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Crack opening stretch concept in relation to low cycle fatigue

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ABSTRACT

This report summarizes the results of an exploratory experimental investigation into the possibility of the use of crack opening stretch concept (COS) in low cycle fatigue studies of high strength materials. A method for estimation of COS and a definition of the range of COS, for double cantilever beam (DCB) specimen configuration has been presented. The usefulness of face grooved DCB specimens for fatigue crack growth rates near and beyond general yielding conditions has been established through correlation of range of COS ($\hat{\delta}$) with optically measured crack growth rates up to the onset of rapid fracture. Thickness fluctuation at the crack tip have been observed to show, within the experimental limitations, a one to one relation to corresponding $\hat{\delta}$ values. Routine fractographic observations, which revealed locations of striation spacing have been reported.

This investigation was carried out as a part of the major program being conducted in Fritz Engineering Laboratory for the Low Cycle Fatigue Evaluation of High Strength Steels.

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1. INTRODUCTION AND BACKGROUND

Recent developments in limit design and need for improved material efficiency have necessitated material evaluation at high stress levels. In particular, studies concerning low cycle fatigue behavior of high strength steels have been given considerable importance in recent times. Among the many approaches to fatigue studies, investigations based upon linear Fracture Mechanics (1) have been found convenient to interprete fatigue behavior of metals in terms of their fracture chacteristics. Fracture Mechanics approach essentially treats the problem on the basis that the crack is small enough so that continuum principles can be applied. A stress field parameter, K, is defined as a function of applied stress and crack length and this represents the elevated stress field in the immediate neighborhood of the crack tip. This approach enables a successful extension of Griffith's Theory (2) of Brittle Fracture, based on surface energies of materials, to most engineering materials in which fracture occurs after substantial yielding at the crack tip. Based on the principles of continuum mechanics Irwin (3) established the following relation between Stress Intensity Factor and Griffith's Strain Energy Release Rate and is given by

$$K^2 = E' \mathcal{G} \qquad (1)$$

where

$E' = E$ for plane stress

$$E' = \frac{E}{1 - \nu^2} \text{ for plane strain}$$

E = elastic modulus

ν = Poisson's Ratio

In the fracture mechanics analyses $\mathcal{G}$ is used as a generalized (3) force and is called "crack extension force". In lieu of rigorous mathematical analysis Irwin and Kies (4) have shown that $\mathcal{G}$ can be estimated experimentally as discussed later in this report. The values of K and $\mathcal{G}$ for the onset of rapid fracturing, namely K_{Ic} or $\mathcal{G}_{Ic}$ in plane strain and K_c or $\mathcal{G}_c$ in plane stress, are the critical values and considered to be invariant toughness property of a given material. However, experimental data revealed dependence of these values on specimen geometry and for a given temperature appear to have a lower bound (K_{Ic}) for thick specimens and an upper bound (K_c) for a specimen thickness intermediate between thick and extremely small thickness. While in practice K_{Ic} is accepted as a measure of plane strain fracture toughness and considerable work has been reported concerning measurement techniques and standards (5), the definition of plane stress fracture toughness is found to be difficult due to complex large scale yielding at the crack tip. Considerable effort has been put in by various investigators to include the effect of large plasticity at the crack tip for the purpose of estimating stress intensity factor.

One of the first but perhaps most widely used plasticity models is due to Irwin (6), schematically shown in Fig. 1. The plastic zone is assumed to be circular whose radius (r_Y) can be defined by the equations

$$r_Y = \frac{1}{2\pi} \left(\frac{K}{\sigma_{YS}} \right)^2 \quad \text{(plane stress)}$$

$$r_Y = \frac{1}{2\sqrt{2\pi}} \left(\frac{K}{\sigma_{YS}} \right)^2 \quad \text{(plane strain)} \qquad (2)$$

where σ_{YS} is the yield strength of the material. The plastic zone size is used as a positive correction factor to crack length in the evaluation of K. A more rational approach to the problem resulted in a number of plasticity models, which can be further simplified for non-linear elastic or elastic perfectly plastic analysis.

1.1 Dougdale Model

The plasticity model due to Dougdale (7) assumes that plastic flow is confined to a narrow region extending straight ahead of the crack tip and has a height of the order of specimen thickness. Schematic representation of the model is shown in Fig. 2. Mathematical analyses of the model by Goodyear and Field (8) and by Rice (9) summarize a number of results for Dougdale type plastic zone. For example, for the single crack of length $2a$, in an infinite plate with tension, σ, normal to the crack, the crack tip displacement (V_a) is given by

$$V_a = \frac{4\,\sigma_{YS}\,a}{\pi E} \ln \sec \left(\frac{\pi\,\sigma}{2\,\sigma_{YS}} \right) \qquad (3)$$

Experimental observations of the crack tip strains (10) in aluminum alloy sheet specimens however, differed considerably from the prediction of Eq. (3) presumably due to experimental limitations and to the fact that the Dougdale model does not include the variation of yield stress along the strip representing the plastic zone. Nevertheless, it is noted that the crack boundary displacement at the crack tip is a function of stresses and strains in the immediate vicinity and can, in principle, be deduced from K parameter. Since the main limitation to the use of stress intensity parameter for large scale yielding conditions, corrected for the same, seem to be the breakdown of the basic assumptions in its definition, the above relation for crack boundary displacement can be used to define a fracture parameter that could be used for small scale and large scale yielding conditions without losing the generality of its definition. The principal advantage in such a parameter is that its definition roughly represents the integrated effect of the plastic strains within the yielded zone, at the same time it can be unpresumptive of the nature of strain distribution. Various other investigations of the concept (11, 12) and other fracture models proposed by Bilby et al (13) Krafft (14) would appear to point out closely similar results regarding the crack boundary displacements, the variations being dependent upon the nature of simplification in their respective approaches.

For small scale yielding conditions, the results from various models are found to be equivalent to those obtained from linear fracture mechanics analysis. It would appear then that for large scale yielding conditions it is appropriate to define the crack boundary displacement, evaluated still from continuum principles, as a characterization parameter for describing strain (or displacement) field at the crack tip. The problem would now be to connect crack boundary displacement to an appropriate fracture parameter.

1.2 Crack Opening Stretch Concept

Although analyses of crack boundary displacements were reported by various investigators, it was Wells (15) who first appreciated the practical meaning of the displacements at the visual crack tip and initiated the so-called crack opening displacement (COD)* concept for a plausible fracture criteria. Interpreting (as a first approximation) that for a plastically deformed region at the crack tip, the energy release rate in the body as a whole corresponds to the integrated sum of the product of the yield stress and plastic strains within the plastically yielded region. Considering that COS is the integrated effect of the plastic strains, neglecting the variations in yield stress Wells obtained the following expression for COS:

$$\delta = \frac{4}{\pi}\frac{\mathcal{G}}{\sigma_{YS}} \approx \frac{\mathcal{G}}{\sigma_{YS}} \tag{4}$$

It is hypothesized that separation of the crack surface will occur when δ reaches a critical value. Wells (16) used the simple proporttionality and found agreement with experimental measurements with a spade shaped foil gage, later developed into a COD meter (17), and a saw-cut representation of the crack tip. Direct measurements by Kanizawa et al (18) using the parallel crack specimens were found to be in agreement with Eq. (4) at near the onset of rapid fracture. From the fracture mechanics point of view the validity of Eq. (3) has been established by Irwin et al (19) for the anti-plane shear

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*Subsequent review of the terminology appear to favor the use of Crack Opening Stretch (COS) instead of COD.

case for applied stresses approaching the yield stress. While the principal criticism of COS is due to lack of rigorous theoretical justification, its claimed advantages are that COS can be considered as a measure of ductility (or lack of it) based upon critical values and can assist in the evaluation of fracture toughness. Having the units of length, COS can be easily conceived as a physical quantity and apparently as a measurable quantity. Within the limitations of its definition, the COS concept is considered (20, 21) as an extension of Linear Elastic Fracture Mechanics with applications beyond small scale yielding conditions. An important outcome of the concept is that the validity of Eq. (4) to fracture analysis should be realized no matter what the state of crack tip conditions are; provided the crack extension force $\mathscr{G}$ is estimated experimentally or otherwise includes the net effect of those crack tip conditions.

Application of the COS concept to low cycle fatigue appears to have an important bearing due to the fact that low cycle fatigue essentially involves large plastic strain cycling at the crack tip. COS can be considered a suitable characterization parameter for comparative studies in fatigue behavior.

For the range of fatigue crack growths corresponding to small scale yielding conditions, several crack propagation laws have been proposed based on experimental data obtained from laboratory tests. In general, most of the data seem agreeable with the crack propagation law (22) of the form

$$\frac{da}{dN} = C\,(\Delta K)^n \tag{5}$$

where

$\frac{da}{dN}$ = crack growth rate (in/cycle)

$\Delta K = (K_{max} - K_{min})$ is the range of applied stress intensity factor

C and N = material constants thought to be invarient of applied load and state of stress at the crack tip

For many engineering materials Eq. (5) in its basic form is found to be applicable only up to a point, where growth rates rather suddenly increase necessitating re-evaluation of the exponent n and the constant C. Considering that such a transition marks the beginning of low cycle fatigue range, Eq. (5) can in principle, be written in terms of an equivalent parameter, namely, range of COS, $(\hat{\delta})$, in the form

$$\frac{da}{dN} = C_1\,(\hat{\delta})^n \tag{6}$$

The definition of $\hat{\delta}$ is obtained from the definition COS, which as mentioned earlier, can be estimated experimentally.

A notable observation of fatigue crack growth rates from experimental data is that the transitional change in crack growth rates appear to occur when incremental crack extensions are of the order of typical microstructural dimensions such as mean grain size. It would appear then that analysis of crack growth rates, in terms of $\hat{\delta}$, which is the net effect of cyclic change of plastic strains within the plastic zone, could lead to an understanding of the effect of microstructure on the toughness characteristics, particularly, in high strength materials.

This investigation, basically exploratory in nature, was intended to study the low cycle fatigue behavior of a typically high strength steel with specifications given in Table 1A. This report presents a plausible method of estimation of COS at high stress levels and, based on experimental evaluation of critical COS, hypothesize a plane stress fracture criteria for the material. For the purpose of correlation with crack growth rates a definition of cyclic range of COS ($\hat{\delta}$) is assumed and the results are presented for different specimen sizes. In addition the results of the attempts of thickness fluxuation (at the crack tip) measurements and routine fractographic observations, which seem to reveal some new scope of further investigation have been presented.

2. A METHOD OF ESTIMATION OF COS

A method for the experimental estimation of crack extension force $\mathcal{G}$ suggested by Irwin and Kies (4) is by the use of compliance characteristics of the test specimen. From simple analysis the relation between $\mathcal{G}$, applied load, and specimen compliance can be obtained in the form

$$\mathcal{G} = \frac{P^2}{2} \frac{dC}{da} \tag{7}$$

where

C = specimen compliance (inverse of stiffness) a function of crack length and specimen geometry

P = applied load

a = crack length

The specimen compliance in general can be obtained in an empirical form (as a function of crack length and specimen geometry) from experimentally obtained load-deflection curves for various crack lengths. Combining Eq. (7) and (4) and retaining the constant $4/\pi$ for the present investigation, we obtain

$$\delta = \frac{2\, P^2}{\pi\, \sigma_{YS}} \frac{dC}{da} \tag{8}$$

For a given material, specimen geometry and crack length, the load at which onset of rapid fracturing occurs would then be defined by Eq. (8), the critical crack opening stretch ($\delta_{critical}$). From Eqs. (4) and (1) it is then possible to obtain K_{Ic} (or K_c) depending upon the nature of loading conditions.

For the use in experiments in plane stress conditions, the specimen should accommodate comparatively large crack lengths (to produce large scale yielding) and at the same time remain stable (being grossly elastic over a range of crack length) for the validity of Eq. (4).

2.1 Double Cantilever Beam (DCB) Specimen

The DCB specimen have been used for studies on the surface energies of brittle materials (23, 24) and for fracture mechanics application to fracture toughness measurements of adhesive joints (25) and metals (26). The principal advantage of a DCB specimen is that, unlike the more conventional edge notched compact specimens, stable test conditions with near general yielding conditions can be realized and fatigue crack growth rates beyond the plane strain to plane stress transitional point could be obtained with great resolution. Since the tests using DCB specimens use relatively large crack lengths, variations of surface energies due to shear effects could be shown to be small and, with symmetrical constraint at the crack tip, Dougdale type plastic strips could be expected to be the actual case.

2.2 Mechanics of DCB Specimens

The DCB specimen, Fig. 3, can be considered as two cantilever beams with one of the surfaces (the plane of symmetry) common to both and with end fixity conditions so defined that the deflections at the built in end are different from zero. Due to the presence of a free boundary (along AB) and crack tip plasticity, the deflections predicted from the simple beam theory are much smaller. Gilman (23) in his simple analysis obtained a correction factor to the solution of the moment-curvature equation

$$\frac{d^2y}{dx^2} = - \frac{M}{E\ I} + (m/AG)\ \frac{dV}{dx} \qquad (9)$$

where

A = the area of cross section (= Bh)

G = shear modulus

M, V = bending moment (= Pa) and shear force respectively at a given cross section

The coefficient m is a numerical factor close to unity. Integrating the above equation by assuming an arbitrary rotation θ at x = a, the equation for slope becomes

$$\frac{dy}{dx} = - \frac{P\ (a^2 - x^2)}{6\ E\ I} - \frac{m\ F}{A\ G} - \theta \qquad (10)$$

Integrating once again, and using the boundary condition y = 0 at x = a, the deflection equation could be easily seen to be

$$y = \frac{P\ (2a^3 - 3a^2x + x^3)}{6\ E\ I} + \frac{m\ P\ (a - x)}{A\ G} + \theta\ (a - x) \qquad (11)$$

The deflection at the load line is obtained by setting x = 0, and substituting E/2(1 + ν), (where ν is Poisson's ratio) for G, we obtain

$$\Delta = \frac{P\ a^3}{3E\ I} + \frac{m\ P\ a}{A\ G} + \theta\ a \qquad (12)$$

Srawley and Gross (27) in a similar analysis compared the deflections to those obtained from the boundary collocation method and solved the arbitrary slope term in terms of crack length and beam height as

$$\theta = \theta\ (a,\ H) = \frac{P}{E\ I}\ (0.7\ a\ h + 0.5\ h^2) \qquad (12)$$

The empirical equation given by Rippling and Mostavoy (25) in the form

$$y_o = \frac{4\ P}{E\ B}\ [(\frac{a + a_o}{h})^3 + \frac{a}{h}] \qquad (13)$$

where

a_o = empirical factor = 0.6 h for specimens of 1/4" to 1" thickness and up to 12" long

was compared to Eq. (11) and found to be in very close agreement for the specimens without face grooves. Since the analysis for face grooved DCB specimen seemed to be more complex, an empirical equation of the form, Eq. (13) will be of reasonable accuracy for the purposes of compliance equation.

Thus the procedure for the estimation of COS will involve mainly the compliance calibration of the specimen, suitably chosen for the test requirements.

2.3 Calibration of Half Circle Face Grooved DCB Specimens

Three double cantilever beam specimens, Fig. 4, one without face groove and two with half circle face grooves were machined from a one inch thick 7075-T6 Aluminum alloy sheet. Starting from an initial saw-cut notch length of about 3 inches, load-deflection data is obtained at various crack lengths at 0.25 inch increments. Before the data is recorded the specimen is pre-cycled at a nominal load (usually 40 percent of the load at which non-linearity in load-deflection is anticipated) for 200 cycles. The slope of the (unloading) portion of the load deflection curve is then calculated for each crack length. Combining the data from three specimens and using the form of Eq. (13), the following empirical equation for specimen compliance (based on total deflection) is obtained.

$$C = \frac{8}{E\ B} \left(\frac{B}{B_N} \right)^{0.22} \left[\left(\frac{a}{h} + 0.4 \right)^3 + \frac{a}{h} \right] \qquad (14)$$

where

B_N = net section thickness

Equation (14) was verified by the compliance measurements made during the progress of the main program specimens (Table 2A). The results are shown in Fig. 5. The close agreement of Eq. (14) to two different materials, namely 7075-T6 Aluminum Alloy (used for calibration) and A514 steel is in fact anticipated as the relative difference between the estimated plastic zone size, compared to the crack lengths, is very small.

Combining Eqs. (14) and (7) we have

$$\delta = \frac{4\ P^2}{\pi\ B\ h\ E\ \sigma_{YS}} \left(\frac{B}{B_N} \right)^{0.22} \left[3 \left(\frac{a}{H} + .0.4 \right)^2 + 1 \right] \qquad (15)$$

It is easily seen from Eq. (15) that for large crack lengths, the main difference from simple beam theory is due to the influence of face groove, which will be about 36 percent for $B/B_N = 4$.

3. A DEFINITION OF RANGE OF COS UNDER CYCLIC LOADING CONDITIONS FOR DCB SPECIMENS

Extending the concepts of fracture mechanics for application to fatigue, it is felt that in low cycle fatigue range the cyclic change of COS ($\hat{\delta}$) corresponding to cyclic loading would control the fatigue crack growth rates. However, at the present state of the art of COS, there appears to be an uncertainity as to the magnitude of the $\hat{\delta}$. Following the approach of linear fracture mechanics (28), it is seen that application of a load level corresponding to a K value results in a crack opening stretch given by Eq. (4) where $\mathcal{G} = K^2/E$. A reduction in stress intensity by an amount corresponding to (Δ K) results in a reduction of COS given by

$$\hat{\delta} = \frac{(\Delta K)^2}{\omega \sigma_{YS} E} \tag{16}$$

where ω is a factor between 1.7 and 2.0 depending upon the extent of compression yielding occurring during unloading of crack tip stresses. For the present investigation, a value of 2 for ω was used with the assumption that for small minimum loads compressive yielding occurs fully and that yield strain in compression and tension are the same. Thus on the basis of Eq. (16), Equation (15) could be used to define the range of COS for a given range of applied load (Δ P). Thus

$$\hat{\delta} = \frac{2 (\Delta P)^2}{\pi B h E \sigma_{YS}} \left(\frac{B}{B_N} \right)^{0.22} \left[3 \left(\frac{a}{H} + 0.4 \right)^2 + 1 \right] \tag{17}$$

A schematic of the relation between δ_{max}, which is COS at maximum load during the first cycle and $\hat{\delta}$ is shown in Fig. 6. Recent experiments at Lehigh (29) using center notched steel specimens indicate the validity of assumption that "ω" is nearly equal to 2. Thus if the sinusoidal loading is so chosen that $\Delta K \sim \frac{1}{2} K_{max}$, then we obtain a simple relation

$$\delta_{max} \sim 2\hat{\delta} \qquad (18)$$

Thus for a given DCB specimen if the crack length and the range of load at the onset of rapid fracturing are known from experiments, by means of the foregoing definition of $\hat{\delta}$, it would be possible to estimate critical value of δ and thus presumably the plane stress intensity factor.

4. CORRELATION OF RANGE OF COS $(\hat{\delta})$ WITH CRACK GROWTH RATES IN LOW CYCLE FATIGUE

The choice of face grooved DCB specimens is found to be useful for conducting fatigue experiments over a range of $(\hat{\delta})$ values from high cycle fatigue range through low cycle fatigue range up to the onset of rapid fracture. For low cycle fatigue tests, with the requirement of plastic zone size in the order of a specimen's net section thickness, the limitations on the maximum loads are governed by the following requirements (19).

1. The beam arm should remain grossly elastic during the loading, i.e.

$$P_{max} < \frac{\sigma_{YS}\; B\; h^2}{6\, a} \left(\frac{B}{B_N} \right)^{\alpha} \qquad (19)$$

where

P_{max} = applied maximum load

a = crack length

B, B_N, h = specimens as shown in Fig. 5

α = a factor to account for stress elevation due to face grooves (= 0.7 to 1.0)

2. For sufficiently large scale yielding at the crack tip

$$\delta > B_N \frac{\sigma_{YS}}{E} \qquad (20)$$

so that crack propagation essentially takes place in a yielded zone and by the first condition unloading would be complete due to the elastic nature of DCB arms.

4.1 Materials and Specimens

a) Materials: ASTM A514 steel in the form of rolled sheets of different thicknesses was used for the investigation. The chemical composition and mechanical properties of the material is listed in Table 2. (For comparison one specimen with specially heat treated A514 steel with specifications RQ100B was also chosen since the material has lower yield stress to yield large $\hat{\delta}$ values).

b) Specimens: Since the specimen geometry has to be chosen subject to restrictions of Eqs. (18) and (19), the parameter that could be used for specimen size effects was the net section thickness. Thus retaining the geometrical similarity to calibration, specimens with four different thicknesses ranging from 0.125 in. to 0.375 in. were chosen. A total number of six specimens as listed in Table 2A were used in this program. Specimen RQ100B-1 having a considerably larger overall dimension, (Table 2B) was intended to obtain a large crack opening stretch if stable crack growths could be obtained up to the onset of rapid fracture.

4.2 Experimental Procedure

From the saw-cut notch of a nominal length a sufficient fatigue crack was obtained by means of high cycle fatigue loading on a MTS fatigue testing machine. For the low cycle fatigue loading a 5-ton Instron testing machine was used. The loading frequencies were between 3 cycles per minute and 10 cycles per minute depending upon the crack length. The general experimental setup is shown in Fig. 7. During the pre-cycling (which was done whenever the experiment started after the overnight stopping), the crack tip was located optically and marked for initial position. After a known number of cycles of loading the final position of the crack tip was located to determine the incremental crack growth. The instrument used was a traveling microscope with a measurement accuracy of 0.0005". The procedure is repeated further, each time measuring the incremental crack growth for a selected number of cycles. The methods of collecting crack growth rates based on incremental crack growth measurements seem to have better accuracy than those based on a single initial reference point. Using the observed (mean) crack lengths between two readings $\hat{\delta}$ was calculated using Eq. (17). For the chosen configuration for DCB specimens it was found possible to test up to $\hat{\delta}$ values larger than 10^{-3} in. and crack growth rates tending to be of the same order could be obtained with considerable resolution. For two of the specimens, testing was continued up to the onset of rapid fracture and corresponding δ_{crit} could be obtained. The results of the experimental data from all the specimens tested were shown in Fig. 8. During the progress of the tests, specimen compliance data using a Linear Voltage Differential Transducer (LVDT) for deflection measurements of the beam arms was obtained for various crack lengths. The data was then used to correlate with Eq. (14) as discussed in Section 2. In addition specific locations where $\hat{\delta}$ and da/dN are

are marked for microscopic observations and the details **were discussed** later in the report.

5. EXPLORATORY MEASUREMENTS OF THICKNESS FLUCTUATIONS IN THE VICINITY OF THE CRACK TIP DURING CYCLIC LOADING

Following the experimental work of Rosenfield et al (30), Gereberich (10), and under plane stress conditions at the crack tip, it is hypothesized that for constant volume (plastic) deformations the magnitude of thickness direction deformations will be of the order of the magnitude of crack opening stretch. Such a hypothesis may tend to be vague due to the lack of uniqueness of gage length to describe the nature of the crack tip (singular) strains and also due to possible three-dimensional nature of the crack front truly plane stress conditions could never be realized for the thicknesses encountered in testing. However anticipating that high $\hat{\delta}$ values a measurable thickness fluctuation will show trends of relationship with respect to $\hat{\delta}$, some exploratory measurements were made.

The experimental set up consists of a Linear Voltage Differential transducer (LVDT), with a measurement resolution of 10^{-5} in. (corresponding to 0.0716 mV DC output) and Hewitt-Packard vacuum tube voltmeter and recorder for measurements. The stem of the LVDT is positioned at the crack tip (see Fig. 9) within a distance of 3/32 inches (visually located) under slight spring tension. The main body of LVDT is attached to an adjustable bracket which is in turn fixed to the back of the specimen. As the crack tip (normal) strains increase, the LVDT stem tends to follow the deforming material in the thickness direction and the output voltage indicated the extent of deformation. During the unloading the material regained its original position (with negligible permanent strains if any) and the reading of the instrument returned to its original position. Thus as the load is applied sinusoidal, the thickness fluctuations follow similar patterns. The output from LVDT is monitored continuously during the fatigue loading and the maximum fluctuation measured as the crack tip approaches the stem position was noted. However, perhaps due to the three-dimensional nature of the crack front, the position at which maximum thickness fluctuation occurs was ambiguous.

The results are compared with the estimated COS and are shown in Fig. 10.

6. FRACTOGRAPHY

Fatigue striation spacings are considered by many investigators (31, 32) to be useful fracture surface features indicative of the fracture mode and the stress conditions which caused the failure. Bates and Clark (33) have reported an empirical relation between striation spacing and the range of the Stress Intensity Factor (ΔK) and observed that each striation is caused by one load cycle, and that striation spacing is a function of the strain in the vicinity of the crack tip. For stress levels corresponding to small scale yielding, striation spacings were reported to be of the same order of magnitude as macroscopic crack growth rates. Recent observations (34) however seem to indicate restrictions on such a correlation for higher stress intensities, when striations begin to lag behind the macroscopic crack growth rate with an increase in difference.

Fractographic examinations were conducted on a typical specimen at three locations where the applied $\hat{\delta}$ and da/dN are known. Electron microscopic specimens were obtained by a stardard two-stage replication method using a cellulose acetate strip. Figure 11. shows the microphotographs of the locations. It is interesting to note that at stress intensities higher than those usually reported in literature for fractographic observations, striation spacing could be observed without elaborate searching of the fracture surface. Even though these may be thought as not representational, the striation spacings were measured to estimate the corresponding microscopic growth rate. The variation of striation spacing with COS are shown in Table 3.

7. DISCUSSION

Wells' definition of COS is a natural extension of concepts of linear fracture mechanics to large scale yielding and is being accepted at least in an engineering sense, as yet another parameter for fracture criteria. But the specific advantage claimed is that it could be defined unpresumptive of the nature of crack tip conditions and at the same time inclusive of the same through the use of crack extension force corrected for plastic zone size effects. The two main uncertainties in fracture analysis are the elastic plastic boundary and the nature of stress and strain distribution within the plastic zone. Even though various simplified analytical approaches do take into account the effect of crack tip conditions the search is still in progress for a unified characterization parameter that will contain the strain singularity at the crack tip. While the stress intensity factor is an adequate characterization parameter so far as small scale yielding is considered, the COS concept could be the basis for the characterization controlling parameter in the large scale yielding.

The proportionality of COS to crack extension force $\mathcal{G}$, and the similarity of expressions obtained by the analysis of various plasticity models appear to suggest that COS can be considered as the net effect of plastic strains in the crack tip vicinity which may or may not be measurable as a physical quantity. However, an estimation of COS by a semi-empirical method is plausible as seen from the present investigations.

The definition of $\hat{\delta}$ adopted for fatigue experiments is substantiated by a close agreement of crack growth rate data obtained by conventional specimen testing (29) as shown in Fig. 8. Since $\hat{\delta}$ is estimated from an empirical equation it may be said that the plasticity effects are implicitly included and extension of the definition to higher stress levels is reasonable.

The application of double cantilever beam specimens for low cycle fatigue testing is helpful as fatigue crack growth rates well above the growth rates generally reported could be obtained. Although due to large deflections at the load lines, the frequency of loading is generally restricted from practical considerations. The experimental versatility could be the definite advantage as from a single specimen test a variety of parameter changes, such as mean load or magnitude of $\hat{\delta}$ over a wide range could be made. By proper choice of specimen geometry the trends of crack growth rate as it approaches the onset of rapid fracture could be observed with a considerable resolution. The presence of face grooves augumented the needed strain elevation in the neighborhood of crack tip. For the purposes of estimation of $\hat{\delta}$, Eq. (17) is found to be representative for a range of specimen sizes. The compliance equation, Eq. (14) being empirical, can be thought to be inclusive of plastic zone corrections.

Within the limitations of the definition of COS and experimental restrictions, the thickness fluctuation measurements, though smaller than the anticipated value by a factor of 10 as seen from Fig. 10 appear to be a potential measurable parameter in relation to fracture process. However the experiment was exploratory and the results reported were meant to indicate an observed trend. It may however be pointed out that the smaller thickness fluctuations could be due to the three-dimensional nature of the crack front, which may mean that plane stress conditions may not have been present and to the fact that the symmetry and increased thickness on either side of the crack plane in the specimen could have restrained normal strains from a distance ahead of the crack tip.

Many engineering materials were found to exhibit a rather sudden transitional change in their fatigue crack growth behavior. Since fatigue data are generally reported as a function of stress intensity parameter analyses of the data beyond the transitional change have not been discussed. It is seen from Fig. 8, that such a transition is observed in the data obtained as a function of estimated (independent of K or ΔK) $\hat{\delta}$ values. The transitional change, generally thought of as a result of plane strain to plane stress transition, seems to occur when COS becomes larger than $(B_N \sigma_{YS}/E)$. A summary of the limited data reported (29, 35) and the data from the present investigation regarding the mean COS at the transitional change and respective mechanical properties of various high strength steels is given in tabulated form, Table 4. Although no apparent trends could be conclusively observed from this limited data, it may be noted that the occurrence of transition at lower COS values may be considered as indicative of comparatively weaker in toughness characteristics and this could form the basis for the design engineer involved in limit design procedures. In addition it is interesting to note that the transition occurs at crack growth rates ranging from 5×10^{-5} in/cycle to 5×10^{-4} in/cycle (in about the same order as toughness) a dimension seen to be of the order of microstructural dimensions. Since a relation between mean grain size and strength properties is in general expected, the foregoing observations could lead to further investigations. In conclusion the following observations may be summarized with regard to the fatigue crack growth rate trends in low cycle fatigue range.

1. For the project A514 steel, and other types of steels considered for discussion, the transitional change in fatigue crack growth rate appear to be rather sudden and occurs when growth rates are of the order of microstructural dimensions generally encountered for such materials.

2. For A514 steel it is seen that stable crack growth could be observed up to the onset of rapid fracture, which occurs at a $\hat{\delta}$ value of about $10 - 14 \times 10^{-2}$ in.

3. The observation by Carman and Katlin (35) that the transition for high strength steels occur at stress intensities at 60-80 percent of K_c value appear to deserve more careful consideration.

4. At the instance of observed final fracture, the corresponding $\hat{\delta}$ value was considered to be a critical value, at least for engineering purposes and corresponding critical stress intensity factor is calculated by using the equation

$$K_c^* = \frac{1}{\sqrt{2}} \sqrt{\pi E \sigma_{YS} \hat{\delta}_c} \qquad (20)$$

For A514 steel with $\hat{\delta}_c \sim 10 \times 10^{-3}$ in. K_c^* is found to be 250 ksi $\sqrt{\text{in}}$. This value in comparison with recent experiments (36) at Lehigh in which using thickness reduction measurements on flat specimens, K_c values of over 500 ksi $\sqrt{\text{in}}$ at room temperature were obtained, would appear to be conservative for design purposes. However it should be mentioned that the DCB type test specimens closely simulate the actual conditions in structural components, and as such from an engineering point, critical $\hat{\delta}$ values obtained by present methods should be useful.

It is anticipated (37) that there will exist a range of $\hat{\delta}$ values, just before the onset of rapid fracture, for which $da/dN \sim \hat{\delta}$. It is interesting to note that in the present investigation the crack growth rate at the observed instability position was found to be equal to δ. However, additional data may clarify the trend anticipated. Experiments using specially heat treated RQ100-B steel to obtain detailed data at high stress intensities were unsuccessful due to extreme crack branching in the material. However, as can be seen from the data, the toughness of the material is reflected by lower crack growth rates for a given $\hat{\delta}$ and a transition (upturn) occurring at a higher $\hat{\delta}$ value.

The microscopic crack growth rates, based on striation spacing were found to lag behind the macroscopic growth rates, Table 5, for the range of δ values investigated. The difference between the two growth rate data was found to increase as $\hat{\delta}$ increases, which appears to be consistent with recent observations in many materials by Bates et al (33), and at Lehigh (36) in A514 steel. Nevertheless, the observation of striation spacing, though not considered representative at these high stress levels, and the lack of appreciable permanent necking up to the onset of rapid growth, as observed, would indicate distinct features of fatigue failure, different from static fracture under monotonically increasing load.

8. CONCLUSIONS

The method using an empirical compliance equation for estimation of COS (or range of COS) is seen to be independent of specimen thickness over the range of specimen sizes used, and appears to have considerable potentiality for use in low cycle fatigue investigations.

For A514 steel, beyond the upturn point, considerable stable crack growth was observed before the final fracture. The value of $\hat{\delta}$ at this point was used, from an engineering design point of view, to determine K_c value for the material.

Thickness fluctuations were observed to be measurable within the experimental limitations, there seems to be a possible one to one relation with $\hat{\delta}$ values.

Striation spacings at high stresses, though not representational, were found during the routine fractographic observations and these appear to indicate that microscopic crack growth rate data based on striation spacing were much smaller than the corresponding mocroscopic observations.

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Professor Ferdinand P. Beer is the Chairman of the Department of Mechanical Engineering and Mechanics.

10. NOMENCLATURE

a	crack length from the loading axis
B	specimen thickness (nominal)
B_N	net section thickness
C	overall compliance of the specimen
DCB	double cantilever beam (specimen)
E	modulus of elasticity
$\mathcal{G}$	crack extension force
h	half the specimen height
I	moment of inertia (= $Bh^3/12$)
K	stress intensity factor
L	specimen length
r_Y	plasticity correction factor to crack length
t	range of thickness reduction at the crack tip
U_a	crack boundary displacement normal to the crack front
y	deflection of the DCB arm at a given location
y_o	deflection of the DCB arm at the load time
α	factor to account for stress elevation due to face groove
ΔK	range of stress intensity factor (= $K_{max} - K_{min}$)
δ	crack opening stretch (COS)
$\hat{\delta}$	range of crack opening stretch ($\delta_{max} - \delta_{min}$)
ν	Poisson's ratio
σ_{YS}	yield stress (σ_Y)
ω	factor to account for the extent of compression yielding occurring during the unloading process

TABLE 1

CHEMICAL COMPOSITION AND MECHANICAL PROPERTIES OF TEST MATERIALS

A. A514 Steel - Chemical Composition (%)

C	Mn	P	S	Si	Ni	Cr	V	Mo	Cu	B
010/0.20	0.060/100	0.035 (max)	0.040 (max)	0.15/0.35	0.70/1.00	0.40/0.65	0.03/0.08	0.60/0.60	0.15/0.50	0.002/0.006

Mechanical Properties

Yield Stress	110 ksi
Ultimate Strength	115/135 ksi
Percent Elongation	16-18
Modulus of Elasticity	30×10^3 ksi
Fracture Toughness (K_{Ic})	160 ksi$\sqrt{in}$

B. RQ100B - Chemical Composition

C	Mn	P	S	Si	Ni	Cr	V	Mo	Cu	B
0.16	.69	0.11	.025	.26	1.37	1.04	--	0.59	--	0.003

Mechanical Properties

Yield Stress	83.5 ksi
Ultimate Strength	110/112 ksi
Percent Elongation	23.5 in 2 in.
Modulus of Elasticity	30×10^3 ksi

TABLE 2

DCB SPECIMENS: MATERIALS AND SPECIMEN DIMENSION

A. A514 Steel

Specimen No.	B (in)	B_N (in)	H (in)	L (in)	B/B_N	L/H	Symbol
T-1.1	1.00	0.25	1.875	11.875	4.00	6.33	●
T-1.2	1.00	0.375	1.875	11.875	2.66	7.38	Δ
T-1.3	1.00	0.375	3.250	24.0	2.66	7.38	+
T-1.4	1.00	0.250	1.855	11.875	4.00	6.33	○
T-1.5	0.375	0.188	2.000	11.750	2.00	5.80	◇
T-1.6	0.656	0.125	1.875	11.875	5.20	6.33	□

B. RQ100B Heat Treated Steel

RQ100B-1	1.00	0.25	6.00	26.2	4.00	4.33	⦶

TABLE 3

COMPARISON OF MICROSCOPIC CRACK GROWTH RATE ESTIMATES FROM STRIATION SPACING WITH MACROSCOPIC DATA

$\hat{\delta} \times 10^3$ (inches)	Microscopic Crack Growth Rate (inch/cycle)	Macroscopic Crack Growth Rate (inch/cycle)
1.0	3.00×10^{-6}	1.0×10^{-5}
4.5	3.62×10^{-6}	1.2×10^{-4}
8.0	5.25×10^{-6}	3.0×10^{-3}

TABLE 4

TABULATIONS OF COS VALUES AT TRANSITION IN CRACK GROWTH RATES

FOR VARIOUS HIGH STRENGTH STEELS

Materials & Specification	Mechanical Properties σ_{YS} (ksi)	σ_{ult} (ksi)	$E \times 10^{-3}$	Specimen Thickness (in)	Material Toughness K_{Ic} (ksi$\sqrt{in}$)	K_c (ksi$\sqrt{in}$)	K_{Ic}/E	K_c/E	COS[1] at Transition in $\times 10^3$	Ref.
H-11	242.0	305.5	30.0	0.078	45.5	63.2	1.51	2.10	0.535	
$D6A_c$	241.5	271.5	30.0	0.160	52.0	73.0	1.73	2.46	0.647	
250 Maraging	252.5	266.0	30.0	0.270	73.6	133.0	2.48	4.44	1.680	(34)
250 Maraging	267.5	275.0	30.0	0.075	70.4	218.4	2.35	7.26	3.050	
A514	110.0	115/135	30.0	0.25[2]	160.0[3]	345.0	5.30	11.50	3.40	
A514	110.0	115/135	30.0	0.265	160.0[3]	--	5.30	--	2.06[4]	(F)

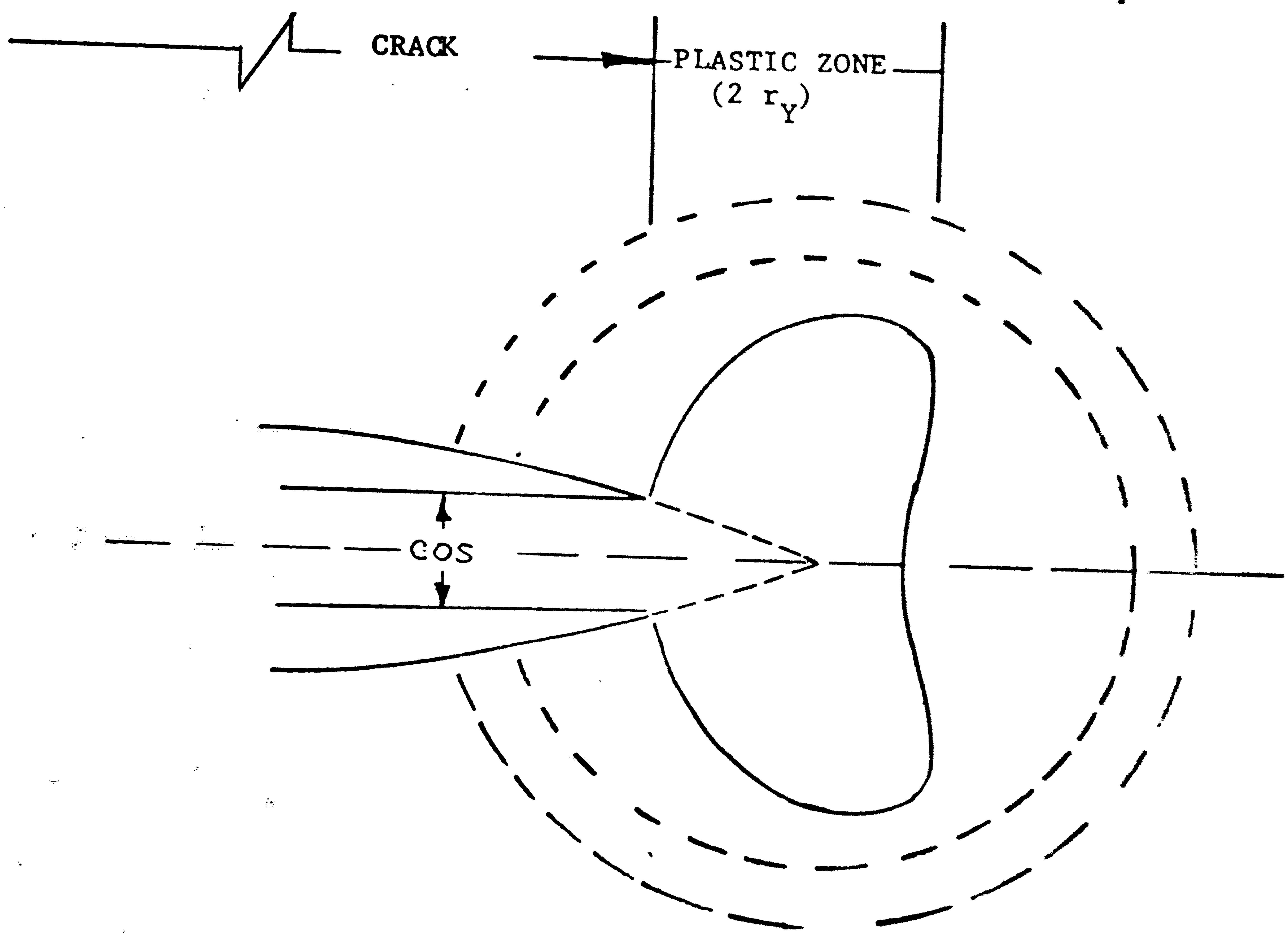
(1) COS = $\frac{4}{\pi} \frac{K^2}{E\,\sigma_{YS}}$

(2) Face Grooved Specimen, net section thickness

(3) From CN Specimen testing

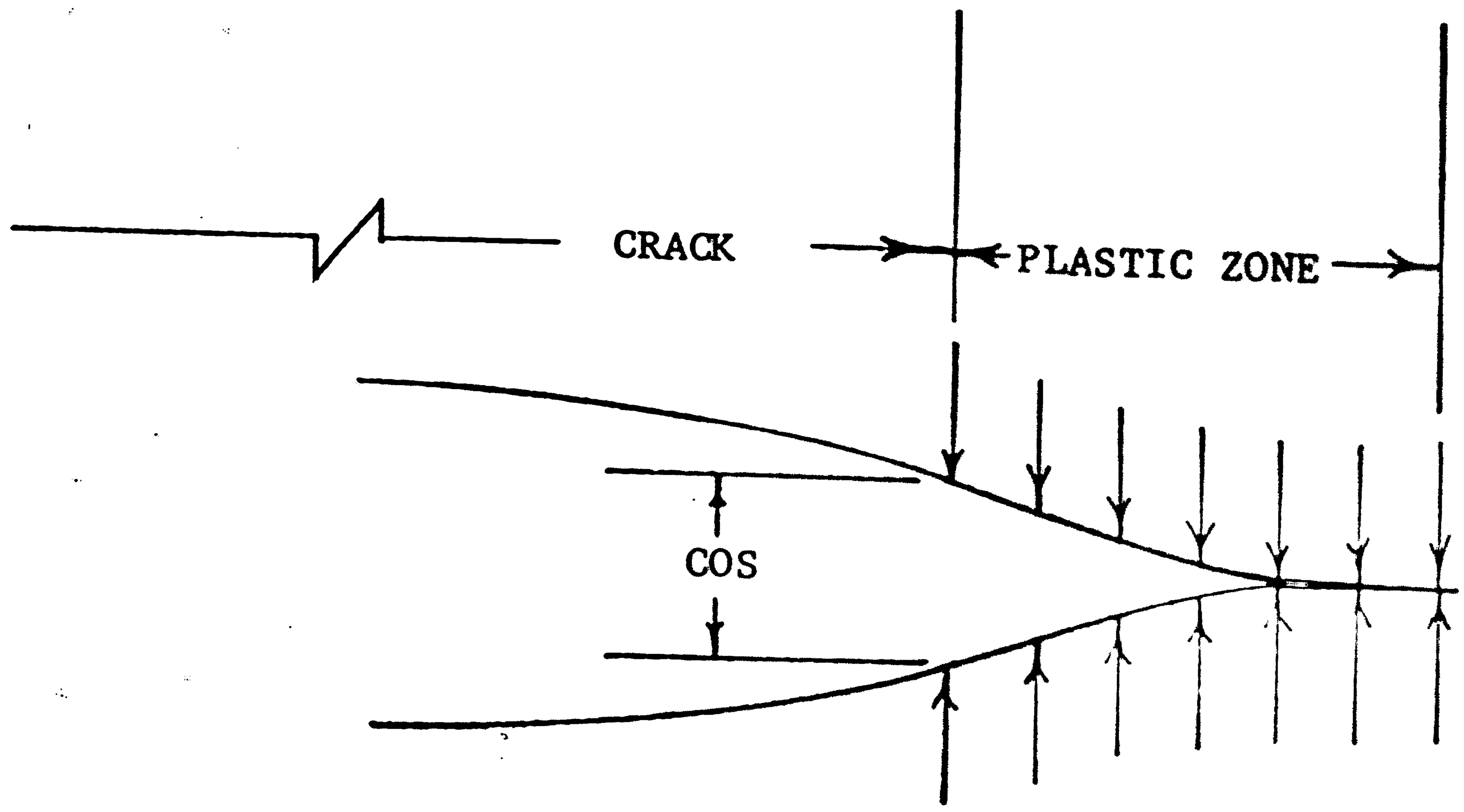
(4) $COS = 2\delta = \frac{2}{\pi} \frac{\Delta K^2}{E\sigma_{YS}}$

(F) Present Investigation



$$COS = \frac{4}{\pi}\,\frac{\mathcal{G}}{\sigma_Y} \qquad 2\,r_Y = \frac{1}{\pi}\,\frac{K^2}{\sigma_Y^{\,2}}$$

Fig. 1 Schematic of Crack Opening Stretch (COS) - Irwin's Plasticity Correction Model



$$\text{COS}\ (\delta) = \frac{\mathcal{G}}{\sigma_Y}$$

Fig. 2 Schematic of Crack Opening **Stretch (COS)** - Plastic Strip Model

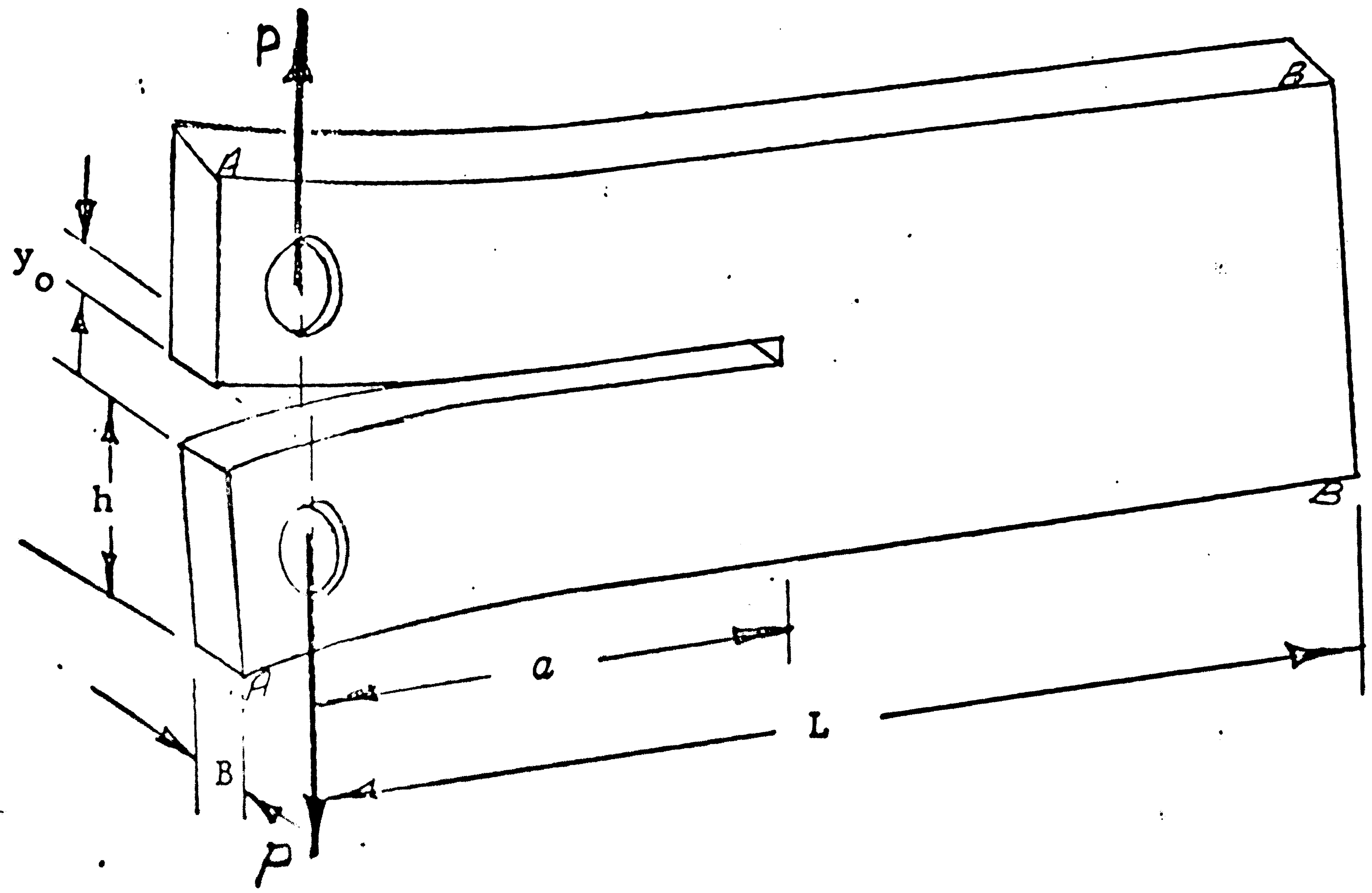


Fig. 3 DCB Specimen

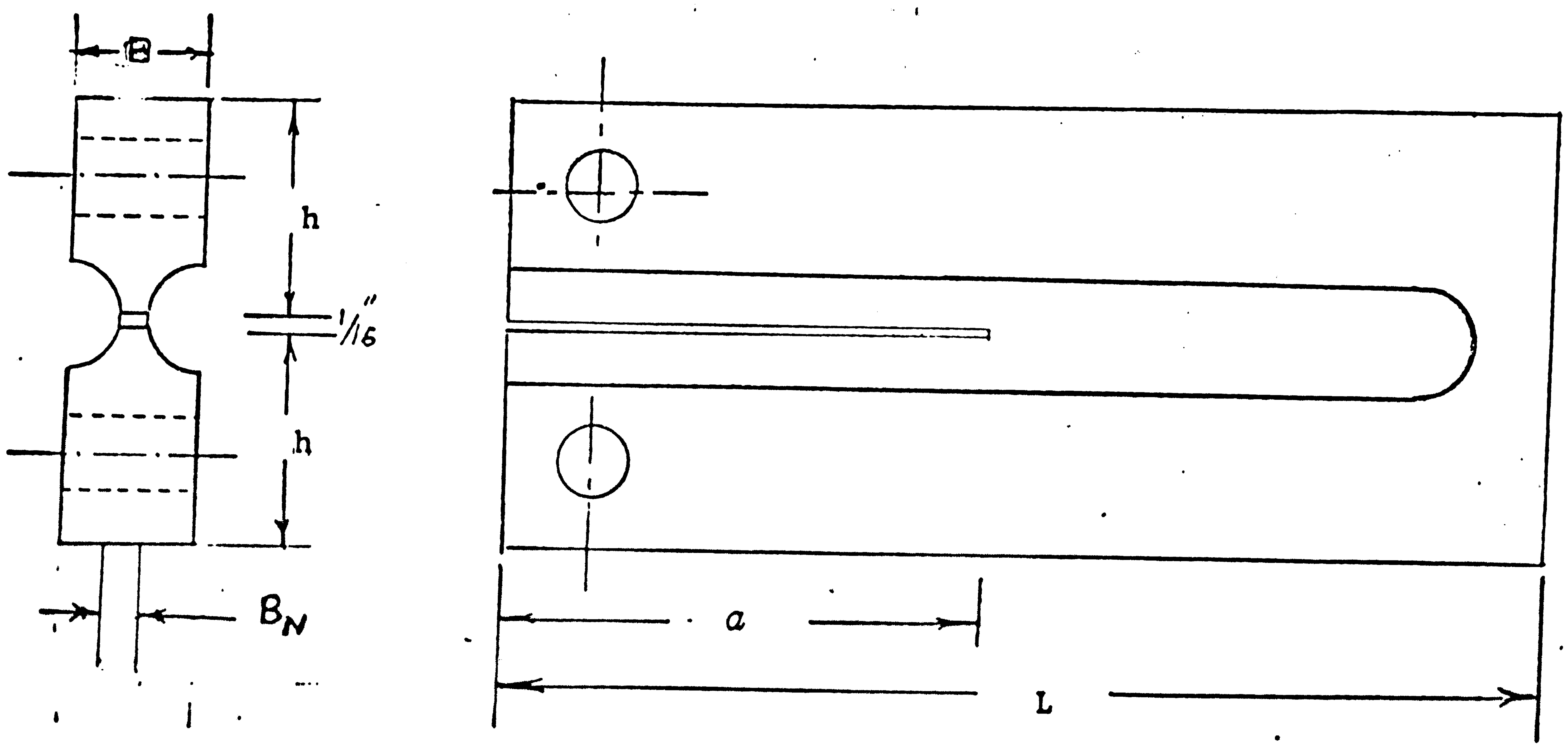


Fig. 4 Half Circle Face Grooved DCB Specimen

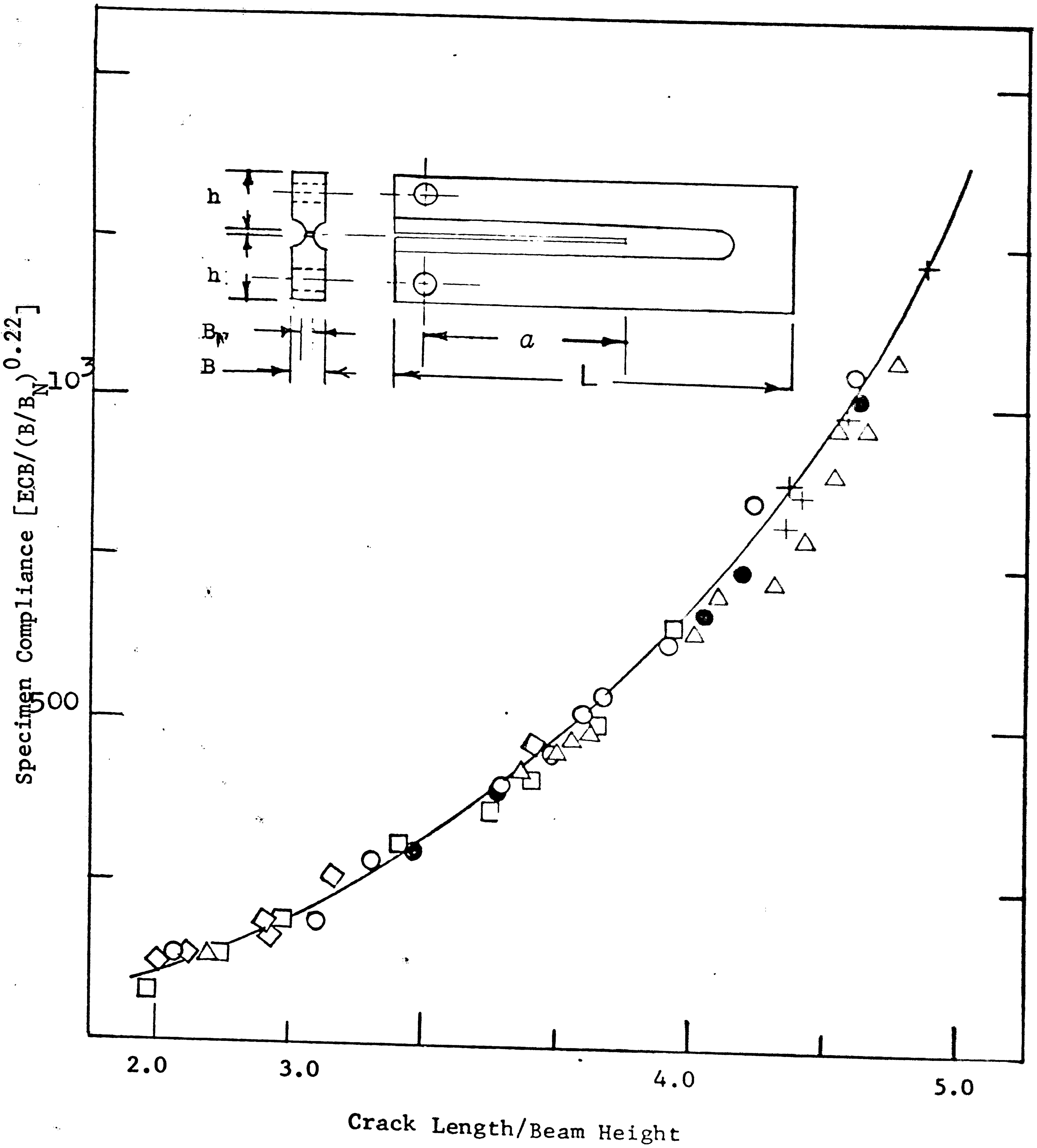


Fig. 5 Solid line shows the dimensionless compliance, EBC, divided by $(B/B_N)^{0.22}$ as a function of a/H, as plotted from Eq. (14) in the test. The specimens providing the indicated data point verifications of that equation are listed in Table 2.

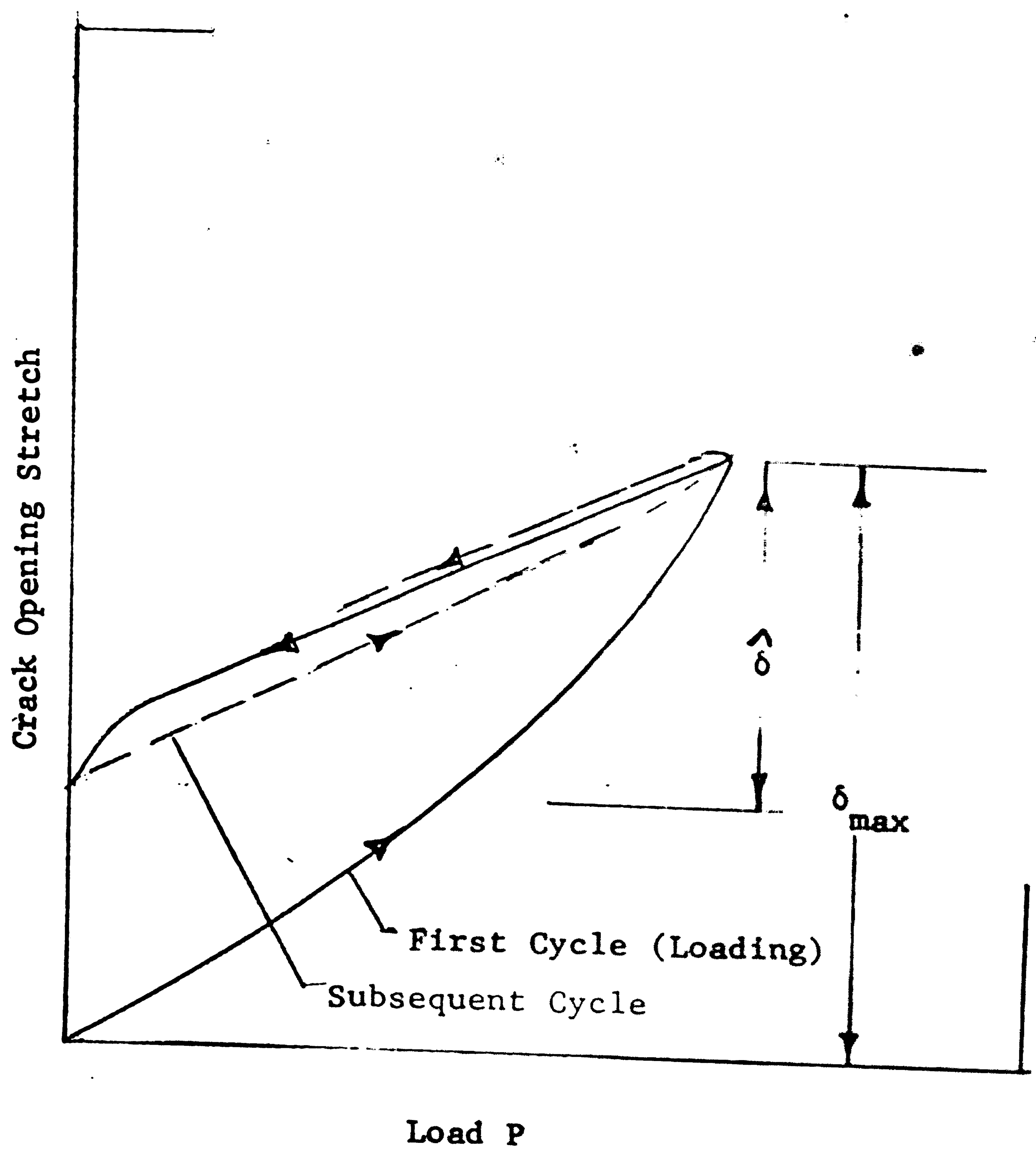


Fig. 6 Schematic of Range of COS (δ) in Cyclic Loading

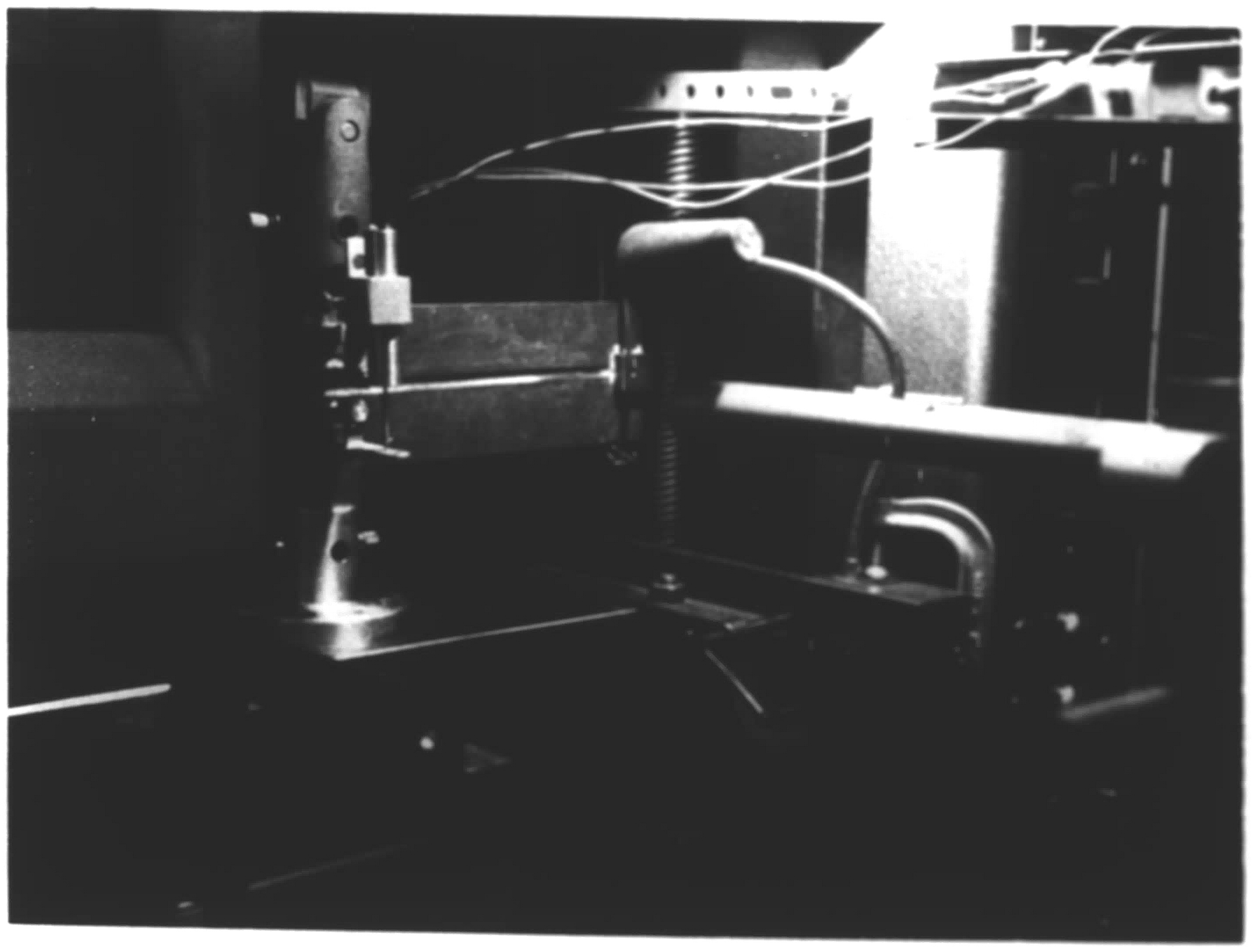


Fig. 7 Experimental Setup Showing the Arrangement of LDVT for Measuring Deflections and Telescope for Crack Growth Measurement

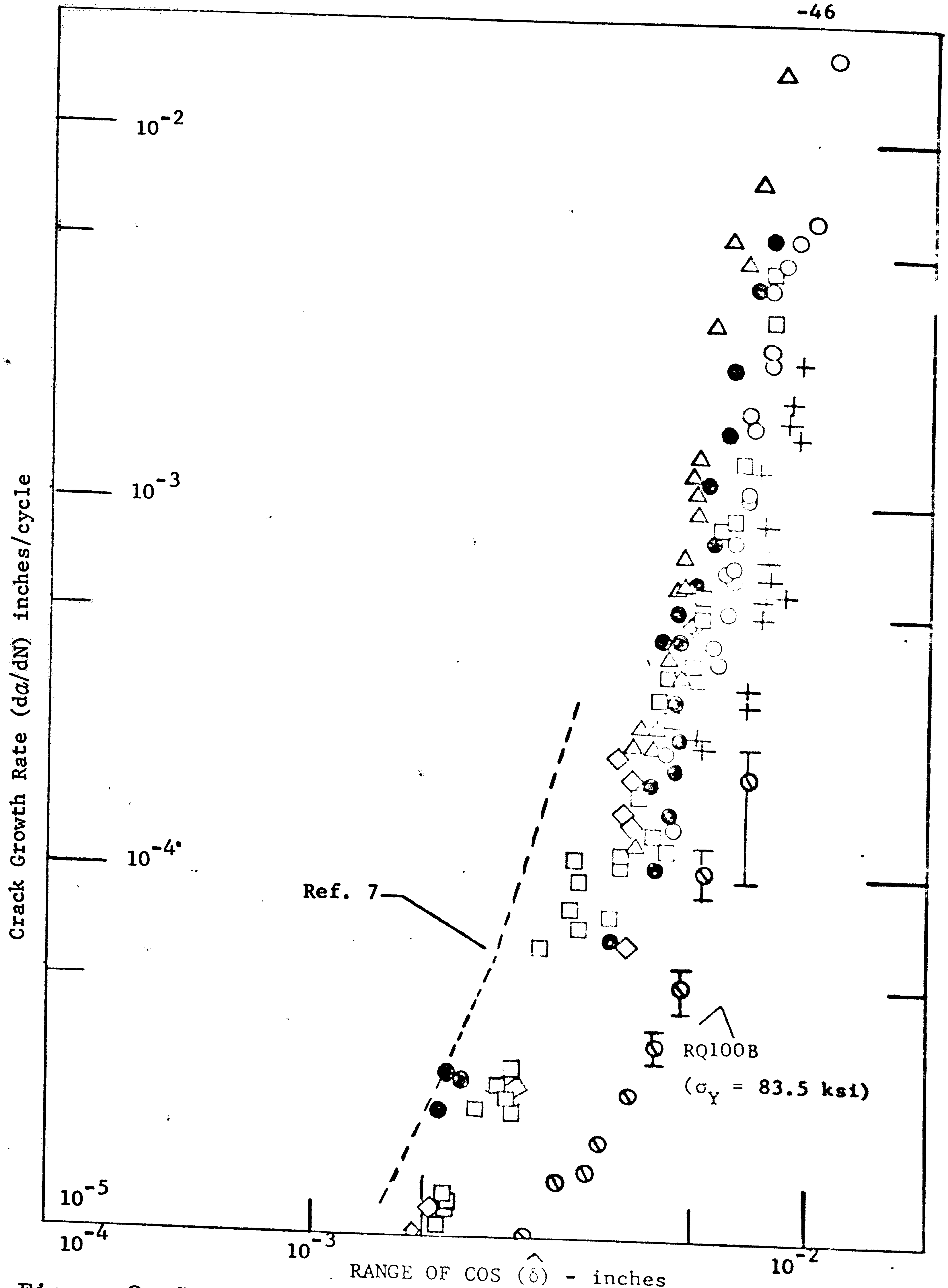


Figure.8. Crack Growth Rate Vs Range Of COS.

Fig. 9 Experimental Setup Featuring LVDT Position for Thickness Fluctuation Measurement

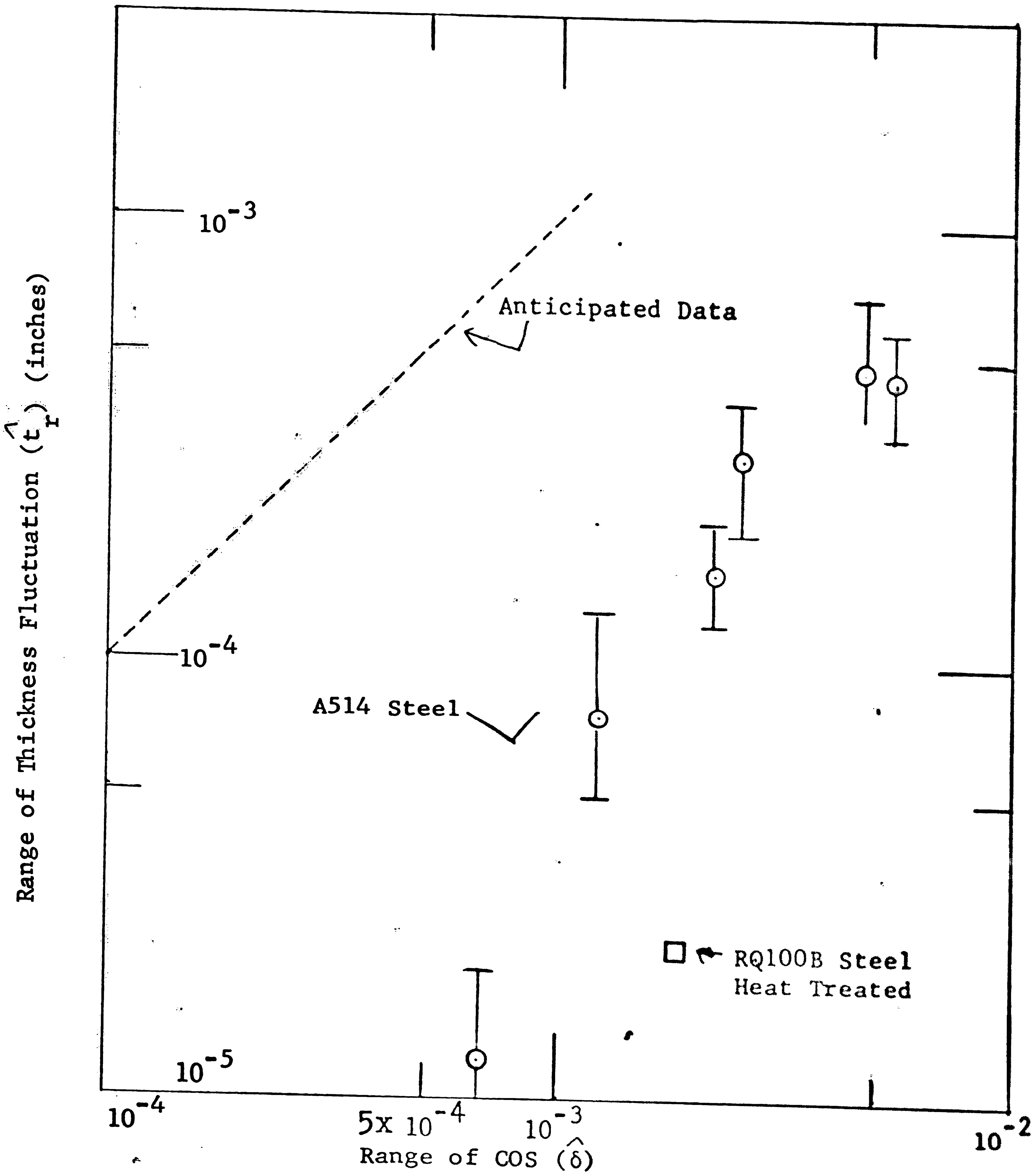
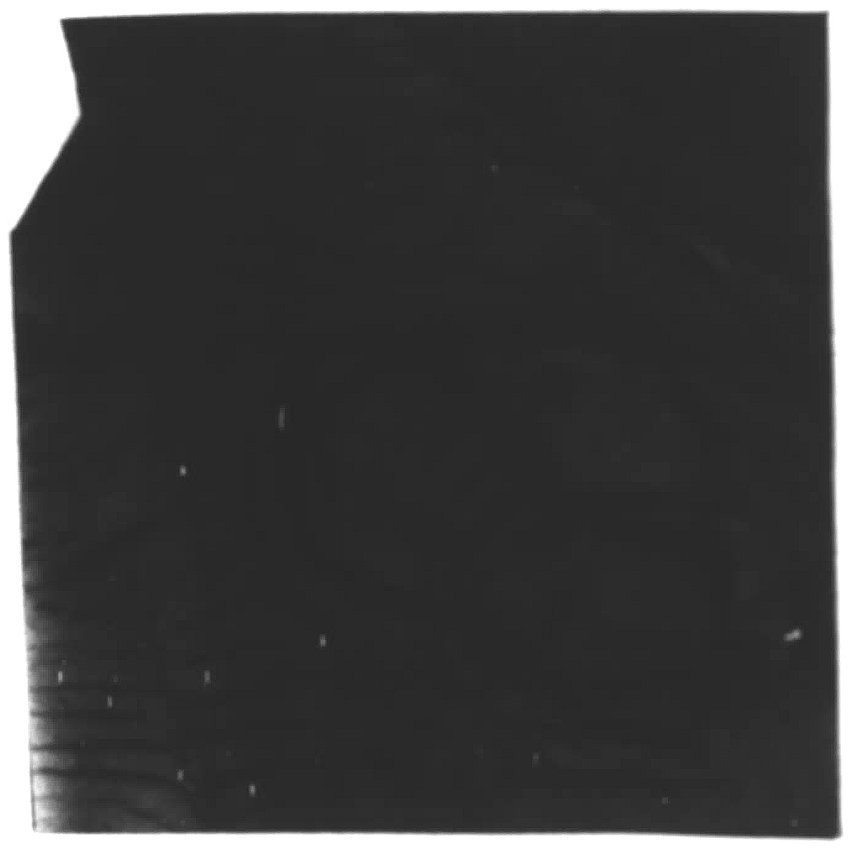
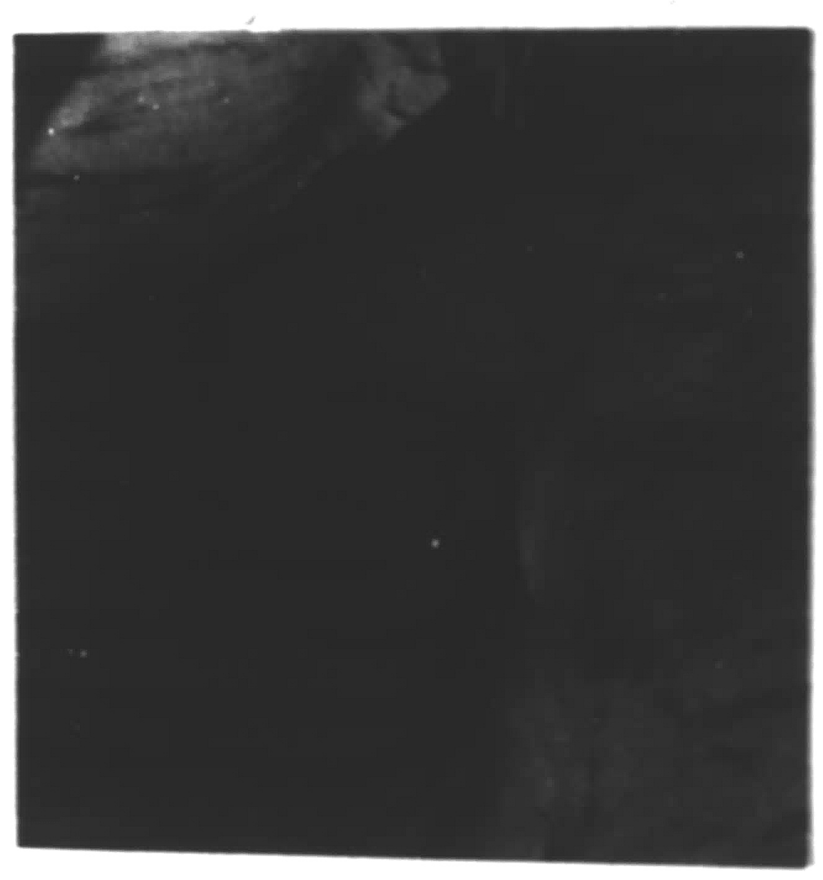


Fig. 10 Plot of Thickness Reduction: **Range ($\hat{t}_r$)** vs. Range of COS ($\hat{\delta}$)



$\hat{\delta} \approx 1.0 \times 10^{-3}$ inches

$\hat{\delta} \approx 4.5 \times 10^{-3}$ inches



$\hat{\delta} \approx 8.0 \times 10^{-3}$ inches

Fig. 11 Fractographic Pictures Showing Striation Spacing at Different $\hat{\delta}$ Values

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