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Numerical representation of time overcurrent relay characteristics.

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NUMERICAL REPRESENTATION OF TIME OVERCURRENT RELAY CHARACTERISTICS

by

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A Thesis
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April 30, 1976
(date)

Professor in Charge

Chairman of Department

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ABSTRACT

Electric utilities are turning to the digital computer for use in calculation of protective relay settings. Common to all computer programs which determine settings for overcurrent relays is a scheme to describe the operating characteristics of the relay to the computer. Specifically needed is the time for the relay to operate as a function of time dial setting and relay current. Characteristics are required for each type of relay in use on the power system.

A frequently cited and commonly used method of representing overcurrent relay characteristics was proposed by George Radke of the Philadelphia Electric Company. The "Radke Curve"[1] method represents the highest time dial curve for a relay with an empirical equation. Remaining curves are assumed to have the same shape as the highest time dial curve. Thus, for a constant relay current, operating time at one half of the highest time dial setting is assumed to be one half of that determined for the highest curve.

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[1] Radke, G. E. "A Method for Calculating Time-Overcurrent Relay Settings by Digital Computer", IEEE Transactions on Power Aparatus and Systems, Vol. 82, 1963, Supplement, pp. 189-205.

Significant errors exist in Radke's representation of the highest time dial curve and in the assumption that remaining curves are related by simple ratios. Inaccuracies in the "Radke Curve" method and other published relay equations are a limiting factor in the acceptance of computer relay setting programs by relay engineers. The engineer must do manual checking of the computer analysis to assure that calculations agree with published relay curves.

R. C. Enders suggests that storage of discrete data points and use of a table lookup scheme is the only viable technique among published methods.[1] Storing tables for each relay type has the disadvantage of requiring an excessive amount of computer storage.

A search is undertaken herein for a method of accurately representing relay characteristics with attention to minimizing associated computer storage and time requirements. An empirical model is developed which affords significantly more accurate results than Radke's method.

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[1] Ender, R. C. "The Numerical Representation of Time-Current Characteristics", Paper presented at the 1973 Engineering Computer Forum of the Edison Electric Institute, Philadelphia, September 24-26, 1973.

INTRODUCTION

THE PROTECTIVE RELAY

A protective relay is a device which monitors power system currents or voltages and gives indication of abnormal operating conditions. Protective relays consist of an operating element and a set of contacts. The operating element performs the measuring operation and translates a critical measurement into motion of the contacts. The state of relay contacts (open or closed) governs operation of power system control devices such as circuit breakers.

Operating elements for protective relays are usually of electromagnetic design. These are further categorized as plunger, hinged armature, induction disk, and induction cup types. Solid state units are available with characteristics similar to their electromagnetic counterparts.

Plunger and hinged armature relays are magnetic attraction devices. An armature is attracted into a coil or to the pole face of an electromagnet. The armature carries a moving contact to meet a fixed contact.

Induction disk and induction cup relays develop torque in a movable rotor. The principle of the induction motor is applied to control the moving contact.

THE POWER SYSTEM

Continuity of power system service is critical in maximizing return on capital investment and in maintaining user satisfaction. Protective relays perform a significant role in assuring continuous service. They are the devices which operate to disconnect a faulted portion of the power system. Thus the faulted portion and remainder of the system are protected from further damage.

To adequately monitor a given device in the power system a relay must operate with prescribed sensitivity, selectivity, and speed. The relay must be sufficiently sensitive to operate reliably for the fault condition which produces the minimum operating tendency. It must be able to select among conditions which require immediate operation, time delay operation, or no operation and it must operate with sufficient speed to minimize damage.

RELAY COORDINATION

The selection of relay types for application at each device to be protected and the adjustment of these relays to optimize overall system reliability is termed relay coordination. The adjustments must be applicable to normal system conditions as well as conditions with equipment out of service. The two critical factors to be considered are minimizing the amount of equipment taken out of service to isolate a fault and clearing the fault as quickly as possible.

To minimize equipment taken out of service the circuit breakers selected for tripping should interrupt each path of current to the fault at the breaker nearest to the fault. Hence relays nearest the fault must operate quickly to isolate the fault before remote relays operate.

To provide rapid clearing of faults on transmission lines instantaneous overcurrent relays are applied at each end of the line. These relays are designed to operate with no intentional time delay and ideally they would be adjusted to operate for any fault between the line's breakers. Due to equipment tolerances and the dynamic nature of the power system the protective zone cannot be established with precision. Rather than risk operation of an instantaneous relay for a fault on an adjacent line, the instantaneous relays are set conservatively to cover only a portion of the line. The gaps at the ends of the line are protected by other relaying schemes with a sacrifice in clearing speed.

THE TIME OVERCURRENT RELAY

A time overcurrent relay is designed such that the time for contact closing varies inversely with the current applied. This is the desired characteristic to permit relays nearest to a fault to operate quickly and isolate the fault before remote relays operate. Frequently, time overcurrent relays are used to cover the gaps between protective zones of instantaneous relays.

Time overcurrent relays are usually of the induction disk variety. A permanent magnet induces a drag on the disk to produce a controlled time delay. The initial contact spacing is adjustable and controls the operating time versus applied current characteristic of the relay. This adjustment is known as the time dial setting.

The input coil of an overcurrent relay is tapped at several points to permit adjustment to the range of operating currents expected. Once the tap has been selected, the minimum current which will cause the relay to operate is fixed. Frequently, relay currents are expressed as multiples of this "pickup" value.

The relay engineer is responsible for selecting types of relays and specifying their settings for each location in the power system to be monitored. He has available magnitudes of fault current in the system for faults at various locations as determined by computer study. Relay characteristics are known from manufacturers' published curves.

While overcurrent relays are of relatively simple design and are economical, they are among the most difficult to apply. The objective is to protect that portion of the remote end of a transmission line where faults are not cleared by instantaneous relays. To guarantee coverage of this line section for various system conditions the relay is adjusted for coverage well beyond the end of the line being protected. To preclude breaker operation for faults on adjacent lines all overcurrent relay settings must coordinate so that relays nearest to the fault operate first.

When the task of coordinating overcurrent relays is undertaken manually, typically an iterative approach is used. Initial settings are assumed for each relay. Then the first relay is checked for coordination with each remaining relay, its setting being adjusted, if necessary, to provide that coordination. An iteration is completed when each relay in turn has been selected as the one to adjust. Since changing a relay's setting means that other relays already checked might no longer coordinate, the process must continue until an iteration is completed with no setting changes.

Failure of this scheme to converge indicates that more sophisticated relays, such as those with impedance characteristics, may be needed. Engineering judgment is required to determine if the situation warrants the expense of additional hardware or if some compromise in relay standards can be tolerated.

The trend toward large, interconnected power systems has been accompanied by the acceptance of the digital computer as a powerful tool in the solution of the relay coordination problem. As the size of the power system increases, the task of manually coordinating relays becomes increasingly complex. The computer can perform the iterative coordination process, eliminating much of the tedium from the relay engineer's task.

Whereas the relay engineer had available manufacturers' curves to describe operating characteristics of various relay types, an alternative scheme is required to describe these characteristics to the computer. Specifically needed is the time for the relay to operate as a function of time dial setting and relay current. Characteristics are required for each type of relay in use on the power system.

THE TEMPLATE METHOD

PUBLISHED RELAY CURVES

Manufacturers specify operating characteristics of overcurrent relays by means of a plot of operating time versus multiples of pickup current. The time dial setting is varied in discrete steps to generate a family of curves such as is illustrated in Figure 1 (below). Although typically only ten to twelve of these surves are plotted, an infinite number of operating curves is available from the relay, since the time dial has a continuous means of adjustment.

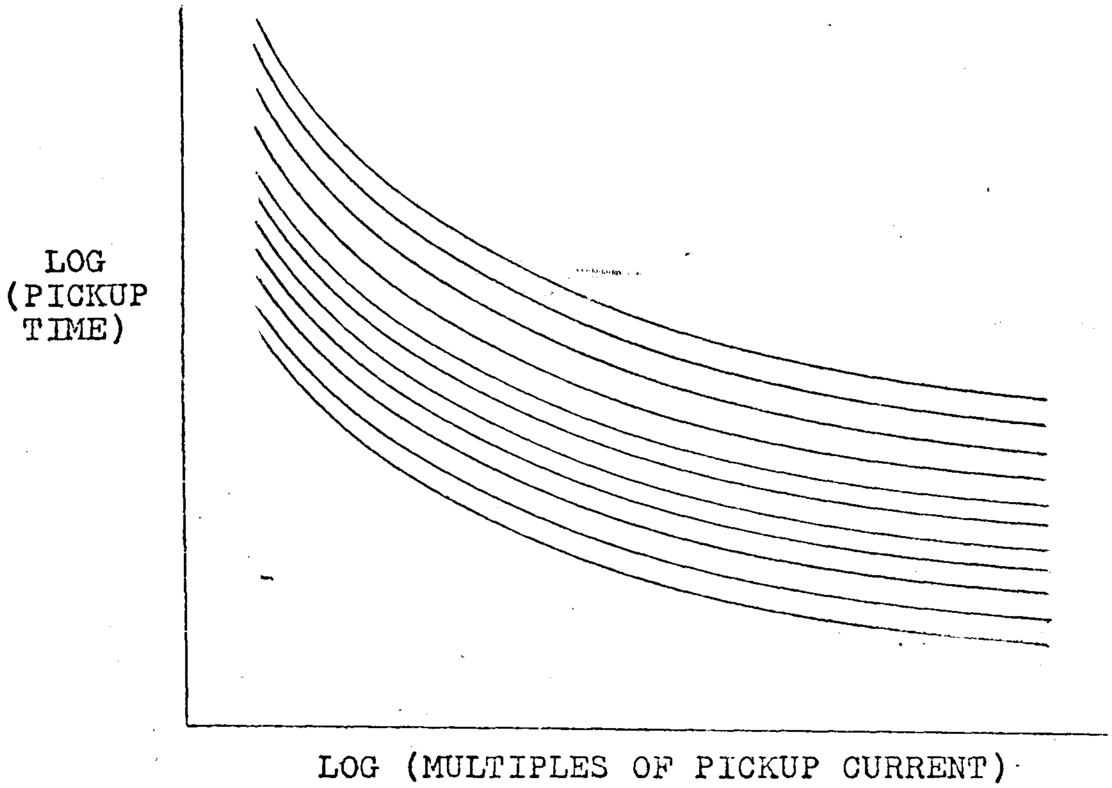

Figure 1. Typical overcurrent relay characteristic.

The relay engineer works under the assumption that the manufacturer's published curves for a given type of relay are the best available data. In practice individual relays of the same type will display slightly different characteristics due to manufacturing tolerances and degradation with time in the field. Measuring and maintaining characteristics for each relay in the power system on an individual basis rather than a type basis would carry a prohibitive cost.

SIMPLIFYING ASSUMPTION

Curves for a given relay type can be assumed to be related to each other by simple ratios of time. Thus, if pickup time for a time dial setting is half that of another time dial setting at a given multiple of pickup current, the pickup time is expected to be half that of the other setting at any constant multiple of pickup current.

If relay curves are plotted with a logarithmic time scale, the assumed relation by simple ratios becomes a relation of constant vertical distance between curves. That is, a change in time dial setting shifts curves in a direction parallel to the time axis. This is a consequence of the mathematical principle that multiplication in the linear domain maps to addition in the logarithmic domain:

$$\log (A \times B) = \log A + \log B$$

The property of the logarithmic time scale of shifting the curves without changing their shape makes possible the use of a transparent plastic template to specify relay characteristics. The template is shaped so as to represent the best average fit to a sample of relay curves for a given relay type. The template is moved vertically on log paper with appropriate dimensions or is used as an aid in interpolating between manufacturers' published curves.

ANALYTIC METHOD

The simplifying assumption which forms the basis for the template method suggests a means of numerically representing relay characteristics. A typical curve can be fitted to a basic equation or represented as a series of discrete points. Then the curve is translated by ratios in the linear domain or by vertical shifts in the logarithmic domain to represent specific relay settings.

MEASURING ACCURACY OF COMPUTED RELAY CURVES

DESIRED ACCURACY

When checking coordination between a relay which is to pickup for a given fault condition and a relay which is not part of the primary protection scheme, the relay engineer wants to include a safety margin in the comparison. His requirement is that the difference between expected time to clear the fault by primary protection and expected operating time for any other relay is large enough to mask any error due to assumptions made in estimating these times. A margin of 0.2 second is a typical requirement.

When the task of coordinating system relays is undertaken manually, manufacturers' published curves are assumed to be the best available data. When the task is modeled on the computer, deviation between the numerical representation of the relay and the published curve is as additional source of error. This error is accomodated by requiring a larger safety margin. Since the requirement of a larger margin tends to increase the number of relays which cannot be coordinated, the incentive to minimize deviation between the numerical representation and the published curves is clear.

A maximum deviation of no more than 0.05 second between a numerical representation and published curve data is desired. Exception to this criterion must be taken for the low current operating region of certain relays which may have operating times as large as 60 seconds. Here the published curve cannot be read to within 0.05 second accuracy. Hence operating points with greater than 0.05 second deviation will be tolerated if the deviation represents less than two percent error.

CURVE FITTING PROGRAM

Fundamentally, the numerical representation of overcurrent relay characteristics is an exercise in curve fitting. A function (f) and a set of parameters (a) are desired, such that f(a,x) will in the sense of least mean squares be a close approximation to a set of experimental data $g(x) \pm e(x)$. For a trial function $f_1(x)$, a measure of "goodness of fit" is the chi-squared statistic:[1]

$$\mathrm{CHI}^2 = \sum_i \left\{ \frac{f_1(a,x_i) - g(x_i)}{e(x_i)} \right\}^2 .$$

A best fit for the function $f_1(x)$ is achieved, by definition, when chi-squared is minimized as a function of the

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[1] Plano, R. Documentation for curve fitting computer program "CHIFIT", Lehigh University Department of Physics Library, p. 6.

parameters (a).

The selection of chi-squared as a statistic for minimization is appropriate to the problem of representing relay characteristics. The normalization of deviation at each data point by the apriori error at that point accomodates the wide range of operating times available with many relays. Data points with large operating times are not weighted more heavily in the analysis than those with small operating times.

Computer curve fitting programs which minimize chi-squared are available from program libraries. A general purpose curve fitting program[1] written in FORTRAN for the CDC 6400 computer at Lehigh University was selected. Modifications have been made to address the specific problem of representing relay characteristics and to run the program on IBM 360 equipment.

The algorithm for chi-squared minimization involves perturbations of trial values for parameters and is applicable to any assumed functional form. Evaluation of the assumed function at small perturbations from current values of parameters permits evaluation of gradients which specify appropriate corrections to improve upon these

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[1] Klenk, Kenneth. Curve fitting computer program "MINNY" and associated documentation, Lehigh University Department of Physics Library.

values. The iterative process is continued until corrections to the current values of parameters become negligible.

RELAY CURVE DATA

Operating characteristics for thirteen relay types were obtained and transcribed to machine readable form. Fifteen data points were taken from each of ten curves for each relay type. These data serve as input to the curve fitting program and as a standard for evaluating accuracy of computed relay characteristics.

Effort was made to read data points from the published curves to three significant figures whenever possible. The curves are scaled such that an error of two percent might be expected in the process of transcribing a selected data point.

SELECTION CRITERIA

Primarily, the selection of a functional form for representation of relay characteristics is to be based on "goodness of fit" to published relay curves. Attention must also be given to suitability of a proposed function to computer application.

Although modern computer systems afford considerably more high speed memory capacity than their predecessors, storage requirement remains an important consideration.

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Were storage not a problem, retaining a large number of data points for each relay type and interpolating among them would be an acceptable solution to the relay representation problem. With consideration to the existence of about one hundred overcurrent relay types in general use, an upper limit of about twenty parameters will be tolerated for fitting each relay type to the assumed function.

A second computer resource to be conserved is processing time. It is desirable that parameters for all relay types be resident in high speed memory during solution of the relay coordination problem. Use of transcendental functions is to be minimized.

DESCRIPTIVE EXAMPLE

The curve fitting program was set up to fit each of ten curves for which data are available for a given relay type to a fourth degree polynomial:

$$T = a_0 + a_1 M + a_2 M^2 + a_3 M^3 + a_4 M^4$$

The independent variable "M" represents multiples of pickup current. "T" is the corresponding time in cycles (on sixty hertz base) for the relay to operate. An apriori error of two percent is assumed for each data point.

Column I of Table 1 (page 17) gives chi-squared, as minimized by the curve fitting program, summed over the

150 sample points for each relay type. These statistics correspond to absolute deviations as large as 60 seconds (90% error).

Fifty parameters are used here for each relay type (five for each of ten time dial settings). In practice, relay engineers specify a given relay curve by its pickup time at a certain multiple of pickup current (typically

TABLE 1

RESULTS OF DESCRIPTIVE EXAMPLE

CHI-SQUARED FOR 150 SAMPLE POINTS

Relay Type	I Linear Fit To Each of 10 Curves	II Log Fit To Each of 10 Curves	III Log Fit To Ratios of Average Curve
CO-2	15098.	78.88	1980.
CO-5	11589.	126.5	729.1
CO-6	7620.	113.8	663.7
CO-7	11788.	128.5	862.5
CO-8	36337.	105.8	496.6
CO-9	38669.	16.44	593.6
CO-10	48949.	119.9	9366.
CO-11	56507.	238.1	5062.
IAC-11	8836.	94.26	555.6
IAC-15	36175.	144.3	3606.
IAC-51	9053.	122.2	1292.
IAC-53	34293.	113.3	6411.
IAC-77	57778.	316.8	8374.
Totals	372692.	1719.	39992.

ten times pickup) and restrict use of curves to those a certain incremental distance apart (typically 0.05 second). About fifty curves are of interest (0.05 to 2.50 in increments of 0.05 second at ten times pickup) so 250 parameters are required to individually represent curves by fourth degree polynomials.

In an attempt to find a better fit for the individual curves, the program was modified to map all data through logarithms before fitting to a fourth degree polynomial:

$$\log T = a_0 + a_1 (\log M) + a_2 (\log M)^2 + a_3 (\log M)^3 + a_4 (\log M)^4$$

Column II of Table 1 (page 17) shows the effect of this mapping. Chi-squared is reduced by about two orders of magnitude for each relay type. The maximum deviation is reduced to 1.0 second (2% error).

Next the program was modified to apply the template method to reduce the number of parameters required for each relay type. All data are mapped through logarithms and lower curves are shifted vertically to coincide with the top curve at ten multiples of pickup. Then a single fourth degree polynomial is fitted to these resulting data.

The rules of the template method are used to reconstruct ten curves having the same pickup time at ten multiples of pickup current as the original data. Column III of Table 1 (page 17) gives chi-squared for the reconstructed curves. A comparison of column II and column III shows that considerable error is introduced through use of the template method.

THE "RADKE CURVE" METHOD

RADKE'S EQUATION

A frequently cited and commonly used method of representing overcurrent relay characteristics was proposed by George Radke of the Philadelphia Electric Company. The "Radke Curve" method represents the highest time dial curve for a relay by the following equation:[1]

$$\log (T - DC) = a_0 + a_1 (\log M) + a_2 (\log M)^2 + a_3 (\log M)^3 + a_4 (\log M)^4$$

"M" represents multiples of pickup current and "T" is the corresponding time in cycles for the relay to operate. "DC" is a constant for each relay type and is referred to as the "delay time" of the relay.

Parameters "a_1" through "a_4" are assumed to be constant for a given relay type. A family of curves is generated by varying "a_0." This variation causes a shift in plots of "log T" versus "log M" similar to that of the template method. In this case the shift is not strictly vertical due to the effect of delay time in Radke's equation.

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[1] Radke, G. E., et al. "A Computer Approach to Setting Overcurrent Relays in a Network", Conference Record of the 1967 Power Industry Computer Applications Conference, May 1967, p. 451.

DELAY TIME

Radke's primary contribution to numerical representation of relay characteristics is his concept of delay time. He describes the concept as follows:[1]

> "When developing either a template or one equation to represent a particular time-overcurrent relay characteristic, the procedure includes testing several characteristic curves for possible use as a template. When a high time dial curve is used, it diverges from the lower time dial curve in a particular manner. If the template and a curve are made to coincide at the low current end, the template falls below the curve with separation increasing toward the high current end. This is the basis for considering the relay operating time as the sum of two distinct time periods, the first in which the relay induction disc accelerates to the steady angular velocity for a particular current, and the second period during which the steady state velocity is maintained until contact closing time. The latter time period coincides with the template or ideal value, while the first, called delay time, accounts for the difference between actual and ideal curves . . ."

The delay time concept is illustrated by example in Table 2 (page 22). Column I shows ratios of pickup time for bottom and top curves of an IAC-51 relay at selected multiples of pickup. The ratios increase monotonically with increasing multiples of pickup. If a constant delay time of 0.08 second is subtracted from the time values before taking ratios (column II), the variation in ratios

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[1] Radke, G. E. "A Method for Calculating Time-Overcurrent Relay Settings by Digital Computer", <u>IEEE Transactions on Power Aparatus and Systems</u>, Vol. 82, 1963, Supplement, p. 193.

TABLE 2

EFFECT OF DELAY TIME ON IAC-51 RELAY

RATIOS OF TIME (SECONDS)

Bottom Curve versus Top Curve

Multiples of Pickup	I Without Delay Time	II With Delay Time
1.5	$\frac{.640}{14.6} = .044$	$\frac{.640 - .08}{14.6 - .08} = .039$
5.0	$\frac{.258}{4.10} = .063$	$\frac{.258 - .08}{4.10 - .08} = .044$
10.0	$\frac{.196}{2.92} = .067$	$\frac{.196 - .08}{2.92 - .08} = .042$
30.0	$\frac{.146}{1.84} = .079$	$\frac{.146 - .08}{1.84 - .08} = .038$
50.0	$\frac{.139}{1.58} = .088$	$\frac{.139 - .08}{1.58 - .08} = .039$

is significantly less.

The effect of delay time for the IAC-51 relay is pronounced and it is used as an example for this reason. The effect is not as significant in many other relay types.

COEFFICIENT EVALUATION

The procedure outlined by Radke for evaluating the constants in his equation is in two steps.[1] The first

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[1] Radke, G. E., et al. "Digital Computer Programs For Determining Time-Overcurrent Relay and Fuse Equation Constants -- User's Guide", Philadelphia Electric Company, pp. A1, B1.

step is a "trial and error" approach to determine an acceptable value of delay time. Then a curve fitting algorithm is used to evaluate the "a_i." Both steps are designed for computer implementation.

The search for an acceptable value of delay time considers values between zero and 30 cycles in increments of 0.05 cycle. Ratios are computed between times for the top and bottom curves and between times for the top and an intermediate curve at two multiples of pickup. Before taking ratios, the present delay time being tested is subtracted from actual times.

The testing process is carried out at 6, 12, and 20 multiples of pickup for the bottom and intermediate curves. Each of the six sample data points is checked by subtracting the delay time being tested from the time value for the top curve at the appropriate multiple of pickup. Then the appropriate ratio is applied, the delay constant added back, and the error calculated.

A test value of delay time is deemed acceptable if each of the six computed times corresponds to an error of less than three cycles or five percent. The average between the smallest and largest acceptable delay times found is used as the final delay constant.

The five coefficients in Radke's equation are determined with a polynomial least squares fitting program.

Input data are seventy points transcribed from the manufacturer's plot for the highest time dial curve. This curve is considered the truest representation of the relay characteristics, since the delay time is negligible compared to the relatively long times associated with it.

Radke published values of delay time and coefficients for various relay types based on this procedure. Column I of Table 3 (page 25) shows error statistics evaluated in comparing data transcribed from manufacturers' curves and values computed using Radke's equation and published coefficients.

COEFFICIENT OPTIMIZATION

A comparison of the chi-squared values in column I of Table 3 with those from the template method (column III of Table 1, page 17) reveals that for most relay types a better fit is obtained using the template method. Since Radke's method differs from the template example only in consideration of delay time, results with Radke's equation should be at least as good. Radke's equation is reduced to that of the template method by setting the delay time to zero.

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[1] Radke, G. E. "A Method for Calculating Time-Overcurrent Relay Settings by Digital Computer", IEEE Transactions on Power Aparatus and Systems, Vol. 82, 1963, Supplement, p. 193.

TABLE 3

ERROR STATISTICS FOR "RADKE CURVE" METHOD

ERROR STATISTICS BASED ON 150 SAMPLE POINTS

	I			II		
	Radke's Coefficients			Optimized Coefficients		
Relay Type	Maximum Deviation (sec)	Average Deviation (sec)	CHI^2 Error	Maximum Deviation (sec)	Average Deviation (sec)	CHI^2 Error
CO-2	.3412	.02077	2465.	.8147	.02298	701.8
CO-5	---	---	---	.1813	.008950	710.0
CO-6	.9929	.08236	6997.	.6106	.04180	433.1
CO-7	3.469	.1202	1445.	1.058	.04720	358.4
CO-8	8.899	.1502	687.7	10.62	.1595	448.6
CO-9	2.977	.1247	735.2	1.435	.05356	317.9
CO-10	---	---	---	6.283	.1972	3904.
CO-11	4.454	.1574	2642.	6.954	.2711	2435.
IAC-11	2.365	.08823	2434.	1.868	.06204	522.4
IAC-15	---	---	---	5.176	.1523	2080.
IAC-51	1.039	.09736	1452.	1.561	.05610	400.4
IAC-53	.8966	.07450	3097.	1.420	.07119	519.0
IAC-77	3.302	.1831	4512.	2.726	.1349	3734.
					Total	16565.

Column II of Table 3 shows the results obtained with Radke's equation when the curve fitting program is used to optimize delay time and coefficients over the entire family of curves. The procedure is similar to that used in evaluating coefficients for the template method with the exception that delay time is included as a sixth parameter. Delay time and coefficients are optimized in a single step, as contrasted to the two step procedure used by Radke. Data for the entire family of curves are used in fitting, whereas Radke used only the highest time dial curve.

A comparison of the chi-squared values in column II of Table 3 (page 25) with those from the template method (column III of Table 1, page 17) shows the inclusion of delay time does improve the fit for all relay types considered. Chi-squared is reduced by an order of magnitude for the IAC-53 relay.

The error statistics in Table 3 show that Radke's method does not meet the 0.05 second criterion for an upper limit on deviation from manufacturers' data. A deviation over 10 seconds (17% error) arises for the CO-8 relay when coefficients are optimized for minimum chi-squared error.

VARIABLE TIME CORRECTION

VARIABLE DELAY TIME

Radke's concept of delay time as an enhancement to the template method has been shown to improve computed representations of relay characteristics significantly. However, a close examination of relay curve data reveals that his use of a single delay constant for each relay type warrants investigation. A. J. McConnell of the General Electric Company suggests:[1]

> "Delay time is actually a variable, depending principally upon the damping constant of the damping magnet which varies, in most induction-disk relays, with time-dial setting. However, the variation of the damping constant is small, and at the lower time-dial settings where the delay time matters most, no appreciable error would be introduced by considering it to be a constant."

DISCRETE TIME CORRECTIONS

An attempt was made to identify variations in delay time with time dial setting and to determine if the accuracy of Radke's model can be improved by treating delay time as a variable. The curve fitting program was modified to determine the optimum delay time associated with

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[1] McConnell, A. J. Discussion appended to G. E. Radke's paper "A Method for Calculating Time-Overcurrent Relay Settings by Digital Computer", <u>IEEE Transactions on Power Aparatus and Systems</u>, Vol. 82, 1963, p. 303.

each of ten curves for given relay types. The family of curves was modeled using ratios to the highest time dial curve, with corrections subtracted from time values before taking ratios, using the following equation:

$$T_{cm} = \left\{ \frac{T_{c10} - TC_c}{T_{t10} - TC_t} \quad x \quad (T_{tm} - TC_t) \right\} + TC_c$$

Where: "T_{cm}" is the operating time for an arbitrary curve "c" at "m" multiples of pickup current

"T_{c10}" is the operating time for curve "c" at ten multiples of pickup current

"T_{t10}" is the operating time for the top curve at ten multiples of pickup current

"T_{tm}" is the operating time for the top curve at "m" multiples of pickup current

"TC_c" is the time correction for curve "c"

"TC_t" is the time correction for the top curve

The program uses the data points transcribed from manufacturers' curves for the "T" and computes optimum values of "TC" for chi-squared minimization. The ten discrete values of time correction are optimized simultaneously so that the value corresponding to the highest time dial curve is not biased to favor any of the remaining nine curves.

Optimum values for "TC" are found to be highly correlated to time dial setting. The values are highest for the top curve and decrease monotonically with decreasing time dial setting. An order of magnitude variation is typical.

MODES OF SOLUTION

Starting values for parameters, step sizes for perturbations, and acceleration factors must be specified in each use of the curve fitting program. In tuning the program to achieve proper convergence for analysis of variable delay time, an alternate solution mode was discovered. While convergence could be obtained with a decreasing series of positive time delays, a good fit could also be achieved with a series of negative numbers.

Column I of Table 4 (page 30) lists optimum time corrections for the CO-2 relay. This solution is consistent with the interpretations of Radke and McConnell regarding delay time as a positive value to be subtracted from an aggregate time before manipulating curves. The wide variation in time correction is a departure from their theories.

Column II of Table 4 (page 30) illustrates the alternate solution mode. For the CO-8 relay the best fit is achieved with negative time corrections. This is a total departure from the physical interpretation of subtractive delay times, since time values are added to data before

TABLE 4

OPTIMUM TIME CORRECTIONS FOR THE CO-2 AND CO-8 RELAYS

DISCRETE TIME CORRECTIONS (SECONDS)

I CO-2		II CO-8	
CURVE (time @ 10xPU)	TIME CORRECTION	CURVE (time @ 10xPU)	TIME CORRECTION
.510	.302	2.87	-1.19
.413	.251	2.23	-.734
.322	.200	1.67	-.517
.287	.182	1.43	-.451
.237	.154	1.17	-.364
.203	.136	.960	-.271
.149	.101	.705	-.187
.106	.0735	.448	-.119
.0617	.0446	.218	-.0588
.0385	.0303	.0865	-.0178

curves are manipulated.

The effect of optimized time corrections on ratios of time for bottom and top curves of the CO-8 relay is shown in Table 5 (page 31). Time values in column I are from the manufacturer's published curve. The resulting uncorrected ratios show considerable variation. The variation is significantly reduced when the additive corrections (taken from Table 4, column II) are applied as in column II.

TABLE 5

EFFECT OF ADDITIVE TIME CORRECTIONS ON THE CO-8 RELAY

RATIOS OF TIME (SECONDS)

Bottom Curve versus Top Curve

Multiples of Pickup	I Uncorrected	II Corrected
1.5	$\frac{1.65}{74.0} = .022$	$\frac{1.65 + .0178}{74.0 + 1.19} = .022$
5.0	$\frac{.156}{4.92} = .032$	$\frac{.156 + .0178}{4.92 + 1.19} = .028$
10.0	$\frac{.0865}{2.87} = .030$	$\frac{.0865 + .0178}{2.87 + 1.19} = .026$
30.0	$\frac{.0618}{1.86} = .033$	$\frac{.0618 + .0178}{1.86 + 1.19} = .026$
50.0	$\frac{.0584}{1.70} = .034$	$\frac{.0584 + .0178}{1.70 + 1.19} = .026$

COEFFICIENT OPTIMIZATION

The curve fitting program in the form used with Radke's model was modified to incorporate variable time correction in place of the delay constant. Five coefficients for the fourth degree polynomial and time corrections for each of ten curves are optimized simultaneously. Error statistics corresponding to this model are given in column I of Table 6 (page 32). The total of chi-squared values for the thirteen relay types is 11% lower than that obtained with strict application of Radke's equation (14768. versus 16565. per Table 3, page 25).

TABLE 6

ERROR STATISTICS FOR MODEL WITH VARIABLE TIME CORRECTION

ERROR STATISTICS BASED ON 150 SAMPLE POINTS

	I			II		
	Discrete Corrections			Computed Corrections		
Relay Type	Maximum Deviation (sec)	Average Deviation (sec)	CHI^2 Error	Maximum Deviation (sec)	Average Deviation (sec)	CHI^2 Error
CO-2	.6188	.01939	642.9	.6247	.01947	644.8
CO-5	.09713	.006500	448.5	.09072	.007330	488.4
CO-6	.3933	.04143	389.3	.4960	.04100	415.3
CO-7	.7760	.04014	290.3	.7976	.04124	300.8
CO-8	6.917	.1129	388.6	7.137	.1139	396.2
CO-9	1.398	.04410	274.2	1.427	.04433	277.8
CO-10	5.440	.1879	3755.	5.530	.1869	3834.
CO-11	5.480	.2201	2073.	5.843	.2313	2132.
IAC-11	1.698	.05711	446.4	1.782	.05914	473.0
IAC-15	6.153	.1490	1901.	6.188	.1514	1921.
IAC-51	1.589	.05493	330.2	1.649	.05534	355.2
IAC-53	.9768	.05003	398.4	1.473	.06197	446.8
IAC-77	2.524	.1383	3430.	2.474	.1395	3445.
		Total	14768.		Total	15130.

CONTINUOUS TIME CORRECTION

Since use of discrete time corrections for each relay curve to be considered is undesirable in terms of storage requirement, an attempt was made to compute these corrections as a function of time dial setting. A fourth degree polynomial was selected for this mapping. The revised model uses a pair of equations to describe the entire family of curves for a given relay type as follows:

$$TC = b_0 + b_1 T_{10} + b_2 T_{10}^2 + b_3 T_{10}^3 + b_4 T_{10}^4 \qquad (1)$$

$$\log (T - TC) = a_0 + a_1 (\log M) + a_2 (\log M)^2 + a_3 (\log M)^3 + a_4 (\log M)^4 \qquad (2)$$

Where: "TC" is the time correction in cycles for the curve to be represented

"TC_{10}" is the pickup time in cycles at ten multiples of pickup current for the curve to be represented

"T" is the pickup time in cycles at "M" multiples of pickup current for the curve to be represented

Use of the equations is according to the following procedure:

1) Coefficients b_0, b_1, b_2, b_3, b_4, a_1, a_2, a_3, and a_4 are constant for the given type of relay
2) "T_{10}" is given as the description of the curve to be represented
3) Evaluate "TC" using equation (1)
4) Compute a_0 for the curve using data at ten multiples of pickup where:

$$\log M = \log 10 = 1 \quad \text{and} \quad T = T_{10}$$

So equation (2) reduces to:

$$a_0 = \log (T_{10} - TC) - (a_1 + a_2 + a_3 + a_4)$$

5) Use established values to compute "T" versus "M" for the curve:

$$T = \text{antilog}\left[a_0 + a_1 (\log M) + a_2 (\log M)^2 + a_3 (\log M)^3 + a_4 (\log M)^4\right] + TC$$

The curve fitting program was used to optimize coefficients for both polynomials simultaneously. Column II of Table 6 (page 32) shows error statistics which resulted. A comparison with column I shows that the degradation associated with computing time corrections rather than storing discrete values is not major. The total of chi-squared values for the thirteen relay types is 9% lower than that obtained with strict application of Radke's equation (15130. versus 16565. per Table 3, page 25).

WESTINGHOUSE METHOD

PROTECTIVE DEVICE COORDINATION PROGRAM

Among the services which the Westinghouse Electric Corporation offers to power utilities is that of performing engineering studies using various computer programs. The Protective Device Coordination Program[1] is one of these and is designed to perform the routine aspects of the relay coordination process.

Since the Westinghouse program is proprietary, details of its design are unavailable. The equation for overcurrent relay representation has been published, but the parameters are not completely described.[2] This equation is:

$$T = \left\{ \sum_{j=m_1}^{m_2} \sum_{i=n_1}^{n_2} a_{ji} \; C^j \; M^i \right\}^k$$

Where: "T" is the operating time

"C" is the time dial setting for the desired curve

"M" is multiples of pickup current

The single equation with two independent variables is designed to represent the entire family of curves.

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[1] Albrecht, R. E., et al. "Digital Computer Protective Device Co-ordination Program: I--General Program Description", IEEE Transactions on Power Aparatus and Systems, Vol. 83 (April 1964), p. 402.

[2] Ibid., p. 404.

DECOMPOSITION

When the Westinghouse equation is expanded from the shorthand notation of two nested summations into a series, it becomes apparent that decomposition into polynomials with one independent variable is possible. Secondary polynomials are themselves the coefficients of a primary polynomial:

Let: $k=1$, $m_1=0$, and $n_1=0$

Then: $T = b_0 + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 \quad . . .$

Where: $b_0 = c_0 + c_1 C + c_2 C^2 + c_3 C^3 + c_4 C^4 \quad . . .$

$b_1 = d_0 + d_1 C + d_2 C^2 + d_3 C^3 + d_4 C^4 \quad . . .$

$b_2 = e_0 + e_1 C + e_2 C^2 + e_3 C^3 + e_4 C^4 \quad . . .$

$b_3 = f_0 + f_1 C + f_2 C^2 + f_3 C^3 + f_4 C^4 \quad . . .$

$b_4 = g_0 + g_1 C + g_2 C^2 + g_3 C^3 + g_4 C^4 \quad . . .$

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It is known from the template method example (pages 16 to 19) that a good fit to a single relay curve can be achieved using a fourth degree polynomial and data transformed by logarithms. However, the optimum coefficients for the various curves in the family are not smooth functions and are not readily fitted to polynomials.

It was determined experimentally that by eliminating the zero subscripted term from the fourth degree polynomial, optimum values for the four remaining coefficients had magnitudes monotonically decreasing with decreasing time dial setting. Loss of accuracy due to elimination of the zero subscripted term was partially compensated by adding fifth and sixth degree terms without affecting the smooth transition of coefficients among curves.

OPTIMIZATION

The curve fitting program was modified to investigate the following variation of the Westinghouse equation:

$$\log T = \sum_{j=1}^{5} \sum_{i=1}^{6} a_{ji} \; C^{j} \; (\log M)^{i}$$

Since the simultaneous optimization of the thirty coefficients requires considerable computer time, only the CO-2 relay was examined. Results based on 150 sample points are as follow:

Maximum deviation -- 1.871 seconds (45% error)
Average deviation -- .07208 second
Chi-squared error -- 6394.

The chi-squared statistic is an order of magnitude larger than that obtained with Radke's method for the CO-2 relay (701.8 per Table 3, page 25).

Investigation of the Westinghouse method was terminated due to unimpressive results obtained in this initial examination and due to lack of available information as to ranges of the parameters m_1, m_2, n_1, n_2, and k used in this method. It appears that the number of coefficients required to achieve satisfactory accuracy would be very large.

HIEBER'S METHOD

HIEBER'S EQUATIONS

A method for overcurrent relay representation has been proposed by John Hieber of the Ohio Edison Company. Hieber observed that variation in the slope of curves for a given relay type is continuous from the lowest to highest time dial settings. The general curve is taken to be a weighted average of the bottom and top curves.

Hieber's method combines functions for bottom and top curves and a function for transforming time dial settings to obtain operating time in terms of multiples of pickup current and time dial setting. Intermediate curves are evaluated by non-linear interpolation between the bottom and top curves. The equations are:[1]

General Curve:

$$T(M,C') = T_t(M) \left[1 - F(C')\right] + T_b(M) \left[F(C')\right]$$

Top Curve:

$$T_t(M) = c \,/\, (M - h + wM^{-2M})^m + k - b\,(M \,/\, 50)^u$$

Bottom Curve:

$$T_b(M) = d \,/\, (M - h + wM^{-2M})^n + f - g\,(M \,/\, 50)^v$$

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[1] Hieber, J. E. "Empirical Equations of Overcurrent Relay Curves for Computer Application", IEEE Conference Paper 31-CP-65-91, 1965, p. 2.

Interpolation Function:

$$F(C') = (C_t - C') / (C_t - C_b)$$

$$C' = C - q\,(C_t - C)\,(C - C_b)$$

Where: "M" is multiples of pickup current

"C'" is equivalent time dial setting

"C" is actual time dial setting

"C_t" is time dial setting for top curve

"C_b" is time dial setting for bottom curve

MODIFICATIONS

Hieber's representation for the top and bottom curves requires twelve constants for a given relay type. Since fourth degree polynomials with data transformed by logarithms are known to fit individual relay curves with a high degree of accuracy, they will be substituted for Hieber's representation. This reduces the required number of coefficients for the top and bottom curves to ten.

Top Curve:

$$T_t(M) = \text{antilog}\left[a_0 + a_1 (\log M) + a_2 (\log M)^2 + a_3 (\log M)^3 + a_4 (\log M)^4\right]$$

Bottom Curve:

$$T_b(M) = \text{antilog}\left[b_0 + b_1 (\log M) + b_2 (\log M)^2 + b_3 (\log M)^3 + b_4 (\log M)^4\right]$$

COEFFICIENT OPTIMIZATION

Hieber's contribution to the study of overcurrent relay representation is in two parts. First is his concept of representing top and bottom curves individually and obtaining intermediate curve values by interpolation.

TABLE 7

ERROR STATISTICS FOR HIEBER'S METHOD

ERROR STATISTICS BASED ON 150 SAMPLE POINTS

	I			II		
	Linear Interpolation			Hieber's Interpolation		
Relay Type	Maximum Deviation (sec)	Average Deviation (sec)	CHI^2 Error	Maximum Deviation (sec)	Average Deviation (sec)	CHI^2 Error
CO-2	.7065	.02124	609.7	.5421	.01809	499.2
CO-5	.1538	.008778	577.6	.1721	.009780	548.0
CO-6	.4987	.03474	344.0	.4178	.03184	286.6
CO-7	.9689	.04498	240.4	1.005	.04558	237.4
CO-8	9.221	.1402	282.3	7.705	.1223	243.9
CO-9	1.287	.06363	246.9	1.191	.05035	209.4
CO-10	2.954	.1065	1080.	2.252	.07741	930.9
CO-11	3.502	.1205	696.0	3.656	.1261	694.9
IAC-11	1.636	.05293	374.6	1.542	.04899	355.9
IAC-15	3.541	.1331	864.7	3.181	.1358	758.8
IAC-51	1.235	.04735	302.1	1.111	.04503	265.1
IAC-53	1.448	.06084	276.6	1.178	.05284	243.8
IAC-77	2.632	.1283	2200.	2.103	.09715	969.8
		Total	8095.		Total	6244.

Second is his mapping of actual time dial setting into an equivalent time dial setting to facilitate interpolation. To isolate the effects of these concepts, two optimization procedures were investigated.

Column I of Table 7 (page 41) lists error statistics for linear interpolation between bottom and top curves. Ten coefficients (five for each curve) are optimized simultaneously to minimize chi-squared over 150 sample points. Actual time dial settings at ten multiples of pickup current are used in interpolating:

$$C' = C$$

The effect of mapping actual time dial setting to an equivalent time dial setting by Hieber's formula is shown in column II of Table 7 (page 41). These statistics are obtained by simultaneous optimization of eleven coefficients (five for bottom curve, five for top curve, and "q" in Hieber's equation for equivalent time dial setting). A comparison of column II with column I shows that a significant reduction in chi-squared error is achieved for most relay types. The total of the chi-squared values for the thirteen relay types is 62% lower than that obtained with strict application of Radke's method (6244. versus 16565. per Table 3, page 25).

MECHANICAL ANALYSIS

An investigation of the mechanical operation of overcurrent relays is relevant to the relay representation problem. A mathematical model based on equations of motion applied to relay mechanisms was examined. This is in contrast to previous investigations which were independent of mechanical considerations.

DRIVING TORQUE

The induction disk relay operates on the principle of the induction motor. Two alternating magnetic fields separated in time phase and in space interact to produce a driving torque. This torque will be proportional to the product of the magnitude of the one flux, that of the current induced in the disk by the other, and the cosine of the phase angle between the current and flux.[1] Since the disk currents are in approximate phase quadrature with and are proportional to the flux producing them, the torque is given by:

$$T = k\ \Phi_1\ \Phi_2\ \sin a$$

where "Φ_1" and "Φ_2" are the two fluxes and "a" is the

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[1] Stubbings, G. W. Automatic Protection of A. C. Circuits, Pittsburgh, Pennsylvania: Instruments Publishing Co., 1935, p. 125.

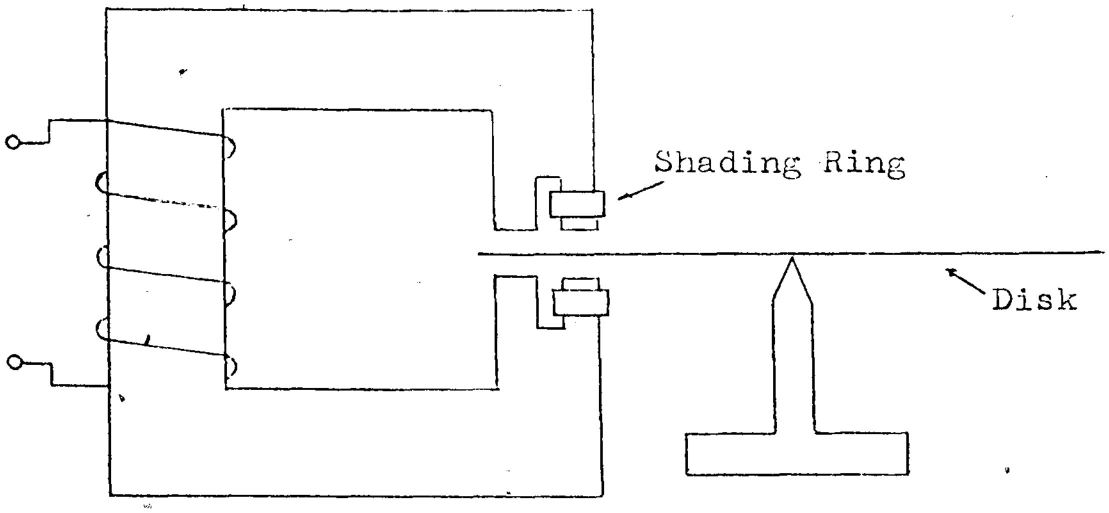

Figure 2. Shaded pole induction relay.

phase angle between them.

The simplest method of obtaining the two required fluxes is found in the "shaded pole" induction relay (see Figure 2). The magnetic circuit is divided into two portions. The ampere turns in the unshaded portion are entirely effective in producing magnetic flux (neglecting iron loss). The ampere turns in the non-ferrous shaded portion are expended partly in producing a magnetic flux and partly in balancing the ampere turns in the shading ring due to this flux. As a result of this interaction the net flux from the shaded portion is shifted in phase. With proper design, the phase angle "a" between the two net fluxes will be 45 degrees (maximum torque condition for shaded pole relay).[1]

Alternate designs not using shading rings are common. Frequently transformers are used to incorporate a phase shift in a portion of the electric circuit. The phase separated fluxes are developed at separate poles.

The magnitude of each of the fluxes is proportional to the current "I" in the relay winding. Hence the driving torque is proportional to the square of this current. In practice it is found that a certain threshold current "I_0" is required to set the disk in motion, due to the effects of friction and inertia. The torque required to offset these effects is proportional to the square of this constant current. The net driving torque is given by:

$$T_{drive} = k_1 \left(I^2 - I_0^2\right)$$

DYNAMIC BRAKING TORQUE

A dynamic braking torque arises due to the movement of the disk in the magnetic fields. This retarding torque results from the attraction between each field and the rotational eddy currents which it induces in the disk. This torque is proportional to the angular speed "w" and the square of the flux magnitudes.[2] Since each flux is proportional to the relay current the retarding torque

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[1] *Ibid.*, p. 127.

[2] *Ibid.*, p. 127.

is given by:

$$T_{retard} = k_2 \; w \; I^2$$

STATIC BRAKING TORQUE

Most induction disk relays contain a permanent magnet to give additional eddy current braking. The braking is desired so that the disk will stop if the fault current supplying the relay is cut off before the relay contacts close. In absence of this braking action the disk might continue to travel, closing the relay contacts and initiating unnecessary tripping action. The damping torque due to a permanent magnet is given by:[1]

$$T_{magnet} = k_3 \; w \; \Phi_m^{\;2}$$

where "Φ_m" is the flux due to the magnet.

RESTRAINING TORQUE

Most induction disk relays include a restraining spring to return the disk to its rest position after operation. In some designs a weight attached to a flexible cord wound around the disk spindle replaces the spring.

In those relays using the cord and weight, the restraining torque is constant. Since this torque is present when the threshold current "I_0" which just initiates

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[1] Knowlton, A. E. Electric Power Metering, New York: McGraw-Hill, 1934, p. 129.

motion of the disk is determined, it is actually a contribution to the constant "$k_1 I_0^2$" torque which was attributed to friction and inertia.

In those relays including a spring, the restraining torque is proportional to the angular displacement of the disk from the position of zero restraining torque. Since the spring is under tension at the rest position of the disk and the displacements to be considered are small, little error is introduced in assuming the spring torque to be constant. This allows the restraining spring to be treated identically to a cord and weight.

ASSUMPTION OF BALANCED TORQUES

If the operation of the relay is assumed to be at constant angular speed, the various torques must balance:

$$T_{drive} = T_{retard} + T_{magnet}$$

$$k_1 (I^2 - I_0^2) = k_2 w I^2 + k_3 w \Phi_m^2$$

The angle "θ" through which the disk must travel to close the relay contacts is related to the angular speed "w" and the operating time "t" through the equation:

$$w = \frac{\theta}{t}$$

Substituting this relation into the previous result and rearranging terms:

$$t = \frac{k_2 \Theta I^2 + k_3 \Theta I_m^2}{k_1 (I^2 - I_0^2)}$$

Define "M" as the multiple of pickup current:

$$M = \frac{I}{I_0}$$

Such that:

$$t = \frac{k_2 \Theta M^2 I_0^2 + k_3 \Theta \Phi_m^2}{k_1 (M^2 I_0^2 - I_0^2)}$$

$$t = \frac{k_2 \Theta M^2 I_0^2 + k_3 \Theta \Phi_m^2}{k_1 I_0^2 (M^2 - 1)}$$

Define new constants:

$$a_1 = k_2 I_0^2$$

$$a_2 = k_3 \Phi_m^2$$

$$a_3 = k_1 I_0^2$$

Then:

$$t = \frac{a_1 \Theta M^2 + a_2 \Theta}{a_3 (M^2 - 1)}$$

COEFFICIENT OPTIMIZATION

An expression has been developed, based on mechanical considerations, which relates operating time to multiples of pickup current and the angular distance which initially separates the relay contacts. Although the final goal is to relate operating time to multiples of pickup current and time dial setting, the "goodness of fit" of the model as developed to manufacturers' data was examined.

The curve fitting program was set up to investigate the equation:

$$t = \frac{a_1' M^2 + a_2'}{a_3 (M^2 - 1)}$$

Each of the ten curves for which data are available for a given relay type was fitted. Since the angular distance "θ" which initially separates the relay contacts is constant for a given curve, it was factored into the parameters "a_1'" and "a_2'":

$$a_1' = a_1 \theta$$

$$a_2' = a_2 \theta$$

Although the parameters "a_1", "a_2", and "a_3" are assumed in the derivation of the equation to be constant for a given relay type, they are optimized independently for each curve in the family.

Error statistics corresponding to these optimizations are listed in Table 8 (page 50). The chi-squared values are an order of magnitude greater than those obtained with other models. Additional error would be introduced in enhancing the model to generate an entire family of curves with a single set of parameters and to relate time dial setting to the initial angular separation of contacts.

TABLE 8

ERROR STATISTICS FOR MODEL BASED ON MECHANICAL ANALYSIS

ERROR STATISTICS BASED ON 150 SAMPLE POINTS

Relay Type	Maximum Deviation (sec)	Average Deviation (sec)	CHI^2 Error
CO-2	.6886	.04845	8120.
CO-5	.2931	.02268	2588.
CO-6	.3986	.03501	374.9
CO-7	3.742	.2910	10762.
CO-8	7.821	.1773	1469.
CO-9	2.667	.1339	1059.
CO-10	1.623	.05690	2807.
CO-11	5.282	.1901	904.7
IAC-11	3.743	.3960	19845.
IAC-15	2.754	.1194	3120.
IAC-51	3.766	.3715	18230.
IAC-53	.4189	.05484	831.9
IAC-77	1.599	.1294	1675.
		Total	71786.

SOURCES OF ERROR

Several assumptions were implicit in the mechanical analysis leading to the equation under consideration. Friction and inertia were assumed to present a constant restraining torque and the disk was assumed to spin at constant speed. Although assumed to be linear, the relation between flux and current is skewed due to saturation in the iron. These assumptions are the sources of error between predicted values and manufacturers' data.

Enhancements to the model intended to minimize error would need to account for the departure of the physical relay from the ideal device postulated. This would require incorporation of additional parameters to compensate for the departure and would of necessity be based more on mathematical analysis than mechanical analysis. Since significantly better results were obtained on a strict mathematical basis than those resulting from mechanical considerations, the model was abandoned.

PROPERTIES OF BEST FITTING EQUATION

BEST FITTING EQUATION

Among the equations which were investigated, the empirical model based on Hieber's method (pages 39 to 42) provides the best fit to manufacturers' data in the sense of minimum chi-squared error. The model uses eleven coefficients to specify variation among relay types (five to represent the top curve, five to represent the bottom curve, and one to map actual time dial settings to equivalent time dial settings). The number of computer operations required in using the model is sufficiently small that processing time limitations do not emerge as a problem.

SENSITIVITY TO NUMBER OF COEFFICIENTS

Fourth degree polynomials were selected for use in the model based on Hieber's method (page 40). It would be expected that inclusion of additional terms in the polynomials would increase the accuracy of the model at the expense of additional storage requirement for coefficients and additional processing time. In order to investigate the sensitivity of chi-squared to the number of coefficients, the curve fitting program in the form previously used with Hieber's equations was modified to represent the top and bottom curves with fifth degree polynomials.

Column I of Table 9 shows the results of optimization of the thirteen coefficients in the model. The total of chi-squared values is 10% lower than that achieved in the

TABLE 9

ERROR STATISTICS FOR HIEBER'S METHOD

ERROR STATISTICS BASED ON 150 SAMPLE POINTS

	I			II		
	Fifth Degree Polynomials and Linear Mapping			Fourth Degree Polynomials and Quadratic Mapping		
Relay Type	Maximum Deviation (sec)	Average Deviation (sec)	CHI^2 Error	Maximum Deviation (sec)	Average Deviation (sec)	CHI^2 Error
CO-2	.5174	.01772	490.8	.5286	.01797	479.0
CO-5	.1497	.009282	481.6	.1578	.008970	497.8
CO-6	.3130	.02728	221.8	.4312	.03146	285.1
CO-7	.8274	.03541	184.1	.9796	.04460	231.4
CO-8	8.102	.1193	235.6	7.829	.1229	241.7
CO-9	1.175	.05016	206.6	1.337	.05036	152.4
CO-10	2.258	.07735	926.3	2.283	.07830	930.4
CO-11	2.703	.1105	601.8	3.519	.1214	686.6
IAC-11	1.460	.04823	335.0	1.477	.04613	322.6
IAC-15	3.473	.1315	724.4	3.211	.1319	747.9
IAC-51	.9268	.03327	191.1	1.114	.04511	265.0
IAC-53	1.306	.05026	217.3	1.147	.05279	234.3
IAC-77	2.842	.1013	821.9	1.896	.08962	861.0
		Total	5638.		Total	5935.

model with fourth degree polynomials (5638. versus 6244. per Table 7, page 41).

Hieber's interpolation formula, which maps actual time dial setting to an equivalent time dial setting, also was subjected to a sensitivity analysis. The equation as considered previously:

$$C' = C - q P$$

where:

$$P = (C_t - C) (C - C_b)$$

was expanded to include a quadratic term:

$$C' = C - (q_1 P + q_2 P^2)$$

Column II of Table 9 (page 53) shows the results obtained with the quadratic mapping function and fourth degree polynomials for the top and bottom curve representations. The total of chi-squared values is 5% less than that achieved in the model with linear mapping and fourth degree polynomials (5935. versus 6244. per Table 7, page 41).

Since significant improvements in chi-squared resulted from an additional term in the polynomials representing the top and bottom curves and from an additional term in the mapping function, the combined effect of these changes was investigated. Column I of Table 10 (page 55) lists statistics corresponding to the model with fourteen

TABLE 10

ERROR STATISTICS FOR HIEBER'S METHOD

ERROR STATISTICS BASED ON 150 SAMPLE POINTS

	I			II		
	Fifth Degree Polynomials and Quadratic Mapping			Sixth Degree Polynomials and Cubic Mapping		
Relay Type	Maximum Deviation (sec)	Average Deviation (sec)	CHI^2 Error	Maximum Deviation (sec)	Average Deviation (sec)	CHI^2 Error
CO-2	.5008	.01760	470.6	.5097	.01759	463.7
CO-5	.1355	.008435	429.8	.1287	.008266	417.2
CO-6	.3233	.02702	220.1	.2984	.02595	208.0
CO-7	.7866	.03437	178.4	.6609	.02967	142.2
CO-8	8.259	.1194	233.5	8.359	.1194	227.8
CO-9	1.340	.05053	149.6	1.281	.05202	141.5
CO-10	2.272	.07771	925.6	2.244	.07732	923.1
CO-11	2.639	.1087	594.2	2.391	.1049	574.6
IAC-11	1.392	.04421	301.3	1.403	.04484	291.0
IAC-15	3.414	.1280	713.5	3.609	.1257	653.5
IAC-51	.9330	.03332	191.1	.8712	.03271	181.4
IAC-53	1.269	.05093	207.9	1.351	.04967	198.1
IAC-77	2.611	.09188	709.2	2.610	.08760	669.8
		Total	5325.		Total	5092.

coefficients which incorporates both enhancements. The total of chi-squared values is 15% lower than that achieved in the model with linear mapping and fourth degree polynomials (5325. versus 6244. per Table 7, page 41).

The 15% improvement in chi-squared at the expense of three additional coefficients was deemed worthwhile and the effect of further expansion was investigated. The curve fitting program was modified to consider sixth degree polynomials for the top and bottom curve representations and a cubic mapping function. Column II of Table 10 (page 55) shows the results of optimization of the seventeen coefficients in the model. The total of chi-squared values is 4% lower than that achieved in the previous model with fourteen coefficients (column I of Table 10).

The 4% improvement in chi-squared at the expense of three additional coefficients is of questionable value. It was decided to retain the additional coefficients on the basis that their effect was significant in certain individual relays (notably the CO-7 and IAC-15). The final form of the model is:

General Curve:

$$T(M,C') = T_t(M) \left[1 - F(C')\right] + T_b(M) \left[F(C')\right]$$

Top Curve:

$$T_t(M) = \text{antilog} \left[a_0 + a_1 (\log M) + a_2 (\log M)^2 + a_3 (\log M)^3 + a_4 (\log M)^4 + a_5 (\log M)^5 + a_6 (\log M)^6\right]$$

Bottom Curve:

$$T_b(M) = \text{antilog}\left[b_0 + b_1 (\log M) + b_2 (\log M)^2 + b_3 (\log M)^3 + b_4 (\log M)^4 + b_5 (\log M)^5 + b_6 (\log M)^6\right]$$

Interpolation Function:

$$F(C') = (C_t - C') / (C_t - C_b)$$

$$C' = C - (q_1 P + q_2 P^2 + q_3 P^3)$$

$$P = (C_t - C)(C - C_b)$$

Where: "M" is multiples of pickup current

"C'" is equivalent time dial setting

"C" is actual time dial setting

"C_t" is time dial setting for top curve

"C_b" is time dial setting for bottom curve

INTERMEDIATE VALUES

Both curve fitting and subsequent error analyses have been based on a single set of 150 sample points from manufacturers' curves for each relay type. The possible existence of spikes or dips between sample points of the predicted curves has not been investigated. To assure the integrity of the final model, the error analysis was expanded to include sample points not considered in the curve fitting phase.

A computer program was written to evaluate the final model using coefficients generated by the curve fitting program. The program computes pickup times for multiples of pickup current between 1.5 and 50.0 in increments of 0.1 at various time dial settings. These results are passed to a computer-driven plotter. The resultant curves were found to be smooth and monotonically decreasing with increasing multiples of pickup current, as was desired.

CONCLUSIONS

The development of equations to represent time overcurrent relay characteristics is a necessary and important step in the development of a computer relay setting program. Such a program can be a powerful tool for the relay engineer in selecting types of relays and specifying their settings.

Of the various methods for relay curve representation which were investigated, best results were obtained with a model based on a method due to John Hieber. This model does not meet the criterion set forth that the maximum deviation from manufacturers' data be less than 0.05 second or two percent error. A deviation of 8 seconds (11% error) arises for the CO-8 relay at one sample point. However, a vast majority of the sample points are within these proposed tolerances. The exceptions are sample points at the low current end of the curves where a slight variation in the position of the curve causes a large change in the time dimension.

Coefficients for the final model are given in Appendix A (pages 66 to 69) for various relay types. These values were optimized in the sense of minimum chi-squared error in comparing computed results to manufacturers' data. The coefficients are scaled for use with time values dimensioned in cycles (on sixty hertz base).

A visual comparison of the results of this final model to manufacturers' data is given in Figures 3 and 4 (pages 61 and 62) for the CO-2 and IAC-11 relays. The plotted curves are from data computed with the empirical model. Sample points indicated by crosses correspond to manufacturers' published data. Within the limitation of the scale of these drawings, the curves pass through the majority of the sample points.

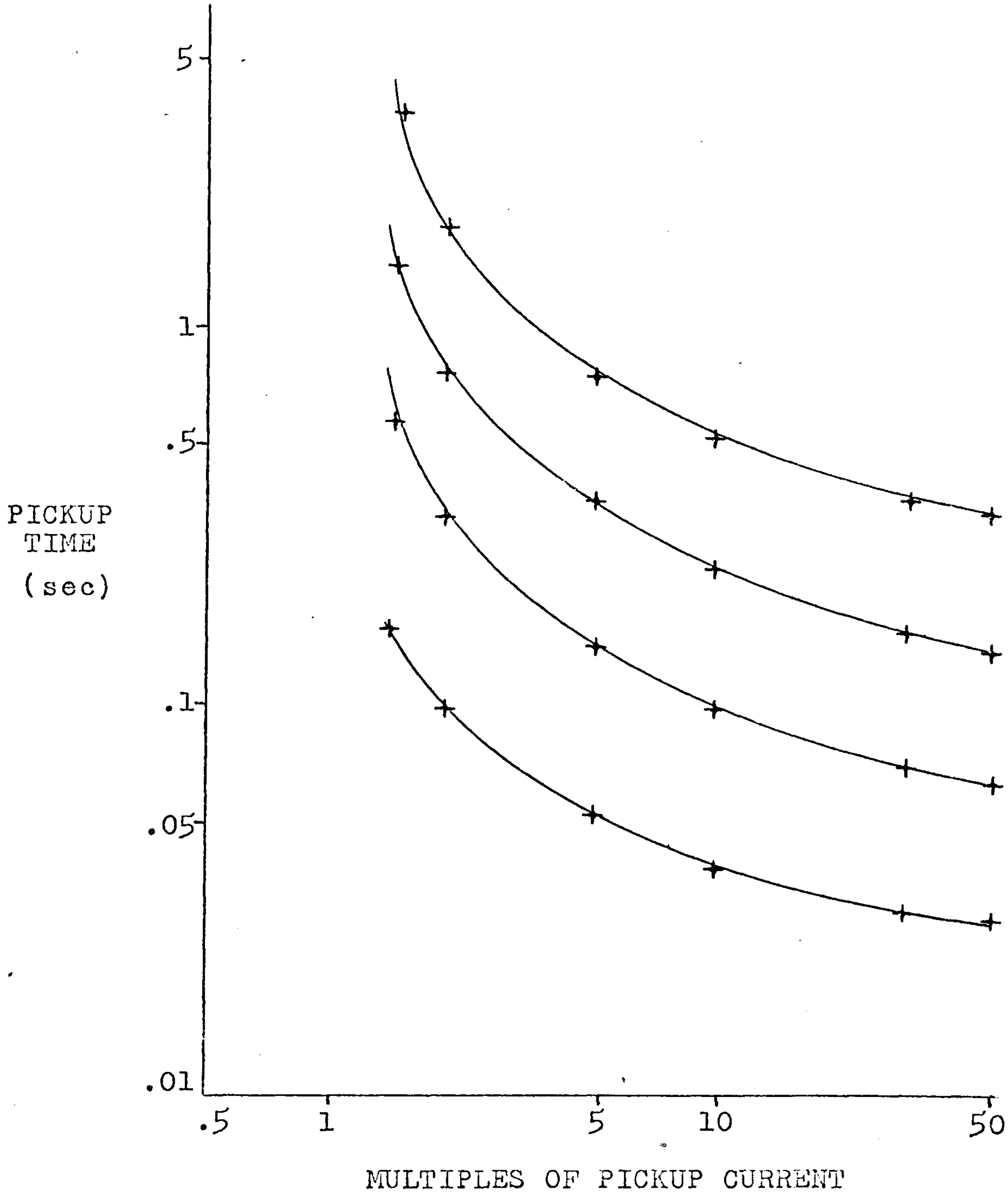

Figure 3. Computed characteristic curves for the CO-2 relay. (Crosses represent data from manufacturer's published curves).

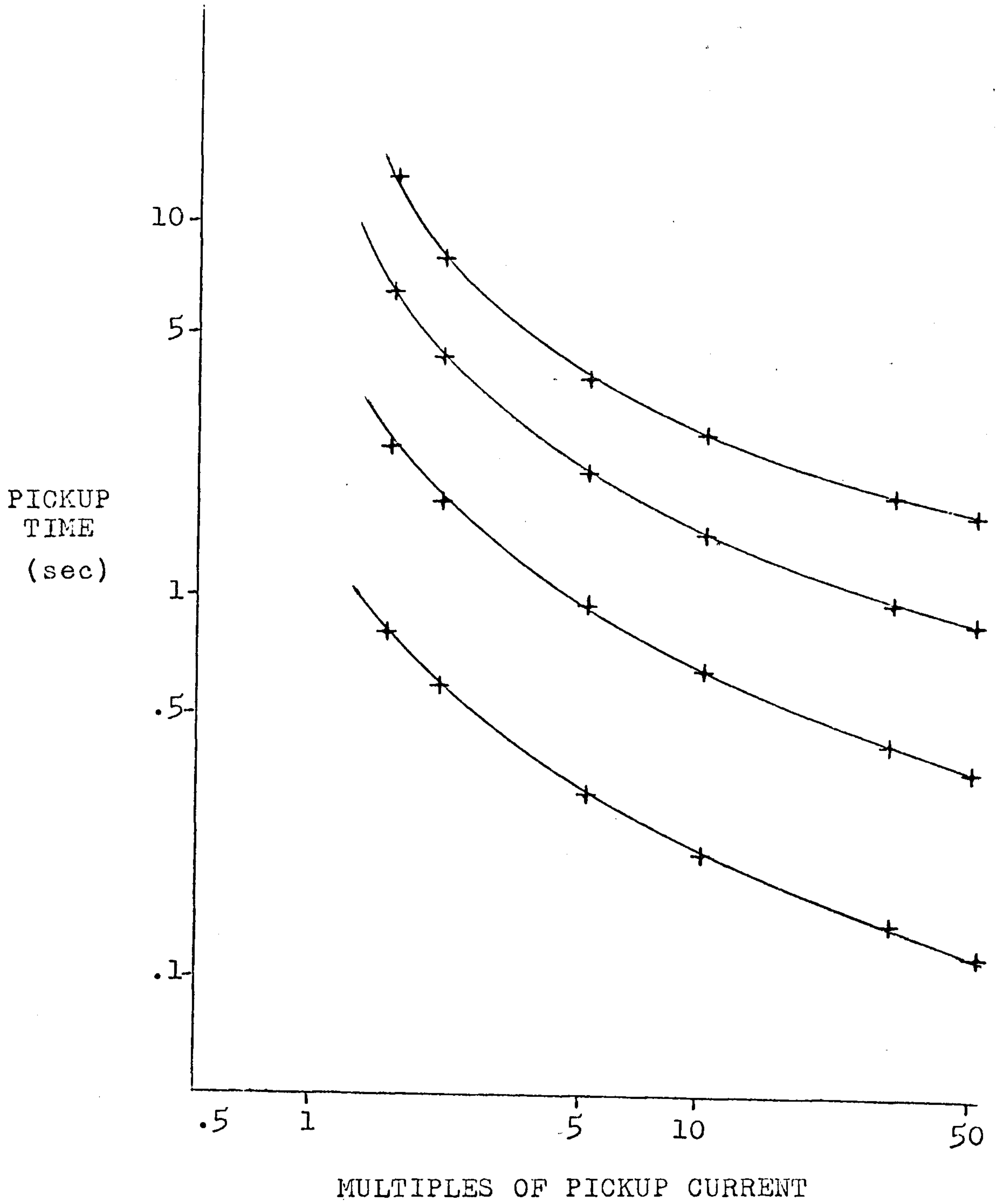

Figure 4. Computed characteristic curves for the IAC-11 relay. (Crosses represent data from manufacturer's published curves).

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APPENDIX A

OPTIMIZED COEFFICIENTS FOR FINAL MODEL BASED ON HIEBER'S EQUATIONS

RELAY

	CO-2	CO-5	CO-6	CO-7
a_0	2.849901E 00	2.782051E 00	3.344507E 00	3.688030E 00
a_1	-3.349548E 00	-5.160757E 00	-5.520572E 00	-6.081888E 00
a_2	2.702259E 00	1.019370E 01	1.213968E 01	1.386726E 01
a_3	6.757953E-01	-1.227093E 01	-1.526929E 01	-1.893825E 01
a_4	-2.728509E 00	8.684289E 00	1.093495E 01	1.457726E 01
a_5	1.695944E 00	-3.285990E 00	-4.113689E 00	-5.845906E 00
a_6	-3.429756E-01	5.108006E-01	6.282701E-01	9.473296E-01
b_0	1.143072E 00	1.328659E 00	1.714789E 00	2.027827E 00
b_1	-6.570478E-01	-5.005988E 00	-2.411434E 00	-3.846599E 00
b_2	-3.513441E 00	9.438403E 00	2.634851E 00	6.989433E 00
b_3	8.608628E 00	-9.937612E 00	-4.400595E-01	-8.356015E 00
b_4	-8.233510E 00	5.412525E 00	-1.268286E 00	6.155953E 00
b_5	3.638698E 00	-1.315267E 00	9.209774E-01	-2.486874E 00
b_6	-6.144286E-01	8.765680E-02	-1.917614E-01	4.157496E-01
q_1	9.880257E-04	-4.702225E-03	3.877901E-04	6.482798E-05
q_2	1.709233E-05	4.137006E-05	1.389716E-07	-1.424493E-07
q_3	-1.245151E-08	-9.022176E-08	-3.515792E-11	2.351866E-11

OPTIMIZED COEFFICIENTS FOR FINAL MODEL BASED ON HIEBER'S EQUATIONS

RELAY

	CO-8	CO-9	CO-10	CO-11
a_0	4.243039E 00	4.174242E 00	3.754701E 00	4.338578E 00
a_1	-3.972998E 00	-4.876146E 00	-3.759114E 00	-5.424426E 00
a_2	1.242786E 00	4.490411E 00	2.272878E 00	6.735065E 00
a_3	3.035461E 00	-2.872796E 00	-1.687322E 00	-6.770239E 00
a_4	-3.604081E 00	1.676852E 00	1.471102E 00	3.631218E 00
a_5	1.535334E 00	-7.216737E-01	-7.237706E-01	-7.440536E-01
a_6	-2.352524E-01	1.356142E-01	1.397267E-01	9.453326E-03
b_0	2.705253E 00	2.662636E 00	2.401025E 00	2.864161E 00
b_1	-5.457782E 00	-5.010036E 00	-5.584355E 00	-5.725945E 00
b_2	9.909526E 00	4.444840E 00	9.879919E 00	7.160604E 00
b_3	-1.453269E 01	-7.989304E-01	-1.227668E 01	-6.982791E 00
b_4	1.280848E 01	-1.796345E 00	8.853970E 00	4.696522E 00
b_5	-5.709185E 00	1.372942E 00	-3.337070E 00	-1.824549E 00
b_6	9.927632E-01	-2.974131E-01	5.130200E-01	2.994334E-01
q_1	2.967829E-04	-5.734337E-04	4.608795E-03	8.449187E-04
q_2	-2.479232E-09	-3.068710E-07	-7.762908E-06	-6.225557E-06
q_3	-1.632129E-12	2.483516E-10	1.790743E-08	7.636565E-09

OPTIMIZED COEFFICIENTS FOR FINAL MODEL
BASED ON HIEBER'S EQUATIONS

RELAY

	IAC-11	IAC-15	IAC-51	IAC-53
a_0	3.313251E 00	3.801733E 00	3.589667E 00	4.087152E 00
a_1	-3.168783E 00	-2.107191E 00	-5.276550E 00	-5.059775E 00
a_2	5.220444E 00	-4.885038E 00	1.116830E 01	5.762060E 00
a_3	-5.670291E 00	1.234703E 01	-1.385017E 01	-5.100397E 00
a_4	3.553699E 00	-1.103122E 01	9.568346E 00	3.308709E 00
a_5	-1.170925E 00	4.542171E 00	-3.456249E 00	-1.236694E 00
a_6	1.565195E-01	-7.218963E-01	5.084319E-01	1.900613E-01
b_0	2.071239E 00	2.655059E 00	1.925550E 00	2.740023E 00
b_1	-2.664545E 00	-2.845043E 00	-2.630234E 00	-6.826877E 00
b_2	3.530855E 00	-5.531229E-01	4.649949E 00	1.371819E 01
b_3	-2.412240E 00	6.331610E 00	-5.228041E 00	-1.739293E 01
b_4	3.962524E-01	-7.655216E 00	3.261295E 00	1.272126E 01
b_5	2.861785E-01	3.873889E 00	-1.032832E 00	-4.831614E 00
b_6	-1.006143E-01	-7.294827E-01	1.296952E-01	7.361813E-01
q_1	-7.697809E-04	8.226458E-04	5.104386E-04	3.024628E-04
q_2	4.260098E-07	-5.579132E-06	-1.069499E-07	-1.337628E-06
q_3	-3.914562E-11	2.976566E-09	1.205255E-11	4.192557E-10

OPTIMIZED COEFFICIENTS FOR FINAL MODEL
BASED ON HIEBER'S EQUATIONS

RELAY

	IAC-77
a_0	3.863255E 00
a_1	-3.058510E 00
a_2	4.260535E-01
a_3	1.840247E 00
a_4	-3.390472E 00
a_5	2.448566E 00
a_6	-5.920754E-01
b_0	2.967396E 00
b_1	-7.543710E 00
b_2	1.472081E 01
b_3	-2.066551E 01
b_4	1.688701E 01
b_5	-7.108502E 00
b_6	1.192505E 00
q_1	3.896907E-03
q_2	-6.187160E-05
q_3	1.161612E-07

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