1-1-1978

The identification and evaluation of techniques for solving a class of plant location problems.

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THE IDENTIFICATION AND EVALUATION OF TECHNIQUES FOR SOLVING A CLASS OF PLANT LOCATION PROBLEMS

by

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A Thesis

Presented to the Graduate Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Industrial Engineering

Lehigh University

1978

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

9/12/78
(Date)

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ACKNOWLEDGEMENT

To Dr. Louis Plebani for his guidance and suggestions, I wish to express my sincere appreciation.

This research was facilitated by the generous support of Air Products and Chemicals, Inc. in providing computer resources.

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ABSTRACT

The problem of interest is the capacitated plant location problem. In this thesis existing plant location models will be reviewed and evaluated, a new model will be proposed, and a sample plant location problem will be formulated and solved using four different models.

Two heuristic plant location methods were reviewed and evaluated. Because of assumptions in the methods themselves and the universal concern about the uncertainty of the quality of any heuristic method, these two models are not tested on the sample problem.

A warehouse location model which used an infinite set of plant locations as its solution space was also reviewed and evaluated. Because of the assumption of unlimited plant capacity, this method was not tested on the sample problem.

A decomposition method which used special features of the problem was reviewed. Because the method was very complex and used a somewhat questionable problem feature which required each customer to be served by only one warehouse, the model was not tested on the sample problem.

A method of tightening the lower bound by Lagrangean Relaxation was reviewed. This particular method was not tested on the sample problem because two alternate methods of tightening the lower bounds were tested.

The casewise linear programming (LP) method was reviewed and tested on the sample problem. For n potential plant locations, this method requires 2^n LP cases, which may not be practical to solve.

The standard mixed integer programming method was reviewed and tested on the sample problem. For large plant location problems, however, the solution times required could be unacceptable.

Two methods which are aimed at tightening the lower bound found by the solution to the relaxed problem (integer value requirements removed) were reviewed and tested on the sample problem. The purpose of tightening the lower bound is the reduction of branching and bounding required. The second method was shown to dominate the first and provide a very tight relaxation.

Tests of the sample problem for the four selected models demonstrated the cumbersomeness of solving 64 separate LP cases for the casewise linear programming method; the simplicity of the mixed integer programming method, where the problem can be solved by a software package within an acceptable amount of time; and the tightness of the relaxation provided by the revised method of Davis and Ray and its consequent improvement in solution time.

In conclusion, to solve this class of plant location problem, first the standard mixed integer programming problem formulation should be applied using the best mixed integer programming software package available (to the user). If the resultant solution times and costs are not acceptable to the user or if the user wants to improve the performance, the problem should be reformulated using the revised method of Davis and Ray to tighten the lower bound found from the solution to the relaxed problem and then solved by using a mixed integer programming package. The new tightly formulated model might even be solved by strictly linear programming with manual branching and bounding if a mixed integer programming package is not readily available.

CHAPTER 1

INTRODUCTION

The problem that is addressed is the capacitated plant location problem. The problem can briefly be described as the decision process of choosing the location or locations of new plants that are being added to a network of existing plants which are producing the same homogenous product or products. Existing facilities can be moved only at extreme expense and hence must be considered fixed in location. Although the replacement of obsolete facilities could require the addition of a new facility, the primary motive for adding new plants is growth in customer demands. Depending upon the growth rate of a company, the plant location decision process could be a frequent one.

The criteria used to evaluate alternate plant locations is the cost effectiveness of the combined production and distribution cost reductions resulting from the new facility. The distribution cost effectiveness of a new plant is a function of how well the new plant or plants combine with the existing network of plants in the distribution of product to customers. The product cost effectiveness is a function of the product cost at the new plant in comparison to the product cost at existing plants. The customer demands are assumed to be known and fixed (at any point in time).

In this plant location problem, there is only a single stage of distribution, which is the shipment of product from the plant directly to the customer. The warehouse location problem has two stages of distribution, the shipment of product from the plant to the warehouse and the subsequent shipment from the warehouse to the customer. In the review of the facility location literature, several warehouse location models were evaluated, because the two stages of distribution in the warehouse location model could be collapsed into a single stage and the location technique can then be applied to the single stage problem.

The purpose of this research is threefold. First, existing methods of locating new plants will be reviewed and evaluated. Next, a new method for locating new plants will be proposed and evaluated. Finally, a sample problem will be formulated and solved using four of the above models to provide a consistent comparison among these four methods.

In the review and evaluation of existing and proposed methods of plant location, the following models will be considered:

1. "A Heuristic Program for Locating Warehouses" [10]
2. "A Multiple Center of Gravity Approach to Warehouse Location with Capacity Constraints" [3]
3. Single and Multiple Plant Location Using an Infinite Solution Space Approach [2]
4. "Multiple Distribution Systems Design by Benders Decomposition" [5]
5. "Lagrangean Relaxation Applied to Capacitated Facilities Location Problems" [7]
6. Casewise Linear Programming (LP)
7. Mixed Integer Programming
8. "A Branch-Bound Algorithm for the Capacitated Facilities Location Problem" [1]
9. The Revised Method of Davis and Ray

The first five models are reviewed and evaluated in this paper, but the sample problem has not been formulated and solved using these models, primarily because of the large amount of effort required to complete this task.

For the final four models, in addition to their review and evaluation, a sample problem was formulated and solved using the models.

Two heuristic models have been reviewed. The general statement that the closeness of the approximation provided by a heuristic to the optimal solution of any particular problem is uncertain applies also to these two heuristics, and should be considered in their evaluation.

CHAPTER 2

REVIEW OF THE LITERATURE AND MODEL EVALUATIONS

2.1 Introduction

The plant location literature is filled with many articles on the topic, and careful selection was required to review and evaluate methods that seemed particularly suitable to the plant location problem previously outlined.

It must be carefully noted and acknowledged that the methods discussed in this chapter, with the exception of the final method, are plant location methods developed by the authors recognized. Frequently the authors' own wording has been used to describe the method. The original notation and symbols have been changed to provide a more consistent paper. The following notation is used in this paper:

Indexes

i - Plant or warehouse (in the single stage problem; the terms plant and warehouse are interchangeable)

j - Customer

Constants

C_{ij} - Per unit production and distribution cost associated with producing and delivering product from plant i to customer j

D_j - Demand at customer j

F_i - Fixed cost of plant i

A_i - Capacity of plant i

C'_{ij} - Total production and distribution cost of serving D_j from plant i

Variables

X_{ij} - Quantity produced and shipped from plant i to customer j

$Y_i = \begin{cases} 1 & \text{if any product is produced at plant } i \\ 0 & \text{otherwise} \end{cases}$

X'_{ij} - Fractional quantity of D_j produced and served by plant i

This notation must be augmented to accommodate the two stages of distribution and multiple products in two of the methods. The augmented notation is:

Indexes

i - Warehouse (Warehouse and plant are not interchangeable in the two stage problem)

j - Customer

k - Plant

l - Product

Constants

C_{lkij} - Per unit production and distribution cost of producing and shipping product l at plant k through warehouse i to customer j

D_{lj} - Demand for product l at customer j

F_i - Fixed cost at warehouse i

A_{lk} - Production capacity of product l at plant k

$\underline{V}_i, \bar{V}_i$ - Minimum and maximum throughputs for warehouse i

B_i - Variable cost of throughput for warehouse i

Variables

X_{lkij} - Quantity produced and shipped of product l at plant k through warehouse i to customer j

$Y_i = \begin{cases} 1 & \text{if any product is shipped from warehouse } i \\ 0 & \text{otherwise} \end{cases}$

$Z_{ij} = \begin{cases} 1 & \text{if warehouse } i \text{ serves customer } j \\ 0 & \text{otherwise} \end{cases}$

In the discussion of plant location methodologies, there is no explicit distinction between existing plants and new plants. This difference is implicitly handled by the value of F_i. For existing plant i, $F_i = 0$.

In the following subheadings, different plant location methods will be described and evaluated.

2.2 "A Heuristic Program for Locating Warehouses" [10]

Method

The following formulation of the warehouse location problem, with the original notation changed for the sake of consistency through this paper is given:

Minimize (1) $\sum_{l}\sum_{k}\sum_{i}\sum_{j} C_{lkij} \; X_{lkij} + \sum_{i} F_i Y_i + \sum_{i} B_i (\sum_{l}\sum_{k}\sum_{j} X_{lkij})$

Subject to (2) $\sum_{k}\sum_{i} X_{lkij} = D_{lj}$ for all l, j

(3) $\sum_{i}\sum_{j} X_{lkij} \leq A_{lk}$ for all l, k

(4) $\sum_{l}\sum_{k}\sum_{j} X_{lkij} \leq \overline{V}_i$ for all i

(5) all $X \geq 0$ all $Y = 0,1$

The interpretation of the equations is as follows:

(1) - Minimize the total production and distribution cost plus the fixed cost of the open warehouses plus the total variable cost of throughput associated with warehouses.

(2) - Customer j's demand for product l must be satisfied.

(3) - The production capacity at plant k for product l must not be exceeded.

(4) - The maximum througput at warehouse i cannot be exceeded.

Although the authors have a cost term associated with a delay of T time units, this term has been excluded from the formulation, because it is not a necessary part of the stated plant location problem.

The following heuristic method of solution has been proposed for the warehouse location problem:

Main Program

Step 1. Locate warehouses one at a time until no additional warehouses can be added to the distribution system without increasing total cost.

Bump and Shift Routine

Step 2. Modify solutions arrived at in Step 1 by evaluating profit implications of dropping individual warehouses or shifting them from one location to another.

The heuristics which are used in main program are:

1. Primary locations will be at or near concentrations of demand.

2. Near optimum system can be developed by locating warehouses one at a time, adding at each stage the warehouse which produces the greatest cost savings for the entire system.

3. Only a small subset of all possible warehouse locations need be evaluated in detail at each stage of the analysis to determine the next warehouse site to be added.

   a. Screen N of M potential sites ($M > N > 1$) to be evaluated in detail.

The following flow diagram is given for the heuristic:

I. INPUT

a) Potential warehouse locations

b) Number of sites N to be evaluated in each cycle and size of buffer

c) Shipping cost from plant to customer

d) Customer demands

e) Variable cost of warehouse and plants

II. Determine and place in buffer the N potential warehouse sites which when considering only their local demands would produce the greatest cost savings if supplied by local warehouses rather than the warehouses currently serving them.

III. Evaluate the cost savings that would result for the total system for each of the distribution patterns resulting from the addition of the next warehouse at each of the N locations in the buffer.

IV. Eliminate from further consideration any of the N sites which do not offer cost savings in excess of fixed cost.

Do any of the N sites offer cost savings in excess of fixed costs?

YES - V Locate a warehouse at site which offers largest savings. Go to II.

NO - VI Have all M potential warehouse locations been either activated or eliminated?

NO - Go to II

YES - VII Bump and Shift Routine

a) Eliminate warehouses which have become uneconomical as a result of placement of subsequent warehouses. Each customer formerly served by such a warehouse will now be served by the lowest cost remaining warehouse.

b) Evaluate the economics of shifting each warehouse located above to other potential site whose local concentration of demand are now served by that warehouse.

Evaluation

The sample problem presented by Keuhn and Hamburger has become a very often used sample problem in the facilities location literature.

The technique used is a heuristic algorithm. It is stated that the heuristic provides near optimal solutions for the warehouse location problem, but no actual comparison between the heuristic and an optimal solution has been reported. It is stated that the heuristic can solve in an economical amount of computer time large problems with several hundred potential warehouse locations and several thousand customer locations. On an IBM 650, 12 sample problems each with 24 potential warehouse locations and 50 customer locations required a total of 132 minutes to solve.

Although this model has two stages of distribution, from plant to warehouse to customer, it can readily be changed to a one stage system. In the problem formulation, plant and warehouse capacity constraints are discussed, but in the published heuristic capacity constraints are not enforced.

Because of limitations on the number of models that could be tested on the sample problem, this method will not be tested on the sample problem. This heuristic is, of course, a candidate for further research.

2.3 "A Multiple Center of Gravity Approach to Warehouse Location with Capacity Constraints" [3]

Method

The center of gravity approach is used to determine the number and location of warehouses which yield a minimum number of delivery miles in distribution subject to certain constraints. The inputs to this model are: the customer locations and demands, the number and capacities, and the warehouse territories. The territories are the customers to be served from each warehouse.

The method of solution is as follows:

1. For each warehouse territory, determine the volume weighted center of gravity. It is stated that the center of gravity will be the location from which delivery miles will be minimized in serving the customers in that territory.

2. Check if any customer is closer to the warehouse location in another territory. If any customer is closer to a warehouse in another territory, switch the customer's assigned source to the closest warehouse. After all switches, recompute the warehouse locations by using the center of gravity technique for any territory with the addition or deletion of customers. Repeat this procedure until there are no further changes in customer territories.

3. After no more reassignments of customer areas from one warehouse territory to another can be based on a reduction in

distribution miles, then warehouse capacity constraints are considered.

a) If a warehouse is over capacity, shed the customers one by one with the least distribution mile penality until the capacity constraint is no longer violated.

b) Revise center of gravity warehouse locations.

c) Repeat (a) and (b) for any other warehouses that are over capacity, except that the initial warehouses which were over capacity cannot be assigned new customers.

Evaluation

This method is heuristic in nature. The goodness of this heuristic has not been discussed. The computational experience in terms of problem size and solution times with this heuristic has also not been discussed. It is stated that the model is inexpensive to use. It is also apparent that the model is rather simple, easy to understand, and straightforward to implement and solve.

The model features are those of the capacitated plant location problem being considered in this thesis.

The quality of this heuristic is reduced by the use of a volume-weighted center of gravity calculation to find the warehouse location from which the distribution miles required to save the customer demands is minimized. The volume-weighted center of gravity is an approximation for the minimum distribution mile warehouse location, but under certain circumstances the approximation is a very poor one. A discussion of the circumstances for which the center of gravity techniques is a poor approximation and a method for locating the minimum distribution mile warehouse by a gradient search technique is given in Distribution Management: Mathematical Modelling and Practial Analysis. [2]

A method such as the previously discussed gradient search technique would find the true minimum distribution location and improve the accuracy of the heuristic.

In the heuristic, customer sourcing patterns and warehouse locations are first computed and then the feasibility is established on the warehouse capacity constraints. The goodness of the warehouse locations could degenerate as feasibility on the capacity constraints is established.

Because of limited time available to test models on a sample problem, this heuristic will be only recommended for future study.

2.4 Single and Multiple Plant Location Using an Infinite Solution Space Approach [2]

Method

The author discusses two different methods of representing new potential plant or warehouse locations, the feasible set method and the infinite set method. In the feasible set method, a number of discrete plant locations are chosen and then evaluated, while in the infinite set method, all points can be viewed as potential plant locations. Models using the infinite set method have been examined in this section.

The first model developed is the single plant location model, which is described as follows:

Define:

(g_o, h_o) - Cartesian coordinate of warehouse location.

(g_j, h_j) - Cartesian coordinate of customer j.

a_j - Cost per weight and per distance from warehouse to customer j.

w_j - Weight transported to customer j.

d_j - Distance from warehouse to customer j.

$$d_j = [(g_o - g_j)^2 + (h_o - h_j)^2]^{1/2}$$

Model:

$$\text{Minimize} \sum_j a_j w_j \, [(g_o - g_j)^2 + (h_o - h_j)^2]^{1/2}$$

A method of solution which finds the minimum value of this function is found by taking the first partial derivatives with respect to the warehouse coordinates g_o and h_o and setting them equal to zero.

The partial derivatives are:

$$\frac{\partial}{\partial g_o} = \sum_j a_j w_j \ (g_o - g_j)/[(g_o - g_j)^2 + (h_o - h_j)^2]^{1/2} = 0$$

$$\frac{\partial}{\partial h_o} = \sum_j a_j w_j \ (h_o - h_j)/[(g_o - g_j)^2 + (h_o - h_j)^2]^{1/2} = 0$$

Since these resultant expressions can be solved in closed form by only numerical methods, an optimum seeking gradient search algorithm is the proposed method of solution.

The basic model given for the multiple warehouse location problem is as follows:

Define:

(g_i, h_i) - Cartesian coordinates of the i warehouses to be located.

(g_j, h_j) - Cartesian coordinates of customer j.

a_j - Cost per weight unit and per distance for customer j.

w_j - Amount transported to customer j.

d_{ij} - Distance from warehouse i to customer j.

$$d_{ij} = [(g_i - g_j)^2 + (h_i - h_j)^2 \ ]^{1/2}$$

$$P_{ij} = \begin{cases} 1 & \text{when customer } j \text{ is served from warehouse } i. \\ 0 & \text{otherwise} \end{cases}$$

Model:

$$\text{Minimize} \sum_{i}\sum_{j} a_j \; w_j \; d_{ij} \; P_{ij}$$

The algorithm for determining the optimal solution is:

1. Choose an initial starting location for each warehouse.

2. Allocate each customer to nearest warehouse, and calculate the value of the resultant function.

3. Calculate new warehouse locations using

$$g_i^* = \frac{\sum_j a_j \; w_j \; g_j \; P_{ij}/d_{ij}}{\sum_j a_j \; w_j \; P_{ij}/d_{ij}}$$

$$h_i^* = \frac{\sum_j a_j \; w_j \; h_j \; P_{ij}/d_{ij}}{\sum_j a_j \; w_j \; P_{ij}/d_{ij}}$$

4. Go to Step 2 and repeat until no further reduction in costs can be made.

Evaluation

The first model which finds the best single plant location to serve a particular set of customer demands is an optimization technique that is solved by few iterations of a gradient search algorithm. For nine problems with 20 customers each the solution required 0.37 minutes on an IBM 7094.

The model was discussed because of prior reference to it in the evaluation of the multiple center of gravity method. The model does not consider the location of a plant or plants to a network of existing plants.

The multiple warehouse location model uses an optimal location process, the gradient search, but the initial location of the warehouses has an impact on the final solution. Thus, the method is not a strict optimization technique. The differences between the solution provided by this technique and the optimal solution are not reported. The computational results were also not reported.

The multiple warehouse location model does not incorporate capacity constraints on the warehouses, which is an essential element of the problem of interest. An area for future investigation is the possibility of using a constrained gradient search technique to incorporate capacity constraints.

Once again, this model will not be tested on the sample problem due to time consideration and the additional research required to incorporate capacity restrictions into the model.

2.5 "Multicommodity Distribution Systems Design by Benders Decomposition" [5]

Method

The problem is to determine which warehouse sites to use, the size of each warehouse, and the customer sourcing patterns. The features of this warehouse location model are:

1. Multiple products.
2. Two stages of distribution; from plant to warehouse to customer.
3. Capacity limits for plants and upper and lower size limits for warehouses.
4. Warehouse economies of scale and fixed costs.
5. Each customer is served by a single warehouse.

The model is stated as follows:

Minimize (1) $\sum_{lkij} C_{lkij} X_{lkij} + \sum_{i} [F_i Y_i + B_i \sum_{lj} D_{lj} Z_{ij}]$

Subject to (2) $\sum_{ij} X_{lkij} \leq A_{lk}$ for all lk

(3) $\sum_{k} X_{lkij} = D_{lj} Z_{ij}$ for all lij

(4) $\sum_{i} Z_{ij} = 1$ for all j

(5) $\underline{V}_i Y_i \leq \sum_{lj} D_{lj} Z_{ij} \leq \bar{V}_i Y_i$ for all i

(6) linear configuration constraints on Y and/or Z

$X \geq 0,\ Z = 0,1,\ Y = 0,1$

The interpretation of the equations is as follows:

(1) Minimize the total production and distribution costs plus the fixed cost of warehouses plus the variable cost of warehouse throughput.

(2) Production capacity for product l at plant k must not be exceeded.

(3) States that demand at customer j for product l must be met (when $Z_{ij} = 1$) and that X_{lkij} must be 0 for all l k when $Z_{ij} = 0$.

(4) Each customer j must be served by only one warehouse i.

(5) Keeps throughput at warehouse i betweein $\underline{V}_i$ and $\overline{V}_i$ or at 0 according to whether the warehouse is open or not ($Y_i = 1,0$). Enforces the correct logical relationship between Y and Z. (That is $Y_i = 1 \Leftrightarrow Z_{ij} = 1$ for same j.)

Benders Decomposition is applied to separate the multiple commodities. When the binary variables are temporarily held fixed so as to satisfy (4) - (6), the remaining optimization in X separates into as many independent classical transportation problems as there are commodities. The transportation problem for the l^{th} commodity is:

(7l) Minimize $\sum_{k}\sum_{j} C_{lk\bar{i}(j)j} \; X_{lk\bar{i}(j)j}$

Subject to $\sum_{j} X_{lk\bar{i}(j)j} \leq A_{lk}$ for all k

$\sum_{k} X_{lk\bar{i}(j)j} = D_{lj}$ for all j

$$X_{lk\bar{i}(j)j} \geq 0 \quad \text{for all } k\ j$$

where $\bar{i}$ (j) is defined for each j as the i index for which $Z_{ij} = 1$ in the temporarily fixed Z array.

In the article, the authors describe an algorithm for the application of Benders Decomposition in the standard fashion and then the actual variant of the pure Benders Decomposition that they have implemented.

Geoffrion [6] also described the decomposition method as having two components, the master problem which is an integer programming problem and the subproblem which is a linear programming problem. The master problem takes all past results (from the solution of the subproblems) into account and selects a trial configuration of facility locations. For the trial configuration of facilities, the optimum transportation flows and resultant system costs are found by solving the linear programming subproblem. The results are sent back to the master problem for the selection of the next trial configuration of facilities.

The master problems are pure 0-1 integer linear programs with a variable for every allowable warehouse and customer combination. The problem is solved by a hybrid branch-and-bound/cutting-plane approach. The subproblems are solved using a primal simplex-based algorithm with factorization developed by Graves and McBride [9].

Evaluation

This method is an optimization process. It is reported that for a problem with 17 products, 14 plants, 0 to 30 potential warehouse locations, and 121 customer zones, solution times of 16 to 191 seconds were required for eight representative runs on an IBM 360/91.

The model uses the two stages of distribution feature, but dropping one of the stages would not cause any difficulties. However, much of the structure of the model is dependent on the requirement that each customer's demand is served from only one warehouse. Surely in the plant location problem addressed in this paper, the single source requirement is not necessary. There is some doubt as to how realistic the single source requirement is in practice. The impact of removing the single source requirement from the model is extremenly unpredictable. This special feature which contributes to the quick solutions may be lost.

This model is rather complex. The development of a model using this basic structure would be an extremely difficult and lengthy task. Therefore, the sample problem will not be tested on this model.

2.6 "Lagrangean Relaxation Applied to Capacitated Facilities Location Problems" [7]

Method

The problem called (P), is formulated as follows:

Minimize (1) $\sum_{i}\sum_{j} C'_{ij} X'_{ij} + \sum_{i} F_i Y_i$

Subject to (2) $\sum_{i} X'_{ij} = 1$ for all j

(3) $\underline{V}_i Y_i \leq \sum_{j} D_j X'_{ij} \leq \overline{V}_i Y_i$ for all i

(4) $E X' + R Y \geq T$

(5) $0 \leq X'_{ij} \leq 1$ for all ij

(6) $Y_i = 0,1$ for all i

where E and R are conformable matrices and T is a column vector. The equations are interpreted as follows:

(1) Minimize the total production and distribution cost plus the fixed cost of open plants.

(2) Since X'_{ij} represents the portion (or fraction) of the demand at customer j served by plant i, this requires that the entire demand at customer j be satisfied.

(3) The total shipments from each plant ($\sum_{j} D_j X'_{ij}$) must be between the minimum and maximum plant capacities when the plant is open ($Y_i = 1$) or zero when the plant is closed ($Y_i = 0$).

(4) Any additional linear constraints which X' and Y must conform to.

(5) Requires that the portion of the demand at customer j served by plant i must be between 0 and 1.

In the branch and bound solution of problem (P) there exists the question of which relaxation to use to form the lower bound in the branching and bounding process. The usual LP relaxation $(\overline{P})$ is obtained by relaxing (6) to (6a) $0 \leq Y_i \leq 1$ for all i.

The aim of Lagrangean relaxation is to find a tighter lower bound for $(\overline{P})$ than the usual LP relaxation so as to reduce the branching and bounding.

The Lagrangean relaxation of (P) relative to any m-vector λ and nonnegative p-vector μ is:

$$(L_{\lambda,\mu}) = \underset{X',Y}{\text{Minimize}} \sum_i \sum_j C'_{ij} X'_{ij} + \sum_i F_i Y_i + \sum_j \lambda_j \left(\sum_i X'_{ij} - 1\right) +$$

$$\mu\,(T - EX' - RY)$$

Subject to (3), (5), (6)

Constraints (2) and (4) are taken into the objective function in a Lagrangean fashion, λ and μ serving as multiplier vectors. $(L_{\lambda,\mu})$ separates into m independent subproblems, designated as $(L^i_{\lambda,\mu})$, one for each i. Each has a single 0-1 variable Y_i and same continuous X'_{ij}. For $Y_i = 0$, the corresponding optimal X'_{ij}'s = 0 and for $Y_i = 1$ the corresponding X'_{ij}'s can be found by solving a simple continuous knapsack-type problem. The optimal value of $(L_{\lambda,\mu})$ can be written as:

$$v(L_{\lambda,\mu}) = \sum_i v\,(L^i_{\lambda,\mu}) - \sum_j \lambda_j + \mu T$$

Some choices of λ and μ will yield a Lagrangean relaxation as tight as (but no tighter than) the following partial convex hull relaxation:

$$(P^*) \ \underset{X',Y}{\text{Minimize}} \ \sum_i \sum_j C'_{ij} X'_{ij} + \sum_i F_i Y_i$$

Subject to (2), (4), and $(X',Y) \in$ Convex Hull [(3), (5), (6)]

The best values of (λ,μ) can be shown to be essentially the LP dual of (P^*). For $D_j > 0$ for all j, the convex hull of solutions to constraints (3), (5), (6) is given by the solutions to (3), (5), (6a) and

(13) $X'_{ij} < Y_i$ for all i j.

If $\overline{V}_i < D_j$ then (5) and (13) can be tightened to:

(5') $0 \leq X'_{ij} \leq \overline{V}_i/D_j$

(13') $X'_{ij} \leq (\overline{V}_i/D_j)Y_i$

Evaluation

This method is an optimization technique. It is reported that for six sample problems with between 7 and 25 potential plant locations and between 40 and 102 customers the solution times for this method were between 3 and 113 seconds on an IBM 370/158.

It is also reported that the difference between the optimal value of (P) and the value of $(\bar{P})$ was 0.61% for their sample problems, using

Langrangean Relaxation to find the value of $(\bar{P})$. This is indeed a very tight relaxation.

When $\underline{V}_i = 0$ for all i and the additional linear constraints (4) do not exist, it is stated that constraints of type (13) have been included in the method of Davis and Ray [1]. Since these two conditions are found in the plant location problem under consideration, the method of Davis and Ray [1] will be tested on the sample problem instead of the more complex Lagrangean Relaxation formulation.

2.7 Casewise Linear Programming[1]

Method

The linear programming model is stated as follows:

Minimize (1) $\sum_i \sum_j C_{ij} X_{ij}$

Subject to (2) $\sum_i X_{ij} = D_j$ for all j

(3) $\sum_j X_{ij} \leq A_i$ for all i

(4) All $X \geq 0$

The use of this model is as follows:

1. Select a set of potential plant locations.

2. Add all combinations of these locations, one combination at a time to the existing system of plants and solve the linear programming model to determine the total system cost for that combination.

3. Add the appropriate fixed costs to the costs found from the solution of the linear programming model.

4. Compare the total cost (including fixed) for each combination of plant locations and choose the best.

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[1]This description is based upon the experience of this writer in the use of linear programming to solve plant location problems at Air Products and Chemicals, Inc.

Evaluation

This method will find the optimal plant location or locations if all possible combinations are evaluated in the model. For problems having approximately 25 plant locations and about 500 customers, solution times of about 50 seconds are typical for each case using an IBM 370/165. Results for the sample problem will also be discussed in the next chapter.

Since linear programming models of the production and distribution system for short range distribution planning are frequently available, the modification of the customer demand scenario to a longer range time frame makes the linear programming approach an easy step into plant location. If the number of potential new plant locations is n, then 2^n LP cases are required to evaluate all possible combinations. For a relatively small value of n, for example 10, the number of LP cases required, 1024 in this example, becomes too large to practically handle.

The methodology was chosen for testing on the sample problem primarily for the purpose of establishing the benchmark solution values.

2.8 Mixed Integer Programming

Method

The standard formulation of the plant location problem is stated by McGinnis [11] as follows:

$$\text{Minimize (1)} \quad \sum_i \sum_j C_{ij} X_{ij} + \sum_i F_i Y_i$$

$$\text{Subject to (2)} \quad \sum_i X_{ij} = D_j \quad \text{for all } j$$

$$\text{(3)} \quad \sum_j X_{ij} \leq A_i Y_i \quad \text{for all } i$$

$$\text{(4)} \quad \text{All } X \geq 0 \quad \text{all } Y = 0,1$$

The interpretation of the equations is as follows:

(1) Minimize the total production and distribution cost plus the fixed cost of the open plants.

(2) The demand at customer j must be satisfied.

(3) The production capacity at plant i must not be exceeded.

There are many commercially available software packages which solve mixed integer programming problems, such as the MPSX-MIP package offered by IBM, UMPIRE, APEX and MPOS to name a few. Each algorithm offers, of course, different performance characteristics, even on the same problem. The basic approach found in these algorithms is the branch and bound approach.

Evaluation

This method is an optimization procedure. Computational experience and comments about the great variability of solution times for the plant location problem formulated as a standard mixed integer programming problem are given by Davis and Ray [1], Geoffrion and Graves [5], Forrest, Hirst, and Tomlin [4], and Geoffrion and Marsten. [8]

Computational experience of this model on the sample problem is reported in Chapter 3.

This model formulation includes all of the problem features. The previously discussed variability of solution times makes the use of this method dependent upon the actual problem. Improvements in computer speeds and mixed integer programming algorithms will make this method increasingly more desirable.

For these reasons, the sample problem was tested using this model.

2.9 "A Branch-Bound Algorithm for the Capacitated Facilities Location Problem" [1] (The Method of Davis and Ray)

Method

The problem (P) is stated as follows:

Minimize (1) $\sum_i \sum_j C_{ij} X_{ij} + \sum_i F_i Y_i$

Subject to (2) $\sum_i X_{ij} = D_j$ for all j

(3) $\sum_j X_{ij} \leq A_i$ for all i

(4) $X_{ij} - \min(D_j, A_i) Y_i \leq 0$ for all i j

(5) $X \geq 0 \quad Y = 0,1$

The equations are interpreted as follows:

(1) Minimize the production and distribution cost plus the fixed cost.

(2) All customer demands must be satisfied.

(3) Plant capacities cannot be exceeded.

(4) Constraints designed to force $Y_i = 1$ if the entire demand at customer j (D_j) is served by plant i or if the entire capacity of plant i (A_i) is shipped to a single customer j.

The method of solution is as follows:

Step 1. Solve the LP relaxation of the stated problem (P), calling the relaxed problem ($\bar{P}$). In the relaxed problem ($\bar{P}$), Y_i is a continuous variable rather than a 0-1 variable. If in the solution of ($\bar{P}$) all $Y_i = 0,1$ the original problem (P) is also solved. (The authors report that the problem (P) is frequently solved at this point.) If not all $Y_i = 0,1$, the value of the objective function is the lower bound to the solution of (P).

Step 2. If in Step 1 some Y_i is not an integer, branch by fixing it first at 1, then at 0 and resolve ($\bar{P}$). A lesser value of the objective function becomes a new lower bound for (P). If all Y_i associated with this lower bound are integers, the problem is solved. Also, if no feasible solution exists when this $Y_i = 0$, then plant i must be open in the final solution.

Step 3. Continue branching and bounding until a solution to (P) is found that is the best lower bound developed.

Evaluation

This method is an optimization procedure. It is stated that the solution of ($\bar{P}$) resulting in all $Y_i = 0,1$ is a frequence occurrence because of the tight formulation resulting from constraint (4) which is

$X_{ij} - \min (D_j, A_i) Y_i \leq 0$ for all i j.

The tightness of this relaxation is verified by an entirely different perspective, the Lagrangean Relaxation, by Geoffrion and McBride. [7]

The motivation for constraint (4) in the model of Davis and Ray was the physical interpretation of (4) as having the effect of forcing $Y_i = 1$ if any customer's entire demand D_j was served by only one plant i or if an entire plant's production capacity A_i was shipped to a single customer j.

The cost of this tight formulation is $m \times n$ additional rows in the model. (Compared to the standard mixed integer programming formulation, where n is the number of potential new plant locations and m is the number of customers.)

From the computational experience reported, the benefit of this tight formulation is much greater than the cost, and this model will be tested on the sample problem.

2.10 The Revised Method of Davis and Ray[1]

Method

The problem (P) is stated as follows:

Minimize (1) $\sum_i \sum_j C_{ij} X_{ij} + \sum_i F_i Y_i$

Subject to (2) $\sum_i X_{ij} = D_j$ for all j

(3) $\sum_j X_{ij} \leq A_i Y_i$ for all i

(4) $X_{ij} - \min (D_j, A_i) Y_i \leq 0$ for all i j

(5) $X \geq 0 \quad Y = 0,1$

This statement of the problem (P) is identical to that proposed by Davis and Ray [1] with the exception of (3). In constraint (3) a redundant inclusion of Y_i is added to the constraint. The motivation for the modification of this constraint (3) by myself is explained in the following interpretation of the equations:

(1) Minimize the production and distribution cost plus the fixed cost.

---

[1]The Revised Method of Davis and Ray was developed by this writer after observations of the performance of the Method of Davis and Ray on the sample problem and was based on the reasoning presented in the text of this method. In the subsequent research into the originality of this method, it has been found that this method was equivalent to the partial convex hull relaxation (described in the Lagrangean Relaxation Method) for the special case when there are no additional constraints between X and Y and the minimum plant production is zero.

(2) All customer demands must be satisfied.

(3) Plant capacities cannot be exceeded. Also, the Y_i was attached to the capacity to force $Y_i = 1$ if plant i's entire capacity is shipped anywhere.

(4) Constraints designed to force $Y_i = 1$ if the entire demand at customer j (D_j) is served by plant i or if the entire capacity of plant i (A_i) is shipped to a single customer j.

The method of solution is then identical to that proposed by Davis and Ray. That is, relax the integrality requirement on Y_i and solve the relaxed problem ($\bar{P}$). If in the solution to ($\bar{P}$) all $Y_i = 0,1$ then (P) is also solved. If some Y_i are not integers, fix the non-integer Y_i to 0 or 1 and begin branching and bounding until a solution to (P) is found that is the best lower bound developed.

Evaluation

This model is an optimization procedure. The redundant inclusion of Y_i in constraint (3) add no new rows or columns to the problem, hence should not cause any drastic change in solution time of the relaxed problem ($\bar{P}$). However, this additional requirement forces $Y_i = 1$ if plant i ships its entire production to any combination of customers. In cases where Y_i is not integer, the value Y_i must take the larger of either the value from (3) or the value from (4), then forcing the higher value of Y_i to be multiplied by F_i. This guarantees that the lower bound on the value of the objective function found by solving ($\bar{P}$) with the revised constraint (3) has to be greater than or equal to the value of ($\bar{P}$) found by the Davis and Ray method.

In addition, from the model formulation, it can be observed that the revised method of Davis and Ray also guarantees a solution which is at least as good as the LP relaxation of the standard mixed integer programming formulation.

In order to determine some computational experience with this proposed model, the sample problem will be tested using this model.

CHAPTER 3

FORMULATION AND SOLUTION OF A SAMPLE PROBLEM FOR SELECTED MODELS

3.1 Introduction

The plant location methods that were tested on the sample problem were: the casewise linear programming method, the standard mixed integer programming method, the Davis and Ray method, and the revised method of Davis and Ray.

The sample problem used was the following variation of the Keuhn and Hamburger sample problem [10]. Since the Keuhn and Hamburger algorithm was designed for the location of warehouses, their two stage system was collapsed to a one stage system. To the plant locations used by Keuhn and Hamburger, which were Indianapolis and Jacksonville, Florida, two additional existing plants were included at Boston and New York. The capacity of each of these four existing plants was set at 15,000. Six potential new plant locations were chosen for evaluation at Chicago, Detroit, Los Angeles, Philadelphia, Pittsburgh, and San Francisco. These locations were chosen because they had customer demands of more than 2000 at the locations themselves. The capacity of the new potential plants was set at 5000 each.

The sales potentials or customer demands were the Keuhn and Hamburger demands, except the six demands points with a demand less than 100 were eliminated to reduce the size of the problem. The customer location and demands which were used are given in Table 1.

TABLE 1

CUSTOMER DATA

Customer Location	Customer Demand
Albuquerque, New Mexico	146
Atlanta, Georgia	672
Baltimore, Maryland	1337
Birmingham, Alabama	559
Boston, Massachusetts	2370
Buffalo, New York	1089
Chicago, Illinois	5495
Cincinnati, Ohio	904
Cleveland, Ohio	1466
Columbia, South Carolina	143
Dallas, Texas	615
Denver, Colorado	564
Des Moines, Iowa	226
Detroit, Michigan	3016
Duluth, Minnesota	253
El Paso, Texas	195
Houston, Texas	807
Indianapolis, Indiana	551
Jacksonville, Florida	304

TABLE 1

CUSTOMER DATA
(Continued)

Customer Location	Customer Demand
Kansas City, Missouri	814
Knoxville, Tennessee	337
Los Angeles, California	4368
Louisville, Kentucky	577
Memphis, Tennessee	482
Miami, Florida	495
Mobile, Alabama	231
Nashville, Tennessee	322
New Orleans, Louisiana	685
New York, New York	12912
Oklahoma City, Oklahoma	325
Omaha, Nebraska	366
Philadelphia, Pennsylvania	3671
Pittsburgh, Pennsylvania	2213
Portland, Oregan	705
Richmond, Virginia	328
St. Louis, Missouri	1681
St. Paul, Minnesota	1117
Salt Lake City, Utah	275

TABLE 1

CUSTOMER DATA
(Continued)

Customer Location	Customer Demand
San Antonio, Texas	500
San Francisco, California	2241
Seattle, Washington	733
Spokane, Washington	222
Washington, D.C.	1464
Wichita, Kansas	222

The cost coefficients, each C_{ij}, were computed by calculating the straight line distance (or airmiles) between the plants and the customer by using the geometric relationship between the latitude and longitude of the two points, and then multiplying the round trip miles by a cost per mile of \$0.025. As is apparent, this cost is for only distribution. The unit production cost could have easily been incorporated in C_{ij} but was fixed at a value of zero, for simplicity.

The levels of fixed cost of new plants (F_i) were 0, \$10,000, \$100,000, \$200,000, \$300,000, and \$400,000. Although some methods were evaluated for all levels of F_i, all methods were evaluated for F_i = 10,000 and F_i = 400,000.

3.2 Case-Wise Linear Programming Method

The number of linear programming cases required to evaluate all combinations of the six potential new plants was $2^6 = 64$. All the linear programming cases were solved using the IBM MPSX linear programming package on an IBM 370/165 using 260K of memory. The results are given in the following Table 2.

To the optimal solution of the LP model (which by definition has $F_i = 0$) the various levels of fixed costs are simply added. This means that for any fixed cost, we can find the best configuration of new plants from Table 2.

TABLE 2

CASEWISE LP SOLUTIONS

PLANTS OPEN (X)						COST IN THOUSANDS OF DOLLARS		
CHICAGO	DETROIT	LOS ANGELES	PITTSBURGH	PHILADELPHIA	SAN FRANCISCO	$F_i = 0$	$F_i = 10$	$F_i = 400$
						1580.6	1580.6	1580.6
X						1441.6	1451.6	1841.6
	X					1440.0	1454.0	1844.0
		X				1048.6	1058.6	1448.6
			X			1469.1	1479.1	1869.1
				X		1514.4	1524.4	1914.4
					X	1068.4	1078.4	1468.4
X	X					1312.3	1332.3	2112.3
X		X				913.7	933.7	1713.7
X			X			1332.2	1352.2	2132.2
X				X		1375.3	1395.3	2175.3
X					X	939.6	959.6	1739.6
	X	X				912.0	932.0	1712.0
	X		X			1349.7	1369.7	2149.7
	X			X		1377.8	1397.8	2177.8
	X				X	934.6	954.6	1734.6
		X	X			937.1	957.1	1737.1
		X		X		982.4	1002.4	1782.4
		X			X	673.6	693.6	1473.6

TABLE 2
CASEWISE LP SOLUTIONS
(Continued)

PLANTS OPEN (X)						COST IN THOUSANDS OF DOLLARS		
CHICAGO	DETROIT	LOS ANGELES	PITTSBURGH	PHILADELPHIA	SAN FRANCISCO	$F_1 = 0$	$F_1 = 10$	$F_1 = 400$
			X	X		1403.4	1423.4	2203.4
			X		X	959.6	976.9	1756.9
				X	X	1002.2	1022.2	1802.2
X	X	X				812.0	842.0	2012.0
X	X		X			1225.3	1255.3	2425.3
X	X			X		1246.0	1276.0	2446.0
X	X				X	838.0	868.0	2038.0
X		X	X			813.2	843.2	2013.2
X		X		X		847.4	877.4	2047.4
X		X			X	576.9	606.9	1776.9
X			X	X		1266.9	1296.9	2466.9
X			X		X	839.2	869.2	2039.2
X				X	X	873.4	903.4	2073.4
	X	X	X			821.7	851.7	2021.7
	X	X		X		845.8	875.8	2045.8
	X	X			X	566.2	596.2	1766.2
	X		X	X		1285.6	1315.6	2485.6
	X		X		X	847.7	877.7	2047.7

TABLE 2
CASEWISE LP SOLUTIONS
(Continued)

PLANTS OPEN (X)						COST IN THOUSANDS OF DOLLARS		
CHICAGO	DETROIT	LOS ANGELES	PITTSBURGH	PHILADELPHIA	SAN FRANCISCO	$F_1 = 0$	$F_1 = 10$	$F_1 = 400$
	X			X	X	868.4	898.4	2068.4
		X	X	X		871.4	901.4	2071.4
		X	X		X	568.8	598.8	1768.8
		X		X	X	607.4	637.4	1807.4
			X	X	X	891.2	921.2	2091.2
X	X	X	X			742.7	782.7	2342.7
X	X	X		X		746.0	786.0	2346.0
X	X	X			X	515.5	555.5	2115.5
X	X		X	X		1163.6	1203.6	2763.6
X	X		X		X	768.6	808.6	2368.6
X	X			X	X	772.0	812.0	2372.0
X		X	X	X		748.0	788.0	2348.0
X		X	X		X	503.8	543.8	2103.8
X		X		X	X	510.7	550.7	2110.7
X			X	X	X	774.0	814.0	2374.0
	X	X	X	X		760.0	800.0	2360.0
	X	X	X		X	492.9	532.9	2092.9
	X	X		X	X	500.1	540.1	2100.1

TABLE 2
CASEWISE SOLUTIONS
(Continued)

PLANT OPEN (X)						COST IN THOUSANDS OF DOLLARS		
CHICAGO	DETROIT	LOS ANGELES	PITTSBURGH	PHILADELPHIA	SAN FRANCISCO	$F_1 = 0$	$F_1 = 10$	$F_1 = 400$
	X		X	X	X	786.0	826.0	2386.0
		X	X	X	X	503.4	543.4	2103.4
X	X	X	X	X		697.2	747.2	2697.2
X	X	X	X		X	451.4	501.4	2451.4
X	X	X		X	X	450.3	500.3	2450.3
X	X		X	X	X	723.2	773.2	2723.2
X		X	X	X	X	445.7	495.7	2445.7
	X	X	X	X	X	447.4	497.4	2447.4
X	X	X	X	X	X	405.9	465.9	2805.9

By inspection from Table 2, when $F_i = 0$, the minimum cost solution is having all six potential new plants open for a cost of $405,903. When F_i = $10,000 the minimum cost solution is having all six new plants open at a total cost of $465,903. When F_i = $400,000, the minimum cost solution is opening only the Los Angeles plant at a total cost of $1,448,624.

For this problem, the case-wise linear approach works and serves as an excellent benchmark for comparison. The IBM MPSX package iterated about 100 times to solve each LP case and used 104.3 CPU seconds to solve the 64 cases. The manual effort to run the cases was substantial, approximately five hours. (This, of course, does not include the development of the basic model.)

The case-wise LP approach has the extremely severe drawback of requiring 2^n cases to completely evaluate n potential plant locations. To evaluate just 10 potential plant locations, 1024 LP cases are required. The hypothetical problem of solving 1024 LP cases is clearly unmanageable.

3.3 Mixed Integer Programming Method

The solution of the sample problem was attempted for the two levels of fixed cost F_i = \$10,000 and F_i = 400,000 using two different mixed integer programming packages and computers. First, the mixed integer programming problem was solved using the IBM MPSX/MIP package on an IBM 370/195 with 260K of main storage allocated. The default package solution branching and bound technique was used.

With F=10,000 the optimal solution was found to have all six potential plants open at a cost of \$465,903. In the case of F_i = \$400,000, the optimal solution was found to open only the Los Angeles plant at a total cost of \$1,448,624. The solution time required for the first problem was 4.14 CPU seconds, and 3.78 CPU seconds for the second problem.

Even though this sample problem is small, the improvements in computer processing speeds and improvements in the programming software point to the standard mixed integer programming solution as a very realistic alternative.

The sample problem was attempted on the MPOS mixed integer programming package at Lehigh University on the CDC 6400, but the amount of central memory required, which was 26,358 and the amount of extended core storage (ECS) required which was 149,850, exceeded the amount available. The point should clearly be made that the MPOS package was not designed to handle large problems. Certainly, there are available software packages for the CDC machine which would handle this problem with ease.

3.4 Davis and Ray Method

The method of Davis and Ray was implemented for the sample problem on an IBM 370/165 using the IBM MPSX linear programming package and 260K of main storage. Because of limited access to mixed integer programming packages, the relaxed problem ($\bar{P}$) was solved using LP and the resolution of non-integer Y_i's was done by subsequent solutions using LP and fixing non-integer Y_i's to 0 or 1. The results are given in Table 3.

TABLE 3

METHOD OF DAVIS AND RAY SOLUTIONS

Value of F_i	Value of Objective	Zero - One Variable Values in $(\bar{P})$ Chicago	Detroit	Los Angeles	Pitts-burgh	Phila-delphia	San Fran-cisco
\$ 10,000	\$ 462,897	0.785	1.0	1.0	1.0	0.773	1.0
\$100,000	\$ 784,285	0.152	0	0.53	0.46	0.278	0.47
\$200,000	\$ 931,286	0.16	0	0.53	0	0	0.47
\$300,000	\$1,043,738	0.015	0	0.53	0	0	0.47
\$400,000	\$1,144,490	0	0	0.53	0	0	0.47

Possibly because of the rather arbitrary existing plant locations and the particular demand distribution found in the sample problem, the initial solution of $(\bar{P})$ had between 2 and 5 non-integer values for the 6 integer variables, depending upon the value of F_i. For F_i = \$10,000, the solution of the relaxed standard mixed integer programming formulation is \$462,526 while the solution to $(\bar{P})$ via the method of Davis and Ray was 462,897 while the optimum solution to (P) was \$465,903. The standard relaxed LP solution was within 0.725% of the optimal, while $(\bar{P})$ was slightly better, within 0.645% of optimal. However, with F_i = \$400,000 the LP relaxation of the standard mixed integer formulation has a value of \$1,372,507, the solution to $(\bar{P})$ via the method of Davis and Ray was \$1,144,490, and the optimal solution to (P) was 1,448,624.

The LP solution of the relaxed standard mixed integer formulation was 5.25% within optimal while the solution of $(\bar{P})$ via the method of Davis and Ray was 20.99% away from the optimal. Clearly this is not good performance for the Davis and Ray model in the F_i = 400,000 case. The solution to the relaxed standard mixed integer formulation is much better.

For the initial LP solution for the five different levels of fixed cost in the sample problem, the problem iterated an average of 245 times and required an average of 12.29 CPU seconds to solve using the Davis and Ray formulation.

The resolution of the non-integer values is reported in Table 4.

TABLE 4

RESOLUTION OF NON-INTEGER INITIAL SOLUTIONS
IN THE METHOD OF DAVIS AND RAY

Value of F_i	Value of Objective	Zero - One Variable Values in ($\bar{P}$): Chicago	Detroit	Los Angeles	Pitts-burgh	Phila delphia	San Fran-cisco
$ 10,000	$ 532,871	0*	1	1	1	0*	1
$ 10,000	$ 497,422	0*	1	1	1	1*	1
$ 10,000	$ 501,352	1*	1	1	1	0*	1
$ 10,000	$ 465,903	1*	1	1	1	1*	1
$400,000	$1,547,732	0.18	0.07	0*	0	0	0*
$400,000	$1,450,259	0.18	0	0*	0	0	1*
$400,000	$1,425,154	0.18	0	1*	0	0	0*
$400,000	$1,473,599	0	0	1*	0	0	1*

*Value was fixed in solution to stated value of 0 or 1.

The resolution of non-integrality was straightforward for F_i = 10,000 but after four cases in F_i = 400,000 the non-integrality had not yet been resolved because plants other than the plants having non-integer values in the first solution, entered the subsequent solutions at small non-integer values. The resolution of non-integrality might be more easily accomplished by using a mixed integer programming package, but nevertheless the non-integer values of other 0,1 variables would increase the branching and bounding required.

Clearly, one would not normally resolve the non-integralities using LP, but in the attempted resolution of the problem with F_i = 10,000, the four cases required an average of 326 iterations and 8.62 CPU seconds each to solve. With F_i = 400,000 the four cases required an average of 227 iterations and 8.40 CPU seconds each to solve.

The particular structure of a plant location problem impacts its ease of solution. The method of Davis and Ray has performed poorly in establishing a lower bound which is close to the value of the optimal solution on this sample problem with F_i = 400,000, despite the fact that in other reported applications, this model performed well.

3.5 The Revised Method of Davis and Ray

The revised method of Davis and Ray implemented for the sample problem on an IBM 370/165 using the IBM MPSX linear programming package with 260K of main storage. As in the previous experiments with the method of Davis and Ray, because of limited access to mixed integer programming packages, the relaxed problem ($\bar{P}$) was solved using linear programming and the resolution of non-integer Y_i's was done by subsequent solutions using LP and fixing non-integer Y_i to 0 or 1. The results of the initial LP solution are given in Table 5.

TABLE 5

REVISED METHOD OF DAVIS AND RAY SOLUTIONS

Value of F_i	Value of Objective	Zero - One Variable Values in $(\bar{P})$ Chicago	Detroit	Los Angeles	Pitts-burgh	Phila-delphia	San Fran-cisco
\$ 10,000	\$ 465,903	1	1	1	1	1	1
\$100,000	\$ 860,549	0	0.63	0.89	0	0	1
\$200,000	\$1,059,481	0	0	0.84	0	0	0.94
\$300,000	\$1,233,694	0	0	0.84	0	0	0.87
\$400,000	\$1,401,102	0	0	0.79	0	0	0.83

The five initial LP solutions of the relaxed problem ($\bar{P}$) required an average of 295 iterations and 12.1 CPU seconds each to solve.

However, the results were much better than the method of Davis and Ray. In the F_i = 10,000 case, an all integer solution was produced from ($\bar{P}$) because of the tight constraints, which means that (P) was also solved with a single LP solution. In the four other cases with F_i = 100,000; 200,000; 300,000; and 400,000 there were only two non-integer values of 0-1 variables in each case while in the method of Davis and Ray there were between 2 and 5 non-integer values of 0-1 variables in each case. The revised methods of Davis and Ray, for F_i = 10,000 yielded a value of \$465,903 as a solution to ($\bar{P}$) which is also the optimal solution to (P). The solution of the relaxed standard mixed integer programming formulation is \$462,526 which is within 0.725% of the optimal solution of (P). For F_i = 400,000, the LP relaxation of the standard mixed integer programming formulation has a value of \$1,372,507. The method of Davis and Ray gives a solution of ($\bar{P}$) of \$1,144,490. The revised method of Davis and Ray gives a solution of ($\bar{P}$) of \$1,401,102. The optimal solution of (P) has a value of \$1,448,624. Thus, the solution of the standard LP relaxation of the mixed integer problem is within 5.25% of the optimal solution of (P); the Davis and Ray method has a solution of ($\bar{P}$) within 20.99% of the optimum value of (P); and the revised method of Davis and Ray produced a solution of ($\bar{P}$) within 3.28% of the optimal solution of (P). These results demonstrate that the revised method of Davis and Ray provides a better solution to ($\bar{P}$) than the method of Davis and Ray.

The resolution of the non-integer values of the 0-1 variables is reported in Table 6. (F_i = 10,000 was initially all integer.)

TABLE 6

THE RESOLUTION OF NON-INTEGER INITIAL SOLUTIONS IN THE REVISED METHOD OF DAVIS AND RAY

		Zero - One Variable Values in $(\bar{P})$					
Value of F_i	Value of Objective	Chicago	Detroit	Los Angeles	Pitts- burgh	Phila- delphia	San Fran cisco
$400,000	$1,580,629	0	0	0*	0	0	0*
$400,000	$1,468,447	0	0	0*	0	0	1*
$400,000	$1,448,627	0	0	1*	0	0	0*
$400,000	$1,473,599	0	0	1*	0	0	1*

*Value was fixed in solution to stated value of 0 or 1.

The resolution of the non-integer value by LP in these four cases for $F_i = 400{,}000$ required 220 iterations and 9.27 CPU seconds for each LP case. This actual computer time required to solve this model is very close to the computer time required to solve the method of Davis and Ray in terms of the initial solution of $(\bar{P})$ and the resolution of non-integer values, when required. But, the revised method of Davis and Ray produces a tighter lower bound on the solution of the relaxed problem.

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

4.1 Conclusions

Based upon the research of existing plant location methods and the implementation of selected models upon a sample problem, new plants should be located using a method that guarantees an optimal solution to the problem. The simplest model which guarantees an optimal solution is the casewise linear programming approach, if all combinations of potential new plant locations are evaluated. Since the number of combinations, which is 2^n where n is the number of new potential plant locations, increases very rapidly as the number of potential plant locations increases, this method in practice is not a good alternative.

The standard mixed integer programming model is the next simplest method. In this model the potential new plant locations have a fixed cost which is included if the plant is open and not included if the plant is closed. If the problem can be solved by a commercially available mixed integer software package within the users' acceptable computer time limits, this method is the simplest. However, experience reported with the standard mixed integer programming formulation of the plant location problem show great variability in solution times.

One approach to reducing the time required to solve the mixed integer programming problem is to tighten the lower bound found by solving the relaxed (integer requirement removed) problem. In order to tighten the lower bound found by relaxing the standard mixed integer programming model, three methods were evaluated. The first was a Lagrangean Relaxation approach. The second was a model formulation proposed by Davis and Ray. The third was a revision of the Davis and Ray model. The model proposed by Davis and Ray performed poorly on the sample problem. In fact, in one case the value of the lower bound found by the method of Davis and Ray was about 21% away from the optional value, while the lower bound found by the relaxation of the standard mixed integer programming model was only about 5% away from the optimal value. The revised method of Davis and Ray produces at least as tight of a lower bound as that of the method of Davis and Ray. In fact, in the sample problem the value was only 3% away from optimal.

In addition, the revised method of Davis and Ray also guarantees a lower bound at least as tight as the lower bound formed from the LP relaxation of the standard mixed integer programming formulation. The tightness of the lower bound formed by the revised method of Davis and Ray has also been verified by a Lagrangean Relaxation approach, and supported by the solution of the sample problem.

Thus, the revised method of Davis and Ray is recommended as the solution procedure if the problem cannot be solved within the acceptable range of time and costs by the standard mixed integer programming formulation.

4.2 Recommendations for Further Study

Because of time restrictions, the heuristic models of Keuhn and Hamburger [10] and Feinberg [3] were not tested on the sample problem. Further work in evaluating the accuracy of these heuristics is suggested.

Also, for the location models described by Eilon [2] further work is suggested in order to incorporate the capacity constraints for the plants, perhaps by using a constrained gradient search solution technique.

The decomposition approach developed by Geoffrion [5] certainly deserves further attention. The required single sourcing of each customer problem feature is somewhat of a restriction, but for a slightly different class of facilities location problem the method offers an excellent method of solution.

Similarly, the Lagrangean Relaxation method [7] offers another excellent solution technique in the capacitated plant location problem with additional linear constraints and minimum production requirements.

Improving the actual branching and bounding process itself is a productive area for future research. The use of a good relaxation and improved branching and bounding procedures would certainly improve the efficiency of solution.

Future research should also be done in evaluating the tightness of the lower bound formed by the revised method of Davis and Ray compared to that of the method of Davis and Ray itself, and to that of the LP relaxation of the standard mixed integer formulation on other sample problems, including actual plant location problems.

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