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ANALOG SIMULATION OF HEAT CONDUCTION IN COATED PLATES

by

Koichi Ogawa

A Thesis

Presented to the Graduate Faculty

of Lehigh University

in Candidacy for the Degree of

Master of Science

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1965

CERTIFICATE OF APPROVAL

This thesis is accepted and approved in partial fulfillment of the requirements for the Degree of Master of Science.

May 20, 1965
(date)

Professor in Charge

Head of the Department

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ABSTRACT

Partial differential equations can be approximated with systems of simultaneous ordinary differential equations by replacing one or more of the partial derivatives by appropriate finite differences. The resulting ordinary differential equations can sometimes be solved directly by an analog computer.

Simulation of the partial differential equation of heat conduction with the PACE TR-10 analog computer and theoretical solution of the problem are described and compared. When the slab of material at temperature T_o ^oC is suddenly immersed into an ice-water bath at 0^o C the slab temperature varies with distance x from the face of the slab. Simulation is shown to be of more practical use than analytical solution for examining the heat conduction.

1. INTRODUCTION

1.1 Purpose

The prime objectives of this investigation are the study of the heat equation for a slab with or without a thermal insulator coating when exposed to heat and the simulation of the equation using the finite difference method by means of an analog computer. The accuracy of the analog solution will be compared to a theoretical solution.

When a slab (coated with or without a thermal insulator) is exposed to heat, the slab temperature change is a function of time, distance from the surface, and the properties of the surface coating. Since the analytical solution of the coated slab is tedious[1], an analog simulation is proposed in this thesis.

1.2 Approach

The analog computer is an important tool for engineering design. Since an analog computer integrates analysis with respect to only one variable, namely time, it is fundamentally limited to the solution of ordinary differential equations. To solve a partial differential equation, it is necessary first to convert the equation to one or more ordinary differential equations. If the partial differential equation is linear, this can often be done by separation of variables, which results in ordinary differential equations of the eigenvalue type.[2,3,4]

The above method of separating variables and obtaining a series type of solution can be carried out fairly efficiently on an analog computer.

Simulation can be done by replacing some of the partial derivatives by finite differences in order to convert the original partial differential equations into a system of ordinary differential equations.[5,6,7,8]

2. PREPARATION AND PROCEDURE

2.1 Basic Heat Equation

The equation of heat flow through a continuous medium involves second order special derivatives and only first order time derivatives. The basic heat equation in the x direction is given by

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k \frac{\partial T}{\partial x}\right] + f \qquad (2.1)$$

where

c = specific heat
ρ = density
k = thermal conductivity
f = heat generated in the material
$T = T_{(x,\ t)}$ temperature at position x and time t in $^{\circ}C$
t = time

The heat flux F across a unit surface normal to the x direction is defined by the equation

$$F = - k \frac{\partial T}{\partial x} \qquad (2.2)$$

In this problem no heat was generated, therefore

$$f = 0 \qquad (2.3)$$

For analog solution the partial differential equation (2.1) must be reduced to a series of ordinary differential equations with constant coefficients. This reduction is ordinarily made by using a finite difference approximation[9] for all derivatives with respect to x.

2.2 Finite-Difference Approximation

The equation becomes

$$c\rho \frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[k \frac{\partial T(x,t)}{\partial x} \right] \qquad (2.4)$$

where

the heat capacity $C = c\rho$

Instead of measuring the temperature T at all distance x we can measure T only at certain stations along x (Fig. 1). Let T_1 be the value of T at the first x station ($x = x_1$), T_2 be the value of T at the second x station ($x = x_2$) and T_N be the value of T at the nth x station ($x = x_N$). Further, let the distance between stations be a constant Δx. Thus T (x,t) is replaced by T_1 (t), T_2 (t), etc. and we can approximate the heat flux $F_{N-1/2}$ at the N - 1/2 station as

$$F_{N-1/2} = -k \left.\frac{\partial T}{\partial x}\right|_{N-1/2} = -k_{N-1/2} \frac{T_N - T_{N-1}}{\Delta x} \qquad (2.5)$$

In fact the limit of (2.5) as $\Delta x \to 0$ is just the definition of the partial x derivative at that point. In the same way we can approximate the gradient of the flux at the nth station as

$$\left.\frac{\partial}{\partial x}\left[k \frac{\partial T}{\partial x}\right]\right|_N = -\left.\frac{\partial F}{\partial x}\right|_N = -\frac{F_{N+1/2} - F_{N-1/2}}{\Delta x} \qquad (2.6)$$

The equation of heat-flow balance at the nth station can now be written. Thus

$$C_N \frac{dT_N}{dt} = k_{N+1/2} \frac{T_{N+1} - T_N}{(\Delta x)^2} - k_{N-1/2} \frac{T_N - T_{N-1}}{(\Delta x)^2} \qquad (2.7)$$

Where C_N is the heat capacity at the nth station. It is now clear that $d\,T_N/d\,t$ is a ordinary derivative and not a partial derivative, since by definition x is fixed for $T = T_N$.

2.3 The Pace TR-10 Analog Computer

The PACE TR-10 Analog Computer is a fully transistorized, general purpose analog computer. Solid state circuit elements are used throughout the computer to eliminate vacuum tubes, thereby achieving a compact design which requires very little power. It is able to operate stably and accurately in normal office surroundings. Reliable with simplicity in functional design, it is easy to use and can be a powerful tool for the individual engineer in the rapid solution of scientific and engineering problems.

To solve mathematical equations, one interconnects the computing components -- the building blocks of the electrical model. These blocks perform the following operations on variable d - c voltages: multiplication by a constant; algebraic summation; integration with respect to time; multiplication of two variables; generation of known functions of a variable; combinations of these operations. Each component has input and output terminations which are readily accessible at the front face of the computer for interconnection by plugs and patch cords.

Below the patching area lies the monitoring and control panel. This contains features which permit (a) switching the computer on and off, (b) controlling the operational mode of the computer, (c) setting the values of problem parameters to tru-place accuracy, (d) reading out stationary values of problem variables, and (e) periodically adjusting the balance of the d - c amplifiers to ensure their accurate operation.

The front face of the computer is divided into three five-inch high rows of computing components and their corresponding inter-connecting terminations. This will be referred to as the "patch panel". In the top row there are attenuators for multiplying voltages by positive constants less than unity. In the bottom row there are high-gain d - c amplifiers uncommitted in their form of operation and capable of performing many tasks. In the middle row there is an assortment of components and terminations - integrator networks for use with the d - c amplifiers, fixed and variable function generators, quarter-square multipliers, comparators, and terminations for additional control panels, mounted attenuators, function switches, the reference voltages of ± 10 volts, and ground potential.

2.4 Simulation of Heat Equation of Slab

For purposes of illustration, let us assume the following boundary conditions for our conducting slab: at $x = 0$ the temperature remains fixed at T_o and at $x = L = \Delta x\ (N + 1/2)$ the heat flow is zero. The space between $x = 0$ and $x = L$ is therefore broken into N cells. Assume the initial condition, which is the initial temperature distribution in the slab, is such that

$$T_1\ (o) = \theta,\ \ T_2\ (o) = \theta_2,\ - - - - - - -,\ T_N\ (o) = \theta_N$$

Thus we have

1. Zero temperature at $x = 0$: $T_0 = 0$
2. Zero heat flux at $x = L = (N + 1/2)\,\Delta x$: $F_{N + 1/2} = 0$
3. $T_1 = \theta_1,\ T_2 = \theta_2 - - - - T_N = \theta_N$ at $t = 0$

Therefore we can write the complete set of differential equations for N cells:

$$
\begin{aligned}
F_{1/2} &= -\,k_{1/2}\,\frac{T_1 - T_0}{\Delta x} \\
F_{3/2} &= -\,k_{3/2}\,\frac{T_2 - T_1}{\Delta x} \\
F_{5/2} &= -\,k_{5/2}\,\frac{T_3 - T_2}{\Delta x} \\
F_{N - 1/2} &= -\,k_{N - 1/2}\,\frac{T_N - T_{N-1}}{\Delta x}
\end{aligned}
\tag{2.8}
$$

For equation (2.7)

$$
\begin{aligned}
C_1\,\frac{d\,T_1}{d\,t} &= k_{3/2}\,\frac{T_2 - T_1}{(\Delta x)^2} - k_{1/2}\,\frac{T_1 - T_0}{(\Delta x)^2} \\
C_2\,\frac{d\,T_2}{d\,t} &= k_{5/2}\,\frac{T_3 - T_2}{(\Delta x)^2} - k_{3/2}\,\frac{T_2 - T_1}{(\Delta x)^2} \\
C_N\,\frac{d\,T_N}{d\,t} &= k_{N/2 + 1}\,\frac{T_{N+1} - T_N}{(\Delta x)^2} - k_{N - 1/2}\,\frac{T_N - T_{N-1}}{(\Delta x)^2}
\end{aligned}
\tag{2.9}
$$

The computer arrangement for solving the difference equations is shown in Fig. 3. Note that the outputs of each successive row of amplifiers are reversed. Thus the temperature T and heat flux F across the slab can be observed directly as a function of time.

Fig. 3 is simpler despite the increased number of amplifiers. To vary the conductivity k or heat capacity C at any station, only the appropriate resistors are varied. Changes in initial temperature distribution across the slab are made by setting the T_1, T_2, - - - - T_N voltages to the desired values.

3. ANALOG SOLUTION FOR SLAB WITH DIFFERENT MATERIAL

3.1 Introduction

Thermal conductivity k, specific heat c, and density ρ are functions of the x coordinate. When a slab is composed of one type of material then $k_{N - 1/2}$ and C_N Eq. (2.9) and Fig. 2 are;

$$\begin{cases} k_{1/2} = k_{3/2} = - - - - - - k_{N - 1/2} = k \\ C_1 = C_2 = - - - - - - - - C_N = C \end{cases} \tag{3.1}$$

where

$$C = \rho c$$

We are only interested in temperature T in different positions of the slab; therefore, we can combine the constants C_N and $k_{N - 1/2}/(\Delta x)^2$; and the new constant is called thermal diffusivity a. Thus we have

$$a = \frac{k}{\rho c} = \frac{k}{C} \tag{3.2}$$

Therefore equation (2.9) becomes

$$\begin{cases} \dfrac{d\ T_1}{d\ t} = \dfrac{a}{(\Delta x)^2} \left\{ (T_2 - T_1) - (T_1 - T_0) \right\} \\ \dfrac{d\ T_2}{d\ t} = \dfrac{a}{(\Delta x)^2} \left\{ (T_3 - T_2) - (T_2 - T_1) \right\} \end{cases}$$

(3.3)

$$\frac{d\ T_N}{d\ t} = \frac{a}{(\Delta x)^2} \quad (T_{N+1} - T_N)\ (T_N - T_{N-1})$$

If the number of cells are 12 then equation (3.3) will be given by 11 ordinary differential equations. The division of slab thickness and the computer arrangement for the 11 ordinary differential equations are shown in Figs. 3 and 4.

3.2 Analog Computer Setup

The following slab materials and their combinations of aluminum, cast iron, aluminum oxide, and asbestos were tested. These values of the thermal conductivity constants are shown in Table 1.

We have the following four cases for analog solutions.

Case 1

The slab having a temperature of 100° C is suddenly immersed in 0° C water (Fig. 5).

Initial condition of the analog computer is 100° C. We have to choose voltages that depend on thermal diffusivity a and the boundary conditions fixed at $x = 0$ and $x = L = \Delta x\ N$.

The space between $x = 0$ and $x = L$ is therefore broken into $N + 1$ cells. In our problems, it was broken into 12 cells with

$\Delta x = 0.1$ cm. Therefore x becomes $x = L = 1.2$ cm and $\frac{a}{(\Delta x)^2} = \frac{a}{0.01}$.

Case 2

One face of the slab having initial temperature of 0^o C is suddenly heated to 100^o C with a side of the face at 0^o C (Fig. 6)

Case 3

The composite slab having initial temperature of 100^o C is suddenly immersed into 0^o C ice-water (Fig. 7). Here we considered one material from $x = 0$ to $x = 0.55$ cm and a second material from $x = 0.55$ cm to $x = 1.2$ cm. The two materials have constants of $\frac{a_1}{(\Delta x)^2}$ and $\frac{a_2}{(\Delta x)^2}$, respectively, where a_1 is thermal diffusivity of one material from $x = 0$ to $x = 0.55$ cm, and a_2 is thermal diffusivity of another material from $x = 0.55$ cm to $x = 1.2$ cm.

Case 4

One face of the composite slab at a temperature of 0^o C is suddenly heated to 100^o C, with the other side of the face remaining at 0^o C (Fig. 8).

3.3 Analog Solutions

All dimensions are expressed in c.g.s. Slab thickness L is 1.2 cm, Δx is 0.1 cm and the number of cells is 12. For values of thermal conductivity constants used, see Table 1.[3,4]

An x - y recorder was used to plot the solutions.

Case 1

Analog solutions for the slab with different materials (aluminum, cast iron, aluminum oxide, and asbestos) are shown in Figs. 9, 10, 11 and 12. In this case temperature change shows symmetry in the right and left sides from the center of the slab. Hence, measurements at only six stations are actually required for this particular problem and the figures show only half of the 12 stations.

Case 2

Analog solutions are shown in Figs. 13, 14, 15 and 16. Temperature changes in 11 stations are shown.

Case 3

Analog solutions are shown in Figs. 17, 18, 19 and 20. In this case temperature change shows no symmetry with respect to the center of the slab since the slab is composite. Measurements in 11 stations are shown.

Case 4

Analog solutions are shown in Figs. 21, 22, 23 and 24. Temperature change of 11 stations is shown.

From the analog solutions of Case 3 we can observe the unsteady state temperature distribution of composite slab against the distance for the time parameter. These graphs are shown in Figs. 30 to 32.

From Cases 2 and 4 we can observe the unsteady-state temperature distribution of single and composite slab. These are shown in Figs. 33 to 41.

3.4 Analog Solutions with 4, 6, 8, 10 and 12 Cells

The accuracy of the finite-difference approximation depends on the number of cells. Let us now examine the error of the analog solution for the problem of Case 1. The slab material was aluminum and the problem was solved with 4, 6, 8, 10 and 12 cells. These analog solutions are shown in Figs. 25 to 29. Digital solutions are also plotted on the same graphs. From these graphs we can observe the temperature distribution against the distance for the time parameter. These graphs are shown in Fig. 42.

Referring to these figures, consider the center of the slab at a time t. We can now plot the percentage error in the temperature as a function of the number of cells. This is shown in Fig. 43. It follows that the accuracy of the analog solution improves with the number of cells. There is obviously a limit in the number of amplifiers available. Thirty-four amplifiers were used in this problem for 12 cells.

4. THEORETICAL SOLUTION OF HEAT TRANSFER EQUATION

4.1 Introduction

In order to evaluate the accuracy of the differenc technique, it is worth-while to solve the partial differential equations of heat flow by separating variables[2,3,4] or using the Laplace method.[1] For simplicity we will solve the problem of the temperature distribution.

Assume that the medium has constant conductivity k and constant specific heat capacity C. Also assume that there are no heat sources within the medium. Using the complete solutions we will show the numerical results obtained by means of the digital computer for Case 1 and Case 2.

4.2 Case 1, Fig. 5

The region $0 < x < L$

Ends kept at zero temperature

Initial temperature 100° C

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a} \frac{\partial T}{\partial t} \qquad 0 \leq x \leq L \qquad (4.1)$$

The thermal diffusivity a which appears in all unsteady-heat-conduction problems is a property of the material. Qualitatively we observe that, in a material that combines a low thermal conductivity with a large specific heat per unit volume, the rate of temperature change will be slower than in a material that possesses a large thermal diffusivity. Since the temperature T must be a function of time t and distance x, we begin by assuming a product solution, i.e., multiplying one function, which only depends on time, $\textcircled{H}$ (t), by another function which only depends on distance X(x).

The nature of these functions is not known at this point, but will be determined as we proceed.

Thus, if

$$T\,(x,t) = X(x)\;\textcircled{H}\;t \qquad (4.2)$$

it follows that

$$\frac{1}{a}\;\frac{\sigma\textcircled{H}}{\sigma\, t} = \textcircled{H}\;\frac{\sigma^2 X}{\sigma\, x^2} \qquad (4.3)$$

We can now separate the variables, i.e., bring all functions which depend on x to one side of the equation and all functions which depend on t to the other.

By dividing both sides of Eq. (4.3) by X $\textcircled{H}$, we obtain

$$\frac{1}{a\textcircled{H}}\;\frac{\sigma\textcircled{H}}{\sigma\, t} = \frac{1}{X}\;\frac{\sigma^2 X}{\sigma\, x^2} \qquad (4.4)$$

Now observe that the left-hand side is a function of t only and, therefore, is independent of x. Similarly the right-hand side is a function of x only and will not change as t varies. Since neither side can change as t and x vary, both sides are equal to a constant which we shall call μ. Hence, we have two ordinary and linear differential equations with constant coefficients.

$$\frac{d \textcircled{H}(t)}{d\ t} = a\ \mu \textcircled{H}(t) \tag{4.5}$$

and

$$\frac{d^2 X}{dx^2} = \mu\ X\ (x) \tag{4.6}$$

The general solution for Eq. (4.5) is

$$\textcircled{H}(t) = C_1\ e^{a\ \mu\ \theta} \tag{4.7}$$

If μ were a positive number, the temperature of the slab would become infinitely high as t increased which is contrary to physical conditions. Therefore, we must reject the possibility that $\mu > 0$. If μ were zero, then we would find that the function expressing the time dependence of the temperature in the slab would be a constant. Again, this possibility must be rejected because it would not be consistent with the physical conditions of the problem. We therefore conclude that μ must be a negative number and for convenience we let $\mu = -\lambda^2_n$.

The time-dependent function, then becomes

$$\textcircled{H}(t) = C_1\ e^{-a\ \lambda^2_n t} \tag{4.8}$$

The general solution of Eq. (4.6) can be written in terms of a sinusoidal function. Since this is a second-order equation, there must be two constants of integration in the solution. The solution to the equation

$$\frac{\partial^2 X\ (t)}{\partial\ x^2} = -\ \lambda^2_n\ X\ (t)$$

is usually written as

$$X\ (x) = C_2 \cos \lambda_n\ x + C_3 \sin \lambda_n\ x \tag{4.9}$$

Returning to the original product solution as expressed by Eq. (4.2), the temperature, as a function of distance and time in the slab, is given by

$$T\,(x,t) = C_1\, e^{-a\lambda_n^2 t}\,(C_2 \cos \lambda_n x + C_3 \sin \lambda_n x)$$
$$= e^{-a\lambda_n^2 t}\,(A_n \cos \lambda_n x + B_n \sin \lambda_n x) \qquad (4.10)$$

where

$$A_n = C_1\, C_2 \text{ and } B_n = C_1\, C_2$$

Both A_n and B_n being constants, they must be evaluated from the boundary and initial conditions. In addition we must also determine the value of the constant λ in order to complete the solution. The initial and boundary conditions, stated in symbolic form, are:

$$\text{At } t = 0,\ T\,(x,0) = T_0 = 100^\circ\,C \text{ (initial condition)}$$
$$(0 \leq x \leq L) \qquad (4.11)$$

At $x = 0$ and $x = L$

$$T\,(0,t) = 0^\circ\,C \qquad (4.12)$$
(boundary condition)
$$T\,(L,t) = 0^\circ\,C \qquad (4.13)$$

Substitute boundary conditions of Eq. (4.12) into Eq. (4.10), then we get for $x = 0$

$$0 = e^{-a\lambda_n^2 t}\,(A_n \cos 0 + B_n \sin 0)$$
$$= A\,e^{-a\lambda_n^2 t}$$

So $\quad A = 0$

The solution for T (x,t) becomes

$$T(x,t) = B_n e^{-a \lambda_n^2 t} \sin \lambda x$$

For x = L this becomes

$$0 = e^{-a \lambda^2 t} B_n \sin \lambda L$$

In order to satisfy the equation, $\sin \lambda_n L$ must be equal to zero, so that

$$\lambda_n L = n \pi$$

and

$$\lambda_n = \frac{n \pi}{L}$$

The general solution is the sum of the solutions corresponding to each characteristic value, or

$$T(x,t) = \sum_{n=1} e^{-a \frac{n^2 a^2}{L^2} t} B_n \sin \frac{n \pi}{L} x \qquad (4.14)$$

The constant B_n is evaluated by substituting the initial condition $T(x,0) = 100^\circ$ C into Eq. (4.14). We have

$$T(x,0) = 100^\circ \text{ C} = \sum_{n=1} B_n \sin \frac{n \pi}{L} x \qquad (4.15)$$

It can be shown that the characteristic functions, $\sin \frac{n \pi}{L} x$, are orthogonal between x = 0 and x = L and therefore

$$\int^{L} \sin \frac{n \pi x}{L} \sin \frac{m \pi x}{L} = 0 \text{ if } m \neq n \qquad (4.16)$$
$$\neq 0 \text{ if } m = n$$

To obtain a particular value of B_n, we multiply both sides of Eq. (4.15) by $\sin \frac{m \pi x}{L}$ and integrate between 0 and L. In accordance with Eq. (4.16) all terms on the right hand side disappear except the one involving the square of the characteristic function, $\sin \frac{n \pi}{L} x$, and we obtain

$$\int_0^L 100 \left(\sin \frac{n \pi}{L} x\right) d x = B_n \int_0^L \left(\sin \frac{n \pi}{L} x\right)^2 d x \qquad (4.17)$$

The left hand side of Eq. (4.17) becomes

$$\int_0^L 100 \sin \frac{n \pi}{L} x \, d x = 100 \frac{L}{n \pi} \left[- \cos \frac{n \pi}{L} x \right]_0^L =$$

$$\frac{-100 L}{n \pi} (\cos n \pi - 1) = \frac{100 L}{n \pi} (1 - \cos n \pi)$$

The right hand side of Eq. (4.17) reduces to

$$B_n \int_0^L \left(\sin \frac{n \pi}{L} x\right)^2 d x = B_n \int_0^L \frac{1 - \cos \frac{2n \pi}{L} x}{2} d x =$$

$$\frac{B_n}{2} \left[x - \frac{L}{2n \pi} \sin \frac{2n \pi}{L} x \right]_0^L = \frac{B_n}{2} L$$

Hence

$$\frac{100 L}{n \pi} (1 - \cos n \pi) = \frac{L}{2} B_n \qquad (4.18)$$

and

$$B_n = \frac{200}{n \pi} (1 - \cos n \pi) \qquad (n = 1, 2, 3, \ldots.)$$

For n even (n = 2, 4, 6 ....), B_n is clearly zero, and for n odd (n = 1, 3, 5 ....)

$$1 - \cos n\pi = 2$$

and hence

$$B_n = \frac{400}{n\pi} \quad \text{for } n = 1, 3, 5, \ldots \tag{4.19}$$

Finally we obtain the complete solution,

$$T(x,t) = \frac{400}{\pi} \sum_{\substack{n=1 \\ (n:\text{odd})}}^{\infty} \frac{1}{n} \sin \frac{n\pi}{L} x \cdot e^{-\frac{a n^2 \pi^2}{L^2} t} \tag{4.20}$$

4.3 Case 2, Fig. 6

The region $0 < x < L$

Ends kept at temperatures 100° C and 0° C

Initial temperature zero

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a} \frac{\partial T}{\partial t} \tag{4.1}$$

The slab is initially at zero temperature

$$T_0 = T(x,0) = 0 \tag{4.2}$$

and for $t > 0$, the end $x = L$ is kept at zero temperature, while the temperature of the end $x = 0$ is varied in a prescribed way with time.

$$T(0,t) = 100^{\circ}\text{ C}, \qquad T(L,t) = 0 \tag{4.3}$$

The temperature distribution throughout the slab is required as a function of x and t. The temperature distribution is expressed as the sum of two distributions, one of which is to represent the limiting steady-state distribution (independent of t) and the other is to represent the transient distribution (which must then approach zero as $t \to \infty$).

$$T(x,t) = T_s(x) + T_T(x,t) \tag{4.4}$$

The function $T_s(x)$ must be a linear function of x datisfying

$$T(0,t) = 100^\circ\ C \quad , \quad T(L,t) = 0^\circ\ C$$

and hence is of the form

$$T_s(x) = 100 - 100\frac{x}{L} \tag{4.5}$$

and $T_T(x,t)$ is a particular solution of (4.1). The function T_T must be determined in such a way that it vanishes when $t \to \infty$

$$T_T(x,\infty) = 0 \tag{4.6}$$

and so that the sum $T_s + T_T$ satisfies the initial condition (4.2). Also, since $T_s(x)$ satisfies (4.3) it follows that T_T must vanish at the ends x = 0 and x = L for all positive values of t.

$$T_T(0,t) = T_T(L,t) = 0 \tag{4.7}$$

Thus the transient distribution satisfies the homogeneous end conditions.

Product solutions of (4.1) satisfying (4.6) and (4.7) are obtained in the form similar to that of Case 1.

$$T_T = A_n \sin\frac{n\pi x}{L}\, e^{-\frac{an^2\pi^2}{L^2}t} \qquad (n = 1, 2, \ldots.)$$

Thus, combining (4.5) and a superposition of solutions of this type, the required function T (x,t) may be assumed in the form

$$T\ (x,t) = 100 - 100\,\frac{x}{L} + \sum_{n=1}^{\infty} A_n \sin \frac{n\,\pi\,x}{L}\, e^{-\frac{an^2\,\pi^2}{L^2}t} \qquad (4.8)$$

We may verify that (4.8) satisfies the end conditions (4.3), and also that this solution approaches the proper steady-state solution as $t \rightarrow \infty$. It remains then to determine the coefficients A_n in such a way that the initial condition (4.2) is satisfied. Hence,

$$T\ (x,0) = 0 = 100 - 100\,\frac{x}{L} + \sum_{n=1}^{\infty} A_n \sin \frac{n\,\pi\,x}{L} e^{-0},$$

or

$$100\,\frac{x}{L} - 100 = \sum_{n=1}^{\infty} A_n \sin \frac{n\,\pi\,x}{L} \qquad (4.9)$$

Multiplying by $\sin \frac{m\,\pi\,x}{L}$ and integrating from $x = 0$ to $x = L$

$$\int_0^L 100\,\left(\frac{x}{L} - 1\right) \sin \frac{m\,\pi\,x}{L}\, dx = \int_0^L \sum_{n=1}^{\infty} A_n \sin \frac{m\,\pi\,x}{L} \sin \frac{n\,\pi\,x}{L}\, d\,x$$

From this, we obtain

$$A_n = \frac{2}{L}\,100 \int_0^L \left(\frac{x}{L} - 1\right) \sin \frac{n\,\pi\,x}{L}\, d\,x$$

$$= \frac{-\,200}{n\,\pi} \qquad (n = 1,\ 2,\ 3\ \ldots\ldots.)$$

Finally we obtain the solution of Eq. (4.1) with initial and boundary conditions (4.2) and (4.3)

$$T\ (x,t) = 100 - 100\,\frac{x}{L} - \frac{200}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\,\pi\,x}{L}\,e^{-\frac{an^2\,\pi^2}{L^2}t}$$

$$(n = 1,\ 2,\ 3\ .\ .\ .\ )$$

5. SUMMARY AND CONCLUSIONS

This thesis is concerned with the simulation of the heat equation by an analog computer, and with the accuracy of the solution. The general form of analog solutions of the temperature distributions of slabs are discussed, both the simple uncoated and also the more complex coated slab situations.

The investigation consists of the following:

1. Simulation of finite differences of the heat equation.
2. Analog solutions of the heat equation under different initial and boundary conditions for the following slab materials: aluminum, cast iron, aluminum-oxide, aluminum with asbestos, cast iron with aluminum-oxide, and cast iron with asbestos.
3. Digital solutions of the heat equation and comparison with the analog solutions.
4. Analog solutions for aluminum slab with 4, 6, 8, 10 and 12 cells to establish accuracy of analog simulations.
5. The temperature distribution curves of the slabs.

The following conclusions are drawn:

The simple and straightforward solutions of the heat equation with the analog computer has been shown. The solution of the heat equation shows good agreement with that obtained by digital computer solutions based on separation of variables.

As far as percent of error is concerned, it is shown that if an error of 5% is tolerable, only four cells need be used, or that if an accuracy of one percent is required, the minimum of ten cells are required.

6. APPENDIX

The methods to get the theoretical solutions used in the preceding cases may be said to be immediate consequences and extensions of Fouries classical work. An alternative method of dealing with the differential equations of applied mathematics follows from the work of Heaviside. This method is adapted to the solution of problems in conduction of heat. All the solutions previously obtained for heat conduction in the unsteady-state can be derived with the Laplace Transformation method. Since, however, the advantage of the method increases with the complexity of the problem it is reasonable to apply it to the most difficult cases, which in this case is the composite solid heat conduction.

Consider the finite slab with one medium $-\ell < x < 0$ and with parameters k, ρ_1, c_ℓ, a_1, T_1 and a second medium $0 < x < a$ and parameters k_2, ρ_2, c_2, a_2, T_2. Zero initial temperature $x = -\ell$ maintained at T_0, constant and x = a at zero for $t > 0$. (Case 4, Fig. 8)

The differential equations to be solved are

$$\frac{\partial^2 T_1}{\partial x^2} - \frac{1}{a_1} \frac{\partial T_1}{\partial t} = 0 \qquad -\ell < x < 0,\ t > 0 \qquad (1)$$

$$\frac{\partial^2 T_2}{\partial x^2} - \frac{1}{a_2}\frac{\partial T_2}{\partial t} = 0 \qquad - \quad 0 < x < a, \ t > 0 \qquad (2)$$

The boundary conditions there are

$$k_1 \frac{\partial T_1}{\partial x} = k_2 \frac{\partial T_2}{\partial x} \quad , \quad x = 0, \quad t > 0 \qquad (3)$$

$$T_1 = T_2 \quad , \quad x = 0, \quad t > 0 \qquad (4)$$

We apply the Laplace transformation ot (1) and (2), that is, multiply by e^{-pt} and integrate with respect to t from . This gives

$$\int_0^{\infty} e^{-pt} \frac{\partial^2 T_1}{\partial x^2} dt - \frac{1}{a} \int_0^{\infty} e^{-pt} \frac{\sigma T}{\sigma t} dt = 0$$

Since

$$L \left\{\frac{\sigma T}{\sigma t}\right\} = P \ L \ (T) - T_0 = P \bar{T} - T_0$$

and

$$L \left\{\frac{\sigma^n T}{\sigma x^n}\right\} = \frac{\sigma^n \bar{T}}{\sigma x^n}$$

where

$$\bar{T} = \bar{T}(P) = L \ (T) = \int_0^{\infty} e^{-Pt} \ T \ dt$$

this gives

$$\frac{d^2 \bar{T}_1}{d x^2} - \frac{P}{a} \bar{T}_1 = - \frac{1}{a} T_0$$

Since initial temperature $T_0 = 0$

$$\frac{d^2 \bar{T}_1}{d x^2} - q_1^2 \bar{T}_1 = 0 \qquad -\ell < x < 0 \qquad (5)$$

and similarly, Eq. (2) gives

$$\frac{d^2 \overline{T}_2}{d x^2} - q_2^2 \overline{T}_2 = 0 \qquad 0 < x < a \qquad (6)$$

where

$$q_1 = \sqrt{\frac{P}{a_1}}$$

$$q_2 = \sqrt{\frac{P}{a_2}} \qquad (7)$$

These have to be solved with

$$k_1 \frac{d \overline{T}_1}{d t} = k_2 \frac{d \overline{T}_2}{d x} , \qquad \overline{T}_1 = \overline{T}_2 , \quad \text{at } x = 0 \qquad (8)$$

$$\overline{T}_1 = \frac{\textcircled{H}}{\overline{P}} , \qquad x = -\ell \qquad (9)$$

$$\overline{T}_2 = 0 , \qquad x = a \qquad (10)$$

A solution of (5) which satisfies (9) is

$$\overline{T}_1 = \frac{\textcircled{H}}{\overline{P}} \cosh q_1 (\ell + x) + A \sinh q_1 (\ell + x)$$

and a solution of (6) which satisfies (10) is

$$\overline{T}_2 = B \sinh q_2 (a - x)$$

The unknowns A and B are found from (8)

$$k_1 \frac{d\,\bar{T}_1}{d\,x}\bigg|_{x\,=\,0} = \frac{k_1\, q_1 \textcircled{H}}{P} \sinh q_1 \ell + A\, q_1\, k_1 \cosh q_1 \ell$$

$$k_2 \frac{d\,\bar{T}_2}{d\,x}\bigg|_{x\,=\,0} = -\,B\, q_2\, k_2 \cosh q_2\, a$$

$$-A\, q_1\, k_1 \cosh q_1 \ell - B\, q_2\, k_2 \cosh q_2\, a = \frac{k_1\, q_1 \textcircled{H}}{P} \sinh q_1 \ell$$

and

$$-A \sinh q_1 \ell + B \sinh q_2\, a = \frac{\textcircled{H}}{P} \cosh q_1 \ell$$

$$A = \frac{\begin{vmatrix} \dfrac{k_1\, q_1 \textcircled{H}}{P} \sinh q_1 \ell & -\,q_2\, k_2 \cosh q_2\, a \\ \dfrac{\textcircled{H}}{P} \cosh q_1 \ell & \sin q_2\, a \end{vmatrix}}{\begin{vmatrix} -\,q_1\, k_1 \cosh q_1 \ell & -\,q_2\, k_2 \cosh q_2\, a \\ -\sinh q_1 \ell & \sin q_2\, a \end{vmatrix}}$$

$$= \frac{\dfrac{k_1\, q_1 \textcircled{H}}{P} \sinh q_1 \ell \sinh q_2\, a + \dfrac{q_2 \textcircled{H}}{P} k_2 \cosh q_1 \ell \cosh q_2\, a}{-q_1\, k_1 \cosh q_1 \ell \sinh q_2\, a - q_2\, k_2 \sinh q_1 \ell \cosh q_2\, a}$$

$$= \frac{-\,\dfrac{q_2 \textcircled{H}}{P} \left\{ \cos q_1 \ell \cos q_2\, a + \dfrac{k_1\, q_1}{q_2} \sinh q_1 \ell \sinh q_2\, a \right\}}{q_1\, k_1 \left\{ \cosh q_1 \ell \sinh q_2\, a + \dfrac{k_2\, q_2}{k_1\, q_1} \sinh q_1 \ell \cosh q_2 a \right\}}$$

$$B = \frac{\begin{vmatrix} -q_1 k_1 \cos h\, q_1 \ell & \frac{k_1 q_1 (H)}{P} \sin h\, q_1 \ell \\ -\sin h\, q_1 \ell & \frac{(H)}{P} \cos h\, q_1 \ell \end{vmatrix}}{-q_1 k_1 \left\{ \cos h\, q_1 \ell \sin h\, q_2 a + \frac{k_2 q_2}{k_1 q_1} \sin h\, q_1 \ell \cos h\, q_2 a \right\}}$$

$$= \frac{\frac{k_1 q_1 (H)}{P} \left\{ -(\sin q_1 \ell)^2 + (\cos h\, q_1 \ell)^2 \right.}{q_1 k_1 \left\{ \cos h\, q_1 \ell \sin h\, q_2 a + \frac{k_2 q_2}{k_1 q_1} \sin h\, q_1 \ell \cos h\, q_2 a \right\}}$$

$$\bar{T}_1 = \frac{(H)}{P} \cos h\, q_1 (\ell + x)$$

$$+ \frac{\frac{k_2 q_2 (H)}{P} \sin h\, q_1 (\ell + x) \left\{ \cos h\, q_1 \ell \cos h\, q_2 a + \frac{k_1 q_1}{k_2 q_2} \sin h\, q_1 \ell \sin q_2 a \right\}}{k_1 q_1 \left\{ \cos h\, q_1 \ell \sin h\, q_2 a + \frac{k_2 q_2}{k_1 q_1} \sin h\, q_1 \ell \cos h\, q_2 a \right\}}$$

$$= \frac{(H)}{P} \; \frac{\cos h\, q_1 x \sin h\, q_2 a - \frac{k_2 q_2}{k_1 q_1} \sin h\, q_1 x \cos h\, q_2 a}{\cos h\, q_1 \ell \sin h\, q_2 a + \frac{k_2 q_2}{k_1 q_1} \sin q_1 \ell \cos h\, q_2 a}$$

and similarly

$$\bar{T}_2 = \frac{(H) \sin h\, q_2 (a - x)}{P \left\{ \cos h\, q_1 \ell \sin h\, q_2 a + \rho \sin h\, q_1 \ell \cos h\, q_2 a \right\}}$$

The solutions obtained from the Inversion Theorem

$$T_1 = \frac{Ⓗ (k_1 a - k_2 x)}{k_1 a + k_2 \ell} -$$

$$2 H \sum_{m=1}^{\infty} \frac{(\cos \beta_m x \sin \alpha a \beta_m - \sigma \sin \beta_m x \cos \alpha a \beta_m)\, e^{-a_1 \beta_m^2 t}}{\beta_m \left\{ (\ell + \sigma \alpha) \sin \beta_m \ell \sin \alpha a \beta_m - (\sigma \ell + \alpha a) \cos \beta_m \ell \cos k a \beta_m \right\}}$$

$$T_2 = \frac{k_1 Ⓗ (a - x)}{k_1 a + k_2 \ell} -$$

$$2 Ⓗ \sum_{m=1}^{\infty} \frac{\sin \alpha \beta_m (a - x)\, e^{-a_1 \beta_m^2 t}}{\beta_m \left\{ (\ell + \sigma \alpha a) \sin \beta_m \ell \sin \alpha \beta_m - (\sigma \ell + \alpha a) \cos \beta_m \ell \cos \alpha a \beta_m \right\}}$$

where

β_m (m = 1, 2, 3, ...) are the roots of

$$\cos \beta \ell \sin \alpha \beta a + \sigma \sin \beta \ell \cos \alpha \beta a = 0$$

and

$$\sigma = \frac{k_2}{k_1} \alpha$$

$$\sigma = \sqrt{\frac{a_1}{a_2}}$$

7. TABLES AND FIGURES

TABLE 1

Thermal Conductivity, Specific Heat, Density and Thermal Diffusivity of Various Materials

Material	k (cal/sec. cm^2 ^oC)	c (cal/gm ^oC)	ρ (gm/cm^3)	a (cm/sec.)
(Metal)				
Aluminum	0.484	0.208	2.71	0.86
Cast Iron	0.124	0.10	7.29	0.171
(Non Metal)				
Aluminum	0.064	0.28	3.20	0.0714
Asbestos	0.00036	0.25	0.58	0.00248

k : thermal conductivity

c : specific heat

ρ : density

a : thermal diffusivity

TABLE 2 DIGITAL SOLUTION OF THE HEAT EQUATION WITH 10 CELLS

CASE 1

Time (sec) \ Station	T_1 (x = 0.12)	T_2 (x = 0.24)	T_3 (x = 0.36)	T_4 (x = 0.48)	T_5 (x = 0.60)
0.05	31.74	58.58	77.62	88.42	91.85
0.10	21.99	41.71	57.20	67.04	70.41
0.15	16.26	30.93	42.55	50.01	52.58
0.20	12.10	23.02	31.69	37.25	39.17
0.25	9.01	17.15	23.60	27.74	29.17
0.30	6.71	12.77	17.58	20.66	21.72
0.35	5.00	9.51	13.09	15.39	16.18
0.40	3.72	7.08	9.75	11.46	12.05
0.45	2.77	5.27	7.26	8.53	8.97
0.50	2.07	3.93	5.41	6.36	6.68

Aluminum $a = 0.86$

$L = 1.2$ cm

$\Delta x = 0.12$ cm

TABLE 3 DIGITAL SOLUTION OF THE HEAT EQUATION WITH 8 CELLS

CASE 1

Time (sec) \ Station	T_1 (x = 0.15)	T_2 (x = 0.30)	T_3 (x = 0.45)	T_4 (x = 0.60)
0.05	39.07	69.15	86.45	91.85
0.10	27.22	50.08	65.16	70.41
0.15	20.14	37.20	48.58	52.58
0.20	14.99	27.70	36.19	39.17
0.25	11.16	20.63	26.95	29.17
0.30	8.31	15.36	20.07	21.72
0.35	6.19	11.44	14.95	16.18
0.40	4.61	8.52	11.13	12.05
0.45	3.43	6.34	8.29	8.97
0.50	2.56	4.73	6.17	6.68

Aluminum $a = 0.86$

$L = 1.20$

$\Delta x = 0.15$

TABLE 4 DIGITAL SOLUTION OF THE HEAT EQUATION WITH 6 CELLS

CASE 1

Station / Time (sec)	T_1 (x = 0.2)	T_2 (x = 0.4)	T_3 (x = 0.6)
0.05	50.41	82.12	91.85
0.10	35.52	61.16	70.41
0.15	26.31	45.54	52.58
0.20	19.59	33.92	39.17
0.25	14.59	25.26	29.17
0.30	10.86	18.81	21.72
0.35	8.09	14.01	16.18
0.40	6.02	10.43	12.05
0.45	4.49	7.77	8.97
0.50	3.34	5.79	6.68

Aluminum $a = 0.86$

$L = 1.20$ cm

$\Delta x = 0.20$

TABLE 5 DIGITAL SOLUTION OF THE HEAT EQUATION WITH 4 CELLS

CASE 1

Station / Time (sec)	T_1 (x = 0.3)	T_2 (x = 0.6)
0.05	69.15	91.85
0.10	50.08	70.41
0.15	37.20	52.58
0.20	27.70	39.17
0.25	20.63	29.17
0.30	15.36	21.72
0.35	11.44	16.18
0.40	8.52	12.05
0.45	6.34	8.97
0.50	4.73	6.68

Aluminum $a = 0.86$

$L = 1.2$ cm

$\Delta x = 0.2$

TABLE 6 DIGITAL SOLUTION OF THE HEAT EQUATION WITH 12 CELLS

CASE 1

Time (sec) \ Station	T_1	T_2	T_3	T_4	T_5	T_6
0.05	26.67	50.41	69.15	82.12	89.48	91.85
0.10	18.43	35.52	50.08	61.16	68.06	70.41
0.15	13.62	26.31	37.20	45.55	50.79	52.58
0.20	10.14	19.59	27.70	33.92	37.83	39.17
0.25	7.55	14.59	20.63	25.26	28.18	29.17
0.30	5.62	10.86	15.36	18.81	20.98	21.72
0.35	4.19	8.09	11.44	14.01	15.62	16.18
0.40	3.12	6.03	8.52	10.43	11.64	12.05
0.45	2.32	4.49	6.35	7.77	8.67	8.97
0.50	1.73	3.34	4.73	5.79	6.46	6.68

Aluminum $a = 0.86$

$L = 1.2$ cm

$\Delta x = 0.1$

TABLE 7 DIGITAL SOLUTION OF THE HEAT EQUATION WITH 12 CELLS

CASE 1

Station / Time (sec)	T_1	T_2	T_3	T_4	T_5	T_6
0.1	41.13	72.05	89.52	98.07	99.30	99.76
0.2	29.78	55.54	74.81	87.23	93.67	95.64
0.4	21.06	40.46	56.78	69.00	76.52	79.05
0.6	16.37	31.59	44.62	54.58	60.82	62.95
0.8	12.91	24.93	35.26	43.18	48.15	49.85
1.0	10.21	19.72	27.89	34.15	38.09	39.44
1.2	8.07	15.60	22.06	27.02	30.13	31.20
1.4	6.39	12.34	17.45	21.37	23.84	24.68
1.6	5.05	9.76	13.80	16.91	18.86	19.52
1.8	4.00	7.72	10.92	13.37	14.92	15.44
2.0	3.16	6.11	8.64	10.58	11.80	12.22

Cast Iron $a = 0.171$

$L = 1.2$ cm

$\Delta x = 0.1$

TABLE 8 DIGITAL SOLUTION OF THE HEAT EQUATION WITH 12 CELLS

CASE 2

Time (sec) \ Station	T_1	T_2	T_3	T_4	T_5	T_6
0.1	80.95	62.96	46.95	33.48	22.80	14.69
0.2	86.46	73.29	60.87	49.46	39.27	30.41
0.3	88.84	77.88	67.29	57.24	47.83	39.14
0.4	90.11	80.32	70.74	61.45	52.51	43.98
0.5	90.80	81.66	72.64	63.77	55.10	46.66
0.6	91.19	82.41	73.69	65.06	56.54	48.15
0.7	91.40	82.82	74.27	65.78	57.34	48.97
0.8	91.52	83.05	74.60	66.17	57.78	49.43
0.9	91.58	83.18	74.78	66.39	58.03	49.68
1.0	91.62	83.25	74.88	66.51	58.16	49.82

Time (sec) \ Station	T_7	T_8	T_9	T_{10}	T_{11}
0.1	9.14	5.36	2.97	1.52	0.63
0.2	2.29	1.66	1.14	7.12	3.41
0.3	3.12	2.39	1.73	1.13	5.54
0.4	3.58	2.81	2.07	1.37	6.78
0.5	3.84	3.04	2.26	1.50	7.47
0.6	3.99	3.17	2.37	1.57	7.85
0.7	4.07	3.24	2.43	1.62	8.07
0.8	4.11	3.28	2.46	1.64	8.19
0.9	4.14	3.31	2.48	1.65	8.25
1.0	4.15	3.32	2.49	1.66	8.29

Aluminum $a = 0.86$

$L = 1.2$ cm

$\Delta x = 0.1$ cm

TABLE 9 DIGITAL SOLUTION OF THE HEAT EQUATION WITH 12 CELLS

CASE 2

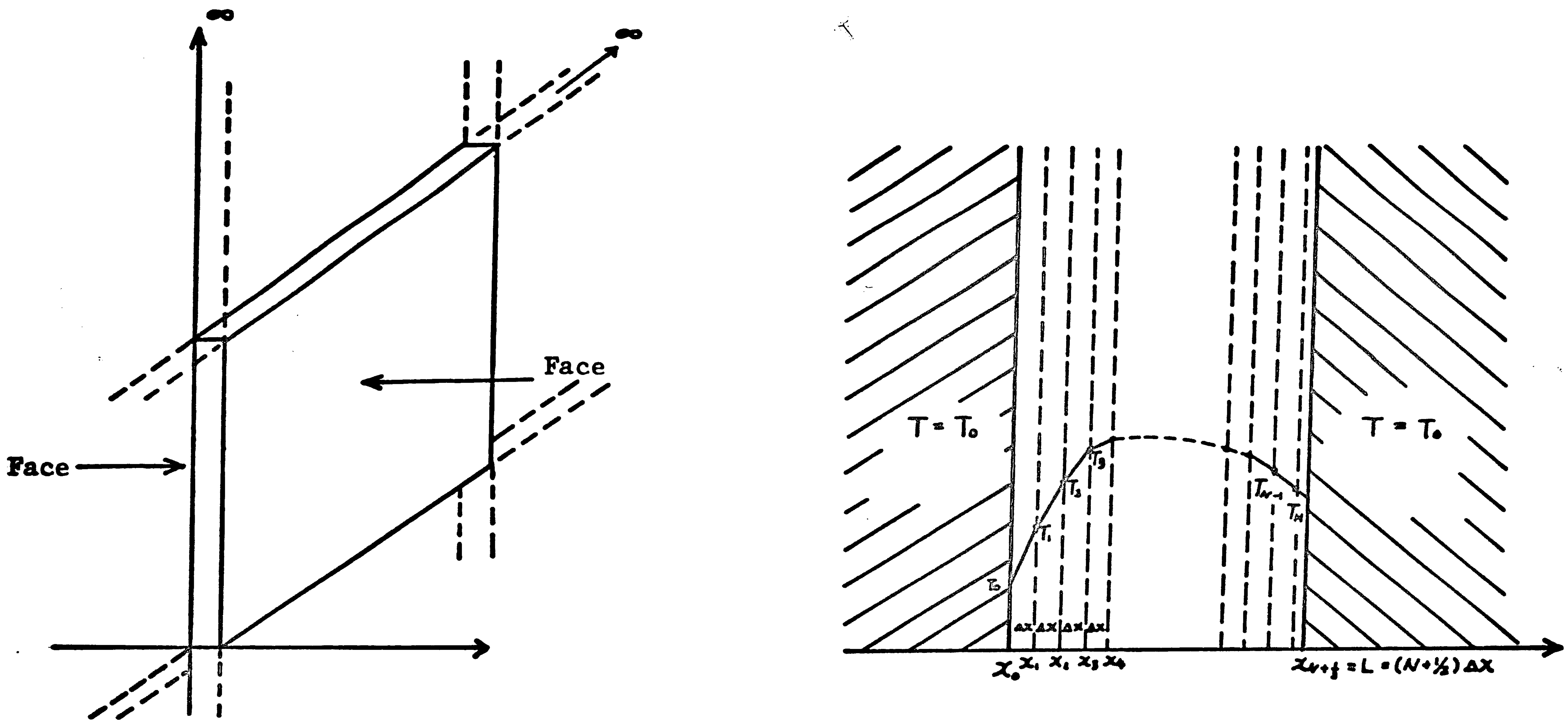
Time (sec) \ Station	T_1	T_2	T_3	T_4	T_5	T_6
0.2	70.22	44.44	25.11	12.26	5.59	0
0.4	78.69	58.87	41.73	27.94	17.64	10.16
0.6	82.53	65.88	50.78	37.72	26.97	18.48
0.8	84.84	70.22	56.62	44.43	33.88	25.07
1.0	86.42	73.22	60.76	49.33	39.14	30.28
1.2	87.57	75.43	63.86	53.05	43.21	34.40
1.4	88.45	77.12	66.23	55.94	46.39	37.66
1.6	89.13	78.44	68.08	58.20	48.90	40.24
1.8	89.66	79.47	69.53	59.97	50.87	42.28
2.0	90.08	80.28	70.68	61.37	52.43	43.89

Time (sec) \ Station	T_7	T_8	T_9	T_{10}	T_{11}
0.2	0	0	0	0	0
0.4	5.84	3.06	1.49	0	0
0.6	12.21	7.70	4.60	2.53	1.11
0.8	17.97	12.39	8.12	4.85	2.25
1.0	22.77	16.51	11.34	7.06	3.38
1.2	26.66	19.92	14.08	8.97	4.35
1.4	29.77	22.69	16.32	10.54	5.16
1.6	32.25	24.90	18.12	11.80	5.82
1.8	34.21	26.65	18.55	12.81	6.34
2.0	35.77	28.05	20.68	13.62	6.75

Cast Iron $a = 0.171$

$L = 1.2$ cm

$\Delta x = 0.1$ cm



(a) Typical Slab

(b) Division of slab thickness for Finite-Difference approximation

Fig. 1 (a) TYPICAL SLAB SHOWING (b) DIVISION OF SLAB THICKNESS FOR FINITE-DIFFERENCE APPROXIMATION

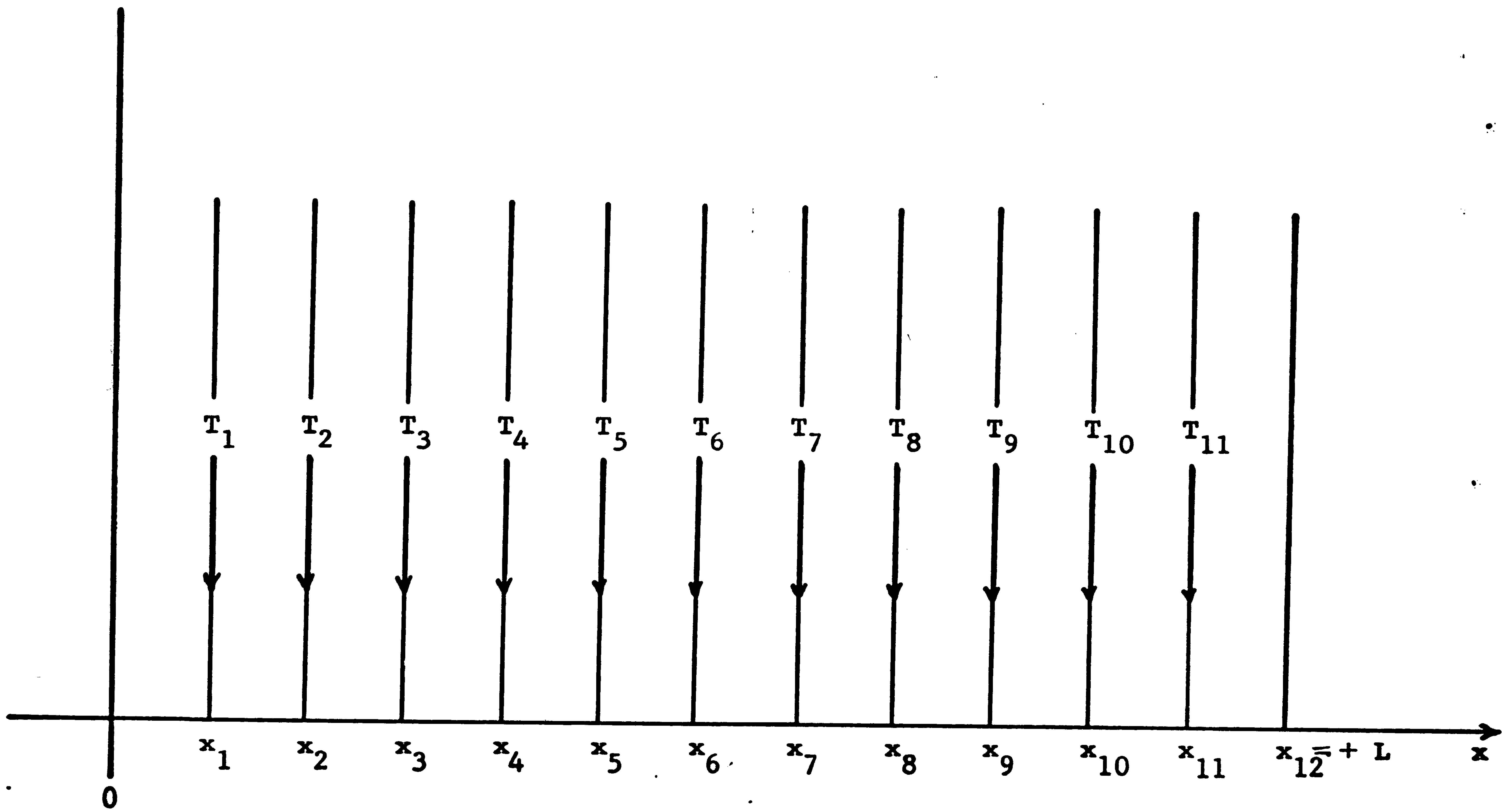


Fig. 2 DIVISION OF SLAB THICKNESS (12 CELLS)

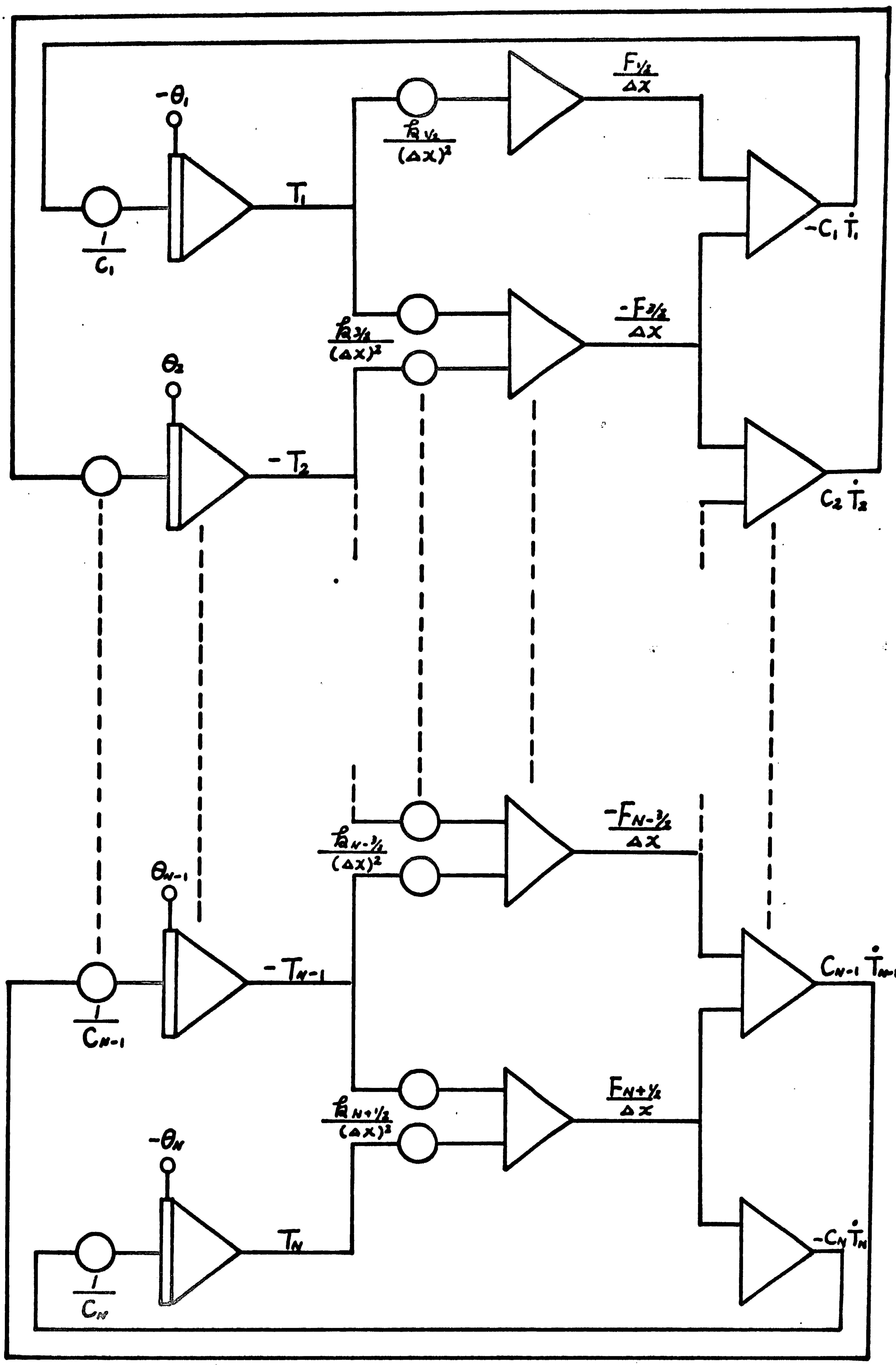


Fig. 3 COMPUTER CIRCUIT FOR SOLVING THE GENERAL HEAT EQUATION

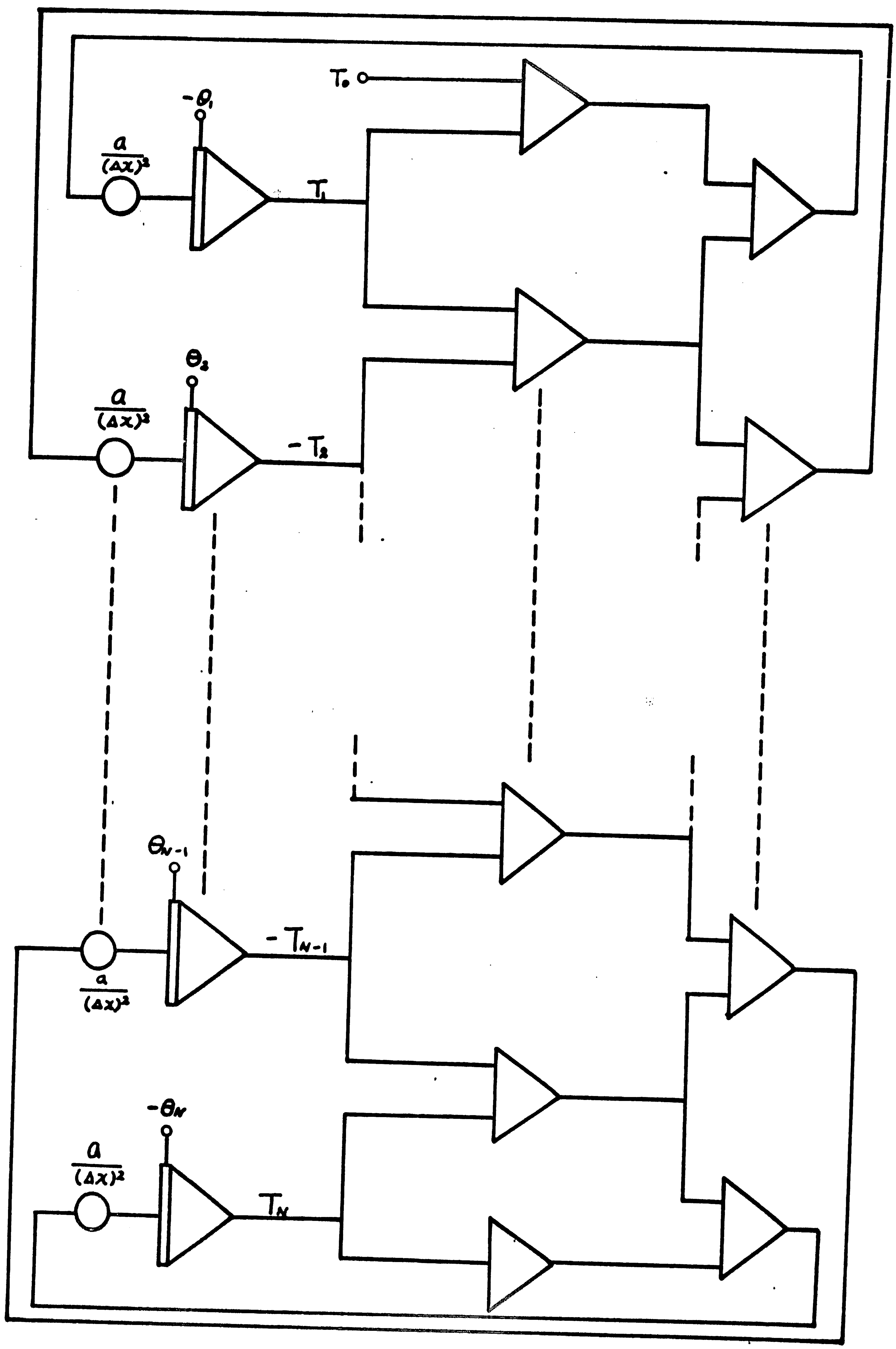
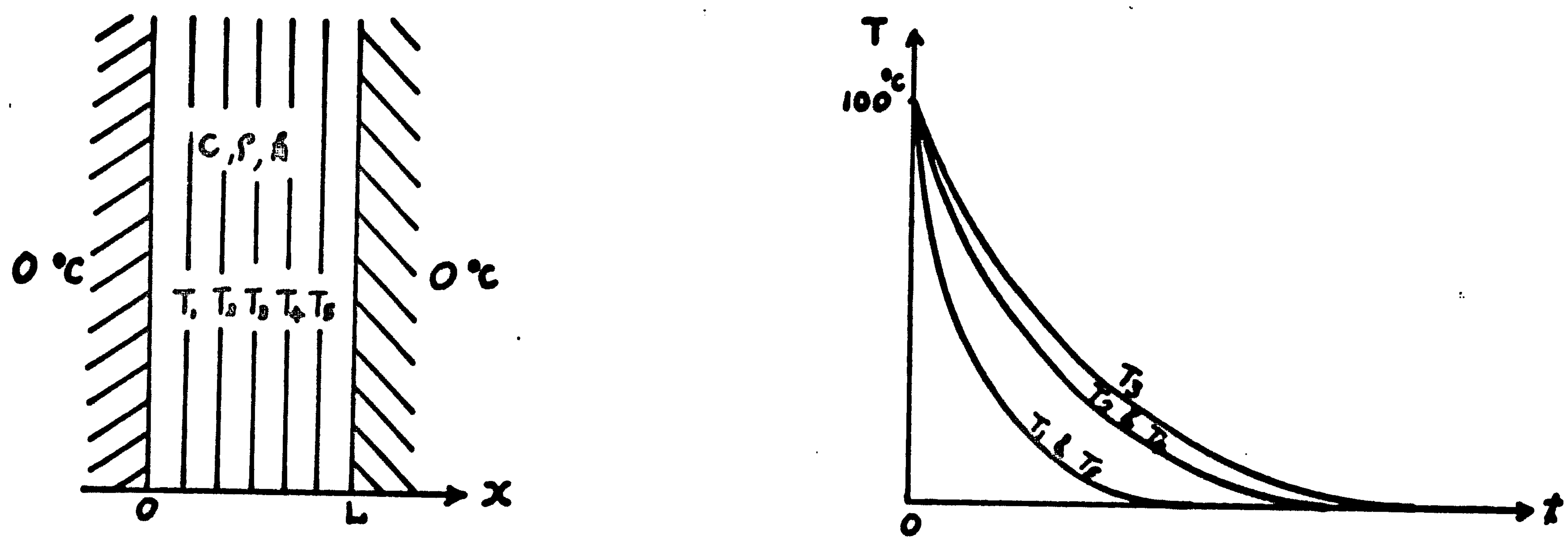


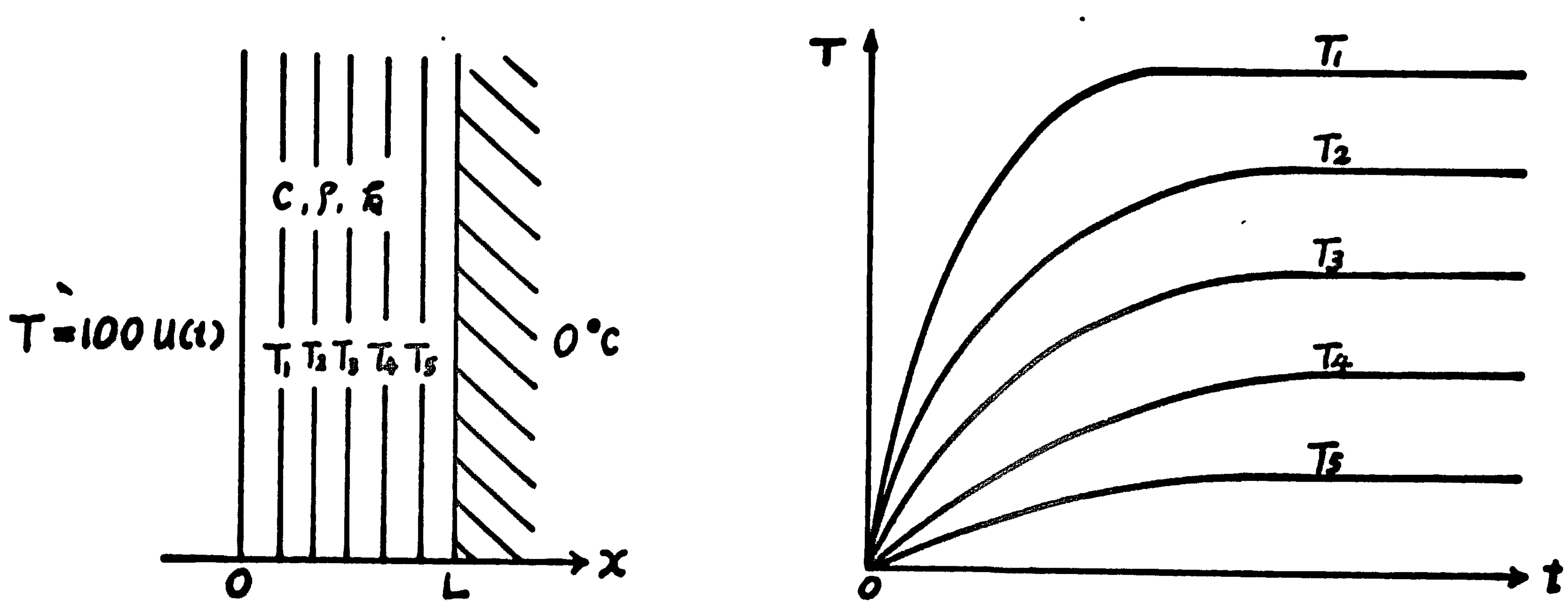
Fig. 4 MODIFIED COMPUTER CIRCUIT FOR SOLVING THE HEAT EQUATION



CASE 1

Fig. 5 CROSS SECTION OF SLAB WITH BOUNDARY CONDITION

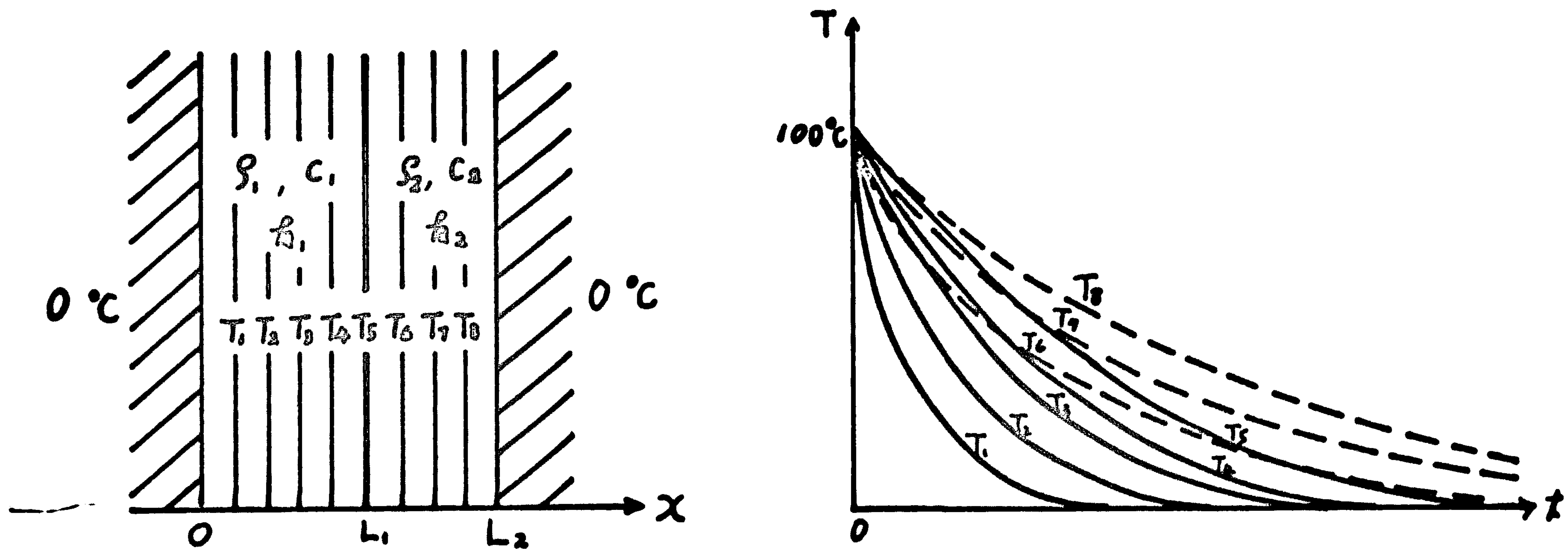
$T_o = T_{n+1} = 0$ at $x = o$, $x = L$



CASE 2

Fig. 6 CROSS SECTION OF SLAB WITH BOUNDARY CONDITIONS

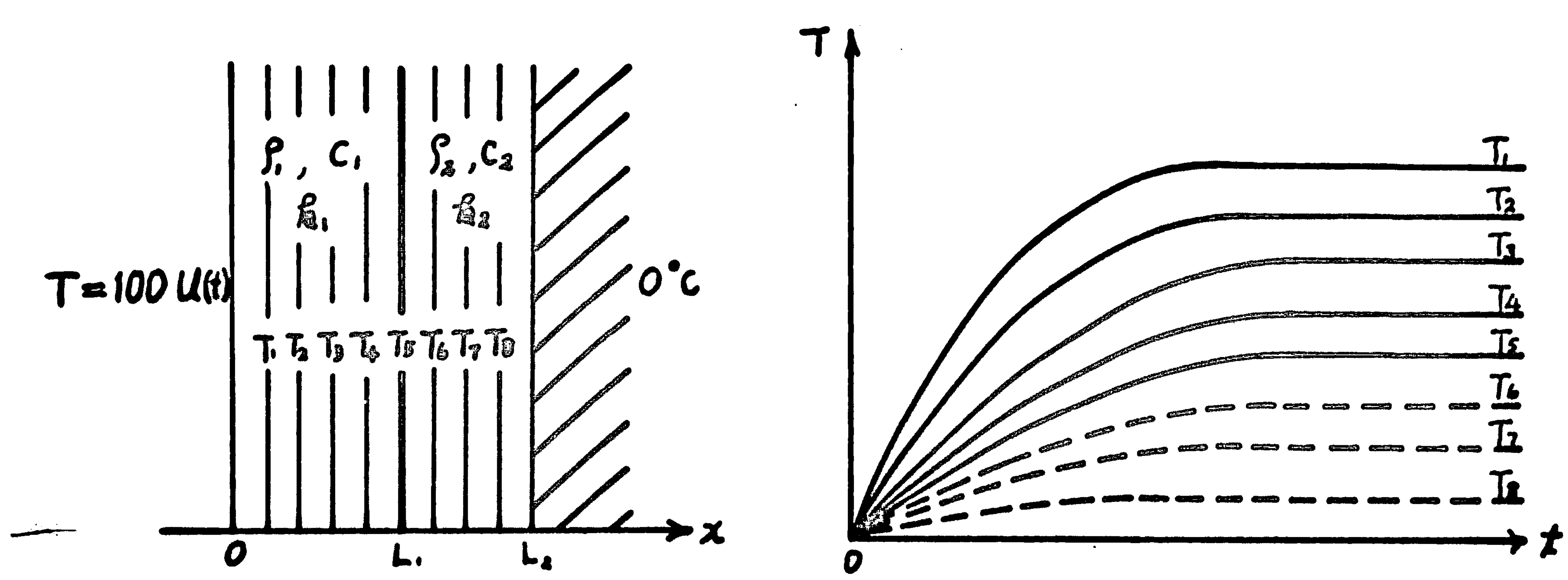
$T_o = 100^o$ C at $x = 0$, $T_{n+1} = 0$ at $x = L$



CASE 3

Fig. 7 CROSS SECTION OF COMPOSITE SLAB WITH BOUNDARY CONDITIONS

$$T_o = T_{n+1} = 0 \text{ at } x = 0,\ x = L_2$$



CASE 4

Fig. 8 CROSS SECTION OF COMPOSITE SLAB WITH BOUNDARY CONDITION

$$T_o = 100^{o}\text{ C at } x = 0,\ T_{n+1} = 0 \text{ at } x = L_2$$

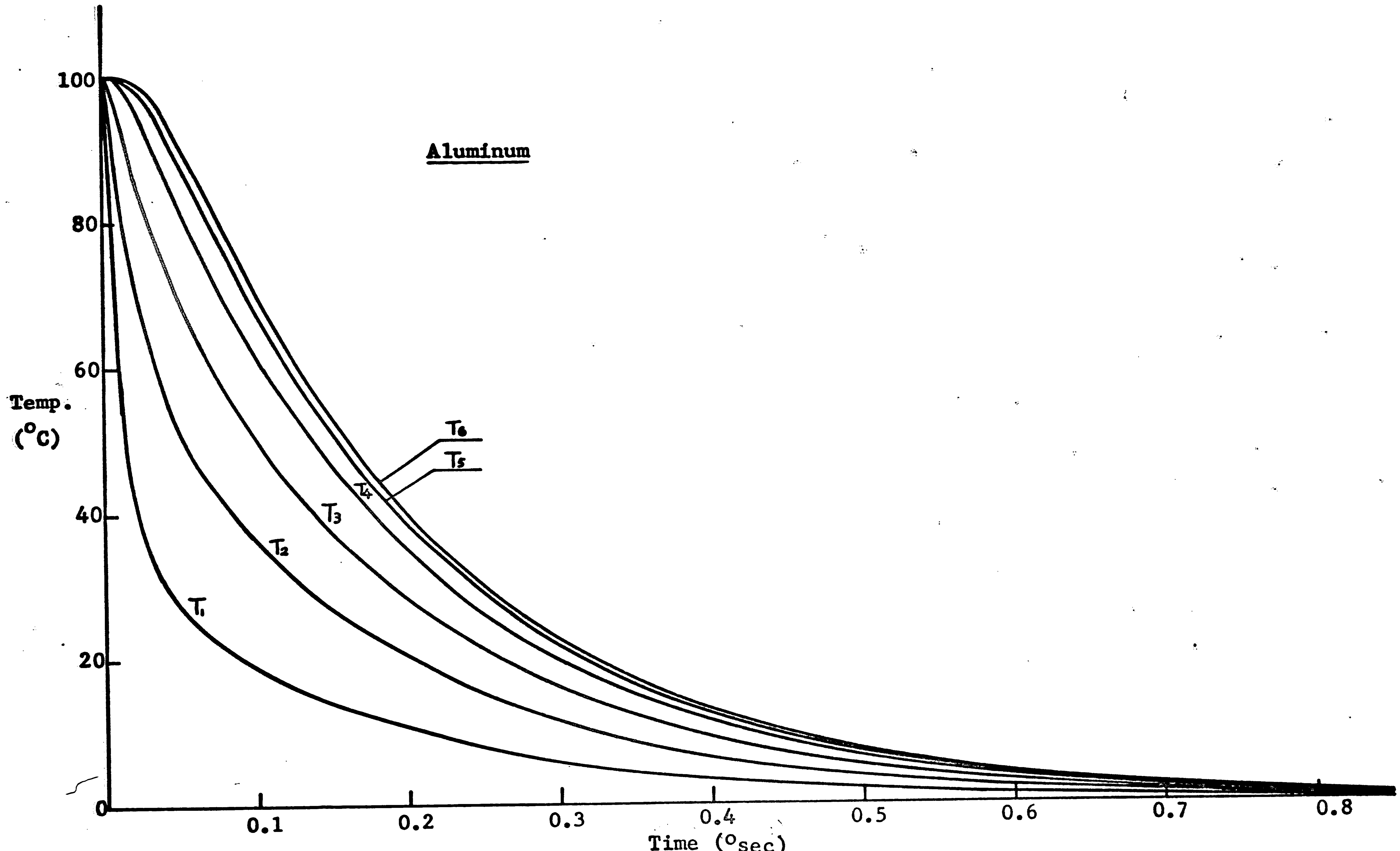


Fig. 9 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 1)

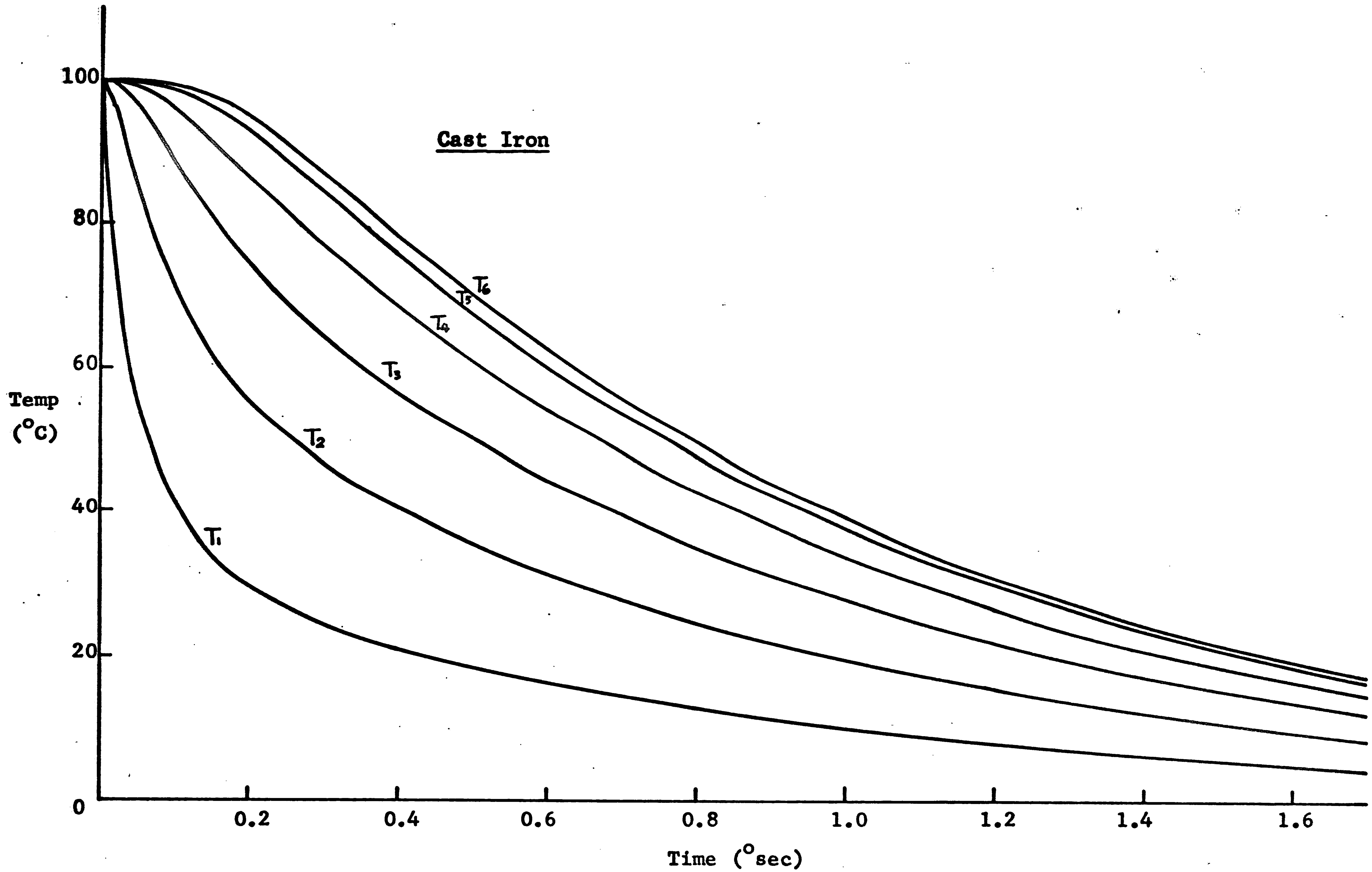


Fig. 10 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 1)

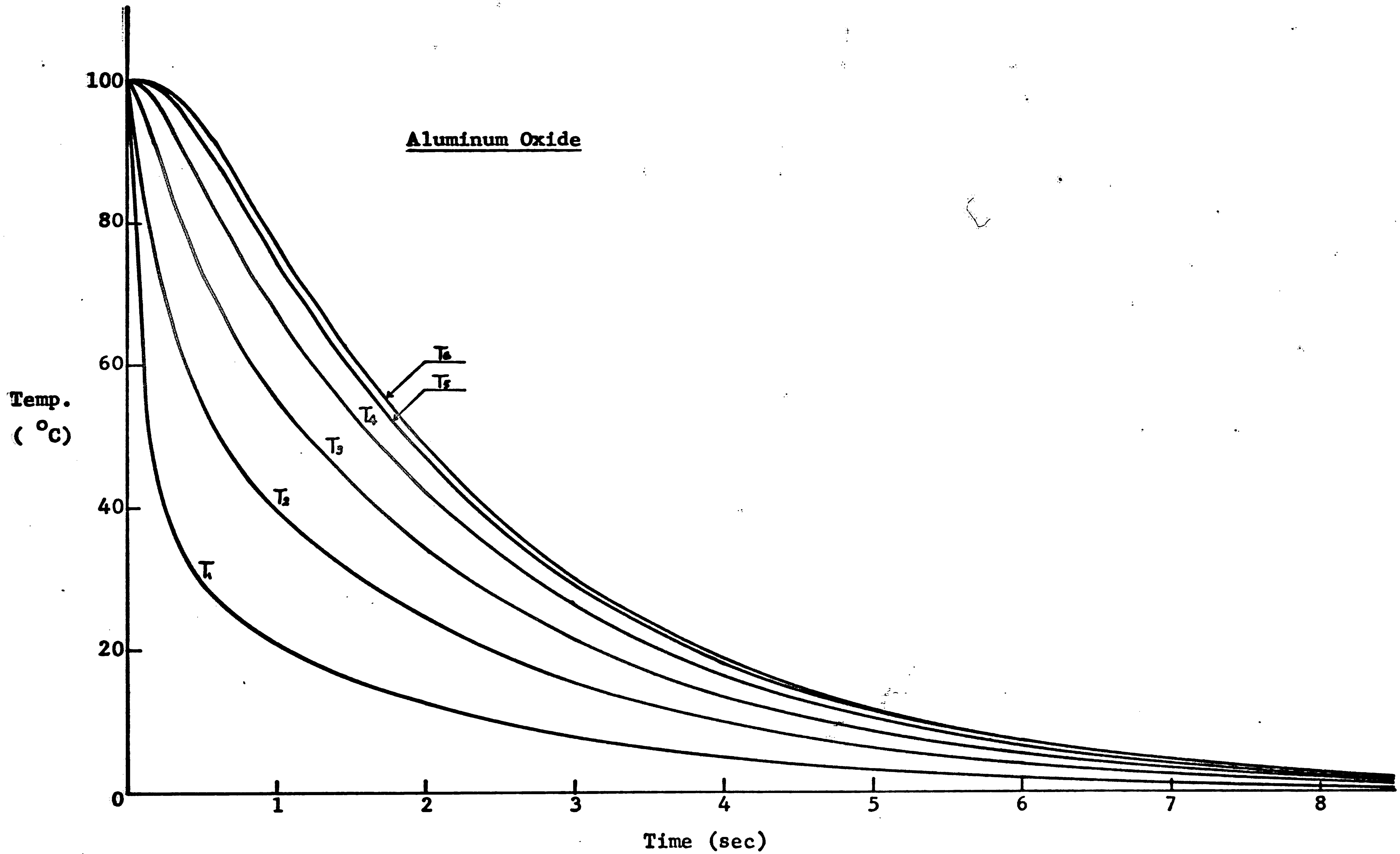


Fig. 11 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 1)

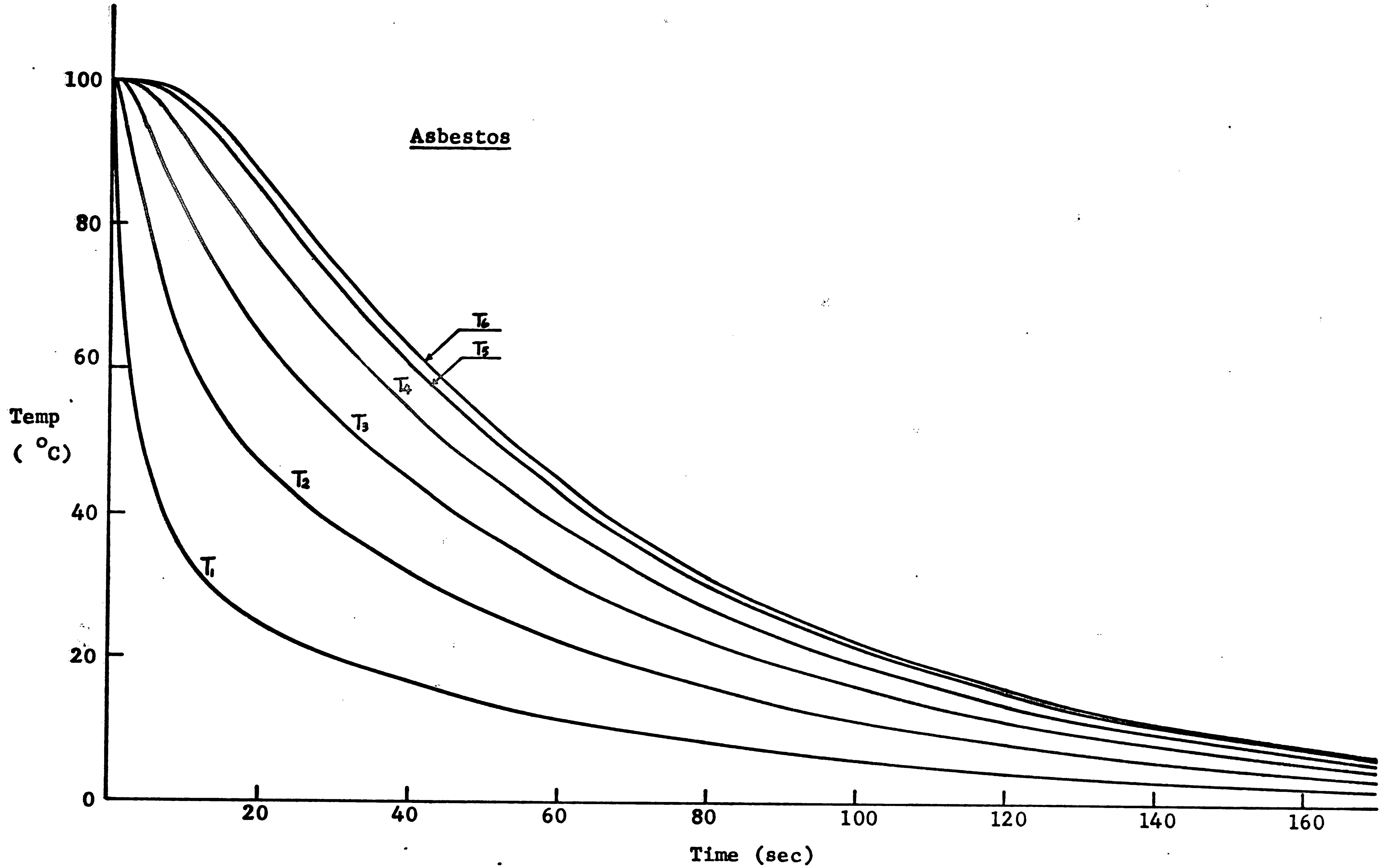


Fig. 12 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 1)

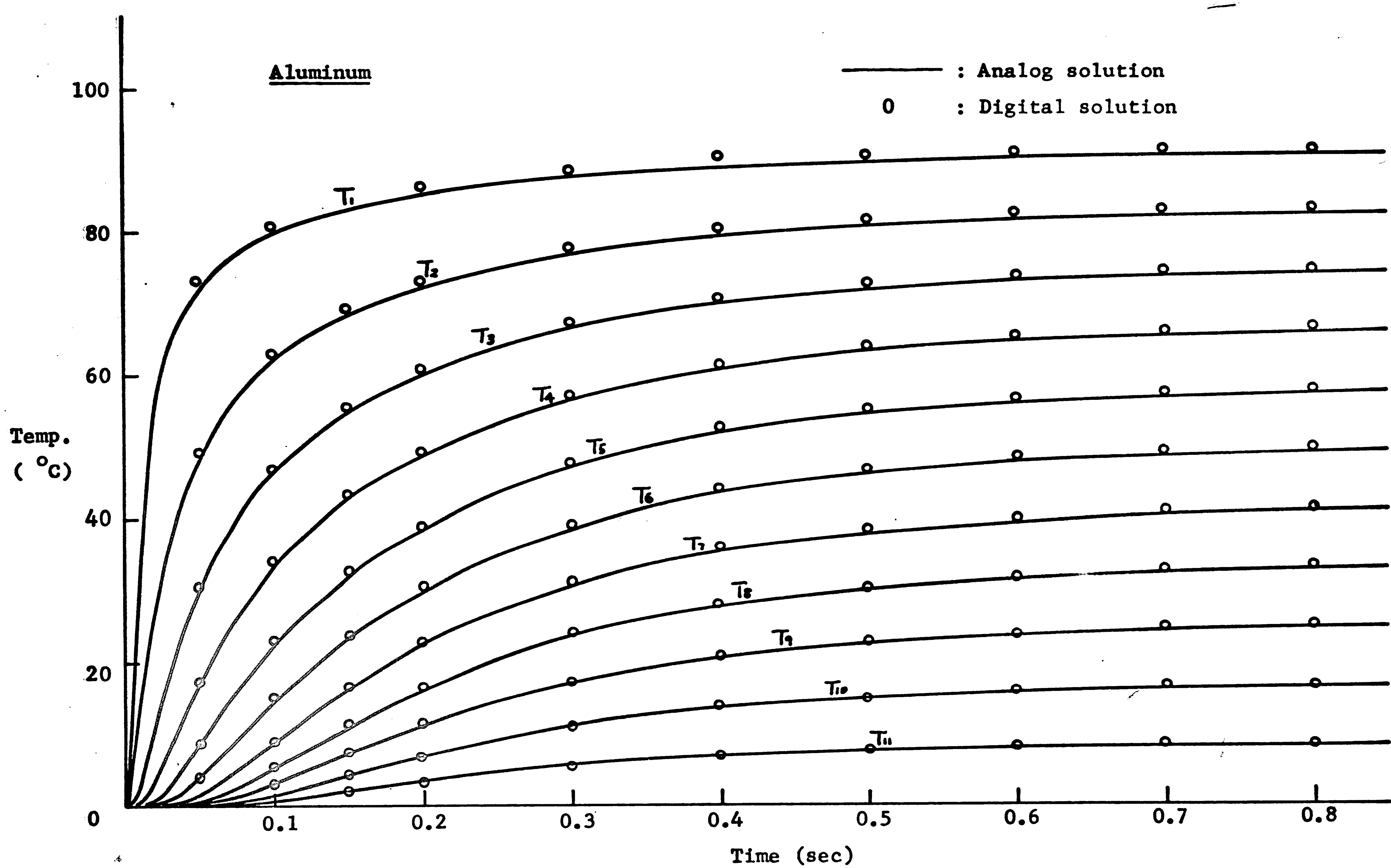


Fig. 13 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 2)

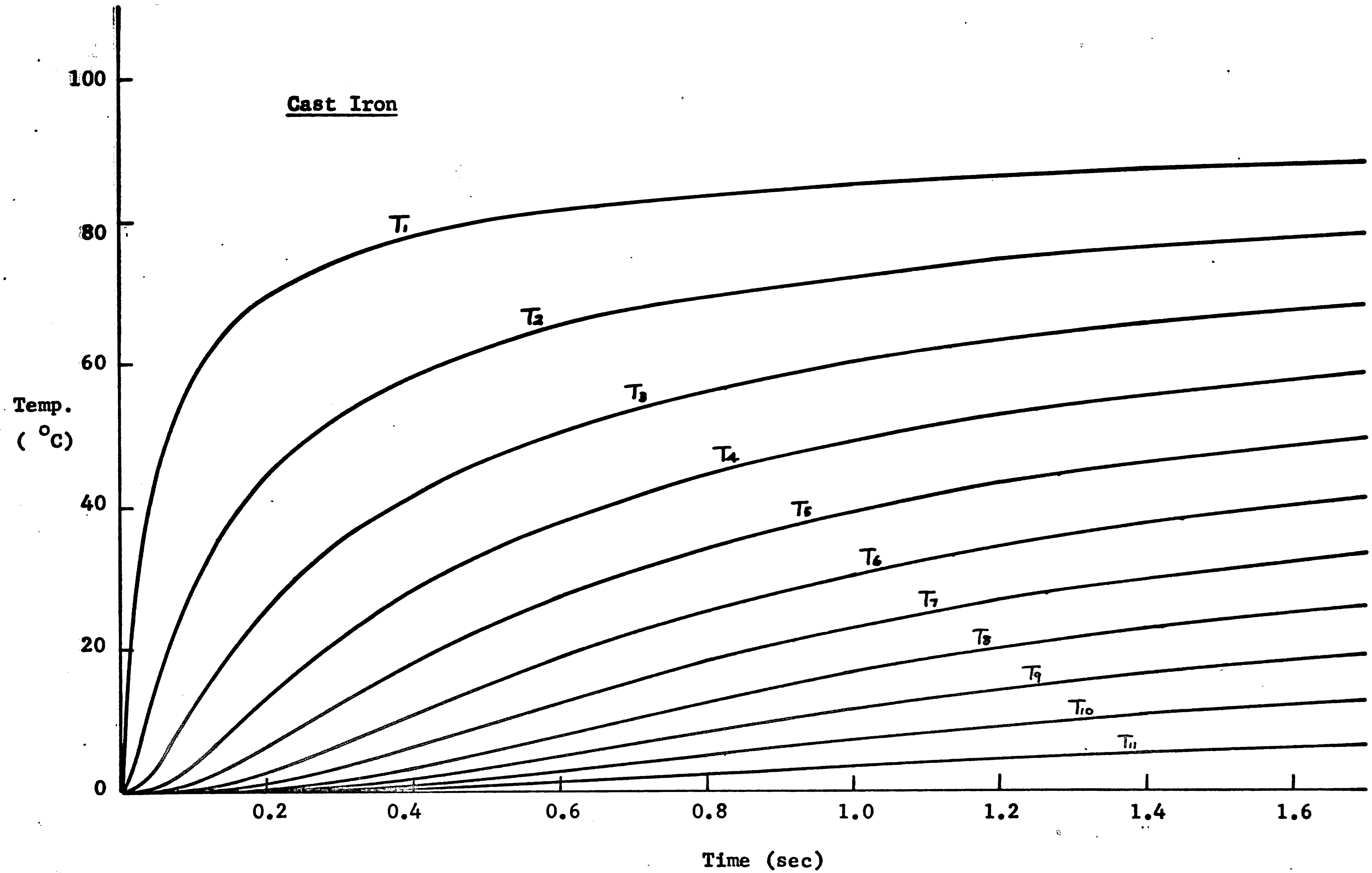


Fig. 14 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 2)

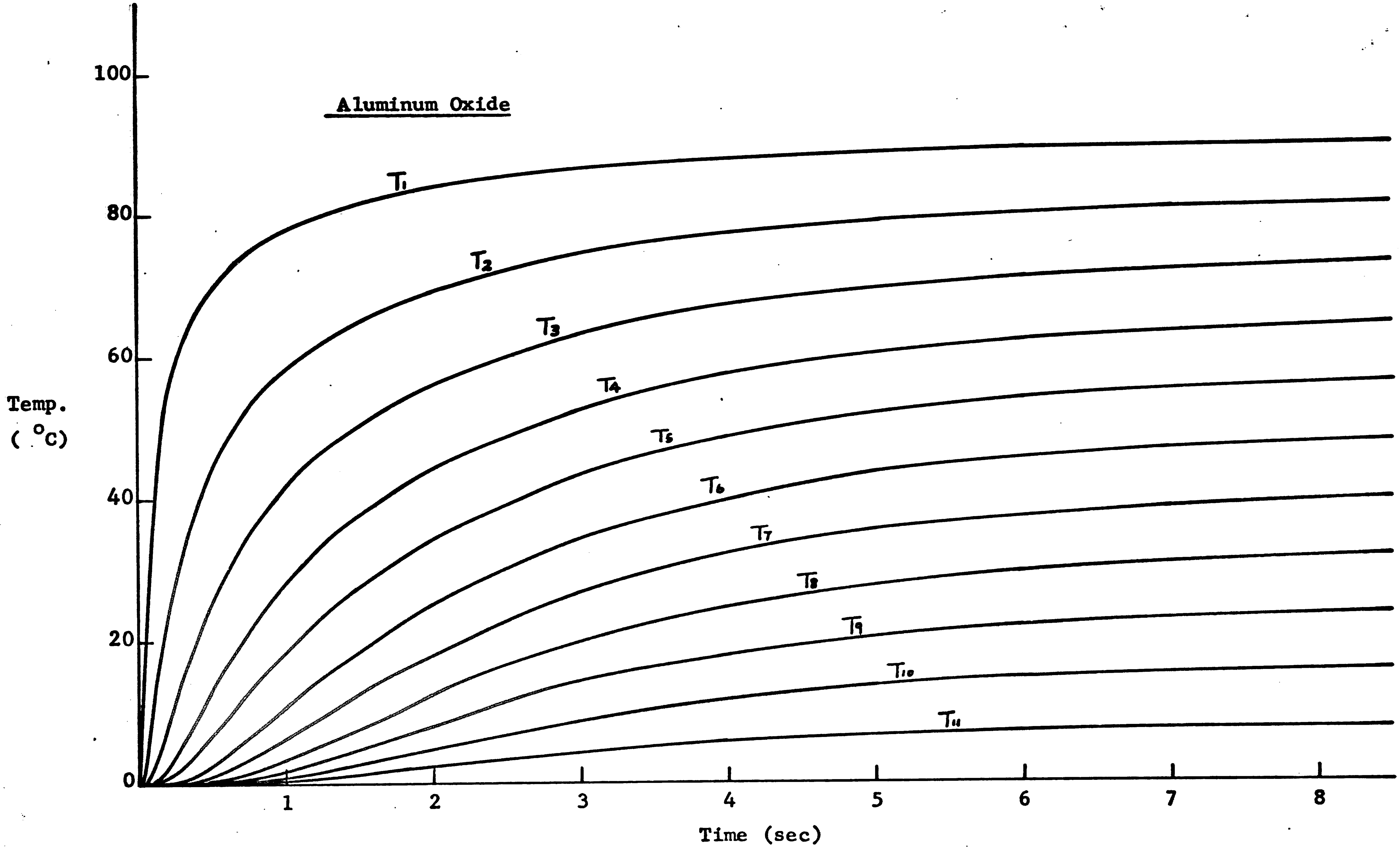


Fig. 15 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 2)

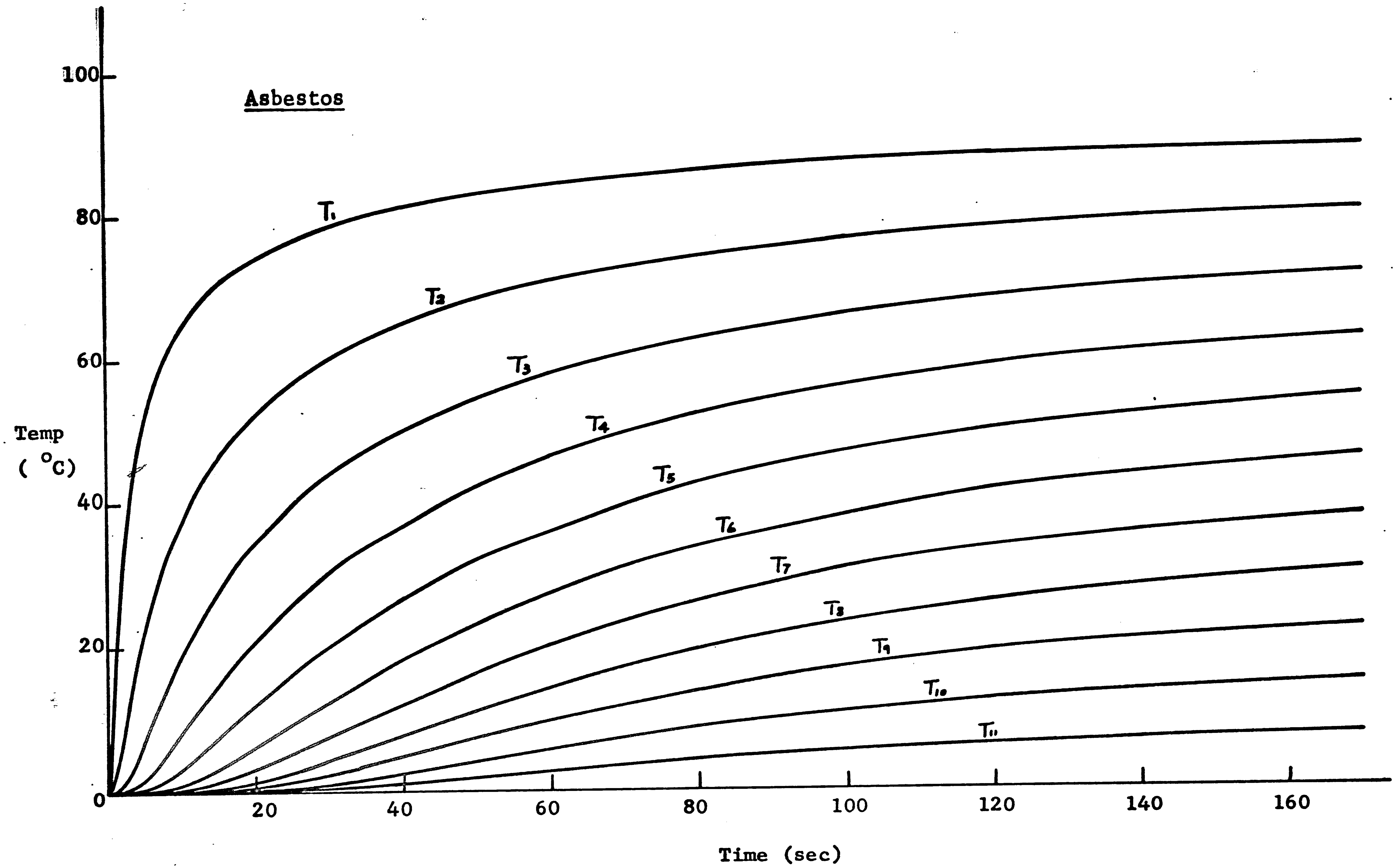


Fig. 16 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 2)

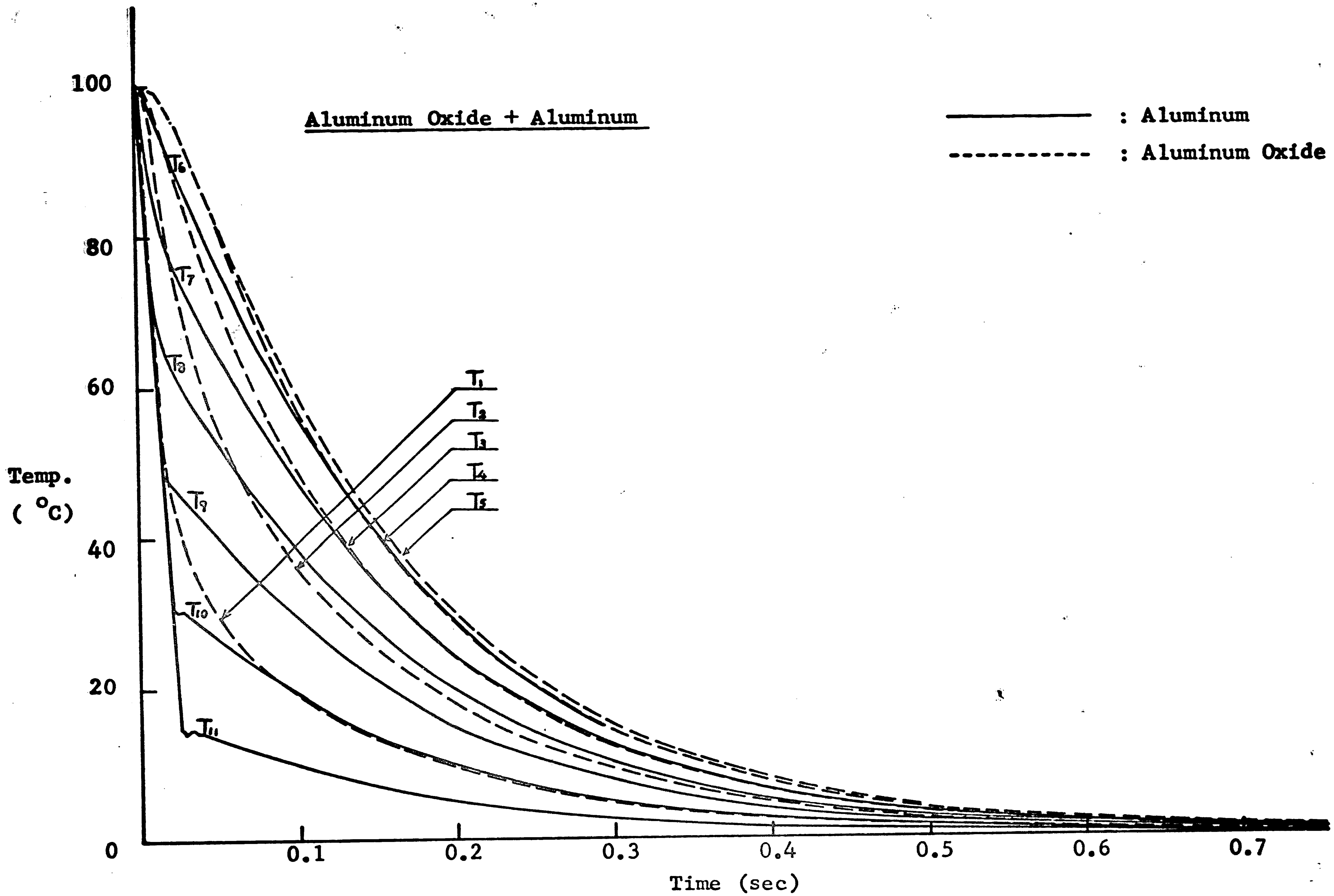


Fig. 17 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 3)

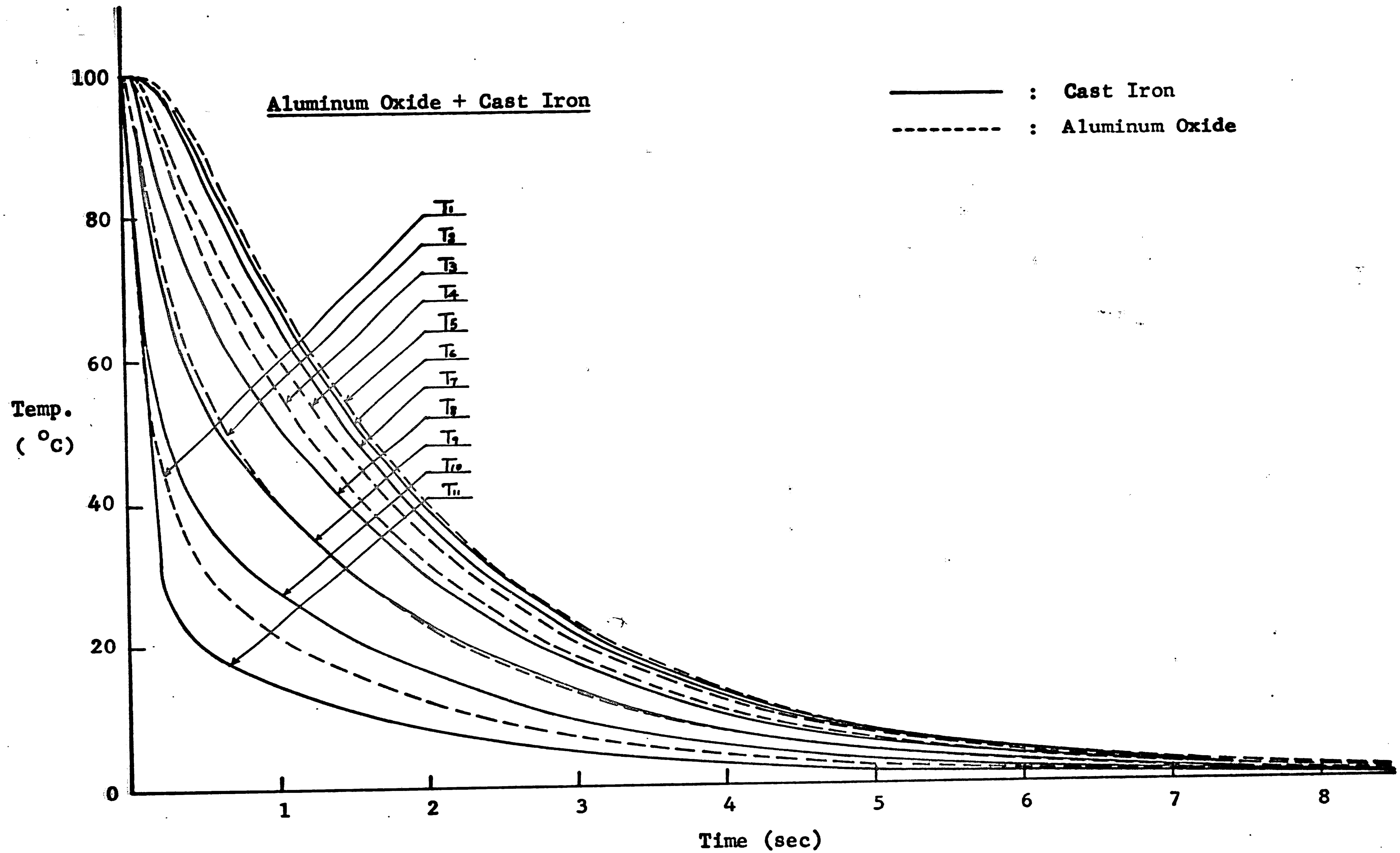


Fig. 18 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 3)

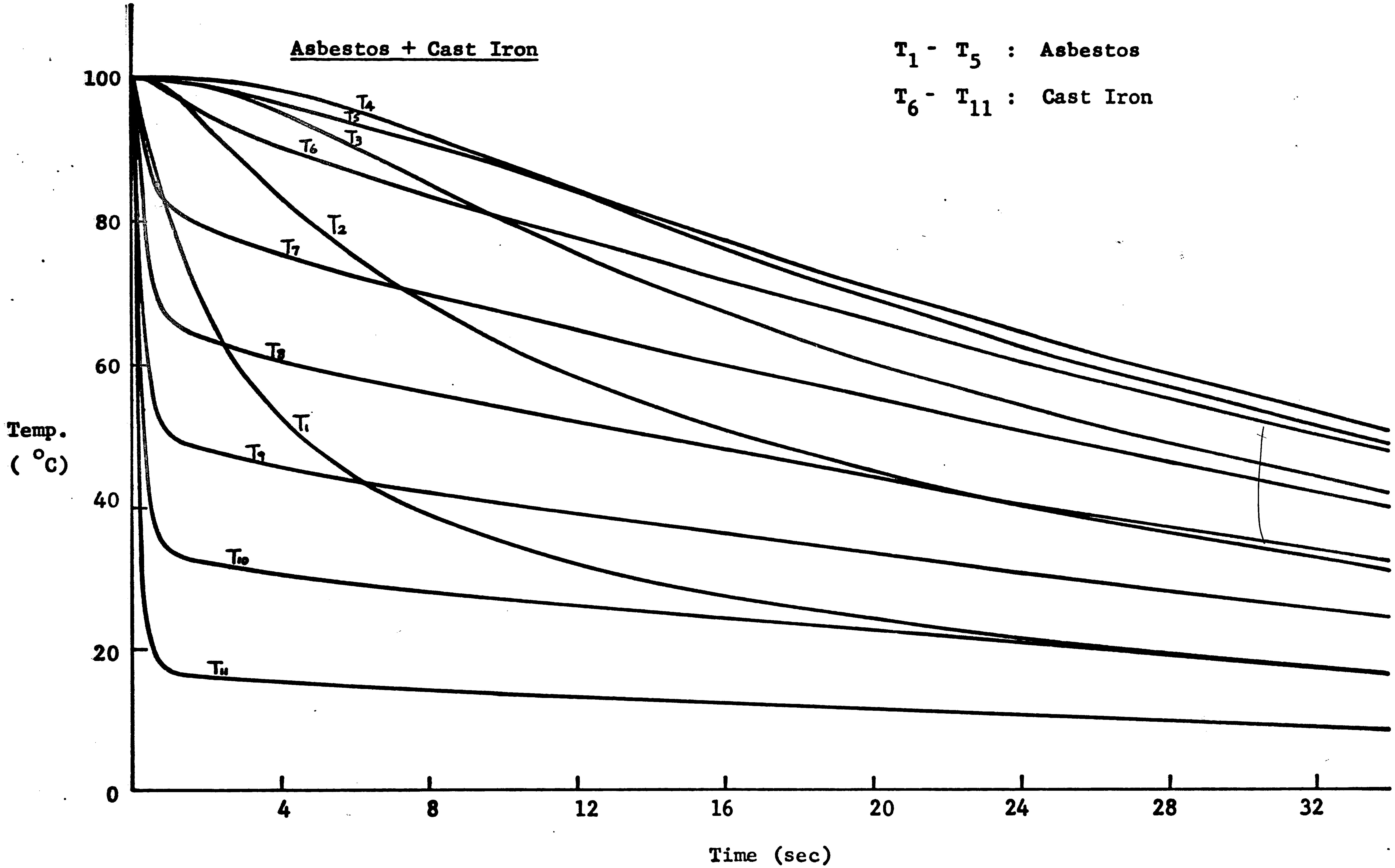


Fig. 19 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 3)

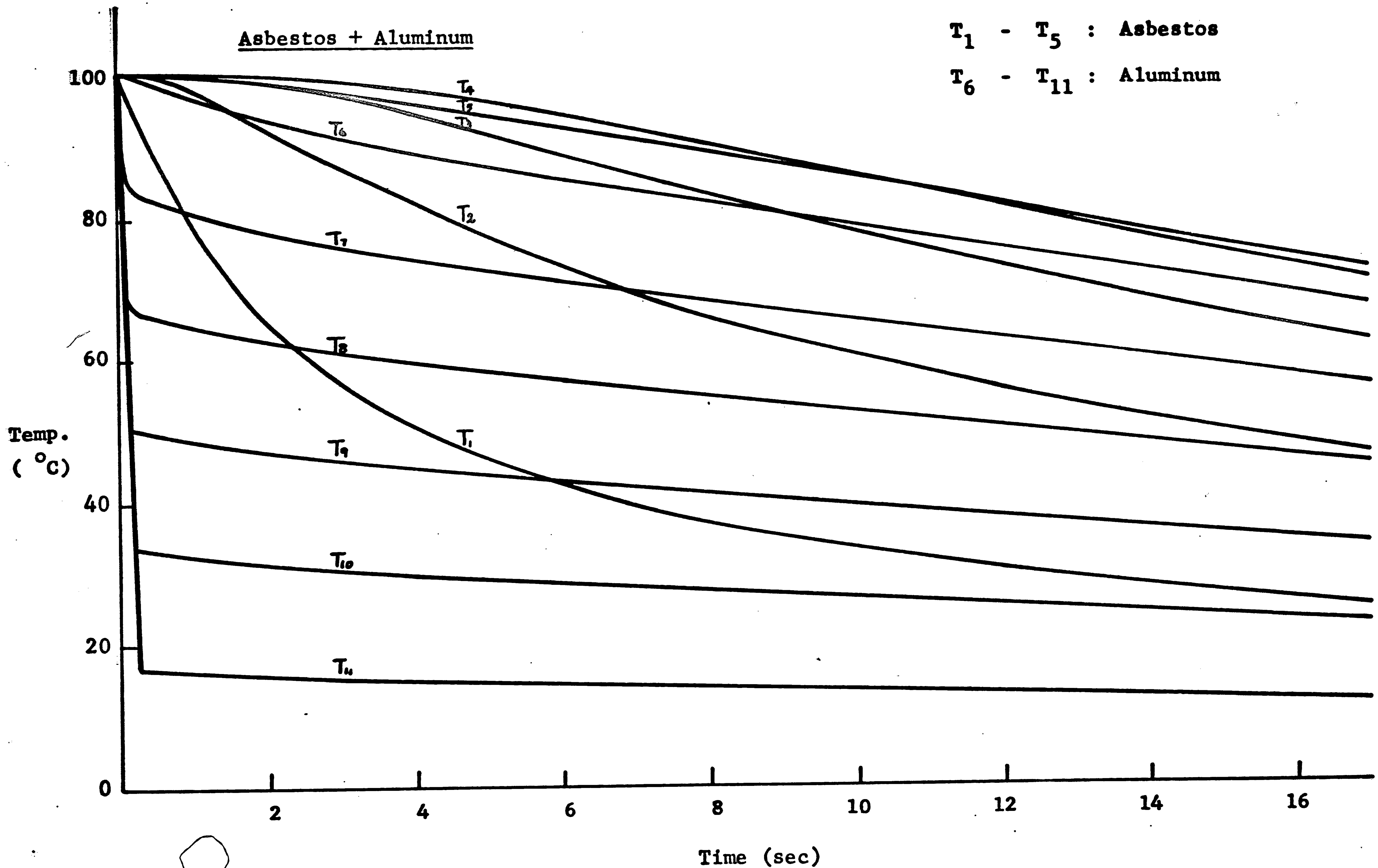


Fig. 20 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 3)

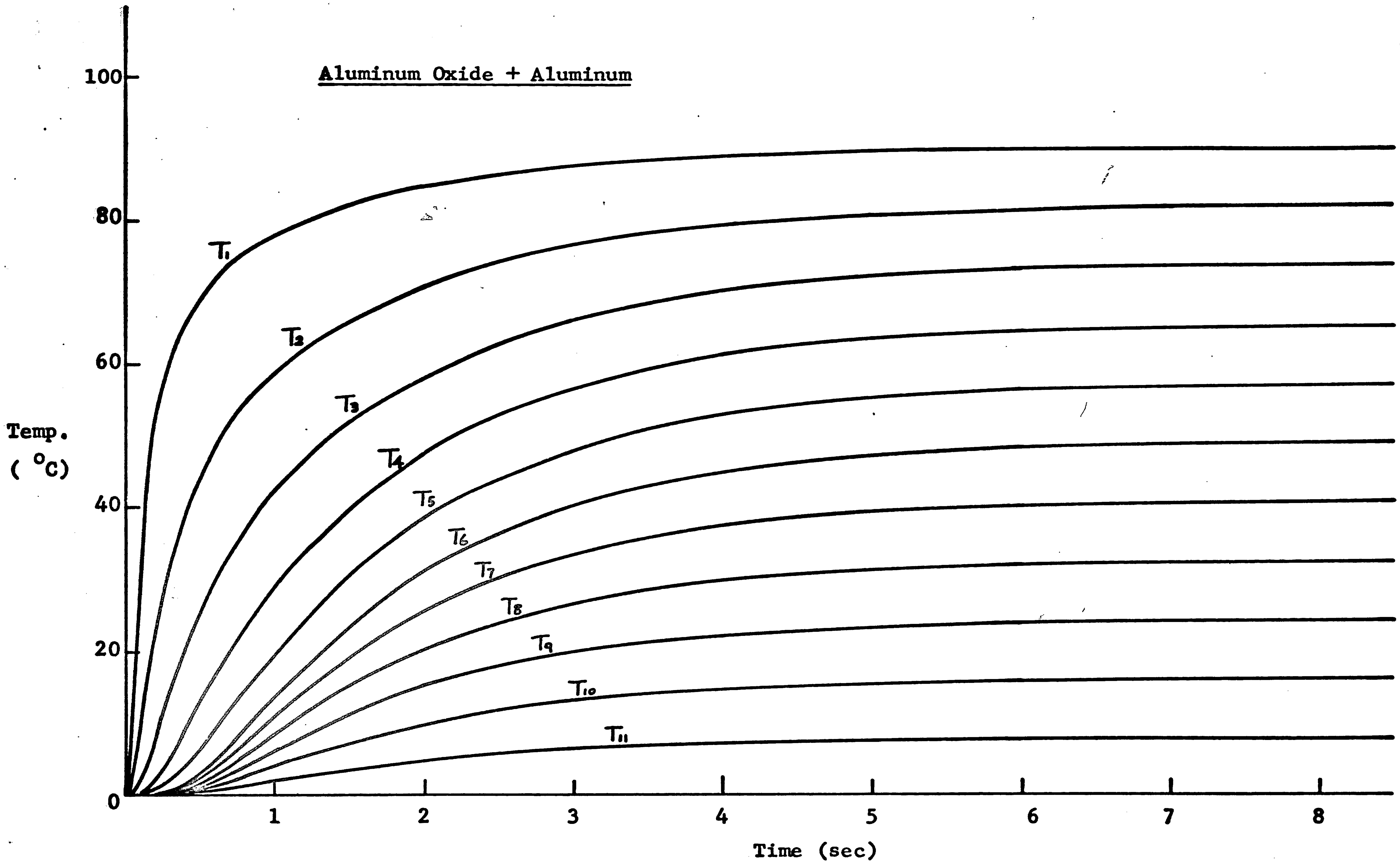


Fig. 21 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 4)

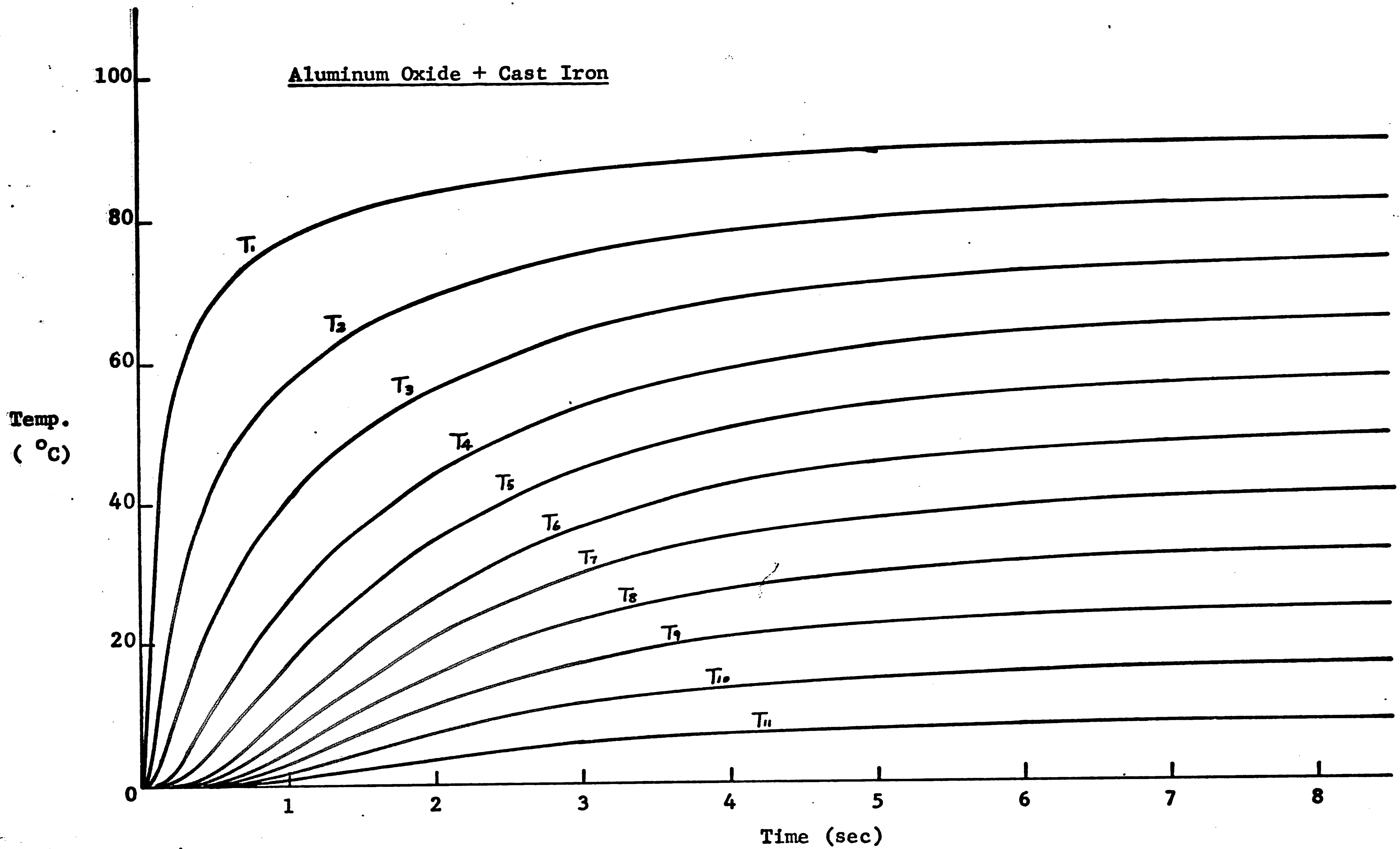


Fig. 22 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 4)

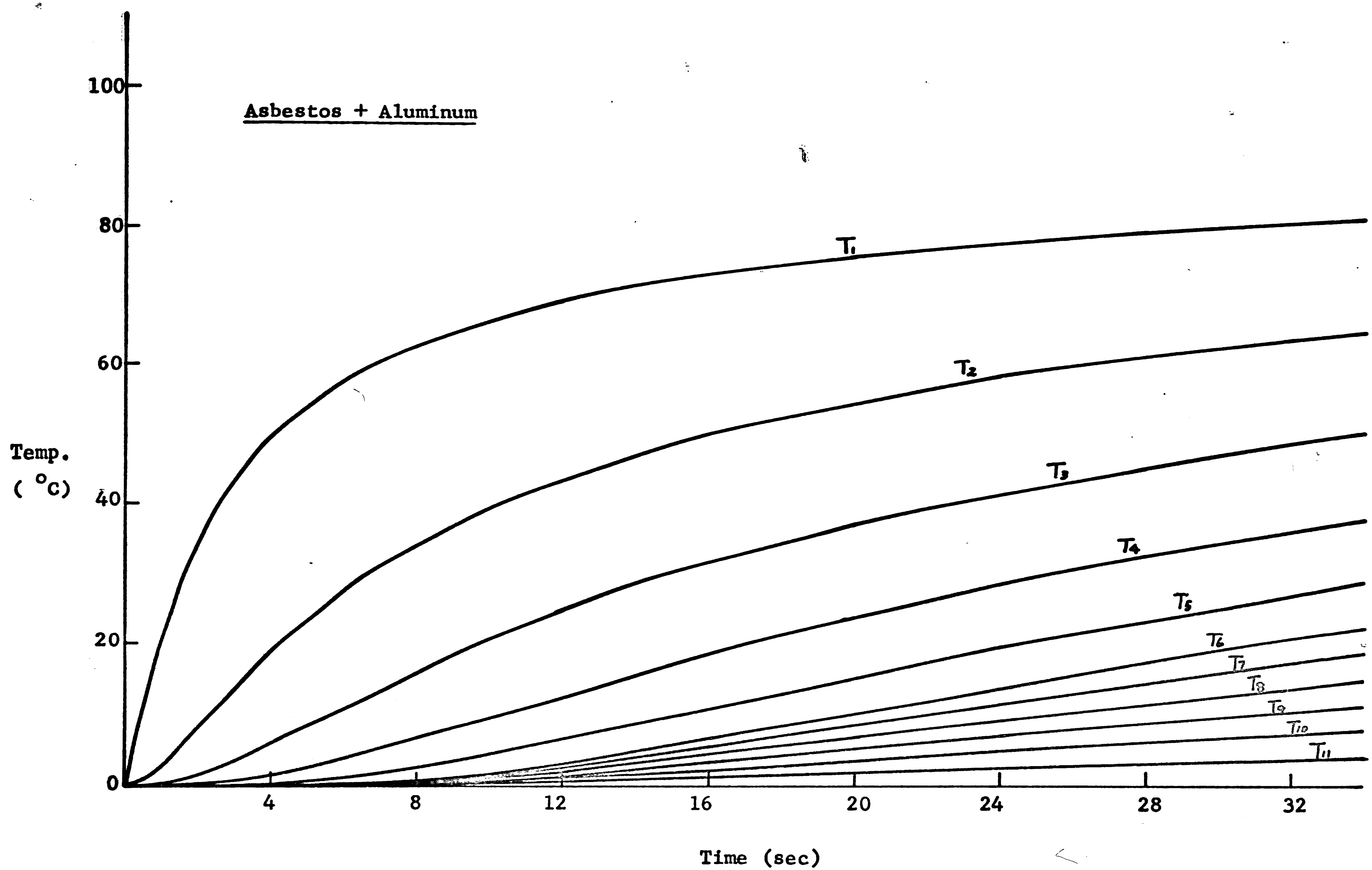


Fig. 23 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 4)

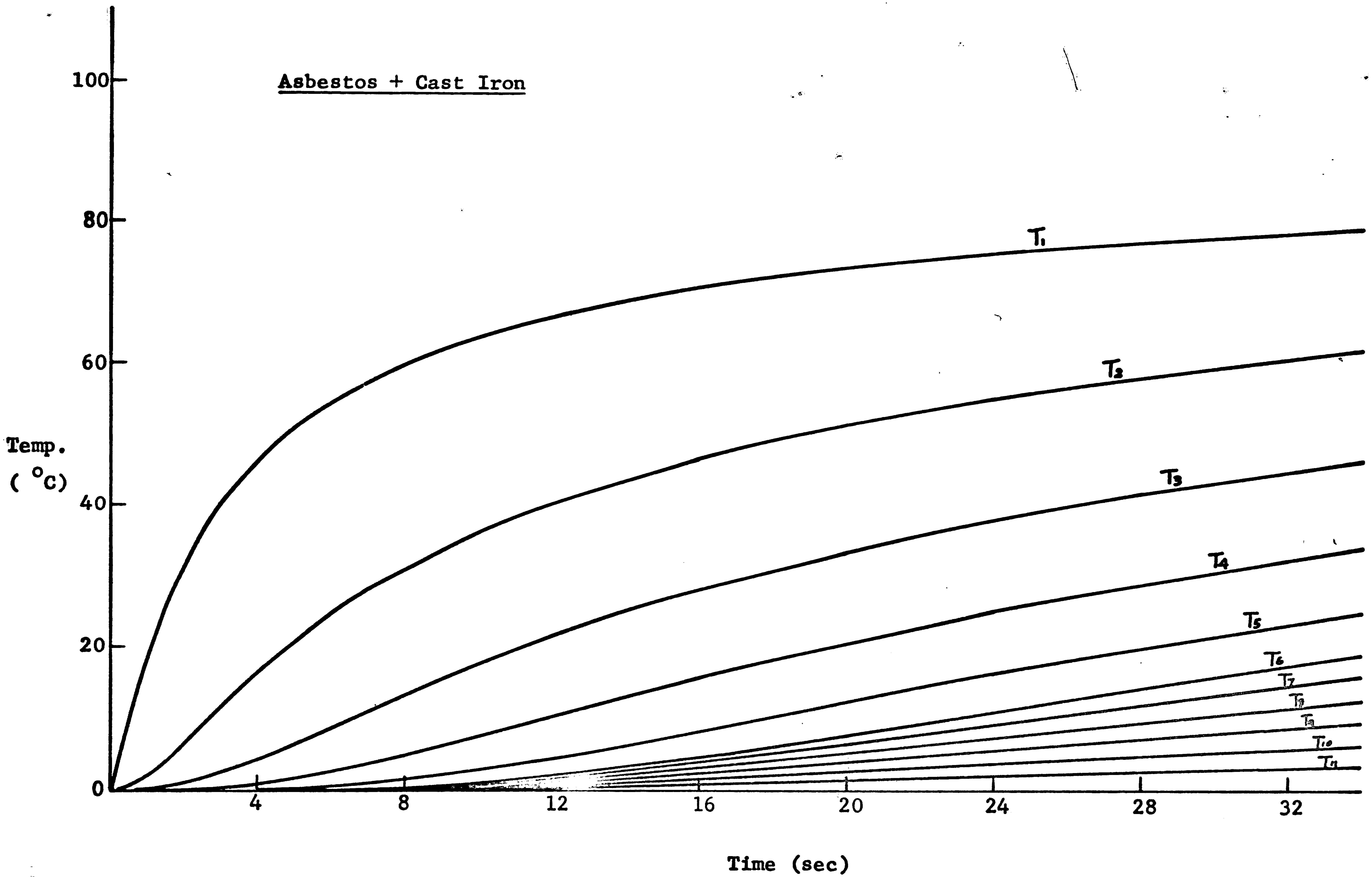


Fig. 24 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 4)

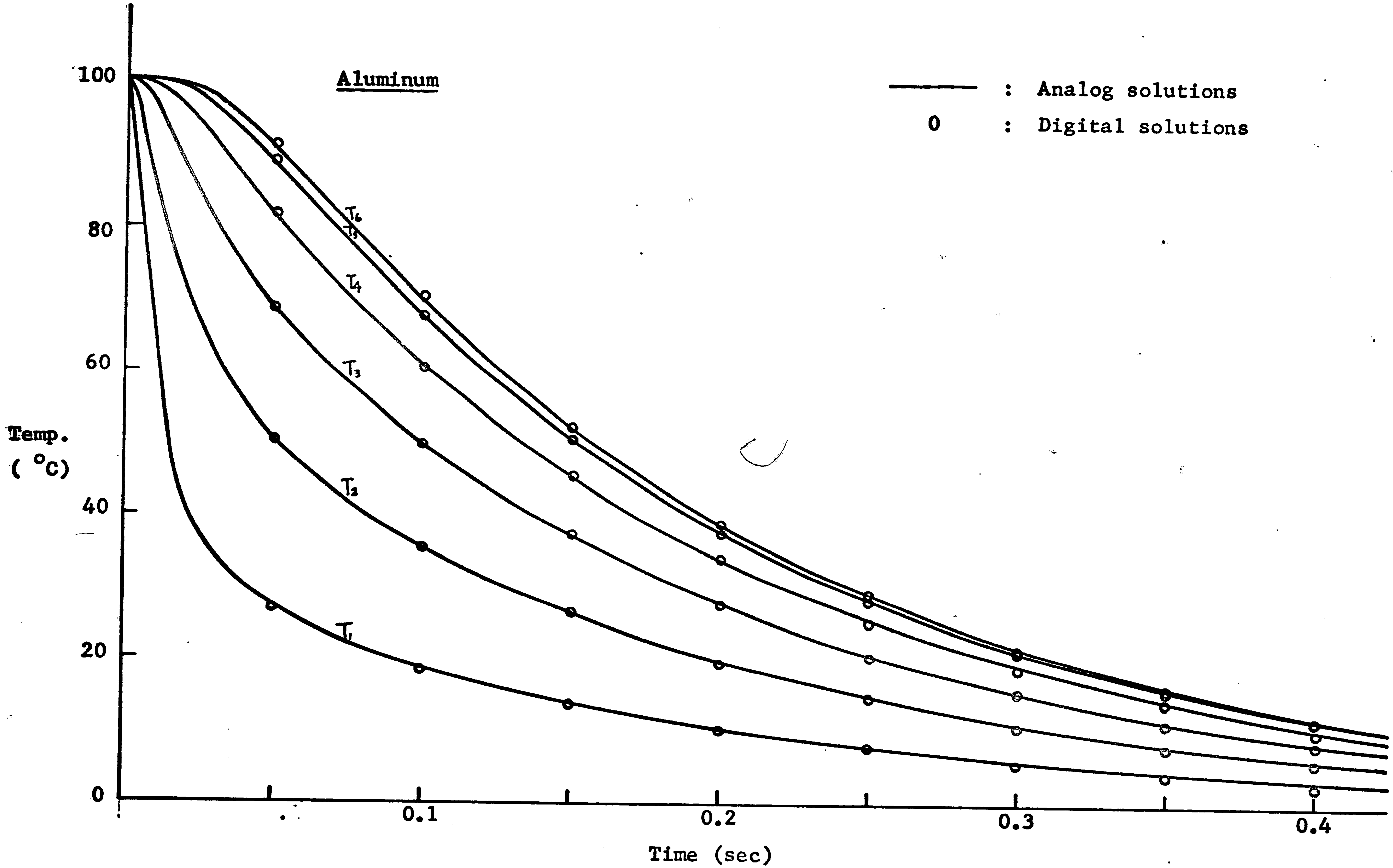


Fig. 25 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 12 CELLS (CASE 1)

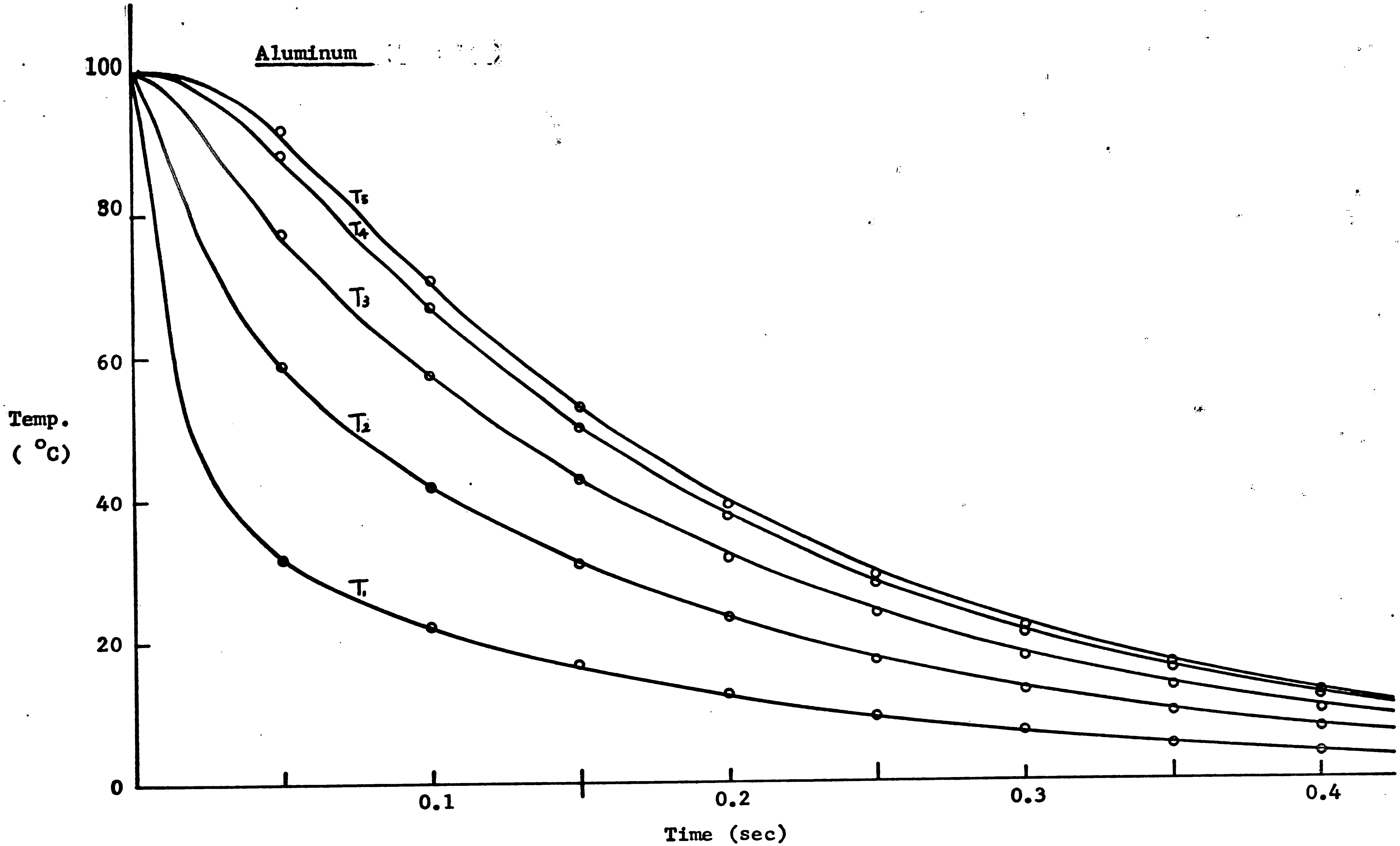


Fig. 26 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 10 CELLS (CASE 1)

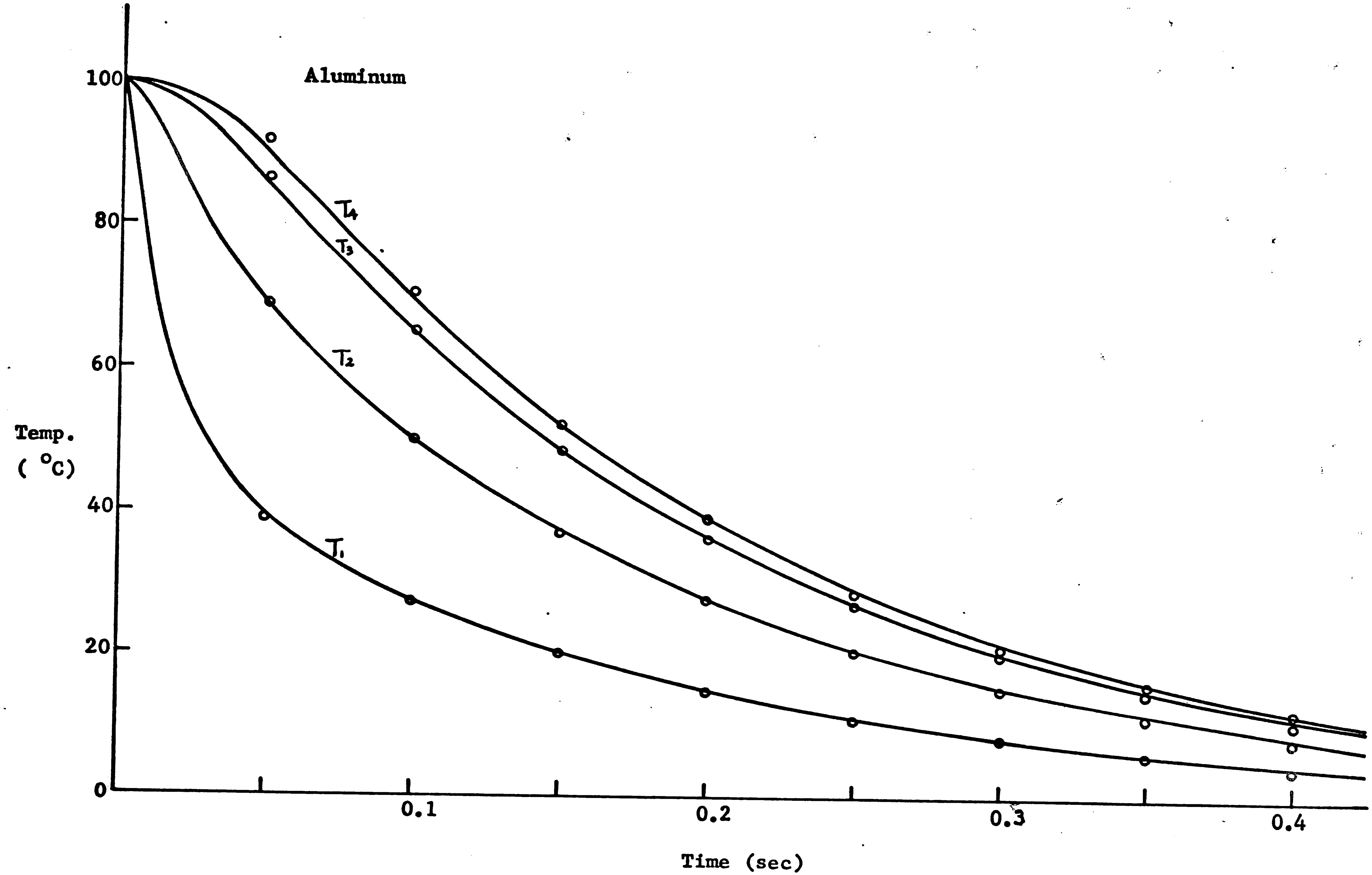


Fig. 27 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 8 CELLS (CASE 1)

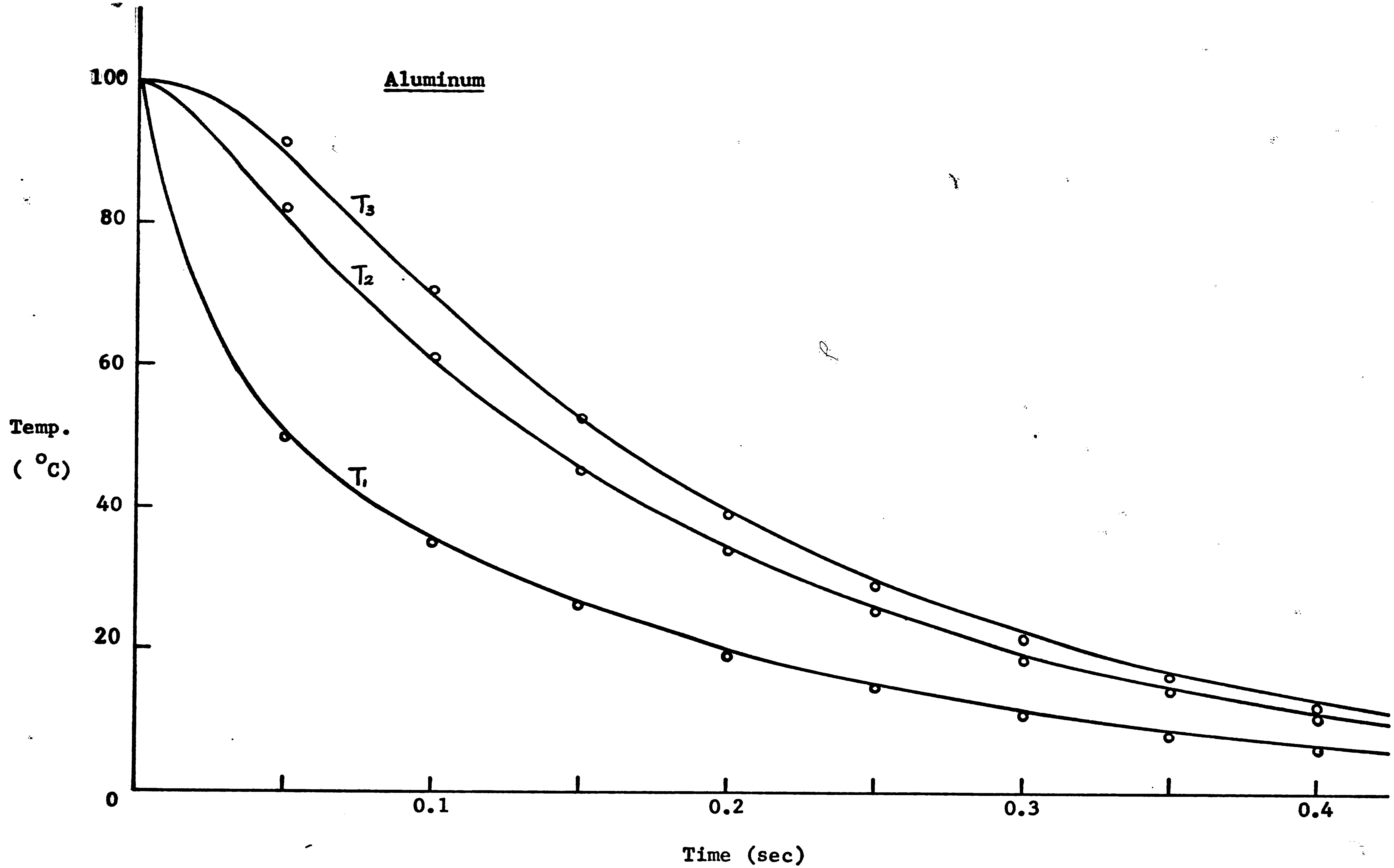


Fig. 28 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 6 CELLS (CASE 1)

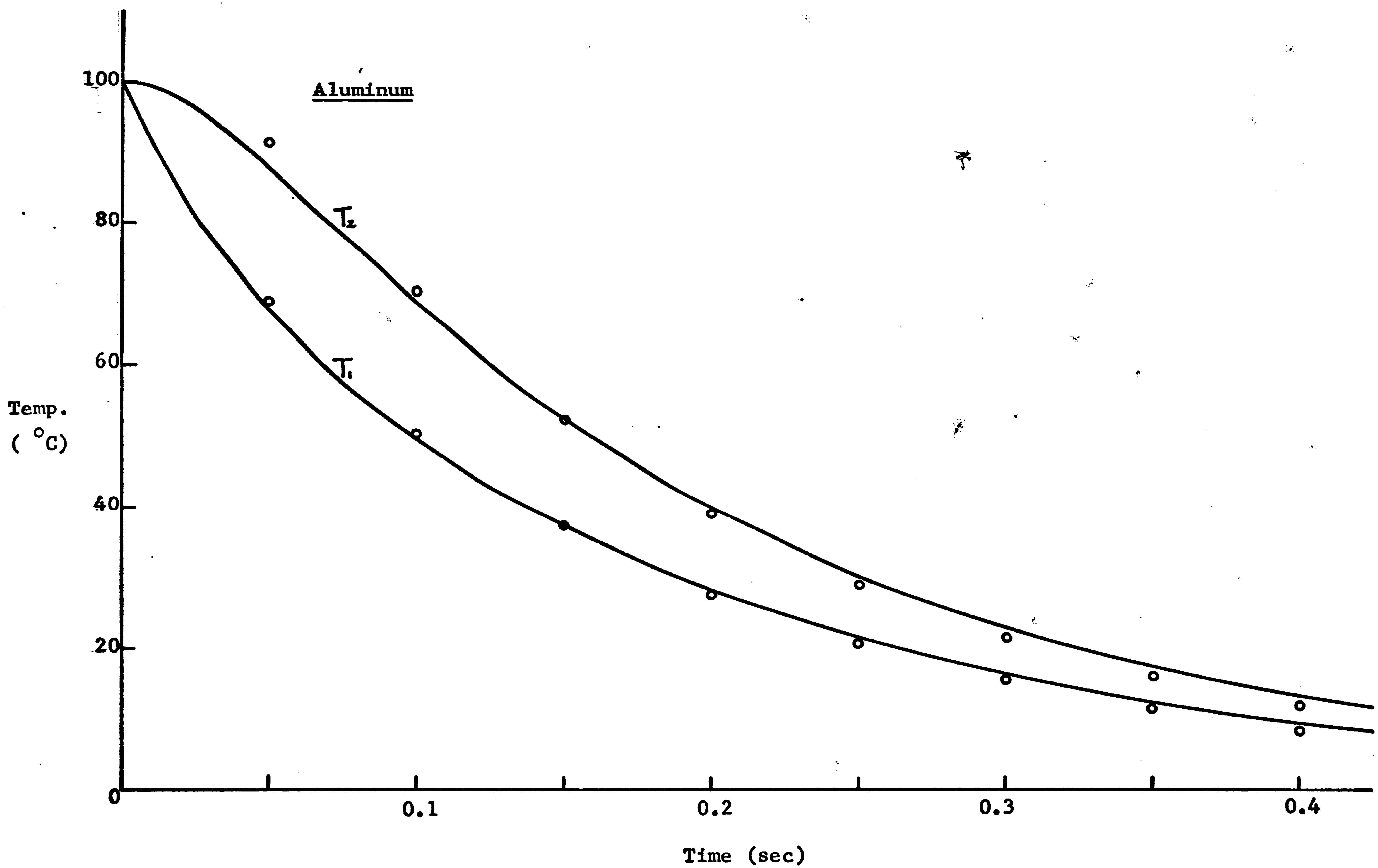


Fig. 29 ANALOG SOLUTIONS OF THE HEAT EQUATION WITH 4 CELLS (CASE 1)

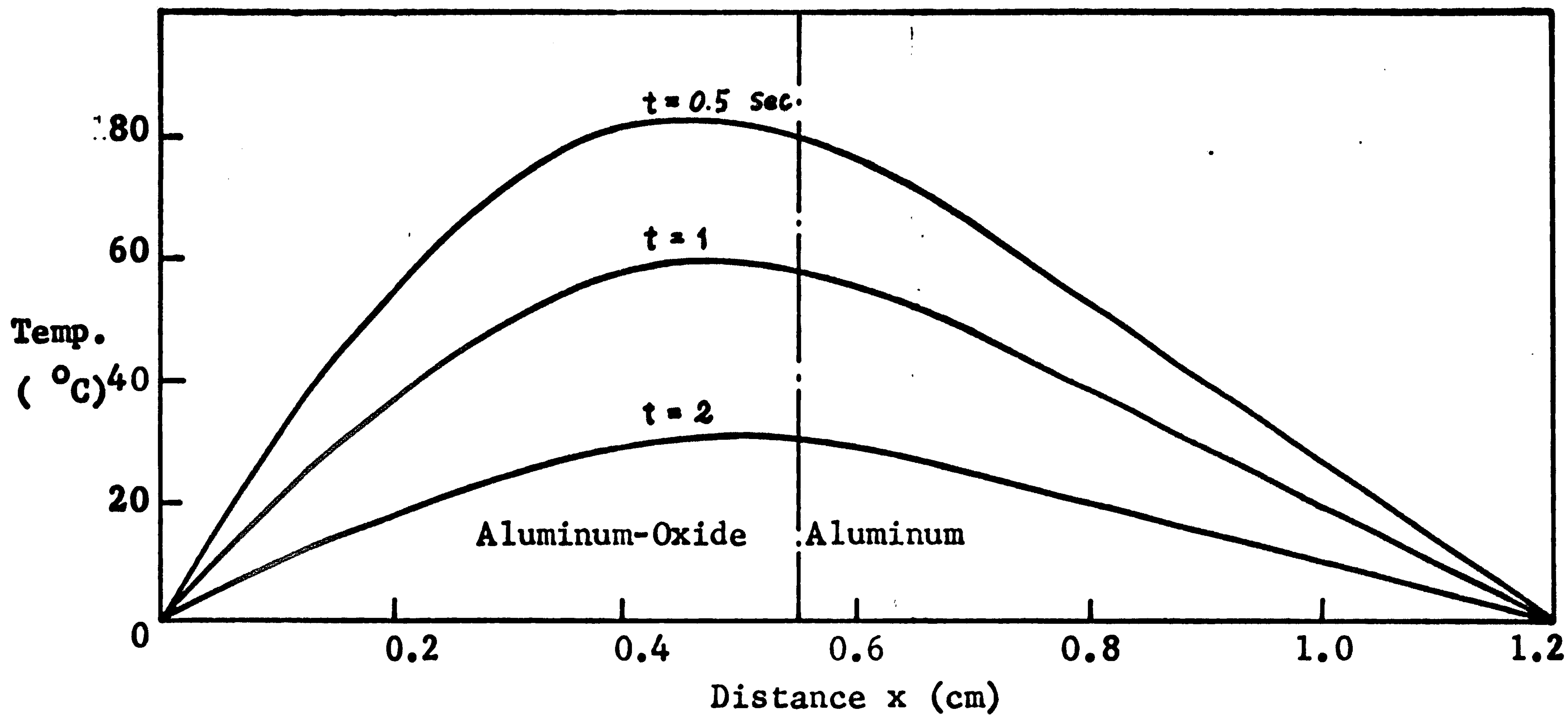


Fig. 30 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF **THE ALUMINUM OXIDE** AND ALUMINUM COMPOSITE SLAB (CASE 3)

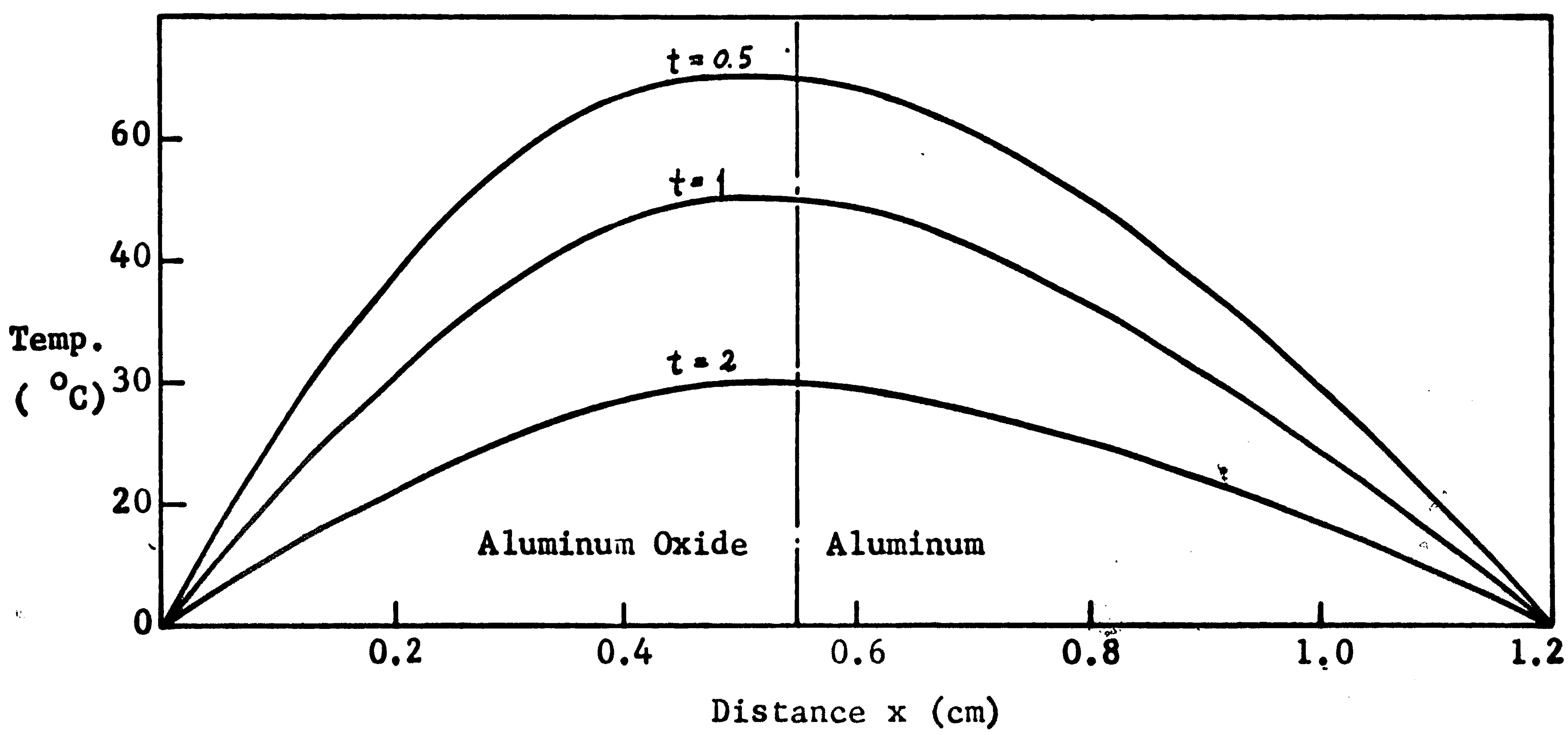


Fig. 31 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF **THE ALUMINUM OXIDE** AND CAST IRON COMPOSITE SLAB (CASE 3)

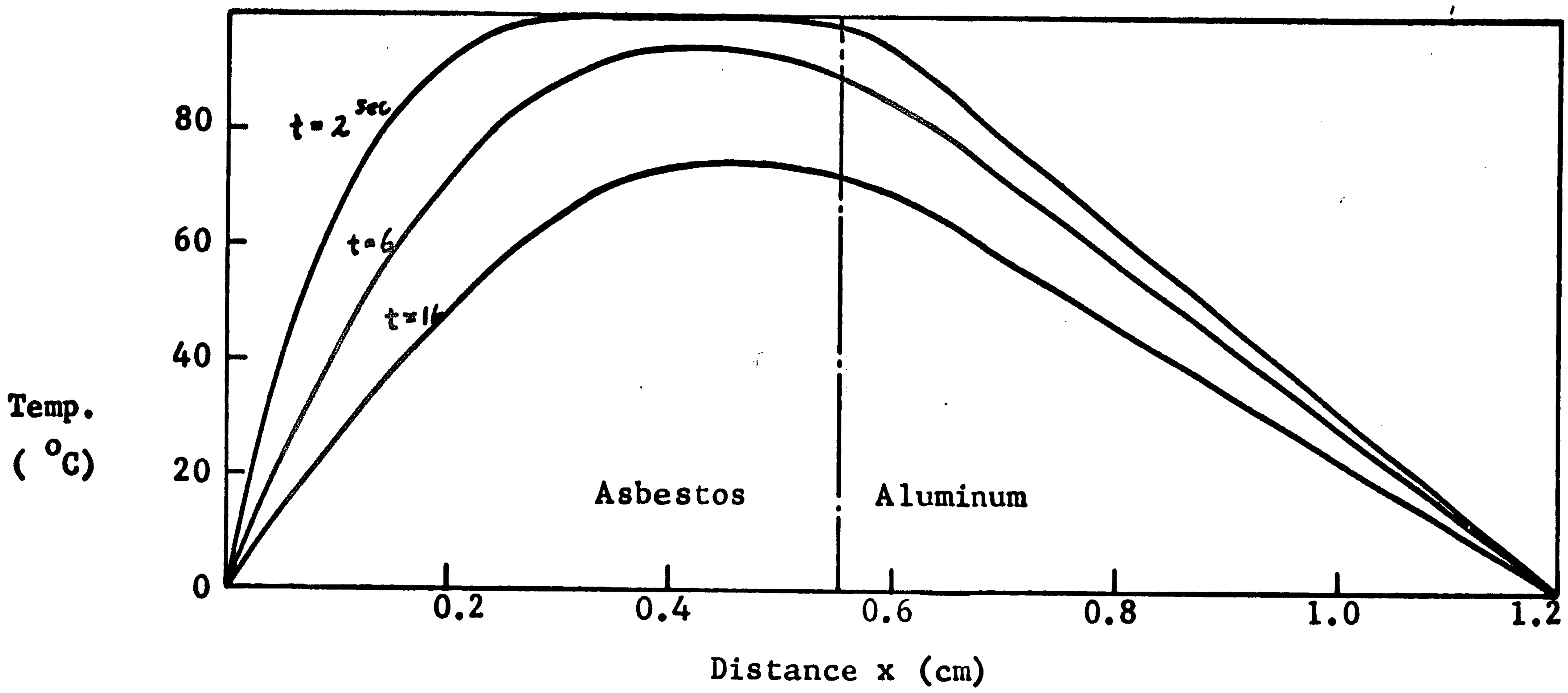


Fig. 32 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF THE ASBESTOS AND ALUMINUM COMPOSITE SLAB (CASE 3)

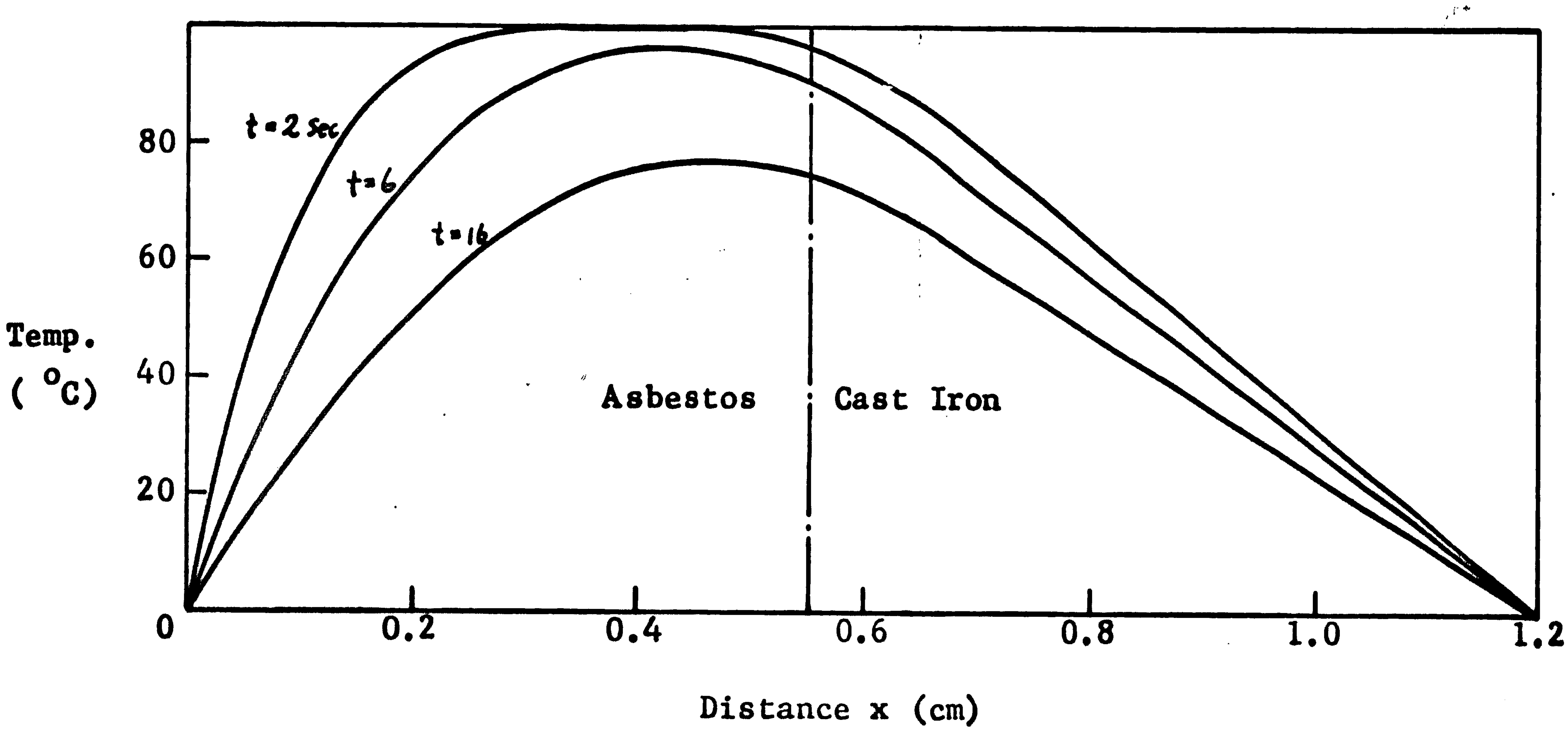


Fig. 33 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF THE ASBESTOS AND CAST IRON COMPOSITE SLAB (CASE 3)

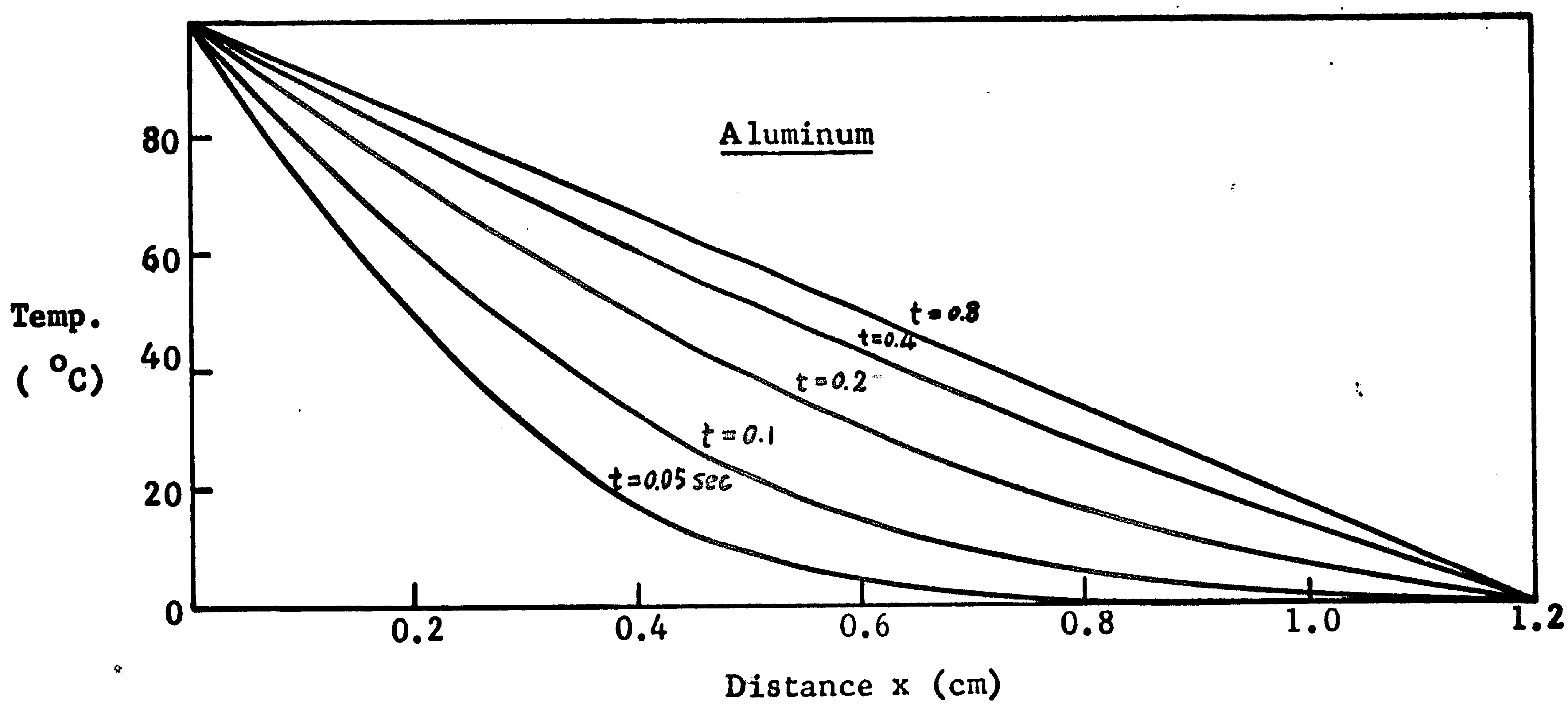


Fig. 34 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF THE ALUMINUM SLAB (CASE 2)

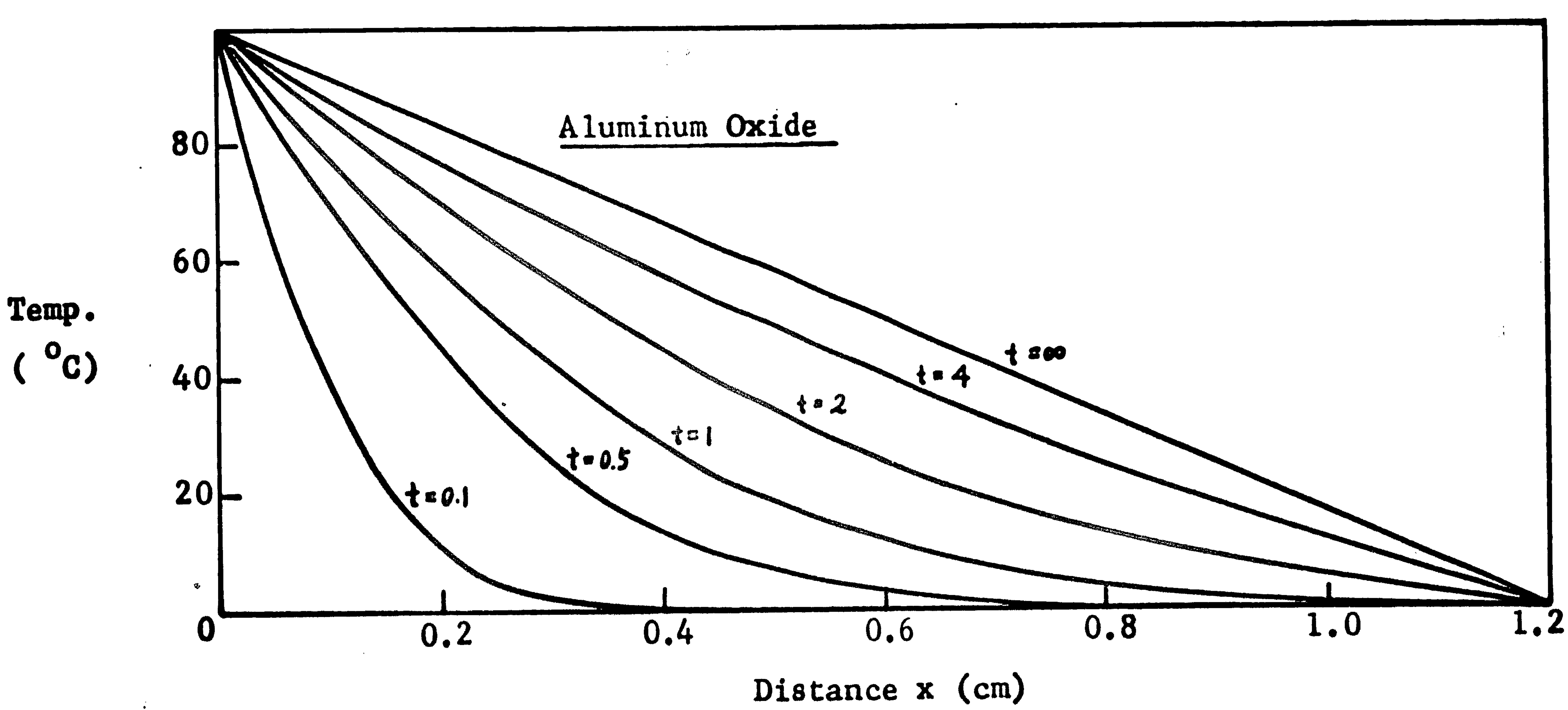


Fig. 35 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF THE ALUMINUM OXIDE (CASE 2)

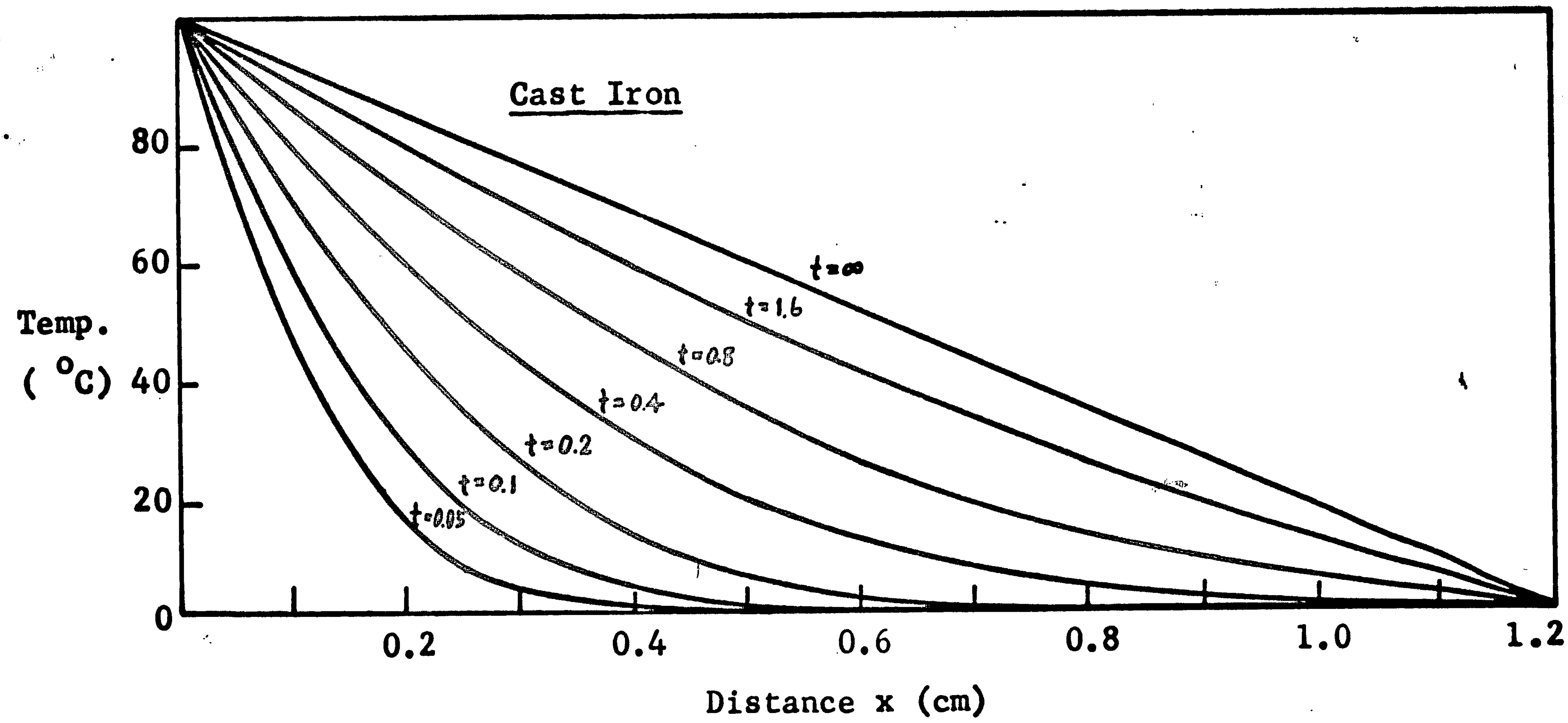


Fig. 36 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF THE CAST IRON
(CASE 2)

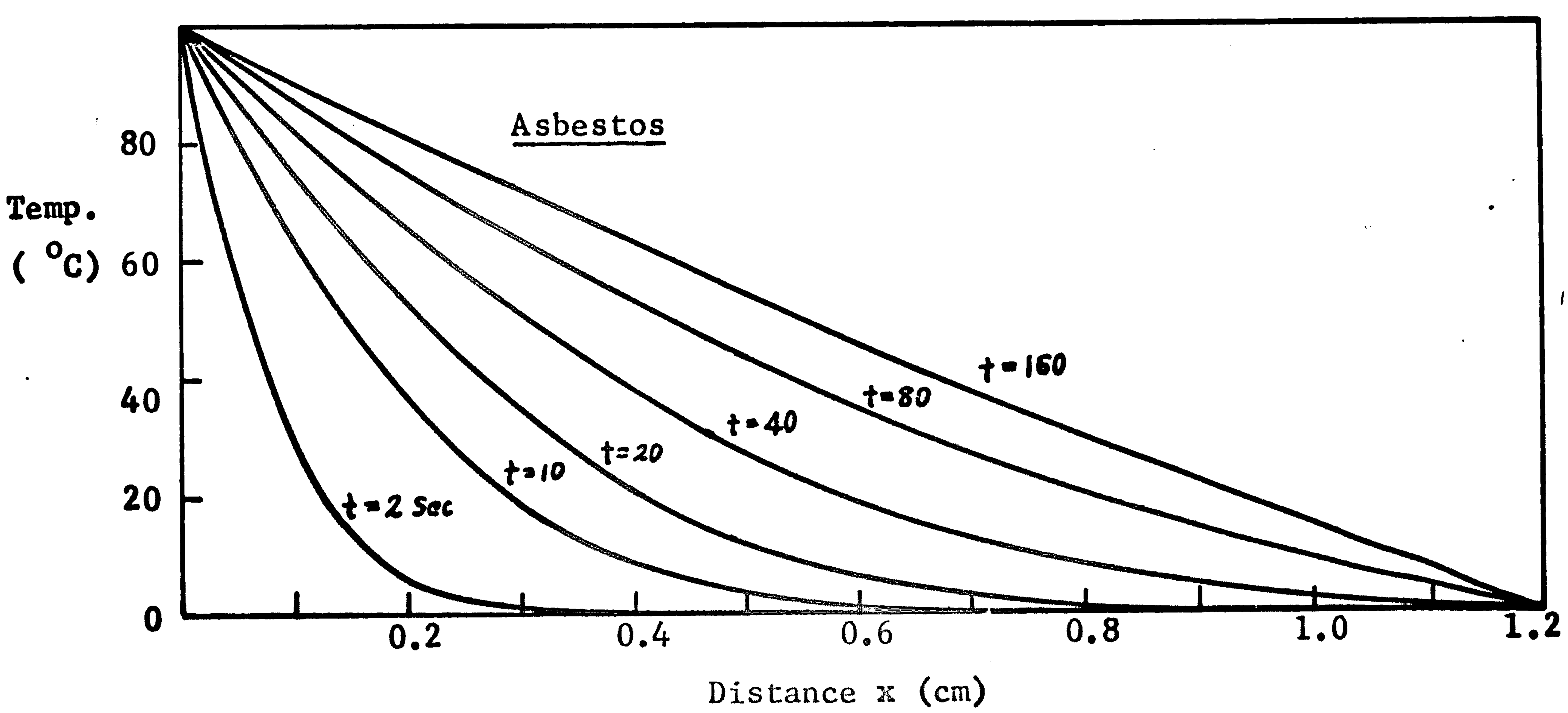


Fig. 37 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF THE ASBESTOS
(CASE 2)

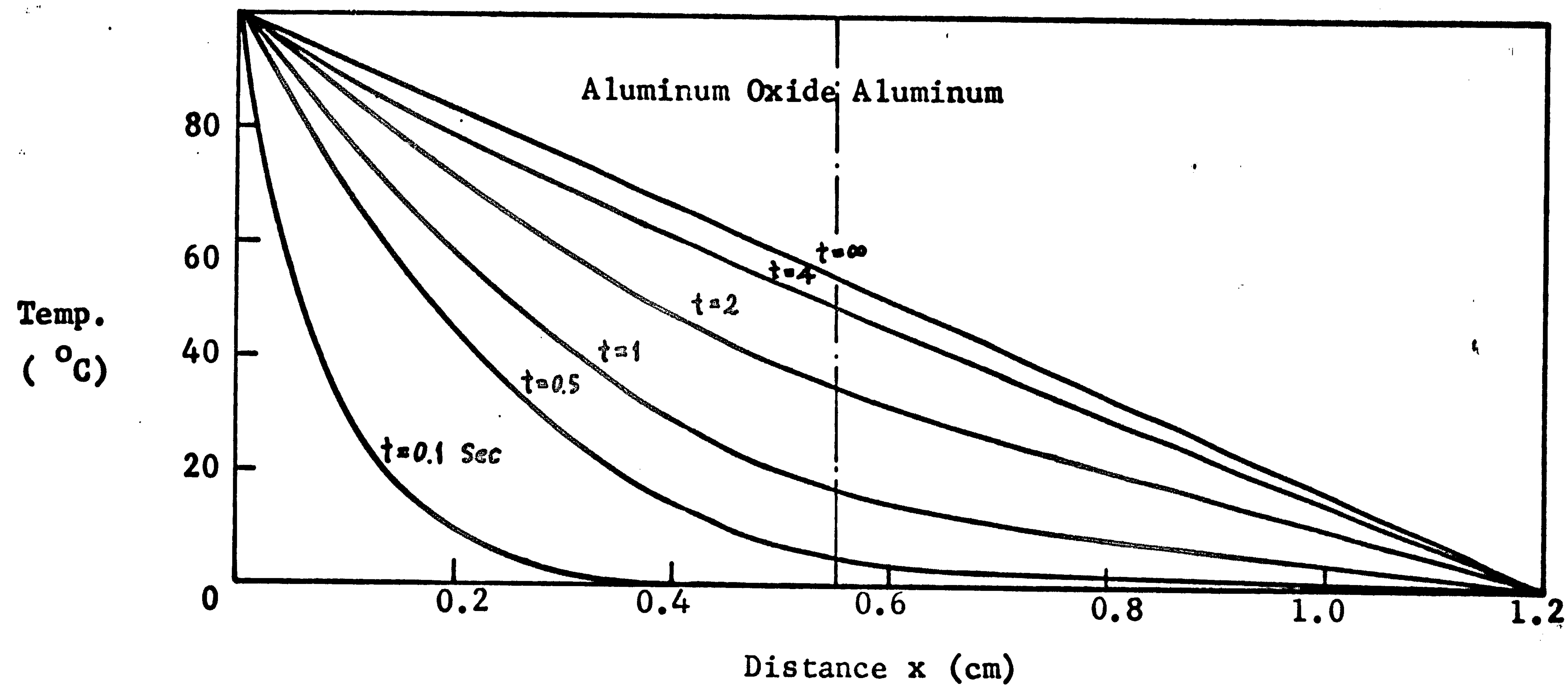


Fig. 38 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF THE ALUMINUM OXIDE AND ALUMINUM COMPOSITE SLAB (CASE 4)

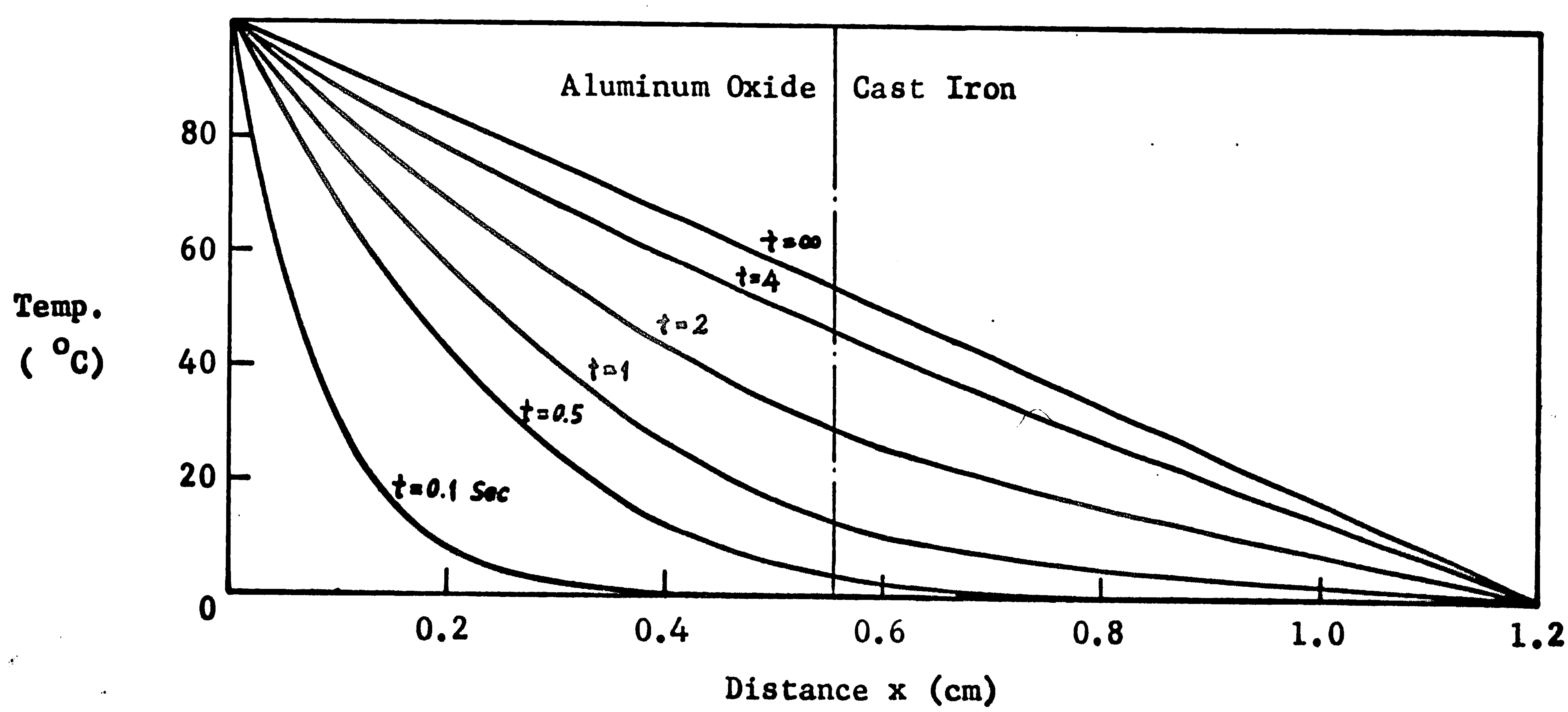


Fig. 39 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF THE ALUMINUM OXIDE AND CAST IRON COMPOSITE SLAB (CASE 4)

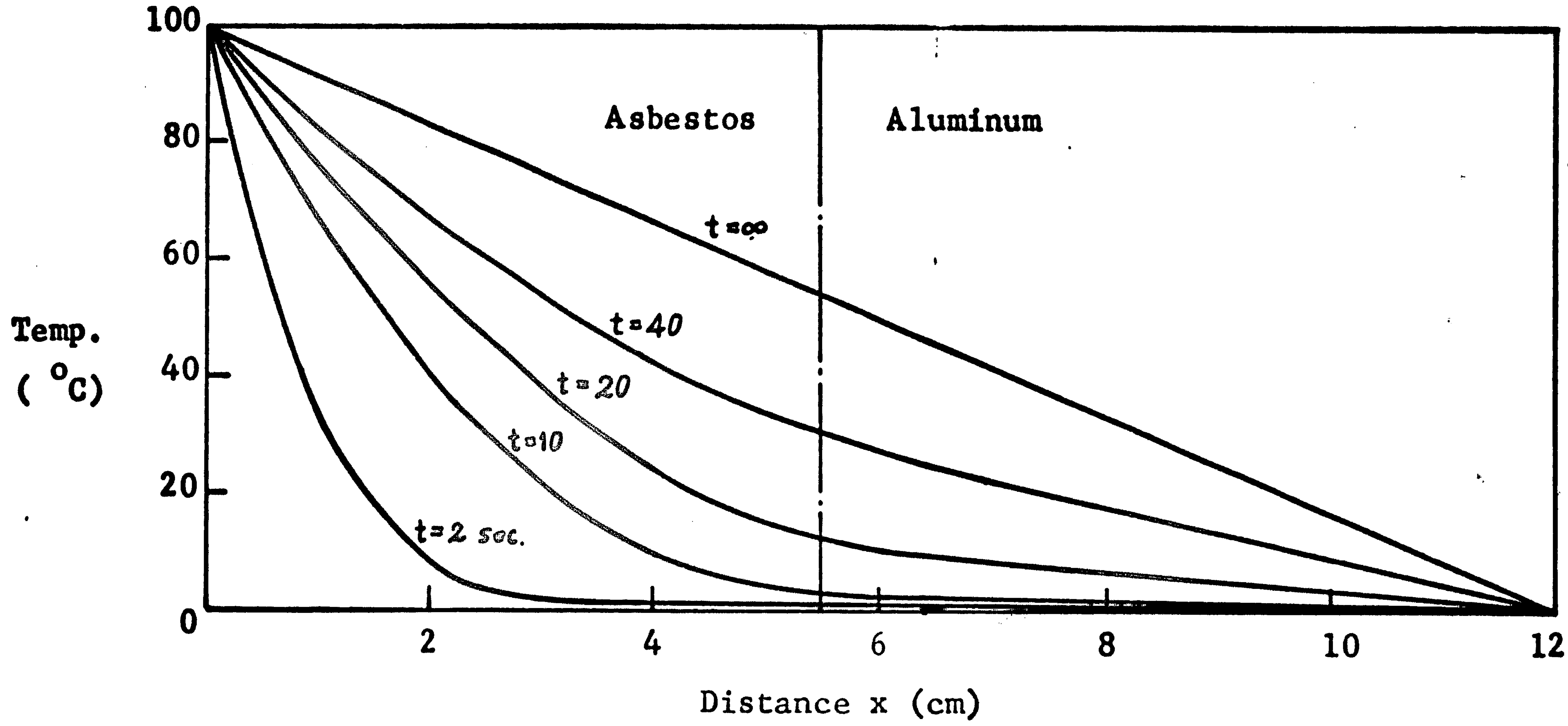


Fig. 40 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF THE ASBESTOS AND ALUMINUM COMPOSITE SLAB (CASE 4)

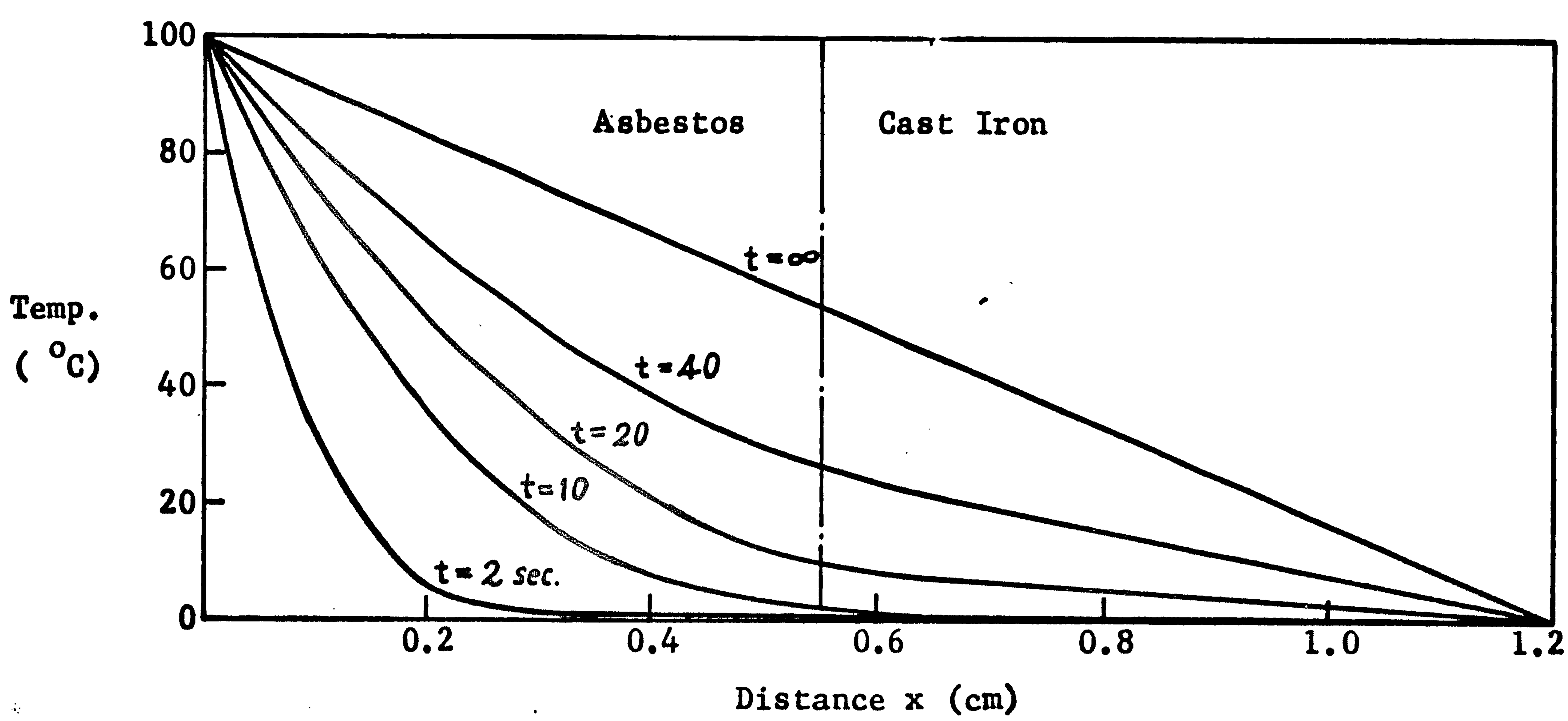


Fig. 41 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF THE ASBESTOS AND CAST IRON COMPOSITE SLAB (CASE 4)

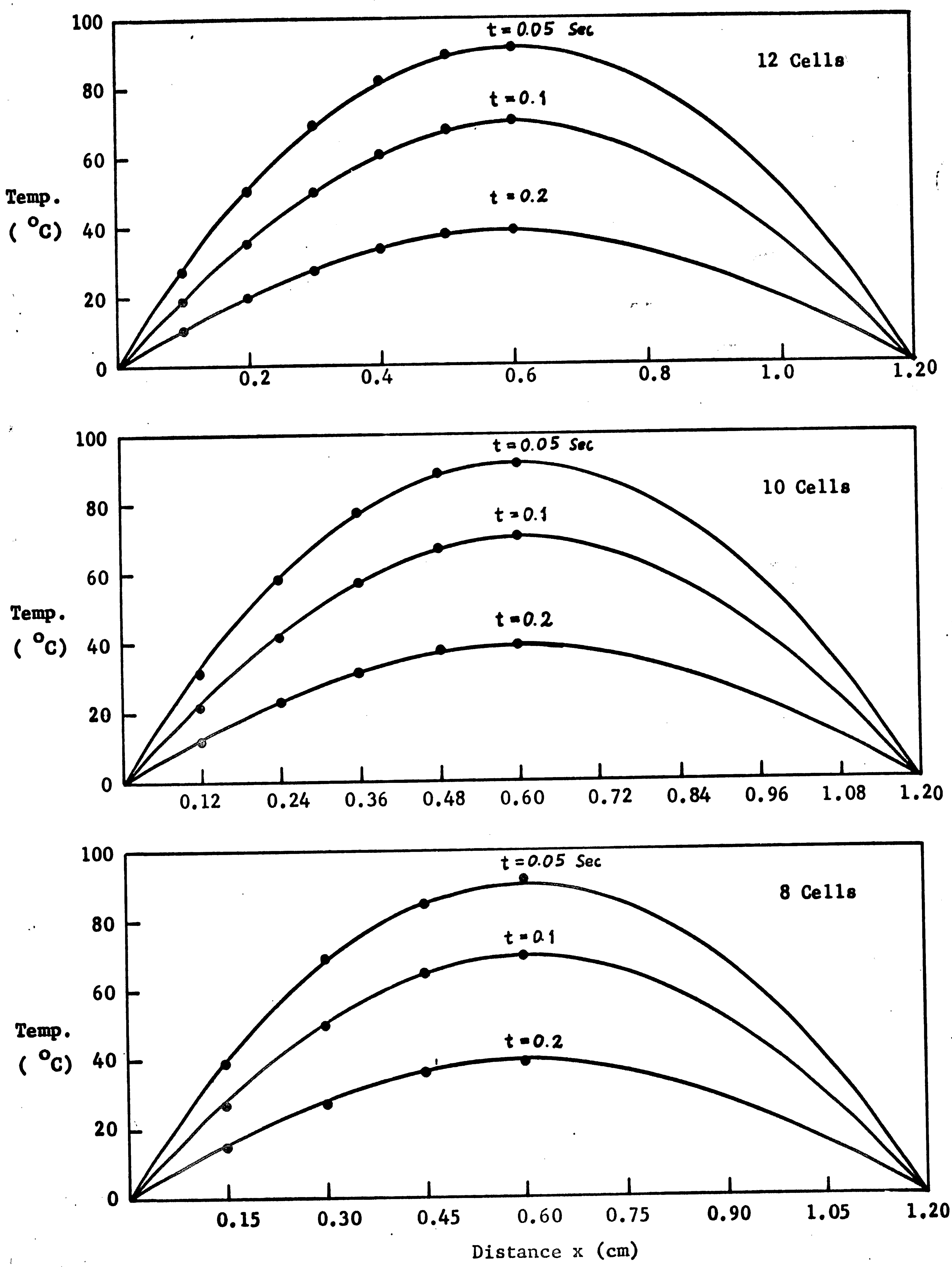
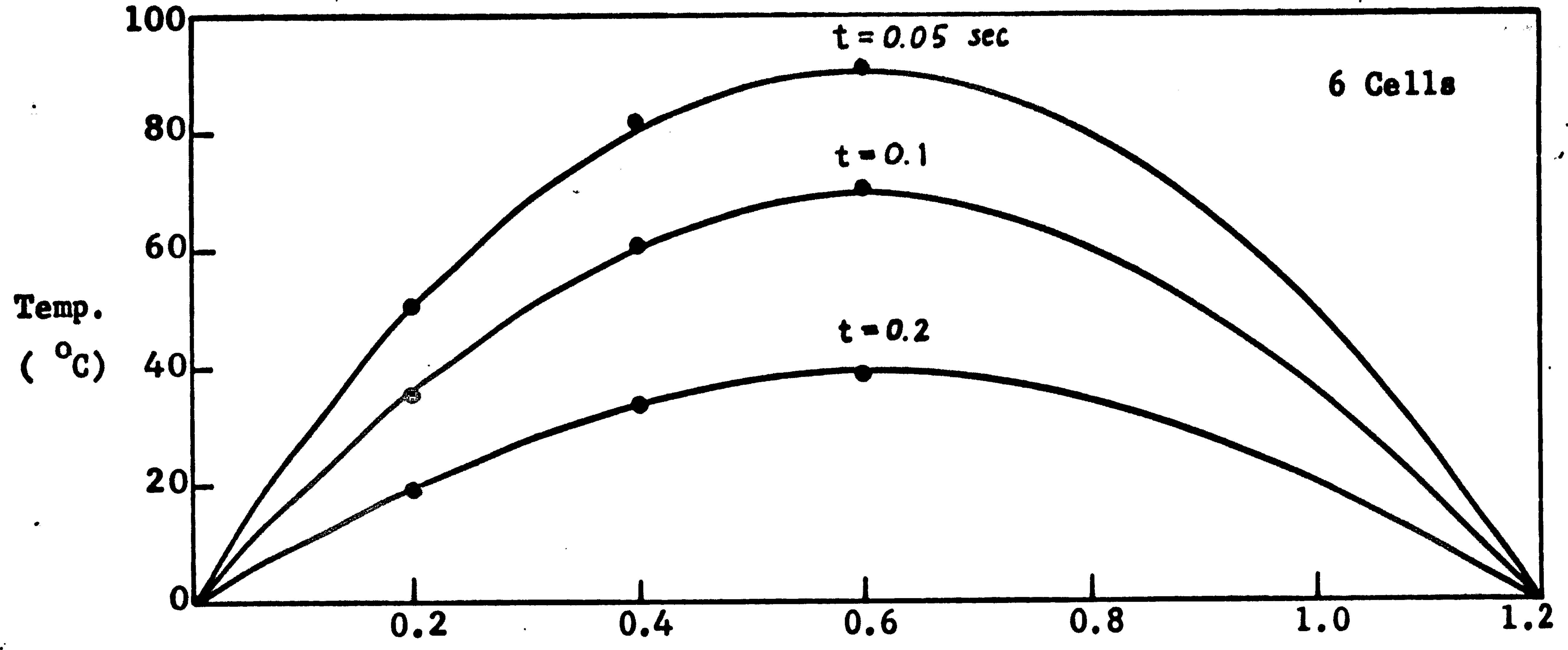


Fig. 42 UNSTEADY-STATE TEMPERATURE DISTRIBUTION OF THE ALUMINUM SLAB WITH DIFFERENT NUMBER OF CELLS



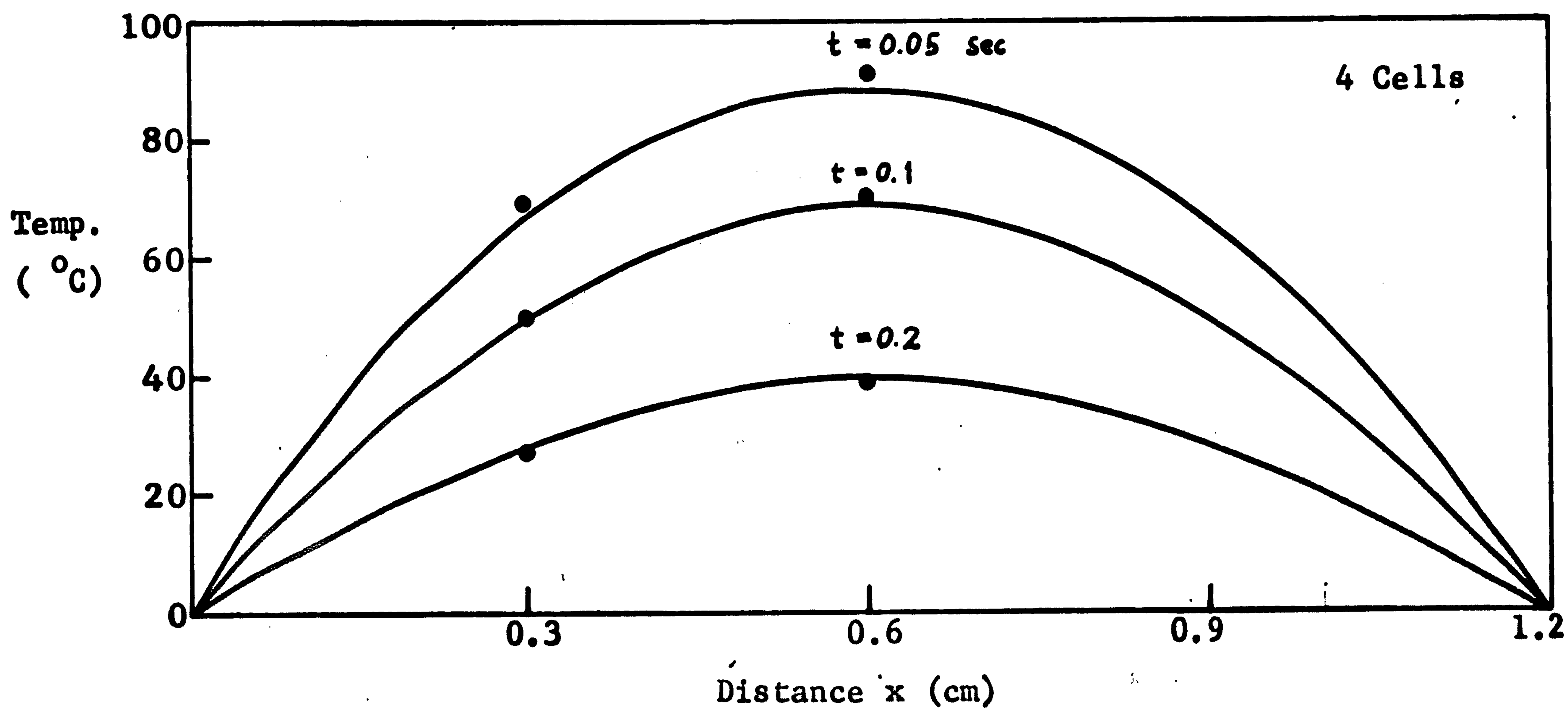


Fig. 42 continued

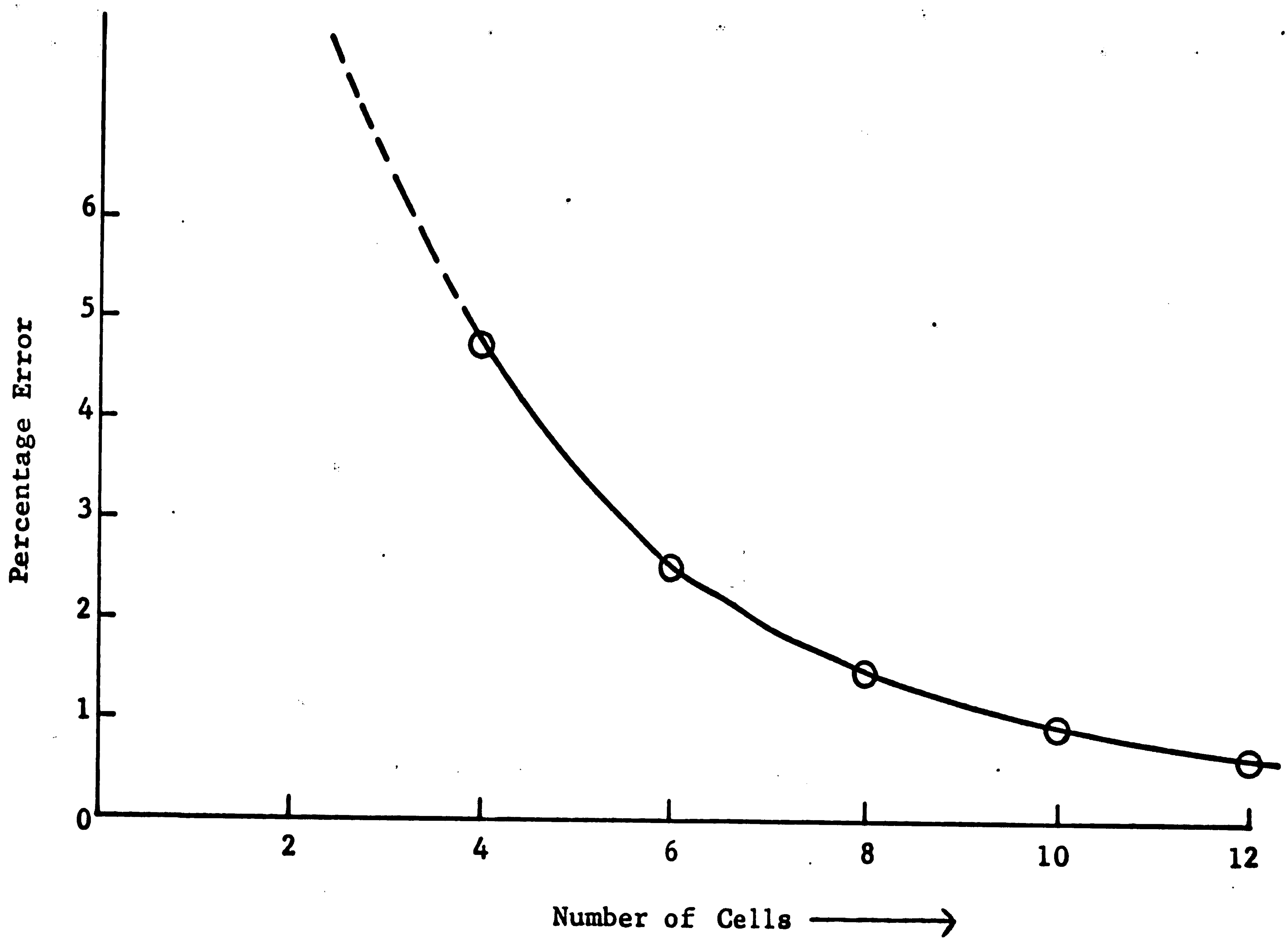


Fig. 43 PERCENTAGE ERROR AS A FUNCTION OF THE NUMBER OF CELLS

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9. VITA

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From 1961 he worked part time as a Research Assistant in the National Aerospace Laboratory in Tokyo. During his undergraduate years he passed the examination of Government Man in 1961.

In September, 1963 he came to Lehigh as a Research Assistant in Civil Engineering to pursue graduate work in Electrical Engineering.