1-1-1976

Some tolerance effects on a controlled displacement compressor.

Frederic H. Abendschein

SOME TOLERANCE EFFECTS ON A CONTROLLED DISPLACEMENT COMPRESSOR

by

Frederic H. Abendschein

A Thesis

Presented to the Graduate Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Mechanical Engineering

Lehigh University

1976

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering.

November 19, 1976
(date)

Professor in Charge

Department Chairman

Acknowledgements

I thank Dr. Russell E. Benner of the Department of Mechanical Engineering and Mechanics for his advice and help and the following people at Borg-Warner:

York Division

Michael J. Blachly
Dr. Kenneth E. Hickman
C. B. Hilliard, Jr.
Zigmund J. Szabo
Dwight L. Tothero
Robert L. Young

Research Center

Dr. Kenneth J. Kountz
Richard W. Roberts
Les Varga

I also thank all of the others who have worked on the CD compressor and the staff of York's Computer Center.

Contents

List of Tables

List of Figures

Abstract

Two effects of tolerances on the operating characteristics of a five cylinder swashplate type compressor whose displacement may be varied and which is intended for an automotive air conditioning application were investigated. The first effect is the variation seen in the top dead center clearance because of the tolerances on the many parts involved in the stackup. The second effect is the variation in the force required on the wobble plate mechanism to hold it at a given displacement caused by changes in the location of the center of gravity of the mechanism because of the many dimensional and weight tolerances.

The first effect was investigated by means of a computer program based on the Monte Carlo method. Each tolerance involved was randomly selected and the resulting parts were assembled and a clearance calculated; this was done 10,000 times and the resulting clearances were arranged to give the expected distribution of clearances. A method was devised then to use this data and selectively choose gaskets of the appropriate size so that the volumetric efficiency of the compressor could be held within acceptable limits.

A similar computer program was developed to study the second effect. Random selection of tolerances on dimensions and weights was followed by a calculation of the location of the center of gravity of the wobble plate subassembly. After repeating this procedure numerous times a distribution of the location of the center of gravity resulted. This data was then used to calculate

the variation in the axial force needed to hold the wobble plate in position. The variation was found to be so small that it does not have to be considered in the design of the control system that supplies this force.

I. Introduction

Reciprocating compressors used in automotive air conditioning applications come in several basic design arrangements: two cylinder in line, two cylinder V, four cylinder radial scotch yoke, and multi-cylinder swashplate. Each of these must supply a minimum capacity at the slowest, or idle, speed of the compressor. However, because of the large range of engine speeds all will have excess capacity at the upper end of their speed range. To prevent this extra capacity from cooling the car down to objectionable levels two methods are used. In the older one the clutch cycles off when the temperatures begin to fall too low and the compressor drive is disengaged. However, this cycling sometimes can be heard inside the car and often, especially with small engines, results in a noticeable change in available power used to drive the car. The other method uses a valve in the suction line which holds the evaporator pressure constant just above where it would begin to ice but drops the suction pressure with increasing speed to force the compressor to work against an increasing pressure difference. While this method avoids the cycling problem it wastes large amounts of power.

In order to retain the best features of these methods while avoiding the associated problems York Division, Borg-Warner, has a new automotive compressor under development. This is a five cylinder swashplate type and is shown in Figure 1. The unique feature of this compressor is the mechanism which allows the swashplate (or wobble plate) angle to be varied with a consequential change in displacement. The control system is designed so that as speed increases the crankcase pressure is incresed and this higher pressure acting on the back of the pistons creates an unbalanced force which causes the wobble plate to pivot and so decrease its angle and the compressor displacement. At the same time the controls maintain the suction pressure constant. The net result is that the horsepower used by the compressor is nearly constant no matter what the speed and thus energy is saved. Also, because the suction pressure is held high enough so that there is no danger of the evaporator freezing no cycling clutch is needed. In winter and periods where air conditioning is not wanted the compressor runs in the minimum stroke position where there is only enough pumping to keep the parts oiled.

In the design of a normal reciprocating compressor the top dead center (hereafter abbreviated TDC) clearance is always made as small as possible to minimize re-expansion effects. However, in the York Controlled Displacement (CD) compressor the TDC clearance varies with the wobble plate angle. A computer program (F1028) was written previously which used the nominal dimensions of parts to calculate the variation of the TDC clearance with the wobble plate angle or stroke. In the design of the latest prototype compressors, the 5187 series, this program was used to select basic linkage dimensions that gave the minimum TDC clearance at half stroke with the clearance being 0,20 mm larger at full stroke and 0,46 mm larger at minimum stroke. This variation is shown in Figure 2 for the 5187-CD4 compressor.

Another contributing factor to the TDC clearance is the number of tolerances involved in the dimensional stackup for the TDC clearance. Because of the mechanism used to vary the stroke more parts and tolerances are involved in the TDC clearance calculation for this compressor than for a normal one. Such a large variation in TDC clearance would result, it was felt, that cylinder gaskets would have to be selectively chosen to maintain the clearance within acceptable bounds; if possible it would be desirable to avoid following this procedure or, at least, to minimize the number of required gaskets. So, the first problem investigated here is the effect of the tolerances on the TDC clearance and the resulting consequences.

Another area where tolerances were felt to have an undesirable effect on the compressor was on the dynamic balance. For a wobble plate compressor of fixed angle it can be shown that the thickness of the rotating parts can be chosen so that the stroke decreasing moment from them will just overcome the stroke increasing moment generated by the reciprocating parts leaving the compressor in dynamic balance so far as changing stroke is concerned. For this to occur three or more pistons are required. A theoretical investigation by L. Varga of Borg-Warner's Roy C. Ingersoll Research Center showed that for a variable angle wobble plate balance could only be achieved at one point; at other angles there will be a moment which tends to change the stroke. Now this assumes the center of gravity of the rotating parts is located at the center of the hinge ball. However, while this ideal location was aimed for it was not reached because of physical constraints on the parts. This tendency to alter the stroke changing moment also varies because of the shifting of the actual center of gravity caused by weight and location variations. Differences in the stroke altering moment from one compressor to another could result in problems with the control system being able to produce the desired responses in the form of proper pressures. Therefore the effect of tolerances on the stroke changing moment (or hinge moment) becomes the second problem to be investigated here.

II. Tolerance Effects on the Top Dead Center Clearance

While there are many parts that go into the stackup of the TDC clearance many may be grouped together for convenience into members as shown in Figure 3 and identified as they were in the computer programs. These members represent the basic mechanism that causes the TDC clearance to vary with the stroke. Each affects the stroke differently as shown in Figure 4.

The members are composed of dimensions and tolerances from the parts as given below:

SUPPL
- Shaft assembly
- Link plate

LINK
- Link
- Front link pin
- Rear link pin

WOBPL
- Wobble plate
- Drive plate
- Hinge ball
- WP radial bearing
- WP radial bearing race
- Shaft assembly

DRIVE and GAMMA
- Wobble plate
- WP front thrust bearing
- Drive plate
- Hinge ball
- WP radial bearing
- WP radial bearing race
- Shaft assembly

ROD
- Rod

CYLRD
- Cylinder

These members are the ones which cause the variation in TDC clearance with wobble plate angle. Three others exist which also add contributions to the TDC clearance caused by tolerances:

CASE
- Bearing housing
- Cylinder-crankcase
- Front gasket
- Cylinder gasket

SABH
- Front thrust bearing
- Shaft assembly

PIST
- Piston

The tolerances given to these parts are as tight as it was possible to hold and still be manufacturable at reasonable cost.

Rather than using the conventional x-y dimensioning system to give these tolerances the geometrical tolerancing system was used. In this system basic dimensions are given to a center (Fig. 5), and a diametrical tolerance for the center is given. The hole placed at the basic center must have its center lie in the given diametrical tolerance. This system is more realistic than the x-y system because a hole center may only be located equidistant in any direction from the basic center. In the x-y system, however, the hole may be further away on the diagonal joining the x-y tolerance extremes than along the x or y axis.

A new computer program was then written to assemble the members and calculate the TDC clearance. However, the program does not merely use the nominal dimensions; instead it repeatedly calculates the TDC clearance using slightly different dimensions each time and, as a good example of the Monte Carlo method, supplies the distributions of these clearances along with the maximum and minimum values.

Before writing the program the type of distribution expected for the dimension of each part involved in the TDC clearance was needed. Two types of distribution were felt to be most likely. The first is a normal distribution for the dimensions. This will occur if the tooling is set up to produce parts which are supposed to be at the nominal dimension. Putting together these parts will give an assembly with normal distribution of TDC clearances. The second distribution is a square one. This comes about if the common manufacturing process is followed of setting the machine at one end of the tolerance range and allowing it through wear to go to the other end of the tolerance range. Assembly of parts with square distributions of dimensions will also result in a normal distribution of TDC clearances although it will be a much broader distribution than that for an assembly whose parts follow a normal curve. A mixture of parts machined in the two ways will result in a TDC clearance distribution lying between the two extremes.

A method is needed to generate these two types of distribution for each part inside the program with only the two items supplied to the computer about each part: its nominal dimension and the tolerance on it. First, for both types of distributions the tolerance given is assumed to represent the 3σ level; 99.73% of all parts should fall within this range. Next, the method used to generate the dimensions for the parts when they follow a normal distribution is a several step process described below:

1. First a random number is supplied by the function RN. This function works as follows:
   a. The first time used the function uses an initial value that is fractional and includes all digits from 0 to 9. Thereafter it starts with the previous value of the random number.
   b. Next this number is multiplied by 29 and the integer part is dropped.
   c. The remaining fractional part is a random number.
2. The random number is then multiplied by 100 and if the

resulting number is larger than 500 it is divided by 2. Then this number, made integer, is used as an address label to randomly go into an array of 500 numbers which are a table of random number deviates and select one. Random selection of a number from this table (which contains plus and minus values) gives one a number which lies in a normal distribution of numbers about zero.

3. This number from the random normal deviate table is multiplied then by the standard deviation of the dimension in question, or in other words, by the tolerance divided by three.
4. This value is added to the nominal dimension and the result is a new value for the dimension randomly selected from a normal distribution.

Finally, the dimensions when the parts follow a square distribution are found as shown below:

1. The random number is generated as before.
2. The number has 0.5 subtracted from it and then the result is dobled to get a random number between -1 and 1.
3. This random number is multiplied by the tolerance being examined.
4. Adding the result to the nominal dimension yields a value that lies in a square distribution.

These two methods were incorporated into the computer program which produces the TDC clearance distributions by following this form:

1. Function RN explained above.

2. Function RND for the random normal deviates explained above. This is only used for the normal distribution.

3. Dimension, format, and initial value statements.

4. Data read. This consists of the nominal dimensions and the tolerance for each part.

5. Distribution array set equal to zero.

6. Main loop entered:

   a. Variation from nominal dimension calculated for each part.

   b. The dimension of each part is found by adding the part's nominal dimension to this variation. These parts are assembled to make members by using the appropriate equations.

   c. The members are in turn assembled by using the equations developed for use in program F1028 and verified by the assembly of the first CD4 compressor to arrive finally at a value for the TDC clearance.

   d. The calculated value of the TDC clearance is then compared to the previous values for the maximum and minimum. If it is larger than the maximum or smaller than the minimum the appropriate one is given the new value.

   e. The distribution array is reviewed and when the correct location is found where the just calculated TDC clearance belongs the value of this array at this location is increased by one.

   f. The main loop is gone through up to the number of times given as part of the data.

7. Finally the output data is printed:

   a. The minimum and maximum values found for the TDC clearance.

   b. The distribution of the TDC clearance. This is given as a value of the TDC clearance and the number of times the calculated TDC clearance was found to lie between it and the next value 0,01 mm away.

The number of times selected for the computer to go through the main loop was 10,000 and this value was chosen for several reasons:

1. The number is large enough that a good distribution should result.
2. The number is small enough that the amount of computer time needed is reasonable.
3. 10,000 represents what would be a reasonable first year trial run production quantity and thus gives an idea on what to expect on these compressors.

This, and all other computer programs used here, were run on York Division's Hewlett-Packard HP3000 computer.

The computer program was run three different ways:

1. The function RND was included as were those cards needed to give the normal distribution to the dimensions of the parts.
2. Next the function RND was removed and some cards inside the main loop were altered to give the square distribution to

the parts' dimensions.

3. Finally all of the data cards were changed so the tolerances were zero and the program was allowed to go through the main loop only once. This gave the nominal value of the TDC clearance and was based on the nominal dimensions of the parts.

The two distributions obtained from the program are presented in Figures 6 and 7. Note they are for the 5187-CD5 compressor which has slightly greater clearance than the CD4. The nominal, minimum, and maximum TDC clearances between the piston top and valve plate bottom at zero stroke as predicted by the program are given below for the square and normal distribution of parts.

Table 1

Top Dead Center Clearance at Zero Stroke

Parts Distribution	Without suction valve With 0,380 gasket TDC Clearance (mm) Minimum	Maximum	Nominal TDC Clearance (mm)
			1,369
Normal	1,259	1,501	
Square	1,153	1,590	

Now to find the TDC clearance between the piston top and suction valve bottom at full stroke and those above 0,305 must be subtracted to account for the suction valve thickness and 0,508 is removed because of the different stroke. This difference totals 0,813 and changes the above table to:

Table 2

Top Dead Center Clearance at Full Stroke

Parts Distribution	With 0,305 suction valve With 0,380 gasket TDC Clearance (mm) Minimum	Maximum	Nominal TDC Clearance (mm)
			0,556
Normal	0,446	0,688	
Square	0,340	0,777	

The effect of this variation in TDC clearance on volumetric efficiency can be estimated from the equation:

$$e = 1+c(1-P_2/P_1)$$

where:
- e: volumetric efficiency
- c: ratio of clearance volume to full stroke (30,18 mm)
- P_1: absolute suction pressure (Pascals)
- P_2: absolute discharge pressure (Pascals)

This equation considers only the effect of the clearance volume and disregards such factors as blowby. However, the relative differences between the calculated volumetric efficiencies indicates the effect of the tolerances. Below is a chart which shows the calculated volumetric efficiencies for the pressure ratio where the compressor is rated -- 7.286:

Table 3

Volumetric Efficiency (Full Stroke)

Parts Distribution	Volumetric Efficiency Minimum	Maximum	Difference
Normal	0,857	0,907	0.050
Square	0.838	0,929	0,091

Now having the effects of the tolerances on the volumetric efficiency a base is needed for reference in order to determine if the variation in the volumetric efficiency is acceptable. Such a reference exists in York's reciprocating compressor for automotive applications which has been in production many years. Before installing the cylinder gasket the difference between the maximum and minimum TDC clearance on the reciprocating compressor is 0,343 mm. Experience showed this to be unacceptable from the standpoint of volumetric efficiency. The solution was to select one of three gaskets; the one chosen depended on the top dead center clearance measured. This process cut the difference to 0,191 mm. Then the VE turned out as shown below:

Table 4

Volumetric Efficiency and TDC Clearance for York 2 Cylinder Reciprocating Compressor

	Minimum	Maximum	Difference	
TDC Clearance	0,584	0,775	0,191	mm
Volumetric Efficiency	0,896	0,923	0,027	

This volumetric efficiency is calculated from the equation given above.

Thus the variation in volumetric efficiency caused by the tolerances should be acceptable if held to under 3% by proper selection of gaskets for the CD compressor. Another constraint is that the minimum gasket thickness allowed is 0,330 mm.

For the normal distribution using the following two gaskets will result in an acceptable variation in the volumetric efficiency.

Table 5

Clearance Ranges and Gasket Selection

TDC Clearance Range (mm) without gasket or suction valve	Gasket(mm) to be used	TDC Clearance Range (mm) with gasket and suction valve	Volumetric Efficiency
0,096	0,432	0,498	0.896 maximum
0,217		0,619	
			0.029 difference
0,217	0,330	0,517	0.882 average
0,338		0,638	0.867 minimum
		0,568 average	

For the square distribution four gaskets should be used:

-0,010	0,635	0,595	0.876 maximum
0,100		0,705	
0,100	0,533	0,603	
0,210		0,713	
			0.028 difference
0,210	0,432	0,612	0.862 average
0,320		0,722	
0,320	0,330	0,620	
0,398		0,728	0.848 minimum
		0,662 average	

The difference in the TDC clearance averages between the distributions is attributable to the 0,330 minimum gasket thickness; in the square distribution not being able to select a thinner gasket forced the average clearance up. To counteract this 0,090 should be removed from the cylinder thickness. This will bring the square distribution TDC clearance down to equal that for the normal distribution. Also the average volumetric efficiencies will then be equal.

Temporarily ignoring the selective choice of gaskets, the two extreme ends of the wider distribution (square distribution of the parts' tolerances) were used in computer program F1029 which calculates the force that must be supplied to the wobble plate to maintain it at a given stroke while compressing gas. No significant difference existed between the two forces predicted for the extreme ends of the TDC clearance range.

A difference between the TDC clearance originally predicted for the nominal CD4 dimensions and that for the nominal CD5 dimensions should be pointed out here: the clearance for the CD5 compressor is 0,231 mm greater at full stroke than that for the CD4. Now 0,074 mm of this is because of changes in nominal dimensions in the crnkcase area. The remainder is because of the more sophisticated method used in calculating the clearance for the CD5. In the CD4 the centers of mating parts that were circular were assumed to be coincident. In the CD5 analysis, however, the actual positions of the parts when the compressor was running were considered. Thus the centers of the links and pins do not coincide nor do those of the many parts nominally centered at the middle of the wobble plate.

As a result the actual dimensions of members such as LINK, WOBPL, and DRIVE change not just because of the tolerances themselves but also because of the play caused by the tolerances. Consequently the TDC clearance when the compressor is running may be up to 0,157 mm more than when it is measured at assembly. The gaskets shown are for the running clearances.

Next to be calculated were the extreme ends of the TDC clearance range by first using Figure 4 as a guide to selecting either the maximum or minimum value for the sizes of the parts which make up the members. These extreme dimensions were used in the computer program to calculate the minimum and maximum TDC clearance. With the 0,380 gasket but without the suction valve the TDC clearances found are shown below:

	TDC Clearance (mm) at Zero stroke	
Distribution of Parts	Minimum	Maximum
Normal	1,259	1,501
Square	1,153	1,590
Extreme dimensions on parts	1,101	1,633

III. Tolerance Effects on the Stroke Altering Moment

The first major step in investigating this problem was to locate the nominal center of gravity (CG) of each major rotating part of the wobble plate subassembly. For symmetrical, simple pieces like the bearings this was an easy task. However, for complicated non-symmetrical parts such as the drive plate a more involved process was used. Each of these parts was first divided into simple geometrical shapes that would approximate the general shape of the body. Next, using the known location of the CG of each simple shape the CG for the entire part was calculated. As a check the CG of the drive plate was determined experimentally. A drive plate was suspended by a thin thread and photographed. The point of suspension was changed and another photograph made. After superimposing the two pictures the line made by each thread was extended until they intersected. The crossing point of the lines was the center of gravity. The experimentally determined CG of the drive plate came within 0,25 mm of the calculated CG. No other parts were checked in this manner. All CG's used are the calculated ones.

Next the two factors that cause the actual CG of the subassembly to vary from its nominal position. First the CG of each part will vary from its nominal position. Second, the weight of each part will vary because of dimensional and density variations. Initially an attempt was made to calculate the variation in the parts' weights. Based on the two extreme sets of dimensions a maximum and minimum volume and then weight were calculated for each part. For simple parts such as the balance weight this method worked fine. However, for a part such as any one of the bearings which were obtained commercially many of the dimensions were unknown and it was felt using estimated values for these dimensions would result in maximum and minimum weights that were not reliable. Consequently another method was used to determine the variations in the weights. The parts going into the wobble plate subassembly that had not been built into compressors yet were collected and weighed with the weights being read to 0,01 gms. The number of each part available for weighing was not constant; generally the smaller the part the more there were to be weighed. The weights for each part were entered into program F2402. This program was written to analyze data using Student's t test where only a small number of samples are available. As output the program supplies the sample mean and standard deviation and, for up to seven confidence levels, the maximum and minimum population mean and range. While the seven confidence levels were entered only the 99.73% level is of interest because it represents the 3σ level for a normal distribution. Another way to describe the output is to say that one is 99.73% sure that the population will fall within the two extremes given for the range at that confidence level based on the number of samples given. Naturally, for a given part at one confidence level the larger the number of samples available the closer the predicted distribution and actual distribution for large number of samples will be. Because of this fact and the larger number of small parts available as compared to the heavy parts available for weighing the final variation arrived at for the location of the center of gravity will be somewhat greater than the actual variation. This will result in a calculated effect on the hinge moment that will be on the safe side; in other words the actual effect will be slightly smaller than that calculated.

No reliable method could be found to either calculate or measure the other factor -- the variation in CG location in each part -- that contributes to the location of the actual CG of the wobble plate subassembly. Suspending each part from the thread would not be accurate enough to find the small variation present. Calculating the CG location variation for each sample would be hampered by the inability of one to measure on some parts such as the bearings all of the needed dimensions without destroying the part. Consequently the following method was used to estimate what this variation would be. Whatever part tolerances were available for each part were reviewed and an average tolerance value was selected. This value was the one assigned to the variation in the CG location about the nominal position for that particular part and this will certainly be of the correct order of magnitude.

Not all parts in the wobble plate subassembly were considered when its CG was calculated. More properly, the CG was found for the rotating parts of the wobble plate subassembly. Listed below are those parts that were included in the CG location calculation:

1. Snap ring
2. Balance weight
3. WP radial bearing race
4. Rear link pin
5. Drive plate
6. ½ WP rear thrust bearing
7. ½ WP front thrust bearing
8. ½ WP radial bearing
9. ½ of each link

Several assumptions that were made affected this list and are given below:

1. Only one half of each bearing is rotating; the other halves are going through the same motion as the wobble plate.
2. One half of each link is supported by the wobble plate while the other half is supported by the front link pin and link plate and does not enter into this calculation.
3. The wobble plate, rods, slipper, and one half of each bearing do not "rotate" but "wobble". This is explained in more detail in Appendix A. The result is that these parts do not contribute significantly to the moment.

This problem is similar to the TDC clearance one in that a Monte Carlo simulation is required to arrive at a solution and consequently a computer program for this was written. Exactly the same function was used to generate the necessary numbers and the rest of the program is similar to that written for the TDC clearance problem as can be seen in the outline below:

CG Variation Program:

1. Function RN to generate random numbers; explained before.
2. Dimensions, input, and output formats. Initial values defined.
3. Function RND (explained previously).
4. Input data read
   a. For each part of the wobble plate subassembly considered as rotating its weight (in grams), weight tolerance (in grams), and any required dimensions and

tolerances are read.

b. The array of random normal deviates is read.

5. The array used to tabulate the distribution of the CG positions is initialized at zero.

6. The main loop is entered:

   a. First each weight and dimensional tolerance is multiplied by a number, randomly selected by using RN, from the random normal deviate array. This results in values that represent how far the weight or dimension varies from its nominal value.

   b. In the case of the weights the amount of deviation is added (the deviation may be plus or minus) to the nominal weight. For the bearings and links the weights are halved based on one assumption given above.

   c. The distance from each part to the vertical axis of the coordinate system is found by summing the appropriate dimensions and their matching deviation from this nominal dimension.

   d. The weights of all the parts are added.

   e. The sum of the products of the weight of each part times its distance from the origin (CG to the vertical axis).

   f. This last sum is divided by the sum of the weights to arrive at a horizontal position of the CG of the sub-assembly.

   g. This value is then compared to the previous minimum and maximum value of the horizontal CG. If it is respectively smaller or larger the new value is substituted to retain a new extreme along with the corresponding weight.

h. The CG horizontal position is then tested to see where, in 0,01 mm wide segments, it lies and, when this is found out, the number lying in this segment is increased by one.

i. Step c is repeated for the parts which are not symmetrical about the shaft axis except the distances to the horizontal axis were used. The symmetrical parts do not have to be considered here.

j. Step e is repeated for the non-symmetrical parts using the distance between the horizontal axis and their CGs.

k. Repeating step f gives the vertical position of the CG.

l. Steps g and h are repeated for the vertical position.

7. The main loop is repeated 10000 times.

8. The final results are printed out:

   a. The minimum and maximum horizontal and vertical positions of the CG of the wobble plate subassembly and the corresponding weights.

   b. The vertical and horizontal CG position distribution.

The distribution obtained from the CG program for the horizontal and vertical position of the wobble plate subassembly's CG are shown in Figures 8 and 9 respectively. As expected both display the normal distribution shape. Notice that the horizontal distribution is much wider than the vertical one. This result is also as expected. Many more distances were involved in the horizontal calculation than in the vertical one because the symmetry of many of the parts about the shaft axis eliminated them from the latter calculations.

If the assumption is made that the width of the CG distribution along the vertical axis can be the minor axis of an ellipse and that along the horizontal axis the major axis can lie parallel then the shape shown in Figure 10 can be constructed. This solid shows what the chances are that the actual CG will be located at any position. Also it is assumed that the CG remains in the XY plane and that there is no significant variation in the Z direction.

Now the moment that will be set up by this non-ideal location of the wobble plat subassembly CG can be found from the following equations:

$$M=Fx$$

$$F=ma=m(y\omega^2)$$

or

$$M=my\omega^2x \quad \text{gm (mm) (rad/sec)}^2 \text{ (mm)= gm mm}^2/\text{sec}^2$$

where:

M: moment

m: mass of rotating parts

x: horizontal distance CG to vertical axis

y: vertical distance CG to horizontal axis

ω: wobble plate subassembly angular velocity

This can be rearranged to:

$$M=(m\omega^2)yx=kmyx$$

For comparative purposes ω may be held constant. Then the product yx at its extremes will determine the extremes of M. Because of the elliptical shape assumed above the extremes should occur close to when y is at its midpoint and x is at its extremes; this assumes the mass is nearly constant. These two extreme points are shown below:

Table 6

WP Subassembly CG Variation and Resulting Moment

x (mm)	y (mm)	m (gm)	M (gm mm^2/sec^2)
4,176	0,943	851,78	$3354\omega^2$ maximum
3,858	0,943	852,61	$3102\omega^2$ minimum

Notice there is only an 8% difference between the extreme moments.

Now an idea is needed on how badly the compressor is out of balance and how large the resultant unbalanced hinge moment is. The equation which can be used to find this hinge moment is:

$$\tfrac{1}{2}Nm_pL^2 = \cos^2\phi(A-B)$$

where:

N: number of pistons

m_p: mass of one piston assembly (gm)

L: ½ of the diameter of the wobble plate circle on which the rods are centered. (mm)

ϕ: wobble plate angle (degrees)

A: sum of the moments of inertia of the WP subassembly's rotating parts; it is about the axis which is at right angles to the WP plane.

B: sum of the moments of inertia of the WP subassembly's rotating parts about an axis in the plane of the wobble plate.

Rearrangement and multiplication by ω^2 gives the hinge moment:

$$M_h = (\tfrac{1}{2}Nm_pL^2 - \cos^2\phi(A-B))\omega^2$$

The sign convention is such that a + sign means a moment which increases stroke.

The first term is the contribution of the reciprocating parts while the second term comes from the rotating parts. Ideally the two should be equal with no resulting hinge moment. However, if B comes close to A in magnitude which happens if the rotating parts are more spherical in shape than disk like, then the terms from the rotating parts will be small and a net positive hinge moment will exist. This is what happens in the 5187 compressor; physical restrictions on size do not allow proper balancing.

When the appropriate numbers are placed in the equation it reduces to:

$$M_h = +56221\omega^2 \quad \text{(at minimum stroke)}$$

The sign on this indicates the moment tends to increase the stroke.

One sign which has not yet been discussed is that from the CG position of the wobble plate subassembly. As can be seen in Figure 10 positioning the CG in quadrants I and III will result in a moment that increases stroke while placing it in the other two gives a stroke decreasing moment. Because the actual CG is located in the number II quadrant the moment decreases the stroke and the sign from this source should be minus.

Calculating the mean and a tolerance on the moment from the wobble plate CG and adding them to the hinge moment from the unbalance existing between the rotating and reciprocating parts gives the following:

$$M_h = (+56221-3228+126)\omega^2$$
$$M_h = (+52993+126)\omega^2$$

Thus the variation in the hinge moment caused by dimensional tolerances is only 0.2%.

A check on this theoretical unbalanced hinge moment was found in a series of tests run at Borg-Warner's Research Center. Using a modified compressor the axial force on the wobble plate subassembly that tends to change the stroke was measured. A multiple regression analysis was run on the resulting data to relate it to RPM^2, discharge, suction, and crankcase pressure, and stroke. The axial force at 1000RPM from this equation was 0,90 (10^6) dynes. Converting the theoretical unbalanced hinge moment to an axial force gives 1,23 (10^6) dynes at the same speed. While the theoretical force is 37% higher it is of the same order of magnitude as the measured force which, when all of the various factors are considered, indicates reasonable agreement exists.

IV. Conclusions about the Tolerance Effects

A. Top Dead Center Clearance

1. The TDC clearance varies with wobble plate angle being a minimum at half stroke, a maximum at minimum stroke, and in between at full stroke. A plot of wobble plate angle versus clearance is parabolic in shape.

2. The TDC clearance is affected significantly by the tolerance stackup of the parts. While the compressor is running the TDC clearance has the following ranges:

Table 7

TDC Clearances at Full Stroke

Distribution of Parts	TDC Clearance (mm) Minimum	Maximum	
Normal	0,446	0,688	0,380 mm gasket
Square	0,340	0,777	0,305 mm suction
Extreme dimensions	0,288	0,820	valve

Notice that the range on both distributions is considerably less than that calculated using the extreme dimensions on the parts entering into the stackup. This emphasizes the fact that the chances of randomly selecting all of the pieces so that the dimensions are at the extremes which would assure extreme clearances is vanishingly small.

3. To insure the volumetric efficiency variation caused by the tolerances is under 3% as it is in the current reciprocating compressors the gaskets shown on the following table are needed. In addition 0,090 should be taken off (figure is in mm) the cylinder-crankcase if the parts are made such that they have square distributions of tolerances.

Table 8

Clearance Ranges and Gasket Selection

Distribution of Tolerances	Full Stroke TDC Clearance Range (mm) without gasket or valve	Gasket (mm)	Full Stroke Running TDC Clearance (mm) with gasket and valve
Normal	0,096 0,217	0,432	0,498 0,619
	0,217 0,338	0,330	0,517 0,638
Square (does not have 0,09 mm taken off)	-0,010 0,100	0,635	0,595 0,705
	0,100 0,210	0,533	0,603 0,713
	0,210 0,320	0,432	0,612 0,722
	0,320 0,398	0,330	0,620 0,728

The gasket should be based on whichever piston has the least amount of TDC clearance.

4. The measured TDC clearance may be up to 0,157 mm less than that indicated above because the compressor is in an unloaded state then while the clearance is measured but it was considered loaded to take out the play when the above clearances were calculated. Tests should be run to measure the TDC clearance when the compressor is running to get an actual correlation between values for the clearance in the loaded and unloaded condition. This would assure the proper choice of the basic thicknesses for the gasket. In any case the differences between the thicknesses for the gaskets will

remain the same and valid.

B. Stroke Altering Moment

1. Variations in the weights and locations of the parts' centers of gravity bring about a variation in the location of the center of gravity of the rotating components of the wobble plate subassembly. Because this CG is not located in the center of the hinge ball a moment is set up that tends to decrease the stroke. The resultant variations in this moment from the tolerance effects are not significant and no changes are needed to account for these in the control system.

2. The above moment was calculated based on the geometry of the subassembly and after it has been added to that moment from the unbalance existing between the reciprocating and rotating parts and the result converted to an axial force, this force was compared to the term arising from the RPM^2 term from a multiple regression analysis. This analysis was done on experimental data from a test on a CD4 compressor where this force was measured. Reasonable agreement exists between theoretical and experimental forces.

3. As an approximation the stroke altering moment from parts moving in a manner such as the wobble plate may be neglected.

Appendix A

Moment From the CD Compressor Wobble Plate

The rotating parts that are connected to the shaft but angled to it (link plate) impart a rotation to the wobble plate about the Y and Z axis. The right hand coordinate system XYZ is attached to the crankcase; see Figure 11. The restraint-slipper-track combination prevents the wobble plate from rotating about the X axis. The xyz axis is attached to the wobble plate. Figure 11 shows the position of the wobble plate in four positions, 90° apart.

Consideration of the wobble plate in these positions shows that its rotation is:

$$\omega_Y = \omega\cos\omega t \qquad \text{where the rotation of the shaft is}$$

$$\omega_Z = -\omega\sin\omega t \qquad \underline{\omega}_{shaft} = +\omega\,\underline{I}$$

$$\theta = \omega t$$

Or the angular velocity of the wobble plate is:

$$\underline{\Omega} = \omega\cos\omega t\,\underline{J} - \omega\sin\omega t\,\underline{K}$$

Assuming the wobble plate is a thin disk:

$$I_x = I_y + I_z$$

$$I_y = I_z$$

$$P_{xy} = P_{yz} = P_{xz} = 0 = P_{XY} = P_{YZ} = P_{XZ}$$

Now:

$$\Sigma\underline{M}_O = (\dot{\underline{H}}_O)_{Oxyz} + \Omega \times \underline{H}_O$$

where:

$$(\dot{\underline{H}}_O)_{Oxyz} = 0$$

shaft angular velocity constant

$$\underline{H}_O: \quad H_X = I_X\omega_X - \cancel{P}_{XY}\omega_Y - \cancel{P}_{XZ}\omega_Z = I_X\omega_X$$

$$H_Y = -\cancel{P}_{YX}\omega_X + I_Y\omega_Y - \cancel{P}_{YZ}\omega_Z = I_Y\omega_Y$$

$$H_Z = -\cancel{P}_{ZX}\omega_X - \cancel{P}_{ZY}\omega_Y + I_Z\omega_Z = I_Z\omega_Z$$

For Position A the direction cosines for each axis are:

X:	Y:	Z:
$\lambda_x = \cos\phi$	$\lambda_x = \sin\phi$	$\lambda_x = 0$
$\lambda_y = \sin\phi$	$\lambda_y = \cos\phi$	$\lambda_y = 0$
$\lambda_z = 0$	$\lambda_z = 0$	$\lambda_z = 1$

Now:

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2P_{xy}\lambda_x\lambda_y - 2P_{yz}\lambda_y\lambda_z - 2P_{zx}\lambda_z\lambda_x$$

And allowing OL to be X, Y, and Z in turn gives:

$$I_X = I_x\cos^2\phi + I_y\sin^2\phi$$
$$I_Y = I_x\sin^2\phi + I_y\cos^2\phi$$
$$I_Z = I_z$$

From $\underline{\Omega}$ when $\omega t = 0$:

$$\omega_X = 0 \qquad H_X = I_X\omega_X = 0$$

$$\omega_Y = 0 \qquad H_Y = I_Y\omega_Y = I_x\omega\sin^2\phi + I_y\omega\cos^2\phi$$

$$\omega_Z = 0 \qquad H_Z = I_Z\omega_Z = 0$$

So:

$$\underline{H}_O = (I_x\omega\sin^2\phi + I_y\omega\cos^2\phi)\underline{J}$$

Finally at position A:

$$\underline{\Omega} = \omega\underline{J}$$

$$\underline{\Sigma M}_O = 0 + (I_x\omega\sin^2\phi + I_y\omega\cos^2\phi)\underline{J} \times \omega\underline{J}$$
$$= 0$$

Therefore the moment from the wobble plate is zero.

Actually the motion of the wobble plate is more complicated than that given by the above _; two conditions describe the true motion of the wobble plate:

1. The line joining the center of the wobble plate to the centerline of the restraint remains in the XY plane.
2. Axis x describes a cone about centerline X.

The result is that the centers of the rod ends in the wobble plate desribe an elongated figure eight. 90° apart the centers will be in the XY plane as the shaft rotates and then they will agree with Figure 11. At other points there will be a slight Z component in the position of the centers. This leads to extremely complicated expressions. Because of the elongated nature of the figure eight it may be approximated as a curve projected on two dimensions to obtain simpler expressions as was done above.

Another approach to this problem which was followed up to this time was to assume one half of the wobble plate rotates while the other reciprocates. These two effects almost cancel leaving only a small moment from the wobble plate which agrees reasonably with the result found above.

VI. References

Articles

1. Galin, R., "Balance a Swashplate Mechanism Used in an Automotive Air Conditioning System Compressor", GM Engineering Journal, First Quarter, 1964.

2. Maurizson, B. H., "Cost Cutting with Statistical Tolerances", Machine Design, November 25, 1971.

Books

1. ASHRAE Guide and Data Book, ASHRAE, New York, 1963.

2. Beer, F. P. and Johnston, E. R., Dynamics, Mc-Graw Hill, New York, 1972.

3. Foster, L. W., A Pocket Guide to Geometric Dimensioning and Tolerancing, Addison-Wesley Publishing Co., Reading, Mass.

4. Lipson, C. and Narenda, S. J., Engineering Experiments, McGraw -Hill, New York, 1973.

5. Natrella, M. G., Experimental Statistics, U.S. Government Printing Office, Washington, D.C., 1963.

6. Siddall, J. N., Analytical Decision Making in Engineering Design, Prentice-Hall, Englewood Cliffs, N.J., 1972.

7. Spiegel, M. R., Theory and Problems of Statistics, McGraw-Hill, New York, 1961.

Computer Programs

1. Abendschein, F. H., F1028, Top Dead Center Clearance of the Controlled Displacement Compressor, 1975.

2. Abendschein, F. H., F1029, Indicator Diagram Values and Component Forces for a Five Cylinder Controlled Displacement Compressor, 1975.

3. Abendschein, F. H., (on cards), Tolerance Effects on the Top Dead Center Clearance in the CD Compressor, 1976.

4. Abendschein, F. H., (on cards), Tolerance Effects on the Wobble Plate Subassembly Center of Gravity, 1976.

5. Groft, G. E., F2402, Statistical Analysis of Experimental Measurement Data, 1969.

Vita

Frederic Henry Abendschein was born on August 11, 1947, in Columbia, Pennsylvania, to Frederic and Lucille Abendschein. Undergraduate work was done at Lehigh University. He graduated with Honors in June, 1969, and received a Bachelor of Science Degree in Mechanical Engineering.

From that time he has worked at York Division, Borg-Warner Corporation on the design of compressors for both whole house air conditioning units and, most recently, for automotive air conditioning applications. He is presently a Principle Engineer.

From September, 1975, to May, 1976, on leave of absence from York he did graduate study at Lehigh University. This thesis was written at York after he returned from the leave of absence and this thesis will also be a report at York.

He is a registered Professional Engineer in Pennsylvania and an Associate Member of the American Society of Mechanical Engineers.

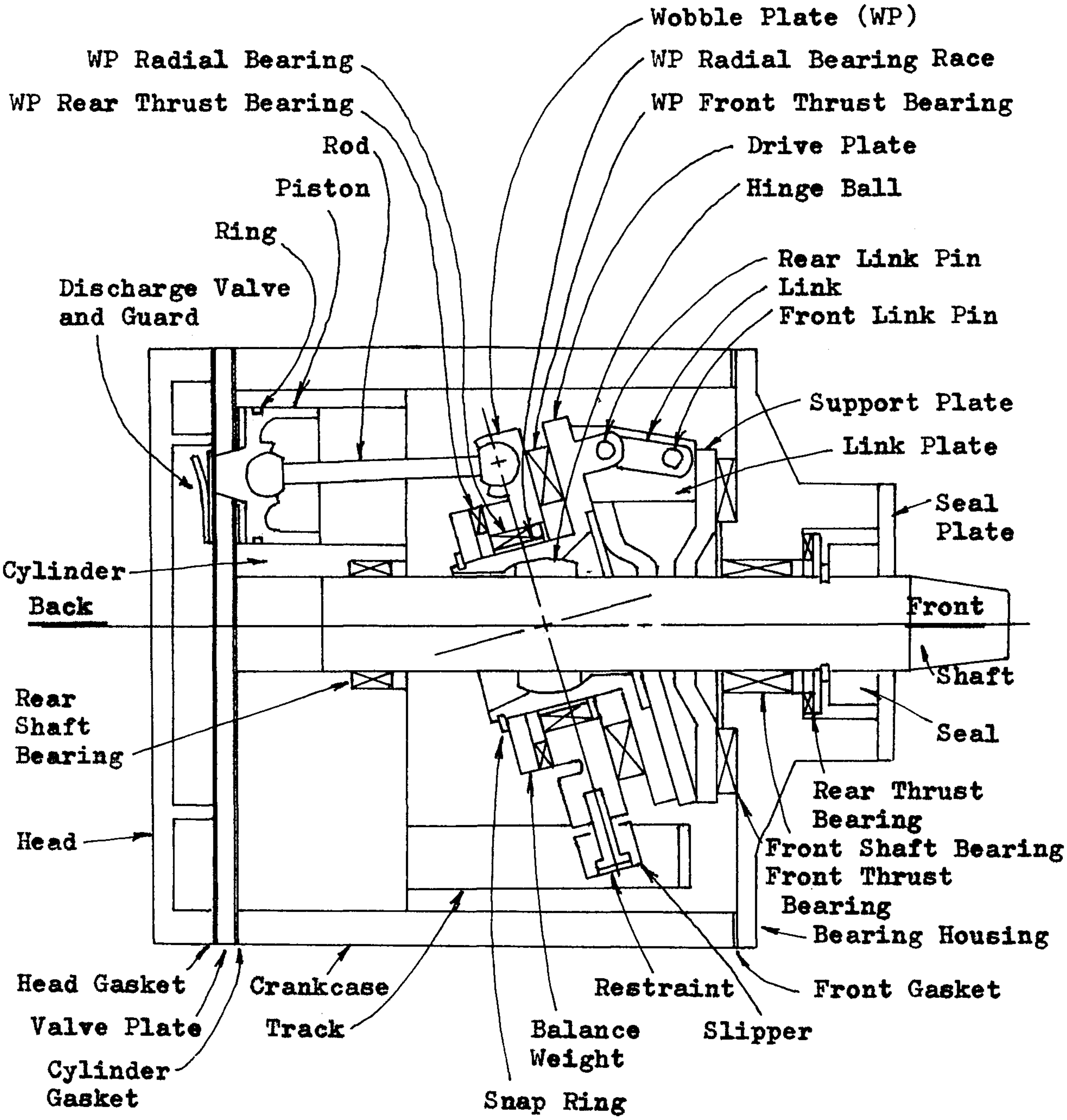

Figure 1
Controlled Displacement Compressor

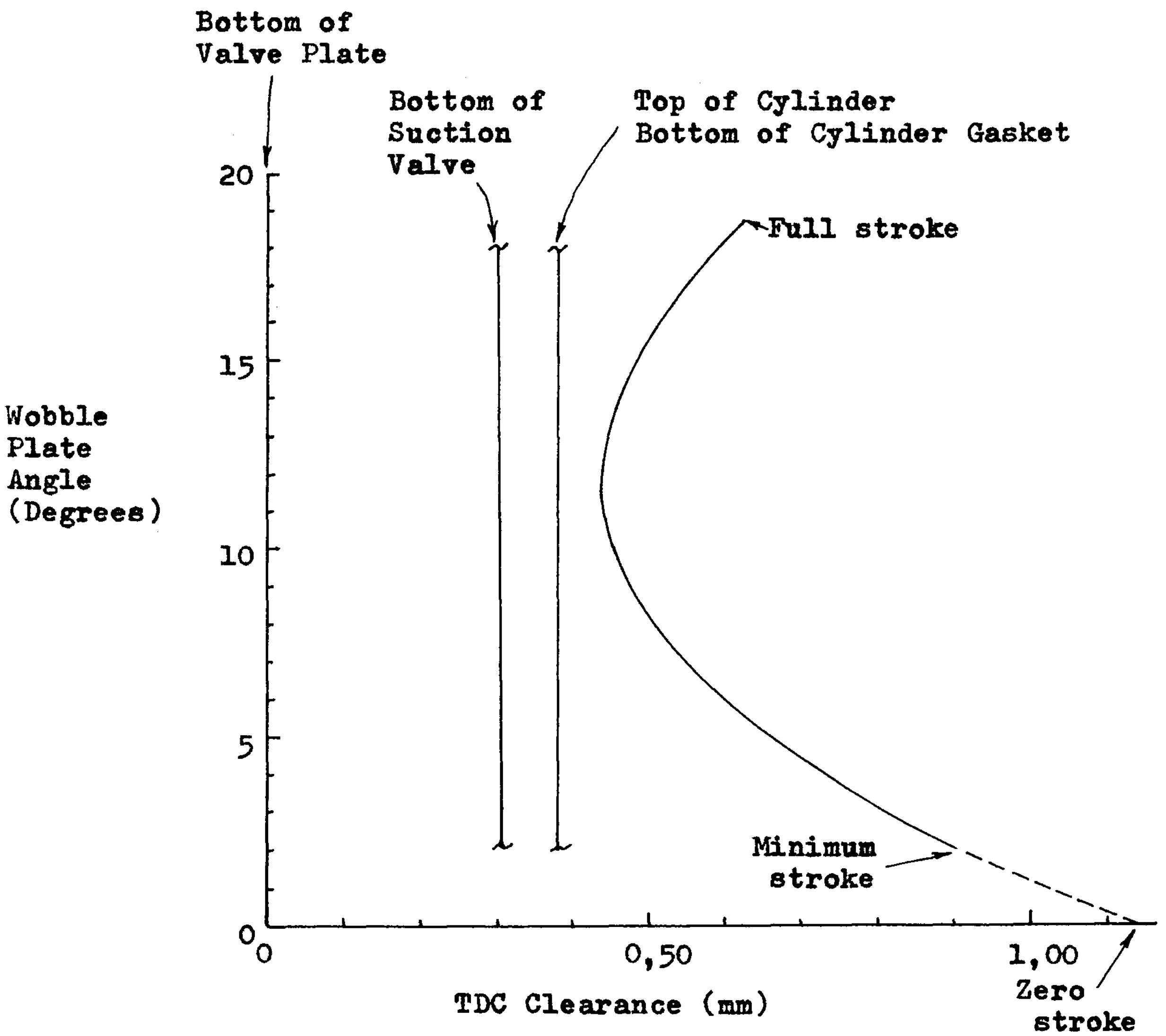

Figure 2
Nominal Top Dead Center Clearance as a Function
of Wobble Plate Angle (5187-CD4)

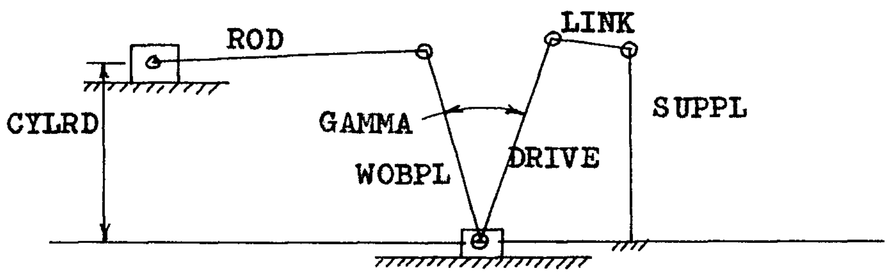

Figure 3
CD Compressor Members

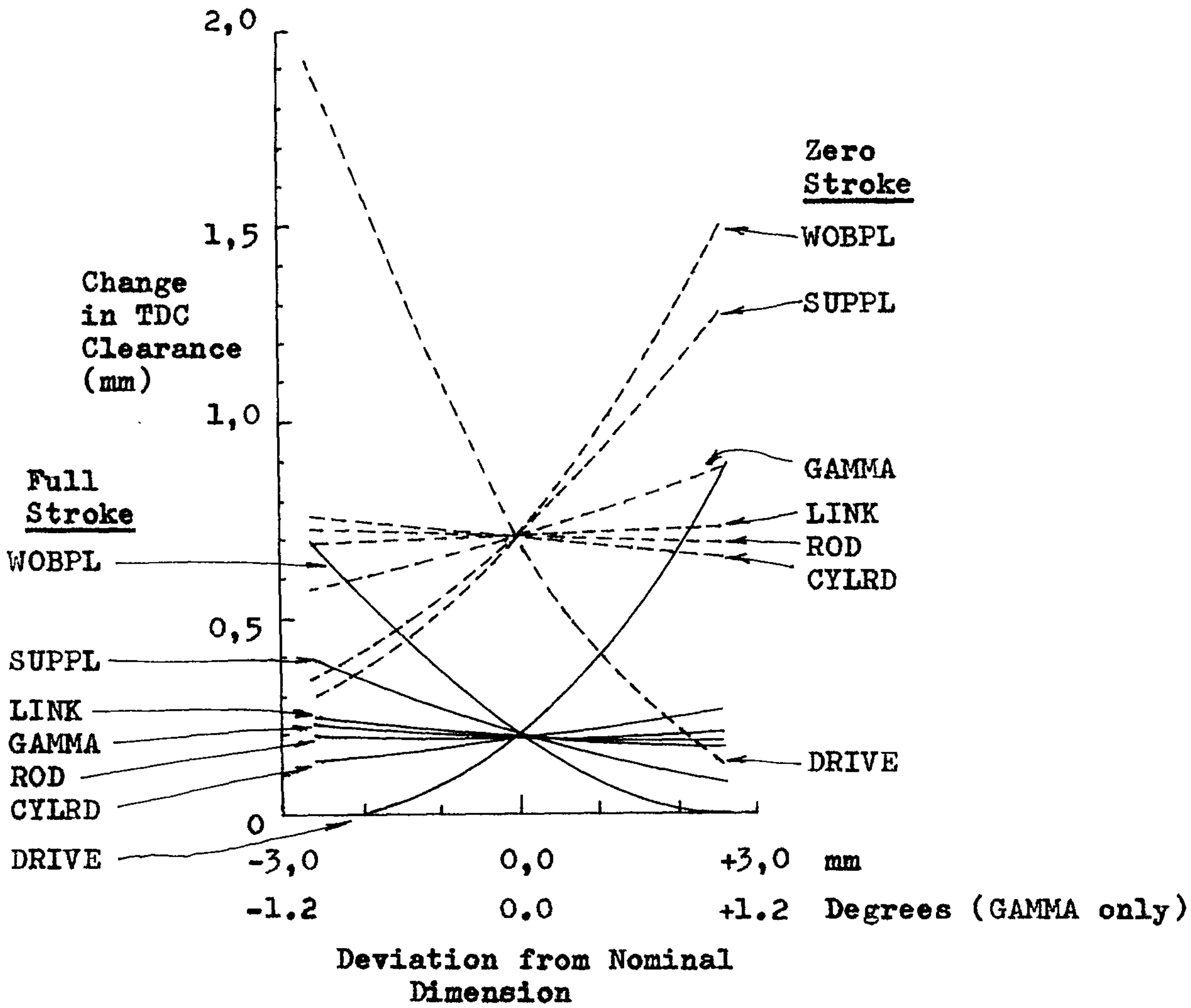

Figure 4
Effects of changes in the Dimensions
of the Members on TDC Clearance

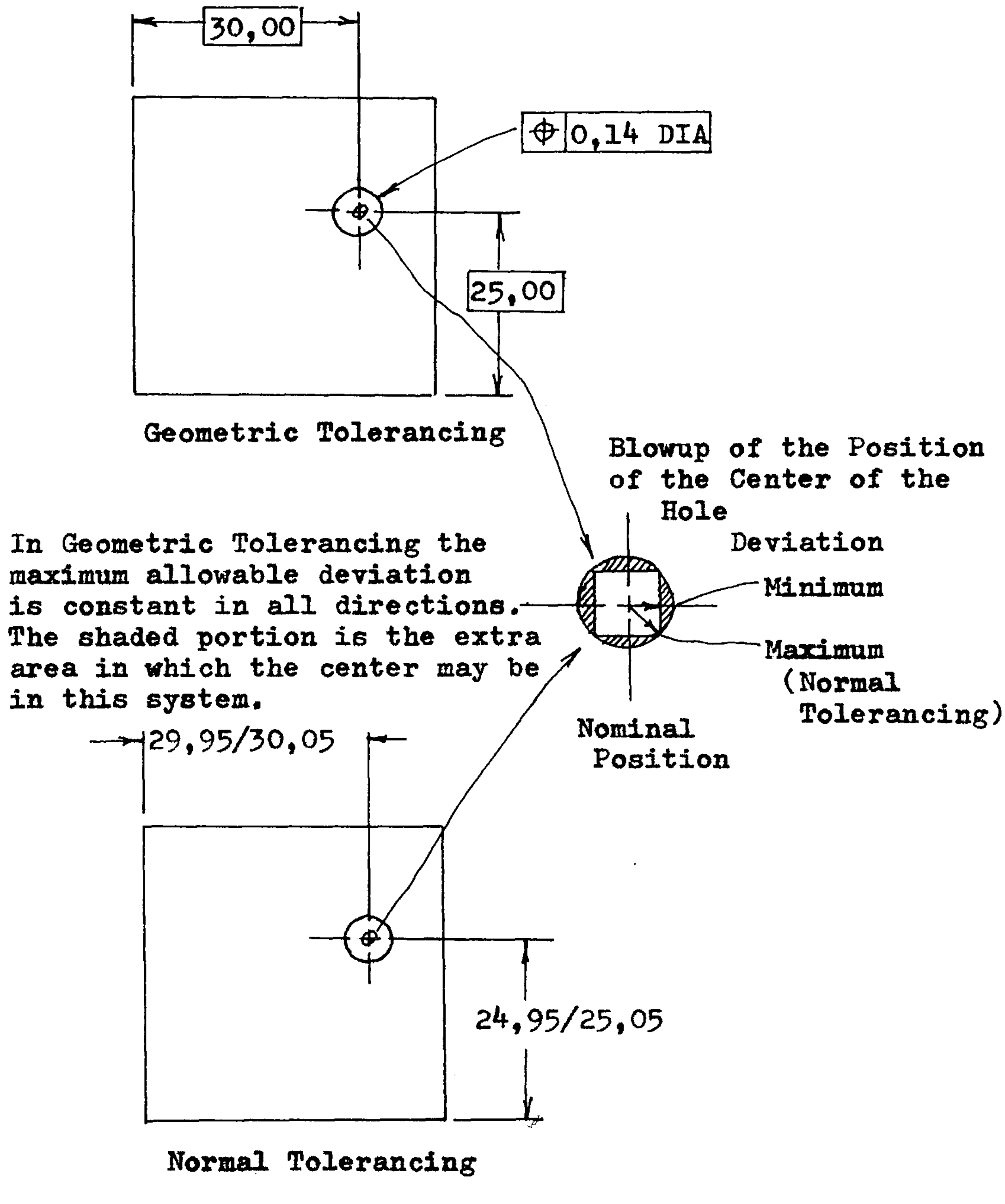

Figure 5
Geometric and Normal Tolerancing

5187-CD5 Compressor
Zero stroke
with 0,380 Gasket (cylinder)
without 0,305 Suction Valve

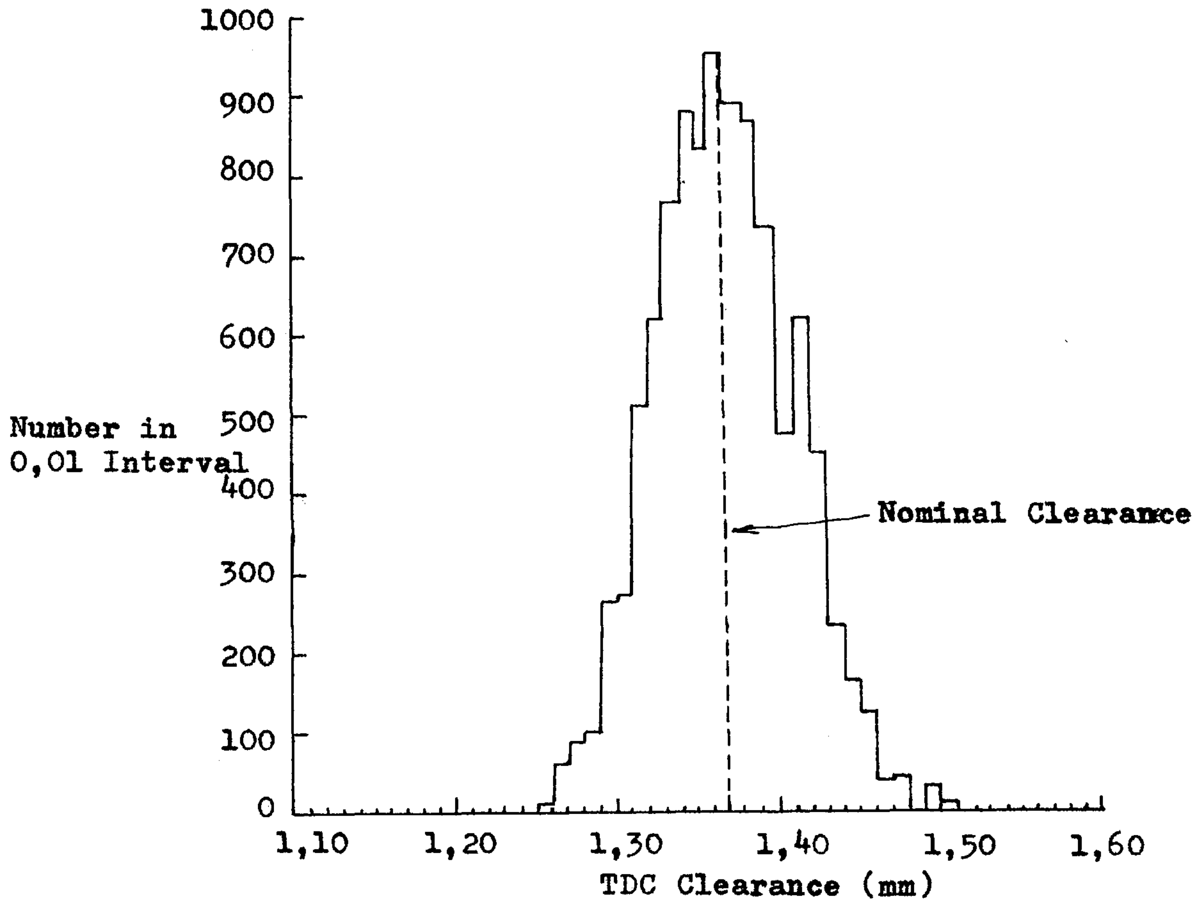

Figure 6
TDC Clearance for Normal Distribution
of Tolerances on the Parts

5187-CD5 Compressor
Zero stroke
with 0,380 Gasket (cylinder)
without 0,305 Suction Valve

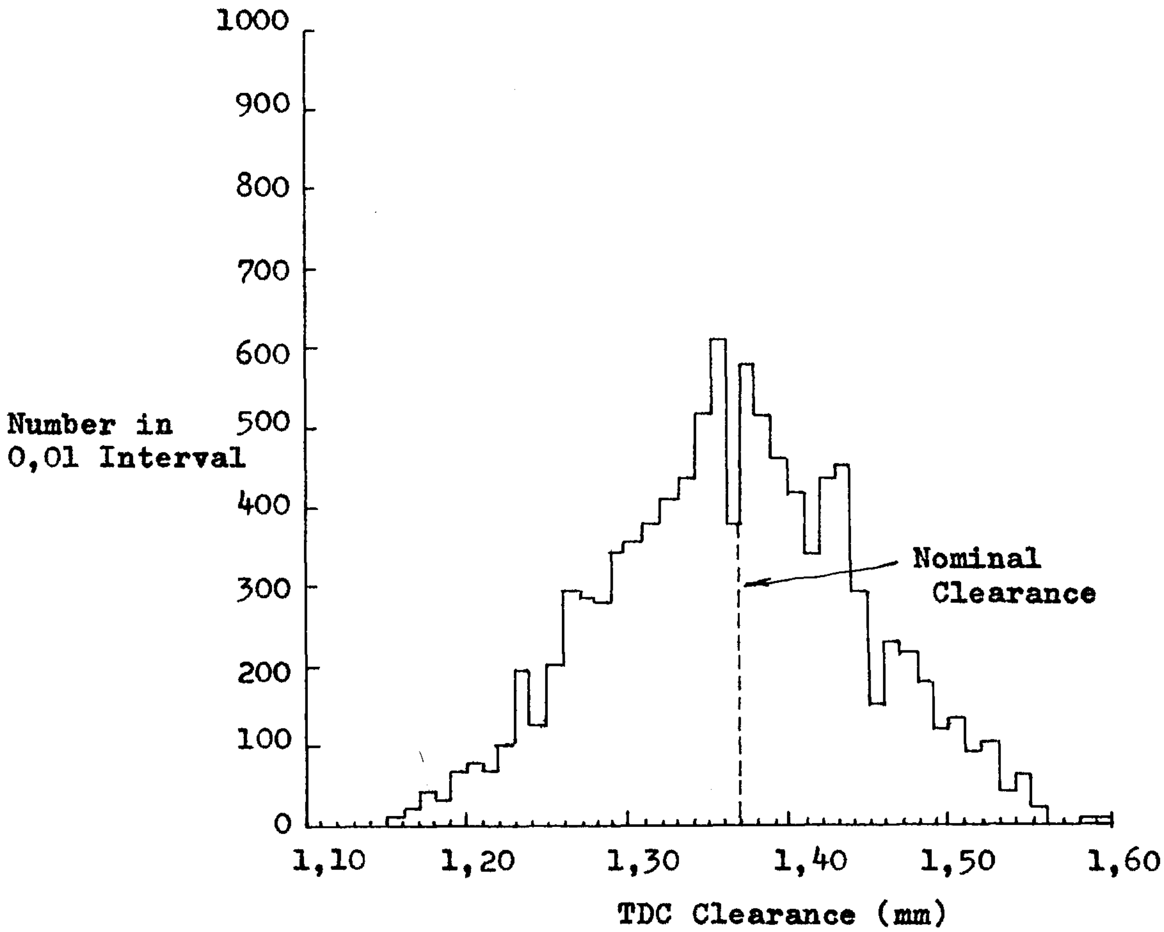

Figure 7
TDC Clearance for Square Distribution
of Tolerances on the Parts

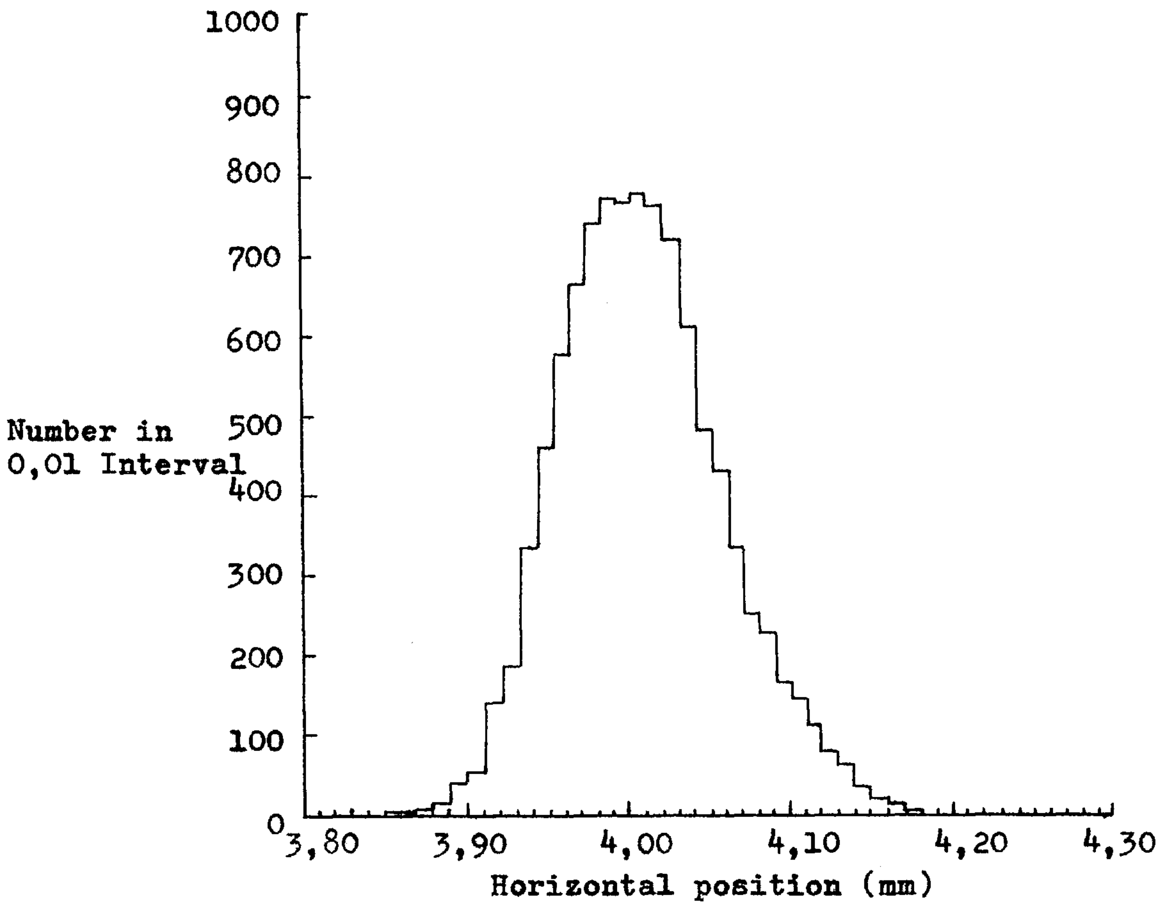

Figure 8
Horizontal Position of the
Wobble Plate Center of Gravity

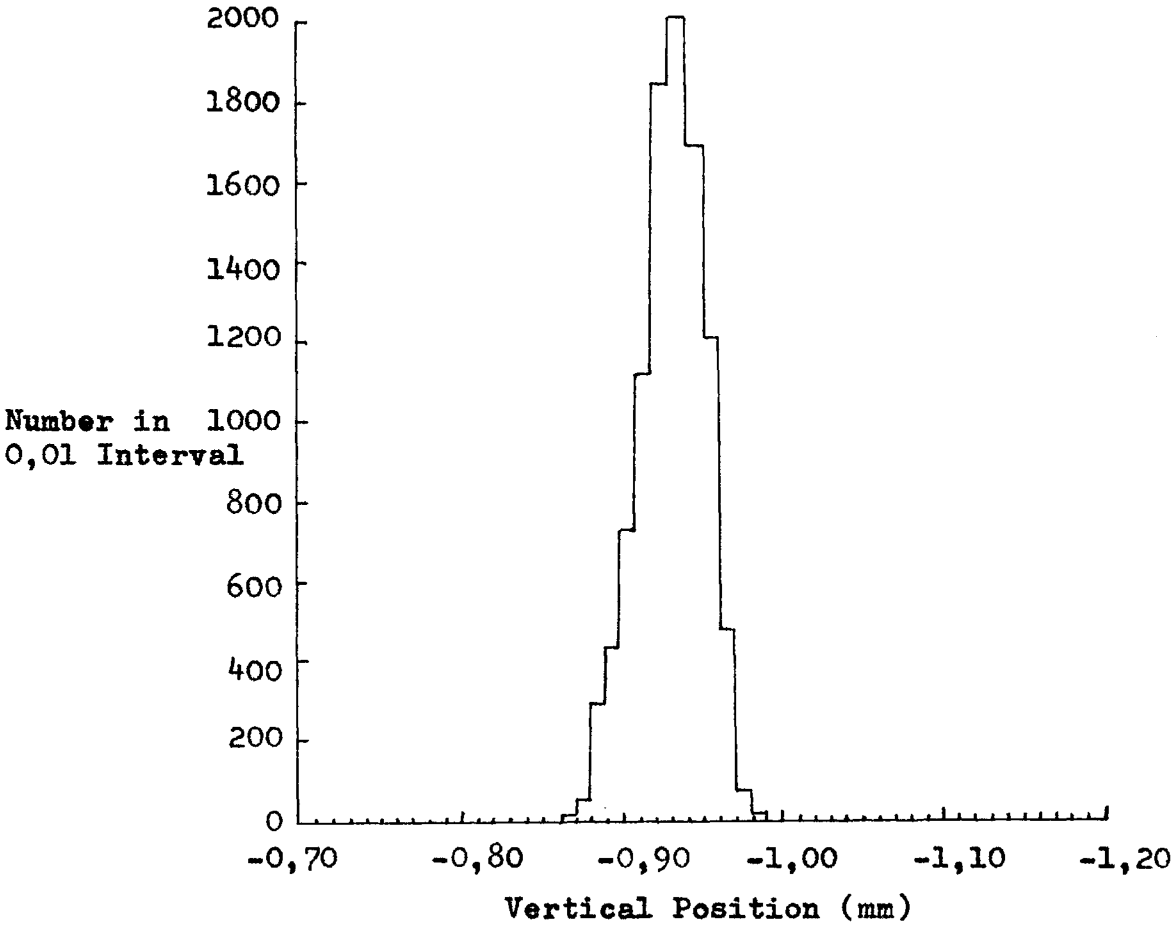

Figure 9
Vertical Position of the Wobble
Plate Center of Gravity

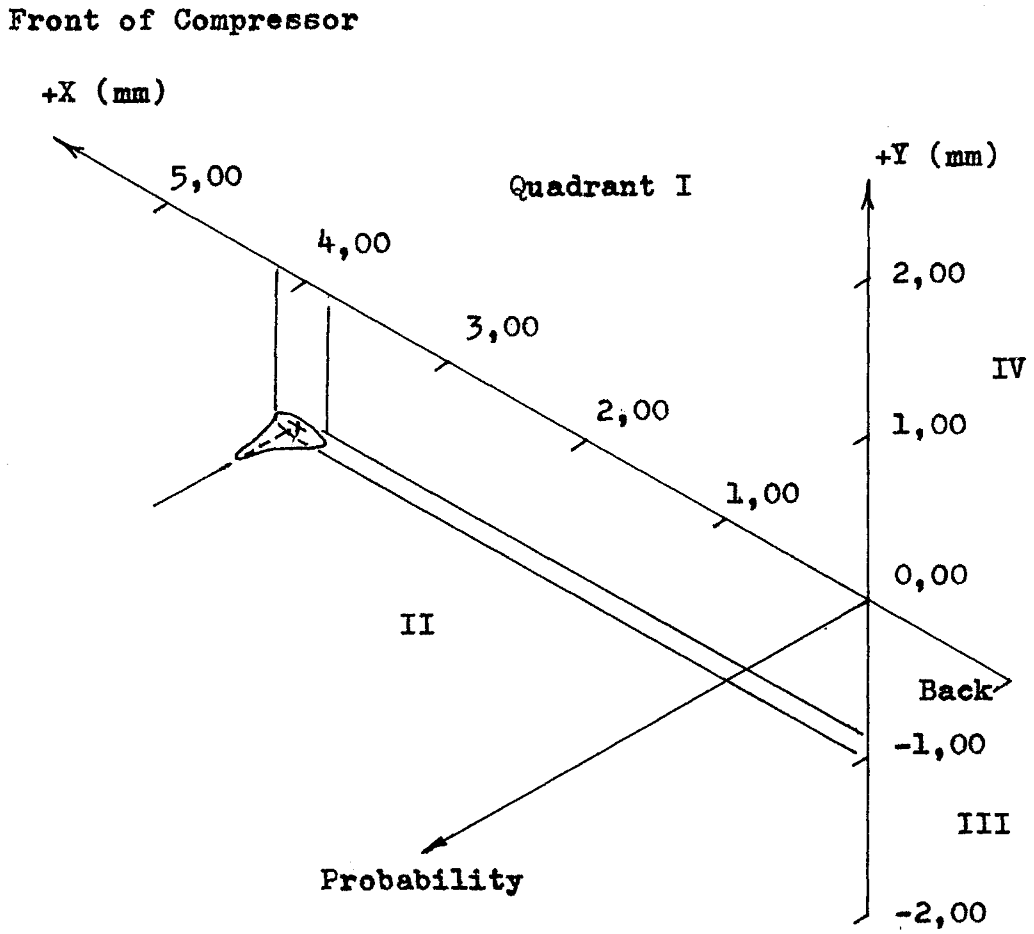

Figure 10
Probable Location of the Wobble
Plate Center of Gravity

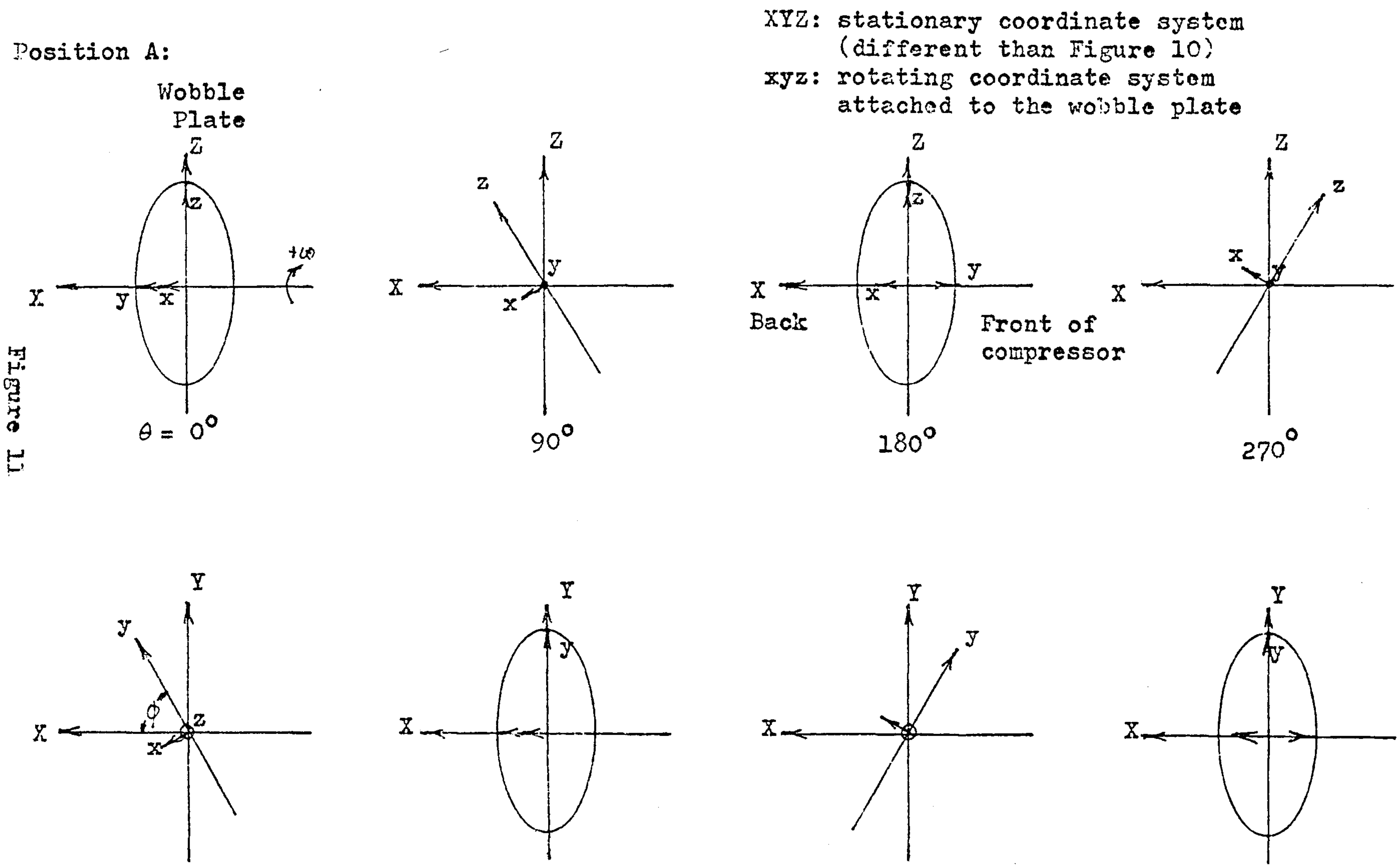

Figure 11
Wobble Plate Motion