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Innovative Applications of Robust Optimization for Long-Term Decision-Making

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Abstract

This dissertation investigates Operations Research (OR) models and methods in the field of health care financing with a focus on preventive care. Our first chapter focuses on Value-based Insurance Design, where insurers encourage positive behavior from patients by designing plan features appropriately, in particular with respect to cost-sharing. We present a Markov Chain framework to determine the optimal copayment in order to minimize the insurer's long-term cost, with uncertain disease occurrence. Our second chapter analyzes via simulation two mathematical modeling frameworks that reflect different managerial attitudes toward upside risk in the context of R&D portfolio selection. The manager seeks to allocate a development budget between low-risk, low-reward projects, called incremental projects, and high-risk, high-reward projects, called innovational projects. We study the differences in strategy and portfolios risk profile that arise between a risk-aware manager, who takes upside risk because he has to for the long-term competitive advantage of his company, and a risk-seeking manager, who will take as big a bet as allowed by the model. The third chapter studies hospitals optimal strategies of building community health program portfolio in order to achieve the maximum potential benefits under a worst case benefit tolerance level. Our model incorporates the fact that hospitals might have tolerances for upside and downside deviation and thus different uncertainty budgets for upside risk and downside risk and analyzes how key parameters influence the optimal portfolio and implement our approach in a numerical example with promising and insightful results. Contributions of this dissertation are:

Value-Based Insurance Design

- Our research investigates optimization models and Markov Chain models in the healthcare finance area. To the best of our knowledge, studies exists applying Operations Research (OR) models and methods in medical decision making, but they focus on medical decisions such as the optimal timing to start a certain treatment. We believe we are the first to incorporate Markov Chain in a health insurance setting.
- We analyze stochastic models of disease progression to estimate the cost of insurers and evaluate the impact of uncertain parameters.
- We quantify the benefit of offering some preventive care for free or at a discounted price while comparing the VBID plan with traditional plan and quantifying savings.

R&D Portfolio Management

- Traditional methods, such as Net Present Value (NPV), use the expectation to represent random variables and select projects. Such methods usually bias towards incremental projects, which can delay the innovational project and fail to achieve revenue goals.
- Our method incorporates the uncertainties and considers the best possible return of each project.
- To the best of our knowledge, we are the first to study the differences in strategy and portfolios risk profile that arise between different attitude towards risk: risk loving and risk tolerance.

Community Benefit Programs

- Most Community Benefit Programs studies focus on descriptive models, applying statistical tests and performing cost-effectiveness analysis. Our use of optimization methods provides a new quantitative tool to exam such programs with a fresh perspective.

- We incorporate robustness techniques to address possible inaccuracy estimation of parameters in the healthcare finance framework.
- We incorporates the fact that hospitals might have tolerances for upside and downside deviation and thus different uncertainty budgets for upside risk and downside risk and analyzes how key parameters influence the optimal portfolio.

Chapter 1

Introduction

The United States leads developed countries in healthcare spending, consuming approximately 18% of GDP. Levi et al. [1] suggested that the cost was more than 3 times higher than the cost in 1990 and more than 8 times higher than in 1980. High health care costs are hurt the economy in many different ways. Appleby et al. [2] estimated that in General Motors (GM) the cost of health care coverage to employees and retirees adds from $1,100 to $1,500 to the cost of per car production, more than the cost of steel in 2005. What's more, the cost of providing health care, according to GM, played a critical role in the decision to cut 25,000 jobs, which could affect up to 175,000 jobs in other areas of the economy.

However, the skyrocketing costs of health care are not associated with high quality care. Reid [3] concluded that the U.S. falls behind most of the world's developed countries in important rankings of access to and quality of medical care. One way to measure the quality of medical treatment is "avoidable mortality", i.e., how good the country is at treating the curable diseases. Nolte et al. [4] concluded that the U.S. was the worst of the rich countries in term of this measure. The number of people dying from curable illness before age 75 was almost twice as high in the U.S. as in countries like France and Japan. In terms of life expectancy, the U.S. again is at the bottom of 23 countries in a 2006 survey by the Commonwealth Fund [5].

It is widely agreed that preventive care is one of the most promising ways to solve the challenge of decreasing cost while improving people's life quality. Kids et al. [6] suggested that 10 percent of total hospital expenditure was spent on preventable hospitalizations. Levi [1] suggested that some of the most expensive conditions include uncomplicated diabetes and high blood pressure (9.4 percent of the U.S. healthcare cost), complicated diabetes and high blood pressure (16 percent of the U.S. healthcare cost), and uncomplicated heart disease (6.2 percent of the U.S. healthcare cost), and that significant numbers of cases of these diseases could be prevented or delayed.

However, Russell [7] and Cohen [8] pointed out that preventive care is not guaranteed to save money. Critical factors include how effective the preventive care is and the population targeted. Our study is to use robust optimization techniques to optimize the healthcare cost and increase people's life quality at the same time.

This dissertation investigates Operations Research (OR) models and methods in the field of health care financing. The first chapter focuses on Value-Based Insurance Design. In traditional insurance design, medical interventions have the same copayments regardless of their different values. The drawback is that patients, especially chronic disease patients, might not adhere to their medication due to financial burden. This will increase the risk of future adverse outcome, such as emergency visits. Value-Based Insurance Design(VBID) refers to a practice among insurers to encourage positive behavior from patients by designing plan features appropriately, in particular with respect to cost-sharing. The high-level aim of VBID is to optimize the trade-off between current costs with future costs and to improve patients life quality at the same time. We first consider the cost sharing of operations and then we consider the medication sharing. Our general assumption is that if patients don't take treatment at the early stage of their disease they will develop into a more serious condition and the medical cost will be higher. We try to decide the optimal cost sharing level so that we can balance off the current cost and the future cost while we improving patient's life quality. We also include sensitivity analysis so see how the key parameters

influence the optimal copayment level and robust technique to deal with the fact that the estimation of parameters may be inaccurate.

In the second chapter, we analyze via simulation two mathematical modeling frameworks that reflect different managerial attitudes toward upside risk in the context of R&D portfolio selection. The manager seeks to allocate a development budget between low-risk, low-reward projects, called incremental projects, and high-risk, high-reward projects, called innovational projects. Because of their highly uncertain nature and significant probability of failure, the expected value of the innovational projects is smaller than that of their incremental projects' counterpart, but the long-term financial health of a company necessitates to take risk in order to maintain growth. We study the differences in strategy and portfolio's risk profile that arise between a risk-aware manager, who takes upside risk because he has to for the long-term competitive advantage of his company, and a risk-seeking manager, who will take as big a bet as allowed by the model.

The third chapter focuses on community benefit programs. Most hospitals in United States are non-profit and federal tax exempt. In order to maintain their tax exempt status, these hospitals must contribute part of their revenue to benefit their communities. Most of the contributions are in the form of direct financial assistance, such as uncompensated healthcare service. If we can shift that amount of money upstream to invest in community-based activities that can prevent disease, we can improve people's health while mitigate the rise in healthcare cost. At the same time, as the Accountable Care Act expands the health insurance coverage, hospitals are able to focus on community benefit investments to reduce unnecessary use of emergency departments and hospital readmission. We create a model where each of the possible action costs money and is aligned with one of the hospitals non-monetary goals to serve the community, the hospital has a budget constraint, and we must decide which action items to select at which time period with unknown/uncertain payoffs that may materialize far into the future.

Chapter 2

Value-Based Insurance Design

In traditional insurance design, medical interventions have the same copayments regardless of their different values. The drawback is that patients, especially chronic disease patients, might not adhere to their medication due to financial burden. This will increase risk of future adverse outcome, such as emergency visits. Value-Based Insurance Design (VBID) refers to a practice among insurers to encourage positive behavior from patients by designing plan features appropriately, in particular with respect to cost-sharing. An example is providing some types of preventive care for free for some patients, for instance patients above a certain age. While in theory it also includes penalizing customers for negative behavior, such as smoking, this aspect has not been implemented by insurers in practice. The high-level aim of VBID is to optimize the trade-off between current costs with future costs and to improve patients' life quality at the same time.

2.1 Literature Review

2.1.1 Traditional Health Insurance

The issue of appropriate cost-sharing has been the focus of significant attention in the healthcare community, from payers, policy-makers and patient advocates alike. A widely

accepted notion is that higher cost sharing can help insurance companies control their costs by providing an incentive for patients to avoid unnecessary or wasteful use of services, and is counterbalanced by lower (more affordable) premiums; however, increases in deductibles, copayments, and coinsurance rates shift a greater part of the financial burden if treatment is needed directly from payers to enrollees for each covered service. Thus, higher cost-sharing may motivate certain patients to postpone receiving needed care, especially for slowly-progressing conditions such as heart disease or chronic illnesses. On the other hand, lower cost-sharing would negatively affect all premiums, including for healthy enrollees, and raise issues of affordability as well. We aim to investigate the potential of differentiated cost-sharing to incentivize patients who would otherwise be reluctant to receive early care due to the financial burden and to maintain the payer's financial stability.

The link between patients' financial cost and health resource usage was investigated in the 1970s in a landmark RAND study known as the Health Insurance Experiment (HIE), which was overseen by Dr. Joseph Newhouse [19, 18]. Specifically, the authors suggest that on average participants with cost sharing had one to two fewer doctor visits each year and 20 percent fewer hospital admissions than those with no financial contribution; participants with 25 percent coinsurance level reduced their spending by 20 percent compared with participants with free care, and those with 95 percent coinsurance reduced their spending by 30 percent. While the study was completed in 1982 and focused on the option of free, universal health care that was of greatest interest at the time, it provides a compelling motivation for the problem we study in this paper.

Many other researchers have attempted to quantify the link between cost-sharing and health resource utilization. Goldman reported that doubling copayments was associated with reductions in use of 8 therapeutic classes; patients with diabetes reduced their anti-diabetes drugs by 23 percent and patients with hypertension reduced their anti-hypertension

medication by 10 percent [20]. Mager and Zeber reached similar conclusions [21][22] . Tamblyn found that when cost sharing increased, elderly and poor people utilized less medication; reduced use of essential medication is associated with a 6.8 percent net increase in adverse events for elderly people and a 12.9 percent net increase for poor people [30]. According to Gibson, higher levels of cost sharing can be associated with treatment disruptions such as lower levels of adherence and thus can negatively affect the outcome of the care [24]. Both Hsu and Chandra found increased emergency room visits, hospital admissions and other negative health outcomes in response to higher cost sharing for physicians and prescription drugs [25] [26].

Chernew caution against using the HIE results to justify higher cost sharing. They argue that the negative effects of higher cost sharing are most pronounced in the case of chronic diseases, for which fewer effective treatment options were available in the 1970s when the HIE was ongoing [27]. Further, given the significant improvements observed in technology over the past decades, diseases that used to be considered acute or untreatable have now become chronic in nature; this leads to an increased adverse effect of higher cost sharing on health outcomes. The authors advocate for a value-based insurance design approach, which reduces the risk of adverse consequences linked to high cost-sharing by keeping copayments or coinsurance levels low for high-value healthcare services. The literature on these Value-Based Insurance Design approaches is presented in the next section.

2.1.2 Value-Based Insurance Design

Value-Based Insurance Design (VBID) was first introduced by Fendrick in 2001 [**?**]. The ultimate goal of VBID is to align the value of the medical services provided with patient incentives to encourage the use of high value services and discourage the use of low value services via appropriate cost-sharing. The logic behind VBID is that by encouraging chronic patients to take high-value medication, it is possible to limit complications and slow the progress of the disease, and thus decrease the need for any future more expensive treatments

such as emergency room visits and inpatient hospital stays. A reduced utilization of unnecessary and low value services will reduce the cost to payers as well and thus strengthen the long-term financial stability of the system. VBID thus has the potential to help decrease cost and improve healthcare outcomes.

Cost sharing vs. Adherence

The economic principle behind VBID is that patients will respond to the price change until their margin utility from using the service is the same as the price of the service. The utilization of medical services and adherence change in response to change in cost sharing has been the focus of several studies in the literature. Goldman et al. investigate the relationship between cost sharing and adherence for cholesterol-lowering drugs [28]. They concluded that as cost sharing goes up, compliance goes down. For high and medium risk patients, when cost-sharing levels increase from $10 to $20, the proportion of full compliance is projected to fall by 6 percent to 10 percent.

Similarly, Chernew et al. explored the impact of reduced cost sharing on five chronic medication classes and found that non-adherence to medications decreased for four of the five medication classes by 7-14 percent [27]. Schmittdiel et al. [38] studied the Medicare gap coverage and conclude that the gap coverage leads to lower total medication costs, higher out-of-pocket spending for patients and lower adherence rate. Maciejewski et al. [37] argued that even a modest cost-sharing increase can be important for medication adherence. They focus on populations of Veterans Afairs (VA) patients with diabetes or hypertension and study the effect of a $2 to $5 cost-sharing increase. They find that depending on the condition and medication, long-term adherence rate declines by 1.9 percent to 10.3 percent. Lee et al. [29] reviewed thirteen studies on VBID and concluded that lower cost sharing is consistently associated with improved adherence, on average of 3.0 percent over one year, in response to cost-sharing reductions that range from 0.5 percent to almost 10 percent.

Cost sharing vs. Emergency Visits and Hospitalizations

A key motivation to decrease cost sharing in order to increase patients' compliance is to decrease the need for future more expensive medical services such as emergency visits and hospitalizations. Goldman et al. [28] showed that compliance is negatively correlated with the use of expensive medical services. The trend is most dramatic for patients with high risk: for every 1000 high-risk patients, those with full compliance have 357 fewer hospitalizations and 168 emergency visits than those not in full compliance. The authors then simulate a policy that would eliminate cost-sharing for high and medium risk patients and increase them from $10 to $22 for low-risk patients, and find that this would result in 80,000 to 90,000 reductions in the number of hospitalizations and 30,000 to 35,000 reductions in the number of emergency visits, leading to a net aggregate savings of more than $1 billion. Tamblyn et al. [30] investigated the effect of cost-sharing for prescription drugs for the elderly and low-income population. They use a 10-month pre-policy control to estimate the impact of cost-sharing on emergency visits and find that emergency visits increase by 14.2 per 10,000 people per month for elderly people and by 54.2 for welfare recipients. These increases were primarily a result of the decrease in the use of essential drugs; however, reductions in the use of less essential drugs are not significantly associated with an increase in adverse events. Fendrick et al. [**?**] showed that for patients with asthma, diabetes or gastric disorders, doubling the cost-sharing leads to a 17 percent increase in emergency visits and a 10 percent increase in hospitalizations. Karaca et al. [42] found that higher cost sharing for asthma medications is associated with a 42 percent increase in asthma hospitalization among children aged 5 years or older.

Cost sharing vs. Total Cost

Lee et al. [29] found that the VBID policies they reviewed are associated with increases in drug spending, ranging from 0.2 percent for diabetes, hypertension, and asthma at Novartis to 61 percent for diabetes medications for the state of Colorado. Sokol et al. [39] examined the effect of adherence level on healthcare cost. The authors conclude that for diabetes

and hypercholesterolemia, improved medication adherence can indeed lower disease-related medical costs. They also find that for diabetes, hypercholesterolemia, and hypertension, higher medication costs are offset by medical cost reductions. Since lowering the cost-sharing improves adherence, this means that lower cost-sharing will lead to lower total expenditures for diabetes and hypercholesterolemia. Rosen et al. [40], in a study of the cost-effectiveness of full Medicare coverage, concluded that Medicare first-dollar coverage of ACE inhibitors for diabetes patients appears to reduce Medicare program costs. Gibson et al.[41] found that participation in both VBID and disease management produces a return of $1.33 for every dollar spent during a three-year period. Karaca et al.[42] found that in a disease management program, decreased cost-sharing for asthma medication results in higher drug spending but asthma-related total medical spending was neutral. Maciejewski et al. [43] examined the VBID program of North Carolina Blue Cross Blue Shield, which eliminates cost sharing for generic medications and reduces cost sharing for brand-name medications. They find that the medication expenditures increased by $6.4 million and non-medication expenditures decreased by $5.7 million. Chernew et al. [44], in an analysis of the fiscal consequences of VBID, found that the program they study was cost-neutral: the use of nondrug health care services was reduced, offsetting the cost associated with increased drug spending.

Heart Disease Treatment

Heart disease is the leading cause of death for both men and women. Every year, heart disease kills about 600,000 people in the United States, which represents 1 in every 4 deaths, and coronary heart disease is the most common type of heart disease, killing nearly 380,000 people annually [49]. There are 720,000 heart attacks in the United States each year, of which 515,000 are a first heart attack for the patient [50]. Risk factors for heart disease and stroke include high blood pressure, high cholesterol and diabetes, among others. While many of these factors may be asymptomatic, most of them are preventable and controllable.

Controlling these risk factors could reduce the risk of heart attack or stroke by more than 80 percent.

Statin is considered an effective medicine to reduce the risk of heart attack. Shepherd et al.[54] found that, in a 4.9 year period, statin lowered low-density lipoprotein cholesterol levels by 26 percent and plasma cholesterol levels by 20 percent. They randomly assigned 6595 men to receive statin or placebo. There were 248 definite coronary events in the placebo group and 174 in the statin group. Deaths from coronary heart disease was reduced by 28 percent. Downs et al.[55], after randomly assigned 5608 men and 997 women to statin treatment or placebo, found that after 5.2 years, the statin group had lower incidence of first acute major coronary events (118 vs 116). According to Simes et al. [53], a statin-based treatment could reduce the all-cause mortality and coronary mortality in patients with and without a history of coronary heart disease. All-cause mortality among patients assigned to the statin group was 7.9 percent compared with 9.8 percent among those assigned to the placebo group. Compared to the control group, coronary mortality decreased by 24 percent in patients receiving the statin treatment. Sokol et al. [39] showed that adhering to statin treatment could reduce hospitalization risk by 12 percent, regardless of patients' risk.

Statin adherence rate is an issue in successfully treating patients. Jackevicius et al. [52] analyzed patients' (aged 66 or older) two-year adherence rate to statins following their first statin prescription. All the medication costs are covered except for a small co-payment per prescription. The authors find that the two-year adherence rates following statin initiation are only 40.1 percent for patients with acute coronary syndrome, 36.1 percent for those with chronic coronary artery disease, and 25.4 percent for primary prevention. According to Benner [58], on average, the proportion of days covered by a statin was 79 percent in the first 3 months of treatment, 56 percent in the second quarter and 42 percent after 120 months; only 25 percent of patients maintained a Proportion of Days Covered of at least 80 percent after 5 years. Schultz et al. [56] showed that a higher prescription cost-sharing is correlated with lower statin adherence rate. Goldman et al.[28] estimated that the proportion of full

compliance will fall by 6 percent to 10 percent for high and medium risk patients when cost-sharing increases from $10 to $20. Ellis et al.[57] find that fifty percent of patients whose average monthly statin cost-sharing was less than $10 discontinued treatment (stopped refilling their medication) by the end of the follow-up period (3.9 years), while fifty percent of those who paid between $10 and $20 or strictly more than $20, respectively, discontinued by 2.2 and 1.0 years.

2.2 Towards Optimizing Co-insurance for Heart Disease Treatment

In this paper, we present a Markov Chain approach to model disease progression and ultimately to minimize the overall total cost to the payer. Markov Chains have been used in multiple studies on medical decision making but to the best of our knowledge have not been implemented in the context of VBID. Maillart et al. [34] deployed an observable Markov Chain Model to compare various breast cancer screening policies and identified efficient policies. Shechter et al. [33] considered the optimal timing of HIV therapy with an infinite-horizon MDP model, when the objective is to maximize quality-adjusted life years (QALYs) over the patient's lifetime. Denton et al. [35] presented a Markov Chain model to find the optimal time to initiate statin for type 2 diabetes patients in order to prevent cardiovascular events. The objective is to maximize the monetary rewards for QALYs minus costs of statin treatment and cardiovascular events and the decision is revisited each year. Further, Denton et al. [36] proposed a Markov Decision process to improve diabetes patients' adherence to medication. The decision variable is the timing to perform adherence-improving interventions. The objective is a combination of maximizing patient's quality-adjusted time to the first adverse health event and minimizing costs of treatment.

Our model contributes to the literature in the following ways:

- Our model focuses on the healthcare financing area, instead of medical operations.

The decision variables are patients' coinsurance levels, i.e., the percentage of the medication cost that they have to pay (the health payer will pay the remainder), and the objective is the total cost from the payer's standpoint.

- We incorporate the fact that people with different risk levels have different probabilities to develop negative events and thus assign its own set of decision variables to each group.
- We discuss how to approach uncertainty on future payoffs and patients' price response function using simple robust optimization techniques and sensitivity analysis.
- We argue that Markov Chains provide an important tool to better understand and design incentives for high-risk heart disease patients through appropriate cost sharing.

We design and analyze two models based on Markov Chains: the first one (the "traditional model") does not consider the risk category of the patients when they enter the system while the second model (the "VBID model") incorporates the fact that patients have different risk levels and thus different transition probabilities between medical states. The traditional model serves as a benchmark to show the advantages and disadvantages of VBID.

2.2.1 Traditional Insurance Design

Disease progression in our simplified model can be described as follows. The patient is diagnosed with being at risk for a heart attack and is prescribed medication. Medication adherence depends on the coinsurance level, denoted x, and other factors, so that medication adherence is never a perfect 100% even if the medication is free. Let $a(x)$ be the proportion of patients that will be adherent, i.e., follow the doctor's prescription and take their medication.

We introduce the following notation. Per time period:

- If patients take their medication, then only a fraction t_1 will indeed have a heart attack in the future, a fraction s_1 will remain at their current "at risk" health status and the remaining fraction $1 - t_1 - s_1$ will have a natural death.

- If patients are not adherent to their medication, then a fraction t_2 will have a heart attack in the future, a fraction s_2 will remain at their current "at risk" health status and the remaining fraction $1 - t_2 - s_2$ will have a natural death.

- If patients have a heart attack, a fraction d of them will decease and $1 - d$ of them will survive and be at risk for another heart attack.

We assume $t_1 < t_2$ (adherence makes a heart attack less likely) and $t_1 + s_1 > t_2 + s_2$ (adherence makes death less likely).

In our simplified model, we model adherence as a yes/no state, although more complex models could incorporate the extent of medication adherence through the proportion of days covered. A more refined model of adherence could for instance include states such as very adherent, moderately adherent, somewhat adherent and non-adherent. While such a model would better capture various degrees of adherence, we feel that a yes/no adherence model is sufficient at this point to investigate the potential of risk-dependent coinsurance levels. Further, we assume that a patient's adherence is static, i.e., does not change over time. In other words, an adherent patient always remains adherent and a non-adherent patient always remains non-adherent. All the patients prescribed medication against heart attacks start in the "at risk" state.

This model of disease progression is represented in Markov Chain format in Figure 2.1.

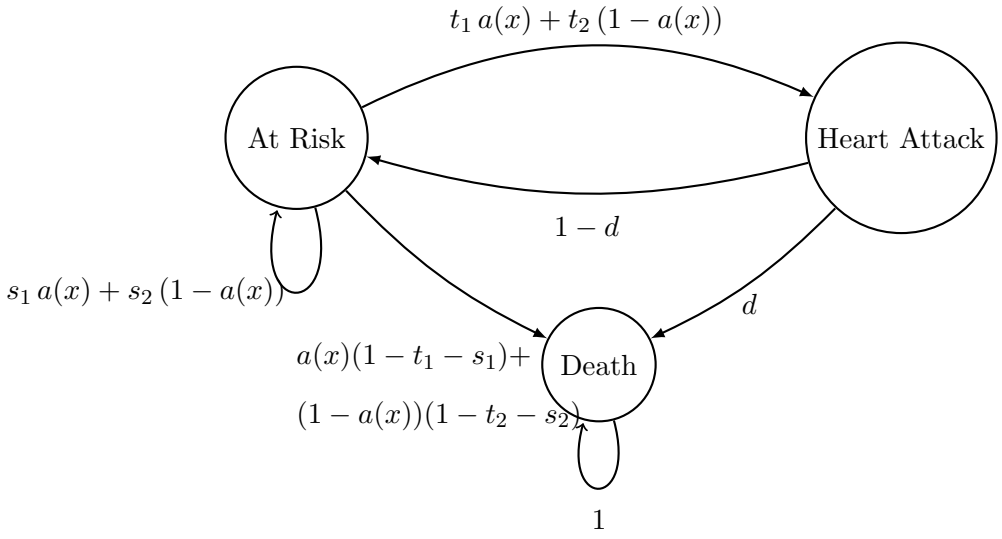


Figure 2.1: Markov Chain of entire-population case.

Our goal is to minimize the steady-state cost to the payer per time period. We assume that patients have at most one heart attack per time period. In addition to the notations above, let us define the following parameters:

c_1**:** the total (patient + payer) drug cost per patient per time period,

c_2**:** the total cost of a heart-attack-related hospital stay per patient per time period,

p**:** the fraction of the cost hospital stay paid by the patient,

r**:** the number, in millions, of people diagnosed with being at risk of a heart attack per time period,

R**:** the number of people at risk of a heart attack in steady state per time period,

T**:** the number of people having a heart attack per time period,

A**:** the penalty factor for a fatal heart attack.

We assume that patients who have a heart attack all go to the hospital and incur the same treatment during their hospital stay, and that non-adherent patients do not take their

medicine at all. The fact that non-adherent patients might take a small amount of medicine and the adherent patients may not be completely adherent can easily be incorporated by redefining the adherence function. For instance, if non-adherent patients take a fraction ϵ of the medication that adherent patients take, then the new adherence function should be redefined as $a(x) := (1-x)\, c_1 \cdot R\,[\epsilon + (1-\epsilon)\, a(x)]$ for the results below to remain valid.

Our objective function is formulated as:

$$\min_x (1-x)\, c_1\, R\, a(x) + (1-p)\, c_2 T + ADT$$

Lemma 2.1 (Payer's Cost in Entire-Population Model) *In the Markov Chain model for the entire population of at-risk patients, the payer's problem can be written as:*

$$\min_{0\le x\le 1} \frac{(1-x)c_1 r\, a(x) + (1-p)c_2\, r\,[t_2 - a(x)(t_2-t_1)]}{1-[s_2 + (1-d)\,[t_2 - a(x)(t_2-t_1)] + a(x)(s_1-s_2)]}.$$

Proof. See appendix.

While the objective function is quiet complex, the minimization problem is only over one variable belonging to a bounded interval and thus can be approached by discretizing the feasible set(interval) and evaluating the objective function at each point. This is particularly true in this health insurance application where practical values of the coinsurance level take only a few discrete values in 5% steps between 0 and 100%.

2.2.2 Value Based Insurance Design Model

Next we consider the fact that different people have different risk of developing heart disease and different transition probabilities between states, and thus the same medicine has different different values to different people. We group people into high risk and low risk and each group has its own cost-sharing level, which is the core idea of Value Based Insurance Design.

Using notations similar to those presented earlier, we assume that a fraction $a^l(x^l)$ of the low risk people will adhere to their medication and $1 - a^l(x^l)$ of them will not, where $a^l(x^l)$ is a function of the cost-sharing level x^l. If patients take their medication, in the next period s_1^l of them will remain low risk, h_1^l of them will become high risk and n_1^l of them will have natural death. If patients do not take their medication, s_2^l of them will remain low risk, h_2^l of them will transit to high risk, t_2^l of them will have a heart attack and n_2^l of them will have natural death. $a^h(x^h)$ of the high risk patients will adhere to their medication and $1 - a^h(x^h)$ of them will not, where $a^h(x^h)$ is a function of the cost-sharing level x^h. If patients take their medication, s_1^h of them will remain as high risk, t_1^h of them will have a heart attack and n_1^h of them will have natural death. If patients do not take their medication, s_2^h of them will remain as high risk, t_2^h of them will have a heart attack and n_2^h of them will have a natural death. This model of disease progression is represented in Markov Chain format in Figure 2.2. Then we can minimize the total average cost in the

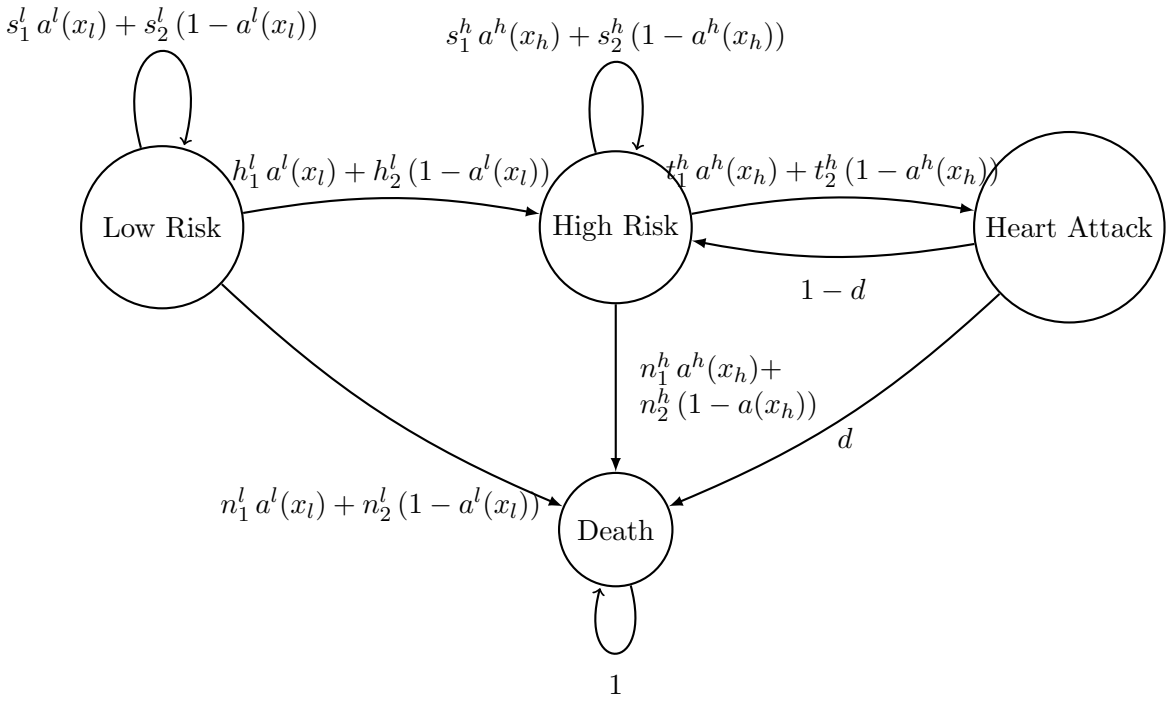


Figure 2.2: Markov Chain of entire-population case.

long run as

$$\min_{\substack{0\leq x^l\leq 1\\ 0\leq x^h\leq 1}} (1-x^l)c_1^l L(x_l,x_h)a^l(x^l)+(1-x^h)c_1^h H(x_l,x_h)a^h(x^h)+(1-p)c_2T(x_l,x_h)+AdT(x_l,x_h)$$

where

c_1^l: the medication cost of low risk people each period

$L(x_l,x_h)$: the number of low risk people in constant state

c_1^h: the medication cost of high risk people

$H(x_l,x_h)$: the number of high risk people in constant state

c_2: the heart attack cost

p: the proportion of heart attack cost paid by the patient

$T(x_l,x_h)$: the number of heart attack in constant state

A: the penalty factor for a fatal heart attack.

Again, while the objective function is quiet complex, the minimization problem discretizing the feasible set $[0,1]*[0,1]$ at reasonable values of the coinsurance levels.

2.2.3 Numerical Experiments

In this section, we illustrate how our model can be applied in practice, using parameter values drawn from the literature review as shown in in Table 2.1.

Grohol (2004) states that:

- 76.2 percent of those patients whose out-of-pocket prescription cost was $20 or more for a month's worth of statin drugs were non-adherent, compared with 49.4 percent of patients whose monthly prescription co-pay was less than $10.

Table 2.1: Parameter Estimation of VBID

Parameters	Estimation	Source	Parameters	Estimate	Source
h_1^l	6.9%	[54]	t_1^h	10.20%	[61]
n_1^l	2%	[80]	n_1^h	2%	[80]
s_1^l	91.10%	(1-5.3%-6.9%-2%)	s_1^h	87.80%	(1-10.2%-2%)
h_2^l	9.9%	[54]	t_2^h	13.20%	[21]
n_2^l	2%	[80]	n_2^h	2%	[80]
s_2^l	88.10%	(1-7.6%-2.3%-2%)	s_2^h	84.80%	(1-13.2%-2%)
c_2	30,000	[43]	c_1^l	432	[42]
d	0.143	[43]	c_1^h	804	[42]

- Response function is the same for both high risk patients and low risk patients.

Thus we can estimate the adherent function as

$$a^l(x^l) = a^h(x^h) = \frac{1}{1 + e^{-0.66+0.0055x^l}}$$

We follow Shepherd [54] for our assumptions on the enrollee pool. Specifically, we assume that a health insurance company has 6595 low risk members and 4159 high risk members. Every year the insurance company adds to its insurance pool 5% of its current population, 330 low risk members and 208 high risk members. If the company uses a traditional insurance model and does not classify their members as high risk and low risk then they will rely on the weighted-average estimates of the parameters for their whole population as shown in Table 2.2.

Table 2.2: Parameter Estimation of Traditional Insurance Design

Parameters	Estimate	Parameters	Estimation
t_1	3.94%	t_2	5.10%
n_1	2%	n_2	2%
s_1	94%	s_2	92.90%
c_1	575.87	c_2	30,000
d	0.143		

The estimated adherence function remains:

$$a(x) = \frac{1}{1 + e^{-0.66+0.0055x}}$$

In our example, after calculating the constant states with Matlab, the optimal cost-sharing level of traditional insurance design is 100, which means patients' needs to pay their medication in full amount, and the optimal cost is 41.5 million dollars. For the VBID insurance design, the optimal cost sharing level for low risk enrollees is 61% and for high risk enrollees is 78%, meaning that low-risk people shoulder a lower relative burden of the medication cost. This suggests that it is more crucial to prevent people from getting sick at the beginning. The optimal cost of VBID design is 38.9 million dollars, which is 6.7% lower than the optimal cost of traditional insurance design. Note that traditional insurance design always gives a higher optimal cost sharing level than VBID, this is because traditional insurance design does not have an accurate estimate of members' possibility to transfer between states when proportions of high risk and low risk members change in each state.

Sensitivity Analysis

In this section, we perform sensitivity analysis on the following factors:

- The cost of adverse outcomes
- Medicine effectiveness at preventing adverse outcomes
- Consumers' responsiveness to lower copayments

We look at how these factors influence VBID cost savings, which is the cost difference between VBID and traditional insurance design when given the same parameters, and the optimal cost sharing levels.

The cost of adverse outcomes

Figure 2.3 and Figure 2.4 show how the cost of adverse outcome influence VBID cost savings and optimal copayment, respectively. As the heart attack cost increases from zero, VBID

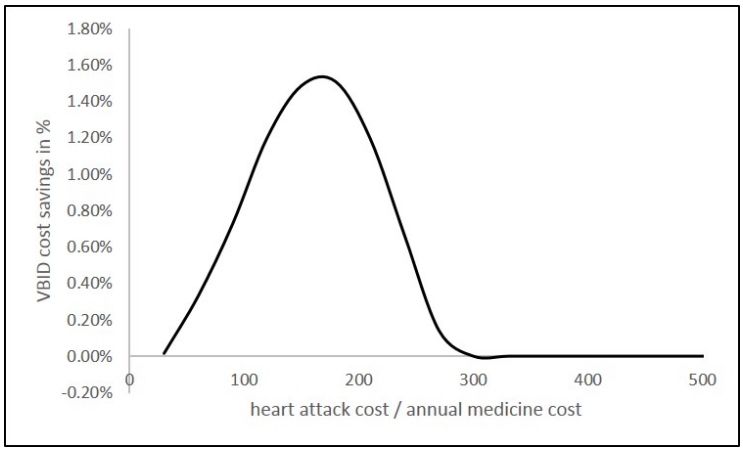


Figure 2.3: Sensitivity Analysis of Heart Attack Cost on VBID Cost Savings

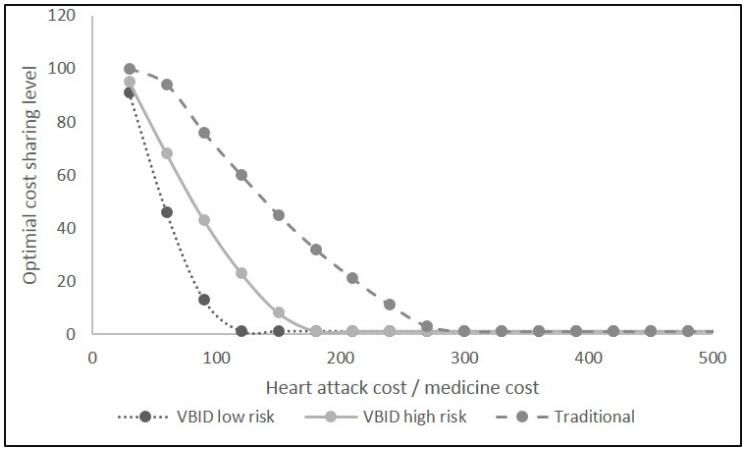


Figure 2.4: Sensitivity Analysis of Heart Attack Cost on VBID Optimal Cost Sharing

cost savings increase too. This is because a higher heart attack cost represents a more severe consequence of not taking medication as prescribed. Since VBID insurance design separates patients as high risk and low risk, and thus relies on more accurate estimates of the transition probabilities between each state and the consequence of non-adherence, it

generates more savings when the heart attack cost begins to increase, at which point traditional insurance design does not emphasize the consequence of non-adherence. However, when the heart attack cost is very high and the adverse outcome has a large impact on the total cost, the traditional insurance model will also encourage patients to take their medication with a low cost sharing level. At this point, it is best to reduce heart attacks as much as possible and thus both high risk patients and low risk patients will have a zero optimal copayment in the VBID model. This explains why VBID cost savings have a negative relationship with the heart attack cost after the heart cost exceeds 180 times the medicine cost and why VBID cost savings become zero after the heart attack cost exceeds 300 times the medicine cost.

Medicine effectiveness at preventing adverse outcome

In our example, medicine effectiveness means how effective the medicine is at reducing the

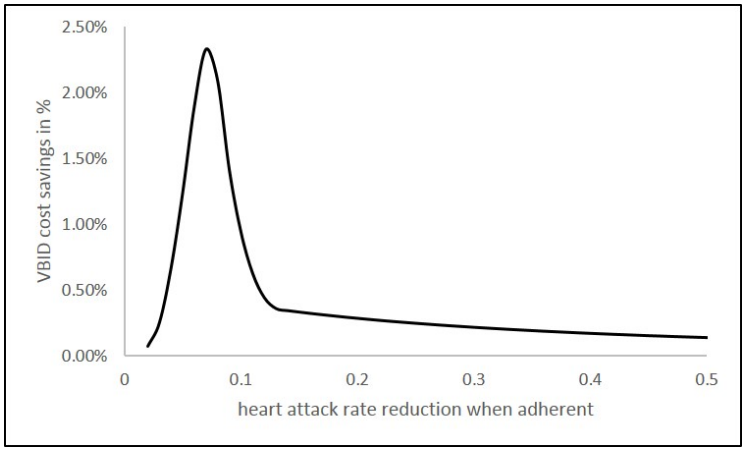


Figure 2.5: Sensitivity Analysis of Medicine Effectiveness on VBID Cost Savings

heart attack rate among high risk patients and is measured by the difference in likelihood of having a heart attack of high-risk adherent and high-risk non-adherent patients. Similar to the heart attack cost, medicine effectiveness has a positive relationship with VBID cost savings at the beginning and then has a negative impact on VBID cost savings. When

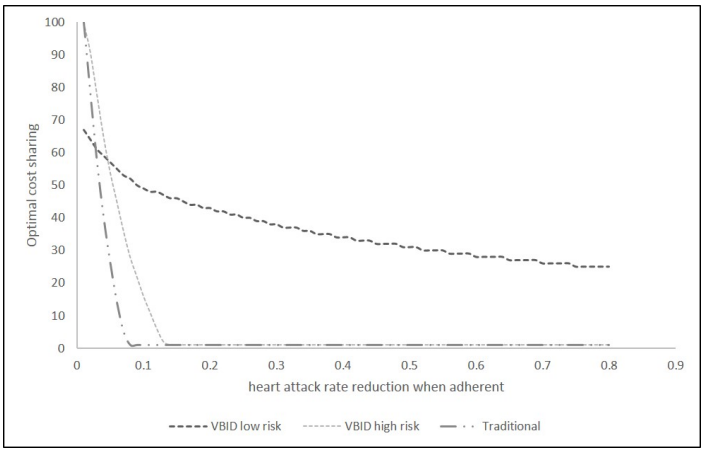


Figure 2.6: Sensitivity Analysis of Medicine Effectiveness on VBID Optimal Cost Sharing

the medicine has no effect on reducing heart attack rate for high risk patients, the optimal cost sharing level for high risk patient is 100%, meaning that high risk patients' need to pay the zero-value medicine in full if they want to purchase it. Since the medicine still can help prevent low risk patients from having their condition worsen and thus turning into high risk patients, the optimal cost sharing level for low risk patients is around 70%. At this point, the optimal cost sharing level in traditional insurance design is 100% and the VBID cost sharing level is very low around 0.07%. As the medicine becomes more effective, both optimal cost sharing levels for high risk and low risk patients decrease and VBID optimal high risk cost sharing level decreases much faster than the VBID optimal low risk cost sharing level, which makes sense since the increasing benefit is for high risk patients. At the same time, the optimal cost sharing level in the traditional model decreases and reaches zero first, compared to all the other optimal cost sharing levels. This is because the traditional insurance model does not differentiate between high risk and low risk patients and does not have an accurate estimate of transition probabilities between states. VBID cost savings reach a maximum of 2.33% when the medication reduces heart attacks by 7% among high risk patients. At this point, the optimal high risk cost sharing level is 36% in VBID and low risk 53% in VBID, while the traditional optimal cost sharing level is 5%.

After that, as the medicine become more effective at preventing heart attacks, traditional optimal cost sharing drops to zero first and then VBID high risk optimal copayment drop to zero, as VBID low risk optimal slowly approaching zero, which results in a reduction in VBID cost saving.

Consumers' responsiveness to lower copayments

As indicated in Figure 2.7 and Figure 2.8, patients' sensitivity towards copayment is mod-

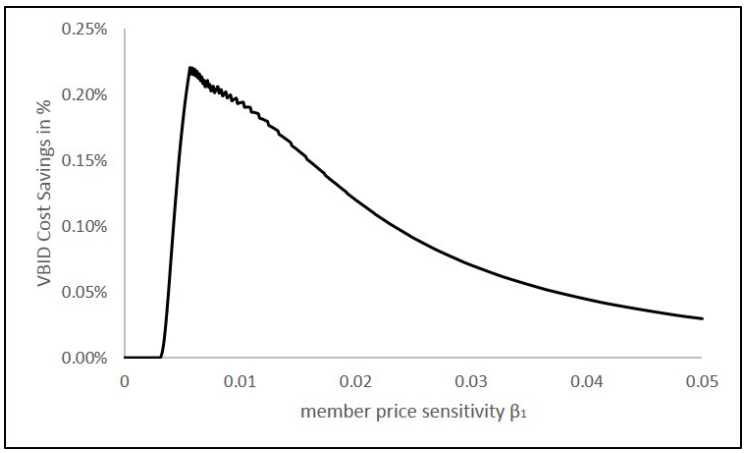


Figure 2.7: Sensitivity Analysis of Price Responsiveness on VBID Cost Savings

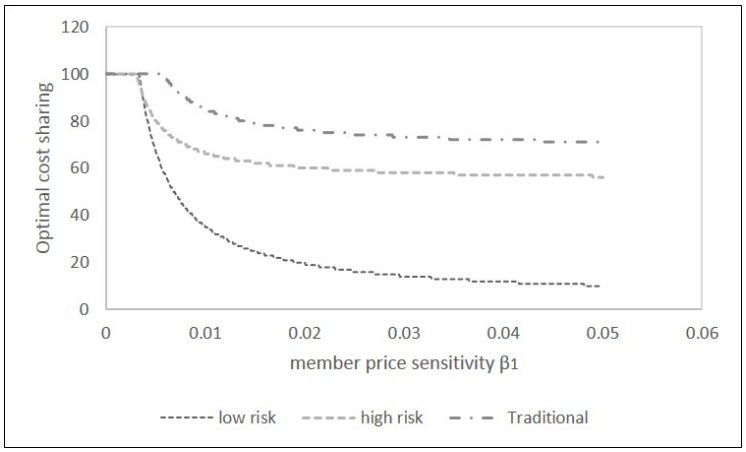


Figure 2.8: Sensitivity Analysis of Price Responsiveness on VBID Optimal Cost Sharing

eled by β_1 in the adherent function $a(x) = \frac{1}{1+e^{\beta_0+\beta_1 x}}$. When consumers are not sensitive to copayment change, there is no point in decreasing copayment since low copayment will not improve the medication adherence rate. Hence both the VBID model and traditional model give an optimal copayment of 100% and the VBID cost saving is zero. As patients become more sensitive to price change, optimal cost sharing levels in the VBID model start to decrease first while the traditional insurance model still advocates a 100 optimal cost sharing level. At this stage, VBID cost savings increase sharply. Then the traditional optimal copayment starts to decrease too and VBID cost savings decreases at the same time. Therefore, if the payer estimates that β_1 is less than 0.02, he should adopt VBID model as the VBID cost savings are large.

2.2.4 Basic Model

We consider the medication copayment as decision variables to minimize the insurer's cost. We assume that patient either adheres to medicine now and pays co-insurance rate x, or nonadherent to their medication and develops a more severe disease later. At that time, he will require a surgery that is A times as expensive as the medication cost in present value. For the surgery cost, the patient pays at copayment rate a, which is a constant. Let $\pi(x)$ be the percentage of patients following their doctor's instructions and adherent to the medication when copayment rate is x. Assume the logit model function for π as

$$\ln\left(\frac{\pi(x)}{1-\pi(x)}\right) = \alpha - \beta x$$

which gives

$$\pi(x) = \frac{e^{\alpha-\beta x}}{1+e^{\alpha-\beta x}}$$

Specifically, we must have $\beta > 0$, for the $\pi(x)$ to be a decreasing function of price. The higher beta is, the more price-responsive the patients are. The objective function is given

as

$$\min c\pi(x)(1-x) + cA(1-a)(1-\pi(x))$$

where $0 \leq x \leq 1$ and c is medication cost at early stage. Obviously, minimum is achieved when derivative of x is 0. Set the derivative to 0 we have

$$\beta[A(1-a)-1]-1 = -\beta x + e^{\alpha-\beta x}$$

Therefore, we can estimate x once we get α and β. α and β can be obtained by estimating the probability of medication adherence.

Lemma 2.2 (α and β Estimation) *α and β are given as*

$$\alpha = \ln\left(\frac{\pi(0)}{1-\pi(0)}\right)$$
$$\beta = \ln\left(\frac{\pi(0)(1-\pi(1))}{\pi(1)(1-\pi(0))}\right)$$

where $\pi(0)$ and $\pi(1)$ are the proportion of people adhering to medication when copayment level is 0 and 1 respectively.

Proof. Since $\ln\left(\frac{\pi(x)}{1-\pi(x)}\right) = \alpha - \beta x$, when $x = 0$ we have

$$\ln\left(\frac{\pi(0)}{1-\pi(0)}\right) = \alpha$$

Note that because of side effects, disease stigma etc, drug adherence is never 100% even when the medication is completely free to patients. Similarly, when $x = 1$ we have

$$\ln\left(\frac{\pi(1)}{1-\pi(1)}\right) = \alpha - \beta$$

then,

$$\beta = \ln\left(\frac{\pi(0)(1-\pi(1))}{\pi(1)(1-\pi(0)}\right)$$

□

Sensitivity Analysis

Since we need to estimate parameters $\pi(1)$, $\pi(0)$ and A, we would like to know how accurate we need to be. We do so by analyzing how the estimators affect the optimal copayment level. There is a bottoming-out relationship between $\pi(0)$ and optimal $x^*(\pi(0))$. In other words, the minimum of optimal copayment is attained for $\pi(0) \in (0,1)$ as shown in Figure 2.11.

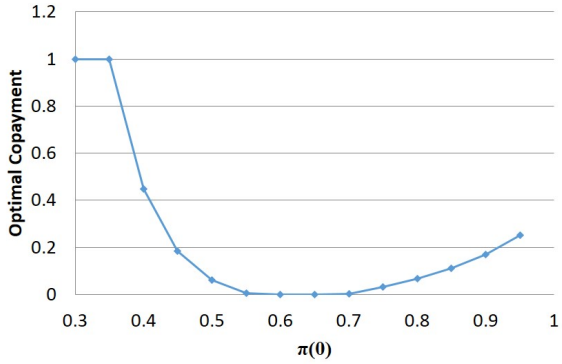


Figure 2.9: Bottoming-out of $x^*(\pi(0))$

Lemma 2.3 (Bottoming-out of $x^*(\pi(0))$) *Let $\pi^c(0)$ and $\pi^c(1)$ be such that*

$$\ln\left(\frac{\pi^c(0)}{1-\pi^c(0)}\right) = \ln[A(1-a)-1]$$
$$\frac{\pi^c(1)}{1-\pi^c(1)} = A(1-a)-1$$

If $\pi(0) > \pi^c(0)$, we observe the bottoming-out in the relationship between x^* and $\pi(0)$, or say $x^*(\pi(0))$, otherwise we don't. If $\pi(1) < \pi^c(1)$, we observe the bottoming-out of $x^*(\pi(0))$, otherwise we don't.

Proof. Taking derivative of the cost function with respect to x and setting the derivative to 0 we have

$$\beta[A(1-a)-1]-1 = -\beta x^* + e^{\alpha-\beta x^*}$$

Plugging in $\ln\left(\frac{\pi(0)}{1-\pi(0)}\right) = \alpha$ and $\ln\left(\frac{\pi(1)}{1-\pi(1)}\right) = \alpha - \beta$, we have

$$x^* = \frac{\ln\left(\frac{\pi(0)}{1-\pi(0)}\right) - \ln[A(1-a)-1]}{\ln\left(\frac{\pi(0)}{1-\pi(0)}\right) - \ln\left(\frac{\pi(1)}{1-\pi(1)}\right)} \tag{2.1}$$

See Appendix A for detail. Obviously, if $A(1-a)-1 < \frac{\pi(1)}{1-\pi(1)}$, then $x^* > 1$; if $\ln\left(\frac{\pi(0)}{1-\pi(0)}\right) < \ln[A(1-a)-1]$, then $x^* < 0$. Hence for these two situations there is no bottoming-out since $0 < x < 1$.

Therefore, the critical $\pi(0)$ will be $\pi^c(0)$ such that

$$\ln\left(\frac{\pi^c(0)}{1-\pi^c(0)}\right) = \ln[A(1-a)-1]$$

If $\pi(0) > \pi^c(0)$, we observe the bottoming-out in the relationship between x^* and $\pi(0)$, or say $x^*(\pi(0))$, otherwise we don't. The critical $\pi(1)$ will be $\pi^c(1)$ such that

$$\frac{\pi^c(1)}{1-\pi^c(1)} = A(1-a)-1$$

If $\pi(1) < \pi^c(1)$, we observe the bottoming-out of $x^*(\pi(1))$, otherwise we don't.

□

If the true $\pi(0)$ is around the bottom point of $x^*(\pi(0))$, then the optimal x is not sensitive to $\pi(0)$ and therefore we don't have to be very precise on the estimation of $\pi(0)$. The situation is illustrated with a numerical example in experiment session.

Robustness

If the estimated $\pi(0)$ is not around bottom point, then we will use a robustness technique to optimize the worst case scenario. Assume we have ranges for true $\pi(0)$ and true $\pi(1)$:

$$\pi^-(0) \leq \pi(0) \leq \pi^+(0)$$

$$\pi^-(1) \leq \pi(1) \leq \pi^+(1)$$

Then we have

$$\ln\left(\frac{\pi^-(0)}{1-\pi^-(0)}\right) \leq \alpha \leq \ln\left(\frac{\pi^+(0)}{1-\pi^+(0)}\right)$$

$$\ln\left(\frac{\pi^-(0)(1-\pi^+(1))}{(1-\pi^-(0))\pi^+(1)}\right) \leq \beta \leq \ln\left(\frac{\pi^+(0)(1-\pi^-(1))}{(1-\pi^+(0))\pi^-(1)}\right)$$

Our goal is to minimize the worst case cost:

$$\min\Big(\max_{\substack{\ln\left(\frac{\pi^-(0)}{1-\pi^-(0)}\right)\leq\alpha\leq\ln\left(\frac{\pi^+(0)}{1-\pi^+(0)}\right) \\ \ln\left(\frac{\pi^-(0)(1-\pi^+(1))}{(1-\pi^-(0))\pi^+(1)}\right)\leq\beta\leq\ln\left(\frac{\pi^+(0)(1-\pi^-(1))}{(1-\pi^+(0))\pi^-(1)}\right)}} \pi(x)(1-x) + A(1-a)(1-\pi(x)))$$

which can be rewritten as

$$\min(\max \frac{e^{\alpha-\beta x}}{1+e^{\alpha-\beta x}}[1-x-A(1-a)] + A(1-a))$$

It's reasonable to assume that $1-x-A(1-a) < 0$, then $\frac{e^{\alpha-\beta x}}{1+e^{\alpha-\beta x}}[1-x-A(1-a)]+A(1-a)$ is maximized when $\alpha = \ln\left(\frac{\pi^-(0)}{1-\pi^-(0)}\right)$ and $\beta = \ln\left(\frac{\pi^+(0)(1-\pi^-(1))}{(1-\pi^+(0))\pi^-(1)}\right)$ (lowest alpha, highest beta). Then the problem becomes

$$\min \frac{e^{\ln\left(\frac{\pi^-(0)}{1-\pi^-(0)}\right)-\ln\left(\frac{\pi^+(0)(1-\pi^-(1))}{(1-\pi^+(0))\pi^-(1)}\right)x}}{1+e^{\ln\left(\frac{\pi^-(0)}{1-\pi^-(0)}\right)-\ln\left(\frac{\pi^+(0)(1-\pi^-(1))}{(1-\pi^+(0))\pi^-(1)}\right)x}}[1-x-A(1-a)] + A(1-a)$$

Numerical Experiment: Sensitivity Analysis

We first take a look at A. The optimal copayment level x decreases when A increases. For example, when $a = 0.1$, $\pi(0) = 0.65$ and $\pi(1) = 0.3$, we have the sensitivity analysis of A shown in Figure 2.12

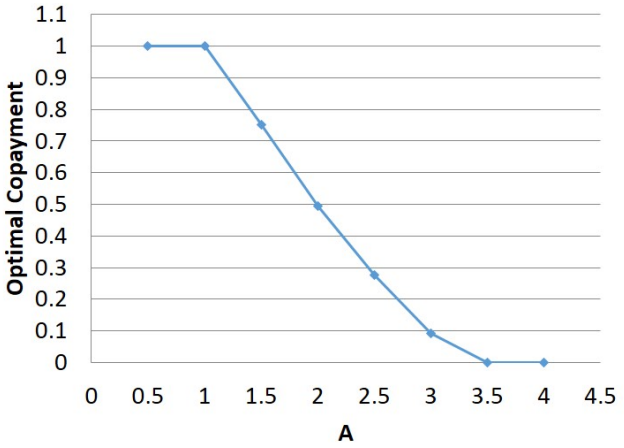


Figure 2.10: Sensitivity Analysis of A

The Optimal co-insurance percentage decreases when later consequences are more severe. This implies that patient should pay less for the high value medicine at the beginning, if a late treatment is much more expensive.

We next give an example of how the estimate of $\pi(0)$ and $\pi(1)$ influences the optimal copayment x when treating the disease at time T is twice as expensive as at time t_0, i.e. $A = 2$. Table 2.5 is the optimal copayment level x in response to different $\pi(0)$ and $\pi(1)$.

Table 2.3: Optimal Copayment as A Function of $\pi(0)$ and $\pi(1)$

$\downarrow \pi(1)$ $\pi(0) \rightarrow$	0.95	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.45	0.4	0.35
0.3	0.43	0.4	0.39	0.39	0.41	0.44	0.49	0.58	0.71	0.92	**1**	**1**	**1**
0.35	0.47	0.45	0.45	0.47	0.51	0.57	0.67	0.81	**1**	**1**	**1**	**1**	
0.4	0.51	0.51	0.53	0.57	0.63	0.74	0.9	**1**	**1**	**1**	**1**		
0.45	0.56	0.57	0.61	0.68	0.79	0.96	**1**	**1**	**1**	**1**			
0.5	0.61	0.65	0.72	0.83	**1**	**1**	**1**	**1**	**1**				

As illustrated in Table 2.5, when the difference between $\pi(0)$ and $\pi(1)$ decreases, the optimal copayment increases. This is because when people are less responsive to their out-of-pocket expenses, it's better for the insurer to raise their copayment level since people will adhere to their medication regardless of the copayment level.

Now let's study $\pi(0)$ and $\pi(1)$ separately. When $A = 3$ and $\pi(0) = 0.65$, sensitivity analysis of $\pi(1)$ is shown in Figure 2.13

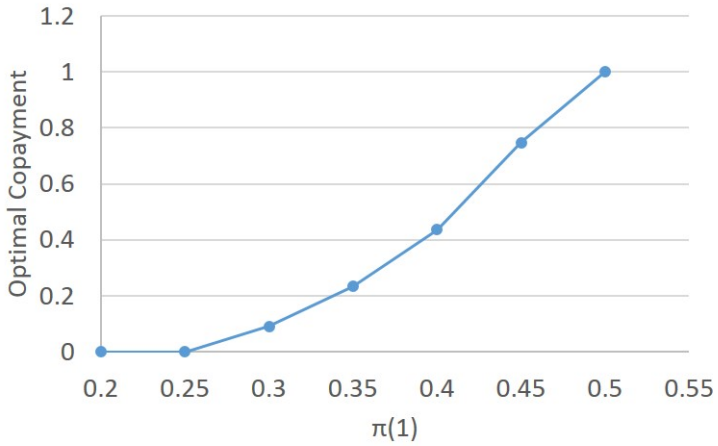


Figure 2.11: Sensitivity Analysis of $\pi(1)$

There's almost a linear relationship between optimal copayment and $\pi(1)$. If patients are willing to adhere to their medicine even if they have to pay the full price, it is beneficial for the insurance company to increase copayment level.

For $\pi(0)$, there is a bottoming-out relationship between the optimal x and $\pi(0)$. For example, when $A = 3$ and $\pi(1) = 0.3$, sensitivity analysis of $\pi(0)$ is shown in Figure 2.14

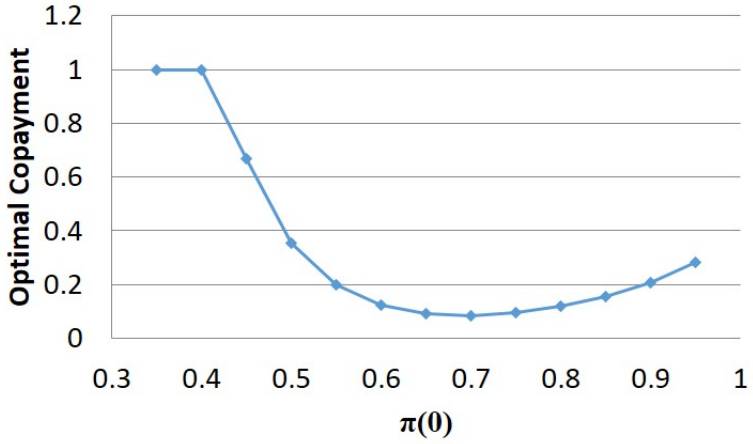


Figure 2.12: Sensitivity Analysis of $\pi(0)$

We next consider that following question: "We assume $\pi(0) = 0.6$ while in real world $\pi(0) = 0.5$. What's the impact of our mistake?" We compare the impact in two situations.

For the first situation, $A = 3$ and $\pi(1) = 0.25$. Then $\alpha_r = 0, \beta_r = 1.1$ and the true optimal $x_r = 0.06$. Then we have

$$\pi(x_r) = \frac{e^{\alpha_r - \beta_r x_r}}{1 + e^{\alpha_r - \beta_r x_r}} = 0.48$$

The true minimum cost is given as

$$\pi(x_r)(1 - \pi(x_r)) + A(1 - a)(1 - \pi(x_r)) = 1.6992$$

However, since we assume $\pi(0) = 0.6$, the optimal x we get is 0 and the minimum cost is actually 1.7.

For the second situation, $A = 3$ and $\pi(1) = 0.3$. Then $\alpha_r = 0$, $\beta_r = 0.85$ and the true

optimal $x_r = 0.35$. Then the real minimum cost is 1.6475. However, since we assume $\pi(0) = 0.6$, the optimal x we get is 0.12 and the minimum cost is actually 1.6856.

Hence, for the first situation the insurer pays 0.0471% more than necessary and for the second situation the insurer pays 2.31% more than necessary. The reason behind this can be illustrated by Figure 2.15 and Figure 2.16

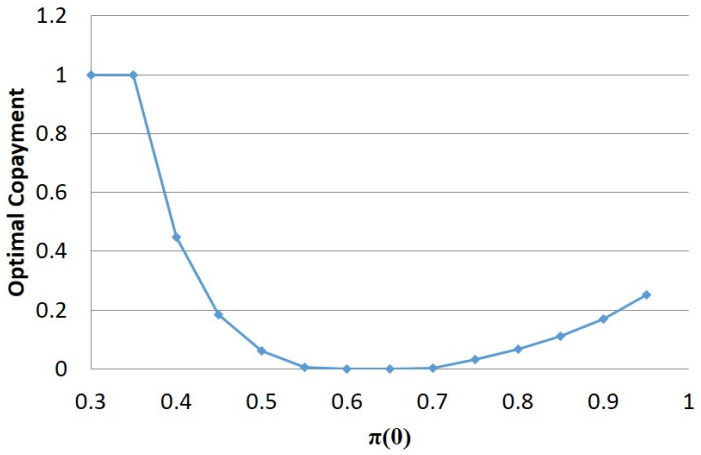


Figure 2.13: Sensitivity Analysis of $\pi(0)$, $\pi(1) = 0.25$

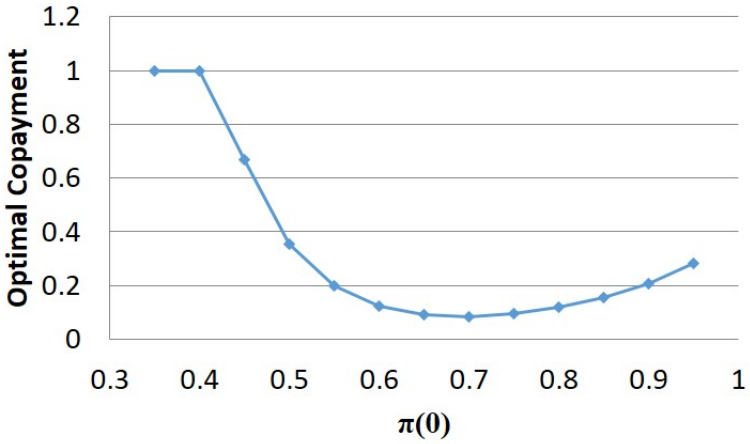


Figure 2.14: Sensitivity Analysis of $\pi(0)$, $\pi(1) = 0.3$

As illustrated in the graphs, for the first case the true $\pi(0)$ is around the bottom point where the optimal x is not sensitive to $\pi(0)$. But for the second situation, the optimal copayment level x is sensitive to the points around estimated $\pi(0)$, so a slight change of $\pi(0)$ will lead to a big change of optimal x. Therefore, if estimated $\pi(0)$ is around bottom point and we believe the true $\pi(0)$ is around the bottom point, then the estimation does not have to be very precise.

Robustness

Let $\pi^-(0) = 0.5, \pi^+(0) = 0.7, \pi^-(1) = 0.2, \pi^+(1) = 0.4$, then $0 \leq \alpha \leq 0.85$, $0.41 \leq \beta \leq 2.23$. Thus, the objective function becomes

$$\min \frac{e^{-2.23x}}{1 + e^{-2.23x}}[1 - x - A(1 - a)] + A(1 - a)$$

where the optimal x is 0 and the minimum worst case cost is 1.7.

What if we simply put in nominal minimum parameter? let $\pi(0) = (0.5 + 0.7)/2 = 0.6$ and $\pi(1) = (0.2 + 0.4)/2 = 0.3$, then $\alpha = 0.41$ and $\beta = 1.25$. Optimal x is 0.12 and the optimal cost is 1.67. If we plug $x = 0.12$ in the worst case scenario, cost will be 1.7411, which is 2.42% higher.

2.3 Operation Copayment

In this section we determine the optimal operation copayment level to minimize insurer's cost. The contribution of this model is that patients decide when to take operation based copayment level. Their decision depend on their income and the cost of the surgery. Therefore the timing of the surgery is part of the decision. We first develop a fixed-operation-time model where patients only have two options: taking an easy operation now or taking a more complex operation at time T. We then expand it to a flexible-operation-time model where patients will take the operation whenever they saved enough money for it.

2.3.1 Fixed-Operation-Time Model

In this model, we assume patients will take the operation at time t if they can afford it, meaning that if the operation cost is less than α percent of his annual income. Otherwise, they will push to time T. We have the following parameters:

- D: deductible
- C: operation cost at time t_0
- x: copayment level
- A: cost multiplying factor at time T
- M: maximum out of pocket expense

Lemma 2.4 (Total Cost Facing Insurer in Fixed-Operation-Time Model) *In the Fixed-Operation-Time model, total cost facing insurance company is:*

- $(1-x)\left[\frac{AC-D}{(1+r)^T} + (C - D - \frac{AC-D}{(1+r)^T})\pi(x_1)\right]$, *if* $x \leq \frac{M}{AC-D}$
- $(1-x)(C-D)\pi(x_1) + \frac{AC-D-M}{(1+r)^T}(1-\pi(x_1))$, *if* $x > \frac{M}{AC-D}$.

where $\pi(x) = pr\left(income \geq \frac{D+x(C-D)}{\alpha}\right) = F\left(\frac{D+x(C-D)}{\alpha}\right)$

Proof. Since patients need to pay the deductible and share part of the excess cost, taking t/he operation at time t_0 will cost patients

$$D + x(C - D)$$

Insurance company pays whatever is left and thus at time t_0 the operation will cost the insurer

$$C - D - x(C - D) = (1 - x)(C - D)$$

2.3. OPERATION COPAYMENT

If the patient delays the operation till time T, the cost of the operation will increase to AC since the condition of the patient gets worse. Then the patient will pay the deductible D plus the co-insurance $x(AC-D)$, or the out-of-pocket maximum M. Therefore the present value of the operation is given as

$$\frac{D+\min(x(AC-D),M)}{(1+r)^T}$$

The present value of the operation cost for insurance company is given as

$$\frac{AC-D-\min(x(AC-D),M)}{(1+r)^T}$$

Assume that patient will choose to take the operation at time t_0 if the operation cost is less than α of his annual income.

Hence, proportion of patients who will take the operation at time t_0 is

$$\pi(x)=pr\left(income\geq\frac{D+x(C-D)}{\alpha}\right)=F\left(\frac{D+x(C-D)}{\alpha}\right)$$

- If $x\leq\frac{M}{AC-D}$, the total cost facing insurance company is

$$\begin{aligned}&(1-x)(C-D)\pi(x)+\frac{(1-x)(AC-D)(1-\pi(x))}{(1+r)^T}\\=&(1-x)\left((C-D)\pi(x)+\frac{(AC-D)(1-\pi(x))}{(1+r)^T}\right)\\=&(1-x)\left(\frac{AC-D}{(1+r)^T}+(C-D-\frac{AC-D}{(1+r)^T})\pi(x)\right).\end{aligned}$$

- If $x>\frac{M}{AC-D}$, the total cost facing insurance company is

$$(1-x)(C-D)\pi(x)+\frac{AC-D-M}{(1+r)^T}(1-\pi(x))$$

□

2.3.2 Flexible-Operation-Time Model

Now in this model, we assume that patients will save money for the surgery. They take the operation as soon as they save enough money for it. They don't need to wait till time T. If a patient take the surgery at t, then the total operation cost will be $\frac{AC}{1+(A-1)e^{-t}}$, where $\frac{A}{1+(A-1)e^{-t}}$ is cost multiplying factor and $A > 1$. Hence at time 0, the value of the operation is still C; when time approaches ∞, the value of the operation is AC.

Lemma 2.5 (Total Cost Facing Insurer in Flexible-Operation-Time Model) *In the Flexible-Operation-Time model, total cost facing insurance company is:*

- $(1-x)\sum_i \frac{\frac{AC}{1+(1-A)e^{-t_i}}-D}{(1+r)^{t_i}}$, *if* $x \le M$ $(\frac{AC}{1+(A-1)e^{-t}} - D)$.
- $\sum_i \frac{\frac{AC}{1+(A-1)e^{-t_i}}-D-M}{(1+r)^{t_i}}$, *if* $x > M$ $(\frac{AC}{1+(A-1)e^{-t}} - D)$.

Proof. • If $x\left(\frac{A}{1+(A-1)e^{-t}}C - D\right) \le M$

It takes the patient t time to save enough money for the surgery, which is given as

$$t\alpha I = D + x\left(\frac{A}{1+(A-1)e^{-t}}C - D\right)$$

where α is the proportion of income patient will put aside for the surgery in unit time. So we need to solve the above equation for t and make sure at the same time that t satisfies $x(\frac{A}{1+(A-1)e^{-t}}C - D) \le M$. Hence, for patient i, the out-of-pocket expense is

$$D + x\left(\frac{A}{1+(A-1)e^{-t_i}}C - D\right)$$

The present value of the surgery cost facing the insurer is

$$\frac{A}{1+(A-1)e^{-t_i}}C - D - x\left(\frac{A}{1+(A-1)e^{-t_i}}C - D\right)$$

Then, summing what the insurer pays for each of the patient we have the insurer's

total cost

$$(1-x)\sum_i \left(\frac{\frac{AC}{1+(1-A)e^{-t_i}} - D}{(1+r)^{t_i}} \right)$$

- If $x\left(\frac{AC}{1+(1-A)e^{-t}} - D\right) > M$

  Time to take operation t satisfies

$$t\alpha I = D + M$$

we can see that in this case, what time to take the operation does not depend on co-payment level x any more. Similarly, present value of total cost to insurance company is

$$\sum_i \left(\frac{\frac{AC}{1+(A-1)e^{-t_i}} - D - M}{(1+r)^{t_i}} \right)$$

□

2.3.3 Operation Co-payment Numerical Experiment

We test for copayment level $x = 0; 0.03; 0.05; 0.15; 0.3; 0.5; 1$. We calculate the total cost of the insurance company at each copayment level and pick the one with the lowest cost.

Fixed-Operation-Time Model

We assume the income of the patients follow a normal distribution with mean equal to 50,000 and standard deviation equal to 35,000 [32]. We assume that at time T the cost of the treatment will be twice as much as the cost of treatment at time t, so A =2. Let the operation cost at time t_0 be \$39,000, so C=39,000; let deductible be \$500, so D=500; let maximum out of pocket expense be \$3000, so M=3000; let the time span be 2 years, so T=2; let the discount rate be 0.24%, so r=0.24%. The result is shown in Figure 2.17

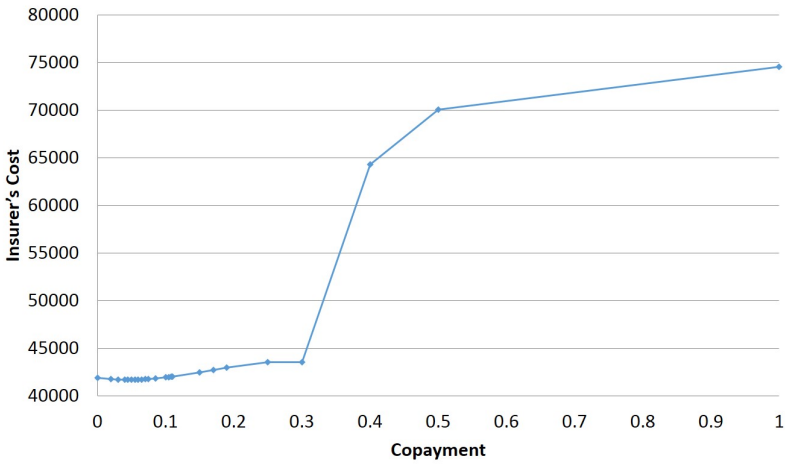


Figure 2.15: Hip Replacement copayment vs cost discrete case

Flexible-Operation-Time Model

We simulate 30 income based on normal distribution with mean 50,000 and standard deviation 35,000. Let $\alpha = 0.2$ and other parameter are the same as in discrete model. Result is given in Figure 2.18

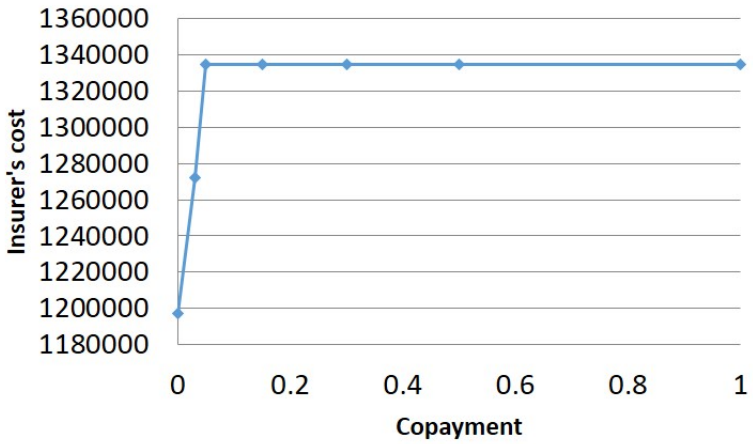


Figure 2.16: Hip Replacement copayment vs cost continuous case

2.4 Conclusion

Markov Chains have been used in multiple studies on medical decision making but to the best of our knowledge have not been implemented in the context of VBID. Our paper investigates the optimal coinsurance levels to minimize the total cost from the payer's standpoint in the presence of different risk groups for the statin treatment of heart disease. We incorporate the fact that people with different risk levels have different probabilities to develop negative events and thus assign its own set of decision variables to each group. We find that:

- Compared to the goal of preventing high risk patients from having a heart attack, given current medical technology and cost, preventing low risk patients from turning into high risk patients is more crucial to total cost.
- There is a concave decreasing relationship between heart attack cost and VBID cost savings. Given current medicine effectiveness and patients' responsiveness to copayment change, if the heart attack cost is more than 300 times the cost of medicine, there is no point in implementing VBID design.
- There is a concave decreasing relationship between medicine effectiveness and VBID cost savings. Given average heart attack cost and patients' responsiveness to copayment change, VBID cost savings and optimal cost sharing levels are very sensitive to medicine effectiveness when heart attack reduction rate less is less than 14% and thus payers might want to put more efforts in estimating the parameters and adopting the VBID model.
- VBID cost savings increase sharply as patients' copayment sensitivity increases from 0 to 0.0058, and then decrease gradually as patients become more sensitive to copayment. If the payer believe that patients' copayment sensitivity is around 0.0058, it should group patients into high risk and low risk pools and adopt the VBID model.

Chapter 3

A Comparison Between the Robust Risk-Aware And Risk-Seeking Managers In R&D Portfolio Management

In this chapter, we analyze via simulation two mathematical modeling frameworks that reflect different managerial attitudes toward upside risk in the context of R&D portfolio selection. The manager seeks to allocate a development budget between low-risk, low-reward projects, called incremental projects, and high-risk, high-reward projects, called innovational projects. Because of their highly uncertain nature and significant probability of failure, the expected value of the innovational projects is smaller than that of their incremental projects' counterpart, but the long-term financial health of a company necessitates to take risk in order to maintain growth. We study the differences in strategy and portfolio's risk profile that arise between a risk-aware manager, who takes upside risk because he has to for the long-term competitive advantage of his company, and a risk-seeking manager,

who will take as big a bet as allowed by the model.

3.1 Introduction

There is a broad consensus that successful portfolio management of research and development projects is crucial condition to a firm's long-term survival. This is particularly the case for pharmaceutical industry because pharmaceutical companies faces high probabilities of failure at every stage of development. According to Rodriguez (1998) [76], twenty percent of the R&D projects fail during Phase I and and only twenty percent of the remaining candidates successfully pass the human efficacy testing. Even if the new drugs are successfully launched in the market, pharmaceutical sales estimate is highly uncertain. Griffin (1997) [77] states that sixty percent of new products fail after launching in the market. What's more, capital requirements for developing and launching drugs are escalating ([78],[82]). Munos (2009)[79] estimates that the cost of developing new molecular entities (NME) have been increasing exponentially 13.4% annually since the 1950s. Garnier (2009) [80] acknowledges that the growing R&D cost is possibly due to the fact that drugs for common disease that are easy to treat have already been invented, while less common diseases require more basic research in order to develop effective drugs and patients are more difficult to recruit for clinical trials. Fifth, many of the branded drugs are coming off patent. When Lipotor® went off patent in 2011, Pfizer, which markets Lipotor®, loses around a sixth of its 2010 revenues, is $ 11 billion.

The combination of these challenges makes it crucial for pharmaceutical companies to have a carefully designed R&D portfolios management strategy. In general, there are two types of R&D projects: (i) low-risk, low-reward or incremental projects and (ii) high-risk, high-reward or innovational projects. Incremental projects require few resources and have little uncertainties. While they are necessary for continuous improvement, they do not generate the growth or competitive advantage that companies seek in the long term. Innovational

projects are highly uncertain but potentially transformative and can profoundly affect an industry's competitive landscape through substantial revenues. In the pharmaceutical industry, Wuyts (2004) [88] define innovational drugs as those which use a distinct core technology and provide substantially greater benefits than existing drugs. Kim (1999) [86] reports that substantial innovations makes up fourteen percent of new-product launches, but account for sixty-one percent of all profit from innovations. While innovational projects are risky and their rewards are often far in the future, too much investment in incremental projects will delay truly innovational projects and fail to achieve revenue goals [87].

Selecting the composition of the portfolio with respect to incremental projects and innovational projects is at the heart of portfolio optimization and a rich literature is devoted to this problem. Existing tools include checklists, scoring models and bubble charts to help managers decide on the appropriate balance between incremental and radial innovation and the right mix of short and long-term developments [89]. Roussel (1991) [90], Wheelwright (1992) [91] and Cooper (2001) [83] develop a case-driven framework to decide on the appropriate balance between incremental and innovational projects, risk and reward, and market and technology risk. These studies summarize best practices to decide the optimal trade-offs and divide resources across a portfolio of new product development endeavors [93].

There have been many attempts to address the project selection problem in the context of multi-objective optimization, where various projects are evaluated based on their weighted average performance on multiple criteria selected by management. Since weights are often difficult to define, a widely used methodology is to utilize a financial metric such as net present value (NPV) or break-even time (BET) to evaluate the projects [94]. Computing an efficient frontier, which characterizes the best possible returns at each given risk level, is very common in practice [92, 93]. Mathematical programming formulations, such as knapsack problems, have been studied extensively and various mixed-integer programming heuristics have been developed to find approximate solutions.

Among industry-focused papers, Brenner (1994) [95] develop a systematic project selection process to help Air Products select criteria, weight those criteria and allocate resources to maximize project progress. Loch (2001) [94] utilize a mathematical model to minimize the gap between target performance and actual performance on selected criteria in BMW. Dickinson (2001) [96] uses a dependence matrix to quantify the interdependence between projects and develop an integer model to prioritize project selection for Boeing Company. Another stream of the literature treats the problem as a dynamic allocation of resources across multiple projects [93]. Ding (2002) [97] develop a multi-stage model that recommends optimal product pipeline structures, considering business value, cost of development and survival probabilities. Their results indicate that pharmaceutical companies tend to underspend on research and development. Chao (2008) [99] use a concept called strategic buckets to address the dynamic resource allocation problem in NPD portfolio management. They assume that the overall budget depends on cash flow generated at different points and show that time commitment for each project is the key input driving the composition of the project portfolio.

In this paper, we consider two types of managers, the risk-aware manager and the risk-seeking manager (defined below), and formulate the mathematical programming problems they seek to optimize when uncertainty is modeled using a robust optimization framework. The robust optimization framework is particularly well-suited to model uncertainty in R&D project cash flows due to the very large uncertainty associated with such cash flows and the difficulty in estimating the underlying probability distributions accurately (see [85] for an overview). We analyze those models' implications with respect to the manager's strategy and the innovation portfolio's risk/reward profile via simulations. To the best of our knowledge, this is the first paper that combines an uncertainty modeling inspired from robust optimization and upside risk in the context of innovation management. In Section 4.2, we present the two mathematical models we will compare and explain how to solve them in a tractable manner. We present numerical experiments in Section 3.3 and conclude

in Section 3.4.

3.2 Mathematical Models

3.2.1 The Risk-Aware Manager

The risk-aware manager takes risk because he understands he has to invest in some innovative projects for the sake of the company's future financial health but has intrinsically a low appetite toward risk. In other words, he is a reluctant risk-taker focusing on upside risk.

Uncertainty Modeling

First, we describe how such a manager computes the risk-aware cash flow for a given project investment decision vector $\mathbf{x} \in \mathcal{X}$ where $\mathcal{X} \subset \{0,1\}^n$ is the set of feasible solutions, with n the total number of projects. We will use the following notations, for $i = 1, \ldots, n$ denoting the project number:

$\overline{CF}_i$ nominal cash flow (i.e., expected value) of project i,

$\widehat{CF}_i$ upward deviation of the cash flow of project i from its nominal value to its best case,

z_i upward scaled deviation of cash flow from its nominal value, $z_i \in [0,1]$,

The risk-aware manager only considers upside deviations of the cash flows from their nominal values. Hence, the uncertain cash flow for project i, CF_i, is then modeled as:

$$CF_i = \overline{CF}_i + \widehat{CF}_i \, z_i,$$

with $0 \leq z_i \leq 1$.

The manager gives himself a risk budget Γ. In a context of being risk-aware, he enforces that:

$$\sum_{i=1}^{n} z_i \geq \Gamma$$

where Γ is here the *budget of upside uncertainty*. Note the sign of the inequality.

Further, we must have:

$$z_i \leq x_i$$

so that the budget of upside uncertainty is only used for the cash flow of a project actually selected.

This leads to the following computation for the uncertain total cash flow with upside risk awareness $\sum_{i=1}^{n} CF_i\, x_i$ for $\mathbf{x}$ feasible, i.e., $\mathbf{x} \in \mathcal{X}$:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} \left(\overline{CF}_i + \widehat{CF}_i\, z_i \right) x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} z_i \geq \Gamma, \\ & 0 \leq z_i \leq x_i, \ \forall i. \end{aligned} \tag{3.1}$$

Tractable Reformulation

The traditional robust optimization methodology invokes strong duality to transform the inner problem from a minimization to a maximization and reinjects the dual problem into the master maximization problem. In order to use strong duality for Problem (3.1), we must make sure the problem has a finite optimal objective. Because the feasible set is bounded, this is equivalent to showing that the feasible set of Problem (3.1) is nonempty.

Lemma 3.1 *The inner problem (3.1) has a nonempty feasible set if and only if* $\sum_{i=1}^{n} x_i \geq \Gamma$.

Proof. The proof is in two parts. (i) Let $\mathbf{z}$ be a feasible solution for Problem (3.1). Then $\sum_{i=1}^{n} x_i \geq \sum_{i=1}^{n} z_i$ from $z_i \leq x_i$ for all i so we must have $\sum_{i=1}^{n} x_i \geq \Gamma$ from $\sum_{i=1}^{n} z_i \geq \Gamma$. (ii) Assume $\sum_{i=1}^{n} x_i \geq \Gamma$. Then $\mathbf{z}$ defined by $z_i = x_i$ for all i is feasible.

Hence, we formulate the risk-aware manager's problem as:

$$\begin{aligned}
\max \quad & \sum_{i=1}^{n} \overline{CF}_i \, x_i + \qquad \min \quad \sum_{i=1}^{n} \widehat{CF}_i \, x_i \, z_i \\
& \qquad\qquad\qquad\qquad\quad \text{s.t.} \quad \sum_{i=1}^{n} z_i \geq \Gamma, \\
& \qquad\qquad\qquad\qquad\quad\quad 0 \leq z_i \leq x_i, \ \forall i, \\
\text{s.t.} \quad & \sum_{i=1}^{n} CD_i \, x_i \leq B \\
& \sum_{i=1}^{n} x_i \geq \Gamma \\
& x_i \in \{0,1\} \ \forall i
\end{aligned} \tag{3.2}$$

Theorem 3.2 (Tractable Reformulation, Risk-Aware Manager) *Problem (3.2) can be solved efficiently as a series of n MIPs, where Problem j is defined as:*

$$\begin{aligned}
(Pj): \ \max \quad & \sum_{i=1}^{n} \overline{CF}_i \, x_i + \widehat{CF}_j \Gamma - \sum_{i=1}^{n} \max(0, \widehat{CF}_j - \widehat{CF}_i) x_i \\
s.t. \quad & \sum_{i=1}^{n} CD_i \, x_i \leq B \\
& \sum_{i=1}^{n} x_i \geq \Gamma \\
& x_i \in \{0,1\} \ \forall i
\end{aligned} \tag{3.3}$$

and keeping the subproblem achieving the highest objective.

Proof. Follows by applying strong duality to the inner minimization in Problem (3.2). Strong duality holds since the feasible set is nonempty and bounded; however, the constraint $z_i \leq x_i$ introduces nonlinearities in the objective when dualized. Specifically, the dual of

the inner minimization problem becomes:

$$\begin{aligned} \max \quad & p\Gamma - \textstyle\sum_{i=1}^{n} q_i x_i \\ \text{s.t.} \quad & p + q_i \leq \widehat{CF}_i x_i \ \forall i \\ & p, q_i \geq 0 \ \forall i \end{aligned}$$

[75] proves that the optimal solution is at the breakpoint of p, thus $p* = \widehat{CF}_j$ for some j and $q_i x_i = max(0, \widehat{CF}_j - \widehat{CF}_i)x_i$, which allows us to conclude.

3.2.2 The Risk-Seeking Manager

Uncertainty modeling

The risk-seeking manager seeks to take as much upside risk as possible within reason. (Intuitively, he knows he has to take risk and make bets to pursue some of the high-risk, high-return projects that could help establish his company's competitive advantage, so he decides to take big bets.) Hence, he solves the best-case problem:

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} \overline{CF}_i \, x_i + \max_{z \in \mathcal{Z}'} \sum_{i=1}^{n} \widehat{CF}_i \, x_i \, z_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} CD_i \, x_i \leq B, \\ & x_i \in \{0, 1\} \ \forall i \end{aligned}$$

where the uncertainty set $\mathcal{Z}'$ is defined as:

$$\sum_{i=1}^{n} z_i \leq \Gamma, \ 0 \leq z_i \leq 1, \forall i.$$

Tractable Reformulation

Theorem 3.3 (Tractable Reformulation, Risk-Seeking Manager) *The risk-seeking manager's problem can be formulated as a MIP:*

$$\begin{aligned}
\max \quad & \sum_{i=1}^{n} \overline{CF}_i \, x_i + \sum_{i=1}^{n} \widehat{CF}_i \, y_i \\
s.t. \quad & \sum_{i=1}^{n} CD_i \, x_i \leq B, \\
& y_i \leq x_i, \ \forall i, \\
& y_i \leq z_i, \ \forall i, \\
& \sum_{i=1}^{n} z_i \leq \Gamma, \\
& 0 \leq z_i \leq 1, \forall i, \\
& y_i \in \{0,1\}, \ \forall i, \\
& x_i \in \{0,1\} \ \forall i
\end{aligned}$$

Proof. This follows from a straightforward linearization of $x_i \, z_i$, defined as y_i for all i, using that the x_i's are binary.

3.3 Numerical Result

The purpose of these experiments are to compare the risk profiles of the R&D portfolios of the risk-aware and risk-seeking managers, using the portfolio optimal in the nominal case as a benchmark. We are particularly interested in understanding the implications of the model choice on upside risk and selecting of high-risk, high-return projects. We consider three different data sets.

3.3.1 High Risk vs Low Risk

We consider 20 projects, among which project 1 to project 12 are low-risk projects (incremental projects), and project 13 to project 20 are high risk projects (innovational projects).

If successful, innovational projects will bring significant profit to the company; however, these projects require more resources and have more uncertainty. With a low success rate, innovational projects have low average cash flow but a high additional uncertain cash flow. Incremental projects have less uncertainty and are more likely to succeed; however, the benefit of these projects is very limited. Therefore, low risk projects can have higher average value with a small deviation from the nominal cash flow. For example, Project 11 is an innovational project and if it succeeds, the revenue can reach 1100. However, with a low success rate, the expected cash flow is only 100. Project 3 is an incremental project. If it succeeds, the maximum revenue is only 150. However, it has a low risk and a high probability to succeed, which results in a higher expected cash flow of 120. We assume that cost development of each project is linear with its best-case cash flow.

We solve the two models with different uncertainty budget and simulate the cash flows. The cash flow of incremental project i is simulated using a uniform distribution $U(\overline{CF}_i - \widehat{CF}_i, \overline{CF}_i + \widehat{CF}_i)$. The cash flow of innovational project follows an extreme value distribution. If successful, its cash flow is $\overline{CF}_i + \widehat{CF}_i$; if it fails, its cash flow is $\overline{CF}_i - \widehat{CF}_i$. Therefore, we simulate the cash flow of innovational projects j as follows: with probability $\frac{\widehat{CF}'_j}{\overline{CF}_j + \widehat{CF}_j}$, $CF_j = \overline{CF}_j + \widehat{CF}_j(1 + \delta\, N(0,1))$, otherwise $CF_j = \overline{CF}_j - \widehat{CF}'_j(1 + \delta N(0,1))$, where $\delta = 0.2$.

Figure 3.1 and Figure 3.2 show the number of incremental and innovational projects selected at each uncertainty level.

We make the following observations:

- As the uncertainty protection level increase, risk aware model tends to choose more high risk projects. Also, starting from $\Gamma = 0$, the number of low risk projects selected decreases. When $\Gamma = 5$, none of the low risk projects are selected. Then the constraint $sum_i x_i \geq \Gamma$ kicks in, forcing the model to select more projects. However, without

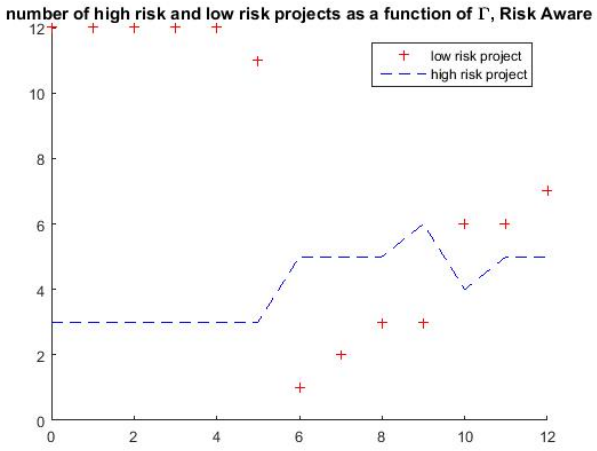


Figure 3.1: Number of high risk and low risk project selected as a function of Γ, risk aware

enough budget, the model couldn't select more high risk projects and hence turn to low risk projects, which explains why the number of low risk projects selected increases after $\Gamma = 5$.

- The risk seeking model first select more high risk project as the protection level increases and then decrease high risk project number. The project it selects also becomes less risky. For example, when Γ is less than 9, project 17, the riskiest project, is always selected; when $\Gamma > 9$, project 17 is never selected again.

Figure 3.3 shows the optimal objective value of risk aware model and risk seeking model. We have the following observations:

- Starting from $\Gamma = 0$ risk aware model selects mostly incremental projects. When Γ is 4, risk aware model doesn't use up all the budget. It gradually adds in more projects, which explains the sharp increase of optimal objective value when Γ is between 5 to 8.

- When Γ is small, risk seeking model gradually adds in more innovational projects as Γ increases. When Γ is larger than 13, risk seeking model reduce the innovational projects and selects more incremental projects.

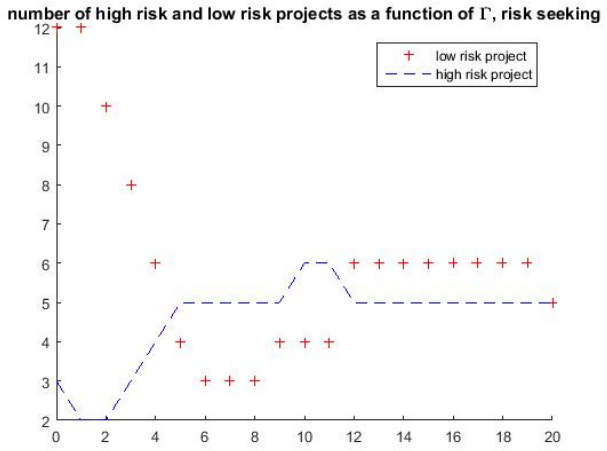


Figure 3.2: Number of high risk and low risk project selected as a function of Γ, risk seeking

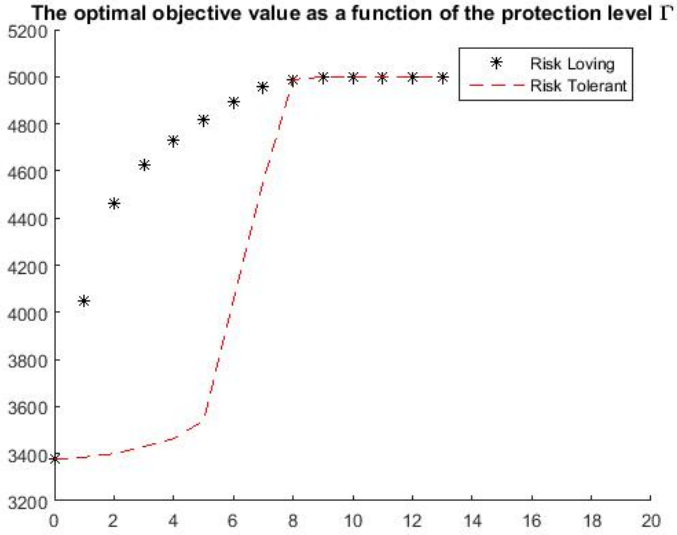


Figure 3.3: Optimal Objective Value as a Function of Protection Level Γ

Figure 3.4 to Figure 3.8 show the uniform distribution simulation result for various protection level.

When $\Gamma = 3$:

- The nominal model selects all the incremental projects and two innovational projects.
- When $\Gamma \leq 3$ risk aware model selects the same projects as in the nominal case.

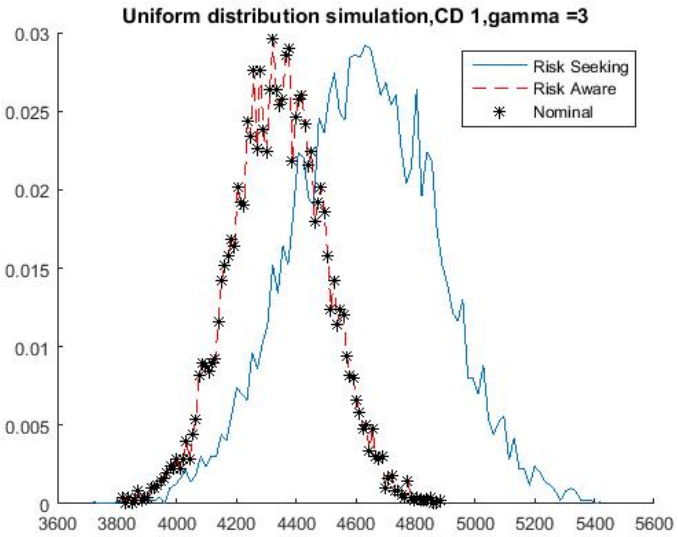


Figure 3.4: Uniform Distribution and Extreme Value Distribution, Gamma = 3

- When $\Gamma = 3$ risk seeking model abandons four incremental projects and choose a completely different set of three innovational projects. These three projects have higher $\widehat{CF}$ and higher $\frac{\widehat{CF}}{\overline{CF}}$ than the innovational projects selected by nominal model. Two of these three projects have higher $\overline{CF}$.

When $\Gamma = 8$:

- Risk seeking model only selects three incremental projects and five innovational projects, which are the three innovational projects selected by risk seeking model when $\Gamma = 3$ plus the two innovational projects selected by nominal model.
- Risk aware model selects the same projects as the risk seeking model.

When $\gamma = 10$:

- Risk seeking model abandons one innovational project and select two different innovational projects, including a project with the highest $\frac{\widehat{CF}}{\overline{CF}}$.
- Risk aware model delete one innovational project and select more incremental projects.

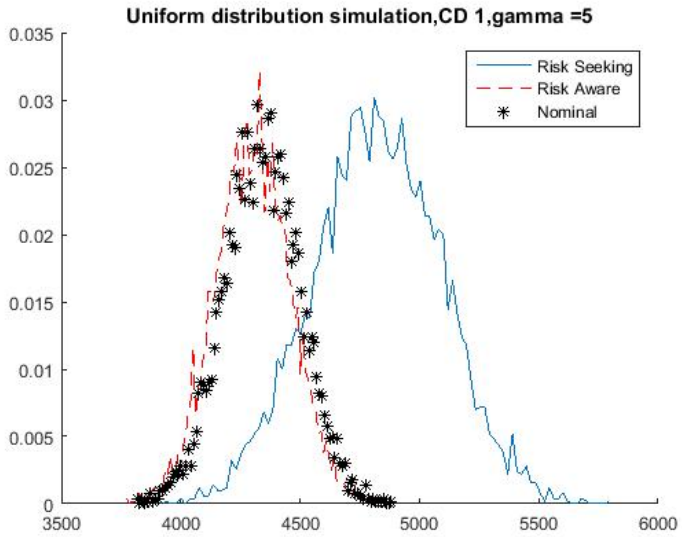


Figure 3.5: Uniform Distribution and Extreme Value Distribution, Gamma = 5

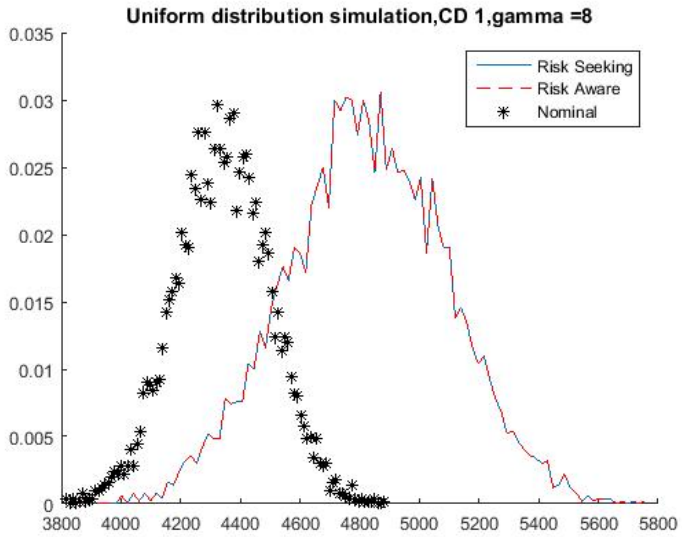


Figure 3.6: Uniform Distribution and Extreme Value Distribution, Gamma = 8

When $\Gamma = 12$:

- Risk seeking model abandons one innovational project and one incremental projects, replacing them with incremental projects.

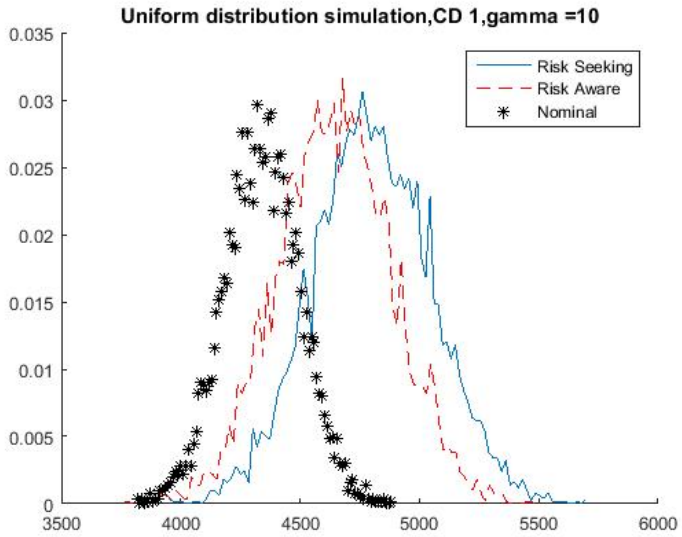


Figure 3.7: Uniform Distribution and Extreme Value Distribution, Gamma = 10

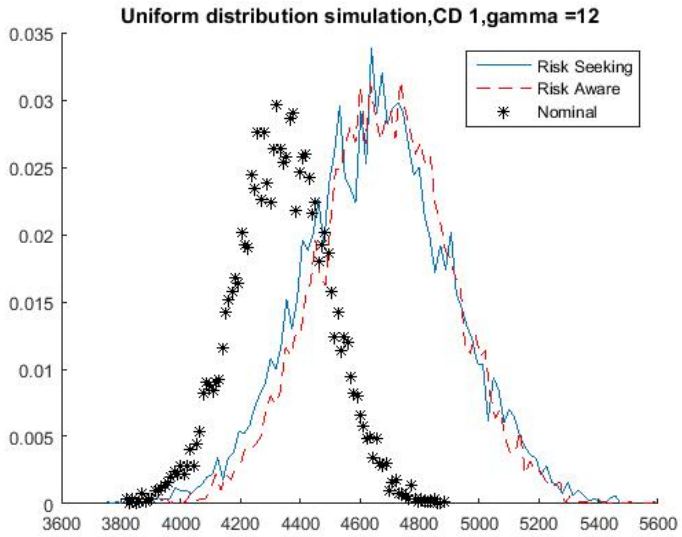


Figure 3.8: Uniform Distribution and Extreme Value Distribution, Gamma = 12

- Risk aware model deletes five incremental projects, replacing them with three incremental projects and two innovational projects.

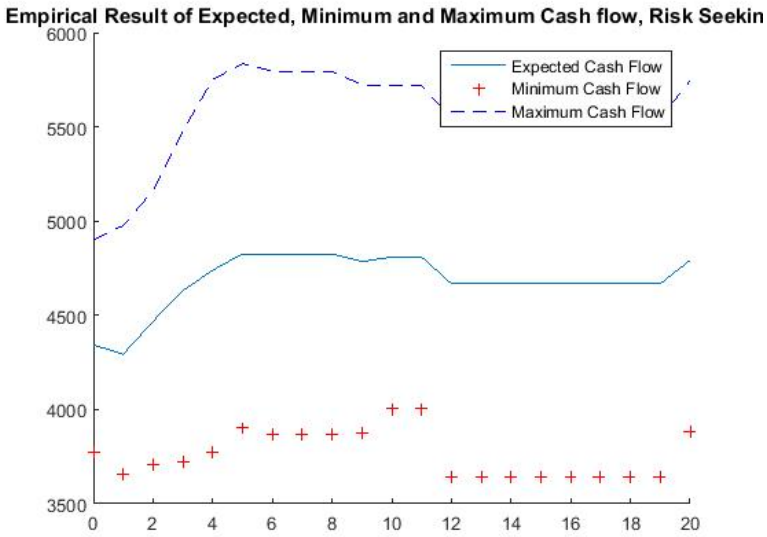


Figure 3.9: Empirical Result of Expected, Minimum and Maximum Cash Flow of Risk seeking Model

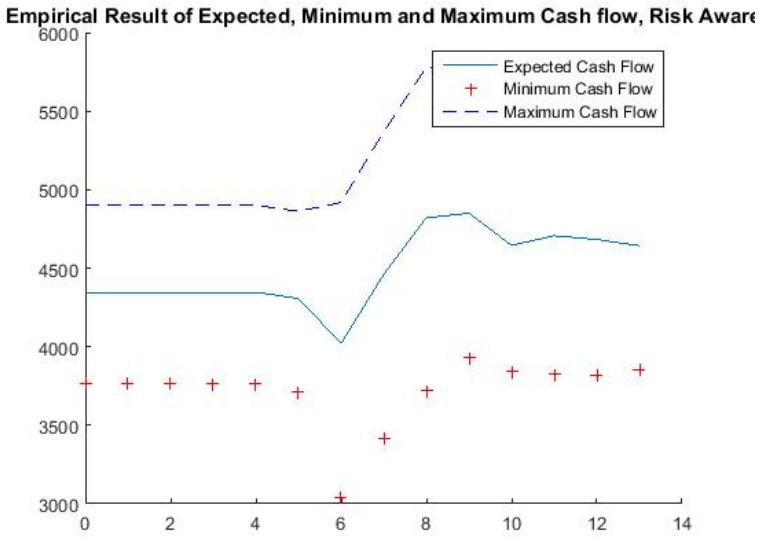


Figure 3.10: Empirical Result of Expected, Minimum and Maximum Cash Flow of Risk Aware Model

Figure 3.9 and Figure 3.10 show the empirical result of expected, minimum and maximum cash flow of risk seeking model and risk aware model. For the risk seeking model:

- Starting from $\Gamma = 0$ the maximum, minimum and expected cash flow increases as Γ increases.
- When Γ is between 4 and 11, the maximum, minimum and expected cash flow are not sensitive to the uncertainty protection level and remain almost constant.
- When Γ is larger than 11, the distance between the maximum cash flow and the minimum cash flow decreases, which indicates that the uncertainty of the model decreases.

For the risk aware model:

- When Γ is between 1 and 3, the risk aware model have the same project selection as the nominal model, hence the maximum, minimum and expected cash flow remain constant.
- When Γ is 5, the model doesn't use up all the budget and thus there is a sharp drop on all the cash flow matrices.
- As Γ increases, the model adds in more projects and the maximum, minimum and expected cash flow all increase and remain almost constant when Γ is between 10 and 14.

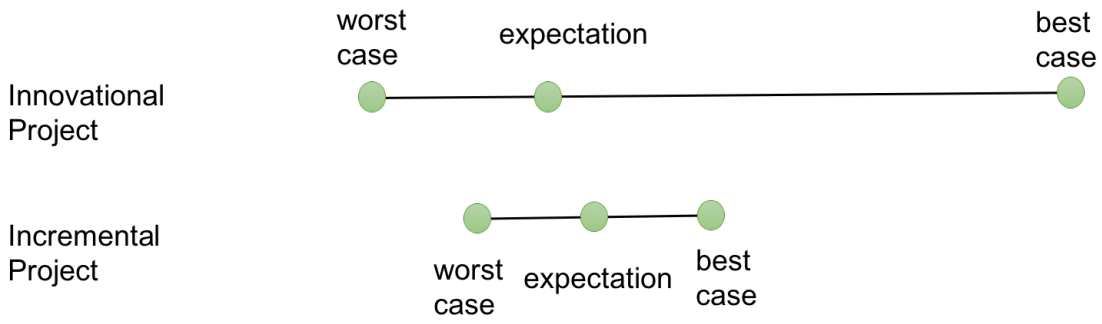
3.3.2 Gradually Increasing Risk

We next construct a data set where the expected value of the projects increase gradually, the upside risk of the projects decrease gradually. Under this method, none of the projects will dominate another project. An illustration is shown in Figure 3.3.2.

More specifically, for project i we assume:

$$\overline{CF}_i = 300 + 4 * (n - i) \tag{3.4}$$

$$\widehat{CF}_i = 0.5i\sqrt{2n(n+1)} \tag{3.5}$$



$$\overline{CF}_i' = \sqrt{2in(n+1)} \tag{3.6}$$

where n is the total number of projects. We set $n = 30$, and consider project 1 to project 20 as the incremental projects and project 21 to project 30 as the innovational projects. Similarly as in the previous experiment, we simulate incremental project cash flow with uniform distribution and innovational project cash flow with extreme value distribution.

The simulation result is demonstrated in Figure 3.11 to Figure3.13. As expected, nominal model chooses the top 16 projects. We have the following observations on risk seeking model:

- When Γ is between 1 to 11, risk seeking portfolio is scholastically dominated by the nominal portfolio. Starting from $\Gamma = 0$, risk seeking model gradually selects more innovational projects. The project it selects also become riskier. When $\Gamma = 7$, risk seeking model starts to reduce the innovational projects and when $\Gamma = 11$ there is only one innovational project in the portfolio.

- When Γ is between 12 to 16, risk seeking model doesn't select any innovational projects and selects several medium-risk project, such as project 18 and project 19. With such decision, risk seeking model outperform nominal model and have a longer tail on the right side.

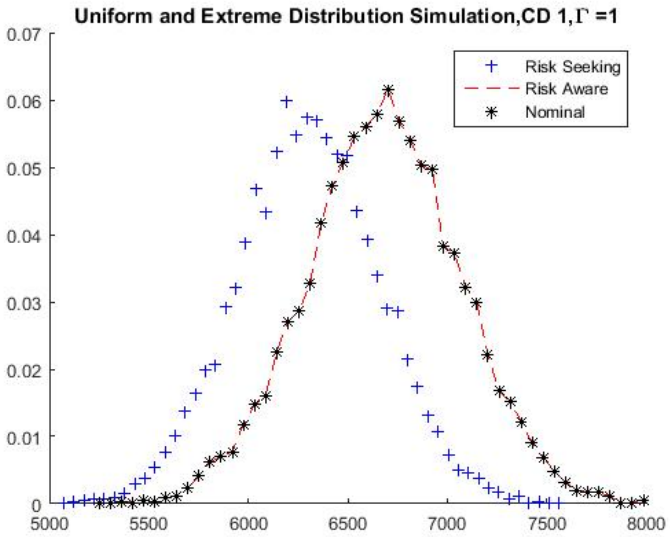


Figure 3.11: Uniform Distribution and Extreme Value Distribution, Gamma = 1

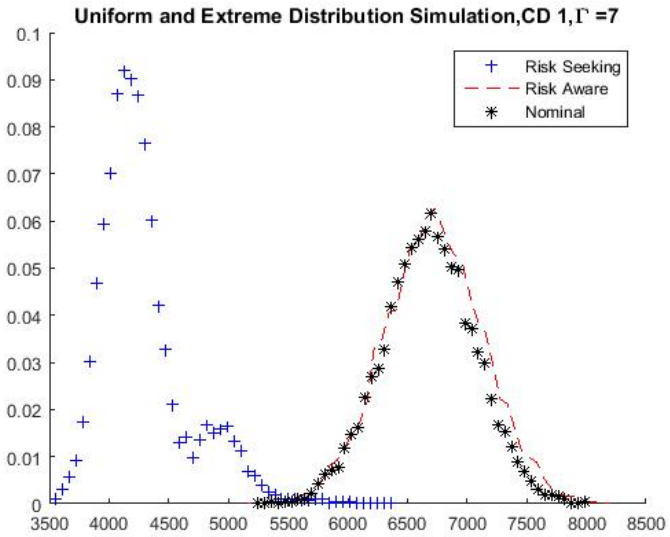


Figure 3.12: Uniform Distribution and Extreme Value Distribution, Gamma = 7

- When Γ is between 17 to 30, risk seeking model deletes several incremental projects and medium-risk projects and add in project 28. Risk seeking model outperforms the nominal model.

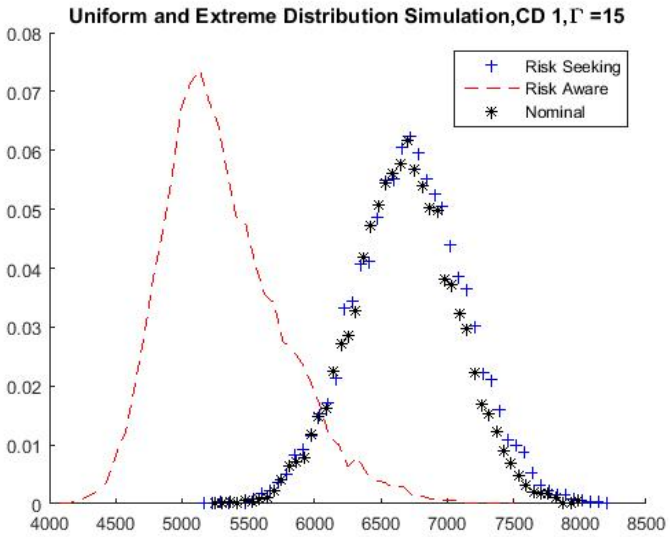


Figure 3.13: Uniform Distribution and Extreme Value Distribution, Gamma = 15

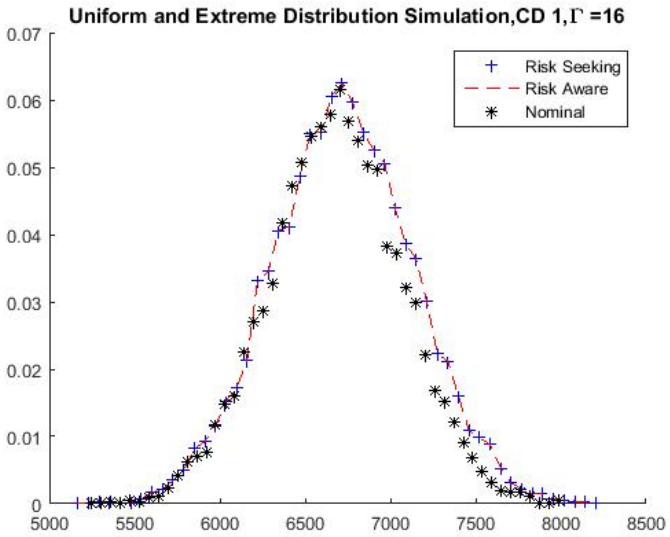


Figure 3.14: Uniform Distribution and Extreme Value Distribution, Gamma = 16

We observe the following things on risk aware model.

- When Γ is between 1 and 4, risk aware optimal portfolio is the same as the nominal optimal portfolio.

- When Γ is between 5 and 7, risk aware model model deletes project 3 and adds project 17. It is slightly better simulation result the the nominal model with a fatter tail on the right.

- When Γ is between 8 and 15, risk aware model performs worse than the nominal model. When Γ is between 8 and 11, most of the project risk aware model selects are innovational projects, which are project 21 to project 30. After that, risk aware model starts to delete innovational projects and add in more incremental projects.

- When $\Gamma = 16$, risk aware model selects the same portfolio as when $\Gamma = 5$ and outperform the nominal model.

On general, robust optimization doesn't show much advantage in this example. One of the reason is that the difference between projects are small and the cost of development of innovational projects are very large. This means that the combination of several incremental projects, especially the medium-risk projects, can outperform one innovational project but use a smaller budget.

3.3.3 Incremental vs Innovational

In this experiment, we increase the difference between incremental projects and innovational projects by using the same data set as in Experiment 2 but without project 11 to project 20. Hence we have 20 projects in this example, where the first 10 projects are incremental projects and the last 10 projects are innovational.
The nominal model still selects the top 12 projects. Again, we simulate the portfolio cash flow at various uncertainty protection level. Incremental projects are simulated with uniform distribution and innovational projects are simulated with extreme value distribution.

We make the following observations regarding the risk seeking model:

- Overall, the risk seeking model outperforms the nominal model.

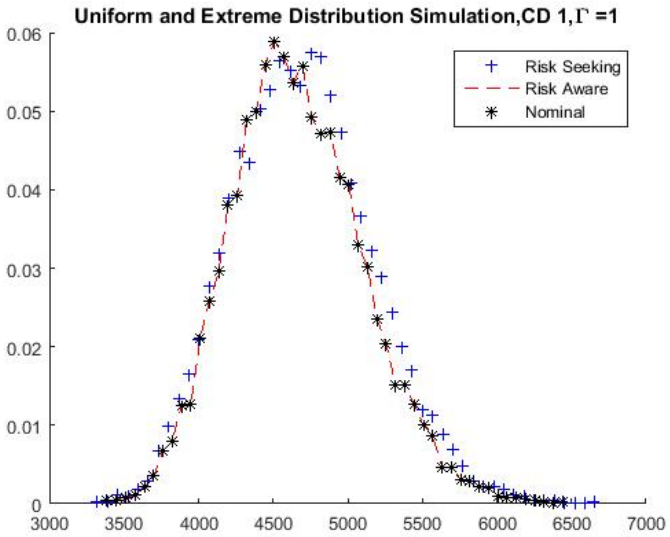


Figure 3.15: Uniform Distribution and Extreme Value Distribution, Gamma = 1

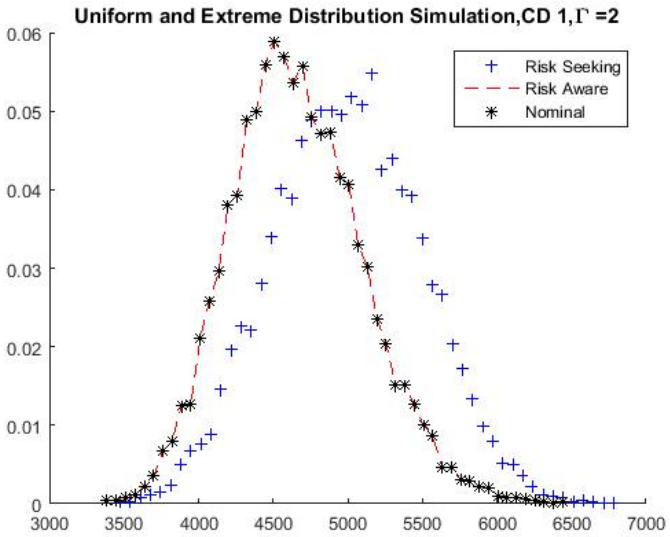


Figure 3.16: Uniform Distribution and Extreme Value Distribution, Gamma = 2

- When Γ is between 2 and 5, the risk seeking model selects 2 to 4 innovational projects in its portfolio, all of which are riskier than the innovational projects selected by nominal model. The advantage of the risk seeking model in this case is large as shown

in Figure 3.16.

- When Γ is between 6 and 12, the risk seeking model selects less innovational projects which are all less risky. Its advantage over risk nominal model decreases as shown in Figure 3.17 and 3.18.

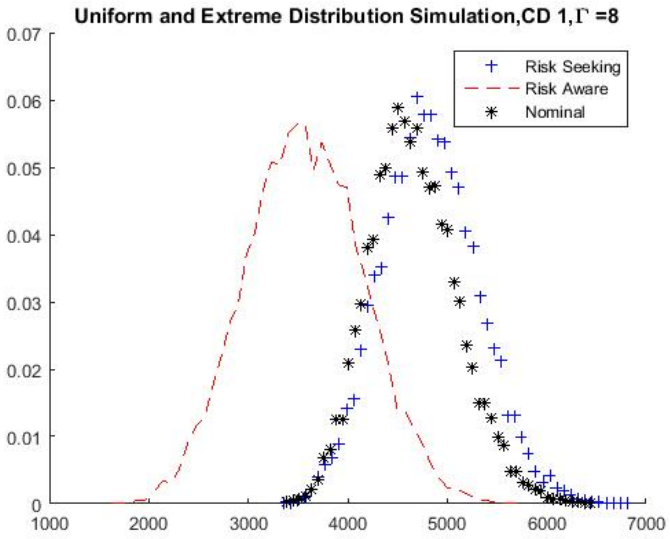


Figure 3.17: Uniform Distribution and Extreme Value Distribution, Gamma = 8

We observe the following about the risk aware model:

- When Γ is between 2 and 5, the risk aware model chooses the same portfolio as the nominal model.

- When Γ is between 6 and 11, the risk aware model outperforms the nominal model. When $\Gamma = 6$, risk aware model only selects the innovational projects, project 14 to project 20, the most risky projects. Then it gradually decrease the number of innovaitonal projects and add in more incremental projects.

- When $\Gamma = 12$ risk aware model selects project 2 to project 17 and slightly outperforms the nominal model.

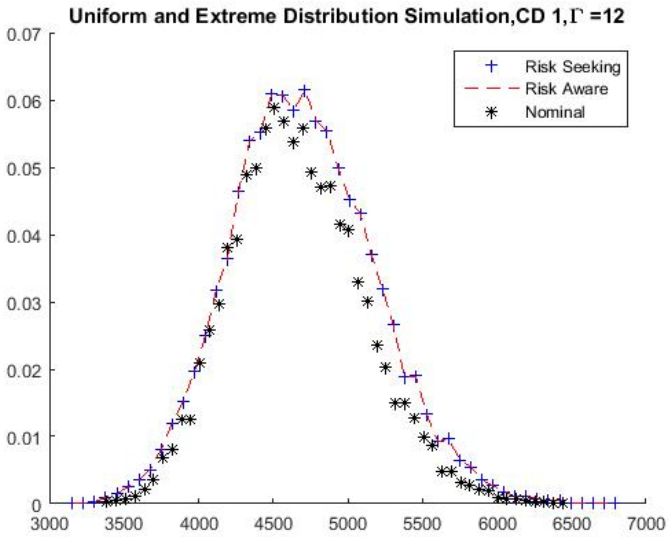


Figure 3.18: Uniform Distribution and Extreme Value Distribution, Gamma = 12

From this example we note that when there is a significant difference between incremental projects and innovational projects, risk seeking model outperforms nominal model a lot more.

3.4 Conclusions

In this paper we have proposed two decision-making models to help companies select R&D projects. Our model takes into account enormous up risk of innovational projects and encourage managers to invest in such projects which can give companies the competitive advantage that companies seek. To the best of our knowledge, this is the first work to compare the performance of risk seeking model and risk aware model. We test our model with simulated data where incremental projects are simulated with uniform distribution and innovational projects are simulated with innovational projects.

Chapter 4

Community Benefit Programs

Most hospitals in the United States are non-profit and are federal tax exempt. In order to maintain their tax exempt status, these hospitals must contribute part of their revenue to benefit their communities. Most of the contributions are in the form of direct financial assistance, such as uncompensated healthcare service. Many believe that if we can shift that amount of money upstream to invest in community-based activities that can prevent disease, we can improve people's health and decrease the rising healthcare cost. At the same time, as the Accountable Care Act expands the health insurance coverage, hospitals are able to invest in community benefit programss to reduce emergency room visits and hospital readmissions.

4.1 Literature Review

4.1.1 Systematic Review of Community-Based Programs

In spite of government's effort to develop community benefit programs, there is still a long way to go. In 2013 Young et al. used tax documents filed by more than 1800 nonprofit hospitals for fiscal year 2009 and other data to study the level and pattern of community benefit hospitals provide [10]. They concluded that in 2009 little was spent on community

benefit programs. Approximately $13 billion, which is 7.5 percent of the nonprofit hospitals' operating expenses, went to community benefits. However, 85 percent of these benefits took the form of discounted care or uncompensated health care services. For example, Medicaid always reimburse hospitals at lower rates than commercial health plans, (sometimes even lower than the cost). Discounted care for Medicaid patients makes up 45 percent of that $13 billion, which is $5.85 billion.

In terms of cost, some studies showed that if we invest in community conditions that could improve health and prevent diseases, we could sustainably and equitably improve USA's health conditions while save health care cost. Levi, Segal and Juliano [1] did a literature review and evaluated 84 studies on community-based disease prevention programs. They found that for every $1 invested in community-based programs that aims to increase physical activity, improve nutrition and decrease tobacco use, there is a return of $5.60. They first identified the most expensive set of disease in the country. They found that the country could save more than $5 billion in healthcare cost if the type 2 diabetes and high blood pressure rate decrease by 5 percent; the saving will rise up to more than $19 billion if heart disease, kidney disease, and stroke prevalence decrease by 5% in addition; the savings will increase to more than $21 billion if additional 2.5 percent of reductions in the prevalence of some forms of cancer, chronic obstructive pulmonary disease (COPD) and arthritis is achieved. Next, they identified 3 of the most important factors that can reduce the most expensive set of diseases: 1) physical activity 2) nutrition (including right nutrition value and quantities) 3) whether or not smoking. Then, they determined how the proven community-based disease prevention programs can improve these three factors and thus decreases the diseases by reviewing a range of evidence-based studies. They found that in community benefit programs targeting at these factors, type 2 diabetes and high blood pressure can decrease by 5 percent in 1 to 2 years; heart disease, kidney disease, and stroke can decrease by 5 percent in 5 years; some forms of cancer, COPD, and arthritis can decrease by 2.5 percent in 10 to 20 years. Next, they evaluated the cost of the prevention programs.

Most of the studies they reviewed had cost in the range of $3-$8 per person per year. They estimated the cost to be $10 per person to yield conservation analysis for savings. They concluded that an investment of $10 per person per year in proven community-based disease prevention could reduce more than $2.8 billion savings annually in health care costs in one to two years, which is 0.96 return of investment (ROI) for every $1 invested; within 5 years, the savings will be more than $16 annually, which is 5.6 ROI; in 10 to 20 years, the savings will be nearly $18 billion annually (in 2004 dollars), which is 6.2 ROI. Three things to be noted. First the return did not include the gains in work productivity and improved life quality. Second the authors only considered the marginal cost and does not reflect the basic infrastructure cost of the intervention programs. Third, the authors focused on low cost primary prevention programs (taking action before a problem arises so that the problem can be avoided entirely) and secondary prevention programs (detecting the problem early to control the problem and minimize the consequences). They didn't review the tertiary prevention programs (reducing further complications of an existing disease by treatment and rehabilitation).

Similarly, in 2013 Trust for America's Health (TFAH), The New York Academy of Medicine (NYAM), and the Urban Institute performed did a systematic review of hundreds of community-based intervention programs. All of the programs are outside of clinical setting. They concluded that such interventions could reduce healthcare cost dramatically [65].

However, preventive care does not always save money. Russell [7] concluded that preventing illness save money in health care in some cases but in other cases can increase health care costs. The reason is that prevention has to delivered all the people identified at risk, usually repeatedly over many years. Of the people receiving prevention care, some will still develop the disease, since prevention is not 100 percent effective; some will not develop the disease even they don't receive the prevention care. The author focused on secondary prevention and tertiary programs and concluded that such prevention programs will add to

health care cost while improve patient's life quality. Similarly, Cohen [8] did a systematic review of the cost-effectiveness of the preventive care and suggested that whether a preventive program could reduce costs depends on factors like how effective the preventive care is and the population targeted. In their summary, intensive tobacco-use prevention program for seventh- and eighth-graders will lead to medical cost of $23,000 per QALY added and screening all 65-year-olds for diabetes will add to medical cost of $590,000 per QALY added. Roux et al. [62] performed a lifetime cost-effectiveness analysis to estimate the costs and health improvement of physical activity interventions. They found that it cost between $14,000 and $69,000 per quality-adjusted life year relative to no intervention.

4.1.2 Preventions and Community-Based Prevention Program Examples

The Trust for America's Health (TFAH) and New York Academy of Medicine (NYAM) [65] summarized six categories of intervention strategies: (1) Cardiovascular Disease,Stroke & Diabetes, (2) Tobacco Use, (3) Asthma, (4) Injuries (including falls), (5) Sexually Transmitted Infections & HIV/AIDS, (6) Alcohol Use. Following are some current community-based prevention program examples.

- Eddy [12] performed a cost-effectiveness analysis of a Diabetes Prevention Program. The program provided individually tailored diet plans and physical training sessions. The author found that for every life quality year gained in the program, medical cost increase by $143,000 in 2000 dollars.

- A community-level program aimed at preventing HIV and other sexually transmitted infections was conducted in five U.S. cities. Activities include HIV risk reduction workshop and community HIV prevention events. Sikkema et al. followed the program for 12 months, they found that the proportion of targeted women who had unprotected intercourse declined from 50 percent to 37.6 percent. [71]

- Richard et al. [60] performed a cost-benefit analysis for the tobacco cessation program

implemented by the state of Massachusetts. The program included pharmacotherapy, counseling, and outreach. The authors found that the program cost $183 per participant. They estimated that the inpatient savings per participant was $571. So the medical savings for each $1 spent was $3.12.

4.2 Mathematical Models

We consider the following problem. A hospital wants to decide which programs to invests in each community. There is uncertainties in the benefits of each program. In each community, the hospital has uncertainty budget for the upside risk and downside risk, and a total budget of B to split in each community. The hospital aims to maximize the total potential benefits with a performance tolerance level. This problem can be formulated as:

$$
\begin{aligned}
\max \quad & \sum_i \sum_j (\overline{CF}_{ij} x_{ij} + \widehat{CF}_{ij} y_{ij}) \\
\text{s.t.} \quad & \sum_j CD_{ij} x_{ij} \leq B \\
& y_{ij} \leq x_{ij} \ \forall i \ \forall j \\
& \sum_{i=1} y_{ij} \leq \Gamma_j \ \forall j \\
& \min \sum_i \sum_j (\overline{CF}_{ij} x_{ij} - \widehat{CF}'_{ij} u_{ij}) \geq M \\
& \sum_i u_{ij} \leq \Gamma'_j \ \forall j \\
& x_{ij} \in 0, 1 \ \forall i \ \forall j \\
& 0 \leq u_{ij} \leq 1 \ \forall i \ \forall j \\
& y_{ij} \geq 0 \ \forall i \ \forall j
\end{aligned}
\tag{4.1}
$$

where

x_{ij}: decision variables of whether investing in program i in community j

y_{ij}: decision variables of whether including of upside risk of program i in community j

u_{ij}:decision variables of whether including of downside risk of program i in community j

$\overline{CF}_{ij}$: expected benefits of program i in community j

$\widehat{CF}_{ij}$: upside risk of program i in community j

$\widehat{CF}'_{ij}$: downside risk of program i in community j

CD_{ij}: cost of developing program i in community j

B: total budget

Γ_j: upside uncertainty budget in community j

Γ'_j: downside uncertainty budget in community j

M: the worst total benefits the hospital is willing to accept

We assume that $\widehat{CF}_{ij}$ and $\widehat{CF}'_{ij}$ are decided by the decision maker. A study done by UPMC Bedford Memorial Hospital provided a possible way to estimate CF [74]. They developed a list of health problems and rated them based on:

- Importance (I)
- Likelihood of making a measurable impact (L)
- Ability to address the problem (A)

Thus a possible way to measure programs benefit is:

$$CF = I * L * A$$

Writing the worst performance constraint as its dual problem, problem (4.2) can reformulated as:

$$\begin{aligned}
\max \quad & \sum_i \sum_j (\overline{CF}_{ij} x_{ij} + \widehat{CF}_{ij} y_{ij}) \\
\text{s.t.} \quad & \sum_j CD_{ij} x_{ij} \leq B \\
& y_{ij} \leq x_{ij} \ \forall i \ \forall j \\
& \sum_{i=1} y_{ij} \leq \Gamma_j \ \forall j \\
& \sum_i \sum_j \overline{CF}_{ij} x_{ij} - \sum_j (\Gamma'_j q_j + \sum_i r_{ij}) \geq M \\
& q_j + r_{ij} \geq \widehat{CF}'_{ij} \ \forall i \ \forall j \\
& x_{ij} \in 0, 1 \ \forall i \ \forall j \\
& y_{ij}, r_{ij}, q_j \geq 0 \ \forall i \ \forall j
\end{aligned} \tag{4.2}$$

4.3 Numerical Experiments

We consider 20 program candidates in 5 communities. Without losing generosity, we assume that program 1 has the least uncertainties and program 20 has the largest uncertainties. Programs with higher risk have larger uncertainties in all the 5 communities. Community 1 is the least risky community and community 5 is the riskiest community. Thus we construct the data set so that the expected value of the programs decrease gradually, the upside risk of the programs increase gradually. Under this method, none of the programs will dominate another program. For ease of implementation, here benefits are generated as vectors. For example, $\overline{CF}_2$ corresponds to $\overline{CF}_{12}$ in our model. Hence for program i we have:

$$\overline{CF}_i = 1000 + 4(n - i)$$

$$\widehat{CF}_i = in(n + 1))$$

$$\widehat{CF}'_i = 0.5\sqrt{in(n + 1)}$$

Cost of development is a geometric progression sequence:

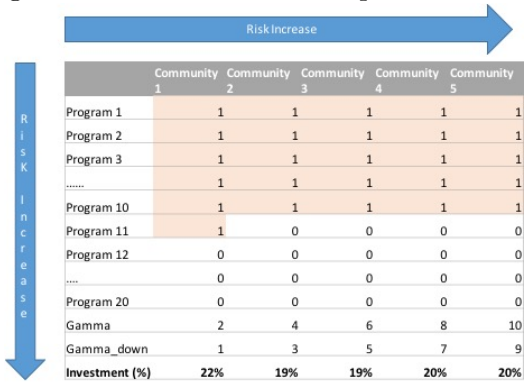
$$CD_i = 500 + 50i$$

The hospital has a budget that can cover half of the median size projects.

$$B = median(CD) * 30$$

As illustrated in Figure (4.1) and Figure (4.2), all the program the nominal model chooses are low risk program-community pairs and the program robust model chooses consists of low risk program-community pairs and high risk program-community pairs.

Figure 4.1: Nominal Model Optimal Solution



	Community 1	Community 2	Community 3	Community 4	Community 5
Program 1	1	1	1	1	1
Program 2	1	1	1	1	1
Program 3	1	1	1	1	1
.....	1	1	1	1	1
Program 10	1	1	1	1	1
Program 11	1	0	0	0	0
Program 12	0	0	0	0	0
....	0	0	0	0	0
Program 20	0	0	0	0	0
Gamma	2	4	6	8	10
Gamma_down	1	3	5	7	9
Investment (%)	**22%**	**19%**	**19%**	**20%**	**20%**

It's interesting that the nominal model evenly distributed its budget across all the communities with different risk levels and different uncertainties protection levels while the robust model prefers riskier markets with higher uncertainty protection levels.

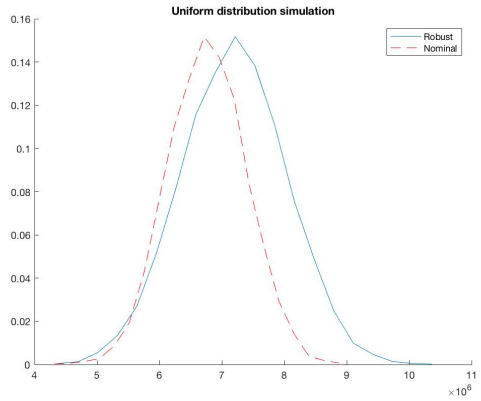
To compare the optimal solutions, we the total benefits of the two models with two simulations. In our first simulation, benefit of program i in community j follows a uniform distribution $CF_{ij} \sim U(\overline{CF}_{ij} - \widehat{CF'}_{ij}, \overline{CF}_{ij} + \widehat{CF}_{ij})$. In our second simulation, program

Figure 4.2: Robust Model Optimal Solution

Risk Increase

Risk Increase

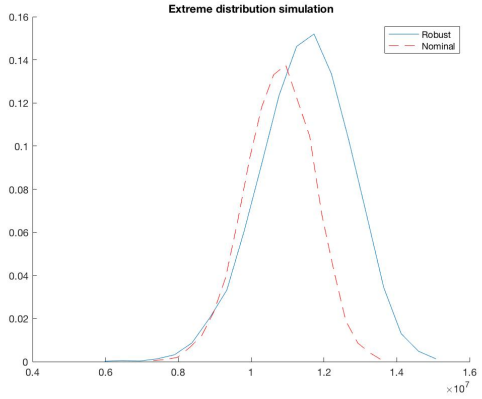
	Community 1	Community 2	Community 3	Community 4	Community 5
Program 1	1	1	1	1	1
Program 2	1	1	1	1	1
Program 3	1	1	0	1	1
Program 4	0	0	1	1	0
Program 5	1	0	1	1	0
Program 6	1	1	0	1	1
Program 7	0	1	0	1	1
Program 8	0	0	0	1	1
Program 9	0	1	0	1	1
Program 10	0	0	0	0	0
Program 11	0	0	0	0	0
Program 12	0	0	0	0	0
Program 13	0	0	0	0	1
Program 14	0	0	0	0	1
Program 15	0	0	1	0	1
Program 16	0	0	1	1	1
Program 17	0	0	1	0	0
Program 18	0	0	0	0	1
Program 19	0	0	1	0	0
Program 20	0	0	0	0	0
Gamma	2	4	6	8	10
Gamma_down	1	3	5	7	9
Investment (%)	**6%**	**10%**	**25%**	**22%**	**37%**



benefit follows an extreme value distribution. As shown in Figure (4.3) and Figure (4.3), robust model has a fatter tail on the upside risk in both cases.

4.3.1 Sensitivity Analysis of Worst Case Tolerance Level

We next analyze how the worst case tolerance level M influences the robust optimal programs portfolio. As shown in Figure (4.4) and Figure (4.3), when $M = 0$ the robust model



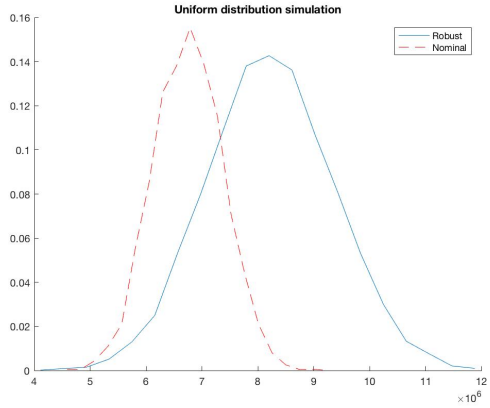


Figure 4.3: Uniform distribution simulation when $M = 0$

Figure 4.4: Robust model optimal solution when $M = 0$

chooses the riskiest program-community pairs. The simulated total benefit of this portfolio has a large deviation with a large upside risk compared to the nominal model. The downside risk is slightly larger than the nominal model.

As M increases to 20,000, as indicated in Figure (4.5) and Figure (4.6), the robust model starts to add in some less risky program-community pairs and the simulated total benefit is less spread-out.

When M increases to 40,000, most of the programs the robust model choose are low-risk programs. As a result, the simulated benefit has a smaller upside deviation. Downside deviation is still slightly larger than the nominal optimal portfolio.

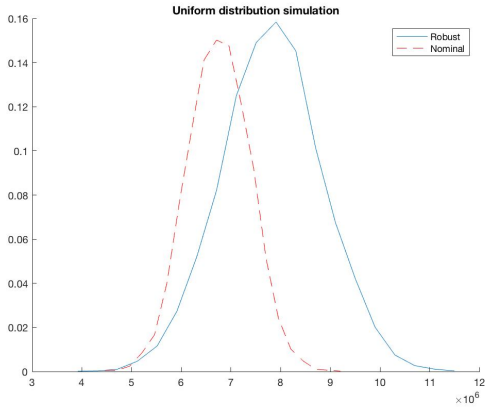
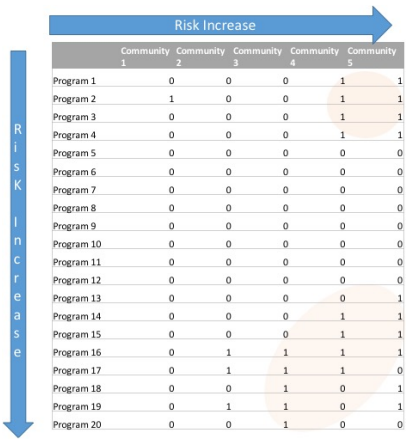


Figure 4.5: Robust model optimal solution when $M = 2000$



	Community 1	Community 2	Community 3	Community 4	Community 5
Program 1	0	0	0	1	1
Program 2	1	0	0	1	1
Program 3	0	0	0	1	1
Program 4	0	0	0	1	1
Program 5	0	0	0	0	0
Program 6	0	0	0	0	0
Program 7	0	0	0	0	0
Program 8	0	0	0	0	0
Program 9	0	0	0	0	0
Program 10	0	0	0	0	0
Program 11	0	0	0	0	0
Program 12	0	0	0	0	0
Program 13	0	0	0	0	1
Program 14	0	0	0	1	1
Program 15	0	0	0	1	1
Program 16	0	1	1	1	1
Program 17	0	1	1	1	0
Program 18	0	0	1	0	1
Program 19	0	1	1	0	1
Program 20	0	0	1	0	0

Figure 4.6: Uniform distribution simulation when $M = 20000$

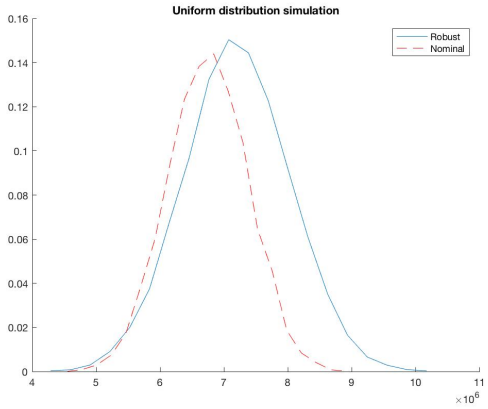
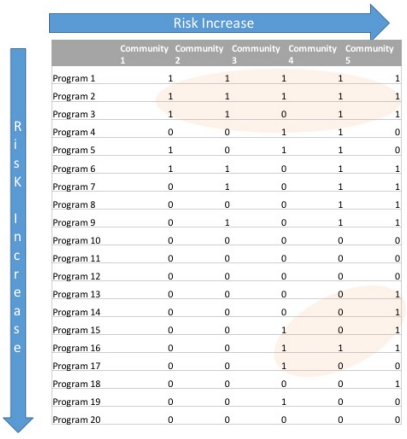


Figure 4.7: Robust model optimal solution when $M = 40000$



	Community 1	Community 2	Community 3	Community 4	Community 5
Program 1	1	1	1	1	1
Program 2	1	1	1	1	1
Program 3	1	1	0	1	1
Program 4	0	0	1	1	0
Program 5	1	0	1	1	0
Program 6	1	1	0	1	1
Program 7	0	1	0	1	1
Program 8	0	0	0	1	1
Program 9	0	1	0	1	1
Program 10	0	0	0	0	0
Program 11	0	0	0	0	0
Program 12	0	0	0	0	0
Program 13	0	0	0	0	1
Program 14	0	0	0	0	1
Program 15	0	0	1	0	1
Program 16	0	0	1	1	1
Program 17	0	0	1	0	0
Program 18	0	0	0	0	1
Program 19	0	0	1	0	0
Program 20	0	0	0	0	0

Figure 4.8: Uniform distribution simulation when $M = 40000$

When M is equal to 50,000, all the programs the robust model chooses are low risk program-community pairs, which leads the upside deviation of the simulated total benefits very small. At the same time, the robust optimal portfolio also has a smaller downside deviation compared to the nominal optimal portfolio.

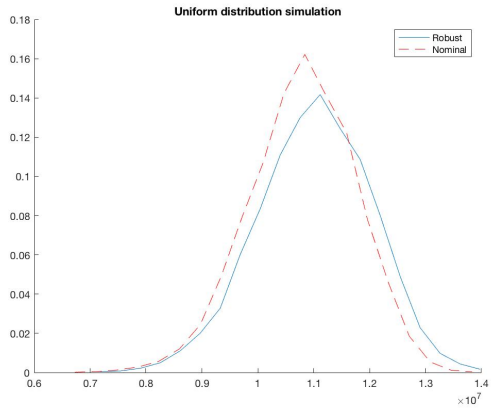
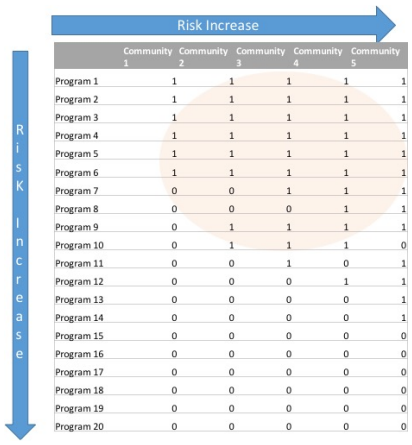


Figure 4.9: Robust model optimal solution when $M = 50000$



	Community 1	Community 2	Community 3	Community 4	Community 5
Program 1	1	1	1	1	1
Program 2	1	1	1	1	1
Program 3	1	1	1	1	1
Program 4	1	1	1	1	1
Program 5	1	1	1	1	1
Program 6	1	1	1	1	1
Program 7	0	0	1	1	1
Program 8	0	0	0	1	1
Program 9	0	1	1	1	1
Program 10	0	1	1	1	0
Program 11	0	0	1	0	1
Program 12	0	0	0	1	1
Program 13	0	0	0	0	1
Program 14	0	0	0	0	1
Program 15	0	0	0	0	0
Program 16	0	0	0	0	0
Program 17	0	0	0	0	0
Program 18	0	0	0	0	0
Program 19	0	0	0	0	0
Program 20	0	0	0	0	0

Figure 4.10: Uniform distribution simulation when $M = 50000$

4.3.2 Sensitivity Analysis of Upside and Downside Uncertainty Budget

In this section we analyze how the upside and downside uncertainty budget influence the optimal solution. We are interested to see as uncertainty budget varies, what types of programs should be chosen and how that will change the realized benefits. We first gradually increase the upside risk budgets in all the market from 1 to 10 and see that there is a clear trend.

As indicated in Figure (4.11) - (4.14), as the upside risk protection increase, robust model choose riskier program-community pairs and the simulated benefits shift towards the right side. This is because when investing in programs with high potential return, larger benefits are realized in the simulation, which also made up for the loss of the downside deviation, leading the overall benefit distribution shifting towards the right hand side.

We next take a look at the downside uncertainty budget. As the downside uncertainty budget increase, the model becomes more and more conservative and gradually adds in low-risk programs and takes away the high-risk programs.

As shown in Figure (4.15)-(4.18), when the downside uncertainty budget in all market is

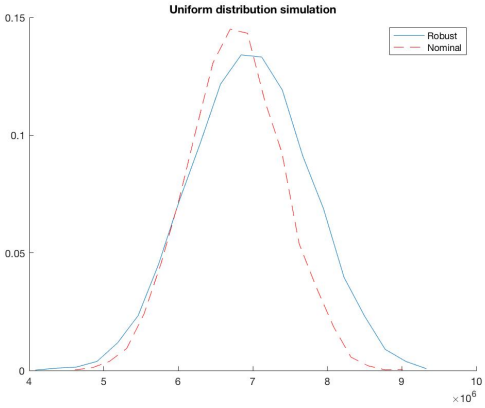
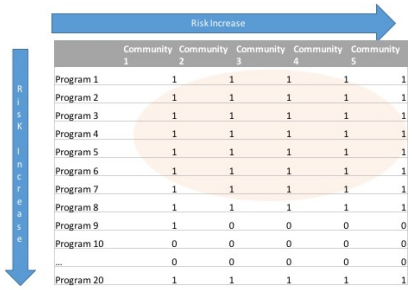


Figure 4.11: Robust model optimal solution when all the upside uncertainty protection level is 1



	Community 1	Community 2	Community 3	Community 4	Community 5
Program 1	1	1	1	1	1
Program 2	1	1	1	1	1
Program 3	1	1	1	1	1
Program 4	1	1	1	1	1
Program 5	1	1	1	1	1
Program 6	1	1	1	1	1
Program 7	1	1	1	1	1
Program 8	1	1	1	1	1
Program 9	1	0	0	0	0
Program 10	0	0	0	0	0
...	0	0	0	0	0
Program 20	1	1	1	1	1

Figure 4.12: Uniform distribution simulation when all the upside uncertainty protection level is 1

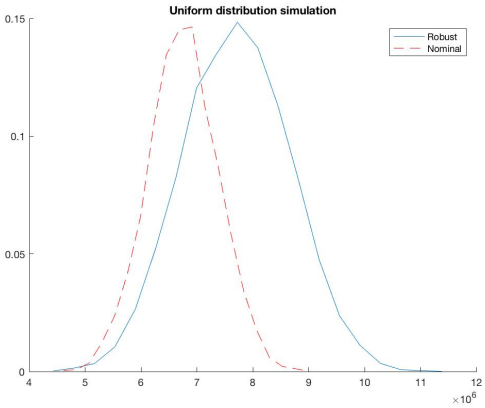
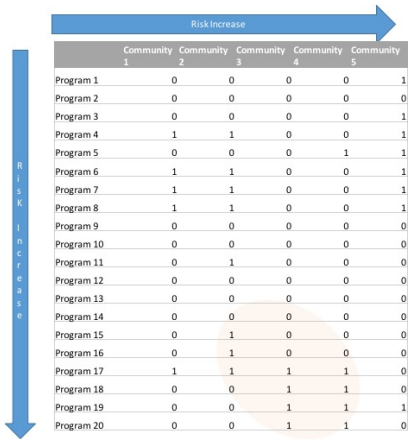


Figure 4.13: Robust model optimal solution when all the upside uncertainty protection level is 10



	Community 1	Community 2	Community 3	Community 4	Community 5
Program 1	0	0	0	0	1
Program 2	0	0	0	0	0
Program 3	0	0	0	0	1
Program 4	1	1	0	0	1
Program 5	0	0	0	1	1
Program 6	1	1	0	0	1
Program 7	1	1	0	0	1
Program 8	1	1	0	0	1
Program 9	0	0	0	0	0
Program 10	0	0	0	0	0
Program 11	0	1	0	0	0
Program 12	0	0	0	0	0
Program 13	0	0	0	0	0
Program 14	0	0	0	0	0
Program 15	0	1	0	0	0
Program 16	0	1	0	0	0
Program 17	1	1	1	1	0
Program 18	0	0	1	1	0
Program 19	0	0	1	1	1
Program 20	0	0	1	1	0

Figure 4.14: Uniform distribution simulation when all the upside uncertainty protection level is 10

1, a lot of the high risk programs are chosen since only one of them will realize the worst benefit in the model, with this solution the simulated benefit of robust optimal portfolio has a slightly larger fatter tails on the downside. When the downside uncertainty is 10, the model becomes a lot more conservative since at most 10 of the programs will realize the worst scenario, and thus in order to satisfy the worst performance tolerance level most of the programs the model choose are the low-risk programs.

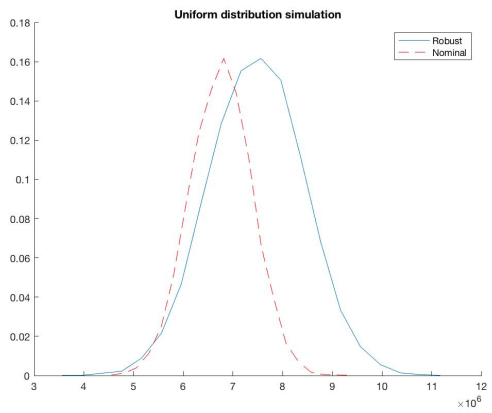
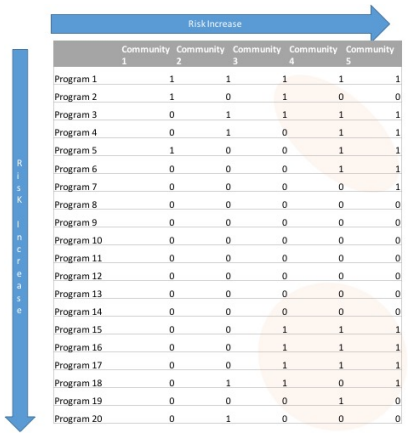


Figure 4.15: Robust model optimal solution when all the downside uncertainty protection level is 1



	Community 1	Community 2	Community 3	Community 4	Community 5
Program 1	1	1	1	1	1
Program 2	1	0	1	0	0
Program 3	0	1	1	1	1
Program 4	0	1	0	1	1
Program 5	1	0	0	1	1
Program 6	0	0	0	1	1
Program 7	0	0	0	0	1
Program 8	0	0	0	0	0
Program 9	0	0	0	0	0
Program 10	0	0	0	0	0
Program 11	0	0	0	0	0
Program 12	0	0	0	0	0
Program 13	0	0	0	0	0
Program 14	0	0	0	0	0
Program 15	0	0	1	1	1
Program 16	0	0	1	1	1
Program 17	0	0	1	1	1
Program 18	0	1	1	0	1
Program 19	0	0	0	1	0
Program 20	0	1	0	0	0

Figure 4.16: Uniform distribution simulation when all the downside uncertainty protection level is 1

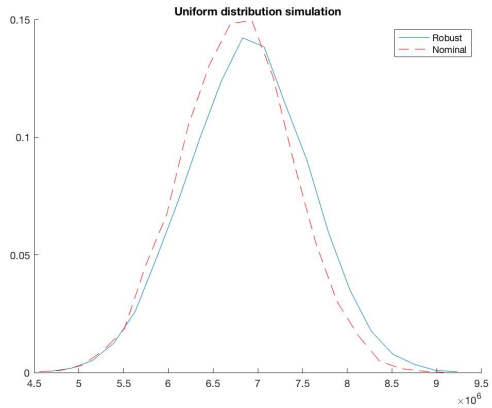
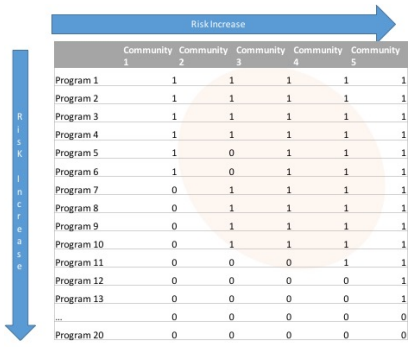


Figure 4.17: Robust model optimal solution when all the downside uncertainty protection level is 10



	Community 1	Community 2	Community 3	Community 4	Community 5
Program 1	1	1	1	1	1
Program 2	1	1	1	1	1
Program 3	1	1	1	1	1
Program 4	1	1	1	1	1
Program 5	1	0	1	1	1
Program 6	1	0	1	1	1
Program 7	0	1	1	1	1
Program 8	0	1	1	1	1
Program 9	0	1	1	1	1
Program 10	0	1	1	1	1
Program 11	0	0	0	1	1
Program 12	0	0	0	0	1
Program 13	0	0	0	0	1
...	0	0	0	0	0
Program 20	0	0	0	0	0

Figure 4.18: Uniform distribution simulation when all the downside uncertainty protection level is 10

4.3.3 Sensitivity Analysis of Actual Upside and Downside Deviation

In this section, we analyze how well the robust model performs, compared to the normal model, when the actual number of projects that deviates from the estimated parameter is different from decision maker's upside and downside deviation tolerance level. Obviously,

when the actual number of deviated programs is larger than the decision makers' estimates, robust model will perform better than the nominal model due to its robust properties. It's unclear that when fewer number of programs deviate from the estimates, upside or downside, how strong the robust model performs compared to the nominal model since the robust model may become too conservative.

For the upside deviation, we let the actual number of deviated programs gradually in-

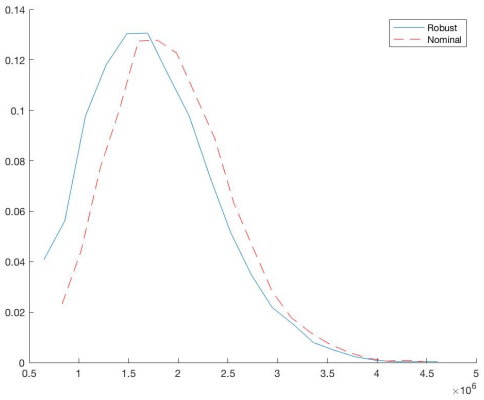


Figure 4.19: Simulated result when [2,2,2,2,2] programs deviate upward

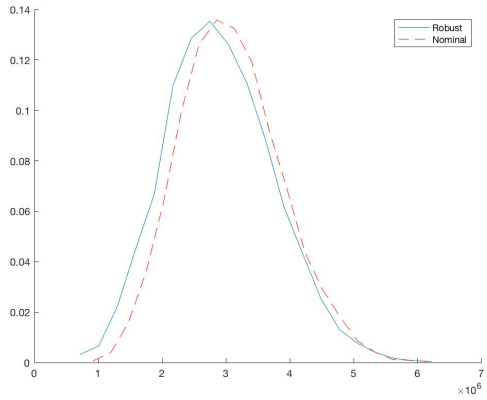


Figure 4.20: Simulated result when [2,4,4,4,4] programs deviate upward

crease from $[2, 2, 2, 2, 2]$ to $[2, 4, 6, 8, 10]$. Here deviation refers to taking extreme values. So when the number of programs deviated in community j is n then n number of programs in community j will take extreme values, which is simulated as $(1 + \delta)(\overline{CF}_{ij} + \widehat{CF}_{ij})N(0, 1)$. The benefit of the rest of the programs follows distribution $(1 + \delta)\overline{CF}N(0, 1)$.

As shown in Figure (4.19) - (4.22), when very few programs actually deviates upward from the expected value, the robust model performs worse than the nominal model with a fatter left tail, lower expected value and thinner right tail. This is because the model choose more high-potential programs which also has a larger developing cost, thus the overall number of programs implemented are lower. When the upside potential is not realized, the robust model has a lower expected value and even lower right hand deviation. As more programs deviate upward from the expected value, the distribution of the total benefits shift on the

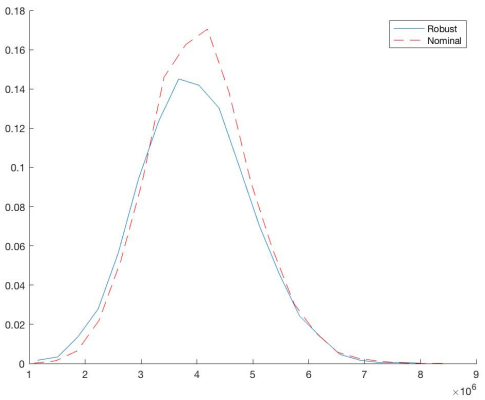


Figure 4.21: Simulated result when [2,4,6,6,6] programs deviate upward

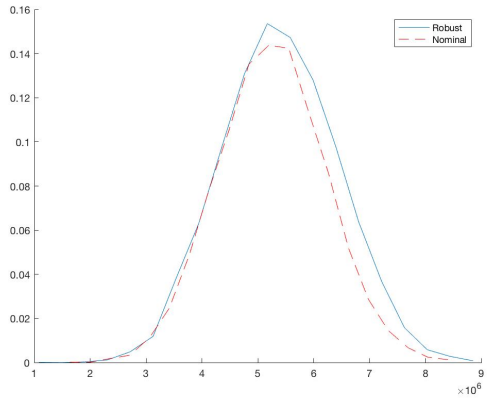


Figure 4.22: Simulated result when [2,4,6,8,10] programs deviate upward

right side and the advantage of the robust model starts to show. When [24666] number of programs deviate upward, which is still lower than the upside risk budget, robust model performs almost the same as the nominal model. When the number of up-deviated programs is equal to the upside risk budget, the robust model performs better than the nominal model.

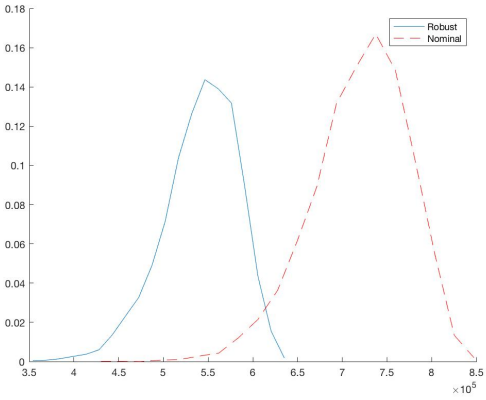


Figure 4.23: Simulated result when [1,1,1,1,1] programs deviate downward

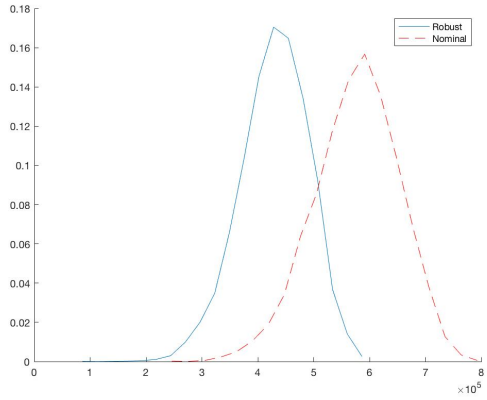


Figure 4.24: Simulated result when [1,3,5,5,5] programs deviate downward

Similarly, for the downside deviation, we let the number of actually deviated programs

increase from $[1, 1, 1, 1, 1]$ towards $[10, 10, 10, 10, 10]$. When the benefit of the program deviates, it follows distribution $(1 + \delta)(\overline{CF}_{ij} - \widehat{CF'}_{ij})N(0, 1)$. The benefit of the rest of the programs follows distribution $(1 + \delta)\overline{CF}N(0, 1)$. We found that the robust model performance is more sensitive to the number of downside deviation and its advantage is slower to show.

As shown in Figure (4.23) to (4.28), when actually only 1 program in each community

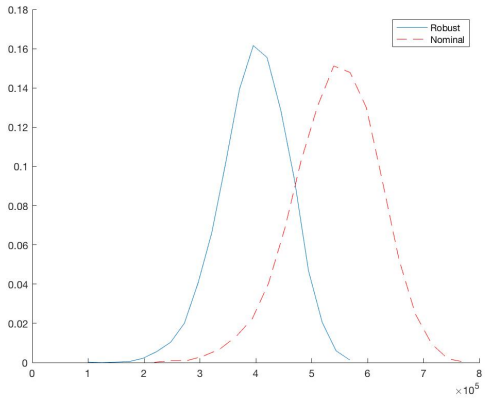


Figure 4.25: Simulated result when [1,3,5,7,9] programs deviate downward

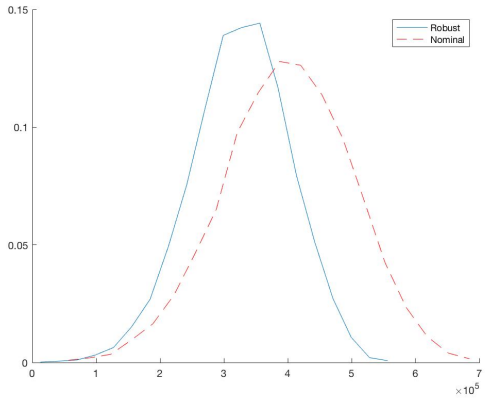


Figure 4.26: Simulated result when [5,5,5,7,9] programs deviate downward

might take the worst case scenario value, robust model performs a lot worse than the nominal model. This is because when most of the benefits equal to the expected value, the robust solution is not optimal and thus have lower overall benefits than the nominal model. Another reason is that we want to solely look at how well the robust model protect the decision makers from the downside risk and thus don't include the benefit of upside risk budget. Therefore, even when the actually downward deviated programs equal to the downside risk budget, the robust model is still stochastically dominated by the nominal model. Only when the downward deviated programs increase to $[9, 9, 9, 9, 9]$, the robust model thinner left hand tail and a higher expectation than the nominal model.

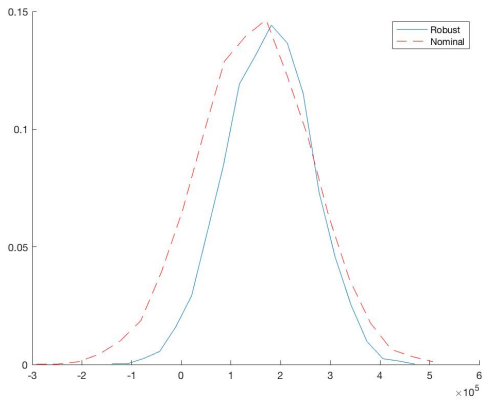


Figure 4.27: Simulated result when [9,9,9,9,9] programs deviate downward

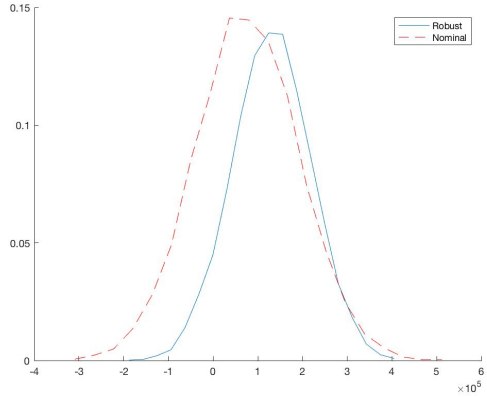


Figure 4.28: Simulated result when [10,10,10,10,10] programs deviate downward

4.4 Conclusion

Due to the tax exempt status, most hospitals are encouraged to focus on community benefit investments to reduce unnecessary use of emergency departments and hospital readmission. In this paper we study the uncertainties of community health programs and build a robust optimization model to help health institutions maximize the total benefits out of their investment under their worst performance tolerance level. Through the method of simulation, we analyze the effect of worst performance tolerance level, upside uncertainty budget and downside uncertainty budget on the optimal program portfolio and the overall performance.

Chapter 5

Conclusions

This dissertation presents innovative applications of robust optimization for long-term decision-making. The first chapter focuses on Value Based Insurance Design (VBID) in the context of heart disease treatment. The high-level aim of VBID is to provide incentives for patients to better align their behavior with the system-level optimum of medication adherence and early (as opposed to late and more expensive) treatment. Our goal is to investigate a VBID approach with different cost-sharing parameters for low risk and high risk patients, in order to achieve a tradeoff for current and future costs for patients while improving patients life quality. The second chapter analyzes via simulation two mathematical modeling frameworks that reflect different managerial attitudes toward upside risk in the context of R&D portfolio selection. The manager seeks to allocate a development budget between low-risk, low-reward projects, called incremental projects, and high-risk, high-reward projects, called innovational projects. We study the differences in strategy and portfolios risk profile that arise between a risk-aware manager, who takes upside risk because he has to for the long-term competitive advantage of his company, and a risk-seeking manager, who will take as big a bet as allowed by the model. The third chapter studies hospitals optimal strategies of building community health program portfolio in order to achieve the maximum potential benefits under a worst case benefit tolerance level. Our

model incorporates the fact that hospitals might have tolerances for upside and downside deviation and thus different uncertainty budgets for upside risk and downside risk and analyzes how key parameters influence the optimal portfolio and implement our approach in a numerical example with promising and insightful results.

Bibliography

[1] Levi, J., Segal, L. M., & Juliano, C. (2008). Prevention for a healthier America: Investments in disease prevention yield significant savings, stronger communities. Trust for America's Health.

[2] Appleby, J., & Carty, S. S. (2005). Ailing GM looks to scale back generous health benefits. USA Today, 24.

[3] Reid, T.R., The Healing of America: A Global Quest for Better, Cheaper, and Fairer Health Care, New York, The Penguin Press, 2009, 277p

[4] Nolte, E., & McKee, C. M. (2008). Measuring the health of nations: updating an earlier analysis. Health affairs, 27(1), 58-71.

[5] Schoen, C., Davis, K., How, S. K., & Schoenbaum, S. C. (2006). US health system performance: a national scorecard. Health Affairs, 25(6), w457-w475.

[6] Kids, A., Breathe, R. T., & First, C. Asthma and Schools.

[7] Russell, L. B. (2007). Prevention's potential for slowing the growth of medical spending. Washington, DC: National Coalition on Health Care.

[8] Cohen, J. T., Neumann, P. J., & Weinstein, M. C. (2008). Does preventive care save money? health economics and the presidential candidates. New England Journal of Medicine, 358(7), 661-663.

[9] Rosenbaum,S., Rieke, A., & Byrnes,M. (2014). Encouraging Nonprofit Hospitals To Invest In Community Building: The Role Of IRS 'Safe Harbors', http://healthaffairs.org/blog/2014/02/11/encouraging-nonprofit-hospitals-to-invest-in-community-building-the-role-of-irs-safe-harbors/

[10] Young, G. J., Chou, C. H., Alexander, J., Lee, S. Y. D., & Raver, E. (2013). Provision of community benefits by tax-exempt US hospitals. New England Journal of Medicine, 368(16), 1519-1527.

[11] Cammisa,M., Wood, E. (2013) Boston Children's Hospital's Approach to Community Health: Using programs to achieve systemic change. Trust for America's Health.

[12] Eddy, D. M., Schlessinger, L., & Kahn, R. (2005). Clinical outcomes and cost-effectiveness of strategies for managing people at high risk for diabetes. Annals of Internal Medicine, 143(4), 251-264.

[13] Haines DJ, L Davis, P Rancour, M Robinson, T Neel-Wilson, and S Wagner. 2007. "A pilot intervention to promote walking and wellness and to improve the health of college faculty and staff." Journal of American College Health: J of ACH. 55 (4).

[14] Sanigorski, A. M., Bell, A. C., Kremer, P. J., Cuttler, R., & Swinburn, B. A. (2008). Reducing unhealthy weight gain in children through community capacity-building: results of a quasi-experimental intervention program, Be Active Eat Well. International Journal of Obesity, 32(7), 1060-1067.

[15] Simon, C., Schweitzer, B., Oujaa, M., Wagner, A., Arveiler, D., Triby, E., ... & Platat, C. (2008). Successful overweight prevention in adolescents by increasing physical activity: a 4-year randomized controlled intervention. International Journal of Obesity, 32(10), 1489-1498.

[16] Gustat, J., Rice, J., Parker, K. M., Becker, A. B., & Farley, T. A. (2012). Effect of

changes to the neighborhood built environment on physical activity in a low-income African American neighborhood. Preventing chronic disease, 9.

[17] Piatt, G. A., Seidel, M. C., Chen, H. Y., Powell, R. O., & Zgibor, J. C. (2012). Two-year results of translating the diabetes prevention program into an urban, underserved community. The Diabetes Educator, 38(6), 798-804.

[18] Joseph P. Newhouse, & Rand Corporation. Insurance Experiment Group (Eds.). (1993). Free for all?: lessons from the RAND health insurance experiment. Harvard University Press.

[19] Brook, R. H., Keeler, E. B., Lohr, K. N., Newhouse, J. P., Ware, J. E., Rogers, W. H., ... & Reboussin, D. (2006). The health insurance experiment: A classic RAND study speaks to the current health care reform debate. Santa Monica, Calif.: RAND Corporation, RB-9174-HHS.

[20] Goldman, D. P., Joyce, G. F., Escarce, J. J., Pace, J. E., Solomon, M. D., Laouri, M., ... & Teutsch, S. M. (2004). Pharmacy benefits and the use of drugs by the chronically ill. Jama, 291(19), 2344-2350.

[21] Mager, D. E., & Cox, E. R. (2007). Relationship between generic and preferred-brand prescription copayment differentials and generic fill rate. American Journal of Managed Care, 13(6), 347.

[22] Zeber, J. E., Grazier, K. L., Valenstein, M., Blow, F. C., & Lantz, P. M. (2007). Effect of a medication copayment increase in veterans with schizophrenia. American Journal of Managed Care, 13(6), 335.

[30] Tamblyn, R., Laprise, R., Hanley, J. A., Abrahamowicz, M., Scott, S., Mayo, N., ... & Mallet, L. (2001). Adverse events associated with prescription drug cost-sharing among poor and elderly persons. Jama, 285(4), 421-429.

[24] Gibson, T. B., Ozminkowski, R. J., & Goetzel, R. Z. (2005). The effects of prescription drug cost sharing: a review of the evidence. Am J Manag Care, 11(11), 730-740.

[25] Hsu, J., Price, M., Huang, J., Brand, R., Fung, V., Hui, R., ... & Selby, J. V. (2006). Unintended consequences of caps on Medicare drug benefits. New England Journal of Medicine, 354(22), 2349-2359.

[26] Chandra, A., Gruber, J., & McKnight, R. (2007). Patient cost-sharing, hospitalization offsets, and the design of optimal health insurance for the elderly (No. w12972). National Bureau of Economic Research.

[27] Chernew, M. E., & Newhouse, J. P. (2008). What does the RAND Health Insurance Experiment tell us about the impact of patient cost sharing on health outcomes. Am J Manag Care, 14(7), 412-414.

[28] Goldman, D. P., Joyce, G. F., & Karaca-Mandic, P. (2006). Varying pharmacy benefits with clinical status: the case of cholesterol-lowering therapy. American Journal of Managed Care, 12(1), 21.

[29] Lee, J. L., Maciejewski, M. L., Raju, S. S., Shrank, W. H., & Choudhry, N. K. (2013). Value-Based Insurance Design: Quality improvement but no cost savings. Health Affairs, 32(7), 1251-1257.

[30] Tamblyn, R., Laprise, R., Hanley, J. A., Abrahamowicz, M., Scott, S., Mayo, N., ... & Mallet, L. (2001). Adverse events associated with prescription drug cost-sharing among poor and elderly persons. Jama, 285(4), 421-429.

[31] Fendrick, A. M., Smith, D. G., Chernew, M. E., & Shah, S. N. (2001). A benefit-based copay for prescription drugs: patient contribution based on total benefits, not drug acquisition cost. American Journal of Managed Care, 7(9), 861-874.

[32] How the "Income" Tax Rate for Poor People is Higher Than the "Income" Tax Rate for Rich People, accessed at http://lifting-the-veil-of-maya.blogspot.com/ on May 22nd, 2014

[33] Shechter, S. M., Bailey, M. D., Schaefer, A. J., & Roberts, M. S. (2008). The optimal time to initiate HIV therapy under ordered health states. Operations Research, 56(1), 20-33.

[34] Maillart, L. M., Ivy, J. S., Ransom, S., & Diehl, K. (2008). Assessing dynamic breast cancer screening policies. Operations Research, 56(6), 1411-1427.

[35] Denton, B. T., Kurt, M., Shah, N. D., Bryant, S. C., & Smith, S. A. (2009). Optimizing the start time of statin therapy for patients with diabetes. Medical Decision Making, 29(3), 351-367.

[36] Mason, J. E., Denton, B. T., Smith, S. A., & Shah, N. D. (2012). Using Electronic Health Records to Monitor and Improve Adherence to Medication.

[37] Maciejewski, M. L., Bryson, C. L., Perkins, M., Blough, D. K., Cunningham, F. E., Fortney, J. C., ... & Liu, C. F. (2010). Increasing copayments and adherence to diabetes, hypertension, and hyperlipidemic medications. The American journal of managed care, 16(1), e20-34.

[38] Schmittdiel, J., Ettner, S. L., Fung, V., Huang, J., Turk, N., Quiter, E., ... & Mangione, C. M. (2009). The Medicare Part D coverage gap and diabetes beneficiaries. The American journal of managed care, 15(3), 189.

[39] Sokol, M. C., McGuigan, K. A., Verbrugge, R. R., & Epstein, R. S. (2005). Impact of medication adherence on hospitalization risk and healthcare cost. Medical care, 43(6), 521-530.

[40] Rosen, A. B., Hamel, M. B., Weinstein, M. C., Cutler, D. M., Fendrick, A. M., & Vijan, S. (2005). Cost-effectiveness of full medicare coverage of angiotensin-converting enzyme inhibitors for beneficiaries with diabetes. Annals of Internal Medicine, 143(2), 89-99.

[41] Gibson, T. B., Mahoney, J., Ranghell, K., Cherney, B. J., & McElwee, N. (2011). Value-based insurance plus disease management increased medication use and produced savings. Health Affairs, 30(1), 100-108.

[42] Karaca-Mandic, P., Jena, A. B., Joyce, G. F., & Goldman, D. P. (2012). Out-of-pocket medication costs and use of medications and health care services among children with asthma. JAMA, 307(12), 1284-1291.

[43] Maciejewski, M. L., Wansink, D., Lindquist, J. H., Parker, J. C., & Farley, J. F. (2014). Value-Based Insurance Design Program In North Carolina Increased Medication Adherence But Was Not Cost Neutral. Health Affairs, 33(2), 300-308.

[44] Chernew, M. E., Juster, I. A., Shah, M., Wegh, A., Rosenberg, S., Rosen, A. B., ... & Fendrick, A. M. (2010). Evidence that value-based insurance can be effective. Health Affairs, 29(3), 530-536.

[45] Mahoney, J. J. (2005). Reducing patient drug acquisition costs can lower diabetes health claims. Am J Manag Care, 11(5 suppl), S170-S176.

[46] Chernew, M. E., Shah, M. R., Wegh, A., Rosenberg, S. N., Juster, I. A., Rosen, A. B., ... & Fendrick, A. M. (2008). Impact of decreasing copayments on medication adherence within a disease management environment. Health Affairs, 27(1), 103-112.

[47] Smith, J., & Matthisen, J. (2010) Comparative Pricing of the ValueBased Essential Benefit Package.

[48] Choudhry, N. K., Avorn, J., Glynn, R. J., Antman, E. M., Schneeweiss, S., Toscano, M., ... & Shrank, W. H. (2011). Full coverage for preventive medications after myocardial infarction. New England Journal of Medicine, 365(22), 2088-2097.

[49] Murphy, S. L., Xu, J., & Kochanek, K. D. (2013). Deaths: final data for 2010. National vital statistics reports, 61(4).

[50] Go, A. S., Mozaffarian, D., Roger, V. L., Benjamin, E. J., Berry, J. D., Blaha, M. J., ... & Turner, M. B. (2014). Heart Disease and Stroke Statistics2014 Update A Report From the American Heart Association. Circulation, 129(3), e28-e292

[51] About Heart Disease & Stroke. Last modified May 20, 2014. http://millionhearts.hhs.gov/abouthds/risk-factors.html

[52] Jackevicius, C. A., Mamdani, M., & Tu, J. V. (2002). Adherence with statin therapy in elderly patients with and without acute coronary syndromes. Jama, 288(4), 462-467.

[53] Simes, J., Furberg, C. D., Braunwald, E., Davis, B. R., Ford, I., Tonkin, A., & Shepherd, J. (2002). Effects of pravastatin on mortality in patients with and without coronary heart disease across a broad range of cholesterol levels. The Prospective Pravastatin Pooling project. European heart journal, 23(3), 207-215.

[54] Shepherd, J., Cobbe, S. M., Ford, I., Isles, C. G., Lorimer, A. R., Macfarlane, P. W., ... & Packard, C. J. (1995). Prevention of coronary heart disease with pravastatin in men with hypercholesterolemia. New England Journal of Medicine, 333(20), 1301-1308.

[55] Downs, J. R., Clearfield, M., Weis, S., Whitney, E., Shapiro, D. R., Beere, P. A., ... & AFCAPS/TexCAPS Research Group. (1998). Primary prevention of acute coronary events with lovastatin in men and women with average cholesterol levels: results of AFCAPS/TexCAPS. Jama, 279(20), 1615-1622.

[56] Schultz, J. S., ODonnell, J. C., McDonough, K. L., Sasane, R., & Meyer, J. (2005). Determinants of compliance with statin therapy and low-density lipoprotein cholesterol goal attainment in a managed care population. Am J Manag Care, 11(5), 306-31

[57] Ellis, J. J., Erickson, S. R., Stevenson, J. G., Bernstein, S. J., Stiles, R. A., & Fendrick, A. M. (2004). Suboptimal statin adherence and discontinuation in primary and secondary prevention populations. Journal of general internal medicine, 19(6), 638-645.

[58] Benner, J. S., Glynn, R. J., Mogun, H., Neumann, P. J., Weinstein, M. C., & Avorn, J. (2002). Long-term persistence in use of statin therapy in elderly patients. Jama, 288(4), 455-461.

[59] Finkelstein, E. A., Khavjou, O., & Will, J. C. (2006). Cost-effectiveness of WISE-WOMAN, a program aimed at reducing heart disease risk among low-income women. Journal of Women's Health, 15(4), 379-389.

[60] Richard, P., West, K., & Ku, L. (2012). The return on investment of a Medicaid tobacco cessation program in Massachusetts. PloS one, 7(1), e29665.

[61] Lightwood, J., & Glantz, S. A. (2013). The effect of the California tobacco control program on smoking prevalence, cigarette consumption, and healthcare costs: 19892008. PloS one, 8(2), e47145.

[62] Roux, L., Pratt, M., Tengs, T. O., Yore, M. M., Yanagawa, T. L., Van Den Bos, J., ... & Buchner, D. M. (2008). Cost effectiveness of community-based physical activity interventions. American journal of preventive medicine, 35(6), 578-588.

[63] Bedimo, A. L., Pinkerton, S. D., Cohen, D. A., Gray, B., & Farley, T. A. (2002). Condom distribution: a costutility analysis. International journal of STD & AIDS, 13(6), 384-392.

[64] Findley, S. E., Thomas, G., Madera-Reese, R., McLeod, N., Kintala, S., Martinez, R. A., ... & Herman, E. (2011). A community-based strategy for improving asthma management and outcomes for preschoolers. Journal of Urban Health, 88(1), 85-99.

[65] Trust for America's Health (TFAH) & The New York Academy of Medicine (NYAM). (2013) A COMPENDIUM OF PROVEN COMMUNITY-BASED PREVENTION PROGRAMS.

[66] Akinbami, O. J., Moorman, J. E., & Liu, X. (2011). Asthma prevalence, health care use, and mortality: United States, 2005-2009. US Department of Health and Human Services, Centers for Disease Control and Prevention, National Center for Health Statistics.

[67] Asthma and Schools, last modified May 25, 2014, http://www.cdc.gov/HealthyYouth/asthma

[68] Weinstock, H., Berman, S., & Cates, W. (2004). Sexually transmitted diseases among American youth: incidence and prevalence estimates, 2000. Perspectives on sexual and reproductive health, 36(1), 6-10.

[69] HIV in the United States, At a Glance, last modified May 25, 2014, http://www.cdc.gov/hiv/statistics/basics/ataglance.html

[70] Male Latex Condoms and Sexually Transmitted Diseases: Fact Sheet for Public Health Personnel, last modified May 25, 2014, http://www.cdc.gov/condomeffectiveness/latex.htm

[71] Sikkema, K. J., Anderson, E. S., Kelly, J. A., Winett, R. A., Gore-Felton, C., Roffman, R. A., ... & Brondino, M. J. (2005). Outcomes of a randomized, controlled community-level HIV prevention intervention for adolescents in low-income housing developments. Aids, 19(14), 1509-1516.

[72] Injury and Violence Prevention, accessed May 27, 2014, http://www.healthypeople.gov/2020/topicsobjectives2020/overview.aspx?topicid=24.

[73] DiMaggio, C., & Li, G. (2013). Effectiveness of a safe routes to school program in preventing school-aged pedestrian injury. Pediatrics, 131(2), 290-296.

[74] "Community Health Needs Assessment And Community Health Strategic Plan of UPMC Bedford Memorial", accessed June 2nd, 2014, http://www.upmc.com/about/community-commitment/Documents/bedford-community-health-needs-assessment.pdf

[75] Bertsimas, D., & Sim, M. (2003). Robust discrete optimization and network flows. Mathematical programming, 98(1-3), 49-71.

[76] Rodriguez, D. (1998). Decisions of pharmaceutical firms for new product development. Industrial Performance Center, Massachusetts Institute of Technology.

[77] Griffin, A. (1997). PDMA research on new product development practices: Updating trends and benchmarking best practices. Journal of product innovation management, 14(6), 429-458.,

[78] DiMasi, J. A., & Grabowski, H. G. (2007). The cost of biopharmaceutical R&D: is biotech different?. Managerial and Decision Economics, 28(45), 469-479.

[79] Munos, B. (2009). Lessons from 60 years of pharmaceutical innovation. Nature Reviews Drug Discovery, 8(12), 959-968.

[80] Garnier, J. P. (2008). Rebuilding the R& D engine in big pharma. Harvard business review, 86(5), 68-70.

[81] Rosenblatt, M. (2013). How Academia and the Pharmaceutical Industry Can Work Together: The Presidents Lecture, Annual Meeting of the American Thoracic Society, San Francisco, California. Annals of the American Thoracic Society, 10(1), 31-38.

[82] Kahn, K. B. (2002). An exploratory investigation of new product forecasting practices. Journal of Product Innovation Management, 19(2), 133-143.

[83] Cooper, R. G. (2012). Winning at new products.

[84] Pharma, I. M. A. P. (2011) Biotech Industry Global Report 2011. IMAP,44.

[85] Gabrel, V., Murat, C., & Thiele, A. (2014). Recent advances in robust optimization: An overview. European Journal of Operational Research, 235(3), 471-483.

[86] Kim, W. C., & Mauborgne, R. (1999). Strategy, value innovation, and the knowledge economy. MIT Sloan Management Review, 40(3), 41.

[87] Day, G. S. (2007). Is it real? Can we win? Is it worth doing. Harvard business review, 85(12), 110-120.

[88] Wuyts, S., Dutta, S., & Stremersch, S. (2004). Portfolios of interfirm agreements in technology-intensive markets: Consequences for innovation and profitability. Journal of marketing, 68(2), 88-100.

[89] Ding, M., Eliashberg, J., & Stremersch, S. (2014). Innovation and marketing in the pharmaceutical industry. Emerging Practices, Research, and Policies. New York: Spring Science. Print.

[90] Roussel, P. A., Saad, K. N., & Erickson, T. J. (1991). Third generation R& D: managing the link to corporate strategy. Harvard Business Press.

[91] Wheelwright, S. C., & Clark, K. B. (1992). Revolutionizing product development: quantum leaps in speed, efficiency, and quality. Simon and Schuster.

[92] Cooper, R. G., Edgett, S. J., & Kleinschmidt, E. J. (2001). Portfolio management for new products. Basic Books.

[93] Kavadias, S., & Chao, R. O. (2007). Resource allocation and new product development portfolio management. Handbook of new product development management, 135-163.

[94] Loch, C. H., Pich, M. T., Terwiesch, C., & Urbschat, M. (2001). Selecting R& D projects at BMW: A case study of adopting mathematical programming models. IEEE Transactions on Engineering Management, 48(1), 70-80.

[95] Brenner, M. S. (1994). Practical R&D project prioritization. Research-Technology Management, 37(5), 38-42.

[96] Dickinson, M. W., Thornton, A. C., & Graves, S. (2001). Technology portfolio management: optimizing interdependent projects over multiple time periods. IEEE Transactions on engineering management, 48(4), 518-527.

[97] Ding, M., & Eliashberg, J. (2002). Structuring the new product development pipeline. Management Science, 48(3), 343-363.

[98] Blanford, G. J. (2004). Dynamic Allocation of R & D Investments Between Uncertainty Resolution and Technology Development. Stanford MS& EE Working Paper.

[99] Chao, R. O., & Kavadias, S. (2008). A theoretical framework for managing the new product development portfolio: when and how to use strategic buckets. Management Science, 54(5), 907-921.

Appendix A

Mathematical Details

A.1 Proof of Equation (2.1)

Set the derivative to 0 we have:

$$\beta[A(1-a)-1]-1 = -\beta x + e^{\alpha-\beta x}$$

Let $\rho = \frac{\pi(0)}{1-\pi(0)}$, $s = \frac{\pi(1)}{1-\pi(1)}$ and $y = \beta x$. This yields

$$(\frac{\pi(0)}{1-\pi(0)} - \frac{\pi(1)}{1-\pi(1)})[A(1-a)-1]-1 = -\beta x + e^{\alpha-\beta x}. \tag{A.1}$$

Let $\rho' = \rho + d\rho$, $y = y + dy$. Then we have

$$\ln \rho' = \ln\left(\rho + \frac{d\rho}{\rho}\right)$$
$$e^{y'} = e^{y} + dy(-e^{-y})$$

Substituting into equation (A.1) and simplifying the equation we obtain:

$$d\rho[\frac{A(1-a)-1}{\rho} - e^{-y}] = -dy(1+\rho e^{-y})$$

Or equivalently:

$$\frac{d\rho}{dy} = \frac{1 + \rho e^{-y}}{e^{-y} - \frac{A(1-a)-1}{\rho}}$$

Since $1 + \rho e^{-y} \geq 0$, x satisfying the following equation

$$\rho e^{-y} = \frac{\phi(0)}{1 - \phi(0)} e^{-\beta x} = A(1-a) - 1$$

is the optimal copayment level x^*. This equation can be rewritten as

$$-\beta x^* = \ln\left(\frac{\phi(0)}{1 - \phi(0)} - [A(1-a) - 1]\right)$$

which leads to:

$$x^* = \frac{\ln\left(\left(\frac{\phi(0)}{1-\phi(0)}\right) - [A(1-a) - 1]\right)}{\ln\left(\frac{\phi(0)}{1-\phi(0)}\right) - \ln\left(\frac{\phi(1)}{1-\phi(1)}\right)}$$

Vita

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