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Fundamentals of list structures and a pascal implementation of basic list processing techniques.

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FUNDAMENTALS OF LIST STRUCTURES AND A PASCAL IMPLEMENTATION OF BASIC LIST PROCESSING TECHNIQUES

BY

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Abstract

The construction of a program requires a well-designed algorithm as well as careful attention to the design of associated data structures. The linked list is a particularly useful structure type.

Let A be a nonempty set of objects called atoms. We distinguish one particular atom, called the NIL atom and designated by Λ. Let $L_0 = A$. We define the sets $L_1, L_2, \ldots, L_n, \ldots$ as follows:

Suppose $L_0, \ldots, L_k$ have been defined, $K \geq 0$. Define L_{k+1} to be the set of all sequences $a_1, \ldots, a_m$, $m \geq 1$, where $a_1, \ldots, a_{m-1} \in (L_0 \cup L_1 \cup \ldots \cup L_k) \sim \{\Lambda\}$ and $a_m = \Lambda$. We call the members of L_n the linear lists of order less than or equal to n.

In order to represent this list structure in computer memory, we utilize and maintain a set of nodes, each including a symbol field and a link to the next node. Since each node of the list contains a pointer to the next node, successive list elements are not required to be consecutive words in computer memory. This ability to utilize arbitrary, disjoint sections of memory is one of the powerful features of lists.

The operations on list structures normally consist of

accessing an element or series of elements, moving them to other lists, replacing them by other series, or processing the entities represented by them.

This paper describes the various types of list structures and explains the concepts behind list processing techniques. Several list processing methods are presented. The PASCAL programming language is used to implement a list processing system, in order that the reader may obtain a working knowledge of this beautifully simple and powerful aspect of programming.

I. Introduction

Computer programs consist of algorithms which transform informational structures. An informational structure consists of a collection of relations and properties on a basic set of elements or atoms. The construction of a program requires, in addition to a well-designed algorithm, careful attention to the design of associated informational structures.

Since computer programs are frequently designed to facilitate the processing of complicated situations, the informational structures required in such programs may be quite intricate. A particular structure type which has been used effectively in the development of informational structures is that of the list.

The use of lists and their manipulation has all too often been restricted to a few specialists in several narrow areas. Moreover, the most frequently used languages, FORTRAN and COBOL, do not permit the easy use of list processing techniques. Despite all this, list processing is capable of a wide area of application and should be known by more programmers.

It is still the case that many programmers feel that list processing techniques are quite complicated. We will see that there is nothing magic or mysterious about the methods of dealing with complex structures. List processing should be one of the many techniques at the disposal of programmers, for use in those parts of programs which require it.

The purpose of this paper is to explain the concepts behind list processing techniques in order that the reader may obtain a working knowledge of this beautifully simple and powerful aspect of programming.

It will be shown how several list processing methods can be easily embedded and used in the language PASCAL.

The basic concepts of list processing may be found in [4], [7], [9], and [13].

II. The List Concept

Let A be a non-empty set of objects. We distinguish one particular object and designate it by Λ. The objects of A are called atoms and in particular Λ is called the nil atom. Let $L_0 = A$. We define the sets $L_1, L_2, \ldots, L_n, \ldots$ as follows:

Suppose $L_0, \ldots, L_k$ have been defined, $k \geq 0$. Define L_{k+1} to be the set of all sequences $a_1, \ldots, a_m$, $m \geq 1$, where $a_1, \ldots, a_{m-1} \in (L_0 \cup L_1 \cup \ldots \cup L_k) \sim \{\Lambda\}$ and $a_m = \Lambda$.

We call the members of L_n the linear lists of order less than or equal to n. A list is said to be of order n if and only if it has order less than or equal to n, but not order less than or equal to $n-1$.

For notational purposes, if $a_1, \ldots, a_p, \Lambda$ is a list, we write it as $(a_1, \ldots, a_p)$. Observe then that if $A = \{a, b, c, \Lambda\}$, then a list of order two might have the form

$$(a, ((a,b), c), (a,b), a).$$

Of course there are infinitely many lists of order n for each $n \geq 1$. The latter is true even when A is finite.

In order to realize the list structure in computer memory, we utilize and maintain a set of nodes. Each node consists of one or more consecutive words of computer memory, divided into named parts called fields. Every node includes a link field and a symbol field. The link component contains the address of the node to be regarded as the successor of the node in question. The symbol component may represent any defined informational structure, e.g. a number, a string of characters, or other information. It may contain the address of another node and thus refer to another sequence of symbols.

Thus, since the items of a list may themselves be lists, the general structure obtained in this manner is called a list structure. Since a list element may contain a pointer to another list, it is possible to build up list structures of arbitrary complexity. Ordinarily these are tree structures, but it is possible to share sublists, build circular structures, etc.

Since each element of a list points to (that is, contains the address of) its successor, successive list elements are not required to be consecutive words in computer memory. This ability to utilize arbitrary, disjoint sections of memory is one of the powerful features of lists.

III. Representations of Lists

For simple programs, the space required for execution is known and allocated prior to execution. Suppose, however, we wish to store a set of numbers the size of which will not be known until the reading is completed. In order to make efficient use of space, the program should allocate space during execution. The techniques of list processing grew in solution to this type of problem.

Consider a program which is intended to read in a sentence, arrange the words in alphabetical order, and then print them in this order. Assume we store each word at a new address in memory. This might appear as follows:

100	FOUR
101	SCORE
102	AND
103	SEVEN
104	YEARS
105	AGO

where the column of numbers on the left indicates the storage location. Arranging the words such that they are ordered alphabetically yields the following:

200	AGO
201	AND
202	FOUR
203	SCORE
204	SEVEN
205	YEARS

An alternate approach eliminates the unnecessary duplication of words. We may create a vector which represents the alphabetical ordering by indicating the address of each of the words:

300	105
301	102
302	100
303	101
304	103
305	104

where the numbers on the right are the contents of the locations numbered on the left. This has no effect upon the cells containing the actual characters. Additional words may be incorporated with this scheme without disturbing those currently present or the vectors already in existence. The idea is that it may be advantageous to manipulate the addresses of quantities rather than the quantities themselves. Such is the fundamental basis of list processing.

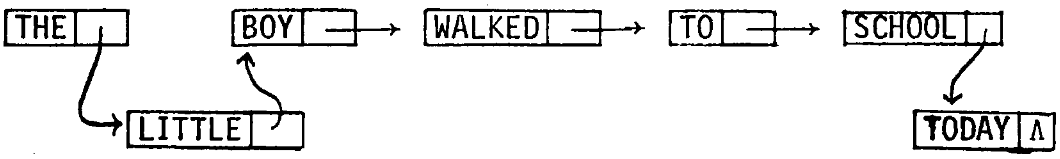
Consider the computer representation of a sentence in storage. The store for each word will also contain the address of the location of the next word in the sentence.

100	THE,	101
101	BOY,	102
102	WALKED,	103
103	TO,	104
104	SCHOOL,	Λ

Recall, the use of the greek letter lambda (Λ) denotes the end of a list. This structure may be represented diagrammatically as follows:

Words can be added to or deleted from this sentence without moving the existing words, since the sequence of stores in which the words occur is insignificant.

100	THE	105
101	BOY,	102
102	WALKED,	103
103	TO,	104
104	SCHOOL,	106
105	LITTLE,	101
106	TODAY,	Λ.



Linked storage representation allows the possibility of random insertions and deletions. These frequently used list operations are thus accomplished through simple manipulation of pointers. With sequential allocation of storage, insertion is particularly difficult since it may involve shifting a large number of elements. This also holds for deletion if we are to utilize deleted storage space. Insertion and deletion are much simpler with linked lists, as we need only alter the appropriate linkages.

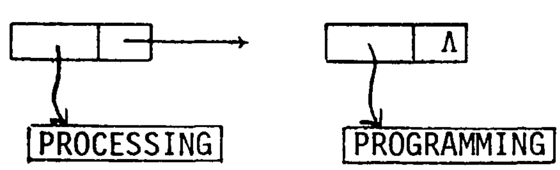
The linked list lends itself immediately to more intricate structures. We can maintain a variable number of variable size lists; any node of the list may be a starting point for another list, the nodes may simultaneously be linked together in several orders, corresponding to different lists.

Suppose the items in the chain are addresses, for example addresses of strings of letters or perhaps addresses of other chains.

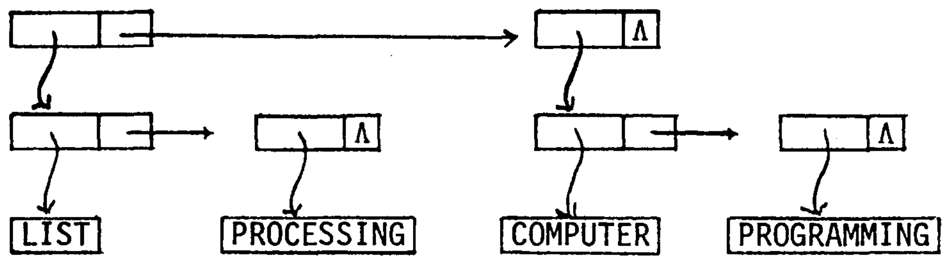
100	LIST
101	PROCESSING
102	COMPUTER
103	PROGRAMMING

200	101,	201
201	103,	Λ
202	203,	205
203	100,	204
204	101,	Λ
205	206,	Λ
206	102,	207
207	103,	Λ

A chain starts at location 200 which consists of two items - a pointer to the word "processing", and a pointer to the word "programming":



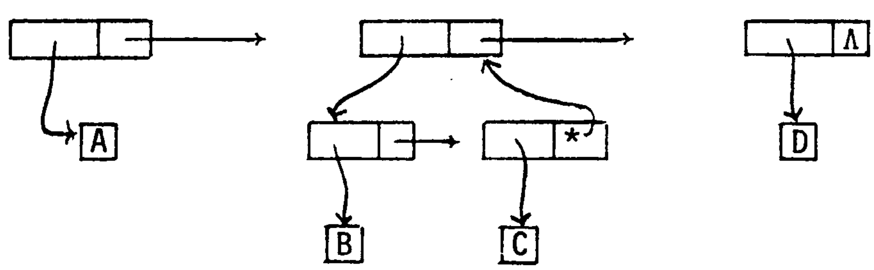
At location 202 begins another chain consisting of just two items. The first item is a chain of two items - a pointer to the word "list" and a pointer to the word "processing". The second item is also a chain of two items - a pointer to the word "computer" and a pointer to the word "programming". This list structure is diagrammed as follows:



The objects which do not have the two-pointer nature represent the atoms. Their structure is not the concern of the particular program which is operating on the list in which they occur.

IV. Types of Lists

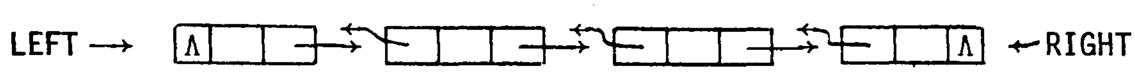
The conventions for the ending of lists may be altered. The space at the end of each sublist can indicate the place in the main list from which the sublist has been referenced. The cell at the end of a sublist must provide an indication that it is an end point, and not a continuation of the sublist. Extra space must be available for storing tag markers to imply this. When the procedure reaches a point in the list that is tagged as end of a sublist, then attention is transferred back to the main list. The following representation of the list (A, (B,C), D) illustrates this concept:



where * denotes the marker for the end of a sublist. Note that the sublist (B,C) points to its referencing node in the main list and, therefore, cannot be a sublist anywhere else. This scheme has the serious disadvantage that a list can only be a sublist of one list, and if required as part of another list, then it must be duplicated. Some problems suffer severely from shortage of store if common sublists do not exist.

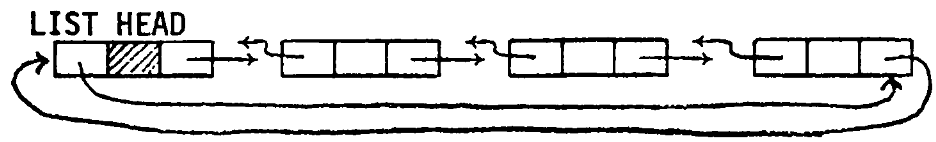
Operations on list structures normally consist of accessing an element or a series of elements, moving them to other lists, replacing them by other series, or processing the entities represented by them. Accessing an element in a list is usually restricted to the first element after a particular given element. Thus it is possible to access any list element, but only by traversing the list from the first element. This is the situation with simple linked lists.

Doubly linked lists. If each node of a list has two links, pointing to the nodes on either side of it, then a more flexible method of handling lists is obtained at the expense of extra storage space for links. This is intended to make movement about the list easily possible in both directions, as is illustrated in the following diagram:



Here, LEFT and RIGHT are pointer variables to the left and right of the doubly linked list. Each node includes two links, called, for example, LLINK and RLINK.

Manipulations of doubly linked lists almost always become much easier if a list head node is part of each list. We have the following typical representation:



The LLINK and RLINK fields of the list head replace LEFT and RIGHT in the previous illustration. If the list is empty, then both link fields of the head point to the head itself.

This representation clearly satisfies the condition that RLINK(LLINK(X)) = LLINK(RLINK(X)) = X where X is the location of any node in the list (including the head). It is for this reason that a list head is desirable.

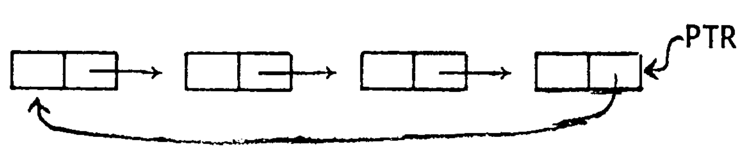
In addition to the obvious advantage of the ability to move in either direction when examining a doubly linked list, one of the important new abilities is that we can delete a node X from the list containing it, given only the value of X. In a simple linked list with only one-directional links, we cannot delete the node X without knowing its predecessor in the chain, since the link of the preceding node requires alteration in performing a deletion of the node X.

Suppose we wish to write a particular routine to search a list of atoms to find the predecessor of a given atom A. With singly linked list structures, it becomes necessary to keep track of two atoms at all times as we search the list. Everytime we compare an atom with A, its predecessor must be known, in the event of a match between the current atom and the atom A. With doubly linked list structures, however, this is not necessary. The desired result can be obtained by first locating A, and then ensue its predecessor pointer. A doubly linked list also permits easy insertion of a node adjacent on either side.

The obvious disadvantages of doubly linked lists are that more memory space is required, and more pointers need by manipulated than with singly linked lists.

<u>Circular lists</u>. A circular list is a list in which every element is the successor of exactly one other element of the list. A circular list possesses the property that its last node links back to the first node, instead of to Λ. There is no need to think of a first or last element. We require only one pointer to the list. The entire circular list may be accessed from any given node of that list.

The following diagram illustrates a (singly-linked) circular list. The nodes have just two fields: INFO and LINK:



Circular lists can represent not only inherently circular structures, but also linear structures. A circular list with one pointer to the rear node is essentially equivalent to a simple linked list with two pointers, one to the front and one to the rear.

In view of the circular symmetry, and since there is no Λ link to signal the end, how do we recognize the end of the list? We must record our starting point, process the list as desired, and stop when we encounter the starting node (assuming that node is still present in the list). An alternate solution is to include a special recognizable node in each circular list, as a convenient stopping point. This list head is quite convenient for applications. An obvious advantage is that the circular list will then never be empty.

Given only X, it is possible to delete the node X in a circular list. This is accomplished by progressing through the entire list in order to locate the predecessor of X. This operation may be inefficient. Some operations, however, become very efficient with circular lists. For example, it is very convenient to move an entire list to become part of another list, or to divide a circular list into two lists.

V. A List Processing System Embedded in PASCAL

While there may exist some programming tasks best solved entirely within some list processing system, most tasks facing the ordinary programmer require the application of a number of distinct techniques. Many programs contain sections which are suitable for list processing. The packaging of a variety of tools within a single tool box seems to be the best way to outfit a worker setting out to solve complex problems.

We shall use the PASCAL programming language and embed in it various procedures to implement a list processing system. Familiarity with the PASCAL language is assumed [6]. The task of understanding these new techniques, then, is that of adding to a vocabulary rather than that of learning an entirely new one. The ideas for the approach taken here come from [11].

PASCAL provides pointer variables as a simple tool for the construction of complicated and flexible data structures. The lists considered here (with the exception of the free list) are circular.

The declaration part of the main program will define the type PTR as a pointer to NODE, where NODE is defined as a record type including a LINK field of type PTR. Also, a variable identifier FREE of type PTR must be declared. Nodes are deleted by moving them to the list containing all freed nodes. FREE will point to this list.

We shall first need a procedure to initialize the free list, which is initially empty. The free list is the only non-circular list being considered in this section. Procedure ORG performs the initialization:

```
PROCEDURE    ORG;
BEGIN
     FREE : = NIL;
END;
```

A very useful function is one which acts on a pointer variable P by finding its antecedent in the list. P remains unchanged. The value of the function is the pointer to the antecedent of P.

```
FUNCTION    ANTE(P : PTR) : PTR;
VAR
   TEMP : PTR;
BEGIN
   TEMP : = P;
   WHILE  TEMP↑. LINK ≠ P  DO
         TEMP : = TEMP↑. LINK;
   ANTE : = TEMP;
END;
```

A node, pointed to by P, which is to be "erased" is moved to the free list by Procedure RELEASE. P is changed to point to its successor in the original list, unless P was from a list of just one element. In this case, P is set to NIL.

```
PROCEDURE   RELEASE   (VAR P : PTR);
VAR
   TEMP, PTI : PTR;
BEGIN
     IF P↑. LINK = P  THEN  PTI : = NIL
     ELSE
       BEGIN
          TEMP : = ANTE (P);
          TEMP↑. LINK : = P↑. LINK;
          PTI : = P↑. LINK;
        END;
  P↑. LINK : = FREE;
  FREE : = P;
   P : = PTI;
END;
```

Procedure RELIST (P,Q) may be used in the same way as RELEASE, the only difference being that it will free the string of nodes starting with that node pointed to by P and ending with the node pointed to by Q. Q remains unchanged. P becomes what was Q↑. LINK unless P through Q was the entire list. In that case, P is set to NIL.

```
PROCEDURE   RELIST (VAR  P:PTR;  Q:PTR);
VAR
  TEMP,PTI:PTR:
BEGIN
     IF  Q↑.LINK = P  THEN   PTI : = NIL
     ELSE
       BEGIN
           TEMP : = ANTE (P);
           TEMP↑. LINK : = Q↑. LINK;
           PTI : = Q↑. LINK;
       END;
   Q↑. LINK : = FREE;
   FREE : = P;
   P : = PTI
END;
```

Procedure ALLOCATE (P) allocates a variable of type PTR and assigns its address to P. This is done utilizing nodes from the free list, if there are any. Storage space is generated dynamically by the procedure new if the free list is empty.

```
PROCEDURE   ALLOCATE (VAR   P : PTR);
BEGIN
  IF   FREE = NIL   THEN   NEW (P)
  ELSE
    BEGIN
       P : = FREE;
       FREE : = FREE↑. LINK;
     END;
END;
```

A new (circular) list of one element may be established by means of the Procedure INIT (P). P then points to that one node.

```
PROCEDURE   INIT (VAR   P : PTR);
BEGIN
     ALLOCATE (P)
     P↑. LINK : = P;
END;
```

The Procedure INSERT (P) creates a variable of type PTR and inserts it as the antecedent of the variable to which P points in the list containing P. P becomes the pointer to this newly created variable.

```
PROCEDURE   INSERT (VAR   P : PTR);
VAR
   TEMP, PTI : PTR;
BEGIN
    ALLOCATE (TEMP);
    PTI : = ANTE (P);
    PTI ↑. LINK : = TEMP;
    TEMP ↑. LINK : = P;
     P : = TEMP;
END;
```

One of the most important processes in list structuring is the moving of list elements from one list to another. The Procedure MOV (P,Q) moves the element to which P points such that it becomes the antecedant of the element to which Q points. Q is set equal to P and P becomes the pointer to what was its successor, unless P is an entire list. In that case, moving the node to which P points eliminates that list and thus, P is set to NIL.

```
PROCEDURE   MOV (VAR   P,Q : PTR);
VAR
  TEMP,PTI,PT2 : PTR;
BEGIN
  IF   P↑. LINK = P    THEN    TEMP : = NIL
   ELSE
     BEGIN
         PTI : = ANTE (P);
         PTI ↑. LINK : = P↑. LINK;
         TEMP : = P ↑. LINK;
       END;
     PT2 : = ANTE (Q);
     PT2 ↑. LINK : = P;
     P ↑. LINK : = Q;
     Q : = P;
     P : = TEMP;
   END;
```

The following example shows a simple use of MOV(P,Q).

```
Before:      (a,b,c,d,e)          (f,g,h,i)
                  ↑                   ↑
                  P                   Q

After:       (a,b,d,e)            (f,g,c,h,i)
                  ↑                   ↑
                  P                   Q
```

Suppose we now call MOV(P,Q) again:

```
Then we have      (a,b,e)          (f,g,d,c,h,i)
                     ↑                  ↑
                     P                  Q
```

Now a call of MOV(Q,P) will yield the following

```
                  (a,b,d,e)        (f,g,c,h,i)
                     ↑                  ↑
                     P                  Q
```

And a second call to MOV(Q,P) brings us back where we started:

```
                  (a,b,c,d,e)      (f,g,h,i)
                     ↑                  ↑
                     P                  Q
```

Procedure INCOR(,Q,R) given below may be used in the same way as MOV, the only difference being that it will move each of the nodes starting with that to which P points and ending with the node to which Q points. This string of elements is then inserted to precede the node to which R points. Q remains unchanged. R is set to P, and P becomes Q ↑. LINK unless

P through Q is an entire list. If this is so, then P is set to NIL.

```
          PROCEDURE   INCOR (VAR  P:PTR; Q:PTR; VAR R:PTR);
          VAR
            TEMP, PTI,PT2 : PTR;
          BEGIN
            IF  Q ↑. LINK = P  THEN  TEMP : = NIL
            ELSE
               BEGIN
                   PTI = ANTE (P);
                   PTI ↑. LINK : = Q ↑. LINK;
                   TEMP : = Q ↑. LINK;
               END;
         PT2 : = ANTE (R);
         PT2 ↑. LINK : = P;
         Q ↑. LINK : = R;
         R : = P;
         P : = TEMP;
    END;
```

The following illustrates the effect of Procedure INCOR (P,Q,R):

```
          Before:      (a,b,c,d,e,f,g)          (h,i,j,k)
                          ↑     ↑                  ↑
                          P     Q                  R

          After:       (a,f,g)               (h,b,c,d,e,i,j,k)
                          ↑                     ↑     ↑
                          P                     R     Q
```

Function ELEM (P,N) will have as its value the pointer to the n-th element after the element to which P points. P remains unchanged.

```
FUNCTION    ELEM (P:PTR,  N:INTEGER) : PTR;
VAR
  I : INTEGER:
BEGIN
   FOR  I : = 1  TO  N  DO
        P : = P ↑. LINK;
   ELEM : = P;
END;
```

Program TEST is included as the Appendix I so that the reader may inspect the performance of a few of these procedures.

As a simple example of the use of these list processing techniques, we consider the dealing of a deck of cards in a bridge game [11]. Declare a node to be a record with three fields: card value, card suit, and a link to the next card in the list. A program to simulate the deal has been written in five sections.

1. INITIALIZE. In this procedure, we seed a random number generator, call Procedure ORG to initialize the free list, and set the symbols J,Q,K,A to represent the jack, queen, king, and ace of each suit.
2. GENDECK. This procedure generates a circular list containing a node for each of the fifty-two different combinations of card values and suits. The pointer variable, DECK, will designate the list by pointing to an arbitrary node.
3. STARTLISTS. Sixteen lists are initialized - four per player (one for each of the four suits). M is a four by four

array containing pointers to the first element for each of the sixteen lists.

4. DEALDECK. In this procedure, a card is randomly chosen from the remainder of the deck. The card is removed from the deck list and placed in one of the sixteen lists initialized in STARTLISTS (which list depends upon the suit of the card drawn, and which player is to receive the card).
5. PRINT. This procedure prints to output the hands of the four players, with cards listed according to suit. That is, the sixteen lists of STARTLISTS are printed. As each card is printed, it is moved from its current list to the deck list. At the conclusion of this procedure, the deck is reconstructed.

The program listing and an actual run are presented in Appendix II in Program DEAL. One deal requires about 0.5 CP seconds.

As another example of the use of list processing techniques, we consider the construction of a Binary Huffman Code [5]. Given a message source of N possible messages, $N \geq 1$, each with its own probability of occurrence, the process is as follows:

1. Organize the possible messages according to probability of occurrence, in descending order.
2. Combine the two messages with lowest probabilities by drawing lines from each to a single point. Label the line

of the more frequent message with a "1" and the line of the other message with - "0".

3. Combine the next two messages with lowest probabilities, and label them.

4. Continue this process until all messages are merged at one point.

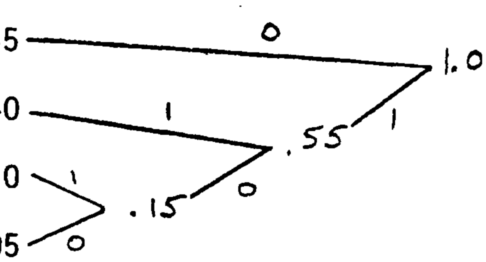
5. Read the labels along the path from the unique point to each symbol for its code.

For example, suppose we have four messages with probabilities of occurrence as follows:

SYMBOL	PROBABILITY
A	.45
B	.40
C	.10
D	.05

Then the following diagram illustrates the construction of a Huffman Code for this message source.

SYMBOL	PROBABILITY	CODE
A	.45	0
B	.40	11
C	.10	101
D	.05	100



The computer program to construct a Huffman Code first reads the symbols and their corresponding probabilities of occurrence. Each symbol and corresponding probability is represented in a node of a circular list. This list is then sorted according to the probability field, in descending order. Then, the two nodes of lowest probability are removed from this list to form a new circular list with head. They are substituted by a single element with a probability equal to the combined probabilities of the other two nodes. This substitute element points at, and is pointed at by the head of the circular list formed by the nodes with low probabiliteis. Now the original list is sorted again, and the two nodes with lowest probabilities are combined as before. This process is repeated until just one element exists in the list.

Now the list structure is complete, and we need only traverse it properly to obtain a Huffman Code. See Program HUFFMAN in Appendix III for the program listing and sample run.

VI. A LISP Interpreter

LISP lists are simple singly linked structures. Each list element contains a pointer to a data item and a pointer to the following list element. The last element of a list points to the special atom NIL. The two pointers in a list element are termed the CAR pointer and the CDR pointer. The CAR pointer indicates the data item while the CDR pointer indicates the successor to that list item. The CAR value of a list item may be a pointer to an atom, or to another list.

Of the elementary LISP operations, CAR and CDR dissect a list, giving as values the left and right pointers, respectively. Suppose X is the list ((A),B,C,(D,E,(F)),G). Diagrammatically, this list may be represented as in figure 1. Then CAR(X) = (A) and CDR(X) = (B,C,(D,E,(F)),G). These operations may be applied successively so, for example, CAR(CDR(X)) = B and CDR(CDR(X)) = (C,(D,E,(F)),G). The functions CAR and CDR are undefined on atomic objects. Note that successive elements of a list X are given by CAR(X), CAR(CDR(X)), CAR(CDR(CDR(X))), CAR(CDR(CDR(CDR(X)))),...

To construct a list, the operator CONS is used. If X is an atom or a list and Y is a list, then CONS(X,Y) has as its value a new list cell whose left pointer indicates X and whose right

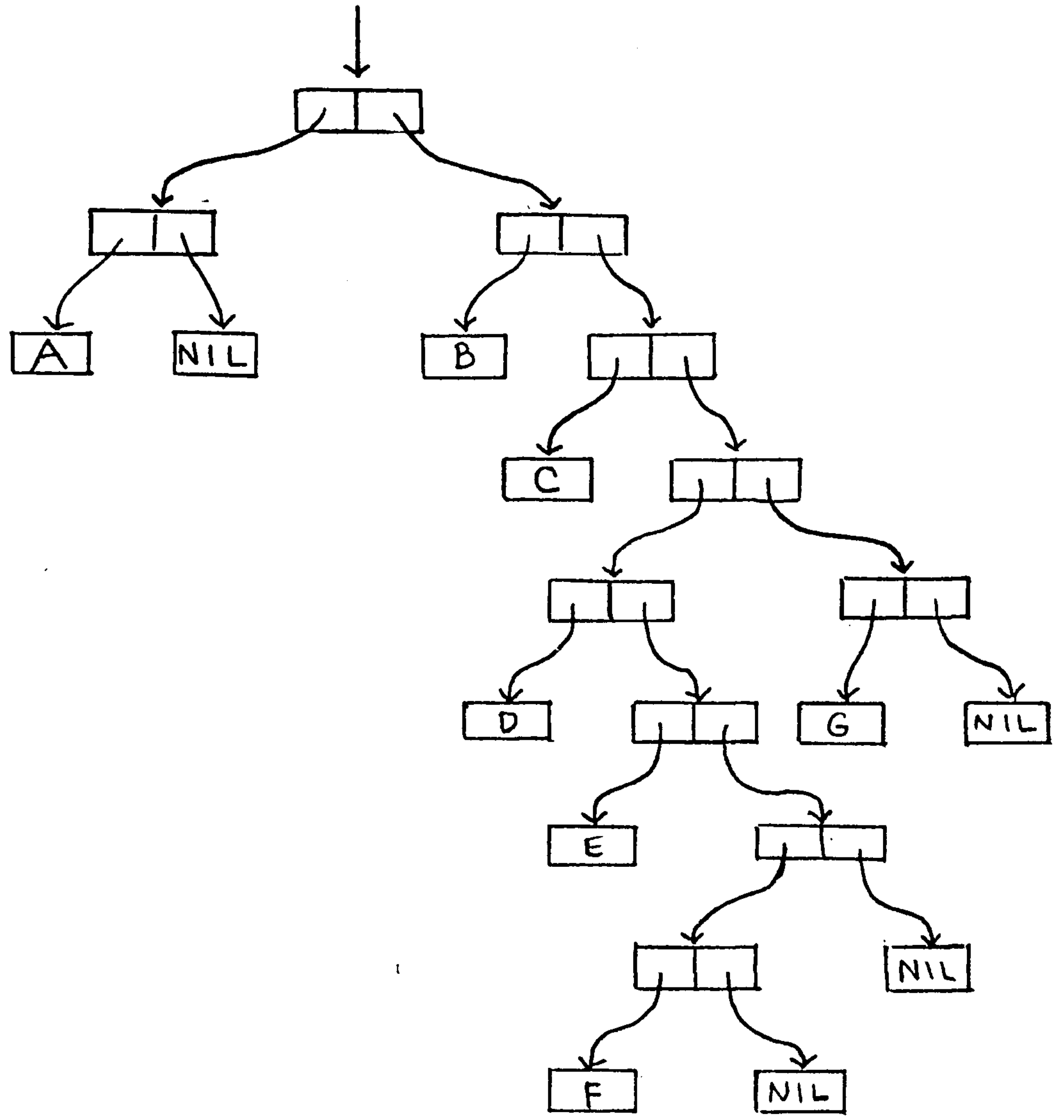


Figure 1

pointer indicates Y. To form a list of one element, say A, we have that CONS(A,NIL) = (A). Although it would be possible to allow the second parameter of CONS to be an atom, we shall not do so but shall preserve the convention that items of a list are shown by the left pointer and that the right pointer (the second parameter) links the remaining list cells. The only exception to this is that the special atom NIL may appear as the second parameter.

Note that CAR(CONS(A,B)) ≡ A and CDR(CONS(A,B)) ≡ B. But CONS(CAR(X), CDR(X)) is a new cell in storage which contains the same pointers as did X. It is a copy of X, not the cell X itself.

The function ATOM(X) has the value *T* (representing "true") if X is an atom and the value NIL otherwise. Function EQ(X,Y) has the value *T* if the two atoms, X and Y, are identical. Otherwise, its value is NIL. EQ(X,Y) is undefined if either X or Y is not an atom.

To implement this system in PASCAL, we create a circular list with head, to keep track of all atoms. This list is initialized to contain nodes with name NIL and *T*. Two types of nodes are considered: atomic and nonatomic. Atomic nodes contain two fields - one for NAME and another for a LINK to other elements of the atomlist. Nonatomic nodes contain two fields, a HEAD and a TAIL, both pointers.

When a list containing atoms is input to the program, the names of the atoms are inserted in the atomlist, unless they already appear there.

TREW is the name of the pointer to the atom whose name is "*T*" and NILL is the name of the pointer to the atom whose name is "NIL". The Function ATOM(L1) assumes either the value TREW or the value NILL, depending upon whether L1 is an atom or not:

```
FUNCTION    ATOM(L1 : PTR) : PTR;
BEGIN
   IF  L1 ↑. STATE = ATOMIC  THEN  ATOM : = TREW
   ELSE        ATOM : = NILL;
END;
```

The value of the Function CONS(L1, L2) is a pointer to the cell whose head is L1 and whose tail is L2:

```
FUNCTION    CONS (L1, L2 : PTR) : PTR;
VAR
    Q : PTR;
BEGIN
     NEW(Q):
     WITH  Q ↑  DO
          BEGIN
                STATE : = NONATOMIC;
               HEAD : = L1;
               TAIL : = L2;
          END;
   CONS : = Q :
END;
```

The Function CAR(L1) assumes as its value a pointer to the head of the list L1. If L1 is an atom, the function is undefined.

```
FUNCTION  CAR(L1 : PTR) : PTR;
BEGIN
   IF  ATOM(L1) = TREW THEN ERROR(1)
   ELSE CAR : = L1  ↑. HEAD;
END;
```

The Function CDR(L1) assumes as its value a pointer to the tail of the list L1. If L1 is an atom, then the function is undefined.

```
FUNCTION  CDR(L1 : PTR) : PTR;
BEGIN
   IF  ATOM(L1) = TREW  THEN  ERROR(2)
   ELSE CDR : =  L1 ↑.  TAIL;
END;
```

Function EQ(L1,L2) takes on the value TREW or NILL, depend- upon whether the atoms L1 and L2 are identical. If either L1 or L2 is not an atom then the function is undefined.

```
FUNCTION  EQ(L1, L2 : PTR) : PTR;
BEGIN
   IF (ATOM(L1) = NILL) or (ATOM(L2) = NILL)
   THEN    ERROR (4)
   ELSE
     IF  L1 = L2  THEN  EQ : = TREW
     ELSE  EQ : = NILL;
END;
```

Function EQUAL (L1, L2) performs exactly as does Function EQ(L1, L2), except that L1, L2 need not be atoms, and the general list structures, L1 and L2, are tested for equality.

```
FUNCTION    EQUAL (L1, L2 : PTR) : PTR;
BEGIN
   IF (ATOM(L1) = TREW)  AND (ATOM(L2) = TREW)
   THEN EQUAL : = EQ(L1, L2)
   ELSE
     IF EQUAL (CAR(L1), CAR(L2)) = TREW
     THEN EQUAL : = EQUAL (CDR(L1), CDR(L2))
     ELSE  EQUAL : = NILL;
END;
```

Thus, we have the basic LISP operations. Let us discover what can be accomplished with these functions.

Consider the Function FLAT(L1, L2) which accepts the general list L1 and creates another list containing the same atoms in the identical order as in L1, but with all atoms on the same level. This flattened version of L1 is placed in front of L2 for the final result. For example, let X = ((A),B,(C,D,(E,F),G)). Then FLAT(X,NILL) is a pointer to the list structure (A,B,C,D,E,F,G). With the use of the techniques defined in this section, Function FLAT is defined with only one program statement.

Another usage of these techniques occurs in Function REV(L1,L2). This function reverses the top level of the list L1, and places it in front of L2. Suppose L1 is the list ((A),B,C,(D,E,F),G). Then REV(L1,NILL) indicates the list (G,(D,E,F),C,B,(A)). Also, the programming for this function requires only one statement.

Function EVALUATE (L1) takes the list L1 to be the Polish notation of an arithmetic expression, and creates the corresponding infix notation for its evaluation. This effort is greatly simplified by the list processing techniques presented here.

Procedure PRINT (L1) performs a preorder traversal of the list L1 (also, a tree) in order to write to output the list corresponding to the internal computer representation.

These programs illustrate the utility of general list processing techniques, and are included in Program LISP in Appendix IV for the reader's inspection.

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Appendix I: Program TEST

```
 (*$U+[W1,56] MARY CAPECE *)
PROGRAM TEST(INPUT, OUTPUT);

TYPE
   PTR = ↑ NODE;
   NODE = RECORD
             VAL: INTEGER;
             NAME: ALFA;
             LINK: PTR;
          END;

VAR
   X: REAL;
   FREE: PTR;
   I: INTEGER;
   J: INTEGER;
   EL: PTR;



PROCEDURE ORG;
(* INITIALIZE THE FREE LIST *)

   BEGIN
      FREE := NIL;
   END (*ORG*);



FUNCTION ANTE(P: PTR): PTR;
(* POINTS TO THE ANTECEDENT OF P *)
(* P REMAINS UNCHANGED *)

   VAR
      TEMP: PTR;

   BEGIN
      TEMP := P;
      WHILE TEMP↑.LINK <> P DO
         TEMP := TEMP↑.LINK;
      ANTE := TEMP;
   END (*ANTE*);



PROCEDURE RELIST(VAR P: PTR; Q: PTR);
(* MOVES THE STRING OF ELEMENTS,
BEGINNING WITH THE ELEMENT TO WHICH P POINTS,
```

```
AND ENDING WITH THE ELEMENT TO WHICH Q POINTS *)
(* INSERTS IT IN THE FREE LIST *)
(* IF P THRU Q IS AN ENTIRE LIST, THEN P:= NIL ELSE P:=
Q↑.LINK; *)
(* Q REMAINS UNCHANGED *)

   VAR
      TEMP: PTR;
      PT1: PTR;

   BEGIN
      IF Q↑.LINK = P
      THEN
         PT1 := NIL
      ELSE
         BEGIN
            TEMP := ANTE(P);
            TEMP↑.LINK := Q↑.LINK;
            PT1 := Q↑.LINK;
         END;
      Q↑.LINK := FREE;
      FREE := P;
      P := PT1;
   END (*RELIST*);


PROCEDURE ALLOCATE(VAR P: PTR);
(* CREATES A VARIABLE OF TYPE PTR, POINTED TO BY P *)
(* UTILIZES SPACE FROM THE FREE LIST, IF THERE IS ANY *)

   BEGIN
      IF FREE = NIL
      THEN
         NEW(P)
      ELSE
         BEGIN
            P := FREE;
            FREE := FREE↑.LINK;
         END;
   END (*ALLOCATE*);


PROCEDURE INIT(VAR P: PTR);
(* ESTABLISHES A NEW CIRCULAR LIST OF ONE ELEMENT *)
(* P POINTS TO THAT ELEMENT *)
```

```
   BEGIN
      ALLOCATE(P);
      P↑.LINK := P;
   END (*INIT*);



PROCEDURE INSERT(VAR P: PTR);
(* CREATES A VARIABLE OF TYPE PTR *)
(* INSERTS IT AS THE ANTECEDENT OF THE VARIABLE TO WHICH
P POINTS *)
(* P BECOMES THE POINTER TO THIS NEWLY CREATED VARIABLE
   *)

   VAR
      TEMP: PTR;
      PT1: PTR;

   BEGIN
      ALLOCATE(TEMP);
      PT1 := ANTE(P);
      PT1↑.LINK := TEMP;
      TEMP↑.LINK := P;
      P := TEMP;
   END (*INSERT*);



FUNCTION ELEM(P: PTR; N: INTEGER): PTR;
(* POINTS TO THE N-TH ELEMENT AFTER THE ELEMENT TO WHICH
P POINTS *)
(* P REMAINS UNCHANGED *)

   VAR
      I: INTEGER;

   BEGIN
      FOR I := 1 TO N DO
         P := P↑.LINK;
      ELEM := P;
   END (*ELEM*);



FUNCTION RANDOM: REAL;
   EXTERN;
```

```
PROCEDURE SKIP(N: INTEGER);

   VAR
      I: INTEGER;

   BEGIN
      FOR I := 1 TO N DO
         WRITELN;
   END (*SKIP*);



PROCEDURE WRITEPTR(PT: PTR);

   BEGIN
      WRITELN(≡PT ≡, ORD(PT): 7, ≡ NAME ≡, PT↑.NAME: 10,
         ≡ VAL ≡, PT↑.VAL: 2, ≡ PT↑.LINK ≡, ORD(PT↑.LINK
         ): 7);
   END (*WRITEPTR*);



PROCEDURE WRITELIST(LIST: PTR);

   VAR
      EL: PTR;
      I: INTEGER;

   BEGIN
      EL := LIST;
      IF EL = NIL
      THEN
         WRITELN(≡ PT = NIL≡)
      ELSE
         BEGIN
            I := 1;
            REPEAT
               WRITE(≡ ≡, I: 3, ≡  ≡);
               WRITEPTR(EL);
               I := I + 1;
               EL := EL↑.LINK;
            UNTIL (EL = LIST) OR (EL = NIL);
         END;
   END (*WRITELIST*);
```

```
PROCEDURE TESTELEM;

   VAR
      PT: PTR;
      I, N: INTEGER;

   BEGIN
      WRITELN(≡ TESTING FUNCTION ELEM≡);
      SKIP(1);
      WRITE(≡ EL:  ≡);
      WRITEPTR(EL);
      SKIP(2);
      FOR I := 1 TO 3 DO
         BEGIN
            N := TRUNC(RANDOM * 7);
            PT := ELEM(EL, N);
            WRITE(≡ N=≡, N: 2, ≡ ≡);
            WRITEPTR(PT);
            SKIP(1);
         END;
   END (*TESTELEM*);


PROCEDURE TESTANTE;

   VAR
      N: INTEGER;
      PT: PTR;

   BEGIN
      WRITELN(≡ TESTING FUNCTION ANTE≡);
      SKIP(1);
      WRITE(≡ EL:  ≡);
      WRITEPTR(EL);
      SKIP(2);
      FOR I := 1 TO 3 DO
         BEGIN
            N := TRUNC(RANDOM * 7);
            PT := ELEM(EL, N);
            WRITE(≡ N=≡, N: 2, ≡ ≡);
            WRITEPTR(PT);
            WRITE(≡ ANTE:≡);
            WRITEPTR(ANTE(PT));
            SKIP(1);
         END;
   END (*TESTANTE*);
```

```
PROCEDURE TESTRELIST;

   VAR
      TEMP: PTR;
      I, N: INTEGER;

   BEGIN
      WRITELN(≡ TESTING PROCEDURE RELIST≡);
      SKIP(1);
      WRITELN(≡ EL - LIST≡);
      WRITELIST(EL);
      SKIP(1);
      WRITELN(≡ FREE - LIST≡);
      WRITELIST(FREE);
      SKIP(3);
      I := 7;
      WRITE(≡ EL:  ≡);
      WRITEPTR(EL);
      N := TRUNC(RANDOM * I);
      TEMP := ELEM(EL, N);
      WRITE(≡ N=≡, N: 2, ≡ ≡);
      WRITEPTR(TEMP);
      SKIP(1);
      RELIST(EL, TEMP);
      WRITELN(≡ EL - LIST≡);
      WRITELIST(EL);
      SKIP(1);
      WRITELN(≡ FREE - LIST ≡);
      WRITELIST(FREE);
      SKIP(3);
      I := I - (N + 1);
      IF I > 0 THEN
         BEGIN
            RELIST(EL, ANTE(EL));
            WRITELN(≡ EL - LIST≡);
            WRITELIST(EL);
            SKIP(1);
            WRITELN(≡ FREE - LIST≡);
            WRITELIST(FREE);
         END;
   END (*TESTRELIST*);


PROCEDURE TESTALLOCATE;
```

```
   VAR
      I: INTEGER;

   BEGIN
      WRITELN(≡ TESTING PROCEDURE ALLOCATE≡);
      SKIP(1);
      WRITELN(≡ FREE - LIST≡);
      WRITELIST(FREE);
      SKIP(3);
      FOR I := 1 TO 10 DO
         BEGIN
            ALLOCATE(EL);
            WRITE(≡ EL:  ≡);
            WRITELN(≡PT ≡, ORD(EL): 7);
            SKIP(1);
            WRITELN(≡ FREE - LIST≡);
            WRITELIST(FREE);
            SKIP(3);
         END;
   END (*TESTALLOCATE*);



BEGIN (*TEST*)
   FOR I := 1 TO CLOCK MOD 750 DO
      X := RANDOM;
   ORG;
   SKIP(4);
   FOR I := 1 TO 7 DO
      BEGIN
         IF I = 1
         THEN
            INIT(EL)
         ELSE
            INSERT(EL);
         FOR J := 1 TO 10 DO
            EL↑.NAME[J] := CHR(I);
         EL↑.VAL := I;
      END;
   WRITELN(≡ EL - LIST≡);
   WRITELIST(EL);
   SKIP(4);
   TESTELEM;
   SKIP(4);
   TESTANTE;
   SKIP(4);
   TESTRELIST;
   SKIP(4);
```

```
    TESTALLOCATE;
    SKIP(4);
END (*TEST*).
```

```
EL - LIST
  1  PT 2100709 NAME GGGGGGGGGG VAL  7 PT↑.LINK 1838569
  2  PT 1838569 NAME FFFFFFFFFF VAL  6 PT↑.LINK 1576429
  3  PT 1576429 NAME EEEEEEEEEE VAL  5 PT↑.LINK 1314289
  4  PT 1314289 NAME DDDDDDDDDD VAL  4 PT↑.LINK 1052149
  5  PT 1052149 NAME CCCCCCCCCC VAL  3 PT↑.LINK  790009
  6  PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  527869
  7  PT  527869 NAME AAAAAAAAAA VAL  1 PT↑.LINK 2100709


TESTING FUNCTION ELEM

EL:  PT 2100709 NAME GGGGGGGGGG VAL  7 PT↑.LINK 1838569


N= 5 PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  527869

N= 4 PT 1052149 NAME CCCCCCCCCC VAL  3 PT↑.LINK  790009

N= 0 PT 2100709 NAME GGGGGGGGGG VAL  7 PT↑.LINK 1838569


TESTING FUNCTION ANTE

EL:  PT 2100709 NAME GGGGGGGGGG VAL  7 PT↑.LINK 1838569


N= 6 PT  527869 NAME AAAAAAAAAA VAL  1 PT↑.LINK 2100709
ANTE:PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  527869

N= 3 PT 1314289 NAME DDDDDDDDDD VAL  4 PT↑.LINK 1052149
ANTE:PT 1576429 NAME EEEEEEEEEE VAL  5 PT↑.LINK 1314289

N= 6 PT  527869 NAME AAAAAAAAAA VAL  1 PT↑.LINK 2100709
ANTE:PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  527869
```

```
TESTING PROCEDURE RELIST

EL - LIST
  1  PT 2100709 NAME GGGGGGGGGG VAL  7 PT↑.LINK 1838569
  2  PT 1838569 NAME FFFFFFFFFF VAL  6 PT↑.LINK 1576429
  3  PT 1576429 NAME EEEEEEEEEE VAL  5 PT↑.LINK 1314289
  4  PT 1314289 NAME DDDDDDDDDD VAL  4 PT↑.LINK 1052149
  5  PT 1052149 NAME CCCCCCCCCC VAL  3 PT↑.LINK  790009
  6  PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  527869
  7  PT  527869 NAME AAAAAAAAAA VAL  1 PT↑.LINK 2100709

FREE - LIST
PT = NIL


EL:  PT 2100709 NAME GGGGGGGGGG VAL  7 PT↑.LINK 1838569
N= 5 PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  527869

EL - LIST
  1  PT  527869 NAME AAAAAAAAAA VAL  1 PT↑.LINK  527869

FREE - LIST
  1  PT 2100709 NAME GGGGGGGGGG VAL  7 PT↑.LINK 1838569
  2  PT 1838569 NAME FFFFFFFFFF VAL  6 PT↑.LINK 1576429
  3  PT 1576429 NAME EEEEEEEEEE VAL  5 PT↑.LINK 1314289
  4  PT 1314289 NAME DDDDDDDDDD VAL  4 PT↑.LINK 1052149
  5  PT 1052149 NAME CCCCCCCCCC VAL  3 PT↑.LINK  790009
  6  PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  131071


EL - LIST
PT = NIL

FREE - LIST
  1  PT  527869 NAME AAAAAAAAAA VAL  1 PT↑.LINK 2100709
  2  PT 2100709 NAME GGGGGGGGGG VAL  7 PT↑.LINK 1838569
  3  PT 1838569 NAME FFFFFFFFFF VAL  6 PT↑.LINK 1576429
  4  PT 1576429 NAME EEEEEEEEEE VAL  5 PT↑.LINK 1314289
  5  PT 1314289 NAME DDDDDDDDDD VAL  4 PT↑.LINK 1052149
  6  PT 1052149 NAME CCCCCCCCCC VAL  3 PT↑.LINK  790009
  7  PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  131071
```

```
TESTING PROCEDURE ALLOCATE

FREE - LIST
  1   PT  527869 NAME AAAAAAAAAA VAL  1 PT↑.LINK 2100709
  2   PT 2100709 NAME GGGGGGGGGG VAL  7 PT↑.LINK 1838569
  3   PT 1838569 NAME FFFFFFFFFF VAL  6 PT↑.LINK 1576429
  4   PT 1576429 NAME EEEEEEEEEE VAL  5 PT↑.LINK 1314289
  5   PT 1314289 NAME DDDDDDDDDD VAL  4 PT↑.LINK 1052149
  6   PT 1052149 NAME CCCCCCCCCC VAL  3 PT↑.LINK  790009
  7   PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  131071


EL:  PT  527869

FREE - LIST
  1   PT 2100709 NAME GGGGGGGGGG VAL  7 PT↑.LINK 1838569
  2   PT 1838569 NAME FFFFFFFFFF VAL  6 PT↑.LINK 1576429
  3   PT 1576429 NAME EEEEEEEEEE VAL  5 PT↑.LINK 1314289
  4   PT 1314289 NAME DDDDDDDDDD VAL  4 PT↑.LINK 1052149
  5   PT 1052149 NAME CCCCCCCCCC VAL  3 PT↑.LINK  790009
  6   PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  131071


EL:  PT 2100709

FREE - LIST
  1   PT 1838569 NAME FFFFFFFFFF VAL  6 PT↑.LINK 1576429
  2   PT 1576429 NAME EEEEEEEEEE VAL  5 PT↑.LINK 1314289
  3   PT 1314289 NAME DDDDDDDDDD VAL  4 PT↑.LINK 1052149
  4   PT 1052149 NAME CCCCCCCCCC VAL  3 PT↑.LINK  790009
  5   PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  131071


EL:  PT 1838569

FREE - LIST
  1   PT 1576429 NAME EEEEEEEEEE VAL  5 PT↑.LINK 1314289
  2   PT 1314289 NAME DDDDDDDDDD VAL  4 PT↑.LINK 1052149
  3   PT 1052149 NAME CCCCCCCCCC VAL  3 PT↑.LINK  790009
  4   PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  131071
```

```
EL:  PT 1576429

FREE - LIST
  1  PT 1314289 NAME DDDDDDDDDD VAL  4 PT↑.LINK 1052149
  2  PT 1052149 NAME CCCCCCCCCC VAL  3 PT↑.LINK  790009
  3  PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  131071


EL:  PT 1314289

FREE - LIST
  1  PT 1052149 NAME CCCCCCCCCC VAL  3 PT↑.LINK  790009
  2  PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  131071


EL:  PT 1052149

FREE - LIST
  1  PT  790009 NAME BBBBBBBBBB VAL  2 PT↑.LINK  131071


EL:  PT  790009
FREE - LIST
PT = NIL

EL:  PT 2362849
FREE - LIST
PT = NIL

EL:  PT 2624989
FREE - LIST
PT = NIL

EL:  PT 2887129
FREE - LIST
PT = NIL
```

Appendix II: Program DEAL

```
 (*$U+[W1,56] MARY CAPECE *)
PROGRAM DEAL(OUTPUT);

TYPE
   COLOR =
      (SPADES, HEARTS, DIAMONDS, CLUBS);
   PTR = ↑ NODE;
   NODE = RECORD
             VAL: 0..14;
             SUIT: COLOR;
             LINK: PTR;
          END;

VAR
   FREE: PTR;
   SYM: ARRAY [11..14] OF CHAR;
   DECK: PTR;
   M: ARRAY [1..4, COLOR] OF PTR;
   X1, X2: INTEGER;



PROCEDURE ORG;
(* INITIALIZE THE FREE LIST *)

   BEGIN
      FREE := NIL;
   END (*ORG*);



FUNCTION ANTE(P: PTR): PTR;
(* POINTS TO THE ANTECEDENT OF P *)
(* P REMAINS UNCHANGED *)

   VAR
      TEMP: PTR;

   BEGIN
      TEMP := P;
      WHILE TEMP↑.LINK <> P DO
         TEMP := TEMP↑.LINK;
      ANTE := TEMP;
   END (*ANTE*);



PROCEDURE RELEASE(VAR P: PTR);
```

```
(* MOVES THE ELEMENT TO WHICH P POINTS, TO THE FREE LIST
   *)
(* IF P IS AN ENTIRE LIST, THEN P:= NIL ELSE P:= P↑.LINK
   *)

   VAR
      TEMP: PTR;
      PT1: PTR;

   BEGIN
      IF P↑.LINK = P
      THEN
         PT1 := NIL
      ELSE
         BEGIN
            TEMP := ANTE(P);
            TEMP↑.LINK := P↑.LINK;
            PT1 := P↑.LINK;
         END;
      P↑.LINK := FREE;
      FREE := P;
      P := PT1;
   END (*RELEASE*);


PROCEDURE RELIST(VAR P: PTR; Q: PTR);
(* MOVES THE STRING OF ELEMENTS,
BEGINNING WITH THE ELEMENT TO WHICH P POINTS,
AND ENDING WITH THE ELEMENT TO WHICH Q POINTS *)
(* INSERTS IT IN THE FREE LIST *)
(* IF P THRU Q IS AN ENTIRE LIST, THEN P:= NIL ELSE P:=
Q↑.LINK; *)
(* Q REMAINS UNCHANGED *)

   VAR
      TEMP: PTR;
      PT1: PTR;

   BEGIN
      IF Q↑.LINK = P
      THEN
         PT1 := NIL
      ELSE
         BEGIN
            TEMP := ANTE(P);
            TEMP↑.LINK := Q↑.LINK;
            PT1 := Q↑.LINK;
```

```
         END;
      Q↑.LINK := FREE;
      FREE := P;
      P := PT1;
   END (*RELIST*);


PROCEDURE ALLOCATE(VAR P: PTR);
(* CREATES A VARIABLE OF TYPE PTR, POINTED TO BY P *)
(* UTILIZES SPACE FROM THE FREE LIST, IF THERE IS ANY *)

   BEGIN
      IF FREE = NIL
      THEN
         NEW(P)
      ELSE
         BEGIN
            P := FREE;
            FREE := FREE↑.LINK;
         END;
   END (*ALLOCATE*);


PROCEDURE INIT(VAR P: PTR);
(* ESTABLISHES A NEW CIRCULAR LIST OF ONE ELEMENT *)
(* P POINTS TO THAT ELEMENT *)

   BEGIN
      ALLOCATE(P);
      P↑.LINK := P;
   END (*INIT*);


PROCEDURE INSERT(VAR P: PTR);
(* CREATES A VARIABLE OF TYPE PTR *)
(* INSERTS IT AS THE ANTECEDENT OF THE VARIABLE TO WHICH
P POINTS *)
(* P BECOMES THE POINTER TO THIS NEWLY CREATED VARIABLE
   *)

   VAR
      TEMP: PTR;
      PT1: PTR;

   BEGIN
```

```
      ALLOCATE(TEMP);
      PT1 := ANTE(P);
      PT1↑.LINK := TEMP;
      TEMP↑.LINK := P;
      P := TEMP;
   END (*INSERT*);


PROCEDURE MOV(VAR P, Q: PTR);
(* MOVES THE ELEMENT TO WHICH P POINTS, SUCH THAT IT IS
THE ANTECEDENT
OF THE ELEMENT TO WHICH Q POINTS *)
(* IF P IS AN ENTIRE LIST THEN P:=NIL ELSE P:=P↑.LINK *)
(* Q NOW POINTS TO WHAT WAS ORIGINALLY POINTED TO BY P
   *)

   VAR
      PT1, PT2: PTR;
      TEMP: PTR;

   BEGIN
      IF P↑.LINK = P
      THEN
         TEMP := NIL
      ELSE
         BEGIN
            PT1 := ANTE(P);
            PT1↑.LINK := P↑.LINK;
            TEMP := P↑.LINK;
         END;
      PT2 := ANTE(Q);
      PT2↑.LINK := P;
      P↑.LINK := Q;
      Q := P;
      P := TEMP;
   END (*MOV*);


PROCEDURE INCOR(VAR P: PTR; Q: PTR; VAR R: PTR);
(* MOVES THE STRING OF ELEMENTS, BEGINNING WITH THE ELEM
ENT TO WHICH
P POINTS, AND ENDING WITH THE ELEMENT TO WHICH Q POINTS
   *)
(* INSERTS IT TO PRECEDE THE ELEMENT TO WHICH R POINTS
   *)
(* IF P THRU Q IS AN ENTIRE LIST, THEN P:=NIL ELSE P:=Q↑
```

```
.LINK *)
(* Q REMAINS UNCHANGED *)
(* R NOW POINTS TO WHAT WAS ORIGINALLY POINTED TO BY P
   *)

   VAR
      PT1, PT2: PTR;
      TEMP: PTR;

   BEGIN
      IF Q↑.LINK = P
      THEN
         TEMP := NIL
      ELSE
         BEGIN
            PT1 := ANTE(P);
            PT1↑.LINK := Q↑.LINK;
            TEMP := Q↑.LINK;
         END;
      PT2 := ANTE(R);
      PT2↑.LINK := P;
      Q↑.LINK := R;
      R := P;
      P := TEMP;
   END (*INCOR*);


FUNCTION ELEM(P: PTR; N: INTEGER): PTR;
(* POINTS TO THE N-TH ELEMENT AFTER THE ELEMENT TO WHICH
P POINTS *)
(* P REMAINS UNCHANGED *)

   VAR
      I: INTEGER;

   BEGIN
      FOR I := 1 TO N DO
         P := P↑.LINK;
      ELEM := P;
   END (*ELEM*);


FUNCTION RANDOM: REAL;
   EXTERN;
```

```
PROCEDURE WRITELIST(LIST: PTR);

   VAR
      PT1: PTR;
      I: INTEGER;

   BEGIN
      PT1 := LIST;
      I := 1;
      REPEAT
         WRITELN(≡ ≡, I: 2, PT1↑.VAL, ORD(PT1↑.SUIT));
         I := I + 1;
         PT1 := PT1↑.LINK;
      UNTIL PT1 = LIST;
   END (*WRITELIST*);


PROCEDURE INITIALIZE;

   VAR
      I: INTEGER;
      X: REAL;

   BEGIN
      FOR I := 1 TO CLOCK MOD 750 DO
         X := RANDOM;
      ORG;
      SYM[11] := ≡J≡;
      SYM[12] := ≡Q≡;
      SYM[13] := ≡K≡;
      SYM[14] := ≡A≡;
   END (*INITIALIZE*);


PROCEDURE GENDECK;

   VAR
      I: INTEGER;
      J: COLOR;

   BEGIN
      INIT(DECK);
      FOR J := SPADES TO CLUBS DO
         FOR I := 2 TO 14 DO
            BEGIN
```

```
            IF (J <> SPADES) OR (I <> 2) THEN
               INSERT(DECK);
            DECK↑.VAL := I;
            DECK↑.SUIT := J;
         END;
   END (*GENDECK*);


PROCEDURE STARTLISTS;

   VAR
      I: INTEGER;
      J: COLOR;

   BEGIN
      FOR I := 1 TO 4 DO
         FOR J := SPADES TO CLUBS DO
            BEGIN
               INIT(M[I, J]);
               M[I, J] ↑.VAL := 0;
            END;
   END (*STARTLISTS*);


PROCEDURE DEALDECK;

   VAR
      I: INTEGER;
      P: PTR;

   BEGIN
      FOR I := 52 DOWNTO 1 DO
         BEGIN
            DECK := ELEM(DECK, TRUNC(RANDOM * I));
            P := M[(I MOD 4) + 1, DECK↑.SUIT];
            REPEAT
               P := P↑.LINK;
            UNTIL P↑.
            VAL < DECK↑.VAL;
            MOV(DECK, P);
         END;
   END (*DEALDECK*);


PROCEDURE PRINT;
```

```
VAR
   ST, NAME: ALFA;
   V, I: INTEGER;
   J: COLOR;
   P: PTR;

BEGIN
   FOR I := 1 TO 4 DO
      BEGIN
         CASE I OF
            1:
               NAME := ≡NORTH     ≡;
            2:
               NAME := ≡EAST      ≡;
            3:
               NAME := ≡SOUTH     ≡;
            4:
               NAME := ≡WEST      ≡;
         END;
         WRITELN;
         WRITELN(≡  ≡, NAME);
         FOR J := SPADES TO CLUBS DO
            BEGIN
               CASE J OF
                  SPADES:
                     ST := ≡SPADES    ≡;
                  HEARTS:
                     ST := ≡HEARTS    ≡;
                  DIAMONDS:
                     ST := ≡DIAMONDS  ≡;
                  CLUBS:
                     ST := ≡CLUBS     ≡;
               END;
               WRITE(≡       ≡, ST);
               P := M[I, J] ↑.LINK;
               V := P↑.VAL;
               WHILE V > 0 DO
                  BEGIN
                     IF V < 11
                     THEN
                        WRITE(≡  ≡, V: 3)
                     ELSE
                        WRITE(≡  ≡, SYM[V]: 3);
                     IF DECK = NIL
                     THEN
                        BEGIN
                           INIT(DECK);
```

```
                                 DECK↑.VAL := V;
                                 DECK↑.SUIT := J;
                                 RELEASE(P);
                              END
                           ELSE
                              MOV(P, DECK);
                           V := P↑.VAL;
                        END;
                     WRITELN;
                  END;
            END;
      END (*PRINT*);


   BEGIN (*DEAL*)
      INITIALIZE;
      GENDECK;
      STARTLISTS;
      WRITELN;
      WRITELN;
      WRITELN;
      WRITELN;
      DEALDECK;
      WRITELN;
      PRINT;
   END (*DEAL*).
```

NORTH						
	SPADES	A	8	7	3	
	HEARTS	Q	9	3		
	DIAMONDS	K	Q	4	3	
	CLUBS	K	J			
EAST						
	SPADES	Q	10	6		
	HEARTS	K				
	DIAMONDS	J	8	7	6	5
	CLUBS	A	Q	7	6	
SOUTH						
	SPADES	4	2			
	HEARTS	A	10	8	7	6
	DIAMONDS	A	10	9	2	
	CLUBS	5	2			
WEST						
	SPADES	K	J	9	5	
	HEARTS	J	5	4	2	
	DIAMONDS					
	CLUBS	10	9	8	4	3

Appendix III: Program HUFFMAN

```
 (*$U+[W1,56] MARY CAPECE *)
PROGRAM HUFFMAN(INPUT, OUTPUT);

TYPE
   PTR = ↑ NODE;
   NODE = RECORD
             LINK: PTR;
             CODE: INTEGER;
             FREQ: REAL;
             CASE CONTINUE: BOOLEAN OF
                TRUE: (MORE: PTR);
                FALSE: (SYMBOL: ALFA);
          END;

VAR
   FREE: PTR;
   FIRSTREAD: BOOLEAN;
   HEAD: PTR;
   P, Q, R: PTR;


PROCEDURE WRITELIST(LIST: PTR);

   VAR
      P: PTR;

   BEGIN
      P := LIST↑.LINK;
      REPEAT
         WRITE(≡ P:≡, ORD(P), ≡CODE≡, P↑.CODE: 3, ≡FREQ≡
            , P↑.FREQ);
         CASE P↑.CONTINUE OF
            TRUE:
               WRITE(≡ MORE≡, ORD(P↑.MORE));
            FALSE:
               WRITE(≡ SYMBOL≡, P↑.SYMBOL);
         END;
         WRITE(≡ P↑.LINK≡, ORD(P↑.LINK));
         WRITELN;
         P := P↑.LINK;
      UNTIL P = LIST;
      WRITELN;
   END (*WRITELIST*);


PROCEDURE ORG;
```

```
(* INITIALIZE THE FREE LIST *)

   BEGIN
      FREE := NIL;
   END (*ORG*);


FUNCTION ANTE(P: PTR): PTR;
(* POINTS TO THE ANTECEDENT OF P *)
(* P REMAINS UNCHANGED *)

   VAR
      TEMP: PTR;

   BEGIN
      TEMP := P;
      WHILE TEMP↑.LINK <> P DO
         TEMP := TEMP↑.LINK;
      ANTE := TEMP;
   END (*ANTE*);


PROCEDURE RELEASE(VAR P: PTR);
(* MOVES THE ELEMENT TO WHICH P POINTS, TO THE FREE LIST
   *)
(* IF P IS AN ENTIRE LIST, THEN P:= NIL ELSE P:= P↑.LINK
   *)

   VAR
      TEMP: PTR;
      PT1: PTR;

   BEGIN
      IF P↑.LINK = P
      THEN
         PT1 := NIL
      ELSE
         BEGIN
            TEMP := ANTE(P);
            TEMP↑.LINK := P↑.LINK;
            PT1 := P↑.LINK;
         END;
      P↑.LINK := FREE;
      FREE := P;
      P := PT1;
   END (*RELEASE*);
```

```
PROCEDURE RELIST(VAR P: PTR; Q: PTR);
(* MOVES THE STRING OF ELEMENTS,
BEGINNING WITH THE ELEMENT TO WHICH P POINTS,
AND ENDING WITH THE ELEMENT TO WHICH Q POINTS *)
(* INSERTS IT IN THE FREE LIST *)
(* IF P THRU Q IS AN ENTIRE LIST, THEN P:= NIL ELSE P:=
Q↑.LINK; *)
(* Q REMAINS UNCHANGED *)

   VAR
      TEMP: PTR;
      PT1: PTR;

   BEGIN
      IF Q↑.LINK = P
      THEN
         PT1 := NIL
      ELSE
         BEGIN
            TEMP := ANTE(P);
            TEMP↑.LINK := Q↑.LINK;
            PT1 := Q↑.LINK;
         END;
      Q↑.LINK := FREE;
      FREE := P;
      P := PT1;
   END (*RELIST*);


PROCEDURE ALLOCATE(VAR P: PTR);
(* CREATES A VARIABLE OF TYPE PTR, POINTED TO BY P *)
(* UTILIZES SPACE FROM THE FREE LIST, IF THERE IS ANY *)

   BEGIN
      IF FREE = NIL
      THEN
         NEW(P)
      ELSE
         BEGIN
            P := FREE;
            FREE := FREE↑.LINK;
         END;
   END (*ALLOCATE*);
```

```
PROCEDURE INIT(VAR P: PTR);
(* ESTABLISHES A NEW CIRCULAR LIST OF ONE ELEMENT *)
(* P POINTS TO THAT ELEMENT *)

   BEGIN
      ALLOCATE(P);
      P↑.LINK := P;
   END (*INIT*);


PROCEDURE INSERT(VAR P: PTR);
(* CREATES A VARIABLE OF TYPE PTR *)
(* INSERTS IT AS THE ANTECEDENT OF THE VARIABLE TO WHICH
P POINTS *)
(* P BECOMES THE POINTER TO THIS NEWLY CREATED VARIABLE
   *)

   VAR
      TEMP: PTR;
      PT1: PTR;

   BEGIN
      ALLOCATE(TEMP);
      PT1 := ANTE(P);
      PT1↑.LINK := TEMP;
      TEMP↑.LINK := P;
      P := TEMP;
   END (*INSERT*);


PROCEDURE MOV(VAR P, Q: PTR);
(* MOVES THE ELEMENT TO WHICH P POINTS, SUCH THAT IT IS
THE ANTECEDENT
OF THE ELEMENT TO WHICH Q POINTS *)
(* IF P IS AN ENTIRE LIST THEN P:=NIL ELSE P:=P↑.LINK *)
(* Q NOW POINTS TO WHAT WAS ORIGINALLY POINTED TO BY P
   *)

   VAR
      PT1: PTR;
      PT2: PTR;
      TEMP: PTR;

   BEGIN
```

```
      IF P↑.LINK = P
      THEN
         TEMP := NIL
      ELSE
         BEGIN
            PT1 := ANTE(P);
            PT1↑.LINK := P↑.LINK;
            TEMP := P↑.LINK;
         END;
      PT2 := ANTE(Q);
      PT2↑.LINK := P;
      P↑.LINK := Q;
      Q := P;
      P := TEMP;
   END (*MOV*);


PROCEDURE INCOR(VAR P: PTR; Q: PTR; VAR R: PTR);
(* MOVES THE STRING OF ELEMENTS, BEGINNING WITH THE ELEM
ENT TO WHICH
P POINTS, AND ENDING WITH THE ELEMENT TO WHICH Q POINTS
   *)
(* INSERTS IT TO PRECEDE THE ELEMENT TO WHICH R POINTS
   *)
(* IF P THRU Q IS AN ENTIRE LIST, THEN P:=NIL ELSE P:=Q↑
.LINK *)
(* Q REMAINS UNCHANGED *)
(* R NOW POINTS TO WHAT WAS ORIGINALLY POINTED TO BY P
   *)

   VAR
      PT1: PTR;
      PT2: PTR;
      TEMP: PTR;

   BEGIN
      IF Q↑.LINK = P
      THEN
         TEMP := NIL
      ELSE
         BEGIN
            PT1 := ANTE(P);
            PT1↑.LINK := Q↑.LINK;
            TEMP := Q↑.LINK;
         END;
      PT2 := ANTE(R);
      PT2↑.LINK := P;
```

```
      Q↑.LINK := R;
      R := P;
      P := TEMP;
   END (*INCOR*);


FUNCTION ELEM(P: PTR; N: INTEGER): PTR;
(* POINTS TO THE N-TH ELEMENT AFTER THE ELEMENT TO WHICH
P POINTS *)
(* P REMAINS UNCHANGED *)

   VAR
      I: INTEGER;

   BEGIN
      FOR I := 1 TO N DO
         P := P↑.LINK;
      ELEM := P;
   END (*ELEM*);


PROCEDURE READLINE;

   VAR
      CH: CHAR;
      NAME: ALFA;


   PROCEDURE NEXTCH;

      BEGIN
         IF FIRSTREAD
         THEN
            FIRSTREAD := FALSE
         ELSE
            GET(INPUT);
         CH := INPUT↑;
      END (*NEXTCH*);


   PROCEDURE GETNONBLANK;

      BEGIN
         NEXTCH;
```

```
         WHILE (CH = ≡ ≡) AND (NOT EOLN(INPUT)) DO
            NEXTCH;
      END (*GETNONBLANK*);



   PROCEDURE GETALFA(VAR NAME: ALFA);

      VAR
         I: INTEGER;

      BEGIN
         FOR I := 1 TO 10 DO
            BEGIN
               NAME[I] := CH;
               NEXTCH;
            END;
      END (*GETALFA*);



   BEGIN (*READLINE*)
      P := HEAD;
      GETNONBLANK;
      WHILE NOT EOLN(INPUT) DO
         BEGIN
            INSERT(P);
            P↑.CONTINUE := FALSE;
            GETALFA(NAME);
            P↑.SYMBOL := NAME;
            READ(P↑.FREQ);
            WRITELN(≡ ≡, P↑.SYMBOL, P↑.FREQ);
            FIRSTREAD := TRUE;
            GETNONBLANK;
         END;
   END (*READLINE*);


PROCEDURE SORT;

   VAR
      V: REAL;
      LAST, NEXT: PTR;

   BEGIN
      LAST := HEAD↑.LINK;
      NEXT := LAST↑.LINK;
```

```
        WHILE NEXT <> HEAD DO
           IF LAST↑.FREQ < NEXT↑.FREQ
           THEN
              BEGIN
                 LAST := NEXT;
                 NEXT := LAST↑.LINK;
              END
           ELSE
              BEGIN
                 P := HEAD;
                 V := NEXT↑.FREQ;
                 REPEAT
                    P := P↑.LINK;
                 UNTIL P↑.
                 FREQ ≥ V;
                 MOV(NEXT, P);
              END;
     END (*SORT*);


  PROCEDURE COMBINE;

     VAR
        I: INTEGER;
        F: REAL;
        TEMP: PTR;

     BEGIN
        REPEAT
           SORT;
           P := HEAD;
           F := 0;
           FOR I := 0 TO 1 DO
              BEGIN
                 P := P↑.LINK;
                 P↑.CODE := I;
                 F := F + P↑.FREQ;
              END;
           INIT(Q);
           TEMP := Q;
           R := HEAD↑.LINK;
           INCOR(R, P, Q);
           INSERT(R);
           R↑.CONTINUE := TRUE;
           R↑.MORE := TEMP;
           R↑.FREQ := F;
           TEMP↑.CONTINUE := TRUE;
```

```
          TEMP↑.MORE := R;
       UNTIL HEAD↑.
       LINK↑.LINK = HEAD;
    END (*COMBINE*);


PROCEDURE ANSWER;

    BEGIN
       WHILE HEAD↑.LINK <> HEAD DO
          BEGIN
             WRITELN('  ');
             Q := HEAD↑.LINK;
             REPEAT
                P := Q↑.MORE;
                Q := P↑.LINK;
                WRITE(Q↑.CODE: 2);
             UNTIL NOT Q↑.
             CONTINUE;
             WRITE(':    ');
             WRITE(Q↑.SYMBOL);
             WRITELN;
             RELEASE(Q);
             WHILE (P = P↑.LINK) AND (P <> HEAD) DO
                BEGIN
                   Q := P↑.MORE;
                   RELEASE(P);
                   P := Q↑.LINK;
                   RELEASE(Q);
                END;
          END;
    END (*ANSWER*);


BEGIN (*HUFFMAN*)
    WRITELN;
    WRITELN;
    ORG;
    FIRSTREAD := TRUE;
    INIT(HEAD);
    WHILE NOT EOF(INPUT) DO
       READLINE;
    COMBINE;
    ANSWER;
    WRITELN;
    WRITELN;
```

```
END (*HUFFMAN*).
```

1	2.0000000000000E-001
2	1.8000000000000E-001
3	1.0000000000000E-001
4	1.0000000000000E-001
5	1.0000000000000E-001
6	6.0000000000000E-002
7	6.0000000000000E-002
8	4.0000000000000E-002
9	4.0000000000000E-002
10	4.0000000000000E-002
11	4.0000000000000E-002
12	3.0000000000000E-002
13	1.0000000000000E-002

0 0: 1

0 1 0: 3

0 1 1: 4

1 0 0: 5

1 0 1 0: 6

1 0 1 1 0 0: 13

1 0 1 1 0 1: 12

1 0 1 1 1: 8

1 1 0: 2

1 1 1 0 0: 11

1 1 1 0 1: 10

1 1 1 1 0: 9

1 1 1 1 1: 7

Appendix IV: Program LISP

```
 (*$U+[W1,56] MARY CAPECE *)
PROGRAM LISP(INPUT, OUTPUT);

TYPE
   SYMBOL =
      (ATOMSYM, LPAREN, RPAREN, COMMA, DOL);
   KIND =
      (ATOMIC, NONATOMIC);
   PTR = ↑ NODE;
   NODE = RECORD
             CASE STATE: KIND OF
                ATOMIC: (NAME: ALFA;
                         LINK: PTR);
                NONATOMIC: (HEAD, TAIL: PTR);
          END;

VAR
   L1, L2: PTR;
   NILL, TREW, ATOMLIST: PTR;
   SYM: SYMBOL;
   IDENT: ALFA;
   XX, YY, ZZ: PTR;



PROCEDURE ERROR(N: INTEGER);

   BEGIN
      WRITELN;
      WRITE(≡ ≡);
      CASE N OF
         1:
            WRITE(≡*** CAR OF ATOM IS UNDEFINED ***≡);
         2:
            WRITE(≡*** CDR OF ATOM IS UNDEFINED ***≡);
         3:
            WRITE(
   ≡*** INPUT TO PROCEDURE WRITEATOM MUST BE ATOMIC ***≡
               );
         4:
            WRITE(≡*** EQ DEFINED ONLY ON TWO ATOMS ***≡
               );
         5:
            WRITE(≡*** ERROR IN INPUT ***≡);
         6:
            WRITE(≡*** ERROR IN LIST ***≡);
      END;
      WRITELN;
```

```
      HALT;
   END (*ERROR*);


FUNCTION ATOM(L1: PTR): PTR;

   BEGIN
      IF L1↑.STATE = ATOMIC
      THEN
         ATOM := TREW
      ELSE
         ATOM := NILL;
   END (*ATOM*);


FUNCTION CONS(L1, L2: PTR): PTR;

   VAR
      Q: PTR;

   BEGIN
      NEW(Q);
      WITH Q↑ DO
         BEGIN
            STATE := NONATOMIC;
            HEAD := L1;
            TAIL := L2;
         END;
      CONS := Q;
   END (*CONS*);


FUNCTION CAR(L1: PTR): PTR;

   BEGIN
      IF ATOM(L1) = TREW
      THEN
         ERROR(1)
      ELSE
         CAR := L1↑.HEAD;
   END (*CAR*);


FUNCTION CDR(L1: PTR): PTR;
```

```
    BEGIN
       IF ATOM(L1) = TREW
       THEN
          ERROR(2)
       ELSE
          CDR := L1↑.TAIL;
    END (*CDR*);


FUNCTION EQ (L1, L2: PTR): PTR;

    BEGIN
       IF (ATOM(L1) = NILL) OR (ATOM(L2) = NILL)
       THEN
          ERROR(4)
       ELSE
          IF L1 = L2
          THEN
             EQ := TREW
          ELSE
             EQ := NILL;
    END (*EQ*);


FUNCTION EQUAL(L1, L2: PTR): PTR;

    BEGIN
       IF (ATOM(L1) = TREW) AND (ATOM(L2) = TREW)
       THEN
          EQUAL := EQ (L1, L2)
       ELSE
          IF EQUAL(CAR(L1), CAR(L2)) = TREW
          THEN
             EQUAL := EQUAL(CDR(L1), CDR(L2))
          ELSE
             EQUAL := NILL;
    END (*EQUAL*);


PROCEDURE ENTER(WRD: ALFA; VAR S: PTR);

    VAR
       Q: PTR;
```

```
   BEGIN
      ATOMLIST↑.NAME := WRD;
      Q := ATOMLIST↑.LINK;
      WHILE Q↑.NAME <> WRD DO
         Q := Q↑.LINK;
      IF Q = ATOMLIST THEN
         BEGIN
            NEW(ATOMLIST);
            ATOMLIST↑.STATE := ATOMIC;
            ATOMLIST↑.LINK := Q↑.LINK;
            Q↑.LINK := ATOMLIST;
         END;
      S := Q;
   END (*ENTER*);


PROCEDURE GETSYM;

   VAR
      I: INTEGER;

   BEGIN
      WHILE INPUT↑ = ≡ ≡ DO
         GET(INPUT);
      IF INPUT↑ = ≡(≡
      THEN
         BEGIN
            SYM := LPAREN;
            GET(INPUT);
         END
      ELSE
         IF INPUT↑ IN [≡A≡ .. ≡Z≡]
         THEN
            BEGIN
               SYM := ATOMSYM;
               I := 0;
               REPEAT
                  I := I + 1;
                  IF I <= 10 THEN
                     IDENT[I] := INPUT↑;
                  GET(INPUT);
               UNTIL NOT (INPUT↑ IN [≡A≡ .. ≡9≡]);
               FOR I := I + 1 TO 10 DO
                  IDENT[I] := ≡ ≡;
            END
         ELSE
            IF INPUT↑ = ≡,≡
```

```
        THEN
           BEGIN
              SYM := COMMA;
              GET(INPUT);
           END
        ELSE
           IF INPUT↑ = ≡)≡
           THEN
              BEGIN
                 SYM := RPAREN;
                 GET(INPUT);
              END
           ELSE
              IF INPUT↑ = ≡$≡
              THEN
                 BEGIN
                    SYM := DOL;
                    GET(INPUT);
                 END
              ELSE
                 ERROR(5);
  END (*GETSYM*);


PROCEDURE GETLIST(VAR L: PTR);

   VAR
      PTSTACK: ARRAY [1..100] OF PTR;
      OPSTACK: ARRAY [1..100] OF SYMBOL;
      PTTOP, OPTOP: INTEGER;

   BEGIN
      GETSYM;
      PTTOP := 1;
      OPTOP := 1;
      WHILE SYM <> DOL DO
         BEGIN
            IF SYM = ATOMSYM
            THEN
               BEGIN
                  PTTOP := PTTOP + 1;
                  ENTER(IDENT, PTSTACK[PTTOP]);
                  GETSYM;
               END
            ELSE
               IF SYM = LPAREN
               THEN
```

```
               BEGIN
                  OPTOP := OPTOP + 1;
                  OPSTACK[OPTOP] := LPAREN;
                  GETSYM;
               END
            ELSE
               IF SYM = COMMA
               THEN
                  BEGIN
                     OPTOP := OPTOP + 1;
                     OPSTACK[OPTOP] := COMMA;
                     GETSYM;
                  END
               ELSE
                  IF SYM = RPAREN
                  THEN
                     BEGIN
                        PTTOP := PTTOP + 1;
                        ENTER(≡NIL        ≡, PTSTACK[
                           PTTOP]);
                        PTSTACK[PTTOP - 1] := CONS(
                           PTSTACK[PTTOP - 1],
                           PTSTACK[PTTOP]);
                        PTTOP := PTTOP - 1;
                        WHILE OPSTACK[OPTOP] = COMMA
                           DO
                           BEGIN
                              PTSTACK[PTTOP - 1] :=
                                 CONS(PTSTACK[PTTOP -
                                 1], PTSTACK[PTTOP]);
                              PTTOP := PTTOP - 1;
                              OPTOP := OPTOP - 1;
                           END;
                        IF OPSTACK[OPTOP] <> LPAREN
                        THEN
                           ERROR(6)
                        ELSE
                           BEGIN
                              OPTOP := OPTOP - 1;
                              GETSYM
                           END
                     END;
      END;
   L := PTSTACK[PTTOP];
END (*GETLIST*);
```

```
PROCEDURE PRINT(L1: PTR);


   PROCEDURE WRITEATOM(L1: PTR);

      VAR
         WRD: ALFA;
         I: INTEGER;

      BEGIN
         IF ATOM(L1) = NILL
         THEN
            ERROR(3)
         ELSE
            BEGIN
               WRITE(≡  ≡);
               WRD := L1↑.NAME;
               I := 1;
               REPEAT
                  WRITE(WRD[I]);
                  I := I + 1;
               UNTIL (I > 10) OR (WRD[I] = ≡ ≡);
               WRITE(≡  ≡);
            END;
      END (*WRITEATOM*);


   PROCEDURE TRAVERSE(L1: PTR);

      BEGIN
         IF L1 = NILL
         THEN
            WRITE(≡)≡)
         ELSE
            WITH L1↑ DO
               BEGIN
                  IF ATOM(HEAD) = NILL
                  THEN
                     BEGIN
                        WRITE(≡ (≡);
                        TRAVERSE(HEAD);
                     END
                  ELSE
                     WRITEATOM(HEAD);
                  TRAVERSE(TAIL);
               END;
```

```
      END (*TRAVERSE*);


   BEGIN (*PRINT*)
      WRITELN;
      IF ATOM(L1) = TREW
      THEN
         WRITEATOM(L1)
      ELSE
         BEGIN
            WRITE(≡ (≡);
            TRAVERSE(L1);
         END;
      WRITELN;
      WRITELN;
   END (*PRINT*);


PROCEDURE INITIALIZEATOMLIST;

   BEGIN
      NEW(ATOMLIST);
      NEW(NILL);
      NEW(TREW);
      ATOMLIST↑.STATE := ATOMIC;
      NILL↑.STATE := ATOMIC;
      TREW↑.STATE := ATOMIC;
      ATOMLIST↑.LINK := NILL;
      NILL↑.LINK := TREW;
      TREW↑.LINK := ATOMLIST;
      NILL↑.NAME := ≡NIL        ≡;
      TREW↑.NAME := ≡*T*        ≡;
   END (*INITIALIZEATOMLIST*);


FUNCTION FLAT(L1, L2: PTR): PTR;

   BEGIN
      IF L1 = NILL
      THEN
         FLAT := L2
      ELSE
         IF ATOM(L1) = TREW
         THEN
            FLAT := CONS(L1, L2)
```

```
         ELSE
            FLAT := FLAT(CAR(L1), FLAT(CDR(L1), L2));
   END (*FLAT*);


FUNCTION REV(L1, L2: PTR): PTR;

   BEGIN
      IF L1 = NILL
      THEN
         REV := L2
      ELSE
         REV := REV(CDR(L1), CONS(CAR(L1), L2));
   END (*REV*);


FUNCTION EVALUATE(L1: PTR): PTR;

   VAR
      M, N: PTR;

   BEGIN
      IF ATOM(L1) = TREW
      THEN
         EVALUATE := L1
      ELSE
         BEGIN
            M := CAR(CAR(CDR(L1)));
            N := CAR(CDR(CAR(CDR(L1))));
            EVALUATE := CONS(EVALUATE(M), CONS(CAR(L1),
               CONS(EVALUATE(N), NILL)));
         END;
   END (*EVALUATE*);


BEGIN (*LISP*)
   INITIALIZEATOMLIST;
   WRITELN;
   GETLIST(XX);
   WRITE(≡ XX≡);
   PRINT(XX);
   GETLIST(YY);
   WRITE(≡ YY≡);
   PRINT(YY);
   WRITE(≡ CAR(YY)≡);
```

```
    PRINT(CAR(YY));
    WRITE(≡ CDR(YY)≡);
    PRINT(CDR(YY));
    WRITE(≡ ATOM(XX)≡);
    PRINT(ATOM(XX));
    WRITE(≡ ATOM(YY)≡);
    PRINT(ATOM(YY));
    WRITE(≡ EQ(NILL,NILL)≡);
    PRINT(EQ (NILL, NILL));
    WRITE(≡ EQUAL(XX,YY)≡);
    PRINT(EQUAL(XX, YY));
    WRITE(≡ CONS(XX,YY)≡);
    PRINT(CONS(XX, YY));
    WRITE(≡ CONS(YY,XX)≡);
    PRINT(CONS(YY, XX));
    WRITE(≡ FLAT(YY,NILL)≡);
    PRINT(FLAT(YY, NILL));
    WRITE(≡ REV(YY,NILL)≡);
    PRINT(REV(YY, NILL));
    GETLIST(ZZ);
    WRITE(≡ ZZ≡);
    PRINT(ZZ);
    WRITE(≡ EVALUATE(ZZ)≡);
    PRINT(EVALUATE(ZZ));
END (*LISP*).
```

```
XX
( ( A ) B )

YY
( ( E ) ( F ( G H )) I )

CAR(YY)
( E )

CDR(YY)
( ( F ( G H )) I )

ATOM(XX)
 NIL

ATOM(YY)
 NIL

EQ(NILL,NILL)
 *T*

EQUAL(XX,YY)
 NIL

CONS(XX,YY)
( ( ( A ) B ) ( E ) ( F ( G H )) I )

CONS(YY,XX)
( ( ( E ) ( F ( G H )) I ) ( A ) B )

FLAT(YY,NILL)
( E F G H I )

REV(YY,NILL)
( I ( F ( G H )) ( E ))

 ZZ
( DIV ( ( PLUS ( A ( MINUS ( B C ))))
      ( TIMES ( ( DIV ( D E )) F ))))

 EVALUATE(ZZ)
( ( A PLUS ( B MINUS C )) DIV
      ( ( D DIV E ) TIMES F ))
```

```
XX
( ( A ) B )

YY
( ( E ) ( F ( G H )) I )

CAR(YY)
( E )

CDR(YY)
( ( F ( G H )) I )

ATOM(XX)
 NIL

ATOM(YY)
 NIL

EQ(NILL,NILL)
 *T*

EQUAL(XX,YY)
 NIL

CONS(XX,YY)
( ( ( A ) B ) ( E ) ( F ( G H )) I )

CONS(YY,XX)
( ( ( E ) ( F ( G H )) I ) ( A ) B )

FLAT(YY,NILL)
( E F G H I )

REV(YY,NILL)
( I ( F ( G H )) ( E ))

 ZZ
( DIV ( ( PLUS ( A ( MINUS ( B C ))))
        ( TIMES ( ( DIV ( D E )) F ))))

 EVALUATE(ZZ)
( ( A PLUS ( B MINUS C )) DIV
        ( ( D DIV E ) TIMES F ))
```

VITA

Mary J. Capece, daughter of Mr. and Mrs. David S. Capece, was born in Philadelphia, Pennsylvania on February 10, 1954. She attended Chestnut Hill College in Philadelphia and graduated Magna Cum Laude, receiving the degree of Bachelor of Science in Mathematics. Ms. Capece is a member of Delta Epsilon Sigma National Scholastic Honor Society. In September 1975, she began working toward the degree of Master of Science in Computer Science at Lehigh University in Bethlehem, Pennsylvania. During that time, Ms. Capece held a teaching assistantship in the Department of Mathematics at Lehigh University.

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May 6, 1977

Dean Robert D. Stout
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Dear Dean Stout:

The computer programs included in the master's thesis of Mary Capece do not have any commercial value nor are there any copyright problems. The programs have purely educational content.

Yours truly,

Samuel L. Gulden

Samuel L. Gulden
Professor of Mathematics

SL:gb