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On the transhipment approach to personnel scheduling with fluctuating demands

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ON THE TRANSHIPMENT APPROACH
TO PERSONNEL SCHEDULING
WITH FLUCTUATING DEMANDS

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ABSTRACT

A particular type of personnel scheduling is concerned with the assignment of employees where the number of jobs vary from period to period during the workday. In the past this class of scheduling problems has been dealt with primarily by the use of Gantt chart techniques. This method is largely a trial-and-error solution and usually very time consuming.

This paper presents a mathematical approach, namely the transhipment method, for use in problems of this type. The steps necessary to transform the scheduling restrictions to conform to the transhipment requirements are shown in detail, and tableau usage is outlined.

Two types of applications (1) scheduling without relief and (2) scheduling with relief are explained, and problems of each types are demonstrated. The results are discussed and comparisons are made to the Gantt chart methods.

I. INTRODUCTION

A large number of organizations are confronted with the task of determining overall requirements and arranging work-schedules for employees. One particular type of manpower scheduling occurs where the requirement for employees fluctuates from one period of the work day to another. The occurrence of this type scheduling is most easily recognized in such cases as telephone operators, toll booth collectors, surveillance tours, exhibit workers, and restaurant and cafeteria employees. However, these examples given do not preclude the existence of the problem in many areas where it is not recognized. For instance, many industrial schedules for a given employee demand are formulated on peak load conditions during the work day. This is done in these cases since the idle time during low demand periods is utilized elsewhere. This, however, might not be an economical approach.

Several published articles [2, 7, 11, 12, 15] illustrate the difficulty in solving a scheduling problem of this type, and point out that the basic approach has been the use of Gantt charts. Churchman, Ackoff, and Arnoff [2] categorize the nature of the problem by saying,

> "The scheduling of manpower in such a manner requires the preparation of a Gantt-type chart for each day, showing the working and idle time for every toll collector. Toll-collector starting times and relief periods must be juggled in an effort to provide exactly the number of collectors needed to give the optimum service each half-hour of the day. This is largely a trial-and-error problem, and preparation of such schedules may be very time-consuming when the objective is to make the schedule as efficient as possible."

The above comment results from work by Edie [12] where he was concerned with scheduling toll-collectors to meet toll collection requirements at various locations for the New York Port Authority. The demands for collectors were derived by a queueing analysis at various bridges and tunnels in New York City. Once the demands were found, it showed that there were peaks and valleys in the requirements. A definite problem existed in optimizing the schedule since a level work force created idle time that could not be utilized due to transportation difficulites. As previously pointed out, the Gantt chart approach was taken. A fixed shift length was decided upon, and the schedule juggled until a feasible solution was found. The efficiency of the solution was computed by using the ratio of minimum toll collector requirements where no idle time existed to the actual number of toll collectors scheduled.

Glieberman's Problem [15] dealt with the scheduling of people at IBM's World Fair Exhibit. The problem included the scheduling of skilled and non-skilled employees. The skilled were required to do certain tasks, but when these tasks were not required, they were allowed to do unskilled work. The exhibit was open from 9:00 A.M. until 10:00 P.M., and the requirement for workers was basically a unimodal function with the peak occurring around the middle of the day. From this information an arbitrary decision was made to use three shifts. Two basic shifts were used, and the third overlapped the first two to meet the peak demands and furnish relief. The first approach Glieberman took was one of using integer programming. Keeping in mind that the number and length of the shifts were predetermined, certain rules were set for allowing relief and dinner breaks. An integer linear program was developed for only half of the day at a time using 15 minute intervals in order that a 15 minute relief period would be allowable. When the restrictive and objective equations were formulated, there were at least 104 constraints in 300 unknowns, which is more than presently available integer linear programming codes can handle. Furthermore, he found that if ordinary linear programming were used, the values of the solution variables were expected to be so small that a significant amount of error would be introduced by rounding to an integral solution. The amount of error thus introduced would likely wipe out the benefits. In connection with this question, the reader may find [19] to be of interest.

The second approach taken by Glieberman was to subdivide the problem and deal with each shift separately. This method admittedly sub-optimized; however, a solution was attainable. Upon experimentation with the formulation, it was found that the frequency of specification changes and resulting lengthy computer runs made this method economically unattractive. In fact, a production run using this technique was never made.

After examination of the two methods mentioned above, Glieberman was forced to use the Gantt chart solution to this problem. By this method a feasible solution could be reached. However, the efficiency of the schedule depended on the amount of juggling done, and furthermore, an optimal solution was not certain unless there was no idle time.

This type scheduling problem is generally different in each case due to the period length, demand in each period, shift length, length of work day, and cost parameters. However, if the problems are examined closely, the above variables are found to exist in each case. By recognizing this, the differences become similarities that link the problems together and allow a general approach.

From the analysis of the preceding discussion, it is evident that a mathematical approach is highly desirable. It is to this problem that the following discussion is directed.

II. STATEMENT OF THE PROBLEM

Consider a typical transit company scheduling problem where a certain bus route requires the number of buses during the work day as shown in Figure 1, and each driver must work eight consecutive hours per day.

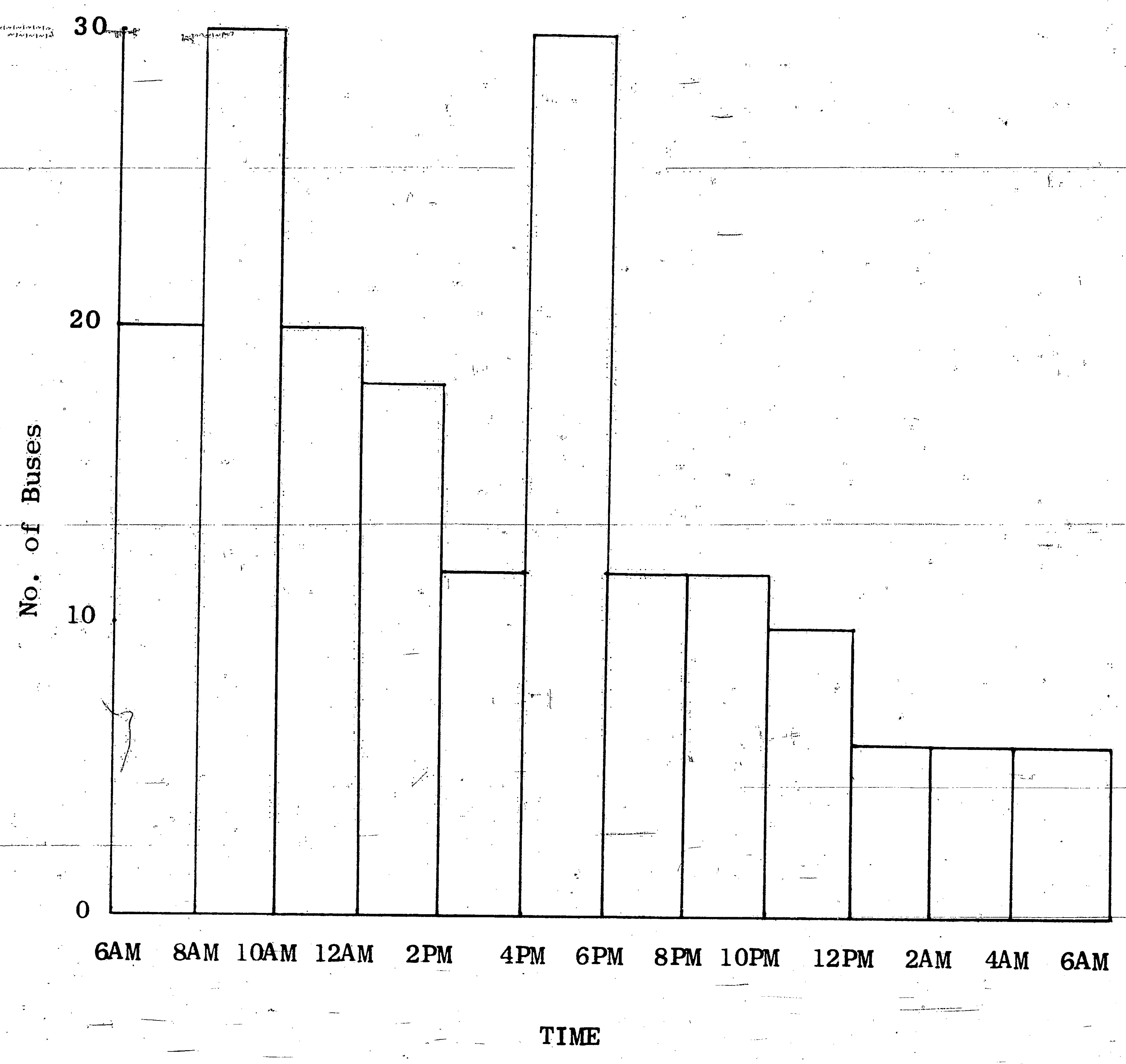


Figure 1

From this information the individual responsible for the schedule would like answers to the following questions:

1. What is the minimum number of people required?
2. Is the cost minimized?
3. What length of time is involved in completing the scheduling task?

In answering the above three questions, this paper presents a new and mathematical approach to the problem. Where there is no relief allowed in the schedule, as in the problem above, the algorithm will:

1. Minimize the number of people.
2. Minimize the cost.
3. Give the scheduler a transportation type model that may be solved by hand or computer.

In the schedule where relief is required, the algorithm will:

1. Show the minimum people to meet the schedule without relief.
2. Show the exact period where idle time exists, if any, and therefore allow its quick utilization for relief.
3. Give the scheduler a transportation type model that may be solved by hand or computer.
4. Minimize the amount of judgment and experience required by the scheduler when the relief is added.

A mathematical model is developed and presented. The methods of solution and interpretation are shown.

The following assumptions are made about the system under study:

1. Deterministic demands.
2. Demand changes from one fixed period to another.

3. The employees may start work at the beginning of any period except those that require him to work a shift shorter than the predetermined shift.

4. Shifts last for a predetermined length of time.

5 Demands must be met.

6. Homogeneous work force.

III. SCHEDULE FORMAT

In the development of a mathematical approach to the scheduling problem there must first be a general schedule format that depicts any problem of this nature. The following discussion lays the foundation for this generalized format.

One assumption in section II is a fluctuating demand for employees during the day. These fluctuations occur from one period of the day to another. The work day may be divided into n periods where there is a change between these periods. It should be pointed out that there may be level requirements for several periods without affecting the solution in any way. Figure 2 illustrates a typical work day divided into n periods where n is equal to twelve. Once the work day is divided into n periods, it is possible to associate each period with the demand for that period. These demands are noted as R_1, R_2, R_3, . . ., R_n and are shown in Figure 2.

Now the work day is divided into the appropriate number of periods as required by the fluctuating demands, and a demand is associated with each period. These demands must be satisfied in a manner as prescribed in the assumptions. The employee must work a predetermined shift length, and he may start work at the beginning of any period as long as beginning at that period does not require him to work a short shift. The predetermined shift length does not restrict the model to one shift length, it only means that the user must decide on a shift length or a number of shift lengths that each employee might work before using the model. Figure 2 presents an instance where only one shift length is permitted. The shifts are denoted as the variable x and in Figure 2 are three periods long.

The subscripts associated with each x, shift, are extremely important due to the large number of permitted shifts in even a small problem. The notation x_{kt} is used. The k indicates the period the shift begins, and the t signifies the beginning of the period following that period in which the shift ends. To illustrate, a shift that begins in period 1 and extends for 2 additional periods would be noted as $x_{1,4}$. This type notation adds a fictitious last period, and it will be shown later that it does not affect the solution.

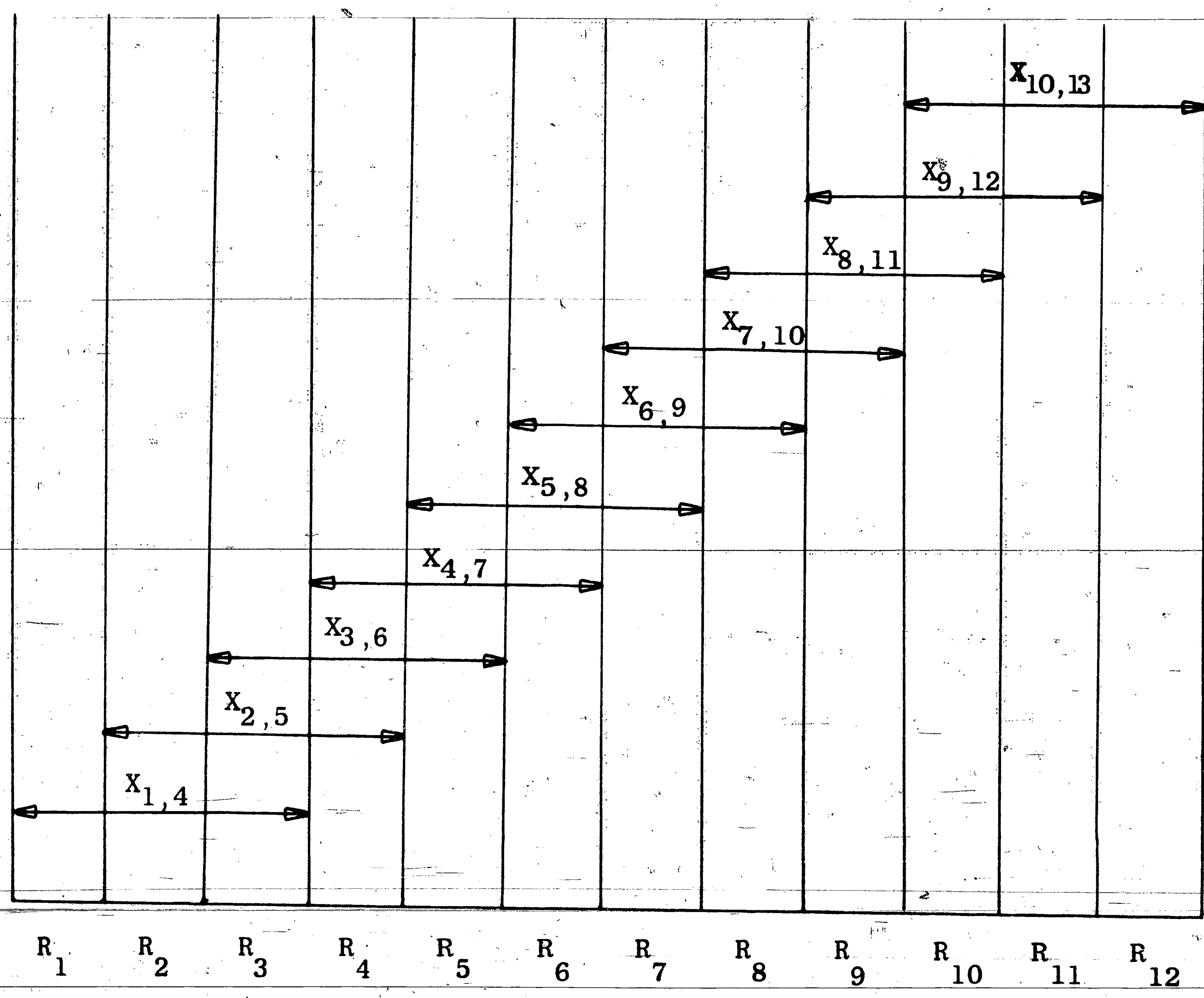


Figure 2

IV. RESTRICTIVE EQUATIONS

Upon examination of the schedule format developed in section III, it can be seen that a set of linear equations may be written to express the schedule in mathematical notation. Considering the assumption that the demand for each period must be met, a slack variable, S, is associated with each period. Summing the variables in each period and setting them equal to or greater than the demand, the following equations may be written:

$$x_{1,4} \geq R_1$$

$$x_{1,4} + x_{2,5} \geq R_2$$

$$x_{1,4} + x_{2,5} + x_{3,6} \geq R_3$$

$$x_{2,5} + x_{3,6} + x_{4,7} \geq R_4$$

$$x_{3,6} + x_{4,7} + x_{5,8} \geq R_5$$

$$x_{4,7} + x_{5,8} + x_{6,9} \geq R_6$$

$$x_{5,8} + x_{6,9} + x_{7,10} \geq R_7$$

$$x_{6,9} + x_{7,10} + x_{8,11} \geq R_8$$

$$x_{7,10} + x_{8,11} + x_{9,12} \geq R_9$$

$$x_{8,11} + x_{9,12} + x_{10,13} \geq R_{10}$$

$$x_{9,12} + x_{10,13} \geq R_{11}$$

$$x_{10,13} \geq R_{12}$$

To rid the equations of the inequalities, the slack variables are added and the following equations then result:

$$x_{1,4} - S_1 = R_1$$

$$x_{1,4} + x_{2,5} - S_2 = R_2$$

$$x_{1,4} + x_{2,5} + x_{3,6} - S_3 = R_3$$

$$x_{2,5} + x_{3,6} + x_{4,7} - S_4 = R_4$$

$$x_{3,6} + x_{4,7} + x_{5,8} - S_5 = R_5$$

$$x_{4,7} + x_{5,8} + x_{6,9} - S_6 = R_6$$

$$x_{5,8} + x_{6,9} + x_{7,10} - S_7 = R_7$$

$$x_{6,9} + x_{7,10} + x_{8,11} - S_8 = R_8$$

$$x_{7,10} + x_{8,11} + x_{9,12} - S_9 = R_9$$

$$x_{8,11} + x_{9,12} + x_{10,13} - S_{10} = R_{10}$$

$$x_{9,12} + x_{10,13} - S_{11} = R_{11}$$

$$x_{10,13} - S_{12} = R_{12}$$

This is now a complete system of equations for a problem where n is equal to 12, and the shift length, L, is equal to three periods.

The objective function for the equations is

$$Z = C_{1,4}\, x_{14} + C_{2,5}\, x_{25} + C_{3,6}\, x_{36} + C_{4,7}\, x_{47} + C_{5,8}\, x_{58} + C_{6,9}\, x_{6,9} + C_{7,10}\, x_{7,10} + C_{8,11}\, x_{8,11} + C_{9,12}\, x_{9,12} + C_{10,13}\, x_{10,13}$$

To give nomenclature to any set of equations of this type, let R_k be the demand during k, where $k = 1, 2, \ldots, n$. Employees starting work at the beginning of period k, available for work during period $k, k+1, k+2, \ldots, k+L-1$, and relinquished at the beginning of period $k+L$ are said to be available for the interval $[k, t]$ where $t = k + L$. $x_{k,t}$ now represents the employees available for work during the interval $[k, t]$.

V. MODEL BACKGROUND

An application for both the transhipment and transportation methods is in the area of assignment problems, e.g., optimal assignment of men to machines. These type problems appear purely combinatorial at first glance, however, they may be fitted into the two above named methods of solution. Upon formulation of a problem of this type, one might suspect that there would be fractional assignment. However, one of the most significant advantages of this technique is always having at least one integer valued optimal solution if the demands and receipts are integers.

If one associates the fluctuating demand as depicted in Figure 1 with the knowledge that there is a need to minimize the number of fixed shifts that employees work, the scheduling problem fits the combinatorial aspects of the transportation formulation. The problem becomes one of examining the number of ways that the demand for employees may be met, and selecting the way that minimizes cost. While remembering both Glieberman's problem [15] of rounding to integral solutions and the property of always having an integral solution with the transportation method, the attractiveness of the transportation method is apparent.

Now that the type of mathematical approach has been decided upon, the problem becomes one of developing a transportation type algorithm that meets the needs of the scheduling problem. Recalling the type scheduling problem under investigation, it is apparent that the standard transportation technique does not satisfy all the requirements. In the scheduling problem an employee's time must be utilized for all periods which he works, not only the originating and terminating periods. This leads directly into the transhipment technique.

VI. TRANSHIPMENT FORMULATION

The transhipment model [3, 5, 6, 16] is a generalized transportation model that allows shipments through intermediate points rather than directly from source to sink. The development of a transhipment model, including slacks, follows.

The material balance equations for each point is given by:

Gross supply = amount shipped in + produced + slack

Gross supply = amount shipped out + consumed + slack

The following notations will be used to identify the components of the material balance equations:

x_{ij} = total quantity shipped from i to j, $i \neq j$

x_{jj}^* = gross supply at j

a_j^* = production at j

b_j^* = consumption at j

S_{ij} = amount shipped from i to j without cost

C_{ij} = cost of shipping from i to j

Now the material balance equations are expressed algebraically with the slack included as a term of ΣX_{ij}. Any time the cost of shipping is zero, the variable is called S_{ij}.

$$x_{jj}^* = \sum_{i \neq j} x_{ij} + a_j^* \qquad j = 1, 2, \ldots, n$$

$$x_{jj}^* = \sum_{k \neq j} x_{jk} + b_j^*$$

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}\, x_{ij} \text{ where}$$

$i \neq j$ and a^*, b^*, and C_{ij} are given parameters.

The gross supply equations may be equated with the following expression resulting:

amount shipped in + produced + slack =

amount shipped out + consumed + slack

or algebraically expressed as:

$$\sum_{i \neq j} x_{ij} + a_j^* = \sum_{k \neq j} x_{jk} + b_j^* \qquad j = (1, 2, \ldots, n)$$

collecting terms gives

$$\sum_{k \neq j} x_{jk} - \sum_{i \neq j} x_{ij} = a_j^* - b_j^* \qquad j = (1, 2, \ldots, n)$$

This gives a complete transhipment matrix for n points. Figure 3 shows the matrix of detached coefficients. It should be noted that each column includes only two non-zero coefficients, + 1 and - 1. This characteristic identifies the transhipment matrix.

Thus far in the formulation, there has been no assumption about cost except between neighboring points. However, the assumption must be made and is valid that the shipping cost between non-neighboring points is the sum of the costs which links the points. The basic formulation of the transhipment model allows both backward and forward shipments, and in most cases the cost for these shipments are the same. In instances where they are not, the formulation remains valid [3].

VI - a. Presentation as the Standard Transportation Tableau.

From the original material balance equations, the row and column equations are:

AMOUNT SHIPPED	$x_{1,2}$	$x_{1,3}$	$x_{1,n}$	$x_{2,1}$	$x_{2,3}$	$x_{2,n}$	$x_{n,1}$	$x_{n,2}$		$x_{n,nd}$	NET PROD. OR CONSUMP.
POINT 1	1	1	1	-1			-1				$a_1^* - b_1^*$
POINT 2	-1			1	1	1		-1			$a_2^* - b_2^*$
POINT 3		-1			-1						$a_3^* - b_3^*$
. . .											. . .
										-1	
POINT n			-1			-1	1	1	. . .	1	$a_n^* - b_n^*$
COST	$c_{1,2}$	$c_{1,3}$	$c_{1,n}$	$c_{2,n}$	$c_{2,3}$	$c_{2,n}$	$c_{n,1}$	$c_{n,2}$		$c_{n,n-1}$	

Figure 3

Row

$$-x_{jj}* + \sum_{k\neq j} x_{jk} = - b_j* \qquad j = 1, 2, \ldots, n$$

Column

$$-x_{jj}* + \sum_{i\neq j} x_{ij} = -a_j* \qquad j = 1, 2, \ldots, n$$

When the above equations are expanded there is a real variable, x_{ij}, as each term. However, the restriction was made previously that all backward shipments would be at zero cost, and where this occurs the real variable, x_{ij}, is replaced by S_{ij}. Figure 4 shows a standard tableau with real variables in the upper-right side and slack variables in the lower-left side, j=1, 2, 3, 4. This signifies that forward shipments are real variables and the backward shipments are slack variables.

ORIGIN i	DESTINATION j: 1	2	3	4	ROW SUM $-b_j*$
1	$-X_{11}$ 0	X_{12} C_{12}	X_{13} C_{13}	X_{14} C_{14}	$-b_1*$
2	S_{21} 0	$-X_{22}$ 0	X_{23} C_{23}	X_{24} C_{24}	$-b_2*$
3	S_{31} 0	S_{32} 0	$-X_{33}$ 0	X_{34} C_{34}	$-b_3*$
4	S_{41} 0	S_{42} 0	S_{43} 0	$-X_{44}$ 0	$-b_4*$
COLUMN SUM $-a_i*$	$-a_1*$	$-a_2*$	a_2*	a_3*	$\Sigma a_j* = \Sigma b_i*$

Figure 4

At this point a transhipment variable must be added to the system to allow shipments via interim points to the destination. In Figure 4 the gross supply variable, x_{jj}^*, is shown along the left to right diagonal, and the new variable is related to it in the following manner.

Gross supply at point j= net transhipment through point j + production at point j + consumption at point j or algebraically.

$$x_{jj}^* = x_{jj} + a_j^* + b_j^*$$

Now the new variable x_{jj} is introduced into the row and column equations.

Row

$$-[x_{jj} + [a_j^* + b_j^*]] + \sum_{k \neq j} x_{jk} = -b_j^*$$

Collecting terms

$$\sum_{k \neq j} x_{jk} - x_{jj} = a_j^* \qquad j = 1, 2, \ldots, n$$

or

total shipped from j including slack - amount transhipped = net production at j

Column

$$-[x_{jj} + [a_j^* + b_j^*]] + \sum_{i \neq j} x_{ij} = -a_j^*$$

Collecting terms

$$\sum_{i \neq j} x_{ij} - x_{jj} = a_j^* \qquad j = 1, 2, \ldots, n$$

or

total amount shipped to j including slack - amount transhipped = net consumption at j

The object equation is:

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$$

$C_{jj} = 0$ for all j, and the cost for the slack = 0

The transhipment variable, x_{jj}, as it appears in the equation is shown as a negative variable. This is not permissible in any transportation model, for all variables must have a lower bound of zero; hence, a new variable must be introduced here to change the sign of the transhipment variable. The new variable $\bar{x}_{jj}$ sets both a lower and upper bound [5].

If the least cost solution for a problem were the shipment of all products through a single point, then the amount of production at that point would have to look as large as the total production. This criteria introduces a need for a fictitious stock pile at each point. This stockpile is as large as the total consumption or production, and is expressed by

$$G \text{ (stockpile)} = \Sigma a_i = \Sigma b_j$$

The addition of this fictitious stockpile, G, does not affect the objective equation since the cost of shipping from a point to itself is zero. The transhipment variable is now redefined as:

$$\bar{x}_{jj} = G - x_{jj}$$

The new stockpile variable, G, is now added to both sides of the defining equations to give:

Row Equation

$$\sum_{k \neq j} x_{jk} - x_{jj} + G = G + b_j^*$$

or

$$\sum_{k \neq j} x_{jk} + \bar{x}_{jj} = G + a_j^* \qquad j = 1, 2, \ldots, n$$

Column Equation

$$\sum_{i \neq j} x_{ij} - x_{jj} + G = G + b_j^*$$

or

$$\sum_{i \neq j} x_{ij} + \bar{x}_{jj} = G + b_j^* \qquad j = 1, 2, \ldots, n$$

The objective equation remains

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$$

$C_{jj} = 0$ for all j, and the cost for slack = 0

VII. MATHEMATICAL FORMULATION

VII - a. Transformation of Restrictive Equations.

In the previous section basically two things were done. The first was the development of the transhipment characteristic matrix of detached coefficients, and the second was the development of the transhipment defining equations where the backward shipments are considered to have zero cost. With these two things kept in consideration, a method must be devised to show that the equations developed from the scheduling problem may be transformed into the transhipment formulation. The general technique described in [19] is found to be appropriate.

First write the equations as they appeared in section IV.

$$x_{1,4} - S_1 = R_1$$

$$x_{1,4} + x_{2,5} - S_2 = R_2$$

$$x_{1,4} + x_{2,5} + x_{3,6} - S_3 = R_3$$

$$x_{2,5} + x_{3,6} + x_{4,7} - S_4 = R_4$$

$$x_{3,6} + x_{4,7} + x_{5,8} - S_5 = R_5$$

$$x_{4,7} + x_{5,8} + x_{6,9} - S_6 = R_6$$

$$x_{5,8} + x_{6,9} + x_{7,10} - S_7 = R_7$$

$$x_{6,9} + x_{7,10} + x_{8,11} - S_8 = R_8$$

$$x_{7,10} + x_{8,11} + x_{9,12} - S_9 = R_9$$

$$x_{8,11}+x_{9,12}+x_{10,13} - S_{10} = R_{10}$$

$$x_{9,12}+x_{10,13} - S_{11} = R_{11}$$

$$x_{10,13} - S_{12} = R_{12}$$

Now if the first equation is left intact and each equation is then subtracted from the next equation, the following system of equations results:

$$\begin{aligned}
&+x_{1,4} &&+s_1 &&= R_1\\
&+x_{2,5} &&-s_1+s_2 &&= R_2 - R_1\\
&+x_{3,6} &&-s_2+s_3 &&= R_3 - R_2\\
&-x_{1,4}+x_{4,7} &&-s_3+s_4 &&= R_4 - R_3\\
&-x_{2,5}+x_{5,8} &&-s_4+s_5 &&= R_5 - R_4\\
&-x_{3,6}+x_{6,9} &&-s_5+s_6 &&= R_6 - R_5\\
&-x_{4,7}+x_{7,10} &&-s_6+s_7 &&= R_7 - R_6\\
&-x_{5,8}+x_{8,11} &&-s_7+s_8 &&= R_8 - R_7\\
&-x_{6,9}+x_{9,12} &&-s_8+s_9 &&= R_9 - R_8\\
&-x_{7,10}+x_{10,13} &&-s_9+s_{10} &&= R_{10} - R_9\\
&-x_{8,11} &&-s_{10}+s_{11} &&= R_{11} - R_{10}\\
&-x_{9,12} &&-s_{11}+s_{12} &&= R_{12} - R_{11}
\end{aligned}$$

At this point the equations' coefficients resemble closely the characteristic transhipment matrix. However, the last column of both the real and slack variables does not have a + 1 and - 1 in it. This must be corrected for the two to be exactly the same.

If the last equation of the original system is multiplied by a minus one, its value is not changed. Now, recalling that a redundant equation may be added to a system of equations without changing the solution, the equation is added to the revised system. The new system meets the requirements for a linear transformation. The matrix of detached coefficients for the system of equations is shown in Figure 5.

Comparing Figure 3 with Figure 5, it is found that the matrix of detached coefficients has the same characteristics in both cases i.e., each column has only a - 1 and a + 1. This now allows the transhipment method to be used as a model for the scheduling problem under study.

VII - b. Transhipment Tableau for the Scheduling Model

In developing the transhipment defining equations, the stipulation was made that any backward shipment has zero cost. These variables are then called slack variables. Referring to Figure 4, the lower left half of the tableau has only slack variables. All of these slacks, except the diagonal immediately below the main left to right diagonal, may be removed without changing the solution. An analysis of Figure 6 explains this.

AMOUNT SHIPPED	$x_{1,4}$	$x_{2,5}$	$x_{3,6}$	$x_{4,7}$	$x_{5,8}$	$x_{6,9}$	$x_{7,10}$	$x_{8,11}$	$x_{9,12}$	$x_{10,13}$	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	NET PROD. OR CONSUMP.
POINT 1	1										1												R_1
2		1									-1	1											$R_2 - R_1$
3			1									-1	1										$R_3 - R_2$
4	-1			1									-1	1									$R_4 - R_3$
5		-1			1									-1	1								$R_5 - R_4$
6			-1			1									-1	1							$R_6 - R_5$
7				-1			1									-1	1						$R_7 - R_6$
8					-1			1									-1	1					$R_8 - R_7$
9						-1			1									-1	1				$R_9 - R_8$
10							-1			1									-1	1			$R_{10} - R_9$
11								-1												-1	1		$R_{11} - R_{10}$
12									-1												-1	1	$R_{12} - R_{11}$
13										-1												-1	$- R_{12}$
COST	$C_{1,4}$	$C_{2,5}$	$C_{3,6}$	$C_{4,7}$	$C_{5,8}$	$C_{6,9}$	$C_{7,10}$	$C_{8,11}$	$C_{9,12}$	$C_{10,13}$	0	0	0	0	0	0	0	0	0	0	0	0	Z

Figure 5

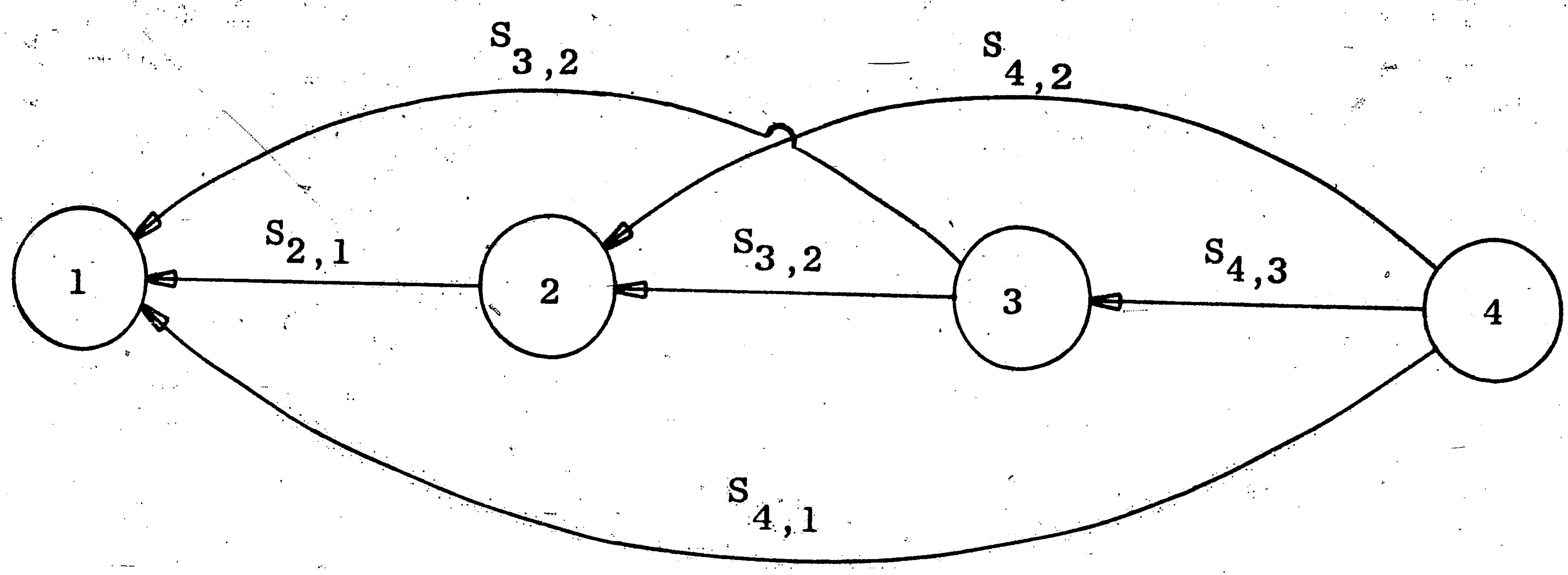


Figure 6

In Figure 6, it is easily seen that when each path has zero cost, flow along any portion of the paths 4 to 3 to 2 to 1 may replace any of the longer paths. This allows all the paths to be removed except the paths between adjacent nodes. The scheduling problem may now be treated as a forward directed net work, excepting the backward links mentioned above. A pure forward directed network only has variables in the upper right side of the tableau.

The transformation of the restrictive equations, and the above analysis allow the scheduling problem to be defined by the following equations:

Column

$$\sum_{o<m<k} x_{mk} + x_{kk} + S_{(k+I)},\ k<(n+1) \quad \begin{cases} = G + R_k - R_{(k-1)} > 0 & ;k \le n \\ = G & ;\ k = n + 1 \end{cases}$$

for $k = 1, 2, \ldots, n, n + 1$

Row

$$\sum_{n+1 \ge t > k} x_{kt} + x_{kk} + S_{k,(k-1)} > 0 \quad \begin{cases} = G & ;\ n \le n \\ = G + R_{k-1} & ;\ k \quad n + 1 \end{cases}$$

for $k = 1, 2, \ldots, n, n + 1$

objective equation

$$Z = \sum_{m=1}^{n} \sum_{k=1}^{n} C_{m,k} \quad x_{m,k} \ .$$

where

$C_{kk} = 0$ for all k

$C_{mk} = \infty$ for all inadmissable variables

$$G = \tfrac{1}{2} \left[R_1 + \sum_{k=2}^{n} R_k - R_{k-1} + D_4\right]$$

In the preceding work, equations have been developed that define the scheduling problem as a hybrid transhipment model. As in standard transportation problems, the defining equations represent a tableau. This tableau gives a pictorial representation of sinks, sources, and available links for shipments, and it allows solution by the tools used in the standard transportation method i.e., stepping stone, modi method, etc. Figure 7 shows a typical tableau for a scheduling problem where there are 12 periods and the shift length is 3 periods.

S'CE	DESTINATION													
	1	2	3	4	5	6	7	8	9	10	11	12	13	
1	$\bar{X}_{1,1}$			$X_{1,4}$										$G+R_1$
2	$S_{2,1}$	$\bar{X}_{2,2}$			$X_{2,5}$									$G+R_2-R_1$
3		$S_{3,2}$	$\bar{X}_{3,3}$			$X_{3,6}$								$G+R_3-R_2$
4			$S_{4,3}$	$\bar{X}_{4,4}$			$X_{4,7}$							$G+R_4-R_3$
5				$S_{5,4}$	$\bar{X}_{5,5}$			$X_{5,8}$						$G+R_5-R_4$
6					$S_{6,5}$	$\bar{X}_{6,6}$			$X_{6,9}$					$G+R_6-R_5$
7						$S_{7,6}$	$\bar{X}_{7,7}$			$X_{7,10}$				$G+R_7-R_6$
8							$S_{8,7}$	$\bar{X}_{8,8}$			$X_{8,11}$			$G+R_8-R_7$
9								$S_{9,8}$	$\bar{X}_{9,9}$			$X_{9,12}$		$G+R_9-R_8$
10									$S_{10,9}$	$\bar{X}_{10,10}$			$X_{10,13}$	$G+R_{10}-R_9$
11										$S_{11,10}$	$\bar{X}_{11,11}$			$G+R_{11}-R_{10}$
12											$S_{12,11}$	$\bar{X}_{12,12}$		$G+R_{12}-R_{11}$
13												$S_{13,12}$	$\bar{X}_{13,13}$	G
	G	G	G	G	G	G	G	G	G	G	G	G	$G+R_{12}$	

Figure 7

VIII. NETWORK RELATIONSHIP

In the development of the scheduling model, the scheduling problem has been transformed into a transhipment problem. At this time it is advantageous to show the analogy between the two.

The schedule with twelve periods is used for illustration. In section VII the system of equations that resulted from the transformation consisted of thirteen equations; therefore a network having thirteen nodes is established. Figure 8 shows the nodes and the linking paths as described by a shift length of three periods.

By subtracting the demands in the manner described, the number of people required in each period is treated as a change in the workforce from one period to the next period. The network, as shown in Figure 8, consists of a node for each restrictive equation with one arc for each activity variable. The network representation is interpreted as the amount of flow along the arc [k,t]. $C_{k,t}$ is the cost for the flow. Also, the slack, S, is the flow from node k to k-1 with a cost of zero.

The number of employees associated with each node, e.g., $R_k - R_{k-1}$, represents the amount by which the flow out of the node exceeds the total flow into the node. With the observation that $R_k - R_{k-1}$ may be negative, the network represents a situation where flows from positive nodes satisfy requirements at negative nodes through the minimal cost paths.

To further point out the analogy between the scheduling problem and the transhipment model, all nodes are considered to be sources except the last, which is a sink. This allows freedom of flow through the network with the final node being the restrictive sink. With the model constructed in this manner, employees beginning work in the first period and working to the beginning of the fifth period are said to flow through arc $x_{1,4}$ in Figure 8. With the zero cost path from node four to node three, etc., the employees along arc $x_{1,4}$ may fill requirement for period two. Even though this is obviously true, the model allows this mathematically and at minimum cost. In regard to this analogy the reader may find [3, 14, 19] to be of interest.

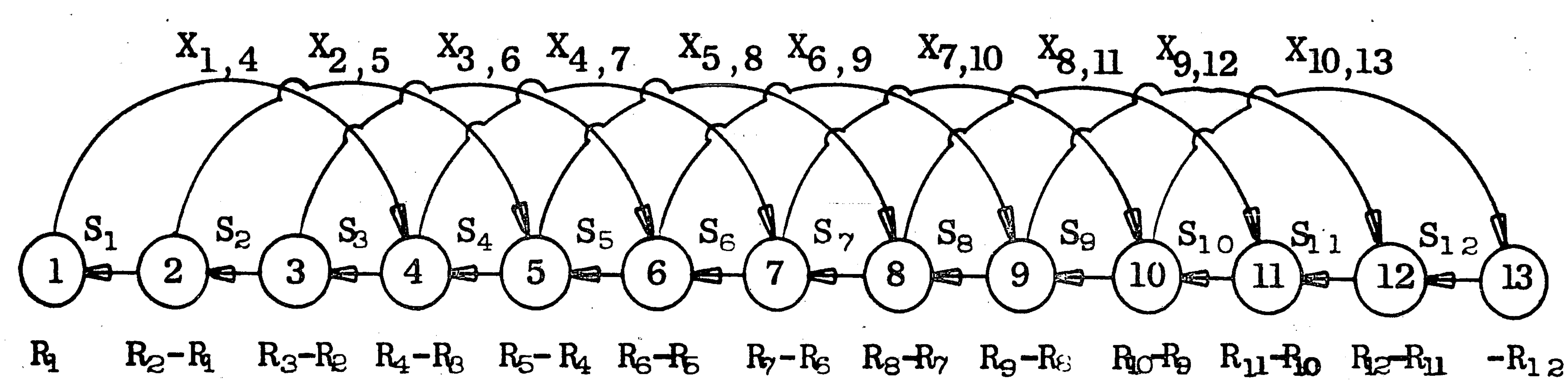


Figure 8

IX. APPLICATION OF THE MODEL

IX - a. General Approach

In the statement of the problem, two types of scheduling problems are pointed out.

1. Scheduling without relief
2. Scheduling with relief

To show the methods of approach and of interpretation in applying the model, problems of each type are used.

The application of the model to type one is illustrated by solving two scheduling problems where relief is not required. The solutions to the problems are developed and explained. A comparison between the Gantt chart solution and the mathematical model is discussed.

The second type, scheduling with relief, is demonstrated by applying the model to an industrial application where the Gantt chart is presently utilized in scheduling. A comparison between the two approaches is made.

IX - b. Scheduling Without Relief

The data for the first problem under consideration is found in Eilon [4]. The problem is typical of a situation that might exist at a transit company. A bus route requires the following number of buses during the work day:

Time	Buses
6 A.M. - 8 A.M.	20
8 A.M. - 10 A.M.	30
10 A.M. - 12 P.M.	20

Time (cont.)	Buses (cont.)
12 A.M. - 2 P.M.	18
2 P.M. - 4 P.M.	12
4 P.M. - 6 P.M.	30
6 P.M. - 8 P.M.	12
8 P.M. - 10 P.M.	12
10 P.M. - 12 P.M.	10
12 P.M. - 2 A.M.	6
2 A.M. - 4 A.M.	6
4 A.M. - 6 A.M.	6

The drivers may start work at the beginning of any 2 hour period, and once at work, they must work 8 consecutive hours. The cost is the same for each driver, and no premium is paid for late shift work.

The tableau for the problem may be formulated directly from the data without referring to the defining equations. It is done using the following steps:

1. There are twelve periods; hence set up a tableau that is 13 x 13.

2. Calculate G by using

   $$G = \frac{1}{2}\left[R_1 + \left|R_k - R_{k-1}\right| + R_k\right]$$

3. The sinks and sources are calculated as prescribed in section VII. Source 1 is $R_1 + G$, 2 through 12 are $R_k - R_{k-1} + G$, and 13 is G. Sinks 1 through 12 are G, and 13 is $G + R_{13}$. Enter these in the tableau.

4 The transhipment variable $\bar{x}_{kk}$ is allowed along the main left to right diagonal.

5. The slack variables are allowed along the left to right diagonal immediately below the main left to right diagonal.

6. The real variables are allowed in positions $x_{1,5}$, $x_{2,6}$, $x_{3,7}$, ..., $x_{9,13}$.

7. All other positions in the tableau are inadmissable.

Appendix 1 shows the solution tableau and the actual schedule for the problem. The cost coefficients are the same for each real variable, therefore any feasible solution is also an optimum solution.

Appendix 1 shows that the minimum number of drivers is 66 with 82 hours of idle time. To reach this solution it took approximately 10 minutes using the mathematical model with hand calculations. The solution is also optimal with drivers and idle time minimized.

The same problem was attempted by three graduate students using the Gantt chart method. Each obtained the solution in times varying from 30 minutes to 1 hour. The trial and error method did produce the correct answer; however, the students had no way of knowing that their solution was optimum, and only stopped when there was no apparent way to improve the solution.

The second example is basically the same problem except the fluctuations occur between one hour intervals and with more pronounced changes. Appendix 1 shows the data, tableau, and final schedule for the problem. The number of drivers required is 68 and the idle time is 173 hours.

The author attempted a Gantt chart solution to this problem, and after approximately one hour, achieved a feasible solution. The solution did not prove to be optimum; however, this was not known until it was compared to the results of the mathematical model.

IX - c. Scheduling With Relief

The data used in this analysis was obtained from a telephone exchange in the New England Telephone and Telegraph Company and exhibited as a case study in [7]. The Company presently forecasts the number of calls requiring operators on a weekly basis. This forecast is then broken down into the number of operators required for each 30 minute period of the day. From these demands for operators a weekly schedule for operator assignments is made. Table 1 shows the shift lengths, number of operators and hourly cost per shift, and total daily cost.

The operator schedule is based upon a Gantt chart method that has been developed over a period of years. The operator work schedule is formulated for any given week, and until a drastic change in the overall work pattern takes place, the schedule for the following weeks is made by scheduling the changes between weeks.

The scheduling is done by an employee with several years experience on the job, and with guide lines derived from his own experience and others that preceded him on the job. With this type scheduler performing the job, the original schedule requires approximately 6 hours scheduling time and the weekly changes about 2 hours.

The Gantt chart approach as utilized by the telephone company produces restrictions slightly different from those used in section VII of this paper. These restrictions allow several basic continuous shifts, but due to the flexibility of the employees, split shifts are allowed which aid in reducing idle time. This condition would usually not exist in other scheduling environments. The final schedule that results from the Gantt chart solution includes the required relief time for each operator. This forces the scheduler to manipulate the schedule for the optimum number of operators and necessary relief, both which require considerable time.

The application of the mathematical model to this problem does the following things:

1. Attempts to develop a schedule which reduces daily operator cost and requires no split shifts.
2. Establishes a set of rules that aid in introducing required relief into the schedule.
3. Shows that the mathematical model is applicable where split shifts are allowed.

To find a lower cost schedule using only continuous shifts in lieu of split shift, two assumptions are made.

1. Employees are paid 105% of standard pay when the shift ends after 7:00 P.M. but before 10:00 P.M. and 110% if the shift ends at 10:00 P.M. or later.
2. The standard rate per hour is $1.92. This rate is the amount now paid for regular day shift work.

Using the above assumptions, shift lengths of six, six and one-half, seven, and seven and one-half hours are introduced into the model. Appendix 2 shows the data, tableau, and the final schedule including relief for the appropriate shift length. The computations were done on an IBM 1620 computer using a transhipment program with indirect addressing. The iterations to optimum solution varied between 41 and 45.

Once the schedule without relief is obtained for each shift length, systematic steps may be devised to minimize the length of time necessary to introduce relief into the schedule. These steps are:

1. Establish relief requirements for each operator.
2. Establish the time frame within which the above may occur.
3. Examine the schedule to find which periods have idle time.
4. If the idle time exists in periods were relief may be taken, assign as much idle time to these as possible.
5. Assign additional employees to the schedule to meet the remaining relief and lunch requirements. These assignments are made one at a time until the requirements are met.

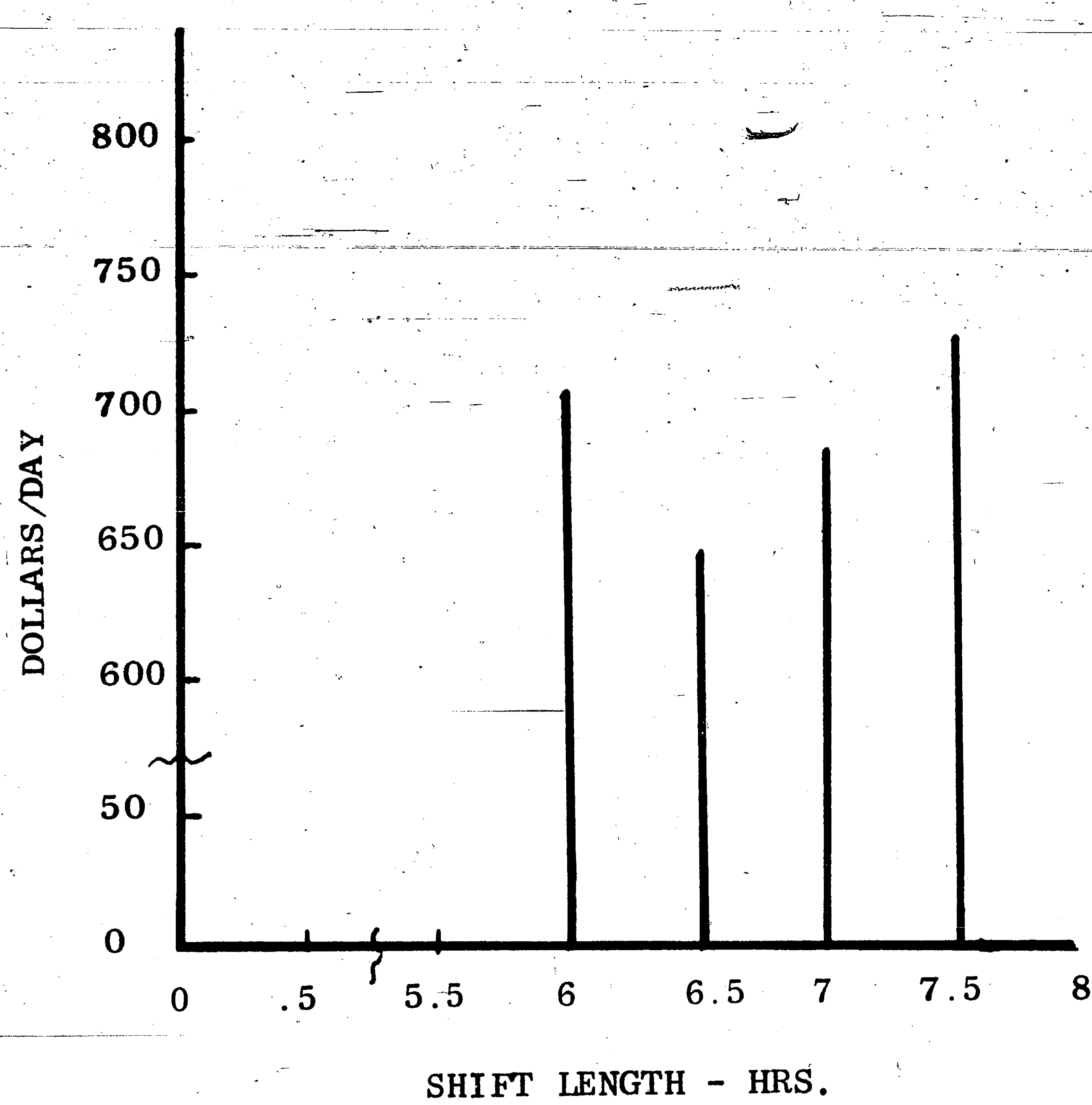
The above rules were used in arriving at a final schedule for each shift length. Appendix 2 illustrates the results of this method for reaching the final schedule.

Shifts of different lengths are introduced into the model to test its sensitivity to this variable and to seek the lowest cost. Graphs in Figure 9 show plots of the number of people and cost versus length of shifts. The graphs point out that the six and one-half hour shift is the optimum shift. It requires one more employee than the seven hour shift; however, the cost is lower due to the difference in paid idle time.

A comparison of Table 1 and Table 2 shows that the cost of the present Gantt chart solution is $727.25 which is $79.56 higher than the six and one-half hour transhipment solution. As stated in the application assumptions, the transhipment solution is based on a $1.92 cost per operator hour. This cost per hour may be increased to $2.15, making the total cost of the two approaches the same while the transhipment solution has the advantage of continuous shifts.

The final test of the transhipment technique is the direct application of the Gantt chart restrictions to the model. Here, multi-length shifts are allowed, and Table 1 shows these lengths. Also, the exact same costs which are used in the Gantt chart solution are introduced into the transhipment model. Appendix 2 shows the tableau and final schedule for this solution.

The results of this experiment are shown in table 2. Comparing them with the Gantt chart results in Table 1, it shows that the number of operators is reduced by three, and the daily cost is reduced from $727.25 to $692.16. This reduction is attributed to an increase in the number of operators working an eight hour morning-afternoon shift, and the deletion of all operators working six hour premium shifts.


800
750
700
650
600
50
0
DOLLARS/DAY
0
.5
5.5
6
6.5
7
7.5
8
SHIFT LENGTH - HRS.

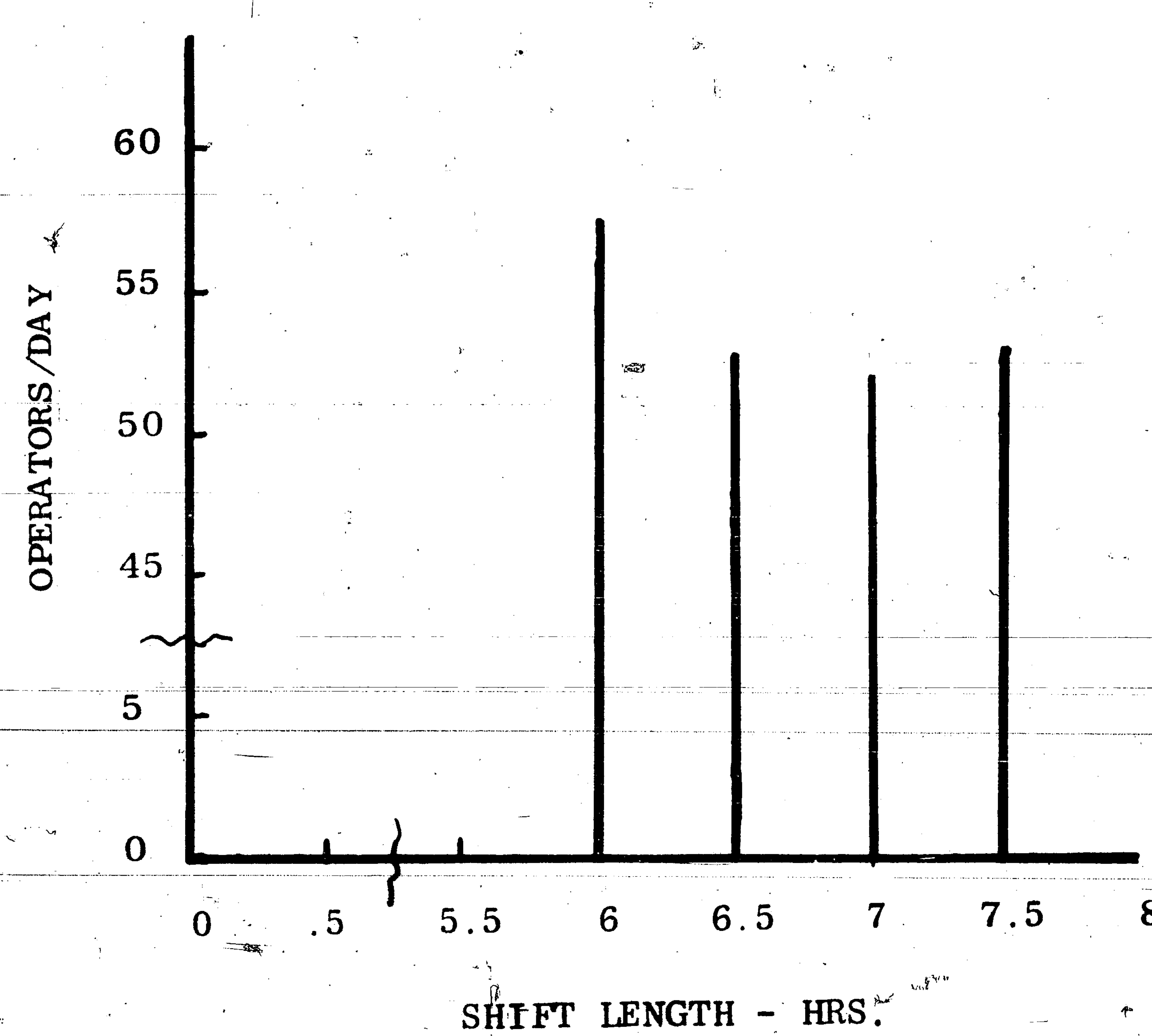

60
55
50
45
5
0
OPERATORS/DAY
0
.5
5.5
6
6.5
7
7.5
8
SHIFT LENGTH - HRS.

Figure 9

X. CONCLUSIONS

The application of the technique provides the user with a new tool for scheduling problems of the type under study. Since the model is transhipment in nature, all the attributes of the transportation method are present. This makes implimentation of the model relatively easy by manual or computer methods.

When the technique is applied to scheduling problems where relief is not required, it does without question minimize the objective function. This is inherent in the technique since the original restrictive equations are transformed to meet the transhipment requirements. This fact alone gives the model a decided advantage over the Gantt chart method, for in the Gantt solution an optimum is not recognized unless no idle time is present or every possible combination is tried.

In the schedule requiring employee relief, the model gives only a basic schedule without relief; however, this schedule is an optimum starting point. The introduction of relief is then left to the discretion of the scheduler. The idle time that exists is immediately displayed in the proper period, and where applicable, may be used to supply relief for the employees. The model allows experimentation with different shift length to find the one most suitable for the demand distribution under consideration. This was demonstrated in section IX, and it proved to be quite valuable in finding the minimum cost solution.

In using a Gantt chart approach to this problem, a scheduler must arrange and rearrange both the shifts and the reliefs until a suitable schedule is effected. The experience and knowledge of the scheduler, along with relaxed restrictions, reduce the scheduling time as illustrated by the New England Telephone and Telegraph Co. In contrast to this, the scheduling model allows the introduction of relief at a point known to be optimum. With a known starting point, the systematic steps outlined in section IX may be used to enter the required relief.

The direct application of the model to the exact restrictions used by the telephone company proved highly successful. The computations were done on an IBM 1620 computer with 102 iterations to solution. The cost and number of employees were reduced and the addition of relief was done in approximately 30 minutes.

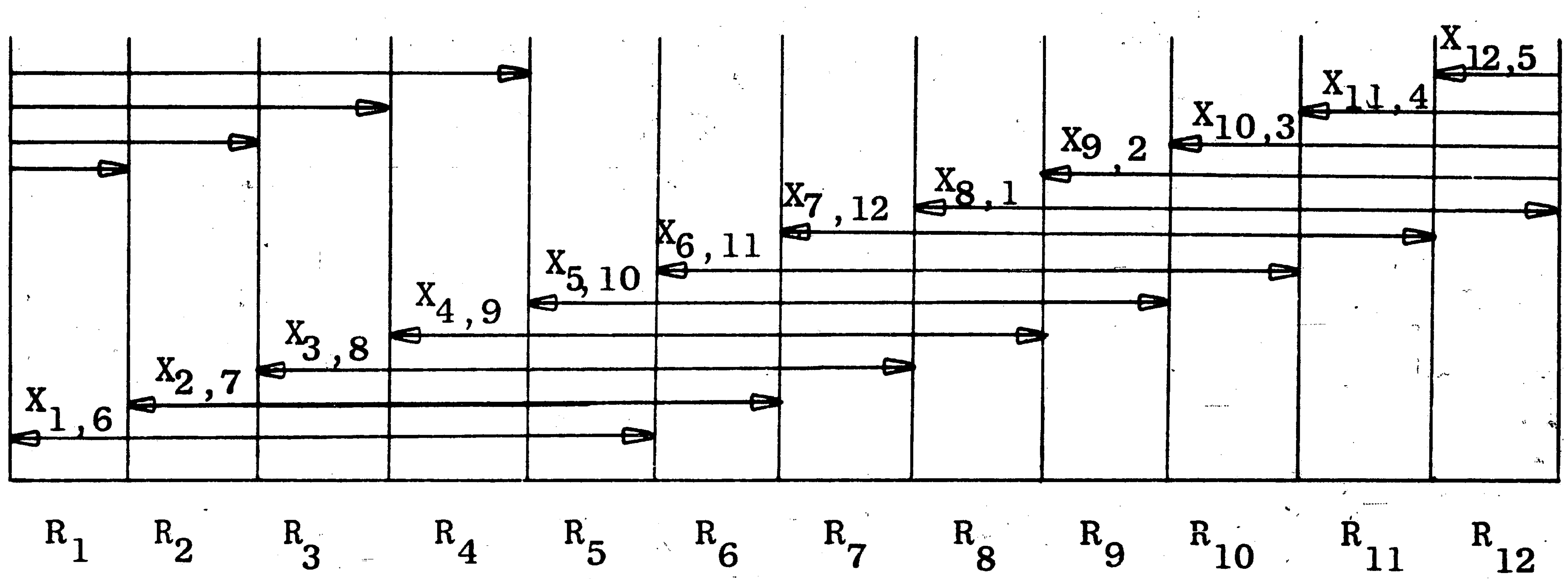
The model proved to be clearly superior to the Gantt chart in both types of application. However, the manual introduction of relief into the schedule does require a minimum amount of judgment and experience by the scheduler. By using the systematic steps outlined in section IX, these factors are judged to be much less than that required by the Gantt chart method.

XI. RECOMMENDATION FOR FURTHER STUDY

The problem under consideration in this paper may be defined as the non-steady state case. This definition means that there is always a beginning and ending restriction, i.e., the first and last equations always have one real variable.

There exists a steady state case where each equation in the system has the same number of real variables. This problem arises where an employee begins work at any period during the day and the shift is allowed to extend into the following day. This situation is best illustrated by an example.

Consider the following schedule format:



By using the technique as outlined in this paper, the system of equations may be transformed to a state where the matrix of detached coefficients closely resembles that of a transhipment model. However, a number of the columns, based on the number of real variables in each equation, contain more than one plus and minus one.

An attempt to develop a suitable transformation should be made. If this were successfully done, the techniques described in this paper could be used in environments where continuous work is required. The reader may find [17] to be of interest in regard to transformations of linear programs to transportation programs.

TABLE 1

RESULTS OF GANTT CHART SOLUTION

Type Shift	No. of oper.	Hrs/ Shift	Idle Time	No. of oper. hrs.	Cost/ Hr.	Total Cost
Morning-Afternoon	16	8	0	128	1.92	245.76
Morning Evening	15(split shift)	7	0	105	2.27	238.35
Afternoon Evening	2	7	0	14	2.27	31.78
Short Hour Evening	11	6	0	66	2.64	174.24
Night	2	8	0	16	2.32	37.12
Total	46		0			727.25

TABLE 2

RESULTS OF TRANSHIPMENT

SOLUTIONS

Length of Shift-Hrs.	No. of oper.	Idle Time-Hrs.	Cost
6:00	58	18	705.86
6:30	53	3.25	647.69
7:00	52	38	686.70
7:30	53	41.5	749.44
Multi-Length	43	1	692.16

APPENDIX 1

DATA, TABLEAU AND FINAL SCHEDULE

for

PROBLEMS WITHOUT RELIEF

DAILY DRIVER REQUIREMENTS FOR PROBLEM 1

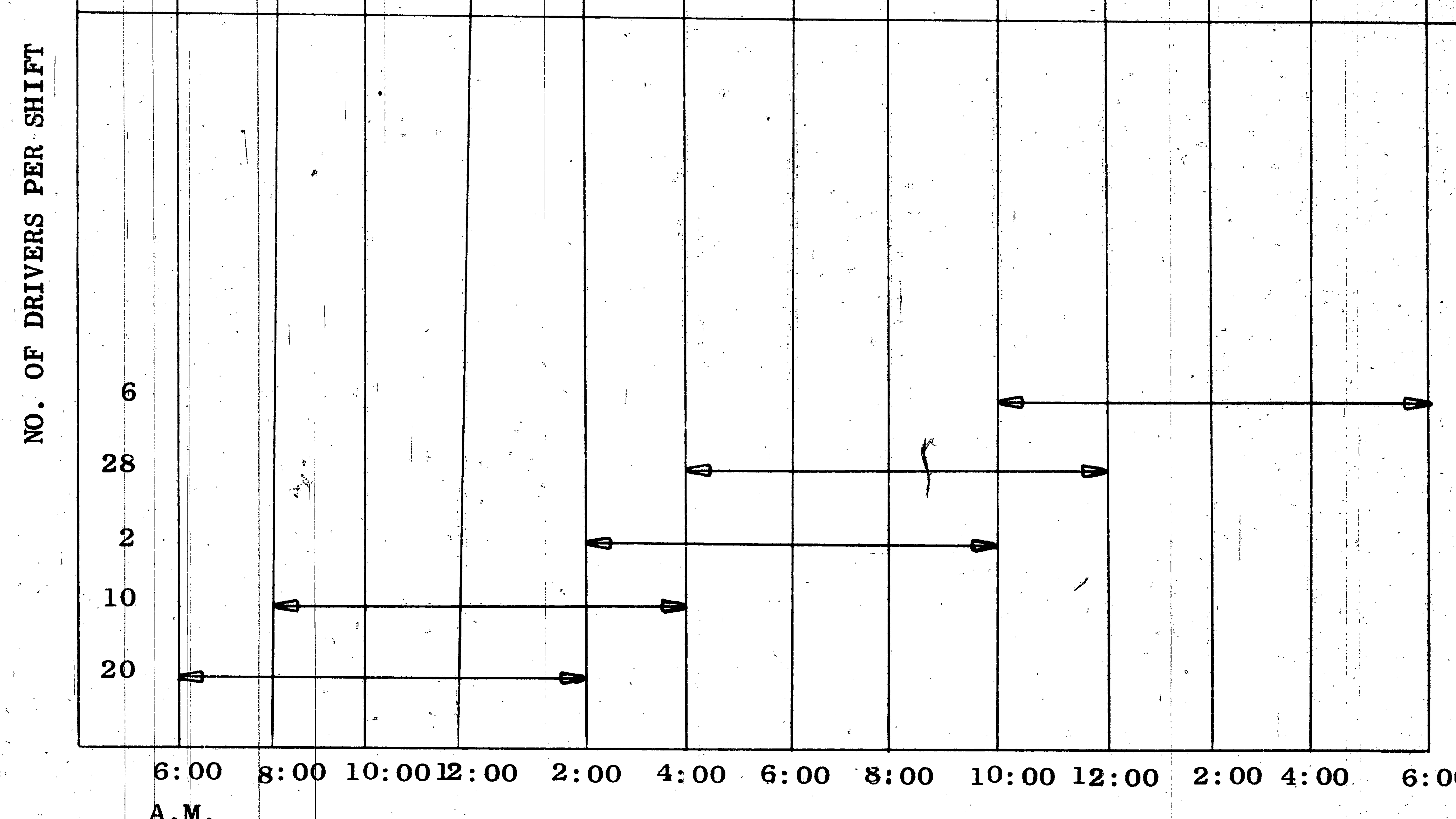
Time	Driver's Req'd	$R_k - R_{k-1}$
6:00 - 8:00	20	20
8:00 -10:00	30	10
10:00 -12:00 Noon	20	-10
12:00 - 2:00	18	- 2
2:00 - 4:00	12	- 6
4:00 - 6:00	30	+18
6:00 - 8:00	12	-18
8:00 -10:00	12	0
10:00 -12:00 Midnight	10	- 2
12:00 - 2:00	6	- 4
2:00 - 4:00	6	0
4:00 - 6:00	6	0

$$G = .5[R + \left| R_k - R_{k-1} \right| + R_k] = 48$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	
1	48				20									68
2	0	48				10								58
3		0	38				0							38
4			10	36				0√						46
5				12	28				2					42
6					0	38				28				66
7						0	30				0			30
8							18	30				0		48
9								18	22				6	46
10									24	20				44
11										0	48			48
12											~~0~~	48		48
13												0	48	48
	48	48	48	48	48	48	48	48	48	48	48	48	54	

TABLEAU FOR PROBLEM 1

DR. REQ.	20	30	20	18	12	30	12	12	10	6	6	6	182
DR. FURN.	20	30	30	30	12	30	30	30	34	6	6	6	264
SLACK	0	0	10	12	0	0	18	18	24	0	0	0	82



FINAL SCHEDULE FOR PROBLEM 1

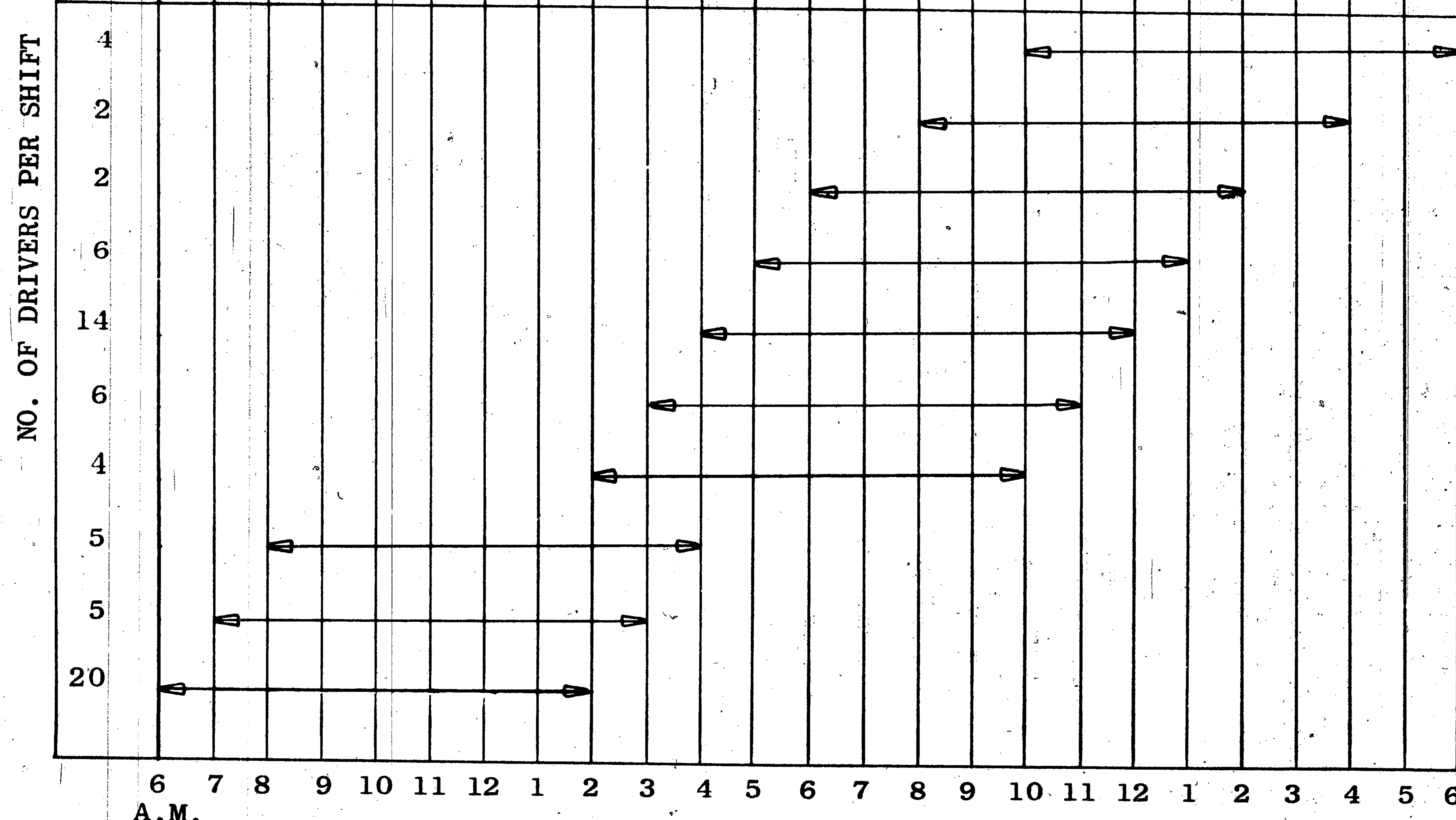
DAILY DRIVER REQUIREMENTS FOR PROBLEM 2

Time	Drivers	$R_k - R_{k-1}$
6:00 - 7:00	20	20
7:00 - 8:00	25	5
8:00 - 9:00	30	5
9:00 - 10:00	26	- 4
10:00 - 11:00	18	- 8
11:00 - 12:00 Noon	22	4
12:00 - 1:00	16	- 6
1:00 - 2:00	18	2
2:00 - 3:00	14	- 4
3:00 - 4:00	12	- 2
4:00 - 5:00	24	12
5:00 - 6:00	30	6
6:00 - 7:00	18	-12
7:00 - 8:00	14	- 4
8:00 - 9:00	12	- 2
9:00 - 10:00	14	2
10:00 - 11:00	16	2
11:00 - 12:00 Midnight	10	- 6
12:00 - 1:00	6	- 4
1:00 - 2:00	8	2
2:00 - 3:00	4	- 4
3:00 - 4:00	6	2
4:00 - 5:00	4	- 2

Time	Drivers	$R_k - R_{k-1}$
5:00 - 6:00	4	0

$$G = .5\left[R_1 + \left| R_k - R_{k-1} \right| + R_n\right] = 62$$

DR. REQ.	20	25	30	26	18	22	16	18	14	12	24	30	18	14	12	14	16	10	6	8	4	6	4	4	371
DR. FURN.	20	25	30	30	30	30	30	30	14	15	24	30	32	32	34	34	34	28	14	8	6	6	4	4	544
SLACK	0	0	0	4	12	8	14	12	0	3	0	0	14	18	22	20	18	18	8	0	2	0	0	0	173



FINAL SCHEDULE FOR PROBLEM 2

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
1	62								20																	82
2	0	62								5																67
3		0	62								5															67
4			0	58								0														58
5				4	50								0													54
6					12	54								0												66
7						8	48								0											56
8							14	50								0										64
9								12	42								4									58
10									0	54								6								60
11										3	57								14							74
12											0	62								6						68
13												0	48								2					50
14													14	44								0				58
15														18	40								2			60
16															22	42								0		64
17																20	40								4	64
18																	18	38								56
19																		18	40							58
20																			8	56						64
21																				0	58					58
22																					2	62				64
23																						0	60			60
24																							0	62		62
25																								0	62	62
	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	66	

TABLEAU FOR PROBLEM 2

APPENDIX 2

DATA, TABLEAU AND FINAL SCHEDULE

for

SOLUTIONS TO TELEPHONE

OPERATOR SCHEDULING PROBLEM

DAILY OPERATOR REQUIREMENTS

Time	Operators Req'd	$R_k - R_{k-1}$
6:30 - 7:00	1	1
7:00 - 7:30	6	5
7:30 - 8:00	10	4
8:00 - 8:30	13	3
8:30 - 9:00	19	6
9:00 - 9:30	22	3
9:30 - 10:00	22	0
10:00 - 10:30	24	2
10:30 - 11:00	24	0
11:00 - 11:30	22	-2
11:30 - 12:00 Noon	21	-1
12:00 - 12:30	18	-3
12:30 - 1:00	21	3
1:00 - 1:30	21	0
1:30 - 2:00	18	-3
2:00 - 2:30	16	-2
2:30 - 3:00	16	0
3:00 - 3:30	16	0
3:30 - 4:00	16	0
4:00 - 4:30	16	0
4:30 - 5:00	16	0
5:00 - 5:30	20	4
5:30 - 6:00	19	-1

Time	Operators Req'd	$R_k - R_{k-1}$
6:00 - 6:30	21	2
6:30 - 7:00	24	3
7:00 - 7:30	24	0
7:30 - 8:00	23	-1
8:00 - 8:30	23	0
8:30 - 9:00	19	-4
9:00 - 9:30	15	-4
9:30 - 10:00	13	-2
10:00 - 10:30	10	-3
10:30 - 11:00	6	-4
11:00 - 11:30	4	-2
11:30 - 12:00 Midnight	1	-3

$$G = .5[R_1 + \left| R_k - R_{k-1} \right| + R_n] = 37$$

Note: There are always 2 operators that work from 11:00 P.M. until 7:00 A.M. There is enough idle time during their shifts for relief.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
1	37												1																								33
2	0	37												5																							42
3		0	35												6																						41
4			2	30												2																					40
5				1	37												5																				43
6					0	37												3																			40
7						0	37												0																		37
8							0	33												6																	39
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10									6	29												0															35
11										8	28												0														36
12											9	25												0													34
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14													8	29												0											37
15														3	31												0										34
16															0	35												0									35
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18																	0	33												4							37
19																		1	36												0						37
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22																					0	37												4			41
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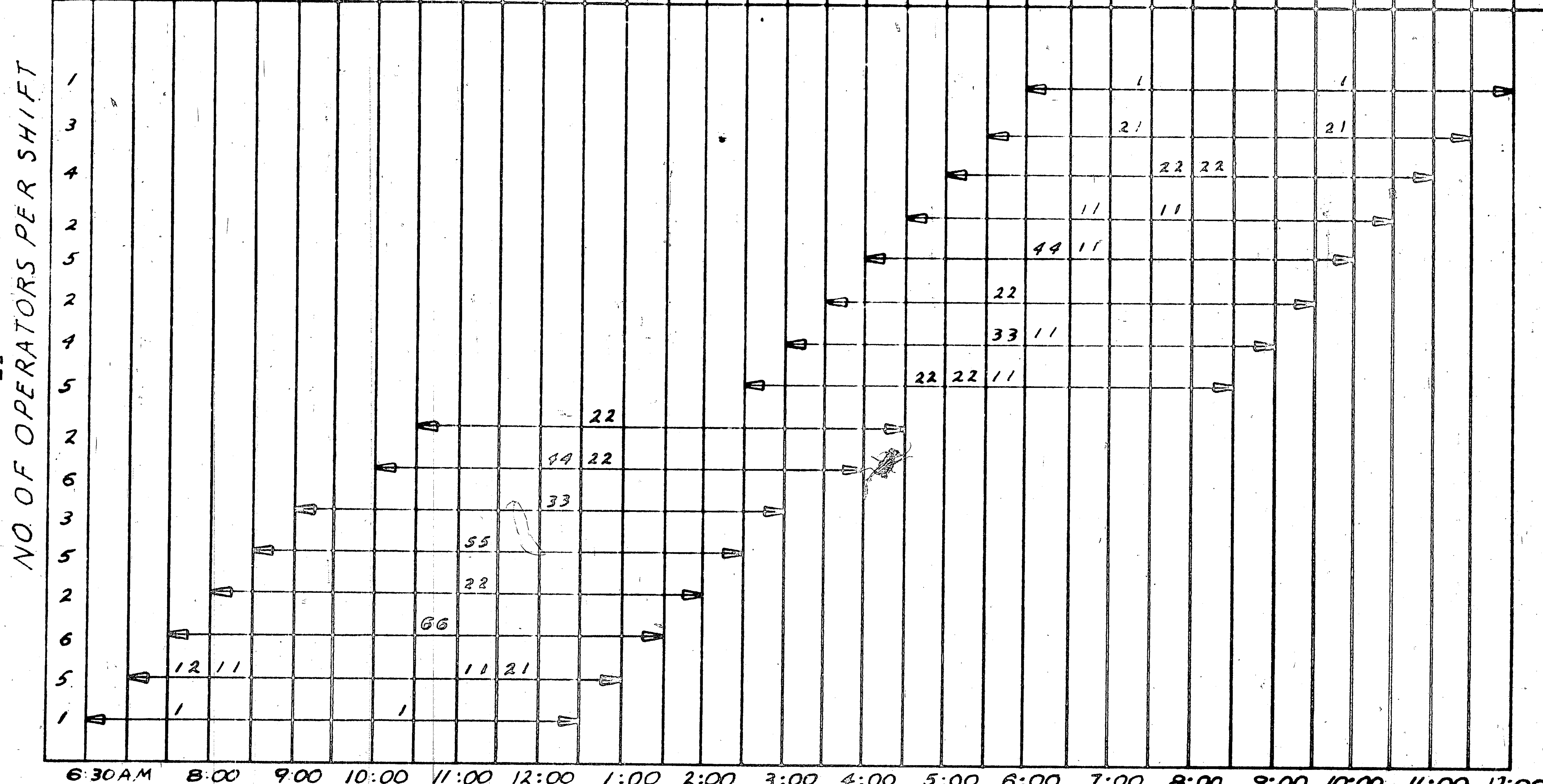
	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	38	

TABLEAU FOR 6 HOUR SHIFT

OP. RQ.	1	6	10	13	19	22	22	24	24	22	21	18	21	21	18	16	16	16	16	16	16	20	19	21	24	24	23	23	19	15	13	10	6	4	1	580
OP. FUR.	1	6	12	14	19	22	22	28	30	30	30	30	29	24	18	16	16	17	19	18	18	22	25	26	26	26	26	26	21	17	15	10	8	4	1	672
ALT. RQ.	0	0	2	1	0	0	0	½	6	8	1½	7	4	0	0	0	0	0	0	0	2	2	6	5	2	2	3	2	0	0	2	0	0	0	0	56
SLK.	0	0	0	0	0	0	0	3½	0	0	7½	5	4	3	0	0	0	1	3	2	0	0	0	0	0	0	0	1	2	2	0	0	2	0	0	36



FINAL SCHEDULE FOR 6 HOUR SHIFT

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
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2	0	37													5																						42
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13												6	34													0											40
14													3	33													1										37
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16															1	30													4								35
17																0	33													4							37
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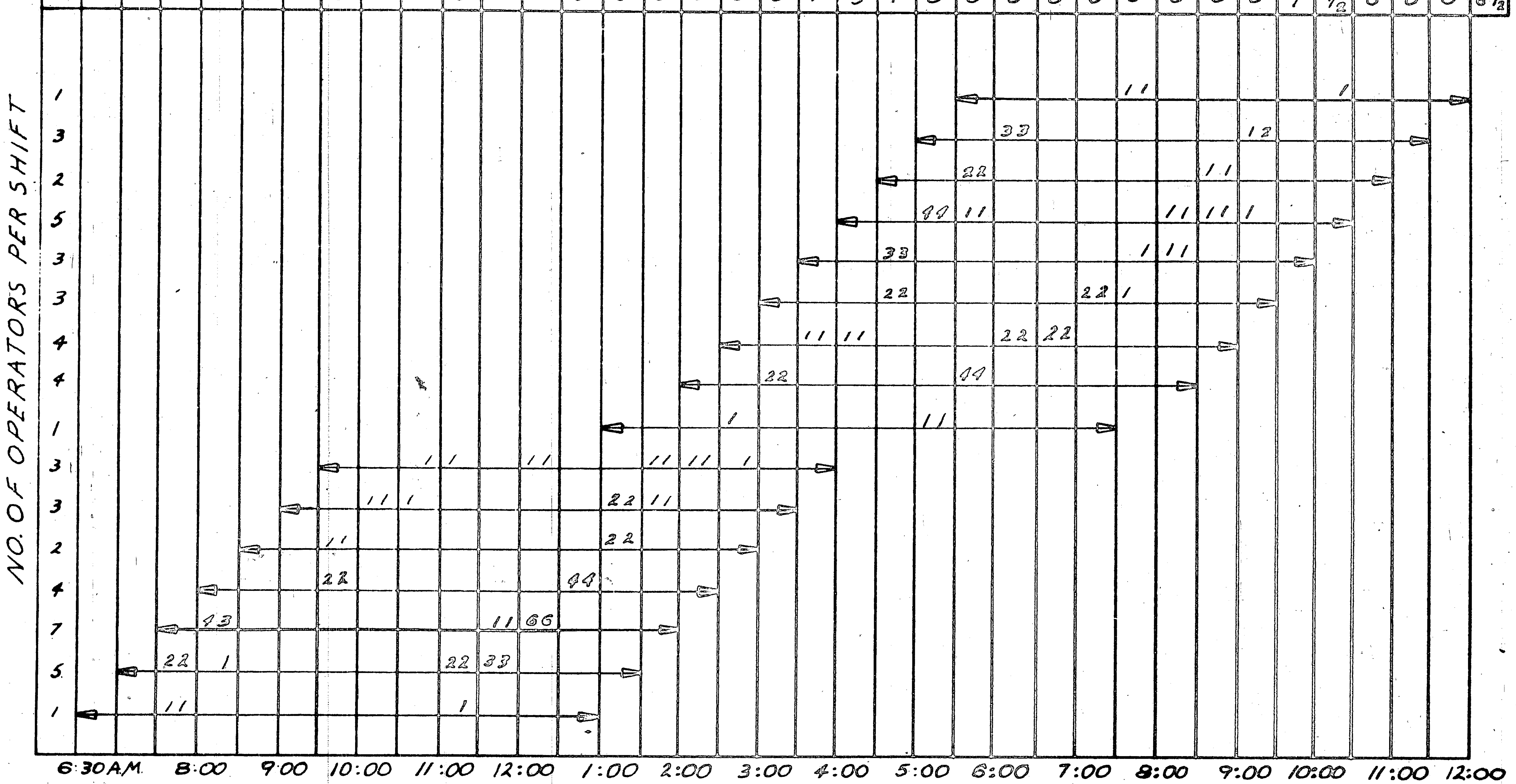
36																																			0	37	37
	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	36	

TABLEAU FOR 6½ HOUR SHIFT

OP. RQ.	1	6	10	13	19	22	22	24	24	22	21	18	21	21	18	16	16	16	16	16	16	20	19	21	24	24	23	23	19	15	13	10	6	4	1	580
OP. FUR.	1	6	13	17	19	22	25	25	25	25	25	25	25	25	20	17	17	18	18	20	22	25	26	26	26	26	25	25	21	17	19	11	6	4	1	663
RLF. RQ.	0	0	3	4	0	0	3	1	1	3	4	7	4	4	2	1	1	2	2	1	5	5	7	5	2	2	2	2	2	2	0	1/2	0	0	0	76 1/2
SLK.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	3	1	0	0	0	0	0	0	0	0	0	1	1/2	0	0	0	6 1/2



FINAL SCHEDULE FOR 6½ HOUR SHIFT

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
1	37														1																						38
2	0	26														16																					42
3		11	26														2																				41
4			9	31														0																			40
5				6	32														5																		43
6					5	35														0																	40
7						2	35														0																37
8							2	37														0															39
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10									0	35														0													35
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12											3	31														0											39
13												6	33														1										40
14													4	33														0									37
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17																0	35														2						37
18																	0	37														0					37
19																		0	30														7				37
20																			2	33														2			37
21																				4	30														3		37
22																					7	33														1	41
23																						4	28														36
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25																								3	37												40
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33																																3	30				33
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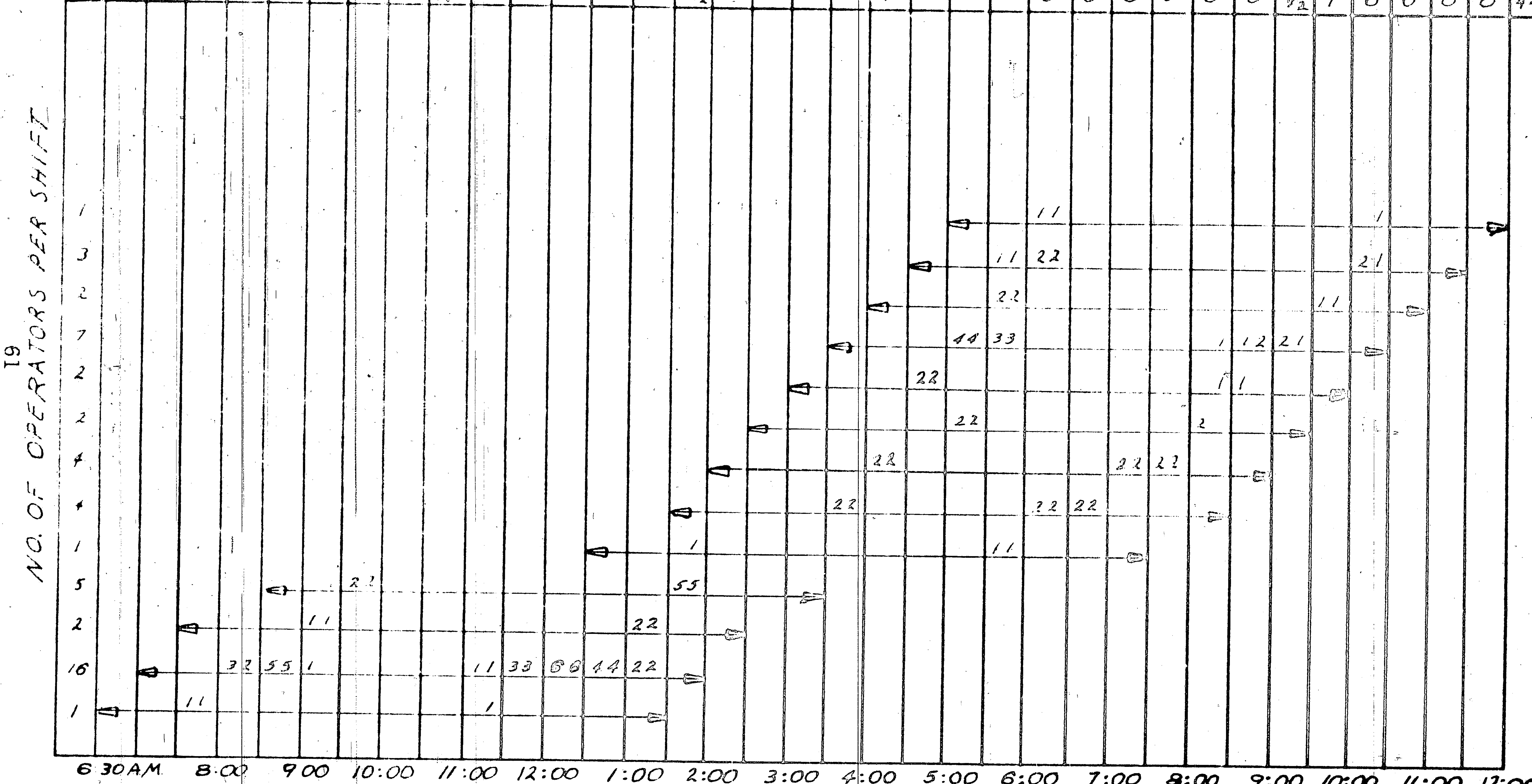
35																																		0	34		34
36																																			0	37	37
	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	38	

TABLEAU FOR 7 HOUR SHIFT

OP. RQ	1	6	10	13	19	22	22	24	24	22	21	18	21	21	18	16	16	16	16	16	16	20	19	21	24	24	23	23	19	15	13	10	6	4	1	580
OP FUR	1	17	19	17	24	24	24	24	24	24	24	24	25	25	28	16	16	18	20	22	25	26	26	26	26	26	25	25	21	17	15	13	6	4	1	700
ALT. RQ	0	0	1	2½	5½	2	0	0	0	½	3	6	4	4	5½	0	0	0	2	2	2	6	7	5	2	2	2	2	2	1½	1	3	0	0	0	76
SLK	0	11	8	3½	0	0	0	0	0	½	0	0	0	0	4½	0	0	2	2	4	7	0	0	0	0	0	0	0	0	½	1	0	0	0	0	44



FINAL SCHEDULE FOR 7 HOUR SHIFT

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
1	19															19																					38
2	18	21															3																				42
3		16	23															2																			41
4			14	26															0																		40
5				11	32															0																	43
6					5	35															0																40
7						2	36															0															37
8							2	37															0														39
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10									0	35															0												35
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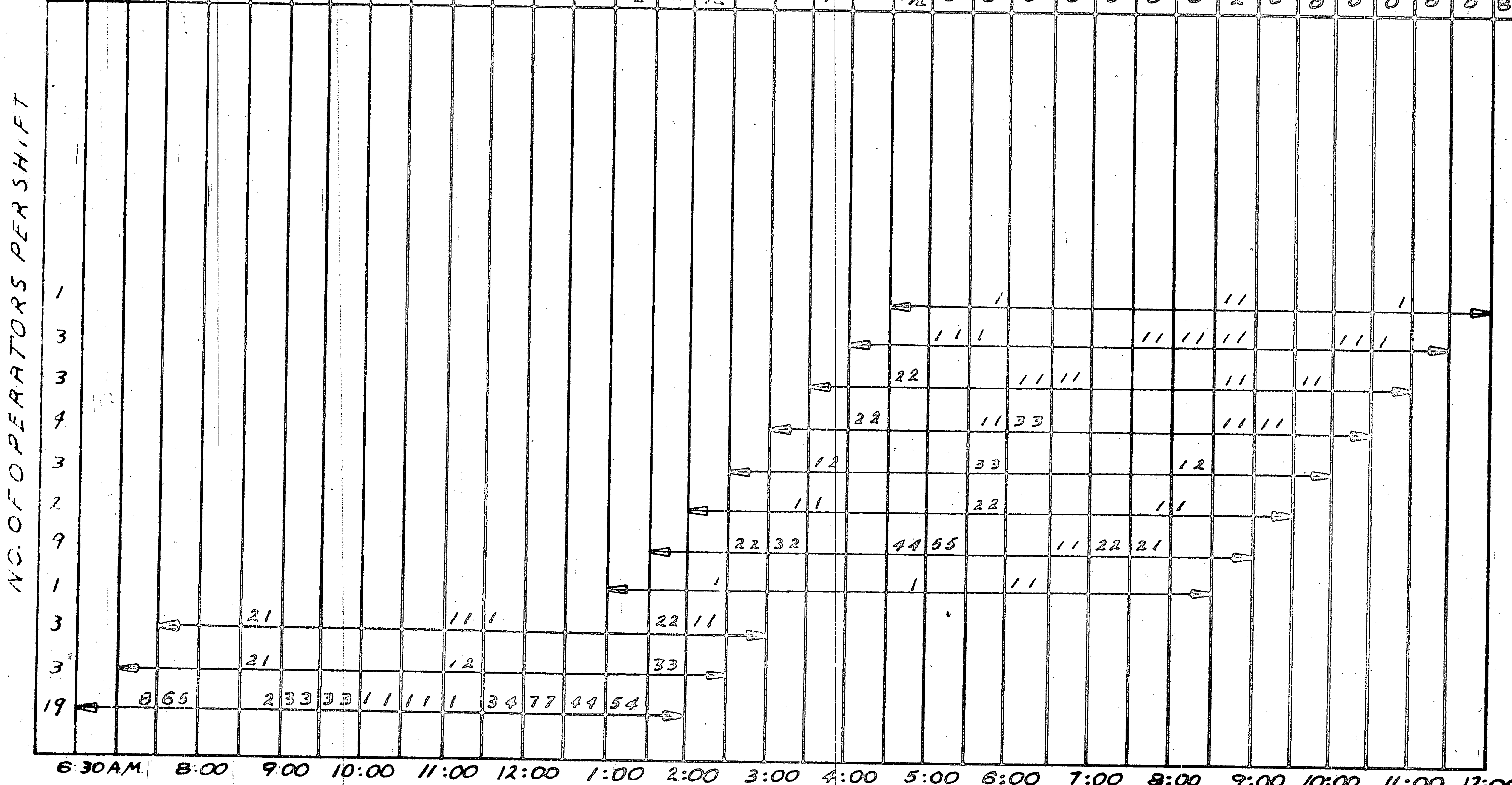
34																																	0	35			35
35																																		0	31		34
36																																			0	37	37
	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	36	

TABLEAU FOR $7\frac{1}{2}$ HOUR SHIFT

OP. RQ.	1	6	10	13	19	22	22	24	24	22	21	18	21	21	18	16	16	16	16	16	18	20	19	21	24	24	23	23	19	15	13	10	6	4	1	580
OP. FUR.	19	22	25	25	25	25	25	25	25	25	25	25	25	26	35	18	18	19	22	26	26	26	26	26	26	26	26	26	25	16	14	11	7	4	1	755
RLF. RQ.	0	4	5½	0	4	3	3	1	1	3	4	7	4	4½	5	1½	2	3	2	2	6½	6	7	5	2	2	3	3	4	1	1	1	1	0	0	102
SLK.	18	12	9½	12	2	0	0	0	0	0	0	0	0	½	12	½	0	0	4	7	3½	0	0	0	0	0	0	0	2	0	0	0	0	0	0	83



FINAL SCHEDULE FOR 7½ HOUR SHIFT

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
1	37					0	0	0	0				0				0		1																		39
2	0	37					0	0	0	0				0				0		5																	42
3		0	37					3	0	0	1				0				0		0																41
4			0	37					0	1	0	2				0				0		0															40
5				0	37					5	0	0	0				0				0		1														43
6					0	37					3	0	0	0				0				0		0													40
7						0	37					0	0	0	0				0				0		0												37
8							0	34					0	3	0	2				0				0		0											39
9								0	37					0	0	0	0				0				0		0										37
10									0	31					4	0	0	0				0				0		0									35
11										0	33					0	0	0	0				0				0		3								36
12											0	34					0	0	0	0				0				0		0							39
13												1	37					0	0	0	0				0				0		2						40
14													0	34					0	0	0	0				0				0		3					37
15														0	33					0	0	0	0				0				0		1				34
16															0	35					0	0	0	0				0				0		0			35
17																0	37					0	0	0	0				0				0		0		37
18																	0	36					0	0	0	0				0				0		1	37
19																		1	36					0	0	0	0				0				0		37
20																			0	32					0	0	0	5				0				0	37
21																				0	37					0	0	0	0				0				37
22																					0	37					3	0	0	1				0			41
23																						0	36					0	0	0	0				0		36
24																							0	37					2	0	0	0				0	39
25																								0	37					3	0	0	0				40
26																									0	37					0	0	0	0			37
27																										0	34					0	0	0	2		36
28																											0	32					3	2	0	0	37
29																												0	32					0	1	0	33
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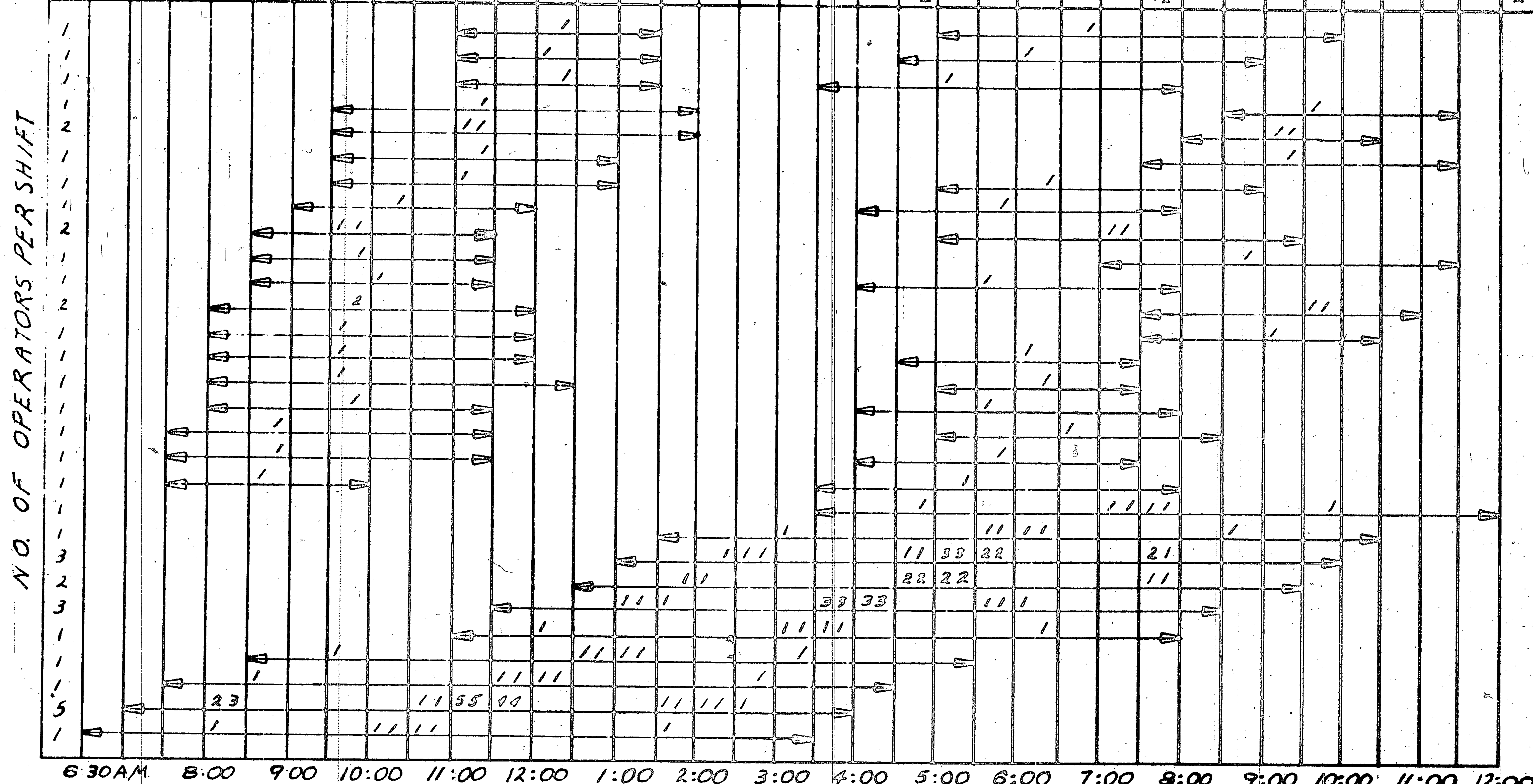
TABLEAU FOR MULTI-LENGTH SHIFTS

OP. RQ	1	6	10	13	19	22	22	24	24	22	21	18	21	21	18	16	16	16	16	16	16	20	19	21	24	24	23	22	19	15	13	10	6	4	1	580
OP. FUR.	1	6	10	16	21	22	27	26	26	30	26	21	22	23	21	18	18	18	20	19	20	26	25	25	25	26	27	23	20	17	15	10	6	4	1	561
ALT. RQ	0	0	0	3	2	0	5	2	2	7½	5	3	1	2	2½	2	2	2	4	3	3½	6	6	4	1	2	3½	0	1	2	2	0	0	0	0	79
SLK	0	0	0	0	0	0	0	0	0	½	0	0	0	0	½	0	0	0	0	0	½	0	0	0	0	0	½	0	0	0	0	0	0	0	0	2



FINAL SCHEDULE FOR MULTI-LENGTH SHIFTS

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VITA

PERSONAL HISTORY

Name:	Douglas D. Padgette
Date of Birth:	June 29, 1934
Place of Birth:	Saluda, South Carolina
Parents	D. Davenport and Gladys W. Padgette
Wife:	Barbara Rogers Padgette
Children:	Stephen Rogers and Mark Douglas Padgette

EDUCATIONAL BACKGROUND

Saluda High School	1947-1951
Clemson University Bachelor of Science	1951-1955
North Carolina State Work in Electrical Engineering	1961-1963
Lehigh University Candidate for Master of Science in Industrial Engineering	1963-1965

HONORS

Cum Laude Graduate from Clemson University

PROFESSIONAL EXPERIENCE

Western Electric Co., Inc.
Burlington, North Carolina
Design Engineer (BTL)
June 1955 to September 1955.

Western Electric Co., Inc.
Burlington, North Carolina
Planning Engineer - Nike Zeus
February 1958 to June 1963.

Western Electric Co., Inc.
Princeton, N. J.
Research Engineer - Candidate in
Lehigh Master's Program
July 1963 to June 1965.