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An investigation into the validity of the breakpoint hypothesis.

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AN INVESTIGATION INTO
THE VALIDITY OF
THE BREAKPOINT HYPOTHESIS

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Professor in Charge

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ABSTRACT

This master's thesis investigates the validity of the breakpoint hypothesis as used in modelling conflict termination. The breakpoint hypothesis is based upon the concept that a combat force will terminate an engagement after sustaining a sufficiently high percentage of personnel casualties. Despite use of the hypothesis in many combat modelling efforts, there has not yet been a satisfactory validation of the technique.

The current investigation incorporates a series of modifications to a working hypothesis published by Helmbold (1971) and utilizes new data for empirical testing. This investigation focuses on the identification and examination of the validity of a series of break curves using an engagement data matrix containing 323 engagements; with 24 data elements available for each engagement. The methodology consists of the use of principle components analysis to reduce the initial set of observed variables to an underlying set of defined factors followed by a test procedure to determine the degree to which Helmbold's Theorem holds for the subset of engagements associated with each uniquely defined factor combination. The results are then confirmed using discriminant analysis.

The breakpoint hypothesis testing results indicate a number of engagement subsets which obey the breakpoint hypothesis reasonably well. In addition, several engagement subsets have been found which obey the breakpoint hypothesis better than the full engagement set. These indicate that a series of break curves, rather than a single break curve, may better account for the break behavior of military forces for the set

of possible engagement types. The discriminant analysis confirms the significance of some of the observed variables and, for the most part, reconciles with the results from the breakpoint hypothesis testing.

This investigation is a preliminary empirical validation of the use of probabalistic break curves to model battle termination. Several modifications to the present methodology are suggested to refine the results. In addition, the specification of break curves for particular groupings of engagement types should be investigated.

Chapter 1

INTRODUCTION

The breakpoint hypothesis is based upon the concept that a combat force will terminate an engagement after sustaining a sufficiently high percentage of personnel casualties. Despite use of the hypothesis in many force-on-force attrition studies, there has not yet been a satisfactory validation of the technique (Taylor, 1980:111). This thesis investigates the validity of the breakpoint hypothesis as used in modelling conflict termination. It incorporates a series of modifications to a previously-examined working hypothesis and utilizes new data for empirical testing.

1.1 Theoretical Framework

The breakpoint hypothesis concerns the relationship of casualties to a force's decision to terminate a battle. It is the assumption that a military force gives up the battle when its personnel casualty fraction reaches a certain level, which may be either a fixed quantity or one determined on a probabilistic basis. Assumptions of this type are commonly used in war games, field maneuvers and computer simulations.

There are two techniques currently used to apply the breakpoint hypothesis: deterministic and probabilistic break curves. These are illustrated below (Helmbold, 1971:2-3).

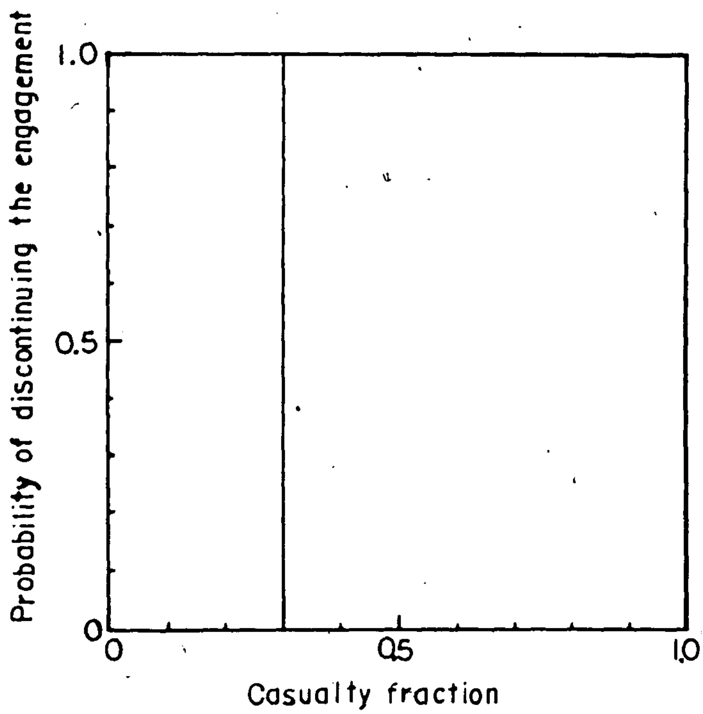

Figure 1.1A

A Deterministic Breakcurve

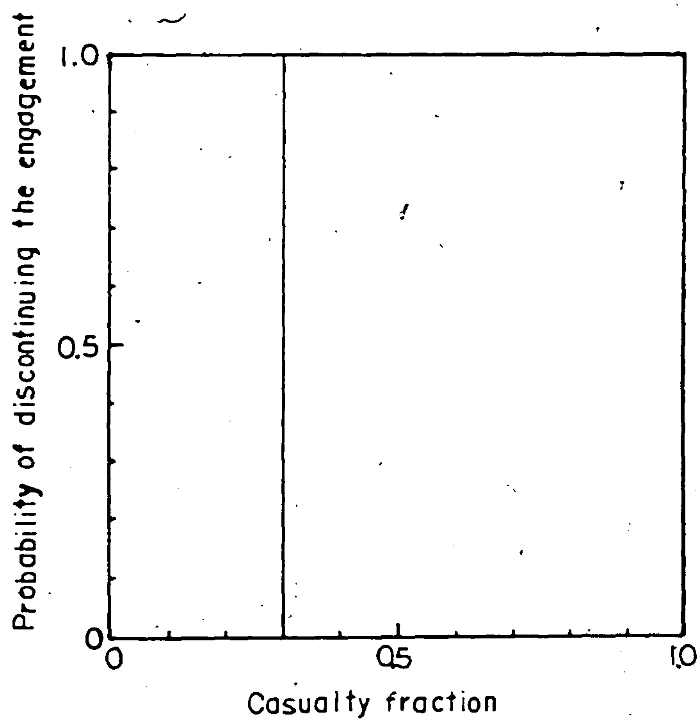

Figure 1.1A

A Deterministic Breakcurve

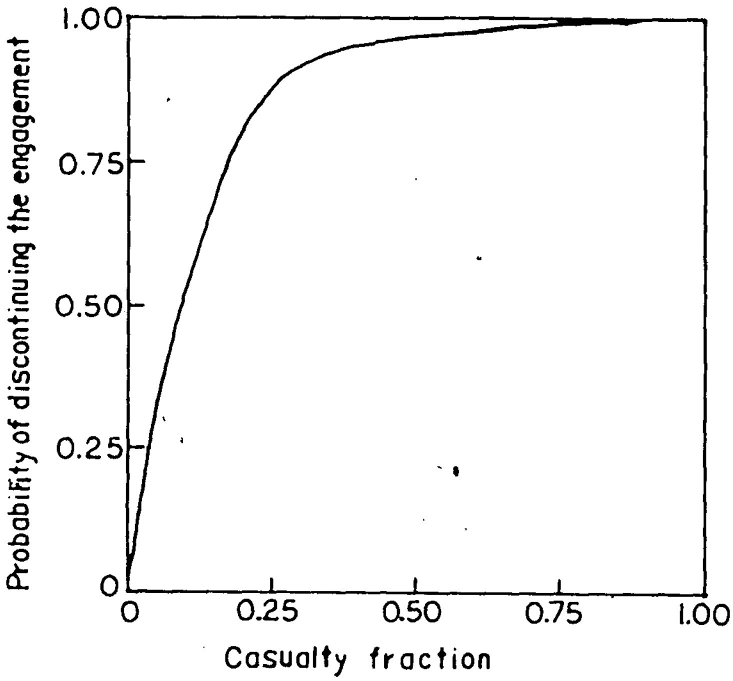

Figure 1.1B

A Probabilistic Breakcurve

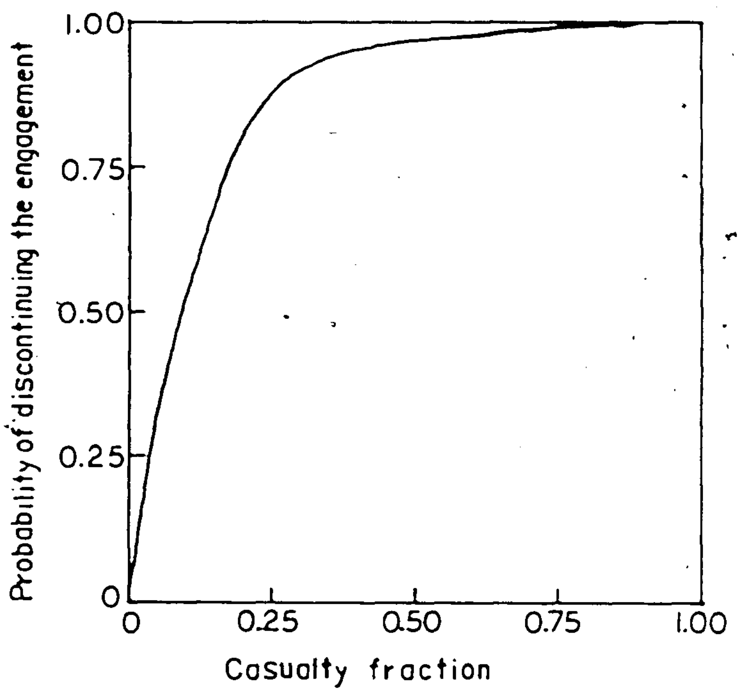

Figure 1.1B

A Probabilistic Breakcurve

A study of the battle termination process performed by Clark (1954) demonstrates that the use of deterministic breakpoints is inconsistent with a set of combat data from World War II. More recently, the validity of probabilistic break curves was investigated by Helmbold (1971). His conclusions state that probabilistic break curves, although inherently more flexible, are not valid as currently employed.

The use of the breakpoint hypothesis in force-on-force attrition modelling can perhaps best be seen by example. The following is an auxilliary model for combat between two homogeneous forces following Lanchester's Equations for Modern Warfare that was presented by Dr. James Taylor in his book, Force-on-Force Attrition Modelling (Taylor, 1980:63-66):

$$\frac{dx}{dt} = \begin{cases} -ay & \text{for } x > x_{BP} \text{ and } y > y_{BP} \\ 0 & \text{otherwise} \end{cases}$$
$$\frac{dy}{dt} = \begin{cases} -bx & \text{for } x > x_{BP} \text{ and } y > y_{BP} \\ 0 & \text{otherwise} \end{cases} \quad (1.1)$$

where:

y = The force level for force y
x = The force level for force x
a = attrition rate coefficient for y forces
b = attrition rate coefficient for x forces
x_{BP} = x's breakpoint force level
y_{BP} = y's breakpoint force level

From this it is observed that y would "win" an engagement if and only if:

$$\frac{x_o}{y_o} < \sqrt{\left\{\frac{a}{b} \cdot \frac{1 - (y_{BP})^2}{1 - (x_{BP})^2}\right\}} \quad (1.2)$$

As illustrated in Table 1.1 below, the breakpoints selected have a very significant effect on simulated engagement outcomes. This sensitivity of engagement outcome to the breakpoint used points out the need for rigorous justification of the values used.

Table 1.1

Influence of Unit Breakpoints on the Outcome of Battle for an Attack by X against Y with Battle Dynamics by Lanchester's Equations for Modern Warfare (Taylor, 1980:66)

CASE	$\frac{x_o}{y_o}$	$\frac{a}{b}$	x_{BP}	y_{BP}	$\sqrt{\left\{\frac{a}{b} \cdot \frac{1 - (y_{BP})^2}{1 - (x_{BP})^2}\right\}}$	WINNER	$\frac{x_f}{x_o}$	$\frac{y_f}{y_o}$
1	3.0	5.0	0.8	0.5	3.23	y	0.8	0.59
2	3.0	5.0	0.7	0.5	2.71	x	0.76	0.50
3	3.0	5.0	$x_{BP} = y_{BP}$		2.24	x	$> y_{BP}$	x_{BP}

Note: x is the attacker.

1.2 Research Question

Is the breakpoint hypothesis as stated below consistent with the set of historical data available for this investigation?

1.3 Breakpoint Hypothesis

The breakpoint hypothesis used for this investigation is listed below. Elements 1, 3a and 3b of the hypothesis come directly from Helmbold's working hypothesis (Helmbold, 1971:7). The second element is unique.

1) Termination of a battle can be considered as governed by the following mechanism, or one that gives the same results: prior to the battle, each side independently and at random selects a casualty-fraction value (breakpoint) from some distribution of casualty fractions. Where either side experiences a casualty fraction equal to the preselected breakpoint, the battle terminates with a loss to the side that broke.

2) The breakpoint distributions (break curves) mentioned above are generally applicable within one or more of the combinations of the <u>observed variables</u> contained within the engagement data matrix available for this investigation.

3a) The losses, and hence equivalently the casualty-fraction, of the forces are deterministically and monotonically related to each other. There is a monotonically increasing function, $\psi(\cdot)$, such that:

$$f_x(t) = \psi \left[f_y(t) \right] \quad 0 \leq t \leq T \qquad (1.3)$$

<u>OR</u>

3b) There is a monotone nondecreasing function ψ such that:

$$f_x = \psi(f_y) \qquad (1.4)$$

when the defender wins, while

$$f_x = \psi^{-1}(f_y) \qquad (1.5)$$

when the attacker wins.

Also: $\psi(s) \geq s$ for all s.

A complete mathematical development of the above relations is contained in subsection 2.3.1.

The modifications embodied in element 2 utilizes many of the factors of combat considered significant in the literature. To a significant extent, the results of this investigation are an important statement on the validity of many of the Department of Defense sponsored combat models currently in use.

The second chapter reviews several applications of the breakpoint hypothesis in force-on-force attrition studies as well as the empirical validation studies performed by Clark (1954) and Helmbold (1971). It also discusses the applicability of historical data in the analysis of present and future combat processes.

Chapter 3 specifies the experimental design used for the current investigation. This includes a description of the contents of the engagement data matrix, the procedure for break curve construction and the methodology used to examine their validity.

Chapter 4 presents and discusses the results of the investigation. The results of the data reduction process via factor analysis are described. Break curves are presented along with statements about their validity. In addition, the results obtained using the Helmbold procedure are then compared with the results obtained via discriminant analysis.

Chapter 5 documents what the results mean in terms of modelling the battle termination process. The limitations of the study results are reviewed and areas for future research are identified.

Chapter 2

LITERATURE REVIEW

The breakpoint hypothesis has been used in many Department of Defense sponsored force-on-force attrition studies for modelling the battle termination process. In spite of its widespread use, however, empirical support for the hypothesis is limited. Further research found only two empirical validation studies. They consist of a study performed on the validity of deterministic break curves (Clark, 1954) and an investigation into the validity of probabilistic break curves (Helmbold, 1971). Both studies found significant problems with formulations of the breakpoint hypothesis currently in use. A significant obstacle to further investigative work is the question concerning the applicability of historical data in the analysis of present and future combat processes.

This chapter provides a brief review of breakpoint hypothesis applications and its validity.

2.1 Sample Applications

Varying formulations of the breakpoint hypothesis have been used in many force-on-force attrition studies. One representative application is the current U.S. Army's procedures for the control of map exercises (Department of the Army, 1973:FM105-5, Appendix D). The functions and values which they use are based on past Army studies and combat experience. These procedures are considered valid for all military combat units up to and including battalion-sized units (Department of the Army, 1973:FM105-5, Appendix D). The method for modelling the battle termination process for defending units is presented below, as taken verbatim from the Army manual. Note that the procedures provide distinct break curves to be used for attacking and defending units, respectively.

Defending Units

a. Casualties in a defending force can cause one of three results:

   (1) The defense is continued without interruption by the survivors.
   (2) The defending unit requires up to 48 hours to recuperate before being able to resume the defense in another position. (It must withdraw to a rear position.)
   (3) The defending unit is totally ineffective and must either be replaced immediately or risk being overrun.

b. If the percentage of casualties is 25% or less, result a(1) occurs. If the percentage of casualties is 40% or greater, result a(3) . . . occurs. If the percentage of casualties is greater than 25% but less than 40% the casualty percentage is used to determine the probability for the unit to continue defending without a delay . . . . This probability is used in conjunction with the table of random numbers to determine whether the event occurs. If the decision is affirmative, the unit can continue to defend without interruption.

c. If the decision is negative, result a(2) above occurs. (Department of the Army, 1973:FM105-5, Appendix D)

This procedure utilizes two breakpoints. Result a(2) is controlled by a probabilistic break curve whose boundary conditions are (see Appendix A for the definitions of the mathematical symbols used):

$$F_z(.25) = 0 \tag{2.1}$$

$$F_z(.40) = 1.0 \tag{2.2}$$

For $.25 \le u \le .40$, the following break curve is used:

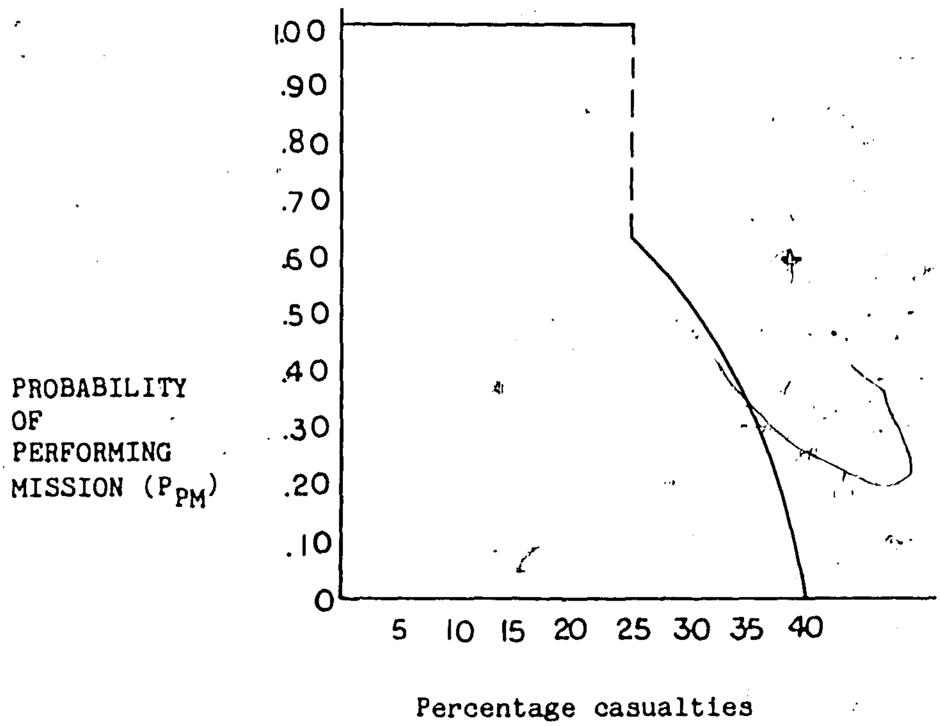

Figure 2.1

Probability of Defending Unit Continuing to Perform Mission (Department of the Army, 1973:FM105-5, Appendix D)

Note that the probability that the unit breaks = 1 - P_{PM}. The discontinuity in the curve occurs at the point where the probability of the defending unit becomes nonzero.

Result a(3) is controlled by a deterministic break curve with the break occurring at 40% casualties. It represents a more serious degradation of the military force.

Another application of deterministic break curves is found in the ATLAS (A Tactical Logistical Air Simulation) model. A key assumption used to model battle termination states:

> a division-sized unit in a defensive position is considered combat ineffective and hence withdrawn from combat when its personnel strength falls below 67%. The level at which an attacking unit becomes combat ineffective is 79%, although it is not withdrawn until the 67% level is reached. (Kerlin & Cole, 1969:17)

As of 1977, this still is one of the most frequently used force-on-force attrition models in Department of Defense studies (Taylor, 1980:105).

In each of these two studies, the break curves are defined differently. The relevant breakpoints are listed in Table 2.1, along with nominal attacker and defender breakpoints identified by Taylor for company-sized units (Taylor, 1980:63,64).

Table 2.1

Typical Breakpoints

Source	Applicable Unit Size	Attacker Breakpoint	Defender Breakpoint
Maneuver Control	battalion or less	N/A	.6
ATLAS	division	.79	.67
Taylor	company	.7	.5

Taylor's values are inconsistent with those used in Maneuver Control, although both are considered valid for company-sized units. The functional forms employed are different as well. The sample break curves from Maneuver Control are both probabilistic and deterministic while ATLAS and Taylor's are deterministic only. These differences are significant, given the demonstrated sensitivity of the results of a battle termination model to the breakpoint values selected (see Table 1.1 and associated text).

A typical justification for the use of battle termination breakpoints is clearly seen in a study performed by Spring and Miller (1970) entitled: "Fast-Val: Relationships among Casualties, Suppression and the Performance of Company-Sized Units". It concerns evaluating the effects of air-delivered munitions during a land combat engagement. With respect to the unit breakpoints used, the study states:

> . . . because of the lack of available data or relevant reference materials, we have relied heavily on the judgement and experience of RAND's military consultants throughout this study . . . while it must be emphasized that there is no

> universal agreement as to the validity of the parameters we have defined, we feel that they serve as a much-needed working starting point . . . these assumptions and relationships are subjective and will undoubtedly undergo modification as empirical data become available. (Spring and Miller, 1970:11)

Both this and other studies identify the requirement for the functional forms and values used to be empirically based whenever possible.

Two empirical investigations into the validity of the functional forms and values used in break curves were found in the literature. The first concerns deterministic break curves while the second investigates probabilistic break curves. The results are documented in the following two sections.

2.2 Deterministic Break Curves

Clark conducted the principle investigation into the validity of deterministic break curves. Specifically, the study's intent was:

> To investigate from actual combat data the validity of the statement that a unit may be considered no longer combat effective when it has suffered N percent casualties. (Clark, 1954:1)

The data compiled for the study consisted of forty-four battalion-level engagements that occurred within seven divisional engagements in the European Theater of Operations during World War II (see Appendix B for Clark's definition of a divisional engagement). Twenty-seven distinct U.S. infantry battalions were used in the analysis.

For each battalion-level engagement, there are eight data elements available. They are listed in Table 2.2.

Table 2.2

Data Elements Contained Within Clark's Engagement Data Base (Clark, 1954:2)

Data Element	Description
1	Date
2	Geographic location of engagement
3	Attacker
4	Defender
5	Initial U.S. force level
6	Daily U.S. casualties
7	Daily U.S. replacements
8	Number of days until breakpoint occurred

Clark collected data from engagements involving battalion-level infantry units only. There are two reasons given for this. It was considered easiest to discern the significance of personnel casualties on combat effectiveness using infantry units since these units typically incurred the highest casualty rate in conventional warfare. In addition, the effects of other combat variables on unit combat effectiveness would be minimized by using data from infantry battalions. Clark notes that this was in contrast to other types of combat battalions. Specifically, "the effectiveness of a tank battalion . . . depends as much on the operability of its tanks as on adequate crews to man them. . . . An artillery battalion may have to change its mission, not because of actual losses of material but because enemy counterbattery fire threatens to produce such losses and the battery must shift its position to safeguard its weapons." (Clark, 1954:8-9). This lack of qualitative correlation between personnel casualties and combat effectiveness in non-infantry combat units could be a significant problem where combat data consists of engagements between combined arms or wholly non-infantry forces.

Clark took the casualty data for the engagements studied from the daily morning reports of battalion HQ, headquarters company, the three infantry companies, and the heavy weapons company. Data contained in these reports included casualties, replacements and men returned from hospital or detachment to another unit (Clark, 1954:12).

There are limitations to Clark's data. To the extent that division-related events that occurred outside the battalion's Area of Operations have an effect on the breaking of an individual infantry battalion, the observations within each divisional engagement are not independent. In addition, no information on enemy force strengths, positions or other significant data are included in the data base. This severe drawback is mitigated by World War II (Europe) being selected by Clark as the source for all engagements, since ". . . German equipment and methods accord ed more closely with the U.S. than did Japanese, North Korean or Chinese" (Clark, 1954:8). Other than the casualties inflicted on the U.S. infantry battalions, however, no quantitative statement can be made about the effect of the enemy on a unit breaking.

Clark examined three categories of deterministic break curves. They are contained in Table 2.3.

Table 2.3

Deterministic Breakpoint Categories (Clark, 1954:11)

Category	Description
I	Attack - Rapid Reorganization - Attack
II	Attack - Defense
III	Defense - Withdrawal by Order to a Quieter Sector

The Category I break represents a temporary cessation of the attack imposed by the defending force. The attack is resumed within a 24 hour period. A Category II break represents a switch from attack to defense for the duration of the divisional engagement. A Category III break constitutes the removal of the unit from the battalion-level engagement. (Clark, 1954:9-12)

In addition, Clark investigated the effect that the distribution of casualties over time has on a unit's breakpoint. This included three alternative methods of calculating casualties and examining the breakpoint data with respect to total engagement length. These three methods of calculating casualties are contained in Table 2.4.

Table 2.4

Methods of Calculating Casualties
(Clark, 1954:17)

No.	Method	Rationale
1	Losses and net losses, in percent, for the day of breakpoint	Based on the present experience being the major demoralizing experience
2	Cumulative losses and cumulative net losses, in percent, for the two days preceeding plus the day of breakpoint	Suggested by the idea that men's memories encompass not only the present, but also the experiences of the very immediate past.
3	Cumulative losses and cumulative net losses, in percent, from the beginning of the engagement to the breakpoint	Represents the battle experience as entirely cumulative and provides as well the best measure of actual unit strength.

The third method of calculation is the most desirable from a practical standpoint since it requires only the initial and final force levels of the units evaluated. (Clark, 1954:17)

The primary result obtained by Clark is negative. The study concludes:

> The statement that a unit can be considered no longer combat effective when it has suffered a specific casualty percentage is a gross oversimplification not supported by combat data. . . . individual differences in the ability of units to carry out their missions cannot be entirely explained on the basis of casualties and replacements alone. (Clark, 1954:3)

Personnel casualties have a significant, but not dominating, influence on a unit's combat effectiveness.

Clark also found that, when calculating casualties for unit breakpoint determination, cumulative casualties is the best of the three methods listed in Table 2.4. This method suffers, however, from the drawback of not considering the time from the start of the engagement to the unit's breakpoint. This varies significantly between the break types. For Category I breaks, the time to breakpoint ranges from 2 to 11 days. Category II times to breakpoint ranges from 2 to 22 days and Category III from 6 to 17 days (Clark, 1954:20).

In addition to the deficiencies mentioned above, Clark found that at least two additional factors need to be accounted for when specifying a breakpoint for a combat unit. These consist of the type and size of unit(s) involved in an engagement and the presence of ". . . widely differing ranges of loss percentages (being) associated with a breakpoint from attack to defense and a breakpoint from defense to withdrawal." (Clark, 1954:3)

The specification of unit size is evident in the applications which the investigator found elsewhere in the literature, although these applications are not always consistent. With respect to unit type, combat units whose effectiveness is not proportional to personnel strength limits the ability of personnel casualties to determine when a unit broke. Evidence that loss percentages are signficantly different for attacker and defender breakpoints indicates that breakpoints should be divided, as a minimum, into these two categories.

Clark also found that, due to the variations in the data,

". . . ranges of loss percentages probabilistic break curves adhering to a uniform distribution must be used to give an accurate description of what happens in actual combat." (Clark, 1954:3) Put differently, deterministic break curves were found to be invalid. Probabilistic break curves are required to account for the variations in historical breakpoint data.

2.3 Probabilistic Break Curves

The second investigation by Helmbold examines the validity of probabilistic break curves. Probabilistic break curves provide the necessary randomness in determining the breakpoint for a combat unit. They constitute a generalized technique for modelling the battle termination process, with the deterministic break curve being contained within the family of potential probabilistic break curves (Helmbold, 1971:3).

Helmbold's work centered upon determining the degree of validity contained within the hypothesis which represents common formulations of probabilistic break curves in Department of Defense sponsored studies. The hypothesis consists of the following three elements:

1) Termination of a battle can be considered as governed by the following mechanism, or one that gives the same results: prior to the battle, each side independently and at random selects a casualty-fraction value (breakpoint) from some distribution of casualty fractions. When either side experiences a casualty fraction equal to the preselected breakpoint, the battle terminates with a loss to the side that "broke."

2) The breakpoint distributions (break curves) mentioned above are generally applicable. That is, they are the same for all battles, irrespective of the size of forces involved or when, where, by whom, or with what the battle was fought.

3) The losses, and hence equivalently the casualty fractions, of the forces are deterministically and monotonically related to each other. That is, there is a monotonically increasing function, $\varphi(\cdot)$, such that

$$f_x(t) = \varphi f_y(t) \ , \quad 0 \leq t \leq T \tag{2.3}$$

(Helmbold, 1971:7-9)

The complete development of Helmbold's breakpoint model is contained in section 2.3.1, as taken verbatim from Helmbold's report. The reader may skip to section 2.3.2 for a recitation and discussion of the more significant points without loss of continuity.

2.3.1 Complete Mathematical Development of Helmbold's Probabilistic Break Curve Model (Helmbold, 1971:9-16)

If the attacker is to win, then we must have

$$f_x(t) < L_x, \qquad 0 \leq t \leq T \tag{2.4}$$

and

$$f_y = f_y(T) = L_y. \tag{2.5}$$

In particular, if the attacker wins, we must have

$$L_x > f_x = \varphi(f_y) = \varphi(L_y). \tag{2.6}$$

Conversely, if

$$\varphi(L_y) < L_x, \tag{2.7}$$

then, since

$$f_y(t) \leq L_y, \qquad 0 \leq t \leq T, \tag{2.8}$$

it follows, by the monotonicity of φ, that

$$\varphi(f_y(t)) \leq \varphi(L_y) < L_x, \qquad 0 \leq t \leq T, \qquad (2.9)$$

and then using $f_x(t) = \varphi(f_y(t))$,

$$f_x(t) \leq \varphi(L_y) < L_x, \qquad 0 \leq t \leq T, \qquad (2.10)$$

and the attacker wins. Thus, the attacker wins if, and only if,

$$\varphi(L_y) < L_x. \qquad (2.11)$$

Since we intend that battle outcomes be . . . well-defined by our model, we could assign victory to the defender when $\varphi(L_y) = L_x$, or we could see to it that this equality has zero probability of occurrence. For some purposes, it may be convenient to adopt the convention that the battle is a toss-up when $\varphi(L_y) = L_x$, and to award victory with equal probability to both sides. In any case, we arrange things so that

$$P(W_x) = 1 - P(W_y), \qquad (2.12)$$

where $P(W_z)$ is the probability of a win for side z, z = x or y.

Let

$$F_z(u) = P\left[L_z \leq u\right] \qquad (2.13)$$

be the break curve for side z. Now, $F_z(0) \neq 0$ would imply that there is some positive probability that side z would break while its casualty fraction was zero, which may physically be interpreted as a refusal to engage in battle on the part of side z. Since we wish to consider only cases where the battle has been joined, we take

$$F_z(0) = 0. \qquad (2.14)$$

Also, $F_z(1) \neq 1$ would imply that side z might not break even when its casualty fraction was unity. This seems to be intuitively unreasonable, and so we assume that

$$F_z(1) = 1. \tag{2.15}$$

[Investigator note: This assumption ignores the situation where a force has had 100% of its personnel killed and wounded but at least a percentage of its wounded continue to fight.]

We now wish to express $P(W_y)$ in terms of the F_z's. To do this we begin by noting that the preceding discussion of the conditions under which the defender wins yields the following relationship

$$P(W_y) = P\left[L_x \leq \varphi(L_y),\ 0 \leq L_y \leq 1\right], \tag{2.16}$$

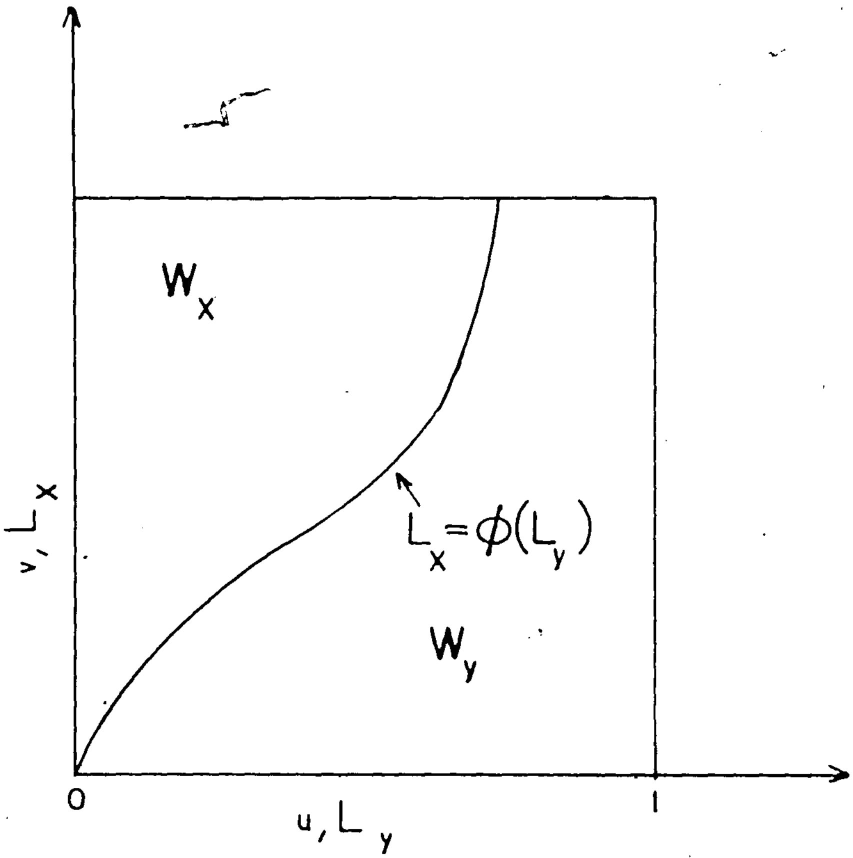

Figure 2.2

Relationship between L_x and L_y

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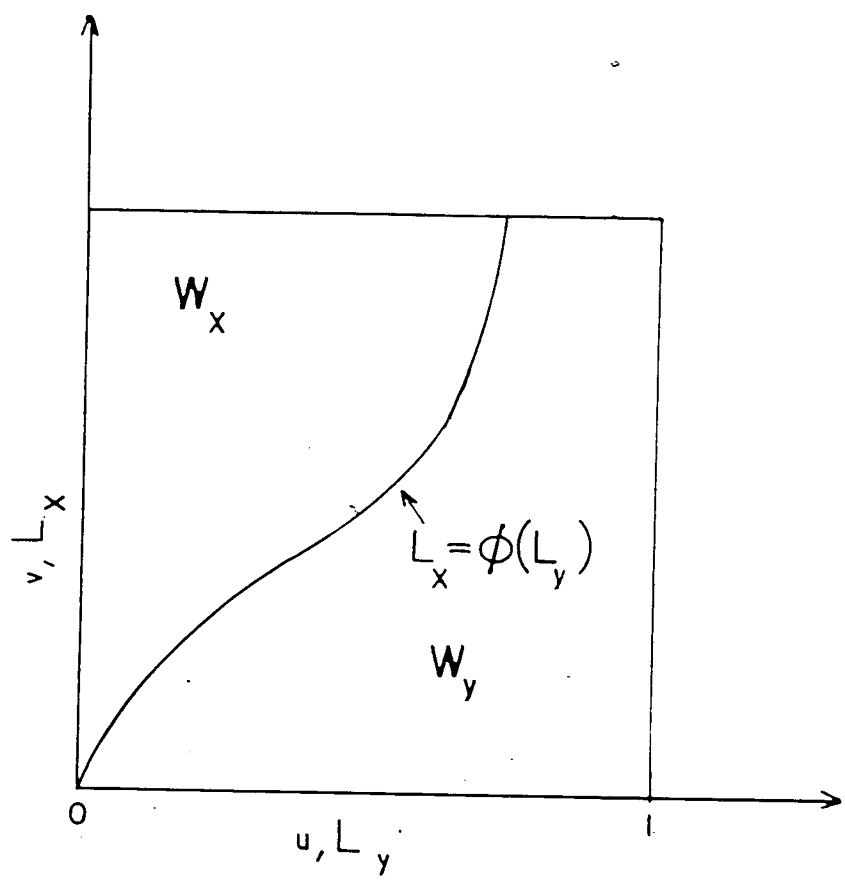


Figure 2.2

Relationship between L_x and L_y

since $L_x > \varphi(L_y)$ if, and only if, the attacker wins. To calculate $P(W_y)$, we consider the schematic diagram shown as [Figure 2.2]. The set of points $(L_y, L_x) = (u, v)$ for which the attacker wins is marked by W_x, and similarly for defender wins by W_y. The joint density of (L_y, L_x) is, by [Element 1 of the Hypothesis], given by

$$dF_x(v)\ dF_y(u),$$

and so,

$$P(W_y) = \int_{u=0}^{1} \int_{v=0}^{\psi(u)} dF_x(v)\ dF_y(u) \tag{2.17}$$

$$= \int_0^1 F_x(\psi(u))\ dF_y(u),$$

where we have truncated $\varphi(u)$ by setting

$$\varphi(u) = \mathrm{Min}\left[\varphi(u),\ 1\right]. \tag{2.18}$$

where $\varphi(u)$ is assumed to be monotonically increasing and defined for all $0 \leq u \leq 1 + Q$.

Similarly,

$$P(W_x) = \int_{v=0}^{1} \int_{u=0}^{\psi^{-1}(v)} dF_x(v)\ dF_y(u), \tag{2.19}$$

which becomes

$$P(W_x) = \int_0^1 F_y(\psi^{-1}(v))\ dF_x(v). \tag{2.20}$$

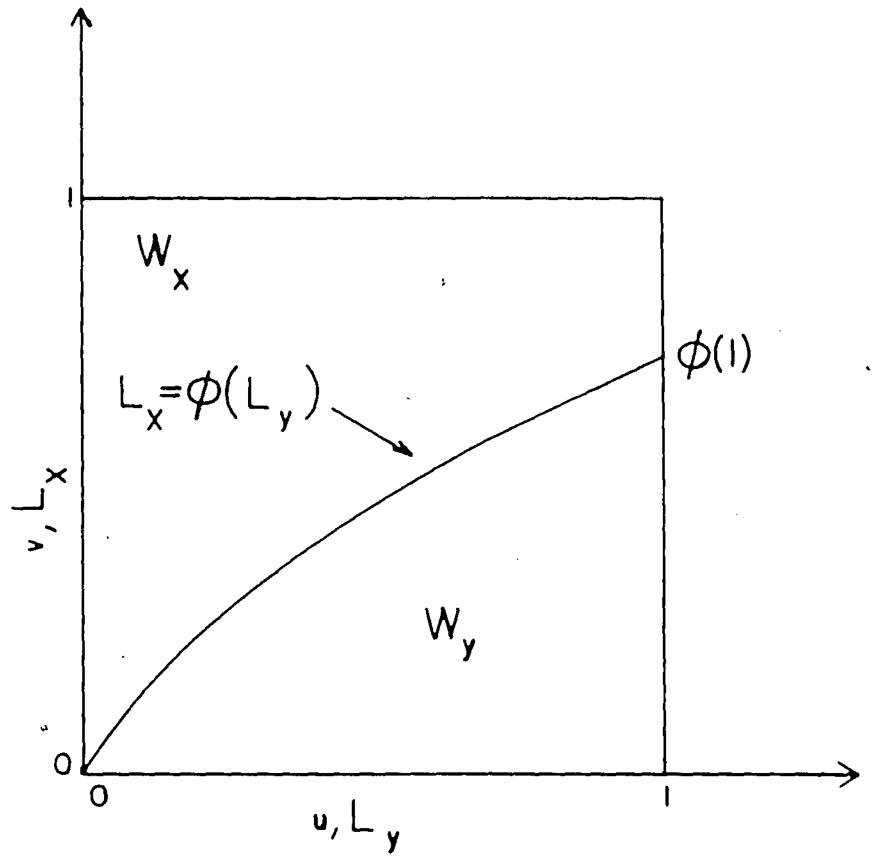


Figure 2.3

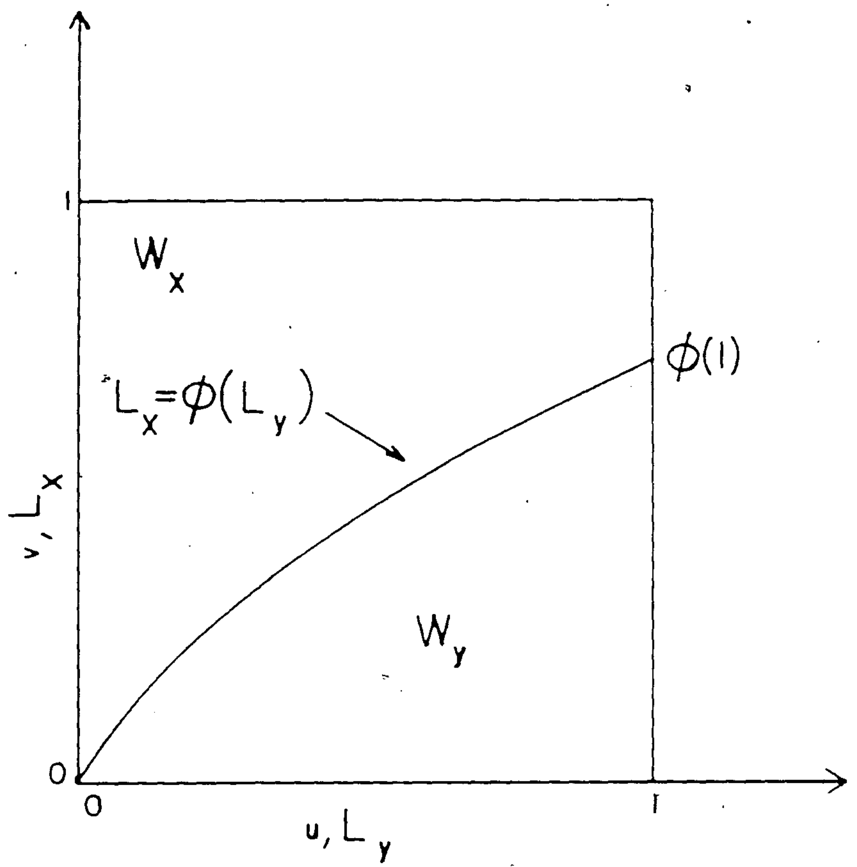
Another possible relation between L_x and L_y

If $\varphi(1) < 1$, as illustrated in [Figure 2.3], then we define $\varphi^{-1}(v) = 1$ for $\varphi(1) \leq v \leq 1$. This manner of defining the inverse function preserves the correctness of the formulae just given.

By integrating in the reverse order with respect to the variables u and v, we obtain formulae equivalent to those that would result from an integration by parts, thus,

$$P(W_y) = \int_{v=0}^{1} \int_{\varphi^{-1}(v)=u}^{1} dF_y(u)\ dF_x(v)$$

$$= \int_{0}^{1} \left[1 - F_y(\varphi^{-1}(v))\right] dF_x(v) \qquad (2.21)$$



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If $\varphi(1) < 1$, as illustrated in [Figure 2.3], then we define $\varphi^{-1}(v) = 1$ for $\varphi(1) \leq v \leq 1$. This manner of defining the inverse function preserves the correctness of the formulae just given.

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$$P(W_y) = \int_{v=0}^{1} \int_{\varphi^{-1}(v)=u}^{1} dF_y(u)\, dF_x(v)$$

$$= \int_0^1 \left[1 - F_y(\varphi^{-1}(v))\right] dF_x(v) \qquad (2.21)$$

and

$$P(W_x) = \int_{u=0}^{1} \int_{\psi(u)=v}^{1} dF_y(u)\ dF_x(v)$$

$$= \int_0^1 \left[1 - F_x(\psi(u))\right] dF_y(u). \qquad (2.22)$$

From Figures 2.2 and 2.3, we see that the conditional joint density of (L_y, L_x), given W_x, is

$$\frac{dF_y(u)\ dF_x(v)}{P(W_x)}, \qquad \text{for } (u, v) \in W_x.$$

So the conditional density of L_y, given W_x, is

$$dD_y(u|W_x) = \int_{v=\psi(u)}^{1} \frac{dF_y(u)\ dF_x(v)}{P(W_x)}$$

$$= \frac{\left[1 - F_x((\varphi(u))\right] dF_y(u)}{(P(W_x)}. \qquad (2.23)$$

Integration of this expression with respect to u from u = 0 to u = 1 and comparing the result with Equation 2.22 shows that it represents a proper probability density.

We now find the conditional distributions of casualty fractions on each side when the attacker wins. We begin by recalling that when x wins, $L_y = f_y$. But we have just found the density of L_y when x wins. Hence,

$$P(f_y < q|W_x) = \int_0^q d\ D_y(u|W_x)$$

$$= D_y(q|W_x). \qquad (2.24)$$

Since $f_x = \psi(f_y)$, the conditional distribution of the attacker's casualty fraction when the attacker wins is:

$$P(f_x < s \mid W_x) = P(\psi(f_y) < s \mid W_x)$$
$$= D_y(\psi^{-1}(s) \mid W_x). \tag{2.25}$$

In similar fashion we find the conditional density of L_x given W_y as

$$d\, D_x(v \mid W_y) = \int_{u=\psi^{-1}(v)}^{1} \frac{dF_y(u)\; dF_x(v)}{P(W_y)}$$

$$= \frac{\left[1 - F_y(\psi^{-1}(v))\right] \; dF_x(v)}{P(W_y)}. \tag{2.26}$$

Since $L_x = f_x$ whenever y wins, the conditional distribution of the attacker's casualty fraction when the defender wins is just:

$$P(f_x < s \mid W_y) = D_x(s \mid W_y). \tag{2.27}$$

Since $f_x = \psi(f_y)$, the conditional distribution of the defender's casualty fraction when he wins is

$$P(f_y < q \mid W_y) = P(f_x < \psi(q) \mid W_y)$$
$$= D_x(\psi(q)\; W_y). \tag{2.28}$$

2.3.2 Review of Helmbold's Probabilistic Break Curve Model

Section 2.3.1 contains a complete development of Helmbold's mathematical framework for the breakpoint hypothesis. This section reviews the important aspects of Helmbold's model.

The three elements of the hypothesis contain certain underlying principles. The first element states that the casualty-fraction is the principle parameter used in the break curve. In addition, Helmbold assumes that the battle is fought with the forces available at the start of the engagement. This is analytically convenient ". . . since this provides a well-defined (if not precise) base for establishing the casualty fradtion" (Helmbold, 1971:7). The limitations to this approach are discussed in Chapter 3 of the current investigation.

With regards to element 2, there is no inherent reason for only one break curve to be included. Helmbold states that:

> The appropriate break curve could be made to depend on any condition that could be known at the time the break curve is sampled, such as whether the force is initially attacking or defending, its state of training, experience, morale, physical weariness, etc. . . . The approach adopted here is in keeping with the spirit of Richardson's Principle to the effect that 'formulae are not to be complicated without evidence'. (Helmbold, 1971:8)

Later in the investigation, Helmbold altered element 2 by defining break curves on the basis of who won the engagement and the nature of the engagement.

Element 3 of the hypothesis is an outgrowth of previous work performed by Weiss (1966). Weiss postulated that the casualty fractions were proportional to one another, or:

$$f_x(t) = Rf_y(t) \tag{2.29}$$

This became a special case contained within Helmbold's hypothesis (Helmbold, 1971:10).

In order to completely specify the probabilistic break curve model outlined in the hypothesis, Helmbold developed the mathematical characteristics of the probabilistic break curve. This includes its boundary conditions, the relationship between each force's probability of winning and the relationship between the break curves of the two opposing sides.

Helmbold specified physical meanings for the upper and lower boundary conditions for the probabilistic break curves. Letting:

$$F_z(u) = P\left[L_z \le u\right] \tag{2.30}$$

be the break curve for force z, he established the lower and upper boundary values for the function. In the case of the lower boundary,

> $F_z(0) = 0$ would imply that there is some positive probability that side z would break while its casualty fraction was zero, which may physically be interpreted as a refusal to engage in battle on the part of side z. Since we wish to consider only cases where the battle has been joined, we take $F_z(0) = 0$. (Helmbold, 1971:11)

The argument for the upper bound was as follows:

> $F_z(1) = 1$ would imply that side z might not break even when its casualty fraction was unity. This seems to be intuitively unreasonable since the unwounded personnel strength of the unit = 0 , and so we assume that $F_z(1) = 1$. (Helmbold, 1971:12)

In addition, Helmbold proved a theorem stating that the two Ψ functions relating the attacker and defender conditional casualty fraction distributions curves to each other are mathematical inverses (Helmbold, 1971:15-16). He also proved that the following two lemmas are true:

<u>LEMMA I</u>. If

$$\Delta_{xx}(u) = \Delta_{yy}(u) \tag{2.31}$$

and

$$\Delta_{yx}(u) = \Delta_{xy}(u) \tag{2.32}$$

then

$$\Psi = \Psi^{-1} = I, \text{ where I is the identity function} \tag{2.33}$$

where:

$$\Delta_{xx}(s) = P(f_x < s | W_x) = D_y(\psi^{-1}(s) | W_x) \quad (2.34)$$

$$\Delta_{yy}(s) = P(f_y < s | W_y) = D_x(\psi(s) | W_y) \quad (2.35)$$

$$\Delta_{yx}(q) = P(f_y\ q | W_x) = D_y(q | W_x) \quad (2.36)$$

$$\Delta_{xy}(q) = P(f_x < q | W_y) = D_x(q | W_y) \quad (2.37)$$

LEMMA II. If $\psi(s) \geq s$ for some s, then:

$$\Delta_{yy}(s) \geq \Delta_{xy}(s), \quad (2.38)$$

and

$$\Delta_{xx}(s) \leq \Delta_{yx}(s). \quad (2.39)$$

Conversely, if $\psi(s) \leq s$ for some s, then:

$$\Delta_{yy}(s) \leq \Delta_{xy}(s), \quad (2.40)$$

and

$$\Delta_{xx}(s) \geq \Delta_{yx}(s) \quad (2.41)$$

The theorem, and to a lesser extent, the lemmas, were incorporated by Helmbold into his test procedure (Helmbold, 1971:18-20).

2.3.3 Test Procedure

Helmbold's test procedure is based upon whether the inverse relationship of the ψ and ψ^{-1} curves is consistent with the observed casualty fraction distributions. Recall that:

$$\psi(u) = \text{MIN}\left[\varphi(u), 1\right] \quad (2.18)$$

where: φ = a strictly increasing monotonic function relating $f_x(t)$ to $f_y(t)$ via the formula:

$$f_x(t) = \varphi\left[f_y(t)\right] \quad (2.3)$$

And:

$$\Delta_{xx}(s) = \Delta_{yx}(\psi^{-1}(s)) \quad (2.42)$$
$$\Delta_{yy}(s) = \Delta_{xy}(\psi(s)) \quad (2.43)$$

The validity of the breakpoint hypothesis would then be dependent upon equations 2.42 and 2.43 holding true for the data set tested. Specifically:

> If ψ and ψ^{-1} obey the inverse functional relationship, then this would tend to support the breakpoint hypothesis. If ψ and ψ^{-1} do not obey the necessary mathematical relationship between inverse functions, then the breakpoint hypothesis would be definitely disproven. (Helmbold, 1971:16)

Helmbold developed a graphical procedure for obtaining the ψ and ψ^{-1} functions independently from the observed casualty fraction distributions. The following subsection describing the procedure is taken verbatim from Helmbold's report.

2.3.4 Graphical Procedure for Obtaining ψ and ψ^{-1} Functions from Casualty-Fraction Distributions (Helmbold, 1971:16-17)

We have set down in explicit terms the breakpoint hypothesis Elements 1, 2 and 3 and have shown how to derive from these elements formulae that purport to describe empirical casualty-fraction distributions. In carrying out this derivation, we have been careful to maintain the essential distinction between a break curve, which is a distribution of L_z breakpoint values and a casualty-fraction distribution, which is a distribution of f_z values. In this paragraph we show how observed casualty-fraction distributions can be used to test the breakpoint hypothesis.

We begin by recalling relations 2.34 and 2.36 , which are

$$P(f_x < s | W_x) = D_y(\psi^{-1}(s) | W_x) = \Delta_{xx}(s), \qquad (2.34)$$

and

$$P(f_y < q | W_x) = D_y(q | W_x) = \Delta_{yx}(q) \qquad (2.36)$$

where the Δ_{yx} and Δ_{xx} notation is introduced as an abbreviation. Combining relations 2.34 and 2.36 yields

$$\begin{aligned}\Delta_{xx}(s) &= P(f_x < s \mid W_x) \\ &= P(f_y < \psi^{-1}(s) \mid W_x) = \Delta_{yx}(\psi^{-1}(s)),\end{aligned} \qquad (2.44)$$

with a dual result obtainable by the usual transposition $x \to y$, $y \to x$, $\psi^{-1} \to \psi$, $\psi \to \psi^{-1}$.

Now suppose that we had a graphical plot of the observed casualty fractions for a collection of battles that were won by the attacker. A hypothetical plot is shown in [Figure 2.4]-- and there will be a dual plot whose labels are obtainable from [Figure 2.4] by the usual transposition, although the curves may, of course, be differently shaped on the dual. We have indicated by the dashed lines how, using [equation 2.44], the value of $\psi^{-1}(q_1)$ can be graphically read off this plot. An exactly analogous procedure applied to the dual plot will yield the value of $\psi(q_1)$. By repeating the process for several values of q_1 and interpolating, it is thus possible to determine suitable approximations to the functions ψ and ψ^{-1}.

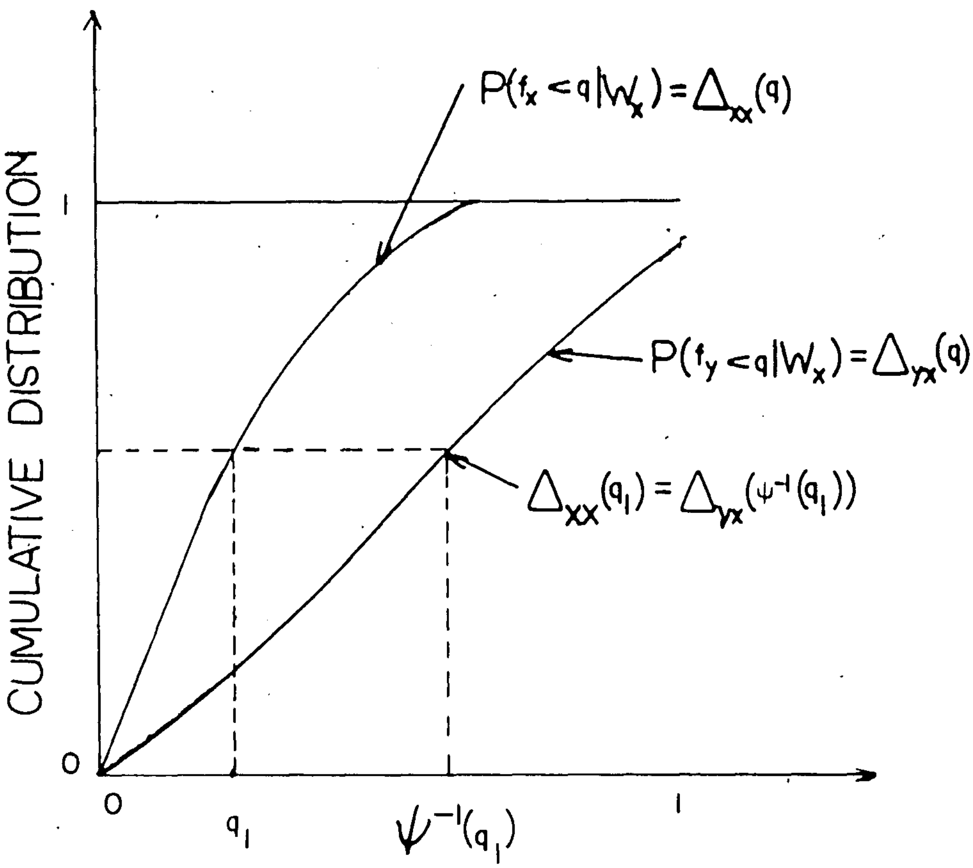

Figure 2.4

Hypothetical Casualty-Fraction Distribution
in Battles Won by the Attacker

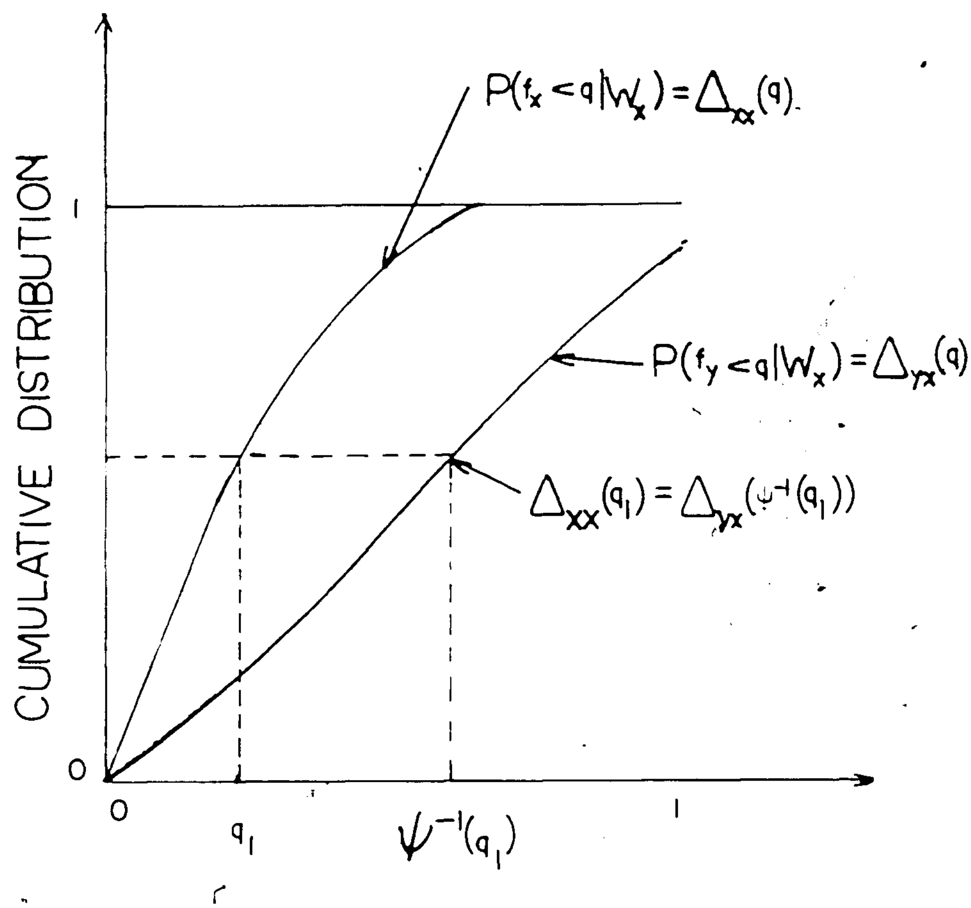

Figure 2.4

Hypothetical Casualty-Fraction Distribution in Battles Won by the Attacker

Now, ψ is the functional relation between f_x and f_y, since from the definition of ψ, we may write without loss of generality

$$f_x(t) = \psi\left[f_y(t)\right]. \quad (2.45)$$

Having determined ψ and ψ^{-1} by the graphical procedure just described, we may plot these functions on a graph and see whether or not they obey the necessary mathematical relationship between inverse functions, that is, whether or not ψ is a reflection of ψ^{-1} in the 45-deg line through the origin, as illustrated in Figure 2.5 . If ψ and ψ^{-1} obey the inverse functional relationship, then this would tend to support the breakpoint hypothesis. If ψ and ψ^{-1} do not obey the necessary mathematical relationship between inverse functions, then the breakpoint hypothesis would be definitely disproven.

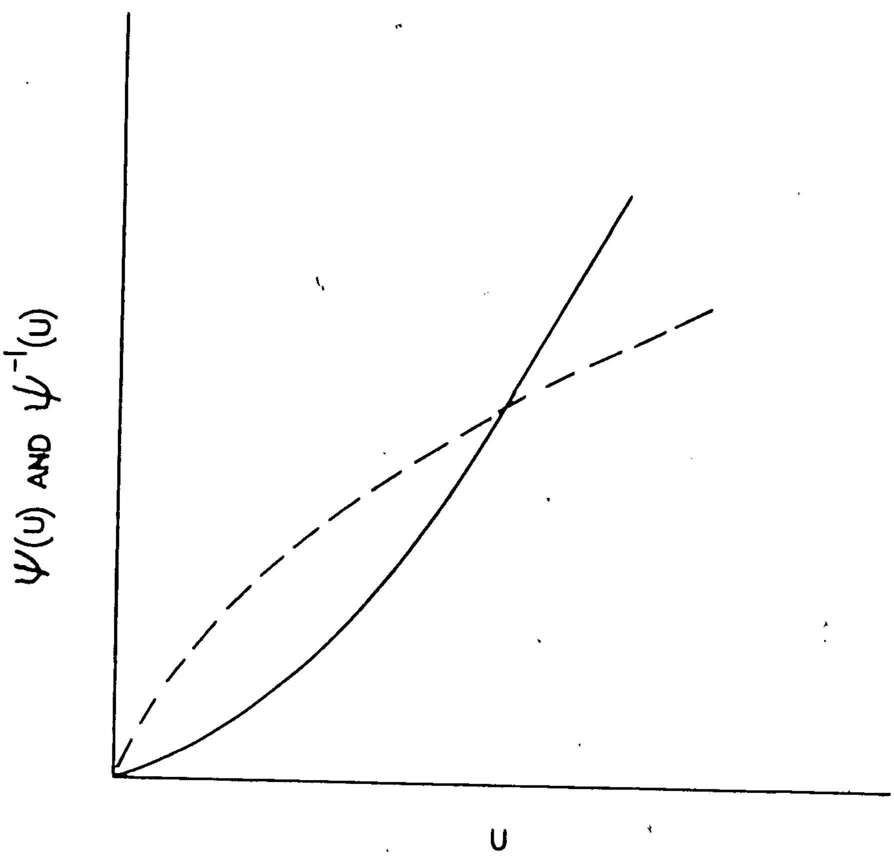

Figure 2.5

Inverse Functional Relationship

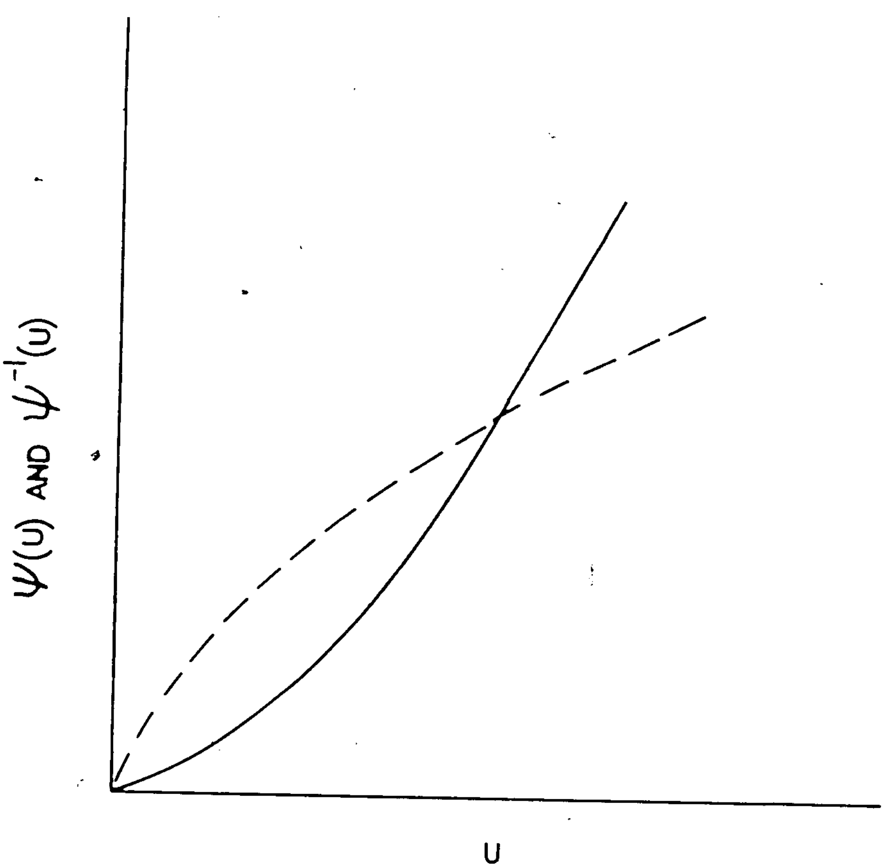

Figure 2.5

Inverse Functional Relationship

2.3.5 Data

The quantity and, more importantly, the type of historical combat data available to Helmbold severely restricted the complexity of the hypotheses capable of being tested. Helmbold used data from two of his earlier studies (Helmbold, 1961 and 1964) as well as a data set extracted from Bodart's Kriegs-Lexicon (Bodart, 1908) by Willard (Willard, 1962). There are 173 engagements from Helmbold's earlier studies and 1,080 engagements in Willard's data set. The data elements in the engagement data matrix for the probabilistic break curve investigation are listed in Table 2.5.

Table 2.5

Data Elements Contained in Helmbold's Engagement Data Matrix (Helmbold, 1971:5-6)

Data Element Number	Description
1	Date
2	Battle
3	Source
4	War or campaign
5	Identification of attacker
6	Identification of defender
7	Initial attacker strength
8	Initial defender strength
9	Attacker casualties
10	Defender casualties
11	Duration (in hours)
12	Nominal victor
13	Engagement type categories*

* Helmbold's engagement types are a slight modification of categories first developed by Willard. Helmbold's categories are as follows: Category I battles denote open battles in the sense that both sides could, with about equal facility, disengage and conduct an orderly withdrawal. Category II battles denote closed battles in the sense that one of the parties in the battle is encircled or otherwise in a position from which an orderly withdrawal can't readily be made, and whose options for maneuver are correspondingly markedly more restricted than those of his opponent. This categorization of the data is not available for the data sets from Helmbold's earlier studies (1961 and 1964).

2.3.6 Results

Helmbold initially tested the data using four groupings of the data. They include:

1) The set of data from Helmbold's earlier studies.

2) Willard's complete data set. This consists of all Category I and Category II engagements.

3) A subset of Willard's data set containing only all Category I engagements.

4) A subset of Willard's data set containing only all Category II engagements.

In none of these groups of the data are the associated ψ and ψ^{-1} curves mathematical inverses of each other (Helmbold, 1971:21-32).

Upon visually examining the casualty-fraction distribution curves for each of the 4 data sets, it appeared to Helmbold that the following might hold true:

$$\Delta_{xx}(u) = \Delta_{yy}(u) \quad (2.46)$$

$$\Delta_{yx}(u) = \Delta_{xy}(u) \quad (2.47)$$

(Helmbold, 1971:26)

According to Lemma I, this signifies that $\psi = \psi^{-1} = I$. However, the ψ and ψ^{-1} curves do not reflect this for any of the four data sets. The presence of relations 2.46 and 2.47 in the data, however, indicates that a force's break behavior might be significantly dependent upon whether it won or lost the engagement (Helmbold, 1971:26).

As a further test of the validity of equations 1 and 2, Helmbold used the Kolmogorov/Smirnov test for testing the equivalence of the appropriate casualty-fraction distributions for each of the four data sets. The results are shown in Table 2.6.

Table 2.6

Test Results on the Validity of Lemma I for the Specified Data Sets (Helmbold, 1971:28-30)

No.	Description	$\Delta_{xx}(u) = \Delta_{yy}(u)$	$\Delta_{yx}(u) = \Delta_{xy}(u)$
1	Data Set from Helmbold's Earlier Studies	.30*	.30
2	Willard's Complete Data Set	.10	.10
3	Category I Battles; Willard's Data Set	.05	.03
4	Category II Battles; Willard's Data Set	.99	.98

* The value indicates the probability that the deviation between the two distributions would be greater than at present, ". . . given that the empirical distribution functions actually are obtained from independent random samples from a common continuous distribution function."

With the exception of the fourth data set, the appropriate casualty-fraction distribution curves were not found by Helmbold to be equivalent for a reasonable level of confidence. The results of the visual examination are not, in general, confirmed by the statistical results (Helmbold, 1971:28-30).

Another inconsistency between the breakpoint hypothesis and Helmbold's data is the failure of the empirical casualty-fraction distribution curves to exhibit the behavior mandated by Lemma II. Helmbold developed an alternative element 3 of the hypothesis to correct for this. It consists of the following:

There is a monotonic non-decreasing function φ such that:

$$f_x = \varphi(f_y) \tag{2.48}$$

when the defender wins, while:

$$f_x = \psi^{-1}(f_y) \tag{2.49}$$

when the attacker wins.

$\psi(s) \geq s$, for all s.

This results in the following significant change in Lemma II:

When the defender wins,

$$\Delta_{yy}(s) \geq \Delta_{xy}(s) \tag{2.50}$$

for all s. When the attacker wins,

$$\Delta_{xx}(s) \geq \Delta_{yx}(s) \tag{2.51}$$

for all s. (Helmbold, 1971:33-35)

The modified Lemma II is consistent with the empirical casualty-fraction distribution curves for Helmbold's data. Despite this improvement, however, the division of the data into winner and loser groupings does not result in the ψ and ψ^{-1} functions being mathematically inverse (Helmbold, 1971:36).

On the basis of his results, Helmbold also demonstrated that the second element of the hypothesis needs to be altered to allow for specified subsets of the universe of possible types of engagements to be handled separately. For Helmbold's set of data, the division of the data into two subsets consisting of Category I and Category II, while an improvement, is not sufficient in defining subsets of data within which a break curve would be universally applicable (Helmbold, 1971:45).

Helmbold listed several properties which he believed should be contained in any theory for the battle termination process. A listing of the properties applicable to this investigation is given below:

1) The theory should have a simplicity and "naturalness" of form in consonance with the principle of "Ocknam's Razor". . . that "multiplicity ought not to be posited without necessity."

2) The theory must address the separate casualty distribution curves observed for the category I and the category II battles.

3) The theory must not produce an estimate of the ψ function relating casualty fractions via the relations:

$$f_x = \psi(f_y) \tag{2.48}$$

<u>or</u>

$$f_l = \psi(f_w) \tag{2.52}$$

that is at variance with the actual relations between these quantities.

4) The theory ought to explain why the loser's and the winner's casualty-fraction distributions are very nearly the same, independent of the attack/defense status of forces. (Helmbold, 1971:60-61)

In regards to the battle termination models in use today, Helmbold concludes that ". . . the soundness of models of combat that make essential use of breakpoint hypotheses must be considered suspect until a better theoretical understanding of the battle termination process is obtained" (Helmbold, 1971:61).

2.4 Data Issues

Other analysts in the profession (Dupuy, 1979:143; Taylor, 1980:110,111) share Helmbold's doubts about the validity of the breakpoint hypothesis currently used, and deplore the lack of empirical work. In particular it is felt that the inadequacies contained within the data currently used in combat models seriously degrade the utility of the output of force-on-force attrition models. According to Stockfish:

> . . . the unverified findings of modelling conducted by one organization can be taken as fact by another organization and used as inputs for the latter's model. Another aspect is that a number or a set of numbers constituting data can be an admixture of subtle concepts, subjective evaluations, and limited but hard evidence based on actual physical testing. The particular testing, however, may have been undertaken for purposes remote from the use that another study makes of the data. (Stockfish, 1975:vi,vii)

The inadequacies of available data is also illustrated by the incompleteness of the data utilized in both empirical validation studies.

The danger of unsuitable data is reinforced by a warning given by Dupuy on the interpretation of historical combat data. He demonstrated how seemingly contradictory statements based on the same set of historical events could each have a basis in fact (Dupuy, 1979:4-18). He concluded by stating:

> To some extent, they illustrate why it is both vain and dangerous to seek immutable lessons from the records of the past; the facts are too contradictory, too specialized, too subject to misinterpretation, to support unequivocal conclusions. Certain generalized principles can be substantiated - usually. But the specifics of combat processes and relationships are elusive. (Dupuy, 1979:18)

As an extrapolation of this, some military operations research analysts believe that ". . . the changes in weapons and technology of the past few decades have made all military history - even that as recent as World War II - irrelevant" (Dupuy, 1979:141). If this is the case, it would be incorrect to attempt to validate a battle termination technique using historical combat data since the combat processes that the data came from would be significantly different from present and future combat processes.

However, there is considerable disagreement with that suggestion. For example, Clark has successfully taken combat data from conventional ground warfare and, admittedly with considerable limitations, applied it to atomic warfare (Clark, 1954:27-28). Dupuy investigated historical trends in weapon lethality and their effect on warfare. He found that the technological change that most affected modern ground warfare, in contradiction to the assumption in the statement above, was the large-scale transition from the smoothbore infantry musket to rifled small arms that occurred between 1850 and 1860 (Dupuy, 1979:6).

The desirability of using historical combat data in investigative work is further indicated by Stockfish, who stated:

> The output or assertions of current campaign models are of questionable worth because of inadequate empirical work, which should consist of both operational testing . . . and empirical study of past wars. . . . Without increased and definitive operational testing and empirical studies, the use of detailed models to treat larger force aggregations is probably of limited value in the analysis of conventional wars. Overall, we are left with faulty concepts, such as the firepower indexes, as empirical inputs for aggregative models, and an abundance of unverified - or only partially verified - detailed models. This condition results from an imbalance between empirical and theoretical endeavor in DoD analysis and study. The image of scientific activity - an image that depicts theories and models as being independently tested by experiment or appeal to experience, with the empirical work in turn casting up new insight that contributes to theoretical advance - does not seem to prevail in the military establishment. (Stockfish, 1975:vi,vii)

In addition to its desirability, Dupuy felt historical combat data remains applicable since ". . . the principle weapon of war is . . . man himself. . . . the nature of warfare has changed only in its details (sometimes dramatically, but always relatively slowly) as man adapts himself and his thinking to new weapons and technology." (Dupuy, 1979:142). Therefore, using historical data to validate combat models is defensible.

2.5 Summary

The breakpoint hypothesis is used in many Department of Defense sponsored force-on-force attrition studies to model the battle termination process. Empirical support for these models, however, is limited.

Clark found that deterministic break curves were not validated by combat data. Individual differences in the ability of units to carry out their missions could not be explained on the basis of casualties and replacements alone. Furthermore, when using casualties as a measure of loss of combat effectiveness the type and size of the unit must be specified, different breakpoints must be specified for attacking and defending forces, and the time basis for loss percentages must be specified.

Helmbold found that probablistic break curves, although inherently more flexible, are not valid as presently used. Six groupings of the data were evaluated. In each case, the theorem and/or lemmas mandated by the breakpoint hypothesis are violated by the empirical data. Suggested improvements included a more detailed categorization of the data when constructing combat unit break curves.

The negative results of past validation studies raise considerable doubt about the validity of current battle termination models. One criticism of these results has been to question the applicability of historical data to present and future combat. On the basis of statements made by Dupuy (1979), Stockfish (1975) and Clark (1954), however, this investigation finds their use in validating combat models to be defensible.

Further empirical work in this area has been, and is, hampered by a relative scarcity of available historical data. A significant amount of historical data has been obtained for the current investigation, however. Chapter 3 contains a list of the data used and a description of its contents, in addition to documenting the experimental design for the current investigation.

Chapter 3

EXPERIMENTAL DESIGN

This investigation focuses on the identification and examination of the validity of a series of break curves using an engagement data matrix. The matrix contains 323 engagements; with 24 data elements available for each engagement. The data elements contained in the engagement data matrix represent those combat factors deemed significant in the literature for which data is available. In addition to the data required to specify an engagement's time and geographic location in history, the data elements consist mostly of environmental and operational factors significant to the combat process. The conditions under which an engagement terminates are listed and described. Other considerations include the size and unity of mission of the respective combat forces in the engagement.

The break curves are identified and their validity examined using the following methodology. Note that a more detailed discussion of the methodology is contained in section 3.7. The data elements from the engagement data matrix are transformed into a set of observed variables in order to obtain variables having an appropriate level of measurement for the techniques to be used. The observed variables are reduced to a set of defined factors using factor analysis. All possible combinations of one or more defined factors are identified with each combination representing a specific break curve. For each possible defined factor combination, a test procedure is performed which determines the degree to which Helmbold's Theorem holds for the combination's associated engagement subset. The break curves whose associated engagement subsets obey Helmbold's Theorem reasonably well are identified. The results of this procedure are then confirmed using discriminant analysis. Discriminant analysis identifies those observed variables which are significant in distinguishing between winners and losers in an engagement using the engagements contained in the engagement data matrix. The observed variables found to be significant are compared to the observed variables which specify the break curves whose associated engagement subsets reasonably well.

3.1 Categorization of the Data

The formulation of break curves is based upon those factors that are significant to the battle termination process. These factors are used in the modification of the second element of the hypothesis, thereby allowing the break curves to vary depending on the battle type under study. Helmbold (1971) first suggested this potential modification. He stated:

> The appropriate break curve could be made to depend on any condition that could be known at the time the break curve is sampled, such as whether the force is attacking or defending, its state of training, experience, morale, physical weariness, etc. (Helmbold, 1971:8)

Those combat factors identified in the literature as significant, and for which relatively accurate and precise data is available, are used in the current investigation.

The number of personnel casualties incurred by a combat force are considered to be a very influential factor. Clark (1954) gave two reasons for the frequent use of personnel casualties in determining the residual combat effectiveness of a unit. They are:

> 1) Casualties are . . . a factor always present in battle, and their magnitude may be assumed to reflect to some degree the magnitude of other less tangible factors.
>
> 2) . . . of all possibly significant factors affecting a unit's combat effectiveness, casualties alone can be directly quantified. (Clark, 1954:7)

Both Clark (1954) and Helmbold (1971), however, found that personnel casualties, while necessary, are not sufficient in explaining combat unit break behavior.

Other factors significant to the combat process are identified in the literature. Clark (1954) offered the list of combat factors given in Table 3.1.

Table 3.1

Additional Combat Factors
(Clark, 1954:2)

Factor Number	Description
1	casualty rate
2	level of training and battle experience of the troops
3	the influence of inclement weather or other unusual environmental stress
4	the importance of the mission
5	troop morale
6	quality of leadership
7	knowledge and intelligence on the enemy's intentions
8	perceived vigor of enemy opposition
9	scale of friendly fire support and troop reinforcement
10	logistical supply situation
11	command, control and communications

More recently, the Historical Evaluation and Research Organization (HERO) compiled a more comprehensive, but nonexhaustive, list of factors which it determined have a significant effect on combat (Dupuy, 1979:33). They are included in Figure 3.1. Figure 3.1 also includes a listing of the data elements contained in the engagement data matrix for the current investigation. These data elements are described in greater detail in section 3.4. A more detailed listing of HERO's factors is provided in Tables 3.2 A through K.

Universe of Possibly Significant Combat Factors

➤

<u>Set of Significant Combat Factors Identified by HERO</u>

1. Weapons Effects
2. Terrain Factors
3. Weather Factors
4. Season Factors
5. Air Superiority Factors
6. Posture Factors
7. Mobility Effects
8. Vulnerability Factors
9. Tactical Air Effects
10. Other Combat Processes
11. Intangible Factors

➤

<u>Data Elements Contained in Engagement Data Matrix</u>

1) War
2) Campaign
3) Battle
4) Engagement
5) Starting Date
6) Duration (in days)
7) Attacker
8) Defender
9) Victor
10) Attacker Initial Strength
11) Defender Initial Strength
12) Attacker Casualties
13) Defender Casualties
14) Willard's Category
15) Engagement Type
16) Terrain
17) Weather
18) Season
19) Morale
20) Presence of Surprise
21) Presence of Setpiece Effect
22) Overall Air Superiority
23) Close Air Support for Attacker
24) Close Air Support for Defender

Figure 3.1

Derivation of Data Elements Contained in the Engagement Data Matrix

Table 3.2A

Weapons Effects

Variable Number	Description
1	Rate of fire
2	Potential targets per strike
3	Relative incapacitating effect
4	Effective range (or muzzle velocity)
5	Accuracy
6	Reliability
7	Battlefield mobility
8	Radius of action
9	Punishment (vulnerability) factor
10-13	Armor performance factors (4)*
14	Helicopter
15-21	Special weapons effects factors (7+)**
22	Dispersion factor

* The armor performance factors consist of the rapidity of fire effect, the fire control effect, the ammunition supply effect and the wheel/halftracks effect.

** HERO accounts for seven types of special weapons effects. They consist of factors for self-propelled artillery, missile guidance, multi-barreled weapons, multiple charge artillery weapons, armored personnel carriers, fixed-wing aircraft and vehicles with amphibious capabilities. HERO indicates that this set is not exhaustive.

Table 3.2B

Terrain Factors

Variable Number	Description
1	Mobility effect
2	Defense posture effect
3	Infantry weapons effect
4	Artillery weapons effect
5	Air effectiveness effect

Table 3.2C

Weather Factors

Variable Number	Description
1	Mobility effect
2	Attack posture effect
3	Artillery effect
4	Air effectiveness effect
5	Tank effect

Table 3.2D

Season Factors

Variable Number	Description
1	Attack posture effect
2	Artillery effect
3	Air effectiveness effect

Table 3.2E

Air Superiority Factors

Variable Number	Description
1	Mobility effect
2	Artillery effect
3	Air effectiveness effect
4	Vulnerability effect

Table 3.2F

Posture Factors

Variable Number	Description
1	Force strength effect
2	Vulnerability effect

Table 3.2G

Mobility Effects

Variable Number	Description
1	Characteristics of mobility
2	Environmental effect

Table 3.2H

Vulnerability Factors

Variable Number	Description
1	Exposure consideration, general
2	Environmental effects, general
3	Across beach
4	Across major unfordable river
5	Across major fordable or minor unfordable river

Table 3.2I

Tactical Air Effects

Variable Number	Description
1	Close air support damage and casualties
2	Close air support morale effect*
3	Interdiction of logistical movement**
4	Interdiction delays on ground movement*
5	Interdiction damage and casualties
6	Interdiction disruption effect*

* Intangible; probably individually incalculable
** Probably calculable; not yet calculated

Table 3.2J

Other Combat Processes

Variable Number	Description
1	Mobility effects of surprise
2	Surpriser's vulnerability effect
3	Surprised's vulnerability effect
4	Other surprise effects*
5	Degradation effects of fatigue and casualties*
6	Casualty-inflicting capability factor
7	Disruption*

* Probably calculable; not yet calculated

Table 3.2K

Intangible Factors

Variable Number	Description
1	Combat effectiveness*
2	Leadership**
3	Training/experience**
4	Morale***
5	Logistics*
6	Time***
7	Space***
8	Momentum***
9	Intelligence
10	Technology***
11	Initiative**

* Sometimes calculable
** Probably calculable; not yet calculated
*** Intangible; probably individually incalculable

The HERO factor set represents the nominal effects on combat of the weaponry used and the environmental and operational factors that affect their employment. The majority of the factors are considered quantifiable at a sufficient level of detail and established validity.

Although HERO's listing is the basis for the factors tested in this investigation, only a limited number of combat factors are represented by the data elements included in the engagement data matrix. In large measure, this is due to the limited amount of verified historical combat data available. The significance of the limitation is seen in a statement by Taylor who felt that ". . . the nature and quality of the available combat data is so extremely poor that (there is) no reliable benchmark against which to "calibrate" our combat models" (Taylor, 1980:110). While a substantial amount of compiled combat data is available for this investigation, it is small when compared to the amount of historical combat data that exists in raw form in the open literature in the United States.

3.2 Data Sources

The data for this investigation consists of 323 engagements, with 24 data elements representing each engagement. A listing of the complete engagement data matrix is given in section 3.3.

Roughly one-half of the engagements are obtained from an historical combat data base compiled by HERO (Dupuy, 1976-1978, 1979). It is the source of all the engagements from World War II, the Korean conflict and the 1948, 1956, 1967 and 1973 Arab/Israeli Wars. It contains sufficient data for each engagement to specify the data elements used in the current investigation. The availability of this data is significant in the determination of the data elements to be included in the engagement data matrix.

Additional compiled data sources of varying degrees of utility are available. They consist of the engagement data base which Helmbold (1961, 1964, 1971) used in his investigations, Clark's (1954) engagement data base and the engagement data base compiled by Willard (1962) from Bodart (1905).

Helmbold's data base consists of 173 engagements contained in two papers published by Helmbold in 1961 and 1964. The engagement data matrix consists of the elements contained in Table 2.5. The primary limitation is the lack of many of the data elements used in the current investigation. Where possible, an engagement's data is augmented from additional sources and included in the current investigation's data matrix. Further details will be given in section 3.3.

Clark's investigation (1954) also provides an historical combat data base. A listing of the component data elements is provided in Table 3.1. Its limitations are severe. It does not include many of the data elements in the current investigation. Data is available for only one side in each engagement. In addition, the independence of some of the engagements is in doubt. Due to these limitations, none of the engagements are included in the current engagement data matrix.

Another compiled data source is that extracted by Willard (1962) from Bodart's Kriegs-Lexicon (1905). As in the Clark data, the data for each engagement does not include many of the data elements for the engagement data matrix used in the current investigation. Extracting additional data from the Kriegs-Lexicon is not possible for the investigator since it has never been translated into English. In addition, HERO feels that ". . . the Bodart data was not necessarily reliable or verifiable from other sources." (Dupuy, 1979:149). For these reasons, the data set is not used in the investigation.

In addition to the HERO and Helmbold data sources, combat data was compiled by the investigator from historical sources for this investigation. Where applicable, classification of the data is based upon guidelines developed by HERO (Dupuy, 1979). These guidelines, together with the meaning and limitations for each data element in the engagement data matrix shown in section 3.3, are discussed in section 3.4.

3.3 Listing of Engagement Data Matrix

This section contains a listing of the data used in the current investigation. The engagement data matrix is followed by documentation of the data encoding scheme, footnotes pertaining to specific data entries and a listing, by war, of the sources used. The utility of each of the data elements with respect to the current investigation is discussed in section 3.4.

Table 3.3

Engagement Data Matrix

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
1.1	Boston	Bunker Hill	Bunker Hill	17 Jun 1775	1	British	American	British	2400	1500	1050	440/A	I	3	6	2	Spring	A-Ex D-Gd	No	No	N/A	N/A	N/A
1.2	Canadian	Quebec	Quebec	31 Dec 1775	1	American	British	British	800	1800	490	18	I	4	14	12	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
1.3	New York	Long Island	Long Island/B	27 Aug 1776	1	British/ Hessian	American	British/ Hessian	20000	3500	390	1500/C	I	2	5	2	Summer	A-Gd D-Gd	Sub-stan-tial (A)	No	N/A	N/A	N/A
1.4	New York	Harlem Heights	Harlem Heights	16 Sept 1776	1	American	British/ Hessian	American	5000	5000	120	270	I	1	2	2	Summer	A-Gd D-Gd	Subs. (A)	No	N/A	N/A	N/A
1.5	New York	White Plains	Chatterton's Hill	28 Oct 1776	1	British/ Hessian	American	British/ Hessian	4000	1600	240	250	I	2	5	2	Fall	A-Gd D-Fr	No	No	N/A	N/A	N/A
1.6	New York	Ft. Washington	Ft. Washington	16 Nov 1776	1	British/ Hessian	American	British/ Hessian	8000	3000	460	150/D	II	3	1	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
1.7	Delaware	Trenton	Trenton	26 Dec 1776	1	American	Hessian	American	2400	1400	4	110/E	II	2	14	12	Winter	A-Gd D-Gd	Major (A)	No	N/A	N/A	N/A
1.8	Delaware	Princeton	Princeton/F	3 Jan 1777	1	American	British	American	5200	1200	40	100/G	I	2	6	3	Winter	A-Gd D-Ex	Minor (A)	No	N/A	N/A	N/A
1.9	Saratoga	Hubbardton	Hubbardton	7 Jul 1777	1	British	American	British	850	700	150	96/H	I	2	4	2	Summer	A-Ex D-Gd	Subs. (A)	No	N/A	N/A	N/A
1.10	Saratoga	Bennington	Bennington/I	16 Aug 1777	1	American	Hessian/ British	American	1500	2500	60	610	II	2	13	2	Summer	A-Gd D-Gd	Minor (A)	Yes	N/A	N/A	N/A
1.11	Philadel-phia	Brandywine	Brandywine/J	11 Sep 1777	1	British/ Hessian	American	British/ Hessian	13000	12000	580	1350	I	2-3	5	2	Summer	A-Gd D-Gd	Minor (A)	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
1.12	Philadel- phia	Paoli	Paoli	19 Sep 1777	1	British	American	British	700/K	1500	7	150	I	2	7	5	Summer	A-Ex D-Ex	Major (A)	No	N/A	N/A	N/A
1.13	Philadel- phia	Germantown	Germantown/L	4 Oct 1777	1	American	British	British	11000	8000	1070	530	I	2	8	8	Fall	A-Gd D-Gd	Subs. (A)	No	N/A	N/A	N/A
1.14	Philadel- phia	Fort Mercer	Fort Mercer	22 Oct 1777	1	Hessian	American	American	2000	400	400	10/M	II	4	9	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
1.15	Saratoga	Freeman's Farm	Freeman's Farm	19 Sep 1777	1	American	British/ Hessian	British/ Hessian	3000	3250	320	600	I	2	4	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
1.16	Saratoga	Bemis Heights	Bemis Heights/L	7 Oct 1777	1	American	British/ Hessian	American	5000	2500/N	150	600	I	2-4	4	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
1.17	Monmouth	Monmouth Courthouse	Monmouth Courthouse/L	28 Jun 1778	1	British/ Hessian	American	American	13000	14000	720/O	720/O	I	2	13	1	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
1.18	Southern	1st Battle of Savannah	1st Battle of Savannah	29 Dec 1778	1	British	American	British	3500	1000	20	250	II	3	8	2	Winter	A-Gd D-Gd	Subs. (A)	No	N/A	N/A	N/A
1.19	Southern	Kettle Creek	Kettle Creek	14 Feb 1779	1	American	Tory	American	300	700	30	120	I	2	6	2	Winter	A-Gd D-Gd	Major (A)	No	N/A	N/A	N/A
1.20	Southern	2nd Battle of Savannah	2nd Battle of Savannah	9 Oct 1779	1	American/ French	British	British	4550	3300	830	170	II	4	13	2	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
1.21	Hudson Highlands	Stony Point	Stony Point	16 Jul 1779	1	American	British	American	1300	700	100	190/P	II	4	13	5	Summer	A-Ex D-Gd	Major (A)	No	N/A	N/A	N/A
1.22	Observation of New York	Paulus Hook	Paulus Hook	Aug 1779	1	American	British/ Hessian	American	350	260	5	210	II	4	13	2	Summer	A-Gd D-Gd	Major (A)	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
1.23	Southern	Mount Pleasant	Mount Pleasant	2 Feb 1780	1	British/ Hessian/ Tory	American	British/ Hessian/ Tory	450	450	20	50/Q	I	3	5	2	Winter	A-Gd D-Gd	Major (A)	No	N/A	N/A	N/A
1.24	Southern	Ramson's Mill	Ramson's Mill	20 Jun 1780	1	American	Tory	American	1200	1300	150	150	I	2	5	2	Spring	A-Gd D-Gd	Major (A)	No	N/A	N/A	N/A
1.25	Carolinas (Gates)	Camden	Camden	16 Aug 1780	1	British	American	British	2300	3050	320 /R	1100 /S	I	2	9	2	Summer	A-Ex D-Gd	No	No	N/A	N/A	N/A
1.26	Carolinas	Fishing Creek	Fishing Creek	18 Aug 1780	1	Tory	American	Tory	160	700	20	150/T	I	2	5	2	Summer	A-Ex D-Gd	Major (A)	No	N/A	N/A	N/A
1.27	Carolinas (Greene)	King's Mountain	King's Mountain	7 Oct 1780	1	American	Tory	American	900	1020	90	320/U	II	2	2	8	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
1.28	Carolinas (Greene)	Cowpens	Cowpens	17 Jan 1781	1	British	American	American	1120	1100	340/V	70	I	2	6	2-3	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
1.29	Carolinas (Greene)	Guilford Courthouse	Guilford Courthouse	15 Mar 1781	1	British/ Hessian	American	British/ Hessian	2000	4300	530	260/W	I	2	5	2	Winter	A-Ex D-Gd	No	No	N/A	N/A	N/A
1.30	Carolinas (Greene)	Hobkirk's Hill	Hobkirk's Hill	25 Apr 1781	1	British	American	British (Marginal)	900	1400	260	270	I	2	6	2	Spring	A-Gd D-Fr	No	No	N/A	N/A	N/A
1.31	Carolinas (Greene)	Seige of Point Ninety-Six	Assault on June 18th	18 Jun 1781	1	American	Tory	Tory	1000	550	150	90	II	4	8	1	Spring	A-Gd D-Ex	No	No	N/A	N/A	N/A
1.32	Carolinas (Greene)	Eutaw Springs	Eutaw Springs	8 Sep 1781	1	American	British	British (Marginal)	2450	2000	520	870	I	2	8	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
1.33	Yorktown	Yorktown	Yorktown/L	30 Sep 1781	20	American/ French	British/ Hessian	American/ French	18000	7800	370	550/X	II	4	13	8	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
2.1	Italy (1796–97)	Lodi	Lodi	10 May 1796	1	French	Austrian	French	25200	10000	350	450/O	I	2	5	2	Spring	A-Ex D-Fr	No	No	N/A	N/A	N/A
2.2	Italy (1796–97)	Castig-lione	Castig-lione	5 Aug 1796	1	French	Austrian	French	28750	25000	1500	2000 /Y	I	2	5	2	Summer	A-Gd D-Gd	Subs. (A)	No	N/A	N/A	N/A
2.3	Italy (1796–97)	Arcola	Arcola	15 Nov 1796	3	French	Austrian	French	20000	17100	4500	7000	I	2	13	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.4	Italy (1796–97)	Rivoli	Rivoli	14 Jan 1797	2	French	Austrian	French	20400	28000	5000	3000 /Z	I	2	2	2	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.5	Egyptian	Aboukir	Aboukir	25 Jul 1799	1	French	Egyptian	French	10000	15000	970	2000 /AA	II	3	10	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.6	Italian (1799–1800)	Marengo	Marengo	14 Jun 1800	1	Austrian	French	French	31000	28000	14000	7000	I	2	5	2	Summer	A-Gd D-Gd	Minor (A)	No	N/A	N/A	N/A
2.7	Third Coalition	Durren-stein	Durren-stein	11 Nov 1805	1	Russian	French	French	40000	5000	4000	3000	II	2	5	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.8	Third Coalition	Austerlitz	Austerlitz	2 Dec 1805	1	French	Austrian/ Russian	French	75000	89000	8820	15000 /BB	I	2	5	8	Fall	A-Gd D-Gd	Subs. (A)	No	N/A	N/A	N/A
2.9	Prussian (1806–7)	Saalfeld	Saalfeld	10 Oct 1806	1	French	Prussian	French	14000	8300	172	900/CC	I	2	4	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.10	Prussian (1806–7)	Jena/ Auerstadt	Jena	14 Oct 1806	1	French	Prussian	French	96000	53000	5000	10000 /DD	I	2	1	2	Fall	A-Ex D-Pr	Minor (A)	No	N/A	N/A	N/A
2.11	Prussian (1806–7)	Jena/ Auerstadt	Auerstadt	14 Oct 1806	1	Prussian	French	Prussian	63500	27000	13000	7000	I	2	4	8	Fall	A-Fr D-Ex	Minor (A)	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
2.12	Prussian (1806-7)	Eylau	Eylau	8 Feb 1807	1	French	Russian	French	75000	76000	25000	15000	I	2	5	12	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.13	Prussian (1806-7)	Heilsburg	Heilsburg	10 Jun 1807	1	French	Russian	Russian	50000	68000	10570	8000	I	3	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.14	Prussian (1806-7)	Friedland	Friedland	14 Jun 1807	1	French	Russian	French	80000	60000	8000	20000 /EE	I	2	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.15	Peninsular (1808)	Pancorbo	Pancorbo	31 Oct 1808	1	French	Spanish	French	21000	19000	200	600	I	2	6	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.16	Peninsular (1808)	Corunna	Corunna	16 Jan 1809	1	French	British	British	19500	15200	1500	800	II	2	5	3	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.17	Danube (1809)	Eckmuhl	Eckmuhl	22 Apr (1809)	1	French	Austrian	French	60000	35000	6000	12000 /FF	I	2	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.18	Danube (1809)	Asper-nessling	Aspern-nessling	21 May 1809	2	Austrian	French	Austrian	95800	70000	23400	21000	I	2	8	2	Spring	A-Gd D-Gd	Minor (A)	No	N/A	N/A	N/A
2.19	Danube (1809)	Wagram	Wagram	5 Jul 1809	2	French	Austrian	French	170500	146600	39500	40000	I	2	5	1	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.20	Russian	Borodino	Borodino	7 Sep 1812	1	French	Russian	French	133000	120000	30000	44000	I	2	2	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.21	Germany (1813)	Lutzen	Lutzen	2 May 1813	1	Prussian	French	French	73000	110000	18000	20000	I	2	4	2	Spring	A-Gd D-Gd	No /GG	No	N/A	N/A	N/A
2.22	Germany (1813)	Bautzen	Bautzen	20 May 1813	2	French	Prussian/ Russian	French	200000	96000	20000	20000	I	3	2	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
2.23	Germany (1813)	Dresden	Dresden	26 Aug 1813	2	French	Allied	French	120000	170000	10000	38000	I	2	14	11	Summer	A-Gd D-Gd	Subs. (A)	No	N/A	N/A	N/A
2.24	Germany (1813)	Kulm	Kulm/HH	30 Aug 1813	1	Allied	French	Allied	54000	32000	11000	16000 /II	II	2	5	2	Summer	A-Fr D-Ex	Subs. (A)	No	N/A	N/A	N/A
2.25	Germany (1813)	Dennewitz	Dennewitz	6 Sep 1813	1	French	Swedish	Swedish	55000	80000	10000	7000	I	3	5	2	Summer	A-Gd D-Gd	Subs. (D)	No	N/A	N/A	N/A
2.26	Germany (1813)	Leipsig	Leipsig (Part I)	16 Oct 1813	1	Allies	French	French	257000	177000	30000	25000	I	2	13	2	Fall	A-Gd D-Gd	Minor (A)	No	N/A	N/A	N/A
2.27	Germany (1813)	Leipsig	Leipsig (Part II)	18 Oct 1813	2	Allies	French	Allies	365000	195000	24000	13000	I	5	14	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.28	France (1814)	Brienne	Brienne	29 Jan 1814	1	French	Allied	French	43000	28000	3000	4000	I	2	8	3	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.29	France (1814)	La Rothiere	La Rothiere	1 Feb 1814	1	Allied	French	Allied	110000	40000	6000	6000	I	2	8	12	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.30	France (1814)	Champ-aubert	Champ-aubert	10 Feb 1814	1	French	Allied	French	24000	5000	200	4000 /JJ	I	2	8	11	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.31	France (1814)	Montmirail	Montmirail	11 Feb 1814	1	Allied	French	French	21000	20000	4000 /KK	2000	I	2	8	11	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.32	France (1814)	Vauchamps	Vauchamps	14 Feb 1814	1	French	Prussian	French	25000	22000	600	7000	I	2	8	11	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.33	France (1814)	Montereau	Montereau	18 Feb 1814	1	French	Allied	French	45000	40000	2500	6000	I	3	5	11	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
2.34	France (1814)	Craonne	Craonne	7 Mar 1814	1	French	Allied	French (Marginal)	37000	30000	5500	5000	I	2	5	11	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.35	France (1814)	Laon	Laon	9 Mar 1814	2	Allied	French	Allied	85000	47600	4000	6000	I	2	5	11	Winter	A-Pr D-Gd	Minor (A)	No	N/A	N/A	N/A
2.36	France (1814)	Arcis-sur-Aube	Arcis-sur-Aube	20 Mar 1814	2	Allied	French	Allied	80000	28000	4000	3000	I	5	5	11	Winter	A-Fr D-Gd	No	No	N/A	N/A	N/A
2.37	100 Days	Ligny	Ligny	16 Jun 1815	1	French	Prussian	French	80000	84000	11000	25000	I	2	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.38	100 Days	Quatre Bras	Quatre Bras	16 Jun 1815	1	French (Nominal)	British/ Dutch	French (Marginal)	24000	36000	4000	4800	I	1	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.39	100 Days	Waterloo	Waterloo	18 Jun 1815	1	French	Allies	Allies	72000	140000 /LL	41000	22000	I	2	5	8	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
2.40	100 Days	Warve	Warve	18 Jun 1815	1	French	Prussian	Prussian/MM	33000	17000	2500	2500	I	5	5	8	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.1	Bull Run	Bull Run	Bull Run	21 Jul 1861	1	Union	Confed-erate	Confed-erate	32000	35000	2710/II	1980	I	2	5	2	Summer	A-Gd D-Ex	No	No	N/A	N/A	N/A
3.2	Missouri (1861)	Wilson Creek	Wilson Creek/NN	10 Aug 1861	1	Union	Confed-erate	Confed-erate	5400	12480	1240	1190	I	2	1	2	Summer	A-Gd D-Gd	Subs. (A)	No	N/A	N/A	N/A
3.3	Tennessee	Ft. Donelson	Ft. Donelson/OO	12 Feb 1862	5	Union	Confed-erate	Union	27000	21000	2830	2000 /PP	II	4	13	2	Winter	A-Gd D-Fr	No	No	N/A	N/A	N/A
3.4	Missouri (1862)	Pea Ridge Arkansas	Pea Ridge Arkansas/QQ	7 Mar 1862	2	Confed-erate	Union	Union	16210	12100	800	1380	I	2	1	2-3	Winter	A-Gd D-Gd	Minor (A)	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
3.5	Valley Campaign (Jackson)	Kernstown	Kernstown	23 Mar 1862	1	Confederate	Union	Union	4200	9000	700	550	I	2	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.6	Valley Campaign (Jackson)	McDowell	McDowell	8 May 1862	1	Union	Confederate	Confederate	4000	9000	250	500	I	2	5	2	Spring	A-Gd D-Gd	Subs. (A)	No	N/A	N/A	N/A
3.7	Valley Campaign (Jackson)	Winchester	Winchester	25 May 1862	1	Confederate	Union	Confederate	16000	7000	350	1800	I	2	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.8	Valley Campaign (Jackson)	Cross Keys/ Port Republic	Cross Keys	8 Jun 1862	1	Union	Confederate	Confederate	15000	6500	650	300	I	2	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.9	Valley Campaign (Jackson)	Cross Keys/ Port Republic	Port Republic	9 Jun 1862	1	Confederate	Union	Confederate	6000	5000	800	1000	I	2	5	8	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.10	Valley Campaign (Jackson)	Front Royal (I)	Front Royal (I)	23 May 1862	1	Confederate	Union	Confederate	16000	1000	50	800/RR	II	2	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.11	Valley Campaign (Jackson)	Front Royal (II)	Front Royal (II)	30 May 1862	1	Union	Confederate	Union	500	300	14	200	I	2	5	2	Spring	A-Gd D-Gd	Minor (A)	No	N/A	N/A	N/A
3.12	Penninsula	Williamsburg	Williamsburg	4 May 1862	2	Union	Confederate	Union	43840	34090	2240	1700	I	2	8	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.13	Penninsula	Fair Oaks	Fair Oaks	31 May 1862	2	Confederate	Union	Confederate (Marginal)	44960	44940	6130	5030	I	2	8	2	Spring	A-Gd D-Gd	Minor (A)	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
3.14	Penninsula	Seven Day's	Mechanics-ville	26 Jun 1862	1	Confed-erate	Union	Confed-erate	17590	16810	1480	360	I	2	5	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.15	Penninsula	Seven Day's	Gaine's Mill	27 Jun 1862	1	Confed-erate	Union	Confed-erate	36790	60410	8750	4040	I	2	8	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.16	Penninsula	Seven Day's	Peach Orchard through Malvern Hill/SS	29 Jun 1862	3	Confed-erate	Union	Confed-erate	92520	105860	9480	8040	I	2	5	2	Summer	A-Gd D-Fr	No	No	N/A	N/A	N/A
3.17	Second Bull Run	Cedar Mountain	Cedar Mountain	9 Aug 1862	1	Union	Confed-erate	Confed-erate	8750	18260	2350	1140	I	2	5	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.18	Second Bull Run	Second Bull Run	Manasas & Chantilly/TT	29 Aug 1862	5	Confed-erate	Union	Confed-erate	81390	54330	16050	9200	I	2	2	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.19	Antietam	South Mountain	South Mountain/UU	14 Sep 1862	1	Union	Confed-erate	Union	30490	19570	1810	2690	I	2	2	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.20	Antietam	Antietam	Antietam/L	17 Sep 1862	1	Union	Confed-erate	Confed-erate	87160	55220	13720	12410	I	2	5	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.21	Confederate Kentucky Offensive	Richmond, Kentucky	Richmond, Kentucky	30 Aug 1862	1	Confed-erate	Union	Confed-erate	7450	6990	450	1050 /VV	II	2	5	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.22	Conferate Kentucky Offensive	Perryville	Perryville	8 Oct 1862	1	Union	Confed-erate	Confed-erate	39720	17200	4210	3400	I	2	5	1	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.23	Iuka/ Corinth	Iuka	Iuka	19 Sep 1862	1	Union	Confed-erate	Confed-erate	13000	17000	790	540	I	2	4	2	Summer	A-Gd D-Gd	Minor (A)	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
3.24	Iuka/ Corinth	Corinth	Corinth	3 Oct 1862	1	Confed-erate	Union	Union	23660	23080	4230	2520	I	2-3	5	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.25	Vicksburg	Chickasaw Bluffs	Chickasaw Bluffs	27 Dec 1862	3	Union	Confed-erate	Confed-erate	40000	14000	1780	210	I	4	2	2	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.26	Arkansas	Prairie Grove	Prairie Grove	7 Dec 1862	1	Confed-erate	Union	Union	11830	13180	1320	1250	I	2	6	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.27	Freder-icksburg	Freder-icksburg	Freder-icksburg/WW	13 Dec 1862	1	Union	Confed-erate	Confed-erate	113990	78510	12650	5310	I	4	2	6	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.28	Murfrees-boro	Murfrees-boro	Murfrees-boro/L	31 Dec 1862	2	Confed-erate	Union	Union	37710	44800	11740	12910	I	2	4	3	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.29	Vicksburg	Arkansas Post	Arkansas Post	11 Jan 1863	1	Union	Confed-erate	Union	31120	4910	1060	110/XX	II	4	13	3	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.30	Chancel-lorsville	Chancel-lorsville	Chancel-lorsville/YY	2 May 1863	3	Confed-erate/ZZ	Union	Confed-erate	60840	104890	12760	16850	I	2	5	2	Spring	A-Ex D-Gd	Major (A)	No	N/A	N/A	N/A
3.31	Vicksburg	Champion's Hill	Champion's Hill	16 May 1863	1	Union	Confed-erate	Union	31700	21510	2440	4360	I	2	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.32	Vicksburg	Seige of Vicksburg	Assault on May 22	22 May 1863	1	Union	Confed-erate	Confed-erate	47170	23800	3200	130	II	4	2	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.33	Vicksburg	Port Hudson	Assault on May 27	27 May 1863	1	Union	Confed-erate	Confed-erate	13980	4790	2000	240	II	4	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.34	Vicksburg	Port Hudson	Assault on June 14	14 Jun 1863	1	Union	Confed-erate	Confed-erate	6000	3750	1790	50	II	4	5	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
3.35	Gettysburg	Gettysburg	Gettysburg /AAA	1 Jul 1863	3	Confed-erate	Union	Union	81840	90620	23050	28060	I	2	5	1	Summer	A-Ex D-Gd	No	No	N/A	N/A	N/A
3.36	Coastal Blockade (1863)	Fort Wagner	Fort Wagner	18 Jul 1863	1	Union	Confed-erate	Confed-erate	5680	1920	1520	170	II	4	8	7	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.37	Chicka-maugua	Chicka-maugua	Chicka-maugua/L	19 Sep 1863	2	Confed-erate	Union	Confed-erate	72760	63420	18470	16170	I	2	4	2	Summer	A-Ex D-Gd	No	No	N/A	N/A	N/A
3.38	Chatta-nooga	Chatta-nooga	Chatta-nooga/BBB	25 Nov 1863	2	Union	Confed-erate	Union	60600	50100	5820	6670	I	4	2	2	Fall	A-Ex D-Gd	No	No	N/A	N/A	N/A
3.39	Red River	Sabine Crossing	Pleasant Hill	9 Apr 1864	1	Confed-erate	Union	Confed-erate	15450	13730	1500	1370	I	2	4	2	Spring	A-Gd D-Gd	Minor (A)	No	N/A	N/A	N/A
3.40	Mine Run	Mine Run	Mine Run	27 Nov 1863	5	Union	Confed-erate	Confed-erate	75760	40750	1190	1650	I	2	4	2	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.41	Coastal Blockade (1864)	Olustee, Florida	Olustee, Florida	20 Feb 1864	1	Union	Confed-erate	Confed-erate	5500	5590	1860	930	I	4	7	2	Winter	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.42	Wilderness/ Spotsyl-vania/Cold Harbor	Spotsyl-vania	Drewry's Bluff	12 May 1864	5	Union	Confed-erate	Confed-erate	16990	19380	4160	3070	I	2	1	8	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.43	Wilderness/ Spotsyl-vania/Cold Harbor	Cold Harbor	Cold Harbor	3 Jun 1864	1	Union	Confed-erate	Confed-erate	50000	34000	6500	1500	I	3	6	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
3.44	Peters-burg (1)	Weldon Railroad	Globe Tavern	18 Aug 1864	4	Confed-erate	Union	Union	15720	21520	1600	4500	I	2	4	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.45	Valley (1864)	Cedar Creek	Cedar Creek/L	19 Oct 1864	1	Confed-erate	Union	Union	19970	34200	2910	5670	I	2	5	5	Fall	A-Gd D-Gd	Major (A)	Yes	N/A	N/A	N/A
3.46	Missis-sippi Area Operations	Brice's Cross Roads	Brice's Cross Roads	10 Jun 1864	1	Confed-erate	Union	Confed-erate	3,000	5400	500	620 /CCC	I	2	5	2	Spring	A-Ex D-Gd	No	No	N/A	N/A	N/A
3.47	Missis-sippi Area Operations	Tupelo	Confed-erate Attack	14 Jul 1864	1	Confed-erate	Union	Union	7100	14000	1300	670	I	2	14	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.48	Franklin-Nashville	Franklin	Franklin	30 Nov 1864	1	Confed-erate	Union	Union	28550	29560	6250	2330	I	4	5	3	Fall	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.49	Franklin-Nashville	Nashville	Nashville	15 Dec 1864	2	Union	Confed-erate	Union	54700	24950	3060	4460 /II	I	3	5	11	Fall	A-Gd D-Pr	No	No	N/A	N/A	N/A
3.50	Atlanta	Buzzard's Roost to New Hope Church	Buzzard's Roost to New Hope Church/DDD	25 May 1864	4	Union	Confed-erate	Confed-erate	117530	72190	11770	9190	I	3	4	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.51	Atlanta	Kenesaw Mountain	Kenesaw Mountain	27 Jun 1864	1	Union	Confed-erate	Confed-erate	17290	19040	2050	440	I	3	2	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.52	Atlanta	Atlanta	Peachtree Creek	20 Jul 1864	1	Confed-erate	Union	Union	20250	21660	2500	1600	I	2	4	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.53	Atlanta	Atlanta	Atlanta/L	22 Jul 1864	1	Confed-erate	Union	Union	40440	36810	8000	3720	I	2	4	2	Summer	A-Gd D-Gd	Minor (A)	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
3.54	Atlanta	Atlanta	Ezra Church	28 Jul 1864	1	Confed-erate	Union	Union	19520	14170	4300	630	I	2	4	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.55	Atlanta	Jonesboro	Jonesboro (1st)	31 Aug 1864	1	Confed-erate	Union	Union	25600	15240	1730	180	I	2	5	2	Summer	A-Fr D-Gd	No	No	N/A	N/A	N/A
3.56	Valley Campaign (1864)	Opequon Creek	Opequon Creek	19 Sep 1864	1	Union	Confed-erate	Union	41300	18910	5020	3920	I	2	5	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
3.57	Valley Campaign (1864)	Monocacy	Monocacy	9 Jul 1864	1	Confed-erate	Union	Confed-erate	8000	8750	700	1800	I	5	5	2	Summer	A-Fr D-Gd	No	No	N/A	N/A	N/A
3.58	Wilmington	Ft. Fisher	Ft. Fisher	15 Jan 1865	1	Union	Confed-erate	Union	10000	2500	1340	500	II	4	11	3	Winter	A-Ex D-Ex	No	No	N/A	N/A	N/A
3.59	Carolinas	Benton-ville	Benton-ville/L	19 Mar 1865	1	Confed-erate	Union	Union	18170	18590	2120	1100	I	2	4	2	Winter	A-Gd D-Gd	Subs. (A)	No	N/A	N/A	N/A
4.1	Natal	Eland-slaagte	Eland-slaagte	21 Oct 1899	1	British	Boer	British	3150	650	280	210 /EFE	I	2	6	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
4.2	Natal	Colenso	Colenso	15 Dec 1899	1	British	Boer	Boer	14100	5500	1120	30	I	3	3	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
4.3	Cape Colony	Modder River	Belmont	23 Nov 1899	1	British	Boer	British	7500	2000	370	100	I	5	6	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
4.4	Cape Colony	Magers-fontein	Magers-fontein/L	10 Dec 1899	2	British	Boer	Boer	11300	8500	970	250	I	3	6	1	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
4.5	Cape Colony	Paarde-berg	Assault on the 18th/FFF	18 Feb 1900	1	British	Boer	Boer	17300	4100	1200	300	II	2	6	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
5.1	Yalu	Yalu River Crossing	Yalu River Crossing	30 Apr 1904	2	Japanese	Russian	Japanese	40000	6100	1100	2500	I	5	2	2	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
5.2	Central Manchuria	Telissu	Telissu	14 Jun 1904	2	Japanese	Russian	Japanese	35000	29500	1160	3480	I	3	3	8	Spring	A-Gd D-Gd	No	No	N/A	N/A	N/A
5.3	Port Arthur	Nanshan	Nanshan	25 May 1904	1	Japanese	Russian	Japanese	30000	3000	4500	1500	I	4	3	2	Spring	A-Gd A-Gd	No	No	N/A	N/A	N/A
5.4	Central Manchuria	Liaoyang	Liaoyang/L	25 Aug 1904	10	Russian	Japanese	Japanese	158000	125000	19000	23000	I	2	3	2	Summer	A-Gd D-Gd	No	No	N/A	N/A	N/A
6.1	Invasion of Italy	Salerno	Port of Salerno	9 Sep 1943	3	British; 46th INF DIV	German; 16th PZ DIV	German	12920	4250	1360	120	I	3	5	2	Summer	A-Gd D-Gd	Minor (A)	No	(A)	No	Yes
6.2	Invasion of Italy	Salerno	Amphitheater	9 Sep 1943	3	British; 56th INF DIV	German; 16th PZ DIV	German	12920	4250	1040	100	I	3	5	2	Summer	A-Gd D-Gd	Minor (A)	No	(A)	No	Yes
6.3	Invasion of Italy	Salerno	Sele-Calore Corridor	11 Sep 1943	1	U.S.; 45th INF DIV	German; 16th PZ DIV	German	12450	8390	240	160	I	2	5	2	Summer	A-Gd D-Gd	Minor (A)	No	(A)	Yes	Yes
6.4	Invasion of Italy	Salerno	Vietri I	12 Sep 1943	3	German; HG PZ DIV	British; 46th INF DIV	British	15000	12920	670	820	II /FFF	2	5	2	Summer	A-Gd D-Gd	No	No	(D)	Yes	Yes
6.5	Invasion of Italy	Salerno	Battipaglia	12 Sep 1943	4	German; 16th PZ DIV	British; 56th INF DIV	British	14730	11230	830	1570	II /GGG	2	5	2	Summer	A-Gd D-Gd	No	No	(D)	Yes	Yes
6.6	Invasion of Italy	Salerno	Tobacco Factory	13 Sep 1943	2	German; 6th & 29th PZ DIV	U.S.; 45th INF DIV	U.S.	14730	12690	630	390	II /GGG	2	5	2	Summer	A-Gd D-Gd	No	No	(D)	Yes	Yes
6.7	Invasion of Italy	Salerno	Vietri II	17 Sep 1943	2	German; HG PZ DIV	British; 46th INF DIV	British	13300	18910	390	250	II /GGG	2	5	2	Summer	A-Gd D-Gd	No	No	(D)	Yes	Yes

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
6.8	Invasion of Italy	Salerno	Battipaglia II	17 Sep 1943	2	British; 56th INF DIV	German; 16th PZ DIV	British	14730	7000	300	110	I	5	5	2	Summer	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.9	Invasion of Italy	Salerno	Eboli	17 Sep 1943	2	U.S.; 45th INF DIV	German; 16th & 26th PZ DIV	U.S.	15580	6700	380	110	I	5	5	2	Summer	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.10	Volturno	Grazzanise	Grazzanise	12 Oct 1943	3	British; 7th Armored DIV	German; 15th PZ GR DIV	British	14560	8070	370	80	I	3	8	2	Fall	A-Gd D-Gd	No	No	(A)	No	No
6.11	Volturno	Capua	Capua	13 Oct 1943	1	British; 56th INF DIV	German; HG PZ DIV	German	16860	8000	420	90	I	3	8	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	No
6.12	Volturno	Triflisco	Triflisco	13 Oct 1943	2	U.S.; 3rd INF DIV	German; HG PZ DIV	U.S.	18480	7250	260	70	I	3	5	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	No
6.13	Volturno	Monte Acero	Monte Acero	13 Oct 1943	2	U.S.; 45th INF DIV	German; 3rd PZ GR & 26 PZ DIV	U.S. (Marginal)	21270	6440	120	130	I	5	2	8	Fall	A-Gd D-Gd	No	No	(A)	No	Yes
6.14	Volturno	Caiazzo	Caiazzo	13 Oct 1943	2	U.S.; 34th INF DIV	German; 3rd PZ GR DIV	U.S.	18210	6440	130	50	I	5	5	8	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.15	Volturno	Castel Volturno	Castel Volturno	13 Oct 1943	3	British; 46th INF DIV	German; 15th PZ GR DIV	British	17770	8160	600	60	I	3	8	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	No
6.16	Volturno	Dragoni	Dragoni	15 Oct 1943	3	U.S.; 34th INF DIV	German; 3rd PZ GR DIV	U.S.	17030	5150	60	50	I	5	5	8	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.17	Volturno	Canal I	Canal I	15 Oct 1943	6	British; 46th INF DIV	German; 15th PZ GR DIV	British	17500	8140	440	280	I	3	5	8	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.18	Volturno	Monte Grande	Monte Grande	16 Oct 1943	2	British; 50th INF DIV	German; HG PZ DIV	British	16400	7240	200	70	I	3	7	11	Fall	A-Gd D-Gd	No	No	(A)	Yes	No

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
6.19	Volturno	Canal II	Canal II	17 Oct 1943	2	British; 7th INF DIV	German; 15th PZ GR DIV	British	14600	8140	130	50	I	3	8	11	Fall	A-Gd D-Gd	No	No	(A)	No	No
6.20	Volturno	Francolise	Francolise	20 Oct 1943	3	British; 7th INF DIV	German; 15th PZ GR DIV	German (Marginal)	14000	8090	80	40	I	3	5	8	Fall	A-Gd D-Gd	No	No	(A)	No	No
6.21	Volturno	Santa Maria Oliveto	Santa Maria Oliveto	4 Nov 1943	2	U.S.; 34th INF DIV	German; 3rd PZ GR DIV	U.S.	16870	6320	400	210	I	3	5	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.22	Volturno	Monte Camino I	Monte Camino I	5 Nov 1943	3	British; 56th INF DIV	German; 15th PZ GR DIV	German	19510	6750	240	20	I	4	2	8	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.23	Volturno	Monte Lungo	Monte Lungo	6 Nov 1943	2	U.S.; 3rd INF DIV	German; 3rd PZ GR DIV	German	16600	6570	350	170	I	4	2	8	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.24	Volturno	Pozzilli	Pozzilli	6 Nov 1943	2	U.S.; 45th INF DIV	German; 3rd PZ GR DIV	German	20120	6570	140	30	I	4	2	8	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.25	Volturno	Monte Camino II	Monte Camino II	8 Nov 1943	5	German; 15th PZ GR DIV	British; 56th INF DIV	German	7940	5200	60	510	I	2	2	8	Fall	A-Gd D-Gd	No	No	(D)	Yes	Yes
6.26	Volturno	Monte Rotondo	Monte Rotondo	8 Nov 1943	3	U.S.; 3rd INF DIV	German; 3rd PZ GR DIV	German	16350	7940	160	60	I	4	2	8	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.27	"Winter Line"	Calabritto	Calabritto	1 Dec 1943	2	British; 46th INF DIV	German; 15th PZ GR DIV	British (Marginal)	17770	7590	250	20	I	4	2	8	Fall	A-Gd D-Gd	No	No	(A)	Yes	No
6.28	"Winter Line"	Monte Camino III	Monte Camino III	2 Dec 1943	5	British; 56th INF DIV	German; 15th PZ GR DIV	British	20740	3290	700	170	I	4	2	8	Fall	A-Gd D-Gd	No	No	(A)	Yes	No
6.29	"Winter Line"	Monte Maggiore	Monte Maggiore	2 Dec 1943	2	U.S.; 36th INF DIV	German; 15th PZ GR & 29 PZ DIV	U.S. (Marginal)	5550	3290	80	20	I	4	2	11	Fall	A-Gd D-Gd	No	No	(A)	Yes	No

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
6.30	Anzio	Aprilia I	Aprilia I	25 Jan 1944	2	British; 1st INF DIV	German; 3rd PZ GR DIV	British	19350	6750	1140	100	I	2	8	11	Winter	A-Gd D-Gd	Minor (A)	No	(A)	No	Yes
6.31	Anzio	The Factory	The Factory	27 Jan 1944	1	German; 3rd PZ GR DIV	British; 1st INF DIV	British	15320	17980	70	60	I	2	8	3	Winter	A-Gd D-Gd	Minor (D)	No	(D)	Yes	Yes
6.32	Anzio	Campoleone	Campoleone	29 Jan 1944	3	British; 1st INF DIV	German; 3rd PZ GR DIV	German	17770	15100	740	220	I	3	8	3	Winter	A-Gd D-Gd	Minor (A)	No	(A)	Yes	Yes
6.33	Anzio	Campoleone Counter-attack	Campoleone Counter-attack	3 Feb 1944	3	German; Combat Command Greizer	British; 1st INF DIV	British	26030	9730	1950	2150	I	2-3	8	8	Winter	A-Gd D-Gd	No	No	(D)	Yes	Yes
6.34	Anzio	Carroceto	Carroceto	7 Feb 1944	2	German; 3rd PZ GR DIV	British; 1st INF DIV	British	26490	4520	340	370	I	2-3	8	3	Winter	A-Gd D-Gd	No	No	(D)	Yes	Yes
6.35	Anzio	Moletta River Defense	Moletta River Defense	7 Feb 1944	3	German; 65th INF Div	U.S.; 45th INF DIV	U.S.	7420	5000	250	120	I	2-3	8	3	Winter	A-Gd D-Gd	No	No	(D)	Yes	Yes
6.36	Anzio	Aprilia II	Aprilia II	9 Feb 1944	1	German; Combat Command Greizer	British; 1st INF DIV	German	27520	17730	230	310	I	2-3	8	3	Winter	A-Gd D-Gd	No	No	(D)	Yes	No
6.37	Anzio	Factory Counter-attack	Factory Counter-attack	11 Feb 1944	2	U.S.; 45th INF DIV	German; 715th LT INF DIV	German	13400	7080	120	210	I	2-3	8	9	Winter	A-Gd D-Gd	No	No	(A)	No	Yes
6.38	Anzio	Bowling Alley	Bowling Alley	16 Feb 1944	4	German; 4 Divisions	U.S.; 45th INF DIV	U.S.	41980	20500	1900	1300	II	2-3	8	3	Winter	A-Gd D-Gd	No	No	(D)	Yes	Yes
6.39	Anzio	Moletta River II	Moletta River II	16 Feb 1944	4	German; 65th INF & 45h PARA DIV	British; 56th INF DIV	German (Marginal)	21480	9760	1430	1670	II	2-3	8	3	Winter	A-Gd D-Gd	Major (A)	No	(D)	Yes	Yes

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
6.40	Anzio	Fioccia	Fioccia	21 Feb 1944	3	German; 114th LT INF DIV	U.S.; 45th INF DIV	U.S.	15640	19610	570	260	II	3	8	9	Winter	A-Gd D-Gd	No	No	(D)	Yes	Yes
6.41	Rome	Santa Maria Infante	Santa Maria Infante	12 May 1944	2	U.S.; 88th INF DIV	German; 94th & 71st INF DIV	U.S.	18700	9250	550	960	I	4	3	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	No
6.42	Rome	San Martino	San Martino	12 May 1944	2	U.S.; 85th INF DIV	German; 94th INF DIV	U.S. (Marginal)	17970	8140	1150	680	I	4	3	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	No
6.43	Rome	Spigno	Spigno	14 May 1944	2	U.S.; 88th INF DIV	German; 94th & 71st INF DIV	U.S.	18310	8220	340	720	I	5	3	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	No
6.44	Rome	Castello-novato	Castello-novato	14 May 1944	2	U.S.; 85th INF DIV	German; 94th INF DIV	U.S.	16460	7500	540	440	I	4	3	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	No
6.45	Rome	Formia	Formia	16 May 1944	3	U.S.; 85th INF DIV	German; 94th INF DIV	U.S.	23190	7630	400	720	I	5	5	2	Spring	A-Gd D-Gd	No	No	(A)	No	No
6.46	Rome	Monte Grande	Monte Grande	17 May 1944	3	U.S.; 88th INF DIV	German; 94th INF DIV	U.S.	13100	4560	300	490	I	2	3	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	No
6.47	Rome	Itri-Fonde	Itri-Fonde	20 May 1944	3	U.S.; 88th INF DIV	German; 94th INF DIV	U.S.	17910	6650	270	380	I	5	3	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.48	Rome	Terracina	Terracina	22 May 1944	3	U.S.; 85th INF DIV	German; 94th INF DIV	U.S.	18030	6650	270	380	I	2	2	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	No
6.49	Rome	Moletta Offensive	Moletta Offensive	23 May 1944	2	British; 5th INF DIV	German; 4th PARA DIV	British	17350	12570	230	470	I	4	8	8	Spring	A-Gd D-Gd	No	No	(A)	Yes	No
6.50	Rome	Anzio-Albano Road	Anzio-Albano Road	23 May 1944	2	British; 1st INF DIV	German; 65th INF DIV	British	17310	11340	190	480	I	4	8	8	Spring	A-Gd D-Gd	No	No	(A)	Yes	No

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
6.51	Rome	Anzio Breakout	Anzio Breakout	23 May 1944	3	U.S.; 1st Armored DIV	German; 3 PZ GR & 362 INF DIV	U.S.	16220	12820	710	1320	I	4	8	8	Spring	A-Gd D-Gd	Subs. (A)	No	(A)	Yes	No
6.52	Rome	Cisterna	Cisterna	23 May 1944	3	U.S.; 3rd INF DIV	German; 362nd INF DIV	U.S.	19970	11930	1520	1590	I	4	8	8	Spring	A-Gd D-Gd	Minor (A)	No	(A)	Yes	No
6.53	Rome	Sezze	Sezze	25 May 1944	3	U.S.; 85th INF DIV	German; 29th PZ DIV	U.S.	17930	6960	160	280	I	6	5	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	No
6.54	Rome	Velletri	Velletri	26 May 1944	1	U.S.; 1st Armored DIV	German; 362nd INF DIV	German	14620	12330	770	1310	I	4	5	2	Spring	A-Gd D-Gd	Minor (D)	No	(A)	Yes	No
6.55	Rome	Campoleone Station	Campoleone Station	26 May 1944	3	U.S.; 45th INF DIV	German; 65th INF DIV	U.S.	19050	10590	530	870	I	4	5	2	Spring	A-Gd D-Gd	No	No	(A)	No	No
6.56	Rome	Villa Crocetta	Villa Crocetta	27 May 1944	2	U.S.; 34th INF DIV	German; 3rd PZ GR DIV	German	18000	13720	310	600	I	4	5	2	Spring	A-Gd D-Gd	No	No	(A)	No	No
6.57	Rome	Ardea	Ardea	28 May 1944	3	British; 5th INF DIV	German; 4th PARA DIV	British	15560	7660	240	380	I	4	5	2	Spring	A-Gd D-Gd	No	No	(A)	No	Yes
6.58	Rome	Lanuvio	Lanuvio	29 May 1944	4	U.S.; 34th INF DIV	German; 3rd PZ GR DIV	German	17300	6110	820	590	I	4	5	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.59	Rome	Campoleone	Campoleone	29 May 1944	3	U.S.; 1st ARM DIV & 45th INF DIV	German; 3rd PZ GR & 65th INF DIV	U.S. (Marginal)	29710	15800	1300	1380	I	4	5	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.60	Rome	Tarto-Tiber	Tarto-Tiber	3 Jun 1944	2	British; 1st & 5th INF DIV	German; 45th PARA DIV	British	38010	10860	570	850	I	4	8	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	No

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
6.61	Normandy Breakout	Seine River	Seine River	23 Aug 1944	3	U.S.; XX CORPS	German; FIRST ARMY	U.S.	40620	15000	230	890	I	3	5	8	Summer	A-Gd D-Gd	No	No	(A)	Yes	No
6.62	Advance to the Westwall	Moselle Metz	Moselle Metz	6 Sep 1944	6	U.S.; XX CORPS	German; FIRST ARMY	U.S.	59630	41500	1650	1670	I	5	5	8	Summer	A-Gd D-Gd	No	No	(A)	Yes	No
6.63	Advance to the Westwall	Metz	Metz	13 Sep 1944	1	U.S.; XX CORPS	German; FIRST ARMY	German	60790	39580	360	210	I	4	5	8	Summer	A-Gd D-Gd	No	No	(A)	Yes	No
6.64	Advance to the Westwall	Chartres	Chartres	16 Aug 1944	1	U.S.; 7th ARMORED DIV	German; FIRST ARMY	U.S.	15650	8330	110	580	I	2	5	5	Summer	A-Gd D-Gd	No	No	(A)	Yes	No
6.65	Advance to the Westwall	Melun	Melun	23 Aug 1944	3	U.S.; 7th ARMORED DIV	German; 48th INF DIV	U.S.	17230	6000	100	1080	I	3	5	8	Summer	A-Gd D-Gd	No	No	(A)	No	No
6.66	Sigfreid	Foret de Chateau-Salins	Foret de Chateau-Salins/HHH	10 Nov 1944	2	German; 11th PZ DIV XIII CORPS	U.S.; 35th, 26th INF DIV & 4th ARM DIV	U.S. (Marginal)	11190	43590	450	720	I	2	5	12	Fall	A-Gd D-Gd	No	No	(D)	Yes	No
6.67	Sigfreid Line	Morhange	Morhange	13 Nov 1944	3	U.S.; 4th ARM DIV & 35th INF DIV	German; 11th PZ DIV, 361st INF DIV	U.S.	25880	7560	1200	200	I	5	5	12	Fall	A-Gd D-Gd	No	No	(A)	No	No
6.68	Sigfreid Line	Bourgal-troff	Bourgal-troff	14 Nov 1944	2	U.S.; 4th ARM DIV & 26th INF DIV	German; 11th PZ DIV, 361st INF DIV	German	21860	6520	390	140	I	5	5	12	Fall	A-Gd D-Gd	No	No	(A)	No	No
6.69	Sigfreid Line	Baerendorf I	Baerendorf I/III	24 Nov 1944	2	German; PZ Lehr DIV 361 INF DIV	U.S.; 4th Armored DIV	U.S.	5370	7940	220	60	I	2	5	6	Fall	A-Gd D-Gd	No	No	(D)	No	No
6.70	Sigfreid Line	Baerendorf II	Baerendorf II	26 Nov 1944	1	U.S.; 4th Armored DIV	German; PZ Lehr DIV	U.S.	15870	7000	60	230	I	3	5	5	Fall	A-Gd D-Gd	No	No	(A)	No	No

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
6.71	Sigfreid Line	Burbach-Durstel	Burbach-Durstel	27 Nov 1944	3	U.S.; 4th ARM DIV	German; PZ Lehr DIV	German	16230	6710	80	220	I	5	5	9	Fall	A-Gd D-Gd	No	No	(A)	No	No
6.72	Sigfreid Line	Sarre-Union	Sarre-Union	1 Dec 1944	2	U.S.; 4th ARM DIV	German; 11th PZ, PZ Lehr, 25th PZ GR DIV	U.S.	19770	6040	280	130	I	3	5	5	Fall	A-Gd D-Gd	No	No	(A)	No	No
6.73	Sigfreid Line	Singling-Bining	Singling-Bining	6 Dec 1944	2	U.S.; 4th ARM DIV	German; 25th PZ GR DIV, 11th PZ DIV	German	15220	5040	160	120	I	4	5	8	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.74	Sigfreid Line	Seille-Nied	Seille-Nied	8 Nov 1944	5	U.S.; XII CORPS	German; XIII & LXXXIX CORPS	U.S.	99580	23590	4280	4870	I	4	5	12	Fall	A-Gd D-Gd	No	No	(A)	Yes	No
6.75	Sigfreid Line	Morhange-Faulque-mont	Morhange-Faulque-mont	13 Nov 1944	4	U.S.; XII CORPS	German; XIII & LXXXIX CORPS	Incon-lusive (Arbi-trarily German)	92390	28380	3320	3670	I	5	5	3	Fall	A-Gd D-Gd	No	No	(A)	No	No
6.76	Sigfreid Line	Serre-St. Avold	Serre-St. Avold	20 Nov 1944	8	U.S.; XII CORPS	German; XIII & LXXXIX CORPS	U.S.	88940	32400	3270	4950	I	5	5	6	Fall	A-Gd D-Gd	No	No	(A)	Yes	No
6.77	Sigfreid Line	Durstel-Farebers-villes	Durstel-Farebers-villes	28 Nov 1944	2	U.S.; XII CORPS	German; XIII & LXXXIX CORPS	German	90080	30710	490	810	I	5	5	9	Fall	A-Gd D-Gd	No	No	(A)	No	No
6.78	Sigfreid Line	Sarre-Singling	Sarre-Singling	5 Nov 1944	3	U.S.; XII CORPS	German; XII & XI CORPS	U.S.	89980	31500	1130	1170	I	5	5	6	Fall	A-Gd D-Gd	No	No	(A)	Yes	No

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
6.79	Kursk	Oboyan Kursk	Oboyan Kursk	4 Jul 1943	9	German; XLVIII PZ CORPS	Soviet; 6th GD & 1st TK ARMY	German	62000	90000	5640	30940	I	4	5	8	Summer	A-Gd D-Gd	No	No	(A)	Yes	Yes
6.80	Gothic Line	Il Giogo Pass	Il Giogo Pass	13 Sep 1944	5	U.S.; 85th INF DIV	German; 12th PARA RGT	U.S.	15720	3700	560	500	I	4	2	8	Summer	A-Gd D-Gd	No	No	(A)	Yes	No
6.81	Ardennes	Ardennes-Sauer	Ardennes-Sauer	16 Dec 1944	2	German; 212th VG DIV	U.S.; 4th INF DIV	German	10000	8630	270	140	I	2	5	6	Fall	A-Gd D-Gd	Major (A)	No	(A)	No	No
6.82	Invasion of France	Sedan	Sedan	13 Mar 1940	2	German; XIX CORPS	French; ELMS 7 & 9 ARMIES	German	48000	45000	800	4500	I	3	5	2	Spring	A-Gd D-Pr	Minor (A)	No	(A)	Yes	Yes
6.83	Kiev (1941)	Rovno (Ukraine 1941)	Rovno (Ukraine 1941)	21 Jul 1941	5	German; 1st PZ GR	Soviet; ELMS Southwest Army Gr (SWAG)	German	132000	150000	3300	87530	I	3	5	2	Summer	A-Gd D-Gd	Minor (A)	No	(A)	Yes	Yes
6.84	Malaya (1941)	Jitra	Jitra	12 Dec 1941	1	Japan; 5th INF DIV	British; 11th INDIAN DIV	Japan	7000	12000	600	1200	I	2	12	7	Fall	A-Ex D-Fr	Minor (A)	No	(A)	No	No
6.85	Leningrad (1943)	Leningrad	Leningrad	12 Jan 1943	7	Soviet; 2nd Armored ARMY	German; Elements 18th ARMY	Soviet	120000	30000	47960	3150	I	4	5	12	Winter	A-Gd D-Gd	No	Yes	(D)	Yes	Yes
6.86	Soviet Summer Offensive	Kharkov-Belgorad	Kharkov-Belgorad	3 Aug 1943	3	Soviet; 53rd ARMY	German; 167 INF DIV	Soviet	70000	15000	11680	410	I	4	5	2	Summer	A-Gd D-Gd	No	Yes	(D)	Yes	Yes
6.87	Normandy Breakout	Cobra (St. Lo)	Cobra (St. Lo)	24 Jul 1944	3	U.S.; VII CORPS	German; LXXXIV CORPS	U.S.	126000	30700	2270	7500	I	4	5	2	Summer	A-Gd D-Gd	No	No	(A)	Yes	No
6.88	Manchurian	Mutangiang (Manchuria)	Mutangiang (Manchuria)	9 Aug 1945	8	Soviet; 5th ARMY	Japan; 5th ARMY	Soviet	147000	75000	10000	36000	I	4	2	2	Summer	A-Gd D-Pr	Minor (A)	No	(A)	Yes	Yes

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
7.1	Dien Bien Phu	Siege of Dien Bien Phu	Assault on Beatrice	13 Mar 1954	1	Vietminth	French	Viet-minth	8000	750	1710	560	II	4	6	8	Winter	A-Gd D-Gd	No	No	(D)	No	Yes
7.2	Dien Bien Phu	Siege of Dien Bien Phu	Assault on Gabrielle/JJJ	14 Mar 1954	1	Vietminth	French	Viet-minth	6800	1950	5800	1000	II	4	6	8	Winter	A-Gd D-Ex	No	No	(D)	No	Yes
7.3	Dien Bien Phu	Siege of Dien Bien Phu	Huguette (4/4)	4 Apr 1954	1	Vietminth	French	French	4000	410	2170	280	II	4	6	8	Spring	A-Gd D-Ex	No	No	(D)	No	Yes
8.1	Invasion of S. Korea	Invasion of S. Korea	Invasion of S. Korea	25 Jun 1950	3	N. Korea; NKPA ELMS	S. Korea; ROKA ELMS	N. Korea	60000	38000	5510	18000	I	2-3	3	2	Summer	A-Gd D-Gd	Minor (A)	No	(D)	Yes	Yes
9.1	Northern	Mishmar-Haemek	Mishmar-Haemek	4 Apr 1948	6	Arab	Israeli	Israeli	1500	1000	50	30	I	3	6	2	Spring	A-Gd D-Gd	Minor (A)	No	None	No	No
9.2	Northern	Haifa	Haifa	21 Apr 1948	3	Israel	Arab	Israel (Marginal)	1500	500	60	25	II	3	6	2	Spring	A-Gd D-Gd	No	No	None	No	No
9.3	Northern	Safad	Safad	28 Apr 1948	15	Israel	Arab	Israel	1500	2100	100	150	I	3	3	8	Spring	A-Gd D-Gd	No	No	None	No	No
9.4	Eastern	Jerusalem (Nachson)	Jerusalem (Nachson)	6 Apr 1948	7	Israel/KKK	Arab	Israel	2000	1500	150	500	I	3	6	2	Spring	A-Gd D-Gd	No	No	None	No	No
9.5	Eastern	Jerusalem (Jebussi)	Jerusalem (Jebussi)	21 Apr 1948	10	Israel	Arab	Israel	3000	3600	375	500	I	3	14	2	Spring	A-Gd D-Gd	No	No	None	No	No
9.6	Eastern	Jaffa (Chametz)	Jaffa (Chametz)	25 Apr 1948	19	Arab/LLL	Israel	Israel	3500	2100	300	240	I	3	14	2	Spring	A-Gd D-Gd	No	No	(A)	No	Yes

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
9.7	Eastern	Kfar Etzion	Kfar Etzion	4,12-14 May 1948	4	Arab	Israel	Arab	1100	930	200	160	II	3	6	2	Spring	A-Gd D-Gd	No	No	None	No	No
9.8	Northern	Jordan Valley	Jordan Valley	15 May 1948	6	Arab	Israel	Israel	2000	1250	200	150	I	4	9	2	Spring	A-Gd D-Gd	No	No	Nei-ther	Yes	Yes
9.9	Northern	Mishmar Hayarden I	Mishmar Hayarden I	6 Jun 1948	5	Arab	Israel	Arab	4000	2500	250	2500	I	3	9	2	Spring	A-Gd D-Gd	Subs. (A)	No	None	No	No
9.10	Northern	Ein Gev	Ein Gev	10 Jun 1948	3	Arab	Israel	Israel	500	100	50	20	I	4	9	2	Spring	A-Gd D-Gd	No	No	None	No	No
9.11	Northern	Gesher	Gesher	16 May 1948	3	Arab	Israel	Israel	600	500	60	30	I	3	9	2	Spring	A-Gd D-Gd	No	No	None	No	No
9.12	Northern	Jenin	Jenin	31 May 1948	5	Israel	Arab	Arab	1500	2000	150	20	I	3	3	1	Spring	A-Gd D-Gd	No	No	(D)	No	No
9.13	Eastern	Jerusalem (Sheik Jarrah-Notre Dame)	Jerusalem (Sheik Jarrah-Notre Dame)	19 May 1948	5	Arab	Israel	Israel	1800	1700	180	100	II	3	14	2	Spring	A-Gd D-Gd	No	No	None	No	No
9.14	Eastern	Latrun I	Latrun I	26 May 1948	2	Israel	Arab	Arab	2000	1000	800	20	I	4	6	1	Spring	A-Gd D-Gd	No	No	None	No	No
9.15	Eastern	Latrun II	Latrun II	30 May 1948	1	Israel	Arab	Arab	2000	1000	570	20	I	4	6	1	Spring	A-Gd D-Fr	No	No	None	No	No
9.16	Southern	Kfar Darom-Nirim	Kfar Darom-Nirim	15 May 1948	2	Arab	Israel	Israel	500	200	270	20	I	3	10	2	Spring	A-Gd D-Gd	No	No	None	No	No

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
9.17	Southern	Yad Mordechai	Yad Mordechai	19 May 1948	6	Arab	Israel	Arab	2000	300	300	45	I	3	10	2	Spring	A-Gd D-Gd	No	No	None	No	No
9.18	Southern	Neqba-Ashdad	Neqba-Ashdad	29 May 1948	6	Israel/MMM	Arab	Arab	1500	2000	150	20	I	3	10	2	Spring	A-Gd D-Gd	No	No	Neither	Yes	Yes
9.19	Southern	Nitzanin	Nitzanin	7 Jun 1948	1	Arab	Israel	Arab	1840	150	18	150	I	3	9	2	Spring	A-Gd D-Gd	No	No	(A)	Yes	No
9.20	Northern	Mishmar Hayarden II	Mishmar Hayarden II	9 Jul 1948	6	Arab	Israel	Arab (Marginal)	3000	2700	250	270	I	2	9	2	Summer	A-Gd D-Gd	No	No	(D)	No	No
9.21	Northern	Sejera	Sejera	11 Jul 1948	6	Arab	Israel	Israel	2000	1000	150	20	I	3	6	2	Summer	A-Gd D-Gd	No	No	(A)	No	Yes
9.22	Northern	Nazareth (Dekel)	Nazareth (Dekel)	15 Jul 1948	2	Israel	Arab	Israel	2500	2000	25	95	I	3	6	2	Summer	A-Gd D-Gd	Subs. (A)	No	None	No	No
9.23	Eastern	Jerusalem Corridor (Dani)	Jerusalem Corridor (Dani)	9 Jul 1948	5	Israel	Arab	Israel	4500	2500	150	250	I	4	3	2	Summer	A-Gd D-Gd	No	No	Neither	Yes	Yes
9.24	Southern	Death to the Invader	Death to the Invader	14 Jul	5	Israel	Arab	Arab (Marginal)	2500	3000	250	300	I	3	6	2	Summer	A-Gd D-Gd	No	No	(D)	No	Yes
9.25	Southern	Tragel-Manshiya-Faluja	Tragel-Manshiya-Faluja	16 Oct 1948	2	Israel	Arab	Arab (Marginal)	2000	2000	200	150	I	3	6	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	No
9.26	Southern	Huleiqat-Suweidan	Huleiqat-Suweidan	20 Oct 1948	2	Israel	Arab	Israel (Marginal)	1500	1000	150	75	I	3	6	2	Fall	A-Gd D-Gd	No	No	None	No	No
9.27	Southern	Hiram	Hiram	28 Oct 1948	4	Israel	Arab	Israel	6000	6000	650	2100	II	3	6	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	No

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
9.28	Southern	Ayin-El Auja	Ayin-El Auja	25 Dec 1948	3	Israel	Arab	Israel	6000	4000	350	600	I	3	10	2	Winter	A-Gd D-Gd	Subs. (A)	No	(A)	Yes	No
9.29	Southern	Ayin-Abu Ageila-Rafah	Ayin-Abu Ageila-Rafah	28 Dec 1948	11	Israel	Arab	Israel	4000	3000	400	600	I	3	10	2	Winter	A-Gd D-Pr	Subs. (A)	No	(A)	Yes	No
10.1	Sinai	Sabha-Kusseima	Sabha-Kusseima	30 Oct 1956	1	Israel	Egypt	Israel	2340	1800	50	420	I	3	3	1	Fall	A-Gd D-Fr	Minor (A)	No	Nei-ther	No	No
10.2	Sinai	Thamad	Thamad	30 Oct 1956	1	Israel	Egypt	Israel	1000	600	10	80	I	4	3	1	Fall	A-Gd D-Fr	Minor (A)	No	Nei-ther	Yes	Yes
10.3	Sinai	Nakhl	Nakhl	30 Oct 1956	1	Israel	Egypt	Israel	750	1100	4	80	I	4	3	1	Fall	A-Gd D-Pr	Minor (A)	No	Nei-ther	No	No
10.4	Sinai	Abu Ageila-m Katef	Abu Ageila-Um Katef	30 Oct 1956	4	Israel	Egypt	Israel	4700	4800	320	3000	I	4	3	1	Fall	A-Gd D-Fr	Minor (A)	No	Nei-ther	Yes	Yes
10.5	Sinai	Mitla Pass	Mitla Pass	31 Oct 1956	1	Israel	Egypt	Israel (Marginal)	1000	1850	190	710	I	3	3	1	Fall	A-Gd D-Fr	No	No	Nei-ther	Yes	Yes
10.6	Sinai	Bir Rud Salim-Bir Gifgafa	Bir Rud Salim-Bir Gifgafa	1 Nov 1956	2	Israel	Egypt	Israel	2670	3300	10	300	I	2-3	3	1	Fall	A-Gd D-Fr	No	No	Nei-ther	Yes	Yes
10.7	Sinai	Rafah-El Arish	Rafah-El Arish	1 Nov 1956	1	Israel	Egypt	Israel	10000	10050	230	3430	I	4	10	1	Fall	A-Gd D-Pr	No	No	Nei-ther	Yes	No
10.8	Sinai	Gaza-Khan Yunis	Gaza-Khan Yunis	2 Nov 1956	1	Israel	Egypt	Israel	4000	6400	120	1990	II	3	14	1	Fall	A-Gd D-Pr	No	No	Nei-ther	Yes	No
10.9	Sinai	Sharm el Sheikh	Sharm el Sheikh	4 Nov 1956	2	Israel	Egypt	Israel	1800	1500	50	1000	II	4	3	1	Fall	A-Gd D-Fr	No	No	Nei-ther	Yes	No

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
11.1	Sinai	Rafah	Rafah	5 Jun 1967	1	Israeli; Tal DIV	Egyptian; 7th INF DIV (+)	Israeli	19520	19520	700	2700	I	4	6	1	Spring	A-Gd D-Gd	Subs. (A)	No	(A)	Yes	Yes
11.2	Sinai	Abu Ageila	Abu Ageila	5 Jun 1967	2	Israeli; Sharon DIV	Egyptian; 2nd INF DIV	Israeli	19280	18450	600	1800	I	4	6	1	Spring	A-Gd D-Gd	Major (A)	No	(A)	No	No
11.3	Sinai	Gaza Strip	Gaza Strip	6 Jun 1967	2	Israeli; Tal DIV (-)	Palestinian; 20th PLA DIV	Israeli	12150	17450	110	630	I	4	14	1	Spring	A-Gd D-Gd	Minor (A)	No	(A)	Yes	Yes
11.4	Sinai	El Arish	El Arish	5 Jun 1967	2	Israeli; Tal DIV (-)	Egyptian; 7th INF DIV (-)	Israeli	6910	12750	270	450	I	4 /NNN	6	1	Spring	A-Gd D-Gd	Minor (A)	No	(A)	Yes	No
11.5	Sinai	Bir Lahfan	Bir Lahfan	5 Jun 1967	2	Israeli; Yoffe DIV	Egyptian; 3rd INF DIV	Israeli	10450	10050	180	2700	I	5	6	1	Spring	A-Gd D-Gd	Minor (A)	No	(A)	Yes	Yes
11.6	Sinai	Jebel Libni	Jebel Libni	6 Jun 1967	1	Israeli; Yoffe DIV	Egyptian; 3rd INF DIV	Israeli	10800	10050	70	450	I	3 /000	6	1	Spring	A-Gd D-Gd	Minor (A)	No	(A)	Yes	No
11.7	Sinai	Mitla Pass	Mitla Pass	7 Jun 1967	1	Israeli; Tal DIV	Egyptian; 3rd INF DIV & 4th ARM DIV	Israeli	10200	13500	80	550	I	5	3	1	Spring	A-Gd D-Pr	Minor (A)	No	(A)	Yes	No
11.8	Sinai	Bir Hama	Bir Hama	7 Jun 1967	1	Israeli; Tal DIV;	Egyptian; 3rd INF DIV & 4th ARM DIV	Israeli	8700	11000	60	550	I	2	6	1	Spring	A-Gd D-Pr	Minor (A)	No	(A)	Yes	No
11.9	Sinai	Bir Hassna	Bir Hassna	7 Jun 1967	2	Egyptian; 3rd INF DIV & 6th INF DIV	Israeli; Yoffe DIV (-)	Israeli	22000	7250	1100	180	I	2	6	1	Spring	A-Pr D-Gd	Minor (D)	No	(D)	Yes	No

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
11.10	Sinai	Bir Gifgafa	Bir Gifgafa	8 Jun 1967	1	Egyptian; 4th ARM DIV (-)	Israeli; Tal Div (-)	Israeli	3500	3600	450	60	I	2	6	1	Spring	A-Gd D-Pr	No	No	(D)	No	No
11.11	Sinai	Nakhl	Nakhl	8 Jun 1967	1	Israeli; Sharon DIV	Egyptian: 6th MCZD DIV	Israeli	18780	18450	60	630	I	2	6	1	Spring	A-Gd D-Pr	No	No	(A)	Yes	No
11.12	West Bank	Jerusalem	Jerusalem	5 Jun 1967	3	Israeli; CENTRAL COMMAND	Jordanian; 27th INF BDE	Israeli	27680	13600	2620	2250	I	4	14	1	Spring	A-Gd D-Gd	Minor (A)	No	(A)	Yes	Yes
11.13	West Bank	Jenin	Jenin	5 Jun 1967	2	Israeli; Peled DIV (-)	Jordanian; 25th INF BDE	Israeli	10900	6160	450	400	I	3	3	1	Spring	A-Gd D-Gd	Minor (A)	No	(A)	Yes	No
11.14	West Bank	Kabatiya	Kabatiya	6 Jun 1967	2	Israeli; Peled DIV	Jordanian; 40th ARM BDE, 25th ARM BDE	Israeli	12800	9900	750	700	I	2	3	1	Spring	A-Gd D-Gd	Minor (A)	No	(A)	Yes	No
11.15	West Bank	Tilfit-Zababida	Tilfit-Zababida	6 Jun 1967	2	Israeli; Ram BDE	Jordanian; 40th ARM BDE, 25th INF BDE	Israeli	5350	5450	500	500	I	2-3	3	1	Spring	A-Gd D-Gd	Minor (A)	No	(A)	Yes	No
11.16	West Bank	Nablus	Nablus	7 Jun 1967	1	Israeli; Peled DIV (-)	Jordanian; 40th ARM BDE, 25th INF BDE	Israeli	10700	10640	380	430	I	2-3	3	1	Spring	A-Gd D-Gd	Minor (A)	No	(A)	Yes	No
11.17	Golan	Zaoura Kola	Zaoura-Kola	9 Jun 1967	1	Israeli; MECH BDE	Syrian; 11th INF BDE (-)	Israeli	5850	8560	230	500	I	4	3	1	Spring	A-Gd D-Pr	No	No	(A)	Yes	No
11.18	Golan	Tel Fahar	Tel Fahar	9 Jun 1967	1	Israeli; Golani BDE	Syrian; 11th INF BDE (-)	Israeli	5380	8160	300	850	I	4	3	1	Spring	A-Gd D-Pr	No	No	(A)	Yes	No

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
11.19	Golan	Rawiyeh	Rawiyeh	9 Jun 1967	1	Israeli; Ram BDE	Syrian; 8th INF BDE (-)	Israeli	5350	4350	150	300	I	4	3	1	Spring	A-Gd D-Pr	Minor (A)	No	(A)	No	No
12.1	Sinai	Suez Canal Assault (N)	Suez Canal Assault (N)	6 Oct 1973	1	Egyptian; 2nd ARMY	Israeli; Mend DIV (-)	Egyptian	29490	4460	400	280	I	3-4 /PPP	6	2	Fall	A-Gd D-Gd	Major (A)	Yes	Neither	Yes	Yes
12.2	Sinai	Suez Canal Assault (S)	Suez Canal Assault (S)	6 Oct 1973	1	Egyptian; 3rd ARMY	Israeli; Mend DIV (-)	Egyptian	22850	3020	230	350	I	3-4 /PPP	6	2	Fall	A-Gd D-Gd	Major (A)	Yes	Neither	Yes	Yes
12.3	Sinai	Second Army Buildup	Second Army Buildup	7 Oct 1973	1	Egyptian; 2nd ARMY	Israeli; SOUTHERN COMMAND (-)	Egyptian	63910	14000	800	450	I	2-3	6	2	Fall	A-Gd D-Gd	Subs. (A)	Yes	Neither	Yes	Yes
12.4	Sinai	Third Army Buildup	Third Army Buildup	7 Oct 1973	1	Egyptian; 3rd ARMY	Israeli; SOUTHERN COMMAND (-)	Egyptian	45160	10980	750	400	I	2-3	6	2	Fall	A-Gd D-Gd	Subs. (A)	Yes	Neither	Yes	Yes
12.5	Sinai	Kantara-Firdan	Kantara-Firdan	8 Oct 1973	1	Israeli; SOUTHERN COMMAND (-)	Egyptian; 2nd ARMY	Egyptian	25850	67440	700	700	I	2	6	2	Fall	A-Gd D-Gd	Minor (D)	No	Neither	Yes	Yes
12.6	Sinai	Egyptian Offensive (N)	Egyptian Offensive (N)	14 Oct 1973	1	Egyptian; 2nd ARMY (-)	Israeli; SOUTHERN COMMAND (-)	Israeli	81160	43400	1620	380	I	2	6	2	Fall	A-Gd D-Gd	No	No	Neither	Yes	Yes
12.7	Sinai	Egyptian Offensive (S)	Egyptian Offensive (S)	14 Oct 1973	1	Egyptian; 3rd ARMY (-)	Israeli; SOUTHERN COMMAND (-)	Israeli	57910	28600	1350	260	I	2	6	2	Fall	A-Gd D-Gd	No	No	Neither	Yes	Yes
12.8	Sinai	Chinese Farm I	Chinese Farm I	15 Oct 1973	2	Israeli; Sharon DIV (+)	Egyptian; 16th (-) & 21st (-) DIV	Israeli	22790	30970	800	1000	I	2-3	6	2	Fall	A-Gd D-Gd	Subs. (A)	No	Neither	Yes	Yes

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
12.9	Sinai	Chinese Farm II	Chinese Farm II	16 Oct 1973	2	Israeli; Adan DIV (+)	Egyptian; 16th (-) & 21st (-) DIV	Israeli	28700	36840	950	2400	I	2-3	6	2	Fall	A-Gd D-Gd	Minor (A)	No	(A)	Yes	Yes
12.10	Sinai	Deversoir West	Deversoir West	18 Oct 1973	1	Israeli; Adan DIV	Egyptian; 2nd ARMY (-)	Israeli	16200	18180	800	230	I	2-3	6	2	Fall	A-Gd D-Gd	Minor (A)	No	(A)	Yes	Yes
12.11	Sinai	Jebel Geneifa	Jebel Geneifa	19 Oct 1973	3	Israeli; Adan DIV	Egyptian; 3rd ARMY (-)	Israeli	16200	35630	300	1650	I	6	6	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
12.12	Sinai	Ismailia	Ismailia	19 Oct 1973	4	Israeli; Sharon DIV	Egyptian; 2nd ARMY (-)	Egyptian (Marginal)	17000	23860	150	1800	I	2-3	6	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
12.13	Sinai	Adabiya	Adabiya	21 Oct 1973	2	Israeli; Magen DIV	Egyptian; 3rd ARMY (-)	Israeli	10900	14630	150	800	I	6	6	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
12.14	Sinai	Shallufa I	Shallufa I	22 Oct 1973	1	Israeli; Adan DIV	Egyptian; 3rd ARMY (-)	Israeli	16200	25600	150	1100	I	2	6	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
12.15	Sinai	Suez	Suez	23 Oct 1973	2	Israeli; Adan DIV (-)	Egyptian; 3rd ARMY (-)	Egyptian (Marginal)	14680	22570	340	1100	II	2-3	14	2	Fall	A-Gd D-Gd	Minor (D)	No	(A)	Yes	Yes
12.16	Sinai	Shallufa II	Shallufa II	23 Oct 1973	2	Israeli; Adan DIV (-)	Egyptian; 3rd ARMY (-)	Israeli	11700	27570	150	1350	I	2	6	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
12.17	Golan	Ahmadiyeh	Ahmadiyeh	6 Oct 1973	2	Syrian; 7th INF BDE (+)	Israeli; 7th ARM BDE (-)	Israeli	22750	5750	700	250	I	3-4	3	2	Fall	A-Gd D-Gd	Subs. (A)	Yes	Neither	Yes	Yes
12.18	Golan	Kuneitra	Kuneitra	6 Oct 1973	2	Syrian; 9th INF BDE (+)	Israeli 7th & 188th ARM BDE (-)	Syrian	17750	3630	350	200	I	3	3	2	Fall	A-Gd D-Gd	Subs. (A)	Yes	Neither	Yes	Yes

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
12.19	Golan	Rafid	Rafid	6 Oct 1973	2	Syrian; 5th INF BDE (+)	Israeli; 188th BDE	Syrian	19530	4960	700	500	I	3-4	3	2	Fall	A-Gd D-Gd	Major (A)	Yes	Nei-ther	Yes	Yes
12.20	Golan	Yehudia-El Al	Yehudia-El Al	7 Oct 1973	2	Syrian; 5th 5th INF (+)	Israeli; Laner DIV (-)	Israeli	21980	6300	1000	300	I	2	3	2	Fall	A-Gd D-Gd	Subs. (A)	No	Nei-ther	Yes	Yes
12.21	Golan	Nafekh	Nafekh	7 Oct 1973	2	Syrian; 1st ARM DIV (+)	Israeli; Ori BDE (+)	Israeli	12500	6950	1000	500	I	2	3	2	Fall	A-Gd D-Gd	Subs. (A)	No	Nei-ther	Yes	Yes
12.22	Golan	Mt. Hermonit	Mt. Hermonit	8 Oct 1973	2	Syrian; 7th INF DIV (+)	Israeli; 7th ARM BDE (-)	Israeli	31350	5230	1200	400	I	2-3	3	2	Fall	A-Gd D-Gd	Minor (A)	No	(D)	Yes	Yes
12.23	Golan	Mt. Hermon I	Mt. Hermon I	8 Oct 1973	1	Israeli; 1st INF BDE (-)	Syrian; PARA BDE (-)	Syrian	2690	1580	50	100	I	3	3	2	Fall	A-Gd D-Gd	Minor (D)	No	(A)	Yes	Yes
12.24	Golan	Hushiniyah	Hushiniyah	8 Oct 1973	3	Israeli; Laner DIV (-)	Syrian; 1st ARM & 9th INF DIV	Israeli	12730	14680	450	1120	I	2	3	2	Fall	A-Gd D-Gd	Minor (D)	No	(A)	Yes	Yes
12.25	Golan	Tel Faris	Tel Faris	8 Oct 1973	3	Israeli; Peled Div (+)	Syrian; 5th INF & 1st ARM DIV (-)	Israeli	23750	17830	450	1500	I	2	3	2	Fall	A-Gd D-Gd	Minor (D)	No	(A)	Yes	Yes
12.26	Golan	Tel Shams	Tel Shams	11 Oct 1973	3	Israeli; Eitan DIV	Syrian; 7th INF & 3rd ARM DIV (-)	Israeli	16100	19400	530	980	I	4	3	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
12.27	Golan	Tel Shaar	Tel Shaar	11 Oct 1973	2	Israeli; Laner DIV	Syrian; 1st, 3rd ARM DIV & 9th INF DIV	Israeli	14700	21500	280	900	I	3	3	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
12.28	Golan	Tel el Hara	Tel el Hara	13 Oct 1973	1	Iraqi; 3rd ARM DIV	Israeli; Laner DIV	Israeli	12500	11000	450	40	I	2	3	2	Fall	A-Gd D-Gd	No	No	(D)	Yes	Yes

Table 3.3 (continued)

Engagement Number	Campaign	Battle	Engagement	Start Date	Duration (Days)	Attacker (A)	Defender (D)	Victor	Attacker Initial Strength	Defender Initial Strength	Attacker Casualties	Defender Casualties	Willard's Category	Engagement Type	Terrain	Weather	Season	Morale	Surprise	Set Piece	Overall Air Superiority	Close Air Support for A	Close Air Support for D
12.29	Golan	Kfar Shams-Tel Antar	Kfar Shams-Tel Antar	15 Oct 1973	1	Israeli; Laner DIV (-)	Iraqi; 3rd ARM DIV	Israeli	11000	12000	100	200	I	2	3	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
12.30	Golan	Naba	Naba	16 Oct 1973	1	Jordanian; 40th ARM BDE	Israeli; Laner DIV (-)	Israeli	11500	11000	450	100	I	2	3	2	Fall	A-Gd D-Gd	No	No	(D)	Yes	Yes
12.31	Golan	Arab Counter-offensive	Arab Counter-offensive	19 Oct 1973	1	Syrian 9th INF & Iraqi 3rd ARM DIV	Israeli; Peled DIV	Israeli	35750	16100	550	160	I	3	3	2	Fall	A-Gd D-Gd	No	No	(D)	Yes	Yes
12.32	Golan	Mt. Hermon II	Mt. Hermon II	21 Oct 1973	1	Israeli; 1st INF BDE	Syrian: PARA BDE (-)	Syrian	5700	4750	80	150	I	4	3	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes
12.33	Golan	Mt. Hermon III	Mt. Hermon III	22 Oct 1973	1	Israeli: 1st INF & 31st PARA BDE	Syrian; PARA BDE	Israeli	11400	4750	100	250	I	4	3	2	Fall	A-Gd D-Gd	No	No	(A)	Yes	Yes

DATA ELEMENT ENCODING SCHEME:

In order to conserve space, the data for three data elements are numerically encoded in the matrix. Their meanings are provided below.

Data Element	State	Code
Engagement Type	Meeting engagement	1
	Attack on a hasty defense	2
	Attack on a prepared defense	3
	Attack on a fortified defense	4
	Attack on a delaying force	5
	Attack on a withdrawing force	6
Terrain	Rugged-heavily wooded	1
	Rugged-mixed (or extra rugged-bare)	2
	Rugged-bare	3
	Rolling-heavily wooded	4
	Rolling-mixed	5
	Rolling-bare	6
	Flat-heavily wooded	7
	Flat-mixed	8
	Flat-bare	9
	Hard; Flat-desert	10
	Rolling dunes	11
	Swamp-jungled	12
	Swamp-mixed or open	13
	Urban	14
Weather	Dry-sunshine-extreme heat	1
	Dry-sunshine-temperate	2
	Dry-sunshine-extreme cold	3
	Dry-overcast-extreme heat	4
	Dry-overcast-temperate	5
	Dry-overcast-extreme cold	6
	Wet-light-extreme heat	7
	Wet-light-temperate	8
	Wet-light-extreme cold	9
	Wet-heavy-extreme heat	10
	Wet-heavy-temperate	11
	Wet-heavy-extreme cold	12

FOOTNOTES:

/A Most casualties were incurred during the retreat of the American force.

/B This action contains four distinct engagements. They consist of the Gowanus Road, Flatbush Pass, Bedford Pass and Jamaica Pass engagements. Sufficient data is not available to represent each engagement separately, however.

/C This includes 1,100 captured personnel, of whom many were wounded.

/D This does not include 2,722 unwounded personnel which were captured in a mass surrender after the breakpoint occurred.

/E This does not include 948 unwounded personnel which were captured in a mass surrender after the breakpoint occurred.

/F This action contains two distinct engagements. They consist of the Stoney Brook and Princeton engagements. Sufficient data is not available to represent each engagement separately, however.

/G This does not include 200 unwounded personnel which were mostly captured in a mass surrender after the breakpoint occurred. No further details as to when the personnel which were captured were found.

/H This does not include 228 unwounded personnel which were captured in a mass surrender after the breakpoint occurred.

/I This action consists of two sequentially distinct engagements. Sufficient data is not available to represent each engagement separately, however.

/J This action contains two distinct engagements. They consist of the Chadd's Ford and Birmingham Meeting House engagements. Sufficient data is not available to represent each engagement separately, however.

/K This is estimated from the typical sizes of the units which made up the British force.

/L This action consists of several distinct engagements. Sufficient data is not available to represent each engagement separately, however.

/M This value is estimated. Casualties were said to be negligible.

/N This value is estimated from the approximate sizes of the units which made up the American force.

/O This value is estimated.

/P This does not include approximately 410 unwounded personnel which were captured in a mass surrender after the breakpoint occurred.

/Q This does not include approximately 80 unwounded personnel which were captured in a mass surrender after the breakpoint occurred.

/R The British are suspected of having significantly understated their casualties.

/S This does not include approximately 500 unwounded personnel which were captured after the breakpoint occurred.

/T This does not include approximately 300 unwounded personnel which were captured after the breakpoint occurred.

/U Some of the casualties occurred after the Tory forces surrendered. The casualty figure does not include approximately 700 unwounded personnel which were captured in a mass surrender after the breakpoint occurred.

/V This does not include approximately 570 unwounded personnel which were captured in mass surrenders after the breakpoint occurred.

/W This is the reported casualty figure. Actual casualties are probably higher.

/X This does not include approximately 8,090 unwounded personnel which were captured in a mass surrender after the breakpoint occurred.

/Y This does not include 1,000 unwounded personnel which were captured after the breakpoint occurred.

/Z This does not include 11,000 unwounded personnel which were captured after the breakpoint occurred.

/AA This includes casualties which occurred during the Egyptian retreat.

/BB This does not include 12,000 personnel (some wounded) which were captured primarily during the Austrian retreat. No further details about this were found.

/CC This does not include 1,800 unwounded personnel which were primarily captured after the breakpoint occurred.

/DD This does not include 6,500 unwounded personnel which were primarily captured after the breakpoint occurred.

/EE This includes casualties incurred during the Russian retreat.

/FF This includes a sizable number of unwounded captured personnel. No further details about this were found.

/GG Both sides achieved surprise to some degree. The effects are assumed to cancel each other out.

/HH This action consists of two distinct engagements (Priesten and Kulm). Sufficient data is not available to represent each engagement separately, however.

/II This includes a significant number of unwounded captured personnel, which may or may not have occurred after the breakpoint was reached. No further details as to when the personnel which were captured were found.

/JJ This includes casualties incurred during the French retreat.

/KK This does not include Yorck's casualties.

/LL This includes Prussian reinforcements.

/MM This is a Prussian victory primarily due to the French being unable to break the resistance of the Prussian force (low French mission accomplishment).

/NN This action contains two distinct engagements. They consist of the Bloody Hill and Sharp's Creek engagements. Data is not available in sufficient detail to represent them separately, however.

/OO This action contains several distinct engagements. These include the Confederate breakout attempt and the Union seige operations. Data is not available in sufficient detail to represent them separately, however.

/PP This does not include approximately 14,620 unwounded captured personnel since these primarily occurred in a mass surrender after the breakpoint was reached.

/QQ This action contains three distinct engagements. The engagements consist of Leetown, Elkhorn Tavern (first day) and Elkhorn Tavern (second day). Data is not available in sufficient detail to represent them separately, however.

/RR The force was overwhelmed. This figure probably includes many personnel who were captured in a mass surrender after the breakpoint occurred.

/SS This action contains five distinct engagements. They consist of the Peach Orchard, Savage Station, White Oak Swamp, Glendale and Malvern Hill engagements. Data is not available in sufficient detail to represent them separately, however.

/TT This action contains several distinct engagements. The engagements include Manasas and Chantilly. Data is not available in sufficient detail to represent them separately, however.

/UU This action consists of several geographically distinct engagements. Data is not available in sufficient detail to represent them separately, however.

/VV This does not include 4,300 unwounded personnel which were captured after the breakpoint occurred.

/WW This action contains two distinct engagements. They consist of Marye's Hill and Deep Run/Massaponax. Data is not available in sufficient detail to represent them separately, however.

/XX This does not include 4,790 unwounded personnel which were captured in a mass surrender after the breakpoint occurred.

/YY This action contains several distinct engagements. They include Jackson's assault, Fredericksburg I and Fredericksburg II. Data is not available in sufficient detail to represent them separately, however.

/ZZ The designation of attacker and defender is not clear. The Union force advanced to contact but the battle primarily consists of the Confederate attacks about Chancellorsville. In the Fredericksburg area, while the Union force drove the Confederate force from their entrenchments, subsequent Confederate attacks forced the Union force to withdraw. Since the decisive attacks during the battle were made by the Confederate Army, it is designated the attacker.

/AAA This action contains several distinct engagements. The engagements include Gettysburg town, Peach Orchard/Devil's Den/Little Round Top, Cemetery Hill, Culp's Hill I, Culp's Hill II and Pickett's Charge. Data is not available in sufficient detail to represent them separately, however.

/BBB This action contains several distinct engagements. The engagements include Lookout Mountain, Missionary Ridge (Nov. 24), Missionary Ridge (Nov. 25; Union left flank) and Missionary Ridge (Nov. 25; center). Data is not available in sufficient detail to represent them separately, however.

/CCC This does not include 1,620 unwounded personnel which were captured after the breakpoint occurred.

/DDD This action contains several distinct engagements. The engagements consist of Buzzard's Roost, Snake Creek Gap and New Hope Church. Data is not available in sufficient detail to represent them separately, however.

/EEE This does not include 180 unwounded personnel which were captured after the breakpoint occurred.

/FFF This action contains several distinct engagements. The engagements consist of the North Bank, the South Bank and the Eastern River Valley. Data is not available in sufficient detail to represent them separately, however.

/GGG Although amphibious withdrawal was possible, it would have been extremely risky in the face of German attacks.

/HHH The engagement starts as an attack by the American force against a fortified German defense, but soon changes to the recorded engagement type.

/III The engagement starts as an attack by the American force against a German hasty defense, but soon changes to the recorded engagement type.

/JJJ The action contains several distinct engagements. These engagements include the Vietminh attack and the French relief attack. Data is not available in sufficient detail to represent them separately, however.

/KKK The engagement is characterized by HERO as an Israeli attack on an Arab prepared defense which later changes to an Arab attack on an Israeli hasty defense. It is treated as an Israeli attack on an Arab prepared defense.

/LLL The engagement is characterized by HERO as an Israeli attack on an Arab fortified defense which later changes to an Arab attack on an Israeli prepared defense. It is treated as an Arab attack on an Israeli prepared defense.

/MMM The engagement is characterized by HERO as an Arab attack on an Israeli prepared defense which later changes to an Israeli attack on an Arab prepared defense. It is treated as an Israeli attack on an Arab prepared defense.

/NNN The engagement is characterized by HERO as a fortified defense which later changes into a delaying action. It is treated as a defense of a fortified position.

/OOO The engagement is characterized by HERO as a delaying action which becomes a defense of a prepared position. It is treated as a defense of a prepared position.

/PPP The attack involves crossing a major water obstacle.

/QQQ The engagement involves an attack on a prepared defense by the Israelis which later changes into a withdrawal in the face of continuing Syrian pressure. The engagement is treated as an attack on a prepared defense.

SOURCES:

War	Author, Title, Date of Source
1. American Revolutionary War	1) Alden, John R., A History of the American Revolution, 1st ed., 1969. 2) Dupuy, T.N., Compact History of the Revolutionary War, 1963.
2. Napoleonic War	1) Britt, Albert Sidney, The Wars of Napoleon, 1973. 2) Chandler, David G., Dictionary of the Napoleonic Wars, 1979. 3) Chandler, David G., The Campaigns of Napoleon, 1966.
3. American Civil War	1) Dupuy, T.N., Compact History of the Civil War, 1960. 2) Livermore, T.L. Numbers and Losses of the Civil War, 1957. 3) Simulations Publications, Inc., Strategy and Tactics Magazine, nos. 67;76;80;86;87;89, 1978-1981.
4. Boer War	1) Belfield, E.M.G., The Boer War, 1975. 2) Pakenham, T., The Boer War, 1979.
5. Russo/Japanese War	1) Prussia. Armee. Grosser Generalstab, The Russo-Japanese War, 1908-1914 (7 vols).
6. World War II	1) Blumenson, Martin, Breakout and Pursuit, 1961. 2) Dupuy, T.N., Numbers, Predictions and War: Using History to Evaluate Combat Factors and Predict the Outcome of Battles, 1979. 3) HERO, Combat Data Subscription Service, vol. 1, no. 2, 1975. 4) Jackson, W.G.F., The Battle for Italy, 1967.
7. Korean War	1) Dupuy, T.N., Numbers, Predictions and War, 1979.
8. First Indochinese War	1) Fall, Bernard B., Hell in a Very Small Place: The Siege of Dien Bien Phu, 1966.

War	Author, Title, Date of Source
9. First Arab/Israeli War (1948)	1) HERO, Combat Data Subscription Service, vol. 1, no. 3, 1975. 2) Kurzman, Dan, Genesis 1948, The First Arab-Israeli War, 1970. 3) Lorch, Netanel, Israel's War of Independence, 1947-1949, 1968.
10. Second Arab/Israeli War (1955)	1) Barker, A.J., Arab-Israeli Wars, 1981. 2) HERO, Combat Data Subscription Service, vol. 1, no.3, 1975. 3) O'Ballance, E., The Sinai Campaign of 1956, 1960.
11. Third Arab/Israeli War (1967)	1) Dayan, David, Strike First. A Battle History of Israel's Six Day War, 1968. 2) Dupuy, T.N., Numbers, Predictions and War, 1979. 3) HERO, Combat Data Subscription Service, vol. 2, no. 1, 1976. 4) Young, Peter, The Israeli Campaign 1967, 1967.
12. Fourth Arab/Israeli War (1973)	1) Dupuy, T.N., Numbers, Predictions and War, 1979. 2) HERO, Combat Data Subscription Service, vol. 2, no. 2, 1977. 3) Herzog, Chaim. The War of Atonement, October, 1973, 1975.

Note: In addition to the sources listed, the following document was used as a general reference for each of the wars listed: Dupuy, R.E. and T.N., Encyclopedia of Military History, 1970.

3.4 Data Elements Contained within the Engagement Data Matrix

Each engagement in the engagement data matrix is represented by 24 data elements. The set contains those factors identified in the literature as being significant. It is further constrained by the availability of suitable data. The data elements this investigation uses are listed in Table 3.4. The sources of all engagement data are included in section 3.3.

Table 3.4

Data Elements Contained in Engagement Data Matrix

Number	Data Element
1	War
2	Campaign
3	Battle
4	Engagement
5	Starting Date
6	Duration (in Days)
7	Attacker
8	Defender
9	Victor
10	Attacker Initial Strength
11	Defender Initial Strength
12	Attacker Casualties
13	Defender Casualties
14	Willard's Category
15	Engagement Type
16	Terrain
17	Weather
18	Season
19	Morale
20	Surprise
21	Setpiece Effect
22	Overall Air Superiority
23	Close Air Support for Attacker
24	Close Air Support for Defender

Table 3.5 gives the variables contained within each of the elements listed above.

Table 3.5

Data Elements and Their Associated Observed Variable Subsets

Data Element No.	Name	Observed Variables
1	WAR	American Revolutionary War Napoleonic Wars American Civil War Boer War Russo/Japanese War World War II First Indochinese War Korean War First Arab/Israeli War (1948) Second Arab/Israeli War (1956) Third Arab/Israeli War (1967) Fourth Arab/Israeli War (1973)
9	VICTOR	Winner, Loser
14	WILLIARD'S CATEGORY	Category I, Category II
15	ENGAGEMENT TYPE	Meeting Engagement Attack on a Hasty Defense Attack on a Prepared Defense Attack on a Fortified Defense Attack on a Delaying Force Attack on a Withdrawing Force
16	TERRAIN	Rugged-Heavily Wooded, Rugged-Mixed (or Extra Rugged-Bare), Rugged-Bare, Rolling-Heavily Wooded, Rolling-Mixed, Rolling-Bare, Flat-Heavily Wooded, Flat-Mixed, Flat-Bare, Hard, Flat-Desert, Rolling-Dunes, Swamp-Jungled, Swamp-Mixed or Open, Urban

Table 3.5 (continued)

Data Element No.	Name	Observed Variables
17	WEATHER	Dry-Sunshine-Extreme Heat Dry-Sunshine-Temperate Dry-Sunshine-Extreme Cold Dry-Overcast-Extreme Heat Dry-Overcast-Temperate Dry-Overcast-Extreme Cold Wet-Light-Extreme Heat Wet-Light-Temperate Wet-Light-Extreme Cold Wet-Heavy-Extreme Heat Wet-Heavy-Temperate Wet-Heavy-Extreme Cold
18	SEASON	Winter, Spring, Summer, Fall
19	MORALE	Excellent, Attacker; Good, Attacker; Fair, Attacker; Poor, Attacker; Panic, Attacker; Excellent, Defender; Good, Defender; Fair, Defender; Poor, Defender; Panic, Defender;
20	SURPRISE	Major, Attacker; Substantial, Attacker; Minor, Attacker; Major, Defender; Substantial, Defender; Minor, Defender; None
21	SETPIECE EFFECT	Present, Absent
22	OVERALL AIR SUPERIORITY	Neither Side, Attacker Defender, None

Table 3.5 (continued)

Data Element No.	Name	Observed Variables
23	CLOSE AIR SUPPORT FOR ATTACKER	Present, Absent
24	CLOSE AIR SUPPORT FOR DEFENDER	Present, Absent

The first element indicates the specific war within which the engagement occurs. It is included since it enables the effect that the different historical periods have on the break behavior of a military force to be analyzed. For example, one distinction between historical periods is the increasing weapon lethality observed over history and the related increase in manpower dispersion on the battlefield (Dupuy, 1979:6). Table 3.6 lists the number of engagements for each war contained in the engagement data matrix.

Table 3.6

Number of Engagements for Each War in the Engagement Data Matrix

War	Number of Engagements
American Revolutionary War	33
Napoleonic Wars	40
American Civil War	59
Boer War	5
Russo/Japanese War	4
World War II	88
First Indochinese War	3
Korean War	1
First Arab/Israeli War (1948)	29
Second Arab/Israeli War (1956)	9
Third Arab/Israeli War (1967)	19
Fourth Arab/Israeli War (1973)	33

Elements 2, 3, 4 and 5 specify the precise time and geographic location in which the engagement takes place. All observations in the data matrix are at the engagement level. One or more engagements are contained within a battle and one or more battles are contained within a campaign (see Appendix B for precise definitions of these terms). The identification of an engagement within a battle is discussed in further detail in section 3.5. Element 5, start date, specifies the starting time of the engagement to the nearest day.

Element 6 specifies the duration of the engagement to the nearest day. The values are relatively imprecise, however. Engagements which occur over portions of a two-day period are recorded as being two days in length. In addition, no distinction is made between continuous and intermittent fighting during multi-day engagements.

Elements 7 and 8 specify the identity of the attacker and defender in the engagement. In addition to the nationality(ies) of the respective forces, the individual units involved are sometimes identified.

Element 9 indicates the victor in the engagement. The victor is determined on the basis of three criteria developed by HERO. The criteria consist of mission accomplishment, spatial effectiveness and casualty effectiveness. Mission accomplishment is a numerical assessment of ". . . the extent to which each side in an engagement accomplished its assigned or perceived mission." (Dupuy, 1979:48). Spatial effectiveness is a quantitative assessment of ". . . the extent to which each side was able to gain or hold ground." (Dupuy, 1979:48). Casualty effectiveness consists of a quantitative comparison of the casualties incurred by one force to those incurred by the opposition. Upon obtaining the three scores for each side, they are summed and compared to determine the victor (Dupuy, 1979:47-49).

This victor determination procedure was rigorously performed by HERO for all engagements contained in the HERO data base. For the remaining engagements, the guidelines are used but the judgements are qualitative.

Elements 10 and 11 contain the initial strengths for the attacking and defending forces. The addition of significant reinforcements and replacements are accounted for by attempting to ensure that this does not occur within a specified engagement. A more detailed discussion of this is contained in section 3.5.

There is error associated with the initial troop strengths for the opposing forces in an engagement. With regards to both available troop strengths and casualty data, Helmbold (1971) stated that:

> . . . there is often much scope for human error and for capriciousness in selecting the forces to be included in establishing troop strength or casualties, as well as in arriving at an accurate inventory of these quantities. (Helmbold, 1971:6)

The initial force levels, as well as other engagement data, obtained from the HERO data base are considered very accurate. Data for the other engagements has been collected by the investigator from those outlined earlier.

Elements 12 and 13 specify the casualties suffered by the attacking and defending forces, respectively. These values include the personnel killed, wounded, captured or otherwise missing prior to the breakpoint in the engagement. Helmbold's statement concerning the inaccuracies in initial force personnel strengths also holds for casualties. In addition:

> . . . personnel casualties consist of not necessarily only those inflicted prior to reaching a breakpoint. In some cases, a portion of the historically reported casualties may have occurred after the break. For example, routs sometimes degenerate into massacres, and on occasion troops that have surrendered may have been slain. (Helmbold, 1971:5)

With respect to the current investigation, attempts are made to ensure that personnel killed, wounded, captured or who become otherwise missing after the breakpoint occurs are not included in the casualty values. In particular, mass surrenders by unwounded personnel after the breakpoint occurs in an engagement are excluded wherever possible.

Element 14 indicates which of the two Willard categories the engagement is classified under. This is a relatively crude categorization of the engagements first developed by Willard (1962:2) and slightly modified by Helmbold (1971:25-26) (see Appendix B for a concise definition of each category).

Element 15 represents a categorization of engagement types on the basis of the postures of the two respective forces in the engagement. The strength of a military force is enhanced by a defensive posture and its vulnerability decreases proportional to its increase in defense readiness. Although the categories used in the investigation are obtained from work published by HERO (Dupuy, 1979:210-211,230), similar categories have been in common usage in the field prior to that time. The categories consist of a meeting engagement, an attack on a hasty defense, an attack on a prepared defense, an attack on a fortified defense, an attack on a delaying force, an attack on a withdrawing force and a holding engagement.

A meeting engagement consists of both forces moving to contact followed by predominantly offensive actions by each side against the other. This category is seldom used since, upon two forces making contact, one force usually assumes a defensive posture. These engagements are properly classified as attacks on a hasty defense.

Attacks on hasty, prepared or fortified defenses consist of an attack by one force against the other in a defensive position. The three categories differentiate among the possible relative strengths of the defensive position, independent of the size of the defending force. It is a function of the concentration of the defending force, its time in the defensive position and the presence of attached engineering troops to aid in the preparation of the defenses. Relatively precise quantitative rules are developed by HERO for distinguishing between these categories (Dupuy, 1979:210-211). The engagements from the HERO data base are classified according to these rules. For the remaining engagements in the engagement data matrix, the guidelines are used but the judgements are qualitative. This represents a significant limitation to 44% of the data contained in element 15 of the engagement data base. This is unavoidable, however, given the scope of the current investigation.

Attacks on a delaying or withdrawing force consist of an attack by one force against a force which is deliberately engaged in permanent retrograde movement. The two categories differ in the degree of resistance offered to the pursuing force. A delaying force actively resists an advancing force while it moves rearward to a new position. A withdrawing force attempts to avoid combat, if possible, during the retrograde movement. It does defend as a coherent military force if attacked but, it attempts to disengage as rapidly as possible.

The seventh category specified by HERO, holding, is not used. Holding is defined as the occupation of a position or area by a military force for the purpose of defending it but in the absence of any significant enemy attack. Contact is maintained with an opposing military force, however. It is an engagement in the sense that both forces incur losses as a result of their interactions. Furthermore, it is possibile for a military force that is holding to break because it is unable to sustain the losses incurred by the low-intensity combat interactions. However, a set of such low-intensity engagements would contain many instances where the breaking of one of the two forces is determined by primarily external events. Due to this drawback, the holding category is not used in the current investigation. This is further discussed in section 3.5.

Element 16 is a categorization of the various types of battlefield terrain. The terrain has an effect on the weapon effectiveness, mobility and posture of a military force. The set of terrain types used were developed by HERO (Dupuy, 1979:228). A complete listing is included in Table 3.5.

Elements 17 and 18 represent the various types of climate which can be present during an engagement. These categories were developed by HERO (Dupuy, 1979:229). A complete listing is included in Table 3.5. The following distinction is made between the two factors. Weather (element 17) pertains to the specific conditions at the time of the engagement while the season indicates the long-term climatic effects that the forces are operating under. Weather has an effect on the weapon effectiveness, mobility and posture of a military force. The season affects weapon effectiveness and reflects the significance of changes in the hours of daylight and darkness in the Temperate Zone.

Element 19 accounts for the various levels of morale which can be present in the two respective forces in the engagement. The categories used were defined by HERO (Dupuy, 1979:231). A listing of the levels is included in Table 3.5.

Due to the imprecision in this behavioral variable, conservative classification rules are adopted. 'Good' morale is assumed for a military force unless historical evidence clearly indicates otherwise. In addition, only two groupings of the five categories are used. The first consists of good and excellent morale while the second contains fair morale, poor morale and panic. Based on these groupings, the presence or absence of significant differences in morale are identified. This is discussed in more detail in section 3.6. These guidelines are adopted in order to ensure that only significant differences in morale are considered.

Element 20 indicates the level of surprise achieved by the attacking or defending force. The levels were developed, and their significance verified, by HERO (Dupuy, 1979:63-64,231). A listing of the surprise levels is included in Table 3.5.

The engagements in the HERO data base are already classified with respect to the level of surprise achieved by the attacking or defending force. Conservative classification rules are adopted for the remaining engagements. Surprise is not recorded unless the historical record clearly indicates that it has been achieved by the attacking or defending force. In addition, only the presence, not the level, of surprise is used in the analysis. This is discussed in further detail in section 3.6.

Element 21 indicates the presence or absence of a setpiece effect. This represents the advantage that an attacking force has as a result of intensive preparations against a specific defensive position. These preparations may include a rehearsal of the assault against dummy positions. This effect is applicable only to an attacking force which is, otherwise, inferior to the defending force. This effect was proposed and verified by HERO (Dupuy, 1979:203).

Element 22 indicates the presence or absence of overall air superiority for one of the two sides in an engagement. The presence of overall air superiority has four effects on the engagement. The achievement of air superiority for a military force:

1) Enhances the effectiveness of its own tactial air support while degrading the opposition's.

2) Slightly enhances the effectiveness of its artillery while degrading the effectiveness of hostile artillery.

3) Slightly enhances its own ground mobility while substantially degrading the mobility of the opposing military force.

4) Reduces the vulnerability of its own force while increasing that of the hostile force. (Dupuy, 1979:77)

Four categories, or conditions, are used in this investigation. The first two conditions, attacker air superiority or defender air superiority, are present when one side's combat airpower dominates. The third condition, air superiority not being achieved by either side in an engagement, exists when, despite the presence of combat airpower on both sides, neither side has achieved overall air superiority. The fourth category indicates that no combat aircraft are available to either military force. This primarily consists of cases where militarily-significant airpower is not present in the time period within which the engagement occurs.

Elements 23 and 24 indicate the presence or absence of close air support for the attacking or defending forces, respectively. On the basis of work performed by HERO (Dupuy, 1979:71-76), this represents only close air support sorties actually flown in support of engaged ground troops.

Data for each of the 24 elements for each engagement is included in the engagement data matrix. The identification of which interactions between military forces constitute engagements is discussed in the following section.

3.5 Engagement Identification

All observations in the data matrix are at the engagement level. The identification of an engagement within a battle is performed on the basis of two factors. A significant degree of independence of the actions of the engaged forces on both sides must exist with respect to the remainder of the battle. In addition, the military units for each side are required to have unity of mission for the engagement's duration.

The termination of an engagement represents a change in the state of at least one of the two military forces involved in the engagement. Five states are identified for a military force involved in active combat operations. The effects of temporary pauses in operations and the addition of reinforcements and replacements to the initial forces are also accounted for.

The following subsection describes the importance of unity of mission in engagement identification.

3.5.1 Unity of Mission

The aggregation of units into an engagement is performed, in part, on the basis of the unity of mission exhibited. For example, a U.S. infantry battalion in World War II is an aggregation of three infantry companies, a headquarters company and a heavy weapons company (Clark, 1954:12). Each of these companies are subdivided into platoons, sections, squads and, lastly, individual soldiers. Within a particular battalion-level operation, if a high degree of dependence is present among the subordinate units in carrying out their respective missions, then their combination into a single battalion-level operation is required. In this instance, the identification of the operation of the subordinate units as separate engagements would bias the results of the analysis since the engagements would contain a high degree of interdependence. The proper level of aggregation is dependent on the nature of the operation being conducted by the overall military force.

It is also necessary to account for changes in military operations which occur during the course of the battle. It is conceivable for a military unit to be involved in a number of operations during a battle, each of which possibly constitutes an engagement. Military units could be involved in a series of engagements each having the same objective or be involved in a series of engagements each having different objectives. The following example illustrates the identification of distinct engagements within a particular battle. The battle consists of two primary operations. The first consists of one component battalion of a three-battalion infantry regiment, force X, having the objective of blocking the line of retreat of a reduced enemy infantry regiment, force Y, consisting of two infantry battalions. Guarding force Y's line of retreat is one component infantry company. Concurrent with this operation, the remainder of force X (two infantry battalions plus support) is to attack the bulk of force Y (two infantry battalions plus support less one infantry company). The encircling attack by force X's infantry battalion is successful and force Y's line of retreat closed. Force X's main assault, however, fails. As a result of this, the battle ends with the encircling battalion of force X being forced to withdraw from its exposed position.

At least two sufficiently independent engagements occur, the first being a success and the second a failure from the perspective of force X. Dependencies such as the bulk of force Y being surrounded as a result of the success of force X's encircling attack are accounted for by classifying force X's main attack as a Willard category II engagement. A possible third engagement, the withdrawal of the encircling force X infantry battalion, is strongly influenced by the failure of force X's main attack. Since this is an external event, the operation is not included as an engagement in the engagement data matrix. If the infantry battalion is forced to withdraw as a result of an attack against it by part or all of force Y, however, then it is treated as a third distinct engagement.

3.5.2 Engagement Termination

In addition to unity of mission, the complete identification of an engagement requires a precise termination point. The termination point used by Clark consists of ". . . the beginning of a period of relative inactivity following definite success or failure in achieving the mission." (Clark, 1954:9). A similar approach was used by Helmbold (1971:1,2). The current investigation uses a more precise structuring of these concepts coupled with the incorporation of the set of engagement types developed by HERO (Dupuy, 1979:230).

The termination of an engagement represents a change in the state of at least one of the two military forces involved in the engagement. Five possible states are identified for a military force involved in active combat operations. These are depicted in Figure 3.2.

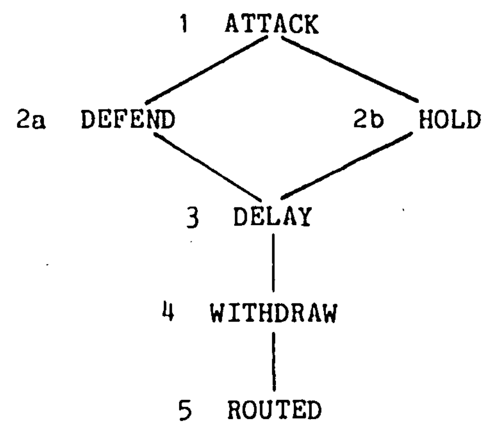

Figure 3.2

Military Force States

A military force is in a state of attack when it is engaged in an offensive operation against an opposing military force. This holds true regardless of the state of the opposing military force.

The second possible state consists of a military force in a fixed defensive posture. The defensive posture is fixed in the sense that permanent retrograde movement by the defending force is not envisioned. A mobile defense, however, is included in this state. The distinction between defend and hold is that a unit defends against an enemy attack while holding consists of the occupation of a position or area for the purpose of defending it but in the absence of any significant enemy attack. In both cases, however, contact is maintained with the opposing military force. The importance of this distinction is made later in this section.

The delay and withdraw states consist of military forces engaged in permanent retrograde movement with respect to an initial position, usually the initial line of contact with the opposing military force. These states differ in the degree of resistance offered to the pursuing force. A delaying force actively resists the opposing military force while it moves rearward. A withdrawing force avoids combat, if possible, during the retrograde movement. If attacked, it defends as a coherent military force, but attempts to disengage as rapidly as possible. In both cases, it is assumed that the opposing military force is attempting to engage the delaying or withdrawing force.

Lastly, the routed state consists of military units which, at least temporarily, have lost all ability to function as a coherent military force. Within this state it is possible for small, relatively insignificant, subsets of the military force to remain combat effective.

In both Clark's and Helmbold's studies, engagement termination is synonymous with one of the two military forces moving from a higher to a lower state. If the victor is the attacker, the defender moves from the defend to a lower state. If the victor is the defender, the attacker moves from the attack to a lower state.

These movements are included in the set of possible state changes for the current investigation. In addition, there are movements associated with the delay, withdraw and routed states. It is not necessary for a military force to drop to the next lower state, given that it does drop. For example, there have been historical engagements where an unsuccessful attack has led directly to the attacking force's withdrawal or rout.

The termination of an engagement could also occur as a result of a military force moving upward in state. This may, or may not, occur with a simultaneous movement downward in state by the opposing military force. The possible state changes are indicated in Table 3.7.

Table 3.7

Possible Transitions to Higher States

Initial State	Final State			
	Withdrawal	Delay	Defend (or hold)	Attack
Routed	x	x	x	x
Withdrawal	N/A	x	x	x
Delay	N/A	N/A	x	x
Defend (or hold)	N/A	N/A	N/A	x

x = possible higher state
N/A = not applicable

If the upward movement of the military force to a state other than the attack state is coupled with a movement downward from the attack state by the opposing military force, then no second engagement occurs. If the upward movement in state of the military force is coupled with no change in the attack state of the opposing military force, this results in the start of a new engagement. The initial engagement is considered terminated and the new engagement begun because the change in posture in one of the military forces significantly changes the nature of the engagement.

There are three additional situations which need to be accounted for when identifying the conditions under which an engagement is terminated. These consist of temporary pauses in operations, the replacement of losses in currently engaged military forces and the addition of reinforcements to one or both military forces in the engagement. The occurrence of any of the three conditions represents a significant change in the engagement. To account for this, guidelines are employed which ensure that none of the three conditions occur within an engagement. The guidelines for each situation are discussed below.

A temporary pause in combat operations of any significant length constitutes a termination of the engagement. This usually occurs in concert with the addition of reinforcements or replacements for the military force.

The addition of significant reinforcements to one or both engaged military forces significantly changes the nature of the engagement. Reinforcements are previously uncommitted military forces with respect to the particular engagement.

To account for this, HERO (1979) employs a rule stipulating that, when significant reinforcements enter into an engagement, in effect it becomes a new engagement, with the original engagement coming to an end. This is in recognition of the occurrence of either of two conditions:

1) A military force moving to a higher state upon the addition of reinforcements. This is already accounted for since an upward movement in state by one of the engaged military forces indicates the termination of the engagement.

2) A military force drops to a lower state if not for the addition of reinforcements. This does not coincide with a change in state. In this event, the addition of reinforcements at least alters the duration of the engagement and the personnel casualties incurred by each side, if not the outcome of the engagement itself.

The third situation concerns the replacement of losses incurred by a military force during an engagement. Clark found that the effects of replacements on a military force conducting military operations are significant. In the case of military units which eventually break, replacements enable the military unit to remain in its initial state for a significantly longer period of time than would otherwise be the case (Clark, 1954:34-35). It appears reasonable to infer that, given less adverse conditions, replacements allow a military force to move to a higher state or maintain its initial state for the duration of the military operation.

The possibility of the replacement of losses incurred by a military force while still engaged in a particular engagement as defined in this study is slim, however. Clark noted that, for the set of World War II data, replacements did not reach the military unit during the first week of operations (Clark, 1954:20). Much faster replacement rates were observed in the 1973 Arab-Israeli conflict (Barker, 1974:123). For example, as of 16 October 1973, the Israeli 14th Armored Brigade had suffered nearly 100% casualties in tanks, along with a percentage of the tanks' personnel, in each of the following three engagements:

7 October 1973, (Egyptian) Third Army Buildup
8 October 1973, Kantara/Firdan
15/16 October 1973, Chinese Farm I.

The brigade was replenished with respect to men and equipment between engagements. However, no instances where losses incurred by a military unit in a particular engagement being replaced during the same engagement were found in the literature for that war. Furthermore, this situation does not occur in any of the engagements included in the engagement data matrix. In the event that significant replacement of losses in a combat unit within an otherwise singular engagement does occur, however, it would be handled in a fashion identical to that for reinforcements.

3.5.3 The Effect of Data Limitations

The ability to identify engagements is constrained by the available historical combat data. In many cases, the engagements in the engagement data matrix contain levels of data aggregation not based on the procedures ennunciated in the previous sections. These are used because sufficient data at a lower level of data aggregation is not available to the investigator. These limitations have an effect on essentially all of the guidelines for identifying military engagements. The engagement data matrix for the current investigation contains 29 cases where unavoidable aggregation of engagements occurred due to aggregated force strength and/or force casualty data alone. For further details concerning these limitations, please refer to section 3.3.

These constraints on the availability of historical combat data affect the type and quantity of data included in the engagement data matrix. The next section discusses its impact on the methodology used for this investigation.

3.6 Transformation of the Data

The research question stated in Chapter 1 for this investigation is answered with respect to the set of engagements as identified by the combat data elements used to specify the engagements. The data elements within the matrix, however, can not be directly used. The set of values within almost all of the data elements are at the nominal level of measurement. This level of measurement is not suitable for the methodology used.

In order to correct for this, the variables used in the investigation are based on the different values contained in a data element. These are referred to as observed variables. The observed variables are indicator variables (taking on values of 0 or 1) by virtue of the fact that they indicate the absence or presence of a particular condition in a specified engagement. The variables are also dichotomous and provide a ratio level of measurement. This is based on the following:

> Although a rank order may not be inherent in the category definitions, either arrangement of the categories satisfies the mathematical requirements of ordering which is the requirement for ordinal-level measurement. The requirement of a distance measure based on equal-sized intervals is also satisfied because there is only one interval naturally equal to itself which is the requirement for interval-level measurement. (SPSS, 1975:5).

The ration level of measurement requires that all the properties of an interval scale be obeyed and, in addition, that the zero point is inherently defined by the measurement scheme. The observed variables contain ratio-level measurements that meet this requirement (a value of 0 for an indicator variable means the absence of the effect). This level of measurement is adequate for all of the statistical techniques used in the investigation, in particular since it is the highest level of measurement in the traditional classification of measurement levels developed by S.S. Stevens (1946).

The group of observed variables contained within a data element is collectively referred to as the subset of observed variables for that data element. The data elements treated in this fashion include war (1-- the number refers to the number of the data element in Table 3.3), victor (9), Willard's category (14), engagement type (15), terrain (16), weather (17), season (18), morale (19), surprise (20), setpiece effect (21), overall air superiority (22), close air support for attacker (23) and close air support for the defender (24). Please refer to Table 3.5 for a complete listing of the observed variables for each data element.

The remaining data elements are employed differently. The campaign (2), battle (3), engagement (4), starting date (5), attacker (7), and defender (8) data elements are used for engagement identification purposes only. The attacker initial strength (10), defender initial strength (11), attacker casualty (12) and defender casualty (13) data elements are used in constructing cumulative casualty fraction distribution curves.

The attacker and defender initial strengths are also used to represent the magnitude of the engagement. This requires two transformations of the data elements. The first concerns the sum of the personnel strengths of the two opposing military forces. The range of the combined personnel strengths are divided into several intervals. The interval boundaries are arrived at by dividing the engagements in the engagement data matrix into five equal groupings. The intervals are listed in Table 3.8.

Table 3.8

Combined Personnel Strength Summation Intervals

Interval	Combined Personnel Strength Range
1	0-20,000
2	20,001-68,000
3	68,001 and up

Observed variables are defined for each interval.

The second transformation concerns the force ratio. It is commonly believed that a personnel strength ratio of 3:1 is sufficient for an attacker to overcome a defender in a wide range of circumstances (Dupuy, 1979:5). In accordance with this, the observed variable subset contains two observed variables. These consist of the force ratio less than 3:1 and the force ration 3:1 or greater. These observed variables are indicator variables as well.

Due to the techniques used in the present analysis, the set of observed variables just described are recombined into a smaller set for the sake of data managability. The set of recombined observed variables is listed and described in Table 3.9. This recombined set of observed variables is the basis for the specification of the break curves tested in the investigation. This is elaborated upon in the following subsections.

Table 3.9

Observed Variables List (Recombined)

Number	Variable Name	Description
1	ENGN1	Indicates that the engagement occurred in the American Revolutionary or Napoleonic wars.
2	ENGN2	Indicates that the engagement occurred in the American Civil, Russo-Japanese, or Boer wars.
3	ENGN3	Indicates that the engagement occurred in WW II, Korean war, First Indochinese war or one of the four Arab/Isreali wars.
4	DURDAY1	engagement lasting one day or less
5	DURDAY2	engagement lasting one day or less
6	DURDAY3	engagement lasting one day or less
7	DURDAY4U	engagement lasting one day or less
8	WILCAT1	indicates a category 1 Williard engagement
9	WILCAT2	indicates a category 2 Williard engagement
10	ENGTYP1	indicates a meeting engagement or an attack on a hasty defense
11	ENGTYP2	indicates an attack on a pepared defense

Table 3.9 (continued)

Number	Variable Name	Description
12	ENGTYP3	indicates an attack on a fortified defense
13	ENGTYP4	indicates an attack on a delaying or withdrawing force
14	TERR1	indicates a rugged-type terrain
15	TERR2	indicates a rolling-type terrain
16	TERR3	indicates a flat-type terrain
17	TERR4	indicates an urban-type terrain
18	TERR5	indicates a very soft type of terrain (i.e. sand dunes, swamp)
19	WEAT1	indicates dry-sunshine-extreme heat weather conditions
20	WEAT2	indicates dry-temperate weather conditions
21	WEAT3	indicates dry-estreme cold weather conditions
22	WEAT4	indicates wet-extreme heat weather conditions
23	WEAT5	indicates wet-temperate weather conditions
24	WEAT6	indicates wet-extreme cold weather conditions
25	SEASN1	indicates that the engagement was fought in the winter
26	SEASN2	indicates that the engagement was fought in the spring
27	SEASN3	indicates that the engagement was fought in the summer
28	SEASN4	indicates that the engagement was fought in the fall

Table 3.9 (continued)

Number	Variable Name	Description
29	MOR1	indicates a significant morale advantage existed for the attacker
30	MOR2	indicates a significant morale advantage existed for the defender
31	MOR3	indicates that no significant morale advantage existed for either the attacker or the defender
32	BTHSURN	indicates that neither side achieved surprise against the other
33	ATTSURY	indicates that the attacker achieved surprise
34	DEFSURY	indicates that the defender achieved surprise
35	SETEFFO	indicates the absence of the setpiece effect
36	STEFF1	indicates the presence of the setpiece effect
37	AIRSPRO	indicates the absence of air superiority for either side in the engagement
38	AIRSPR1	indicates the presence of air superiority for the attacker
39	AIRSPR2	indicates the presence of air superiority for the defender
40	AIRSPR3	indicates that military-significant airpower was not a factor in the historical period within which the engagement took place
41	CAIRO	indicates that close air support was not available to either side in the engagement

Table 3.9 (continued)

Number	Variable Name	Description
42	CAIR1	indicates that close air support was available to the attacker in the engagement
43	CAIR2	indicates that close air support was available to the defender in the engagement
44	CAIR3	indicates that military-significant airpower was not present in the historical period within which the engagement took place
45	TPERSTR1	indicates that the combined personnel strength of both sides in the engagement totaled less than 20,000 men
46	TPERSTR2	indicates that the combined personnel strength of both sides in the engagement totaled between 20,001 and 68,000 men
47	TPERSTR3	indicates that the combined personnel strength of both sides in the engagement totaled more than 68,000 men
48	FRATIO1	indicates that the attacking forces in the engagement had a personnel strength greater than or equal to triple that of the defending forces
49	FRATIO2	indicates that the attacking forces in the engagement had a personnel strength less than triple that of the defending forces

3.7 Methodology

An outline of the procedure followed in the analysis is given in Table 3.10. Following that is a more detailed discussion of the steps taken.

Table 3.10

Research Methodology Steps

Step	Description	Name of Program(s) Where This is Implemented
1	The reduction of the set of observed variables to a simplified set of defined factors using factor analysis.	SSPS Subprogram FACTOR, FACTCOM
2a	The identification of all possible combinations of one or more factors together with their associated engagement subsets.	HLMBD
2b	Construction of the attacker and defender cumulative casualty fraction distributions for each engagement subset.	HLMBD
2c	Generation of data points for the ψ_{EMP} and ψ^{-1}_{EMP} functions.	HLMBD
2d	Using regression, fit the best functional form to the ψ_{EMP} data set.	HLMBD
2e	Obtain the inverse to the ψ_{CALC} curve obtained in step 2d. Defined as ψ^{-1}_{CALC}.	HLMBD
2f	Determine the probability that the set of discrete data points for ψ^{-1}_{EMP} are contained within the continuous distribution represented by ψ^{-1}_{CALC}. Identify those factor combinations whose associated engagement subsets obey Helmbold's Theorem reasonably well.	HLMBD

In this investigation it is necessary to examine all possible combinations of one or more types of factor combinations within the particular types of engagements identified by the observed variables. Unfortunately, this results in the identification of a very large number of observed variable combinations. Figure 3.3 depicts the number of possible combinations as a function of the number of observed variables.

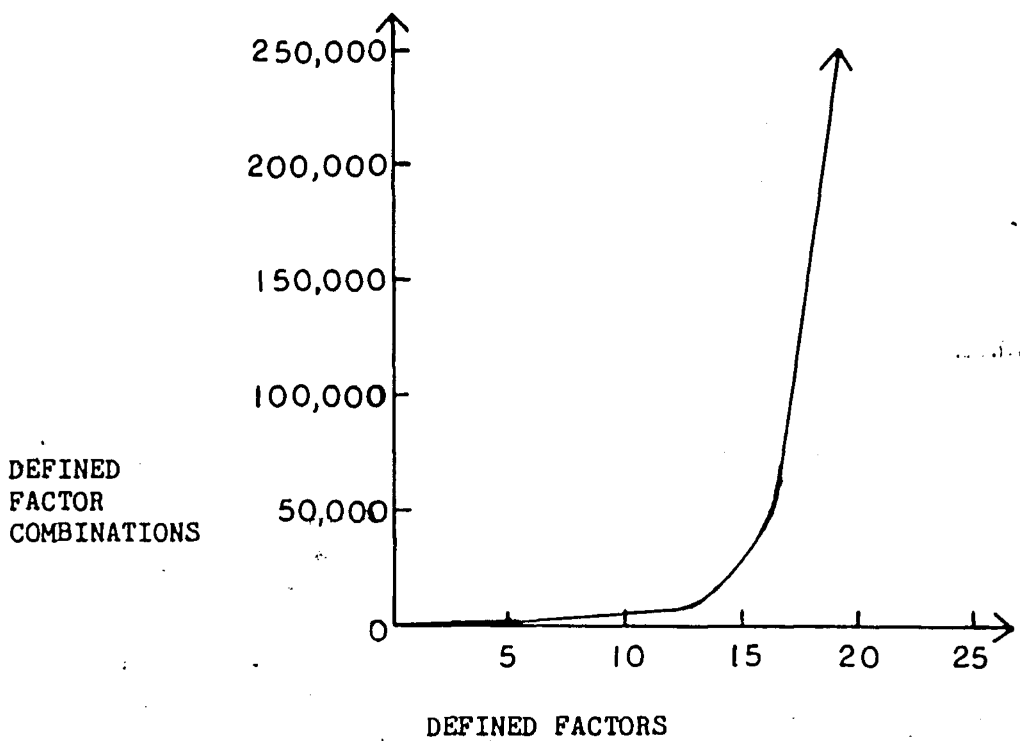

Figure 3.3

Possible Combinations of Common Factors

This problem is alleviated by using factor analysis to achieve the reduction of the set of observed variables to a set of underlying factors which accounts for most of the variation present in the data. The factor analysis is performed using the Statistical Package for the Social Sciences (SPSS, 1975). The initial extraction of factors is performed using the principle components method. The primary motivation for this is that the observed variables are each dichotomous, and that dichotomous variables should not be used in classical factor analysis (Kim and Mueller, 1978:74). Nevertheless, it is acceptable to use dichotomous variables in principle components analysis, since it is an exact mathematical transformation of the data and does not hypothesize factorial causation and the specification of common and unique components.

The rotation to a terminal solution is performed using one of the three methods of orthogonal rotation available in SPSS. These methods consist of Quartimax, Varimax and Equimax. Quartimax minimizes the complexity of the observed variables with respect to the number of defined factors upon which it loads. Varimax minimizes the complexity of the defined factors with respect to the number of observed variables which each contains. Equimax is a compromise between the first two. The method used is the one which best represents the significant differences between the engagements in the engagement data matrix.

The rotated factor matrix is then reduced via the identification of those observed variables which load significantly on the rotated factor matrix. This is accomplished for each observed variable by identifying the smallest set of defined factors which ensures that at least 75% of the variance accounted for in the original rotated factor matrix be accounted for in the reduced rotated factor matrix. This is performed in program FACTCOM. FACTCOM then explicitly relates the defined factor set to its significant component variables and the engagements within which they are presented via the Defined Factor/Observed Variable matrix and the Defined Factor/Engagement matrix.

These matrices are then input to program HLMBD where the main portion of the analysis is performed. Program HLMBD evaluates all potential combinations of the defined factors, or factor combinations, that have been identified as a result of the factor analysis. Each factor combination is identified and undergoes preliminary screening to determine its suitability. These matrices are then input to program HLMBD where the main portion of the analysis is performed. Program HLMBD evaluates all unique combinations of the defined factors, or defined factor combinations, that have been identified as a result of the factor analysis. Each defined factor combination is identified as a result of the factor analysis. Each defined factor combination is identified and undergoes preliminary prescreening to determine its suitability.

There are two reasons for the prescreening of the unique defined factor combinations that are identified. The first is the removal from consideration those defined factor combinations whose evaluation adds little to the results of the analysis. The second is the reduction in defined factor combinations to a number which is realistic from the standpoint of computer resources. There are three prescreening techniques identified for use in this investigation. All three are based on the associated engagement subset being key to the evaluation of a defined factor combination. Note that, regardless of the prescreening techniques used, the set of all engagements is evaluated to serve as a baseline against which to compare the results of the evaluation of the other selected defined factor combinations.

The first prescreening technique involves the rejection of all defined factor combinations whose associated number of engagements meets or exceeds a maximum number of engagements specified as a fraction of the total engagement set. This technique is based upon the logic that, beyond a certain engagement subset size, the results of its evaluation differs little from the results obtained by evaluating the entire engagement set. This is the initial prescreening technique to be employed and uses a cutoff value of 0.80, or 80% of the total engagement set.

The second prescreening technique consists of the rejection of all defined factor combinations whose associated number of engagements is less than a specified minimum number of engagements. The basis for this technique is that, below a certain number of engagements, the ψ_{CALC} equation fits and their associated descriptive statistics become invalid. This technique is used only if the first prescreening technique is not sufficient in reducing the number of defined factor combinations with respect to available computer resources.

The third prescreening technique rejects any defined factor combination whose associated engagement subset is not unique with respect to the other defined factor combinations to be evaluated. This prescreening technique is the most demanding in terms of computer resources and is used only if the first two prescreening techniques are not sufficient.

For each defined factor combination which is not rejected by the prescreening process, the following procedure is performed. The attacker and defender cumulative casualty fraction curves are generated for the attacker-win and defender-win portions of the associated engagement subset. The attacker and defender cumulative casualty fraction curves for the attacker-win portion are used to generate data points for the $\psi^{-1}{}_{EMP}$ data set. The attacker and defender cumulative casualty fraction curves for the defender-win portion are used to generate data points for the ψ_{EMP} data set.

Given the ψ_{EMP} and $\psi^{-1}{}_{EMP}$ data sets, the test is made to determine whether the inverse is, in fact, inverse. Using bivariate regression, the best available functional form is then fit to the ψ_{EMP} data set. The three functional forms evaluated for this investigation are listed in Table 3.11.

Table 3.11 Functional Forms Fit to ψ_{EMP} Data Set

Form	Mathematical Description
linear	$y = b_0 + b_1 x$
power	$y = B_0 X^{b1}$
log	$y = b_0 + b_1 (\log x)$

The best of the three functional forms is then selected on the basis of each fitted equation's multiple correlation coefficient squared and its associated F ratio. The multiple correlation coefficient squared shows the amount of variance in the ψ^{-1}_{EMP} data set accounted for by the fitted equation. The F ratio judges the significance of the value of the multiple correlation coefficient squared statistic. The fitted equation is defined as ψ_{CALC}.

The mathematical inverse to ψ_{CALC}, or ψ^{-1}_{CALC}, is then examined to determine the degree to which the ψ^{-1}_{CALC} represents the variation present in the ψ^{-1}_{EMP} data set. The basic quality of the fit is again performed using the multiple correlation coefficient squared and associated F ratio statistics relating the ψ^{-1}_{CALC} equation and the ψ^{-1}_{EMP} data set. The subset of defined factor combinations are identified whose associated engagement subsets best obey Helmbold's Theorem. In particular, the results obtained for each associated engagement subset is compared to the full engagement set evaluation results.

The results are then checked using discriminant analysis. The discriminant analysis is performed using the subprogram DISCRIMINANT available in SPSS. The full set of observed variables are entered into a single run of subprogram DISCRIMINANT. Using two groupings; the winners and losers of the engagements contained in the engagement data matrix, the subset of observed variables which are significant in determining the winner and loser in an engagement are identified. Observed variable selection is performed using a stepwise procedure using th Wilks lambda criterion.

This criterion is described as follows:

> The Wilks criterion is the overall multivariate F ratio for the test of differences among the group centroids. The variable which maximizes the F ratio also minimizes Wilks' lambda, a measure of group descrimination. This test takes into consideration the differences between all centroids and the cohesion (homogeneity) within the groups (SPSS, 1975:447).

Coupled with this criterion is the specification of the minimum level of significance for an observed variablle to be designated (and remain designated) significant during the stepwise procedure. This is indicated by a minimum partial F ratio, which is:

> the likelihood ratio of equality of the test variable over all groups, given the distribution produced by the variables already entered. Expressed another way, this is a test for the statistical significance of the amount of centroid separation added by this variable above and beyond the separation produced by the previously entered variables (SPSS, 1975:453).

For this investigation the default minimum partial F value = 1.0 is used. This is associated with a minimum significance level of 0.50. This ensures that any observed variables with discriminatory power is included in the set of significant observed variables.

The set of significant observed variables found using discriminant analysis is compared to the original results. The degree to which the observed variables used in the construction of good breakcurves are found to be significant in deterimining the winner and loser in a specified type of engagement strengthens the validity of the initial results.

3.8 Summary

This chapter describes the basis for the current investigation. The type, quantity and limitations of the data contained in the engagement data matrix are discussed. In addition, the analytical steps contained in the methodology are described. The next chapter documents the results of the analysis. This includes the results of the data reduction process, the construction, testing and identification of meaningful breakcurves, their aggregation into breakcurve sets and the independent examination of their validity using discriminant analysis.

Chapter 4

RESULTS

In this chapter the results obtained by following the procedure contained in chapter 3 are described and discussed. Factor analysis is used to reduce the set of 49 observed variables specifying each engagement to a total of 21 underlying defined factors. Of the potential defined factor combinations, prescreening results in the identification of 125 defined factor combinations for further evaluation of their associated engagement subsets. With respect to hypothesis testing, it is found that many of the defined factor combinations' associated engagement subsets have $\psi^{-1}{}_{CALC}$ curves which fit the $\psi^{-1}{}_{EMP}$ data set reasonably well. Using the set of all engagements as a baseline for comparison, 22 defined factor combinations achieved $\psi^{-1}{}_{CALC}$ curve fits which are better on the basis of their multiple correlation coefficient squared and associated F statistics. The observed variables contained within these defined factor combinations are identified and the implications of their presence briefly discussed. Discriminant analysis to distinguish between winners and losers in an engagement using the engagement data matrix results in some agreement, but some differences as well, in the observed variables which are significant in determining the outcome of the engagement. These areas of agreement and differences are discussed.

4.1 Factor Analysis for Data Reduction

Factor analysis is performed on the observed data set in order to reduce the number of combinations to be evaluated during the testing of the breakpoint hypothesis. As mentioned in section 3.7, in order to meet the requirement that the observed variables entered into the factor analysis be independent, then for the set of observed variables there must be a subset of joint reference category variables identified and kept separate. The factor analysis is, therefore, performed in two segments. The first segment consists of the reduction of the subset of 14 joint reference category variables to 6 defined factors. The second segment consists of the reduction of the remaining 35 observed variables to 15 defined factors. With regards to which factor rotation method to use, for each of the two factor runs the VARIMAX and QUARTIMAX rotational methods are used to rotate the factors initially extracted to a terminal solution. Within each of the two segments the rotated factor matrix produced by the factor analysis is the same. Given this, the VARIMAX rotational method is selected and used since it is the default method for subprogram FACTOR.

The rotated factor matrix produced for each of the two observed variable segments are then separately input into the factor combination program (program FACTCOM). A listing of the computer source code for FACTCOM, written in the FORTRAN IV computer language, is available in the Industrial Engineering department office located at Lehigh University. As stated in section 3.7, program FACTCOM reduces the rotated defined factor matrix by identifying the least number of defined factors which, for a specified observed variable, account for most of the observed variable's variance. For this investigation, it is required that 75% of the variance accounted for in the original rotated factor matrix be accounted for in the reduced rotated factor matrix.

For each segment, at the conclusion of this reduction process, two matrices are generated to be used later in the analysis. These include the Defined Factor/Observed Variable matrix and the Defined Factor/Engagement matrix.

The Defined Factor/Observed Variable matrix is a direct result of the reduction process of the rotated factor matrix discussed above. The Defined Factor/Observed Variable matrices for each of the two observed variable segments are shown in Tables 4.1 and 4.2, respectively. A number one in the cell for a particular observed variable/defined factor pair indicates that the observed variable is represented by that defined factor. A blank in the cell for a particular observed variable/defined factor pair indicates that the observed variable is not represented by that defined factor. The Defined Factor/Observed Variable matrices for each of the two observed variable segments contain the net reduction of the data in the observed variable set that is achieved through factor analysis.

The observed variable/defined factor relationships identified in the Defined Factor/Observed Variable matrices are used to generate the Defined Factor/Engagement matrix for each of these two observed variable segments. As mentioned in section 3.7, engagements are identified with specific defined factors according to the following

Table 4.1

Defined Factor/Observed Variable Matrix for the First Observed Variable Segment

Observed Variable	Defined Factor 1	2	3	4	5	6
ENGN1	1					
DURDAY1	1					
WILCAT1				1		
ENGTYP1	1			1		
TERR2						1
WEAT2			1		1	
SEASN2					1	
MOR3			1			
BTSURN		1		1		1
SETEFFO		1				
AIRSPR3	1					
CAIRO			1			1
TPERSTR1				1	1	
FRATIO1		1		1		

Table 4.2

Defined Factor/Observed Variable Matrix for the Second Observed Variable Segment

Observed Variable	Defined Factor 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
ENGN2	1			1											
ENGN3	1			1											
DURDAY2	1												1		
DURDAY3	1												1		
DURDAY4U											1				
WILCAT2	1									1	1	1		1	1
ENGTYP2								1	1						
ENGTYP3								1							
ENGTYP4									1						
TERR1		1	1	1				1		1				1	
TERR3	1	1							1						
TERR4														1	
TERR5										1					
WEAT1			1												
WEAT3		1													
WEAT4										1					
WEAT5			1		1		1	1				1			
WEAT6		1							1					1	
SEASN1		1													
SEASN3							1								
SEASN4				1			1								
MOR1			1												
MOR2						1					1		1		1
ATTSURY			1	1	1		1				1				1
DEFSURY															1
SETEFF1					1										
AIRSPR0			1	1	1		1								
AIRSPR1	1														
AIRSPR2		1		1								1			1
CAIR1	1			1											
CAIR2												1			
CAIR3				1											
TPERSTR3						1									
TPERSTR2			1			1									
FRATIO2					1										

rule:

> If any of the observed variables that are present in an engagement are represented by a given defined factor, then that engagement is included in that defined factor's associated engagement subset.

A sampling of the Defined Factor/Engagement matrix generated for the second segment of observed variables is given in Table 4.3. The contents of this matrix are used by program HMBLD in identifying the associated engagement subsets for each of the factor combinations evaluated. A complete listing of the Defined Factor/Engagement matrices for each of the two observed variable segments is not provided since the relationship between an engagement and a defined factor can be identified using one of the Defined Factor/Observed Variable matrices, the relationships between observed variables and combat factors (Table 3.9), and the engagement data matrix (Section 3.3).

4.2 Breakpoint Hypothesis Testing

This portion of the analysis involves the identification and evaluation of the validity of the breakpoint hypothesis for all unique defined factor combinations, subject to defined factor prescreening requirements. This is performed using program HMBLD. A listing of the computer source code, written in the FORTRAN IV computer language, is also located at the Industrial Engineering department office at Lehigh University. Using the Defined Factor/Engagement matrix generated in program FACTCOM, program HMBLD identifies, prescreens and evaluates the unique defined factors in accordance with the methodology presented in section 3.7. All unique defined factor

Table 4.3

Sample of the Factor/Engagement Matrix Generated for Segment 2

	Factor														
Engagement	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1					1			1	1						
2		1			1			1	1					1	
3			1	1	1	1	1				1				1
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
321	1	1	1	1	1	1	1	1	1	1		1		1	1
322	1	1	1	1	1		1	1		1				1	
323	1	1	1	1	1		1	1		1				1	

Table 4.3

Sample of the Factor/Engagement Matrix Generated for Segment 2

	Factor														
Engagement	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1					1			1	1						
2		1			1			1	1					1	
3			1	1	1	1	1				1				1
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
321	1	1	1	1	1	1	1	1	1	1		1		1	1
322	1	1	1	1	1		1	1		1				1	
323	1	1	1	1	1		1	1		1				1	

combinations are identified and examined to at least a certain extent.

The extent to which a defined factor combination is examined is dependent upon whether or not it passes the initial prescreening. As discussed in section 3.7, three types of prescreening are identified for potential use in this investigation. The use of one of the prescreening techniques, the specification of a maximum number of engagements (as a fraction of the total engagement set) contained in any defined factor combination to be defined further, results in the number of defined factor combinations to be evaluated indicated in Table 4.4.

Table 4.4

Defined Factor Combination Prescreening

Observed Variable* Segment	Potential Defined Factor Combinations	Defined Factor Combinations Selected For Further Evaluation
1	63	1
2	32, 767	123

* In addition, the set of all engagements is evaluated to serve as a baseline for comparison.

This degree of prescreening is achieved using a maximum engagement cutoff of 80% of the total engagement set.

Given this reduction in the number of defined factor combinations to be evaluated to a reasonable number, no further reduction of the number of combinations to be evaluated need be made from the point of view of available computer resources. Also, additional reductions in the number of defined factor combinations to be evaluated resulting from the remaining two prescreening techniques appears to be negligible. Based on the number of engagements associated with each of the individual factors listed in Table 4.5, a requirement for a reasonable minimum number of engagements would result in no additional defined factor combinations being dropped. Additional testing for the uniqueness of each defined factor combination's associated engagement subset is not profitable, either, given that the results shown in Appendix C indicate that only a few defined factor combinations are redundant from the standpoint of their associated engagement subsets.

The results of evaluating the associated engagement subsets of the prescreened set of defined factor combinations show the breakpoint hypothesis being obeyed reasonably well. When evaluating the entire engagement set, the ψ^{-1}_{CALC} equation fitted the ψ^{-1}_{EMP} data set with a multiple correlation coefficient of .8653 and an associated F statistic of 292.00 with 98 degrees of freedom. The ψ_{CALC} equation from which ψ^{-1}_{CALC} is obtained has a multiple correlation coefficient of 0.9876 with an associated F statistic of 3696.40 with 98 degrees of freedom.

The all-engagement case is used as a baseline with which to compare the results of the associated engagement subsets of the remaining 124 defined factor combinations. Twenty-two of the combinations have associated engagement subsets which result in ψ^{-1}_{CALC} equation fits to ψ^{-1}_{EMP} data sets that have multiple correlation coefficient squared greater than 0.8653. The relevant

Table 4.5

Number of Engagements Associated with Each Defined Factor for the Two Observed Variable Segments

Observed Variable Segment	Factors 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	220	321	305	321	258	278									
2	283	171	257	291	299	215	237	211	150	118	156	112	127	132	149

data concerning each of these "good" defined factor combinations is listed in Table 4.6a. Table 4.6b presents the same results, but explicitly displays the observed variables present in each of the 22 "good" defined factor combinations. The results for all evaluated defined factor combinations are listed in Appendix C. Table 4.7 lists the component observed variable for each of the defined factor combinations which are better than the case where all engagements are evaluated. The implications of these results are discussed in section 4.4.

Table 4.6a

Breakpoint Hypothesis Test Results--
Identification of "Good" Defined Factor
Combination Subsets and Associated Equation Fit Results

Rank Order	Factors Present: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total No. of Engag.	Won by Attk.	Won by Def.	Functional Form*	ψ_{CALC}: b_o	b_1	mult. corr. coeff.^{2}	F	ψ^{-1}_{CALC}: mult. corr. coeff.^{2}	F
1										1			1	1		171	107	64	2	0.1531	0.9486	0.9870	3690.95	0.9241	572.77
2										1		1	1	1		215	134	81	2	0.1686	0.9364	0.9828	2771.53	0.9184	528.26
3												1	1	1		237	153	84	2	0.1680	0.9343	0.9828	2781.70	0.9182	526.24
4												1	1			211	139	72	2	0.1577	0.9665	0.9861	3463.24	0.9181	525.90
5										1				1		150	100	50	2	0.1555	0.9463	0.9847	3127.43	0.9170	517.74
6										1		1		1		118	75	43	2	0.1868	0.9531	0.9743	1832.87	0.9167	515.49
7										1		1	1			156	102	54	2	0.1577	0.9721	0.9864	3519.01	0.9164	513.57
8														1		112	63	49	2	0.1526	0.9345	0.9830	2811.08	0.9146	501.14
9										1		1				127	86	41	2	0.1565	0.9822	0.9876	3887.66	0.9099	471.29
10													1	1		132	83	49	2	0.0471	0.9131	0.9893	4512.27	0.9097	470.37
11									1				1			149	93	56	2	0.3669	1.0183	0.9764	1999.23	0.9010	422.57
12						1										247	157	90	2	0.2284	1.0474	0.9892	4453.24	0.8987	411.32
13												1				228	147	81	2	0.1376	0.9477	0.9867	3598.96	0.8959	398.40
14												1		1		192	120	72	2	0.0572	0.9251	0.9871	3711.17	0.8942	390.99
15						1							1			243	155	88	2	0.2347	1.0195	0.9870	3688.09	0.8929	385.54
16										1						210	131	79	2	0.0757	0.9673	0.9913	5564.88	0.8903	374.65
17										1			1			222	146	76	1	0.0294	0.9644	0.9984	30060.25	0.8773	327.59
18		1							1							195	123	72	2	0.1945	0.9355	0.9773	2084.12	0.8717	310.22
19		1							1				1			234	150	84	2	0.1749	0.9411	0.9862	3487.80	0.8717	310.12
20		1						1								238	146	92	2	0.2053	0.9147	0.9679	1452.60	0.8702	305.66
21								1					1			242	163	79	2	0.2088	0.9148	0.9708	1604.19	0.8673	297.44
22								1	1							227	145	82	2	0.2066	0.9043	0.9611	1187.77	0.8667	295.79
23	All Engagements Included															323	203	120	2	0.1620	0.9510	0.9870	3696.40	0.8653	292.00

* 1--Linear Form; 2--Power Form; 3--Log Form

Table 4.6b

Breakpoint Hypothesis Test Results--
Component Observed Variables for Subsets
of "Good" Defined Factor Combinations

Rank Order	Multiple Correlation Coefficient^2	ENGN2	ENGN3	DURDAY2	DURDAY3	DURDAY4U	WILCAT2	ENGTYP2	ENGTYP3	ENCTYP4	TERR1	TERR3	TERR4	TERR5	WEAT1	WEAT3	WEAT4	WEAT5	WEAT6	SEASN1	SEASN3	SEASN4	MOR1	MOR2	ATTSURY	DEFSURY	SETEFF1	AIRSPRO	AIRSPR1	AIRSPR2	CAIR1	CAIR2	CAIR3	TPERSTR3	TPERSTR2	FRATIO2
1	0.9241			1	1		1				1		1	1			1		1					1												
2	0.9184			1	1		1				1		1	1			1	1	1					1						1		1				
3	0.9182			1	1		1				1		1					1	1					1						1		1				
4	0.9181			1	1		1											1						1						1		1				
5	0.9170						1				1		1	1			1		1																	
6	0.9167						1				1		1	1			1	1	1											1		1				
7	0.9164			1	1		1				1			1			1	1						1						1		1				
8	0.9146						1				1		1						1																	
9	0.9099						1				1			1			1	1												1		1				
10	0.9097			1	1		1				1		1						1					1												
11	0.9010			1	1			1		1		1							1					1												
12	0.8987																							1										1	1	
13	0.8959						1											1												1		1				
14	0.8942						1				1		1					1	1											1		1				
15	0.8929			1	1																			1										1	1	
16	0.8903						1				1			1			1																			
17	0.8773			1	1		1				1			1			1							1												
18	0.8717							1		1	1	1				1			1	1										1						
19	0.8717			1	1			1		1	1	1				1			1	1				1						1						
20	0.8702							1	1		1	1				1		1	1	1										1						
21	0.8673			1	1			1	1		1							1						1												
22	0.8667							1	1	1	1	1						1	1																	

Table 4.7

Observed Variables Contained in Those Defined Factor Combinations Whose Associated Engagement Subsets Produced ψ^{-1}_{CALC} Fits Which are Superior to that for the Entire Engagement Set

Nominal Order	Variable Name
1	DURDAY2
2	DURDAY3
3	WILCAT2
4	ENGTYP2
5	ENGTYP3
6	ENGTYP4
7	TERR1
8	TERR3
9	TERR4
10	TERR5
11	WEAT3
12	WEAT4
13	WEAT5
14	WEAT6
15	SEASN1
16	MOR2
17	AIRSPR2
18	CAIR2
19	TPERSTR3
20	TPERSTR2

4.3 Discriminant Analysis

The results of the Helmbold hypothesis testing are partially checked using discriminant analysis on the full engagement data set. As discussed in section 3.7, discriminant analysis is used to determine which observed variables are significant in determining the winner and loser in an engagement. SSPS subprogram DISCRIMINANT is used to perform the stepwise selection of observed variables using the WILKS lambda criterion. The default value for minimum selection criteria for variable selection is used to ensure that all observed variables having discriminatory power are included. The significant variables identified are contained in Table 4.8. In section 4.4, this set of significant observed variables is compared to the observed variables found in order to identify the associated engagement subsets which obey the breakpoint hypothesis best.

Table 4.8

Observed Variables Found to be Significant in Distinguishing Between Winners and Losers in Engagements

Nominal Ordering	Observed Variable
1	ENGN2
2	ENGN3
3	DURDAY1
4	ENGTYP4
5	WEAT1
6	WEAT3
7	WEAT6
8	SEASN1
9	MOR1
10	ATTSUR1
11	DEFSUR1
12	AIRSPR1
13	AIRSPR2
14	CAIR1
15	CAIR2
16	FRATIO1

4.4 Discussion of Results

The discussion of the results is comprised of several parts. The first concerns the degree to which the various associated engagement subsets, as well as the complete engagement set, obey, or do not obey, element three of the breakpoint hypothesis. Secondly, the implications of those existing subsets of the engagement data base which obey the breakpoint hypothesis to a greater extent are discussed. Lastly, as a partial check of the validity of the breakpoint hypothesis testing, the set of observed variables found to be significant in the discriminant analysis are compared with those observed variables contained within the defined factor combinations whose associated engagement subsets obeyed element three of the breakpoint hypothesis to a better extent than the complete engagement data set.

The results presented in section 4.2 show that element three of the breakpoint hypothesis is obeyed reasonably well by the complete engagement data set and the breakpoint hypothesis obeyed better by 22 of the 124 defined factor combinations that are evaluated. The multiple correlation coefficient squared for those 22 defined factor combinations ranged from a high of 0.9241 to a low of 0.8667. All of the defined factors which fall in this "good" category come from the second observed variable segment. Note that, except for the combination ranked 17th, the power functional form is consistently selected as best fitting the ψ^{-1}_{EMP} data set. This holds true for the vast majority of the remainder of the 124 defined factor combinations.

The existence of a number of defined factor combinations whose associated engagement subsets obey element three of the breakpoint hypothesis better than the entire engagement data matrix, suggests that a series of break curves should be used to account for the universe of possible engagement types. The observed variables contained in those defined factor combinations which fit well are candidates for specifying the types of engagements grouped within the individual break curves.

Comparison between the observed variables found to be significant in distinguishing between winners and losers in an engagement, and the observed variables inferred to be significant in determining the outcome of an engagement by virtue of their being contained in "good" defined factor combinations, however, results in differences between the two sets of observed variables. Only 6 of the 16 observed variables found to be significant in the discriminant analysis are contained in the "good" groups of defined factor combinations. In addition, 14 variables inferred to be significant in the results for the breakpoint hypothesis testing are not found to be significant by the discriminant analysis for the complete engagement data set. Many of these discrepancies, however, can be attributed to drawbacks in the prescreening technique used for defined factor combinations as well as reduction of the set of observed variables to a smaller set of defined factors which represent more than one observed variable each.

The impact of the prescreening technique on denying or hampering the ability of observed variables to enter into fully-evaluated factor combinations is significant. Table 4.9 lists the observed variables effectively denied being explicitly included in any fully evaluated defined factor combination. Table 4.10 lists the observed variables hampered in any fully evaluated defined factor combination. Table 4.11 presents a matching of the two sets of observed variables and shows how 13 of the 16 observed variables found via the discriminant analysis can be accounted for. Of the remaining three observed variables, two of them (ENGN1 and DURDAY1) are in the joint reference category observed variable segment where the number of potential combinations they can enter into is limited. The last observed variable, DEFSURY, may be deemed insignificant due to the small number of engagements (3%) where this observed variable is

Table 4.9

Observed Variables Effectively Denied Being Explicitly Included in Any Fully Evaluated Defined Factor Combination

First Observed Variable Segment	Second Observed Variable Segment
WILCAT1	ENGN2
TERR2	ENGN3
WEAT2	WEAT1
SEASN2	MOR1
MOR3	SETEFF1
BTHSURN	AIRSPR1
SETEFF0	CAIR1
CAIR0	CAIR3
TPERSTR1	FRATIO2
FRATIO1	

Table 4.10

Observed Variables Hampered in Being Explicitly Included in Any Fully Evaluated Defined Factor Combination

First Observed Variable Segment	Second Observed Variable Segment
ENGTYP1	DURDAY2
	DURDAY3
	WILCAT2
	TERR1
	TERR3
	WEAT5
	SEASN4
	ATTSURY
	AIRSPR0
	AIRSPR2
	TPERSTR2

Table 4.11

Comparison Between Observed Variables Found to be Significant via Helmbold Hypothesis Testing and via Discriminant Analysis

Significant Variables via Discriminant Analysis	Subset of Good Defined Factor Combinations			
	Present	Denied Access	Hampered	Other
ENGN1				X
ENGN2		X		
DURDAY1				X
ENGTYP4	X			
WEAT1		X		
WEAT3	X			
WEAT6	X			
SEASN1	X			
MOR1		X		
ATTSURY			X	
DEFSURY				X
AIRSPR1		X		
AIRSPR2	X			
CAIR1		X		
CAIR2	X			
FRATIO2		X		

* X = indicates yes

present.

There are additional observed variables found in the subset of good defined factor combinations which are not considered significant by the discriminant analysis. These can be partially explained by the reduction of the observed variables to a set of defined factors during the factor analysis step. Table 4.12 shows that the presence of half of the 14 observed variables can be explained in this fashion. The presence of the other seven observed variables (coming from five combat factors) can not be readily explained and should be the subject of further research.

Table 4.12

Additional Observed Variables Found in Subset of Good Defined Factor Combinations

Additional Observed Variables Found in Subset of Good Defined Factor Combinations	Located in Same Defined Factor as Observed Variable Deemed Significant via Factor Analysis
DURDAY2	
DURDAY3	
WILCAL2	YES
ENGTYP2	YES
ENGTYP3	
TERR1	YES
TERR3	YES
TERR4	YES
TERR5	
WEAT4	
WEAT5	YES
MOR2	YES
TPERSTR3	
TPERSTR2	

4.5 Summary

Results from testing of the breakpoint hypothesis have been presented here. They clearly indicate a number of engagement subsets which obey element three of the breakpoint hypothesis reasonably well. In addition, engagement subsets have been found which obey element three of the breakpoint hypothesis better than the full engagement set. These indicate that a series of break curves, rather than a single break curve, may better account for the break behavior of military forces for the universe of possible engagement types. The discriminant analysis confirmed the significance of some of the observed variables and, for the most part, reconciles with the results from the breakpoint hypothesis testing. The differences which remain concern the determination of which observed variables are significant in determining the outcome of an engagement and should be considered when specifying multiple break curves. Those differences do not diminish the degree to which the breakpoint hypothesis has been seen to hold. Additional areas of research indicated by these results are identified in the next chapter.

Chapter 5

CONCLUSIONS

This chapter includes a reaffirmation of the degree of validity found for the breakpoint hypothesis and, by extension, the validity of the use of probabalistic break curves in modelling probibalistic break curves. In addition, areas for future research are discussed.

5.1 Results Summary

The Helmbold Theorem appears to be obeyed reasonably well by the full engagement data base, as well as being better in certain additional subsets of the engagement data base.

The presence of specific subsets of the engagement data base which result in better fits by the $\psi^{-1}{}_{CALC}$ curve are an indication that a series of break curves might be better able to deal with the battle termination behavior of the universe of possible engagement types.

The observed variables found to be significant by discriminant analysis and those inferred as being significant by virtue of their being contained in the subset of good defined factor combinations in the Helmbold Theorem testing are in many cases different. However, many of these differences can be accounted for by the fact that the prescreening technique used for hypothesis testing precluded or hampered many of those observed variables from being in the defined factor combinations which survived the prescreening. In addition, a number of the observed variables inferred as being significant are contained in the same defined factors as observed variables which are found to be significant by the discriminant analysis. Therefore, it is not possible to distinguish between which of the observed variables in the defined factor is significant in identifying the associated engagement subset which produced the good fit.

5.2 Areas for Future Research

This investigation can be viewed as a preliminary empirical validation of the use of probabilistic break curves to model battle termination. Additional research needs to be performed in order to refine these results as well as investigate the specification of break curves for particular groupings of engagement types.

One improvement would be the use of discriminant analysis to identify significant observed variables followed by testing of the breakpoint hypothesis using combinations of the significant variables. The use of discriminant analysis for significant observed variable selection would eliminate the need for segmenting the observed variable set and provide the ability to examine the observed variables directly during the testing of the breakpoint hypothesis. Prescreening of the combinations could be performed by more restrictive variable selection in the discriminant analysis, which would overcome the drawbacks encountered in the current prescreening problems.

Another improvement would be to use additional functional forms for fitting the Ψ_{EMP} data. This could permit greater precision in identifying the degree to which particular engagement subsets obeyed element three of the breakpoint hypothesis.

The validity of specifying multiple breakcurves, as opposed to a single breakcurve, to properly model the battle termination process of all possible engagement types should also be investigated. Two points can be made concerning the specification of such a set of breakcurves.

1) Each observed variable combination within a breakcurve set must contain at least one observed variable whose exclusion is mandated by each of the other observed variable combinations within the breakcurve set. This precludes the possibility of an engagement being contained within the engagement subsets of two or more observed variable combinations. The occurence of this would infer that the break behavior of a combat force in a specified engagement could be represented by two or more breakcurves. For modelling purposes, however, it is necessary to ensure that only one unique breakcurve is associated with a combat force in a specified situation.

2) It is essential that a fully specified breakcurve set is capable of accounting for all possible types of engagements.

Lastly, the addition of more engagements to the engagement data base used in any future research, as well as the quantification of additional combat factors, would add to the validity of any results achieved in that research.

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Appendix A

MATHEMATICAL GLOSSARY OF TERMS

a	=	attrition rate coefficient for y forces
b	=	attrition rate coefficient for x forces
$C_z = C_z(T)$	=	total casualties suffered by side z during the combat encounter ($C_z = z_o - z$)
$C_z(t)$	=	casualties sustained by side z as of time t into the combat encounter; $C_z(t) = z_o - z(t)$
$D_x(v \mid W_y)$	=	conditional distribution of L_x, given W_y
$D_y(u \mid W_x)$	=	conditional distribution of L_y, given W_x
$F_z(u)$	=	break curve for side z, given the probability that the side's breakpoint threshold, L_z, will not exceed u; $Pr\left[L_z < u\right]$
$f_z = f_z(T)$	=	casualty fraction sustained by side z in the combat encounter
$f_z(t)$	=	casualty fraction for side z as of time t into the combat encounter; ($f_z(t) = C_z(t)/z_o$)
L_z	=	preselected (breakpoint) casualty-fraction level which, if met or exceeded, results in side z's losing the combat encounter
P_{PM}	=	the probability of a military force performing a specified mission
$P(W_x)$	=	probability that force x wins a specified combat encounter
$P(W_y)$	=	probability that force y wins a specified combat encounter
R	=	a proportionality constant relating the force y casualty fraction to the force x casualty fraction at time t
T	=	duration of the combat encounter
x	=	the force level for force x

x_{BP} = force x's breakpoint force level

x_o = the initial force level for force x

y = the force level for force y

y_{BP} = force y's breakpoint force level

y_o = the initial force level for force y

z = general symbol denoting a value of either x or y, depending on context

$z = z(T)$ = surviving troop strength of side z at the end of the combat encounter

z_o = initial troop strength of side z at the start of the combat encounter

$z(t)$ = surviving troop strength of side z as of time t into the combat encounter

$\Delta_{zz'}(u)$ = $P(f_z < u | W_{z'})$

φ = a strictly increasing monotonic function relating $f_x(t)$ to $f_y(t)$ via the formula $f_x(t) = \varphi f_y(t)$

$\psi(u)$ = $\mathrm{Min}\left[\varphi(u), 1\right]$

ψ^{-1}_{CALC} = the ψ function associated with the defender cumulative casualty-fraction distribution, which is obtained by taking the inverse to a fitted curve representing the ψ function for the attacker cumulative casualty-fraction curve. This curve, in turn, is obtained by fitting an appropriate functional form to the data points contained in ψ_{EMP}.

ψ_{EMP} = the set of data points which lie on the ψ curve associated with the attacker cumulative casualty-fraction distribution

ψ^{-1}_{EMP} = the set of data points which lie on the ψ^{-1} curve associated with the defender cumulative casualty-fraction distribution

dual = the result of applying the usual transposition to a formula, expression, etc.

"usual transposition" $= x \to y,\ y \to x,\ \psi^{-1} \to \psi,\ \psi \to \psi^{-1}$

Appendix B

NONMATHEMATICAL GLOSSARY OF TERMS

Active combat operations - refers to a military force engaged in combat activities against an opposing military force.

Area of operations - the area that a specified military force is responsible for in a particular military operation. This area either changes during the course of an operation or remains constant for more than one operation.

Associated engagement subset - the engagements in the engagement data matrix which are classified under a specified factor combination. These engagements constitute the subset of engagements which is associated with the factor combination.

ATLAS - an acronym representing the title "A Tactical, Logistical, Air Simulation." This is a combat model which represents theater-level operations (Taylor, 1980:11).

Attacker - the military force in an engagement which conducts primarily offensive operations.

Attack on a positional defense - An attack by a military force against an opposing military force in a defensive posture. The defense is positional, or fixed, in the sense that permanent retrograde movement by the defender is not envisioned. The defending military force occupies one of three operational states. These consist of, in the order of increasing strength, the Hasty Defense, the Prepared Defense and the Fortified Defense. The differences between these states are a function of the concentration of the defending force, time in the defensive position and the presence of attached engineering troops to aid in the preparation of the defenses (Dupuy, 1979:210-211).

Attack on a delaying force - an attack by a military force against an opposing military force which attempts to delay it. The delaying military force actively resists the advancing force while it moves rearward to a new position.

Attack on a withdrawing force - an attack by a military force against an opposing military force which attempts to withdraw. The withdrawing force attempts to avoid combat during the retrograde movement. It defends as a coherent military force if attacked, but attempts to disengage as rapidly as possible.

Auxilliary model - a model which is a simplification of a large-scale operational model. It is used to investigate the system dynamics of the more complex model by considering alternative assumptions and data estimates (Taylor, 1980:15-17).

Battle - a combat encounter between two opposing military forces. Each force has opposing aims or objectives (assigned or implicit) and each seeks to impose its will on the opponent by achieving its objective, while preventing the enemy from achieving his. A battle terminates when one side or the other clearly achieves its objective or when one side (or both) clearly fails to achieve its objective. It is considered possible for modern battles between large forces to last many days (Dupuy, 1979:187).

Battle termination process - the militarily-significant events and activities in a battle which result in its conclusion.

Break curve - a curve which indicates the probability that a military force will discontinue the engagement as a function of the personnel casualties it sustains (Helmbold, 1971:2).

Break curve set - an aggregation of break curves which is capable of accounting for the universe of possible engagement types.

Breakpoint - the casualty fraction at which a specified military force discontinues the engagement. Alternatively, the casualty fraction at which a specified military force 'breaks,' or moves to a lower operational state (Helmbold, 1971:2).

Campaign - usually an aggregation of several battles. A campaign usually lasts longer than a battle and encompasses a larger geographic area. It concludes when either a strategic objective is achieved or when a lull or stalement in combat operations occurs (Dupuy, 1979:187).

Casualty effectiveness - a quantitative comparison of the casualties incurred by the opposing military forces in an engagement (Dupuy, 1979:49).

Close air support - aircraft (fixed-wing or helicopter) used against ground targets in direct support of friendly ground forces. This represents only close air support sorties actually flown in support of engaged ground troops (Dupuy, 1979:72).

Coherent military force - a force capable of fighting in an organized fashion, as opposed to a military force for which the command and control structure has broken down.

Combat ineffective - a condition which exists in a military force when it is unable to carry out its mission (Clark, 1954:9).

Common factor - an unmeasured (or hypothetical) underlying variable which is the source of variation in at least two observed variables under consideration (Kim and Mueller, 1978:76).

Conflict termination - the end of a specified military operation.

Current investigation - refers to the thesis work performed by the author.

Data elements - the distinct types of data for each engagement contained in the engagement data matrix.

Defender - the military force in an engagement which conducts primarily defensive operations.

Defensive position - refers to the geographic area which a defending force occupies (either temporarily or permanently) while in a defensive posture.

Defensive posture - the manner in which a defending (and coherent) military force is organized to resist an attack. Possible defenses includes a positional defense of specified strength, a delay or a withdrawal.

Defined factor combination - a set of one or more defined factors. Each combination provides the basis for identifying a break curve to be tested.

Defined factors - a set of variables, or principle components, which is obtained via the mathematical transformation of an original variable set. The transformed variables are orthogonal to each other. No assumptions are made about the general structure of the original variable set.

Deterministic break curve - a break curve which contains one breakpoint to be used for any condition for which the curve is defined. This constitutes a special case of the probabilistic break curve.

Discriminant analysis - a data analysis technique which calculates the effects of a collection of interval-level independent variables on a nominal dependent variable (classification). The linear combinations of independent variables that best distinguish between cases in the categories of the dependent variable are then found (SPSS, 1975:435).

Divisional engagement - the period of combat during which a division fights to carry out a specific mission, the termination being marked by the beginning of a period of relative inactivity following definite success or failure in achieving the mission (Clark, 1954:8).

Engagement - a combat encounter between forces smaller than an army. A battle between two armies usually contains several engagements involving subordinate units. The duration of an engagement is usually shorter than a battle. A modern divisional engagement rarely lasts more than two or three days (Dupuy, 1979:187).

Engagement data matrix - the data base which contains the set of engagement data used in the current investigation.

Engagement identification - the specification of an engagement on the basis of geographic location, start date, duration, termination and the unity of mission within each of the two respective military forces.

Engagement magnitude - the size of the engagement with respect to the number of personnel engaged. For the current investigation, an engagement's magnitude is represented by the total number of personnel engaged and the ratio of attacker to defender personnel, or force ratio.

Engagement type - the classification of engagements on the basis of the postures of the opposing military forces (Dupuy, 1979:230).

Environmental factors - those physical conditions which affect the effectiveness of weapons. These include terrain, weather and seasonal factors (Dupuy, 1979:34).

Factor analysis - a variety of statistical techniques for the location and definition of dimensional space among a relatively large group of variables. Its primary uses include the location of a smaller number of valid dimensions, clusters, or factors contained in a larger set of independent items or variables and the determination of the degree to which a given variable or several variables are part of a common underlying phenomenon (SPSS, 1975:469).

Factor combination - any possible combination of one or more of the defined factors obtained from the factor analysis in the current investigation.

Fixed defensive posture - a defensive posture where permanent retrograde movement by the defender is not envisioned. This includes a mobile defense.

Force-on-force attrition modelling - the representation of combat between two military forces with respect to temporal force levels and engagement outcome (Taylor, 1980:1).

Helmbold's Theorem - a theorem stating the expected mathematical behavior between related attacker and defender break curves first developed by Helmbold (Helmbold, 1971:7-9). Refer to section 2.3 for details.

Holding - the occupation of a position or area by a military force for the purpose of defending it but in the absence of any significant enemy attack. Contact is maintained with an opposing military force, however.

Initial strength - the number of personnel in a military force at the start of an engagement.

Investigator - the author of the thesis.

Mass surrender - the capitulation of large numbers of personnel relative to the initial strength of the military force.

Meeting engagement - an engagement which consists of both military forces moving to contact followed by predominantly offensive actions by each side against the other.

Mission accomplishment - an assessment of the extent to which each side in an engagement accomplishs its assigned or perceived mission (Dupuy, 1979:48).

Morale - a sense of common purpose or a degree of dedication to a common task regarded as characteristic of or dominant in a particular group or organization (Webster's, 1971:1469).

Nominal level of measurement - a measurement scheme where no assumption has been made about the values assigned to the data. Each value represents a distinct category and no assumptions are made concerning the ordering or distances between categories (SPSS, 1975:4).

Observed variable - a variable which partially specifies an engagement and whose values are at the ratio level of measurement.

Observed variable subset - the observed variable associated with a specified common factor combination.

Operational factor (combat) - a factor which influences the employment of weapons and military forces. The set of operational factors includes air superiority, tactical air, force posture and mobility factors (Dupuy, 1979:34).

Open literature - literature whose distribution is not significantly restricted.

Operational state - one of five states which categorizes the activities of a military force during active combat operations. These consist of the attack, defend/hold, delay, withdraw and routed states.

Orthogonal rotation - in factor analysis, the operation through which a simple structure is sought under the restriction that the factors must be orthogonal (uncorrelated). Factors obtained through this rotation are by definition uncorrelated (Kim and Mueller, 1978:78).

Postulate of factorial causation - the assumption that the observed variables are linear combinations of underlying factors, and that the covariation between observed variables is solely due to their common sharing of one or more of the common factors (Kim and Mueller, 1978:78).

Probabilistic break curve - a break curve contains a set of breakpoints, each having a nonzero probability of occurrence.

Quartimax - a method of rotation to a terminal solution in factor analysis, the emphasis being on the simplification of the rows of the factor pattern matrix. In other words, the emphasis is on reducing the complexity of the observed variables. For further details, please consult Statistical Package for the Social Sciences, 2nd edition (SPSS, 1975:484-485).

Ratio level of measurement - a measurement where the distances between the categories is defined in terms of fixed and equal units and where the zero point is inherently defined by the measurement scheme (SPSS, 1975:5).

Reinforcements - the addition of previously uncommitted military units to an ongoing military operation. To be classified as reinforcements, it is required only that the military units be uncommitted with respect to the particular operation.

Replacements - the replacement of losses incurred by military units during a military operation.

Rotation to a terminal solution - in factor analysis, the simplification of the structure once the initial set of common factors have been obtained in the factor-extraction phase (Kim and Mueller, 1978:49).

Rout - an operational state which represents military units which, at least temporarily, have lost all ability to function as a coherent military force. Within this condition, however, it is possible for small, relatively insignificant, subsets of the military force to remain combat effective.

Season - a data element indicating the climatic season within which the engagement took place. The season affects weapon effectiveness and reflects the significance of changes in the hours of daylight and darkness in the Temperate Zone (Dupuy, 1979:35).

Setpiece effect - a data element which represents the advantage that an attacking force has as a result of intensive preparations against a specified defensive position. These preparations may include a rehearsal of the assault against dummy positions. This effect is applicable only to an attacking force which is otherwise inferior to the defending force (Dupuy, 1979:203).

Spatial effectiveness - a quantitative assessment of the extent to which each side is able to gain or hold ground (Dupuy, 1979:48-49).

Surprise - a data element which indicates the level of surprise achieved by the attacking or defending force (Dupuy, 1979:63).

Termination point - the point in time at which an engagement ended. This is indicated by one or more of the following conditions occurring: a change in the state of at least one of the two military forces involved in the engagement, a temporary pause in operations, and the addition of reinforcements or replacements to one or both military forces in an engagement.

Terrain - a data element which indicates the type of terrain upon which the engagement is fought. Terrain has an effect on the weapon effectiveness, mobility and the posture of a military force (Dupuy, 1979:34-35,228).

Unity of mission - the degree of interdependence among military units in carrying out their respective missions. This is one of the criteria used in engagement identification.

Victor - the military force which wins the engagement. This is determined on the basis of three criteria. Each military force was evaluated and compared on the basis of mission accomplishment, spatial effectiveness and casualty effectiveness (Dupuy, 1979:47-49).

Vulnerability - the vulnerability of a force to hostile firepower. The considerations which affect a force's vulnerability include its personnel strength, combat deployment exposure (in terms of terrain and posture), the relative firepower of the opposing forces, the presence or absence of air superiority, and increased exposure in amphibious and river crossing situations (Dupuy, 1979:36-37).

Weapon effectiveness (lethality) - the inherent capability of a given weapon to kill personnel or to make material ineffective for a given period of time. Capability includes weapon range, rate of fire, accuracy, radius of effects, and battlefield mobility factors (Dupuy, 1979:19).

Weather - a data element which indicates the specific climatic conditions at the time of the engagement. Weather has an effect on the weapon effectiveness, mobility and the posture of a military force (Dupuy, 1979:35,229).

Willard's Category I engagement - a category which contains open engagements in the sense that both sides could, with about equal facility, disengage and conduct an orderly withdrawal (Willard, 1962:2; Helmbold, 1971:25).

Willard's Category II engagement - a category which contains closed engagements in the sense that one of the forces in the engagement is encircled or otherwise in a position from which an orderly withdrawal can not readily be made, and whose options for maneuver are correspondingly markedly more restricted than those of his opponent (Willard, 1962:2; Helmbold, 1971:25).

Appendix C

BREAKPOINT HYPOTHESIS TEST RESULTS

Table C.1	Identification of All Evaluated Defined Factor Combination Subsets and Associated Equation Fit Results (Second Observed Variable Segment)
Table C.2	Component Observed Variables for Subsets of All Evaluated Defined Factor Combinations (Second Observed Variable Segment)
Table C.3	Identification of All Evaluated Defined Factor Combination Subsets and Associated Equation Fit Results (First Observed Variable Segment)
Table C.4	Component Observed Variables for Subsets of All Evaluated Defined Factor Combinations (First Observed Variable Segment)

Table C.1

Breakpoint Hypothesis Test Results--
Identification of All Evaluated Defined Factor
Combination Subsets and Associated Equation Fit Results (Second Observed Variable Segment)

Rank Order	Factors Present 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total No. of Engag.	Won by Attk.	Won by Def.	Functional Form*	ψ_{CALC} b_o	b_1	mult. corr. coeff.^{2}	F	ψ^{-1}_{CALC} mult. corr. coeff.^{2}	F
1										1			1	1		171	107	64	2	0.1531	0.9486	0.9870	3690.95	0.9241	572.77
2										1		1	1	1		215	134	81	2	0.1686	0.9364	0.9828	2771.53	0.9184	528.26
3												1	1	1		237	153	84	2	0.1680	0.9343	0.9828	2781.70	0.9182	526.24
4												1	1			211	139	72	2	0.1577	0.9665	0.9861	3463.24	0.9181	525.90
5										1				1		150	100	50	2	0.1555	0.9463	0.9847	3127.43	0.9170	517.74
6										1		1		1		118	75	43	2	0.1868	0.9531	0.9743	1832.87	0.9167	515.49
7										1		1	1			156	102	54	2	0.1577	0.9721	0.9864	3519.01	0.9164	513.57
8														1		112	63	49	2	0.1526	0.9345	0.9830	2811.08	0.9146	501.14
9										1		1				127	86	41	2	0.1565	0.9822	0.9876	3887.66	0.9099	471.29
10													1	1		132	83	49	2	0.0471	0.9131	0.9893	4512.27	0.9097	470.37
11									1				1			149	93	56	2	0.3669	1.0183	0.9764	1999.23	0.9010	422.57
12						1										247	157	90	2	0.2284	1.0474	0.9892	4453.24	0.8987	411.32
13												1				228	147	81	2	0.1376	0.9477	0.9867	3598.96	0.8959	398.40
14												1		1		192	120	72	2	0.0572	0.9251	0.9871	3711.17	0.8942	390.99
15						1							1			243	155	88	2	0.2347	1.0195	0.9870	3688.09	0.8929	385.54
16										1						210	131	79	2	0.0757	0.9673	0.9913	5564.88	0.8903	374.65
17										1			1			222	146	76	1	0.0294	0.9644	0.9984	30060.25	0.8773	327.59
18		1							1							195	123	72	2	0.1945	0.9355	0.9773	2084.12	0.8717	310.22
19		1							1				1			234	150	84	2	0.1749	0.9411	0.9862	3487.80	0.8717	310.12
20		1						1								238	146	92	2	0.2053	0.9147	0.9679	1452.60	0.8702	305.66
21								1					1			242	163	79	2	0.2088	0.9148	0.9708	1604.19	0.8673	297.44
22								1	1							227	145	82	2	0.2066	0.9043	0.9611	1187.77	0.8667	295.79
23										1	1	1		1		229	144	85	2	0.1859	0.9737	0.9783	2181.26	0.8647	290.49
24										1	1	1		1	1	245	160	85	2	0.1854	0.9705	0.9786	2218.98	0.8643	289.26
25											1		1			232	147	85	2	0.1613	1.0007	0.9895	4602.36	0.8634	287.01
26											1	1		1	1	229	149	80	2	0.1854	0.9705	0.9786	2218.98	0.8631	286.19
27											1	1		1		235	153	82	2	0.1845	0.9696	0.9784	2197.08	0.8627	285.31
28									1	1		1				203	128	75	2	0.1823	0.9509	0.9756	1931.01	0.8624	284.53
29									1	1		1		1		205	134	71	2	0.1811	0.9464	0.9761	1976.63	0.8616	282.49
30										1	1			1		229	150	79	2	0.1643	0.9671	0.9854	3272.11	0.8614	281.77
31									1			1	1			234	151	83	2	0.1584	0.9454	0.9850	3187.39	0.8612	281.34
32		1							1	1		1				199	131	68	2	0.1644	0.9517	0.9850	3184.59	0.8606	279.72
33										1			1	1	1	169	106	63	2	0.1541	0.9423	0.9861	3459.66	0.8604	279.25
34													1	1	1	200	132	68	2	0.1541	0.9423	0.9861	3459.66	0.8604	279.25
35										1	1		1			136	85	51	2	0.1528	0.9860	0.9894	4554.29	0.8602	278.88
36											1	1	1			193	125	68	2	0.1632	0.9802	0.9864	3524.74	0.8601	278,55
37										1		1		1	1	198	124	74	2	0.1811	0.9420	0.9780	2157.96	0.8598	277.95
38		1						1				1				237	158	79	2	0.1694	0.9356	0.9816	2591.08	0.8596	277.39

* 1--Linear Form; 2--Power Form; 3--Log Form

Table C.1 (cont.)

Breakpoint Hypothesis Test Results--
Identification of All Evaluated Defined Factor
Combination Subsets and Associated Equation Fit Results (Second Observed Variable Segment)

Rank Order	Factors Present 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total No. of Engag.	Won by Attk.	Won by Def.	Functional Form*	ψ_{CALC} b_o	b_1	Mult. Corr. $Coeff.^2$	F	ψ^{-1}_{CALC} Mult. Corr. $Coeff.^2$	F
39									1			1		1		208	134	74	2	0.1787	0.9402	0.9762	1983.11	0.8596	277.32
40										1				1	1	174	106	68	2	0.1591	0.9582	0.9862	3483.14	0.8596	277.31
41		1						1						1		192	124	68	2	0.1679	0.9311	0.9817	2597.23	0.8587	275.20
42		1						1				1		1		178	112	66	2	0.1679	0.9311	0.9817	2597.23	0.8587	275.20
43		1							1			1				188	119	69	2	0.1617	0.9442	0.9849	3163.01	0.8587	275.12
44		1							1			1		1		205	136	69	2	0.1605	0.9406	0.9849	3172.00	0.8580	273.40
45										1	1	1			1	222	146	76	2	0.1653	0.9875	0.9860	3421.31	0.8580	273.34
46												1		1	1	204	130	74	2	0.1797	0.9391	0.9781	2166.30	0.8579	273.25
47										1	1	1				253	160	93	2	0.1645	0.9882	0.9857	3354.69	0.8578	273.03
48		1							1	1						255	161	94	2	0.1535	0.9563	0.9869	3661.77	0.8578	272.89
49														1	1	243	156	87	2	0.1571	0.9546	0.9862	3470.47	0.8577	272.83
50								1								248	160	88	2	0.2169	0.8808	0.9496	898.58	0.8574	271.96
51									1				1	1		250	163	87	2	0.1461	0.9532	0.9886	4227.70	0.8572	271.58
52		1							1	1				1		243	157	86	2	0.1523	0.9520	0.9869	3666.19	0.8570	270.97
53										1			1		1	245	156	89	2	0.1497	0.9640	0.9896	4634.92	0.8567	270.36
54								1		1		1		1		214	133	81	2	0.1900	0.9195	0.9666	1393.59	0.8565	269.74
55								1		1				1		239	157	82	2	0.1900	0.9195	0.9666	1393.59	0.8564	269.68
56									1	1						199	125	74	2	0.1557	0.9682	0.9872	3749.83	0.8563	269.43
57		1														236	151	85	2	0.1974	0.9381	0.9767	2026.98	0.8558	268.29
58									1	1				1		255	162	93	2	0.1549	0.9650	0.9875	3852.16	0.8558	268.22
59												1	1		1	245	156	89	2	0.1613	0.9578	0.9859	3393.14	0.8558	268.17
60										1		1			1	246	155	91	2	0.1630	0.9761	0.9861	3448.31	0.8557	268.03
61									1		1				1	246	161	85	2	0.1637	0.9707	0.9857	3359.66	0.8552	266.71
62		1							1					1		215	135	80	2	0.1496	0.9452	0.9867	3604.72	0.8550	266.41
63									1			1				251	160	91	2	0.1736	0.9280	0.9737	1789.02	0.8546	265.36
64													1		1	239	157	82	2	0.1500	0.9613	0.9890	4372.15	0.8545	265.15
65									1					1		238	152	86	2	0.1545	0.9625	0.9875	3847.52	0.8542	264.60
66								1						1		254	166	88	2	0.1893	0.9164	0.9664	1385.08	0.8541	264.17
67									1			1			1	233	146	87	2	0.1749	0.9383	0.9774	2092.04	0.8539	263.71
68								1		1						236	149	87	2	0.1907	0.9223	0.9665	1390.89	0.8536	263.24
69											1	1			1	237	148	89	2	0.1694	0.9952	0.9858	3372.52	0.8530	261.84
70								1				1		1		244	158	86	2	0.1869	0.9115	0.9666	1393.25	0.8528	261.38
71											1	1				255	167	88	2	0.1677	0.9944	0.9857	3341.44	0.8525	260.59
72								1		1		1				233	152	81	2	0.1875	0.9163	0.9667	1396.77	0.8521	259.77
73									1						1	252	159	93	2	0.1579	0.9518	0.9850	3197.02	0.8505	256.26
74		1											1			244	156	88	2	0.1682	0.9240	0.9853	3263.37	0.8505	256.15

* 1--Linear Form; 2--Power Form; 3--Log Form

Table C.1 (cont.)

Breakpoint Hypothesis Test Results--
Identification of All Evaluated Defined Factor
Combination Subsets and Associated Equation Fit Results (Second Observed Variable Segment)

	Factors Present																			ψ_{CALC}				ψ_{CALC}^{-1}	
Rank Order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total No. of Engag.	Won by Attk.	Won by Def.	Functional Form*	b_o	b_1	Mult. Corr. Coeff.^{2}	F	Mult. Corr. Coeff.^{2}	F
75								1				1				239	154	85	2	0.1849	0.9098	0.9665	1390.90	0.8486	252.19
76		1									1	1				243	158	85	2	0.1737	0.9721	0.9842	3026.44	0.8484	251.74
77		1								1		1			1	250	159	91	2	0.1731	0.9599	0.9837	2923.68	0.8474	249.65
78										1	1				1	253	166	87	2	0.1139	0.9951	0.9926	6584.41	0.8469	248.66
79												1			1	230	149	81	2	0.1647	0.9738	0.9859	3410.52	0.8466	248.06
80		1								1		1				254	166	88	2	0.1653	0.9572	0.9851	3218.22	0.8461	246.99
81		1								1		1	1			210	135	75	2	0.1575	0.9434	0.9853	3269.28	0.8460	246.80
82		1								1		1		1		211	133	78	2	0.1646	0.9550	0.9851	3222.74	0.8459	246.47
83		1								1						222	145	77	2	0.1602	0.9710	0.9863	3505.08	0.8456	245.99
84										1	1					182	114	68	2	0.1135	0.9867	0.9920	6052.95	0.8453	245.35
85		1										1	1			220	144	76	2	0.1562	0.9371	0.9854	3277.43	0.8449	244.50
86		1										1		1	1	208	137	71	2	0.1695	0.9512	0.9839	2961.30	0.8448	244.29
87		1										1			1	242	159	83	2	0.1695	0.9512	0.9839	2961.30	0.8447	244.04
88		1								1		1	1	1		206	131	75	2	0.1554	0.9361	0.9855	3308.51	0.8446	243.78
89		1										1				252	165	87	2	0.1629	0.9505	0.9849	3169.35	0.8444	243.39
90		1								1	1				1	237	152	85	2	0.1587	0.9719	0.9877	3899.02	0.8443	243.21
91		1										1	1	1		204	126	78	2	0.1547	0.9329	0.9854	3282.10	0.8440	242.59
92		1									1			1	1	247	159	88	2	0.1587	0.9719	0.9877	3899.02	0.8437	242.02
93		1									1				1	219	136	83	2	0.1587	0.9719	0.9877	3899.02	0.8437	241.99
94		1								1	1					225	148	77	2	0.1576	0.9696	0.9877	3900.06	0.8437	241.99
95		1								1	1			1		240	158	82	2	0.1571	0.9682	0.9877	3919.87	0.8434	241.43
96		1										1		1		229	149	80	2	0.1612	0.9438	0.9850	3204.18	0.8433	241.25
97		1									1					248	163	85	2	0.1576	0.9696	0.9877	3900.06	0.8430	240.77
98		1								1	1			1	1	255	161	94	2	0.1566	0.9655	0.9878	3952.50	0.8429	240.42
99		1									1			1		252	162	90	2	0.1571	0.9682	0.9877	3919.87	0.8428	240.21
100		1												1		247	159	88	2	0.1554	0.9561	0.9863	3505.86	0.8422	239.04
101		1								1				1	1	251	162	89	2	0.1561	0.9549	0.9868	3635.25	0.8418	238.27
102		1												1	1	247	157	90	2	0.1565	0.9561	0.9867	3608.44	0.8414	237.54
103		1													1	248	156	92	2	0.1565	0.9561	0.9867	3608.44	0.8412	237.24
104											1				1	249	162	87	2	0.1188	1.0047	0.9927	6659.00	0.8408	236.37
105		1								1			1	1		219	137	82	2	0.1406	0.9352	0.9873	3793.12	0.8401	235.06
106		1								1			1			253	161	92	2	0.1406	0.9352	0.9873	3793.12	0.8400	234.89
107										1					1	242	158	84	2	0.1004	0.9613	0.9922	6170.77	0.8383	231.66
108		1											1	1		240	153	87	2	0.1380	0.9270	0.9868	3635.05	0.8382	231.59
109											1					248	156	92	2	0.1166	0.9928	0.9921	6151.94	0.8377	230.49
110							1									248	162	86	2	0.1564	0.9947	0.9903	4972.64	0.8256	209.82

* 1--Linear Form; 2--Power Form; 3--Log Form

Table C.1 (cont.)

Breakpoint Hypothesis Test Results--
Identification of All Evaluated Defined Factor
Combination Subsets and Associated Equation Fit Results (Second Observed Variable Segment)

Rank Order	Factors Present 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total No. of Engag.	Won by Attk.	Won by Def.	Functional Form*	ψ_{CALC} b_o	b_1	Mult. Corr. Coeff.^{2}	F	ψ^{-1}_{CALC} Mult. Corr. Coeff.^{2}	F
111								1	1					1		254	162	92	2	0.0672	0.9001	0.9805	2438.34	0.8214	203.18
112									1		1	1				241	150	91	2	0.0654	0.9245	0.9833	2856.18	0.8190	199.61
113										1	1			1	1	247	160	87	2	0.0173	0.9458	0.9949	9548.32	0.8163	195.72
114											1		1		1	239	153	86	2	0.0190	0.9541	0.9957	11255.57	0.8156	194.78
115											1			1	1	233	149	84	2	0.0162	0.9435	0.9948	9418.23	0.8141	192.58
116											1			1		221	137	84	2	0.0176	0.9392	0.9944	8606.36	0.8130	191.13
117									1		1					228	149	79	2	0.0341	0.9411	0.9939	7899.82	0.8091	185.80
118									1							231	150	81	2	0.0895	0.8904	0.9713	1631.32	0.8086	185.05
119		1								1					1	249	163	86	2	0.0622	0.9174	0.9854	3272.37	0.8030	177.85
120													1			240	152	88	2	0.0621	0.9761	0.9897	4667.96	0.7928	165.76
121															1	250	157	93	1	0.0123	0.9837	0.9996	100000 +	0.7846	156.91
122									1	1			1			251	163	88	1	0.0465	0.9493	0.9963	13039.99	0.7582	132.51
123		1								1				1		242	153	89	1	0.0418	0.9532	0.9962	12964.84	0.7286	110.87

* 1--Linear Form; 2--Power Form; 3--Log Form

Table C.2

Breakpoint Hypothesis Test Results--
Component Observed Variables for Subsets
of All Evaluated Defined Factor Combinations (Second Observed Variable Segment)

Rank Order	Multiple Correlation Coefficient^2	ENGN2	ENGN3	DURDAY2	DURDAY3	DURDAY4U	WILCAT2	ENGTYP2	ENGTYP3	ENGTYP4	TERR1	TERR3	TERR4	TERR5	WEAT1	WEAT3	WEAT4	WEAT5	WEAT6	SEASN1	SEASN3	SEASN4	MOR1	MOR2	ATTSURY	DEFSURY	SETEFF1	AIRSPRO	AIRSPR1	AIRSPR2	CAIR1	CAIR2	CAIR3	TPERSTR3	TPERSTR2	FRATIO2
1	0.9241			1	1		1				1		1	1			1		1					1												
2	0.9184			1	1		1				1		1	1			1	1	1					1						1		1				
3	0.9182			1	1		1				1		1					1	1					1						1		1				
4	0.9181			1	1		1											1						1						1		1				
5	0.9170						1				1		1	1			1		1																	
6	0.9167						1				1		1	1			1	1	1											1		1				
7	0.9164			1	1		1				1			1			1	1						1						1		1				
8	0.9146						1				1		1						1																	
9	0.9099						1				1			1			1	1												1		1				
10	0.9097			1	1		1				1		1						1					1												
11	0.9010			1	1			1		1		1							1					1												
12	0.8987																							1										1	1	
13	0.8959						1											1												1		1				
14	0.8942						1				1		1					1	1											1		1				
15	0.8929			1	1																			1										1	1	
16	0.8903						1				1			1			1																			
17	0.8773			1	1		1				1			1			1							1												
18	0.8717							1		1	1	1				1			1	1										1						
19	0.8717			1	1			1		1	1	1				1			1	1				1						1						
20	0.8702							1	1		1	1				1		1	1	1										1						
21	0.8673			1	1			1	1		1							1						1												
22	0.8667							1	1	1	1	1						1	1																	
23	0.8647					1	1				1		1	1			1	1	1					1	1					1		1				
24	0.8643					1	1				1		1	1			1	1	1					1	1	1				1		1				
25	0.8634			1	1	1	1																	1	1											
26	0.8631					1	1				1		1					1	1					1	1	1				1		1				
27	0.8627					1	1				1		1					1	1					1	1					1		1				
28	0.8624						1	1		1	1	1		1			1	1	1											1		1				
29	0.8616						1	1		1	1	1	1	1			1	1	1											1		1				
30	0.8614					1	1				1		1	1			1		1					1	1											
31	0.8612			1	1		1	1		1		1						1	1					1						1		1				
32	0.8606						1	1		1	1	1		1		1	1	1	1	1										1		1				
33	0.8604			1	1		1				1		1	1			1		1					1	1	1				1						
34	0.8604			1	1		1				1		1						1					1	1	1				1						

Table C.2

Breakpoint Hypothesis Test Results--
Component Observed Variables for Subsets
of All Evaluated Defined Factor Combinations (Second Observed Variable Segment)

Rank Order	Multiple Correlation Coefficient[2]	ENGN2	ENGN3	DURDAY2	DURDAY3	DURDAY4U	WILCAT2	ENGTYP2	ENGTYP3	ENGTYP4	TERR1	TERR3	TERR4	TERR5	WEAT1	WEAT3	WEAT4	WEAT5	WEAT6	SEASN1	SEASN3	SEASN4	MOR1	MOR2	ATTSURY	DEFSURY	SETEFF1	AIRSPR0	AIRSPR1	AIRSPR2	CAIR1	CAIR2	CAIR3	TPERSTR3	TPERSTR2	FRATIO2
1	0.9241			1	1		1				1		1	1			1		1					1												
2	0.9184			1	1		1				1		1	1			1	1	1					1						1		1				
3	0.9182			1	1		1				1		1					1	1					1						1		1				
4	0.9181			1	1		1											1						1						1		1				
5	0.9170						1				1		1	1			1		1																	
6	0.9167						1				1		1	1			1	1	1											1		1				
7	0.9164			1	1		1				1			1			1	1						1						1		1				
8	0.9146						1				1		1						1																	
9	0.9099						1				1			1			1	1												1		1				
10	0.9097			1	1		1				1		1						1					1												
11	0.9010			1	1			1		1		1							1					1												
12	0.8987																							1										1	1	
13	0.8959						1											1												1		1				
14	0.8942						1				1		1					1	1											1		1				
15	0.8929			1	1																			1										1	1	
16	0.8903						1				1			1			1																			
17	0.8773			1	1		1				1			1			1							1												
18	0.8717							1		1	1	1				1			1	1										1						
19	0.8717			1	1			1		1	1	1				1			1	1				1						1						
20	0.8702							1	1		1	1				1		1	1	1										1						
21	0.8673			1	1			1	1		1							1						1												
22	0.8667							1	1	1	1	1						1	1																	
23	0.8647					1	1				1		1	1			1	1	1					1	1					1		1				
24	0.8643					1	1				1		1	1			1	1	1					1	1	1				1		1				
25	0.8634			1	1	1	1																	1	1											
26	0.8631					1	1				1		1					1	1					1	1	1				1		1				
27	0.8627					1	1				1		1					1	1					1	1					1		1				
28	0.8624						1	1		1	1	1		1			1	1	1											1		1				
29	0.8616						1	1		1	1	1	1	1			1	1	1											1		1				
30	0.8614					1	1				1		1	1			1		1					1	1											
31	0.8612			1	1		1	1		1		1						1	1					1						1		1				
32	0.8606						1	1		1	1	1		1		1	1	1	1	1										1		1				
33	0.8604			1	1		1				1		1	1			1		1					1	1	1				1						
34	0.8604			1	1		1				1		1						1					1	1	1				1						

Table C.2 (cont.)

Breakpoint Hypothesis Test Results--
Component Observed Variables for Subsets
of All Evaluated Defined Factor Combinations (Second Observed Variable Segment)

Rank Order	Multiple Correlation Coefficient^2	ENGN2	ENGN3	DURDAY2	DURDAY3	DURDAY4U	WILCAT2	ENGTYP2	ENGTYP3	ENGTYP4	TERR1	TERR3	TERR4	TERR5	WEAT1	WEAT3	WEAT4	WEAT5	WEAT6	SEASN1	SEASN3	SEASN4	MOR1	MOR2	ATTSURY	DEFSURY	SETEFF1	AIRSPRO	AIRSPR1	AIRSPR2	CAIR1	CAIR2	CAIR3	TPERSTR3	TPERSTR2	FRATIO2
35	0.8602			1	1	1	1				1			1			1							1	1											
36	0.8601			1	1	1	1											1						1	1					1		1				
37	0.8598						1				1		1	1			1	1	1					1	1	1				1		1				
38	0.8596						1	1	1		1	1				1		1	1	1										1		1				
39	0.8596						1	1		1	1	1	1					1	1											1		1				
40	0.8596						1				1		1	1			1		1					1	1	1				1						
41	0.8587						1	1	1		1	1	1			1		1	1	1										1						
42	0.8587						1	1	1		1	1	1			1		1	1	1										1		1				
43	0.8587						1	1		1	1	1				1		1	1	1										1		1				
44	0.8580						1	1		1	1	1	1			1		1	1	1										1		1				
45	0.8580					1	1				1			1			1	1						1	1	1				1		1				
46	0.8579						1				1		1					1	1					1	1	1				1		1				
47	0.8578					1	1				1			1			1	1						1	1					1		1				
48	0.8578						1	1		1	1	1		1		1	1		1	1										1						
49	0.8577						1				1		1						1					1	1	1				1						
50	0.8574							1	1		1							1																		
51	0.8572			1	1		1	1		1	1	1	1						1					1												
52	0.8570						1	1		1	1	1	1	1		1	1		1	1										1						
53	0.8567			1	1		1				1			1			1							1	1	1				1						
54	0.8565						1	1	1		1		1	1			1	1	1											1		1				
55	0.8564						1	1	1		1		1	1			1	1	1																	
56	0.8563						1	1		1	1	1		1			1		1																	
57	0.8558										1	1				1			1	1										1						
58	0.8558						1	1		1	1	1	1	1			1		1																	
59	0.8558			1	1		1											1						1	1	1				1		1				
60	0.8557						1				1			1			1	1						1	1	1				1		1				
61	0.8552					1	1	1		1		1							1					1	1	1				1						
62	0.8550						1	1		1	1	1	1			1			1	1										1						
63	0.8546						1	1		1		1						1	1											1		1				
64	0.8545			1	1		1																	1	1	1				1						
65	0.8542						1	1		1	1	1	1						1																	
66	0.8541						1	1	1		1		1					1	1																	
67	0.8539						1	1		1		1						1	1					1	1	1				1		1				
68	0.8536						1	1	1		1			1			1	1																		
69	0.8530					1	1											1						1	1	1				1		1				
70	0.8528						1	1	1		1		1					1	1											1		1				

Table C.2 (cont.)

Breakpoint Hypothesis Test Results--
Component Observed Variables for Subsets
of All Evaluated Defined Factor Combinations (Second Observed Variable Segment)

Rank Order	Multiple Correlation Coefficient[2]	ENGN2	ENGN3	DURDAY2	DURDAY3	DURDAY4U	WILCAT2	ENGTYP2	ENGTYP3	ENGTYP4	TERR1	TERR3	TERR4	TERR5	WEAT1	WEAT3	WEAT4	WEAT5	WEAT6	SEASN1	SEASN3	SEASN4	MOR1	MOR2	ATTSURY	DEFSURY	SETEFF1	AIRSPR0	AIRSPR1	AIRSPR2	CAIR1	CAIR2	CAIR3	TPERSTR3	TPERSTR2	FRATIO2
35	0.8602			1	1	1	1				1			1			1							1	1											
36	0.8601			1	1	1	1											1						1	1					1		1				
37	0.8598						1				1		1	1			1	1	1					1	1	1				1		1				
38	0.8596						1	1	1		1	1				1		1	1	1										1		1				
39	0.8596						1	1		1	1	1	1					1	1											1		1				
40	0.8596						1				1		1	1			1		1					1	1	1				1						
41	0.8587						1	1	1		1	1	1			1		1	1	1										1						
42	0.8587						1	1	1		1	1	1			1		1	1	1										1		1				
43	0.8587						1	1		1	1	1				1		1	1	1										1		1				
44	0.8580						1	1		1	1	1	1			1		1	1	1										1		1				
45	0.8580					1	1				1			1			1	1						1	1	1				1		1				
46	0.8579						1				1		1					1	1					1	1	1				1		1				
47	0.8578					1	1				1			1			1	1						1	1					1		1				
48	0.8578						1	1		1	1	1		1		1	1		1	1										1						
49	0.8577						1				1		1						1					1	1	1				1						
50	0.8574							1	1		1							1																		
51	0.8572			1	1		1	1		1	1	1	1						1					1												
52	0.8570						1	1		1	1	1	1	1		1	1		1	1										1						
53	0.8567			1	1		1				1			1			1							1	1	1				1						
54	0.8565						1	1	1		1		1	1			1	1	1											1		1				
55	0.8564						1	1	1		1		1	1			1	1	1																	
56	0.8563						1	1		1	1	1		1			1		1																	
57	0.8558										1	1				1			1	1										1						
58	0.8558						1	1		1	1	1	1	1			1		1																	
59	0.8558			1	1		1											1						1	1	1				1		1				
60	0.8557						1				1			1			1	1						1	1	1				1		1				
61	0.8552					1	1	1		1		1							1					1	1	1				1						
62	0.8550						1	1		1	1	1	1			1			1	1										1						
63	0.8546						1	1		1		1						1	1											1		1				
64	0.8545			1	1		1																	1	1	1				1						
65	0.8542						1	1		1	1	1	1						1																	
66	0.8541						1	1	1		1		1					1	1																	
67	0.8539						1	1		1		1						1	1					1	1	1				1		1				
68	0.8536						1	1	1		1			1			1	1																		
69	0.8530					1	1											1						1	1	1				1		1				
70	0.8528						1	1	1		1		1					1	1											1		1				

Table C.2 (cont.)

Breakpoint Hypothesis Test Results--
Component Observed Variables for Subsets
of All Evaluated Defined Factor Combinations (Second Observed Variable Segment)

Rank Order	Multiple Correlation Coefficient^2	ENGN2	ENGN3	DURDAY2	DURDAY3	DURDAY4U	WILCAT2	ENGTYP2	ENGTYP3	ENGTYP4	TERR1	TERR3	TERR4	TERR5	WEAT1	WEAT3	WEAT4	WEAT5	WEAT6	SEASN1	SEASN3	SEASN4	MOR1	MOR2	ATTSURY	DEFSURY	SETEFF1	AIRSPRO	AIRSPR1	AIRSPR2	CAIR1	CAIR2	CAIR3	TPERSTR3	TPERSTR2	FRATIO2
71	0.8525					1	1											1						1	1					1		1				
72	0.8521						1	1	1		1			1			1	1												1		1				
73	0.8505						1	1		1		1							1					1	1	1				1						
74	0.8505			1	1						1	1				1			1	1				1						1						
75	0.8486						1	1	1		1							1												1		1				
76	0.8484					1	1				1	1				1		1	1	1				1	1					1		1				
77	0.8474						1				1	1		1		1	1	1	1	1				1	1	1				1		1				
78	0.8469					1	1				1			1			1							1	1	1				1						
79	0.8466						1											1						1	1	1				1		1				
80	0.8461						1				1	1		1		1	1	1	1	1										1		1				
81	0.8460			1	1		1				1	1		1		1	1	1	1	1				1						1		1				
82	0.8459						1				1	1	1	1		1	1	1	1	1										1		1				
83	0.8456						1				1	1		1		1	1		1	1										1						
84	0.8453					1	1				1			1			1							1	1											
85	0.8449			1	1		1				1	1				1		1	1	1				1						1		1				
86	0.8448						1				1	1	1			1		1	1	1				1	1	1				1		1				
87	0.8447						1				1	1				1		1	1	1				1	1	1				1		1				
88	0.8446			1	1		1				1	1	1	1		1	1	1	1	1				1						1		1				
89	0.8444						1				1	1				1		1	1	1										1		1				
90	0.8443					1	1				1	1		1		1	1		1	1				1	1	1				1						
91	0.8440			1	1		1				1	1	1			1		1	1	1				1						1		1				
92	0.8437					1	1				1	1	1			1			1	1				1	1	1				1						
93	0.8437					1	1				1	1				1			1	1				1	1	1				1						
94	0.8437					1	1				1	1		1		1	1		1	1				1	1					1						
95	0.8434					1	1				1	1	1	1		1	1		1	1				1	1					1						
96	0.8433						1				1	1	1			1		1	1	1										1		1				
97	0.8430					1	1				1	1				1			1	1				1	1					1						
98	0.8429					1	1				1	1	1	1		1	1		1	1				1	1	1				1						
99	0.8428					1	1				1	1	1			1			1	1				1	1					1						
100	0.8422						1				1	1	1			1			1	1										1						
101	0.8418						1				1	1	1	1		1	1		1	1				1	1	1				1						
102	0.8414						1				1	1	1			1			1	1				1	1	1				1						
103	0.8412						1				1	1				1			1	1				1	1	1				1						
104	0.8408					1	1																	1	1	1				1						
105	0.8401			1	1		1				1	1	1	1		1	1		1	1				1						1						
106	0.8400			1	1		1				1	1		1		1	1		1	1				1						1						

Table C.2 (cont.)

Breakpoint Hypothesis Test Results--
Component Observed Variables for Subsets
of All Evaluated Defined Factor Combinations (Second Observed Variable Segment)

Rank Order	Multiple Correlation Coefficient[2]	ENGN2	ENGN3	DURDAY2	DURDAY3	DURDAY4U	WILCAT2	ENGTYP2	ENGTYP3	ENGTYP4	TERR1	TERR3	TERR4	TERR5	WEAT1	WEAT3	WEAT4	WEAT5	WEAT6	SEASN1	SEASN3	SEASN4	MOR1	MOR2	ATTSURY	DEFSURY	SETEFF1	AIRSPR0	AIRSPR1	AIRSPR2	CAIR1	CAIR2	CAIR3	TPERSTR3	TPERSTR2	FRATIO2
71	0.8525					1	1											1						1	1					1		1				
72	0.8521						1	1	1		1			1			1	1												1		1				
73	0.8505						1	1		1		1							1					1	1	1				1						
74	0.8505			1	1						1	1				1			1	1				1						1						
75	0.8486						1	1	1		1							1												1		1				
76	0.8484					1	1				1	1				1		1	1	1				1	1					1		1				
77	0.8474						1				1	1		1		1	1	1	1	1				1	1	1				1		1				
78	0.8469					1	1				1			1			1							1	1	1				1						
79	0.8466						1											1						1	1	1				1		1				
80	0.8461						1				1	1		1		1	1	1	1	1										1		1				
81	0.8460			1	1		1				1	1		1		1	1	1	1	1				1						1		1				
82	0.8459						1				1	1	1	1		1	1	1	1	1										1		1				
83	0.8456						1				1	1		1		1	1		1	1										1						
84	0.8453					1	1				1			1			1							1	1											
85	0.8449			1	1		1				1	1				1		1	1	1				1						1		1				
86	0.8448						1				1	1	1			1		1	1	1				1	1	1				1		1				
87	0.8447						1				1	1				1		1	1	1				1	1	1				1		1				
88	0.8446			1	1		1				1	1	1	1		1	1	1	1	1				1						1		1				
89	0.8444						1				1	1				1		1	1	1										1		1				
90	0.8443					1	1				1	1		1		1	1		1	1				1	1	1				1						
91	0.8440			1	1		1				1	1	1			1		1	1	1				1						1		1				
92	0.8437					1	1				1	1	1			1			1	1				1	1	1				1						
93	0.8437					1	1				1	1				1			1	1				1	1	1				1						
94	0.8437					1	1				1	1		1		1	1		1	1				1	1					1						
95	0.8434					1	1				1	1	1	1		1	1		1	1				1	1					1						
96	0.8433						1				1	1	1			1		1	1	1										1		1				
97	0.8430					1	1				1	1				1			1	1				1	1					1						
98	0.8429					1	1				1	1	1	1		1	1		1	1				1	1	1				1						
99	0.8428					1	1				1	1	1			1			1	1				1	1					1						
100	0.8422						1				1	1	1			1			1	1										1						
101	0.8418						1				1	1	1	1		1	1		1	1				1	1	1				1						
102	0.8414						1				1	1	1			1			1	1				1	1	1				1						
103	0.8412						1				1	1				1			1	1				1	1	1				1						
104	0.8408					1	1																	1	1	1				1						
105	0.8401			1	1		1				1	1	1	1		1	1		1	1				1						1						
106	0.8400			1	1		1				1	1		1		1	1		1	1				1						1						

Table C.2 (cont.)

Breakpoint Hypothesis Test Results--
Component Observed Variables for Subsets
of All Evaluated Defined Factor Combinations (Second Observed Variable Segment)

Rank Order	Multiple Correlation Coefficient^2	ENGN2	ENGN3	DURDAY2	DURDAY3	DURDAY4U	WILCAT2	ENGTYP2	ENGTYP3	ENGTYP4	TERR1	TERR3	TERR4	TERR5	WEAT1	WEAT3	WEAT4	WEAT5	WEAT6	SEASN1	SEASN3	SEASN4	MOR1	MOR2	ATTSURY	DEFSURY	SETEFF1	AIRSPRO	AIRSPR1	AIRSPR2	CAIR1	CAIR2	CAIR3	TPERSTR3	TPERSTR2	FRATIO2
107	0.8383						1				1			1			1							1	1	1				1						
108	0.8382			1	1		1				1	1	1			1			1	1				1						1						
109	0.8377					1	1																	1	1											
110	0.8256																	1			1	1			1			1								
111	0.8214						1	1	1	1	1	1	1					1	1																	
112	0.8190					1	1	1		1		1						1	1					1	1					1		1				
113	0.8163					1	1				1		1	1			1		1					1	1	1				1						
114	0.8156			1	1	1	1																	1	1	1				1						
115	0.8141					1	1				1		1						1					1	1	1				1						
116	0.8130					1	1				1		1						1					1	1											
117	0.8091					1	1	1		1		1							1					1	1											
118	0.8086							1		1		1							1																	
119	0.8030						1				1	1		1		1	1		1	1				1	1	1				1						
120	0.7928			1	1																			1												
121	0.7846						1																	1	1	1				1						
122	0.7582			1	1		1	1		1	1	1		1			1		1					1												
123	0.7286						1				1	1	1	1		1	1		1	1										1						

Table C.2 (cont.)

Breakpoint Hypothesis Test Results--
Component Observed Variables for Subsets
of All Evaluated Defined Factor Combinations (Second Observed Variable Segment)

Rank Order	Multiple Correlation Coefficient^2	ENGN2	ENGN3	DURDAY2	DURDAY3	DURDAY4U	WILCAT2	ENGTYP2	ENGTYP3	ENGTYP4	TERR1	TERR3	TERR4	TERR5	WEAT1	WEAT3	WEAT4	WEAT5	WEAT6	SEASN1	SEASN3	SEASN4	MOR1	MOR2	ATTSURY	DEFSURY	SETEFF1	AIRSPR0	AIRSPR1	AIRSPR2	CAIR1	CAIR2	CAIR3	TPERSTR3	TPERSTR2	FRATIO2
107	0.8383						1				1			1			1							1	1	1				1						
108	0.8382			1	1		1				1	1	1			1			1	1				1						1						
109	0.8377					1	1																	1	1											
110	0.8256																	1			1	1			1			1								
111	0.8214						1	1	1	1	1	1	1					1	1																	
112	0.8190					1	1	1		1		1						1	1					1	1					1		1				
113	0.8163					1	1				1		1	1			1		1					1	1	1				1						
114	0.8156			1	1	1	1																	1	1	1				1						
115	0.8141					1	1				1		1						1					1	1	1				1						
116	0.8130					1	1				1		1						1					1	1											
117	0.8091					1	1	1		1		1							1					1	1											
118	0.8086							1		1		1							1																	
119	0.8030						1				1	1		1		1	1		1	1				1	1	1				1						
120	0.7928			1	1																			1												
121	0.7846						1																	1	1	1				1						
122	0.7582			1	1		1	1		1	1	1		1			1		1					1												
123	0.7286						1				1	1	1	1		1	1		1	1										1						

Table C.3

Breakpoint Hypothesis Test Results--
Identification of All Evaluated Defined Factor Combination
Subsets and Associated Equation Fit Results (First Observed Variable Segment)

Rank Order	Factors Present 1	2	3	4	5	6	Total No. of Engag.	Won by Attk.	Won by Def.	Functional Form*	ψ_{CALC} b_0	b_1	Mult. Corr. Coeff.^{2}	F	ψ^{-1}_{CALC} Mult. Corr. Coeff.^{2}	F
1	1						220	133	87	2	0.1299	0.9076	0.9890	4362.03	0.8377	230.66

* 1--Linear Form; 2--Power Form; 3--Log Form

Table C.4

Breakpoint Hypothesis Test Results--
Component Observed Variables for Subsets of All Evaluated
Defined Factor Combinations (First Observed Variable Segment)

Rank Order	Multiple Correlation Coefficient^2	ENGN1	DURDAY1	WILCAT1	ENGTYP1	TERR2	WEAT2	SEASN2	MOR3	BTSURN	SETEFFO	AIRSPR3	CAIRO	TPERSTR1	FRATIO1
1	0.8377	1	1		1							1			

Table C.4

Breakpoint Hypothesis Test Results--
Component Observed Variables for Subsets of All Evaluated
Defined Factor Combinations (First Observed Variable Segment)

Rank Order	Multiple Correlation Coefficient[2]	ENGN1	DURDAY1	WILCAT1	ENGTYP1	TERR2	WEAT2	SEASN2	MOR3	BTSURN	SETEFFO	AIRSPR3	CAIRO	TPERSTR1	FRATIO1
1	0.3377	1	1		1							1			

VITA

The investigator was also an undergraduate of Lehigh University. He received his Bachelor of Science in Industrial Engineering in 1976, also completing the AFROTC program there. He was born in New York City, New York on May 8, 1954, the son of Robert and Margaret (nee Hayes) Greene, and presently resides in Virginia. Mr. Greene has worked for the Santa Fe Corporation and the Adler Corporation as a military operations research analyst. His assignments at the Adler Corporation included the development of several tactical and campaign-level naval engagement models and the evaluation of naval weapon, sensor and other ship systems. At the Santa Fe Corporation his project work included the development of several campaign-level simulation models to evaluate alternative future U.S. naval nuclear force structures, a discrete time-event simulation for the analysis of drydock utilization, and a comprehensive resource allocation model for analyzing ship and aircraft resource requirements. Currently he is employed as a System Engineer at the MITRE Corporation in their Command, Control and Communications Division.

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