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AFTER-FRACTURE REDUNDANCY OF STEEL BRIDGES:
A Review Of Published Research

by

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TABLE OF CONTENTS

ABSTRACT

The purpose of this report is to review published research pertaining to after-fracture redundancy of steel bridges. Sixteen articles and technical reports are reviewed. While they do not constitute an all-inclusive literature survey, these references are representative samples of deterministic and probabilistic redundancy research conducted over the past ten years. They were selected for review because they have some direct or indirect application to the specific topic of after-fracture redundancy of steel bridges, as defined in the AASHTO Specification.

The reviewed articles and reports indicate that deterministic research has shown significant, steady progress since 1979. Early efforts were primarily concerned with qualitative discussions of redundancy and with establishing a frame of reference for further research. Initial quantitative studies consisted of computer analyses of simple, idealized structural models. These were followed by a succession of increasingly sophisticated finite element analyses, several of which realistically modeled actual bridges. The most successful studies have been performed by assuming worst-case fracture scenarios, then analyzing the response of the bridge at various levels of load. The authors of these studies generally agree that after-fracture redundancy can only be evaluated via *analysis of the full three-dimensional structural system in the damaged condition.*

Probabilistic redundancy research has not progressed as rapidly as has deterministic research, in the same period of time. Several reliability-based studies of redundancy have been performed; however they are all based on definitions of redundancy which are not consistent with the definition provided in the AASHTO Specification. Moreover, probabilistic methods have only been applied to analyses of

simple, two-dimensional structural models.

The following conclusions are drawn from the published research reviewed in this report:

(1) There is general disagreement regarding an appropriate definition for the term "redundancy". All future redundancy research should establish a clear, precise definition.

(2) In certain respects, deterministic redundancy research cannot progress significantly beyond the current state-of-the-art. The reason is that bridge behavior (and redundancy in particular) is characterized by a significant degree of inherent variability and uncertainty.

(3) Probabilistic methods offer the *potential* to deal rationally with these forms of uncertainty. As of yet, that potential has not been fully realized.

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1. INTRODUCTION

1.1 Background

In recent years, the importance of structural redundancy has been dramatically illustrated by the failures of several steel two-girder highway bridges. The I-79 Back Channel Bridge near Pittsburgh, Pennsylvania, and the Lafayette Street Bridge in St. Paul, Minnesota, are noteworthy examples. Both experienced brittle fracture of one girder, but did not collapse. Both were subsequently repaired at relatively low cost and returned to service. Had they collapsed, lives might have been lost, and expensive replacement structures would have been required.

The American Association of State Highway and Transportation Officials (AASHTO) Standard Specifications for Highway Bridges recognizes the need for redundancy by specifying reduced allowable fatigue stress ranges for "nonredundant load-path structures" [1]. *Redundant* load-path structures are defined as "structure types with multi-load paths where a single fracture in a member cannot lead to collapse." Yet the specification classifies two-girder bridges as nonredundant, despite the demonstrated redundancy of in-service two-girder bridges. Furthermore, the specification provides no specific guidelines for design and evaluation of redundancy in bridges. It does not, for example, specify how much (if any) live load a redundant bridge ought to be capable of carrying after experiencing a fracture. It is apparent that specific provisions for after-fracture redundancy should be incorporated into future editions of the AASHTO Specification. Such provisions must be the result of an extensive research effort.

A significant amount of research has already been conducted in the general area of redundancy. Only a small portion of that research has been specifically oriented toward after-fracture redundancy of steel bridges, however.

All existing redundancy research can be generally characterized as either *deterministic* or *probabilistic*. Probabilistic methods take into account the inherent uncertainty in load and resistance variables and in the process of predicting structural behavior. Deterministic methods ignore this uncertainty or assume it to be insignificant.

1.2 Purpose and Scope

The purpose of this report is to review published research pertaining to after-fracture redundancy of steel bridges.

Sixteen articles and technical reports are reviewed. These references do not constitute an all-inclusive literature survey. Rather, they are representative samples of deterministic and probabilistic redundancy research conducted over the past ten years. They were selected for review because they have some direct or indirect application to the specific topic of after-fracture redundancy of steel bridges. Several of these references are not directly concerned with redundancy, but make reference to redundancy within the context of a more general area of research (e.g., highway loads).

1.3 Objectives

The objectives of this literature review are as follows:

(1) Summarize the key points of each piece of published material.

(2) For each piece of published material, evaluate the applicability of the reported research to future studies of after-fracture redundancy in steel bridges.

(3) For each piece of published material, identify shortcomings or limitations in the reported research, which might be addressed in future studies of after-fracture

redundancy in steel bridges.

(4) Based on the reviewed literature, identify general trends in redundancy research, evaluate the relative merits of deterministic and probabilistic methods, and identify important research needs.

Deterministic research and probabilistic research are presented in Chapters 2 and 3, respectively. Within each chapter, published research is reviewed in chronological order, from oldest to most recent. Each article or report is described individually.

For each article or report, a brief summary is provided. Wherever applicable, the summary includes a definition of the term *redundancy* (or *redundant structure*), as stated in the reviewed article or report. The definition is emphasized because there is widespread disagreement about what redundancy really is. A comparison of various authors' definitions clearly illustrates the many possible interpretations of the term. Many of these interpretations are inconsistent with the definition provided in the AASHTO Specification. (See Section 1.1.)

Following the summary, a discussion of the reported research is presented. The discussion consists largely of subjective observations. Applicability to future studies of after-fracture redundancy is emphasized. Throughout this report, the AASHTO definition of redundancy is used as a frame of reference for evaluation of published research. In several cases, articles are judged to be largely inapplicable to future studies of after-fracture redundancy, simply because of an inconsistent or overly broad definition. Topics which warrant further study are noted.

2. DETERMINISTIC RESEARCH

2.1 Sweeney, R.A.P., "Importance of Redundancy in Bridge Fracture Control" [2]

2.1.1 Summary

Sweeney provides examples of structures which experienced fractures in main load-carrying elements, but did not collapse. He illustrates his concept of *structural redundancy* by examining several cases where bridges were damaged by fire. These "statically determinate" structures—a simple truss and a three-hinged arch—experienced deck fires; after the fires were brought under control, during cooling of the superstructure, several members fractured. Neither bridge collapsed. Sweeney postulates that the phenomenon occurred because the structures were not really statically determinate. His theory is that, as members of the superstructure expanded due to heating, restraint provided by adjacent members caused local compression yielding. As the bridges cooled, large tensile residual stresses developed, resulting in widespread fracturing. He notes that a true statically determinate structure would never develop such residual stresses, because members would be free to expand without restraint. Thus, he says, these structures were "redundant", and the existence of this redundancy explains why the bridges did not collapse.

Sweeney further discusses his concept of *component redundancy* or *internal member redundancy.* Component redundancy is exhibited by riveted members because they are made of discrete built-up components—web plate, flange plates, flange angles, and cover plates. If a crack develops in one component, it cannot propagate into adjoining components. If one component fractures, the girder still retains significant load-carrying capacity. Sweeney notes that welded members do not exhibit component redundancy because cracks can run freely through welded joints.

The author presents a case study involving a series of riveted railroad bridges, in which corroded members were repaired with welded patches. The bridges all experienced significant cracking within ten years of the repair. He concludes that design of welded bridges must compensate for their lack of component redundancy with increased structural redundancy. He emphasizes the need for rigid quality control in fabrication and inspection of welded bridges.

2.1.2 Discussion

(1) Sweeney does not provide a consistent definition of redundancy. In his discussion of structural redundancy, he makes several references to "multi-load-path structures", but does not fully explain or develop the concept. His use of fire-damaged bridges as examples suggests that redundancy is related to the ability of a structure to undergo a fracture of a main load-carrying member without collapsing; yet his discussion of those examples clearly indicates that he equates *redundancy* and *static indeterminacy*. This definition is inconsistent with the AASHTO definition, and is too broad to be of practical use. Virtually all actual structures are statically indeterminate. For example, a simple truss bridge might be represented as a statically determinate structure for analysis or design; but the actual three-dimensional structure is likely to be highly indeterminate. Sources of indeterminacy include rotational restraint at connections, additional members in the floor systems and secondary bracing systems, and restraint at bearings. If redundancy were, in fact, equivalent to static indeterminacy, one would logically conclude that virtually all structures are redundant. Such a conclusion would be erroneous and potentially dangerous. Static indeterminacy alone does not guarantee that a structure which has experienced brittle fracture of a member will not collapse.

(2) Sweeney suggests that there are two fundamentally different types of after-

fracture redundancy—component redundancy and structural redundancy. Component redundancy is the capacity of a *member* to carry load following a fracture of one *component*. Structural redundancy is the capacity of a *structural system* to carry load following the fracture of a *member*. The distinction between the two forms of redundancy is certainly valid. There is, however, potential for confusion regarding Sweeney's terminology. He uses the terms *component redundancy* and *internal member redundancy* interchangeably. Other references use the term *internal redundancy* to describe the same property. Some standardization is needed.

(3) The most significant limitation of Sweeney's paper is his failure to fully address after-fracture behavior of the damaged bridges. While he observes that none of the example bridges collapsed after fractures occurred, he does not discuss the alternate load paths which prevented the bridges from collapsing.

2.2 Haaijer, G., Schilling, C.G., Carskaddan, P. S., "Bridge Design Procedures Based on Performance Requirements" [3]

2.2.1 Summary

The authors suggest new bridge design procedures, based on the AASHTO Load Factor Design provisions. These procedures are defined in terms of four distinct performance requirements, with corresponding limit states, as follows:

(1) Service load
(2) Overload
(3) Maximum load
(4) "Fail-safe load"

For the service load performance requirement, a proposed fatigue design truck is described. This hypothetical truck is used to calculate the fatigue life of the components of a bridge. For the overload performance requirement, a procedure called "auto-stress design" is described. The procedure considers the beneficial effects of inelastic deformation in the negative moment regions of continuous spans. For the maximum load requirement, the authors propose that formation of a full plastic mechanism be allowed for continuous spans. For the fail-safe load requirement, a test for after-fracture redundancy is proposed. The authors define a fail-safe structure as a simplified model of the actual structure, with a "through-separation at any section where the design life is less than the specified value." The fail-safe requirement is stated as "adequate load-carrying capacity when a bridge has one separated element." The proposed fail-safe requirement would only apply to members whose design is governed by fatigue.

The authors define a redundant structure as a one for which "stress resultants and reactions cannot be found from equilibrium alone." This definition suggests that redundancy is equivalent to static indeterminacy.

2.2.2 Discussion

(1) Discussions of service load, overload, and maximum load performance requirements have little or no applicability to redundancy. Conversely, the general concept of the proposed fail-safe load requirement is very applicable. By proposing a test for adequacy of a full structure after one principal member has failed, the authors have effectively defined after-fracture redundancy. Note, however, that their explicit definition of a redundant structure equates redundancy with static indeterminacy, just as Sweeney's did.

(2) The authors' emphasis on selecting the *appropriate limit state* for each performance requirement is also noteworthy. Unfortunately, the authors do not actually define a limit state for their fail-safe load test. Rather, they refer only to

"adequate load-carrying capacity." Clearly, any practical procedure for evaluating the after-fracture redundancy of a bridge must precisely define the limit state. What is appropriate? Is it collapse, yielding, fracture of other members, excessive deflections, a combination of these, or something else? Haaijer, Schilling, and Carskaddan do not attempt to answer these questions.

(3) The authors make no attempt to define appropriate loads for evaluation of the fail-safe structure.

(4) The proposal to use the fail-safe criterion only for members whose design is governed by fatigue is potentially unconservative. A bridge girder whose design is governed by something other than fatigue (maximum deflection, for example) might still be susceptible to brittle fracture. Nonetheless, the general principal behind the authors' proposal is fully valid. The proposal suggests that *evaluation of redundancy should focus on members which are susceptible to brittle fracture.*

2.3 Csagoly, P. F., "Multi-Load-Path Structures for Highway Bridges" [4]

2.3.1 Summary

Csagoly's article is primarily intended to establish a frame of reference for discussion of redundancy. It presents appropriate nomenclature, extracted from the Ontario Highway Bridge Design Code [18]. Csagoly also discusses the need for specific exclusion of single-load-path structures from design codes. He establishes the probability of failure of a multi-load-path structure as virtually zero. He also provides historical examples of bridge failures—the Silver Bridge, the Lafayette Street Bridge, the I-79 Bridge, the Ontario-35 Bridge, the Ontario-33 Bridge, and several simple truss bridges. Of these, all but the Silver Bridge behaved as multi-load-path structures, several unintentionally.

Csagoly presents the following definitions, all of which are taken from the Ontario Bridge Code:

> COLLAPSE - A major change in the geometry of a structure, which makes it unserviceable.
>
> COMPONENT - A structural element or group of elements which require individual design consideration.
>
> FAILURE - A state in which the load carrying capacity of a component or connection has been exceeded.
>
> MULTI-LOAD-PATH STRUCTURE - A structure in which the failure of a component or connection does not result in collapse of the structure.

Note that, according to these definitions, a *component* can *fail* without causing *collapse* of the structure. Note also that a bridge which *collapses* does not necessarily fall down; it only undergoes a "major change in geometry". Thus a large (but repairable) deflection due to a fractured girder could be classified as a collapse.

Csagoly characterizes single-cell steel box girders as single-load-path structures, and recommends that they be forbidden in American codes, as they are in the Ontario Code. He notes that monolithic concrete bridges can still be regarded as multi-load-path structures because reinforcing bars can be considered to be individual components.

The author does not, at any time, use the word "redundancy". Rather, in keeping with the terminology of the Ontario Bridge Code, he uses the term "multi-load-path structure."

2.3.2 Discussion

(1) Csagoly's article provides much useful background information and does, in fact, establish a good frame of reference for redundancy research. It does not provide the results of research.

(2) The Ontario Bridge Code's definition of *collapse* may be useful for design purposes; but for evaluation of existing bridges it is overly broad. In assessing the redundancy of existing structures, it is quite important to distinguish between a bridge that has been destroyed and a bridge which has undergone large deflections, but remains intact. Csagoly's broad definition of collapse does not provide the needed distinction.

2.4 Heins, C. P., and Hou, C. K., "Bridge Redundancy: Effects of Bracing" [5]

2.4.1 Summary

Heins and Hou discuss the use of finite element analysis to evaluate the redundancy of two- and three-girder bridges. Their finite element models are space frames composed entirely of beam elements. The models are simplified representations of the full three-dimensional structural systems for hypothetical two- and three-girder bridges. The models include diaphragms and lateral bracing. To evaluate redundancy, the top and bottom flanges of one girder are assumed to be fractured. The fracture is modeled by using a very short beam element with negligible stiffness at the fracture location. For both bridges, analyses are performed with:

(1) no fractures, no diaphragms, no lateral bracing
(2) no fractures, diaphragms, no lateral bracing
(3) no fractures, diaphragms, lateral bracing
(4) fractured flanges, no diaphragms, no lateral bracing
(5) fractured flanges, diaphragms, no lateral bracing
(6) fractured flanges, diaphragms, lateral bracing

"Design live loads" are used in all cases, though this term is not fully explained. Based on computed deflections, it is concluded that diaphragms and lateral bracing contribute significantly to redundancy in two- and three- girder bridges.

Heins and Hou do not explicitly define redundancy. Indirectly, they refer to it as "residual strength of a bridge structure to withstand repetitive loadings after a crack has developed."

2.4.2 Discussion

(1) The general concept of this analysis is quite appropriate. After-fracture redundancy is evaluated by comparing the results of analyses of the three-dimensional bridge structure in the undamaged and damaged conditions. Execution of the concept, however, is probably too simplistic to be of direct practical use.

(2) There is no indication that the finite element models are based on actual bridges. Furthermore, the simple space frame models used are, at best, very coarse representations of actual bridges. While there is a justifiable need for such simple models in engineering practice, it is important that the behavior of those models be correlated with the behavior of actual bridges. Heins and Hou apparently do not attempt to establish that correlation.

(3) The modeling of the girder fractures in this study is unrealistic. The compression flange of the girder is assumed to be severed, while the girder web is assumed to remain intact. In practice, the top flange is unlikely to fracture because it is in compression. In a welded bridge, a fracture of the bottom flange would most likely penetrate at least partially through the web as well.

(4) "Design loads" are not defined. There does not appear to be any consideration of what loads might be appropriate for evaluation of after-fracture redundancy.

(5) Comparisons of behavior are based solely on deflections. Stresses are not considered, nor is the potential for instability in diaphragms and lateral bracing members.

2.5 Heins, C. P., and Kato, H., "Load Redistribution of Cracked Girders" [6]

2.5.1 Summary

Heins and Kato use finite element analysis to investigate the load distribution of a two-girder bridge with an induced crack in one girder. A "typical two-girder system" is modeled as a space frame similar to the one used in the earlier work by Heins and Hou [5]. Girder flanges and webs, floor beams and stringers, and bottom lateral bracing are all modeled with beam elements. Two different span lengths are used. The bridge is analyzed in three configurations: (1) intact, (2) with fractured girder but without bottom lateral bracing, and (3) with fractured girder and with bottom lateral bracing. The fracture is assumed to sever the bottom flange and half of the depth of the web. AASHTO HS 20-44 truck and lane loads are applied to the model. Based on a comparison of computed deflections and flange stresses, the authors conclude that bottom lateral bracing has a significant influence on load redistribution.

To check the validity of the three-dimensional finite element model, Heins and Kato also compare the results of a series of two-dimensional analyses of a single girder. Five different finite element models are used. One is composed entirely of beam elements, just as the three-dimensional model is. The other four use 3-D shell elements or 2-D plane stress elements to model the web. The authors conclude that the space frame model is adequate for evaluation of load redistribution, but that a more sophisticated model is required for analysis of local stresses in the vicinity of the crack.

2.5.2 Discussion

(1) This research overcomes some of the shortcomings of the previous work by Heins and Hou. Loads are better defined, and stresses are included in the comparison of results. Modeling of the fracture is more realistic.

(2) In this analysis, the authors have attempted to validate the use of their simple space frame model. Note, however, that they have done so with a series of *two-dimensional* analyses. It would have been more conclusive to perform a single three-dimensional analysis with a more sophisticated model, and compare those results with those of the space frame analyses.

(3) The most significant shortcoming of this research is that it merely presents the raw results of analyses. There is no attempt to draw conclusions about *why* lateral bracing improves load redistribution, to evaluate the viability of using lateral bracing as an alternate load path, or to define guidelines for the design of these members for optimum load redistribution.

2.6 Task Committee on Redundancy of Flexural Systems of the ASCE-AASHTO Committee on Metals of the Structural Division, "State-of-the-Art Report on Redundant Bridge Systems" [7]

2.6.1 Summary

The report describes the concept of "redundant load paths" and its application to both the macro level (member configurations) and the micro level (connection configuration). Welded structures are described as significantly less redundant than bolted ones, because welded material is continuously connected. In a bolted connection, cracks in one component (connection plate) cannot propagate into adjacent components.

The report describes three methods of preventing collapses:

(1) Reduce element stresses (a method which is often uneconomical)

(2) Improve quality control of elements (in accordance with the AASHTO Fracture Control Plan)

(3) Design the structural system such that, if one component fails, adjacent components remain intact.

A structure is described as redundant if it allows for redistribution of loads from damaged members along alternate load paths *of similar stiffness*. The report indicates that two-girder systems are susceptible to sudden failure, because of the small relative stiffness of the alternate load path, in the event of a girder fracture. Nonetheless, it is acknowledged that two-girder bridges have demonstrated significant redundancy in recent failures.

The report makes it very clear that redundancy can only be evaluated in a meaningful way if the entire three-dimensional structural system is modeled. Finite element analysis is described as the best, though most expensive, method of performing such analyses. The following guidelines for modeling are presented:

(1) The overall behavior of skewed bridges is not substantially different from those with no skew, provided that the skew angle is less than 60°.

(2) Bridges designed with composite decks in accordance with the AASHTO specification can be expected to demonstrate full composite action to the elastic limit.

(3) Diaphragms and lateral bracing contribute significantly to redundancy and should be modeled.

(4) The concrete deck of a steel girder bridge is the first element of the structure to exhibit nonlinearity.

(5) The decision to use nonlinear analysis should be based on the required load level and the likelihood of inelastic behavior. In performing nonlinear analyses, it is generally sufficient to consider material nonlinearity only (small deformations); while

consideration of geometric nonlinearity (large deflections) is more accurate, the large increase in cost is normally not justified.

(6) Where appropriate, stress concentrations and displacement-induced stresses should be considered.

(7) Stability may need to be considered, particularly when large lateral deflections of girders are involved.

The state-of-the-art report defines redundancy as "the existence of simultaneous load paths, which ensure reliable structural behavior in instances of damage to some of the elements." This definition is not used consistently throughout the article, however.

2.6.2 Discussion

(1) This paper contains several key concepts which should be considered in future research. In the definition of redundancy and in all subsequent discussion, emphasis is placed on the existence of *alternate load-paths*. Recognition that redundancy can only be evaluated through three-dimensional analysis of an entire structural system in the damaged condition is also very important.

(2) Discussion of the relative stiffness of alternate load-paths is noteworthy, in that the concept has not been mentioned in any previous research.

(3) The report acknowledges that two-girder bridges can have substantial redundancy, but makes no attempt to reconcile the apparent disagreement with the AASHTO Specification, which classifies all two-girder bridges as "nonredundant load path structures".

(4) A significant shortcoming of the state-of-the-art paper is the lack of any

specific discussion concerning appropriate loads for evaluation of redundancy.

(5) It is apparent that, as of 1985, there was still no generally accepted frame of reference for discussion of bridge redundancy. In the state-of-the-art paper, terminology is generally not adequately defined and is often used inconsistently.

2.7 Daniels, J. H., Wilson, J. L., and Chen, S. S., "Redundancy of Simple Span and Two-Span Welded Steel Two-Girder Bridges" [8]

2.7.1 Summary

Daniels, Wilson, and Chen perform highly detailed analyses of three actual in-service two-girder bridges—a simple span right bridge, a simple span skew bridge, and a two-span continuous right bridge. Upper bound elastic-plastic analyses are performed for all three bridges. Lower bound analyses are performed for the simple span right bridge and two-span continuous bridge, using a sophisticated three-dimensional finite element model. Loads are applied incrementally, so that inelastic behavior and instability of members can be taken into account.

Based on these analyses, the authors conclude that two girder bridges can, in fact, have a significant degree of redundancy. They identify the extent to which secondary members contribute to redundancy and propose guidelines for redundancy design. They demonstrate conclusively that studies of after-fracture redundancy in two-girder bridges require three-dimensional analytical models.

The definition of redundancy which forms the basis for this study is identical to the one provided in Section 10.3.1 of the AASHTO Specification [1]. Redundant structures are "structures with multi-load-paths where a single fracture in a member cannot lead to collapse."

2.7.2 Discussion

(1) This research is significant in that actual in-service bridges are analyzed in great detail. Thus while the results may not be universally applicable to *all* two-girder bridges, they are reasonably valid for a large number of similarly configured structures. This report is a significant improvement over earlier studies which used simplified models of hypothetical bridges, without correlation to actual structures.

(2) This research is also significant in that it goes well beyond mere presentation of raw results. Significant, quantitative conclusions are drawn and proposed design guidelines are developed.

2.8 Parmelee, R. and Sandberg, H., "Redundancy—A Design Objective" [9]

2.8.1 Summary

The authors describe how redundancy was incorporated into the design of a major highway bridge, the US Route 31 crossing of the Saint Joseph River in Michigan. When the designers elected to change the originally proposed bridge design from a four-girder to a three-girder configuration, Michigan Department of Transportation required proof that the new design was redundant. This was accomplished through a series of computer analyses of the structure, with an assumed girder fracture.

The bridge is modeled as a grid (a planar structure with out-of-plane loading and displacements). The fracture is modeled by placing a hinge in the appropriate girder element. The computer model is loaded with the bridge's dead load, plus one HS 20-44 truck load placed directly over the fractured girder. Based on these analyses, Parmelee and Sandberg identify three sources of redundancy:

(1) Cantilever action of the fractured girder. (The bridge is 3-span continuous.)

(2) Support from the composite deck slab.

(3) Cross frames (diaphragms).

Of these, (3) is found to be the most effective in enhancing redundancy. The authors note, however, that excessive yielding did occur in the cross frames nearest girder fracture. Their solution is to redesign the structure with a heavy (redundant) cross frame at every third cross frame location. Intermediate cross frames are designed with light angles and channel sections which carry service loads in tension, but buckle as soon as a girder fracture occurs. Buckling of intermediate frames allows loads to be redistributed through the stiffer cross frames without causing yielding. Parmelee and Sandberg also point out that, for these analyses, computed after-fracture deflections were large, but not excessive. The authors discuss the concept of an "interactive structure"—one for which after-fracture deflections are large enough to provide an obvious visual indication that damage has occurred. They conclude that better criteria are needed for evaluating redundancy and recommend that AASHTO permit increased allowable stresses for structures which have been *demonstrated to be redundant.*

The authors define a redundant structure as one which has "two or more paths by which the loads acting on the structure can be supported."

2.8.2 Discussion

(1) Parmelee and Sandberg take a realistic, rational approach to redundancy. They perform their analyses on an *actual* bridge design and use a realistic loading condition (though no specific rationale is given for the use of a single HS 20-44 truck). They vary the position of the girder fracture and redesign secondary members accordingly. Perhaps their most significant observation is that the *number* of girders in a bridge is not as important as the *interaction* between those girders, through

secondary members.

(2) The article deals with redundancy in the design of a new bridge. The designers were able to achieve substantial redundancy because they were free to configure the secondary bracing system to achieve the desired results. Such would not be the case, had the bridge already been built. This suggests that there must be a clear distinction between *designing for redundancy* and *evaluating the redundancy of existing structures.*

(3) The authors' emphasis on deflection as a "warning signal" for detection of girder fracture is important. It suggests that the *capacity for detection of damage* is a key element of redundancy evaluation. The authors do not, however, attempt to define the actual magnitude of "large, but not excessive" deflections.

(4) Parmelee and Sandberg only perform linear elastic analyses. In fact, their principal design criterion seems to have been prevention of yielding in any member after girder fracture. This requirement may be overly restrictive. Certainly some local yielding could be tolerated, as long as the structure does not collapse.

(5) In this study, a girder fracture is modeled as loss of the tension flange and one-half of the web. The girder is assumed to retain its full capacity to transmit shear. The validity of this assumption is questionable, and definitely warrants further study.

2.9 Seim, Charles, "Increasing the Redundancy of Steel Bridges" [10]

2.9.1 Summary

Seim recommends the use of high-strength steel cables in parallel arrangement with main tensile elements of a bridge to achieve redundancy. The concept is applied to welded plate girders and tied arch bridges. It is described as being highly economical, because the working stress level for these cables is approximately 160 ksi. Seim presents examples of several structures constructed in this manner. As a rule, the designs are such that the cables could carry the entire tensile load if the main member fractured.

Seim also presents the results of a computer study of a three-span continuous two-girder bridge with one fractured girder. The computer model is loaded with the structure's dead load plus one HS 20-44 truck load. The results indicate that two-girder bridges do, in fact, have a significant degree of redundancy. Seim suggests that the after-fracture redundancy of a two-girder bridge may be largely dependent on the torsional stiffness of the damaged structure.

Though he uses the term quite freely, Seim never provides a definition of redundancy.

2.9.2 Discussion

(1) Use of parallel cables to achieve redundancy is fundamentally different than the approaches used in all previous research. While the concept certainly has potential, Seim fails to discuss its most significant limitation: unless the parallel cables are slack, they are subjected to large stress ranges, just as the main load carrying members are. Thus they are also subject to fatigue damage and possible fracture. In short, parallel cables do not necessarily provide a "fail-safe" alternate load path.

(2) Emphasis on the torsional stiffness of fractured two-girder systems is noteworthy; further research in this area is warranted.

2.10 Lenox, T. A. and Kostem, C. N., "The Overloading Behavior of Damaged Steel Multigirder Bridges" [11]

2.10.1 Summary

Lenox and Kostem perform detailed linear and nonlinear finite element analyses of a simple-span six-girder bridge with a damaged exterior girder. The bridge configuration is taken from the Federal Highway Administration's Standard Plans for Highway Bridges. Girder damage is assumed to be a severed bottom flange at midspan. Analyses are performed for AASHTO HS 20-44 loading, 204 kip PennDOT Permit Vehicle loading, and 128 kip Dolly Vehicle loading. Deflections and stresses are computed and compared for the six girders. The influence of the composite deck in load redistribution is studied as well. Use of nonlinear analysis permits accurate determination of the extent of cracking and crushing in the deck slab, as well as yielding of steel components.

The authors conclude that steel multi-girder bridges have a large degree of redundancy. Their research indicates that, even under overload conditions, the effects of the damage are significant only on the side of the bridge corresponding to the fractured girder. On the opposite side of the longitudinal centerline, deflections and stresses are virtually unaffected by the damage. Lenox and Kostem also note the substantial roles of cross bracing and the deck slab in load redistribution.

2.10.2 Discussion

(1) Like the work of Daniels, Wilson, and Chen, this research is significant in that it performs sophisticated three-dimensional analyses of a realistic bridge configuration.

(2) The work of Lenox and Kostem is unique in its employment of nonlinear finite element analysis. In particular, by incorporating the nonlinear material properties of concrete, the authors are able to demonstrate the significant contribution of the deck slab to redundancy of composite multi-girder structural systems.

(3) Modeling the damaged girder with only the bottom flange severed is questionable, unless the authors intended to represent a riveted structure. In a welded girder, the fracture could be expected to penetrate at least partially through the web as well.

(4) The most significant value of this research is that it quantifies the very large degree of redundancy inherent in multi-girder bridges.

2.11 Daniels, J. H., Kim, W., and Wilson, J. L., "Guidelines for Redundancy Design and Rating of Two-Girder Steel Bridges" [12]

2.11.1 Summary

Daniels, Kim, and Wilson present a comprehensive system for redundancy design, rating, and retrofit of steel two-girder bridges. The research is based on three-dimensional finite element analyses and a thorough compilation of experience with after-fracture behavior of actual in-service bridges. It is shown that a fractured two-girder structure carries dead and live loads as a "pseudo space truss." Based on this conclusion, simple procedures for design and rating of two-girder bridges are presented. These procedures focus on the need for a "redundant bracing system", composed of properly configured diaphragms, and lateral bracing. The contribution of the deck slab in composite bridges is also accounted for. Equations and worked examples are provided for design of new and retrofitted redundant bracing systems. Guidelines include suggested design and rating loads, allowable stresses, load factors, serviceability criteria, probable fracture locations, strength of connections, and allowable fatigue stresses. Procedures for computer modeling are presented as well.

The definition of redundancy used in this study is a recommended modification of the AASHTO definition. *Redundant load path structures* are defined as "new, existing, or rehabilitated steel highway bridges where at least one alternate load path exists and is capable of safely supporting the specified dead and live loads and maintaining serviceability of the deck following fracture of a main load-carrying member."

2.11.2 Discussion

(1) This research is a comprehensive, practically-oriented study of after-fracture redundancy. It effectively combines the results of computer analyses, experience with in-service bridges, and previous research in a format oriented toward practicing engineers.

(2) A very significant aspect of this research is that it is organized, performed, and presented *within the context of the AASHTO Specification and AASHTO bridge rating procedures.*

(3) In many respects, the work of Daniels, Kim, and Wilson represents the current state-of-the-art in two-girder bridge redundancy research. However, the work is not without limitations, most of which are acknowledged in the report. These include the following:

- The authors recognize that the most appropriate limit state for evaluation of after-fracture redundancy is *serviceability of the deck*, rather than collapse; however, they do not fully address this limit state in the redundancy design and rating procedures presented in the report.

- The live loads and allowable stresses specified for redundancy rating are based solely on subjective judgement. Though they are well-defined and reasonable, they have no explicit rational basis.

- All discussion of redundant bracing systems is based on the assumption that the *second* girder will not fail when the first girder fractures. Likewise, it is assumed that a fracture will not occur in the redundant bracing system itself. These assumptions, though reasonable, ignore the possibility of significant accumulation of fatigue damage in the intact girder and bracing system.

- The authors do not fully address the effects of the girder fracture on the dynamic response of the bridge. Such effects would include a significant change in the natural frequency of the structure and enhanced impact effects due to increased deflection of the deck.

- The research does not attempt to establish the shear capacity of a fractured girder.

(4) It is important to note that the first four limitations listed above are concerned with quantities which are inherently variable and highly uncertain. Such uncertainty can only be rationally dealt with via probabilistic methods. Thus in many respects, the research of Daniels, Kim, and Wilson represents the practical limit of deterministic redundancy research.

3. PROBABILISTIC RESEARCH

3.1 Galambos, T. V., "Probabilistic Approaches to the Design of Steel Bridges" [13]

3.1.1 Summary

Galambos uses "first-order" probability theory to assess the reliability implicit in the 1977 AASHTO bridge design specification, for both allowable stress design (ASD) and Load Factor Design (LFD). He begins with a discussion of basic probabilistic concepts. To illustrate the application of those concepts, he computes the safety index, β, for a typical multi-stringer bridge. The limit state is assumed to be flexural capacity of the stringers. β is computed using both ASD and LFD provisions, as a function of the dead load-to-live load ratio. The results show clearly that the ASD safety index increases significantly as the dead load-to-live load ratio increases, while the LFD safety index remains virtually constant. It is concluded that the AASHTO LFD provisions result in bridge designs with more consistent reliability.

The paper concludes with a discussion of procedures for development of a probability-based bridge design code and summarizes the currently available statistical data base for resistance of various members and connections.

3.1.2 Discussion

(1) Galambos' paper does not deal directly with redundancy; however, many of his ideas are fully applicable to the subject. His discussion emphasizes the variability inherent in both loads and resistance in bridges. More importantly, he demonstrates that even very simple probability theory can be used to draw very powerful conclusions about structural reliability.

(2) Any probabilistic study of bridge redundancy would rely heavily upon experimentally derived statistics describing the resistance of various structural components and connections. Thus Galambos' compilation of the currently available data base provides a valuable reference.

3.2 Moses, F., "Probabilistic Approaches to Bridge Design Loads" [14]

3.2.1 Summary

Moses investigates the use of probabilistic concepts to define design loads for bridges. He discusses both railroad and highway loads, but emphasizes vehicle-induced loads for short and medium spans. Fatigue loading and maximum lifetime loading are considered separately.

Moses discusses the inherent difficulties in developing probabilistic models for highway loading. He notes that the load history of a highway bridge is characterized by millions of independent load applications, each with unique magnitude and configuration. These loads are affected by gross vehicle weight, axle spacing, load distribution, number of lanes, impact effects, and vehicle spacing. Moreover, traffic load spectra tend to vary with location, time of day, season, economic factors, government regulation, and enforcement. Moses discusses various attempts to model highway loads, ranging from purely analytical (e.g., stochastic processes) to semiempirical methods. He recommends the use of the *convolution approach* and *Monte Carlo simulation.*

The author illustrates a straightforward method for determining a safety index, β, for fatigue loading. He notes, however, that there are significant limitations involved in applying the same methods to the ultimate strength limit state. The problem is that maximum load magnitudes tend to increase with time, so that past history is not necessarily a good predictor of future loads. Furthermore, the statistical data base on extreme vehicle loads is very limited.

As an alternative, Moses suggests a system-reliability model for strength design of highway bridges. He notes that present design procedures focus on the behavior of individual elements. Recognizing the large uncertainty in the magnitude of maximum lifetime loads, he recommends that future specifications consider the behavior of structural systems beyond the load levels which cause failure of individual elements. He illustrates the concept through the use of *damage cost vs. load* curves. These curves are employed to demonstrate that redundant structures are better able to survive overload, because they incur damage at a lower rate than nonredundant structures. Moses concludes by demonstrating that damage cost vs. load curves can be operated on with probabilistic descriptions of load and resistance uncertainties to calibrate optimum load factors.

Moses does not explicitly define redundancy; he uses the term in referring to the capacity of a structure to carry additional load (i.e., overload) beyond the first yield of a single element.

3.2.2 Discussion

(1) Moses' research is concerned primarily with definition of bridge design loads; it is only peripherally concerned with redundancy. Furthermore, his discussion of redundancy is focused, not on after-fracture behavior, but on the reserve capacity of a structure under overload conditions. All girders are assumed to fail by yielding. Thus Moses' paper is further evidence of the general lack of agreement concerning a practical definition of redundancy and the limit state with which redundancy should be associated. Nonetheless, the paper is significant in that it demonstrates the general

potential for reliability-based study of redundancy.

(2) Moses' research in probabilistic modeling of highway loads would have significant applicability to probabilistic research in after-fracture redundancy. Probabilistically-defined loads, derived from measured traffic load spectra, would provide a rational basis for evaluation of after-fracture behavior.

(3) For application to a study of after-fracture redundancy, Moses' probabilistic load models would require some modification. Those models are intended to define the *maximum lifetime load* for a given bridge. Thus they are defined in terms of the design life of the bridge. Appropriate loads for evaluation of after-fracture redundancy would be defined in terms of a much shorter period of time; e.g., the elapsed time between occurrence of fracture and detection of the damage. Nonetheless, Moses' research shows that probabilistic methods can be used to rationally account for this form of time-dependence.

3.3 Gorman, M. R.,"Structural Redundancy" [15]

3.3.1 Summary

Gorman discusses the theoretical relationship between structural redundancy and system reliability. He defines structural redundancy as the degree of static indeterminacy, and notes that the degree of indeterminacy has two counteracting effects. In general, reliability of a structural system can be improved by increasing the number of elements that must fail for the structure to fail; however, increasing the number of elements also rapidly increases the number of possible failure modes. Gorman illustrates the relative effects of these two factors by examining a series of optimally designed trusses with varying degrees of indeterminacy. He concludes that increasing structural redundancy increases system reliability, but that reliability is only slightly improved for highly redundant structures.

3.3.2 Discussion

Gorman's paper is an interesting mathematical study, but has little application to a realistic investigation of after-fracture redundancy. The entire analysis is based on an assumption that redundancy is the same as static indeterminacy. As discussed in Section 2.1.2, this definition is too broad to be of practical use. The focus of Gorman's research is on the reserve capacity of an intact structure, rather than the ability of a damaged structure to redistribute applied loads.

3.4 Moses, F. and Ghosn, M., "Reliability-Based Design and Evaluation of Highway Bridges" [16]

3.4.1 Summary

The authors describe the problems inherent in managing the inspection, rehabilitation, and replacement of the nation's highway bridges. They note that, as time passes, highway loads applied to a given bridge tend to increase in magnitude while the resistance of the structure tends to deteriorate. The result is a continuing decrease in reliability. The authors present several applications of structural reliability theory which can be used to assess the risk levels of bridges over an extended life span.

Both fatigue and ultimate load are considered. The discussion of fatigue is essentially a summary of Moses' previous work in this area, as described in Reference 14. The discussion of ultimate load includes a new scheme for modeling extreme load events, which uses a *modified filtered Markov renewal process* to account for the presence of trucks in more than one lane.

The authors conclude with a discussion of redundancy and ductility in structures. *Damage vs. load* curves for hypothetical two- and five-girder bridges are used to illustrate the importance of redundancy. These curves are similar to the *damage cost vs. load* curves presented in Reference 14, except that a nondimensionalized damage index (0=no damage, 1=collapse) is used on the vertical axis. The curves are used as the basis for probabilistic computation of *expected damage* for each bridge; the significantly larger expected damage for the two-girder bridge is cited as evidence of the importance of redundancy.

As was the case in Reference 14, redundancy is not explicitly defined, but the term is used in reference to excess capacity of the structure beyond first yield.

3.4.2 Discussion

(1) Like Moses' earlier work, this research has little direct application to after-fracture redundancy because it assumes that all girders fail by yielding, under overload conditions.

(2) The use of expected damage as a measure of redundancy provides a convenient means of comparing different structural systems. With modification to account for different limit states, the concept may be useful in probabilistic studies of after-fracture redundancy.

3.5 Frangopol, D. M. and Curley, J. P., "Effects of Damage and Redundancy on Structural Reliability" [17]

3.5.1 Summary

Frangopol and Curley investigate the effects of damage and redundancy on the reliability of structural systems. The investigation is based on a definition of structural redundancy which includes both system reliability and damage assessment concepts. Practical applications are illustrated by means of analyses of bridges for different damage scenarios.

Initially, the authors equate redundancy with static indeterminacy; subsequently, they discuss the limitations of this definition and propose alternative measures of redundancy. Using a simple two-dimensional truss model as an example, they show that redundancy (referring to indeterminacy) is a poor measure of overall system strength. They demonstrate that the ultimate capacity of a structure is substantially influenced by the ultimate behavior (ductile or brittle) of its members, and by the sequence of member failure.

The authors introduce the concept of *structural damage*, defined as any strength deficiency introduced during design, construction, or subsequent service of the structure. Structural damage forms the basis for definition of a series of redundancy factors, which relate the collapse loads of a structure in the intact and damaged states. Use of these factors is demonstrated in a series of deterministic examples. It is concluded that the redundancy factors provide a realistic means to evaluate the overall system strength of a damaged structure. In particular, they are useful for identifying particular members which are critical to system performance.

The authors discuss the need to account for the random nature of loads and strengths. Probabilistic methods are proposed as a means of assessing the affect of structural damage on system reliability. A simple indeterminate plane truss is used to illustrate the concept. Applied load and all member strengths are defined as random variables with assumed means and variances. Structural geometry is assumed to be deterministic. Member and system reliabilities are computed as a function of mean applied load for various damage scenarios. It is concluded that system reliability methods provide a valuable means of examining the structural behavior of damaged structures beyond single element failure.

3.5.2 Discussion

(1) Frangopol and Curley define redundancy in terms of *the relationship between the strength of a damaged structure and the strength of the corresponding intact structure.* This definition is considerably more flexible than previous definitions which equated redundancy with static indeterminacy. This flexibility is due, in part, to the fact that Frangopol and Curley allow for both brittle and ductile failure of individual members. The authors' procedures are particularly applicable to evaluation of the degree of redundancy of existing (in-service) structures, because any form of observed structural damage (e.g., fabrication errors, fatigue damage, corrosion) can easily be accounted for.

(2) In many respects, Frangopol and Curley have demonstrated the powerful potential for application of probabilistic concepts to studies of redundancy. At the same time, their use of simplistic analytical models clearly suggests that probabilistic methods have yet to be applied to realistic three-dimensional structural geometries.

4. SUMMARY AND CONCLUSIONS

4.1 Summary

This report consists of a review of selected research pertaining to after-fracture redundancy in steel highway bridges. Deterministic and probabilistic research are discussed separately.

Deterministic research has shown significant, steady progress since 1979. Early efforts were primarily concerned with qualitative discussions of redundancy and with establishing a frame of reference for further research [2,3,4]. Initial quantitative studies consisted of computer analyses of simple, idealized structural models [5,6]. These were followed by a succession of increasingly sophisticated finite element analyses, several of which realistically modeled actual bridges [8,9,11]. The most successful studies have been performed by assuming a worst-case fracture scenario, then analyzing the response of the bridge at various levels of load [8,9,11,12]. The most consistent aspect of all these studies is the general recognition that after-fracture redundancy can only be evaluated via *analysis of the full three-dimensional structural system in the damaged condition.*

Deterministic research has demonstrated conclusively that multi-girder bridges possess a high degree of after-fracture redundancy, and that two-girder bridges can have substantial redundancy only if their secondary bracing systems are properly configured. Guidelines have been developed for design and rating of redundant bracing systems for two-girder bridges [12].

Probabilistic redundancy research has not progressed as rapidly as has deterministic research, in the same period of time. Published probabilistic research can be loosely grouped into two categories: probabilistic modeling of highway loads and reliability-based assessment of structural redundancy. Significant progress has been made in modeling of loads, primarily by Moses [14]. Reliability studies, however, are still lacking in practical application. Without exception, reliability-based procedures have only been applied to the analysis of simple, two-dimensional structural models [15,17]. Moreover, much of this research attempts to establish the reserve capacity of an intact bridge, under extreme loading conditions, beyond the first yielding of a member [16,17]. All members are assumed to fail in a ductile manner. While this research addresses an important aspect of bridge behavior, it does not address *after-fracture* redundancy. Of the probabilistic research reviewed herein, only Reference 17 considers the reliability of structures with members that are subject to brittle failure.

4.2 Conclusions

The following conclusions are drawn from the published research reviewed in this report:

(1) The term "redundancy" means many things to many people. Future redundancy research should establish a clear, precise definition.

(2) In certain respects, deterministic redundancy research cannot progress significantly beyond the current state-of-the-art. The reason is that bridge behavior (and redundancy in particular) is characterized by a significant degree of inherent variability. Appropriate highway loads and member resistance cannot be rationally determined via deterministic methods.

(3) Probabilistic methods offer the *potential* to deal rationally with these forms

of uncertainty. As of yet, that potential has not been fully realized. All of the probabilistic tools exist; they only need to be combined in a consistent, practically-oriented format.

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