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Effects of dynamic loads on plate girder panels

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EFFECTS OF DYNAMIC LOADS ON
PLATE GIRDER PANELS

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ABSTRACT

This paper presents a brief theoretical and experimental study of the effects of dynamic loads on plate girder panels. Dynamic response of plate girders, expressed in terms of a dynamic load factor, was evaluated for sinusoidally varying loads. These loads were applied to test girders and the resulting dynamic edge forces were computed for panels under bending. The influence of the loads applied dynamically, rather than statically, was found to be practically negligible. Consequently, the dynamic forces on web panels were calculated as if they were applied statically.

The finite difference technique was then used to compute lateral web deflections for limiting cases of boundary restraints. Measured lateral deflections were found to be between those predicted by assuming the stiffener-to-web joint to be fixed and simply supported. These web deflections were then used to compute the corresponding plate bending stresses at the stiffener-to-web joint. The plate bending stresses agreed well with the measured distribution, but differed slightly in magnitude.

It is suggested that further work be done in this area to facilitate the development of design recommendations for bridge plate girders.

1. INTRODUCTION

It is well known that thin-webs of plate girders deflect laterally under load[1]. When plate girders are subjected to repeated loading, the resulting lateral deflections cause fatigue cracks along the boundary of web panels[2]. An attempt appears to be successful in correlating the occurrence of cracks with the plate bending stresses through experimentally obtained web deflections[3].

The purpose of this study is to review and evaluate analytically the effects of dynamic loads on thin-web plate girder panels. The dynamic response of girders subjected to sinusoidal loads are first examined, the possibility of predicting lateral deflections of web plates is explored, and corresponding plate bending stresses are compared with the experimentally obtained values.

In the prediction of web deflections under load, a semi-empirical approach is used and the finite difference method is employed. Although only web panels subjected to bending are examined, the procedure developed should also be applicable to web panels under shear or the combination of bending and shear. It is hoped that the results of this study will help in the formulation of new design recommendations for bridge girders.

2. THEORETICAL CONSIDERATIONS

2.1 Dynamic Response of Plate Girders

For the analysis of dynamic response, the applied load on plate girders are assumed to be sinusoidal. If the maximum applied load is F_1, the magnitude of which is determined by actual condition, then the equation of motion for a single span girder can be expressed as[4]

$$M \ddot{y} + c \dot{y} + k y = F_1 \sin \Omega t \qquad (1)$$

where M is the girder mass

y is the deflection of the girder

$\dot{y}$ is the change in deflection with respect to time, or velocity

$\ddot{y}$ is the change in velocity with respect to time, or acceleration

c is a numerical damping constant

k is the spring constant of the girder

F_1 is the maximum magnitude of the applied sinusoidal load

Ω is the frequency of the applied force

t is the time in seconds

The solution of equation (1) gives the deflection of the girder.

$$y = e^{-\beta t} (C_1 \sin \omega_d t + C_2 \cos \omega_d t) + \frac{(F_1/k)[(1 - \Omega^2/\omega^2) \sin \Omega t - 2 (\beta\Omega/\omega^2)] \cos \Omega t}{(1 - \Omega^2/\omega^2) + 4 (\beta\Omega/\omega^2)^2} \qquad (2)$$

where ω is the natural frequency of the girder without considering damping

$\omega_d = \sqrt{\omega^2 - \beta^2}$, is the natural frequency of the girder considering damping

$\beta = c/2M$, is the damping coefficient

If a plate girder is assumed to be of uniform mass throughout its length, then its lowest natural frequency may be computed by the expression

$$\omega = \frac{\pi^2}{L^2}\sqrt{\frac{E\,I}{m}} \qquad (3)$$

where E is the modulus of elasticity of the girder material

I is the moment of inertia of the girder cross section about its horizontal axis

m is the mass per unit length of the girder

L is the girder span length

The first term on the right hand side of equation (2) represents the contribution of the free vibration of the girder and becomes negligible after a few cycles of load application. By considering only the second term in equation (2), and rearranging, the deflection of a girder may be rewritten as

$$y = \frac{(F_1/k)\,\left[(1 - \Omega^2/\omega^2)^2 + 4\,(\beta\Omega/\omega^2)^2\right]^{\frac{1}{2}}\,\sin(\Omega\,t + \theta)}{(1 - \Omega^2/\omega^2)^2 + 4\,(\beta\Omega/\omega^2)^2} \qquad (4)$$

where θ is merely a phase angle and does not effect the maximum deflection.

It is apparent that this expression is a maximum when the sine is equal to unity. If dynamic load factor (DLF) is defined as the ratio of the dynamic deflection to the deflection which would have resulted from the static application of the load[4], F_1/k, the maximum value of the dynamic load factor is readily computed to be

$$(DLF)_{max} = \frac{1}{[(1 - \Omega^2/\omega^2)^2 + 4\ (\beta\Omega/\omega^2)^2]^{1/2}} \qquad (5)$$

As long as the girder is subjected to loads which do not cause inelastic behavior, the deflection and stresses of the girder are all proportional. The dynamic load factor thus may also be applied to stresses for the evaluation of the effects of dynamic loads.

2.2 Lateral Web Deflection

The dynamic forces applied to a girder induce bending and shearing stresses in the plane of the web equal to those created by static loads multiplied by the dynamic load factor.

$$\sigma_x = (DLF)\,\frac{M\,c}{I}$$
$$\tau_{xy} = (DLF)\,\frac{V\,Q}{I\,t_w} \qquad (6)$$

where x is the horizontal axis, or abscissa

y is the vertical axis, or ordinate

σ_x is the normal stress in the x-direction

τ_{xy} is the shearing stress on the x and y planes

M is the bending moment at a particular cross section

c is the distance from the neutral axis of a cross section to its extreme fiber

V is the vertical shearing force at a particular cross section

Q is the static moment of area

t_w is the web thickness

When the web deflects laterally, in a direction perpendicular to its plane, additional stresses are generated. To estimate these additional stresses necessitates the computation of the lateral deflections of the web.

2.2.1 Basic Equations

The basic equations of equilibrium and compatibility for plates with initial deflections is[5]

$$\nabla^4 w_1 = \frac{\partial^4 w_1}{\partial x^4} + \frac{\partial^4 w_1}{\partial x^2 \partial y^2} + \frac{\partial^4 w_1}{\partial y^4} \tag{7}$$

$$= \frac{1}{D} \left[q + N_x \frac{\partial^2 (w_o + w_1)}{\partial x^2} + N_y \frac{\partial^2 (w_o + w_1)}{\partial y^2} + 2 N_{xy} \frac{\partial^2 (w_o + w_1)}{\partial x \partial y} \right]$$

where w_o is the initial lateral deflection of a plate

w_1 is the additional deflection of a plate

$D = \frac{E t_w^3}{12 (1 - \mu^2)}$, is the flexural rigidity of a plate

μ is Poisson's ratio (0.3 for steel)

q is the intensity of a distributed lateral load, applied perpendicular to the plane of the plate

N_x, N_y are the normal forces per unit length of sections of a plate perpendicular to the x and y-directions, respectively (for example, $N_x = \sigma_x t_w$)

N_{xy} is the shearing force per unit length of a section of a plate perpendicular to the x-axis.

For a plate girder panel, there is no laterally applied load, therefore q = 0. Considering only panels under pure bending as an example, $N_y = N_{xy} = 0$.

Equation (7) now reduces to

$$\frac{\partial^4 w_1}{\partial_x^4} + 2\frac{\partial^4 w_1}{\partial_x^2 \partial_y^2} + \frac{\partial^4 w_1}{\partial_y^4} = \frac{N_x}{D}\left[\frac{\partial^2 w_o}{\partial_x^2} + \frac{\partial^2 w_1}{\partial_x^2}\right] \tag{8}$$

2.2.2 Finite Difference Solution

If initial lateral deflections of a web plate are known at discrete (mesh) points, the finite difference method may be employed to solve for deflections under load. In the case of a plate panel subjected to pure bending, the $\nabla^4 w_1$ operator of equation (8) can be approximated at mesh points, by[6]

$$\begin{aligned}\nabla^4 w_1 = \frac{1}{d_2^4}\Big[&\gamma^4 (w_{m+2\,n} + w_{m-2\,n}) + w_{m\,n+2} + w_{m\,n-2} \\ &+ (6\gamma^4 + 8\gamma^2 + 6)\, w_{m\,n} \\ &+ 2\gamma^2 (w_{m-1\,n-1} + w_{m-1\,n+1} + w_{m+1\,n-1} + w_{m+1\,n+1}) \\ &- 4\gamma^2 (1+\gamma^2)\,(w_{m-1\,n} + w_{m+1\,n}) \\ &- 4\,(1+\gamma^2)\,(w_{m\,n+1} + w_{m\,n-1})\Big]\end{aligned} \tag{9}$$

where $w_{mn} = w_1$ at mesh point m, n the intersection point of the m^{th} row and n^{th} column (An example of mesh point designation is shown in Fig. 1)

$\gamma = d_2/d_1$

d_1 = mesh spacing between rows

d_2 = mesh spacing between columns

and $\partial^2 w_1/\partial x^2$ or $\partial^2 w_o/\partial_x^2$ can be approximated by

$$\frac{\partial^2 w}{\partial_x^2} = \frac{1}{d_2^2}\,(w_{m\,n+1} - 2\,w_{m\,n} + w_{m\,n-1}). \tag{10}$$

Equation (8) can be written in the finite difference form, as expressed in Equations (9) and (10), for every interior mesh point on the plate, thereby generating a set of linear simultaneous equations which can be solved for the w_1 value at each mesh point. This procedure is facilitated by finite difference operators for $\nabla^4 w_1$ and $\partial^2 w/\partial x^2$, shown in Figs. 2 and 3 as $d_2^{\ 4} (\nabla^4 w)$ and $d_2^{\ 2} (\partial^2 w/\partial x^2)$ respectively. The centers of these finite difference operators are placed over a mesh point and the coefficients of the operators are assigned to the corresponding unknown values of w_1 at all mesh points covered by the operator. By applying this procedure to all of the mesh points inside the plate, the set of simultaneous equations can be obtained.

When the finite difference operators are applied to mesh points near the edge of the plate, imaginary mesh points outside the plate must be established for the solution of the simultaneous equations. The deflections of these imaginary mesh points are related to those of the points inside the plate by boundary conditions. For a simply supported vertical edge (Fig. 4), the moment at the edge is zero. Thus $\partial^2 w/\partial y^2 = 0 = 1/d_2^{\ 2} (w_1 - 2 w_o + w_r)$. Since w_o is zero, it follows that $w_1 = - w_r$. Similarly, for fixed vertical edges, $w_1 = w_r$.

With these conditions defined, there are as many equations as unknown values of w_1. The solution of simultaneous equations can easily be obtained through the use of computers. The final deflections of the plate girder web under load are the algebraic sum of the initial deflections w_o, and the computed values w_1.

It must be pointed out that the dynamic loads (N_x) at the boundary of the plate may cause lateral deflections larger than those computed for static loads of the same magnitude. However, without knowing the exact deflection shape of the web plate, it is difficult to estimate the dynamic load factor for the web deflection. Since the loads N_x have already included the dynamic effects of the loads on the girder, any additional effects on the web deflections can be regarded as secondary.

2.3 Plate Bending Stresses

It has been demonstrated that lateral deflections of web plates may cause fatigue cracks along web boundaries and the most significant stresses are the plate bending stresses[2,3]. After the prediction of lateral deflections of the web, the pate bending stresses along the web-boundaries may be estimated by again using the finite difference method. The procedure is described in detail in Ref. 3.

For the computation of stresses along vertical stiffeners, the procedure includes the following steps:

1. With web deflections known at certain points on a web, a double interpolation was made to approximate web deflections at desired mesh points.

2. With deflections under load known at mesh points of adjacent web panels, a finite difference technique is used to estimate at points along the vertical stiffener, the first and second order finite differences.

3. Since the finite differences approximate the derivatives of the web deflection shape and the approximation is better when the mesh spacing is smaller, an extrapolation is made for the determination

of derivatives at points along the stiffener for an infinetely small mesh spacing.

4. As the first and second order derivatives are the slope and curvature, respectively, of the web deflection shape, a web deflection curve perpendicular to the stiffener can be described mathematically. (The coordinates and configuration of a web section are shown in Fig. 5.)

5. The plate bending stresses at the toe of the weld are then computed by

$$\sigma_{bx} = \frac{-E\ t_w/2}{1-\mu^2} \left(\frac{d^2w}{dx^2} \right) \Bigg|_x \tag{11}$$

and the average stress over a short length, such as a strain gage, is calculated from the average curvature of the length:

$$\sigma_{gage} = \frac{-E\ t_w/2}{1-\mu^2} \quad \frac{\int_{x_1}^{x_2} (d^2w/dx^2)\ dx}{x_3 - x_2} \tag{12}$$

A computer program has been developed for the computation of plate bending stresses following the above steps. For a girder with given geometry, material properties, and an estimated initial deflection, the dynamic response of its panels can then be evaluated through the computation of the dynamic load factor, the prediction of web deflections under load, and the estimation of plate bending stresses.

3. TEST SPECIMENS AND RESULTS

In an experimental study of girder behavior under repeated loading[2], thin-web girders were designed according to ultimate strength procedure and loaded beyond the theoretical web buckling strength. For a comparison between theoretically predicted values and experimentally obtained response, brief summaries of tests are given below.

3.1 Specimens

The elevation of two girders, F6 and F10, are shown in Fig. 6 as examples. All their component dimensions are indicated. Both girders were made of structural steel (ASTM A36); both had a two point loading set up with the center panel under uniform bending moment; and both were subjected to sinusoidal loads of 250 cycles per minute. The applied loads were repeated between 5 and 94 kips for girder F6 and 41 and 82 kips for girder F10.

For reference of locations, a Cartesian coordinate system is used, as sketched in girder F10. The origin of the system is at the center of the web.

3.2 Measurements

Various means were employed to monitor static and dynamic behavior of web panels during testing. Measurements of the out-of-plane movement of the web were made during preliminary static loadings by a vertically placed dial gage rig. The rig consisted of a rigid frames upon which Ames dial gages were securely mounted at different y-ordinates. The gages had a least scale division of one one-thousandth of an inch and a stroke of 1 in. Calibrations made throughout the test indicated maximum changes in the order of two or three thousandths of an inch. During testing, the rig was moved from section to section to measure the deflected shape at any desired x-coordinate. The bottom of the rig was supported on the lower flange and against the web of the girder, while the top was attached to the girder web immediately below the compression flange with a magnet. Throughout testing, the flanges remained straight within the range of loads applied. Therefore, the points at which the rig were supported were relatively fixed in space.

Web strains were measured in the preliminary static tests at selected locations using electric resistance (SR4) strain gages. Under repeated load, strain gages were connected to a six-channel Brush Recorder which continuously plotted the measured dynamic strains.

Girder vertical deflections under static loading were measured with an engineer's level and strip scales mounted on girder stiffeners at the mid-depth of the girder. Movements of all loading and reaction stiffeners were so noted. Thus, all support settlements could be determined and girder deflections obtained. The vertical deflections of the girders under dynamic loading were monitored by a "slip gage" which employed an Ames dial gage and indicated the maximum deflections of the girders to the nearest thousandth of an inch[2].

It is important to know how the dynamic response of girders were measured. First, predetermined static loads were applied and the corresponding deflections noted. Then the dynamic loads were applied and gradually increased until the dynamic deflections equaled those of the static load. The dynamic load corresponding to these deflections was then recorded.

3.3 Results

In all cases of testing ten girders in fatigue, the static and dynamic loads which produced the same vertical deflection of a girder were identical as read from the load indicator of the machine. This indicates that the influence of the dynamic load on girders was less than the accuracy of the machine indicator, about two per cent.

Results of measurements of web deflections from an imaginary perfect plane are listed in Tables 1 and 2 for mesh points of the bending panels of girders F6 and F10, respectively. (Mesh points of girder F10 are shown in Fig. 1). Both initial deflections (w_o) and deflections under maximum static loads (w_m) are indicated in the tables. Double interpoltion was used to approximate the deflections where the mesh points were not exactly the points of measurement. For lack of accuracy, no web deflections were measured under dynamic loading.

The web stresses from recorded web strains under static and fatigue loads are reported in Ref. 3. The deviations of the dynamic strains from the static values are only slight, again within the accuracy of the recording instruments.

5. COMPARISON OF RESULTS

4.1 Dynamic Effects on Girders

By employing equation (3) for girders F6 and F10, the natural frequencies are:

for girder F6:

$$\omega_1 = \frac{\pi^2}{(31 \times 12)^2} \sqrt{\frac{30 \times 10^6 \times 11.66 \times 10^3}{\frac{3500}{32.2 \times 12} \times 31 \times 12}} = 270 \text{ rad/sec} \qquad (13)$$

for girder F10:

$$\omega_1 = \frac{\pi^2}{(32.5 \times 12)^2} \sqrt{\frac{30 \times 10^6 \times 23.4 \times 10^3}{\frac{5350}{32.2 \times 12} \times 32.5 \times 12}} = 290 \text{ rad/sec} \qquad (14)$$

The frequency of the applied forces are 250 cpm or 4.17 rad/sec.

If the dynamic load factors are computed neglecting damping, that is, $\beta = 0$, then the equation for the maximum dynamic load factor becomes

$$(DLF)_{max} = \frac{1}{1 - \Omega^2/w^2} \qquad (15)$$

Thus, for girder F6:

$$(DLF)_{max} = \frac{1}{1 - (4.17)^2/(270)^2} = 1.025 \qquad (16)$$

and for girder F10:

$$(DLF)_{max} = \frac{1}{1 - (4.17)^2/(290)^2} = 1.020 \qquad (17)$$

These values of the dynamic load factor indicate that the effect of pulsating loads upon girder deflections without damping is a two per cent and a two and one-half per cent increase of deflection over that of the static loads for girders F6 and F10, respectively. With damping, the dynamic load factors are smaller. Furthermore, if Eq. 15 is adjusted for loading conditions confirming to the actual situation of fluctuating loads, the dynamic load factors are even smaller, probably less than one per cent. Dynamic effects of this magnitude are not measurable as observed in the tests, and therefore are practically negligible.

4.2 Web Deflections

By using the initial web deflections from measurements (Fig. 7) and assuming the plate boundary forces

$$N_x = \sigma_x t_w = (DLF) \frac{M c}{I} t_w = (1.0) \frac{M c}{I} t_w \qquad (18)$$

lateral web deflections under maximum loads for girders F6 and F10 are predicted for two boundary conditions. In one case, the top and bottom edges of the plate girder panels are considered fixed and the two vertical edges simply supported (w_s); in the other case all four edges are considered fixed (w_f). The predicted increase of web deflection under load and the total deflections are given in Table 1 and 2.

Also listed in the tables are the measured deflections at corresponding mesh points. Since the actual boundary condition is somewhere between the two assumed cases, it would be expected that the measured values fall between the predicted ones. That this is so clearly shown in Fig. 8 for the bending panel of girder F6. (Note that the scale for lateral deflection is very large).

For girder F10, the deflections are much smaller than those of girder F6. Although the predicted deflections do not agree very well with the measured values, they are nevertheless of the same order of magnitude. This result is regarded as satisfactory, considering the accuracy of the measurements and the approximation in computation, as well as the unusual measured deflection shape.

In order to predict deflections more accurately, the restraints to deflection provided by the stiffeners and the flanges must be known. It is sufficient here to say that, under dynamic loads, girder webs deflect laterally to the same order of magnitude as under static loads.

4.3 Stresses

For the estimation of plate bending stresses at web boundaries, the average of predicted web deflections is used in the procedure of Section 2.3. The resulting stresses at the toe of the weld along a stiffener of girder F6 are plotted in Fig. 9. For comparison, the corresponding stresses obtained from measured web deflections are shown. Quantitatively, the predicted plate bending stresses are higher than the "measured" ones. Qualitatively, the predicted and the measured stress distribution agree well with other and with the deflections. This indicates that the prediction of plate bending stresses can be accomplished but that improvement of accuracy must be made.

More important is the result that recorded stresses under static and dynamic load are the same for all practical purposes. If the accuracy of predicting static stresses is improved, the plate bending stresses under dynamic load can be estimated. Any possible crack of girder webs due to repeated loads can then be prevented beforehand.

5. CONCLUSIONS

In conclusion, it can be said that the effect of dynamic loads on plate girder panels are negligible for all practical purposes. This is concluded through the comparison of results of brief analysis and of testing thin-web plate girders. The following items are the results:

1. The dynamic load factors for the thin-web test girders are practically unity. Consequently, the dynamic forces on the web panels can be assumed equal to those by static loading.

2. Lateral deflection of web plates may be estimated by the finite difference method when the initial deflection and the boundary restraint are known. For thin-web girders with fairly large web deflections, measured lateral deflections lie between those computed assuming fixed and simply supported panel edges.

3. By assuming a web deflection shape halfway between the fixed and the simply supported condition, web plate bending stresses along stiffeners are estimated. The estimated stresses are higher than those obtained through measurements of web deflections, but with the same distribution shape. Better results should be possible if exact boundary restraints are known.

4. In experimental investigation, the recorded strains under static and dynamic loads were almost the same, indicating that the dynamic effects are negligible for the girders studied.

Further work may now be done to estimate web plate bending stresses under dynamic load for practical plate girder panels, to compare the stresses with the static and fatigue properties of the girder material, and to consider the influence of those stresses on design of girders.

6. NOMENCLATURE

c	damping coefficient
d_1	vertical spacing between mesh points
d_2	horizontal spacing between mesh points
D	flexural rigidity of a plate
DLF	Dynamic Load Factor
E	modulus of elasticity
F_1	maximum magnitude of applied sinusoidal load
k	spring constant
m	mass per unit length
M	total mass; bending moment
N_x, N_y	normal forces per unit length of sections of a plate perpendicular to the x and y-directions, respectively
N_{xy}	shearing force in the direction of the y-axis per unit length of section of a plate perpendicular to the x-axis
P	load
Q	static moment of area
q	intensity of distributed lateral load
t	time, in seconds
t_w	thickness of the web
V	vertical shearing force at a particular cross section
y	deflection
$\dot{y}$	velocity
$\ddot{y}$	acceleration
w	lateral deflection of web

w_o	initial lateral deflection
w_1	additional web deflection due to applied load
β	damping coefficient
γ	d_2/d_1
μ	Poisson's ratio
σ_{bx}	plate bending stress in the x-direction
σ_{gage}	plate bending stress at strain gage
σ_x	normal stress in the x-direction
τ_{xy}	shearing stress in the x and y-planes
ω	natural frequency of the undamped system
ω_d	natural frequency of the damped system
Ω	frequency of the applied sinusoidal load

TABLES & FIGURES

TABLE 1

WEB DEFLECTIONS AT MESH POINTS, in 10^{-3} in.
GIRDER F6 (P = 94 kips)

Mesh Point	Initial W_o	Additional (W_1) S	Additional (W_1) F	Total W_s	Total W_m	Total W_f
1,1	- 93	-105	- 61	-198	-173	-154
1,2	-108	-144	-105	-252	-229	-213
1,3	- 90	- 77	- 52	-167	-150	-142
1,4	- 85	- 13	0	- 98	- 84	- 85
1,5	- 68	- 29	- 6	- 97	- 81	- 74
2,1	-138	-222	- 71	-260	-232	-209
2,2	-168	-116	- 58	-284	-252	-226
2,3	-125	-117	- 69	-242	-209	-194
2,4	- 96	- 33	- 7	-129	-114	-103
2,5	- 55	- 3	- 6	- 58	- 52	- 49
3,1	-169	- 13	25	-182	-156	-144
3,2	-179	- 38	4	-217	-190	-175
3,3	-112	- 34	- 9	-146	-130	-121
3,4	- 78	- 21	- 2	- 99	- 87	- 80
3,5	- 32	- 8	0	- 40	- 34	- 32
4,1	- 90	51	41	- 49	- 43	- 39
4,2	-103	41	53	- 62	- 54	- 50
4,3	- 58	22	32	- 34	- 28	- 26
4,4	- 55	8	20	- 47	- 39	- 35
4,5	- 30	19	22	- 11	- 10	- 8
5,1	- 29	19	21	- 10	- 9	- 8
5,2	- 45	29	33	- 16	- 13	- 12
5,3	- 15	0	2	- 15	- 16	- 13
5,4	- 30	5	13	- 25	- 20	- 17
5,5	- 24	22	23	- 2	- 1	- 1

TABLE 2

WEB DEFLECTIONS OF MESH POINTS
GIRDER F10 (P = 82 kips)

Mesh Point	Initial W_o	Additional (W_1) S	Additional (W_1) F	Total W_s	Total W_m	Total W_f
1,1	17	18	3	35	16	20
1,2	31	28	12	59	50	43
1,3	40	28	19	68	63	59
1,4	35	19	15	54	65	50
1,5	34	16	15	50	70	49
1,6	25	8	8	33	45	32
1,7	10	- 1	- 2	9	9	8
2,1	7	31	4	38	11	11
2,2	20	46	17	66	48	37
2,3	25	46	26	71	66	51
2,4	24	37	26	61	76	50
2,5	24	29	24	53	83	48
2,6	13	17	13	30	56	26
2,7	- 5	2	0	- 3	9	- 5
3,1	2	33	4	35	- 6	6
3,2	- 6	43	13	37	2	7
3,3	-12	44	22	32	0	10
3,4	-19	39	25	20	- 8	7
3,5	-16	31	22	15	0	6
3,6	-18	21	14	3	- 8	- 4
3,7	-19	10	5	- 9	-19	-14
4,1	- 7	16	2	9	-10	- 5
4,2	-15	25	8	10	-23	- 7
4,3	-28	28	14	0	-44	-14
4,4	-46	30	19	-16	-66	-27
4,5	-40	24	16	-16	-55	-24
4,6	-35	19	12	-16	-53	-23
4,7	-36	16	10	-20	-40	-26
5,1	- 4	4	0	0	-13	- 3
5,2	-17	9	4	- 8	-28	-13
5,3	-24	11	6	-13	-41	-18
5,4	-33	13	9	-20	-50	-24
5,5	-27	10	6	-17	-43	-21
5,6	-29	10	7	-19	-41	-22
5,7	-42	14	10	-28	-40	-32

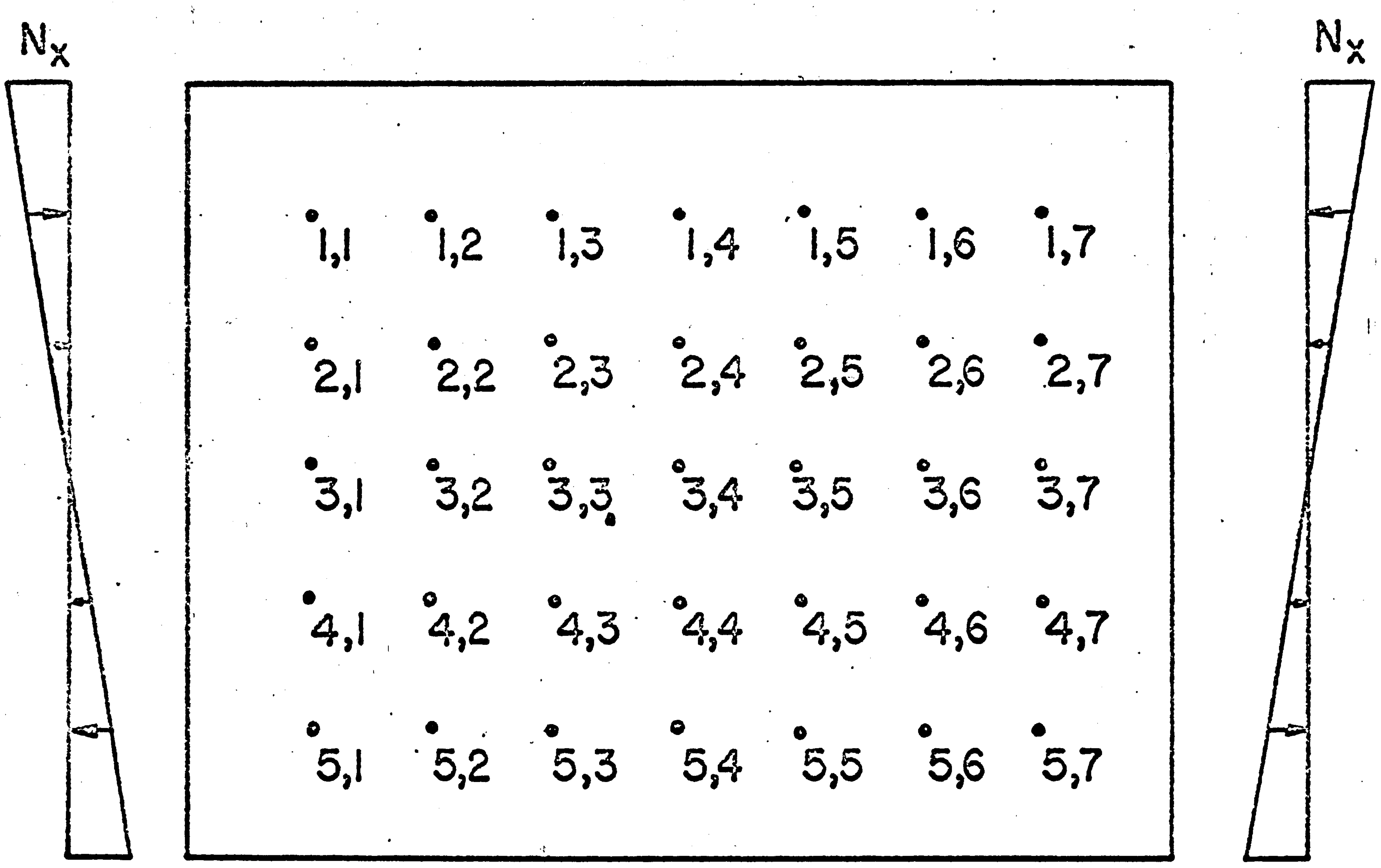


Fig. 1 Designation of Finite Difference Mesh Points, Panel In Bending

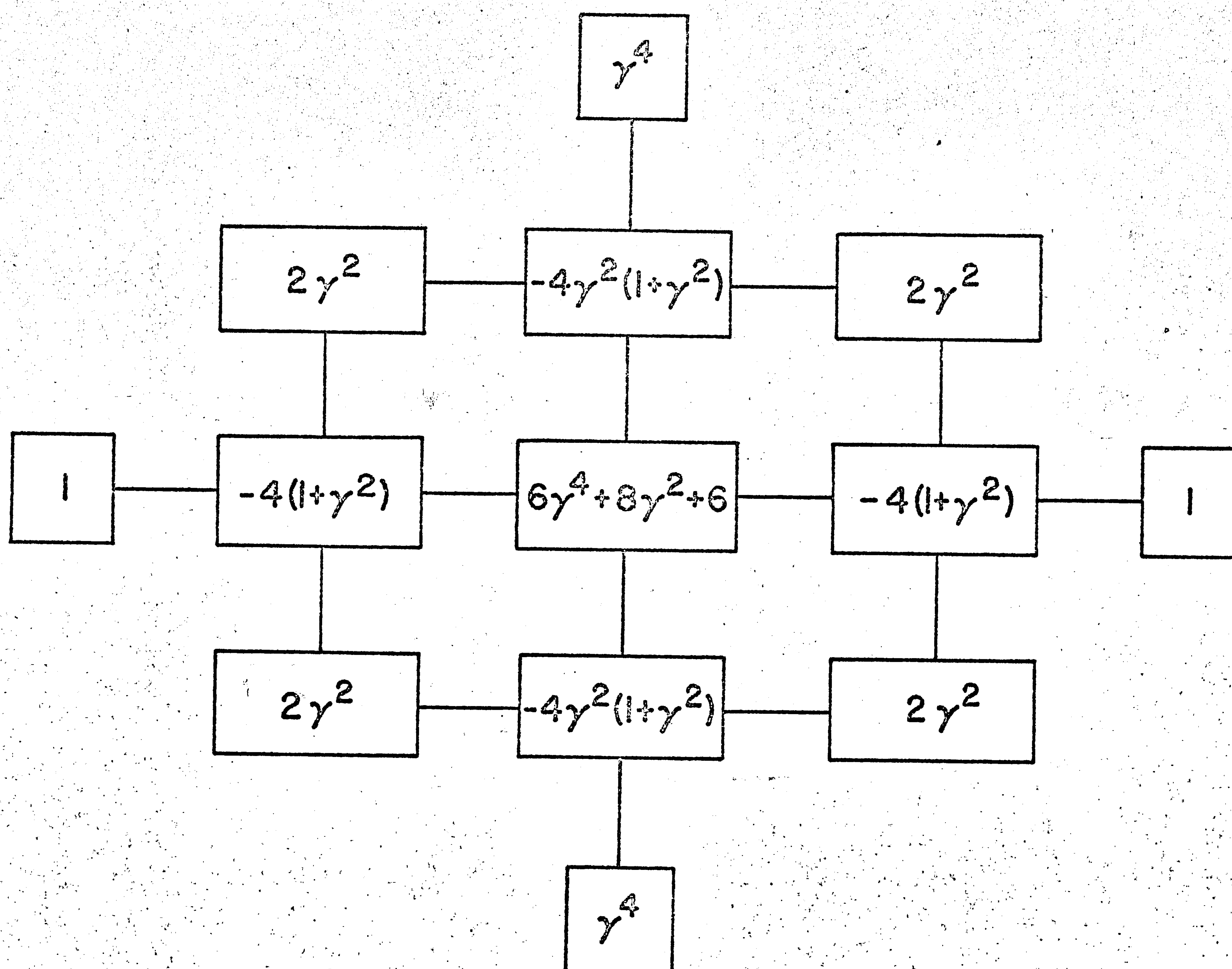


Fig. 2 Finite Difference Operator, $d_2^{\ 4}$ $(\nabla^4 w)$

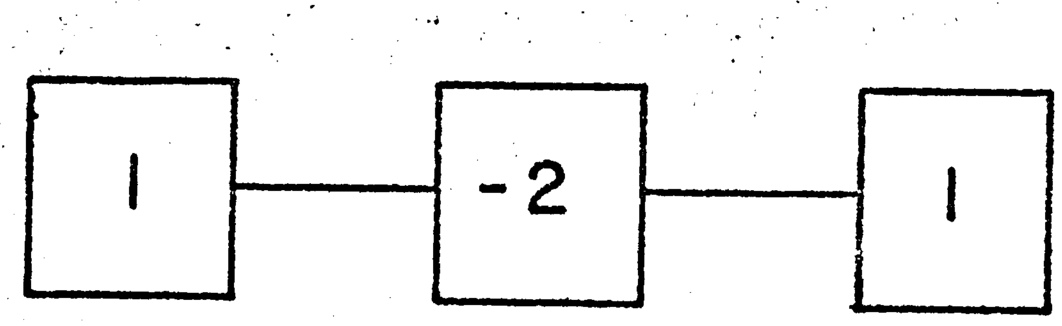
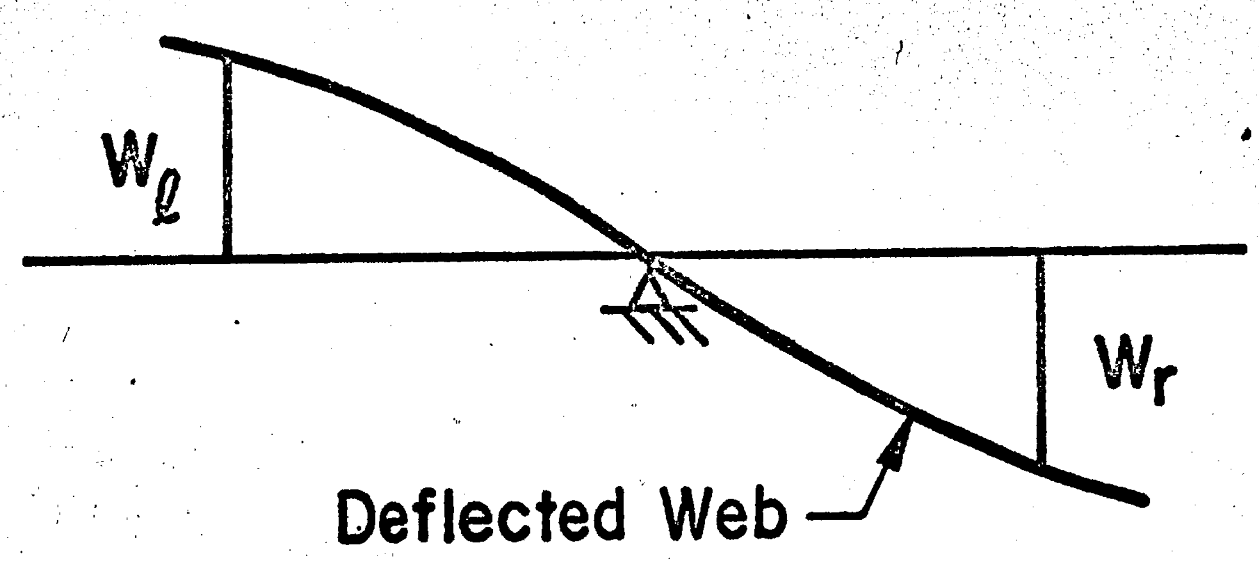
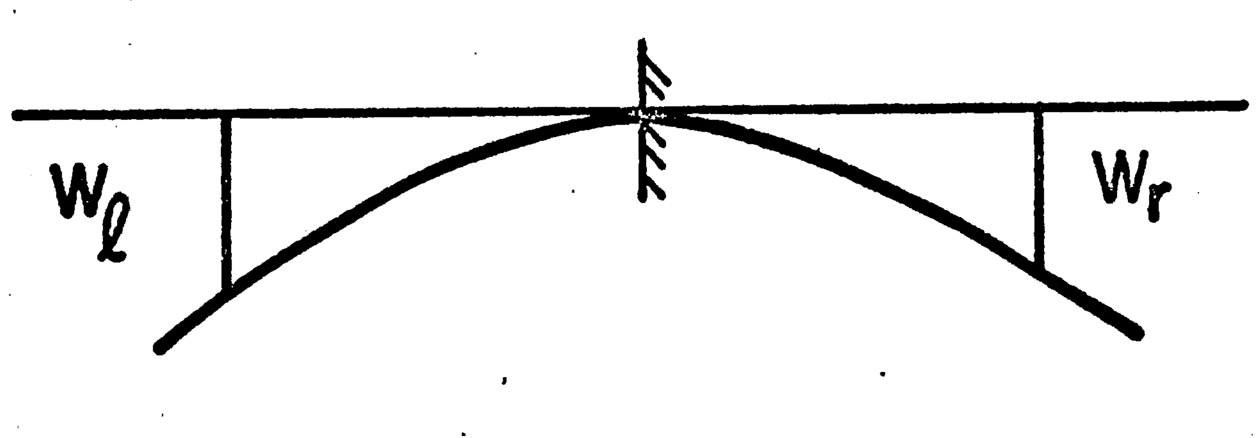


Fig. 3 Finite Difference Operator, $d_2^{\ 2}$ $(\partial^2 w/\partial x)$



(a) Simply Supported Edge $W_\ell = -W_r$



(b) Fixed Edge $W_\ell = W_r$

Fig. 4 Web Deflections at Plate Boundary

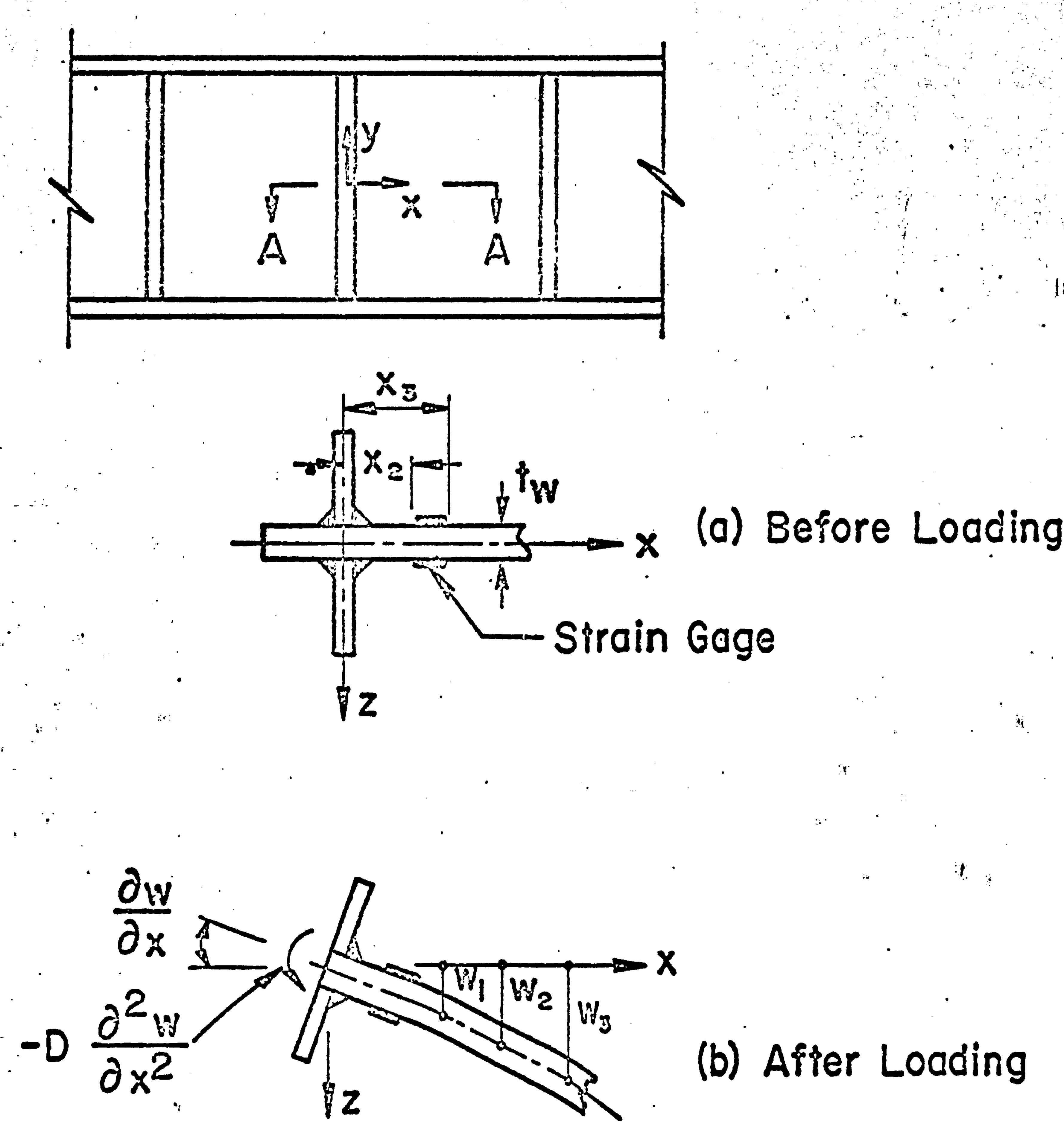


Fig. 5 Section Through Stiffener-to-Web Joint

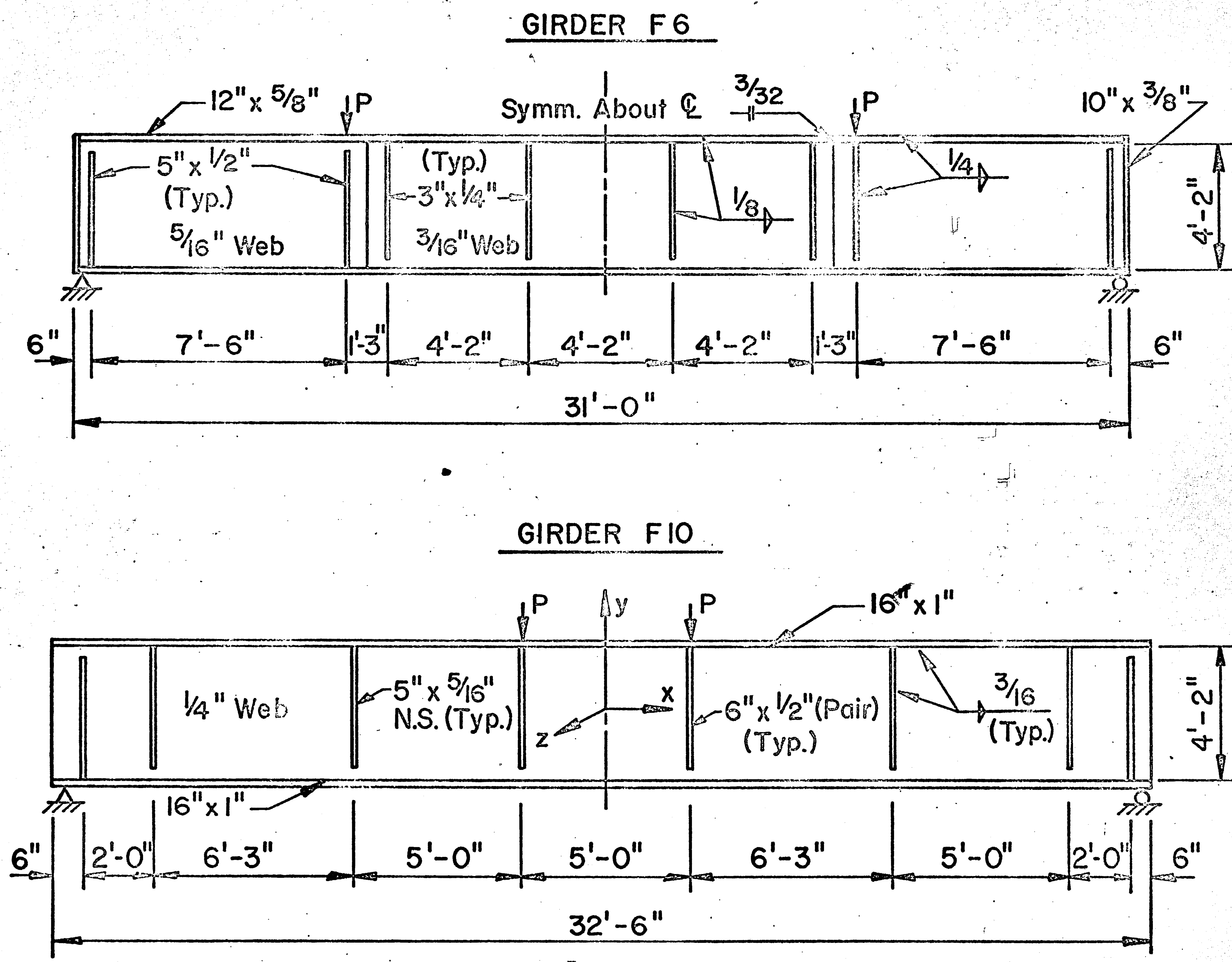
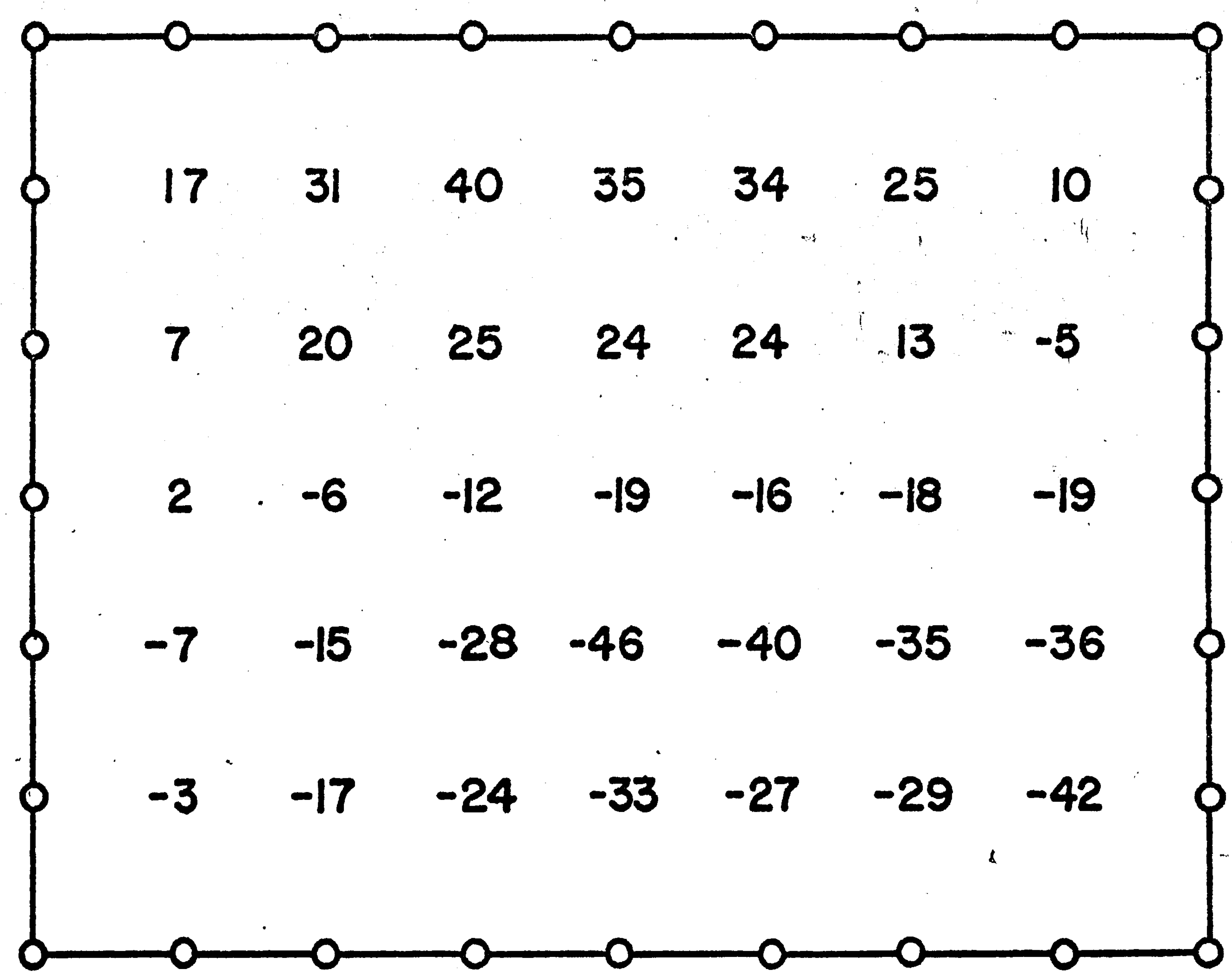
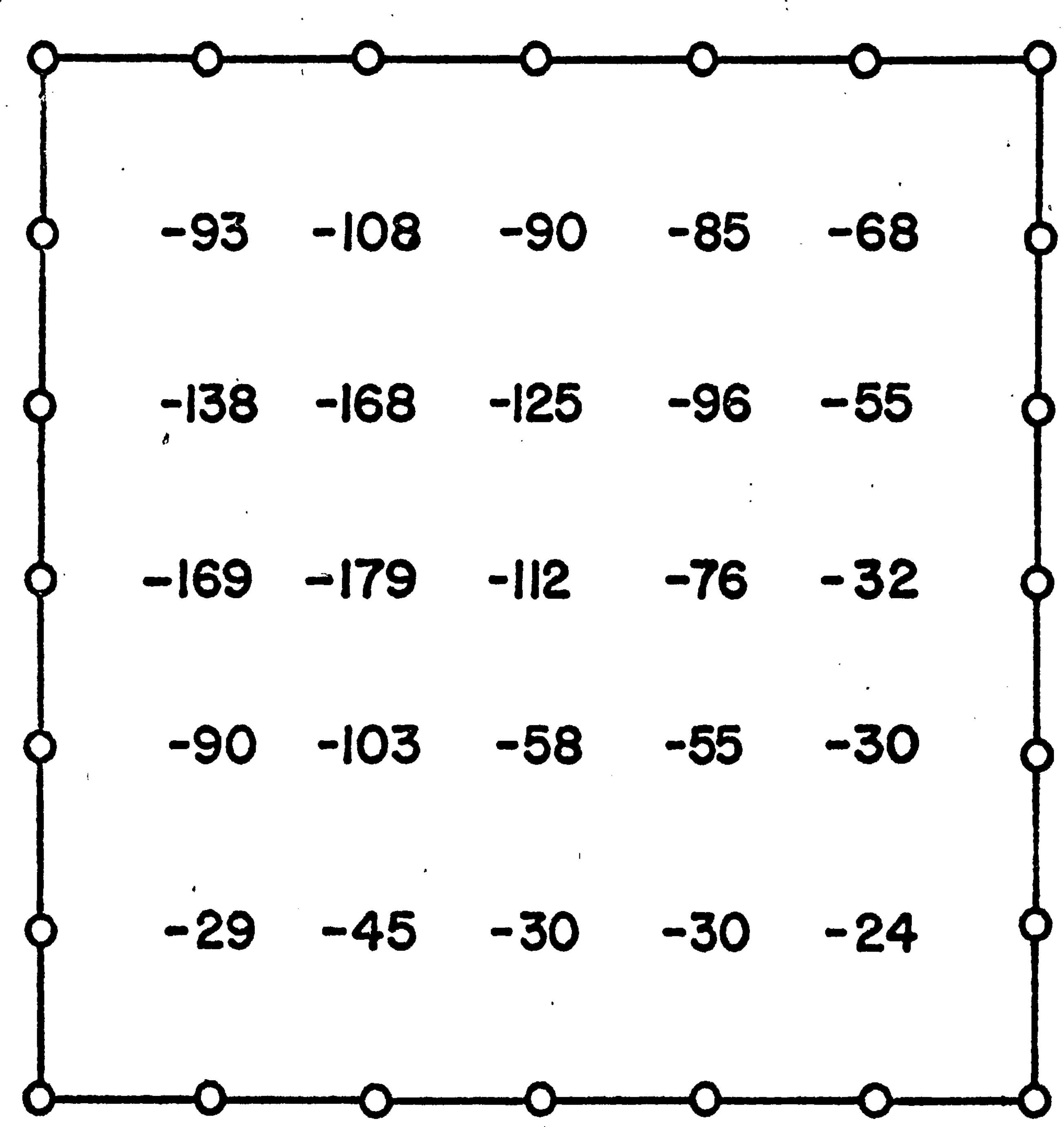


Fig. 6 Geometry of Plate Girders



GIRDER F10 - PANEL 3



GIRDER F6 - PANEL 3

Fig. 7 Initial Web Deflections of Girder Webs

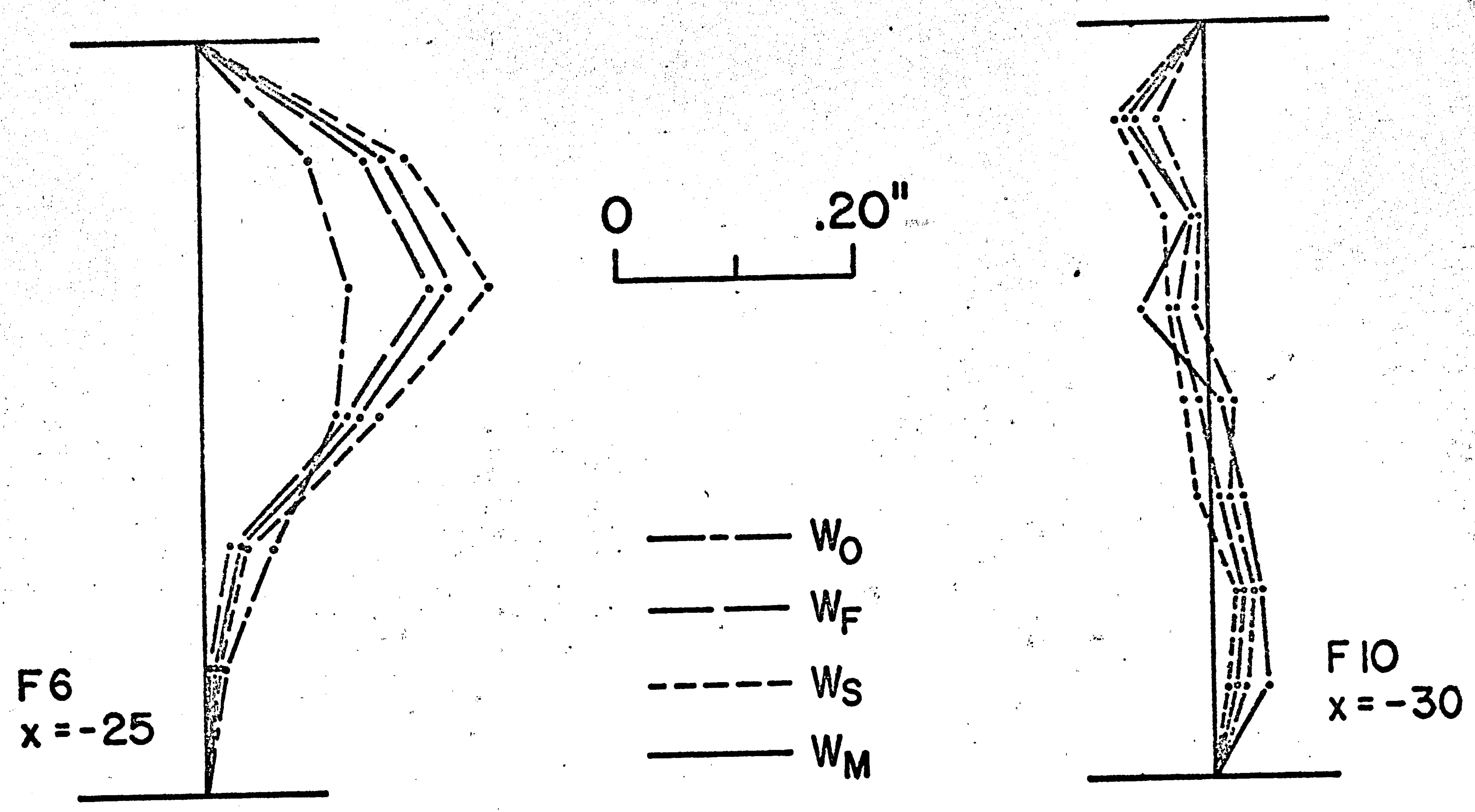


Fig. 8 Lateral Deflections of Girder Webs

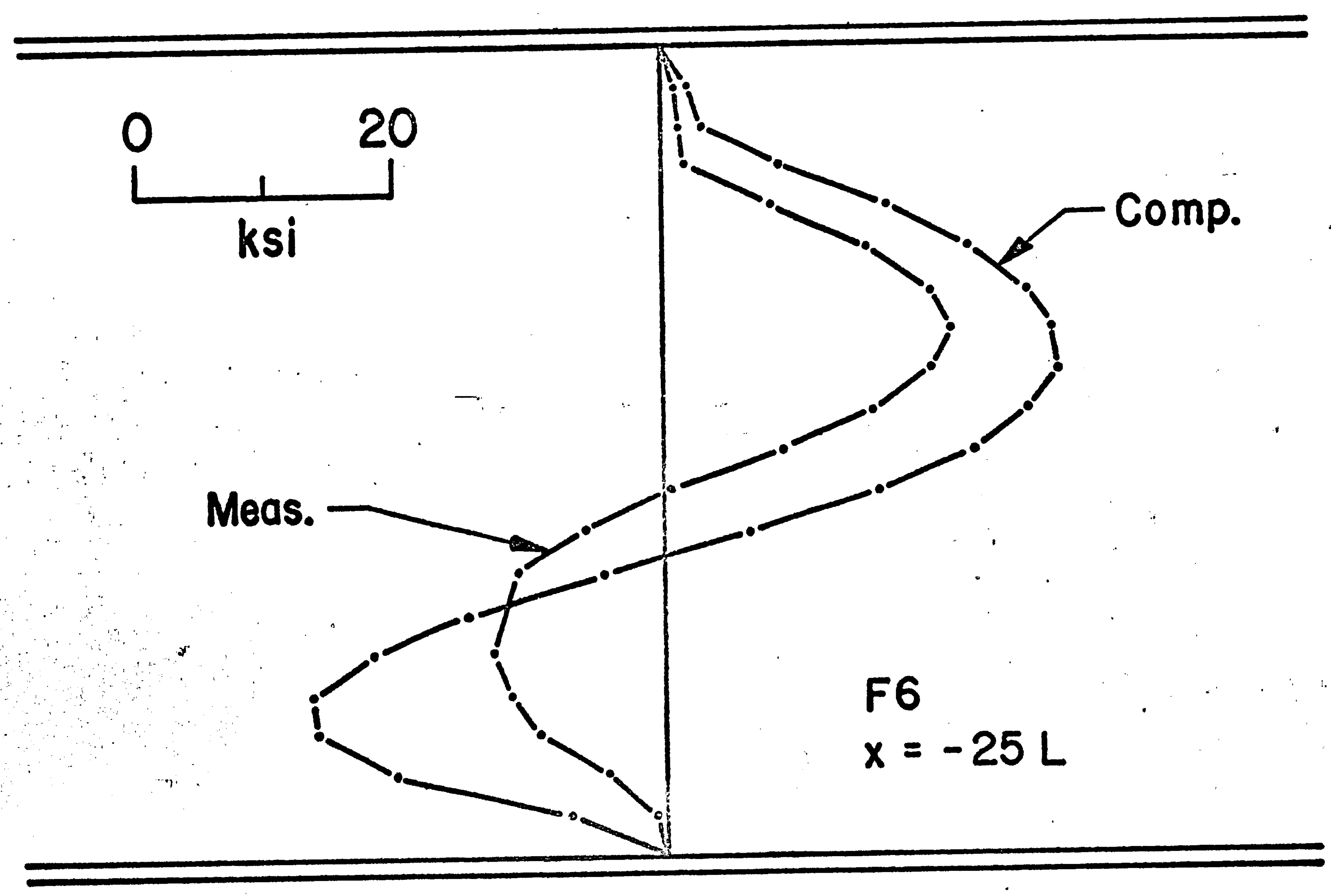


Fig. 9 Web Plate Bending Stresses at Stiffener

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