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SOME ELASTIC PROPERTIES OF A LEDGE IN AN EDGE DISLOCATION WALL

by

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ABSTRACT

An analytical expression for the shear stress field of a semi-infinite dislocation wall and the stress contours are presented. The first order approximation of the expression is identical to the approximate formula previously reported by Amelinckx and Li. The expression of the shear stress field of a ledged dislocation wall is obtained by the superposition of two separated semi-infinite walls. The stress contours are illustrated.

The dilatation field of a ledged dislocation wall is analyzed. With the aid of the characteristics of the dilatation contours, an analytical approach to the volume change of the dilatation field is presented. Both approximate and accurate expressions for the volume exchange of a ledged dislocation wall are given. Based on these analyses, the solute saturation effect around the ledge of a dislocation wall is quantitatively estimated. The results indicate that the width of the ledge plays a dominant role in solute saturation and that the solute saturation in a ledged grain boundary may be higher by 1~2 orders of magnitude than that of a perfect ledge-free grain boundary.

I. Introduction

It is well known that a small angle tilt grain boundary can be treated as an edge dislocation wall. Based on first order elastic continuum theory, analytical solutions of the stress fields of an infinite dislocation wall were obtained.[1] It became evident that because the fields of individual dislocations cancel each other, there are no long range stress and strain fields for an infinite dislocation wall. Some approximate expressions of the stress fields at a large distance for a finite and a semi-infinite dislocation wall have been reviewed by Li.[2] The interaction and saturation of solutes with an infinite dislocation wall have been discussed by Webb.[3] These fundamental studies were conducted in order to provide a better understanding of the mechanical and chemical behavior of the grain boundary.

Actual grain boundaries, however, are seldom straight. Often many additional line defects are present, such as, grain boundary dislocations and grain boundary ledges. The modes of line defects of grain boundaries have been described and summarized by Balluffi.[4] Direct experimental observations of grain boundary ledges via TEM have been reported by several authors.[5] It is expected that the formation of ledges in a grain boundary, i.e., a dislocation wall, will distort the stress and strain fields of the dislocation wall, especially in the region around the ledge. The greater the ledge width, the stronger the distortion. In view of the fact that a large number of ledges are often present along a grain boundary (e.g., a density of $10^4 \sim 10^5$ (number/cm) in purified iron),[6] the mechanical and chemical behavior of a grain boundary with a high density of ledges might be quite different from those of a grain boundary without ledges. For example, the energy of the grain boundary and the saturation limitation of solute atoms may correspondingly increase if more ledges are present. From this point of view, a real grain boundary may no longer be considered as a straight dislocation wall, but as a wall with ledges. Thus, it is essential to provide fundamental study concerning a dislocation wall with ledges. The objectives of the present work include:

(1) the analysis of the shear stress field of a ledged dislocation wall, based on a detailed analysis of the shear stress field of a semi-infinite dislocation wall, and,

(2) the study of the characteristics of the dilatation fields, volume change, and solute saturation of a dislocation wall with ledges, based on both analytical and numerical approaches.

II. The shear stress field of a ledged dislocation wall

1. The shear stress field of a semi-infinite dislocation wall

Because a ledged dislocation wall can be considered as a superimposition of two semi-infinite walls, a study of a semi-infinite dislocation wall can be useful. The stress fields of a semi-infinite wall can be expressed as

$$\sigma_{xx} = -\frac{\mu b}{2\pi(1-\nu)} \sum_{n=0}^{\infty} \frac{y_n(3x^2-y_n^2)}{(x^2-y_n^2)^2} \tag{1}$$

$$\sigma_{yy} = \frac{\mu b}{2\pi(1-\nu)} \sum_{n=0}^{\infty} \frac{y_n(x^2-y_n^2)}{(x^2+y_n^2)^2} \tag{2}$$

$$\sigma_{zz} = -\frac{\mu b}{2\pi(1-\nu)} \sum_{n=0}^{\infty} \frac{y_n}{x^2+y_n^2} \tag{3}$$

$$\sigma_{xy} = \frac{\mu b}{2\pi(1-\nu)} \sum_{n=0}^{\infty} \frac{x(x^2+y_n^2)}{(x^2+y_n^2)^2} \tag{4}$$

where μ is the shear modulus

ν is the Poisson's ratio

$y_n = y + nh$, h is the spacing between two neighboring dislocations

b is the Burger's vector.

Apparently, the summation of Eq (1) ~ (3) is divergent. On the other hand, as shown in Eq (4), the summation for shear stress σ_{xy} is convergent. A general solution of this summation is considered to be difficult[7][11] and will be discussed in detail in the latter part of this paper. However, in some simple cases, when y takes some special values of mh, or $(\frac{1}{2} + m)h$, (m being an integer), Eq (4) can be simplified and expressed as follows:

For y=0, i.e., along the abscissa,

$$\sigma_{xy}(x,0) = \frac{\mu b}{2\pi(1-\nu)} \sum_{n=0}^{\infty} \frac{x[x^2-(nh)^2]}{[x^2+(nh)^2]^2}$$

$$= \frac{\mu b}{2\pi(1-\nu)} \left[- \frac{1}{2x^2} - \frac{\pi}{2h^2} \operatorname{csch}^2 \frac{\pi x}{h} \right] \qquad \ldots \quad (5)$$

For y = mh m = ±1, ±2, . . .

$$\sigma_{xy}(x,mh) = \frac{\mu b}{2\pi(1-\nu)} \sum_{n=0}^{\infty} \frac{x^2-(mh+nh)^2}{[x^2-(mh+nh)^2]^2}$$

$$= \sigma_{xy}(x,0) - \frac{\mu bx}{2\pi(1-\nu)} \sum_{n=0}^{m-1} \frac{x^2-(nh)^2}{[x^2+(nh)^2]^2} \qquad (6)$$

Because the second term of this expression is only a finite summation, the shear stress field can be computed precisely.

Similarly, if $y=\frac{h}{2}$ and $y=(\frac{1}{2}+m)h$, m = ±1, ±2, . . .

$$\sigma_{xy}(x,\tfrac{h}{2}) = \frac{\mu bx}{2\pi(1-\nu)} \sum_{n=0}^{\infty} \frac{x^2- (\frac{h}{2}+nh)^2}{[x^2+(\frac{h}{2}+nh)^2]^2} \qquad (7)$$

$$= \frac{\mu bx}{2\pi(1-\nu)} \left[- \frac{\pi^2}{2h} \operatorname{sech}^2 \frac{\pi x}{h} \right] \qquad (8)$$

$$\sigma_{xy}\left[x,\left(\frac{h}{2}+m\right)h\right] = \sigma_{xy}\left(x,\frac{h}{2}\right) - \frac{\mu bx}{2\pi(1-\nu)} \sum_{n=0}^{m-1} \frac{x^2-\left(nh+\frac{1}{2}h\right)^2}{\left[x^2+\left(nh+\frac{1}{2}h\right)^2\right]^2} \qquad (9)$$

In order to obtain a general solution of Eq (4), a trigamma function, $\psi'(z)$, was first introduced, defined by[8]

$$\psi'(z) = \frac{d}{dz}\psi(z) = \frac{d^2}{dz^2}\log\Gamma(z) = \sum_{n=0}^{\infty}\frac{1}{(n+z)^2} \qquad (10)$$

where z is a complex argument and n is an integer.

Eq (4) can then be expressed as

$$\sigma_{xy}(x,y) = \frac{\mu bx}{2\pi(1-\nu)}\sum_{n=0}^{\infty}\frac{x^2-(y+nh)^2}{[x^2+(y+nh)^2]^2}$$

$$= \frac{-\mu bx}{2\pi(1-\nu)}\sum_{n=0}^{\infty}\left[\frac{1}{[(nh+y)+xi]^2}+\frac{1}{[(nh+y)-xi]^2}\right]$$

$$= \frac{\mu bx}{2\pi(1-\nu)}\left(-\frac{1}{2h^2}\right)\left[\psi'\left(\frac{y}{h}+\frac{x}{h}i\right)+\psi'\left(\frac{y}{h}-\frac{x}{h}i\right)\right] \qquad (11)$$

By setting $\quad z = \frac{y}{h}+\frac{x}{h}i$ and $\bar{z} = \frac{y}{h}-\frac{x}{h}i$, $\qquad (12)$

$$\sigma_{xy}(x,y) = \frac{-\mu bx}{2\pi(1-\nu)}\frac{1}{2h^2}\left[\psi'(z)+\psi'(\bar{z})\right] \qquad (13)$$

since $\quad \psi(\bar{z}) = \overline{\psi(z)}$ [9]

$$\psi'(\bar{z}) = \frac{d}{dz}\psi(\bar{z}) = \frac{d}{dz}\overline{\psi(z)} = \overline{\frac{d}{dz}\psi(z)} = \overline{\psi'(z)} \quad , \qquad (14)$$

we have

$$\psi'(z)+\psi'(\bar{z}) = \psi'(z)+\overline{\psi'(z)} = 2\mathrm{Re}[\psi'(z)] \quad ,$$

and Eq (11) becomes

$$\sigma_{xy}(x,y) = \frac{-\mu bx}{2\pi(1-\nu)} \frac{1}{h} \mathrm{Re}[\psi'(z)] \tag{15}$$

Equation (15) is the general expression for the shear stress field of a semi-infinite dislocation wall. Here a special function, $\psi'(z)$, the trigamma function in complex arguments, is involved. Because the values of trigamma function $\psi'(z)$ in complex arguments have not been published, some data of $\mathrm{Re}[\psi'(z)]$ and $\mathrm{Im}[\psi'(z)]$ are computed and tabulated in Table 7 in the appendix. On the other hand, $\psi'(z)$ can be expanded in the form of the asymptomic formula and the recurrence formula.

The asymptomic formula of $\psi'(z)$ is:[9]

$$\psi'(z) \sim \frac{1}{z} + \frac{1}{2z} + \sum_{k=1}^{\infty} \frac{(-1)^{k+1} B_k}{z^{2k+1}} \tag{16}$$

$$\sim \frac{1}{z} + \frac{1}{2z} + \frac{1}{6z^3} - \frac{1}{30z^5} + \frac{1}{42z^7} - \frac{1}{30z^9} + \cdots \tag{17}$$

$$(z \to \infty\ ,\ |\arg z| < \pi)$$

where B_k denotes the Bernoulli number.

The recurrence formula of $\psi'(z)$ is given by[9]

$$\psi'(z) = \psi'(z+m) + \sum_{n=1}^{m} \frac{1}{z+n-1} \qquad m = 1, 2, 3 \ldots \tag{18}$$

More detailed discussion about trigamma function $\psi'(z)$ and its asymptomic and recurrence formula will be found in the appendix. By substituting Eq (17) and Eq (18) for Eq (15), the explicit expressions for the shear stress field of a semi-infinite dislocation wall are obtained as:

$$\sigma_{xy}(x,y) \sim \frac{-\mu bx}{2\pi(1-\nu)} \frac{1}{h^2} \mathrm{Re}\left(\frac{1}{z} + \frac{1}{2z^2} + \frac{1}{6z^3} - \frac{1}{30z^5} + \ldots\right) \quad (19)$$

$$\sigma_{xy}(x,y) \sim \frac{-\mu bx}{2\pi(1-\nu)} \frac{1}{h^2} \mathrm{Re}\left[\left(\frac{1}{z+m} + \frac{1}{2(z+m)^2} + \ldots\right) + \left(\sum_{n=1}^{m} \frac{1}{(z+n-1)^2}\right)\right] \quad (20)$$

$$m = 1, 2, 3, \ldots$$

For a relatively large value of $|z|$, Eq (19) provides a good approximation and can be used for computing the values of σ_{xy}. Note that Eq (19) is not valid when $|z| \leq 1$ and converges very slowly when $|z| \sim 1$. In these cases, an appropriate value of positive integer m can be selected to make $|z+m|$ relatively large, and the values of σ_{xy} can be computed using Eq (20).

Equation (19) and Eq (20) are more useful than Eq (4) for computing the shear stress of a semi-infinite wall because of rapid convergence. Further analysis of the deviation of these expressions will be discussed in the appendix.

Some years ago, Amelinckx [11] and Li [12] derived a formula for the shear stress field of a semi-infinite dislocation wall which is valid only for the region far from the origin, i.e., for large $|z|$ value. Their formula is as follows:

$$\sigma_{xy} = \frac{-\mu bx}{2\pi(1-\nu)} \cdot \frac{y}{h(x^2+y^2)} \tag{21}$$

It is interesting to note that Eq (21) is identical to the first term of Eq (18), that is

$$\begin{aligned} \sigma_{xy} &= \frac{-\mu bx}{2\pi(1-\nu)} \cdot \frac{1}{h^2} \, \mathrm{Re}\left(\frac{1}{z}\right) \\ &= \frac{-\mu bx}{2\pi(1-\nu)} \, \frac{1}{h^2} \, R_e \cdot \left[\frac{h(y-xi)}{y^2+x^2}\right] \\ &= \frac{-\mu bx}{2\pi(1-\nu)} \cdot \frac{y}{h(x^2+y^2)} \end{aligned} \tag{22}$$

Based on Eq (15), Eq (19), and Eq (20), the contour of the shear stress field of a semi-infinite dislocation wall is computed and plotted in Figure 1. A part of the data for the stress field is tabulated in Table 8 in the appendix. It is seen that the σ_{xy} field in the upper half plane where no dislocation exists is similar to that of a single dislocation,[1] but has a greater strength. The σ_{xy} field of the lower half region containing the dislocation wall is similar to that of an infinite wall.[1] At a large distance from the origin, the contour lines tend to straighten out, in agreement with the approximate solution of Eq (21) given by Li. In fact, for the contours of σ_{xy} = constant, Eq (21) can be written as

$$\sigma_{xy} = \frac{-\mu bx}{2\pi(1-\nu)} \frac{y}{h(x^2+y^2)} = \frac{-\mu bx}{2\pi(1-\nu)} \left(\frac{1}{2} \sin 2\theta\right) = \text{const.}$$

Thus, when θ is constant, it indicates that the contours are straight line at a large distance from the origin.

2. The shear stress field of a ledged dislocation wall.

Figure 2 shows a ledged dislocation wall which consists of two semi-infinite dislocation walls I and II. The shear stress of this ledged wall can be expressed as the sum of the two semi-infinite walls,

$$\sigma_{xy} = (\sigma_{xy})_{I} + (\sigma_{xy})_{II} \tag{23}$$

If another semi-infinite dislocation wall, III, denoted by the dashed lines in Figure 2, is introduced, Eq (23) becomes

$$\sigma_{xy} = (\sigma_{xy})_{I} + (\sigma_{xy})_{III} + (\sigma_{xy})_{II} - (\sigma_{xy})_{III} \tag{24}$$

Here, wall I and wall II constitute an infinite wall, i.e.,

$$(\sigma_{xy})_{I} + (\sigma_{xy})_{III} = \frac{\mu bx}{2\pi(1-\nu)} \cdot \frac{\pi^2}{h^2} \cdot \frac{\cosh\left[\frac{2\pi(x+\epsilon)}{h}\right] \cdot \cos\left(\frac{2\pi y}{h}\right) - 1}{\left[\sinh^2 \frac{(\pi+\epsilon)}{h} + \sin^2 \frac{\pi y}{h}\right]^2} \quad \ldots \tag{25}$$

where E is the half width of the ledge. Combining Eq (25), Eq (15), and Eq (24) yields

$$\sigma_{xy} = \frac{\mu bx}{2\pi(1-\nu)} \frac{1}{h^2} \left\{ \frac{\pi^2\left(\cosh\left[\frac{2\pi(x+\epsilon)}{h}\right] \cos\frac{2\pi y}{h} - 1\right)}{\left[\sinh^2\frac{\pi(x+\epsilon)}{h} + \sin^2\frac{\pi y}{h}\right]^2} + \mathrm{Re}\left[\psi'\left(\frac{y}{h} + \frac{x-\epsilon}{h}i\right)\right] - \mathrm{Re}\left[\psi'\left(\frac{y}{h} + \frac{x-\epsilon}{h}i\right)\right]\right\} \tag{26}$$

Equation (26) is the expression for the shear stress field of a dislocation wall with a ledge of width 2ϵ. Figure 3 shows the shear stress contours of such a ledged dislocation wall with ledge width 2ϵ equal to h.

III. Dilatation field and volume change around a ledge in a dislocation wall

1. Dilatation field and volume change of an infinite dislocation wall

The dilatation field of an infinite dislocation wall can be expressed as the sum of the dilatation fields of the individual dislocations[3]

$$\left(\frac{\Delta V}{V}\right)_{inf} = \sum_{-\infty}^{\infty} \frac{b(1-2\nu)(-y_n)}{2\pi(1-\nu)(x^2+y_n^2)}$$

$$= \frac{b(1-2\nu)}{2\pi(1-\nu)} \frac{\pi}{h} \frac{\cos\frac{\pi y}{h} \cdot \sin\frac{\pi y}{h}}{\sin^2\frac{\pi y}{h}+\sinh^2\frac{\pi x}{h}} \tag{27}$$

The volume change of this dilatation field per unit area of the wall was set forth by Webb[3] as follows:

$$(\Sigma\Delta V)_{inf} = \frac{1}{h}\int_0^1 \int_{-\infty}^{\infty} \int_0^{h/2} \frac{\Delta V}{V}\, dxdydz$$

$$= \frac{-b(1-2\nu)}{8(1-\nu)} \approx 0.79\, \frac{-b(1-2\nu)}{2\pi(1-\nu)} \qquad (28)$$

2. Dilatation field of a ledged dislocation wall

Similar to the discussion of the shear stress field, the dilatation field of a ledged dislocation wall, as shown in Figure 2, can be expressed as

$$\frac{\Delta V}{V} = \left(\frac{\Delta V}{V}\right)_I + \left(\frac{\Delta V}{V}\right)_{III} + \left(\frac{\Delta V}{V}\right)_{II} - \left(\frac{\Delta V}{V}\right)_{III}$$

$$= \frac{b(1-2\nu)}{2\pi(1-\nu)} \left\{ \left[\frac{\pi}{h} \frac{\sin\left(\frac{\pi y}{h}\right) \cdot \cos\left(\frac{\pi y}{h}\right)}{\sin^2\left(\frac{\pi y}{h}\right) + \sinh^2 \frac{\pi(x+\epsilon)}{h}} \right] + \left[\sum_{n=0}^{\infty} \left(\frac{y_n}{(x-\epsilon)^2 + y_n^2} - \frac{y_n}{(x+\epsilon)^2 + y_n^2} \right) \right] \right\}$$

$$= \frac{b(1-2\nu)}{2\pi(1-\nu)} \left[\frac{\pi}{h} \frac{\sin\frac{\pi y}{h} \cdot \cos\frac{\pi y}{h}}{\mathrm{Sin}^2\left(\frac{\pi y}{h}\right) + \mathrm{Sinh}^2 \frac{\pi(x+\epsilon)}{h}} + \sum_{n=0}^{\infty} \frac{4\epsilon x y_n}{(x^2+\epsilon^2+y_n^2)^2 - (2\epsilon x)^2} \right] \qquad (29)$$

To find an explicit expression for the second term in Eq (29) is difficult. However, this summation converges rapidly because its denominator is 3 orders of magnitude higher than its numerator and can be computed numerically by taking a finite number of terms (e.g., n=100). Plots of dilatation contours for ledged dislocation walls with the width ϵ equal to 0.25h, 0.5h, and 1.0h, are illustrated in Figures 4, 5, and 6, respectively.

3. Characteristics of zero lines of the dilatation field of a ledged dislocation wall

Figure 7 shows the zero dilatation lines of an infinite dislocation wall. Figure 8 shows the computed zero dilatation lines of ledged dislocation walls with ϵ = 0.5h, 1.0h, and 2.0h, respectively. For an infinite wall, i.e., $\epsilon = 0$, the zero dilatation lines are parallel horizontal lines at $y=0, \pm\frac{h}{2}, \pm h \; . \; . \; . \; \pm\frac{mh}{2} \; . \; . \; . \; .$ Thus, regions of dilatation and compression alternate in the parallel horizontal strips with a width of h/2, as shown in Figure 7. The field of the same sign is discontinuous. From Figures 8(a) and (b), it is seen that for small ledge widths (ϵ=0.5h and 1.0h), the zero lines contract into a sinusoidal-shaped curve. From Figures 8(b) and (c), it is noted that as the ledge width ϵ increases, the zero lines strongly tend to be straighten and coincide with the y axis in the region near the ledge. For example, when ϵ=h [Figure 8(b)], the zero line is identical with the y axis within the range of $-4h \leq y \leq 4h$, and when ϵ

increases to 2h [Figure 8(c)], the zero line coincides with the y axis in the whole range of ±10h. Hence, it can be concluded that the wider the ledge width, the greater the coincidence range between the y axis and the zero line. Data on the coincidence ranges between the y axis and zero line for different values of ϵ are listed in Table 1.

From the characteristics of these dilatation zero lines it is clear that the region of the same sign, i.e., either dilatation region or compression region, appears to be continuous. Thus, it can be considered that, with the exception of those small shaded regions near the dislocation cores, the field is negative in x>0 (e.g., region of compression) and positive in x<0 (e.g., region of dilatation). Such a consideration would be helpful in simplifying the analysis given in the next section for the volume change of the dilatation field of a ledged dislocation wall.

TABLE 1
THE COINCIDENCE RANGES BETWEEN THE Y AXIS AND THE ZERO LINE

Ledge width ϵ/h	0.8	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Coincidence range between y axis and zero line(1/h)	1	4	17	68	402	703	2142	6300

4. Approximate expression for the volume change of a ledged dislocation wall

From the above discussion, it is clear that in a certain range of a ledged wall (near the ledge) the y axis can be considered as the zero-line of the dilatation field, provided the ϵ is not very small. Following the analysis of Webb, the volume change of the dilatation field per unit area of the ledged wall, in the region of the same sign, $0 \leq y \leq h$ and $0 \leq x \leq \ell$ (the shaded strips in Figure 2), can be expressed approximately as

$$(\Sigma\Delta Vo)_{appr} = \frac{1}{h}\int_o^1\int_o^\ell\int_o^h \frac{\Delta V}{V}dxdydz$$

$$= \frac{1}{h}\int_o^1\int_o^\ell\int_o^h \frac{b(1-2\nu)}{2\pi(1-\nu)}\left[\frac{\pi}{h}\,\frac{\cos\left(\frac{\pi y}{h}\right)\cdot\ \sin\left(\frac{\pi y}{h}\right)}{\sin^2\left(\frac{\pi y}{h}\right)+\ \sinh^2\left[\frac{\pi(x+\epsilon)}{h}\right]}\right.$$

$$\left. + \sum_{n=0}^{\infty}\frac{4\epsilon x y_n}{(x^2+\epsilon^2+y_n^2)^2-(2\epsilon x)^2}\right] dxdydz \qquad (30)$$

Because the integration of the first term is zero, it follows that

$$(\Sigma\Delta V)_{appr} = \frac{b(1-2\nu)}{2\pi(1-\nu)}\,\frac{1}{h}\int_o^\ell\int_o^h\left[\frac{4\epsilon x(y+h)}{(x^2+\epsilon^2+y^2)^2-(2\epsilon x)^2}\right.$$

$$+ \frac{4\epsilon x(y+h)}{[x^2+\epsilon^2+(y+h)^2]-(2\epsilon x)^2} + \ldots$$

$$\left. + \frac{4\epsilon x(y+mh)}{[x^2+\epsilon^2+(y+mh)^2]^2-(2\epsilon x)^2} + \ldots\right] dxdy$$

$$= \frac{b(1-2\nu)}{2\pi(1-\nu)} \frac{1}{h} \int_0^{\ell} \lim_{m \to \infty} \int_{x^2+\epsilon^2}^{x^2+\epsilon^2+(mh)^2} \frac{2\epsilon x}{t^2-(2\epsilon x)^2} \, dt dx \qquad (31)$$

$$= \frac{b(1-2\nu)}{2\pi(1-\nu)} \frac{1}{h} \int_0^{\ell} \lim_{m \to \infty} \left\{ \frac{1}{2} \left[\ln \left| \frac{(x+\epsilon)^2-(mh)^2}{(x-\epsilon)^2+(mh)^2} \right| - \ln \left| \frac{(x+\epsilon)^2}{(x-\epsilon)^2} \right| \right] \right\} dx \qquad . \; . \; . \; (32)$$

when $m \to \infty$

$$\ln \left| \frac{(x+\epsilon)^2+(mh)^2}{(x-\epsilon)^2+(mh)^2} \right| \longrightarrow 0.$$

and Eq (32) becomes

$$(\Sigma \Delta Vo)_{appr} = \frac{-b(1-2\nu)}{2\pi(1-\nu)} \frac{1}{h} \int_0^{\ell} \ln \left| \frac{x+\epsilon}{x-\epsilon} \right| dx$$

$$= \frac{-b(1-2\nu)}{2\pi(1-\nu)} \frac{1}{h} \left[\ell \cdot \ln \left| \frac{\ell+\epsilon}{\ell-\epsilon} \right| + \epsilon \ln \left| \frac{\ell^2-\epsilon^2}{\epsilon^2} \right| \right] \qquad (33)$$

For $\ell >> \epsilon$, Eq (33) can be expanded into Taylor's series. By neglecting the terms of higher orders, Eq (33) can be simplified as

$$(\Sigma \Delta Vo)_{appr} = \frac{-b(1-2\nu)}{2\pi(1-\nu)} \frac{2\epsilon}{h} \left[\ln \left(\frac{\ell}{\epsilon} \right) - 1 \right] \qquad (34)$$

Similarly, for an arbitrary region of $kh \leq y \leq (k+1)h$, the approximate expression for the volume change per unit area of a ledged wall can be expressed as

$$(\Sigma\Delta V_K)_{appr} = \frac{b(1-2\nu)}{2\pi(1-\nu)} \frac{1}{h} \int_0^1 \int_0^\ell \int_{kh}^{(k+1)h} \sum_{n=0}^{\infty} \frac{4\epsilon x y_n}{(x^2+\epsilon^2+y_n^2)^2-(2\epsilon x)^2}$$

$$dxdydz = \frac{-b(1-2\nu)}{2\pi(1-\nu)} \frac{1}{2h} \left\{ \ell \cdot \ln \frac{(\ell+\epsilon)^2+(kh)^2}{(\ell-\epsilon)^2+(kh)^2} \right.$$

$$+ \epsilon \ln \frac{[(\ell+\epsilon)^2+(kh)^2][(\ell-\epsilon)^2+(kh)^2]}{[\epsilon^2+(kh)^2]^2}$$

$$\left. + 2kh(\tan^{-1} \frac{\ell-\epsilon}{kh} - \tan^{-1} \frac{\ell+\epsilon}{kh} - 2\tan^{-1} \frac{\epsilon}{kh}) \right\} \quad (35)$$

where $k = 0, \pm 1, \pm 2 \ldots$

In the case where $\ell \gg \epsilon$, Eq (35) becomes

$$(\Sigma\Delta V_K)_{appr} = \frac{-b(1-2\nu)}{2\pi(1-\nu)} \frac{1}{2h} \left[\ell \cdot \ln \frac{(\ell+\epsilon)^2+(kh)^2}{(\ell-\epsilon)^2+(kh)^2} - 4kh \tan^{-1}\frac{\epsilon}{kh} \right.$$

$$\left. + \epsilon \ln \frac{\ell^2+(kh)^2}{\epsilon^2+(kh)^2} \right] \quad (36)$$

Equation (35) or Eq (36) can be considered as the general expression for the volume change per unit area of a ledged wall for an arbitrary range of $kh \leq y \leq (k+1)h$. At k=0, Eq (35) and Eq (36) are identical to Eq (33) and Eq (34), respectively.

From Eq (35) or Eq (36), it is noted that the value of volume change per unit area, $\Sigma\Delta V_K$, is no longer constant for a ledged dislocation wall as compared with that of an infinite wall, which has a constant value of

$0.79 \frac{-b(1-2\nu)}{2\pi(1-\nu)}$, [Eq (28)]. It is, therefore, useful to introduce an average value of the volume change over the range of y from 0 to nh,

$$(\overline{\Sigma\Delta V})_{appr} = \frac{1}{(n+1)} \sum_{k=0}^{n} (\Sigma\Delta Vk)_{appr}$$

$$= \frac{1}{(n+1)} \sum_{k=0}^{n} \frac{b(1-2\nu)}{2\pi(1-\nu)} \frac{1}{2h} [\ell \cdot \ell n \frac{(\ell+\epsilon)^2+(kh)^2}{(\ell-\epsilon)^2+(kh)^2}$$

$$-4kh\tan^{-1} \frac{\epsilon}{kh} + \epsilon \; \ell n \frac{\ell^2+(kh)^2}{\epsilon^2+(kh)^2} \tag{37}$$

for $\epsilon/h << n << \ell/h$,

$$\sum_{k=0}^{n} \ell \cdot \ell n \frac{(\ell+\epsilon)^2+(kh)^2}{(\ell-\epsilon)^2+(kh)^2} \approx 4n\epsilon$$

and

$$\sum_{k=0}^{n} 4kh \tan^{-1} \frac{\epsilon}{kh} \approx 4n\epsilon \; .$$

The sum of the first two terms of Eq (37) then equals zero, i.e.,

$$(\overline{\Sigma\Delta V})_{appr} \approx \frac{-b(1-2\nu)}{2\pi(1-\nu)} \frac{1}{2(n+1)h} \sum_{k=0}^{n} \epsilon \ell n \frac{\ell^2+(kh)^2}{\epsilon^2+(kh)^2}$$

For $\epsilon/h << n << \ell/h$

$$(\overline{\Sigma\Delta V})_{appr} \approx \frac{-b(1-2\nu)}{2\pi(1-\nu)} \frac{2\epsilon}{2(n+1)h} [(n+1) \; \ell n \frac{\ell}{n} - \ln n!] \tag{38}$$

5. More accurate expression for the volume change of a ledged dislocation wall

It is emphasized that Eq (33) - Eq (38) provide a good approximation only insofar as the zero dilatation line is nearly identical with the y axis. However, if the ledge width ϵ is relatively small or the k value is relatively large, these equations might not be valid because of the sinusoidal shaped curve of the zero line. For example, in Figure 8(a), $\epsilon = 0.5h$ is relatively small, and, in Figure 8(b), at the region of $k \geq 5$, the zero line does not coincide closely with the y axis. Therefore, the additional volume change contributed by the small shaded area, which is located between the y axis and the sinusoidal curve of the zero line, must be taken into account. Thereafter, the approximate expressions of Eq (33)~(38) should be modified by adding a calibration term, A_k, where A_k is defined as the total volume change contributed by the shaded area within the range of $kh \leq y \leq (k+1)h$, that is

$$A_k = \iint_C \frac{\Delta V}{V} d(C_k) - \iint_{D_k} \frac{\Delta V}{V} d(D_k) \qquad (39)$$

$$k = 0, \pm 1, \pm 2, \ldots$$

where C_k and D_k denote the two shaded areas between the y axis and the zero line as shown in Figure 8(a). A more accurate expression of the volume change, therefore, can be written as

$$(\Sigma\Delta V_k)_{acc} = (\Sigma\Delta V_k)_{appr} + A_k \tag{40}$$

The computed results for the relationship of $\Sigma\Delta V_o$ and ϵ/h from both approximate and accurate expressions are plotted in Figure 9. It is seen that Eq (33) provides a good approximation when $\epsilon \geq 0.2h$. If $\epsilon < 0.2h$, calibration should be taken into account. It is also seen that the value of ledge width ϵ strongly influences the local volume change around the ledge. For instance, when ledge $\epsilon = 10h$, its $\Sigma\Delta V_o$ value is almost 100 times that of an infinite dislocation wall ($\epsilon = 0$).

Similary, Eq (37) or Eq (38) can be modified as

$$(\overline{\Sigma\Delta V})_{acc} = (\overline{\Sigma\Delta V_{appr}}) + \frac{1}{n} \sum_{n=1}^{n} A_k \tag{41}$$

It is difficult to obtain an explicit expression for A_k because an explicit expression for the dilatation field given by Eq (39) has not been developed. However, since the areas of C_k and D_k are finite, the values of A_k for different ϵ and k are not difficult to obtain by numerical integration. Some of these values, from A_0 to A_{1000}, for different ϵ, were computed and are tabulated in Table 6 in the appendix.

6. Solute saturation around a ledged dislocation wall

The volume change of the dilatation field will cause the interaction between the solute atoms and the ledged dislocation wall. On the basis of the foregoing discussion, it is not difficult to estimate the saturation effect of solute atoms around a dislocation wall which contains a certain ledge density.

It is known that the volume change ∂V, associated with a solute atom of radius r', can be expressed in terms of the misfit factor $\epsilon = (r'-r)/r$ as

$$\partial V \text{ (solute)} = 4\pi r^3 \epsilon$$

The number of solute atoms required to saturate the dilatation field of one sign associated with unit length of the ledged wall can be expressed as

$$N_o = (\overline{\Sigma \Delta V})/4\pi r^3 \epsilon \tag{43}$$

It is assumed that a tilt grain boundary can be simulated by a ledged edge dislocation wall. Let m be the ledge density, ℓ the grain diameter, and n x m the density of dislocation in the wall. It will be found that by substituting Eq (38) into Eq (43), the total number of solute atoms required to saturate a unit length of dislocation wall (or grain boundary) is

$$(N_o)_{appr} = \frac{1}{4\pi r^3 \epsilon} \frac{-b(1-2\nu)}{2\pi(1-\nu)} \frac{2\epsilon}{(n+1)h} \left\{ (n+1) \ \ell n \frac{\ell}{n} - \ell n \ n! \right\} \quad (45)$$

and

$$(N_o)_{acc} = (N_o)_{appr} + \frac{1}{4\pi r^3 \epsilon} \frac{1}{n+1} \sum_{k=0}^{n} A_k \quad (46)$$

For example, if $m = 10^4 \sim 10^5$, the tile angle $\theta = 6°$, $12°$, $18°$ and other parameters are the same as Webb's,[3] namely:

$b = 3 \times 10^{-8}$cm, $\nu = 1/3$, $\epsilon = 0.2$, $r = 1.5 \times 10^{-8}$cm
$\ell = 10\mu$m.

The values of n for different θ and m can be calculated. The results are listed in Table 2.

TABLE 2

THE VARIATION OF n WITH θ AND m

θ	h (cm)	n	
		$m=10^4$	$m=10^5$
6°	3×10^{-7}	333	33
12°	1.5×10^{-7}	667	67
18°	1.0×10^{-7}	1000	1000

For different values of ϵ, n, and θ, the number of solute atoms required to saturate unit length of grain boundary, N_0, was computed from Eq (45) and the values are tabulated in Table 3.

Figure 10 and Table 4 show the comparison of the approximate and accurate values of N_o, computed at $\theta = 18°$ from Eqs (45) and 46), respectively. It is seen from Figure 10 that when $\epsilon > 1.0h$, the linear relation formula of Eq (45) provides a good approximation of the values of N_o. Thus the ledge width strongly influences the level of saturation of solute atoms. For example, for ledge density of $10^4 \sim 10^5$, width $\epsilon = 1.0h$, the value of N_o is one order of magnitude higher than that of an infinite wall ($\epsilon = 0$). For $\epsilon = 10h$, N_o is of two orders of magnitude higher. Since the ledge width in a real grain boundary is often of the order of several h, it may be expected that the saturation limit of solutes for a real grain boundary with ledges may be $1 \sim 2$ orders of magnitude higher than that for a perfect ledge-free grain boundary.

TABLE 3

COMPUTED VALUES OF N_o FROM EQ (45)

ϵ/h	$(N_o)_{appr} \times 10^{-14}$					
	$m=10^4$			$m=10^5$		
	$\theta=6°$	$\theta=12°$	$\theta=18°$	$\theta=6°$	$\theta=12°$	$\theta=18°$
0.025	0.62	0.52	0.46	0.97	0.85	0.79
0.050	1.24	1.05	0.93	1.94	1.71	1.60
0.100	2.49	2.09	1.86	3.89	3.43	3.19
0.250	6.22	5.23	4.66	9.74	8.59	7.98
0.500	12.40	10.50	9.32	19.50	17.20	15.90
1.000	24.90	20.90	18.60	38.90	34.40	31.90
2.000	49.80	41.90	37.30	77.90	68.70	63.80
4.000	99.60	83.70	74.50	155.00	137.00	127.00

TABLE 4

COMPUTED VALUES OF N_o FROM EQ (45) AND EQ (46) AT θ=18°

ϵ/n	$N_o \times 10^{-14}$					
	n=100 (θ=18°, m=10^5)			n=1000(θ=18°, m=10^4)		
	$\sum_1^n A_k/n$	$(N_o)_{appr}$	$(N_o)_{acc}$	$\sum_1^n A_k/n$	$(N_o)_{appr}$	$(N_o)_{acc}$
0	2.2					
0.025	2.20	0.79	3.00	2.20	0.46	2.66
0.100	2.14	3.19	5.33	2.15	1.86	4.00
0.250	2.10	7.98	10.1	2.14	4.66	6.80
0.500	2.06	15.90	18.00	2.11	9.32	11.40
1.000	2.01	31.90	33.90	2.09	18.60	20.70
2.000	0.67	63.8	64.50	1.96	37.30	39.30
4.000	0.00	127.7	127.7	0.00	74.50	74.50

IV. Summary

1. For a semi-infinite edge dislocation wall, only the shear stress field σ_{xy} is finite. The general solution of the shear stress σ_{xy} is given by Eq (15) in terms of trigamma function $\psi'(z)$ with complex argument. The values of shear stress can be calculated directly with required accuracy by means of Eq (19) and Eq (20), which converge more rapidly than Eq (4). The approximate formula, Eq (21), of shear stress for a semi-infinite wall at a large distance, derived by Amelinckx and Li, is identical to the first order approximation of the complete expression. Contours in the half plane of the shear stress field where no dislocation exists is similar to that of a single dislocation, while that of the other half (containing the semi-infinite wall) is similar to that of an infinite dislocation wall. The shear stress field of a ledged dislocation wall can be obtained by superposition of the two separated semi-infinite walls.

2. The dilatation field of a ledged dislocation wall is analyzed. From the computed results and plotted contours, it is noted that the zero dilatation lines of a ledged dislocation wall tend to coincide with the y axis over a certain range near the ledge as the width of the ledge increases. Approximately, the half plane on the right side of the y axis can be considered as the com-

pression field and the other half as the dilatation field.

3. Quantitatively, the volume change per unit area of the ledge wall, $\Sigma\Delta V_k$, as well as the saturation number of solute atoms, N_0, are linearly proportional to the width of the ledge, provided the width is relatively large (e.g., $\epsilon > h$). If the ledge width is relatively small, the value of $\Sigma\Delta V_k$ and No apparently deviate from the linear relations, and calibrations should be taken into account.

4. The width of the ledge plays an important role in the saturation of solute atoms along the grain boundary. The saturation limit of a real ledged grain boundary may be one or two orders of magnitude higher than that of a perfect ledge-free grain boundary.

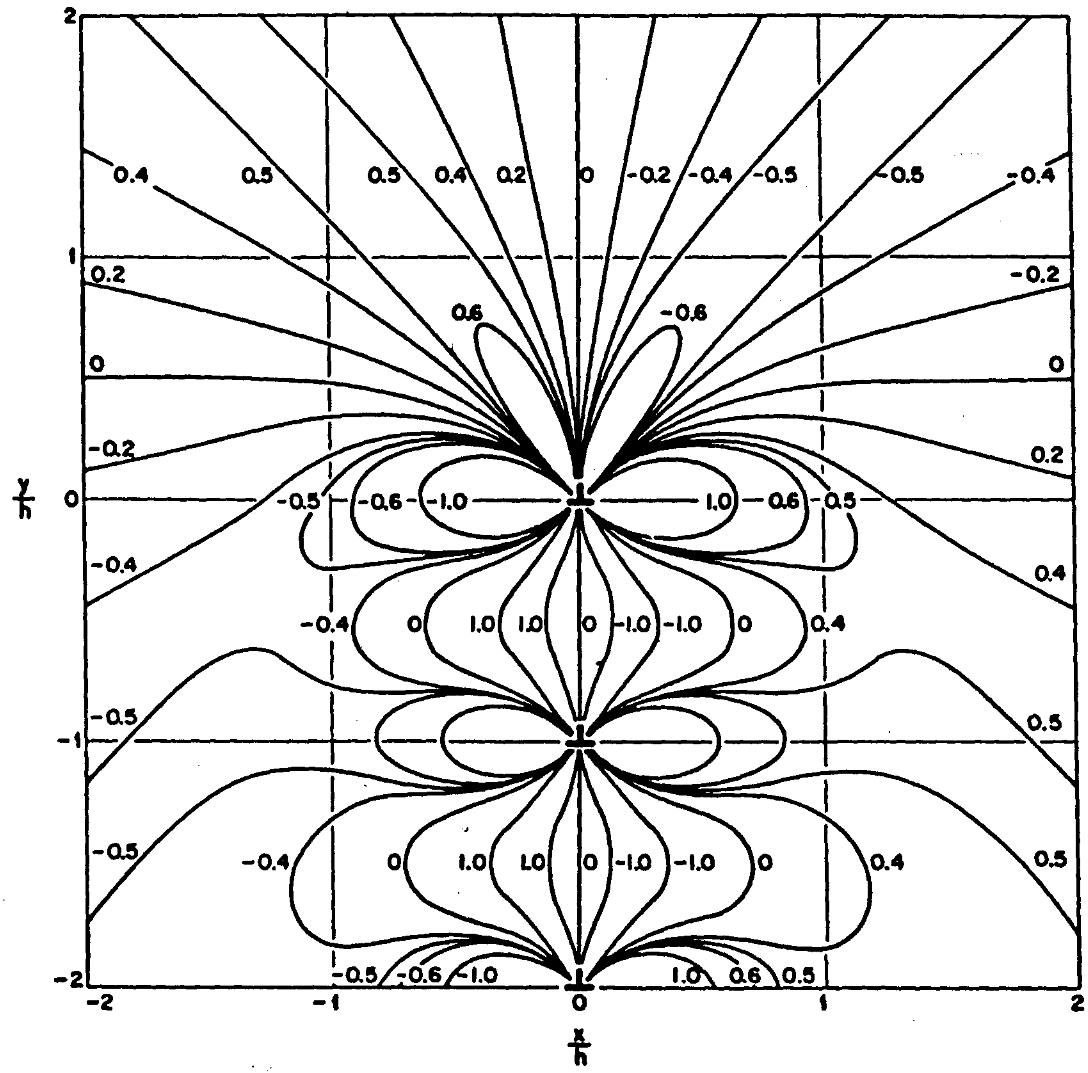

Figure 1. The stress field of a semi-infinite edge dislocation wall.

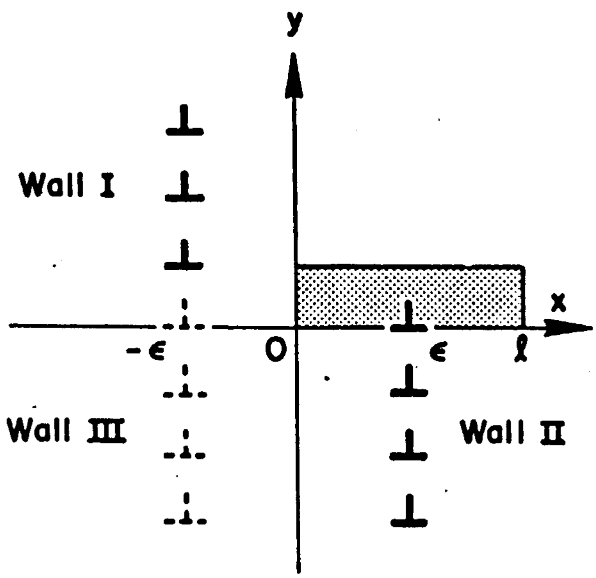

Figure 2. A dislocation wall with a ledge.

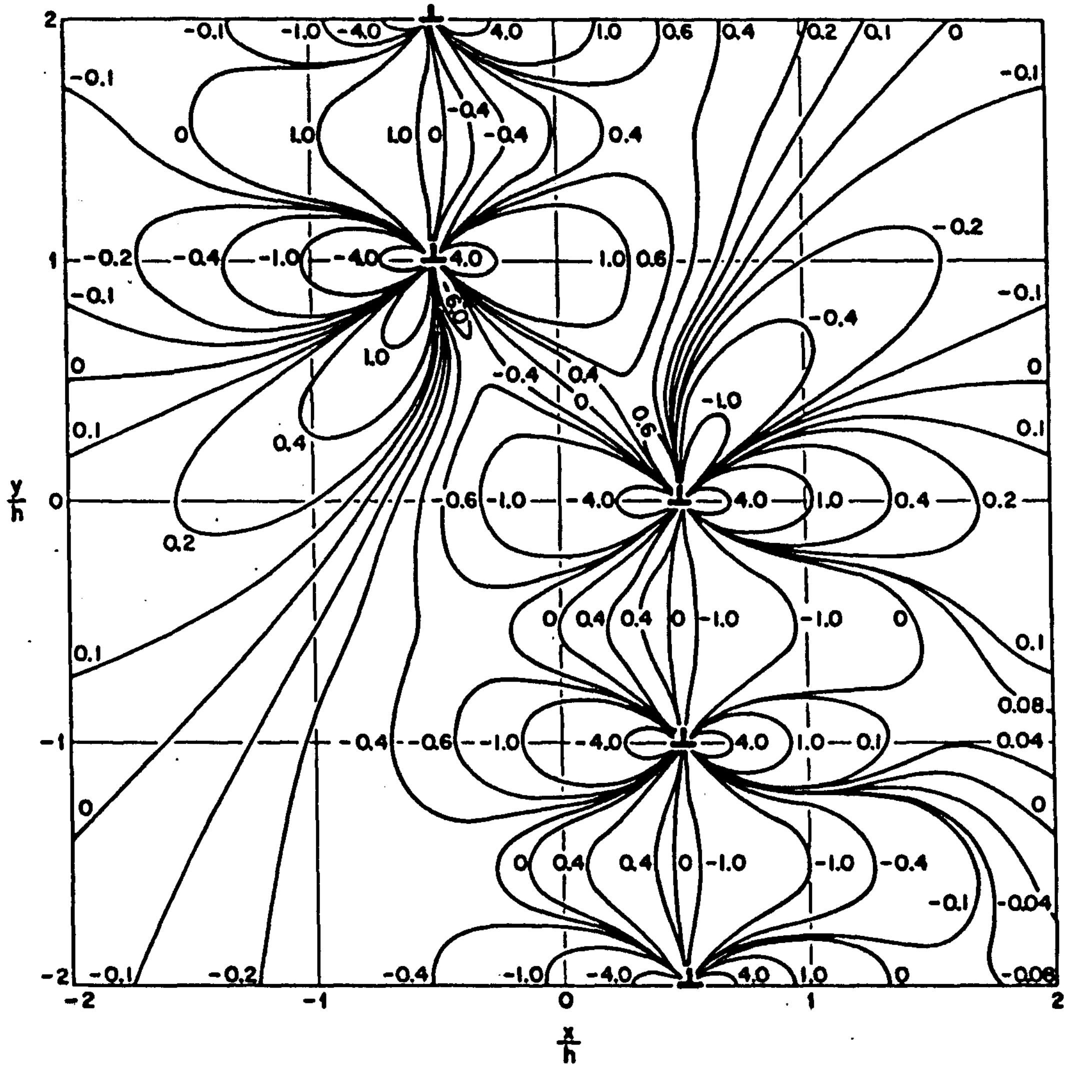

Figure 3. The shear stress field of a ledged dislocation wall ($2\epsilon = h$).

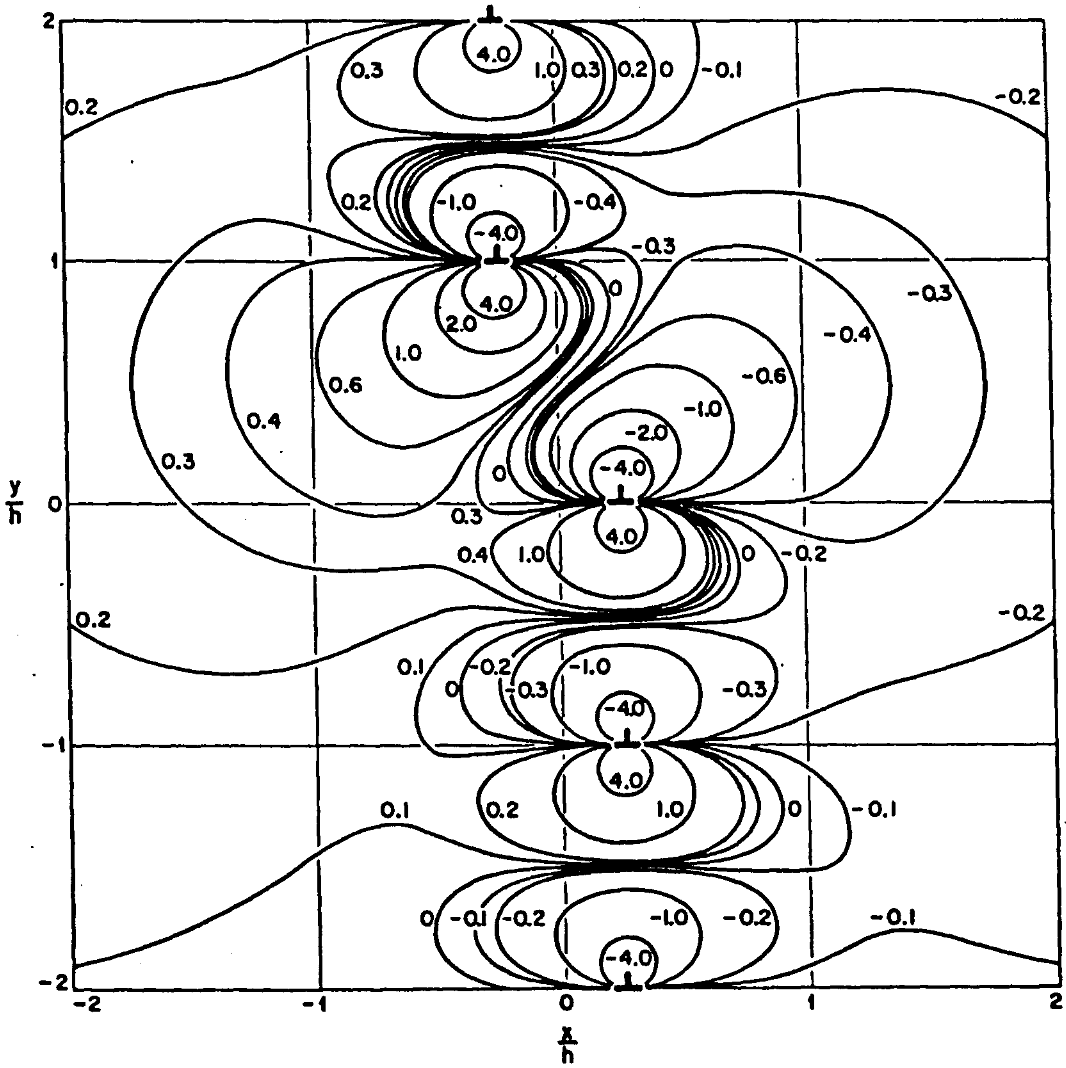

Figure 4. The dilatation field of a ledged dislocation wall ($2\epsilon = 0.5h$).

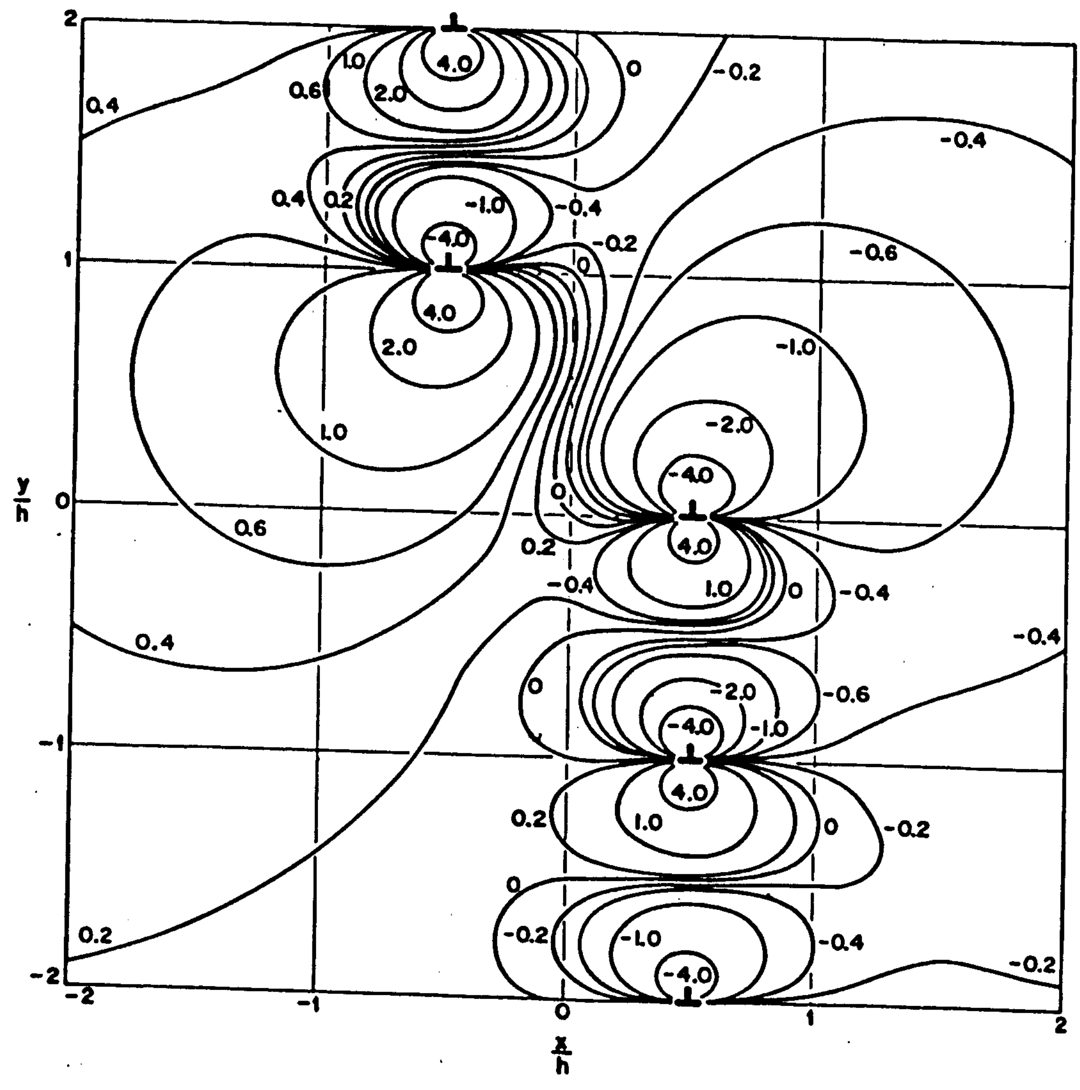

Figure 5. The dilatation field of a ledged dislocation wall ($2\epsilon = h$).

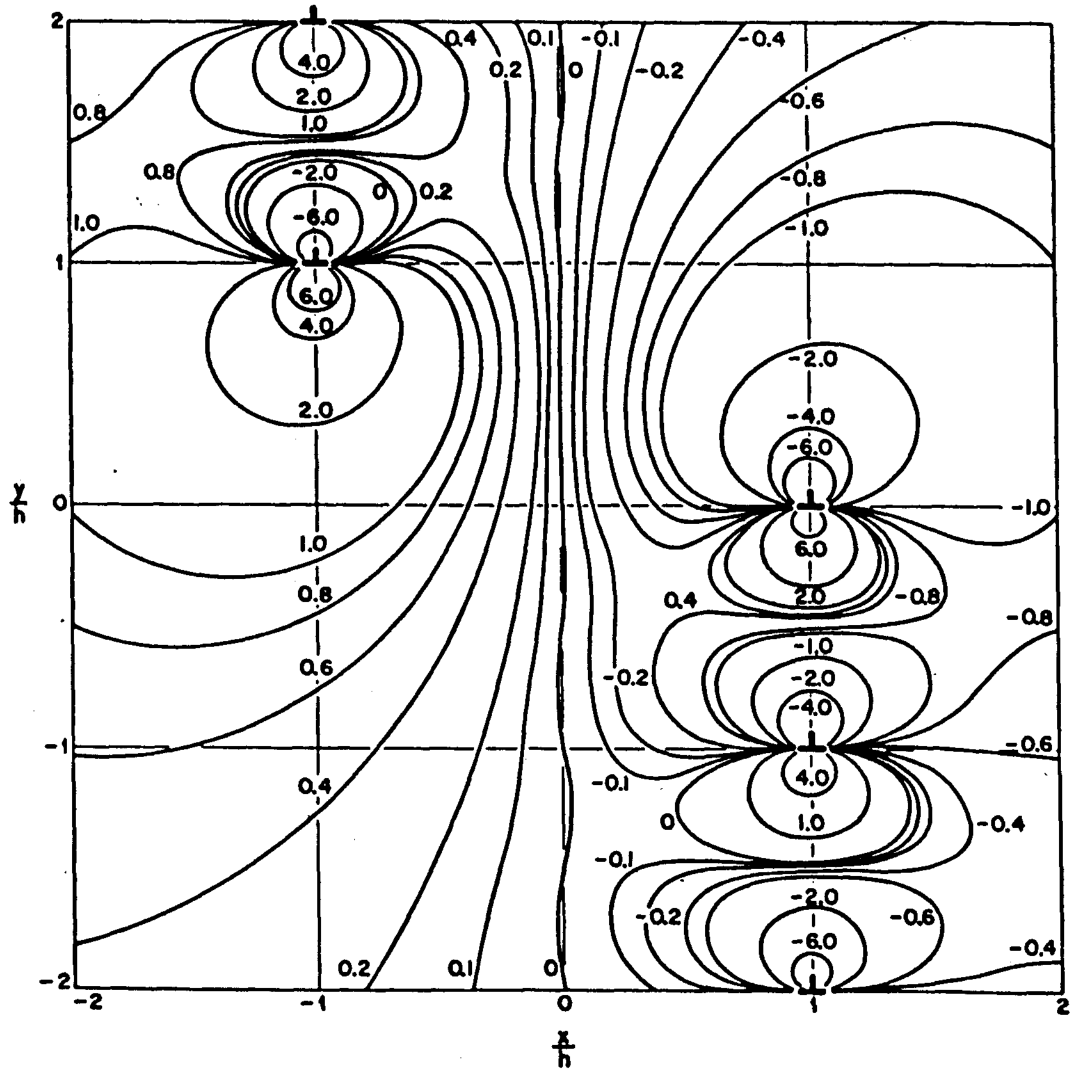

Figure 6. The dilatation field of a ledged dislocation wall (2ϵ = 2h).

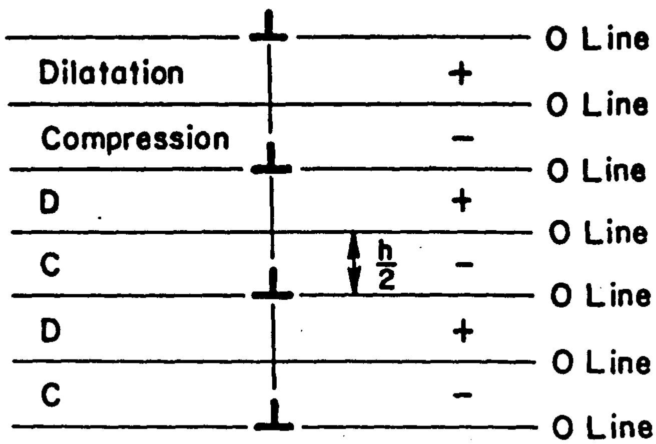

Figure 7. Zero dilatation line as well as the distribution of dilatation and compression regions for an infinite dislocation wall.

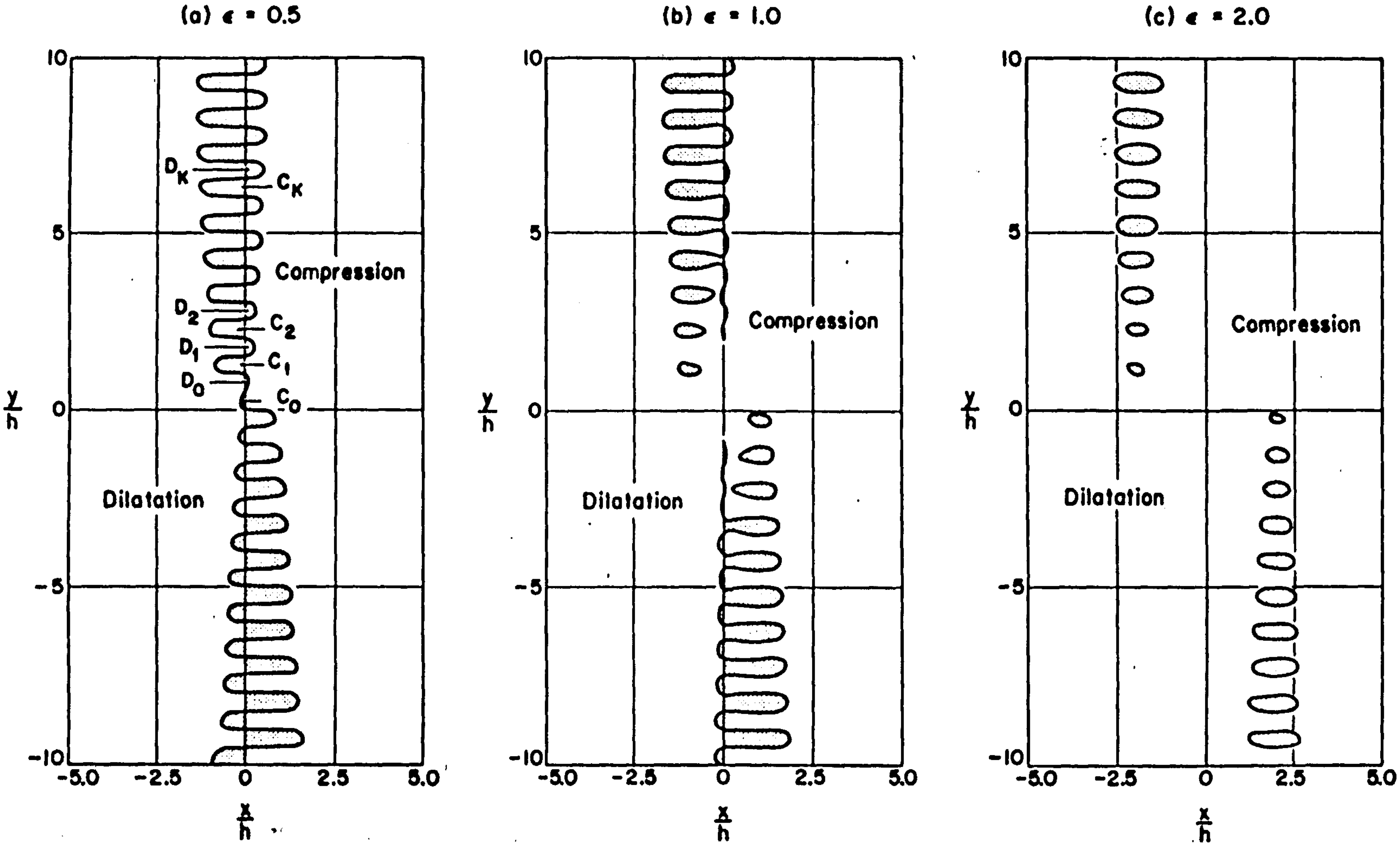

Figure 8. The zero contour lines of dilatation fields of different ledged dislocation walls.

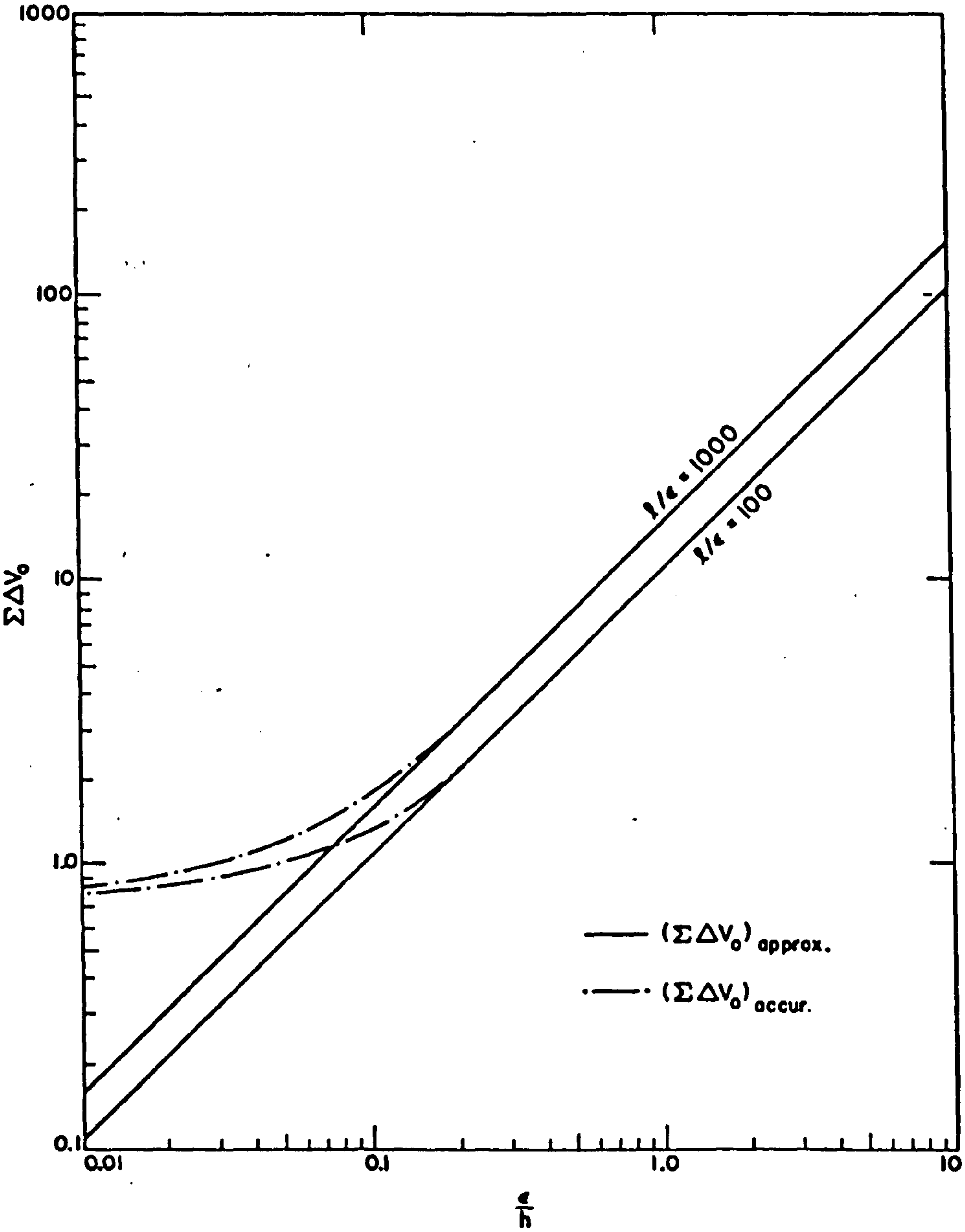

Figure 9. Approximate and accurate values of volume change per unit area of a ledged dislocation wall ($\Sigma\Delta V_o$).

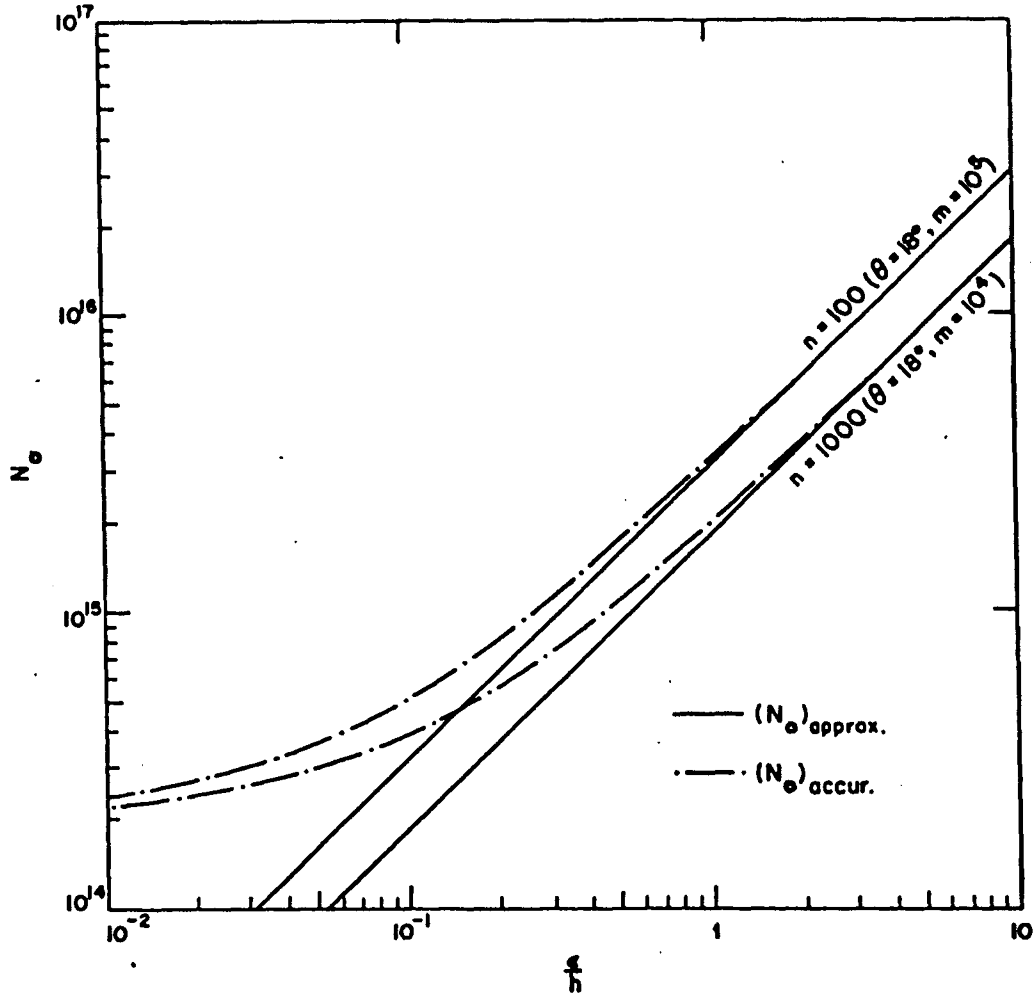

Figure 10. The relationship between N_o and ϵ/h, computed from both approximate and accurate expression (Eq (45) and (46)) at $\theta = 18°$.

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Appendix

Evaluation of trigamma function $\psi'(z)$ and its asymptomic, recurrence formulas.

Because the values of $\psi'(z)$ are not found in mathematical tables, they have been numerically revaluated. Table 7 lists the data for the range of $0 \leq y \leq 4$ with an interval of 0.1.

The asymptomic formula given by Eq (17) gives an approximation for large values of $|z|$, even when $\mathrm{Re}|z|$ is small. The error of Eq (17) for n terms is numerically less than the absolute value of the (n+1) term, provided $|\arg z| \leq \pi/4$.[10] For example, for $|z| \geq 4$, the error of $\psi'(z)$ in taking the first 4 terms of Eq (17) will be less than the 5th term, that is

$$\Delta \leq \frac{1}{42}\frac{1}{z^7} = \frac{1}{42}\frac{1}{4^7} \sim 10^{-6}$$

As a comparison, if the first 100 terms in Eq (4) are summed, the error equals

$$\Delta = \sum_{n=101}^{\infty} \frac{x^2-y_n^2}{(x^2+y_n^2)^2} \sim \sum_{n=101}^{\infty} \frac{1}{n^2} > 10^{-4}$$

Apparently, the error in taking 4 terms of Eq (17) is much less than that in 100 terms of Eq (4). It is advantageous to use Eq (19) instead of Eq (4) for the shear stress σ_{xy} of a semi-infinite wall because of its more rapid convergence and less deviation. Table 5 gives a further comparison of the accuracies of the shear stress data, which are computed from Eq (4), Eq (19) and Eq (20), and Eqs (5) ~ (8), respectively. There, the accurate values from Eq (5) and Eq (8) act as specific key points to evaluate the deviation of the approximate expression, Eq (4) and Eq (19) and Eq (20).

It must be noted that Eq (17) does not converge if $|z| \leq 1$, and converges slowly if $|z| \sim 1$. In these cases, the recurrence formula Eq (18) is introduced. One can select an appropriate value of positive integer m, to make $|z+m|$ relatively large. In that case, the combination of Eq (17) and Eq (18) yields

$$\sigma_{xy}(x,y) \sim \frac{-\mu bx}{2\pi(1-\nu)} \frac{1}{h^2} \mathrm{Re}\left[\left(\frac{1}{z+m} + \frac{1}{2(z+m)} + \ldots\right) + \left(\sum_{n=1}^{m} \frac{1}{(z+n-1)^2}\right)\right]$$

$$m = 1, 2, 3 \ldots$$

(This is Eq (20) which is used for relatively small $|z|$ value: $|z| \leq 1$, $|z| \sim 1$.

TABLE 5

COMPARISON OF THE ACCURACY OF COMPUTED VALUES OF σ_{xy}

y/h	x/h	σ_{xy} from Eq (5) Eq (8)	σ_{xy} from Eq (4), first 1000 terms		σ_{xy} from Eq (19), Eq (20) first 5 terms	
		Accurate Value	Calculated Value	Error	Calculated Value	Error
0.0	0.10	9.8387033898	9.83880	10^{-4}	9.838703379	10^{-8}
	0.50	1.4659010545	1.46640	10^{-3}	1.465901030	10^{-8}
	1.00	0.5369999034	0.53800	10^{-3}	0.536999951	10^{-8}
	2.00	0.2501376757	0.25214	10^{-3}	0.250137726	10^{-7}
0.5	0.10	0.4478098633	0.44771	10^{-4}	0.447809867	10^{-9}
	0.50	0.3919012478	0.39140	10^{-3}	0.391901258	10^{-8}
	1.00	0.0367245519	0.03573	10^{-3}	0.036724541	10^{-8}

Note: The constant of $\frac{-\mu b}{2\pi(1-\nu)}$ is not taken into account in the calculation.

TABLE 6
COMPUTED CALIBRATION CONSTANT A_k FOR
ACCURATE EXPRESSION OF EQ (40)

A_k	ϵ/n										
	0.00	0.025	0.10	0.25	0.50	1.00	1.50	2.00	2.50	3.00	4.00
A_0	0.79	0.60	0.24	0.07	0.01						
A_1		0.77	0.73	0.69	0.57						
A_2	⋮	0.78	0.75	0.72	0.67						
A_3	⋮	0.79	0.76	0.73	0.68						
A_4	⋮		0.77	0.75	0.73	0.68					
A_5		⋮	⋮	⋮	0.74	0.70					
⋮		⋮	⋮	⋮	⋮	⋮					
A_{10}		⋮	⋮	0.76	0.75	0.74			0.0		
⋮						⋮					
A_{17}				⋮	⋮	0.75	0.74				
⋮				⋮	⋮	⋮	⋮				
A_{68}						⋮	⋮	0.75			
⋮						⋮	⋮	⋮			
A_{100}								⋮			
⋮								⋮			
A_{402}									0.75		
⋮									⋮		
A_{703}									⋮	0.75	
⋮									⋮	⋮	
A_{1000}	0.79	0.79	0.78	0.77	0.76	0.75	0.75	0.75	0.75	0.75	0
$\frac{1}{10}\sum_{0}^{9} A_k$		0.77	0.71	0.67	0.64	0.43	0.00	0.00	0.00	0	0
$\frac{1}{100}\sum_{0}^{99} A_k$	0.79	0.79	0.76	0.75	0.74	0.72	0.24	0.00	0.00	0	0
$\frac{1}{1000}\sum_{0}^{999} A_k$		0.79	0.77	0.76	0.75	0.75	0.70	0.45	0.22	0	0

TABLE 7 TRIGAMMA FUNCTION FOR COMPLEX ARGUMENTS

x \ y	Re Ψ'(Z) 0.0	Im Ψ'(Z) 0.0	Re Ψ'(Z) 0.1	Im Ψ'(Z) 0.1
0.00			101.43330	0.00000
.10	-98.38703	-.23632	1.41102	-50.18358
.20	-23.47723	-.44948	-10.65254	-16.35244
.30	-9.72179	-.62237	-6.74857	-6.49509
.40	-5.01822	-.74721	-4.05561	-3.37339
.50	-2.93180	-.82504	-2.54112	-2.16110
.60	-1.86569	-.86284	-1.67192	-1.60451
.70	-1.26927	-.87009	-1.15193	-1.30902
.80	-.91248	-.85615	-.82796	-1.12975
.90	-.68687	-.82889	-.61833	-1.00736
1.00	-.53700	-.79423	-.47771	-.91559
1.10	-.43293	-.75628	-.38009	-.84201
1.20	-.35772	-.71769	-.31007	-.78040
1.30	-.30146	-.68307	-.25829	-.72738
1.40	-.25809	-.64430	-.21893	-.68097
1.50	-.22382	-.61080	-.18827	-.63989
1.60	-.19616	-.57971	-.16386	-.60324
1.70	-.17346	-.55101	-.14407	-.57035
1.80	-.15456	-.52457	-.12777	-.54070
1.90	-.13863	-.50023	-.11415	-.51384
2.00	-.12507	-.47783	-.10265	-.48943

x \ y	Re Ψ'(Z) 0.2	Im Ψ'(Z) 0.2	Re Ψ'(Z) 0.3	Im Ψ'(Z) 0.3
0.00	26.26738	0.00000	12.24536	0.00000
.10	13.25132	-16.14612	9.12235	-6.11870
.20	1.20514	-12.78252	4.04648	-7.33135
.30	-1.82437	-7.50186	1.03432	-5.88615
.40	-1.95396	-4.49746	-.15349	-4.25439
.50	-1.54844	-2.94753	-.49418	-3.07559
.60	-1.15108	-2.11825	-.52374	-2.30663
.70	-.84952	-1.64371	-.45927	-1.80970
.80	-.63473	-1.35124	-.37849	-1.48048
.90	-.48378	-1.15727	-.30593	-1.25386
1.00	-.37710	-1.01966	-.24705	-1.09111
1.10	-.30061	-.91622	-.20104	-.96926
1.20	-.24472	-.83474	-.16553	-.87456
1.30	-.20304	-.76820	-.13808	-.79854
1.40	-.17128	-.71237	-.11672	-.73588
1.50	-.14659	-.66457	-.09989	-.68310
1.60	-.12702	-.62302	-.08646	-.63786
1.70	-.11124	-.58648	-.07560	-.59854
1.80	-.09831	-.55404	-.06670	-.56399
1.90	-.08758	-.52502	-.05932	-.53333
2.00	-.07856	-.49890	-.05313	-.50591

x \ y	Re Ψ'(Z) 0.4	Im Ψ'(Z) 0.4	Re Ψ'(Z) 0.5	Im Ψ'(Z) 0.5
0.00	7.27536	0.00000	4.93490	0.00000
.10	6.20664	-2.86628	4.47810	-1.56159
.20	3.99003	-4.19155	3.40437	-2.53936
.30	2.06874	-4.11629	2.25903	-2.82900
.40	.89573	-3.47441	1.36828	-2.67739
.50	.29937	-2.78885	.78390	-2.35189
.60	.02991	-2.23084	.43479	-2.00793
.70	-.07774	-1.81470	.23691	-1.70711
.80	-.11138	-1.51140	.12783	-1.46270
.90	-.11357	-1.28912	.06861	-1.26938
1.00	-.10392	-1.12309	.03672	-1.11707
1.10	-.09100	-.99601	.01963	-.99612
1.20	-.07833	-.89619	.01048	-.89878
1.30	-.06714	-.81583	.00559	-.81918
1.40	-.05768	-.74969	.00299	-.75304
1.50	-.04984	-.69419	.00159	-.69724
1.60	-.04337	-.64684	.00085	-.64951
1.70	-.03802	-.60590	.00045	-.60819
1.80	-.03359	-.57007	.00024	-.57202
1.90	-.02988	-.53841	.00013	-.54007
2.00	-.02676	-.51020	.00007	-.51162

x \ y	Re Ψ'(Z) 0.6	Im Ψ'(Z) 0.6	Re Ψ'(Z) 0.7	Im Ψ'(Z) 0.7
0.00	3.63621	0.00000	2.83405	0.00000
.10	3.40952	-.94649	2.70881	-.62009
.20	2.83671	-1.63739	2.37779	-1.11511
.30	2.14424	-1.97791	1.94399	-1.42150
.40	1.51671	-2.03156	1.50844	-1.54816
.50	1.03272	-1.91754	1.13278	-1.54484
.60	.59303	-1.73443	.83776	-1.46652
.70	.46681	-1.54045	.61879	-1.35506
.80	.31988	-1.36235	.46126	-1.23614
.90	.22507	-1.20881	.34942	-1.12301
1.00	.16347	-1.08002	.27005	-1.02115
1.10	.12280	-.97297	.21326	-.93190
1.20	.09530	-.88388	.17203	-.85462
1.30	.07619	-.80926	.14155	-.78790
1.40	.06251	-.74618	.11856	-.73020
1.50	.05241	-.69232	.10087	-.68007
1.60	.04474	-.64585	.08698	-.63624
1.70	.03876	-.60537	.07538	-.59768
1.80	.03398	-.56979	.06685	-.56353
1.90	.03009	-.53826	.05949	-.53308
2.00	.02687	-.51012	.05317	-.50578

x \ y	0.8 Re Ψ'(Z)	0.8 Im Ψ'(Z)	0.9 Re Ψ'(Z)	0.9 Im Ψ'(Z)
0.00	2.29947	0.00000	1.92254	0.00000
.10	2.22450	-.43084	1.87478	-.31334
.20	2.02035	-.79490	1.74202	-.58845
.30	1.73780	-1.05158	1.55104	-.79925
.40	1.43301	-1.19500	1.33410	-.93720
.50	1.14808	-1.24449	1.11879	-1.00869
.60	.90495	-1.22889	.92312	-1.02823
.70	.70931	-1.17504	.75549	-1.01205
.80	.55746	-1.10290	.61736	-.97423
.90	.44186	-1.02528	.50623	-.92521
1.00	.35454	-.94938	.41798	-.87199
1.10	.28852	-.87877	.34825	-.81882
1.20	.23826	-.81477	.29308	-.76805
1.30	.19958	-.75755	.24923	-.72080
1.40	.16944	-.70669	.21410	-.67746
1.50	.14561	-.66154	.18569	-.63801
1.60	.12650	-.62141	.16248	-.60224
1.70	.11096	-.58562	.14334	-.56982
1.80	.09817	-.55358	.12737	-.54041
1.90	.08750	-.52478	.11394	-.51369
2.00	.07852	-.49877	.10254	-.48935

x \ y	1.0 Re Ψ'(Z)	1.0 Im Ψ'(Z)	1.1 Re Ψ'(Z)	1.1 Im Ψ'(Z)
0.00	1.64493	0.00000	1.43330	0.00000
.10	1.61297	-.23632	1.41102	-.18358
.20	1.52277	-.44948	1.34746	-.35244
.30	1.38932	-.62237	1.25143	-.49509
.40	1.23178	-.74721	1.13470	-.60522
.50	1.06820	-.82504	1.00918	-.68181
.60	.91209	-.86284	.88469	-.72796
.70	.77155	-.87009	.76807	-.74902
.80	.65002	-.85615	.66316	-.75105
.90	.54770	-.82889	.57144	-.73967
1.00	.46300	-.79423	.49273	-.71953
1.10	.39352	-.75628	.42614	-.69420
1.20	.33672	-.71769	.37008	-.66625
1.30	.29026	-.68007	.32303	-.63742
1.40	.25212	-.64430	.28353	-.60882
1.50	.22063	-.61080	.25030	-.58115
1.60	.19446	-.57971	.22222	-.55479
1.70	.17256	-.55101	.19838	-.52992
1.80	.15408	-.52457	.17803	-.50662
1.90	.13838	-.50023	.16057	-.48485
2.00	.12493	-.47783	.14548	-.46455

x \ y	1.2 Re $\psi'(Z)$	1.2 Im $\psi'(Z)$	1.3 Re $\psi'(Z)$	1.3 Im $\psi'(Z)$
0.00	1.26738	0.00000	1.13425	0.00000
.10	1.25132	-.14612	1.12235	-.11870
.20	1.20514	-.28252	1.08790	-.23076
.30	1.13421	-.40127	1.03432	-.33059
.40	1.04604	-.49746	.96651	-.41439
.50	.94858	-.56941	.88990	-.48044
.60	.84892	-.61825	.80959	-.52886
.70	.75248	-.64691	.72979	-.56118
.80	.66285	-.65920	.65360	-.57975
.90	.58197	-.65900	.58295	-.58720
1.00	.51048	-.64984	.51888	-.58610
1.10	.44819	-.63462	.46168	-.57873
1.20	.39443	-.61560	.41118	-.56699
1.30	.34827	-.59445	.36690	-.55236
1.40	.30872	-.57237	.32826	-.53600
1.50	.27484	-.55016	.29459	-.51873
1.60	.24576	-.52835	.26527	-.50116
1.70	.22074	-.50727	.23970	-.48368
1.80	.19913	-.48712	.21737	-.46659
1.90	.18039	-.46798	.19780	-.45006
2.00	.16406	-.44989	.18061	-.43418

x \ y	1.4 Re $\psi'(Z)$	1.4 Im $\psi'(Z)$	1.5 Re $\psi'(Z)$	1.5 Im $\psi'(Z)$
0.00	1.02536	0.00000	.93480	0.00000
.10	1.01632	-.09811	.92780	-.08230
.20	.99003	-.19155	.90734	-.16124
.30	.94874	-.27629	.87495	-.23384
.40	.89573	-.34941	.83289	-.29785
.50	.83477	-.40931	.78380	-.35189
.60	.76955	-.45569	.73041	-.39546
.70	.70332	-.48926	.67519	-.42880
.80	.63862	-.51140	.62020	-.45273
.90	.57726	-.52389	.56791	-.46839
1.00	.52034	-.52856	.51672	-.47707
1.10	.46843	-.52715	.46999	-.48008
1.20	.42167	-.52119	.42713	-.47863
1.30	.37990	-.51196	.38821	-.47377
1.40	.34282	-.50049	.35310	-.46639
1.50	.31001	-.48758	.32159	-.45724
1.60	.28103	-.47383	.29340	-.44688
1.70	.25545	-.45970	.26821	-.43577
1.80	.23285	-.44550	.24572	-.42424
1.90	.21285	-.43147	.22564	-.41255
2.00	.19513	-.41775	.20768	-.40089

x \ y	Re Ψ'(Z) 1.6	Im Ψ'(Z) 1.6	Re Ψ'(Z) 1.7	Im Ψ'(Z) 1.7
0.00	.85843	0.00000	.79323	0.00000
.10	.85291	-.06993	.78881	-.06009
.20	.83671	-.13739	.77579	-.11832
.30	.81090	-.20013	.75493	-.17299
.40	.77706	-.25642	.72737	-.22271
.50	.73711	-.30507	.69451	-.26653
.60	.69303	-.34554	.65782	-.30389
.70	.64674	-.37782	.61879	-.33465
.80	.59988	-.40235	.57873	-.35902
.90	.55380	-.41985	.53877	-.37745
1.00	.50949	-.43123	.49977	-.39055
1.10	.46764	-.43745	.46239	-.39903
1.20	.42863	-.43944	.42707	-.40360
1.30	.39267	-.43805	.39405	-.40494
1.40	.35978	-.43405	.36346	-.40367
1.50	.32986	-.42808	.33530	-.40035
1.60	.30276	-.42066	.30951	-.39545
1.70	.27828	-.41223	.28596	-.38936
1.80	.25620	-.40312	.26451	-.38240
1.90	.23630	-.39360	.24500	-.37484
2.00	.21836	-.38387	.22728	-.36689

x \ y	Re Ψ'(Z) 1.8	Im Ψ'(Z) 1.8	Re Ψ'(Z) 1.9	Im Ψ'(Z) 1.9
0.00	.73697	0.00000	.68797	0.00000
.10	.73338	-.05214	.68501	-.04564
.20	.72278	-.10286	.67627	-.09018
.30	.70571	-.15085	.66215	-.13258
.40	.68301	-.19500	.64327	-.17198
.50	.65571	-.23451	.62039	-.20769
.60	.62495	-.26889	.59439	-.23927
.70	.59184	-.29791	.56614	-.26649
.80	.55746	-.32165	.53650	-.28933
.90	.52272	-.34038	.50623	-.30793
1.00	.48839	-.35450	.47598	-.32256
1.10	.45506	-.36452	.44628	-.33357
1.20	.42317	-.37098	.41753	-.34138
1.30	.39299	-.37441	.39003	-.34640
1.40	.36470	-.37533	.36398	-.34903
1.50	.33837	-.37419	.33948	-.34966
1.60	.31400	-.37141	.31658	-.34865
1.70	.29152	-.36734	.29527	-.34630
1.80	.27087	-.36228	.27552	-.34288
1.90	.25193	-.35647	.25726	-.33863
2.00	.23458	-.35014	.24042	-.33375

x \ y	Re Ψ'(Z)	Im Ψ'(Z)	Re Ψ'(Z)	Im Ψ'(Z)
	2.0		2.1	
0.00	.64493	0.00000	.60685	0.00000
.10	.64247	-.04026	.60479	-.03577
.20	.63519	-.07966	.59866	-.07084
.30	.62339	-.11736	.58870	-.10456
.40	.60753	-.15258	.57527	-.13636
.50	.58820	-.18504	.55881	-.16577
.60	.56607	-.21405	.53985	-.19244
.70	.54183	-.23949	.51893	-.21615
.80	.51617	-.26127	.49662	-.23681
.90	.48971	-.27946	.47341	-.25442
1.00	.46300	-.29423	.44978	-.26909
1.10	.43652	-.30584	.42614	-.28098
1.20	.41062	-.31458	.40283	-.29032
1.30	.38561	-.32076	.38010	-.29735
1.40	.36168	-.32472	.35817	-.30232
1.50	.33897	-.32678	.33717	-.30550
1.60	.31755	-.32722	.31720	-.30713
1.70	.29746	-.32632	.29832	-.30744
1.80	.27868	-.32432	.28055	-.30664
1.90	.26119	-.32143	.26387	-.30492
2.00	.24493	-.31783	.24827	-.30245

x \ y	2.2		2.3	
0.00	.57293	0.00000	.54254	0.00000
.10	.57118	-.03197	.54104	-.02874
.20	.56598	-.06338	.53659	-.05702
.30	.55751	-.09370	.52933	-.08441
.40	.54604	-.12246	.51947	-.11052
.50	.53193	-.14926	.50729	-.13502
.60	.51558	-.17381	.49311	-.15765
.70	.49744	-.19589	.47729	-.17822
.80	.47794	-.21541	.46019	-.19661
.90	.45752	-.23233	.44215	-.21280
1.00	.43657	-.24672	.42353	-.22679
1.10	.41544	-.25868	.40461	-.23866
1.20	.39443	-.26838	.38566	-.24852
1.30	.37379	-.27598	.36690	-.25650
1.40	.35370	-.28171	.34852	-.26278
1.50	.33432	-.28577	.33066	-.26750
1.60	.31576	-.28835	.31343	-.27084
1.70	.29808	-.28966	.29691	-.27297
1.80	.28131	-.28988	.28114	-.27404
1.90	.26548	-.28917	.26615	-.27419
2.00	.25057	-.28769	.25196	-.27356

x \ y	Re Ψ'(Z) 2.4	Im Ψ'(Z) 2.4	Re Ψ'(Z) 2.5	Im Ψ'(Z) 2.5
0.00	.51515	0.00000	.49036	0.00000
.10	.51386	-.02596	.48924	-.02357
.20	.51003	-.05155	.48592	-.04683
.30	.50377	-.07641	.48047	-.06948
.40	.49523	-.10021	.47304	-.09124
.50	.48465	-.12267	.46380	-.11189
.60	.47229	-.14356	.45296	-.13122
.70	.45842	-.16273	.44076	-.14909
.80	.44335	-.18004	.42743	-.16538
.90	.42738	-.19546	.41323	-.18003
1.00	.41077	-.20899	.39838	-.19304
1.10	.39380	-.22065	.38312	-.20443
1.20	.37668	-.23053	.36764	-.21423
1.30	.35964	-.23874	.35213	-.22254
1.40	.34282	-.24539	.33674	-.22943
1.50	.32637	-.25062	.32159	-.23502
1.60	.31040	-.25455	.30680	-.23941
1.70	.29498	-.25734	.29244	-.24273
1.80	.28018	-.25911	.27857	-.24507
1.90	.26604	-.25999	.26524	-.24656
2.00	.25256	-.26010	.25248	-.24729

x \ y	Re Ψ'(Z) 2.6	Im Ψ'(Z) 2.6	Re Ψ'(Z) 2.7	Im Ψ'(Z) 2.7
0.00	.46781	0.00000	.44721	0.00000
.10	.46683	-.02149	.44636	-.01966
.20	.46393	-.04271	.44381	-.03911
.30	.45918	-.06343	.43963	-.05813
.40	.45267	-.08341	.43391	-.07652
.50	.44456	-.10244	.42675	-.09411
.60	.43501	-.12036	.41830	-.11075
.70	.42422	-.13703	.40872	-.12632
.80	.41238	-.15235	.39817	-.14073
.90	.39971	-.16626	.38683	-.15393
1.00	.38640	-.17874	.37497	-.16586
1.10	.37265	-.18979	.36245	-.17654
1.20	.35863	-.19944	.34973	-.18599
1.30	.34451	-.20774	.33685	-.19422
1.40	.33041	-.21477	.32393	-.20131
1.50	.31646	-.22061	.31108	-.20731
1.60	.30276	-.22535	.29840	-.21230
1.70	.28939	-.22908	.28596	-.21635
1.80	.27642	-.23189	.27382	-.21953
1.90	.26388	-.23389	.26205	-.22194
2.00	.25182	-.23515	.25066	-.22365

x \ y	Re Ψ'(Z) 2.8	Im Ψ'(Z) 2.8	Re Ψ'(Z) 2.9	Im Ψ'(Z) 2.9
0.00	.42833	0.00000	.41096	0.00000
.10	.42758	-.01806	.41030	-.01665
.20	.42533	-.03594	.40831	-.03313
.30	.42164	-.05345	.40503	-.04931
.40	.41658	-.07043	.40053	-.06503
.50	.41023	-.08673	.39488	-.08017
.60	.40272	-.10222	.38818	-.09461
.70	.39418	-.11678	.38054	-.10825
.80	.38475	-.13035	.37208	-.12102
.90	.37457	-.14285	.36291	-.13287
1.00	.36379	-.15425	.35317	-.14375
1.10	.35255	-.16455	.34297	-.15365
1.20	.34099	-.17374	.33244	-.16258
1.30	.32922	-.18186	.32168	-.17053
1.40	.31737	-.18894	.31079	-.17755
1.50	.30552	-.19503	.29987	-.18367
1.60	.29378	-.20018	.28899	-.18894
1.70	.28221	-.20447	.27823	-.19340
1.80	.27087	-.20795	.26764	-.19711
1.90	.25982	-.21070	.25726	-.20013
2.00	.24908	-.21278	.24715	-.20251

x \ y	Re Ψ'(Z) 3.0	Im Ψ'(Z) 3.0	Re Ψ'(Z) 3.1	Im Ψ'(Z) 3.1
0.00	.39493	0.00000	.38010	0.00000
.10	.39434	-.01539	.37956	-.01427
.20	.39257	-.03064	.37798	-.02842
.30	.38965	-.04563	.37537	-.04234
.40	.38563	-.06022	.37177	-.05592
.50	.38059	-.07432	.36724	-.06907
.60	.37458	-.08780	.36185	-.08169
.70	.36772	-.10060	.35567	-.09370
.80	.36010	-.11263	.34879	-.10506
.90	.35183	-.12386	.34129	-.11570
1.00	.34300	-.13423	.33327	-.12559
1.10	.33373	-.14374	.32483	-.13471
1.20	.32412	-.15238	.31604	-.14305
1.30	.31426	-.16015	.30700	-.15061
1.40	.30425	-.16707	.29779	-.15741
1.50	.29417	-.17318	.28847	-.16346
1.60	.28409	-.17850	.27912	-.16880
1.70	.27407	-.18308	.26990	-.17345
1.80	.26418	-.18696	.26055	-.17746
1.90	.25445	-.19019	.25143	-.18086
2.00	.24493	-.19283	.24247	-.18369

y → / x ↓	Re Ψ'(Z) 3.2	Im Ψ'(Z) 3.2	Re Ψ'(Z) 3.3	Im Ψ'(Z) 3.3
0.00	.36632	0.00000	.35350	0.00000
.10	.36585	-.01326	.35307	-.01236
.20	.36442	-.02643	.35179	-.02463
.30	.36208	-.03939	.34968	-.03673
.40	.35884	-.05206	.34676	-.04857
.50	.35477	-.06434	.34308	-.06008
.60	.34990	-.07617	.33868	-.07119
.70	.34432	-.08748	.33361	-.08183
.80	.33808	-.09819	.32795	-.09196
.90	.33128	-.10828	.32176	-.10154
1.00	.32398	-.11771	.31510	-.11052
1.10	.31627	-.12645	.30804	-.11890
1.20	.30822	-.13450	.30066	-.12664
1.30	.29991	-.14184	.29301	-.13376
1.40	.29142	-.14849	.28517	-.14026
1.50	.28280	-.15447	.27719	-.14613
1.60	.27412	-.15979	.26913	-.15141
1.70	.26544	-.16448	.26104	-.15610
1.80	.25680	-.16857	.25297	-.16024
1.90	.24825	-.17209	.24494	-.16385
2.00	.23982	-.17508	.23701	-.16696

y → / x ↓	Re Ψ'(Z) 3.4	Im Ψ'(Z) 3.4	Re Ψ'(Z) 3.5	Im Ψ'(Z) 3.5
0.00	.34154	0.00000	.33036	0.00000
.10	.34115	-.01155	.33001	-.01081
.20	.34000	-.02302	.32896	-.02155
.30	.33809	-.03433	.32722	-.03216
.40	.33544	-.04542	.32482	-.04257
.50	.33211	-.05622	.32179	-.05272
.60	.32811	-.06667	.31816	-.06256
.70	.32351	-.07671	.31396	-.07204
.80	.31835	-.08629	.30926	-.08112
.90	.31270	-.09538	.30408	-.08975
1.00	.30661	-.10395	.29850	-.09792
1.10	.30014	-.11197	.29256	-.10560
1.20	.29335	-.11942	.28631	-.11277
1.30	.28631	-.12631	.27980	-.11943
1.40	.27906	-.13264	.27309	-.12558
1.50	.27166	-.13840	.26623	-.13121
1.60	.26417	-.14361	.25926	-.13634
1.70	.25662	-.14828	.25222	-.14098
1.80	.24907	-.15245	.24515	-.14514
1.90	.24155	-.15612	.23809	-.14885
2.00	.23408	-.15932	.23107	-.15211

x \ y	Re $\Psi'(Z)$	Im $\Psi'(Z)$	Re $\Psi'(Z)$	Im $\Psi'(Z)$
	3.6		3.7	
0.00	.31988	0.00000	.31004	0.00000
.10	.31956	-.01014	.30975	-.00953
.20	.31860	-.02022	.30888	-.01901
.30	.31703	-.03018	.30744	-.02838
.40	.31484	-.03997	.30544	-.03760
.50	.31208	-.04953	.30292	-.04662
.60	.30876	-.05881	.29988	-.05539
.70	.30493	-.06778	.29637	-.06387
.80	.30062	-.07638	.29242	-.07204
.90	.29588	-.08459	.28806	-.07985
1.00	.29075	-.09238	.28334	-.08729
1.10	.28528	-.09974	.27830	-.09433
1.20	.27951	-.10663	.27297	-.10096
1.30	.27350	-.11307	.26740	-.10717
1.40	.26728	-.11903	.26163	-.11295
1.50	.26091	-.12453	.25570	-.11831
1.60	.25441	-.12957	.24964	-.12325
1.70	.24784	-.13415	.24350	-.12776
1.80	.24122	-.13829	.23730	-.13187
1.90	.23459	-.14201	.23107	-.13559
2.00	.22798	-.14532	.22485	-.13892

x \ y	3.8		3.9	
0.00	.30078	0.00000	.29206	0.00000
.10	.30051	-.00897	.29181	-.00847
.20	.29972	-.01790	.29108	-.01689
.30	.29840	-.02674	.28988	-.02523
.40	.29658	-.03543	.28820	-.03345
.50	.29426	-.04395	.28608	-.04150
.60	.29148	-.05225	.28352	-.04936
.70	.28826	-.06029	.28055	-.05700
.80	.28462	-.06805	.27721	-.06437
.90	.28061	-.07549	.27351	-.07146
1.00	.27626	-.08259	.26948	-.07825
1.10	.27160	-.08934	.26517	-.08471
1.20	.26667	-.09571	.26060	-.09084
1.30	.26150	-.10170	.25590	-.09662
1.40	.25614	-.10730	.25081	-.10204
1.50	.25062	-.11252	.24566	-.10711
1.60	.24496	-.11734	.24038	-.11182
1.70	.23922	-.12178	.23590	-.11618
1.80	.23340	-.12585	.22954	-.12019
1.90	.22755	-.12954	.22404	-.12386
2.00	.22169	-.13289	.21852	-.12719

TABLE 8 COMPUTED VALUES OF THE SHEAR STRESS OF A SEMI-INFINITE DISLOCATION WALL

σ_{xy}, unit: $\frac{\mu b}{2\pi(1-\nu)} \cdot \frac{1}{h^2}$

x/h \ y/h	0.0	0.1	0.2	0.3	0.4
0.00	I	0.00000	0.00000	0.00000	0.00000
.10	-9.83870	.14110	1.32513	.91224	.62066
.20	-4.69545	-2.13051	.24103	.80930	.79801
.30	-2.91654	-2.02457	-.54731	.31030	.62062
.40	-2.00729	-1.62224	-.78158	-.06139	.35829
.50	-1.46590	-1.27056	-.77422	-.24709	.14969
.60	-1.11941	-1.00315	-.69065	-.31425	.01794
.70	-.88849	-.80635	-.59466	-.32149	-.05442
.80	-.72999	-.66237	-.50778	-.30279	-.08910
.90	-.61818	-.55650	-.43540	-.27534	-.10221
1.00	-.53700	-.47771	-.37710	-.24705	-.10392
1.10	-.47622	-.41810	-.33067	-.22114	-.10010
1.20	-.42927	-.37208	-.29367	-.19863	-.09400
1.30	-.39190	-.33578	-.26395	-.17951	-.08728
1.40	-.36132	-.30650	-.23979	-.16340	-.08075
1.50	-.33572	-.28240	-.21988	-.14983	-.07475
1.60	-.31386	-.26217	-.20323	-.13833	-.06939
1.70	-.29489	-.24492	-.18911	-.12852	-.06464
1.80	-.27821	-.22998	-.17697	-.12006	-.06046
1.90	-.26340	-.21689	-.16640	-.11271	-.05678
2.00	-.25014	-.20530	-.15712	-.10626	-.05352

x/h \ y/h	0.5	0.6	0.7	0.8	0.9
0.00	0.00000	0.00000	0.00000	0.00000	0.00000
.10	.44781	.34095	.27088	.22245	.18748
.20	.68087	.56734	.47556	.40407	.34840
.30	.67771	.64327	.58320	.52134	.46531
.40	.54731	.60668	.60338	.57321	.53364
.50	.39190	.51636	.56639	.57404	.55939
.60	.26087	.41582	.50265	.54297	.55387
.70	.16584	.32676	.43316	.49652	.52885
.80	.10227	.25591	.36901	.44597	.49389
.90	.06175	.20256	.31448	.39768	.45561
1.00	.03672	.16347	.27005	.35454	.41798
1.10	.02159	.13508	.23458	.31737	.38307
1.20	.01258	.11436	.20643	.28591	.35170
1.30	.00727	.09905	.18401	.25946	.32400
1.40	.00418	.08752	.16599	.23721	.29974
1.50	.00239	.07862	.15131	.21841	.27853
1.60	.00136	.07159	.13918	.20239	.25997
1.70	.00077	.06589	.12900	.18863	.24367
1.80	.00044	.06117	.12033	.17670	.22927
1.90	.00025	.05718	.11286	.16625	.21649
2.00	.00014	.05375	.10634	.15703	.20507

x/h \ y/h	1.0	1.1	1.2	1.3	1.4
0.00	0.00000	0.00000	0.00000	0.00000	0.00000
.10	.16130	.14110	.12513	.11224	.10163
.20	.30455	.26949	.24103	.21758	.19901
.30	.41680	.37543	.34026	.31030	.28462
.40	.49271	.45388	.41842	.38661	.35829
.50	.53410	.50459	.47429	.44495	.41738
.60	.54725	.53081	.50935	.48575	.46173
.70	.54008	.53765	.52673	.51086	.49232
.80	.52001	.53053	.53028	.52288	.51090
.90	.49293	.51429	.52377	.52466	.51953
1.00	.46300	.49278	.51048	.51888	.52034
1.10	.43287	.46876	.49301	.50785	.51528
1.20	.40406	.44409	.47332	.49341	.50600
1.30	.37734	.41993	.45275	.47697	.49387
1.40	.35296	.39694	.43221	.45956	.47994
1.50	.33094	.37544	.41225	.44188	.46501
1.60	.31114	.35555	.39322	.42443	.44965
1.70	.29335	.33725	.37526	.40749	.43426
1.80	.27734	.32046	.35843	.39126	.41912
1.90	.26291	.30518	.34273	.37582	.40442
2.00	.24986	.29097	.32813	.36122	.39026

x/h \ y/h	1.5	1.6	1.7	1.8	1.9
0.00	0.00000	0.00000	0.00000	0.00000	0.00000
.10	.09278	.08529	.07888	.07334	.06850
.20	.18147	.16734	.15516	.14456	.13525
.30	.26249	.24327	.22648	.21171	.19865
.40	.33315	.31083	.29095	.27321	.25731
.50	.39190	.36855	.34725	.32786	.31020
.60	.43825	.41582	.39469	.37497	.35663
.70	.47263	.45272	.43316	.41429	.39630
.80	.49616	.47991	.46299	.44597	.42920
.90	.51031	.49842	.48489	.47045	.45561
1.00	.51672	.50949	.49977	.48839	.47598
1.10	.51699	.51440	.50863	.50057	.49090
1.20	.51256	.51436	.51248	.50780	.50104
1.30	.50467	.51047	.51227	.51089	.50704
1.40	.49434	.50369	.50885	.51059	.50957
1.50	.48239	.49479	.50295	.50756	.50921
1.60	.46944	.48442	.49521	.50239	.50652
1.70	.45596	.47308	.48613	.49559	.50196
1.80	.44230	.46117	.47612	.48757	.49594
1.90	.42871	.44897	.46551	.47867	.48880
2.00	.41536	.43671	.45455	.46916	.48083

x/h \ y/h	2.0	2.1	2.2	2.3	2.4
0.00	0.00000	0.00000	0.00000	0.00000	0.00000
.10	.06425	.06048	.05712	.05410	.05139
.20	.12704	.11973	.11320	.10732	.10201
.30	.18702	.17661	.16725	.15880	.15113
.40	.24301	.23011	.21842	.20779	.19809
.50	.29410	.27941	.26597	.25365	.24233
.60	.33964	.32391	.30935	.29587	.28337
.70	.37928	.36325	.34821	.33410	.32090
.80	.41293	.39729	.38235	.36815	.35468
.90	.44074	.42607	.41177	.39794	.38464
1.00	.46300	.44978	.43657	.42353	.41077
1.10	.48017	.46876	.45698	.44507	.43318
1.20	.49275	.48339	.47332	.46279	.45202
1.30	.50130	.49413	.48592	.47697	.46753
1.40	.50636	.50143	.49518	.48793	.47994
1.50	.50846	.50575	.50149	.49600	.48955
1.60	.50809	.50752	.50522	.50149	.49663
1.70	.50568	.50715	.50673	.50474	.50147
1.80	.50162	.50498	.50636	.50605	.50433
1.90	.49625	.50135	.50440	.50569	.50547
2.00	.48986	.49654	.50114	.50392	.50512

x/h \ y/h	2.5	2.6	2.7	2.8	2.9
0.00	0.00000	0.00000	0.00000	0.00000	0.00000
.10	.04892	.04658	.04464	.04276	.04103
.20	.09718	.09279	.08876	.08507	.08166
.30	.14414	.13775	.13189	.12649	.12151
.40	.18922	.18107	.17356	.16663	.16021
.50	.23190	.22228	.21337	.20512	.19744
.60	.27178	.26100	.25098	.24163	.23291
.70	.30853	.29695	.28610	.27593	.26638
.80	.34194	.32991	.31854	.30780	.29766
.90	.37190	.35974	.34815	.33711	.32662
1.00	.39838	.38640	.37487	.36379	.35317
1.10	.42143	.40992	.39870	.38781	.37727
1.20	.44117	.43036	.41968	.40918	.39893
1.30	.45777	.44786	.43790	.42799	.41818
1.40	.47143	.46257	.45350	.44431	.43511
1.50	.48239	.47469	.46662	.45829	.44981
1.60	.49088	.48442	.47743	.47005	.46239
1.70	.49714	.49197	.48613	.47976	.47299
1.80	.50143	.49755	.49288	.48757	.48175
1.90	.50396	.50137	.49789	.49365	.48880
2.00	.50496	.50364	.50132	.49816	.49430

x/h \ y/h	3.0	3.1	3.2	3.3	3.4
0.00	0.00000	0.00000	0.00000	0.00000	0.00000
.10	.03943	.03796	.03658	.03531	.03412
.20	.07851	.07560	.07288	.07036	.06800
.30	.11690	.11261	.10862	.10490	.10143
.40	.15425	.14871	.14354	.13870	.13418
.50	.19029	.18362	.17738	.17154	.16605
.60	.22475	.21711	.20994	.20321	.19587
.70	.25741	.24897	.24102	.23353	.22646
.80	.28808	.27903	.27047	.26236	.25468
.90	.31664	.30716	.29815	.28958	.28143
1.00	.34300	.33327	.32398	.31510	.30661
1.10	.36710	.35731	.34789	.33884	.33015
1.20	.38894	.37925	.36986	.36079	.35202
1.30	.40854	.39910	.38989	.38091	.37220
1.40	.42596	.41690	.40798	.39924	.39068
1.50	.44126	.43271	.42420	.41579	.40749
1.60	.45455	.44659	.43860	.43061	.42267
1.70	.46593	.45865	.45125	.44377	.43626
1.80	.47552	.46900	.46225	.45534	.44833
1.90	.48346	.47772	.47167	.46539	.45894
2.00	.48986	.48494	.47964	.47403	.46817

x/h \ y/h	3.5	3.6	3.7	3.8	3.9
0.00	0.00000	0.00000	0.00000	0.00000	0.00000
.10	.03300	.03196	.03097	.03005	.02918
.20	.06579	.06372	.06178	.05994	.05822
.30	.09817	.09511	.09223	.08952	.08696
.40	.12993	.12594	.12218	.11863	.11528
.50	.16090	.15604	.15146	.14713	.14304
.60	.19089	.18526	.17993	.17489	.17011
.70	.21977	.21345	.20746	.20178	.19639
.80	.24740	.24050	.23394	.22770	.22177
.90	.27368	.26629	.25926	.25255	.24616
1.00	.29850	.29075	.28334	.27626	.26948
1.10	.32181	.31381	.30613	.29876	.29169
1.20	.34357	.33542	.32757	.32000	.31272
1.30	.36374	.35555	.34762	.33996	.33254
1.40	.38233	.37420	.36628	.35860	.35114
1.50	.39934	.39136	.38355	.37593	.36849
1.60	.41481	.40706	.39943	.39194	.38461
1.70	.42877	.42132	.41395	.40667	.39950
1.80	.44127	.43419	.42714	.42012	.41318
1.90	.45237	.44572	.43904	.43235	.42568
2.00	.46213	.45596	.44969	.44338	.43703

Vita

Tian Yong-Lai was born on July 7, 1944 in Shanghai, China. He attended high school at Shi-Xi High School, Shanghai. In September of 1961 he entered Tsing-hua University, Peking, China, earning a Bachelor of Science degree in Metallurgical Engineering in 1967. Subsequently, he joined the Dalian Spring Factory, Dalian, China, as an engineer. In September of 1978, he entered the Graduate School of Dalian Institute of Technology, Dalian, where he specialized in the field of Fracture Mechanics.

In January of 1980, he entered the Department of Metallurgy and Materials Engineering, Lehigh University. He is a student member of American Society for Metals.

In 1972, Mr. Tian Yong-Lai married. He and his wife have two children.