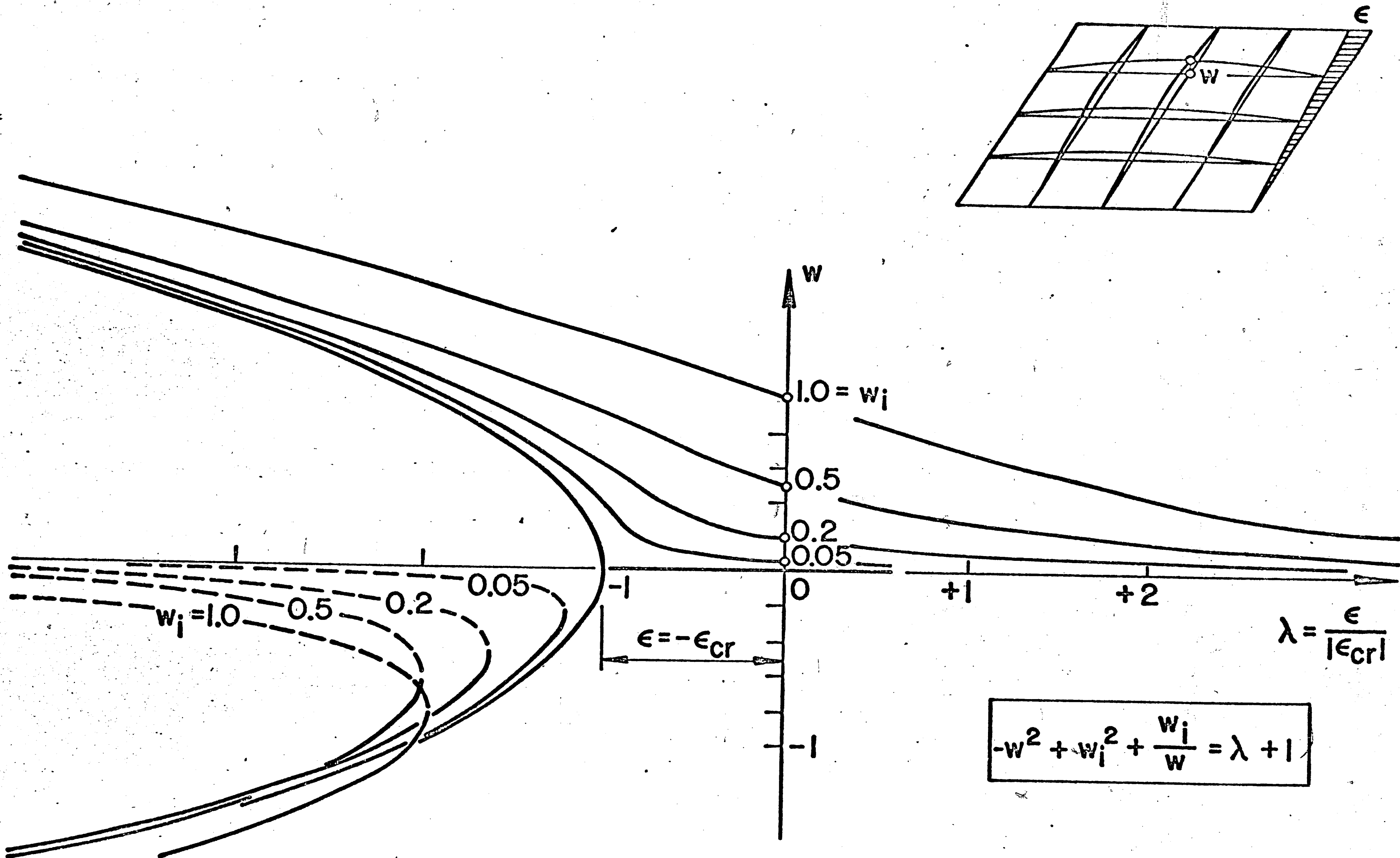
Girder	$\alpha = \frac{d_o}{D}$	$\frac{D}{t_w}$	* F_s (kips)	A_s (sq.in.)	$\frac{F_s}{A_s}$	σ_s (ksi)	$\frac{\sigma_s}{F_s/A_s}$
G6	1.50	259	-36.8	2.0	-18.8	-10.2	0.555
			-36.8	2.0	-18.8	- 8.3	0.451
G7	1.00	255	-42.8	2.0	-21.2	- 6.2	0.290
G8	1.50	254	-40.0	2.0	-20.0	- 6.5	0.325
			-40.0	2.0	-20.0	- 5.4	0.270
G9	1.50	382	-33.8	2.0	-16.9	- 7.5	0.444
E1	1.50	131	-51.2	2.0	-25.6	- 6.6	0.258
E2	1.50	99	- 7.3	2.0	- 3.7	0	0
E4	1.50	128	-48.5	2.0	-24.3	- 3.6 Δ	0.148
E5	0.75	128	-20.0	2.0	-10.0	0 Δ	0
			-26.2	2.0	-13.1	0 Δ	0

* Theoretical value by tension field action

Δ Measured stress on stiffener not adjacent to failed panel

Table 2. FORCES ON TRANSVERSE STIFFENERS
(PARTICIPATION OF WEB)

Girder	$12 t_w^2$ (sq.in.)	A $= A_s + 12 t_w^2$ (sq.in.)	$\frac{F_s}{A}$	$\frac{\sigma_s}{F_s/A}$
G6	0.45	2.45	-15.0	0.68
			-15.0	0.55
G7	0.46	2.46	-17.4	0.36
G8	0.47	2.47	-16.2	0.40
			-16.2	0.33
G9	0.21	2.21	-15.3	0.49
E1	1.75	3.75	-13.7	0.48
E2	3.09	5.09	- 1.4	0
E4	1.84	3.84	-12.6	0.29
E5	1.84	3.84	- 5.2	0
			- 6.8	0



$$-w^2 + w_i^2 + \frac{w_i}{w} = \lambda + 1$$

Fig. 1 Lateral Deflection vs. Applied Strain

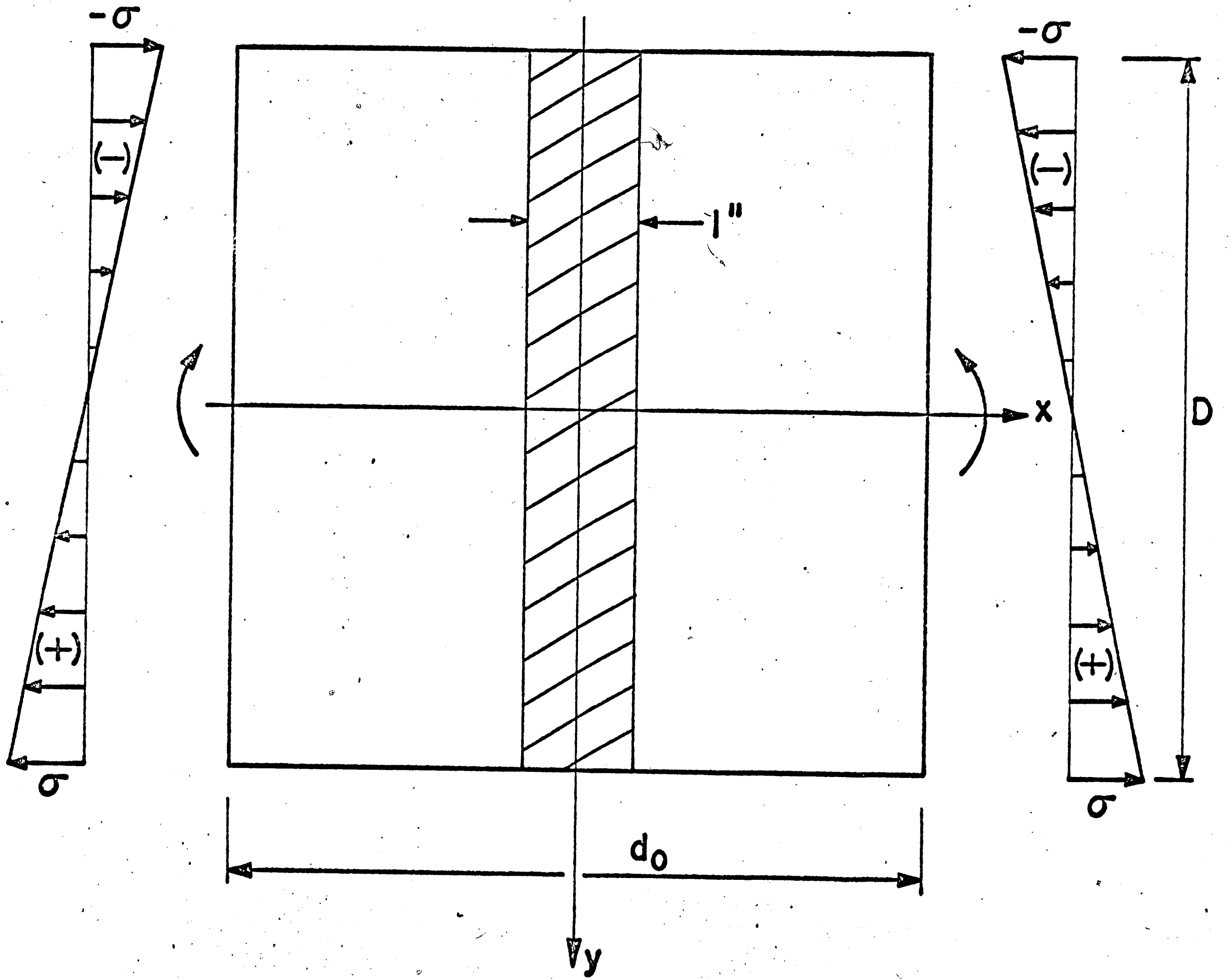


Fig. 2 Plate Buckling Due to Pure Bending

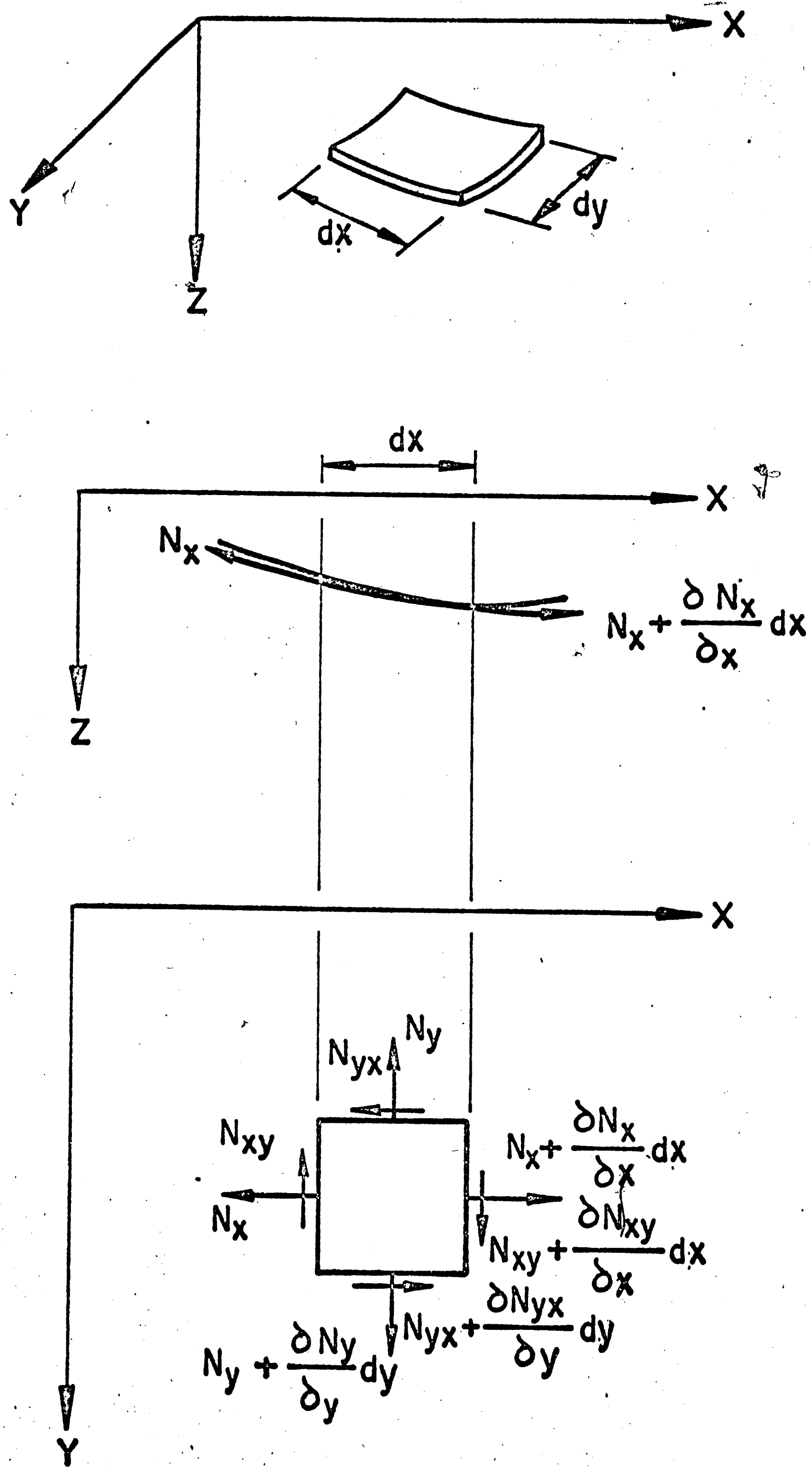


Fig. 3 Membrane Forces Acting on Plate Element

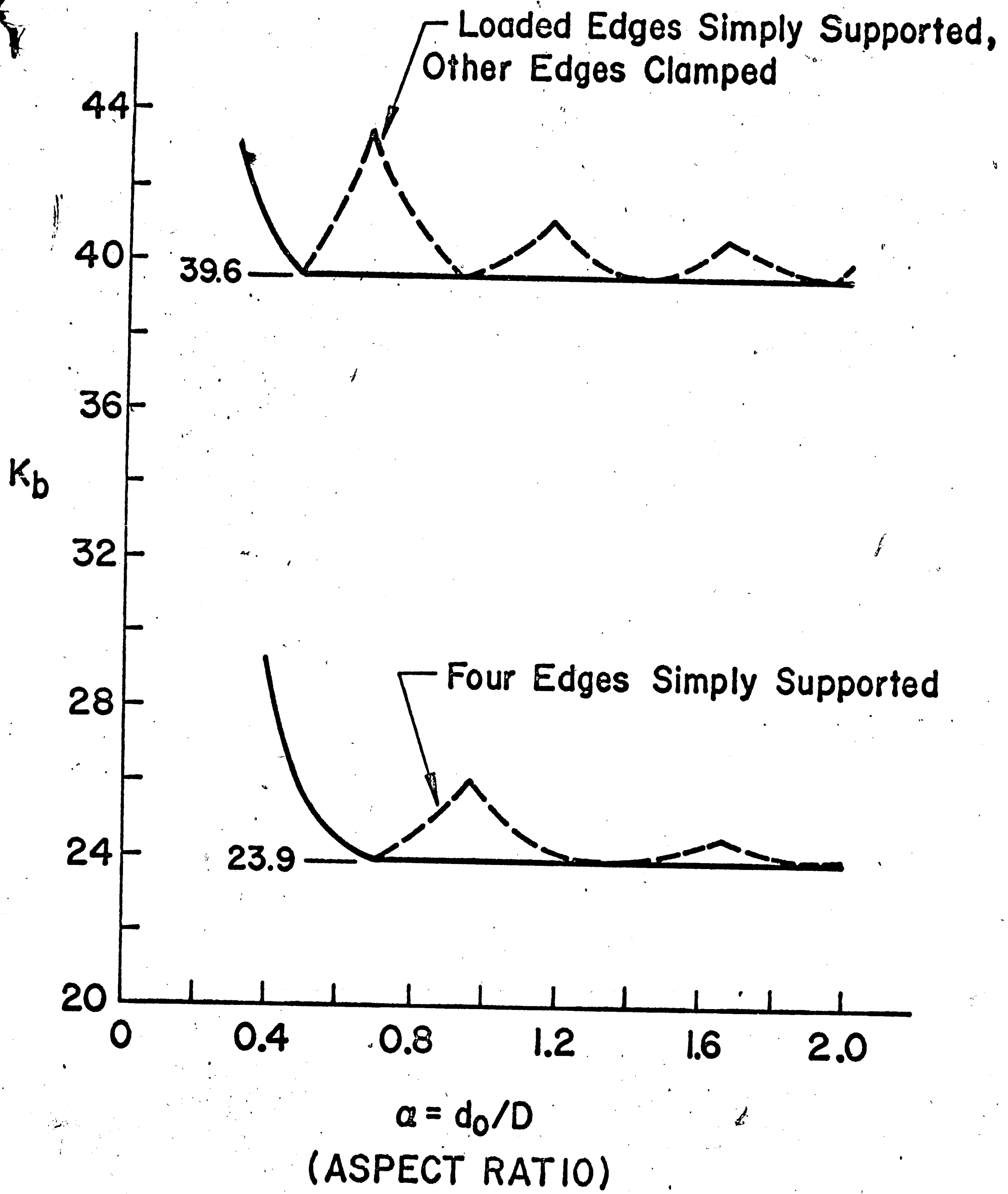


Fig. 4 Buckling Coefficients for Plates in Pure Bending

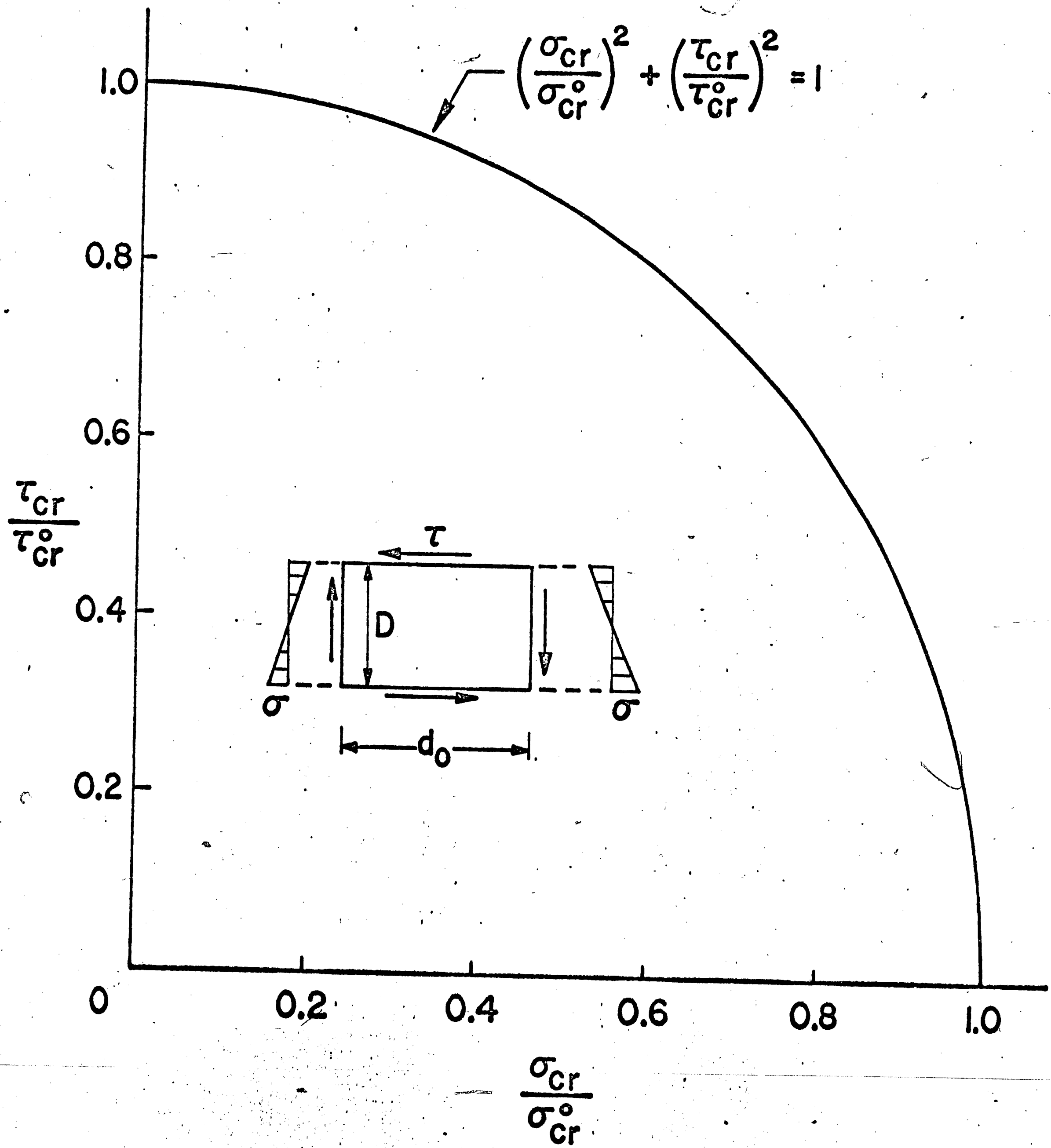


Fig. 5 Simply Supported Plate Buckling Under Combined Bending and Shear

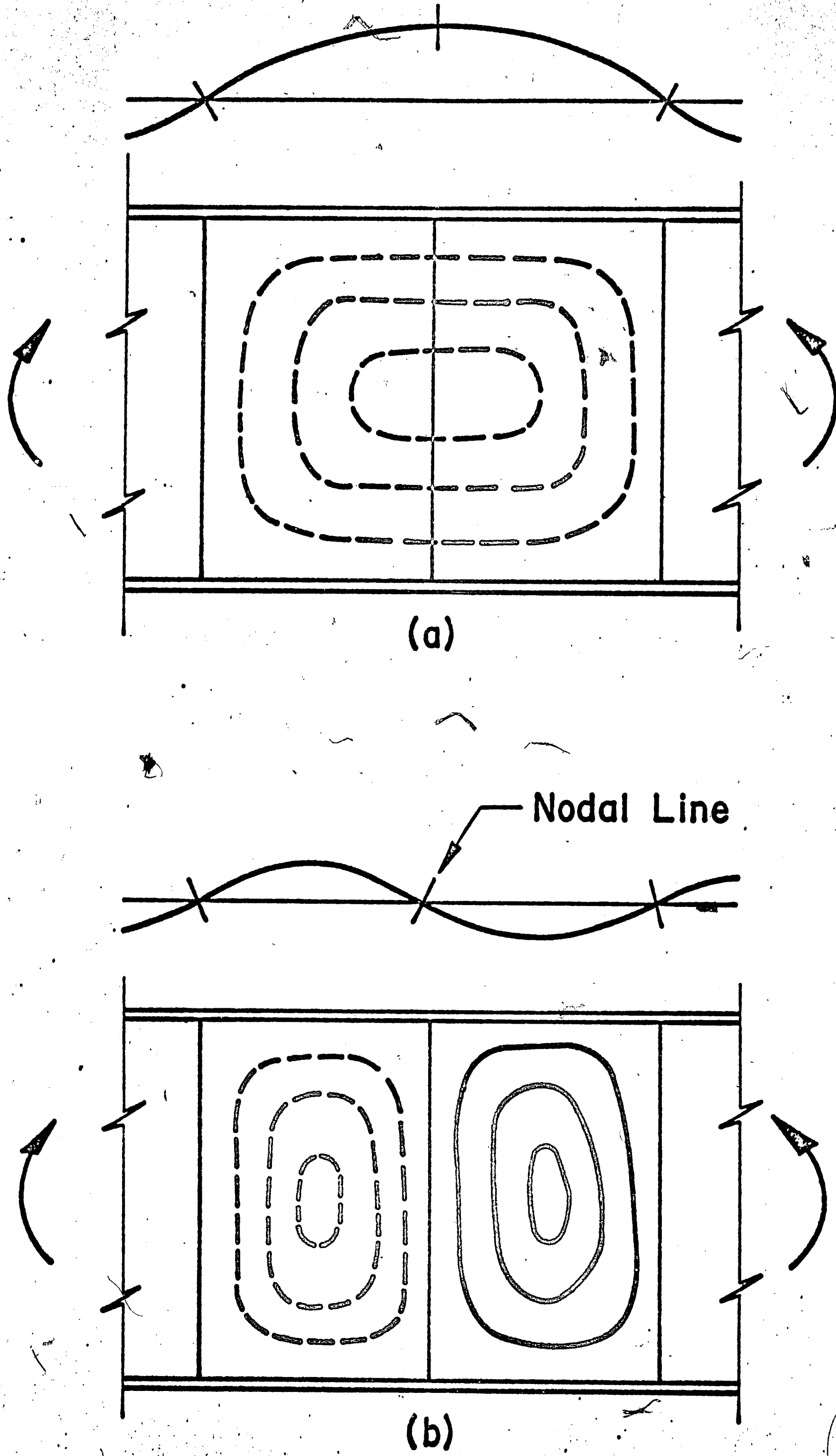


Fig. 6 Rigidity of Transverse Stiffeners

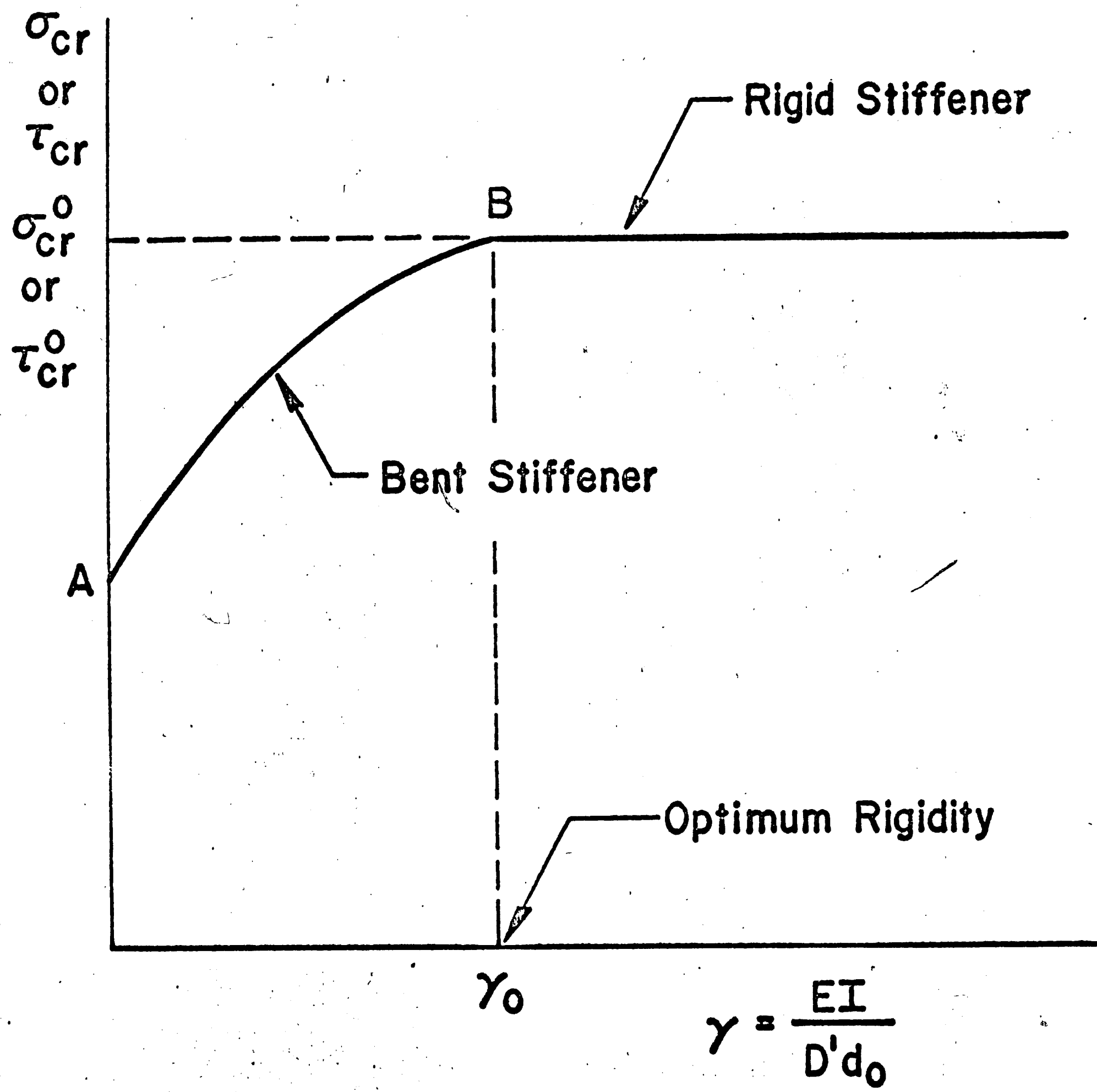


Fig. 7 Relative Rigidity of Transverse Stiffeners vs. Buckling Stress

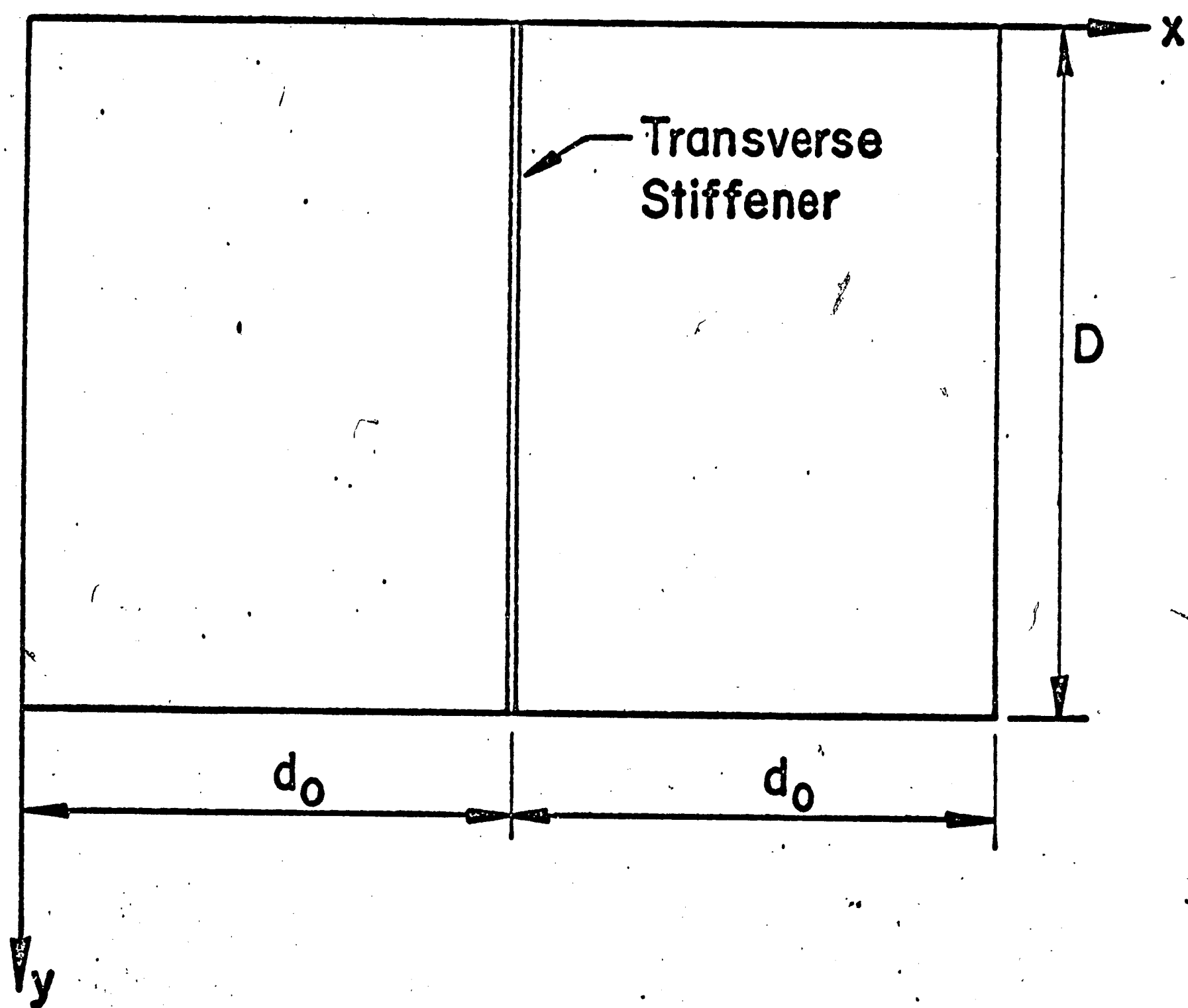


Fig. 8 Web Plate Stiffened Transversely

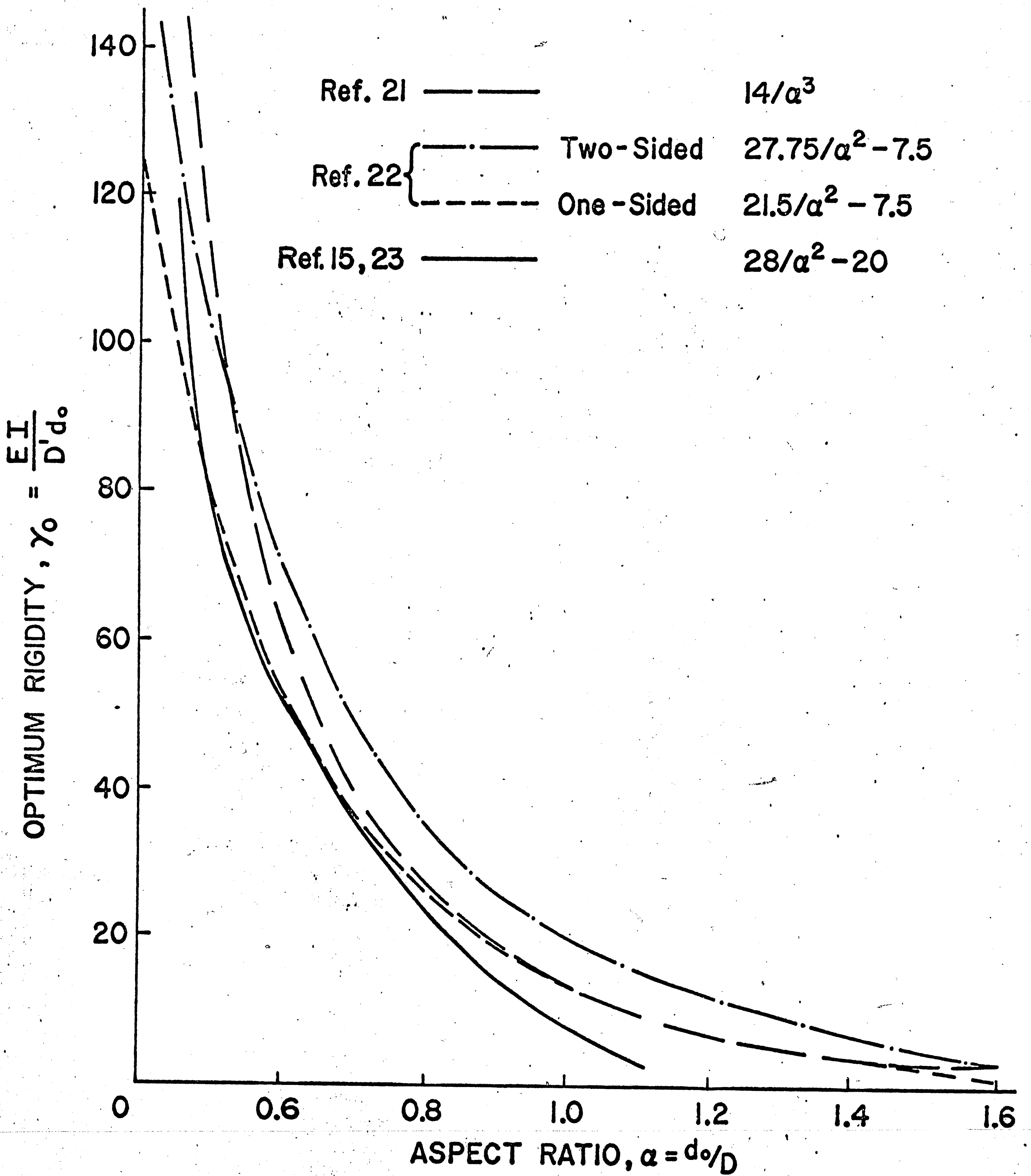
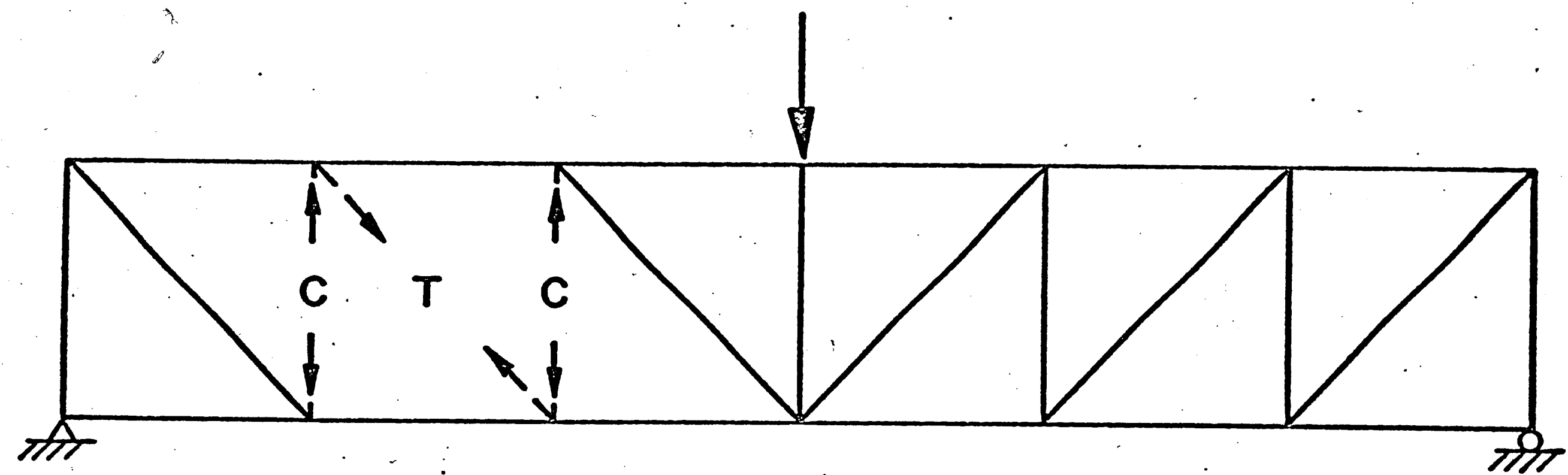


Fig. 9 Optimum Rigidity for Transverse Stiffeners vs. Aspect Ratio



PRATT TRUSS

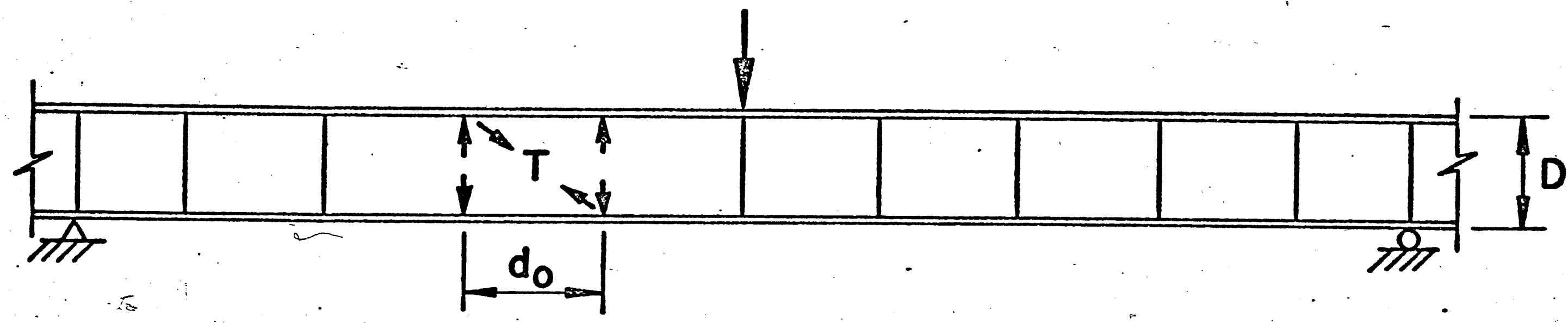


PLATE GIRDER

Fig. 10 Pratt Truss Analogy of Plate Girders

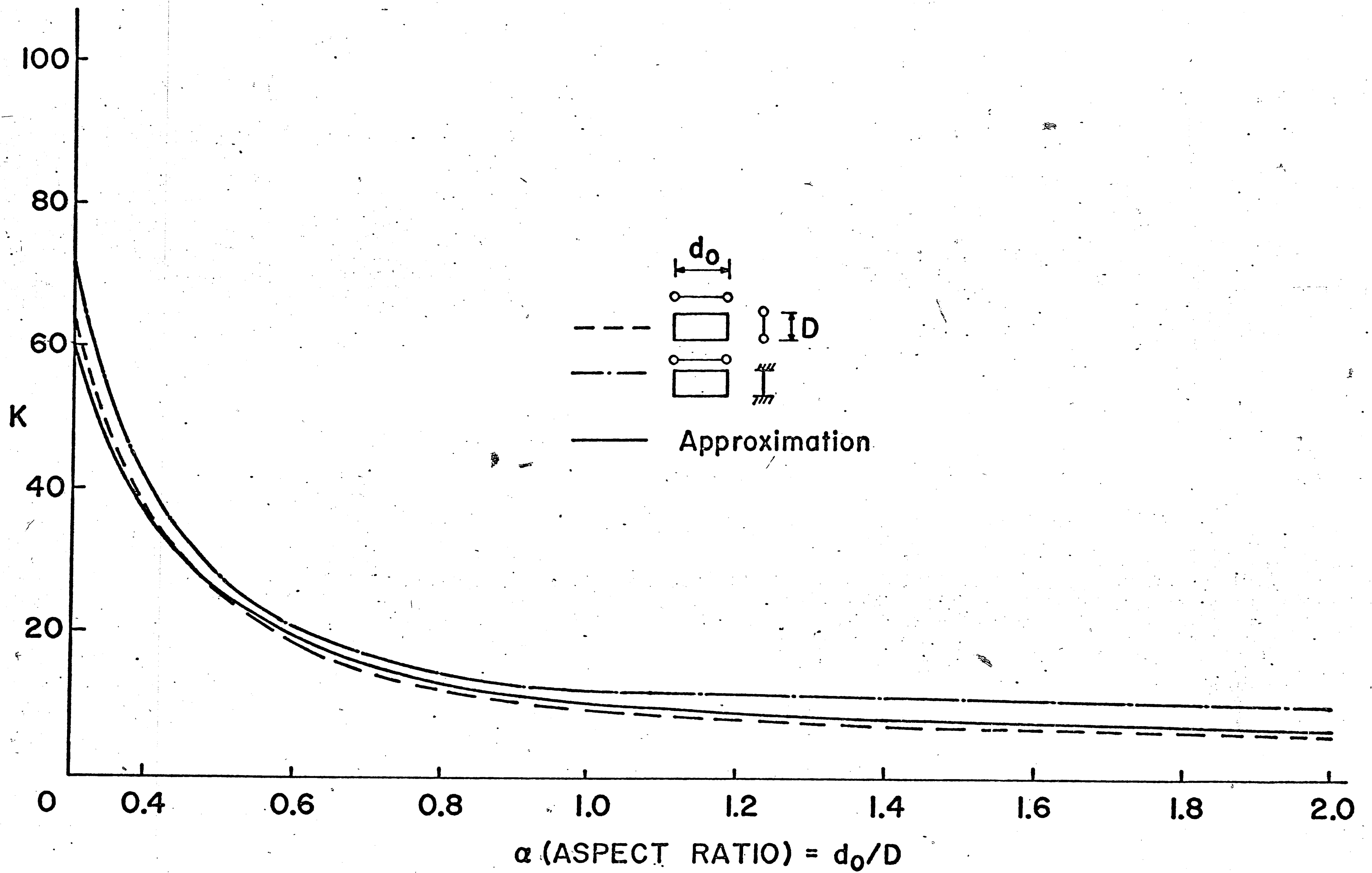


Fig. 11. Shear Buckling Coefficient vs. Aspect Ratio

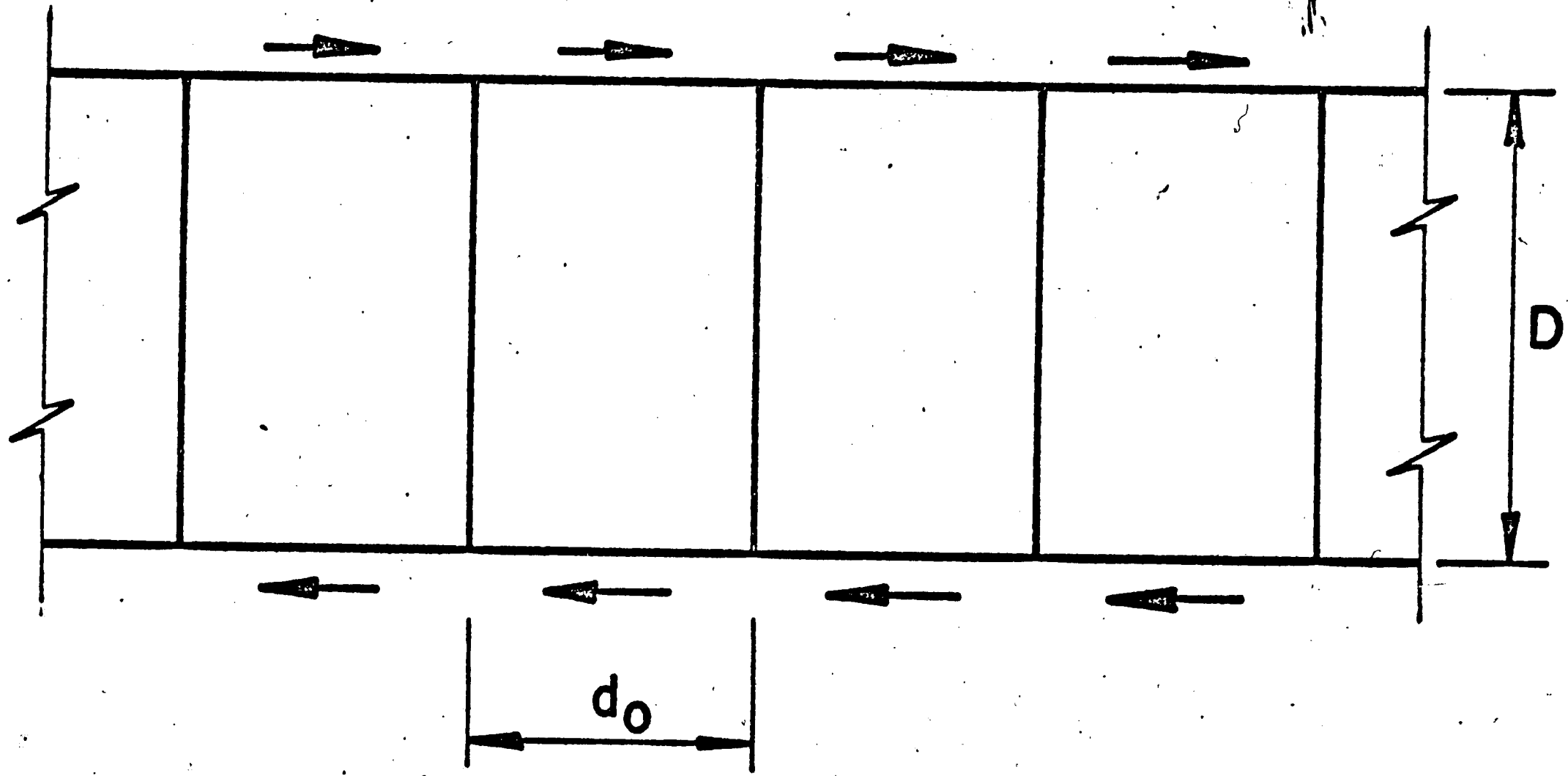


Fig. 12 Infinitely Long, Simply-Supported Plate, With Equidistant Transverse Stiffeners, Under Shear

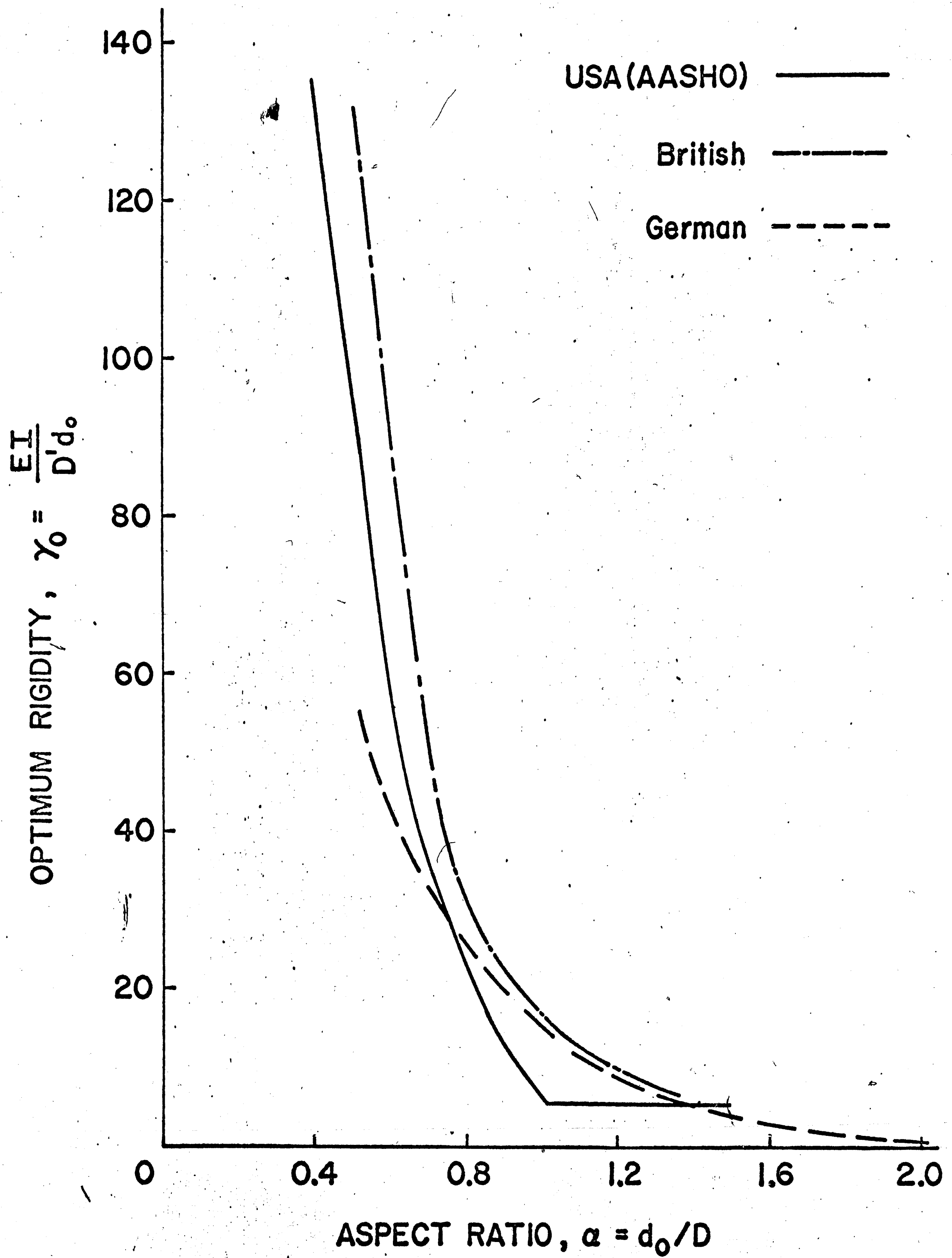


Fig. 13 Rigidity Requirements for Transverse Stiffeners

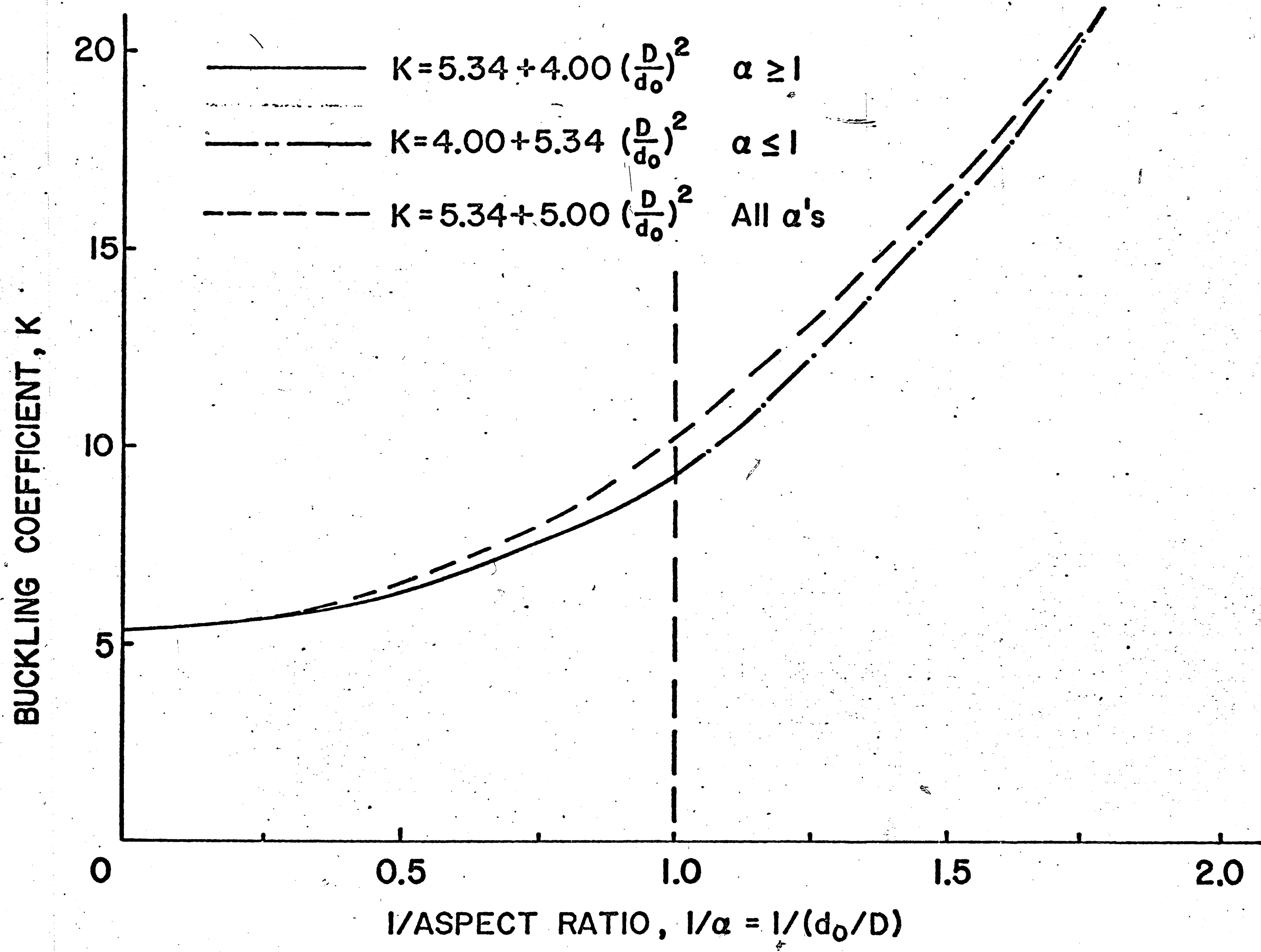


Fig. 14. Shear Buckling Coefficients k vs. Aspect Ratio

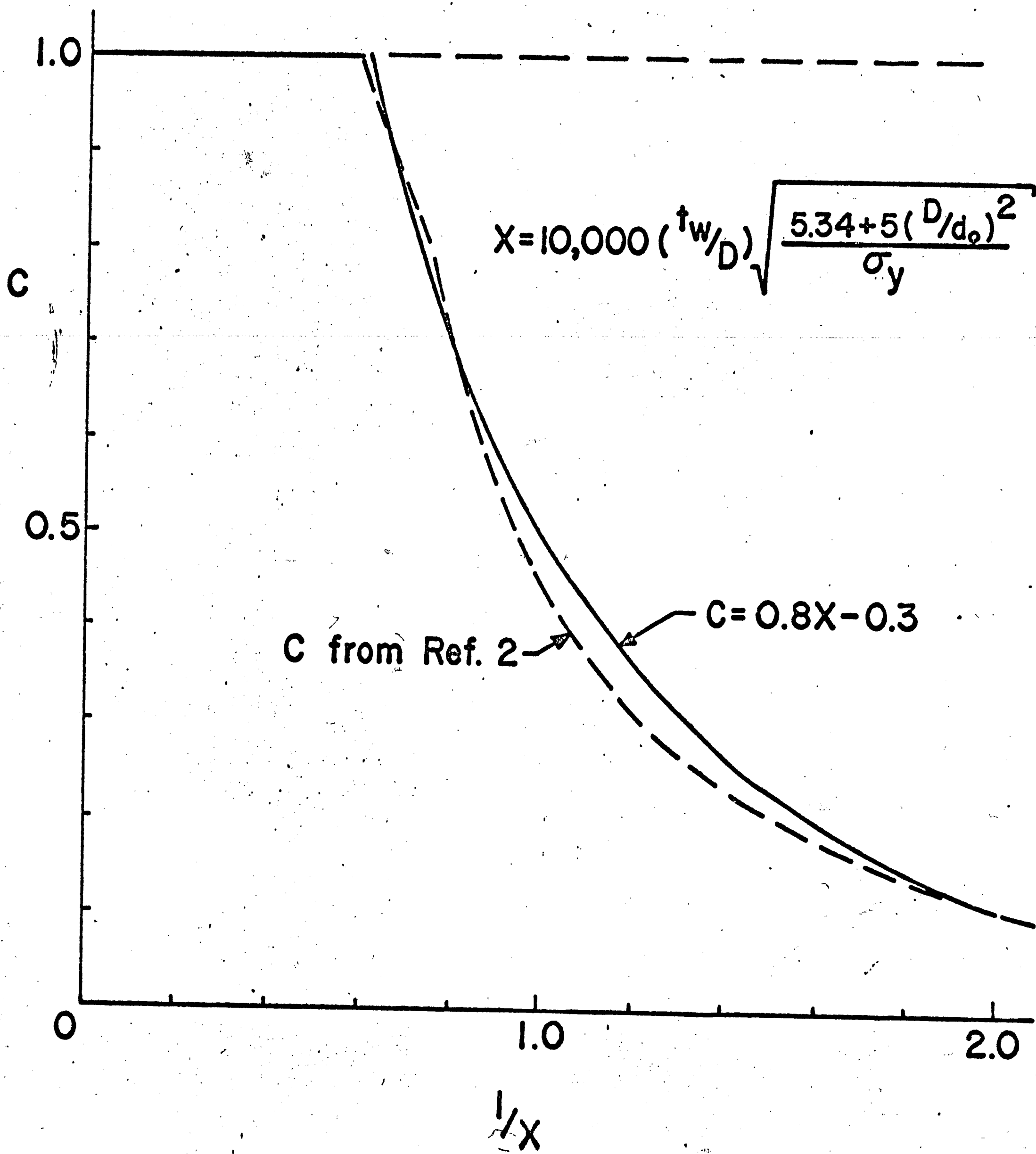


Fig. 15 Approximation for C

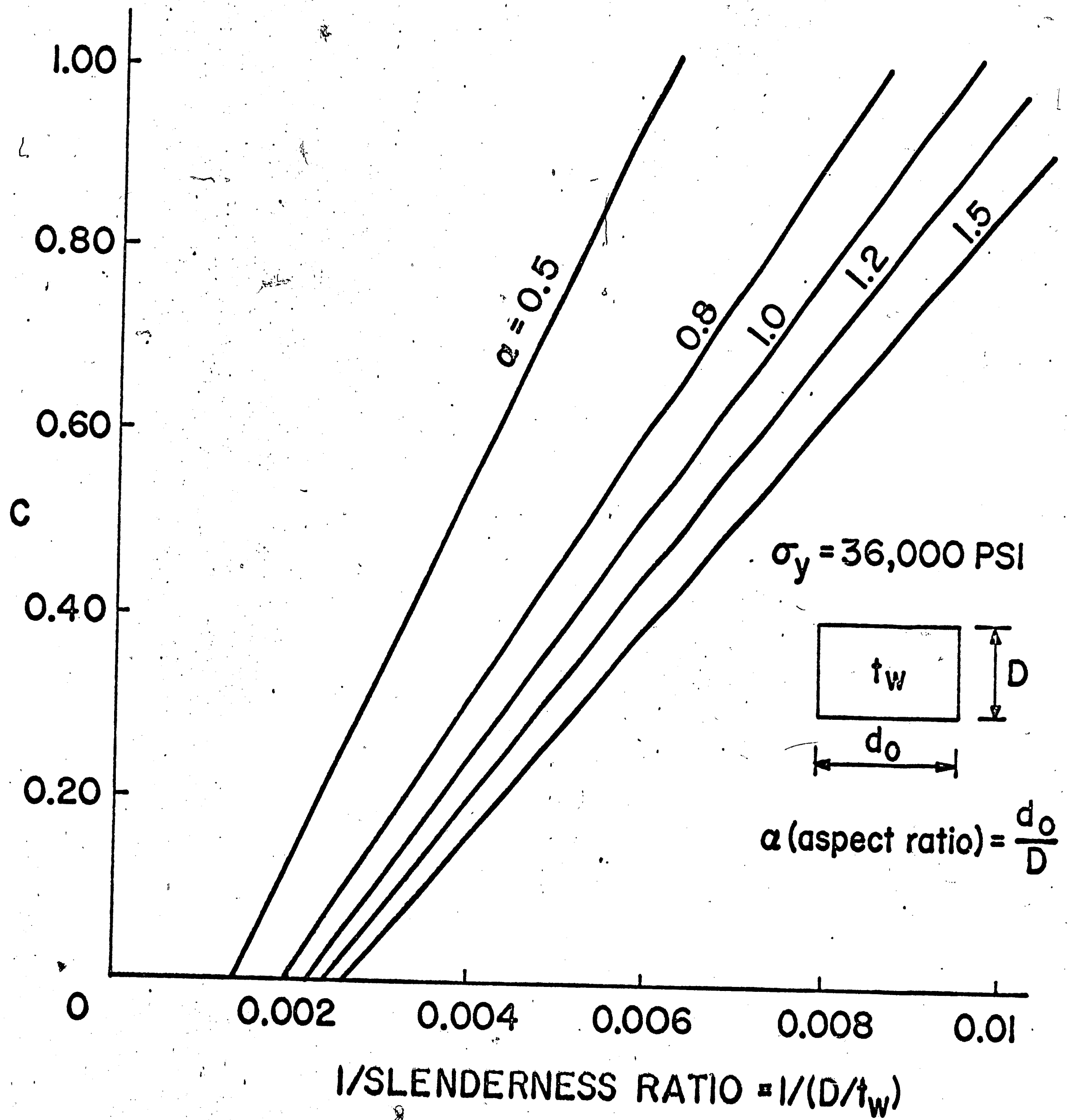


Fig. 16 C vs. t_w/D

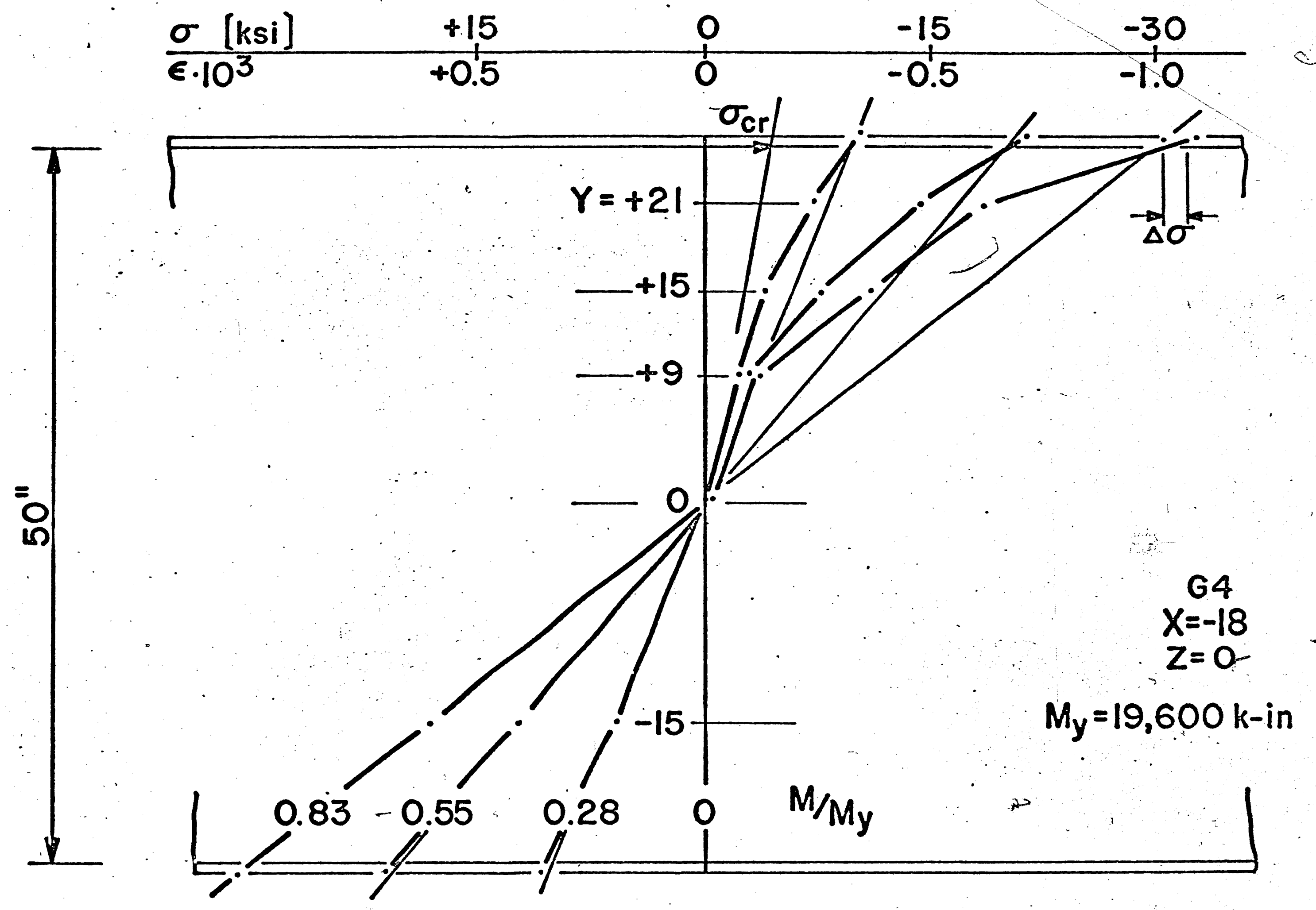
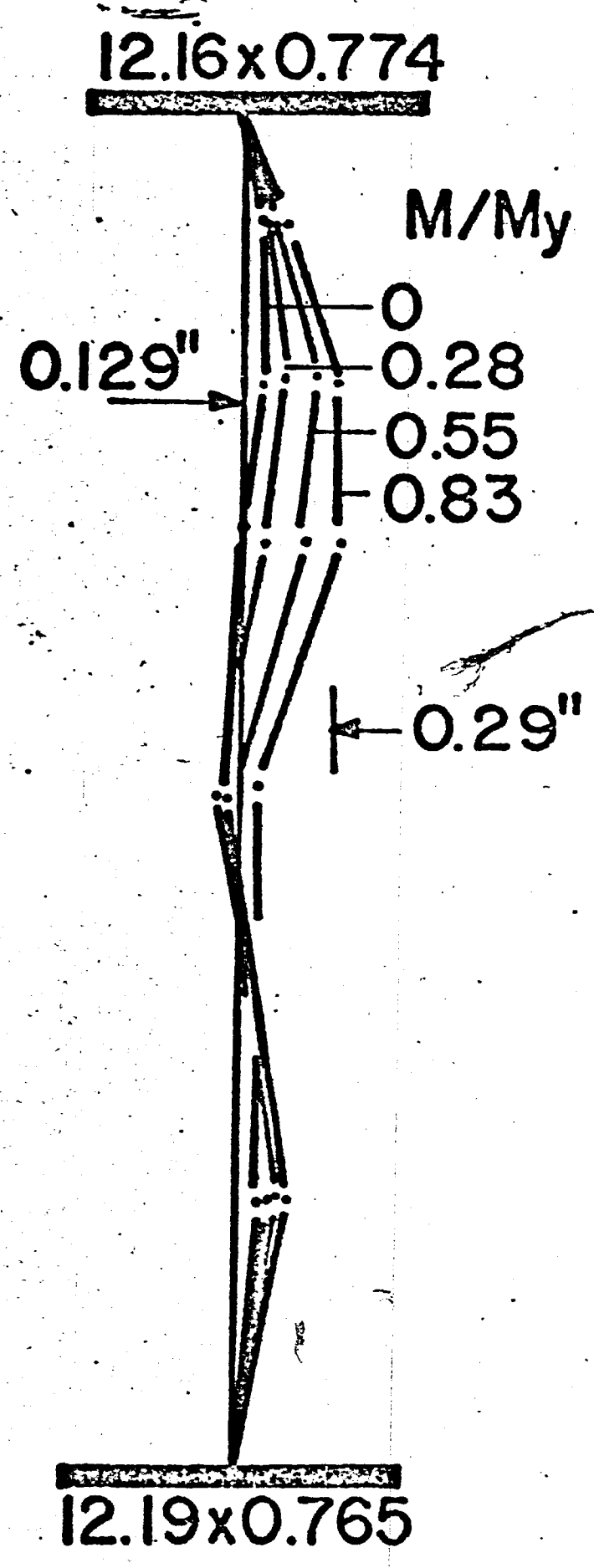


Fig. 17 Measured Web Deflections and Flexural Stresses

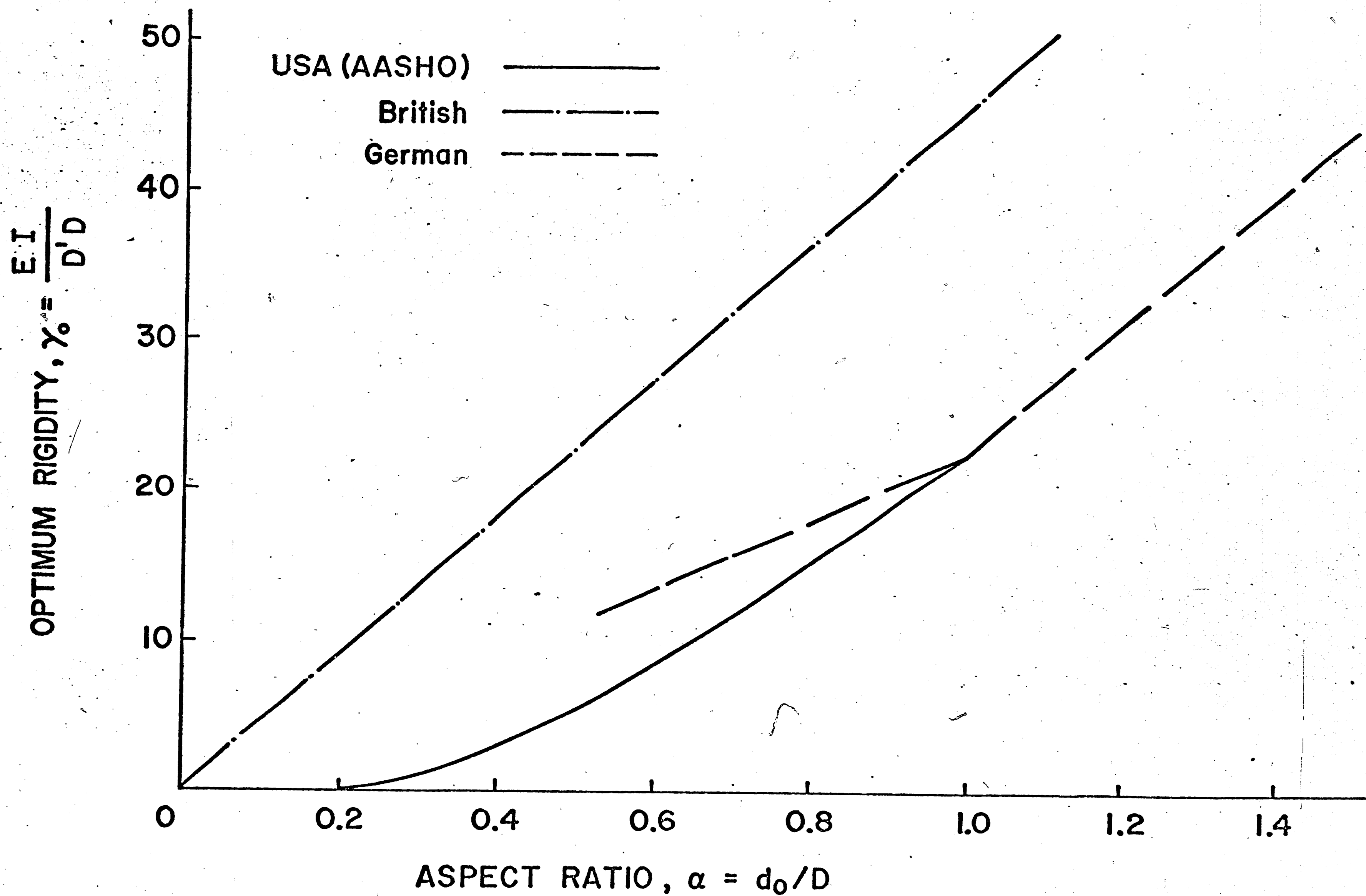


Fig. 18 Rigidity Requirements for Longitudinal Stiffeners

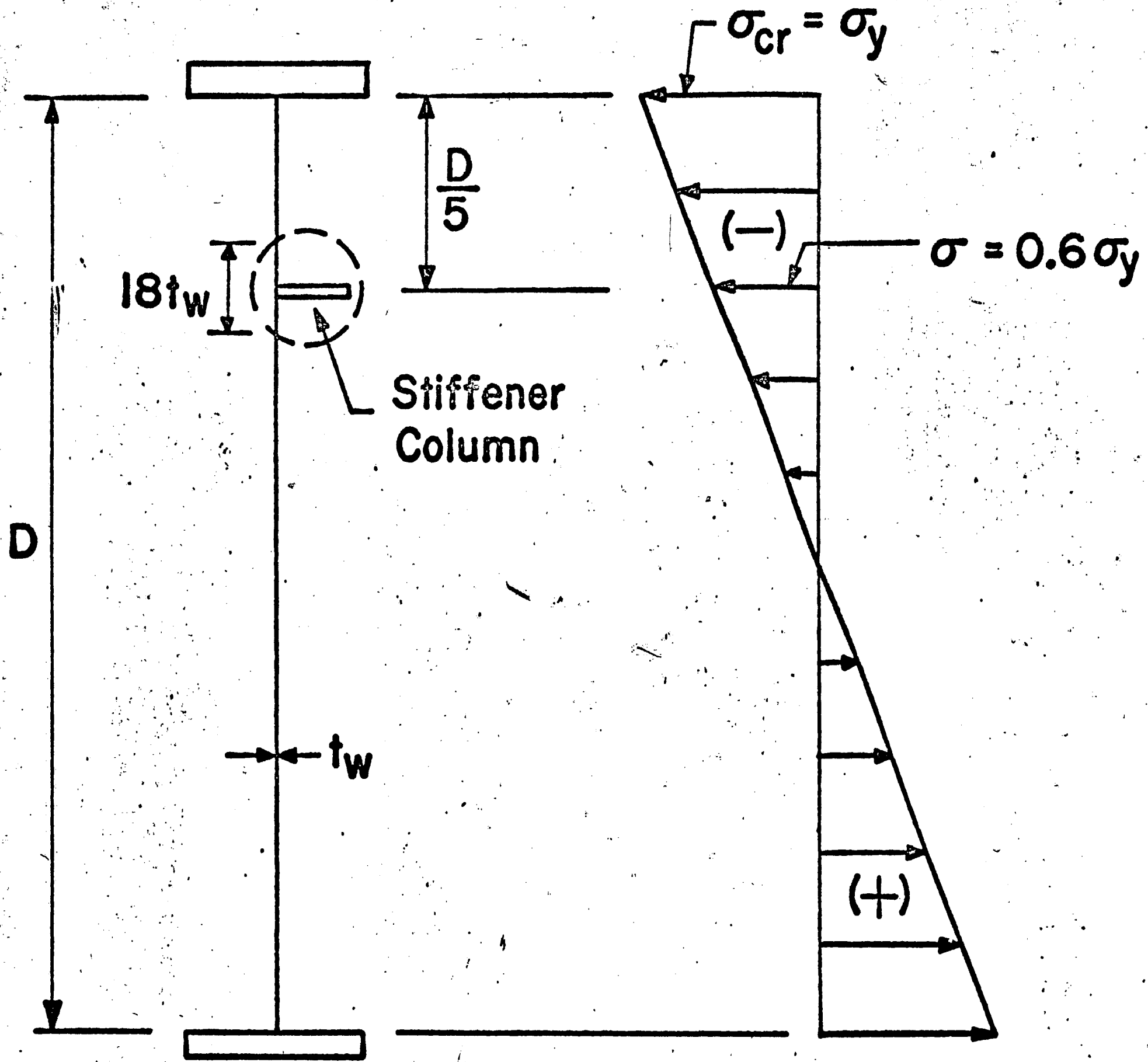
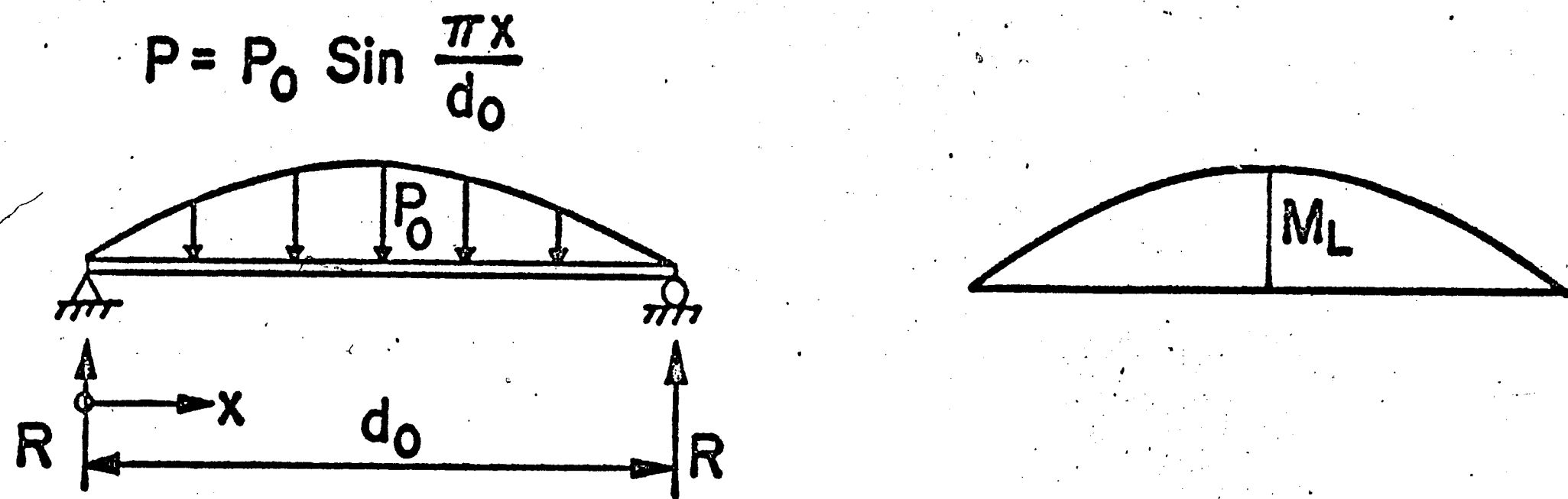


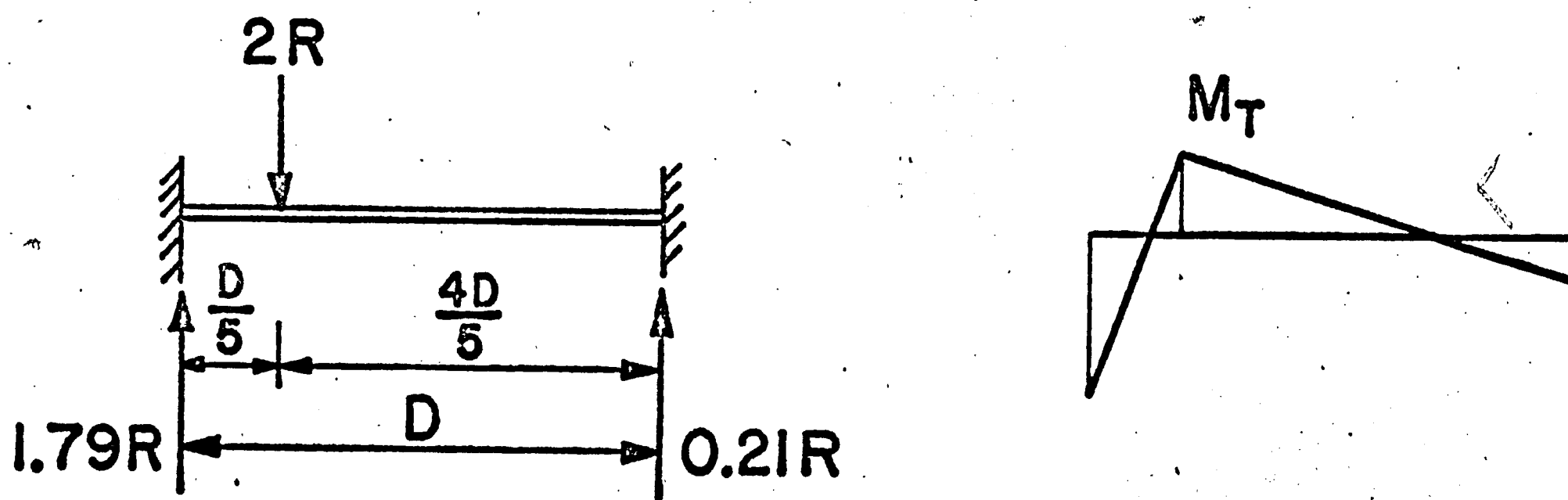
Fig. 19 Stress at Longitudinal Stiffener

Loading

Moment Diagram

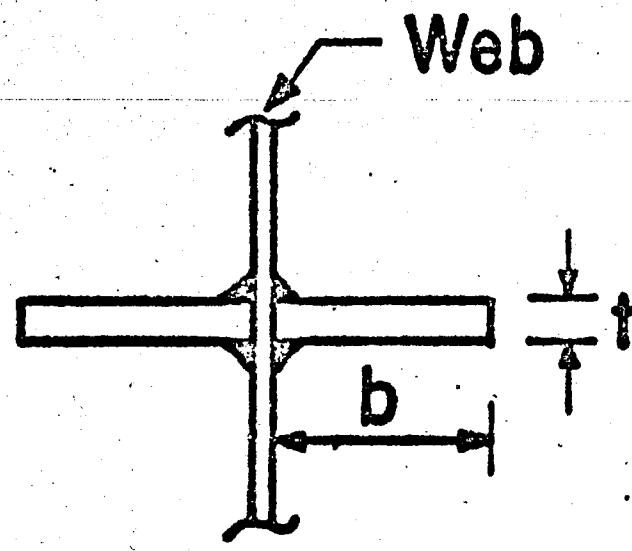


(a) Longitudinal Stiffener

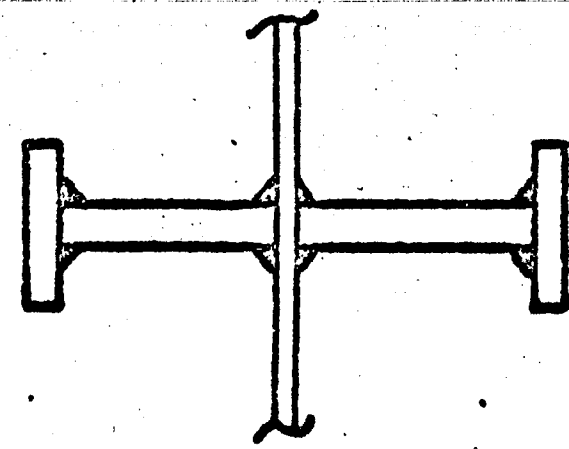


(b) Transverse Stiffeners

Fig. 20 Relationship Between the Section Moduli of Transverse Stiffeners and Longitudinal Stiffeners

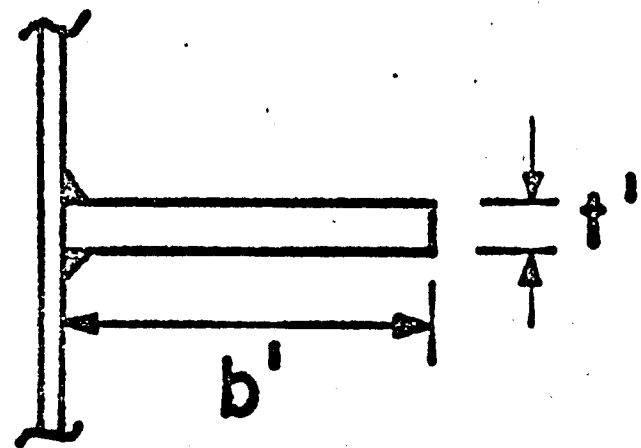


(a)

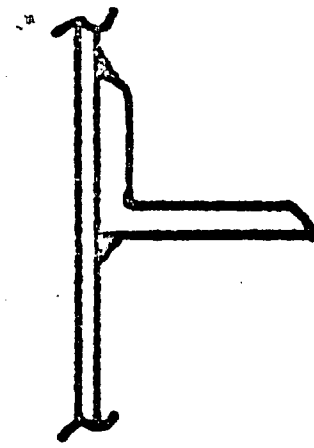


(b)

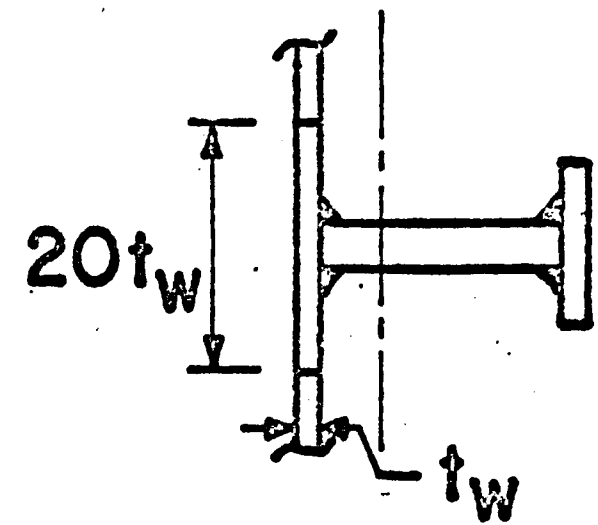
STIFFENER PAIRS



(c)

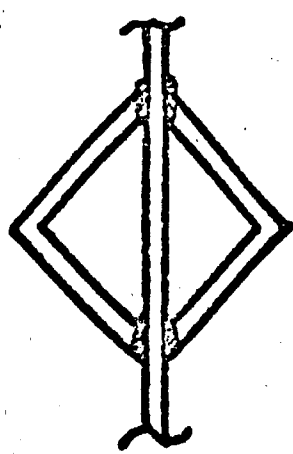


(d)

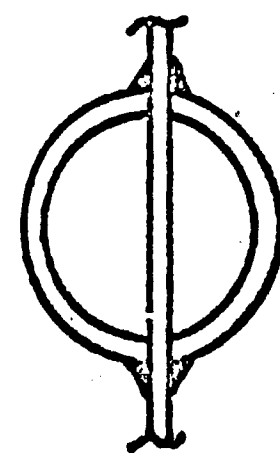


(e)

ONE - SIDED STIFFENERS



(f)



(g)

TUBULAR STIFFENERS

Fig. 21 Stiffener Arrangements

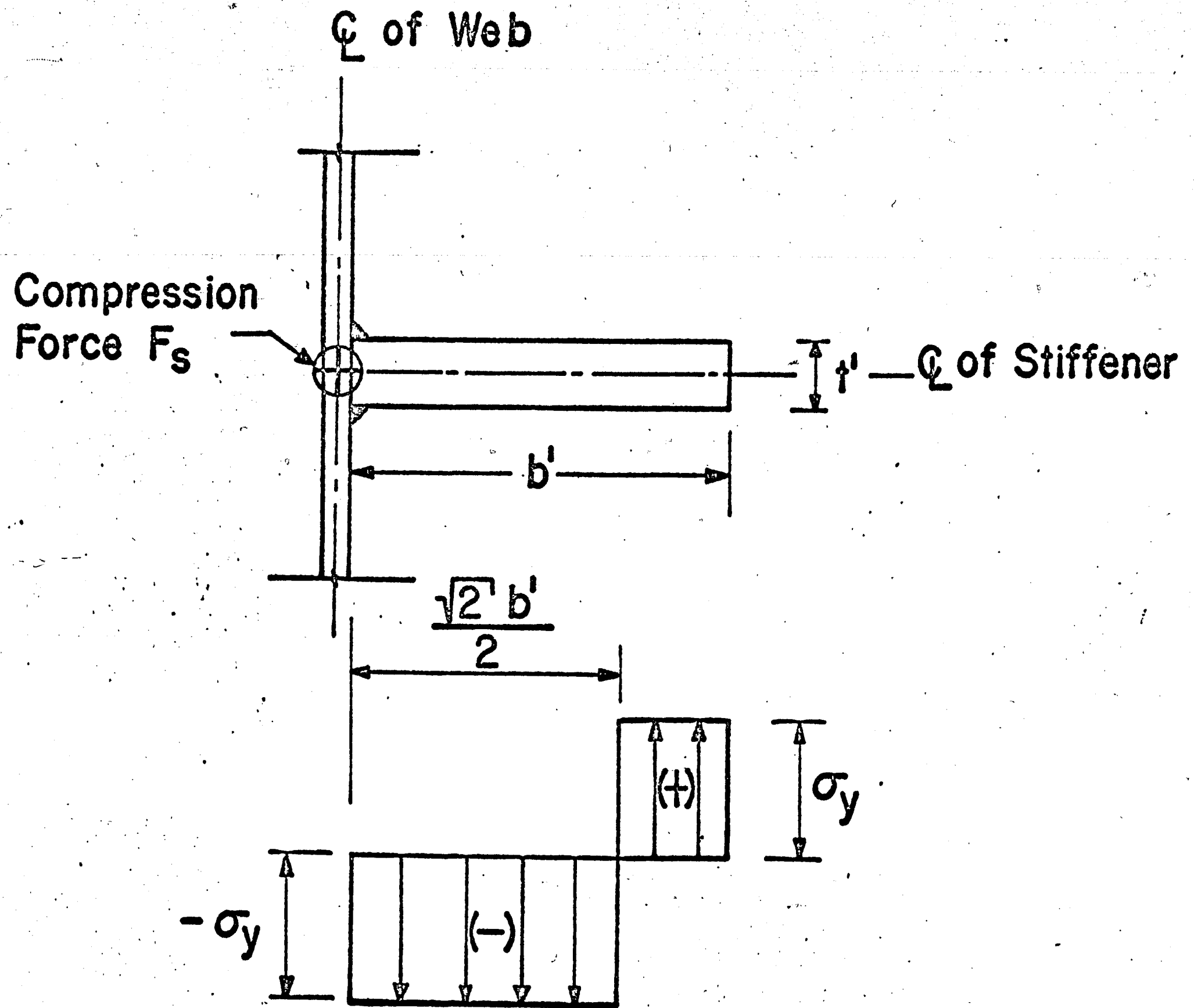


Fig. 22 Stress at One-Sided Stiffener
Due to Compression Force F_s

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