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The development of an economic dispatch methodology taking into account voltage constraints.

Donald St Allen

THE DEVELOPMENT OF AN ECONOMIC DISPATCH METHODOLOGY TAKING INTO ACCOUNT VOLTAGE CONSTRAINTS

By

Donald St. C. Allen

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Chairman of Department

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NOMENCLATURE

V_i: Complex Voltage at node i.

$|V_i|$: Magnitude of voltage at node i.

δ_i: Phase angle of voltage at node i in radians.

S_{Gi}: Complex power generation at node i.

S_{Di}: Complex power demand at node i.

S_i: Complex net power injected at node i.

P_{Gi}: Real power generation at node i.

Q_{Gi}: Reactive power generation at node i.

P_{Di}: Real Power demand at node i.

Q_{Di}: Reactive power demand at node i.

P_i: Real power injected at node i.

Q_i: Reactive power injected at node i.

$\bar{x}$: Column vector of state variables defined as the voltage magnitude and phase angle at each node on the system.

$\bar{p}$: Column vector of disturbance variables defined as the real and reactive power demand at each node on the system.

$\bar{u}$: Column vector of control variables defined as the real and reactive power generation at each node on the system.

C(x,u,p): Cost function or objective function of x,u, and p which is to be minimized.

h(x,u,p): Equality constraint function of x,u and p which must be observed.

g(x,u,p): Inequality constraint function of x,u, and p be which must be observed.

$\frac{\partial}{\partial x_i}$: Partial derivative of a function with respect to a variable x_i

J_i: Complex current injected into node i (or sum of current flowing out of node i).

X_c: Capacitive reactance of one half of a transmission line.

X_L: Inductive reactance of a transmission line.

Z_{ser}: Series impedance of a transmission line.

Y_{SH}: Shunt admittance of a transmission line.

j: $\sqrt{-1}$ used to denote imaginary component of complex number.

Y_{BUS}: Bus admittance matrix for a network.

λ: Lagrangian multiplier

L(x,u,p): Lagrangian function of x,u and p (chapter III).

∇f: Gradient vector f (chapter III).

J: Jacobian (Chapter VI).

ABSTRACT

Economic dispatch of electric generation to meet load has to date been accomplished without too much regard to electrical constraints on transmission systems. Constraints due to reliability and system integrity were considered only from a planning standpoint. That is, transmission systems were planned to ensure that the load could be met without violating thermal limits or limits on state variables such as voltage (magnitude and phase angle) and reactive power flow.

Recent developments such as the rapid escalation in oil prices have resulted in unusually large power transfers in one direction in an eastern power pool causing unforeseen violations of transmission limitations on this system. Working with present available tools and methodologies requires the dispatcher to rely on advance off-line AC load flows to determine the transfer capabilities of the system, translate the reactive limitations into megawatt transfer levels which can then be included in the classic economic dispatch program. Ideally, the dispatcher would like to be able to know that his generation is dispatched at the economic minimum within the electrical constraints of the system.

This thesis proposes a method to determine the economic dispatch within the voltage constraints imposed on the system using current optimization techniques and will document the difficulties which arise. This method can be used by the dispatcher to verify that a new generation schedule which has been chosen to remove a voltage constraint violation is the most economic.

CHAPTER I

INTRODUCTION

1.1 General Considerations

Economic dispatch of generating plants to serve a utility's load is of paramount importance in minimizing its cost to serve load. The production costs represent a significant portion of the utilities total cost of providing its service and almost all of its variable cost. Consequently, savings in operating cost can most directly be achieved by minimizing the cost of fuel to serve load.

To ensure that customer demand is met in the most economic manner, the fuel burned to serve the real power demand is minimized. Implicit in this method, however, is the assumption that, given the generation schedule thus developed, the configuration of the transmission and distribution facilities is such that the generation can be transmitted to the demand without violating constraints imposed by reliability and system integrity considerations. That is in the steady state the system can be operated within stability limits and any possible perturbations (such as loss of a transmission line or a generating unit) will not cause the system to go unstable.

The motivation for choosing this topic was the observation of the extent to which violation of transmission constraints affect the economic operation of an eastern United States Interconnection. In 1981 off-economic dispatch to remain within transmission limits resulted in additional costs of about $60 million [21]. Although this reflects only a small fraction of the total costs of generation it is still a significant amount. This Eastern Interconnection is one of the largest power pools in the United States which operates under centralized dispatch. It is made up of 11 investor owned utilities serving customers in a 50,000 square mile area covering 5 states and Washington D.C. with over 45,000 megawatts of generating capacity or roughly 9 percent of the national total.

The dispatch philosophy of the pool is that the total customer load will be served by the most economical generation available, regardless of ownership, with internal billing and accounting for power transfers between companies done after the fact. Until recently this concept has been mutually acceptable and took place without strains on the transmission system or the members.

Rapid escalation of oil prices over the last seven years, however, has placed the utilities in the eastern section of the pool which have a large proportion of oil-fired capability into a constant buying position, resulting in large power transfers from west to east. The resulting strain on the transmission system has required splitting the system into up to 5 areas each with its own cost signals. The most frequent is a two area split in which the western companies operate on one cost signal and the eastern companies on another.

The criteria on which the system is split are based essentially on the locations of limitations identified in the transmission system.

This thesis suggests a method for combining an economic dispatch method with the solution of the static load flow equations to develop an optimum (minimum cost) dispatch with these constraints considered. Obviously, the utility of such a method would depend on its adaptability to larger systems than the 5 bus system studied here, and also on the computation time required to obtain a solution on a digital computer. Some load flow programs currently in use have the capability of handling 4,000 Buses or more and up to 7500 lines and still require approximations. The adaptation of this method to systems of this order must be evaluated. The development and testing of such a large scale model is beyond the scope of this work. The need for the development of such a tool is becoming increasingly more evident and should lead to a feasible solution soon.

1.2 The Problem

The difficulty in developing a method of economic dispatch which simultaneously satisfies the constraints imposed by the load flow is two-fold. First, the cost associated with generation is the cost of real power generation and, as can be demonstrated by sensitivity analyses, [2] is only weakly linked to voltage magnitudes and reactive power flow - the constraints which are most readily violated. Second, some of the more popular algorithms which are currently used to solve the static load flow equations (such as Newton-Raphson) require the finding and inversion of the Jacobian Matrix (Reference [3]). Matrix inversion - requires a significant amount of computer time for a system of appreciable size.

In addition to this, the solution of the load flow sometimes requires an iterative technique which may or may not converge for each attempt [2]. The result is that any loop which includes solution of the load flow is going to be dependent on a timely convergence of the load flow algorithm.

Utilities with generation of greatly differing costs spread throughout the system and with constantly varying available generation and transmission lines will sooner or later incur situations which require off-economic operation to support the integrity of the system. There are and will always be situations in which the off-economic generation chosen by the system operator is physically unavoidable. The objective of this thesis is to seek a sub-optimal solution in which and the final dispatch chosen to relieve a potentially unstable operation due to voltage constraint violation will also be the next best economic choice.

This is achieved using an available load flow program coupled with an optimization package available at the Lehigh University Computer Center capable of linear and non-linear optimizations with constraints on dependent as well as independent variables.

A situation where violation of defined voltage constraints will occur at the minimum cost dispatch is used as a test. Off-economic dispatch is chosen to remove the violation and the program determines the minimum cost dispatch while remaining within the voltage constraints.

CHAPTER II

BACKGROUND INFORMATION

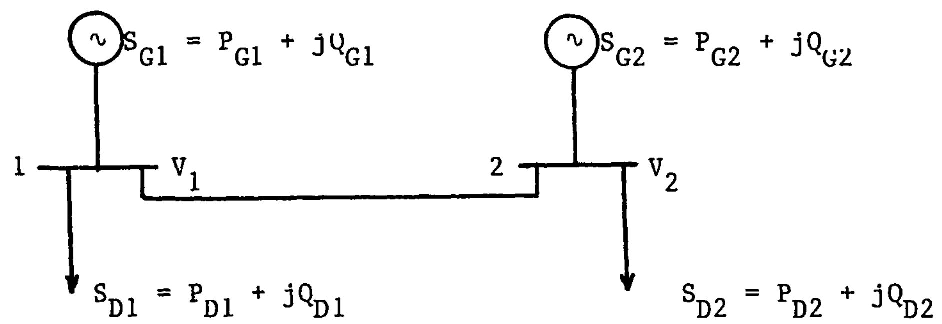
It is necessary to review some of the fundamental concepts of both economic dispatch and load flow prior to discussing the combination of the two.

2.1 Load Flow

In order to review the development and solution method for the static load flow equations, a 2-bus system will be first shown and solved, then a 3-bus system and finally the general method stated for an N-bus system. The explanation used is adapted from reference [2].

2.1.1 Two Bus System

The two bus system shown in Figure 2.1 will be the example.



Two Bus Example

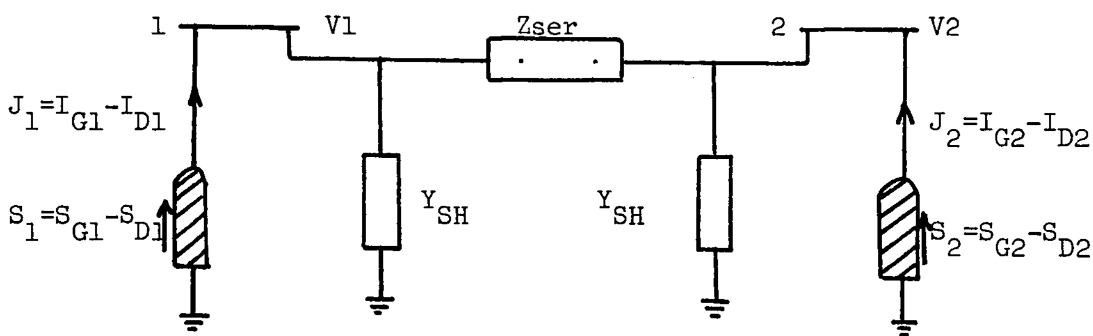
Figure 2.1

A generator is connected to each bus supplying real and reactive power to the bus. Real and reactive loads are tapped from each bus in the amounts S_{D1} and S_{D2}. Both buses are connected by a transmission line whch can be characterized by a series impedance and two shunt admittances. The voltage at the buses are designated V_1 and V_2 respectively.

If we define for convenience the net power S_1 and S_2 being injected into the bus such that

$$S_1 = P_1 + jQ_1 \triangleq P_{G1} - P_{D1} + j(Q_{G1} - Q_{D1}) \quad \ldots.2.1.i$$
$$S_2 = P_2 + jQ_2 \triangleq P_{G2} - P_{D2} + j(Q_{G2} - Q_{G2})$$

the model can be simplified as shown in Figure 2.2.



π Equivalent of Two Bus Example

Figure 2.2

For current balance at Bus 1

$$\frac{S_1^*}{V_1^*} = V_1 Y_{SH} + \frac{V_1 - V_2}{Z_{SER}} \quad \ldots\ldots\ldots\ldots 2.1.iia$$

and at Bus 2

$$\frac{S_2^*}{V_2^*} = V_2 Y_{SH} + \frac{V_2 - V_1}{Z_{SER}} \quad \ldots\ldots\ldots\ldots 2.1.iib$$

The symbol * indicates the complex conjugate of the quantity.

The shunt admittance can be considered purely capacitive since the shunt conductance is usually of negligible magnitude and no reliable formula exists for its determination [2].

$$Y_{SH} = \frac{j}{X_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.1.iii$$

where X_c is the capacitive reactance of half the line. The series impedance can be shown as

$$Z_{SER} = R + jX_L \quad \text{..................2.1.iv}$$

and a loss factor α is defined as [2]

$$\alpha \triangleq \frac{R}{X_L} \quad \text{........................2.1.v}$$

since $\alpha << 1$

$$Z_{ser} \cong X_L e^{j(\pi/2 - 2)} \quad \text{.............2.1.vi}$$

The voltages can be expressed in complex exponent form as

$$V_1 = |V_1|e^{j\delta_1} \quad \text{...................2.1.vii}$$

$$V_2 = |V_2|e^{j\delta_2}$$

Substituting into the current balance equations 2.1.ii and separating real and imaginary components, the static load flow equations become

$$P_{G1} - P_{D1} - \frac{|V_1|^2}{X_L} \sin \alpha + \frac{|V_1||V_2|}{X_L} \sin [\alpha - (\delta_1 - \delta_2)] = 0$$

$$P_{G2} - P_{D2} - \frac{|V_2|^2}{X_L} \sin \alpha + \frac{|V_1||V_2|}{X_L} \sin [\alpha + (\delta_1 - \delta_2)] = 0$$

$$Q_{G1} - Q_{D1} + \frac{|V_1|^2}{X_c} - \frac{|V_1|^2}{X_L} \cos \alpha + \frac{|V_1||V_2|}{X_L} \cos [\alpha - (\delta_1 - \delta_2)] = 0$$

$$Q_{G2} - Q_{D2} + \frac{|V_2|^2}{X_c} - \frac{|V_2|^2}{X_L} \cos \alpha + \frac{|V_1||V_2|}{X_L} \cos [\alpha + (\delta_1 - \delta_2)] = 0$$

.....2.1.viii

The equations are non-linear and are difficult to solve analytically. There are 12 unknowns and only 4 equations which means that assumptions must be made to specify 8 of the unknowns to make solutions possible. This is done by categorizing buses into three types. Since the equations do not allow the solution of individual phase angles but only their differences a reference bus is defined whose phase angle is set to zero. This bus is conventionally bus number 1. The magnitude of the voltage at this bus is also specified. This is referred to as a Type 3 bus.

A Type 1 bus is one at which the power injected at the bus is specified. P_{Di} and Q_{Di} are known and P_{Gi} and Q_{Gi} are specified. A load bus *with* no generation attached falls into this type.

A Type 2 bus is called a voltage control bus and here P_{Di} and Q_{Di} are known and $|V_i|$ and P_{Gi} are specified.

For the two bus example if we specify the following characteristics [2]

$$X_L = 0.1 \text{ PU} \qquad P_{D1} = P_{D2} = 20 \text{ PU}$$
$$X_c = 10 \text{ PU} \qquad Q_{D1} = Q_{D2} = 10 \text{ PU}$$
$$\alpha = 0.1$$

and set bus 1 as the reference bus:

$$|V_1| = 1.0 \text{ p.u.} \qquad \text{Type 3}$$
$$\delta_1 = 0$$

and bus 2 as a voltage control bus:

$$|V_2| = 1.0 \text{ p.u.} \qquad \text{Type 2}$$
$$P_{G2} = 15 \text{ p.u.}$$

Then the equations can be solved, yielding P_{G1}, δ_2, Q_{G1} and Q_{G2}

$$P_{G1} - 20 - \frac{1}{.1} \sin (.1) + \frac{1}{.1} \sin (.1 + \delta_2) = 0$$

$$15 - 20 - \frac{1}{.1} \sin (.1) + \frac{1}{.1} \sin (.1 - \delta_2) = 0$$

$$Q_{G1} - 10 + \frac{1}{10} - \frac{1}{.1} \cos (.1) + \frac{1}{.1} \cos (.1 + \delta_2) = 0$$

$$Q_{G2} - 10 + \frac{1}{10} - \frac{1}{.1} \cos (.1) + \frac{1}{.1} \cos (.1 - \delta_2) = 0$$

Giving

$\delta_2 = -.5433$ rad $= -31.1°$

$P_{G1} = 25.2875$ where $P_{LOSS} = .2875$

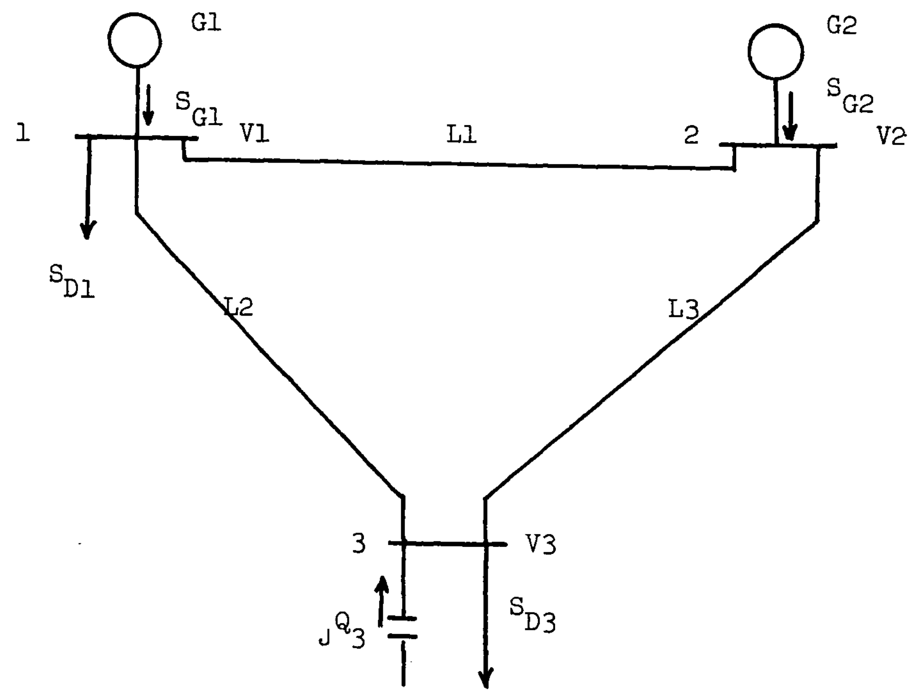
$Q_{G1} = 10.8166$

$Q_{G2} = 11.849$ $Q_{G1} + Q_{G2} = 22.6656$

2.1.2 3 Bus System

In moving to a higher order system it becomes necessary to utilize numerical methods to arrive at a solution. A detailed step by step development of a three bus system is helpful before generalizing to an n-bus system.

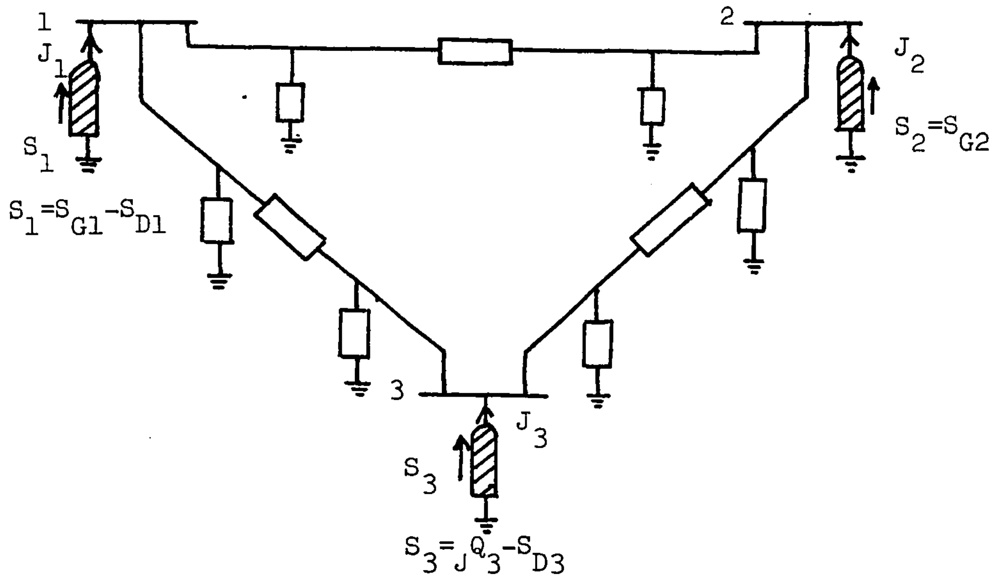
With simple enough assumptions, the 3-bus system shown in Figure 2.3 can be solved analytically but in order to show the general n-bus computer solution method it will be developed for a computer solution.



3 Bus Example

Figure 2.3

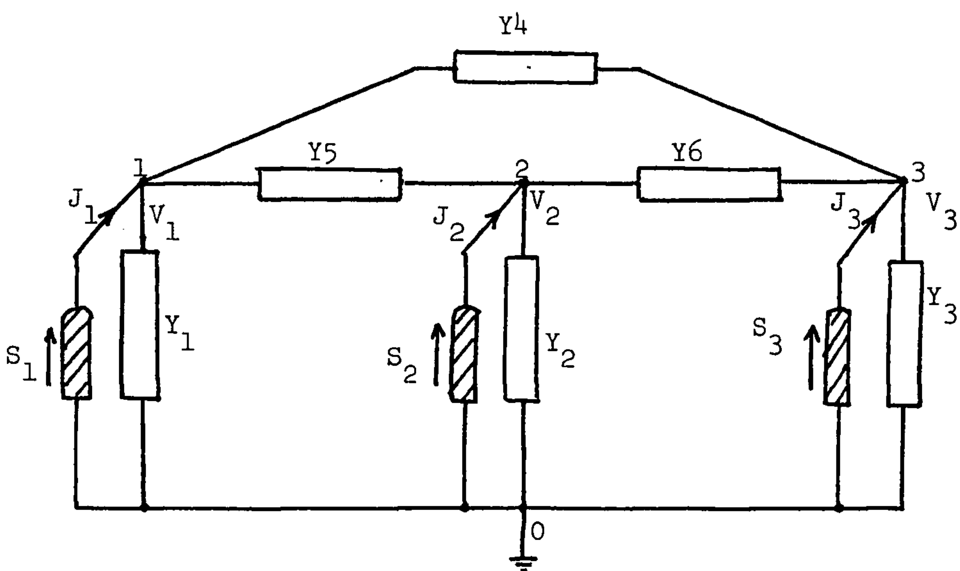
By representing the lines by the π equivalents and summing the generation and the demand into a bus power source the network in Figures 2.4 and 2.5 is obtained.



π Equivalent of 3 Bus Example

Figure 2.4

or



π Equivalent Lumped and Redrawn

Figure 2.5

By Kirchoff's Current Law (KCL) the sum of the currents entering the three nodes must be zero.

$$J_1 = V_1Y_1 + (V_1 - V_2)Y_5 + (V_1 - V_3)Y_4$$

$$J_2 = V_2Y_2 + (V_2 - V_1)Y_5 + (V_2 - V_3)Y_6 \quad ....2.1.ix$$

$$J_3 = V_3Y_3 + (V_3 - V_1)\, Y_4 + (V_3 - V_2)Y_6$$

by defining the following admittances [2]

$$y_{11} \triangleq Y_1 + Y_4 + Y_5$$

$$y_{22} \triangleq Y_2 + Y_5 + Y_6$$

$$y_{33} \triangleq Y_3 + Y_4 + Y_6 \quad ....2.1.x$$

$$y_{23} = y_{32} \triangleq - Y_6$$

$$y_{13} = y_{31} \triangleq - Y_4$$

$$y_{12} = y_{21} \triangleq - Y_5$$

Equation 2.1.ix becomes:

$$J_1 = y_{11}V_1 + y_{12}V_2 + y_{13}V_3$$
$$J_2 = y_{21}V_1 + y_{22}V_2 + y_{23}V_3 \qquad \text{.................. 2.1.xi}$$
$$J_3 = y_{31}V_1 + y_{32}V_2 + y_{33}V_3$$

In general form these equations can be written

$$J_{BUS} = Y_{BUS}V_{BUS}$$

where,

$$J_{BUS} \triangleq \begin{bmatrix} J_1 \\ \cdot \\ \cdot \\ \cdot \\ \\ J_n \end{bmatrix}$$ Bus current vector 2.1.xii

$$V_{BUS} \triangleq \begin{bmatrix} V_1 \\ \cdot \\ \cdot \\ \cdot \\ V_n \end{bmatrix}$$ Bus voltage vector..........2.1.xiii

$$Y_{BUS} \triangleq \begin{bmatrix} y_{11} & y_{in} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ y_{ni} & y_{nn} \end{bmatrix} \quad \text{Nodal Bus Admittance Matrix} \qquad \ldots\ldots 2.1.xiv$$

The complex conjugate of each bus current can be represented by the quotient of the injected power at the bus and the voltage at the bus or $J_i^* = S_i/V_i$.

Each component of the general equations can be represented in the form

$$\frac{P_i - jQ_i}{V_i^*} = y_{i1} V_1 + y_{i2}V_2 + \ldots\ldots y_{in}V_n \quad i = 1,2 \ldots n \qquad 2.1.xv$$

or

$$P_i - jQ_i - y_{i1}V_1V_i^* - y_{i2}V_2V_i^* - \ldots\ldots - y_{in}V_nV_i^* = 0 \quad i = 1, 2, \ldots n \qquad \ldots. 2.1.xvi$$

The Y_{BUS} matrix is developed using the rule that [2] the diagonal elements are the algebraic sum of all admittances incident to that node and the off diagonal elements $y_{ij} = y_{ji}$ are the negative of the admittance connecting node i to node j.

In order to compute the Y_{BUS} matrix from the primitive admittance Matrix Y, a bus incidence matrix A is developed by establishing a tree from a linear network graph of the system [2]. This would give an independent set of equations. The A matrix would be a bxt matrix where b is the number of branches of the linear network graph and t is the number of tree branches. The elements are chosen as follows:

$a_{ij} = 1$ if the i^{th} branch is incident to and oriented away from the j^{th} node or bus

$a_{ij} = -1$ if the i^{th} branch is incident to and oriented toward the j^{th} node or bus

$a_{ij} = 0$ if the i^{th} branch is not incident to the j^{th} node or bus

From the 3-bus example above, based on the orientation of branches from low to high numbered buses except in the reference bus case (branches incident to the reference or ground bus numbered 0 is oriented toward that bus), the bus incidence matrix A becomes

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

This Y_{BUS} matrix can be calculated from the primitive admittance matrix Y by the relationship

$$Y_{BUS} = A^T YA \;\ldots\ldots\ldots\ldots 2.1.xvii$$

so that the general equation can be rewritten

$$A^T YA \; V_{BUS} = J_{BUS} \ldots\ldots\ldots 2.1.xviii$$

Thus the computer can be used to develop the Y_{BUS} matrix by inputting the primitive admittance matrix and the bus incidence matrix which can be obtained by inspection.

Within the satisfaction of these load flow equations is the requirement that voltage magnitudes be maintained within a certain deviation or that

$$V_{imin} \leq V_i \leq V_{imax} \;\ldots\ldots\ldots\; 2.1.xix$$

that the phase angles are within a given range

$$\delta_{imin} \leq \delta_i \leq \delta_{imax} \quad \text{..........2.1.xx}$$

The real power output of each generating unit must also lie within upper and lower limits

$$P_{Gimin} \leq P_{Gi} \leq P_{Gimax} \quad \text{.......2.1.xxi}$$

The power transferred across lines must also lie within a specified range or

$$-T_{ijmax} \leq T_{ij} \leq T_{ijmax} \quad \text{......2.1.xxii}$$

Where T_{ijmax} is the thermal limit or other transfer capability limit or the transmission line. Reactive power generation is also restricted to a specified range or

$$Q_{Gimin} \leq Q_{Gi} \leq Q_{Gimax}.$$

These constraints constitute all the inequality constraints of concern in determing the optimum power flow.

The computer can be used to develop the Y_{Bus} matrix by inputting the primitive admittance matrix and the bus incidence matrix which can be obtained by inspection.

By making simplifying assumptions the 3-bus example can be solved analytically but for real parameters a numerical solution must be obtained. One of the more common numerical algorithm in use by utilities for planning load flow models is the Newton-Raphson Method. This method converges faster than other methods such as Gauss-Seidel [2] and has little risk of divergence. This method has the drawback however, of finding and inverting the Jacobian matrix which can require significant computer time for a system as large as the pool in question. Recursive techniques exist [20] which minimize the computer time required for inversion but the task is still a formidable one.

The step of actually using a program to solve the load flow equations will be postponed until chapter VII. The classical economic dispatch will now be discussed and an example for the simple 3-bus network with two generators shown. The integration of the dispatch and load flow constraints will be discussed in the chapters VI.

2.2 Present Econonic Dispatch Methodology

The objective of economic dispatch is to minimize the total production costs of the utility and can be expressed in general form [2] as the minimization of a cost function C, where

$$C = C\ (x,\ u,\ p) \qquad \text{2.2.i}$$

and simultaneously satisfying the constraint

$$h(x,\ u,\ p) = 0 \qquad \text{2.2.ii}$$

and/or the inequality constraint

$$g(x,\ u,\ p) \leq 0 \qquad \text{2.2.iii}$$

The state variables, $\bar{x}$, are the voltage magnitudes and phase angles on each individual bus in the system.

$$\bar{x} = \begin{bmatrix} \delta_1 \\ |V|_1 \\ \vdots \\ \delta_i \\ |V|_i \\ \vdots \\ \partial_n \\ |V_n \end{bmatrix} \qquad \text{2.2.iv}$$

The control variables in the Vector $\bar{u}$ are the real and reactive power generation at each bus

$$\bar{u} = \begin{bmatrix} P_{G1} \\ Q_{G1} \\ \vdots \\ P_{Gi} \\ Q_{Gi} \\ \vdots \\ P_{Gn} \\ Q_{Gn} \end{bmatrix} \quad \text{.............................2.2.v}$$

The disturbance variable $\bar{p}$ is the power demand experienced at each bus.

$$\bar{p} = \begin{bmatrix} P_{D1} \\ Q_{D1} \\ \vdots \\ P_{Di} \\ Q_{Di} \\ \vdots \\ P_{Dn} \\ Q_{Dn} \end{bmatrix} \quad \text{.............................2.2.vi}$$

In considering the costs which can be controlled (variable cost) the fuel cost at the generating plant is readily identified [2]. Reactive power, although strongly impacting electrical stability, does not directly affect operating costs. The cost function is therefore assumed to be only a function of real power output P_{Gi} or expressed

$$c_i = c_i(P_{Gi}) \text{ ....2.2.vii}$$

and the total cost C in equation 2.2.i can be expressed as

$$C = \sum c_i(P_{Gi}) \text{ ....2.2.viii}$$

The assumption that reactive power does not enter into the cost reduces the equality constraint in equation 2.2.ii (which is the solution of the static load flow equations) to the requirement that

$$\sum P_{Gi} - P_D - P_L = 0 \text{ ....2.2.ix}$$

or that the total real power output must equal the real power demand plus losses. This can be seen by referring to equations 2.1.viii for the two bus system. The last two equations with reactive terms are eliminated leaving the first two and the last two terms of these equations (terms including V) are the loss terms. P_L is the real losses on the transmission system and $P_D \triangleq \sum P_{Di}$. If losses are ignored since they represent only a small portion of costs and/or they are included in dispatch costs through the use of penalty factors, then the equation 2.2.ii becomes

$$h(P_{G1} \text{ ....}, P_{Gn}) \triangleq \sum P_{Gi} - P_D = 0 \text{ ....2.2.x}$$

Each generator must operate between a minimum power output and a maximum output or

$$P_{Gi} - P_{Gimax} \leq 0 \; \ldots\ldots\ldots\ldots\ldots\, 2.2.xi$$
$$P_{Gimin} - P_{Gi} \leq 0$$

Other constraints on Q_{Gi} and $|V_i|$ shown earlier (2.1.xix to 2.1.xxii) must also be observed but are not considered explicitly in the present methodology.

The solution to this simplified economic dispatch is the minimization of the cost function $C = \sum c_i(P_{Gi})$ with the constraint that the load be met and the individual unit is not be dispatched outside their physical limits [2]. The minimization of the augmented function

$$C^* \triangleq C_1 + C_2 + \ldots + C_n - \lambda \left(\sum P_{Gi} - P_D\right) \ldots\ldots 2.2.xii$$

(where λ is called the Lagrangian).

is found when $\dfrac{\partial Ci^*}{\partial P_{Gi}} = \lambda$ for i = 1, 2, ..., n......2.2.xiii

and $\dfrac{\partial C^*}{\partial \lambda} = \sum P_{Gi} - P_D = 0$ 2.2.xiv

Each generator must operate between a minimum power output and a maximum output or

$$P_{Gi} - P_{Gimax} \leq 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots 2.2.xi$$

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$$C^* \triangleq C_1 + C_2 + \ldots + C_n - \lambda\,(\sum P_{Gi} - P_D) \ldots\ldots 2.2.xii$$

(where λ is called the Lagrangian).

is found when $\dfrac{\partial Ci^*}{\partial P_{Gi}} = \lambda$ for $i = 1, 2, \ldots, n$ 2.2.xiii

and $\dfrac{\partial C^*}{\partial \lambda} = \sum P_{Gi} - P_D = 0$ .... 2.2.xiv where $\frac{\partial ci}{\partial P_{Gi}}$ is the incremental cost curve of each unit as determined by the fuel costs and the derivative of the input/output curve of the unit plus any variable operating and maintenance costs. Equation 2.2.xiv is a restatement of the equality constraint.

In essence this methodology reduces the economic dispatch to the scheduling of the real power generation to meet real power demand plus losses and relegates the responsibility of satisfying transmission constraints to the transmission planning arena.

<u>Classical Economic Dispatch Example</u>

As discussed previously, classical economic dispatch considers the optimum dispatch of real power to meet demand and does not explicitly consider electrical constraints as imposed by physical and reliability requirements.

In the three bus example discussed earlier, the objective would be to minimize the cost associated with providing the real demands P_{D1} plus P_{D3} and any real losses with the real generation from buses 1 and 2 (P_{G1} + P_{G2}).

For this situation we want to minimize

$$C = C_1 + C_2$$

where C_1 is the cost of generation P_{G1} or $C_1(P_{G1})$
and C_2 is the cost of generation P_{G2} or $C_2\,(P_{G2})$

and ensure that (ignoring losses)

$$P_{G1} + P_{G2} - P_D = 0$$

where

$$P_D = P_{D1} + P_{D3}$$

This is accomplished by requiring

$$\frac{\partial C^*}{\partial P_{Gi}} = \frac{\partial C^*}{\partial P_{G2}} = 0$$

where

$$C^* \triangleq C_1 + C_2 - \lambda\,[P_{G1} + P_{G2} - P_D]$$

by substitution

$$\frac{\partial C_1}{\partial P_{G1}} = \frac{\partial C_2}{\partial P_{G2}} = \lambda$$

and λ is identified as the incremental cost of the unit.

For example, lets define the cost characteristics of the two generating units as follows:

$$C(P_{G1}) = 300 + P_{G1}(10.0 + .005P_{G1}) \qquad \$/hr$$

$$C(P_{G2}) = 210 + P_{G2}(\ 9.5 + .004P_{G2}) \qquad \$/hr$$

Where P_{G1} and P_{G2} are in megawatts.

Assume we wish to serve a demand of P_{D1} = 200 MW and P_{D2} = 300 MW or P_D = 500 MW. We can find the most economic dispatch point by finding

$$\frac{\partial C(P_{G1})}{\partial P_{G1}} \text{ and } \frac{\partial C(P_{G2})}{\partial P_{G2}},$$

setting them equal and solving with the constraint that $P_{G1} + P_{G2} = 500$. The two equations become:

$$10 + .01P_{G1} = 9.5 + .008P_{G2}$$

$$P_{G1} + P_{G2} = 500$$

yielding

$$P_{G1} = 194.45$$

$$P_{G2} = 305.55$$

For a total production cost of 5919.82 \$/hr. An evaluation of any other possible combinations of meeting the 500 MW load would lead to a higher production cost.

For practical applications the solution of simultaneous equations to determine the economic dispatch point becomes too tedious. Instead a composite curve or schedule is developed indicating the available generation at various incremental cost. The demand is then matched to the generation and all available units at or below the corresponding incremental cost or "lambda" is scheduled for operation.

At any given time the dispatcher has available the incremental cost for all probable load levels. As load changes during the day appropriate signals are sent to the generating stations to raise or lower generation to match the given incremental cost level.

Presently this methodology is used to determine the schedule for generating units for dispatch. If the $P_{Gi's}$ thus obtained then results in a violation of previously stated inequality constraints, the dispatcher must then adjust the P_{Gis} to remove the violation. The question now becomes how can this be done in an optimum manner?

In the pool in question if power flow along a particular line is the cause of a voltage drop at a particular bus, then from precalculated factors the dispatcher knows the generating units which have the most impact on flows on that line. Using this information and the associated incremental costs of these units, the dispatcher then decides which units should change output and by how much. The drawback is that once this has been done there is no verification that this final dispatch selected is optimal.

The objective of this thesis is to let the computer determine an optimum dispatch within the constraint.

The simplest procedure would be to compute a dispatch based on real power transfer, using this generation compute the solution to the load flow equations. If a violation is noted, recompute a dispatch based on a penalty assigned to the violation. Repeat the process if a violation still exists, increase the penalty and recompute a dispatch.

This method could take considerable time and an optimum solution can not be guaranteed in real time.

The method used here assumes an initial generation dispatch schedule. A load flow solution and the resulting production cost is determined for these initial conditions. The optimization routine then minimizes the production costs by varying the generation while keeping voltages within the constraint imposed.

For each new schedule computed by the optimization routine, a load flow is solved and tested to ensure that voltages are within bounds.

The method does not use equal incremental costs as the criteria for economic dispatch (although gradients of the function are computed). Instead numerical procedures in mathematical programming methods are relied on to seek the minimum production costs within the non-linear constraints. Some of these methods are discussed in chapter IV.

CHAPTER III

STATE OF THE ART

The discovery of the equal incremental cost criteria for economic dispatch dates back as early as the 1930s [10]. The use of loss formulae was developed later by E. E. George in 1943 [11]. The use of digital load flow analysis did not develop until the 1950s. It was during this time that efforts began to improve on the classical economic dispatch method and develop an optimal power flow method.

Most of the work on optimal power flow since then has been based on the formulation of work by Squires and Carpentier [12], [13] and [14]. Carpentier's work served to place optimal power flow on a firm mathematical basis and lead to a general formulation of the economic dispatch problem based upon the Kuhn Tucker Theory of nonlinear programming.

The most significant work since Carpentier has been that published by Dommel and Tinney in 1968 [15]. The methodology and notation used earlier is based on their work. They divided the variables into three groups: unknowns (x) which consist of V and θ (δ) on (P,Q) buses, and θ on (P,V) buses fixed parameters (p) P, Q on (P,Q) buses, and θ on the slack bus; and control parameters as voltage magnitudes on generator buses, generator real powers P, transformer tap ratios and denoted by 'u'.

The basic proposal by Dommel and Tinney is summarized as follows. The minimum of

$$f(x,u) \quad \ldots\ldots\ldots\ldots\ldots \quad 3.i$$

subject to the constraints of the load flow stated as

$$g(x,u,p) = 0 \ldots\ldots\ldots\ldots \quad 3.ii$$

is found by determining the Lagrangian function expressed as

$$L(x,u,p) = f(x,u) + [\lambda]^T \cdot [g(x,u,p)] \ldots\ldots\ldots \quad 3.iii$$

The set of necessary conditions for a minimum are

$$\frac{\partial L}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial x}^T \cdot \lambda = 0 \ldots\ldots\ldots\ldots \quad 3.iv$$

and

$$\frac{\partial L}{\partial u} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial u}^T \cdot \lambda = 0 \ldots\ldots\ldots\ldots \quad 3.v$$

$$\frac{\partial L}{\partial \lambda} = [g(x,u,p)] = 0 \ldots\ldots\ldots\ldots\ldots \quad 3.vi$$

where $\partial L/\partial\lambda$ would essentially be the solution of the load flow equations. The first of the conditions contains the transpose of the Jacobian and can be solved for λ.

$$\lambda = -\left[\frac{\partial g}{\partial x}^{T}\right]^{-1} \frac{\partial f}{\partial x} \qquad 3.\text{vii}$$

The second condition represents the gradient vector ∇f

$$\nabla f = \frac{\partial F}{\partial u} + \frac{\partial g}{\partial u}^{T}\lambda \qquad 3.\text{viii}$$

The computational process involves assuming a set of control parameters 'u', solving the load flow by Newton's method, repeat a solution for λ, then calculate ∇f and find a correction in 'u'

$$\Delta u = -c\nabla f \qquad 3.\text{ix}$$

The major difficulty is in selecting the factor c which affects the speed or convergence and oscillation.

Since the work by Dommel and Tinney [14], much work has been done on the subject by several authors. With the growth and advances of optimization techniques and development of improved software packages great strides are being made in this area. At the IEEE winter power meeting in 1982 several papers were presented on the subject of optimal power flow.

Some of the recent contributers include H. H. Happ, J. Carpentier, B. A. Murtagh, M. A. Saunders, K. A. Wirgau and R. C. Burchett to name a few.

Optimization techniques in popular use include the revised simplex method, the reduced Hessian and shifted penalty functions [16], [6], and [5]. Reference [4] provides an excellent survey of work which has been done in the area of optimal power flow up until 1977.

CHAPTER IV

OPTIMIZATION METHODS:

A BRIEF SYNOPSIS

The invention of the calculus in the 17th century provided a means of analytically determining the extremum (maximum or minimum) of a function. Even with the calculus, problems existed which were too complex to solve by analytical methods. In such cases numerical methods had to be applied.

Optimization methods as these optimum seeking algorithms are called were probably started around the turn of the 19th century in works by Gauss and Legendre. Although the algorithms developed then worked and produced good results, the tedium of the iterations involved in numerical analysis impeded the progress and limited the application of these methods to more complex problems.

With the development of the digital computer, areas that were previously closed are now open. With the capability to delegate the drudgery of repetitive computation to the machines it became possible to develop algorithms which would seek the minimum or maximum of a function through numerical methods.

Industry provided the incentive to develop optimization techniques to minimize costs of operation.

This chapter will discuss some of the methods presently in use.

4.1 Necessary Condition for Local Maximum or Minimum

A necessary condition for a local maximum or minimum of a function f at x = a is that

$$f'(a) = 0$$

If $f''(a) < 0$ or $f''(a) > 0$ then f has a local maximum or minimum respectively at $x = a_1$. [1]

Similarly a necessary condition for a function of several variables f at $(a_1, a_2 \ldots\ldots, a_n)$ is that

$$\frac{\partial f}{\partial x_1} = \frac{\partial f}{\partial x_2} = \frac{\partial f}{\partial x_3} = \ldots = \frac{\partial f}{\partial x_n} = 0 \qquad 4.1.i$$

at $(a_1, a_2, \ldots\ldots, a_n)$.

For a two variable function if [1]

$$J = \begin{vmatrix} f_{x_1x_1} & f_{x_1x_2} \\ f_{x_2x_1} & f_{x_2x_2} \end{vmatrix} > 0 \text{ and } \begin{Bmatrix} f_{x_1x_1} < 0 \\ f_{x_1x_1} > 0 \end{Bmatrix} \qquad 4.1.ii$$

then the function has a local

$$\begin{Bmatrix} \text{maximum} \\ \text{minimum} \end{Bmatrix}$$

J is referred to as the Jacobian or Hessian J [1]. If constraints are now added to the problem then Lagrange's method of undetermined multipliers can be used to determine the solution. The basic statement of this is that a necessary condition for a local extremum of the function $F(x_1, x_2, \dots\, x_n)$ subject to $f_i\ (x_i, x_2, \dots\, x_n) = 0$, $i = 1, 2 \dots m$, $(m < n)$ is given by

$$\frac{\partial F}{\partial x_1} + \lambda_1 \frac{\partial f_1}{\partial x_1} + \dots + \lambda_m \frac{\partial fm}{\partial x_1} = 0 \qquad 4.1.iii$$

$$\frac{\partial F}{\partial x_2} + \lambda_1 \frac{\partial f_1}{\partial x_2} + \dots + \lambda_m \frac{\partial fm}{\partial x_2} = 0 \qquad 4.1.iv$$

$$\frac{\partial F}{\partial x_n} + \lambda_1 \frac{\partial f_1}{\partial x_n} + \dots + \lambda_m \frac{\partial fm}{\partial x_n} = 0 \qquad 4.1.v$$

along with the satisfaction of the constraint equations [1].

Optimization methods generally apply some of these principles in a numerical method to seek the optimum point of a function.

4.2 Selected Optimization Methods

4.2.1 Bracketing

The objective of this method is to 'bracket' the maximum (or minimum) of a function by evaluating the function at several points starting at a given point and adding an increasing increment to that point until $f(x_i)>f(x_i-1)$ <u>and</u> $f(x_i)>f(x_i+1)$. The bracket for the maximum is then $(x_i-1,\ x_i+1)$. If the gradient of the function is known the bracket can also be determined if the gradient changes sign from point to point [1].

Once a bracket is found the actual maximum (or minimum) can be determined by other means. The success of this method lies in choosing a good starting point and adequate step sizes.

4.2.2 Fibonacci Type Search

This class of methods is known as the class of direct search methods [1] and is used to optimize when the derivative is not known. Once a bracket is known the objective is to reduce the length of the bracket until it is within a prescribed limit. If (a_1, a_2) brackets a required maximum of $f(x)$ the a_3 and a_4 are symmetrically placed within the interval in the prescribed manner

$$\begin{aligned} a_3 &= (1-\alpha)a_1 + \alpha a_2 \\ & \qquad\qquad 0<\alpha<\tfrac{1}{2} \qquad\qquad 4.2.i \\ a_4 &= \alpha a_1 + (1-\alpha)a_2 \end{aligned}$$

$f(a_i)$ is computed and a new bracket is chosen and the process repeated [1].

4.2.3 Simplex Method

This method is used when the function is a function of several variables and the derivative unknown [1]. This method was first devised by Spendley and coauthors (1962) but it has since been modified (Revised Simplex) to make it a highly efficient routine and has been used in some of the papers published on Optimal Power Flow.

The first step of this method is to set up a regular simplex in an n-dimensional space, that is (n+1) points $x_o, x_1, \ldots, x_n$ all equidistant from each other. In two dimensions a regular simplex is an equilateral triangle and in three dimensions a regular simplex is a regular tetrahedron. If the minimum of the function is required at each vertex $f(x_i)$ is evaluated and the vertices reordered so that $f_o \geq f_1 \geq .... \geq f_n$. Since f_o is the worst value the method makes a move as far away from that point by making a step equal to twice the difference between x_o and the mean value of all the other x_i. The new f(x) is either greater than or less than $f(x_1)$. If it is greater than $f(x_1)$ then it still is the worst, and applying the same procedure would reproduce the same value. In this case the next largest $f(x_i)$ is choosen and the procedure applied. If on the other hand the new f(x) is less than $f(x_1)$ the reordering is done and the same procedure is applied to the worst value.

This procedure continues until the function approaches the minimum [1].

4.2.4 Alternating Variable Method

This method essentially holds all other variables constant and applies one of the single variable search methods to one variable. It is straight forward and simple, but very slow [1].

4.2.5 Gradient Methods

These methods use the fact that the direction in which one should move towards a maximum is in the direction of the steepest gradient from that point. Most useful in unconstrained optimization, the oldest method is Cauchy's optimal gradient method (1847) called 'steepest descent' or ascent depending on whether one is minimizing or maximizing.

This method converges rapidly but does not perform very well near the boundaries in constrained problems.

Much work has been done with this method to determine the best choice of direction and step length.

Work by Davidon (1959), and later by Fletcher and Powell (1963) increased the success of the gradient methods [1] and continued work on these methods [17] make them one of the more widely used.

Reference [17] is a collection of papers presented at a symposium on optimization techniques held by the Institute of Mathematics and its Applications at the University of Keele, England in 1968. This work has contributions by some of the more renowned authors on the subject such as Abadie, Beale, Carpentier, Davidon, Davies, Fletcher, Murtagh, Pearson, Powell, Spendley and Wolfe. The papers and discussions provide a good overview of the basic methods existing then. In particular, References [18] (Fletcher) and [19] (Davidon) provide excellent summaries of methods in unconstrained optimization and variance algorithms for minimization.

CHAPTER V

LOPER: A LEHIGH OPTIMUM PARAMETER ROUTINE

The optimization software used to develope the methodology was written for the Lehigh University computer library by Richard K. Greene in 1970 [22]. The package called LOPER is an optimum parameter routine which searches for the optimum parameter values of a model or function of up to twenty variables, subject to both independent and dependent non-linear constraints. The package contains six search mechanisms: spider (which is essentially the simplex method), steepest ascent, conjugate gradients, rotate axis, parallel tangents and random.

LOPER locates the values of the parameters (XNOW) which lie within their upper and lower constraints (XUP and XLOW), and maximize the performance function in subroutine MODEL (PMODEL) while maintaining dependent variables (DEPV) within upper and lower limits (DEPUP and DEPLOW). A minimum is found by negating the performance function in subroutine MODEL. Only the search methods which were actually used in this thesis will be discussed.

5.1 Defining the Performance Function

For this purpose the performance function which is of interest is the production cost of the system and the parameters are the real power generation of all the units except on the reference bus since generation on this bus cannot be specified in solving the load flow equations. The dependent variables are the voltage magnitudes at all buses except the reference bus, and the sum of the real power generation which must equal the real power demand plus losses.

5.2 The Spider Method

The spider method either reads or generates a set of n+1 non-colinear points to develop a simplex formation. By comparing the performance at each point it finds which point is the worst in the formation. This point is then shifted to the opposite side of the formation and the worst point or the new formation is found. If the shift does not improve the performance of the worst point, then it is placed at the center of gravity of the formation. Successful moves expand the spider while unsuccessful moves compress it. When a parameter boundary is encountered, the point is placed on the boundary.

5.3 Line Search

The a' [redacted]njugate gradients, and rotate axis search methods all use a line search which seeks the optimum on a given line through parameter space. LOPER does this by a combination of accelerated jumping movements and a cubic curve fitting system which is executed at the first failure of a jump to improve performance if it occurs after two other jumps have been made. This ensures there are enough points to determine a cubic curve and that there is a maximum within the fitted span [22].

During a line search, the constraints on the parameters themselves and the dependent variables are enforced by reducing the jump distance until the jump remains within bounds, in addition, if a parameter is still beyond a constraint after three reductions of the length of the jump length, it is placed on the boundary and the direction of the line is turned aside from the boundary by setting the violating directional components to zero. If a line search is unable to meet a constraint on a dependent variable after three reductions in jump distance, LOPER automatically switches to a spider search [22].

5.4 Steepest Ascent Method

This method finds the gradient vector, which aims in the local direction of greatest improvement. A line search is conducted along that direction to find the best point on that line, which is then the base point for the next gradient calculation.

This method has two weaknesses. It oscillates back and forth across a sharp ridge when there is strong interaction between parameters. Secondly, when it drives into a boundary condition, it continues to drive into it, while spider progresses along it [22].

This was one of the methods in use in optimal flow problems in the late 1960's early 1970's. When used in this effort however, the second weakness made it unapplicable. That is, it continued to reduce generation even though violations in the constraints occurred.

5.5 Conjugate Gradients

The first weakness of the steepest ascent method is overcome by using conjugate gradients. The mathematical justification for conjugate gradients uses the fact that the new gradient is perpendicular to the previous line direction. The form used in LOPER begins the line search in the direction of the gradient. Successive search directions are found by adding the new gradient vector to the product of the old direction vector and the ratio of the squares of the lengths of the new to the old gradient vectors.

The conjugate gradient method converges rapidly but shares the difficulty at boundaries with the ascent method.

5.6 Partan Search Method

Partan or parallel tangents begins by calculating a gradient at the initial point. A line search is then conducted in the direction of the gradient to the best point of the line. After this initial line search, partan carries out an iterative procedure which consists of two line searches. The first is along the gradient measured at P(M) to the linear maximum point, P(M+1), where M=1,3,5... and the second line search of an iteration starts at P(M+1) and searches along the vector between P(M+1) and P(M-1). The line search along the gradient aligns the search to the ridge line and the second line search is along the ridge line [22].

The other search routines within loper were not used since the were not easily applicable to the problem.

CHAPTER VI

COMBINED LOAD FLOW AND ECONOMIC DISPATCH PROGRAM

6.1 Introduction

Many methods to solve this problem were proposed and rejected by this author for one reason or another. The most significant difficulty encountered was the determination of a cost function or objective function which contained the parameters which were constrained. No physical cost function having real meaning or dollar value could be attached to such variables as voltage or reactive power.

Among those started and abandoned was the development, through a least squares fit procedure, of a cost function by taking samples over a period of time of system characteristic and cost. The intention was to take measurement of voltages, phase angle and power on the system at any given time as well as the production cost during the sample interval. Various order polynomial relationships would be postulated and a least squares fit used to determine the coefficients. The best fit would have been chosen as the model.

Initial attempts at this indicated that this would require a significant amount of data gathering and there was some doubt as to whether a model thus developed would be truly representative of the system under study.

Another proposed methodology was to assign penalty functions such as suggested by some authors [5]. This was abandoned also due to the complexity involved in the assignment of penalty functions and the uncertainties in choosing the right penalty factors.

The final methodology chosen reflects the tools readily available and the authors limited understanding of optimization techniques.

6.2 Available Tools

The primary tool used in this method is the LOPER optimum parameter routine obtained from the Lehigh University computer library and described in Chapter V. This routine has the capability of finding the maximum of an input function taking into consideration limits imposed on the independent variables as well as dependent variables. It was decided to adapt the problem to use this routine. This was particularly helpful since voltage could be treated as a dependent variable and thus not appear explicitly in the cost function.

In addition, a load flow program using the Gauss Iterative Technique was taken from Elgerd's 'Electric Energy Systems' [2] and modified slightly for adaptation to use with LOPER. The load flow program has the added advantage of solving directly for voltage.

The basic Gauss iterative technique can best be explained by an illustrative example. Given a function

$$f(x) = X^2 - 6x + 5 = 0$$

which we wish to solve (the solution can be verified as x=5 and x=1). A new function is defined such that

$$x = F(x) = \frac{x^2+5}{6}.$$

Starting from an initial point $x^{(i)}$, the iterative procedure is defined as:

$$x^{(i+1)} = F(X^{(i)}).$$

If we start with an initial assumption

$$x^{(0)} = 2$$

then $$x^{(1)} = \frac{2^2+5}{6} = 1.5$$

$$x^{(2)} = \frac{1.5^2+5}{6} = 1.2083$$

$$x^{(3)} = \frac{(1.2083)^2+5}{6} = 1.0767$$

$$x^{(4)} = 1.0265$$

etc.

and the solution slowly approaches 1.0. The iterations are stopped when the change from one iteration to the next is within a prescribed limit. Besides being slow, this procedure has the added drawback that it might not converge at all.

This basic procedure is extended to functions of several variables and improvements such as the addition of an acceleration factor and the Gauss-Seidel process of updating the value of each of the variables as soon as the new value is calculated are used in the load flow analysis. In this case the load flow equations are expressed in the form

$$V = F(P,Q,V)$$

6.3 Method

The method proposed here starts with the same objective function (or performance function) as the traditional method which is the total production cost C or the sum of the generation costs at each generating station as expressed by equation 2.2.viii (p. 28). The equality constraint expressed in equation on 2.2.ix that the sum of the real generation must be equal to the total real demand plus losses is also in effect as well as the inequality constraint on the real power generation on each unit (equation 2.1.xxi). Unlike the traditional method, however, the inequality constraint on voltage magnitude (equation 2.1.xix) is also imposed in this method as a constraint on a dependent variable. (Treatment of voltage as a dependent variable circumvents the need to have it appear in the objective function).

In contrast to the traditional equal incremental cost solution this method uses the optimization process to directly seek the minimum of the function within the constraints. If no violation of the voltage constraint occurs, the solutions are identical as is demonstrated in the first case study in Chapter VII.

The optimization process varies the real power generation at each bus as prescribed by the method in use. The load flow equations are solved for each set of valves of generation, the production cost computed and the voltage magnitudes checked to determine if they are within bounds.

The method uses the subroutine LOPER to determine the minimum production cost of the system while keeping the voltages (dependent variables) within limits. LOPER requires the use of three subroutines: a main routine (main program) which calls LOPER and indicates the number or parameters, the number of dependent variables, the search method and other option switches; subroutine MODEL which supplies the value of the performance (PMODEL) and the dependent variables; and subroutine BOUND which supplies information on the upper and lower limits of the parameters and dependent variables.

The Gauss Technique was considered most adaptive to this method since it solves the load flow equations for voltage.

The source code of the routines written to use LOPER and the modified load flow subroutine is shown in Appendix A. These would be attached to the LOPER Routine. The relationship of the subroutines are shown in Figure 6.1.

The basic flow of the computation is as follows:

1. Program MAIN identifies the number of parameters, dependent variables, maximum number of evaluations, options in effect including search method, and initial values of the parameter and passes these to subroutine LOPER. In this case, limits on the parameters and dependent variables were also read by MAIN and placed into common for use by subroutine BOUND. These values are usually assigned in subroutine BOUND.

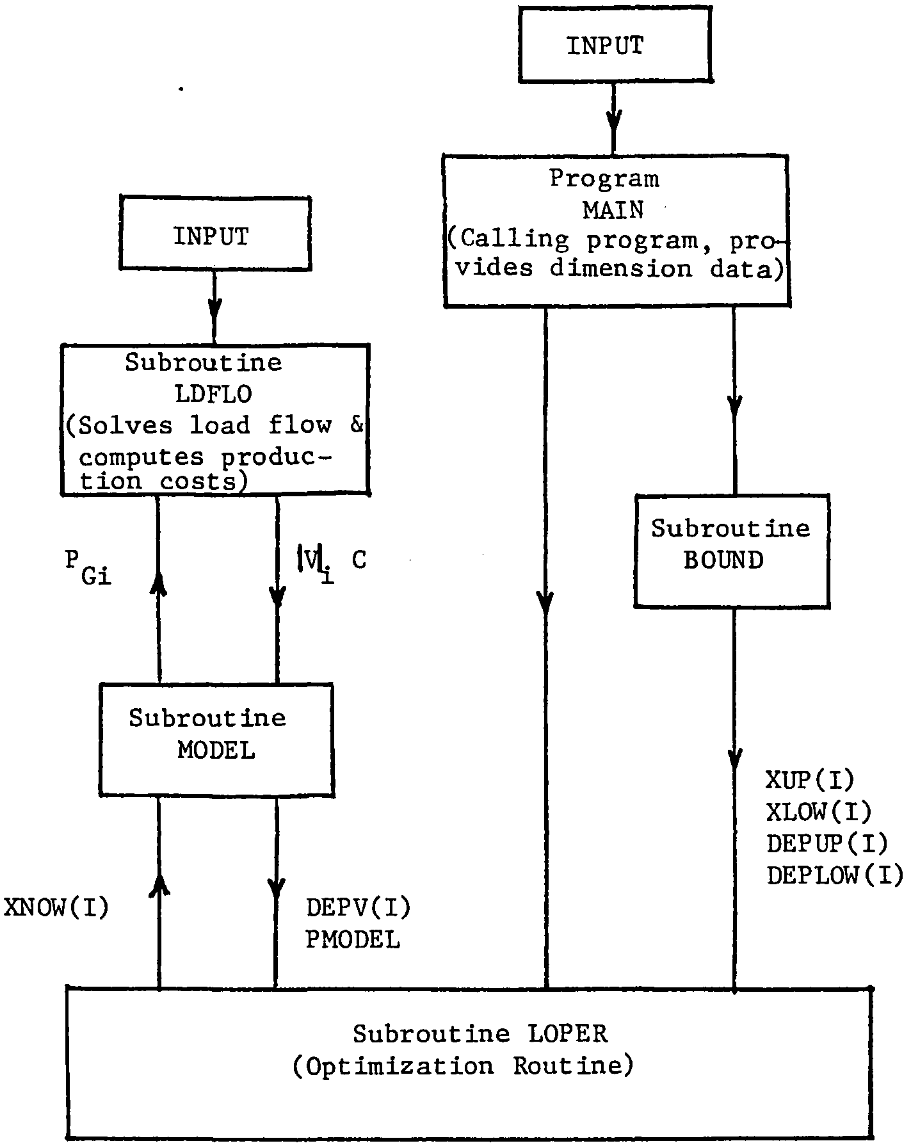
2. Subroutine MODEL gets the values of the parameters (generation at each bus) and passes these to subroutine LDFLOW which is the Gauss Iterative Load Flow Program modified to also calculate the production costs which is the performance to be minimized. This routine solves the load flow equations, computes the production cost

and passes the values of the dependent variables (voltage magnitudes), and the performance (production costs) to subroutine MODEL. Since LOPER seeks a minimum, subroutine MODEL changes the sign of the performance (PMODEL) and passes these valves to LOPER.

3. LOPER then calls subroutine BOUND for the limits on the parameters and dependent variables and checks to see if they are violated.

4. If no violations have occurred, LOPER adjusts the parameters according to the search method selected and again calls subroutine MODEL to find the value of the performance. If violations exist then the type of search being conducted will determine how well LOPER reacts.

5. The search will continue until one of the following occurs: a) No significant change in performance occurs after three iterations, or b) the maximum number of evaluations is reached. If a search method being used fails to move a parameter or dependent variable in bounds after 3 moves, LOPER automatically switches to a spider search. If spider also fails to bring it within bounds after three trials it stops.



RELATIONSHIP OF SUBROUTINES

FIGURE 6.1

Major Difficulties

The first problem encountered was the necessity to input the equality constraint (that generation must equal demand plus losses) into the form of an inequality constraint since LOPER handles only upper and lower limits. This was overcome by setting this limit as a lower limit on a dependent variable. This was felt to be adequate since the minimization of the function should move generation downwards all the time. When this was done it was found that methods such as the conjugate gradient, steepest descent and parallel tangents would keep driving past the bound. This was presumably because the driving force in these searches is the gradient which would keep moving them lower. As indicated in Chapter V, these methods were determined by the author of LOPER to behave badly at boundaries.

Switching to spider search slowed convergence significantly but stayed within bounds. It was necessary to start the spider search at a point close to the known solution to get it to converge swiftly.

The next problem to emerge was the limitations imposed by the load flow equations. Since a reference bus must be assigned and the power at that bus is unknown until the final flow is calculated, the generation at this bus could not be treated as an independent variable. In the problem used to test this method, it was assumed that generation at that bus would indeed affect the production cost. In normal circumstances this bus might be chosen so as not to have much impact on the system under study (sometimes even out of the system under study).

In addition, since the load flow solution process used is iterative and a solution is not guaranteed, any input information which results in non-convergence after the maximum iterations will result in invalid optimization results.

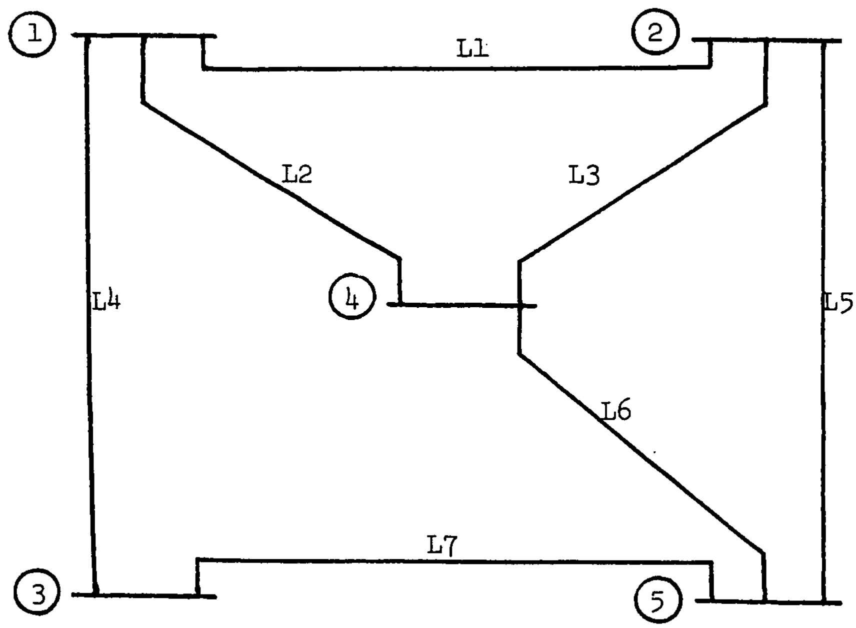
Finally it was found that if the initial values of the parameters resulted in violations of the boundary conditions the optimization process could terminate without bringing it back in bounds.

It should also be noted that two sets of maximum iterations exist for this method - one for the optimization and one for the load flow. The maximum total iterations possible is the product of the two.

Sample System

The system used to test the method is a 5 - bus system shown in Figure 6.2 with line characteristics shown in Table 6.1 and 6.2. The characteristics for this sample were taken from Reference [2]. The real power demand, voltage and reactive power injection at each bus is also shown in Table 6.2. Real power generation at each bus except the reference bus (No. 1) is input at an initial value in the main program and varied by the optimization process to determine the minimum dispatch. Real power generation on bus number 1 is determined after the load flow solution. The constraints imposed on voltage is that the magnitude on any bus be between 0.95 a 1.05 per unit.

The cost characteristics $C(P_{Gi})$, of each generating unit is coded into the load flow portion of the program. After solution of the load flow the calculation of the total production costs associated with the generation is computed and this value is the performance which is then negated and passed on to LOPER to be optimized.



SAMPLE SYSTEM

Figure 6.2

TABLE 6.2

BUS ADMITANCE MATRIX

1.3508	-j10.2446	-0.4087	j3.1294	-0.4002	j3.0641	-0.5419	j4.1489	0.0	0.0
-0.4087	j3.1294	1.5721	-j11.9501	0.0	j0.0	-0.7052	j5.3997	-0.4582	j3.5081
-0.4002	j3.0641	0.0	j0.0	0.6926	j5.2178	0.0	j0.0	-0.2924	j2.2389
-0.5419	j4.1489	-07052	j5.3997	0.0	j0.0	2.1874	-j16.6859	-0.9403	j7.1996
0.0	j0.0	-0.4582	j3.5081	-0.2924	j2.2389	-0.9403	j7.1996	1.6909	-j12.8506

BUS DATA

BUS	REACTIVE POWER	VOLTAGE MAGNITUDE	BUS TYPE
1	UNSPECIFIED	1.0	3
2	UNSPECIFIED	1.0	2
3	UNSPECIFIED	1.0	2
4	0.3	UNSPECIFIED	1
5	0.3	UNSPECIFIED	1

<u>TABLE 6.1</u>

LINE DATA

LINE	SB	EB	LENGTH	SHUNT ADMITTANCE P.U.		SERIES IMPEDANCE P.U.	
1	1	2	70.4	0.0	j0.0704	0.0410	j0.3142
2	1	4	53.1	0.0	j0.0531	0.0310	j0.2370
3	2	4	40.8	0.0	j0.0408	0.0238	j0.1821
4	1	3	71.9	0.0	j0.0719	0.0419	j0.3209
5	2	5	62.8	0.0	j0.0628	0.0366	j0.2803
6	4	5	30.6	0.0	j0.0306	0.0178	j0.1366
7	3	5	98.4	0.0	j0.0984	0.0574	j0.4392

CHAPTER VII

CASE STUDIES

The results of cases studied using the methodology developed will be summarized in this chapter. Table 7.1 identifies the cases and the characteristics assumed in each.

Case 1

Case 1 assumes all 5 generating units are identical in their cost characteristics. The real load on Buses 2 through 5 is assumed to be 1.25 P.U. on a 100 MVA base, no load is on Bus 1. The initial generation on Buses 2 through 5 is assumed to be 1.0 P.U. The cost characteristics of the units are defined as

$$c(P_{Gi}) = P_{Gi}(3.0 + .025\ P_{Gi}) \quad \$/hr$$

Where P_{Gi} is the real generation in megawatts at the i^{th} bus.

From Chapter II we know that the minimum cost to serve this load of 500 MW would require all 5 units to operate at 100 MW each since they all share the same cost characteristic and thus the same incremental cost characteristic. If losses are taken into account, then other than equal dispatch would occur since losses are dependent on dispatch.

TABLE 7.1

CASE IDENTIFICATION

Case No.	Unit Cost Characteristic	Demand P.U. P_{D1}	P_{D2}	P_{D3}	P_{D4}	P_{D5}	Q4
1	P_{Gi} (3.0 + .025 P_{Gi}) \$/HR All Buses	0.0	1.25	1.25	1.25	1.25	-0.3
2	Same as Case 1	0.0	0.5	1.0	2.25	1.25	-0.3
3	Same as Case 1	0.0	0.5	1.0	2.25	1.25	-0.3
4	Same as Case 1	0.0	1.25	1.25	1.25	1.25	-0.6
5	Same as Case 1	0.0	1.25	1.25	1.25	1.25	-0.6
6	P_{Gi} (3.0 + .025 P_{Gi}) \$/HR Units 1,2, & 3 P_{Gi} (4.0 + .035 P_{Gi}) \$/HR Units 4 & 5	0.0	1.25	1.25	1.25	1.25	-0.3
7	Same as Case 6	0.0	1.0	1.0	2.0	1.0	-0.3

Assuming no losses, the cost of dispatching each unit at 100 MW can be found as

$$5[100\ (3.0 + 0.025 \times 100)] = 2750.0 \quad \$/hr$$

On the initial pass through the load flow subroutine the generation on Bus 1 was found to be 1.0126 p.u. or 101.26 MW to cover losses of 1.26 MW for an initial cost of 2760.12 $/hr. Subsequent passes through the optimization routine yielded only minimal savings by changing the way in which the dispatch was divided. A final dispatch of 97.5 MW on Unit 1, 100.9 MW on Unit 2, 100.9 MW on Unit 3, 100.8 MW on Unit 4 and 101.1 MW on Unit 5 resulted in a cost of 2759.61 $/hr or minimal savings due to a slight reduction in losses.

Changing the initial values of generation (without violating constraints initially) still resulted in convergence to the same values as expected. The results of the computer analysis is shown in Appendix B, Tables B.1 and B.2.

Case 1 represents an essentially ideal case where the load and generation are evenly distributed and would not require any significant power flow. The next few cases will show what happens when imbalances occur.

Case 2

Case 2 redistributed the same 500 MW load such that 50 MW was now Bus 2, 100 MW on Bus 3, 225 MW on Bus 4 and 125 MW still on Bus 5. Bus 1 was assumed to carry no load. Generation on Buses 2 through 5 were again initialized at 100 MW. Again, after solution, the generation on Bus 1 will be the additional 100 MW plus losses.

In the situation because of the amount of power flow into Bus 4, (63 MW from bus 1, 47 from bus 2 and 15 from bus 5) the voltage fell beyond the prescribed 95% lower limit. Under these circumstances the dispatcher would raise generation in this area to relieve the situation. The optimization routine could not start with a violation on the voltage constraint at the initial conditions. The resulting production costs was 2771.10 $/hr for a dispatch of 102.6 MW on Unit 1, 100 MW each on Units 2 through 5. However, the voltage on Bus 4 was out of bounds. The results are tabulated in Appendix B, Table B.3.

Case 3

Case 3 is the same as case 2 except that we now increase generation (as the dispatcher would) on Bus 4. The initial generation points are 50 MW on Unit 2, 100 MW on Units 3 and 5 and 200 MW on Unit 4, and 50.4 MW on Unit 1 after solution of the load flow. At this level the production costs are 3127.07 $/hr. The final dispatch after using the methodology was 99.4 MW on Unit 1, 95.0 MW on Unit 2, 92.9 MW on Unit 3, 109.0 MW on Unit 4 and 106.0 MW on Unit 5 for a production cost of 2772.76 $/hr. or a savings of $355.71 $/hr from the arbitrary starting point.

The difference in cost from Case 2 is only 1.66 $/hr but the new configuration without voltage constraint violation is completely different. These results are also summarized in Appendix B, Tables B.4 and B.5.

Case 4

Case 4 Table 3.6 presents a situation where all initial points are as in Case 1 except that the reactive load on Bus 4 is increased by 30 MVARs. This also results in voltage drops which violate the constraints. Again since the initial points in this case resulted in violation on constraints, the optimization routine did not continue.

The production costs in this situation was 2763.57 $/hr for a dispatch of 101.7 MW on Unit 1, 100 MW each on Units 2 through 5. The voltages on Bus 4 and 5 were below the specified 0.95 p.u. minimum.

Case 5

This case is the same as case 4 except generation is shifted to Bus 4 to improve the load factor. The initial generations input were 50 MW on Bus 2, 75 MW on Bus 3, 200 MW on Bus 4 and 125 MW on Bus 5. After solution of the load flow the initial generation on Bus 1 was 51.85 MW, for an initial production cost of 3166.53 $/hr.

With these initial conditions, the voltages were within the initial constraints (Table B.7).

The resulting optimized dispatch within the constraints (Table B.8) was 55 MW on Bus 1, 57.4 MW on Bus 2, 77.1 MW on Bus 3, 188.9 MW on Bus 4 and 123.2 MW on Bus 5 for a total production cost of 3083.06 $/hr.

Summary of Case 1 Through 5

All of the above cases represent serving a real load of 500 MW with 5 generating units with the same cost characteristics. Ideally, with no losses, the minimum cost would be 2750.00 $/hr with each unit generating at 100 MW. With losses included, the minimum cost was computed as 2759.61 $/hr as shown in Case 1.

Cases 2 & 3 represent a situation not uncommon where a large portion of the load is concentrated on one bus requiring a significant power flow to that bus. When this flow exceeds the surge impedance loading of the line, voltage drop occurs at the bus, resulting in violation of the lower limit on the voltage constraint.

Under normal operating circumstances, the procedure for correcting this violation is to raise the generation in the vicinity of the bus with the violation. This was arbitrarily done in Case 3 for an initial starting point. After optimization, the resulting minimum cost point resulted in smaller shifts on all 5 units for a total cost of 13.15 $/hr more than the ideal.

In Cases 4 and 5, a violation in the voltage constraints on Bus 4 was again created. This time the reactive load at the bus was increased. Here again the assumed method of increasing the voltage was to increase the real power generation at that bus. The optimized dispatch without voltage violations resulted in a cost of 3083.06 $/hr.

Case 6

In this case and the next case, the cost characteristics of the generating units on buses 4 and 5 were assumed to be higher than those on buses 1 through 3. The cost characteristics of Units 1, 2 and 3 are assumed to be as before

$$C(P_{G(I)}) = P_{G(I)}(3.0 + .025\ P_{G(I)})\ \$/hr\quad I = 1,2,3$$

and the cost characteristics on Units 4 and 5 are now

$$C(P_{G(I)}) = P_{G(I)}(4.0 + .035\ P_{G(I)})\ \$/hr\quad I = 415.$$

The optimum dispatch in this case to serve 500 MW of load would now be 125.8 MW on Units 1, 2, & 3, and 61.3 MW on Units 4 and 5 for a total production cost of 3072.5 \$/hr assuming no losses.

This distribution of generation results in violation of voltage constraints.

The optimized dispatch computed for this system to serve the 500 MW load plus losses was 116.7 MW on Unit 1, 117.0 MW on Unit 2, 116.5 MW on Unit 3, 72.9 MW on Unit 4, and 78.9 MW on Unit 5, for a total cost of 3083.94 \$/hr. Further reductions in generation on Units 4 and 5 would result in violation of the voltage constraint.

Case 7

Case 7 is similar to Case 2 in that load was shifted to Bus 4. In this case, it was increased by 75 MW with 25 MW reductions on buses 2, 3, and 5 to keep the total real demand at 500 MW.

For this case the final dispatch computed by the program was 104.4 MW on Bus 1, 115.2 MW on Bus 2, 114.9 MW on Bus 3, 83.8 MW on Bus 4 and 84.2 MW on Bus 5. The total production cost was computed as 3103.56 $/hr.

Table 7.2 summarized the results of all 7 cases. More detailed information is shown in Appendix B.

TABLE 7.2

FINAL DISPATCH BY COMPUTER PROGRAM

CASE No.	Demand (MW)					Generation (MW)					Production Cost $/HR
	P_{D1}	P_{D2}	P_{D3}	P_{D4}	P_{D5}	P_{G1}	P_{G2}	P_{G3}	P_{G4}	P_{G5}	
1	0.0	125	125	125	125	97.5	100.9	100.9	100.8	101.1	2759.61
2	0.0	50	50	225	125	102.6	100.0	100.0	100.0	100.0	2771.10*
3	0.0	50	50	225	125	99.4	95.0	92.9	109.0	106.0	2772.76
4	0.0	125	125	125	125	101.7	100.0	100.0	100.0	100.0	2763.57*
5	0.0	125	125	125	125	55.0	57.4	77.1	188.9	123.2	3083.06
6	0.0	125	125	125	125	116.7	117.0	116.5	72.9	78.9	3083.94[1]
7	0.0	100	100	200	100	104.4	115.2	114.9	83.8	84.2	3103.56[1]

*Not optimized due to constraint violation at initial points.

[1]Production cost reflect higher cost characteristics of generators on Bus 4 and 5.

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

The use of an optimization technique coupled with the solution of the static load flow equations has been demonstrated to be capable of determining a minimum economic dispatch for a 5-bus system within constraints imposed on the voltage magnitude of the buses. The extension of this method to a system of larger size has not been investigated here. It can be concluded from the work done here that:

- It is possible to use available mathematical techniques to determine an optimum economic dispatch without violating voltage limits.

- A method such as this can be useful as a verification tool to determine if the new dispatch chosen by a dispatcher to alleviate a voltage violation is the least cost schedule.

- Since the method loops through a load flow solution, checks could also be made on other constrained variables such as line flow and phase angles, and the dispatch changed to bring them within the limits specified.

Recommended Areas For Further Study

Much work has been done in the area of optimal power flow [4]. The major efforts seem to concentrate on developing an "exact" solution through the Lagrangian method introduced by Carpentier. The author believes that some concentration should be given to optimum seeking mathematical methods which do not necessarily use an incremental cost method but starts with the total cost function and proceeds to find that minimum through numerical methods.

The equal incremental cost method should still be the mainstay of economic generation dispatch until less uncertainties exist in the numerical methods but a different method may yield quicker results when constraints causes aberrations.

A major concern of the dispatcher is the assurance that the system is operating as predicted. Ideally then, he/she would like the use of an on-line computerized method which would not only determine the optimal power flow in real time but could also use actual data obtained from telemetering.

A considerable amount of work still remains to be done in the areas of on-line load flow solution as well as telemetering of system parameters.

With increasing strides being made in the field of optimization and increasing digital computing capability, it seems that the only major difficulty in the increased use of optimization techniques in the electric utility industry is general acceptance by engineers in the industry. This general acceptance can be overcome by a "demystifying" of the concepts behind optimization methods and a closer alliance between the academic world and the industry.

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APPENDIX A

COMPUTER PROGRAM

```
      PROGRAM MAIN
      COMMON /CX/ NPAR, XNOW (20)
      COMMON /AAA/ XUPD(20), DEUP(20),DLOW(20)
      COMMON /AA/ KBC
      COMMON /CT/ RANGE(20), STEP
      COMMON /CC/ IPATH, LPRINT, MAXCNT, IOPT(20)
      COMMON /CD/ NDEPV, DEPV(20)
C     THE NUMBER OF PARAMETERS, DEPENDENT VARIABLES, THE MAXIMUM
C     NUMBER OF EVALUATIONS, AND THE PRINT OPTION FROM LOPER ARE
       READ
       READ (5,100) NPAR,NDEPV,MAXCNT,LPRINT
  100 FORMAT(4I5)
C     THE INITIAL VALUES OF THE PARAMETERS XNOW (REAL POWER
       GENERATION)
C     ARE READ IN HERE,ALONG WITH THE RANGE IN WHICH THEY ARE
C     EXPECTED TO VARY, THE UPPER AND LOWER LIMITS ON THE
       PARAMETERS
C     AND THE DEPENDENT VALUES ARE READ AND PLACED INTO COMMON AAA
C     DEPENDENT VARIABLE NO. 1 IS THE TOTAL REAL GENERATION, THE
       LIMIT
```

```
C     IS THE SUM OF THE REAL DEMAND PLUS REAL LOSSES COMPUTED IN
      LDFLOW
          READ(5,101)(XNOW(I),I=1,NPAR)
          READ(5,101)(RANGE(I),I=1,NPAR)
          READ(5,101)(XUPD(I),I=1,NPAR)
          READ(5,101)(XLOWD(I),I=1,NPAR)
          READ(5,101)(DEUP(I),I=2,NDEPV)
          READ(5,101)(DLOW(I),I=2,NDEPV)
  101     FORMAT(8F10.3)
          KBC = 0
          IOPT(9)=1
          IOPT(10)=0
          IOPT(7)=1
C     IPATH INDICATES THE TYPE SEARCH TO BE USED, 1 IS SPIDER, 4 IS
      CONJ.
C     GRADIENTS, 2 IS STEEPEST ASCENT.
          IPATH=4
          CALL LOPER
          IPATH=1
          CALL LOPER
          STOP
          END
          SUBROUTINE LDFLO
          (XX1,XX2,XX3,XX4,PM,DEP1,DEP2,DEP3,DEP4,DEP5,
          1DELOW1)
```

```
COMMON /CX/ NPAR, XNOW(20)
COMMON /AA/ KBC
COMPLEX YSHT,YSER,SERY,SHTY,Y,A,B,V,VI,VII,SUM,S,R,DX,
ZSER,SERZ
1VN
INTEGER SB,EB,NB,NL,MB,MAXIT
REAL LENGTH,MAGV,ALPHA,PM,DEP,DEP2,DEP3,XX
DIMENSION
LINE(14),SB(14),EB(14),LENGTH(14),YSHT(14),YSER(14),
1SERY(14),SHTY(14),A(14),B(8,8),V(14),P(14),Q(14),
 QMAX(14),
2QMIN(14),VSPEC(14),Y(8,8),ZSER(14),SERZ(14),VN(8),
 PG(14),PD(14)
C    READ NUMBER OF BUSES, NUMBER OF LINES,NUMBER OF VOLTAGE
      CONTROL
C    BUSSES INCLUDING SLACK BUS,ACCELERATING FACTOR
      (DEFAULT 1.0)
C    UNLESS THIS IS SECOND OR MORE LOOP FROM LOPER.
      DELOW1=0.0
      PM = 0.0
      IF(KBC.GT.0) GO TO 950
      READ(5,100) MAXIT
      READ(5,100)NB,NL,MB,ALPHA
      DO 1 I = 1,NB
      DO 1 J = 1,NB
```

```
    1 Y(I,J) = CMPLX(0.0,0.0)
      DO 2 I = 1,NL
C     READ LINE NUMBER, STARTING BUS, END BUS, LENGTH, SHUNT
      ADMITTANCE
C     AND SERIES IMPEDANCE IN P.U PER UNIT LENGTH
      READ(5,100)LINE(I),SB(I),EB(I),LENGTH(I),YSHT(I),ZSER(I)
  100 FORMAT(3I5,F5.1,4E10.3)
      SHTY(I) = YSHT(I)*LENGTH(I)
      SERZ(I) = ZSER(I)*LENGTH(I)
      SERY(I) = 1.0/SERZ(I)
C     ASSEMBLE THE BUS ADMITTANCE MATRIX
      L=SB(I)
      M = EB(I)
      Y(L,L) = Y(L,L) + SERY(I) + SHTY(I)/2.
      Y(M,M) = Y(M,M) + SERY(I) + SHTY(I)/2.
      Y(L,M) = Y(L,M) - SERY(I)
    2Y(M,L) = &(M,L) - SERY(I)
C     WRITE OUT THE INPUT LINE DATA AND THE Y BUS MATRIX.
      WRITE(6,101)
  101 FORMAT('1',T38,'LINE DATA'//T8,'LINE',T15,'SB',T19,
      'EB',T24,
      1'LENGTH',T36,'SHUNT ADMITTANCE',T58,'SERIES
       IMPEDANCE'/)
      DO 3 I = 1,NL
```

```
  3 WRITE(6,102)LINE(I),SB(I),EB(I),LENGTH(I),SHTY(I),
    SERZ(I)
102 FORMAT(' '.T6,3I5,F8.1,4X,2F9.4,4X,2F9.4)
    WRITE(6,103)
103 FORMAT(////T10,'BUS AMITTANCE MATRIX'//)
    DO 4 I = 1,NB
  4 WRITE(6,104)(Y(I,J),J=1,NB)
104 FORMAT (2(4(F9.4,1X,F9.4,3X)/))
    K = MB + 1
C     READ IN SPECIFIED BUS DATA, REAL POWER, REACTIVE POWER,
        REFERENCE
C     VOLTAGE V1, VOLTAGE CONTROL BUS MAGNITUDES AND
C     REACTIVE POWER LIMITS.
      READ(5,105)(PD(I),I=2,NB)
      READ(5,105)(Q(I),I=K,NB)
      READ(5,105)V(1),VSPEC(I),I=2,MB)
      READ(5,105)(QMIN(I),QMAX(I),I=2,MB)
  105 FORMAT(8F10.3)
  950 KBC= KBC+ 1
C   INITIALIZE UNKNOWN VOLTAGES AND REACTIVE POWERS.
      PD(1) = 0.0
      PG(2) = XNOW(1)
      PG(3) = XNOW(2)
      PG(4) = XNOW(3)
      PG(5) = XNOW(4)
```

```
      DO 5 I = 2,NB
      P(I) = PG(I) - PD(I)
      IF(I.LT.K) Q(I) = 0.0
      V(I) = CMPLX(1.0,0.0)
C  CALCULATE NECESSARY CONSTANTS A(I) AND B(I)
      IF(I.GT.MB)A(I) = (CMPLX(P(I),-Q(I)))/Y(I,I)
      DO 5 J = 1,NB
      IF(I.NE.J) B(I,J) = Y(I,J)/Y(I,I)
    5 CONTINUE
C  INITIALIZE CONSTANTS AND BEGIN VOLTAGE ITERATIOS
      N = 0
    6 DVMAX = 0.0
      I = 2
    7 VII = V(I)
      IF(I-MB)8,8,15
C  FOR VOLTAGE CONTROL BUSES ADJUST VOLTAGE TO SPECIFIED
     MAGNITUDE
C  AND CALCULATE REACTIVE POWER, IF Q LIMITS ARE EXCEEDED SET
     Q EQUAL
C  TO THE LIMIT AND RETURN VOLTAGE TO PREVIOUS VALUE,
C  CALCULATE A(I)
    8 V(I) = (V(I)/CABS(V(I)))*VSPEC(I)
      SUM = CMPLX(0.0,0.0)
      DO 9 L = 1,NB
    9 SUM - SUM + Y(I,L)*V(L)
```

```
      Q(I) = - AIMAG(SUM*CONJG(V(I)))
      IF(Q(I)-QMAX(I))10.14,11
   10 IF(Q(I)-QMIN(I))12,14,14
   11 Q(I)=QMAX(I)
      GO TO 13
   12 Q(I) = QMIN(I)
   13 V(I) = VII
   14 A(I) =(CMPLX(P(I),-Q(I)))/Y(I,I)
C     CALCULATE NU + 1 VOLTAGES.
   15 SUM = CMPLX(0.0,0.0)
      VI = V(I)
      DO 16 L = 1,NB
      IF(L.NE.I) SUM=SUM+B(I,L)*V(L)
   16 CONTINUE
      VN(I) = A(I)/CONJG(V(I)) - SUM
      DX = VN(I) -VI
      VN(I) = VI + ALPHA*DX
C   DETERMINE MAXIMUM VOLTAGE DIFFERENCES BETWEEN ITERATIONS.
      DELV = CABS(VN(I) - VII)
      IF(DELV.GE.DVMAX)DVMAX=DELV
      I = I + 1
      IF(I.LE.NB) GO TO 7
C   UPDATE VOLTAGES BY ONE ITERATION
      DO 17 I = 2,NB
   17 V(I) = VN(I)
```

```
      N = N + 1
C  COMPARE MAXIMUM VOLTAGE DIFFERENCE AGAINST CONVERGENCE
C  CRITERIA
      IF(DVMAX.LE.1.0E-04) GO TO 19
C  LIMIT ITERATIONS AS PROTECTION AGAINST DIVERGENCE
      IF(N.LT.MAXIT) GO TO 18
      WRITE(6,106) N
  106 FORMAT(////,T10,'CONVERGENCE NOT OBTAINED
      IN',I3,'ITERATIONS')
      GO TO 23
   18 TO TO 6
C  CONVERGENCE OBTAINED --- CALCULATE SLACK BUS POWER.
   19 SUM = CMPLX(0.0,0.0)
      DO 20 I = 1,NB
   20 SUM = SUM + Y(1,I)*V(I)
      P(1) = REAL(SUM*CONJG(V(1)))
      Q(1) = -AIMAG(SUM*CONJG(V(1)))
C  CALCULATE AND WRITE OUT LINE FLOWS
      WRITE(6,107) N
  107 FORMAT('1',T6,'GAUSS ITERATIVE TECHNIQUE CONVERGED
      IN',I3,
     1'ITERATIONS'//T6,'BUS',T16,'VOLTAGE',T30,'MAGNITUDE',
      T42,
     2'DELTA(DEGS)',T57,'REAL POWER',T69,'REACTIVE POWER'/)
      DO 21 I = 1,NB
```

```
      DELTA = ATAN2(AIMAG(V(I)),REAL(V(I)))*57.29578
      MAGV = CABS(V(I))
   21 WRITE(6,108) I,V(I),MAGV,DELTA,P(I),Q(I)
  108 FORMAT(' ',I7,2X,2F8.4,4X,F7.4,4X,F9.5,6X,F8.4,6X,
      F8.4)
C   CALCULATE AND WRITE OUT LINE FLOWS
      WRITE(6,109)
  109 FORMAT(////T25,'LINEFLOW'//TB,'LINE',T15','SB',T20,
      'EB',T27,
     1'REAL POWER",T39,'REACTIVE POWER'/)
      DO 22 I = 1,NL
      L = SB(I)
      M = EB(I)
      S = V(L)*CONJG((V(L)-V(M))*SERY(I)+V(L)*(SHTY(I)/2.))
      R = V(M)*CONJG((V(M)-V(L))*SERY(I)+V(M)*(SHTY(I)/2.))
      DELOW1 = DELOW1 + R + S
      WRITE(6,110)LINE(I),L,M.S
   22 WRITE(6'110)LINE(I),M.L,R
  110 FORMAT(' ',T7,3I5,5X,F8.4,6X,F8.4)
      DO 30 I = 1,NB
      PG(I) = P(I) + PD(I)
      DEP1 = DELOW1 + PD(I) - 0.1
C  ALLOW ERROR OF .1 FOR DEPENDENT VARIABLE 1
C  COMPUTE THE PRODUCTION COST
      PM = PM + PG(I)*(3.0 + .25*PG(I))
```

```
30 WRITE(6,111) I,PG(I)
   DEP2 =CABS(V(2))
   DEP3 = CABS(V(3))
   DEP4 =CABS(V(4))
   DEP5 = CABS(V(5))
   XX1= PG(2)
   XX2= PG(3)
   XX3= PG(4)
   XX4= PG(5)
   WRITE(6,112) PM
111 FORMAT (/1X,'GENERATION ON BUS ',I3,1X,'IS',1X,F8.4)
112 FORMAT (/1X,'TOTAL PRODUCTION COST IS ..$',T28,F8.4)
 23 RETURN
    END
    SUBROUTINE MODEL
    COMMON /CX/ NPAR, XNOW(20)
    COMMON /CD/ NDEPV, DEPV(20)
    COMMON /AA/ KBC
    COMMON /CP/ PMODEL, ICOUNT, IEND
    COMMON /CB/ XUP(20), XLOW(20), DEPUP(20), DEPLOW(20),
     INORD
    CALL LDFLO(XNOW(1),XNOW(2),XNOW(3),XNOW(4),PMODEL,
     DEPV(1),DEPV(2),
   1DEPV(3),DEPV(4),DEPV(5),DEPLOW(1))
    PMODEL = -PMODEL
```

```
      KBC = KBC + 1
      IF(LPRINT.EQ.4) PRINT 910,PMODEL
      RETURN
  910 FORMAT(10H PMODEL = ,E14.6)
      END
      SUBROUTINE BOUND
      COMMON /CX/ NPAR, XNOW(20)
      COMMON /CD/ NDEPV, DEPV(20)
      COMMON /CB/ XUP(20), XLOW(20), DEPUP(20), DEPLOW(20),
       INORD
      COMMON /AAA/ XUPD(20),XLOWD(20),DEUP(20),DLOW(20)
      IF(INORD.EQ.2) GO TO 21
      DO 20 I=1,NPAR
      XUP(I)=XUPD(I)
      XLOW(I)=XLOWD(I)
   20 CONTINUE
      RETURN
   21 DO 23 I=2,NDEPV
      DEPLOW(I)= DLOW(I)
      DEPUP(I) = DEUP(I)
   23 CONTINUE
      RETURN
      END
```

APPENDIX B
CASE STUDY RESULTS

TABLE B.1

CASE 1

LOAD FLOW SOLUTION AND TOTAL GENERATION COSTS FOR INITIAL INPUT CONDITIONS

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	1.0126	-0.0356
2	0.9950	-0.0997	1.0000	-5.72324	-0.2500	0.2888
3	0.9952	-0.0975	1.0000	-5.59448	-0.2500	0.0505
4	0.9612	-0.0908	0.9655	-5.39853	-0.2500	-0.3000
5	0.9551	-0.1096	0.9614	-6.54467	-0.2500	-0.3000

TABLE B.1 (Cont'd)

LINEFLOW

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.3141	-0.0603
1	2	1	-0.3100	0.0211
2	1	4	0.3979	0.0850
2	4	1	-0.3926	-0.0959
3	2	4	-0.0052	0.1696
3	4	2	0.0061	-0.2025
4	1	3	0.3006	-0.0603
4	3	1	-0.2968	0.0176
5	2	5	0.0661	0.0980
5	5	2	-0.0653	-0.1525
6	4	5	0.1376	-0.0017
6	5	4	-0.1372	-0.0239
7	3	5	0.0470	0.0328
7	5	3	-0.0465	-0.1236

GENERATION ON BUS 1 IS 1.0126
GENERATION ON BUS 2 IS 1.0000
GENERATION ON BUS 3 IS 1.0000
GENERATION ON BUS 4 IS 1.0000
GENERATION ON BUS 5 IS 1.0000
TOTAL PRODUCTION COST IS .... $ 2760.120

CASE 1

TABLE B.2

LOAD FLOW SOLUTION AND TOTAL GENERATION COSTS AFTER OPTIMIZATION

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	0.9748	-0.0356
2	0.9954	-0.0960	1.0000	-5.50788	-0.2410	0.2837
3	0.9956	-0.0939	1.0000	-5.38719	-0.2413	0.0475
4	0.9619	-0.0874	0.9658	-5.18938	-0.2422	-0.3000
5	0.9559	-0.1052	0.9617	-6.28146	-0.2386	-0.3000

TABLE B.2 (Cont'd)

LINEFLOW

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.3023	-0.0599
1	2	1	-0.2985	0.0184
2	1	4	0.3831	0.0844
2	4	1	-0.3782	-0.0980
3	2	4	-0.0049	0.1680
3	4	2	0.0057	-0.2009
4	1	3	0.2894	-0.0600
4	3	1	-0.2859	0.0151
5	2	5	0.0631	0.0973
5	5	2	-0.0624	-0.1520
6	4	5	0.1313	-0.0011
6	5	4	-0.1310	-0.0248
7	3	5	0.0448	0.0324
7	5	3	-0.0443	-0.1233

GENERATION ON BUS 1 IS 0.9748
GENERATION ON BUS 2 IS 1.0090
GENERATION ON BUS 3 IS 1.0087
GENERATION ON BUS 4 IS 1.0078
GENERATION ON BUS 5 IS 1.0114
TOTAL PRODUCTION COST IS ....$ 2759.609

CASE 2

TABLE B.3

LOAD FLOW SOLUTION AND TOTAL
GENERATION COSTS FOR INITIAL* INPUT CONDITIONS

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	1.0262	0.0408
2	0.9977	-0.0680	1.0000	-3.89638	0.5000	0.3317
3	0.9984	-0.0567	1.0000	-3.24768	-0.0000	0.0369
4	0.9384	-0.1458	0.9496	-8.83236	-1.2500	-0.3000
5	0.9430	-0.1242	0.9512	-7.50540	-0.2500	-0.3000

*INITIAL CONDITIONS RESULT IN A VIOLATION OF 0.95 P.U. VOLTAGE MINIMUM ON BUS 4. NO OPTIMIZATION WAS DONE.

TABLE B.3 (Cont'd)

LINEFLOW

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.2136	-0.0557
1	2	1	-0.2117	-0.0002
2	1	4	0.6383	0.1502
2	4	1	-0.6248	-0.0967
3	2	4	0.4792	0.2130
3	4	2	-0.4724	-0.2001
4	1	3	0.1742	-0.0537
4	3	1	-0.1729	-0.0084
5	2	5	0.2333	0.1189
5	5	2	-0.2304	-0.1572
6	4	5	-0.1518	-0.0032
6	5	4	0.1522	-0.0209
7	3	5	0.1731	0.0453
7	5	3	-0.1709	-0.1219

GENERATION ON BUS 1 IS 1.0262
GENERATION ON BUS 2 IS 1.0000
GENERATION ON BUS 3 IS 1.0000
GENERATION ON BUS 4 IS 1.0000
GENERATION ON BUS 5 IS 1.0000
TOTAL PRODUCTION COST IS ......$ 2771.099

CASE 3

TABLE B.4

LOAD FLOW SOLUTION AND TOTAL GENERATION COSTS FOR INITIAL INPUT CONDITIONS

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	-0.0	0.5038	-0.0097
2	0.9988	-0.0495	1.0000	-2.83629	0.0000	0.2376
3	0.9995	-0.0307	1.0000	-1.75731	-0.0000	0.0046
4	0.9648	-0.0567	0.9665	-3.36253	-0.2500	-0.3000
5	0.9596	-0.0664	0.9619	-3.95585	-0.2500	-0.3000

TABLE B.4 (Cont'd)

LINEFLOW

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.1554	-0.0516
1	2	1	-0.1543	-0.0112
2	1	4	0.2543	0.0887
2	4	1	-0.2518	-0.1216
3	2	4	0.0716	0.1545
3	4	2	-0.0707	-0.1874
4	1	3	0.0942	-0.0468
4	3	1	-0.0938	-0.0223
5	2	5	0.0835	0.0943
5	5	2	-0.0826	-0.1483
6	4	5	0.0735	0.0089
6	5	4	-0.0734	-0.0365
7	3	5	0.0940	0.0269
7	5	3	-0.0931	-0.1152

GENERATION ON BUS 1 IS 0.5038
GENERATION ON BUS 2 IS 0.5000
GENERATION ON BUS 3 IS 1.0000
GENERATION ON BUS 4 IS 2.0000
GENERATION ON BUS 5 IS 1.0000
TOTAL PRODUCTION COST IS ....$ 3127.065

CASE 3

TABLE B.5

LOAD FLOW SOLUTION AND TOTAL GENERATION COSTS AFTER OPTIMIZATION

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	0.9939	0.0284
2	0.9980	-0.0637	1.0000	-3.65263	0.4498	0.3103
3	0.9978	-0.0658	1.0000	-3.77222	-0.0713	0.0396
4	0.9425	-0.1350	0.9521	-8.15062	-1.1603	-0.3000
5	0.9468	-0.1142	0.9536	-6.87609	-0.1896	-0.3000

TABLE B.5 (Cont'd)

LINEFLOW

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.2002	-0.0549
1	2	1	-0.1985	-0.0028
2	1	4	0.5912	0.1389
2	4	1	-0.5795	-0.1002
3	2	4	0.4390	0.2013
3	4	2	-0.4333	-0.1961
4	1	3	0.2025	-0.0556
4	3	1	-0.2007	-0.0030
5	2	5	0.2100	0.1119
5	5	2	-0.2077	-0.1537
6	4	5	-0.1466	-0.0037
6	5	4	0.1470	-0.0208
7	3	5	0.1296	0.0426
7	5	3	-0.1281	-0.1255

GENERATION ON BUS 1 IS 0.9939

GENERATION ON BUS 2 IS 0.9498

GENERATION ON BUS 3 IS 0.9287

GENERATION ON BUS 4 IS 1.0897

GENERATION ON BUS 5 IS 1.0604

TOTAL PRODUCTION COST IS ....$ 2772.760

CASE 4

TABLE B.6

LOAD FLOW SOLUTION AND TOTAL GENERATION COSTS FOR INITIAL* INPUT CONDITIONS

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	1.0169	0.0721
2	0.9949	-0.1012	1.0000	-5.80794	-0.2500	0.4838
3	0.9952	-0.0983	1.0000	-5.64137	-0.2500	0.0845
4	0.9357	-0.0868	0.9398*	-5.29998	-0.2500	-0.6000
5	0.9404	-0.1081	0.9465*	-6.55544	-0.2500	-0.3000

*INITIAL CONDITIONS RESULT IN VIOLATION OF MINIMUM VOLTAGE CONSTRAINTS OF 0.95 p.u. ON BUSES 4 AND 5. NO OPTIMIZATION WAS DONE.

TABLE B.6 (Cont'd)

LINEFLOW

LINE	SB	[illegible]	REAL POWER	REACTIVE POWER
1	1	2	0.3188	-0.0605
1	2	1	-0.3146	0.0222
2	1	4	0.3950	0.1930
2	4	1	-0.3886	-0.1946
3	2	4	-0.0025	0.3108
3	4	2	0.0051	-0.3293
4	1	3	0.3031	-0.0604
4	3	1	-0.2993	0.0182
5	2	5	0.0678	0.1507
5	5	2	-0.0665	-0.1997
6	4	5	0.1345	-0.0761
6	5	4	-0.1341	0.0523
7	3	5	0.0495	0.0663
7	5	3	-0.0486	-0.1527

GENERATION ON BUS 1 IS 1.0169
GENERATION ON BUS 2 IS 1.0000
GENERATION ON BUS 3 IS 1.0000
GENERATION ON BUS 4 IS 1.0000
GENERATION ON BUS 5 IS 1.0000
TOTAL PRODUCTION COST IS ....$ 2763.571

CASE 5

TABLE B.7

LOAD FLOW SOLUTION AND TOTAL GENERATION COSTS FOR INITIAL INPUT CONDITIONS

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	0.5185	0.0675
2	0.9975	-0.0698	1.0000	-4.00411	-0.7500	0.4707
3	0.9939	-0.1105	1.0000	-6.34363	-0.5000	0.1100
4	0.9512	0.0165	0.9513	0.99343	0.7500	-0.6000
5	0.9543	-0.0261	0.9546	-1.56799	-0.0000	-0.3000

TABLE B.7 (Cont'd)

LINEFLOW

LINE	SB.	EB	REAL POWER	REACTIVE POWER
1	1	2	0.2195	-0.0561
1	2	1	-0.2175	0.0009
2	1	4	-0.0420	0.1849
2	4	1	0.0434	-0.2245
3	2	4	-0.4106	0.3203
3	4	2	0.4174	-0.3074
4	1	3	0.3410	-0.0614
4	3	1	-0.3361	0.0270
5	2	5	-0.1212	0.1493
5	5	2	0.1229	-0.1960
6	4	5	0.2901	-0.0682
6	5	4	-0.2884	0.0536
7	3	5	-0.1637	0.0830
7	5	3	0.1663	-0.1576

GENERATION ON BUS 1 IS 0.5185
GENERATION ON BUS 2 IS 0.5000
GENERATION ON BUS 3 IS 0.7500
GENERATION ON BUS 4 IS 2.0000
GENERATION ON BUS 5 IS 1.2500
TOTAL PRODUCTION COST IS ....$ 3166.528

CASE 5

TABLE B.8

LOAD FLOW SOLUTION AND TOTAL GENERATION COSTS AFTER OPTIMIZATION

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	0.5499	0.0654
2	0.9976	-0.0694	1.0000	-3.97789	-0.6758	0.4610
3	0.9940	-0.1090	1.0000	-6.25871	-0.4793	0.1063
4	0.9506	0.0076	0.9506	0.45615	0.6391	-0.6000
5	0.9538	-0.0320	0.9543	-1.92191	-0.0178	-0.3000

TABLE B.8 (Cont'd)

LINEFLOW

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.2181	-0.0560
1	2	1	-0.2161	0.0007
2	1	4	-0.0046	0.1826
2	4	1	0.0060	-0.2228
3	2	4	-0.3600	0.3135
3	4	2	0.3657	-0.3084
4	1	3	0.3364	-0.0613
4	3	1	-0.3317	0.0259
5	2	5	-0.0989	0.1467
5	5	2	0.1004	-0.1951
6	4	5	0.2684	-0.0689
6	5	4	-0.2669	0.0525
7	3	5	-0.1474	0.0803
7	5	3	0.1496	-0.1574

GENERATION ON BUS 1 IS 0.5499
GENERATION ON BUS 2 IS 0.5742
GENERATION ON BUS 3 IS 0.7707
GENERATION ON BUS 4 IS 1.8891
GENERATION ON BUS 5 IS 1.2322
TOTAL PRODUCTION COST IS ....$ 3083.063

CASE 6

TABLE B.9

LOAD FLOW SOLUTION AND TOTAL GENERATION COSTS FOR INITIAL INPUT CONDITIONS

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	1.0126	-0.0356
2	0.9950	-0.0997	1.0000	-5.72324	-0.2470	0.2882
3	0.9952	-0.0975	1.0000	-5.59448	-0.2500	0.0504
4	0.9612	-0.0908	0.9655	-5.39853	-0.2500	-0.3000
5	0.9551	-0.1096	0.9614	-6.54467	-0.2500	-0.3000

TABLE B.9 (Cont'd)

LINEFLOW

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.3126	-0.0603
1	2	1	-0.3086	0.0208
2	1	4	0.3967	0.0850
2	4	1	-0.3915	-0.0961
3	2	4	-0.0043	0.1694
3	4	2	0.0051	-0.2023
4	1	3	0.3002	-0.0603
4	3	1	-0.2964	0.0175
5	2	5	0.0666	0.0979
5	5	2	-0.0659	-0.1524
6	4	5	0.1374	-0.0016
6	5	4	-0.1371	-0.0240
7	3	5	0.0466	0.0329
7	5	3	-0.0461	-0.1236

GENERATION ON BUS 1 IS 1.0095
GENERATION ON BUS 2 IS 1.0030
GENERATION ON BUS 3 IS 1.0000
GENERATION ON BUS 4 IS 1.0000
GENERATION ON BUS 5 IS 1.0000
TOTAL PRODUCTION COST IS ....$ 3160.120

CASE 6

TABLE B.10

LOAD FLOW SOLUTION AND TOTAL GENERATION
COSTS AFTER OPTIMIZATION

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	1.1673	-0.0112
2	0.9934	-0.1143	1.0000	-6.56265	-0.0800	0.3322
3	0.9966	-0.0824	1.0000	-4.72591	-0.0847	0.0447
4	0.9508	-0.1269	0.9593	-7.60225	-0.5212	-0.3000
5	0.9436	-0.1464	0.9549	-8.82153	-0.4611	-0.3000

TABLE B.10 (Cont'd)

LINEFLOW

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.3603	-0.0614
1	2	1	-0.3550	0.0320
2	1	4	0.5532	0.1086
2	4	1	-0.5431	-0.0828
3	2	4	0.1228	0.1880
3	4	2	-0.1214	-0.2166
4	1	3	0.2538	-0.0585
4	3	1	-0.2511	0.0074
5	2	5	0.1530	0.1122
5	5	2	-0.1514	-0.1599
6	4	5	0.1445	-0.0006
6	5	4	-0.1441	-0.0243
7	3	5	0.1666	0.0373
7	5	3	-0.1646	-0.1159

GENERATION ON BUS 1 IS 1.1673
GENERATION ON BUS 2 IS 1.1700
GENERATION ON BUS 3 IS 1.1653
GENERATION ON BUS 4 IS 0.7288
GENERATION ON BUS 5 IS 0.7889
TOTAL PRODUCTION COST IS ....$ 3083.941

CASE 7

TABLE B.11

LOAD FLOW SOLUTION AND TOTAL GENERATION COSTS FOR INITIAL INPUT CONDITIONS

THIS IS CALL NO. 1 FROM LOPER

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	1.0184	0.0095
2	0.9954	-0.0962	1.0000	-5.51984	-0.0000	0.3201
3	0.9989	-0.0468	1.0000	-2.68077	-0.0000	0.0170
4	0.9474	-0.1309	0.9564	-7.86842	-1.0000	-0.3000
5	0.9528	-0.1029	0.9584	-6.16435	-0.0000	-0.3000

TABLE B.11 (Cont'd)

LINEFLOW

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.3029	-0.0600
1	2	1	-0.2991	0.0186
2	1	4	0.5717	0.1208
2	4	1	-0.5609	-0.0890
3	2	4	0.2429	0.1917
3	4	2	-0.2405	-0.2118
4	1	3	0.1437	-0.0513
4	3	1	-0.1429	-0.0139
5	2	5	0.0569	0.1098
5	5	2	-0.0561	-0.1635
6	4	5	-0.1977	0.0008
6	5	4	0.1984	-0.0230
7	3	5	0.1431	0.0309
7	5	3	-0.1415	-0.1135
GENERATION ON BUS 1 IS			1.0184	
GENERATION ON BUS 2 IS			1.0000	
GENERATION ON BUS 3 IS			1.0000	
GENERATION ON BUS 4 IS			1.0000	
GENERATION ON BUS 5 IS			1.0000	
TOTAL PRODUCTION COST IS ......$				3164.783

CASE 7

TABLE B.12

LOAD FLOW SOLUTION AND GENERATION COSTS FOR
INTERMEDIATE POINT IN OPTIMIZATION
SHOWING VIOLATION OF VOLTAGE CONSTRAINT

THIS IS CALL NO. 9 FROM LOPER

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	1.5165	0.0497
2	0.9894	-0.1449	1.0000	-8.33111	0.0259	0.4217
3	0.9972	-0.0751	1.0000	-4.30447	0.0160	0.0486
4	0.9282	-0.1901	0.9475	-11.57497	-1.2556	-0.3000
5	0.9338	-0.1727	0.9497	-10.47904	-0.2624	-0.3000

TABLE B.12 (Cont'd)

LINEFLOW

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.4577	-0.0614
1	2	1	-0.4491	0.0570
2	1	4	0.8277	0.1683
2	4	1	-0.8053	-0.0474
3	2	4	0.3276	0.2336
3	4	2	-0.3235	-0.2410
4	1	3	0.2311	-0.0573
4	3	1	-0.2289	0.0027
5	2	5	0.1482	0.1311
5	5	2	-0.1464	-0.1773
6	4	5	-0.1257	-0.0115
6	5	4	0.1260	-0.0136
7	3	5	0.2450	0.0459
7	5	3	-0.2411	-0.1091
GENERATION ON BUS 1 IS			1.5165	
GENERATION ON BUS 2 IS			1.0259	
GENERATION ON BUS 3 IS			1.0160	
GENERATION ON BUS 4 IS			0.7444	
GENERATION ON BUS 5 IS			0.7376	
TOTAL PRODUCTION COST IS........$				3140.867

CASE 7

TABLE B.13

LOAD FLOW SOLUTION AND TOTAL GENERATION COSTS AFTER OPTIMIZATION

THIS IS CALL NO. 53 FROM LOPER

BUS	VOLTAGE		MAGNITUDE	DELTA(DEGS)	REAL POWER	REACTIVE POWER
1	1.0000	0.0	1.0000	0.0	1.0438	0.0357
2	0.9951	-0.0984	1.0000	-5.64535	0.1521	0.3501
3	0.9996	-0.0269	1.0000	-1.54399	0.1486	0.0139
4	0.9400	-0.1491	0.9518	-9.01568	-1.1619	-0.3000
5	0.9455	-0.1221	0.9534	-7.35705	-0.1579	-0.3000

TABLE B.13 (Cont'd)

LINEFLOW

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.3098	-0.0602
1	2	1	-0.3059	0.0201
2	1	4	0.6513	0.1415
2	4	1	-0.6373	-0.0848
3	2	4	0.3373	0.2093
3	4	2	-0.3333	-0.2179
4	1	3	0.0827	-0.0456
4	3	1	-0.0824	-0.0241
5	2	5	0.1215	0.1206
5	5	2	-0.1201	-0.1699
6	4	5	-0.1901	0.0027
6	5	4	0.1909	-0.0250
7	3	5	0.2313	0.0379
7	5	3	-0.2278	-0.1050

GENERATION ON BUS 1 IS 1.0438
GENERATION ON BUS 2 IS 1.1521
GENERATION ON BUS 3 IS 1.1486
GENERATION ON BUS 4 IS 0.8381
GENERATION ON BUS 5 IS 0.8421
TOTAL PRODUCTION COST IS ....$ 3103.560

VITA

[redacted]aica, Wes[redacted]ber[redacted] to Donald St. C. and Enid A. Allen and subsequently immigrated to New York City, New York in 1967.

The author graduated from Peter Stuyvesant High School in June, 1971, and from the Columbia University School of Engineering and Applied Science in New York City, New York, in May, 1976, with a B.S. Degree in Electrical Engineering.

The author has worked in the System Planning Department of Pennsylvania Power and Light Company since 1976 and is currently a Project Engineer in the Energy Assessment and Capacity Planning Section. In this capacity, the author is involved in various planning functions including the economic evaluation of alternate energy supply options and the estimation of future energy production costs for various scenarios.

The author was married in November, 1978, to Maxine D. Gyles of Kingston, Jamaica; and they currently reside in Allentown, Pennsylvania, with their daughter.

DA:lm

AB:1