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# Column strength of cold-formed tubular sections.

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COLUMN STRENGTH OF COLD-FORMED  
TUBULAR SECTIONS

by  
Robert James McDermott

A Thesis  
Presented to the Graduate Committee  
of Lehigh University  
in Candidacy for the Degree of

Master of Science  
in  
Civil Engineering

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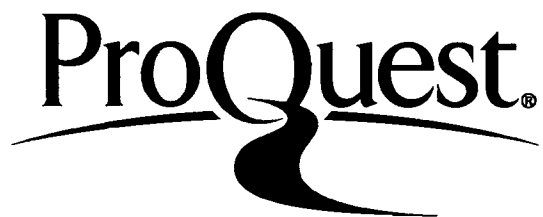
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This thesis is accepted and approved in partial fulfillment  
of the requirements for the degree of Master of Science in Civil  
Engineering.

5-8-81

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## ABSTRACT

Test data presently available on cold-formed tubes exhibit considerable variability. This can be attributed to different forming processes. Two tubes manufactured from identical material, but formed into the same cross-section by different methods, may have a significant difference in column strength.

Recognizing this fact, a testing program was conducted at Fritz Engineering Laboratory to study the parameters that affect column strength of tubes manufactured by Dominion Foundries and Steel, Ltd. The testing program included verification of material properties, residual stress measurements, stub column tests, and long column tests.

The sectioning method, used to obtain longitudinal residual stress distribution of the cross-section, was unsatisfactory. As an alternative results from the stub column test were used to obtain a theoretical prediction of column strength.

Two approaches were considered. Stub column properties were given to each element of the cross-section in an ultimate strength computer analysis. The results underestimate column strength by as much as 33 percent. The second approach used the stub column stress-strain curve to predict the tangent modulus load of the column. These results provide better agreement with test data and can be considered a lower bound for column strength.

## 1. INTRODUCTION

### 1.1 General

In recent years there has been increased interest in the use of cold-formed tubular sections as main load carrying members in structures. Tubular columns have particular advantages over other members because they are extremely effective in carrying compressive and torsional loads. The series of stiffened elements in tubular shapes make these sections resistant to lateral torsional buckling. The cold-working process increases the yield strength of the material to levels comparable to high strength steels without alloying or quench and tempering. In addition, because of highly automated methods of manufacturing, the cold-formed process has significant economic advantages over fabrication of tubes by welding hot-rolled plates. The advantages of a closed section, when coupled with beneficial increase in yield stress from cold forming, result in a highly desirable member for use in structural framing systems.

Although there are advantages to be gained by using cold-formed sections as columns, there are also disadvantages. Unfavorable residual stress patterns may develop from some forming processes. Also at present there exists no reliable analytical model for cold-formed column behavior. Based on the AISI Specification<sup>1</sup>, the design for compression members uses the effective width method of unstiffened compression elements. The Q factor, strength reduction

factor for unstiffened elements, is equal to 1 for tubular members, and therefore reflects no reduction in column strength. Two problems are immediately evident. First, can this specification developed for thin sections be applicable to thicker members. Second; how does welding and cold-forming effect the column strength<sup>2</sup>. The AISI specification recognizes the possible effects of such conditions and requires full section tests when physical or mechanical properties cannot be analytically evaluated. Reference 3 recognizes these differences by developing Class A and B design curves for tubular members, reflecting the influence of a residual stress pattern detrimental to column strength in cold-formed sections.

Based on tests of tubular members conducted primarily in Europe, Class A and B design curves were founded on a relatively clear distinction in behavior of hot and cold-formed tubes. Forming, weld stress relieving and straightening all influence the behavior of a cold-formed member as a column. In light of these facts, it is evident that the manufacturing process significantly influences the material properties of a cold-formed section. Therefore, the column curves developed from European test results, may not be applicable to columns formed by North American processes. Limited data are available for these products, hence research was undertaken to provide a data base for columns manufactured by cold-forming processes.

Research on this subject was completed in 1977 at the University of Alberta for cold-formed Class B tubes<sup>4</sup>. Tests were

conducted on eight sections produced by the same manufacturing process with the goal of developing a set of design criteria for these members. Stub column, long column, and material mechanical property tests were undertaken in an attempt to understand the strength and column behavior of these tubes. The data obtained added significantly to the overall statistical population, but still left many unanswered questions. The effects of member out-of-straightness, the magnitude of residual stress, coupled with the relatively small number of tests performed indicated the need for further research in this area.

The only reasonable approach to understanding the behavior of these members was to undertake a study formulated on a theoretical approach to the problem. Few carefully controlled tests to evaluate the performance on an analytical model would be more significant than only relying on a small statistical sampling of column tests. A study was undertaken at Fritz Engineering Laboratory and has brought to the researchers attention the significant difference in behavior from hot-rolled members<sup>5</sup>. It is the intent of this thesis to introduce the reader to these differences and develop an understanding of the factors that influence the behavior of cold-formed members in order that a sufficiently accurate analytical model may be developed. Factors to be considered are: variation of yield stress in the cross-section, effects of cold working and residual stress, non-linear material stress-strain relationships, initial out-of-straightness, and effects of column end restraint.

## 1.2 Fritz Engineering Laboratory Testing Program

The testing program sponsored by Dominion Foundries and Steel, Ltd. proceeded in a manner similar to previous work conducted on hot-formed members<sup>6,7</sup>. Yield strength, residual stress, and initial out-of-straightness are the parameters that influence column strength of hot-formed members and were therefore considered in this investigation of cold-formed sections. Column test data for the same sections was available for comparison, hence a study of these parameters on each section would be redundant. Consequently, this investigation was limited to two sections, 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) and 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.70 mm) being representative of the extremes in column sizes manufactured by the sponsor.

See Table 1 for the dimensions and properties of these sections. A brief description of the testing performed and presentation of results has been included in the text.



## 2. FACTORS INFLUENCING COLUMN STRENGTH

### 2.1 Material Properties

#### 2.1.1 Cold-working

Material properties play a crucial role in the strength and behavior of structural members. Although dependent upon residual stress and initial imperfections, cold-formed column strength is influenced primarily by the material yield strength except where elastic local buckling or overall buckling is dominant<sup>6</sup>. The stress-strain characteristics (i.e. modulus of elasticity, tangent modulus, and ultimate strength) are other properties that cannot be considered separate from yield strength and have a significant effect on the behavior of the member. Another important property that must be considered is ductility. Other properties that are normally of lesser importance in consideration of column strength are weldability, fatigue strength, and toughness.

Cold-forming changes the material properties of a steel from the original properties as a sheet, strip, plate, or bar before forming. This condition, before cold-working the material, is defined as the virgin state of the steel. The effect of cold-working on the virgin properties depend on many factors including: type of steel, amount of cold-working, ability of the steel to

strain-harden, and the direction of applied stress with respect to the direction of cold-working.

The yield strength of a material is the most basic quantity needed to be defined for both design and analytical purposes. Yield strength of steels can be explicitly defined through close inspection of the stress-strain relationship. The two general types of stress-strain curves for steel are shown in Figs. 1 and 2. The sharp-yielding type shown in Fig. 1 is exhibited in steels produced by hot-rolling. The yield point of this type of steel is defined as the level at which the curve becomes horizontal. Steels that are cold-worked or cold-reduced have stress-strain relationships that are gradually yielding in nature. The yield level of these materials cannot be as readily determined. An offset or extension under load methods are normally used to define the yield point.<sup>7</sup>

The tensile stress-strain relationship for material taken from the "flats" of both sections tested at Fritz Engineering Laboratory are presented in Fig. 3. Note the gradual yielding stress-strain relationship that is normally exhibited by cold-worked material and that the definition of yield is based on the offset method. The yield strength of all materials presented in this analysis are defined in this manner.

The magnitude of the effects from cold-working vary with the method of manufacturing, type of steel, and chemical composition. The most obvious influence is from the manufacturing process. The degree and direction of cold-work will affect material properties.

The virgin properties of a steel determine how the steel will respond to cold-working. In Fig. 4 the stress-strain relationship of a virgin material, as shown by curve one, can be easily obtained from a simple tensile test. Assume the specimen was loaded to point A in the strain hardening range. Upon unloading the material will follow curve two. The process of loading in the strain hardening range and unloading the material is analogous to cold-working.

The cold-forming process is very severe and always stretches the material into the strain hardening range. The material instantly unloads after forming with loss of ductility. This stretching into the strain-hardening range introduces an increase in yield strength when the specimen is reloaded.

All material that is immediately reloaded will have the stress-strain relationship described by curve three. The material will load up to point A on the virgin stress-strain curve and continue to follow the virgin curve to ultimate. The intersection with curve one will not normally be as sharp as shown in Fig. 4, but will deviate from the linear portion of curve 3 at a lower load and gradually intersect curve 1 at a load and strain higher than at the point of unloading. The stress-strain relationship is no longer sharp yielding, necessitating the yield for cold-worked material to be defined by the offset or strain under load methods.

The stress-strain relationship of a strain aged material is shown by curve four. Strain aging is allowing a period of time to pass before reloading a material that has been plastically deformed

into the strain hardening range. This effect is exhibited by all types of steels, except for cold-reduced killed steel. Four distinct effects occur from strain aging. There is an increase in yield and ultimate strength along with a decrease in ductility. Also the well defined yield plateau of the virgin material may be recovered for mild steels.

The more a material is cold-worked the greater the change in yield strength. The effects of cold-stretching on cold-reduced rimmed, cold-reduced killed, and hot-rolled, semi-killed steels are compared in Fig. 5. Sheet steel samples were stretched to a permanent strain of 500, 1250, 2500, 5000  $\mu\text{mm}/\text{mm}$  and tensile tests were made in the direction of cold-stretching at each respective level.<sup>8</sup> All the steels included in this figure show an increase in yield strength as a function of cold-stretching. However, with cold-reduced killed steel the increase in yield strength is substantially less than the other materials. This further substantiates the significant influence of the type of material on the effects of cold-working.

Besides the type of material, one of the most important parameters that influences the effects of cold-working is the ratio of ultimate strength ( $F_u$ ) to yield strength ( $F_y$ ).<sup>8</sup> Figure 6 compares the effect of cold-stretching to the longitudinal tensile yield strength of two hot-rolled semi-killed steels. Material A is 10 gage with  $F_u/F_y$  equal to 1.55 for the virgin steel and B is 16 gage material with  $F_u/F_y$  equal to 1.30. A comparison of the test

results shows a greater increase in yield strength for material A than for material B at any degree of cold-working. Therefore the higher  $F_u/F_y$ , the greater capacity the material has for strain-hardening, producing an increase in yield stress when cold-stretched.

Cold-working of a material changes the stress-strain characteristics of the material. The yield strength will increase in the direction of stretching. However, the degree to which a particular material responds to cold-working is highly variable among different types of steels. The ratio of ultimate strength to yield strength, the effects of strain aging, and the type of steel (killed, semi-killed, or rimmed) all influence what new material properties will result from cold-working.

#### 2.1.2 Bauschinger Effect

To this point the discussion on the effects of cold-stretching have been limited to tensile behavior of the material in the direction of cold-working. However, cold-working also effects compressive properties of the material in the direction of cold-stretching as well as tensile and compressive properties perpendicular to cold-stretching<sup>9</sup>. The compression properties perpendicular to cold-working are of particular interest in cold-formed tubular columns, where the member is loaded perpendicular to the direction of cold-working during manufacturing.

The phenomenon that results in a change in yield strength and proportional limit as a result of plastically deforming a material and reloading is called the Bauschinger Effect. The Bauschinger

Effect increases the tensile yield while lowering the compression yield of a longitudinally cold-worked material. This phenomenon is displayed in Fig. 7 for a cold-reduced killed material.

The properties of the same material in the transverse direction (perpendicular to cold-working) are shown in Fig. 8. Plastically stretching in the longitudinal direction will introduce a permanent transverse strain, if the material is free to deform. This phenomenon will hereafter be called the Inverse Bauschinger Effect. Therefore the effects on the transverse properties of the material can be equivalently viewed as those of a longitudinally cold-compressed specimen. In the direction of equivalently induced cold-compression, there is a significant increase in compression yield over the tensile yield.

All steels exhibit this phenomena whether cold-reduced, hot rolled, killed, semi-killed or rimmed. The effects of strain-aging on a material may cause an increase in compression in the longitudinal direction or tension in the transverse direction. This occurs inspite of the Bauschinger Effect causing increase of one property and decrease of the other. For example, the tensile yield was expected from the Inverse Bauschinger Effect. Although the tensile yield of this material was raised, the proportional increase was much less than the proportional increase in compressive yield for equivalent amount of cold-stretching.

Previous research work at Cornell University has been conducted on the corner properties of thin cold-formed steel shapes<sup>10</sup>.

Test of corners in both tension and compression showed no difference between tension and compression yield, although specimens did exhibit an increase in yield strength from the virgin state. The cold-forming process used to fabricate corners result in a transverse plastic tensile strain on the exterior and transverse plastic compressive strain on the interior of the corner, as shown in the upper portion of Fig. 9. The material properties of these corner specimens were measured normal to the direction of cold-working, hence one would expect the material properties in this direction to be influenced by the Inverse Bauschinger Effect. However, as a consequence of the compressive and tensile plastic strains in the same area of the section, no net Inverse Bauschinger Effect exists.

Nevertheless an area of the member that is either uniformly cold-compressed or cold-stretched will exhibit the Bauschinger Effect. The material properties in an area of a cold-formed section that has been uniformly stretched, as shown in the lower portion of Fig. 9, will be affected in the normal direction. A greater increase in compression yield than tension yield would be expected.

This theory has been supported by limited data obtained on the material properties of the flats of the 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) section. Figure 10 shows the area where tension and compression coupons were removed from the specimen. The coupons were taken from the same area of the cross-section, removed from the influence of weld metal, and should therefore have been exposed to equivalent amounts of cold-working. The stress-strain diagram in

Fig. 11 shows the significant increase in compression yield over tension yield for this area of the cross-section subjected to uniform stretching.

### 2.1.3 Variation in Yield Strength

The yield strength of a cold-formed member will vary with location due to initial variation in yield strength of the virgin material and from the effects of cold-working. The variation due to cold-working is of prime concern in this investigation. Each area of the cross-section subject to cold-forming undergoes plastic deformation and will be subjected to a change in yield. Consequently an increase in yield strength of the material should vary across the section as a function of the cold-forming performed at that location.

Figures 12 and 13 show the variation in yield strength as obtained from the tests of tensile coupons taken from the corresponding area of the respective 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) and 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.70 mm) cross-sections<sup>4</sup>. The yield stress levels for each specimen were divided into three ranges of 34.5 MPa, to aid in distinguishing areas of low, medium and high yield stress.

Examination of these figures reveals that the lowest yield was at the flats. Furthermore the highest yields are located at or adjacent to the corners. Influence of the weld metal is also manifested in the flats by medium range yield levels. This data is in agreement with the degree of cold-work received at each particular



area of the section. Areas that receive a greater amount of cold-working, primarily the corners, exhibited the highest yield. Similarly areas that undergo the least cold-working, the flats, display lower yield stress values.

ASTM A-500 Grade C material is used in the manufacturing of these tubes<sup>11</sup>. The virgin yield and tensile strength of this material is 345 and 427 MPa respectively. In spite of different material thickness, it is interesting to note a significant difference in the yield level of the two specimens formed from the same virgin material. Moreover, the yield strength of the corners of the material has been increased by as much as 70 percent for the 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.70 mm) and 29 percent for the 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) section. Obviously, because similar materials were used in each section, the cold-forming process is the only factor that could cause such a significant increase in tensile yield.

The research work outlined in Ref. 10 pointed out that in addition to the  $F_u$  to  $F_y$  ratio, corner properties of the material are influenced by member geometry. For example, Fig. 14 shows that the increase in yield strength is proportional to the ratio of inside corner radius to material thickness ( $R/t$ ). Consequently from theoretical considerations and extensive testing of cold-formed corners the equation:

$$F_{ye} = B_e F_y / (R/t)^m \quad (1)$$

was derived for calculating the corner yield strength,  $F_{ye}$ . Two factors  $B_e$  and  $m$  are functions of  $(F_u/F_y)$  and are defined as follows:

$$B_e = 3.69 (F_u/F_y) - 0.819 (F_u/F_y)^2 - 1.79 \quad (2)$$

$$m = 0.192 (F_u/F_y) - 0.068 \quad (3)$$

These equations can be applied with equal accuracy to killed, semi-killed, or rimmed steels and have been adopted for calculating the increase in yield gained through cold-working corners in the AISI Cold Formed Steel Design Manual.<sup>2</sup> This work was derived primarily for thin material. However, test results show that the equation is also applicable to thick (up to 1 in. (2.5 cm)) steel sheets and plates having sharp yielding stress-strain curves or cold-rolled sheets and strip.<sup>12</sup>

Equation 1 provides good agreement with yield strengths obtained for the sections under consideration. Values of  $R/t$  given in Table 1 can be used along with the virgin material ultimate and yield strength to calculate the predicted yield strength after cold-working. For the 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.70 mm) section 586.7 MPa was calculated from Eq. 1 and 606.4 MPa was obtained from testing the corners. Because the corners of the 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) section were too sharp, no tensile coupons were taken from this area. However, coupons taken adjacent to the corners averaged 444 MPa while Eq. 1 predicted 489 MPa. In spite of a lack of data to support the predicted

value, the results agree in principle when compared to the yield strength of material adjacent to the corners.

#### 2.1.4 Method of Manufacture

The most inconsistent parameter that influences the material properties of a section is the manufacturing process. The amount of cold-working the section has been subjected to, consequently defines the material properties of the section. There are many different processes used to fabricate cold-formed tubes and each step of the process must be viewed individually to sum the effects of that process on the material properties of the entire cross-section. Two examples of manufacturing an 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) tube will be discussed to point out the wide range in yield that can be obtained from different manufacturing methods using the same material.

The first method, Process A, consists of only bending the corners to form the cross-section as shown in Fig. 15. This manufacturing process does not cold-work the flats and therefore does not effect the material properties in these areas. Only the material properties at the welds are different than in the virgin state.

The effects on yield strength of the entire cross-section can be determined by the following equation:

$$F_{ya} = C F_{yc} + (1 - C) F_{yf} \quad (4)$$

where  $C$  is the ratio of corner area to total area,  $F_{yc}$  is the yield strength of cold-formed corners as determined by Eq. (1) and  $F_{yf}$  is the weighted average tensile yield of the flats. This procedure is described in detail in Ref. 2. The yield strength of the section formed by Process A is 360 MPa, calculated as per Eq. (4).

The second manufacturing method involves cold-working the entire cross-section. Process B, shown in Fig. 15, is the basic method used to manufacture the cold-formed tubes studied in this investigation. As the steel sheet or plate is first rolled into a circular section, the material is uniformly cold-worked. Next, the section is welded and locally heat-treated in the vicinity of the weld. The manufacturing process proceeds to cold-compress and stretch the circular section to the desired rectangular size and shape.

Two problems arise with using Eq. (4) to calculate the yield of the entire cross-section. First, the forming process has cold-formed the flats, changing the yield. Unfortunately the yield for the flats cannot be analytically determined, hence testing must be performed if one is to utilize the increased capacity from cold-working the flats. Second the Inverse Bauschinger Effect will be apparent with uniform cold-stretching of the flats in the transverse direction. This factor can only be determined by testing. The yield of the entire cross-section calculated from Eq. (4) using Eq. (1) for calculating yield of the corners, compression properties of the flats as determined by tests, and ignoring the effect of the

weld results in a yield value of 459 MPa. The yield strength is almost 100 MPa higher than Process A.

It is clearly evident from these examples that the manufacturing process significantly influences the yield strength of the entire cross-section in direct proportion to the cold-working the material is subjected. The results obtained from Eq. 4 have been compared with test data from full section stub column tests.<sup>13</sup> These data obtained on various thin-walled cold-formed members with negligible residual stress provide good agreement with Eq. (4). However, stub column tests performed as part of this investigation, determined the yield of these sections to be 400 MPa. This discrepancy between test and theory may be attributed to the influence of residual stress.

## 2.2 Residual Stress

### 2.2.1 Residual Stress from Cold-Working

Research has shown that the main factor influencing the strength of centrally loaded columns is the distribution of residual stress within the column cross-section.<sup>14,15</sup> The compression residual stresses are of primary concern for columns. If a section were free of residual stress the stress-strain curve of the entire cross-section would be identical to a coupon taken from anywhere in that section.

This condition seldom exists and as a member is loaded fibers of highest compression residual stress will yield at a load below the yield of the coupon representative of that material. This locked-in fiber stress changes the stress-strain characteristics of the entire cross-section, significantly influencing the behavior of the member as a column. However, sections with tensile residual stress yield later than sections with no residual stress.

Residual stress occurs as a direct result of plastic deformations introduced from the rolling, manufacturing, or fabrication process to which a member is subjected. In addition, residual stress can be classified into two broad categories. The categorization has as a basis, the method in which the section was produced. The first, thermal residual stress is created by plastic deformation resulting from differential cooling of elements after hot-rolling, welding, or oxygen cutting. The second, deformational residual stress, is primarily induced by operations like cold-straightening and cold-working. The deformational residual stresses created during the cold-forming process are of primary concern in this section.

The residual stress patterns of cold-formed sections are three-dimensional in nature. The effects of cold-working on residual stress are as directionally dependent as material properties. In cold-formed shapes, the magnitude of residual stress is greatest in the direction transverse to the bending axis.<sup>16</sup> This is the direction of cold stretching or cold compression. There are also significant residual stresses in the longitudinal direction. These

stresses are introduced from the Poisson effect of strain in the transverse direction. In addition differential cold-working on the interior and exterior surface, especially at corners, results in a residual stress variation through the thickness.

### 2.2.2 Sectioning Method

There are numerous methods of determining longitudinal residual stress among which the most economical is the sectioning method. This method is based on the principle that cutting the specimen into many strips of smaller cross-section relieves the internal stresses. Furthermore it is assumed the cutting process along produces no appreciable strain. It is important to note that the analysis of residual stress assumes transverse stresses are negligible. Nevertheless transverse stresses do exist in cold-formed sections influencing the accuracy of results.<sup>17</sup>

The sectioning method consists of cutting the member into longitudinal strips and measuring the change in length of these strips. The stress distribution over the cross-section can then be determined from strain measurements and Hooke's Law, assuming elastic unloading of fibers.

The cold-forming process does not equally strain the inside and outside surface of the tube. It was therefore of interest to measure the strains on both surfaces, to see the effect of through thickness residual stress. For the sections considered in this investigation the interior dimension did not allow measurements with a Whittemore gauge. Consequently the sectioning was modified

based on Ref. 18 to facilitate taking readings on the interior of the section.

Figure 16 shows the slicing sequence and strip numbering sequence for the 8 x 8 x 1/2 (203.2 mm x 203.2 mm x 6.35 mm) cross-section. First, a 68.58 cm section was cut from a longer member, sliced and 10 in. (254 mm) gage lengths layed-out, numbered and holes drilled. Next, initial readings were made with a Whittemore gauge. Longitudinal cut 2 was then made at a section of predicted low residual stress, splitting the section into two pieces.<sup>18</sup> This procedure facilitated taking readings on the interior of the specimen without disturbing the distribution of residual stress. However, there was noticeable displacement of the cross-section in the transverse direction during cutting. This indicated a redistribution of residual stress in spite of no change in the measured gage lengths on the exterior of the specimen.

The sectioning procedure continued by laying-out and drilling the gage lengths for each strip on the interior of the section, followed by initial measurements of gage length. Cuts 3A and 3B were then made reducing the section to 27.94 cm. The section was then cut longitudinally into 60 strips (28 for the 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.7 mm)).

Figure 17 illustrates the significant displacement observed during the sawing process, indicating high residual stresses. All of the longitudinal strips exhibited considerable curvature as shown in Fig. 18 indicating significant variation in residual stress through



the thickness. The curvature required two corrections to be made to the strain measurements. First, because the Whittemore gauge measures direct distance, the true strain lies along the arc length of the specimen rather than the chord. This necessitates a curvature correction. The second correction was required for misalignment of holes. The mathematical expressions for these corrections can be found in Refs. 17 and 19.

### 2.2.3 Longitudinal Residual Stress Distribution

The longitudinal residual stress distributions obtained by the sectioning method are shown in Fig. 19 for the 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) section and in Fig. 20 for the 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.70 mm) section averaged 345 MPa compression. The residual stress distribution on the exterior varied considerably between sections. The 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) section exhibits a variable residual stress pattern with extremely high (Max. 830 MPa) residual stress adjacent to the corners and a lower uniform residual stress averaging 415 MPa in the flats. In contrast, the 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.70 mm) section shows a more uniform tensile residual stress pattern averaging 520 MPa.

An electric resistance welding process was used to fabricate these sections. Both sections show no effect on residual stress from the welding process. There are two explanations for this occurrence. First, the welds are locally heat treated to reduce the residual stress. Second, these results are consistent with the literature

showing cooling residual stress from the electric-resistance welding process to be negligible.<sup>20</sup>

There is a significant difference in the magnitude and distribution of residual stress obtained from cold-formed sections when compared to either hot-rolled tubes or welded box sections. The residual stress distribution for a 3-1/2 x 3-1/2 x 5/16 (88.9 mm x 88.9 mm x 7.94 mm) hot-formed tube is shown in Fig. 21. The distribution in the hot-formed tube and the 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.70 mm) section are uniform. However, the magnitude of stress in the hot-formed tube is approximately equal on the inside and outside surface, but of much lower magnitude than the cold-formed section. Results on other hot-formed tubes show uniform residual stress patterns of low magnitude.<sup>21</sup>

The welded box section shown in Fig. 22 shows variable residual stress on the exterior of the section and low uniform residual stress on the interior. The extreme change in residual stress adjacent to the corners are a result of high tensile residual stresses caused by welding. These sections also have compression residual stress on the inner surface.

Only limited results are available for comparison with residual stress measurements on other cold-formed tubes.<sup>19</sup> The stress distributions measured on the exterior surface are comparable to the results from the 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) tube. Uniform tensile residual stresses are found at the flats, and tensile residual stress greater than yield adjacent to the corner.

Measurements were not taken on the interior, however. The strips cut from Sherman's member also exhibited significant curvature. After annealing the same section does not exhibit curvature. This indicates the curvature is not a result of the sectioning method, but of the state of stress in the specimen.

Based on a linear stress distribution through the thickness of the material, the residual stress distribution is not in mathematical equilibrium. The average of all stress measurements on the interior and exterior of the specimen, indicate a net tension residual stress. However, the residual stresses must be in equilibrium, therefore the through-thickness residual stress distribution must be other than linear. In addition, errors could be introduced into the analysis from the influence of transverse residual stress.

Kayto has studied the residual stresses in cold-formed circular tubes, analyzed on the basis of mathematical plasticity.<sup>20</sup> This study shows the effect of each step in the cold-forming process on longitudinal and circumferential residual stress. On the basis of this analysis, the residual stress distribution through the thickness is not linear but similar to the distribution shown in Fig. 23. Note these results are for circular tubes that are not as severely cold-worked as the corners of square and rectangular tubes.

Alternate procedures have been considered for determining the residual stresses in the longitudinal direction.<sup>22</sup> Since curvature of the specimen effects the surface strain and is due

primarily to the through-thickness stresses, removing this stress gives the distribution in the longitudinal direction. This is accomplished by re-introducing the stress distribution through the thickness, or alternatively applying a moment to balance the internal forces. A simpler method of introducing through-thickness stresses is to physically apply the moment by elastically flattening the specimen and measuring the surface strains, obtaining longitudinal strain directly.

The data obtained by this procedure for one side of the 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) section shows considerable reduction in longitudinal stress. Two problems are associated with applying this method. Plastic deformation is introduced into the specimen when bending the strip back to the flat position. These results indicate extremely high through-thickness residual stress. Also for thicker sections this method is not practical because of the high forces required to straighten the specimen causing large plastic deformation. Further experimentation and data are required to substantiate the validity of longitudinal surface strains measured by this method.

Residual stress distribution, as determined by the sectioning method, does not give a true picture of the longitudinal residual stresses that influence column behavior. First, significant transverse stresses exist in cold-worked members that have not been accounted for in the data analysis. Second, the non-linear through-thickness variation of stress cannot be readily determined by

elementary means. Finally, the validity of measuring curved strips in the flat condition to determine longitudinal residual stress is unknown. As a result, the significance of the residual stress data in this testing program is uncertain.

## 2.3 Geometrical Considerations

### 2.3.1 Initial Out-of-Straightness

Small initial deflections or initial out-of-straightness effects the ultimate column strength. Even small initial out-of-straightness has shown a drastic decrease in ultimate strength.<sup>23</sup> In all cases the maximum load for an initially crooked inelastic column is less than the load for a straight column. Depending upon slenderness ratio, magnitude of initial curvature, residual stress, and shape of the stress-strain curve, load for an initially crooked column may be below the tangent modulus load.

Cold-formed tubes, by the nature of the forming process produces straight columns without requiring such overall straightening as rotorizing or gaging. The testing program examined in Ref. 4 shows these columns to be extremely straight. The average out-of-straightness for 30 specimens was 1/6384 from measurements made on two axes. It is apparent that the effect of the initial out-of-straightness of these members is insignificant compared to the specification limiting value of 1/1000.

Initial out-of-straightness measurements made on the two sections are shown in Figs. 24 and 25. The maximum

out-of-straightness was 1/1700 for the 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) and 1/2400 for the 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.7 mm) section, greater than the same sections tested in Ref. 4.

### 2.3.2 Column End Restraint

Initial crookedness and residual stress have long been recognized as the parameters that significantly reduce the strength of pin-ended columns. The development of the multiple column curve concept is based on the influence of these parameters and the load-deflection approach. In spite of recognizing the influence of residual stress, and initial imperfections, limited information indicates the effects of a small amount of initial crookedness can be offset by the introduction of modest amounts of end restraint.<sup>24</sup>

Square sections used in this study have equal moment of inertia on both major axes and on the diagonal. The spherical bearing shown in Fig. 26 was used as an end fixture in long column tests to allow the column to buckle in any direction. In both the long column tests performed at Fritz Engineering Laboratory, the columns failed on a diagonal, with deflections of one of the major axes being predominant.

The procedure used to test these columns can be found in Technical Memorandum No. 4.<sup>25</sup> Instrumentation included rotation gages at the bearings, strain gages on all sides of the cross-section at 3 locations along the column, and measurements of deflection at midheight. These measurements could then be used to

determine if restraint was being introduced from the spherical bearing.

A review of the data obtained during testing indicated that some restraint was provided by the bearings. To correct for effects on column strength, it was necessary to determine the effective column length rather than the actual length as a pin-ended column. The procedure outlined in Ref. 26 for determining the effective length was used. The curvature distribution along the member is plotted in Fig. 27 for loads approaching bifurcation. Effective column length is taken as the length between points of zero curvature. The effective column length was reduced from 72.9 to 61.8 for the 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) section.

### 3. THEORETICAL PREDICTION OF COLUMN STRENGTH

#### 3.1 Tangent Modulus

The tangent modulus load is the buckling or bifurcation load of the inelastic column. Tangent modulus theory assumes no strain reversal on the convex side of the bent column when it passes from straight to the deflected configuration. This assumption is incorrect for maximum column loads, for there must be unloading of fibers. Other theories (such as Reduced Modulus and Shanley Theory) take into account unloading of fibers. The reduced modulus load is generally considered an upper bound for ultimate column strength. In contrast, the tangent modulus load provides a good approximation of the bifurcation load, representing a lower bound on column strength.

The slope of the stress-strain curve of a material in the inelastic range is defined as the tangent modulus. For column application the tangent modulus ( $E_T$ ) is normally determined from a stub column test of the material in compression. The "effective" modulus of the entire cross-section, determined from the stub column stress-strain curve, is modified by the non-homogeneity of both material properties and residual stress.

The solution for column load at bifurcation by tangent modulus is very simple. The tangent modulus load is the load



providing the level of stress that satisfies the modified Euler equation in the inelastic range. The tangent modulus equation for stress is the same as Euler's equation with  $E_T$  substituted for Young's Modulus  $E$ . The equation is given as:

$$\sigma_c = \frac{\pi E_T}{\left(\frac{KL}{r}\right)^2} \quad (5)$$

To solve for the tangent modulus load a trial stress is chosen,  $E_T$  for that stress level substituted into Eq. 5 to solve for  $\sigma_c$ . If  $\sigma_c$  calculated from Eq. 5 is equal to the stress level for the corresponding value for  $E_T$ , the solution is correct, otherwise another value must be tried.

Tangent modulus theory has been used as standard practice for the design of aluminum and magnesium members, as well as other metals with non-linear stress-strain relationships. Because of the variation in yield strength of cold-formed members, the same procedure for determining column strength has been suggested by Ref. 1. The application of tangent modulus theory to cold-formed members will be defined further in Section 4.1.

## 3.2 Ultimate Strength

### 3.2.1 Computer Program Analytical Approach

A computer program was used to analyze the ultimate strength of the 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) column. This program has been recently developed at Fritz Engineering Laboratory by Z. Y. Shen.<sup>27</sup> The program can be used to analyze any wide flange

or box columns where failure occurs about a principle axis. The column cross-section may be input with number of elements having distinct mechanical properties and residual stress. Column out-of-straightness, initial eccentricity, and end restraint are all important parameters included in the analysis.

The program uses an incremental numerical technique for column analysis. For this procedure the member must satisfy two equilibrium equations in addition to meeting compatibility conditions.

An initial curvature at the boundary is assumed. The program then proceeds to solve a series of simultaneous equations calculating the curvature and displacement of the next longitudinal column segment. This procedure continues until boundary conditions at the other end of the column are encountered. If the horizontal displacement at the boundary condition is not equal to zero, the assumed deflected shape is incorrect. Consequently, a new initial curvature is assumed and the above procedure reinitiated.

When the boundary conditions are satisfied, the column is in equilibrium and the associated load can be supported by the column. The load on the column is then incrementally raised until the compatibility and equilibrium conditions can no longer be satisfied. The condition at this load is termed divergent, indicating a condition of instability. The ultimate column load is defined as the maximum load the column can support before the solution becomes divergent.

### 3.2.2 Input Data

The computer program developed by Shen<sup>27</sup> has been used to accurately predict the ultimate strength of columns. For all cases studied, the input information was well documented and the information provided adequately accurate to proceed with a successful computer analysis. The parameters included in these analyses were; variation in yield strength through the cross-section, residual stress distribution, initial out-of-straightness, column length, eccentricity, and end restraint.

One objective of the testing program was to define these parameters for cold-formed columns in order that a computer based ultimate load analysis could be made. In spite of these efforts to accurately define the residual stress and material properties, the testing program did not supply the necessary data with sufficient accuracy to use in an analytical prediction of maximum strength. The sectioning method was unable to obtain a longitudinal residual stress distribution in equilibrium. Also due to insufficient information about the behavior of material properties in compression, it was not feasible to include these parameters in a computer analysis.

It was still necessary to obtain an ultimate strength prediction of column strength taking into account initial out-of-straightness. The input parameters obtained directly from the testing program, however could not be entered into the computer program. Nevertheless, the composite effects of material properties

and residual stress were available in a more simple form from the stub column stress-strain curve. This information could be used in conjunction with out-of-straightness, load eccentricity, and end restraint, as determined by the long column test, to predict the ultimate column load.

All elements of the cross-section were given the material properties of the stub column with zero residual stress. The stub column curve was approximated by three straight line segments as shown in Fig. 20 for input as the material properties in the computer program. The initial out-of-straightness given in Fig. 24 was also used.

Three cases were included in this initial phase of the investigation. Two levels of column length were used, one based on actual column length, the other on effective column length as defined between points of zero curvature in Section 2.3.2. The solution for actual column length was also investigated with an initial load eccentricity that balanced the column out-of-straightness. Results of the computer analysis can be found in Section 4.2.

#### 4. COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

##### 4.1 Significance of Stub Column

A stub column is defined as a column long enough to contain the original magnitude of residual stress in the section, but short enough to prevent overall column failure before yield of the entire cross-section.<sup>28</sup> Stub column tests are performed in order to obtain an average stress-strain curve for the cross-section which takes into account effects of residual stress and variation of material properties across the section.

Stub column tests were performed on both the 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) and 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.7 mm) sections. Figures 29 and 30 present the stub column stress-strain curves for each specimen. The 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) column exhibited local buckling of the sides followed by a sudden drop in load. The test was continued until the load dropped to 1/2 the maximum load. The 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.7 mm) section, however, exhibited strain-hardening in the plastic range. The test was discontinued when the load level reached 25 percent above the yield strength defined by a 0.2 percent strain offset.<sup>25</sup>

The tensile stress-strain relationship of coupons taken from the flats are plotted in these figures for each respective stub

column. A comparison of the stub column and coupon curves will help illustrate how the factors that influence column strength effect the behavior of these members.

First, the coupon taken from the flats is not representative of the entire cross-section. It is apparent that the yield strength of the stub column, which exhibits an average of material properties of the entire cross-section, is higher than the yield strength of the flats. Material from the flats has the lowest yield strength of any area of the cross-section. Therefore it is expected that the yield strength of the entire cross-section is greater than the material represented by a tensile coupon taken from the flats.

Second, there is little effect from residual stress. The difference in proportional limit of the two curves defines the magnitude of compression residual stress for hot-rolled sections.<sup>29</sup> In contrast cold-formed products exhibit an increase in proportional limit as well as yield strength as a consequence of cold-working the material. Therefore, the magnitude of compression residual stress cannot be applied to these cold-formed members.

As discussed in Section 3.1 the stub column stress-strain curve can be used to obtain the tangent modulus load for a column of the same dimensions, material, and manufacturing method. This procedure can be taken one step further and an entire column curve constructed.<sup>15</sup>

Tangent modulus curves were constructed for both sections and are presented in Figs. 31 and 32. For comparison, long column test results from this investigation and Ref. 4 are plotted along with these curves. In all cases the tangent modulus curve is a conservative prediction of column strength. Furthermore, there appears to be better agreement with higher values of  $\lambda$  where results are closer to elastic column behavior.

In order to explore the utility of this method for predicting column strength, Table 2 presents the ratio  $P_u/P_y$  (ultimate column load to stub column yield load) from tangent modulus theory and the corresponding  $P_u/P_y$  ratio calculated from test data. Test results from this study and Ref. 4 are included in this table.

The data shows good agreement with the predicted value for sections with 6.35 mm wall thickness and low values of  $\lambda$ , for 12.7 mm wall thickness and low values of  $\lambda$  there is also good agreement between test and predicted values. For higher values of  $\lambda$  the predicted values become more conservative. Predicted values are always below test values except for columns that have high values of  $P_y$  and  $\lambda$  with 1/2 in. (12.7 mm) wall thickness.

#### 4.2 Ultimate Strength Prediction

The only reasonable assessment of material properties and residual stress available from this testing program was the stub column stress-strain curve. Consequently theoretical predictions

of ultimate column strength were based on this data and initial out-of-straightness measured on the specimen.

Test results, however, do not adequately predict the ultimate strength of the column. A computer analysis with  $\lambda$  equal to the effective column length, predicts an ultimate column load of 1290 kN, 23 percent below the actual load of 1677 kN.

The load deflection curves for test results and theory are shown in Fig. 33. Besides the obvious discrepancy in ultimate load, there is also a difference in behavior. The theoretical model predicts column buckling to take place in the direction of out-of-straightness. The tested column shows a different behavior. The column initially buckles in the direction of initial curvature, changes direction and finally fails on a diagonal. The largest component was  $180^{\circ}$  to the direction of initial curvature and is plotted as such in Fig. 33. There are two possible explanations for the shift in direction. Initial out-of-straightness of the column could have been introduced elastically. When the column was loaded, the initial out-of-straightness was removed and the specimen free to buckle in any direction. Also the spherical bearings might have developed some restraint in the out-of-straightness direction, causing the specimen to fail in another mode.

These results show the inadequacy of this model, based on the stub column test results, to predict ultimate column strength. It is important to point out that the ultimate strength prediction based on this analysis falls below the tangent modulus load.



Tangent modulus theory predicts 1557 kN which is in reasonable agreement with test results.

## 5. SUMMARY AND CONCLUSIONS

This study was initiated to seek a means of developing a theoretical approach for predicting the behavior of cold-formed columns as manufactured by Dominion Foundries and Steel Ltd. Parameters included in this investigation were: variation in yield strength of the cross-section, residual stress distribution in the longitudinal direction, initial out-of-straightness, and end restraint.

The results of this investigation could not be directly applied to the computer program available for an ultimate strength prediction of column strength because the data required for the computer program is not readily available. This research did not provide a complete solution to the question at hand, but did serve as a preliminary investigation and will help to set the direction for future research.

The following conclusions can be made from this investigation:

1. The manufacturing process affects the amount of cold-working introduced into a particular cross-section. The amount of cold-working in turn affects the material properties of the cross-section.

2. The material is cold-worked to varying degrees throughout the cross-section. Tensile yield strength of the material can be related to the amount of cold-working that areas of the section have been exposed to. Areas that receive a large amount of cold-working, such as corners, show the highest yield strength. The lowest yield strength is exhibited at the flats, where the material is cold-worked the least.
3. Equation 1, developed from research on thin-walled cold-formed members, provides a satisfactory estimate of material yield strength at the corners. This is confirmed by test results given in Ref. 12.
4. Material taken from the flats exhibits different yield strength in tension and compression. These areas of the cross-section have been uniformly cold-stretched transversely. Cold-stretching in the transverse direction increases the compression yield while decreasing the tension yield of the material in the longitudinal direction. This behavior is explained by the Inverse Bauschinger Effect. The Inverse Bauschinger Effect does not influence the material properties of the corners.<sup>10</sup>

5. The residual stress pattern produced in cold-formed members is three-dimensional in nature. Deformational residual stresses are produced from cold-working in the transverse direction, therefore transverse residual stress is predominate. Longitudinal residual stresses are introduced from the Poisson effect of transverse strains. Through-thickness residual stresses are caused from unequal straining of the material on the inside and outside surface of the tube.
6. The residual stress distribution is not in equilibrium based on a linear distribution of stress through the thickness. Other work on circular cold-formed tubes produced by a similar manufacturing process support a through-thickness stress distribution other than linear.<sup>20</sup>
7. The sectioning method does not produce a reliable distribution of residual stress. The assumption that transverse residual stress is negligible cannot be applied to the analysis performed on cold-formed sections.
8. Column strength predictions can be made from tangent modulus theory. The tangent modulus load derived from the stub column test provides a lower bound for column strength. At present the only means

that is available to evaluate the effects of residual stress on column strength is through the stub column test curve.

9. Attempts to make ultimate strength predictions, based on verified computer program analysis, were not successful. The major roadblock to attaining a solution is the unreliability of the measured residual stress distribution. The stub column stress-strain curve was used for input to account for the effects of material properties and residual stress. Initial out-of-straightness was also considered. The analytical results underestimate column strength by as much as 1/3.
- 10 Similar sections manufactured by a different process may not produce columns of comparable strength. An understanding of the cold-forming process is therefore necessary before proceeding with an investigation on the column behavior of cold-formed members.

## 6. SUGGESTIONS FOR FUTURE RESEARCH

The results of this research program cannot be directly applied to a well defined analytical model for the prediction of column strength. Rather the results should be viewed as those from a preliminary investigation of the parameters that influence column strength. Particular emphasis is placed on the effects of the manufacturing process and how it affects material properties, residual stress distribution in the longitudinal direction, and use of the tangent modulus theory for prediction of column strength.

It is particularly advantageous to develop a theoretical prediction of column strength. The tangent modulus theory gives satisfactory results for the present but requires full section tests in compression. It is neither feasible nor economical to test every section produced by an individual manufacturer and assign each section to a column curve. The only reasonable approach is to develop the capability to predict the effects of the manufacturing process on column strength.

Any further study should start with a visit to the manufacturing facility. The researchers could then see the steps in the forming process and evaluate the extent of cold-working. With a basic understanding of how cold-working affects material properties and knowledge of the forming process one would be better able to proceed in an investigation.

The first phase of any future research should focus on accurately predicting the yield strength of various areas in the cross-section. Without such data the present numerical methods for calculating column strength will probably significantly underestimate the column strength. Equation 1 provides a good estimate for the corners but information about the flats must presently come from test data. Two areas should be investigated. First, the significance of the Bauschinger Effect on column strength should be addressed. Second, an effort should be made in developing a simple method of determining the yield strength in the cross-section.

The next phase in the investigation should be to determine the significance of residual stress on the behavior of cold-formed members. The differences in behavior of "as manufactured" tubes and fully stress relieved tubes from the same heat would be of particular utility in ascertaining the affects of residual stress. It is extremely important to document the material properties in both conditions to eliminate the affects of this variable.

The third phase of effort should be launched in the area of residual stress determination. Accurate prediction of the longitudinal and through thickness residual stress and how it affects column strength are essential. The hole drilling technique could be used to obtain data. This information if viewed in conjunction with residual stress distribution predictions from the sectioning method can be used to assess the validity of applying this method

to cold-formed sections. The method of flattening strips to remove curvature caused by through thickness residual stress could also be evaluated if this information were available.

Care should be exerted in conducting the testing program. As with any investigation the person conducting the test should be aware of the factors that significantly influence the test results. In particular, for long column tests, end fixtures should be carefully selected to provide a pin-ended connection or sufficient data taken during the test to determine the effective column length.

A research program such as the one suggested would take a tremendous effort but is a necessary approach to attain a solution to cold-formed column behavior which is better than the tangent modulus approach.



## NOMENCLATURE

A	cross-sectional area
$B_e$	dimensionless multiplier (see Eq. 2)
C	ratio of corner area to cross-sectional area
E	Young's Modulus
$E_T$	Tangent Modulus
$F_y$	yield strength of material
$F_{ya}$	equivalent yield strength of cross-section including the effects of cold-working
$F_{y(cw)}$	yield strength of cold-worked material
$F_{ye}$	yield strength of material at cold-formed corners
$F_{yf}$	equivalent yield of material in flats including the effects of cold-working
$F_{y(vg)}$	yield strength of virgin material
$F_u$	ultimate strength of material
m	dimensionless multiplier (see Eq. 3)
$P_u$	ultimate column load
$P_y$	stub column yield load
Q	effective width factor for thin cold-formed sections
R	inside corner radius
r	radius of gyration
t	thickness of material
w	width of flats
$\lambda$	slenderness function $\frac{1}{\pi} \sqrt{\frac{F_y}{E}} \left(\frac{L}{R}\right)$

TABLE 1 CROSS-SECTION PROPERTIES

	<u>8 x 8 x 1/4<sup>*</sup></u>	<u>4 x 4 x 1/2<sup>**</sup></u>
A (mm <sup>2</sup> )	4581	4142
r (mm)	80.01	34.54
w/t (mm/mm)	31.10	5.10
R/t (mm/mm)	1.56	0.44

<sup>\*</sup>  


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(203.2 mm x 203.2 mm x 6.35 mm)

<sup>\*\*</sup>  
(101.6 mm x 101.6 mm x 12.7 mm)

TABLE 2 SUMMARY OF STUB COLUMN TEST DATA

Member Size (mm)	$\lambda$	Test Value $P_{\max}/P_y$	Predicted Value $P_{\max}/P_y$	$P_y$ (MPa)
50.8 x 50.8 x 6.35	0.50	0.90	0.89	541
	0.99	0.70	0.64	541
	1.34	0.51	0.47	541
101.6 x 101.6 x 6.35	0.46	0.93	0.91	474
	0.93	0.72	0.61	474
	1.21	0.60	0.45	474
101.6 x 101.6 x 12.7	0.50	0.96	0.84	543
	1.00	0.79	0.49	543
	1.00	0.75	0.49	543
	1.00	0.88	0.49	543
	1.22	0.76	0.43	528
	1.34	0.45	0.35	543
	1.75	0.31	0.33	543
152.4 x 152.4 x 6.35	0.32	0.98	0.98	407
	0.83	0.86	0.72	407
	1.21	0.67	0.56	407
152.4 x 152.4 x 12.7	0.48	0.90	0.89	508
	0.96	0.68	0.55	508
	1.29	0.51	0.38	508
	1.89	0.28	0.26	508
203.2 x 203.2 x 6.35	0.44	0.96	0.93	427
	0.88	0.83	0.73	427
	1.02	0.96	0.63	394
	1.29	0.58	0.57	428
203.2 x 203.2 x 12.7	0.47	0.88	0.77	490
	0.95	0.81	0.68	490
	1.42	0.44	0.50	490

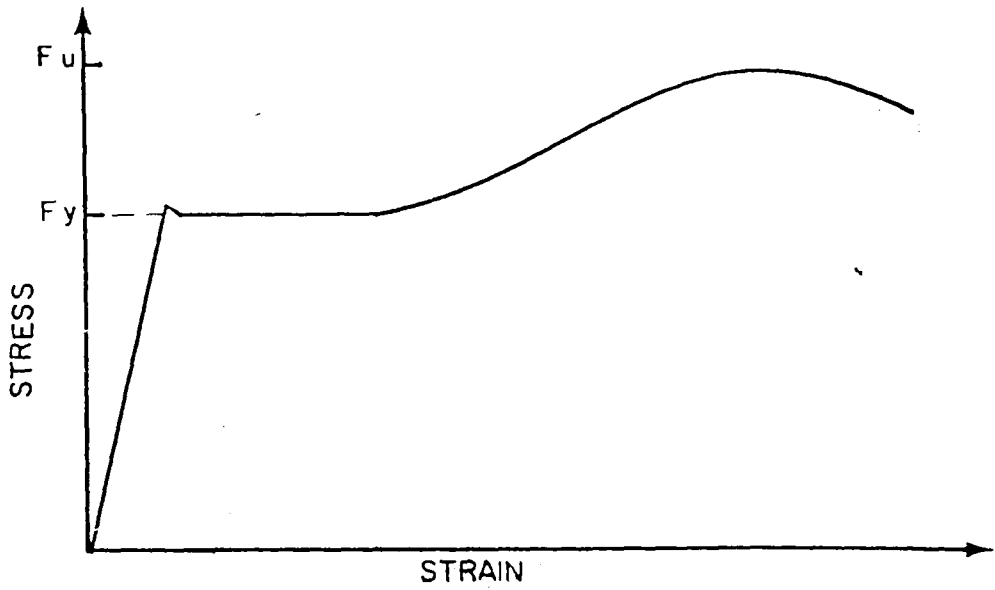


Fig. 1 Sharp-Yielding Stress-Strain Relationship

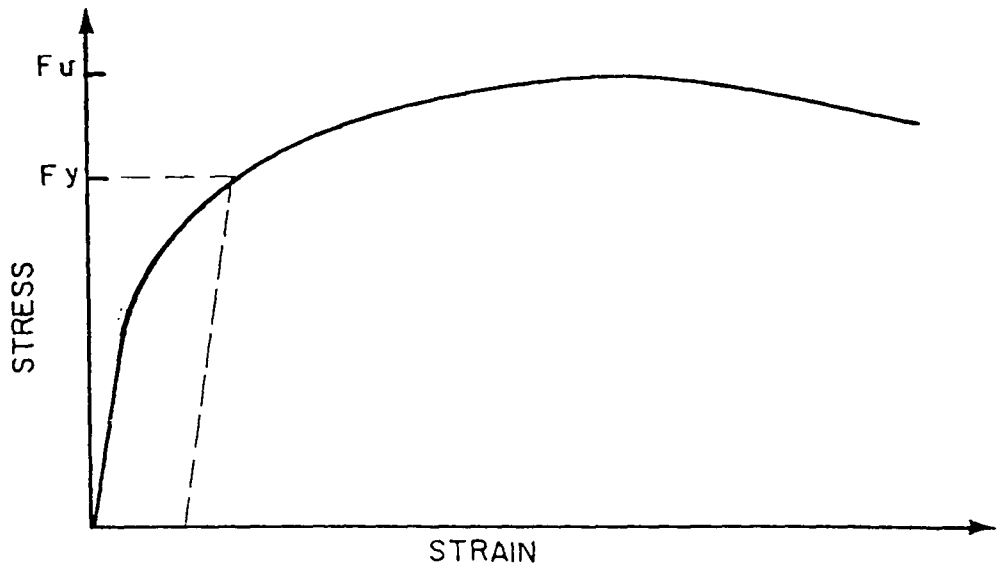


Fig. 2 Gradual-Yielding Stress-Strain Relationship

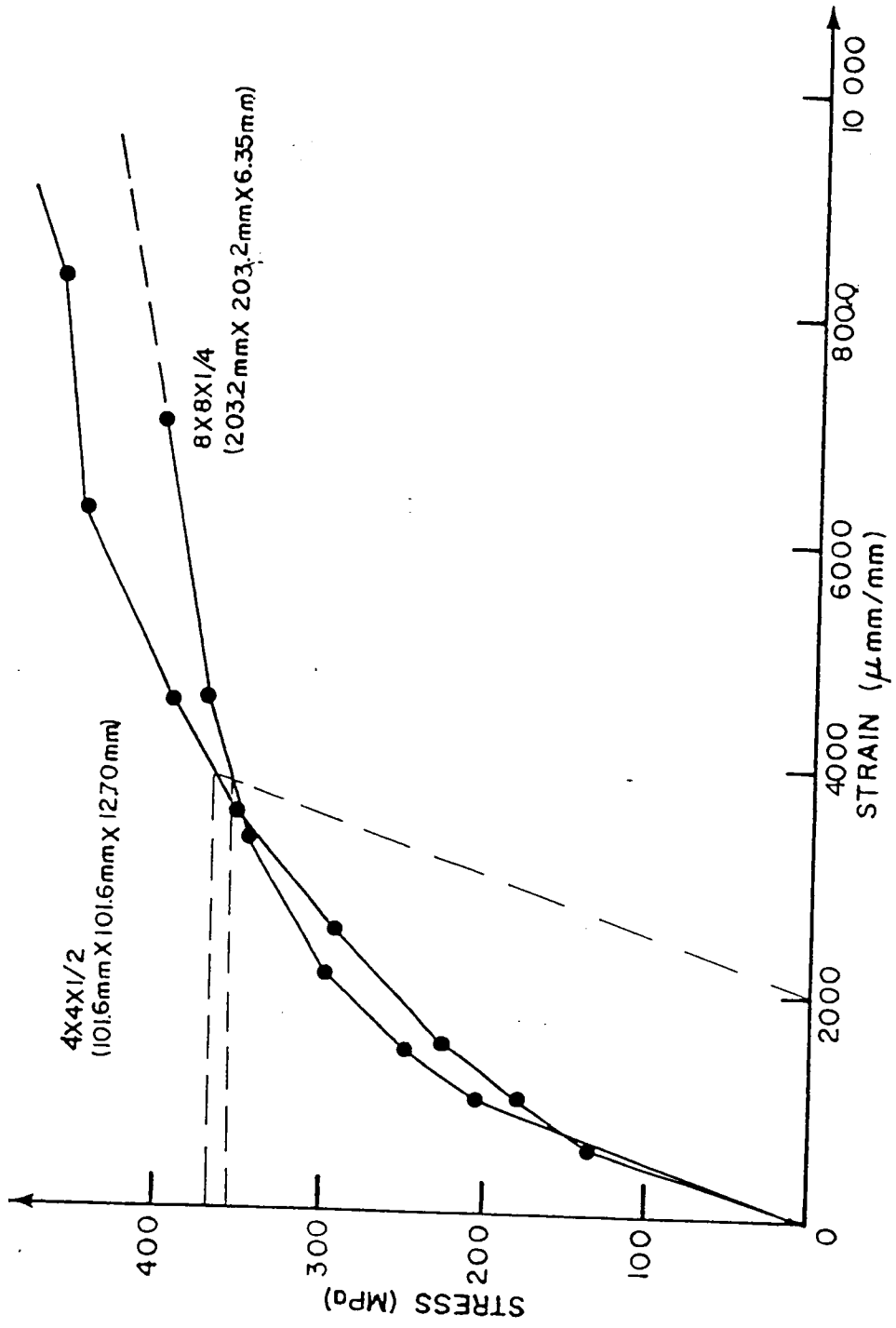
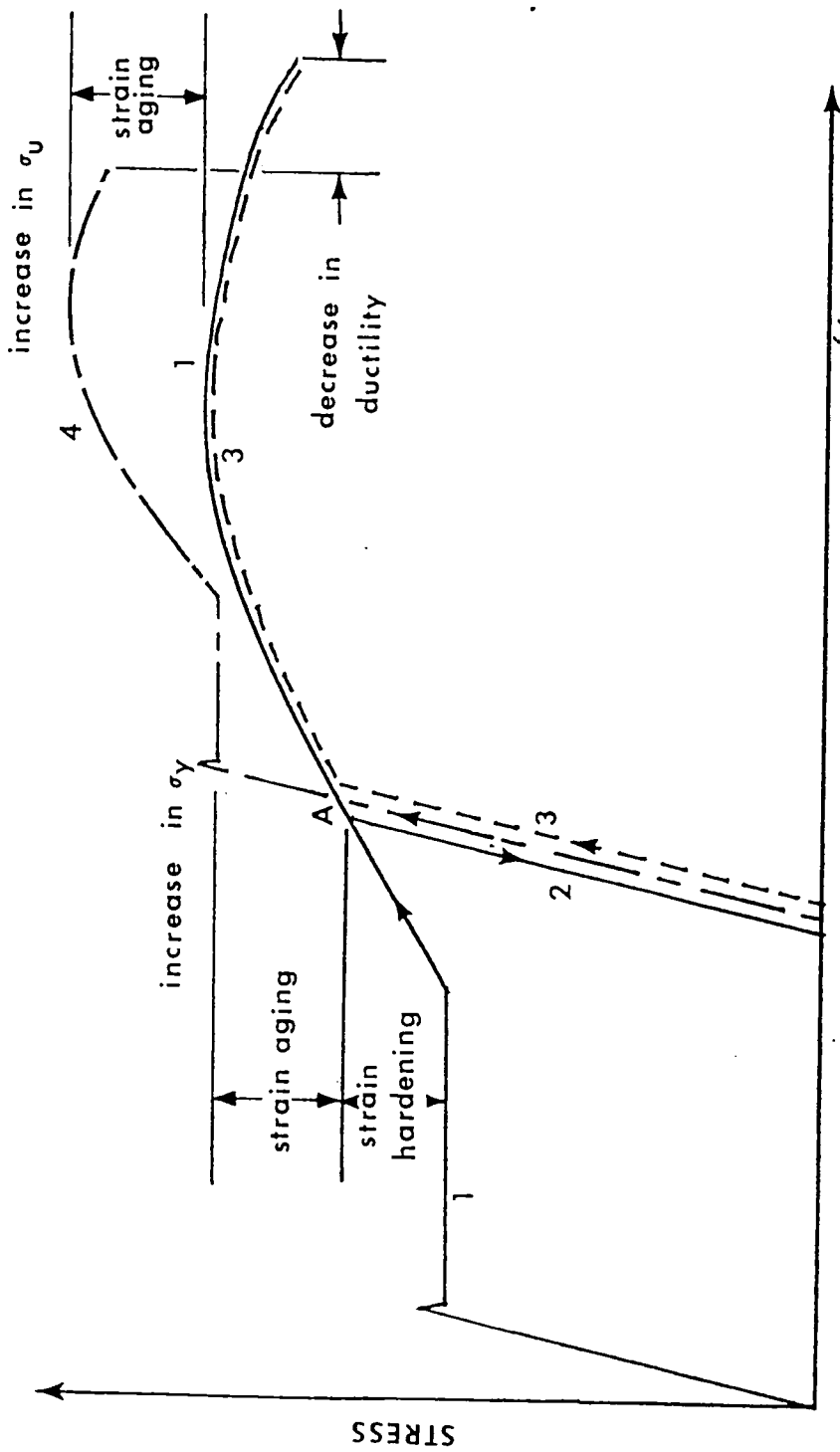


Fig. 3 Tensile Stress-Strain Curves for Material from Flats



### STRAIN

Fig. 4 Effects of Strain-Hardening and Strain Aging on the Stress-Strain Properties of Steel

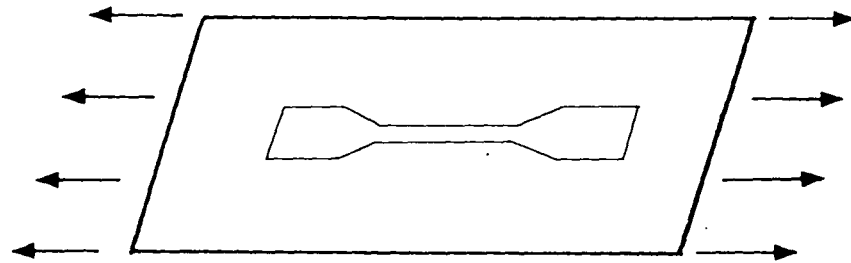
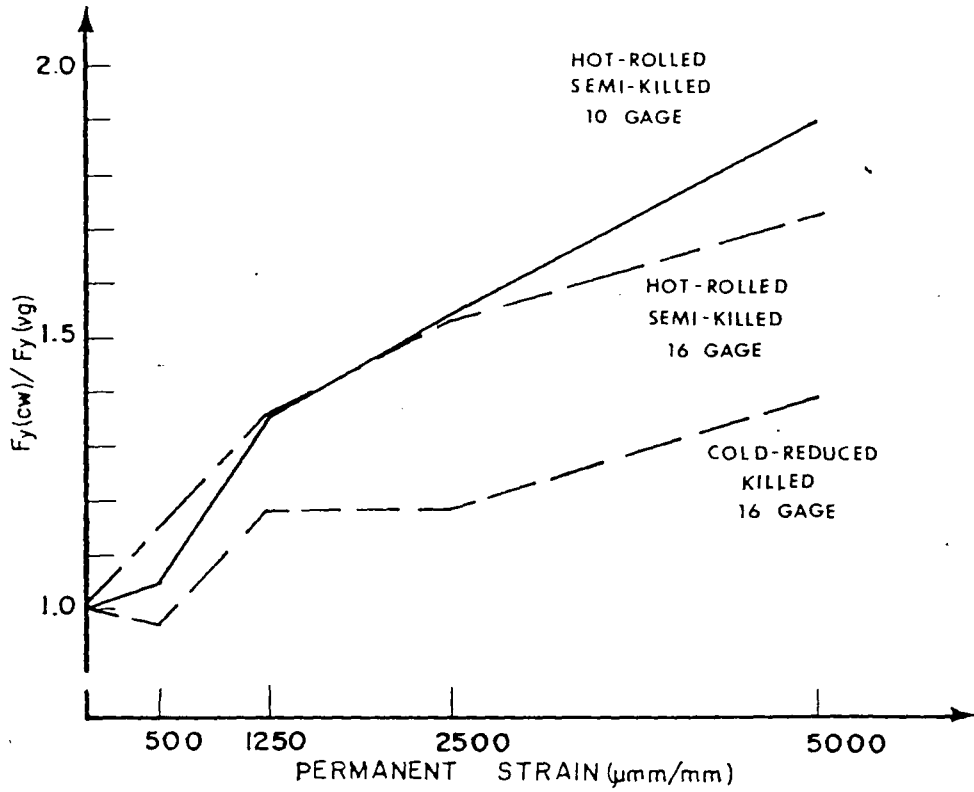


Fig. 5 Influence of Cold Stretching on Yield Strength of Steel

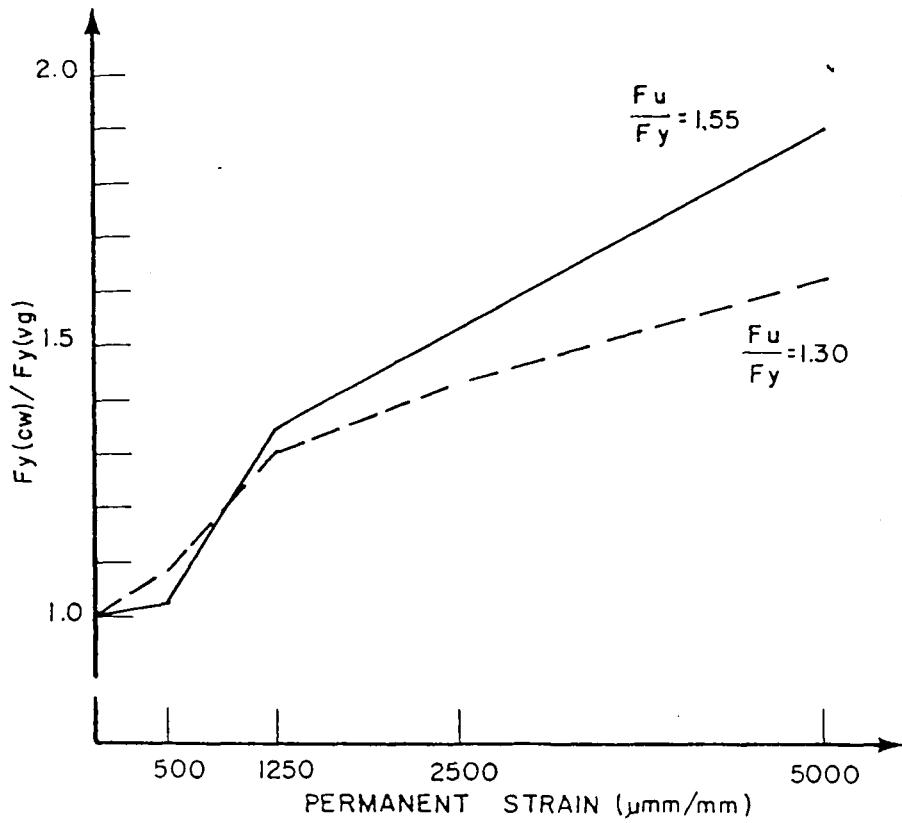


Fig. 6 Effect of  $(F_u - F_y)$  on Yield Strength of Steel



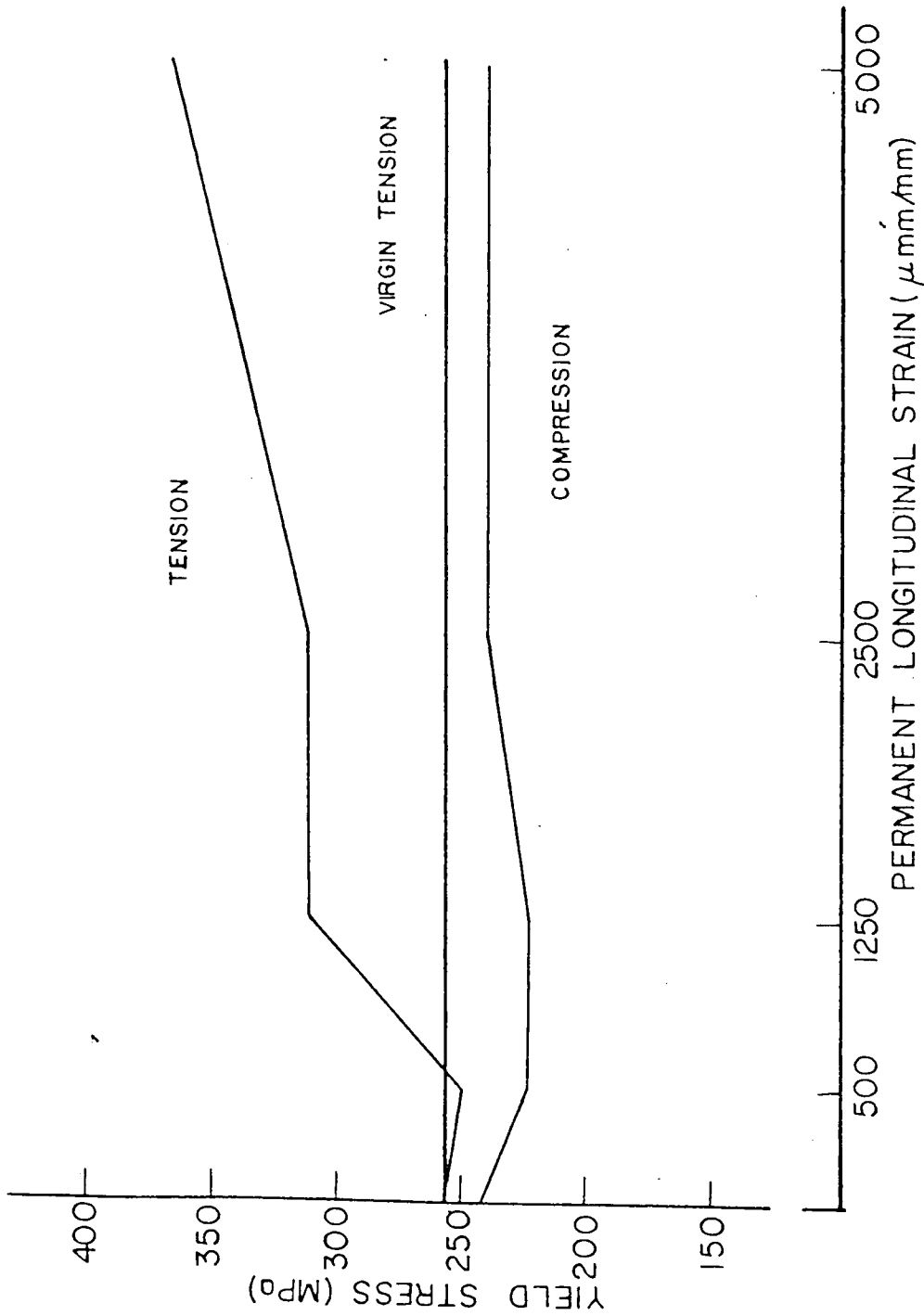


Fig. 7 Effect of Cold-Straining in Longitudinal Direction on Tensile and Compressive Stress in Longitudinal Direction

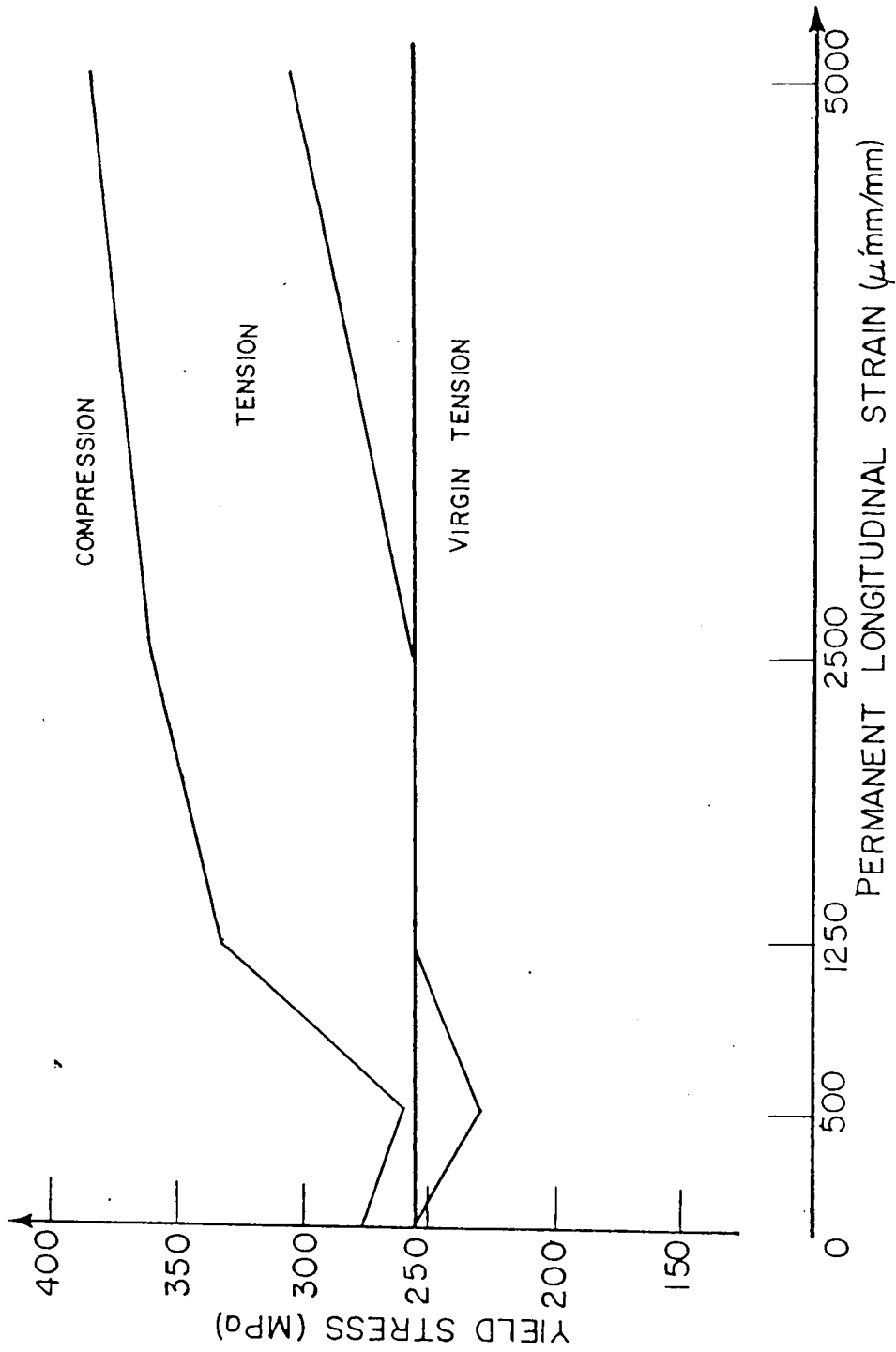
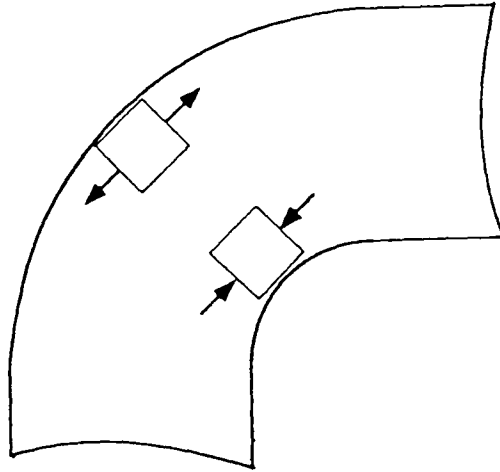
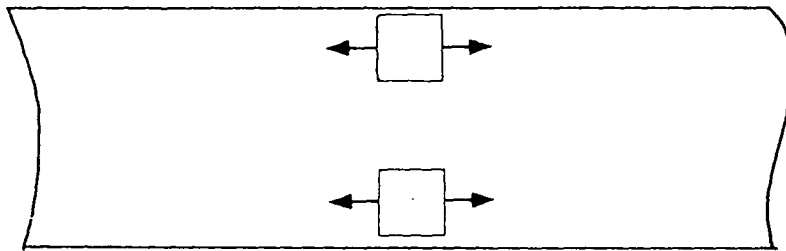


Fig. 8 Effect of Cold-Straining in Longitudinal Direction on Tensile and Compressive Stress in Transverse Direction



CORNER



FLAT

Fig. 9 Transverse Cold-Working at Corners and Flats

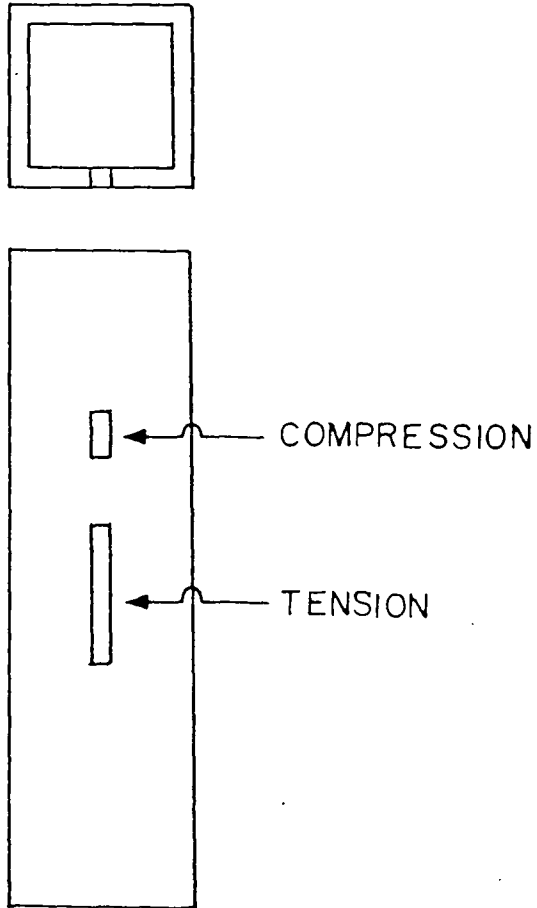


Fig. 10 Location of Coupons Removed from Specimen

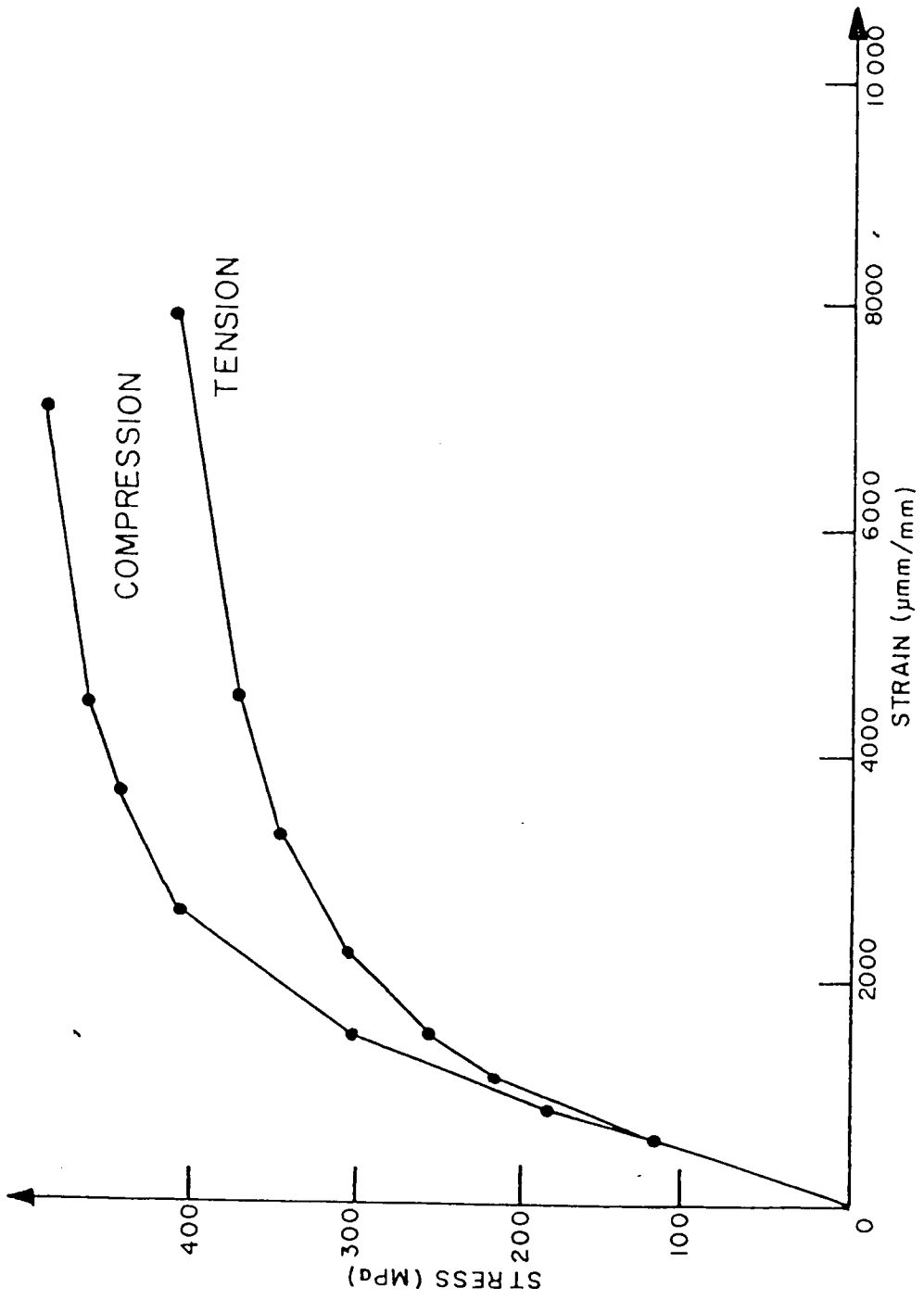
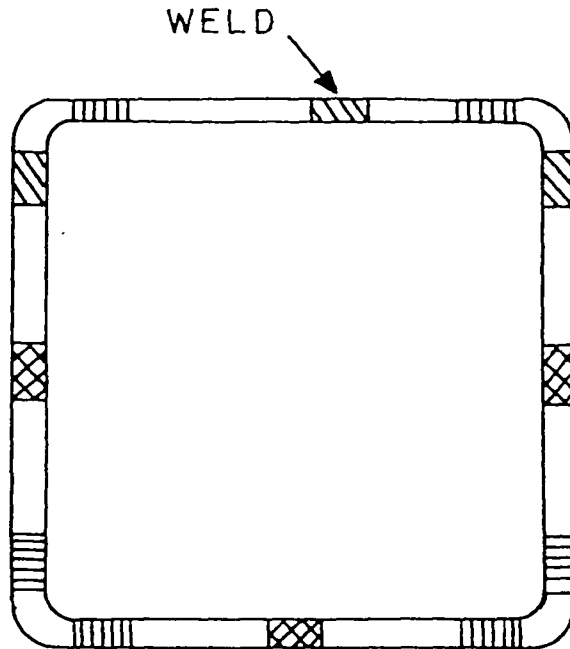




Fig. 11 Stress-Strain Curves for Compression and Tensile Coupons Taken from Flats of  
 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm)



TENSILE YIELD

 — 372 – 393 (MPa)

 — 400 – 421


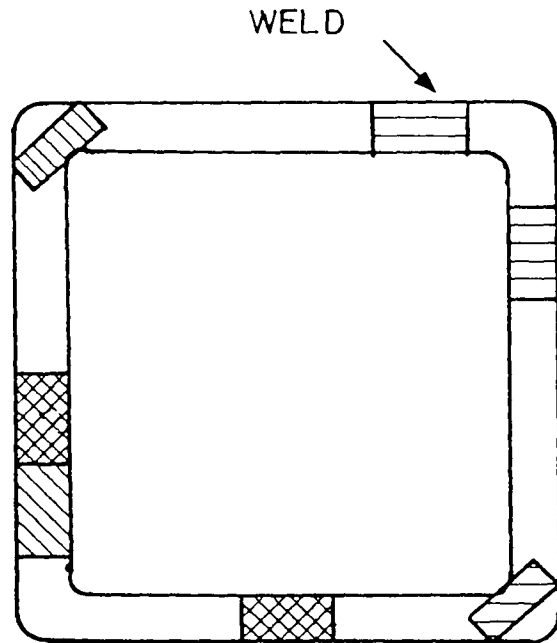


 — 428 – 448

Fig. 12 Variation in Tensile Yield Strength for  
 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm)  
 Section



TENSILE YIELD

 — 483-517 (MPa)

 — 524-552


 — 559-586

Fig. 13 Variation in Tensile Yield Strength for  
 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.7 mm)  
 Section

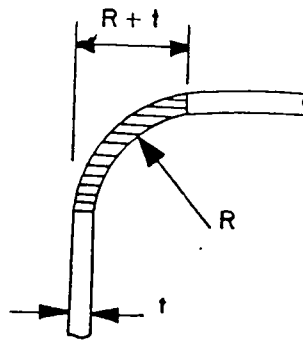
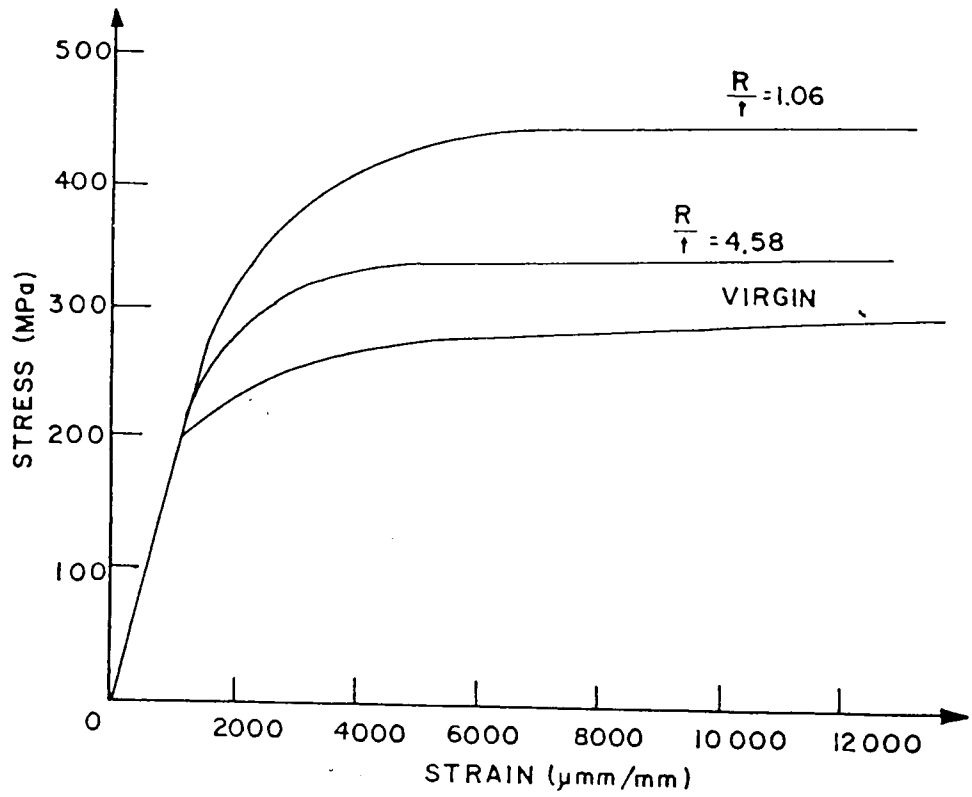
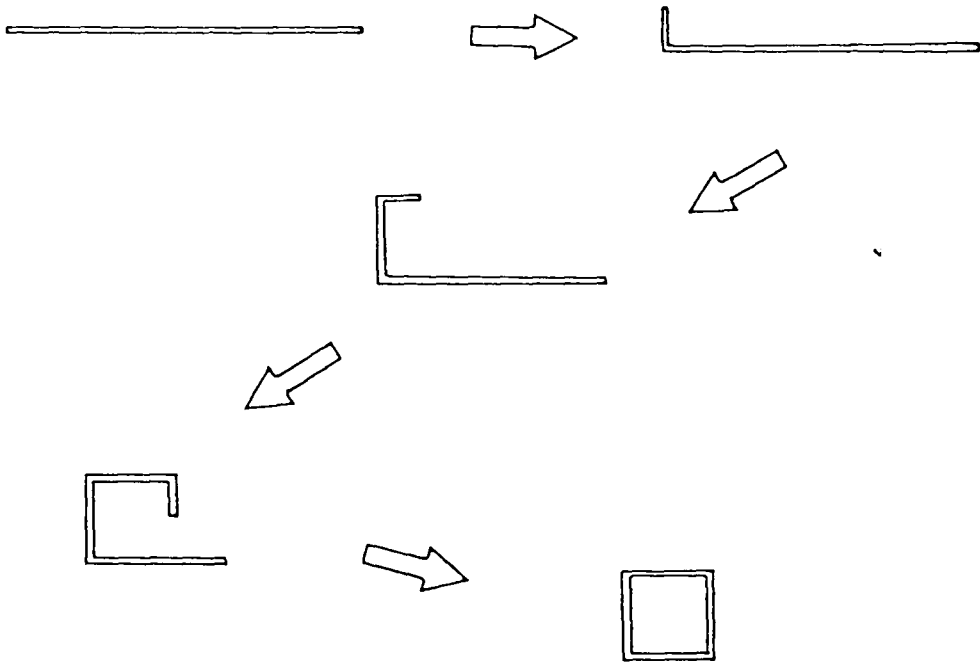
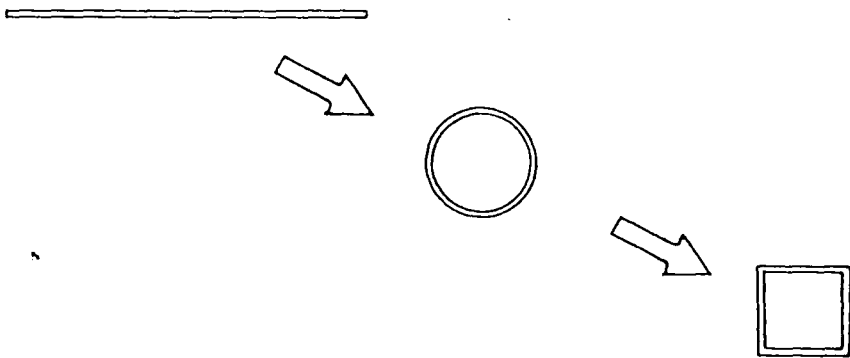


Fig. 14 Influence of  $(R/t)$  on Yield Strength of Cold-Rolled Killed Corners





PROCESS A



PROCESS B

Fig. 15 Cold-Forming Process

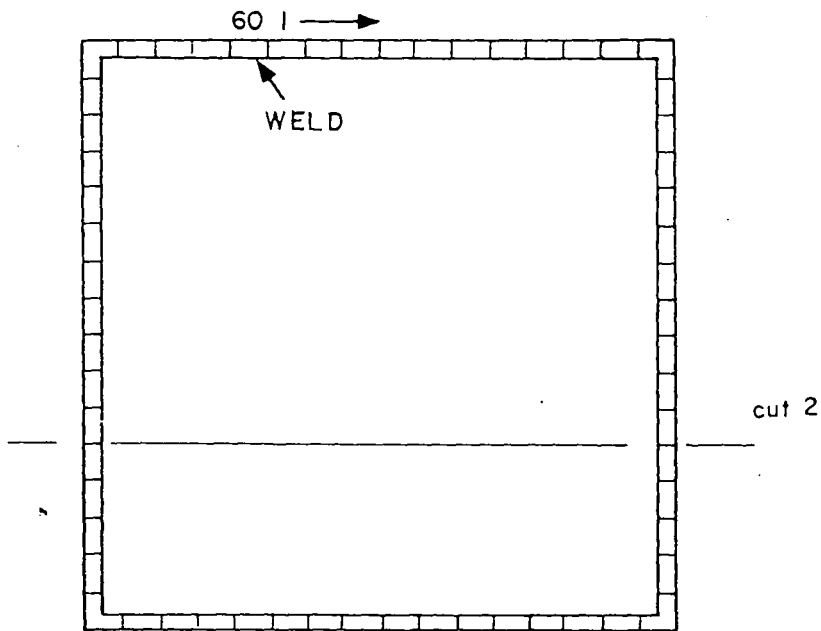
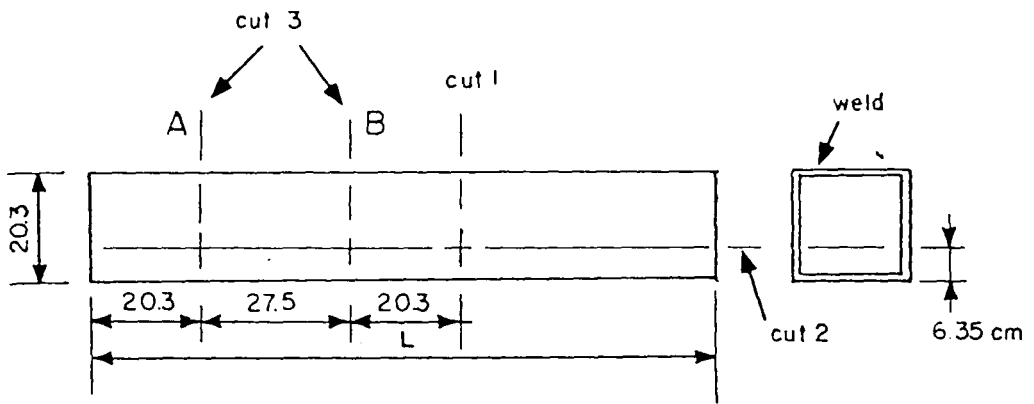


Fig. 16 Sectioning of 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) Cross-Section



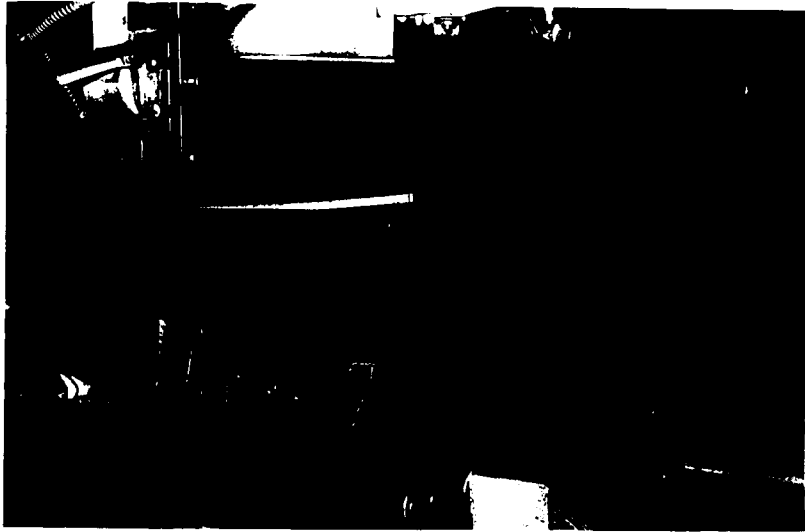


Fig. 17 Sawing Cross-Section into  
Longitudinal Strips

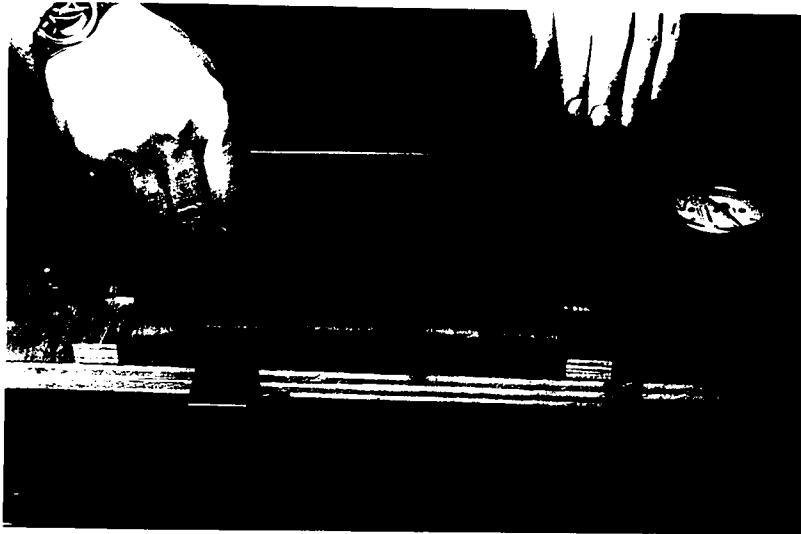


Fig. 18 Residual Stress Measurements  
with Whittemore Gauge

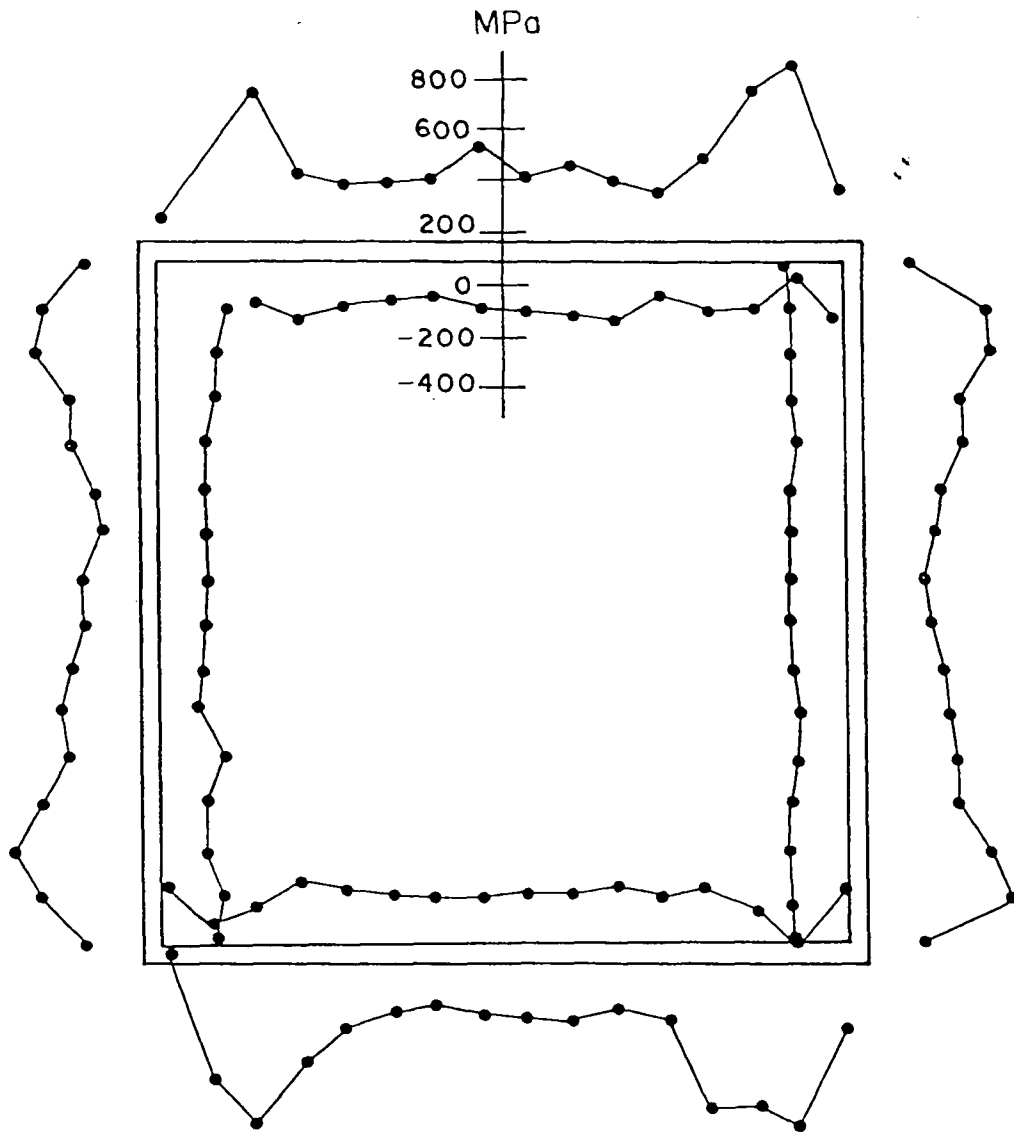


Fig. 19 Residual Stress Distribution for 8 x 8 x 1/4  
(203.2 mm x 203.2 mm x 6.35 mm) Cross-Section

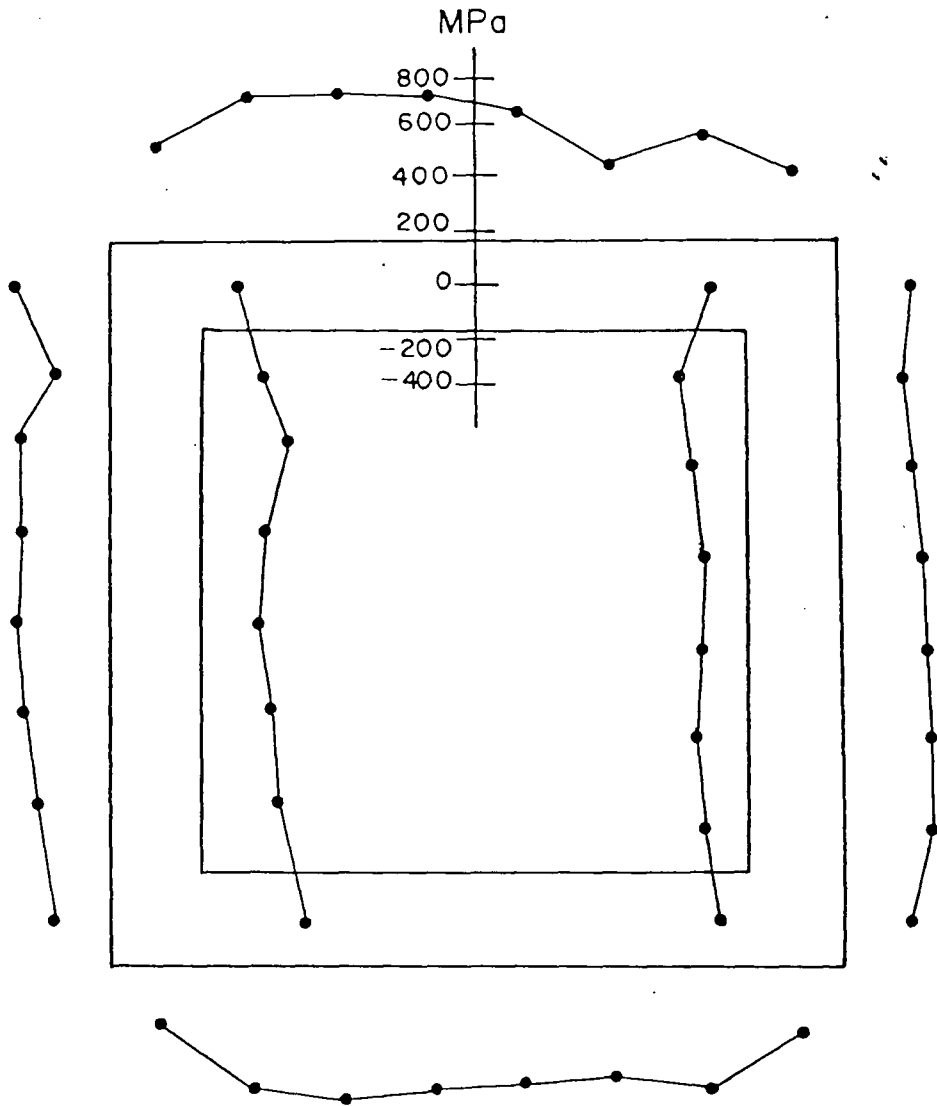


Fig. 20 Residual Stress Distribution for 4 x 4 x 1/2  
(101.6 mm x 101.6 mm x 12.7 mm) Cross-Section

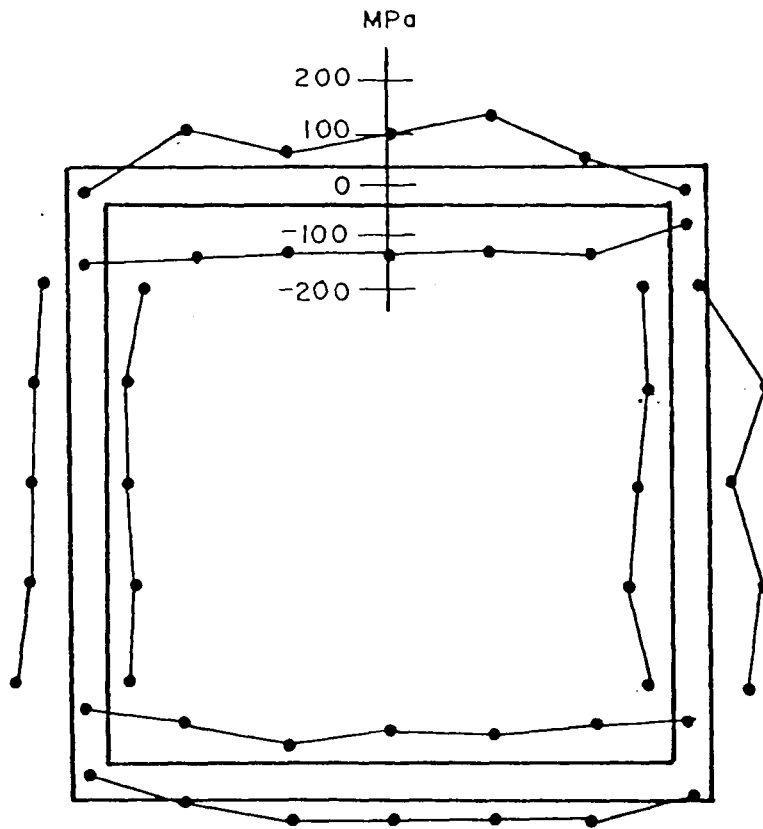


Fig. 21 Residual Stress Distribution for 3-1/2 x  
 3-1/2 x 5/6 (88.9 mm x 88.9 mm x 7.94 mm)  
 Hot-Rolled Tube



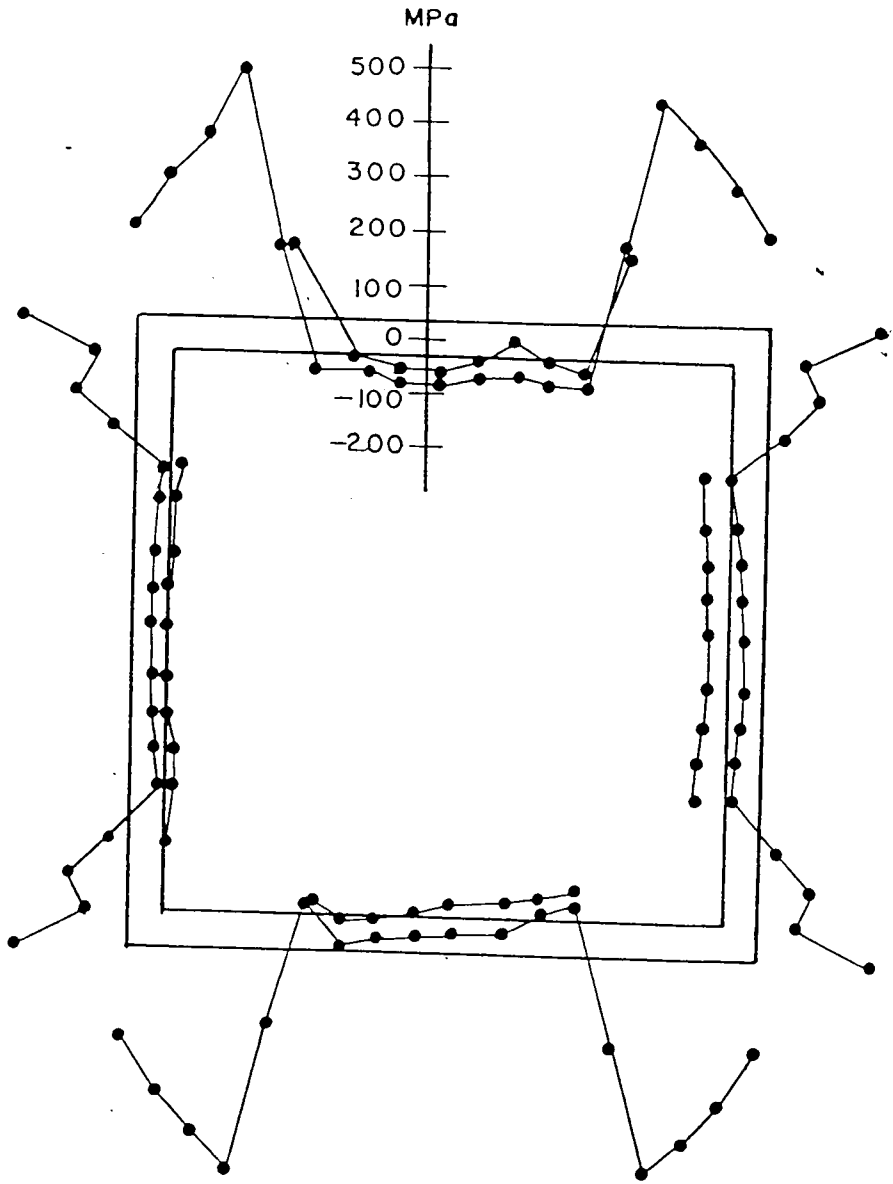


Fig. 22 Residual Stress Distribution for 24 x 774  
(288 mm x 3443 N) Box Section

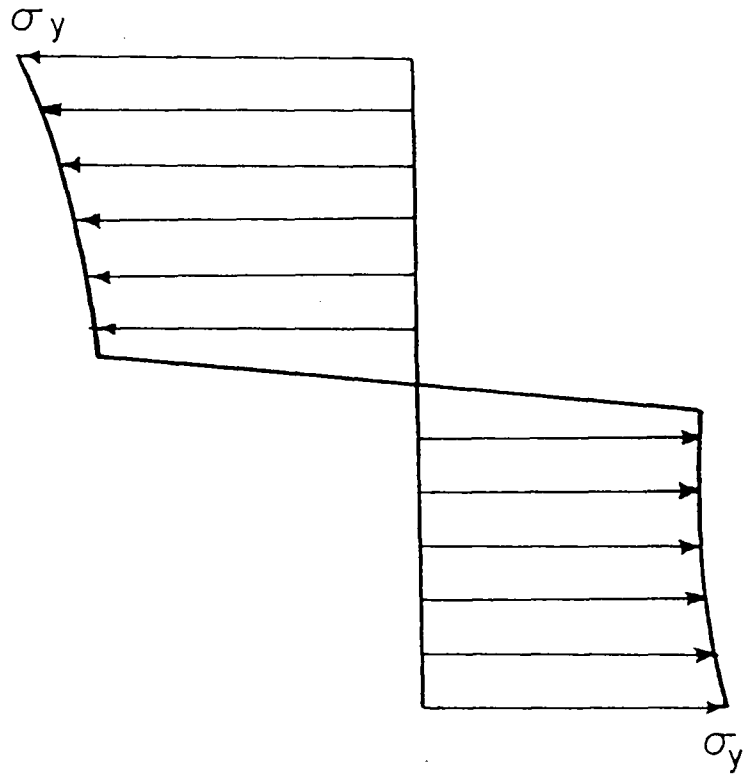


Fig. 23 Through Thickness Residual Stress Distribution

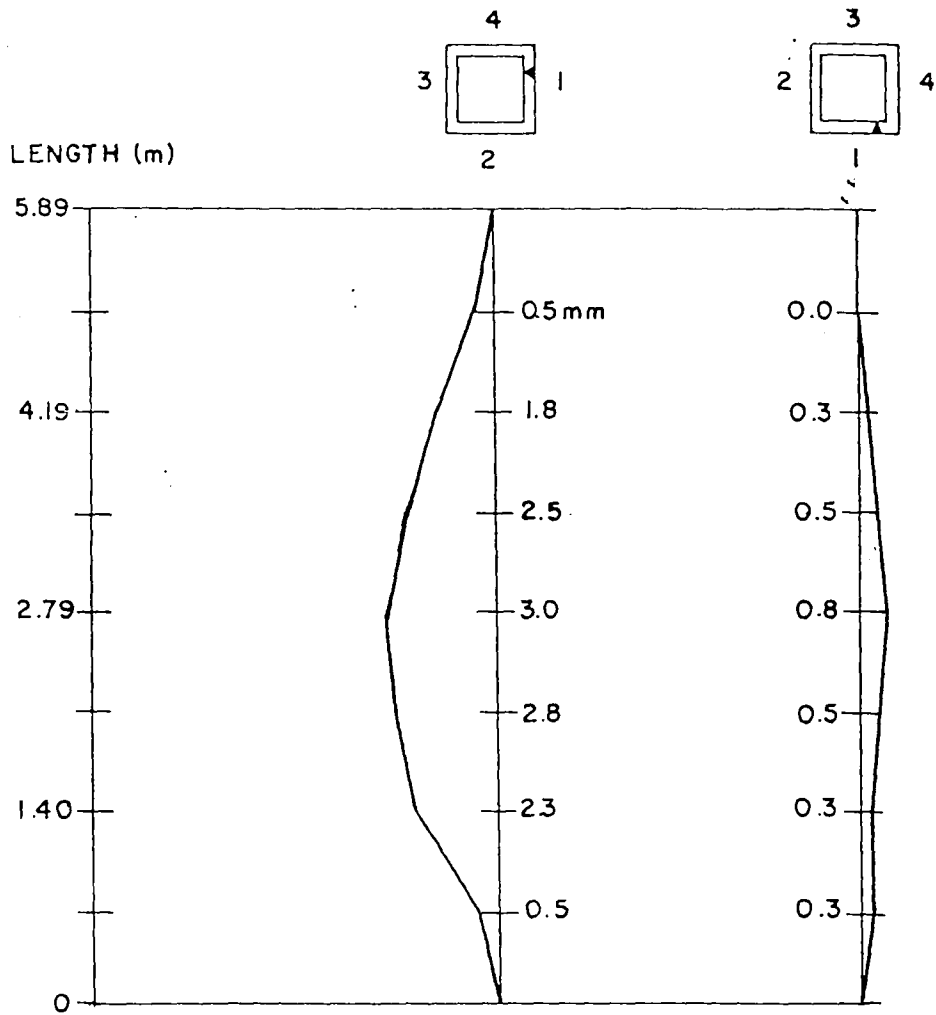


Fig. 24 Out-of-Straightness Measurements for 8 x 8 x 1/4  
(203.2 mm x 203.2 mm x 6.35 mm) Column

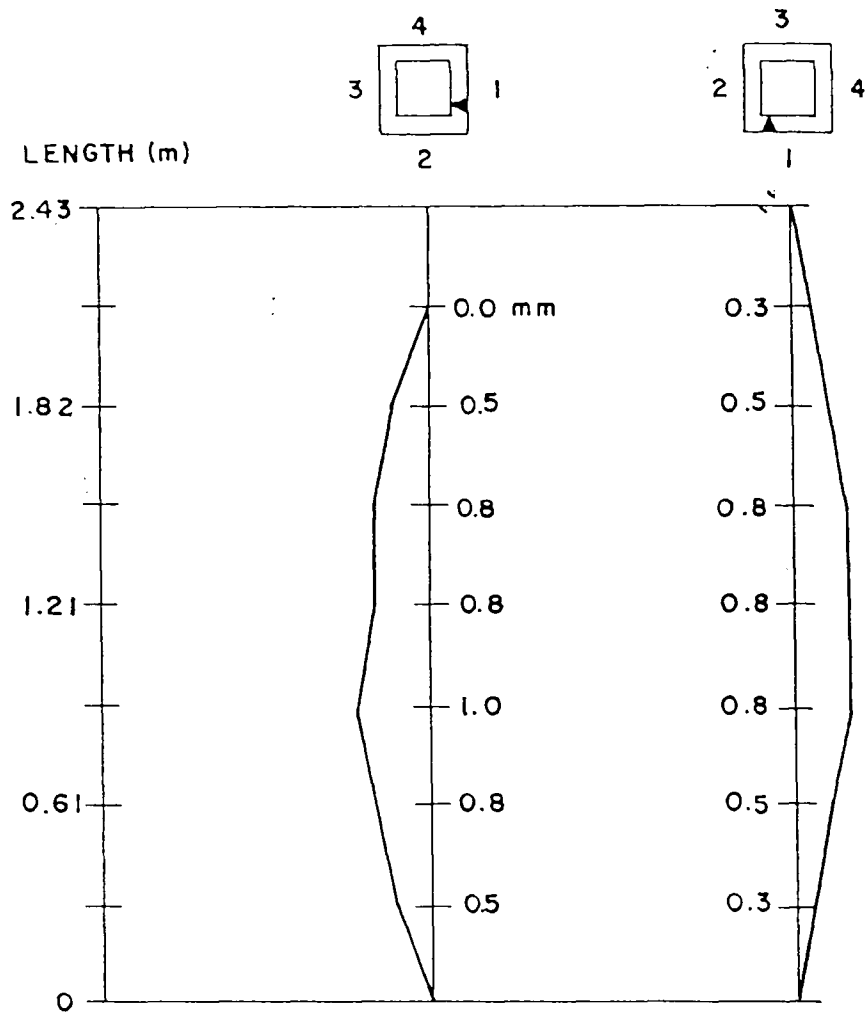


Fig. 25 Out-of-Straightness Measurements for 4 x 4 x 1/2  
(101.6 mm x 101.6 mm x 12.7 mm) Column

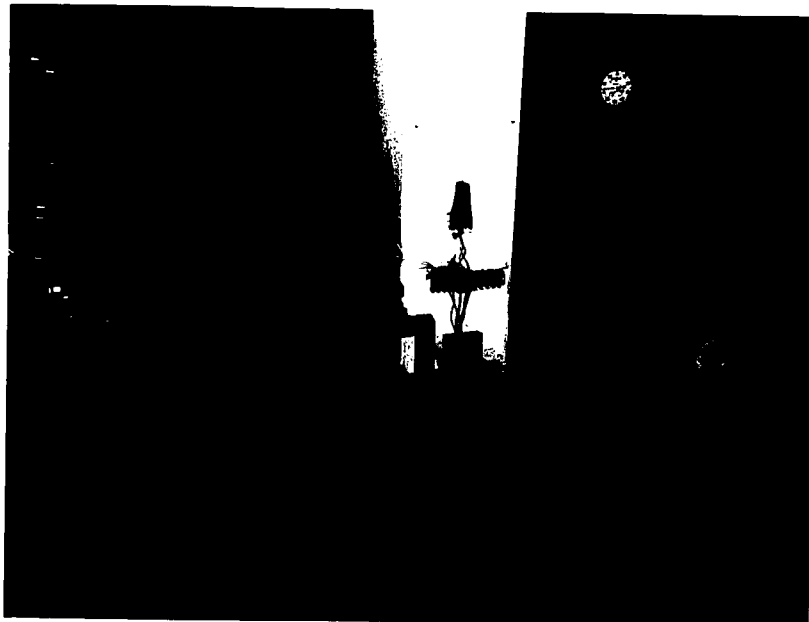


Fig. 26 Spherical Bearing and  
Rotation Gauges

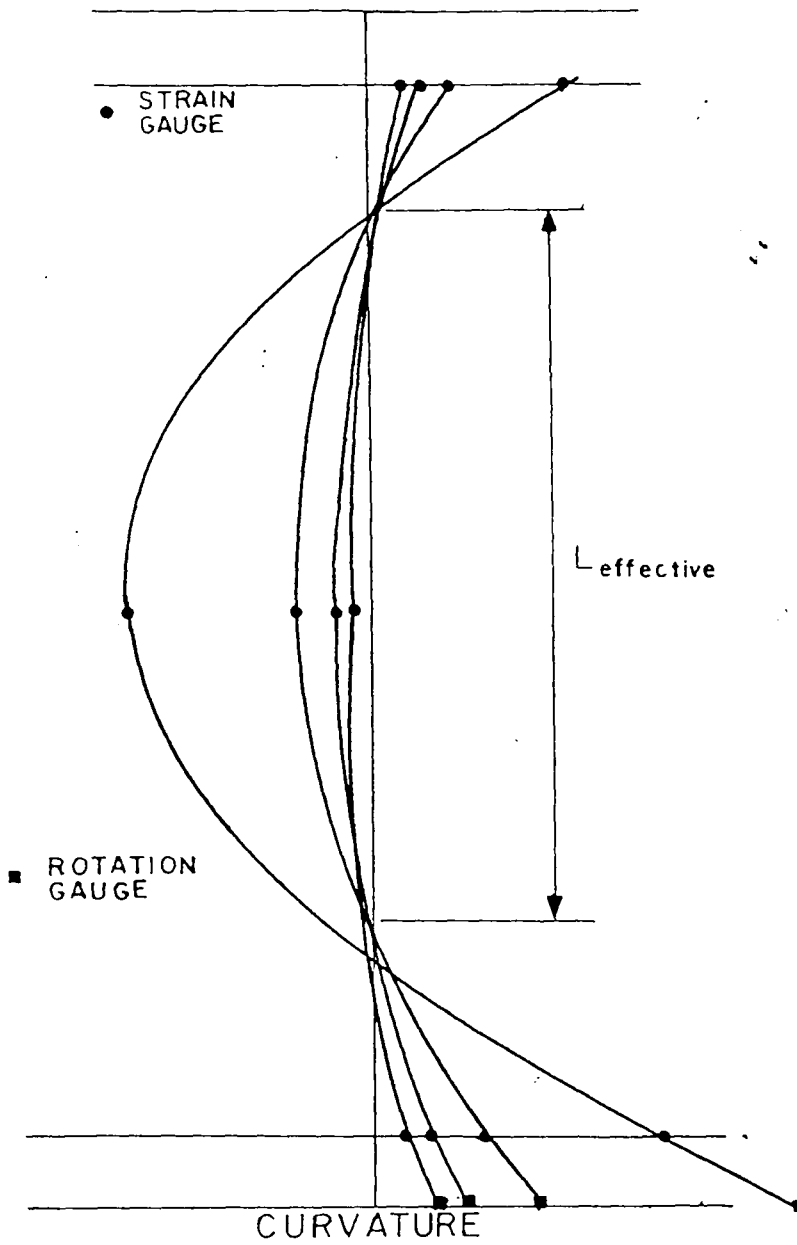


Fig. 27 Effective Column Length

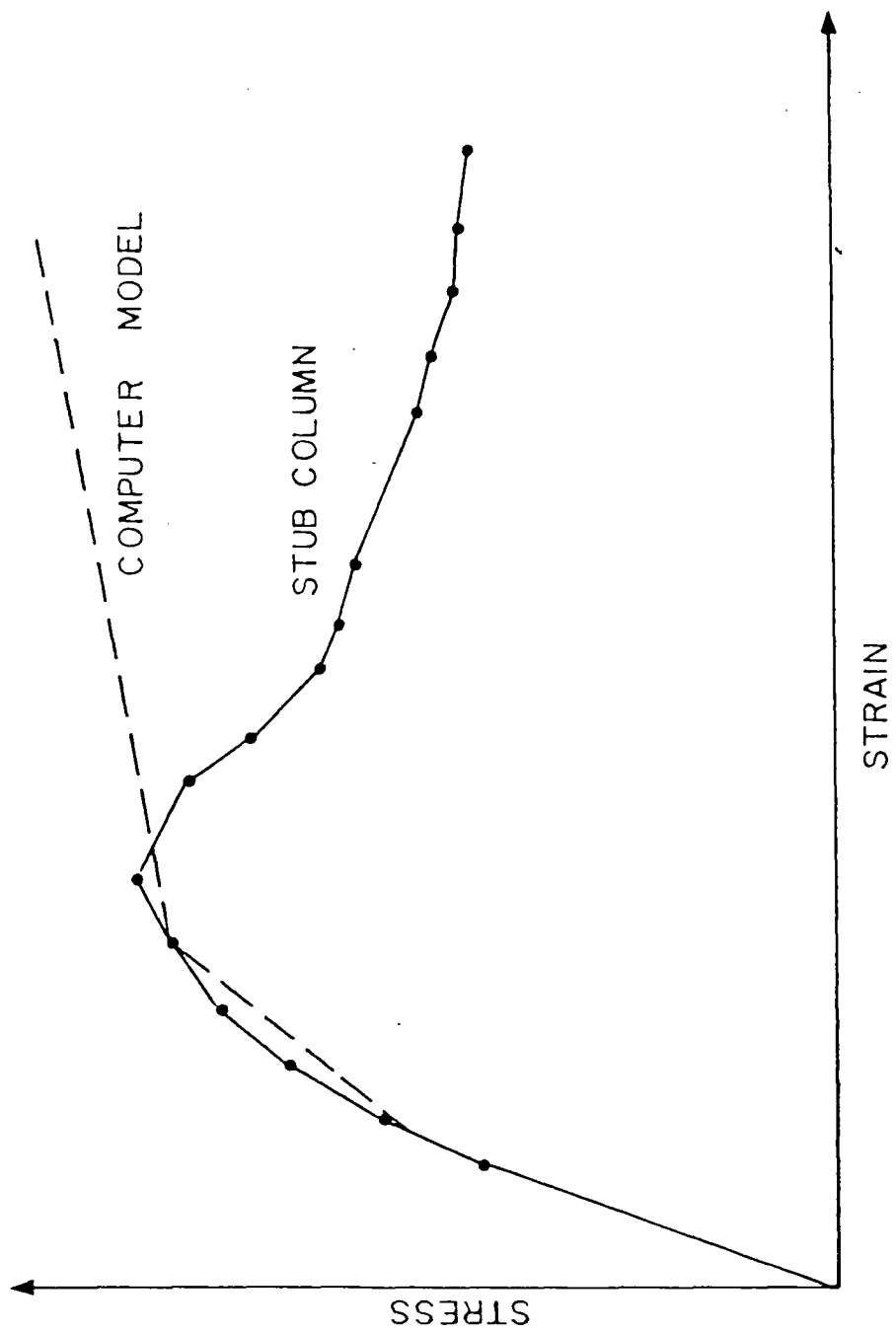


Fig. 28 Stress-Strain Approximation of Stub Column

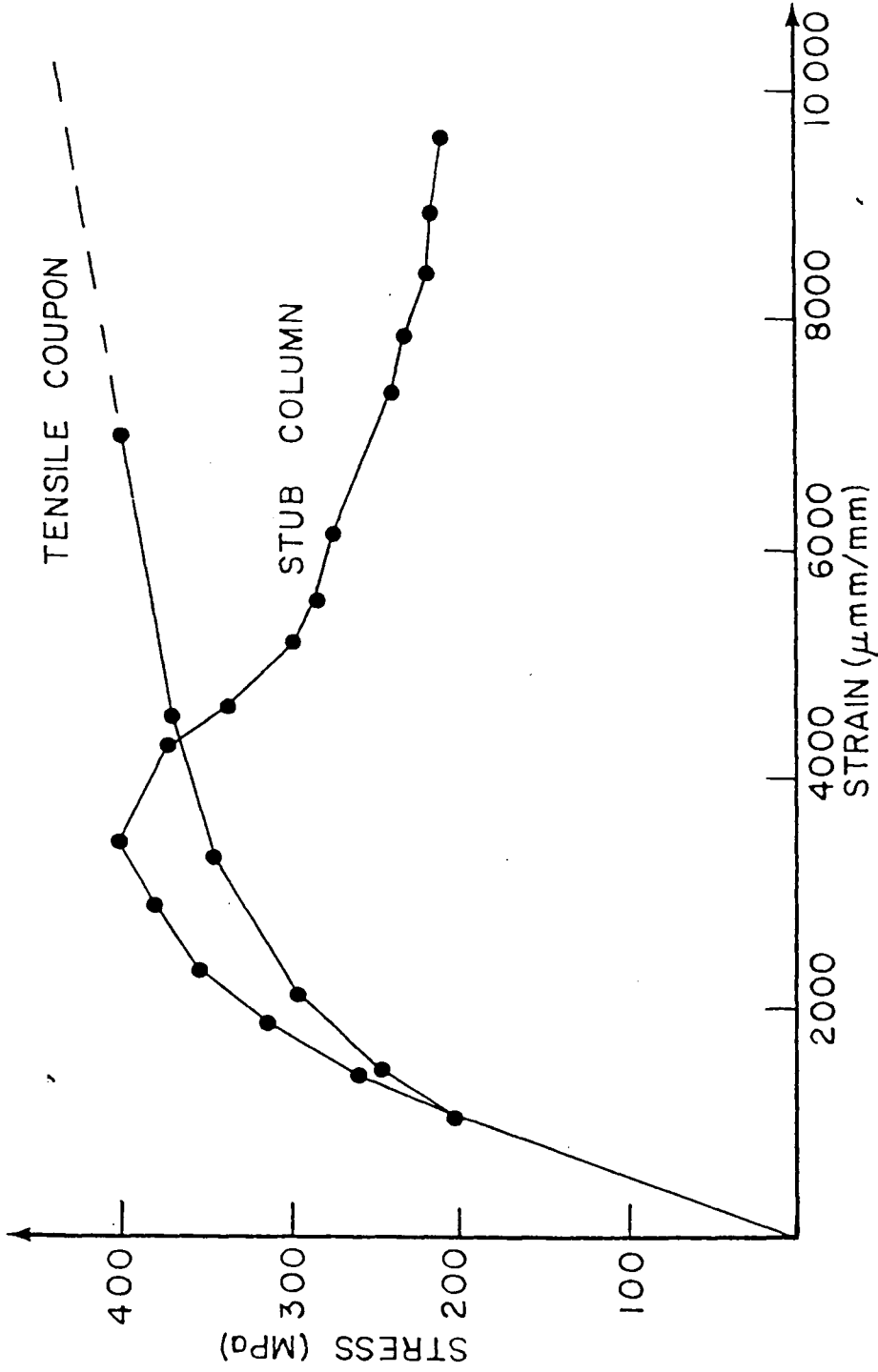


Fig. 29 Stub Column and Tensile Coupon Stress-Strain Curves for 8 x 8 x 1/2  
(203.2 mm x 203.2 mm x 6.35 mm) Section



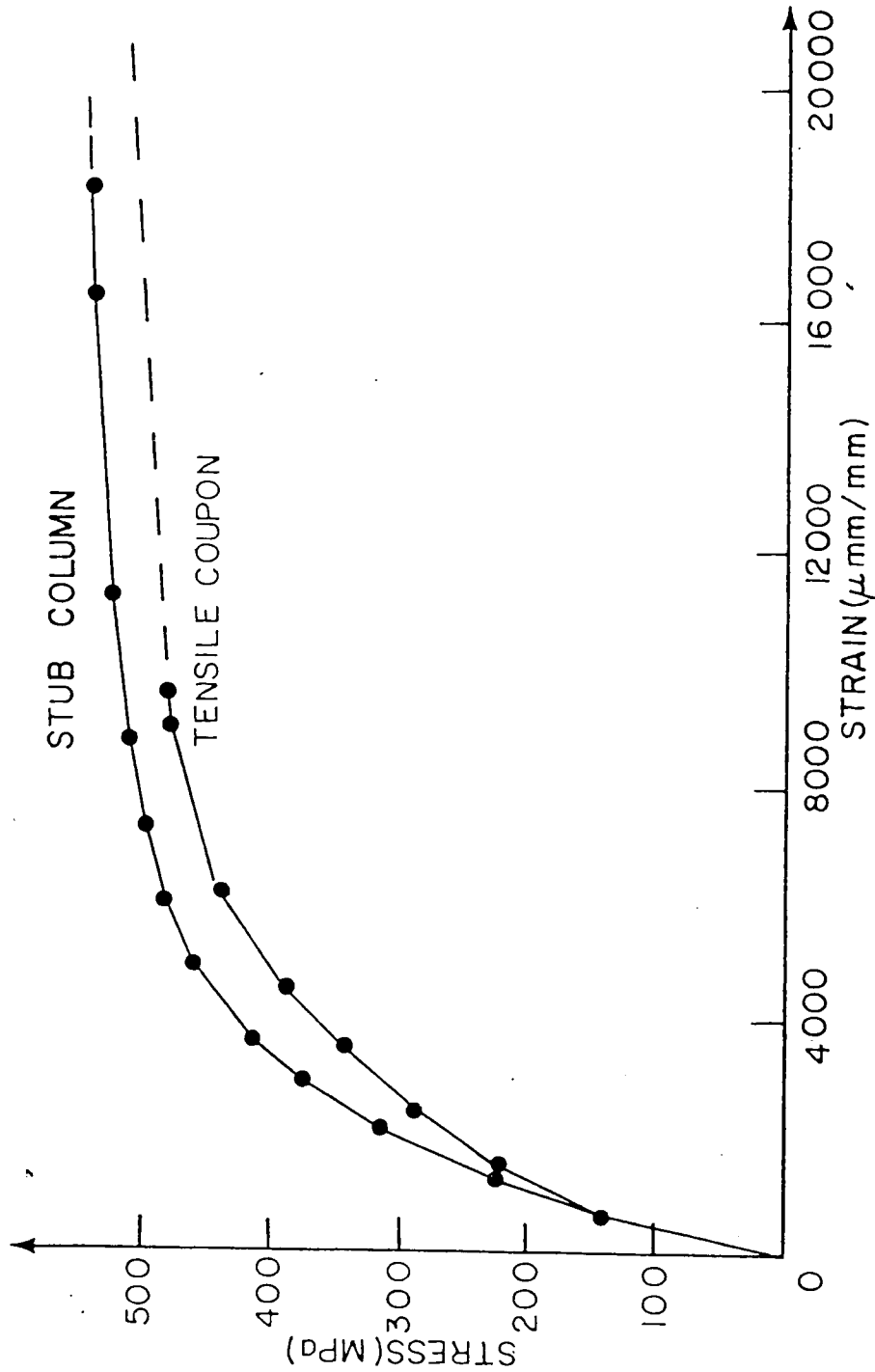


Fig. 30 Stub Column and Tensile Coupon Stress-Strain Curves for 4 x 4 x 1/2  
 (101.6 mm x 101.6 mm x 6.35 mm) Section

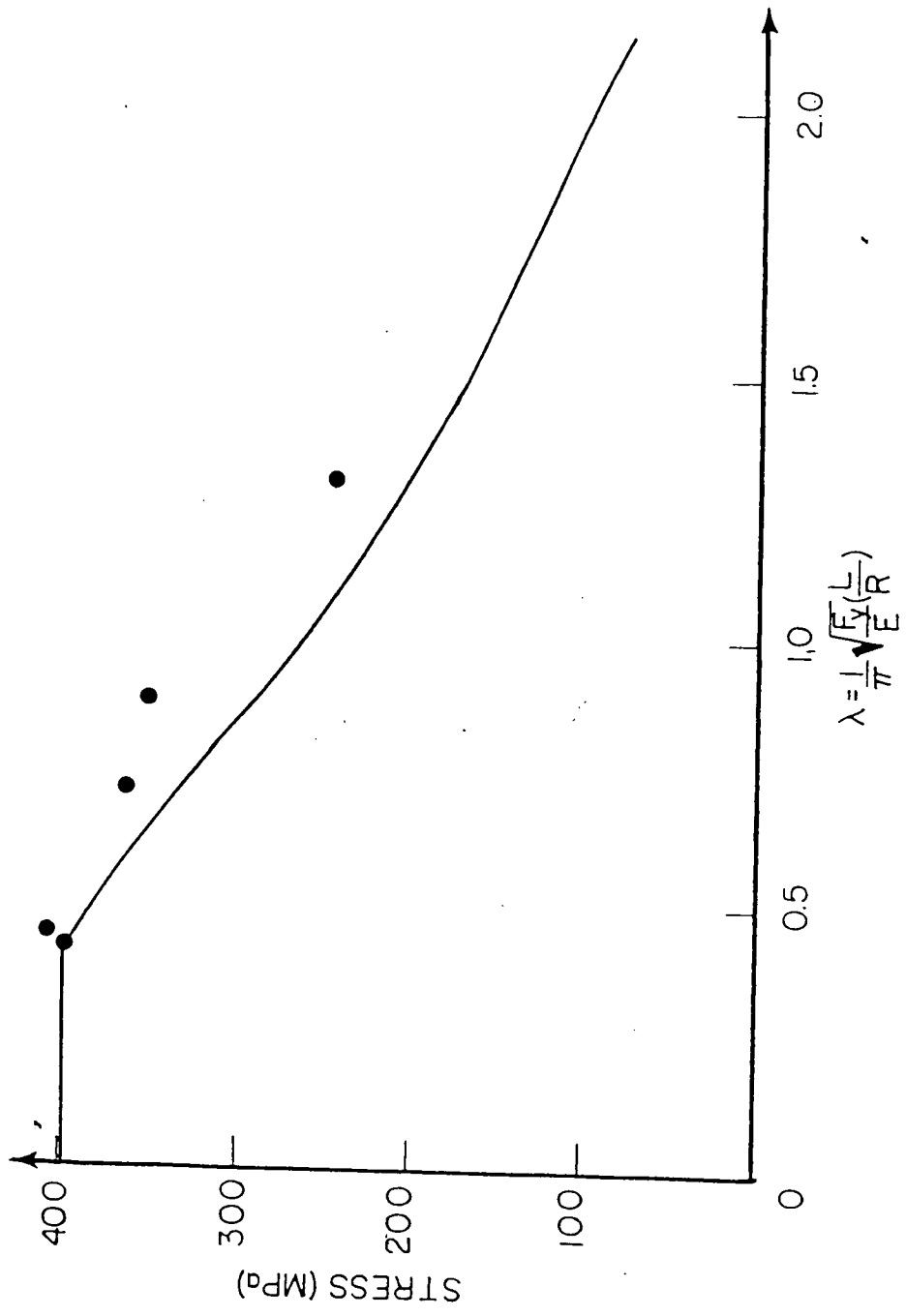


Fig. 31 Tangent Modulus Curve for 8 x 8 x 1/4 (203.2 mm x 203.2 mm x 6.35 mm) Column

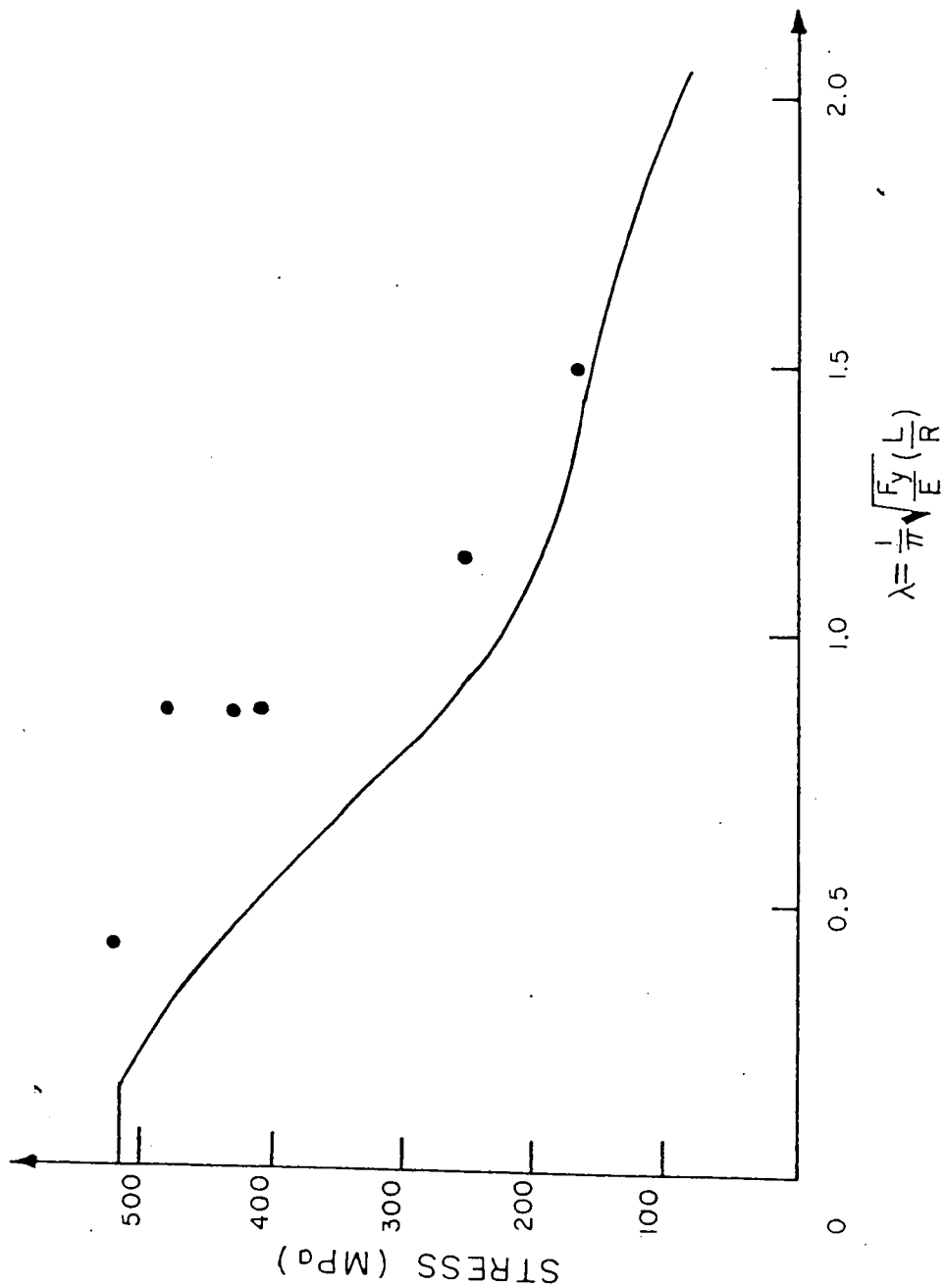


Fig. 32 Tangent Modulus Curve for 4 x 4 x 1/2 (101.6 mm x 101.6 mm x 12.7 mm) Column

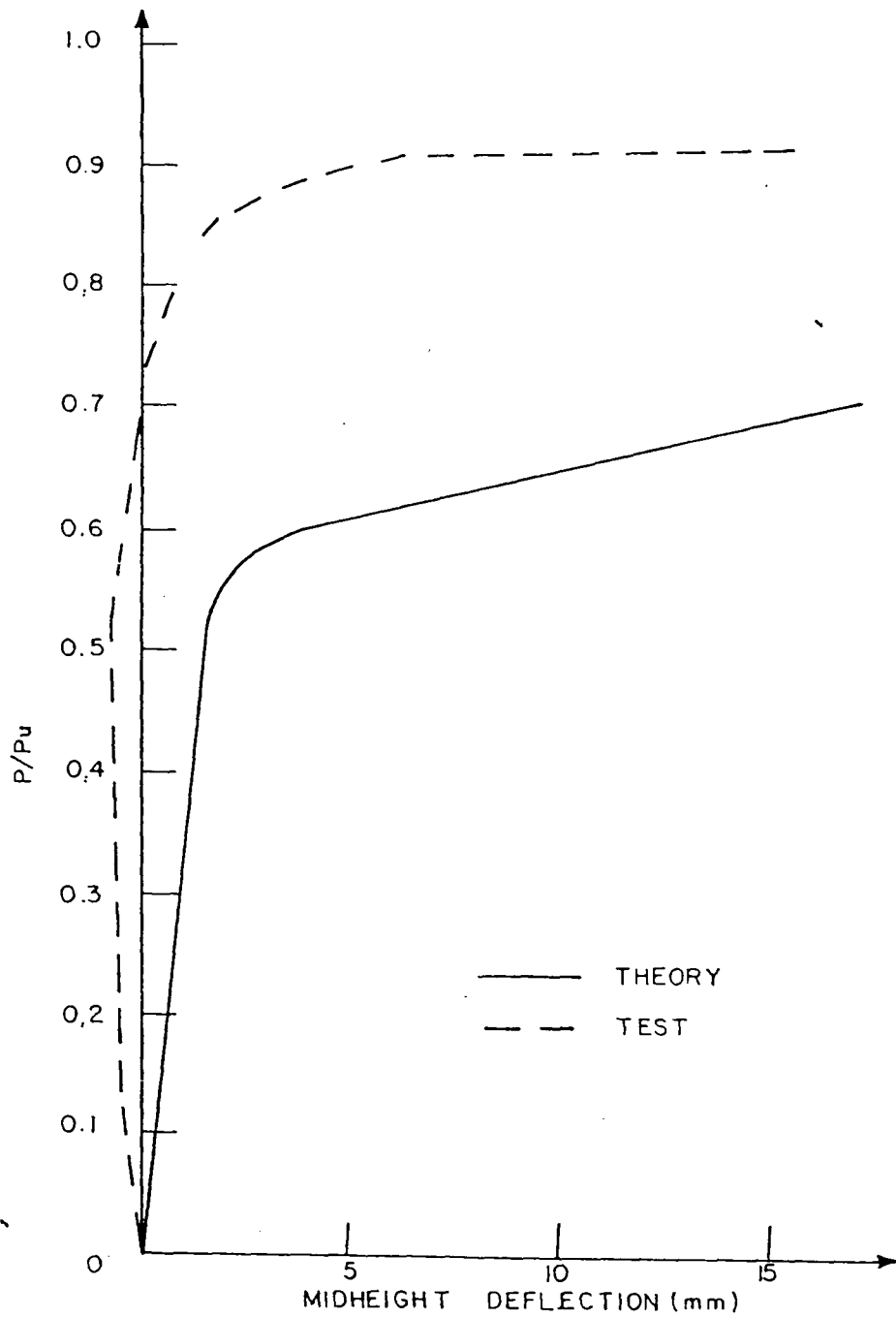


Fig. 33 Theoretical and Experimental Load Deflection Curves for Long Column -  $\lambda = 0.781$

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In August 1979 he accepted a Research Assistantship in the Operations Division of Fritz Engineering Laboratory, Lehigh University. He received his Master of Science in Civil Engineering from Lehigh in May 1981.